Supplementary appendix to "Rhetoric in legislative bargaining with asymmetric information" (*Theoretical Economics*, 2013)

Ying Chen and Hülya Eraslan

Uniqueness in the uniform-quadratic setup

Recall that $z^1(\theta_1) = (y^1(\theta_1); x^1(\theta_1))$ denotes the chair's optimal proposal under complete information when she faces only legislator 1, and $V(\theta_1) = u_0(z^1(\theta_1))$.

Lemma A.1. *Suppose $v(y, \hat{y}_i) = -(y - \hat{y}_i)^2$, and θ_1 is uniformly distributed on $[\underline{t}_1, \bar{t}_1] \subseteq \Theta_1$, where $\bar{t}_1 > \underline{t}_1$. Let G_1 be the cumulative distribution function of θ_1 and let*

$$W(\theta_1) = V(\theta_1) G_1(\theta_1) + u_0(s)(1 - G_1(\theta_1)).$$

(i) If $\hat{y}_1 < \tilde{y}$, then $\bar{t}_1$ is the unique solution to $max_{\theta_1 \in [\underline{t}_1, \bar{t}_1]} W(\theta_1)$.

(ii) If $\hat{y}_1 \geq \tilde{y}$ and $c > 0$, then the solution to $\max_{\theta_1 \in [\underline{t}_1, \bar{t}_1]} W(\theta_1)$ is generically unique in the following sense: Fix all the parameters except for c. Then, there exists at most one value of c for which the solution to $\max_{\theta_1 \in [\underline{t}_1, \bar{t}_1]} W(\theta_1)$ is not unique.

Proof. Without loss of generality, let $\tilde{y} = 0$. Note that

$$W'(\theta_1) = \frac{V'(\theta_1)\theta_1 - V'(\theta_1)\underline{t}_1 + V(\theta_1) - u_0(s)}{\bar{t}_1 - \underline{t}_1}. \tag{A.1}$$

First we show that $V_1'(\theta_1) \leq 0$. When $v(y, \hat{y}_i) = -(y - \hat{y}_i)^2$, straightforward calculation shows that

$$\begin{aligned} y^1(\theta_1) &= \min\{(\theta_0 \hat{y}_0 + \theta_1 \hat{y}_1)/(\theta_0 + \theta_1), e(\hat{y}_1)\}, \\ x_1^1(\theta_1) &= \theta_1 \left[v(\tilde{y}, \hat{y}_1) - v(y^1(\theta_1), \hat{y}_1)\right]. \end{aligned}$$

Hence, if $(\theta_0 \hat{y}_0 + \theta_1 \hat{y}_1)/(\theta_0 + \theta_1) \geq e(\hat{y}_1)$, then $V(\theta_1) = c - \theta_0(e(\hat{y}_1) - \hat{y}_0)^2$, and $V'(\theta_1) = 0$; and if $(\theta_0 \hat{y}_0 + \theta_1 \hat{y}_1)/(\theta_0 + \theta_1) < e(\hat{y}_1)$, then

$$\begin{aligned} V(\theta_1) &= c - x_1^1(\theta_1) + \theta_0 v_0(y^1(\theta_1), \hat{y}_0) \\ &= c - \theta_1 \left[v(\tilde{y}, \hat{y}_1) - v(y^1(\theta_1), \hat{y}_1)\right] + \theta_0 v_0(y^1(\theta_1), \hat{y}_0). \end{aligned}$$

In this case

$$V'(\theta_1) = -\frac{x_1^1(\theta_1)}{\theta_1} = -\left[v(\tilde{y}, \hat{y}_1) - v(y^1(\theta_1), \hat{y}_1)\right]$$

by the envelope theorem. Since $v(y^1(\theta_1), \hat{y}_1) < v(e(\hat{y}_1), \hat{y}_1)$ and $v(\tilde{y}, \hat{y}_1) = v(e(\hat{y}_1), \hat{y}_1)$, it follows that $V'(\theta_1) < 0$.

Part (i): Suppose $\hat{y}_1 < \tilde{y}$. It suffices to show that $W'(\theta_1) > 0$ for all $\theta_1 \in [\underline{t}_1, \bar{t}_1]$.

Since $V'(\theta_1) \leq 0$, to show that $W'(\theta_1) > 0$, we only need to show that $V'(\theta_1)\theta_1 + V(\theta_1) - u_0(s) > 0$. If $(\theta_0\hat{y}_0 + \theta_1\hat{y}_1)/(\theta_0 + \theta_1) > e(\hat{y}_1)$, then $V'(\theta_1)\theta_1 + V(\theta_1) - u_0(s) > 0$ since $V'(\theta_1) = 0$ and $V(\theta_1) - u_0(s) > 0$. If $(\theta_0\hat{y}_0 + \theta_1\hat{y}_1)/(\theta_0 + \theta_1) \leq e(\hat{y}_1)$, then $y^1(\theta_1) = (\theta_0\hat{y}_0 + \theta_1\hat{y}_1)/(\theta_0 + \theta_1) \leq e(\hat{y}_1)$ and

$$\begin{aligned} V'(\theta_1)\theta_1 + V(\theta_1) - u_0(s) &= c - 2x_1^1(\theta_1) + \theta_0 v(y^1(\theta_1), \hat{y}_0) - \theta_0 v(\tilde{y}, \hat{y}_0) \\ &= c - 2\theta_1 y^1(\theta_1)[y^1(\theta_1) - 2\hat{y}_1] - \theta_0 y_1^1(\theta_1)[y^1(\theta_1) - 2\hat{y}_0] \\ &= c + \theta_0(y^1(\theta_1))^2 + 2\theta_1 y^1(\theta_1)\hat{y}_1 \end{aligned}$$

where the last equality uses $y^1(\theta_1) = (\theta_0\hat{y}_0 + \theta_1\hat{y}_1)/(\theta_0 + \theta_1)$.

Since $c \geq 0, \theta_0 > 0$, $\theta_1 > 0$, and $y^1(\theta_1) \leq e(\hat{y}_1) < \hat{y}_1 < \tilde{y} = 0$, it follows that $V'(\theta_1)\theta_1 + V(\theta_1) - u_0(s) > 0$ and therefore $W'(\theta_1) > 0$ and $\bar{t}_1$ is the unique maximizer of $W(\theta_1)$ on $[\underline{t}_1, \bar{t}_1]$.

Part (ii): Suppose $\hat{y}_1 > \tilde{y}$ and $c > 0$. Note that if $V'(\hat{\theta}_1) = 0$, then $V'(\theta_1) = 0$ for any $\theta_1 > \hat{\theta}_1$. Since $c > 0$, if $V'(\theta_1) = 0$, then by equation (A.1), $W'(\theta_1) > 0$. It follows that if $V'(\theta_1) = 0$ and $\theta_1 \neq \bar{t}_1$, then $\theta_1 \notin \arg\max W(\theta_1)$. We next show that any θ_1 such that $V'(\theta_1) < 0$ is not a maximizer of $W(\theta_1)$ on $[\underline{t}_1, \bar{t}_1]$ except for at most one value of c. To do this, we first show that for θ_1 such that $V'(\theta_1) < 0$, the second derivative of $W(\theta_1)$ crosses 0 only once and from below. It is straightforward to verify that

$$\begin{aligned} W''(\theta_1) &= \frac{V''(\theta_1)(\theta_1 - \underline{t}_1) + 2V'(\theta_1)}{\bar{t}_1 - \underline{t}_1} \\ &= \frac{2\theta_1(\hat{y}_1)^2(3\theta_0\theta_1 + 3(\theta_0)^2 + (\theta_1)^2) + C}{(\theta_0 + \theta_1)^3(\bar{t}_1 - \underline{t}_1)} \end{aligned}$$

where C does not depend on θ_1. Hence, if $W''(\theta_1) = 0$, then $W''(\theta_1') > 0$ for any $\theta_1' > \theta_1$, i.e., $W''(\theta_1)$ crosses 0 at most once and from below. Consider the following two possibilities.

(a) Suppose $W''(\theta_1) > 0$ for all $\theta_1 \in [\underline{t}_1, \bar{t}_1]$ such that $V'(\theta_1) < 0$. Then $W(\theta_1)$ does not have an interior maximum. Since $c > 0$ and the proposal $(\tilde{y}; c, 0, 0)$ is accepted by every type, it follows that $W(\bar{t}_1) > u_0(s) = W(\underline{t}_1)$, and therefore $\bar{t}_1$ is the unique maximizer of $W(\theta_1)$ on $[\underline{t}_1, \bar{t}_1]$.

(b) Suppose $W''(\theta_1) = 0$ for some $\theta_1 \in [\underline{t}_1, \bar{t}_1]$ such that $V'(\theta_1) < 0$. Then there is at most one interior maximum of $W(\theta_1)$ at $\tilde{\theta}_1$ where $W'(\tilde{\theta}_1) = 0$ and $W''(\tilde{\theta}_1) < 0$. If $W(\tilde{\theta}_1) = W(\bar{t}_1)$, then both $\tilde{\theta}_1$ and $\bar{t}_1$ are solutions to $\max W(\theta_1)$ on $[\underline{t}_1, \bar{t}_1]$. In what follows, we show that generically $W(\theta_1)$ has only one maximum.

With some abuse of notation, let $V(\theta_1, c) = u_0(z^1(\theta_1)) = c - x_1^1(\theta_1) - \theta_0(y^1(\theta_1) - \hat{y}_0)^2$ and $W(\theta_1, c) = V(\theta_1, c) G_1(\theta_1) + u_0(s)(1 - G_1(\theta_1))$. Since $\partial^2 W/\partial c \partial \theta_1 = dG_1/d\theta_1 > 0$, the function W satisfies the strict increasing difference in (θ_1, c). Then, results from monotone comparative statics literature (see, e.g., Theorem 4′ in Milgrom and Shannon (1994)) imply that for any $\theta_1' \in \arg\max_{\theta_1 \in [\underline{t}_1, \bar{t}_1]} W(\theta_1, c')$ and $\theta_1'' \in \arg\max_{\theta_1 \in [\underline{t}_1, \bar{t}_1]} W(\theta_1, c'')$, we have $\theta_1'' \geq \theta_1'$ if $c'' > c'$. This implies that if for some c', $\arg\max_{\theta_1 \in [\underline{t}_1, \bar{t}_1]} W(\theta_1, c') = \{\tilde{\theta}_1, \bar{t}_1\}$ where $\tilde{\theta}_1 < \bar{t}_1$, then for any $c'' > c'$, $\arg\max_{\theta_1 \in [\underline{t}_1, \bar{t}_1]} W(\theta_1, c'') = \{\bar{t}_1\}$. Hence the solution to $\max_{\theta_1 \in [\underline{t}_1, \bar{t}_1]} W(\theta_1)$ is generically unique. ■

Proof of Lemma 4

Suppose z is elicited in an equilibrium with $x_1 = x_2 = 0$.

Part (i): Since $e(\hat{y}_1) \leq \hat{y}_1 \leq \hat{y}_2$ and $v(y, \hat{y}_i)$ is strictly increasing in y when $y < \hat{y}_i$, it follows that if $y < e(\hat{y}_1)$, then both legislators 1 and 2 reject z, and if $e(\hat{y}_1) \leq y \leq \tilde{y}$, then at least legislator 1 accepts z. Since $v(y, \hat{y}_0)$ is strictly decreasing in y when $y \geq e(\hat{y}_1) > \hat{y}_0$, it is optimal to propose $y = e(\hat{y}_1)$.

Part (ii): Since $y = e(\hat{y}_1)$ and $x_1 = 0$, it follows that $u_1(z, \theta_1) = u_1(s, \theta_1)$ for all $\theta_1 \in \Theta_1$ by the definition of $e(\hat{y}_1)$.

Part (iii): Since $x_2 = 0$, if $e(\hat{y}_1) = e(\hat{y}_2)$, then $u_2(z, \theta_2) = u_2(s, \theta_2)$ for all $\theta_2 \in \Theta_2$.

Part (iv): Since $x_2 = 0$, $e(\hat{y}_2) \leq \hat{y}_2$, and $v(y, \hat{y}_2)$ is strictly increasing in y for $y < \hat{y}_2$, if $e(\hat{y}_1) < e(\hat{y}_2)$, then $u_2(z, \theta_2) < u(s, \theta_2)$ for all $\theta_2 \in \Theta_2$. It immediately follows that legislator 2 rejects z and legislator 1 is pivotal with respect to z. ■

Extension to more than three players

Suppose the set of legislators other than the chair is $N = \{1, \ldots, n\}$ where $n \geq 3$ and the voting rule requires $(\kappa + 1) \geq 2$ votes for a proposal to pass. Since we can assume without loss of generality that the chair votes for any proposal she makes, a proposal needs κ out of the n legislators to vote for it to pass. Call this the κ voting rule. Assume that $1 \leq \kappa < n$. If $\kappa = \min\{x \in N : x \geq n/2\}$, then the voting rule is the majority rule, but our results apply to more general voting rules. Suppose only legislators 1 and 2 have private information on their types, that is, for legislator $i \in \{3, \ldots, n\}$, the distribution of θ_i, still denoted by F_i, is degenerate. We maintain the same assumptions on the players' preferences as in the main text. Denote this game by Γ^N. The definition of equilibrium is analogous to that in the main text and is omitted.

Assume that $\hat{y}_0 < \tilde{y}$. If at least κ legislators prefer $\hat{y}_0$ to $\tilde{y}$, the chair's optimal proposal is $(\hat{y}_0; c, 0, \ldots, 0)$. If $k < \kappa$ legislators prefer $\hat{y}_0$ to $\tilde{y}$, then the analysis of Γ^N with the κ voting rule is similar to that of the game in which the chair faces the $(n - k)$ legislators who strictly prefer $\tilde{y}$ to $\hat{y}_0$ and the voting rule is the $(\kappa - k)$ voting rule. Henceforth, we assume that every legislator $i \in N$ strictly prefers $\tilde{y}$ to $\hat{y}_0$, and as before, let $e(\hat{y}_i) = \min\{y : v(y, \hat{y}_i) = v(\tilde{y}, \hat{y}_i)\}$.

For an integer $1 \leq q \leq n$, say that a proposal $(y; x)$ is a *q-transfer proposal* if $x_i > 0$ for exactly $q \geq 1$ legislators in N. Say that a proposal $(y; x)$ is a no-transfer proposal if $x_i = 0$ for all $i \in N$. As in the main text, we focus on connected equilibria (defined analogously). Suppose the prior F_i $(i = 1, 2)$ has a differentiable density function f_i. In the following lemma, we show that if F_1 and F_2 satisfy IHRP, then no proposal elicited in a connected equilibrium is a q-transfer proposal where $q > \kappa$. Denote the legislators other than i by $-i$.

Lemma A.2. *Suppose F_1 and F_2 satisfy IHRP. Fix a connected equilibrium of Γ^N and let $z^* = (y^*; x^*)$ be a proposal elicited in this equilibrium. (i) The proposal z^* is either a no-transfer proposal or a q-transfer proposal where $q \leq \kappa$. (ii) If any legislator $i \in \{1, 2\}$ is included in z^*, then at most κ legislators vote for z^* in this equilibrium.*

Proof. Part (i): Suppose z^* is a q-transfer proposal. Note that for $i \in \{3, \ldots, n\}$, if $x_i^* > 0$, then $u_i(z^*) \geq u_i(s)$ and legislator i votes for z^*. This is because if $u_i(z^*) < u_i(s)$, then

there exists a proposal $z' = (y'; x')$ where $y' = y^*$, $x'_0 = x^*_0 + x^*_i$, $x'_{-i} = x^*_{-i}$ and $x'_i = 0$ such that the same set of legislators vote for z^* and z' with the same positive probability. Since $x'_0 > x^*_0$, we have $u_0(z') > u_0(z^*)$ and it is strictly better to propose z' than z^*, a contradiction. It follows immediately that z^* includes at most κ legislators in $\{3, \ldots, n\}$. Consider the following possibilities.

(a) Suppose z^* includes κ legislators in $\{3, \ldots, n\}$. Since every such legislator votes for z^*, the proposal z^* does not include legislators 1 and 2. It follows that $q = \kappa$.

(b) Suppose z^* includes $k \leq \kappa - 1$ legislators in $\{3, \ldots, n\}$. Let l be the number of legislators in $\{3, \ldots, n\}$ who vote for z^*. As established earlier, any legislators in $\{3, \ldots, n\}$ who is included in z^* votes for z^* and therefore $k \leq l$.

If $l \geq \kappa$, then $x^*_1 = x^*_2 = 0$ and $q = k < \kappa$.

If $l \leq (\kappa - 2)$, then $k \leq \kappa - 2$ and $q \leq k + 2 \leq \kappa$.

Suppose $l = \kappa - 1$. Then $k \leq (\kappa - 1)$. Suppose, towards a contradiction, that $q > \kappa$, which implies that $k = (\kappa - 1)$ and $x^*_1 > 0$ and $x^*_2 > 0$. Suppose z^* is induced by m, and for $i = 1, 2$, let G_i denote the chair's posterior on legislator i's type when receiving m_i and let g_i denote the associated density. Recall that for $i = 1, 2$ and any proposal z, $t_i(z)$ denotes the highest type of legislator i who is willing to accept z. Let $\beta(z) = 1 - (1 - G_1(t_1(z)))$ $(1 - G_2(t_2(z)))$, the probability that at least one of legislators 1 and 2 votes for z. Since z^* is optimal, and $l = \kappa - 1$ legislators in $\{3, \ldots, n\}$ accept z^*, it follows that x^* is a solution to the problem

$$\max_{x \in X} (c - \sum_{i=1}^{n} x^*_i + \theta_0 v(y^*, \hat{y}_0)) \beta(y^*; x) + \theta_0 v(\tilde{y}, \hat{y}_0)(1 - \beta(y^*; x))$$

subject to $x_1 + x_2 = x^*_1 + x^*_2$ and $x_i = x^*_i$ for $i = 3, ..., n$. But as the proof of Lemma 3 shows, this is impossible in a connected equilibrium if F_i satisfies IHRP for $i = 1, 2$. Hence $q \leq \kappa$.

Part (ii): Suppose $x^*_1 > 0$ and $x^*_2 > 0$. Then, as shown in the proof of part (i), at most $(\kappa - 2)$ legislators in $\{3, \ldots, n\}$ vote for z^* and therefore at most κ legislators vote for z^*.

Suppose $x^*_1 > 0$ and $x^*_2 = 0$ (the argument is similar if $x^*_1 = 0$ and $x^*_2 > 0$). Then, as shown in the proof of part (i), this happens only if $l \leq (\kappa - 1)$. Suppose $(\kappa - 1)$ legislators in $\{3, \ldots, n\}$ and legislator 2 vote for z^*. Then there exists $z' = (y'; x')$ with $y' = y^*$,

$x'_0 = x^*_0 + x^*_1$, $x'_1 = 0$ and $x'_{-1} = x^*_{-1}$ such that κ legislators vote for z'. Since $u_0(z') > u_0(z^*)$, this contradicts the optimality of z^*. Hence, at most $(\kappa - 1)$ legislators other than legislator 1 vote for z^* and therefore at most κ legislators vote for z^*. ■

Lemma A.2 implies that in a connected equilibrium, if legislator $i \in \{1, 2\}$ is included in z^*, then his payoff is at least as high as his status quo payoff when the chair proposes z^*. This is because by part (ii) of Lemma A.2, if legislator i votes against z^*, it will fail to pass, that is, legislator i has veto power with respect to z^*. This is analogous to the result in $\Gamma^{\{1,2\}}$ that legislator i is pivotal with respect to a proposal z^* if z^* includes legislator i.

Define simple connected equilibrium of Γ^N analogously as in $\Gamma^{\{1,2\}}$. The following Proposition says that in Γ^N, each legislator still can convey at most whether he will "fight" or "compromise," just like in $\Gamma^{\{1,2\}}$.

Proposition A.1. *Suppose F_1 and F_2 satisfy IHRP. Fix a simple connected equilibrium (μ, γ, π) in Γ^N. Suppose legislator $i \in \{1, 2\}$ is informative in this equilibrium. Then there exist m_i^c, $m_i^f \in M_i$ such that $q_i(m_i^c) > 0$, $q_i(m_i^f) = 0$. Moreover, μ_i is equivalent to a size-two message rule μ_i^{II} with the property that there exists $\theta_i^* \in (\underline{\theta}_i, \bar{\theta}_i)$ such that $\mu_i^{II}(\theta_i) = m_i^c$ for $\theta_i < \theta_i^*$ and $\mu_i^{II}(\theta_i) = m_i^f$ for $\theta_i > \theta_i^*$.*

The proof is similar to that of Proposition 3. Specifically, we extend Lemma 6 and Lemma 7 to Γ^N and we use Lemma A.2 in place of Lemma 3 and Lemma 5 in the proofs of the extensions of Lemma 6 and Lemma 7. (We also replace the phrase "legislator i is pivotal with respect to z" with the phrase "legislator i has veto power with respect to z" in the relevant places.)

Define $\hat{y}^\kappa \in \{\hat{y}_1, \ldots, \hat{y}_n\}$ implicitly by $\#\{i \in N : \hat{y}_i \leq \hat{y}^\kappa\} \geq \kappa - 1$ and $\#\{i \in N : \hat{y}_i \geq \hat{y}^\kappa\} \geq n - (\kappa - 1)$. If the κ voting rule is the majority rule, then $\hat{y}^\kappa$ is the median position.

Proposition A.2. *Suppose F_1 and F_2 satisfy IHRP. Fix a simple connected equilibrium (μ, γ, π) in Γ^N. If $e(\hat{y}_i) > e(\hat{y}^\kappa)$, then legislator i is uninformative in this equilibrium.*

The proof is similar to that of Proposition 4 (ii). Applied to the majority rule, Proposition A.2 says that if the median legislator wants to move the policy in the same direction as

the chair does, then any legislator whose position is to the right of the median position is uninformative in any SCE.

Unlike part (i) of Proposition 4, which says that at most one legislator is informative in an SCE of $\Gamma^{\{1,2\}}$, it is possible in Γ^N that both legislators 1 and 2 are informative in an SCE. Below, we provide an example (part (iii) of Example A.1) that illustrates what an SCE looks like when both legislators 1 and 2 are informative. In this example, one of them is the median and the other is to the left of the median. (Note that this does not arise in $\Gamma^{\{1,2\}}$ since it is necessarily the case that one legislator is the median and the other is to the right of the median in $\Gamma^{\{1,2\}}$.) Part (i) of Example A.1 illustrates an SCE in which only the median is informative. Part (ii) of Example A.1 illustrates an SCE in which only the legislator to the left of the median is informative. This happens under certain conditions (roughly, when the position of the legislator to the left of the median is sufficiently close to that of the median).

Example A.1. *Suppose* $n = 3$, $\kappa = 2$ *(majority rule),* $\tilde{y} = 0$, $\hat{y}_0 = -1$, $\hat{y}_1 < 0$, $\hat{y}_3 = -0.3$, *and* $c = 1$. *Assume that* $u_i(z, \theta_i, \hat{y}_i) = x_i - \theta_i (y - \hat{y}_i)^2$ *for* $i = 0, 1, 2, 3$, $\theta_0 = 1$, $\theta_3 = 1$, *and* θ_1, θ_2 *are both uniformly distributed on* $[1/4, 4]$.

(i) Suppose $\hat{y}_3 < \hat{y}_1 < \hat{y}_2$. Then legislator 1 is the median. By Proposition A.2, legislator 2, whose position is to the right of the median, is uninformative in any SCE in $\Gamma^{\{1,2,3\}}$. There are SCE in which legislator 1 is informative. For instance, let $\hat{y}_1 = -0.2$ (so $e(\hat{y}_1) = -0.4$) and suppose $\mu_1(\theta_1) = m_1^c$ if $\theta_1 \in [1/4, 1]$, $\mu_1(\theta_1) = m_1^f$ if $\theta_1 \in (1, 4]$, and $\mu_2(\theta_2) = m_2^*$ for all θ_2. Given the message rules, when the chair receives m_1^f, she infers that $\theta_1 \in (1, 4]$ and proposes $(e(\hat{y}_1); c, 0, 0, 0) = (-0.4; 1, 0, 0, 0)$. Legislators 1 and 3 accept the proposal and legislator 2 rejects the proposal. When the chair receives m_1^c, she infers that $\theta_1 \in [1/4, 1]$ and proposes $(y; c - x_1, x_1, 0, 0)$ where $y = -0.6 < e(\hat{y}_1)$ and $x_1 = 0.12 > 0$. Again, only legislators 1 and 3 accept the proposal.

(ii) Suppose $\hat{y}_1 < \hat{y}_3 < \hat{y}_2$. Then legislator 3 is the median. Again, legislator 2, whose position is to the right of the median, is uninformative in any SCE of $\Gamma^{\{1,2,3\}}$. Whether it is possible for legislator 1, whose position is to the left of the median, to be informative in some SCE depends on how close $\hat{y}_1$ is to the chair's position relative to the median's position.

For example, suppose $\hat{y}_1 = -0.31$ (so that $e(\hat{y}_1) = -0.62$). Consider an SCE in which $\mu_1(\theta_1) = m_1^c$ if $\theta_1 \in [1/4, 1]$, $\mu_1(\theta_1) = m_1^f$ if $\theta_1 \in (1, 4]$, and $\mu_2(\theta_2) = m_2^*$ for all θ_2. When the chair receives m_1^f, she infers that $\theta_1 \in (1, 4]$ and proposes $(e(\hat{y}_1); c - x_3, 0, 0, x_3)$ where $x_3 = 0.0061$. When the chair receives m_1^c, she infers that $\theta_1 \in [1/4, 1]$ and proposes $(y; c - x_1 - x_3, x_1, 0, x_3)$ where $y = -0.65 < e(\hat{y}_1)$, $x_1 = 0.0195$ and $x_3 = 0.0325$. In both cases, legislators 1 and 3 accept the proposal and legislator 2 rejects the proposal.

Suppose instead $\hat{y}_1 = -0.4$ (so that $e(\hat{y}_1) = -0.8$). Because $\hat{y}_1$ is sufficiently close to the chair's position relative to the median's position, there exists no SCE in which legislator 1 is informative. To see this, suppose there is an SCE in which legislator 1 follows a size two message rule: $\mu_1(\theta_1) = m_1^c$ if $\theta_1 < \theta_1^*$ and $\mu_1(\theta_1) = m_1^f$ if $\theta_1 > \theta_1^*$ for some θ_1^* such that legislator 1 is included when sending m_1^c and excluded when sending m_1^f. (By Proposition A.1, if legislator 1 is informative in an SCE, then μ_1 is equivalent to such a message rule.) Straightforward calculation shows that conditional on excluding legislator 1, the chair's optimal proposal is $(y; c - x_3, 0, 0, x_3)$ where $y = -0.65$, $x_3 = 0.0325$, and legislators 1 and 3 accept this proposal. Since $y = -0.65 > e(\hat{y}_1)$, legislator 1 gets a payoff strictly higher than his status quo payoff when sending m_1^f. Recall that legislator 1 is included when sending m_1^c. Arguments similar to Lemma 6 show that the threshold type θ_1^* gets a payoff equal to his status quo payoff by sending m_1^c (followed by his optimal acceptance rule). Hence type θ_1^* strictly prefers sending m_1^f to m_1^c, a contradiction. So there exists no SCE in which legislator 1 is informative.

(iii) Suppose $\hat{y}_1 < \hat{y}_2 < \hat{y}_3$. Then legislator 2 is the median legislator. There may exist an SCE in which both legislators 1 and 2 are informative. For example, let $\hat{y}_1 = -0.35$, $\hat{y}_2 = -0.325$ (so that $e(\hat{y}_1) = -0.7$ and $e(\hat{y}_2) = -0.65$) and suppose $\mu_i(\theta_i) = m_i^c$ if $\theta_i \in [1/4, 1/2]$ and $\mu_i(\theta_i) = m_i^f$ if $\theta_i \in (1/2, 4]$ for $i = 1, 2$. When the chair receives m with $m_2 = m_2^f$, she infers that $\theta_2 \in (1/2, 4]$. In this case, independent of m_1, she proposes $(e(\hat{y}_2); c, 0, 0, 0) = (-0.65; 1, 0, 0, 0)$. When the chair receives (m_1^f, m_2^c), she infers that $\theta_1 \in (1/2, 4]$ and $\theta_2 \in [1/4, 1/2]$ and proposes $(e(\hat{y}_1); c - x_2, 0, x_2, 0)$ where $x_2 = 0.0175$. When the chair receives (m_1^c, m_2^c), she infers that $\theta_1 \in [1/4, 1/2]$ and $\theta_2 \in [1/4, 1/2]$ and proposes $(y; c - x_1 - x_2, x_1, x_2, 0)$ where $y = -0.775 < e(\hat{y}_1), x_1 = 0.029$ and $x_2 = 0.048$. In all four

cases, legislators 1 and 2 vote for the proposal and legislator 3 vote against the proposal.

Intuitively, when the median legislator sends the "fight" message, the chair makes no transfers and moves the policy so that the median legislator is just willing to accept. When the median legislator sends the "compromise" message, the chair's proposal depends on the message of the closer legislator. If the closer legislator says "fight", then the chair moves the policy so that the closer legislator is just willing to accept without getting any transfer, and compensates the median legislator accordingly. When the closer legislator also says "compromise", the chair moves the policy even closer to her ideal, and compensates both legislators.

References

[1] Paul Milgrom and Chris Shannon. "Monotone Comparative Statics," Econometrica, 62, 1, 157-180, 1994.