Adverse Selection and Unraveling in Common-Value Labor Markets*

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Abstract

We investigate a common-value labor setting in which firms interview a worker prior to hiring. When firms have private information about the worker's value and interview decisions are kept private, many firms may enter the market, interview, and hire with positive probability. When firms' interview decisions are revealed, severe adverse selection arises. As a result, all firms except for the highest-ranked firm are excluded from the hiring process.

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1 Introduction

The hiring process in many firms includes several stages, at the end of which employment offers are made. This process often begins by conducting an initial evaluation of a potential worker's resumé and other credentials. If the evaluation proves favorable, the worker proceeds to the next stage, which may consist of an interview, a fly-out, or a step of an administrative nature such as a "short list." At the end of the process, the firm may offer the worker a job. Many professionals, including academic economists, newly minted MBAs, law school graduates, and to some extent medical residents, are hired in this way.

We ask how making firms' intermediate decisions known to other firms affects the hiring process and its outcome. For example, in the academic job market for economists an online resource called Econjobmarket[1] started listing universities' interview and fly-out decisions, nearly in real time. Because a university's intermediate decisions (whether or not to interview or fly out a candidate) contain some information about the candidate, making them known has the potential to improve other universities' hiring decisions. This, in turn, could lead to a better hiring outcome. But some universities may not benefit from having their intermediate decisions revealed. Moreover, if universities expect their intermediate decisions to be revealed, they may change their interview or fly out policy. Therefore, it is not immediately apparent which universities and candidates benefit from such information revelation, and whether it should be facilitated or prohibited.

Similar considerations arise whenever firms' intermediate decisions contain some information about a potential worker. In particular, if firms anticipate that their intermediate decisions and those of other firms will be revealed, they may adjust their intermediate decisions for two reasons. First, a firm may advance a marginal candidate in the hiring process in the hopes of learning more about the candidate from other firms' intermediate decisions. Second, a firm may "give up" on a seemingly good candidate because it expects that a positive intermediate decision will result in an offer from a firm that is more attractive to the candidate.[2] Because the amount of information revealed by firms' intermediate decisions is determined endogenously, the overall effect of revealing these decisions is unclear.

To analyze these issues, we investigate a three-stage model in which several privately-informed firms may be interested in hiring a worker. To make the analysis of the strategic interaction among firms manageable, we suppose that the value of the worker is common to all firms, that the worker has a known, strict ranking over firms, and that firms do not compete in wages (so the worker always prefers an offer from a higher-ranked firm).

[1] http://bluwiki.com/go/Econjobmarket.

[2] Thus, it is not clear whether seeing a low-ranked firm interviewing a candidate constitutes "good news" or "bad news" about the candidate from a high-ranked firm's point of view.

While restrictive, these assumptions are not unusual,[3] and may be particularly reasonable for entry-level labor markets. In particular, our own experience and conversations with business school graduates and medical residents suggest that a clear ranking among firms (or small sets of firms) often exists (overall or within field), and any offer from a more prestigious firm is typically accepted.[4]

The timing of the model is as follows. In the first stage, each firm decides whether to pay a small cost to "enter" and participate in the hiring process. Each entering firm obtains a private signal about the value of the worker. Firms' signals are weakly affiliated, and may be drawn from an asymmetric distribution. Entry decisions are made simultaneously and are observable, and a firm that does not enter cannot later interview or hire the worker. In the second stage, all entering firms simultaneously choose whether to pay a small cost to "interview" the worker. This decision is based on each firm's private signal. An interview is an indication that the firm is willing to proceed with the hiring process, and may correspond to a show of interest in the worker, placing the worker on a "short list," an administrative step in the hiring process, a fly-out, or an actual interview. A firm that does not interview the worker cannot later hire the worker. Since we are interested in the informational content of firms' decisions whether to interview the worker, and since information obtained in the course of an interview cannot be revealed by the fact of the interview, for expositional simplicity we assume that "interviews" reveal no additional information about the worker.[5] If an interview does reveal additional information to the interviewing firm, then as long as all interviewing firms obtain the same information from the interview, the results of the paper hold without change. In the third stage, after all interviews have taken place, all firms simultaneously decide whether to make employment offers to the worker. The worker accepts the highest-ranked offer among those he receives.

Because the worker's value is a function of all firms' private information, each firm can make better hiring decisions if it has access to even coarse measures of other firms'

[3] The economics literature on auctions, mechanism design, and labor markets is replete with common-value models (a few examples include Engelbrecht-Wiggans, Milgrom, and Weber (1983), Hendricks and Porter (1988), Harstad (1990), Biais, Martimort, and Rochet (2000), Greenwald (1986), and Montgomery (1991)). Ordinal preferences with no wage competition are often assumed in the matching literature (key examples include Gale and Shapley (1962) and Roth and Sotomayor (1990)).

[4] In addition, in some settings firms cannot adjust their wages as part of the hiring process. One example is Israeli universities, in which salaries are essentially fixed for each academic rank. Another example is the market for American federal court clerkships, about which Roth and Xing (1994, page 1001) wrote "Salary for these positions is fixed, and not subject to negotiation between judges and candidates."

[5] This is in fact the case when an "interview" corresponds to placing the worker on a "short list" or to an administrative step in the hiring process.

private information. This, in turn, is determined by whether firms' interview decisions are revealed before employment offers are made. When interview decisions are not revealed (no revelation), no learning takes place between the interviewing and the hiring stages, so an interview is always followed by an offer. With no revelation, lower-ranked firms may be able to enter and make use of their private information. Theorem 1 describes the unique equilibrium in this case. With two firms and independent signals, for example, the strong firm interviews and hires the worker if its signal is sufficiently high. The weak firm also interviews the worker when it observes a sufficiently high signal. Because the weak firm is able to hire only when the strong firm does not make an offer, the signal the weak firm observes must be sufficiently high to offset the "bad news" that the strong firm did not interview. Section 4.1 describes a setting in which for any n all n firms enter and with positive probability interview and profitably hire the worker.

When interview decisions are revealed (revelation), each firm can condition its hiring decision on the interview decisions of the other firms. The additional information can potentially improve the hiring decisions of all interviewing firms. This is the first effect of revelation. On the other hand, firms anticipate the impact of their interview decisions on subsequent offers, and this typically leads to changes in interviewing behavior, which could counteract the first effect.

In fact, revelation creates increased adverse selection for all but the highest-ranked firm, and this erases all the benefits of revelation. The main result of the paper, Theorem 2, shows that in equilibrium only the highest-ranked firm enters the market, and therefore no other firm interviews or hires. Compared to no revelation, all firms and the worker are weakly worse off. Any firm $2, \ldots, n$ that enters with no revelation is strictly worse off. Firm 1 obtains none of the benefits of revelation, and is no better off than with no revelation. If the worker is hired by one of the firms $2, \ldots, n$ with no revelation, he is unemployed and strictly worse off with revelation. Despite the fact that interviews only provide a coarse measure of a firm's private information, in equilibrium no firm can make use of its private information (except for the highest-ranked firm). Consequently, firms' entry choices are made as if they expect all their private information to be revealed. This is true even if a low-ranked firm has much better (or worse) private information about the worker's value than any other firm, and holds regardless of the number of firms.[6]

The intuition for the result is as follows. Consider the weakest entering firm, other than the highest-ranked firm. This firm enters because for one or more signals it expects to be

[6] While this result is reminiscent of Milgrom and Stokey's (1982) "no-trade" theorem, the setting is quite different. In particular, the initial allocation in our setting is not Pareto efficient relative to payoff-relevant parameters, since no firm initially employs the worker (an efficient allocation would have firm 1 employing the worker if and only if his value conditional on all firms' signals is positive.)

able to profitably hire the worker. Consider the lowest such signal, and a combination of the other firms' interview decisions conditional on which the firm expects to be able to profitably hire the worker. The strongest of the other interviewing firms must sometimes not make an offer, otherwise the weak firm could not hire the worker. But the worker's value, evaluated by the strong firm when it sees the highest signal for which it interviews and does not make an offer, is higher than the worker's value evaluated by the weak firm when it sees the lowest signal at which it interviews. Therefore, the strong firm should make the worker an offer, and the weak firm should not be able to profitably hire the worker.[7]

When the common value assumption is relaxed, the exclusion result with revelation will generally not hold. Indeed, suppose that for certain values of the worker a low-ranked firm is interested in hiring the worker but higher-ranked firms are not. Then, even if the value of the worker is commonly known, the low-ranked firm is not excluded. If, however, the value of the worker weakly increases in a firm's ranking, our exclusion result holds. The result is also robust to the hiring of multiple workers, provided there is sufficient separability across workers.

Nevertheless, we view our contribution not as a complete description of a labor market outcome but as an exposition of basic frictions that necessitate the emergence of stabilizing institutions or conventions observed in practice. One prevalent convention is an orderly sequencing of interviews in which stronger firms move earlier than weaker firms. As we discuss in our concluding section, sequential interviews can serve to mitigate the adverse selection from revelation and to produce the same entry and hiring outcomes as in the setting with no revelation. Indeed, such a convention can arise endogenously if firms have flexibility in timing their interviews and hiring decisions. Stronger firms move first; weaker firms have an incentive to wait for a second round, at which point they can observe and react to the interview decisions of stronger firms. This is consistent with the observation that many professional labor markets, including the academic job market for economists, seem to clear in multiple stages. With these ideas in mind, it would be interesting to explore the effect of the new information available through websites such as Econjobmarket on the hiring strategies of employers at various points in the pecking order.

We emphasize that there are many other features of real world labor markets that go beyond the limits of our model and may alleviate the inefficiency we are highlighting here. For example, firm-specific preferences for heterogeneous workers add private-value elements

[7] The formal proof is considerably more involved, because in addition to considering the worker's expected value from the perspective of different firms, the proof also takes into account that firms may use mixed strategies and that different firms may attribute different probabilities to the same event, because they have different information.

to the model. When firms have multiple vacancies there may arise complementarities across workers, which could make weaker firms endogenously more competitive for workers with crucial skills. Finally, incomplete information about worker preferences may weaken the market power of the stronger firms. Each of these features is present in the market for academic economists, and this complicates the effect of publicizing information about interviews.

There is an extensive literature on two-sided matching, beginning with the seminal work of Gale and Shapley (1962).[8] The novelty of our paper is the focus on a specific hiring process and information revelation when firms have incomplete information about the value of the worker, which is common to all firms. Such a hiring process is often used in practice, and leads to new strategic considerations that influence firms' behavior.[9] Several recent papers examine models in which related considerations arise. Lee and Schwarz (2008) consider a two-sided market with incomplete information on both sides, in which both sides learn their preferences through costly interviews. In contrast to our model, agents do not base their interview decisions on private information. All agents on each side of the market are ex-ante identical, values are private, and signals are independent and fully informative. The main issue is how coordination on which workers each firm should interview influences outcomes. Coles, Kushnir, and Niederle (2009) study the effect of signaling in a two-sided matching market with incomplete information. Lee (2009) studies early admissions, and suggests that because students can apply early to only one university, early admissions help mitigate the adverse selection that universities face during regular admissions. Masters (2009) studies hiring with interviews but does not consider revelation and the resulting interaction among firms. Josephson and Shapiro (2009) investigate a multistage hiring model with multiple workers and firms, each of which can hire at most one worker. Workers have identical, known preferences over firms, but firms do not initially know workers' qualities. In each of several rounds, workers are randomly assigned to firms, each firm can discover the quality of its assigned worker by paying a cost, and each firm can then make its assigned worker an offer. A worker that is not assigned to a firm cannot be hired by the firm, which may lead to a "congestion" inefficiency that does not arise in our model. An additional inefficiency,"information-based unemployment," arises when

[8] A large literature beginning with Greenwald (1986) studies settings in which employers have private information about the ability of their workers. Adverse selection in secondhand labor markets arises in such settings, because firms tend to retain higher-ability workers. Montgomery (1991) considers a setting in which new workers may be referred by old workers of similar ability.

[9] These considerations do not arise in many existing models of two-sided matching, both those that postulate complete information and those that postulate incomplete information of agents' preferences (see, for example, Roth and Sotomayor (1990), Sönmez (1999)).

a firm does not pursue a worker whose quality it infers to be insufficiently high because he did not receive an offer from a firm to which he was previously assigned. The adverse selection underlying this result is related to the one underlying our exclusion result. In contrast to our result, however, information-based unemployment relies on distributional assumptions on the three possible quality levels of a worker, on the sequential assignment of workers to firms, on all firms obtaining the same information about a worker's quality in the discovery process, and on firms' discovery costs being sufficiently large.

A more closely related framework is that of Chakraborty, Citanna, and Ostrovsky (2010), who study the more general problem of stable one-to-one matching mechanisms with multiple firms and multiple workers and incomplete information. The connection to our paper is that the matching mechanism induced by the equilibrium outcome of our no-revelation game satisfies their definition of weak stability. More importantly, they demonstrate a decentralized mechanism that implements that stable matching. This mechanism resembles an extension of our game with revelation in which offers are timed endogenously instead of being made simultaneously. This observation, when contrasted with the extreme inefficiency of the equilibrium we study, shows how market timing can be an endogenous response to an underlying adverse selection problem. We discuss this connection in more detail in the concluding section of the paper.

The rest of the paper is organized as follows. Section 2 introduces the model and related notation. Section 3 conducts a preliminary analysis. Section 4 explores the setting with no revelation. Section 5 explores the setting with revelation, and states and proves the main result. Section 6 discusses extensions to informative interviews, multiple workers, heterogeneous worker value, and sequential interviews. Section 7 concludes. The Appendix contains the proof of Theorem 2 and related preliminaries.

2 The Model and Notation

There are n risk-neutral firms and one worker. The set of firms $\{1, \ldots, n\}$ is denoted by N. The worker is characterized by a vector of weakly affiliated signals, one for each firm. The set of possible signal realizations for firm i, denoted S_i, is finite and linearly ordered, with generic element s_i'. The vector of firms' signals is drawn from a distribution F on $S = \times_i S_i$ with full support. The random variable whose realization is an element in S_i is denoted by s_i, so $s_1, \ldots, s_n$ are weakly affiliated.

The worker can work for only one firm, and has a commonly-known strict ranking over firms. Firm 1 is the highest-ranked firm, firm 2 is the second highest-ranked firm, etc. The net value of employing the worker, in monetary units, is common to all firms. This value, denoted by v, is a function of all firms' signals, and strictly increases in each firm's

signal. Together with the generality of the signal structure, which includes, for example, independent and conditionally independent signals, this specification of v accommodates heterogeneity in the extent to which different firms are informed about the value of the worker. The value of not hiring the worker is normalized to 0.

The timing of the market is as follows. First, before observing their signals, all firms simultaneously choose whether to enter the market. The cost of entry to firm i is $e_i > 0$. A firm that does not enter the market cannot participate in subsequent stages of the market. Entry decisions are public. After the entry stage, all entering firms observe their private signals, and then simultaneously decide whether to interview the worker. The cost of an interview to firm i is $c_i > 0$. A firm that does not interview the worker cannot later hire him. For simplicity, an interview reveals no new information about the worker to the interviewing firm.[10] We analyze the model under two different information structures: no revelation of interview decisions, and full revelation of interview decisions. At the next stage, each firm decides whether to make an employment offer to the worker. The offers are made simultaneously. The worker accepts the offer made by the highest-ranked firm among those that made him an offer. An entering firm i's payoff from hiring a worker with signals $s'_1, \ldots, s'_n$ is $v(s'_1, \ldots, s'_n) - c_i - e_i$. If the firm interviews but does not hire a worker, either because the firm does not make him an offer or because the worker does not accept the firm's offer, then the firm's payoff is $-c_i - e_i$. The payoff of a firm that does not enter is 0. Firms' entry and interview costs are commonly known.

Positive entry costs and interview costs guarantee that entry and interview decisions are not "cheap talk." Because we are interested in the informational effects of signals and interviews, we will consider low (but positive) entry and interview costs. By precluding "cheap talk," low entry and interview costs lead to significantly different predictions than do costs of 0. We maintain the assumption that there is some $M > 1$ such that the ratio of any two firms' interview costs is no more than M. The larger the bound M, the lower costs have to be for Theorem 2 to hold. We analyze the game using the solution concept of sequential equilibrium (henceforth: equilibrium).

3 Preliminary Analysis

As a preliminary exercise, suppose that firms $2, \ldots, n$ do not enter. Then, conditional on entering, firm 1 interviews the worker (and later makes an offer that will be accepted) if and only if its signal s'_1 satisfies

$$E[v|s_1 = s'_1] \geq c_1, \tag{1}$$

[10] Section 6 discusses informative interviews.

with possible mixing between interviewing and not interviewing if the inequality is an equality. Because the worker's value increases in every firm's signal and firms' signals are affiliated, a higher signal makes firm 1 more optimistic about the worker's expected value.[11] This fact and the fact that the worker always accepts the firm's offer if it is made imply that the firm employs a threshold interviewing (and hiring) strategy. As the cost c_1 of interviewing decreases, the threshold decreases and the firm's expected profit increases. For the remainder of the paper, we make the following assumption.

A1 *(1) holds with a strict inequality for at least one signal when* $c_1 = 0$.

Assumption A1 guarantees that for low (but positive) interview costs, firm 1's post-entry expected profit is positive. Therefore, when the entry and interview costs are low (but positive), firm 1 will enter and make positive profits even if it is the only entering firm.

4 No Revelation

With no revelation, a firm will interview if and only if it plans to make an offer. Because the worker accepts the highest-ranked offer, we can solve for equilibrium by moving from firm 1 to firm n and identifying each firm's interviewing strategy given those of all higher-ranked firms. Firm 1 behaves as described in Section 3. Conditional on entering, it employs a threshold interviewing strategy, interviewing and hiring for every signal above the lowest signal s_1' that satisfies (1) (if such signals exist), with possible mixing at the lowest signal if the inequality is an equality.

Given firm 1's interviewing strategy, its entry decision depends on whether its expected profits conditional on entering offset the entry costs.[12] When the expected profits conditional on entering equal the entry cost, the firm may mix between entering and not entering. For low entry and interviewing costs e_1 and c_1, however, firm 1 has a unique optimal strategy. To see this, denote by T_1 the lowest signal for which (1) holds with a strict inequality when $c_1 = 0$ (such a signal exists by Assumption A1). Then, for low e_1 and c_1, firm 1's unique optimal strategy is to enter with probability 1 and interview and hire with probability 1 at all signals greater than or equal to T_1. The following result shows that for low entry and interviewing costs, there is in fact a unique equilibrium, in which every entering firm employs a threshold interviewing strategy.

[11] To see this, apply Lemma 4 in the Appendix with $Z_{-i} = S_{-i} \times \Omega_{-i}$.

[12] If firm 1 mixes at the lowest signal for which it interviews with positive probability, then it makes 0 profits there, so its behavior there does not affect the profitability of entry.

Theorem 1 *For low* $\max_{i\in N} e_i$ *and* $\max_{i\in N} c_i$ *there is a unique equilibrium, which is in pure strategies. In this equilibrium, every entering firm* i *interviews for all signals greater than or equal to some signal* T_i*. The equilibrium can be found by iterated elimination of strictly dominated strategies.*

Proof. We prove the following claim by induction: for any $i \in N$, for low $\max_{j\leq i} e_j$ and $\max_{j\leq i} c_j$, every firm $j \leq i$ has a strictly dominant (pure) strategy once the strictly dominated strategies of higher-ranked firms have been iteratively eliminated. As we have seen, the claim is true for $i = 1$, because for low e_1 and c_1 firm 1 has a strictly dominant threshold interviewing strategy with threshold T_1. Now suppose that the claim is true for $i - 1 \geq 1$. Then, for low $\max_{j\leq i-1} e_j$ and $\max_{j\leq i-1} c_j$, in any equilibrium firms $1, \ldots, i-1$ play the strategies identified by the induction hypothesis. Given the strategies of firms $1, \ldots, i-1$, what should firm i do? Conditional on entering, firm i will succeed in hiring the worker when it makes an offer if and only if firms $1, \ldots, i-1$ do not make make an offer or, equivalently, do not interview. Because firms $1, \ldots, i-1$ use pure strategies, the event that none of these firms interview the worker is the set $B = \times_{j\in N} B_j$, where

$$B_j = \begin{cases} \text{signals in } S_j \text{ for which firm } j \text{ does not interview} & j \leq i-1 \\ S_j & j > i-1 \end{cases}. \tag{2}$$

Note that $B_j = S_j$ if firm j does not enter. Conditional on entering, firm i's net profit if it interviews and makes an offer to a worker at signal s_i' is

$$\Pr(B|s_i = s_i')\, E[v|B, s_i = s_i'] - c_i. \tag{3}$$

Conditional on entering, for low c_i it is uniquely optimal for firm i to interview with probability 1 at precisely all signals s_i' for which the expression in (3) is strictly positive when c_i is replaced with 0. If there is at least one such signal s_i', then for low e_i it is strictly optimal for firm i to enter. If there are no such signals, then it is strictly optimal for firm i not to enter. This shows that the induction hypothesis holds for i. Moreover, if the the expression in (3) is strictly positive for some signal s_i' when c_i is replaced with 0, then the expression is also strictly positive for all signals $s_i'' > s_i'$. This is because (i) by affiliation and because v is strictly increasing, $E[v|B, s_i = s_i']$ increases with s_i' (Lemma 4 in the Appendix) and (ii) $\Pr(B|s_i = s_i')$ is strictly positive for all signals s_i' if it is strictly positive for one signal s_i' (F has full support). Therefore, if firm i enters for low e_i, then for low c_i it interviews for all signals greater than or equal to some signal T_i. ■

Costs and strategies of firms ranked lower than i do not appear in (2) and (3). Therefore, the unique equilibrium when entry and interview costs are low can be solved for by iteratively applying the process described in the proof of Theorem 1, proceeding from firm

1 to firm N. Note that the interview threshold of every firm $2, \ldots, n$ is higher than if would be if the firm was the only one in the market. Firms compensate for the adverse selection they experience from higher-ranked firms by increasing their interview threshold.[13]

The requirement that interview costs be low is necessary to conclude that firms employ threshold interviewing strategies. To see this, suppose that e_1 and c_1 are low enough for firm 1 to use a threshold interviewing strategy with threshold T_1. Consider firm 2 and (3). As s_2' increases, $\Pr(s_1 < T_1 | s_2 = s_2')$ decreases (by affiliation) and $E[v | s_1 < T_1, s_2 = s_2']$ strictly increases (v is strictly increasing and affiliation - see Lemma 4 in the Appendix). Therefore, the expression in (3) may be a non-monotonic function of s_2'. This means that the set of signals for which firm 2 interviews need not correspond to a threshold interviewing strategy when c_2 is not low.

Nevertheless, when firms' signals are independent, each firm employs a threshold interviewing strategy, regardless of interview costs. This is because when firms' signals are independent, $\Pr(B | s_i = s_i')$ is independent of s_i', so the expression in (3) strictly increases in s_i'.[14]

4.1 No Revelation Example

Suppose each firm's signal is drawn uniformly and independently from the set $\left\{-\frac{1}{2} + \varepsilon, \frac{1}{2} - \varepsilon\right\} \cup \left\{-\frac{1}{2} + \frac{i}{2^k} - \varepsilon, -\frac{1}{2} + \frac{i}{2^k} + \varepsilon : i = 1, \ldots, 2^k - 1\right\}$ for some $k \geq n$ and positive $\varepsilon < \frac{1}{2^{k+2}}$ (this approximates the uniform distribution on $\left[-\frac{1}{2}, \frac{1}{2}\right]$). Figure 1 illustrates the set of signals for $k = 2$ and small $\varepsilon > 0$ by depicting the signals as circles.

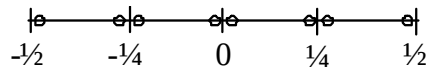

Figure 1: The set of signals for each firm in Example 1 for $k = 2$ and small $\varepsilon > 0$

Suppose that $v = \sum_{i=1}^{n} s_i$. Then, if entry and interview costs are low, for a firm operating alone in the market it is uniquely optimal to interview and hire at any signal greater than or equal to ε, because the expected value of other firms' signals is 0. With no revelation and low entry and interview costs, firm 1 enters, and interviews and hires at any signal greater than or equal to ε. Therefore, the expected value of firm 1's signal conditional on not interviewing is $-\frac{1}{4}$. As a result, for low entry and interview costs, firm 2 enters, and interviews and hires at any signal greater than or equal to $\frac{1}{4} + \varepsilon$. Proceeding

[13] Lee (2009) obtain a similar result when comparing regular admissions to early admissions.

[14] If this expression equals 0 for some signal s_i', then the firm may mix between interviewing and not interviewing at s_i'.

in this way, we see that with no revelation and low entry and interview costs there is a unique equilibrium. In this equilibrium, all firms enter, and every firm i interviews and hires at all signals greater than or equal to $T_i = \frac{1}{2} - \frac{1}{2^i} + \varepsilon$.

5 Revelation

With revelation, each firm can condition its hiring decision on all other firms' interview decisions. As a result, a firm's interview strategy may depend on its beliefs about all other firms' interview strategies, and not only on those of higher-ranked firms. This typically implies that the no-revelation equilibrium identified in Theorem 1 is not an equilibrium with revelation. To see why, consider the example of Section 4.1 with two firms. For low entry and interview costs, with no revelation the unique equilibrium is for both firms to employ threshold interviewing strategies. Firm 1's interview threshold is ε and that of firm 2 is $\frac{1}{4} + \varepsilon$. With revelation, this is no longer an equilibrium. Indeed, suppose that firm 2 maintained the same interviewing strategy. This implies that seeing firm 2 interview is good news about firm 2's signal. Firm 1 would then find it beneficial to interview for some signals lower than ε, and for those signals to make an offer only if it sees firm 2 interview. Firm 2's response to this behavior by firm 1 would be to change the set of signals for which it interviews. More generally, with revelation a firm may choose to interview because it expects to learn something about the worker's value from the other firms' interview decisions. But the firm also knows that the other firms will learn something from its interview decision, which may affect its probability of hiring the worker and the value of the worker conditional on hiring.

More concretely, suppose that m firms enter in an equilibrium with revelation, and assume for simplicity that firms use pure strategies, so that each firm has an "interview set" of signals for which it interviews. Consider the behavior of an entering firm. After interviewing, the firm can condition its hiring decision on each of the 2^{m-1} possible combinations of the other entering firms' interview decisions. For each such combination, the firm determines a "hiring set" of signals for which it makes an offer after interviewing. These hiring sets, which depend on the other firms' interview sets, determine the firm's interview set. Because of this interdependence, all firms' hiring sets for each combination of the other firms' interview decisions, and all interview sets, are determined jointly. A sequential procedure like the the one described in Theorem 1 can therefore not be used to solve for an equilibrium.

The analysis is further complicated by considering mixed strategies (which may depend on entry and interview costs) and non-threshold interview and hiring sets. An argument like the one used in Theorem 1 to show that firms use pure strategies and that entering

firms use threshold interviewing strategies, all of which are independent of costs when costs are low, does not work with revelation. And if firms use non-threshold interview strategies, then seeing another firm interview is not necessarily "good news" about the worker's value. Despite these complications, the following result fully characterizes equilibrium behavior with revelation.

Theorem 2 *With revelation, there is at least one equilibrium. Moreover, for low entry and interview costs, in any equilibrium the only firm that enters is firm* 1.[15]

To illustrate the proof of Theorem 2, suppose that interview costs are 0, and that a firm interviews only if there is a positive probability that it can hire the worker and conditional on hiring the worker the firm makes positive profits. By Assumption A1, firm 1 enters and interviews for some signals, because it can always ignore the other firms. Suppose that firm $i > 1$ is the lowest-ranked firm that enters, and consider the lowest signal s_i' at which firm i interviews. When firm i observes this signal, there is a combination of the other firms' interview decisions, or "interview schedule," that arises with positive probability and at which firm i can profitably hire the worker with positive probability (otherwise firm i would not interview at s_i').

The first step of the proof shows that this interview schedule consists of only firm i interviewing. To see why, suppose that such a schedule $\mathcal{I}$ exists in which other firms interview, and denote by j the highest-ranked firm that interviews in $\mathcal{I}$. Because firm i successfully hires at $\mathcal{I}$, there are signals at which firm j interviews but does not make an offer at $\mathcal{I}$. Suppose that firm i observes s_i', firm j observes the highest signal s_j' for which it interviews but does not make an offer at $\mathcal{I}$, and the schedule $\mathcal{I}$ arises. Then firm j's expectation of the worker's value is higher than that of firm i: firm j observes s_j' and knows only that firm i's signal is greater than or equal to s_i', whereas firm i observes s_i' and knows only that firm j's signal is at most s_j'. Because firm i is willing to hire the worker when it observes s_i' and $\mathcal{I}$, firm j would be willing to hire the worker when it observes s_j' and $\mathcal{I}$. But then firm j should deviate and make an offer at s_j'. This is illustrated in Figure 2 below.

The second step of the proof shows that if firm i is willing to interview when it observes s_i' and make an offer when it is the only firm that interviews, then some higher-ranked firm l can profitably deviate and interview at some signal at which it is not supposed to interview, and then make an offer if it sees that the only other firm that interviews is firm i. Because interview costs are in fact positive, when firm l considers deviating to interview it

[15] When there are only two firms ($n = 2$), the restriction on the ratio between firms' interview costs assumed in Section 2 is not needed for the result.

considers not only the worker's expected value, but also the probabilities of certain events. These probabilities differ from the point of view of different firms, and may also depend on firms' interview costs. Therefore, the second step of the proof is considerably more involved than the first one. The proof of Theorem 2, which is in the Appendix, formalizes these steps and extends them to accommodate mixed strategies.

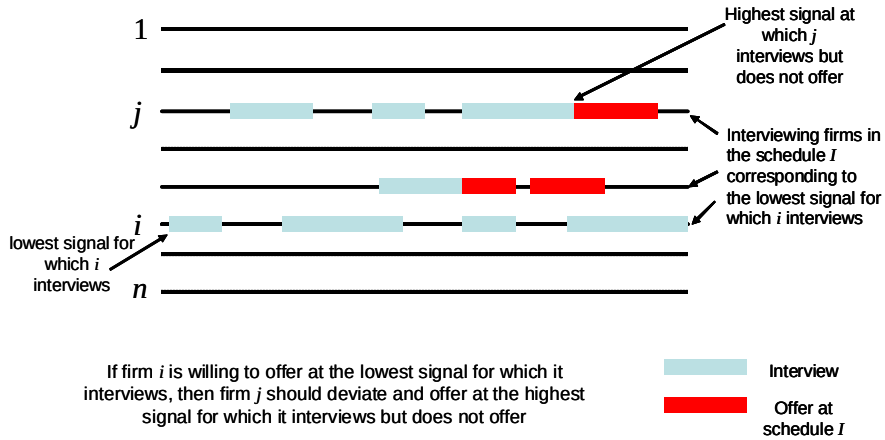

Figure 2: A profitable deviation for firm j

5.1 Discussion

Theorem 2 implies that when entry and interview costs are low, revelation leads to unraveling that excludes all but the highest-ranked firm from interviewing and hiring. The outcome is as if the set of firms included only firm 1. In this outcome, firms $2, \ldots, n$ are just as disadvantaged as they would be if all of their private information were revealed, and firm 1 does not benefit.

Compared to the setting with no revelation, no firm is better off with revelation, and any firm $2, \ldots, n$ that enters with no revelation is strictly worse off with revelation. In the setting of Section 4.1 above, in which all firms enter with no revelation, revelation makes firms $2, \ldots, n$ strictly worse off because they are excluded. Theorem 2 also implies that the worker is no better off with revelation. Firm 1 hires the worker for the same set of signal profiles with revelation and with no revelation, and whenever the worker is hired by some firm $2, \ldots, n$ with no revelation, he is unemployed with revelation. Thus, revelation lowers virtually any measure of welfare and efficiency when entry and interview costs are low.

Because the function v is not assumed symmetric, it may be that the impact of one firm's signal on the worker's value is high, while that of another firm is low. Thus, how informative a firm's signal is may vary across firms. In particular, when the number of

firms is large, it may seem that at least some firms' interview decisions would not be so informative, which may allow these firms to participate in the market. Theorem 2 shows that firms $2, \ldots, n$ cannot make any use of their private information with revelation, regardless of n.

The detrimental effects of revelation are due to firms' strategic modification of their behavior when they anticipate that their interview decisions will be revealed. Indeed, suppose that firms believe that their interview decisions will be kept private, but these decisions are nevertheless revealed before firms make offers. In this case, all firms are weakly better off, and the worker is weakly worse off, compared to the no-revelation setting. This is because with no revelation an interviewing firm hires if and only if no higher-ranked firm interviews, and each firm can replicate this outcome when firms' interview decisions are unexpectedly revealed.

6 Extensions

6.1 Informative Interviews

Suppose that an interview conveys additional information about the worker to the interviewing firm. As long as all interviewing firms obtain the same information from the interview, the results of the paper hold without change. Formally, this information is an additional signal s_0 that is affiliated with the other signals, such that $s_0, \ldots, s_n$ has full, finite support, and v increases in s_0. The realization of s_0 is revealed during the interview. Of course, if firms obtain different information when they interview the worker, the exclusion result may not hold.[16]

6.2 Multiple workers

The analysis applies to multi-worker markets in which all interviews are conducted simultaneously before all offers are made, as long as there is enough separability across workers. For this we require that the vectors of signals for each worker be independent across workers and that the value of the workers hired by a firm be additively separable across workers (so there is no minimal or maximal number of positions to fill). These assumptions imply that with no revelation we can analyze firms' interviewing (and hiring) decisions for each

[16] For example, if firms obtain all their information during the interview, then we are essentially in the no revelation setting, in which many firms may enter.

worker separately, so Theorem 1 holds.[17]

With revelation things are more delicate, because a firm's decision whether to interview a worker could depend on the signals it observes about other workers. To see why, suppose that the other firms believe that firm i decides whether to interview worker k based on the signals firm i observes about other workers. Then, the other firms will infer something about the value of workers other than k from firm i's interview decision regarding worker k. If firm i were to then decide whether to interview worker k based only on the signal it observes about worker k, then the other firms' hiring decisions regarding the other workers may be affected to the detriment of firm i. Thus, firm i may optimally condition its interview decision regarding worker k on the signals it observes about other workers.[18]

We would like to rule out such behavior, because the signal a firm observes for one worker contains no information about the value of other workers. We therefore consider *separable equilibria*, in which each firm's decision whether to interview a worker does not depend the firm's signals for other workers. When analyzing separable equilibria we need only consider strategies in which a firm's hiring decision regarding worker k does not depend on the other firms' interview decisions regarding the other workers.[19] Because firms' signals are independent across workers and the value of the workers to a firm is additively separable across workers, the continuation of any separable equilibrium conditional on entry is the conjunction of sequential equilibria conditional on entry in markets with one worker, one market for each of the workers in the original market. In particular, for low entry costs a firm finds it optimal to enter in a sequential equilibrium of the original market if and only if it finds it profitable to enter in the sequential equilibrium of at least one of the markets corresponding to a single worker. Therefore, Theorem 2 characterizes all separable equilibria when entry and interview costs are low.

[17] In principle, a firm could choose whether to interview a worker based on the signals it observes for other workers. These signals, however, do not tell the firm anything about the worker's value, so the firm could optimally use them only as a randomization device. For low enough entry and interview costs, Theorem 1 rules out such behavior because it shows that firms use pure strategies.

[18] Such behavior does not arise with no revelation, because no firm observes the other firms' interview decisions when it makes its hiring decisions.

[19] If a firm is indifferent between hiring and not hiring a worker, it may use the other firms' interview decisions regarding the other workers as a randomization mechanism. This does not change the statement or proof of Theorem 2.

6.3 Heterogeneous Worker Value

The assumption that the worker's value is common to all firms is key to the analysis. If lower-ranked firms have a positive probability of hiring the worker when all firms' signals are made public, then revelation will generally not exclude these firms. With this in mind, the common value assumption can be replaced with the assumption that the value of the worker to firm j is weakly higher than the value to firm i whenever $j < i$.[20] This maintains the property that if all firms' signals are public, a higher-ranked firm wants to hire the worker if a lower-ranked firm does.

With only two firms and independent signals, we can compare the setting with revelation to that with no revelation, even if the value of the worker to firm 2 is higher than his value to firm 1. Specifically, let the value of the worker be $v(s_1, s_2)$ to firm 1 and $v(s_1, s_2) + w_2$ to firm 2, for some $w_2 > 0$. If v can take negative values lower than w_2, then it may be that firm 2 wants to hire the worker and firm 1 does not. With no revelation, Theorem 1 and its proof hold without change. With revelation, because signals are independent, each firm employs a threshold interviewing strategy. Therefore, seeing the worker interviewed by firm 2 is "good news" and seeing the worker not interviewed by firm 2 is "bad news" for firm 1 about the worker's value, regardless of firm 2's interviewing threshold. This implies that firm 1 is better off, and its interview threshold is weakly lower, with revelation than with no revelation. The opposite is true for firm 2.[21] With more than two firms, firm 1 is always weakly better off with revelation than with no revelation, but the effect of revelation on lower-ranked firms is no longer unambiguous.[22]

6.4 Sequential Interviews

The adverse effects of revelation can be mitigated by having firms interview the worker one after the other instead of simultaneously, with each firm's interview decision being revealed before the next firm makes its interview decision. It can be shown that for small entry and

[20] One example is adding a firm-specific constant to v, with higher-ranked firms having a higher constant.

[21] In particular, if the worker is hired by firm 2 with revelation, then he is hired by firm 2 with no revelation, but the reverse may not be true.

[22] In particular, revelation may make a low-ranked firm better off. To see this, consider three firms, with firm 3's signal so uninformative that firms 1 and 2 ignore firm 3's interview decision when they make their hiring decisions. It is easy to construct a two-firm setting in which sometimes the worker hired by firms 1 and 2 with no revelation is unemployed with revelation. If the value of the worker to firm 3 is then high enough, then firm 3 would like to hire the worker when it is not hired by firms 1 and 2, so revelation may make firm 3 better off.

interview costs each exogenous sequencing of firms induces a unique outcome. When firms are ordered from the lowest-ranked firm to the highest-ranked firm, the entry and hiring outcome is the same as in the setting with revelation (only the highest-ranked firm enters); when firms are ordered from the highest-ranked firm to the lowest-ranked firm, the entry and hiring outcome is the same as in the setting with no revelation. Firms' expenditures on interviews, however, are lower than with no revelation. Therefore, the right sequencing of interviews, combined with revelation, (slightly) improves upon no revelation.

7 Conclusion

This paper has investigated a model in which privately-informed firms interview a worker before making their hiring decisions, and the value of the worker is common to all firms. When firms' interview decisions are kept private, each firm can make use of its private information, even though all but the highest-ranked firm face adverse selection akin to a "winner's curse." When firms' interview decisions are revealed, the adverse selection becomes so strong that regardless of the number of firms only the top firm can make use of its private information - all other firms stay out of the market. Revelation of firms' interview decisions, which has the potential to improve market outcomes through the sharing of private information, leads to complete unraveling and less usage of information than with no revelation. The outcome with revelation is worse than with no revelation according to virtually any efficiency or social welfare criterion.

The effect of revelation may be less pronounced when complementarity/substitution among workers, private value components, and other real-world features of labor markets are introduced. We view our result as indicative of the potential for adverse selection in common-value markets with intermediate, coarse information disclosure, even when there are many firms and the information structure is fairly general. For example, suppose that the worker's preferences are only partially known, but the other assumptions of the model are maintained. Our analysis indicates that when firms know that the worker strictly prefers any firm in a known subset of firms to the other firms, with revelation those firms that are not preferred would be excluded from the market.

Our exclusion result relies on the modeling feature that all firm make interview decisions simultaneously and hiring decisions simultaneously. As discussed in Section 6, having the firms make their interview decisions sequentially, from the highest-ranked firm to the lowest-ranked firm, restores the no-revelation outcome. More realistically, the timing of each firm's interview (and possibly hiring) decision may be chosen by the firm. This too could overturn the exclusion result. For example, suppose that there are two firms and two

rounds in which interview decisions can be made.[23] The firms simultaneously and publicly decide whether to make their interview decision in the first round, and any firm that does not make its interview decision in the first round makes the decision in the second round. Hiring takes place at the end of the second round. Then, it is an equilibrium for the strong firm to make its interview decision in the first round, and for the weak firm to make its decision in the second round, with the weak firm not interviewing in the second round if the strong firm either interviews in the first round or deviates and delays its interview decision to the second round. The equilibrium interview and hiring decisions then coincide with those described in Theorem 1, and the induced matching is weakly stable in the sense of Chakraborty, Citanna, and Ostrovsky (2010).[24] Thus, the endogenous sequencing of firms' decisions may be viewed as a response to the adverse selection that arises with simultaneous decisions.

Two additional modeling assumptions, in addition to simultaneous decisions, suggest avenues for future research. The first is that firms' hiring decisions are binary. This assumption may be suitable for studying certain entry-level labor markets or university admission processes, in which competition in wages does not seem to be a dominant factor. A model with wage competition would fit other settings but presents significant technical challenges.[25] A second modeling assumption is that whether firms' interview decisions are revealed is determined exogenously. An interesting question is under what circumstances we would expect revelation to arise endogenously.

[23] We thank a referee for suggesting this example.

[24] Chakraborty, Citanna, and Ostrovsky (2010) also observe that keeping weak firms' private information from being inferred by stronger firms is in general necessary for the existence of a weakly stable matching.

[25] Klemperer (1998) describes related "almost common value" takeover settings in which disadvantaged firms may be excluded from bidding.

A Theorem 2

A.1 Preliminaries and Notation

To model mixed strategies, we assume that each firm i observes the outcome of a uniform lottery over $\Omega_i = [0,1]$, and denote by ω_i the realization of this lottery. The lotteries of different firms are statistically independent, and are also independent of all firms' signals.

We use the following notation for post-entry interviewing and hiring mixed strategies parameterized by k, that is, strategies that take the set of entering firms as given. Firm i chooses a measurable set $\tilde{I}_i^k \subset S_i \times \Omega_i$ following whose elements it interviews the worker. We define $\sigma_i^k(s_i) = \text{Prob}(\{\omega_i : (s_i, \omega_i) \in \tilde{I}_i^k\})$ as the probability that firm i interviews after observing the signal s_i. For each subset $\mathcal{I} \subset \{1, \ldots, n\}$ such that $i \in \mathcal{I}$, firm i chooses a measurable set $\tilde{O}_{i,\mathcal{I}}^k \subset S_i \times \Omega_i$ following whose elements it makes an offer if it interviewed and observed interview schedule $\mathcal{I}$ (that is, if it observed precisely the firms in $\mathcal{I}$ interviewing). For every interview schedule $\mathcal{I}$ such that $i \in \mathcal{I}$, we define $\tau_i^k(s_i; \mathcal{I}) = \text{Prob}\{\omega_i : (s_i, \omega_i) \in \tilde{O}_{i,\mathcal{I}}^k\}$ as the probability that firm i makes an offer if it both (i) interviewed after observing signal s_i and (ii) observed interview schedule $\mathcal{I}$. We denote by $\underline{s}_i^k = \min\{s_i : \sigma_i^k(s_i) > 0\}$ the lowest signal for which firm i interviews with positive probability, by $\bar{s}_i^k = \max\{s_i : \sigma_i^k(s_i) < 1\}$ the highest signal for which firm i interviews with probability less than one, and by $\bar{s}_{i,\mathcal{I}}^k = \max\{s_i : \tau_i^k(s_i; \mathcal{I}) < 1, \sigma_i^k(s_i) > 0\}$ the highest signal for which firm i interviews with positive probability and makes an offer with probability less than one after interviewing and observing interview schedule $\mathcal{I}$.

Let $I_i^k = \tilde{I}_i^k \times \prod_{j \neq i}(S_j \times \Omega_j)$ and $O_{i,\mathcal{I}}^k = \tilde{O}_{i,\mathcal{I}}^k \times \prod_{j \neq i}(S_j \times \Omega_j)$. For a set of firms $\mathcal{I}$, we denote by $\hat{\mathcal{I}} = \cap_{j \in \mathcal{I}} I_i^k \cap_{j \notin \mathcal{I}} \neg I_j^k$ the event that exactly this set of firms interviews. The set $\Phi_{i,\mathcal{I}}^k = \cap_{j \in \mathcal{I}, j < i} \neg O_{j,\mathcal{I}}^k$ is the event at which firm i could possibly have its offer accepted if precisely the firms in $\mathcal{I}$ interview (because all stronger interviewing firms do not make offers).

A.2 Technical Lemmas

Denote by G_i the uniform CDF on $\Omega_i = [0,1]$. Endow $\Omega = \times_{i \in N} \Omega_i$ with the product CDF $G = \times G_i$. Denote by μ^G the probability measure on Ω induced by G, by μ_i^G the probability measure on Ω_i induced by G_i, and by μ_{-i}^G the probability measure on Ω_{-i} induced by G_{-i}, where $-i$ is the set of indices other than i. Consider the probability space defined by $S \times \Omega$ and the probability measure $\mu^{F \times G}$ induced by $F \times G$. Denote by $\mu_i^{F \times G}$ and $\mu_{-i}^{F \times G}$ the induced probability measures on the measurable spaces $S_i \times \Omega_i$ and $S_{-i} \times \Omega_{-i}$.

Lemma 1 *Every measurable set $Z_i \subseteq S_i \times \Omega_i$ can be represented uniquely as $\cup_{s_i' \in S_i} (\{s_i'\} \times A(s_i'))$, where $A(s_i')$ are measurable subsets of Ω_i.*

Proof. The set $\Delta_i = \left\{\cup_{s_i' \in S_i} (\{s_i'\} \times A(s_i')) : A(s_i') \text{ are measurable subsets of } \Omega_i\right\}$ is a σ-algebra: $S_i \times \Omega_i$ is an element of Δ_i, the complement of an element of Δ_i is in Δ_i, and a countable union of elements in Δ_i are in Δ_i. Moreover, Δ_i is the smallest σ-algebra of $S_i \times \Omega_i$

with respect to which the projection mappings $\pi_1 : S_i \times \Omega_i \to S_i$ and $\pi_2 : S_i \times \Omega_i \to \Omega_i$ are continuous. To see that the projection mappings are continuous, note that for any $B \subseteq S_i$,

$$\pi_1^{-1}(B) = \cup_{s_i' \in B} (\{s_i'\} \times \Omega_i) \cup_{s_i' \notin B} (\{s_i'\} \times \phi),$$

and for any $C \subseteq \Omega_i$,

$$\pi_2^{-1}(C) = \cup_{s_i' \in S_i} (\{s_i'\} \times C).$$

Now consider some σ-algebra $\tilde{\Delta}_i$ of $S_i \times \Omega_i$ with respect to which the projection mapping are continuous. By continuity, for any $s_i' \in S_i$ and measurable $B \subseteq \Omega_i$, the sets $\pi_1^{-1}(\{s_i'\}) = \{s_i'\} \times \Omega_i$ and $\pi_2^{-1}(B) = \cup_{s_i' \in S_i} (\{s_i'\} \times B)$ are elements of $\tilde{\Delta}_i$. Because $\tilde{\Delta}_i$ is closed under finite intersections, $(\{s_i'\} \times \Omega_i) \cap \cup_{s_i' \in S_i} (\{s_i'\} \times B) = \{s_i'\} \times B$ is an element of $\tilde{\Delta}_i$. Because $\tilde{\Delta}_i$ is closed under countable unions, $\Delta_i \subseteq \tilde{\Delta}_i$. By definition as the smallest σ-algebra with respect to which the projection mappings are continuous, the product σ-algebra on $S_i \times \Omega_i$ is therefore Δ_i, so every measurable subset of $S_i \times \Omega_i$ is an element of Δ_i. Uniqueness of the representation follows from the fact that every $s_i' \in S_i$ appears only once in the representation. ■

Consider sets $Z_1, \ldots, Z_n$ such that for every $i \in N$, Z_i is a positive-measure subset of $S_i \times \Omega_i$. Let

$$\tilde{S}_i = \left\{ s_i' \in S_i : \mu_i^G (A(s_i')) > 0 \right\},$$

where $A(s_i')$ is such that $\{s_i'\} \times A(s_i')$ appears in the unique representation of Z_i from Lemma 1. The set $\tilde{S}_i$ is comprised of the signals in S_i that appear in Z_i with positive probability. Let $\tilde{S} = \times_{i \in N} \tilde{S}_i$, and for every $s' = (s_1', \ldots, s_n') \in \tilde{S}$, let

$$\delta(s') = f(s') \Pi_{i \in N} \mu_i^G (A(s_i')) > 0.$$

For every $s' \in \tilde{S}$, let $h(s') = \frac{\delta(s')}{\sum_{s'' \in \tilde{S}} \delta(s'')}$. Then h induces a probability distribution on $\tilde{S}$. Denote the CDF of this probability distribution by H. For every $i \in N$, let $\tilde{s}_i$ be the random variable induced by H on $\tilde{S}_i$.

Lemma 2 *If the random variables $s_1, \ldots, s_n$ are affiliated (under F), then so are $\tilde{s}_1, \ldots, \tilde{s}_n$ (under H).*

Proof. Choose s', s'' in $\tilde{S} \subseteq S$. Because $\tilde{S} = \times_{i \in N} \tilde{S}_i$, $(s' \vee s'') \in \tilde{S}$ and $(s' \wedge s'') \in \tilde{S}$, where $\vee$ is the component-wise maximum and $\wedge$ is the component-wise minimum. It remains to show that $h(s' \vee s'') h(s' \wedge s'') \geq h(s') h(s'')$. We have

$$h(s' \vee s'') h(s' \wedge s'') = \frac{\delta(s' \vee s'') \delta(s' \wedge s'')}{\left(\sum_{\bar{s} \in \tilde{S}} \delta(\bar{s})\right)^2}$$

$$= \frac{1}{\left(\sum_{\bar{s} \in \tilde{S}} \delta(\bar{s})\right)^2} f(s' \vee s'') \Pi_{i \in N} \mu_i^G (A(\max(s_i', s_i''))) f(s' \wedge s'') \Pi_{i \in N} \mu_i^G (A(\min(s_i', s_i''))).$$

$$= \frac{f(s' \vee s'') f(s' \wedge s'')}{\left(\sum_{\bar{s} \in \tilde{S}} \delta(\bar{s})\right)^2} \Pi_{i \in N} \mu_i^G (A(s_i')) \mu_i^G (A(s_i''))$$

$$\geq \frac{f(s')\,f(s'')}{\left(\sum_{\bar{s}\in\tilde{S}}\delta(\bar{s})\right)^2}\Pi_{i\in N}\mu_i^G\left(A\left(s_i'\right)\right)\mu_i^G\left(A\left(s_i''\right)\right)=h(s')\,h(s''),$$

where the inequality follows from affiliation under F. ■

In what follows, we will use the following well-known property of affiliation.

Lemma 3 *If $s_1,\ldots,s_n$ are affiliated and $v(s_1,\ldots,s_n)$ is non-decreasing in each of its arguments, then $E(v(s_1,\ldots,s_n)|s_1=s_1')$ is non-decreasing in s_1'.*

Proof. Milgrom and Weber (1982), Theorem 5 (page 1100). ■

Corollary 1 *If $s_1,\ldots,s_n$ are affiliated and $v(s_1,\ldots,s_n)$ is strictly increasing in each of its arguments, then $E(v(s_1,\ldots,s_n)|s_1=s_1')$ is strictly increasing in s_1'.*

Proof. Let $s_1''\geq s_1'$. We have

$$E(v(s_1=s_1',\ldots,s_n)|s_1=s_1')\leq E(v(s_1=s_1',\ldots,s_n)|s_1=s_1'')$$

$$<E(v(s_1=s_1'',\ldots,s_n)|s_1=s_1''),$$

where the first inequality is an application of the lemma, and the second inequality follows because it holds for every realization of $s_2,\ldots,s_n$. ■

Suppose that a firm has some conjecture about the realization of other firms' signals. The next lemma shows that regardless of this conjecture, seeing a higher signal makes the firm more optimistic about the value of the worker.

Lemma 4 *Suppose that $s_1,\ldots,s_n$ are affiliated, and $v(s_1,\ldots,s_n)$ strictly increases in each of its arguments. Let $i\in N$, and for every $j\neq i$ let Z_j be a positive-measure subset of $S_j\times\Omega_j$. If s_i' and s_i'' are elements of S_i such that $s_i''\geq s_i'$ and A and B are positive-measure subsets of Ω_i, then $E(v|\{s_i''\}\times A,Z_{-i})\geq E(v|\{s_i'\}\times B,Z_{-i})$. If the first inequality is strict, then so is the second.*

Proof. Because G_i is statistically independent of F and G_{-i}, and v is not a function of Ω_i, we have $E(v|\{s_i''\}\times A,Z_{-i})=E(v|s_i'',Z_{-i})$ and $E(v|\{s_i'\}\times A,Z_{-i})=E(v|s_i',Z_{-i})$. Let $Z_i=\{s_i',s_i''\}\times\Omega_i$, and define $\tilde{S}$ from (Z_i,Z_{-i}) as described above. By Lemma 2 and Corollary 1, $E_H(v|\tilde{s}_i=s_i'',\tilde{s}_{-i})\geq E_H(v|\tilde{s}_i=s_i',\tilde{s}_{-i})$, with a strict inequality if $s_1''>s_1'$. Therefore, it suffices to show that $E(v|s_i',Z_{-i})=E_H(v|\tilde{s}_i=s_i',\tilde{s}_{-i})$ and $E(v|s_i'',Z_{-i})=E_H(v|\tilde{s}_i=s_i'',\tilde{s}_{-i})$. We will show the first equality; the second follows by replacing s_i' with s_i''. Using the notation introduced above, we have

$$E_H(v|\tilde{s}_i=s_i',\tilde{s}_{-i})=\frac{1}{\sum_{s_{-i}'\in\tilde{S}_{-i}}h\left(s_i',s_{-i}'\right)}\sum_{s_{-i}'\in\tilde{S}_{-i}}h\left(s_i',s_{-i}'\right)v\left(s_i',s_{-i}'\right)$$

$$=\frac{\sum_{s'\in\tilde{S}}\delta(s')}{\sum_{s_{-i}'\in\tilde{S}_{-1}}\delta\left(s_i',s_{-i}'\right)}\sum_{s_{-i}'\in\tilde{S}_{-i}}\frac{\delta\left(s_i',s_{-i}'\right)}{\sum_{s'\in\tilde{S}}\delta(s')}v\left(s_i',s_{-i}'\right)$$

$$= \frac{1}{\sum_{s'_{-i}\in\tilde{S}_{-1}} \delta\left(s'_i, s'_{-i}\right)} \sum_{s'_{-i}\in\tilde{S}_{-i}} \delta\left(s'_i, s'_{-i}\right) v\left(s'_i, s'_{-i}\right)$$

$$= \frac{1}{\sum_{s'_{-i}\in\tilde{S}_{-1}} f\left(s'_i, s'_{-i}\right) \mu_i^G \underbrace{\left(A\left(s'_i\right)\right)}_{\Omega_i} \Pi_{j\neq i}\mu_j^G\left(A\left(s'_j\right)\right)}.$$

$$\sum_{s'_{-i}\in\tilde{S}_{-i}} f\left(s'_i, s'_{-i}\right) \mu_i^G \underbrace{\left(A\left(s'_i\right)\right)}_{\Omega_i} \Pi_{j\neq i}\mu_j^G\left(A\left(s'_j\right)\right) v\left(s'_i, s'_{-i}\right)$$

$$= \frac{1}{\mu^{F\times G}\left(\{s'_i\}\times\Omega_i\times Z_{-i}\right)} \sum_{s'_{-i}\in\tilde{S}_{-i}} \mu^{F\times G}\left(\left\{s'_i, s'_{-i}\right\}\times\Omega_i\times_{j\neq i} A\left(s'_j\right)\right) v\left(s'_i, s'_{-i}\right)$$

$$= E(v|s'_i, Z_{-i}).$$

■

For any $i \in N$ and $s'_i \in S_i$, denote by $f_i\left(s'_i\right) = \sum_{s'_{-i}\in S_{-i}} f\left(s'_i, s'_{-i}\right)$ the marginal probability of s'_i. For any $s' \in S$, let $\widetilde{f}\left(s'\right) = \prod_{i\in N} f_i\left(s'_i\right) > 0$, and denote by $\widetilde{F}$ the CDF on S corresponding to $\widetilde{f}$. Denote by $\mu^{\widetilde{F}\times G}$ the measure on $S\times\Omega$ induced by $\widetilde{F}\times G$. By definition, under $\mu^{\widetilde{F}\times G}$ the measurable events in $S_i\times\Omega_i$ are statistically independent of those in $S_j\times\Omega_j$, for any $i\neq j$. Clearly, a set $X\subseteq S\times\Omega$ is $\mu^{F\times G}$-measurable if and only if it is $\mu^{\widetilde{F}\times G}$-measurable. By definition, for any measurable subset $Z_i\subseteq S_i\times\Omega_i$ we have $\mu^{F\times G}\left(Z_i\times S_{-i}\times\Omega_{-i}\right) = \mu^{\widetilde{F}\times G}\left(Z_i\times S_{-i}\times\Omega_{-i}\right)$. For any $s'\in S$, let $\phi\left(s\right) = \frac{f(s')}{\widetilde{f}(s')} > 0$. Let $\phi_{\min} = \min_{s'\in S}\phi\left(s'\right)$ and $\phi_{\max} = \max_{s'\in S}\phi\left(s'\right)$.

Lemma 5 *If X is a measurable subset of $S\times\Omega$, then*

$$\phi_{\min}\mu^{\widetilde{F}\times G}\left(X\right) \leq \mu^{F\times G}\left(X\right) \leq \phi_{\max}\mu^{\widetilde{F}\times G}\left(X\right).$$

Proof. For any $s'\in S$ and every measurable set $A\subseteq\Omega$, we have

$$\mu^{F\times G}\left(\{s'\}\times A\right) = f\left(s'\right)\mu^G\left(A\right) = \phi\left(s'\right)\widetilde{f}\left(s'\right)\mu^G\left(A\right) = \phi\left(s'\right)\mu^{\widetilde{F}\times G}\left(\{s'\}\times A\right). \quad (4)$$

A proof similar to that of Lemma 1 shows that every measurable subset of $S\times\Omega$ can be represented uniquely as $\cup_{s'\in S}\{s'\}\times A\left(s'\right)$, where $A\left(s'\right)$ are measurable subsets of Ω. This observation, together with equation (4) implies the result. ■

Corollary 2 *Suppose $X_1, X_2, ...$ is sequence of measurable subsets of $S\times\Omega$. Then $\mu^{F\times G}\left(X_k\right) \underset{k\to\infty}{\to} 0$ if and only if $\mu^{\widetilde{F}\times G}\left(X_k\right) \underset{k\to\infty}{\to} 0$.*

Proof. Immediate from Lemma 5. ■

Corollary 3 *A measurable subset X of $S \times \Omega$ has positive measure under $\mu^{F\times G}$ if and only if it has positive measure under $\mu^{\widetilde{F}\times G}$. For such a positive-measure set,*

$$\phi_{\min} \leq \frac{\mu^{F\times G}(X)}{\mu^{\widetilde{F}\times G}(X)} \leq \phi_{\max}.$$

In particular, if $X = \times_{i\in N} Z_i$ for positive-measure sets $Z_i \subseteq S_i \times \Omega_i$, then

$$\phi_{\min} \leq \frac{\mu^{F\times G}(X)}{\prod_{i\in N} \mu^{F\times G}(Z_i \times S_{-i} \times \Omega_{-i})} \leq \phi_{\max}.$$

Proof. The first two claims are immediate from Lemma 5 and Corollary 2. The last claim follows from the definition of $\mu^{\widetilde{F}\times G}$. ■

Corollary 4 *Suppose $X_1, X_2, \ldots$ and $Y_1, Y_2, \ldots$ are sequences of measurable subsets of $S \times \Omega$, and $\mu^{F\times G}(Y_k)$ is bounded away from 0 for all k. Then (i) $\mu^{\widetilde{F}\times G}(Y_k)$ is bounded away from 0 for all k, and (ii) $\mu^{F\times G}(X_k|Y_k) \underset{k\to\infty}{\to} 0$ if and only if $\mu^{\widetilde{F}\times G}(X_k|Y_k) \underset{k\to\infty}{\to} 0$.*

Proof. Part (i) is immediate from Lemma 5. For part (ii), let $C_k = X_k \cap Y_k$. Let $\mu^{F\times G}(X_k|Y_k) = \frac{\mu^{F\times G}(C_k)}{\mu^{F\times G}(Y_k)}$ and $\mu^{\widetilde{F}\times G}(X_k|Y_k) = \frac{\mu^{\widetilde{F}\times G}(C_k)}{\mu^{\widetilde{F}\times G}(Y_k)}$. Because both $\mu^{F\times G}(Y_k)$ and $\mu^{\widetilde{F}\times G}(Y_k)$ are at most 1 and are bounded away from 0, $\mu^{F\times G}(X_k|Y_k) \underset{k\to\infty}{\to} 0$ if and only if $\mu^{F\times G}(C_k) \underset{k\to\infty}{\to} 0$, and $\mu^{\widetilde{F}\times G}(X_k|Y_k) \underset{k\to\infty}{\to} 0$ if and only if $\mu^{\widetilde{F}\times G}(C_k) \underset{k\to\infty}{\to} 0$. Now apply the Corollary 2 to the sequence $C_1, C_2, \ldots$. ■

A.3 Proof of Theorem 2

A sequential equilibrium exists because the game is finite. Now, recall that the ratio between any two firms' interview costs is at most some $M > 1$. Choose some $M > 1$, and consider a sequence of strictly positive interviewing costs $c^k = \left(c_1^k, \ldots, c_n^k\right)$ whose maximal element approaches 0 and which satisfy $\max_{i,j\in N} \frac{c_i^k}{c_j^k} < M$. Choose a sequence of strictly positive entry fees $e^k = \left(e_1^k, \ldots, e_n^k\right)$ (that need not approach 0). Choose the entry fees and interviewing costs low enough so that firm 1 enters in any equilibrium with revelation. Such costs exist by the following result.

Lemma 6 *For low entry and interviewing costs e_1 and c_1, with revelation it is strictly optimal for firm 1 to enter with probability 1, regardless of other firms' strategies.*

Proof. With no revelation, Assumption A1 guarantees that for low interviewing costs firm 1 enters with probability 1. With revelation, firm 1 is weakly better off conditional on entering than with no revelation, regardless of other firms' strategies (because its offer is always accepted, it can mimic its no-revelation outcome by ignoring other firms' interview decisions). Therefore, for low interviewing costs firm 1 enters with probability 1 with revelation. ■

Because signals are affiliated and v is increasing, a higher signal is good news about a worker's value for any interview schedule of the other firms. This implies the following result.

Lemma 7 *For any $s_i'' > s_i'$ such that $\sigma_i^k(s_i') > 0$ and $\sigma_i^k(s_i'') > 0$, If $\tau_i^k(s_i'; \mathcal{I}) > 0$, then $\tau_i^k(s_i''; \mathcal{I}) = 1$.*

Proof. Because $\sigma_i^k(s_i') > 0$ and $\tau_i^k(s_i'; \mathcal{I}) > 0$, conditional on observing s_i', interviewing, and observing interview schedule $\mathcal{I}$, firm i weakly prefers making an offer to not making an offer. Therefore, $E(v|\hat{\mathcal{I}}, \Phi_{i,\mathcal{I}}^k, s_i = s_i') \geq 0$. By Lemma 4, $s_i'' > s_i'$ implies that $E(v|\hat{\mathcal{I}}, \Phi_{i,\mathcal{I}}^k, s_i = s_i'') > 0$, so conditional on observing s_i'', interviewing, and observing interview schedule $\mathcal{I}$, firm i is strictly better off making an offer than not making an offer. ■

We will show by reverse induction on $i \in \{2, \ldots, n\}$ that for low maximal interviewing costs (large enough k) firm i enters with probability 0 in any equilibrium with revelation, entry costs e^k, and interviewing costs c^k. This will prove Theorem 2. Choose $i \in \{2, \ldots n\}$, and suppose that for large enough k all firms $j > i$ enter with probability 0 in any equilibrium with revelation, entry costs e^k, and interviewing costs c^k. It suffices to show that for large enough k firm i enters with probability 0. Suppose in contradiction that there exists a subsequence of interviewing costs, without loss of generality the sequence itself, such that for any e^k and c^k in the sequence there exists a corresponding equilibrium q^k with revelation in which firm i enters with positive probability. Because entry is costly, for every k and equilibrium q^k in the sequence, firm i must make strictly positive expected profits conditional on entering.

Consider the following preliminary observation: If, given a set of entering firms, a firm interviews with sufficiently small probability, which depends only on the distribution F of the signals, then there is no signal conditional on which the firm interviews with probability 1. In particular, interviewing is not a strict best reply for any signal, so conditional on interviewing the firm expects a profit of 0. This observation is true because F has finite, full support. Because firm i makes strictly positive expected profits conditional on entering in q^k, the preliminary observation means that for every k there is some set J^k of firms that enter in q^k with positive probability, with $i \in J^k$, such that when the set of firms that enter is precisely J^k, firm i interviews with a probability that is uniformly bounded away from 0 for all k.

Consider firm i's strategy in the equilibrium q^k when the set of entering firms is the set J^k specified above. By Lemma 6, $1 \in J^k$. Because firm i interviews with positive probability at signal $\underline{s}_i^k$, there is an interview schedule $\mathcal{I}$ with $i \in \mathcal{I}$ such that conditional on $\underline{s}_i^k$ (i) with positive probability precisely the firms in $\mathcal{I}$ interview and all firms in $\mathcal{I}$ ranked higher than i do not hire and (ii) conditional on this event, firm i's expected value of the worker is positive. Formally, $\Pr\left(\hat{\mathcal{I}} \cap \Phi_{i,\mathcal{I}}^k | s_i = \underline{s}_i^k\right) > 0$ and

$$E(v|\hat{\mathcal{I}}, \Phi_{i,\mathcal{I}}^k, s_i = \underline{s}_i^k) > 0. \tag{5}$$

If not, then conditional on interviewing with $\underline{s}_i^k$, firm i could not cover its interviewing costs. We now show that this $\mathcal{I}$ can only be the singleton $\{i\}$. Let $j = \min \mathcal{I}$ be the

highest-ranked firm in $\mathcal{I}$, and suppose $j \neq i$. Because $\Pr\left(\hat{\mathcal{I}} \cap \Phi_{i,\mathcal{I}}^k\right) > 0$, the signal $\bar{s}_{j,\mathcal{I}}^k$ is well defined (there is at least one signal for which firm j interviews with positive probability and hires with a probability less than 1 when precisely the firms in $\mathcal{I}$ interview). Because firms' signals are affiliated and v is increasing,

$$\begin{aligned} 0 &< E(v|\hat{\mathcal{I}}, \Phi_{i,\mathcal{I}}^k, s_i = \underline{s}_i^k) && (6)\\ &\leq E(v|\hat{\mathcal{I}}, \Phi_{i,\mathcal{I}}^k) && (7)\\ &\leq E(v|\hat{\mathcal{I}}, \Phi_{i,\mathcal{I}}^k, s_j = \bar{s}_{j,\mathcal{I}}^k) && (8)\\ &\leq E(v|\hat{\mathcal{I}}, s_j = \bar{s}_{j,\mathcal{I}}^k). && (9) \end{aligned}$$

The first inequality between conditional expectations follows from the definition of $\underline{s}_i^k$ as i's lowest signal consistent with $\hat{\mathcal{I}}$,[26] the second from the definition of $\bar{s}_{j,\mathcal{I}}^k$ as the highest signal of j consistent with $\Phi_{i,\mathcal{I}}^k$,[27] and the third from the fact that $\Phi_{i,\mathcal{I}}^k$ is bad news about the worker's value (Lemma 7).[28]

The inequality $0 < E(v|\hat{\mathcal{I}}, s_j = \bar{s}_{j,\mathcal{I}}^k)$ implies that in the positive-probability event in which firm j sees signal $\bar{s}_{j,\mathcal{I}}^k$ and interview schedule $\mathcal{I}$ (at which firm j interviews, because $j \in \mathcal{I}$), firm j would profit from hiring the worker. Because j is the strongest firm in $\mathcal{I}$, it would hire the worker if it made him an offer. Thus, j strictly prefers to make an offer at $\bar{s}_{j,\mathcal{I}}^k$, whereas by definition it makes an offer at $\bar{s}_{j,\mathcal{I}}^k$ with a probability less than 1, a contradiction. This shows that $j = i$, so $\mathcal{I} = \{i\}$ and $\Pr\left(\Phi_{i,\mathcal{I}}^k\right) = 1$.

Because $\mathcal{I} = \{i\}$ is the only schedule satisfying (5), this schedule arises with positive probability conditional on firm i seeing the signal $\underline{s}_i^k$, as discussed above. This means that every entering firm $j \in J^k \setminus \{i\}$ interviews with probability less than 1. Recall that $\bar{s}_j^k$ is the highest signal for which firm j interviews with probability less than 1. From (5) and because $\Pr\left(\Phi_{i,\mathcal{I}}^k\right) = 1$, for any $j \in J^k \setminus \{i\}$ we have

$$\begin{aligned} 0 &< E(v|\hat{\mathcal{I}}, s_i = \underline{s}_i^k) && (10)\\ &\leq E(v|\hat{\mathcal{I}}) \\ &\leq E(v|\hat{\mathcal{I}}, s_j = \bar{s}_j^k). \end{aligned}$$

These inequalities follow, as above, from the assumption that firms' signals are affiliated and v is increasing.

Lemma 8 *There exists some $\delta > 0$ and a subsequence, without loss of generality the sequence itself, such that for all large enough k,*

$$E(v|\hat{\mathcal{I}}, s_j = \bar{s}_j^k) > \delta \tag{11}$$

for some $j \in J^k \setminus \{i\}$.

[26] For every s_i^k with $\sigma_i^k\left(s_i^k\right) > 0$, apply Lemma 4 with $Z_{-i} = \hat{\mathcal{I}}_{-i} \cap \Phi_{i,\mathcal{I}}^k$, $s_i' = \underline{s}_i^k$, $A = \sigma_i^k\left(\underline{s}_i^k\right)$, $s_i'' = s_i^k$, and $B = \sigma_i^k\left(s_i^k\right)$.

[27] For every s_j^k with $\sigma_j^k\left(s_j^k\right) > 0$ and $\tau_j^k(s_j^k; \mathcal{I}) < 1$, apply Lemma 4 with $Z_{-j} = \hat{\mathcal{I}}_{-j} \cap \Phi_{i,\mathcal{I},-j}^k$, $s_i' = s_j^k$, $A = \{\omega_j : (s_j^k, \omega_j) \notin \tilde{O}_{j,\mathcal{I}}^k\}$, $s_i'' = \bar{s}_{j,\mathcal{I}}^k$, and $B = \{\omega_j : (\bar{s}_{j,\mathcal{I}}^k, \omega_j) \notin \tilde{O}_{j,\mathcal{I}}^k\}$.

[28] Apply Lemma 4 iteratively, for every $l \in \mathcal{I} \setminus \{i\}$, as in the previous footnote.

Proof. By (10), the claim is clearly true if there exists some $\delta > 0$ and a subsequence such that for all large enough k either $E(v|\hat{\mathcal{I}}, s_j = \bar{s}_j^k) - E(v|\hat{\mathcal{I}}) \geq \delta$ for some $j \in J^k$, or $E(v|\hat{\mathcal{I}}) - E(v|\hat{\mathcal{I}}, s_i = \underline{s}_i^k) \geq \delta$. Suppose to the contrary that for every $\delta > 0$ there exists some $R(\delta)$ such that for all $k > R(\delta)$ and every firm $j \in J^k \backslash \{i\}$ we have (i) $E(v|\hat{\mathcal{I}}, s_j = \bar{s}_j^k) - E(v|\hat{\mathcal{I}}) < \delta$ and (ii) $E(v|\hat{\mathcal{I}}) - E(v|\hat{\mathcal{I}}, s_i = \underline{s}_i^k) < \delta$. The inequality (i) implies that for every firm $j \in J^k$

$$\frac{\Pr\left(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k\right)}{\Pr\left(\hat{\mathcal{I}} \cap s_j = \bar{s}_j^k\right)} \underset{k\to\infty}{\to} 0,$$

because whenever $\Pr\left(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k\right) \neq 0$

$$E(v|\hat{\mathcal{I}}) = \frac{\Pr\left(\hat{\mathcal{I}} \cap s_j = \bar{s}_j^k\right)}{\Pr\left(\hat{\mathcal{I}} \cap s_j = \bar{s}_j^k\right) + \Pr\left(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k\right)} E(v|\hat{\mathcal{I}}, s_j = \bar{s}_j^k) \tag{12}$$

$$+ \frac{\Pr\left(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k\right)}{\Pr\left(\hat{\mathcal{I}} \cap s_j = \bar{s}_j^k\right) + \Pr\left(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k\right)} E(v|\hat{\mathcal{I}}, s_j \neq \bar{s}_j^k) \tag{13}$$

so

$$E(v|\hat{\mathcal{I}}, s_j = \bar{s}_j^k) - E(v|\hat{\mathcal{I}}) = \frac{\Pr\left(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k\right)}{\Pr\left(\hat{\mathcal{I}} \cap s_j = \bar{s}_j^k\right) + \Pr\left(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k\right)} \left(E(v|\hat{\mathcal{I}}, s_j = \bar{s}_j^k) - E(v|\hat{\mathcal{I}}, s_j \neq \bar{s}_j^k)\right)$$

and v is strictly increasing. Moving on, by Corollary 3

$$\frac{\Pr\left(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k\right)}{\Pr\left(\hat{\mathcal{I}} \cap s_j = \bar{s}_j^k\right)} = R \frac{\Pr\left(\neg \tilde{I}_j^k \backslash \left(\bar{s}_j^k \times \Omega_j\right)\right)}{\Pr\left(\neg \tilde{I}_j^k \cap \left(\bar{s}_j^k \times \Omega_j\right)\right)}$$

for some constant $R > 0$ that depends only on the distribution F. Therefore,

$$\frac{\Pr\left(\neg \tilde{I}_j^k \backslash \left(\bar{s}_j^k \times \Omega_j\right)\right)}{\Pr\left(\neg \tilde{I}_j^k \cap \left(\bar{s}_j^k \times \Omega_j\right)\right)} \underset{k\to\infty}{\to} 0. \tag{14}$$

Similarly, (ii) implies that

$$\frac{\Pr\left(\tilde{I}_i^k \backslash \left(\underline{s}_i^k \times \Omega_i\right)\right)}{\Pr\left(\tilde{I}_i^k \cap \left(\underline{s}_i^k \times \Omega_i\right)\right)} \underset{k\to\infty}{\to} 0. \tag{15}$$

For every $l \in J^k \backslash \{i\}$, by (repeatedly) decomposing $E(v|\hat{\mathcal{I}}, s_l = \bar{s}_l^k)$ as we did $E(v|\hat{\mathcal{I}})$ in (12) and applying Corollary 3 using (14) for all $j \in J^k \backslash \{i, l\}$ and (15), we obtain

$$E(v|\hat{\mathcal{I}}, s_l = \bar{s}_l^k) \underset{k\to\infty}{\to} E(v|s_{-i} = \bar{s}_{-i}^k, s_i = \underline{s}_i^k), \tag{16}$$

where $-i$ is the set of indices $J^k \setminus \{i\}$. Now consider two possibilities. The first is that for some subsequence, without loss of generality the sequence itself,

$$E(v|s_{-i} = \bar{s}^k_{-i}, s_i = \underline{s}^k_i) \underset{k\to\infty}{\to} x, x \leq 0.$$

Then, because the number of signals is finite, $E(v|s_{-i} = \bar{s}^k_{-i}, s_i = \underline{s}^k_i) \leq 0$ holds for all large enough k. But then for large enough k we have

$$E(v|\hat{\mathcal{I}}, s_i = \underline{s}^k_i) \leq E(v|s_{-i} = \bar{s}^k_{-i}, s_i = \underline{s}^k_i) \leq 0,$$

a contradiction to (10). The second possibility is that for some subsequence, without loss of generality the sequence itself,

$$E(v|s_{-i} = \bar{s}^k_{-i}, s_i = \underline{s}^k_i) \to x, x > 0.$$

Then, because the number of signals is finite, $E(v|s_{-i} = \bar{s}^k_{-i}, s_i = \underline{s}^k_i) > 2\delta$ holds for some fixed $\delta > 0$ for all large enough k. This, together with (16), implies that (11) holds for large enough k and any $j \in J^k \setminus \{i\}$. ■

Now suppose that for the $j \in J^k$ specified in Lemma 8, $\Pr\left(I^k_i \cap_{m\in J^k\setminus\{i,j\}} \neg I^k_m | s_j = \bar{s}^k_j\right)$ is bounded away from 0 along some subsequence, without loss of generality the sequence itself. Lemma 8 above shows that for some $\alpha > 0$ and all large enough k we would have

$$\Pr\left(I^k_i \cap_{m\in J^k\setminus\{i,j\}} \neg I^k_m | s_j = \bar{s}^k_j\right) E(v|\hat{\mathcal{I}}, s_j = \bar{s}^k_j) \geq \alpha.$$

But for large enough k, $c^k_j < \alpha$, so it is strictly optimal for firm j to interview with probability 1 at $\bar{s}^k_j$ (and make an offer, which will be accepted, when firm i interviews and all other firms do not interview). This contradicts the definition of $\bar{s}^k_j$ as the highest signal for which firm j interviews with probability less than 1.

Therefore,

$$\Pr\left(I^k_i \cap_{m\in J^k\setminus\{i,j\}} \neg I^k_m | s_j = \bar{s}^k_j\right) \underset{k\to\infty}{\to} 0 \tag{17}$$

for some $j \in J^k$. The fact that $\Pr\left(I^k_i\right)$ is bounded away from 0 and (17) imply that $\Pr\left(\neg I^k_l\right) \to 0$ for some firm $l \in J^k \setminus \{j, i\}$.[29] (if $J^k = \{i, j\}$, which happens for example if $n = 2$, we have a contradiction and we are done.) For this firm $l \in J^k \setminus \{j, i\}$, therefore, $\frac{\Pr\left(I^k_i\right)}{\Pr\left(\neg I^k_l\right)} \underset{k\to\infty}{\to} \infty$. By definition of conditional expectation and Corollary 3,

$$\frac{\Pr\left(I^k_i \cap_{m\in J^k\setminus\{i,l\}} \neg I^k_m | s_l = \bar{s}^k_l\right)}{\Pr\left(\cap_{m\in J^k\setminus\{i\}} \neg I^k_m | s_i = \underline{s}^k_i\right)} = \frac{\Pr\left(s_i = \underline{s}^k_i\right)}{\Pr\left(s_l = \bar{s}^k_l\right)} \frac{\Pr\left(I^k_i \cap_{m\in J^k\setminus\{i,l\}} \neg I^k_m \cap s_l = \bar{s}^k_l\right)}{\Pr\left(\cap_{m\in J^k\setminus\{i\}} \neg I^k_m \cap s_i = \underline{s}^k_i\right)}$$

$$\geq \frac{\Pr\left(s_i = \underline{s}^k_i\right)}{\Pr\left(s_l = \bar{s}^k_l\right)} \frac{\Pr\left(I^k_i\right) \Pi_{m\in J^k\setminus\{i,l\}} \Pr\left(\neg I^k_m\right) \Pr\left(s_l = \bar{s}^k_l\right)}{\Pr\left(\neg I^k_l\right) \Pi_{m\in J^k\setminus\{i,l\}} \Pr\left(\neg I^k_m\right) \Pr\left(s_i = \underline{s}^k_i\right)} R = \frac{\Pr\left(I^k_i\right)}{\Pr\left(\neg I^k_l\right)} R$$

for some constant $R > 0$ that depends only on the distribution F. We conclude that

$$\frac{\Pr\left(I^k_i \cap_{m\in J^k\setminus\{i,l\}} \neg I^k_m | s_l = \bar{s}^k_l\right)}{\Pr\left(\cap_{m\in J^k\setminus\{i\}} \neg I^k_m | s_i = \underline{s}^k_i\right)} \underset{k\to\infty}{\to} \infty\,. \tag{18}$$

[29] To see why, apply Corollary 4 to Equation (17), and then use Corollary 2.

Because firm i interviews at $\underline{s}_i^k$ with positive probability,

$$\Pr\left(\cap_{m\in J^k\setminus\{i\}}\neg I_m^k|s_i=\underline{s}_i^k\right)\mathbf{E}(v|\hat{\mathcal{I}},s_i=\underline{s}_i^k)\geq c_i^k. \tag{19}$$

Together with (10) for $j=l$ and the fact that $\frac{c_l^k}{c_i^k}<M$, (19) implies that

$$\Pr\left(\cap_{m\in J^k\setminus\{i\}}\neg I_m^k|s_i=\underline{s}_i^k\right)E(v|\hat{\mathcal{I}},s_l=\bar{s}_l^k)>\frac{c_l^k}{M}.$$

For large enough k, (18) gives us

$$\Pr\left(I_i^k\cap_{m\in J^k\setminus\{i,l\}}\neg I_m^k|s_l=\bar{s}_l^k\right)E(v|\hat{\mathcal{I}},s_l=\bar{s}_l^k)>c_l^k.$$

But then, for large enough k, it is strictly optimal for firm l to interview with probability 1 at $\bar{s}_l^k$. This contradicts the definition of $\bar{s}_l^k$, and therefore shows that for large enough k there is no equilibrium with revelation and costs c^k in which firm i enters with positive probability. This completes the proof of Theorem 2.

Theorem 2 describes the equilibrium outcome, but does not specify firms' off-path behavior. To get an idea of off-path behavior that supports the equilibrium outcome, suppose first that firm 1 does not enter, and choose some set of entering firms. This is a finite proper subgame, and so has a sequential equilibrium. Any sequential equilibrium will do, because firm 1 will not find a deviation to not entering attractive, regardless of what other firms do. Now suppose that at least three firms enter, including firm 1. This is a finite proper subgame in which any sequential equilibrium will do, because no firm can reach this subgame by deviating unilaterally. Finally, suppose that two firms enter, firm 1 and firm $j\neq 1$, and consider a sequential equilibrium of this proper subgame. The proof of Theorem 2 applied to $n=2$ shows that firm j cannot make strictly positive profits net of entry costs in the subgame. Therefore, firm j makes non-positive profits net of entry costs in this subgame, and will not deviate to entering (because entry is costly).[30]

[30] In the subgame, both firms will interview with a positive probability that is strictly less than 1. Firm j will interview with low probability, and will interview with probability strictly less than 1 at any signal. It will therefore have expected profits of 0 net of entry costs.

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