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Fragility of reputation and clustering of risk-taking

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Reputation concerns in credit markets restrain borrowers' temptations to take excessive risk. The strength of these concerns depends on the behavior of other borrowers, rendering the reputational discipline fragile and subject to breakdowns without obvious changes in economic fundamentals. Furthermore, at an aggregate level, breakdowns are clustered among borrowers who have intermediate and good reputations, magnifying otherwise small economic shocks.

Keywords. Reputation, global games, risk-taking, fragility.

JEL classification. D82, E44, G01, G32.

1. Introduction

Financial markets are fragile: small shocks usually do not create large problems, but sometimes they do. The recent crisis is a clear example: the subprime shock was not large, but the crisis was.[1] The literature has identified a series of elements that contribute to fragility in different segments of financial markets. Diamond and Dybvig (1983) explain bank runs as an outcome of multiple equilibria in the presence of sequential withdrawing. Diamond and Rajan (2001) rationalize this particular fragility as a commitment device adopted by banks. Lagunoff and Schreft (1999) argue that fragility is generated by linked portfolios across financial agents. Gorton and Ordoñez (2012) explore how small shocks can trigger information acquisition and asymmetric information about assets used as collateral.

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[1] The Financial Crisis Inquiry Commission (FCIC) Report (2011, pp. 228–229) notes with respect to subprime mortgages, "Overall, for 2005 to 2007 vintage tranches of mortgage-backed securities originally rated triple-A, despite the mass downgrades, only about 10% of Alt-A and 4% of subprime securities had been 'materially impaired'—meaning that losses were imminent or had already been suffered—by the end of 2009." Park (2011) examines the trustee reports from February 2011 for 88.6% of the notional amount of AAA subprime bonds and shows the realized principal losses on the $1.9 trillion of AAA/Aaa-rated subprime bonds issued between 2004 and 2007 to be just 17 basis points as of February 2011.

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The current Federal Reserve chairman, Ben Bernanke, however, highlighted the collapse of financial markets' discipline as the key element of crises: *"Market discipline has in some cases broken down, and the incentives to follow prudent lending procedures have, at times, eroded."*[2] In this paper, I explore a novel mechanism that explains why reputation concerns, usually believed to be the main source of discipline in financial markets, are *fragile*: their breakdown induces a sudden coordinated change in individual risk-taking by large and reputable firms in response to small and not obvious changes in aggregate conditions.

I study a model of credit markets in which firms borrow to invest and then decide the risk of their investment. All firms can invest in "risky" projects, while only some of them (*strategic firms*) can also invest in safer projects, with higher probabilities of success but lower private benefits to managers. Lenders cannot observe which project a firm chooses, but they prefer that strategic firms choose "safe" ones, which implies a lower probability of default.

A firm's *reputation* is defined as the probability that the firm is strategic. Reputation is updated by lenders after observing both the firm's continuation—a signal about the firm's decision—and the aggregate default in the economy—a signal about the average decision of similar firms. Strategic firms want to distinguish themselves from nonstrategic ones so that they can pay lower rates for funds in the future. The fear of losing reputation, therefore, leads strategic firms to take less risks.

I assume firms' temptation to take risks varies monotonically with a stochastic aggregate fundamental. When such fundamental is observable, the model delivers multiple equilibria based on multiple firms' beliefs about similar firms' behavior. There is a range of the fundamental for which two equilibria coexist. At the one extreme, if a strategic firm believes no other strategic firm takes risks, then it does the same. Firms know that in this case the aggregate default rate will be low, lenders will believe no strategic firm takes risks, and firms continuation will be attributed at least partially to their good behavior, thereby improving their reputation. At the other extreme, if a strategic firm believes all other strategic firms take risks, then it does the same. In this case, firms know the aggregate default rate will be high, that lenders will believe strategic firms take risks, and their continuation will be attributed solely to good luck, not improving their reputation at all. Reputation concerns clearly reduce risk-taking in the first equilibrium but not in the second.

To obtain a unique equilibrium, which is robust to small perturbations of information, I use techniques from the global games literature. I assume that after negotiating the loan, but before making a decision about risk-taking, firms do not observe the fundamental but just an independent private noisy signal about it. The model thus becomes a nonstandard dynamic global game in which strategic complementarities are not just assumed, but are rather obtained endogenously from the concerns behind reputation formation. Uniqueness is characterized by a *cutoff* in signals about fundamentals, for each reputation level, around which strategic firms change their decision to take risks. Fundamentals, that is, do not only affect the temptation, but also become a coordination device for risk-taking.

[2]Statement, Board of Governors, December 18, 2007.

This equilibrium selection generates the first of two sources of reputation fragility. If signals about fundamentals are precise, small changes of fundamentals around the cutoff of risk-taking produce a clustering of behavior among firms with the same reputation.

The second source of fragility arises at an aggregate level when firms with different reputation are compared. This source of fragility is independent of the equilibrium selection and depends only on primitive learning properties. Risky projects have lower probabilities of continuation, which generate two types of incentives for not taking risks. One type, *continuation* incentives, increases with reputation; firms with better reputations face lower interest rates in the future, have higher expected future profits, and have more to gain from surviving. The other type of incentives, *reputation formation* incentives, is low for extreme reputations and high for intermediate ones; because of learning, priors are harder to change when reputation is either too high or too low. The combination of these two types of incentives induces firms with different reputation to behave similarly.

- *Poor* reputation firms have incentives to take risks because their continuation value is low and, if they survive, their reputation cannot improve much.
- *Intermediate* reputation firms do not have incentives to take risks, not because their continuation value is high; rather, if they survive, they can improve reputation a lot.
- *Good* reputation firms do not have incentives to take risks either, but not because they can gain much reputation if they survive; rather, their continuation value is high and they can lose that value if they die.

Hence, intermediate and good reputation firms have similar cutoffs for different reasons. They switch to risk-taking under similar conditions, clustering their behavior. Furthermore, since the distribution of reputation tends to be biased toward intermediate and good reputations, what these firms do strongly affects the aggregate level of risk-taking in the market.

My work shows that reputation concerns are beneficial because they reduce and stabilize aggregate risk-taking and default, but they are also fragile because they generate sudden reactions in risk-taking and default without obvious changes in fundamentals. Sudden clusters of risk-taking and default are not only well documented during the recent financial crisis (Taylor 2009, Bernanke 2009), but have been also well documented during previous crises (Campbell et al. 2001, Das et al. 2007).

Related literature: My work here primarily mixes two strands of literature: reputation and global games. With regard to the reputation strand, my model is most closely related to the models of Diamond (1989) and Mailath and Samuelson (2001), who analyze the ability of reputation to deter opportunistic behavior in the presence of both adverse selection and moral hazard. Unlike their work, which is focused on reputation incentives for a single agent living in a state-invariant environment, my work here explicitly introduces a cross section of firms in an environment that evolves stochastically, which allows study of the interplay between reputation incentives and economic conditions in determining aggregate behavior.

As in their work, my model also exhibits multiple equilibria, but in my case the multiplicity cannot be simply assumed away by using the arguments developed by Stiglitz and Weiss (1981), under which lenders compete à la Bertrand for borrowers and coordinate in the equilibrium with the lowest interest rates. While Diamond (1989) focuses on extreme equilibria and Mailath and Samuelson (2001) focus on the most efficient one, I select a unique equilibrium by exploiting economic conditions as a coordination device. Finally, unlike Mailath and Samuelson (2001), here firms' behavior affects the probability of their continuation, a signal to update reputation, and unlike Diamond's (1989) model, mine is flexible enough to include the use of additional signals correlated to projects, breaking the perfect correlation between age and reputation.

My work contributes to the literature on herding. The clustering I highlight in this paper exists in all markets where reputation plays a role, but it is particularly relevant in financial markets, whose operations are inconsistent with the assumptions of other standard herding mechanisms. The work pioneered by Scharfstein and Stein (1990) and Banerjee (1992) obtains herding from agents who sequentially observe the actions of other agents and mimic them, disregarding their own private information. In contrast, in my setting, agents cannot observe other agents' actions and instead pay a lot of attention to their own private information to coordinate behavior.

These assumptions are natural in financial markets. Consider the case where firms correspond to banks that raise funds from others and use these to generate loans that can be either risky or safe. There is an extensive literature on relationship lending that suggests lenders have access to private information about borrowers. But this precludes the type of herding story that has been emphasized in the literature: banks cannot observe the types of projects other banks choose to finance so as to mimic them; neither do those who invest funds with banks. Over the counter (OTC) trading, which grew exponentially before the recent crisis, is another example of the lack of transparency that can characterize financial markets. These exchanges are based on bilateral contracts of nonstandard derivatives and by construction restrict information about the risk-taking of other traders. Hence, in contrast to standard herding models, my mechanism is relevant in understanding the fragility of the new, more complex, financial system, critically characterized by a lack of information about other investors' positions.

An additional critical difference with this standard literature of herding is that in other papers, agents do not know their own types. In contrast, in my paper, agents know their own types and this is what creates reputation concerns. Closer to my mechanism are Zwiebel (1995) and Ottaviani and Sørensen (2006), who obtain herding based on career concerns and simultaneous actions. As in my work, when reputation is based on relative performance, concerns about losing reputation induce managers to behave as they expect others to behave. In these papers, however, the notion of fragility is nonexistent. Since I link individual behavior both to other agents' behavior and to aggregate variables, agents coordinate on different herding attitudes when facing different aggregate conditions. In my model, firms choose risky projects not because they want to be indistinguishable from others, but because the choices of others affect the increase in reputation a firm can expect if it chooses a safe project or, more accurately, if its choice generates an outcome that is more likely to occur under a safe project.

My paper also reveals fragility as a negative aspect of reputation concerns, in addition to other negative aspects discussed in the literature, such as preventing information aggregation (Dasgupta and Prat 2006, 2008) or amplifying price volatility in financial markets (Guerrieri and Kondor 2012).

My work also adds to the dynamic global games literature, such as Morris and Shin (2003), Toxvaerd (2008), Giannitsarou and Toxvaerd (2012), and Chassang (2010), exploiting novel properties that come from the endogenous reputational generation of strategic complementarities. In particular, the range of fundamentals with multiple equilibria depends on the initial reputation of the firm and this is useful in characterizing the schedule of cutoffs for different reputation levels. Also, since interest rates evolve endogenously with reputation, I endogenously determine the value functions that enter into the solution of dynamic global games.

Finally, my work contributes to the scarce literature on learning in global games as well. While most of that literature studies situations in which players learn about a policymaker or a status quo (as in, for example, the work of Angeletos et al. 2006 and 2007), my model deals with the opposite situation, in which the market learns about players' types, generating coordination problems. My work here is the first to exploit fundamental-driven incentives to create a reputation global game and select a unique equilibrium.

In the next section, I show how, with incomplete information about economic fundamentals, a dynamic global game analysis of reputation formation in credit markets delivers a unique equilibrium. In Section 3, I show that firms' concerns about reputation impose a discipline that is fragile. To conclude, in Section 4, I discuss potential extensions.

2. Selecting a unique reputation equilibrium in credit markets

In this section, I show that reputation concerns create strategic complementarities across firms, generating equilibrium multiplicity, and discuss the role of imperfect information in selecting a unique equilibrium. First, I describe the general model. Second, I illustrate the main results using a single period version, with specific distribution and payoff functions and exogenous interest rates and continuation values. Finally, I extend the results to a finite-horizon game, generalizing distribution and payoff functions and endogenizing interest rates and continuation values.

2.1 *The model*

Here I describe my model of reputation concerns in credit markets and the timing of its events.

2.1.1 *Description* Credit markets are composed of a continuum of long-lived, risk-neutral firms (with mass 1) and an infinite number of short-lived risk-neutral lenders who provide funds to those firms.

Each firm runs a unique project per period. The project can be safe (s) or risky (r). As in Fama and Miller (1972) and Jensen and Meckling (1976), a safe project is one that has a higher probability of success and firms' continuation (c).

Assumption 1. *Safe projects make firms' continuation more likely,* $\Pr(c|s) = p_s > \Pr(c|r) = p_r$.

If the firm does not continue, or dies, then current and future cash flows are zero. If the firm chooses a risky project and continues, the project just delivers pledgeable cash flows $\Pi_r = K$ at the end of the period. If the firm chooses a safe project and continues, the project not only delivers pledgeable cash flows K, but also nonpecuniary costs or benefits to managers θ, such that $\Pi_s = K + \theta$ at the end of the period. The single-dimensional variable $\theta \in \mathbb{R}$ is common to all firms (we call it a fundamental), is independently and identically distributed over time, and is normally distributed $\theta \sim \mathcal{N}(\mu, 1/\alpha)$, with mean μ and precision α in each period.[3]

One interpretation is that a manager can increase the probability of success (from p_r to p_s) by performing extra activities that generate a nonpecuniary cost (or benefit) that is purely determined by an aggregate variable θ. If $\theta < 0$, for example, the extra activity requires a costly effort. In contrast, if $\theta > 0$, the extra activity not only increases the probability of success, but also generates nonpecuniary benefits to the manager. Given this interpretation, and depending on the specification behind θ, risky projects are tempting when the opportunity cost of the effort required by the extra activity is large or when the nonpecuniary benefits from performing the extra activity are low.

To fix ideas, consider an example in which the firms that make investment decisions correspond to banks or other financial intermediaries. That is, banks raise funds from outside investors—depositors, other banks, bank-holders—and then turn around and use these funds to earn profits by investing in assets or issuing loans, either to firms or still other banks. It is in this capacity that we can interpret a borrower in my model as a financial intermediary. Once a bank raises funds from outside investors, it can choose to use these funds to generate loans, monitoring and studying either their profitability (safe loans) or their nonprofitability (risky loans). Defining θ as the complexity of financial instruments in which banks can invest, it may be more tempting to make safe loans or to undertake safe investments, determining their profitability, when the projects in which banks can invest using the funds they secure are standard and transparent.

There are two types of firms or borrowers (financial intermediaries in the previous example), defined by their access to projects. *Strategic* firms $\mathcal{S}$ can choose between safe and risky projects, hence they have the possibility of increasing the probability of success from p_r to p_s. *Risky* firms $\mathcal{R}$ can endeavor only in risky projects, which have a low probability of success. Firms know their own type, but lenders (investors in the previous example) do not. Firm's reputation is defined by $\phi = \Pr(\mathcal{S})$, the probability the firm is strategic.[4]

[3] The simple structure of payoffs and fundamentals just simplifies the exposition. We relax the assumptions of linearity in cash flows and normality in the Appendix.

[4] The introduction of these two types is based on my (maybe pessimistic) belief that all firms can endeavor in projects that have a low probability of success, but not all of them have the knowledge to increase the probability of success, potentially at a cost. An alternative assumption is that nonstrategic firms are those that have zero discount factor and simply do not care about continuation or increasing the probability of success.

To run a project, each firm needs external funds (normalized to one per period), which can be provided by lenders, whose outside option is an alternative investment in a risk-free bond that pays $\overline{R} > 1$.[5] Failure to repay loans (*default*) triggers a bankruptcy procedure that destroys the firm's pledgeable output. This is a straightforward way to introduce truth-telling by firms: when pledgeable cash flows are greater than debts, firms always find it optimal to repay the loan and get the positive differential than to default and get nothing. I assume that, conditional on continuation, firms are always solvent and pay their loans (specifically $K > 1/p_r$, where $1/p_r$ is the largest possible face value of debt). To the contrary, if a firm dies, cash flows are zero and the firm has to default.[6]

2.1.2 *Timing* These decisions are repeated during a finite number of periods. The order of events in each period t is the same in all periods $t = \{0, 1, \dots, T\}$ and is given as follows.

- Firms and lenders meet. Lenders do not observe the firm's type, just its continuation, its last period reputation (ϕ_{t-1}), and the last period aggregate default among all firms with the same reputation. Based on this information, each lender revises beliefs about the firm to ϕ_t. New firms start with exogenous ϕ_0.
- Each firm acquires a loan of 1 at a rate that depends on its new reputation, $R(\phi_t)$.
- Firms observe the fundamental θ_t (nonpecuniary costs or benefits of safe projects).
- Strategic firms decide between using safe (s) or risky (r) projects. Risky firms endeavor only on risky projects (r).
- Production occurs and the firms either continue or die.
- If a firm dies, it defaults on its loan. If a firm continues, it pays lenders $R(\phi_t)$ and consumes the remaining cash flows.

2.1.3 *Reputation updating* When updating a continuing firm's reputation from ϕ_t to ϕ_{t+1}, lenders should have a belief about how strategic firms behaved, which they can infer from the realized default of firms that have the same reputation. Let $D_t(\phi_t, \widehat{x}_t)$ denote the aggregate default of all firms ϕ_t at period t, where $\widehat{x}_t(\phi_t, \theta_t)$ is the fraction of strategic firms ϕ_t that took risks at t, given fundamentals θ_t:

$$D_t(\phi_t, \widehat{x}_t) = [(1 - p_r)\widehat{x}_t + (1 - p_s)(1 - \widehat{x}_t)]\phi_t + (1 - p_r)(1 - \phi_t).$$

[5]Since lenders are the long side of the market, there is no competition for funds. The introduction of such competition makes reputation more important and magnifies the results. An alternative assumption is that fundamentals not only affect cash flows (Π), but also affect the probability of continuation (p_s) and/or the risk-free interest rate ($\overline{R}$). Such alternatives reinforce my main results. See Ordonez (2008).

[6]For simplicity I rule out equity, asset accumulation, and other alternative forms of financing. I also assume θ is noncontractible. All these possibilities are likely to reduce the importance of reputation in the first place. However, as long as we believe reputation is an important element in financial relations, the goal is to understand how reputation concerns and aggregate conditions interact in a market with short-term debt contracts, not the optimality of such contracts. Studying other contracting forms is interesting, but outside the scope of this paper.

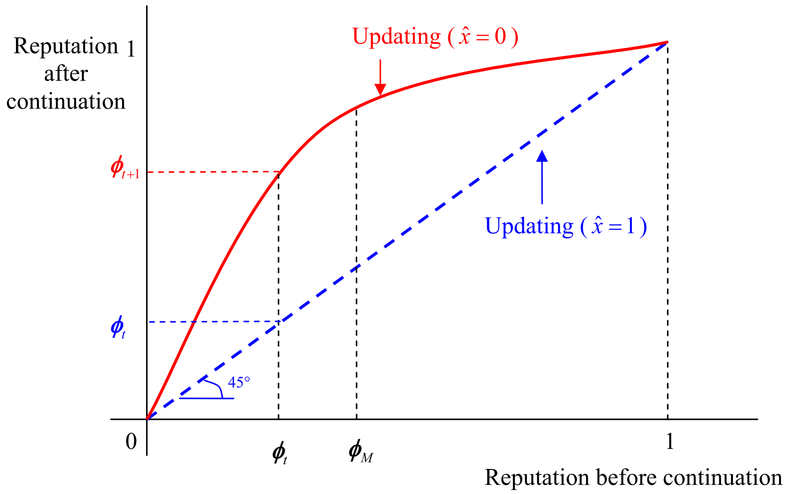

FIGURE 1. Reputation updating for different strategic risk-taking ($\widehat{x}$).

From this equation, lenders can infer $\widehat{x}_t(\phi_t, \theta_t)$ and update the reputation of a single continuing firm using Bayes' rule,

$$\Pr(\mathcal{S}|c) = \phi_{t+1}(\phi_t, \widehat{x}_t) = \frac{[p_r\widehat{x}_t + p_s(1-\widehat{x}_t)]\phi_t}{1-D_t(\phi_t, \widehat{x}_t)}. \tag{1}$$

Note that reputation is nondecreasing in age (continuation) and reputation increases less when many similar firms die. This seemingly counterintuitive result arises because a "good" firm is one that has the chance to behave well, but not necessarily does. This is why a high aggregate default is not good news for surviving firms, since continuation is not assigned to quality but to luck. Note that, for $\phi_t \in (0,1)$, $\phi_{t+1} = \phi_t$ when $\widehat{x}_t = 1$ and $\phi_{t+1} > \phi_t$ when $\widehat{x}_t < 1$, with the gap $(\phi_{t+1} - \phi_t)$ increasing as $\widehat{x}_t$ goes to 0.

Graphically, firms' reputation evolves as in Figure 1. Reputation priors ϕ_t are represented on the horizontal axis; reputation posteriors ϕ_{t+1} are on the vertical axis. For any prior ϕ_t, the following is true.

- Reputation changes less when more strategic firms of the same reputation take risks.[7]
  - If lenders infer that no strategic firm takes risks (that is, if $\widehat{x}_t = 0$), then the gap $\phi_{t+1} - \phi_t$ represents the gains to the firm, in terms of reputation, from continuing.

[7] This is supported empirically by Nickell et al. (2000), Bangia et al. (2002), Altman and Rijken (2006), and Ordonez (2008) using data on corporate credit-rating transitions. This pattern was also observed during the recent financial crisis. The July 2008 Moody's credit policy report, for example, shows that the rating volatility decreased significantly, almost 50%, with respect to historical averages in periods of high risk-taking. Among syndicated loans, Gopalan et al. (2011) also find that beliefs do not change dramatically in years in which several other lead arrangers also experience borrower bankruptcies.

  - If lenders infer that all strategic firms take risks (that is, if $\widehat{x}_t = 1$), then $\phi_{t+1} = \phi_t$ and firms do not gain, in terms of reputation, from continuing.

- Reputation changes more for firms with intermediate reputation.[8]
  - Regardless of $\widehat{x}_t$, updating is weaker when priors are stronger (that is, close to $\phi_t = 0$ or $\phi_t = 1$). In particular, regardless of $\widehat{x}_t$, $\phi_{t+1} = \phi_t$ for $\phi_t = 0$ and $\phi_t = 1$. The maximum gap, $(\phi_{t+1} - \phi_t)$ is obtained at an intermediate reputation level ϕ_M.

2.2 *A single period version*

In this section, I introduce many simplifying assumptions that are useful to highlight the essence of reputational multiplicity and the role of imperfect information in selecting a unique equilibrium. We assume a *single period*. Firms start with a given reputation ϕ and lenders charge an *exogenous interest rate* $R(\phi)$ (decreasing in ϕ) for the loan. The timing described above proceeds. At the end of the period, lenders observe aggregate default and update the reputation of continuing firms up to ϕ'. Finally an *exogenous continuation value* $V(\phi')$ (increasing in ϕ') is transferred to each continuing firm. In the next sections, I relax these simplifications, endogenizing interest rates and repeating the game an arbitrarily large number of periods to endogenize continuation values, showing that the properties we assume here hold in equilibrium.

To eliminate equilibria that require an implausible degree of coordination between the firm's behavior and its beliefs about other firms' behavior, I restrict attention to *Markovian* strategies, such that $x(\phi, \theta)$ is the probability that a firm with reputation ϕ that observes fundamentals θ takes risks.[9]

Given the monotonicity of payoffs on θ, I also focus on equilibria in *cutoff* strategies,[10] in which a firm with reputation ϕ decides to choose risky projects if fundamentals are below a certain cutoff point, $k(\phi)$, and to choose safe projects if fundamentals are above that cutoff:

$$x(\phi, \theta) = \begin{cases} 0 & \text{if } \theta > k(\phi) \\ 1 & \text{if } \theta < k(\phi). \end{cases}$$

Definition 1. A Markov perfect equilibrium in cutoff strategies consists of a (symmetric) strategy for the firms $k(\phi) = \theta^*(\phi) : [0, 1] \to \mathbb{R}$ and posteriors $\phi'(\phi, \widehat{x}) : [0, 1] \times [0, 1] \to [0, 1]$, for all $\phi \in [0, 1]$, such that the following statements hold.

- The equality $k(\phi) = \theta^*(\phi)$ defines the $x^*(\phi, \theta) \in \arg\max_{x\in[0,1]} U(\phi, \theta, x|\widehat{x})$ for all θ, where

$$U(\phi, \theta, x|\widehat{x}) = (1 - x)p_s\big[K + \theta - R(\phi) + \beta V(\phi'(\phi, \widehat{x}))\big] + xp_r\big[K - R(\phi) + \beta V(\phi'(\phi, \widehat{x}))\big]$$

[8]Using data on syndicated loans, Gopalan et al. (2011) show that borrower bankruptcies seem to have little impact on lead arrangers who have very dominant or very poor market positions, where market positions are positively correlated with reputation.

[9]See the discussion in Mailath and Samuelson (2006).

[10]We show later that these are the only Markovian strategies that survive the global game refinement.

and β is the discount factor.

- The posterior $\phi'(\phi, \widehat{x})$ is obtained using Bayes' rule (1), where $\widehat{x}(\phi, \theta) = x^*(\phi, \theta)$ for all θ, and is the updating rule that lenders must use if their beliefs are to be correct (this is consistent with equilibrium strategies of a continuum of firms ϕ, which determine aggregate default $D(\phi, x^*)$).

2.2.1 *Multiple equilibria with complete information* Now I show that in this baseline model, when firms perfectly observe the fundamental, a multiplicity of Markovian perfect equilibria exists in monotone cutoff strategies. First I discuss properties of the firms' differential gains from taking safe projects relative to risky projects, which characterize each firm's decisions. Then I show how these properties interact with firms' beliefs about other firms' actions to create multiple equilibria.

Define by $\Delta(\phi, \theta|\widehat{x}) = U(\phi, \theta, x = 0|\widehat{x}) - U(\phi, \theta, x = 1|\widehat{x})$ the differential gains to firms from taking safe projects relative to risky projects when a firm with reputation ϕ observes a fundamental θ, conditional on beliefs $\widehat{x}(\phi, \theta)$ (expected fraction of strategic risk-taking that lenders will recover from the end of period's aggregate default). A firm chooses safe projects if $\Delta(\phi, \theta|\widehat{x}) > 0$ and risky projects if $\Delta(\phi, \theta|\widehat{x}) < 0$:

$$\Delta(\phi, \theta|\widehat{x}) = (p_s - p_r)\Big[\overbrace{K - R(\phi)}^{\textit{short-term}} + \overbrace{\frac{p_s}{p_s - p_r}\theta}^{\textit{MH}} + \overbrace{\beta V(\phi)}^{\textit{cont}} + \overbrace{\beta\big[V(\phi'(\phi, \widehat{x})) - V(\phi)\big]}^{\textit{reputation formation}}\Big]. \quad (2)$$

Equation (2) displays the four essential components of these differential gains:

- *Short-term* captures the differential gains (in expected net pledgeable cash flows from a higher probability of continuation) from choosing safe projects.
- *MH* refers to moral hazard. It captures the relative temptation to take risks. This is the only part of the differential gains that depends on θ.
- *Cont* captures the idea that taking safe projects increases the probability of the firm's continuation, which has a value that depends on reputation ϕ.
- *Reputation formation* captures the idea that taking safe projects also increases the probability of reputation improvement from ϕ to ϕ'.

It is clear that risk-taking is less tempting as fundamentals increase ($\partial\Delta(\phi, \theta|\widehat{x})/(\partial\theta) = p_s > 0$) and as strategic risk-taking declines ($\partial\Delta(\phi, \theta|\widehat{x})/(\partial\widehat{x}) \leq 0$, since $\partial(\phi' - \phi)/(\partial\widehat{x}) \leq 0$ and $\partial V(\phi')/(\partial\phi') > 0$). It is also important to highlight that the marginal value of reputation combines the accumulated value of reputation (continuation incentives) and the change in reputation (reputation formation incentives). In case of default, firms stop operating, losing not only the possibility of increasing reputation, but also all the reputation acquired until that point. As I show, this implies that reputation incentives increase monotonically with reputation.

Before discussing multiplicity, I assume *uniform limit dominance*, which defines ranges of fundamentals for which, regardless of other firms' actions, a firm chooses risky

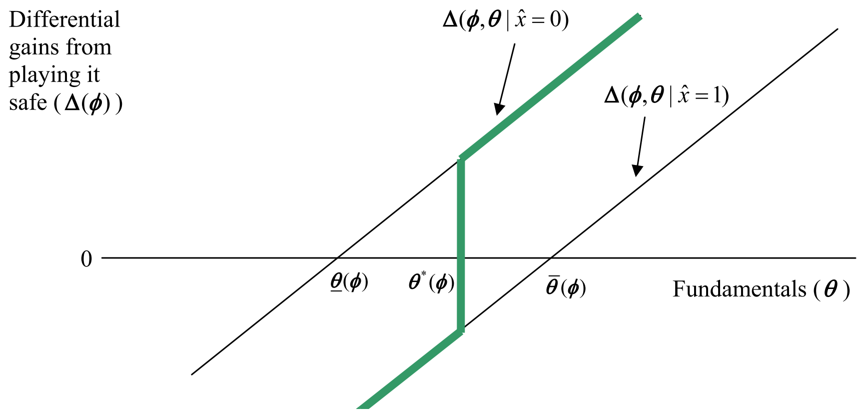

FIGURE 2. Equilibria multiplicity with complete information about fundamentals.

projects (fundamentals below a lower bound $\underline{\theta}$) or safe projects (fundamentals above an upper bound $\overline{\theta}$).

ASSUMPTION 2 (Uniform limit dominance in a single period model).

- *For each ϕ, there is a lower bound $\underline{\theta}(\phi)$ such that $\Delta(\phi, \underline{\theta}|\widehat{x} = 0) = 0$.*
- *For each ϕ, there is an upper bound $\overline{\theta}(\phi)$ such that $\Delta(\phi, \overline{\theta}|\widehat{x} = 1) = 0$.*

In this simple single period version of the model,

$$\underline{\theta}(\phi) = -\frac{p_s - p_r}{p_s}\big[K - R(\phi) + \beta V(\phi'(\phi, \widehat{x} = 0))\big]$$

$$\overline{\theta}(\phi) = -\frac{p_s - p_r}{p_s}[K - R(\phi) + \beta V(\phi)].$$

The gap $\overline{\theta}(\phi) - \underline{\theta}(\phi) = \beta((p_s - p_r)/p_s)[V(\phi'(\phi, \widehat{x} = 0)) - V(\phi)] \geq 0$ (equal to zero for $\phi = 0$ and $\phi = 1$) and achieves the maximum at the intermediate reputation level ϕ_M. This is simply a mapping from the updating gap $\phi'(\phi, \widehat{x} = 0) - \phi$ (illustrated in Figure 1) into value functions that are increasing in reputation ϕ.

The next proposition formalizes the multiplicity of equilibria in this model.

PROPOSITION 1 (Multiplicity in a single period model). *For all reputation levels $\phi \in (0, 1)$, all $\theta \in [\underline{\theta}(\phi), \overline{\theta}(\phi)]$ are equilibrium strategy cutoffs $\theta^*(\phi)$. Only for reputation levels $\phi = 0$ and $\phi = 1$ is there a unique equilibrium cutoff, $\theta^*(0)$ and $\theta^*(1)$, respectively.*

The proof is given in the Appendix, but Figure 2 provides a graphical intuition of multiplicity. Consider a particular risk-taking cutoff $\theta^*(\phi)$ for some firm with reputation $\phi \in (0, 1)$, such that $\theta^*(\phi) \in [\underline{\theta}(\phi), \overline{\theta}(\phi)]$. Then the equilibrium differential gain $\Delta(\phi, \theta|x^*)$ for different levels of fundamentals is the bold function with a discrete jump at $\theta^*(\phi)$. This is an equilibrium because it is a best response for any realization of the fundamental θ such that firms' beliefs about other firms' actions are correct. Playing it safe is optimal for all $\theta \geq \theta^*(\phi)$ (since $\Delta(\phi, \theta|x^* = 0) \geq 0$ for all $\theta \geq \theta^*(\phi)$), and taking risks is optimal for all $\theta \leq \theta^*(\phi)$ (since $\Delta(\phi, \theta|x^* = 1) \leq 0$ for all $\theta \leq \theta^*(\phi)$).

In words, for a fundamental to be a cutoff in equilibrium, it should be the case that three equilibria coexist at exactly that cutoff. At the one extreme, if firms believe no other strategic firm will take risks and aggregate default will be low, it is in the firms' best interest to choose safe projects. Firms know that in this case their continuation and loan repayment will be attributed at least partly to their good behavior, thereby improving their reputation. At the other extreme, if firms believe all other strategic firms will take risks and aggregate default will be high, it is in the firms' best interest to take risks. Under these beliefs, firms know that their continuation and repayment will be attributed solely to good luck, not improving their reputation at all. A third equilibrium is one in which strategic firms are indifferent between taking safe and risky actions. Since the difference of payoffs between the two extremes is strictly positive, a continuum of fundamentals fulfills this condition.

The multiplicity I have described here thwarts attempts to draw conclusions about the effectiveness of reputation to reduce risk-taking, leaving a big role to self-fulfilling beliefs and payoff irrelevant sunspots. However, what really creates the multiplicity is the assumption of complete information about fundamentals, which at the same time requires an implausible degree of coordination and prediction of other firms' behavior in equilibrium. This orients us to what to do next to move toward the selection of a unique equilibrium.

2.2.2 *A unique equilibrium with incomplete information* What I do is modify the assumption that information about fundamentals is complete. I assume instead that firms observe a private noisy signal about the fundamental before deciding whether to take safe or risky projects.[11] This noise, when small, leads to the selection of a unique equilibrium. What creates the multiplicity is the strategic complementarity across firms, which works through lenders beliefs. With complete information, each equilibrium is sustained by different fulfilling expectations about what other firms do, hence in equilibrium firms can perfectly forecast each other's actions and coordinate on multiple courses of action. With incomplete information, private signals serves as an anchor for firm's actions that avoid the indeterminacy of expectations about other firms' actions and hence avoid the indeterminacy of beliefs lenders will use to update reputation.

Formally, the new assumption about the information structure is stated as follows.

Assumption 3. *Each firm i observes a signal about economic fundamentals $z_i = \theta + \epsilon_i$, which is identically and independently distributed across i. The noise $\epsilon_i \sim \mathcal{N}(0, \sigma^2)$ is unbiased and has a variance $\sigma^2 \equiv 1/\gamma$ (precision γ).*

[11] The assumption about the timing is important. If interest rates reveal, through the market's aggregation of information, the true fundamental before production occurs, we go back to complete information and a unique equilibrium cannot be pinned down by introducing heterogeneity through signals. See Atkeson (2000).

I extend the proof to more general distributions in the Appendix. Signals are useful not only to infer θ, but also to infer other firms' actions and the aggregate default lenders will use to update reputation. Given this revised, incomplete information structure, the firm uses a cutoff strategy defined over the set of signals rather than over the set of fundamentals. For a current signal z_i, a strategy of a firm ϕ is a real number $k_z(\phi)$ such that the firm uses safe technologies ($x(\phi, z_i) = 0$) for $z_i > k_z(\phi)$ and risky ones ($x(\phi, z_i) = 1$) for $z_i < k_z(\phi)$. The strategic risk-taking that lenders infer from aggregate default ($\widehat{x}(\phi, \theta)$) still depends on the fundamental. Firms use their signal z_i to take expectations about $\widehat{x}(\phi, \theta)$.

We can now define $\Delta(\phi, z|k_z)$ as firm ϕ's *expected* differential gains from choosing safe projects if they receive a signal z and other strategic firms ϕ use a cutoff $k_z(\phi)$. Formally,

$$\Delta(\phi, z|k_z) = E_{\theta|z}(\Delta(\phi, \theta|\widehat{x})|k_z). \tag{3}$$

The next proposition states that under this incomplete information structure, when signals are precise enough ($\sigma \to 0$), there exists a unique Markovian perfect Bayesian equilibrium in monotone cutoff strategies for each reputation level ϕ.

Proposition 2 (Uniqueness in a single period model). *For a given ϕ, as $\sigma \to 0$, there exists a unique cutoff signal $k_z(\phi) = z^*(\phi)$ in equilibrium such that $\Delta(\phi, z|z^*) = 0$ for $z = z^*(\phi)$, $\Delta(\phi, z|z^*) > 0$ for $z > z^*(\phi)$, and $\Delta(\phi, z|z^*) < 0$ for $z < z^*(\phi)$, where $z^*(\phi)$ is given by*

$$z^*(\phi) = -\frac{p_s - p_r}{p_s}\big[K - R(\phi) + \beta V(\phi'(\phi, \widehat{x} = 0.5))\big]. \tag{4}$$

Proof. Conditional on observing a signal z_i, firm i's expected θ is

$$\widehat{\theta}_i = E(\theta|z_i) = \frac{\alpha\mu + \gamma z_i}{\alpha + \gamma}.$$

Given this update on θ, the conditional distribution of signals of another firm j is

$$z_j|\widehat{\theta}_i \sim \mathcal{N}\left(\widehat{\theta}_i, \frac{1}{\alpha + \gamma} + \frac{1}{\gamma}\right).$$

The expected fraction of other firms having a signal smaller than z_i (and hence taking risks if firm i is indifferent at z_i) is

$$E(\widehat{x}(\phi, \theta)|z_i) = \Pr(z_j < z_i|\widehat{\theta}_i) = \Phi[\sqrt{\zeta}(\widehat{\theta}_i - \mu)],$$

where Φ is the standard normal cumulative distribution function (c.d.f.) and

$$\zeta = \frac{\alpha^2(\alpha + \gamma)}{\gamma(\alpha + 2\gamma)}.$$

The equilibrium cutoff $z^*(\phi)$ is given by the signal at which firms will be ex ante indifferent between taking safe or risky projects, when other firms also follow $z^*(\phi)$, such that

$$\Delta(\phi, z^*|z^*) = 0$$

$$E_{\theta|z^*}(\Delta(\phi, \theta|\widehat{x})|z^*) = (p_s - p_r)\left[K + \frac{p_s}{p_s - p_r}E(\theta|z^*) - R(\phi) + \beta E[V(\phi'(\phi, \widehat{x}|z^*))]\right] = 0.$$

As $\sigma \to 0$ (or $\gamma \to \infty$), $E(\theta|z^*) \to z^*$ and $E[V(\phi'(\phi, \widehat{x}(\phi, \theta)|z^*))] \to V(\phi'(\phi, \widehat{x} = 0.5))$ (since $\zeta \to 0$). Then we have (4). □

Intuitively, relaxing the assumption of complete information about fundamentals and making signals very precise, we can use the approach provided by global games to select a unique equilibrium by iterated deletion of dominated strategies. Assume, for example, that a strategic firm ϕ uses a cutoff strategy $k_z(\phi) = \underline{\theta}(\phi)$, which we know is an equilibrium sustained by $\widehat{x} = 0$ under complete information. If signals are very precise, it means that a firm that observes $z_i = \underline{\theta}(\phi)$ believes that around 50% of other strategic firms ϕ that use the same cutoff observe a signal below $\underline{\theta}(\phi)$ and will decide to take risky projects. Since there is a continuum of firms with reputation ϕ, lenders will observe 50% of the firms taking risks and will update reputation using $\widehat{x} = 0.5$. However, with updating based on $\widehat{x} = 0.5$, the firm would not be indifferent between risky and safe projects at $\underline{\theta}(\phi)$, strictly preferring to take risky projects.

The only cutoff in equilibrium is the signal at which a firm is indifferent between taking safe and risky projects when the expected fraction of strategic risk-taking that lenders use to update beliefs is $\widehat{x} \approx 0.5$, as in Proposition 2. Morris and Shin (2003) show that these Laplacian beliefs, "following Laplace's (1824) suggestion that one should apply a uniform prior to unknown events from the principle of insufficient reason," arise endogenously in global games for a player observing the threshold signal.

Here, fundamentals not only affect incentives, but also become a coordination device. If a firm observes a low signal, it believes the fundamental is low with high probability, which directly induces the firm to take risks. On top of that, the firm also believes that other similar firms have observed a low signal and will take risks as well, which indirectly induces the firm to take risks. This is why, fundamentals, through the generation of signals, pin down firms' expectations about other firms' strategic risk-taking and hence coordinate firms' actions.

Interestingly, under the alternative assumption that only one firm exists and the lender observes fundamentals instead of aggregate default, multiplicity still arises from reputation formation because lenders beliefs about the firm's actions should be correct in equilibrium. Even though this multiplicity does not rely on complementarities with other firms, a unique equilibrium can still be selected by using global games, under the assumption that both the firm and the lender observe the fundamental with noise.

2.3 *The general model*

In this section, I generalize the previous results by making interest rates and continuation values endogenous.[12] In each case, I discuss how multiplicity and the selection of

[12] In the Appendix, we also extend the results to more general payoff functions, distributions of fundamentals, and noise structures.

a unique equilibrium apply. This generalization is relevant because we are ultimately interested in comparing the cutoffs that firms with different reputation follow. To make such a comparison, it is key to obtain continuation values endogenously, and this can be done only by endogenizing interest rates first and then solving the full game as $T \to \infty$.

2.3.1 *Endogenous interest rates* In this section, I relax the assumption of exogenous interest rates in a single period. This is a first step to endogenizing value functions later by repeating the game many periods.

Since lenders are competitive and have a fixed outside option $\overline{R}$, at the time the loan is negotiated, they charge an interest rate that pays in expectation the same return as the outside option. This is an additional condition in the definition of equilibrium. Specifically, in the case of complete information about fundamentals, interest rates are defined by the risk-free interest rate $\overline{R}$ divided by the expected probability of firm's continuation,

$$R(\phi|k) = \frac{\overline{R}}{\Pr(c|\phi, k)}, \tag{5}$$

where

$$\Pr(c|\phi, k) = (1-\phi)p_r + \phi\big[p_r\mathcal{N}(k) + p_s(1-\mathcal{N}(k))\big],$$

$k(\phi)$ are lenders' beliefs about the cutoff that strategic firms ϕ follow, and $\mathcal{N}(k)$ is the expected strategic risk-taking by those firms (normal cumulative density up to $k(\phi)$). Since interest rates depend on $k(\phi)$, expected gains from choosing safe projects (2) under complete information can be rewritten as

$$\Delta(\phi, \theta|k, \widehat{x}) = (p_s - p_r)\bigg[K + \frac{p_s}{p_s - p_r}\theta - R(\phi|k) + \beta V(\phi'(\phi, \widehat{x}))\bigg].$$

To discuss the complications endogenous interest rates introduce, assume for a moment that reputation cannot change (that is, $\widehat{x} = 1$ always and ϕ is never updated). In the previous section, this would eliminate the reputation source of multiplicity. However, endogenous interest rates still may generate a finite number of equilibrium cutoffs $\theta^*(\phi)$. For example, there may be three equilibria, with high, intermediate, and low $\theta^*(\phi)$ and $R(\phi|\theta^*)$.

To be more explicit, without reputation formation, the equilibrium with complete information is characterized by,

$$\Delta(\phi, \theta|k = \theta^*, \widehat{x} = 1) = (p_s - p_r)\bigg[K + \frac{p_s}{p_s - p_r}\theta^* - R(\phi|\theta^*) + \beta V(\phi)\bigg] = 0. \tag{6}$$

To guarantee there is just one equilibrium, we need a unique best response to $k(\phi)$ (this is $\partial\theta^*/(\partial k) < 1$). Taking derivatives with respect to k, $p_s\,\partial\theta^*/(\partial k) = (p_s - p_r)\,\partial R(\phi)/(\partial k)$, and considering $\partial R(\phi)/(\partial k) = \overline{R}(p_s - p_r)^2\phi n(k)/(\Pr(c|\phi, k)^2)$ (where $n(k)$ is the normal density evaluated at $k(\phi)$), the condition for uniqueness is then

$$\frac{\overline{R}(p_s - p_r)^2\phi n(k)}{p_s\Pr(c|\phi, k)^2} < 1. \tag{7}$$

This condition is the most stringent at $\phi = 1$ and $\Pr(c|\phi, k) = p_r$. Since $n(\theta) \leq \sqrt{\alpha/(2\pi)}$ for all θ, a sufficient condition for uniqueness is

$$\alpha < 2\pi\left[\frac{p_s p_r^2}{\overline{R}(p_s - p_r)^2}\right]^2. \tag{8}$$

This condition basically requires a precision of fundamentals that is low enough so that interest rates do not change too quickly with changes in beliefs about the cutoffs $k(\phi)$ that firms follow. If condition (7) is not fulfilled, then we may have a finite multiplicity of equilibria created just by endogenous interest rates.

As discussed by Stiglitz and Weiss (1981), it is possible to select a unique equilibrium in this case by assuming just Bertrand competition, in which lenders first offer a rate and then firms choose the best offer. Assume there are two equilibria and all lenders charge the highest rate. In this case, there are incentives for a single lender to deviate, offering the lower rate, attracting firms, and still breaking even. Then the lenders who effectively provide loans are the optimistic ones. This refinement rationalizes as the unique equilibrium the one with the lowest possible rate.

Now, consider again the environment with reputation formation. Since complementarity is introduced, the problem of multiplicity grows from a possible finite multiplicity to a certain continuum of multiple equilibria. Can we still apply the selection mechanism proposed by Stiglitz and Weiss (1981)? Yes, but only if the lenders who update reputation next period are the same as those who provide loans in the current period. Only in this uninteresting case, in which a firm obtains financing only from a single lender all its life and information is not revealed to other lenders, is there no meaningful complementarity problem.

However, if the lenders who set interest rates in the current period are different than the lenders who provide funds in the following period (or at least there is some chance lenders are not the same), then interest rates cannot be used to select an equilibrium. Assume again that all lenders charge a high rate and then firms take risks with high probability. A single lender does not have incentives to deviate and charge a lower interest rate (contrary to the Bertrand intuition) because the firm taking his loan still would be induced to take risks, knowing that future lenders will likely observe a high aggregate default and will not update its reputation. Hence, even though Bertrand competition can solve multiplicity generated by pure moral hazard, it cannot solve the multiplicity created by complementarity in reputation formation.

In what follows, I assume that the finite static multiplicity generated by interest rates is not an issue (condition (7) holds), so I can focus on the more interesting multiplicity created by reputation formation. Since, for each pair ϕ and $k(\phi)$, there is a unique $R(\phi|k(\phi))$, the uniform limit dominance assumption (Assumption 2) should hold for every pair ϕ and $k(\phi)$, extending the Δ function from (6) by adding the cutoff as an argument. With endogenous interest rates, $\underline{\theta}(\phi|k) < \overline{\theta}(\phi|k)$ for all ϕ and $k(\phi)$, but the range $[\underline{\theta}(\phi|k=\underline{\theta}), \overline{\theta}(\phi|k=\overline{\theta})]$ is wider with endogenous interest rates than with exogenous interest rates.[13] Given this range of multiplicity, we can still obtain uniqueness by assuming imperfect information about fundamentals with small noise and applying global games techniques as before.

2.3.2 *Endogenous value functions* To endogenize value functions, I develop now the full-fledged finite-horizon model as the terminal period T goes to infinity ($T \to \infty$) with endogenous interest rates. After introducing some important new notation, I discuss multiplicity under complete information and uniqueness under incomplete information when continuation values are endogenous. Since the main intuition for the multiplicity and the uniqueness refinement is the same as that in the previous section, here I just discuss the new challenges and show the main extended propositions, deferring the formal definitions and proofs to the Appendix.

With complete information, total discounted profits for a firm with reputation ϕ, that observes a fundamental θ, takes risks with probability $x(\phi, \theta)$, and believes strategic firms follow a cutoff $k(\phi)$ (which generates $\widehat{x}(\phi, \theta)$) for all θ are an extended version of $U(\phi, \theta, x|\widehat{x})$ from Definition 1, which now includes the cutoff k as an argument:

$$\begin{aligned}\widetilde{V}(\phi, \theta, x|k, \widehat{x}) = (1-x)\big[p_s[K+\theta-R(\phi|k)] + \beta p_s \mathbf{V}(\phi'(\phi, \widehat{x}))\big] \\ + x\big[p_r[K-R(\phi|k)] + \beta p_r \mathbf{V}(\phi'(\phi, \widehat{x}))\big].\end{aligned}$$

Define

$$V(\phi, \theta|k, \widehat{x}) = \max_{x\in[0,1]} \widetilde{V}(\phi, \theta, x|k, \widehat{x})$$

and assume $\mathbf{V}(\phi'(\phi, \widehat{x})) = \int_{-\infty}^{\infty} V(\phi', \theta'|k', \widehat{x}')\, d\mathcal{N}(\theta')$ is the expected continuation value for ϕ', where the expectation is over all possible θ' next period. The value $\mathbf{V}(\phi'(\phi, \widehat{x}))$ is an element of a given set of expected continuation values $\Upsilon' = \{\mathbf{V}(\phi')\}_{\phi'=0}^{1}$ for all ϕ.

The definition of equilibrium in a game in which firms live for T periods, with endogenous interest rates and continuation values, is the same as in Definition 1, but with two additional conditions. First, lenders have to obtain, in expectation, a return equal to the risk-free rate every period, which determines lending rates for each ϕ. Second, imposing that terminal values are $\mathbf{V}_{T+1}(\phi) = 0$ for all ϕ, continuation values are endogenously determined by backward induction. The formal definition is given in the Appendix.

The full-fledged model with complete information naturally inherits the multiplicity from single period models. However, the range of multiple equilibria cutoffs in each period widens. Since multiplicity exists in every single period, multiple streams of future expected continuation values (consistent with multiple equilibria in future periods) can be used to construct the differential of taking safe projects. By introducing

[13] It is straightforward to show that

$$\overline{\theta}(\phi|k=\overline{\theta}) - \underline{\theta}(\phi|k=\underline{\theta}) = \overline{\theta}(\phi) - \underline{\theta}(\phi) + \frac{p_s - p_r}{p_s}\beta(R(\phi|k=\overline{\theta}) - R(\phi|k=\underline{\theta})) > \overline{\theta}(\phi) - \underline{\theta}(\phi).$$

extreme streams of continuation values determined by the highest ($\overline{\Upsilon}'$) and the lowest ($\underline{\Upsilon}'$) probability of risk-taking in all future periods for all reputation levels, we can construct extreme bounds $\underline{\theta}(\phi|\overline{\Upsilon}') < \underline{\theta}(\phi|\Upsilon')$ and $\overline{\theta}(\phi|\underline{\Upsilon}') > \overline{\theta}(\phi|\Upsilon')$ such that the region of multiplicity in a given period is wider when considering the multiplicity of equilibria in future periods.

Again, by assuming incomplete information about fundamentals, I show uniqueness in this full model, which is characterized by a unique sequence of equilibrium cutoffs as signals become very precise. Also, as the last period goes to infinity, there is a unique limit to the sequence of cutoffs that characterize the perfect Markovian equilibrium in the finite game.

The following proposition states that, based on the boundary condition $\mathbf{V}_{T+1}(\phi) = 0$ for all ϕ, expected continuation values $\mathbf{V}_t(\phi)$ are well defined in each period t for all reputation levels ϕ and then a unique equilibrium exists in the finite-horizon game as $\sigma \to 0$. To solve this finite dynamic global game, I follow the literature from Morris and Shin (2003), Toxvaerd (2008), Steiner (2008), and Giannitsarou and Toxvaerd (2012).

Proposition 3 (Uniqueness in a finite-horizon game). *For each reputation ϕ, in each period t, as $\sigma \to 0$, a unique cutoff signal $z_t^*(\phi)$ exists such that the* expected *differential gains from choosing safe projects $\Delta_t(\phi, z_t|z_t^*) = 0$ for $z_t = z_t^*(\phi)$, $\Delta_t(\phi, z_t|z_t^*) > 0$ for $z_t > z_t^*(\phi)$, and $\Delta_t(\phi, z_t|z_t^*) < 0$ for $z_t < z_t^*(\phi)$, where $\Delta_t(\phi, z_t|z_t^*)$ and $z_t^*(\phi)$ (as defined in Proposition 2) depend on $\mathbf{V}_{t+1}(\phi)$ and $\mathbf{V}_{t+1}(\phi'(\phi, x_t^* = 0))$. Continuation values $\mathbf{V}_t(\phi)$ are well defined and, given the boundary condition $\mathbf{V}_{T+1}(\phi) = 0$, are recursively determined by*

$$\begin{aligned}\mathbf{V}_t(\phi) = &\int_{-\infty}^{z_t^*(\phi)} p_r[K - R_t(\phi|z_t^*) + \beta \mathbf{V}_{t+1}(\phi)] v(\theta_t)\, d\mathcal{N}(\theta_t) \\ &+ \int_{z_t^*(\phi)}^{\infty} p_s\big[K + \theta_t - R_t(\phi|z_t^*) + \beta \mathbf{V}_{t+1}(\phi'(\phi, x_t^* = 0))\big] v(\theta_t)\, d\mathcal{N}(\theta_t).\end{aligned} \tag{9}$$

The Appendix contains a proof for any prior distributions of fundamentals with a c.d.f. $\mathcal{V}(\theta)$ strictly increasing over a connected set that includes both dominance regions from the uniform limit dominance assumption and any distribution of signals with a c.d.f. strictly increasing over a connected set.

The next proposition establishes that under certain conditions, in particular, when the variance of the fundamentals distribution is large enough, it is possible to treat the infinite-horizon game as a limit of the finite-horizon game.

Proposition 4. *If $\mathbf{V}_T(\phi) \to \overline{\mathbf{V}}(\phi)$ as $T \to \infty$ and $\sigma \to 0$, then a cutoff $z^*(\phi)$ exists for each ϕ that is a unique limit to the sequence of cutoffs $\{z_t^*(\phi)\}_{t=0}^T$ of the finite-horizon Markov perfect equilibrium described in Proposition 3.*

Proof. In the Appendix, I show the sufficient condition for $\mathbf{V}_T(\phi) \to \overline{\mathbf{V}}(\phi)$ for all ϕ as $T \to \infty$ is $\alpha < 2\pi[p_s p_r^2/(\overline{R}(p_s - p_r)^2)]^2[(1 - \beta p_s)/(\beta p_s)]^2$ for all $\theta \in \mathbb{R}$.[14] Having shown

[14]Note that the sufficient condition for convergence is more stringent than the sufficient condition for uniqueness (8) when $\beta p_s > 0.5$.

uniqueness for an arbitrary finite-horizon T, I must show that the same reasoning is extended as $T \to \infty$. First, note that the values of taking safe and risky projects are bounded and well behaved monotone functions of T, as they converge to a fixed point $\overline{\mathbf{V}}(\phi)$ when $T \to \infty$. Second, as defined in (2), $\Delta_t(\phi, z_t|z_t^*)$ also converges to a unique limit as $T \to \infty$. Then $\lim_{T\to\infty} z_t^*(\phi|T) = z^*(\phi)$ for all t, where the dependence on T indicates the length of the game. □

Intuitively, if we solve backward from some T by using as a boundary condition the fixed point $\mathbf{V}_{T+1}(\phi) = \overline{\mathbf{V}}(\phi)$, rather than $\mathbf{V}_{T+1}(\phi) = 0$, then we obtain a unique $z^*(\phi)$ for each ϕ in all periods $t < T$. This matters because, as $\sigma \to 0$, in a period t far enough from the last period $T \to \infty$, unique cutoffs and ex ante probabilities of risk-taking are constant over time for each reputation level ϕ.

3. Establishing the fragility of reputation concerns

Now I use the unique equilibrium from the last proposition to show how concerns about reputation impose discipline and reduce risk-taking, and how this discipline is fragile and can suddenly break down with small and nonobvious changes in economic fundamentals. Furthermore, when discipline collapses, it collapses for a bunch of different firms with intermediate and good reputation, generating a clustering of risk-taking that can have far-reaching aggregate negative consequences. I demonstrate these results first in a formal abstract analysis and then in an illustrative numerical simulation of the model.

3.1 *The formal analysis*

Reputation concerns impose discipline on strategic firms, discouraging them from taking inefficiently risky projects. This discipline is, however, fragile and may suddenly collapse, creating clustering of risk-taking in the aggregate.

3.1.1 *Reputation imposes discipline* The next proposition shows why firms are concerned about constructing and maintaining good reputations. A better reputation for a firm implies a lower ex ante probability of risk-taking, hence the firm has to pay lower interest rates and enjoys higher continuation values. I focus on the case $T \to \infty$ and $\sigma \to 0$ (as in Proposition 4), in which cutoffs, interest rates, and value functions are time independent functions of ϕ. The proof is given in the Appendix.

Proposition 5 (Reputation, risk-taking, lending rates, and continuation values). *As a firm's reputation ϕ improves, the following changes occur.*

- *(i) Its ex ante probability of risk-taking monotonically decreases (that is, $d\mathcal{N}(z^*(\phi))/(d\phi) < 0$ for all $\phi \in [0, 1]$).*
- *(ii) The interest rate it faces monotonically decreases (that is, $dR(\phi)/(d\phi) < 0$ for all $\phi \in [0, 1]$).*

(iii) *Its continuation value monotonically increases (that is,* $d\mathbf{V}(\phi)/(d\phi) > 0$ *for all* $\phi \in [0, 1]$*).*

Evidence that more reputable firms take less risks and pay lower rates than less reputable firms is discussed by John and Nachman (1985) for investment behavior and, more recently, by Chatterjee et al. (2011) for households' credit. More generally, there is evidence for a positive correlation between reputation, performance, and compensation among investment banks (Miles and Miller 2000, Jackson 2005, Fang 2005), initial public offering (IPO) underwriting (Carter et al. 1998, An and Chan 2008), and syndicated loans (Tykvová 2007, Gopalan et al. 2011). This last paper, for example, finds that a large scale bankruptcy filing by a lead arranger's borrower is indicative of poor performance by the lead arranger, damaging its reputation. In the years after a loss of reputation, the lead manager keeps larger fractions of the loans it arranges, it is less likely to syndicate a new loan, and it is less likely to attract lenders to participate in its syndicates.

Even though lenders always prefer firms to choose safe projects, since $p_s > p_r$ independently of θ, sometimes it is efficient for firms to take risks because the opportunity private costs of taking safe projects may be too large.

Lemma 1 (Inefficient risk-taking). *There is always a region of inefficient risk-taking* $\theta \in [\theta_E, z^*(\phi)]$, *where* $\theta_E \equiv -((p_s - p_r)/p_s)[K + \beta V(1)]$ *is the cutoff above which it is efficient that strategic firms take safe projects.*

If actions and types are observable, risk-taking is efficient only if $p_s(K + \theta - 1/p_s + \beta V(1)) < p_r(K - 1/p_r + \beta V(1))$. Then there is a cutoff $\theta_E \equiv -((p_s - p_r)/p_s)[K + \beta V(1)]$ above which it is inefficient for strategic firms to take risky projects. Information frictions add a moral hazard component that induces excessive risk-taking. It is straightforward to see, from (4), that $z^*(\phi) \geq \theta_E$ for all ϕ. This implies that there is always a region of inefficient risk-taking $\theta \in [\theta_E, z^*(\phi)]$.

We show next that reputation concerns relax this inefficiency by reducing $z^*(\phi)$. In the next proposition, I prove that reputation concerns shorten the region $\theta \in [\theta_E, z^*(\phi)]$ by comparing an environment with reputation to an artificial environment in which firms are not concerned at all about their reputation, simply because their reputation cannot change.[15]

Proposition 6 (Reputation concerns reduce inefficient risk-taking). *Define* $\widetilde{z}^*(\phi)$ *as the risk-taking cutoffs when reputation is not a concern (that is, when reputation cannot change). Reputation concerns reduce the ex ante probability of risk-taking (that is,* $z^*(\phi) < \widetilde{z}^*(\phi)$*) for all* $\phi \in (0, 1)$ *and do not change it (that is,* $z^*(\phi) = \widetilde{z}^*(\phi)$*) for* $\phi = \{0, 1\}$.

Proof. With reputation concerns, $z^*(\phi)$ is determined by (4) in the manner

$$z^*(\phi) = -\frac{p_s - p_r}{p_s}\big[K - R(\phi|z^*) + \beta\mathbf{V}(\phi'(\phi, \widehat{x} = 0.5))\big].$$

[15] For example, credit histories are erased or lenders cannot observe the age of the firm.

Without reputation concerns, $\widetilde{z}^*(\phi)$ is determined by

$$\widetilde{z}^*(\phi) = -\frac{p_s - p_r}{p_s}[K - R(\phi|\widetilde{z}^*) + \beta \mathbf{V}(\phi)]$$

since the restriction that reputation cannot change is exactly the same as assuming $\widehat{x} = 1$.

Take $\phi \in (0,1)$. Since $\mathbf{V}(\phi'(\phi, \widehat{x} = 0.5)) > \mathbf{V}(\phi)$ from Proposition 5, then $z^*(\phi) < \widetilde{z}^*(\phi)$. This implies $R(\phi|z^*) < R(\phi|\widetilde{z}^*)$, which reinforces that $z^*(\phi) < \widetilde{z}^*(\phi)$. For $\phi \in \{0,1\}$, $\mathbf{V}(\phi'(\phi, \widehat{x} = 0.5)) = \mathbf{V}(\phi)$ and $z^*(\phi) = \widetilde{z}^*(\phi)$. □

3.1.2 *The reputational discipline is fragile* Now I can go further and establish that the existence of reputation concerns may suddenly collapse by creating sudden changes in aggregate risk-taking behavior without obvious changes in fundamentals. First, we can characterize firms' equilibrium cutoffs $z^*(\phi)$ for different reputation levels ϕ. The next lemma shows that the concern for reputation formation convexifies the schedule of those cutoffs.

Lemma 2. *Reputation concerns convexify the schedule of cutoffs (that is, $d^2 z^*(\phi)/(d\phi^2) > d^2\widetilde{z}^*(\phi)/(d\phi^2)$ for all $\phi \in [0,1]$, where $\widetilde{z}^*(\phi)$ are the cutoffs without reputation concerns). Furthermore, there are always signals about the firm's type that are precise enough (p_s/p_r high enough) such that the schedule of cutoffs is strictly convex (that is, $d^2 z^*(\phi)/(d\phi^2) > 0$) for all ϕ.*

The proof is given in the Appendix, but I use Figure 3 to convey the intuition. At every period, each firm is a point in the graph, a combination of reputation ϕ and a signal z, and follows a cutoff $z^*(\phi)$. Assume, for example, that without reputation concerns, the schedule of cutoffs ($\widetilde{z}^*(\phi)$) is linear in ϕ (as in the figure). As we know from Proposition 6, reputation concerns reduce risk-taking (that is, reduce cutoffs from $\widetilde{z}^*(\phi)$ to $z^*(\phi)$) for all ϕ. However, the strength of this force is not the same across reputation levels and depends on reputation formation incentives. I start discussing $\phi = 0$ and gradually proceed for higher levels of ϕ.

Firms with reputation $\phi = 0$ cannot change their reputation, which means that the cutoff for risky behavior is the same with and without reputation concerns ($z^*(0) = \widetilde{z}^*(0)$). As we consider higher levels of ϕ, firms have higher concerns for reputation formation, which rapidly reduce cutoffs. This effect achieves the maximum at ϕ_M, where reputation changes the most. After this point, further increments in ϕ reduce the role of concerns for reputation formation. At the extreme $\phi = 1$, reputation cannot improve further, so the cutoff is again the same with and without reputation concerns ($z^*(1) = \widetilde{z}^*(1)$). Still, for firms with reputation $\phi = 1$, maintaining their high reputation is still a concern, and this is why $z^*(1) < z^*(0)$. For firms with poor reputation, two types of incentives—continuation and reputation formation—reinforce each other in reducing risk-taking. For firms with good reputation, while continuation effects increase with ϕ, reputation formation effects become less important.

To be more concise, classify firms into three bins: firms with poor, intermediate, and good reputation. *Poor* reputation firms are prone to take risks because their gains

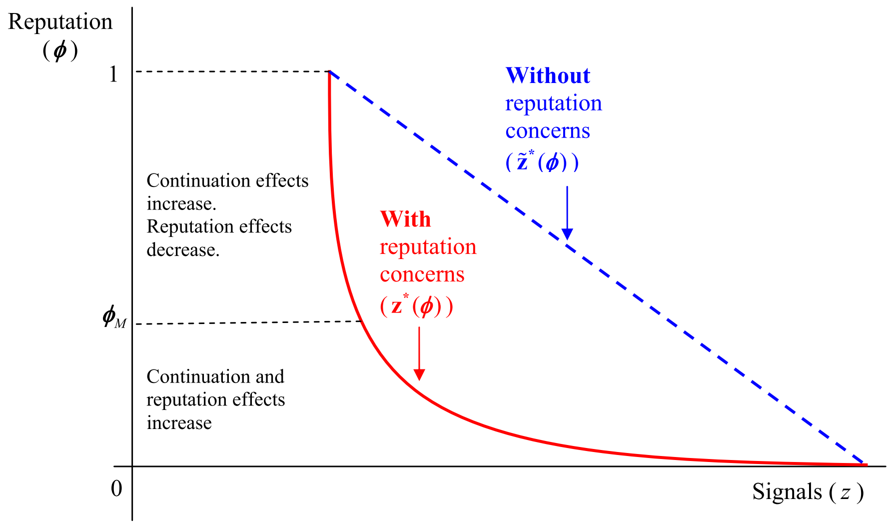

Figure 3. Reputation and cutoffs for risk-taking behavior.

from surviving are low (they will have to pay high interest rates in the future) and they cannot change their reputation much. *Intermediate* reputation firms want to take safe projects, not because they can lose a lot if they die, but because they can improve their reputation a lot if they survive. *Good* reputation firms want to take safe projects for the reverse reason, not because they can improve their reputation a lot if they survive, but because they can lose a lot if they die.[16]

This leads us to a final, crucial proposition.

Proposition 7 (Fragility of reputation and clustering of risk-taking). *The following statements hold for highly precise signals about fundamentals ($\sigma \to 0$), when the reputational distribution is held fixed.*

(i) Reputation is fragile at a firm level: There are small drops in fundamentals (θ_1 and θ_2 such that $\theta_1 - \theta_2$ is arbitrarily small, $\theta_1 > z^*(\phi)$, and $\theta_2 < z^*(\phi)$) that induce sudden risk-taking (from $x(\phi, \theta_1) \approx 0$ to $x(\phi, \theta_2) \approx 1$). This change is clustered among all firms with the same reputation level ϕ.

(ii) Reputation is fragile at an aggregate level: Under Lemma 2, as fundamentals θ decline, firms in an increasingly large range of reputation levels start taking risks. Formally, define $\phi_H(\theta)$ as the highest reputation level that takes risk under fundamental θ. Then $d^2\phi_H(\theta)/(\partial\theta^2) > 0$.

The first part of this proposition is just a result from global games. If $\theta < z^*(\phi)$, then when the signal noise goes to zero, almost all firms with reputation level ϕ receive a signal $z < z^*(\phi)$ and decide to take risks. Hence, reputation concerns are fragile in the

[16]This fragility is robust to continuous types and continuous actions as long as supports are bounded.

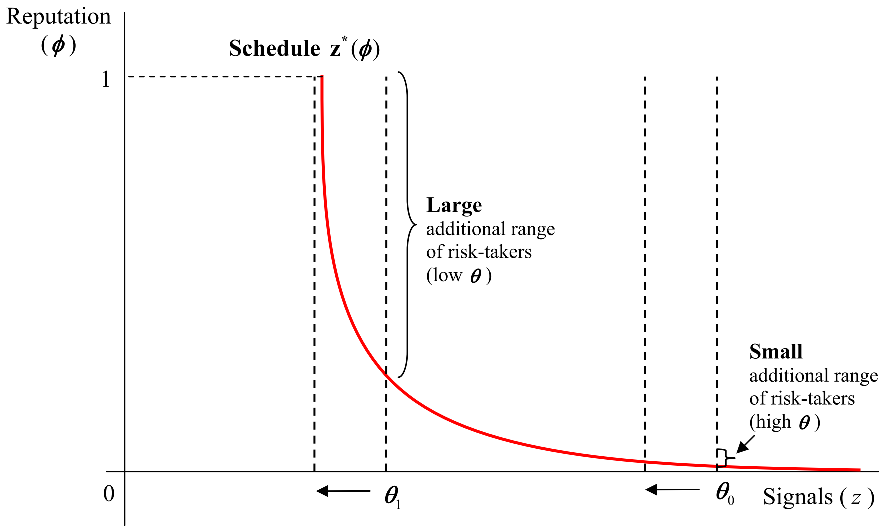

FIGURE 4. Individual and aggregate clustering of risk-taking.

sense that small changes in fundamentals around $z^*(\phi)$ induce sudden shifts in risk-taking for firms with reputation ϕ, from taking to not taking risks, depending on the direction of the change. Our equilibrium selection creates a clustering of risk-taking among firms with the same reputation level.

The second part of Proposition 7 is a corollary of Lemma 2 for a given distribution of reputation levels. When fundamentals are strong enough (high θ), small variations do not induce firms of different reputation to modify their risk-taking behavior. To the contrary, when fundamentals are weak enough (low θ), small variations may induce firms of different reputation to modify their risk-taking behavior under the same shock. Changes around weak fundamentals generate clustering of risk-taking among firms in a larger range of reputation levels. This fragility is generated at an aggregate level by learning primitives. Figure 4 illustrates these two sources of fragility, both at a firm and at an aggregate level.

Even though firms care about both the continuation and the formation of reputation, the first type of concerns determines that firms with good reputations are less likely to take risks (this effect is captured by the negative slope of cutoffs), while the second type of concerns leads to a convexification of cutoffs and then to aggregate fragility (this effect is captured by the curvature of cutoffs). In what follows, I compare situations with and without reputation concerns. In doing such comparisons, I compare situations where both continuation and formation are concerns versus situations where only continuation is a concern.

While reputation concerns reduce excessive risk-taking, these effects are fragile and their breakdown can lead to sudden and isolated clusters of risk-taking, large spikes in aggregate default, and large losses for lenders. Campbell et al. (2001) and Das et al. (2007) provide evidence that risk-taking and defaults cluster sometimes to an extent that cannot be explained just from weakening in fundamentals. Furthermore, Menkhoff et al. (2006) document that high reputation fund managers cluster more in their projects, while decisions of less reputable managers are less correlated, which is consistent with the second part of Proposition 7.

Before illustrating reputation fragility with a simulation exercise (which also shows how to solve the model numerically), it is important to highlight that clustering depends not only on the convex schedule of cutoffs, but also on the distribution of reputation in the market. In particular, a distribution with a large mass of intermediate and good reputation firms strengthens the results.[17] In the numerical exercise below, I derive the endogenous stationary distribution of reputation and show it is indeed skewed toward intermediate and good reputation levels, where cutoffs are similar, reinforcing the aggregate effects of clustering.

3.2 *A numerical exercise*

Here I present a numerical example that illustrates how a market in which firms care about their reputation can be fragile: aggregate default is insensitive to changes in fundamentals, being low and stable even when fundamentals fluctuate, but sometimes small changes in those fundamentals induce sudden risk-taking and a jump in aggregate default. I proceed in three steps. First, I describe the exercise. Then I show that the effects of reputation concerns in reducing risk-taking can be large and I discuss which parameter values can amplify these effects. Finally, I show the fragility implications of reputation concerns in the aggregate, also discussing which parameters affect clustering of risk-taking. The full discussion of this numerical exercise and additional results are given in the Appendix.

3.2.1 *The exercise* As in the model, payoffs are linear and risky projects are more tempting for low values of θ. Short-term cash flows are $\Pi_s = K + \theta$ and $\Pi_r = K$, where $\theta \sim \mathcal{N}(\mu, 1/\alpha)$. I assume $K = 1.5$, $\mu = -0.4$, and $\alpha = 25$, which means the return of the risky project is 50% in case of success. It also implies the average return of the safe project in case of success, both from pledgeable and nonpledgeable sources, is just 10% and covers a range, at 2 standard deviations of fundamentals, of $[-50\%, 70\%]$. Additionally, I assume the probability of continuation is $p_s = 0.9$ for safe projects, $p_r = 0.7$ for risky projects, a discount factor $\beta = 0.95$, and a risk-free interest rate of $\overline{R} = 1$. These parameters guarantee conditions for uniqueness and convergence, and are justified in the Appendix.

I compare the results of the basic model (with reputation concerns) with an artificial situation in which the firm's reputation cannot change (without reputation concerns). First, reputation induces a discipline that reduces inefficient risk-taking and can be large. Second, this discipline is fragile—subject to a sudden aggregate collapse without obvious changes in fundamentals. To make these points, I compute the limit of the schedule of cutoffs, with and without reputation concerns, for different reputation levels, as $T \to \infty$ and $\sigma \to 0$. Then I simulate realizations of fundamentals for 100 periods, and I aggregate risk-taking and default of the firms that follow those cutoffs. The computational procedure is described in the Appendix.

[17]This reputational distribution is independent of cutoffs $z^*(\phi)$, and depends purely on entry assumptions, such as in Atkeson et al. (2012). The theoretical derivation of an endogenous distribution of reputation is feasible and interesting, but beyond the scope of this paper.

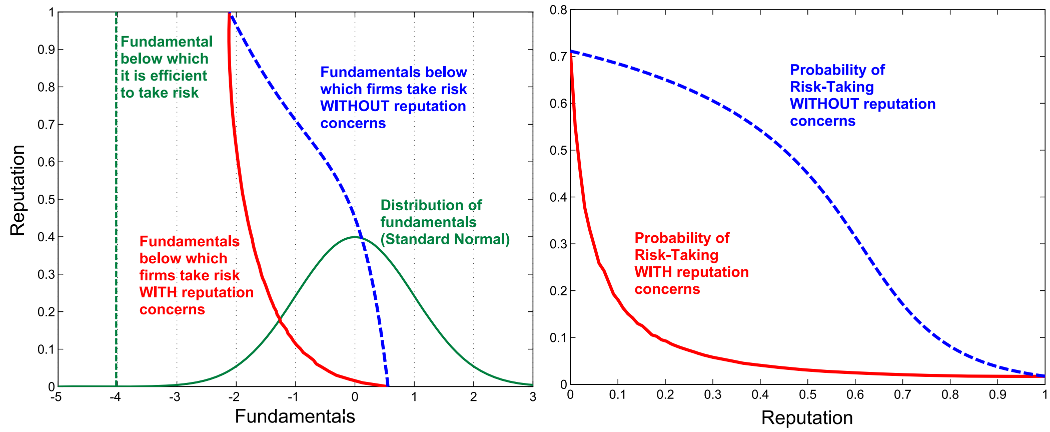

FIGURE 5. Reputation concerns reduce the ex ante probability of risk-taking.

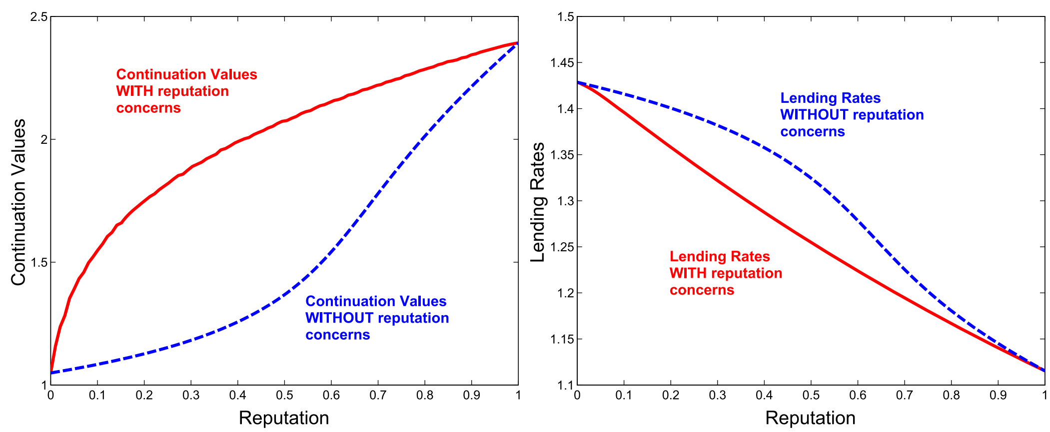

FIGURE 6. Reputation concerns increase continuation values and reduce lending rates.

3.2.2 *Discipline from reputation* Figure 5 shows, for each reputation level ϕ, cutoffs with and without reputation concerns ($z^*(\phi)$ and $\widetilde{z}^*(\phi)$, respectively) and ex ante probabilities of risk-taking ($\mathcal{N}(z^*(\phi))$ and $\mathcal{N}(\widetilde{z}^*(\phi))$) that apply to any period when $T \to \infty$. As stated in Proposition 5, the ex ante probability of risk-taking decreases with reputation. Furthermore, as stated in Proposition 6, for all reputation levels, the probability that firms take risks is lower with reputation concerns. For example, the ex ante probability that a firm with a reputation level $\phi = 0.4$ takes risks is only 4% with reputation concerns but 50% without reputation concerns. By construction, risk-taking is almost never efficient, so the gap between the two curves in the second plot of Figure 5 shows the power of reputation in reducing the ex ante probability of inefficient risk-taking by firms of different reputation levels.

Figure 6 shows, as stated in Proposition 5, that continuation values increase and lending rates decrease with reputation, which introduce the discipline for not taking risks and improving reputation. Furthermore, firms have higher expected continuation values and pay lower interest rates for all reputation levels when reputation is a concern.

Even though the parameters are not calibrated, default rates and lending rates are not unrealistic. Depending on the definition of fundamentals, this exercise shows that reputation concerns can have large effects in reducing risk-taking. More importantly, the exercise is informative about which parameters are most likely to be critical for the quantitative relevance of the mechanism. In particular, a larger ratio p_s/p_r, a larger difference $p_s - p_r$, a larger β, and a lower $\bar{R}$ make reputation concerns more relevant in imposing discipline.

First, as shown in Lemma 2, the ratio p_s/p_r is critical for the convexifying forces of reputation formation. When this ratio is high, the informativeness of continuation about the likelihood a firm has chosen a safe project, and then is strategic, is also high. This increases the incentives to chose safe actions, widening the gap between the cutoffs with reputation concerns (solid curve in the first panel of Figure 5) and the cutoffs without reputation concerns (dotted curve) for all reputation levels, except for the extremes $\phi = 0$ and $\phi = 1$.

Second, the continuation values are the main source of discipline. These values are endogenous and determine both the slope and the curvature of the schedule of cutoffs in the first panel of Figure 5. This schedule is flatter when continuation values are more sensitive to reputation levels. This occurs when interest rates are more sensitive to reputation, which critically depends on the difference between p_s and p_r: interest rates are low for firms with good reputations when p_s is high and are high for firms with bad reputations when p_r is low.

Finally, cutoffs are lower (all cutoffs in the first panel of Figure 5 move to the left) when continuation values are larger or more important for all reputation levels. On the one hand, when β is high, firms assign more weight to the future, and reputation incentives become more important, both because continuation values are large and firms care more about those large continuation values. On the other hand, when risk-free rates $\bar{R}$ are low, interest rates are also low, which reduces the short-term incentives to take excessive risks—less moral hazard—and reduces future interest rates, increasing continuation values.

3.2.3 *Fragility of reputation and clustering of risk-taking* Next, I use the simulation to assess the behavior of aggregate variables in the model. In particular, I simulate fundamental realizations for 100 periods and compute aggregate risk-taking and default in the model. Over the simulated time frame, risk-taking is never efficient (no normalized fundamental realization is below -4 in any period). Reputation concerns that convexify the schedule of cutoffs make aggregate default stable and insensitive to movements in fundamentals most of the time. However, sometimes these concerns collapse, inducing a wave of risk-taking and defaults, without obvious observable changes in those fundamentals.

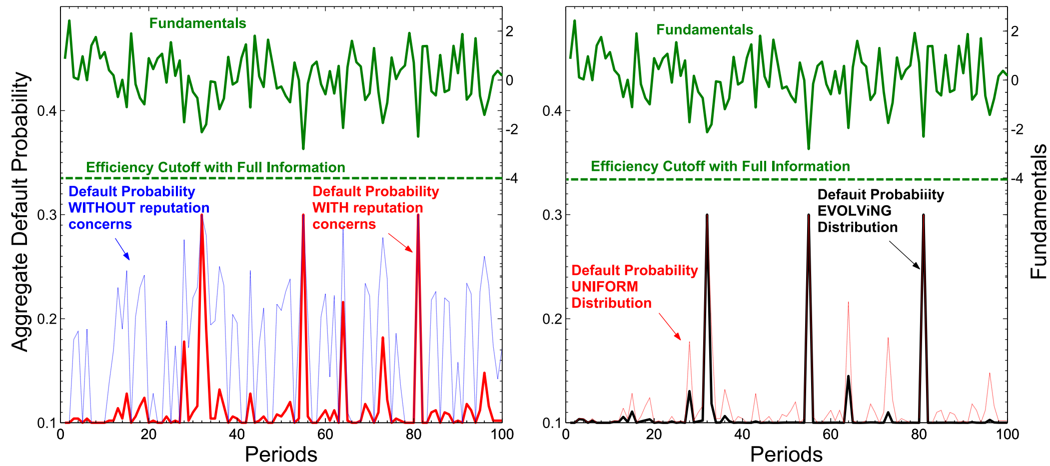

FIGURE 7. Simulated default probability with fixed and evolving reputation distribution.

The first panel of Figure 7 shows aggregate default during the 100 periods for a fixed uniform reputation distribution.[18] Without reputation concerns, aggregate default closely follows changes in fundamentals. With reputation concerns, aggregate default is less sensitive to changes in fundamentals. Aggregate default is low, in general, and increases only when conditions weaken enough. An economy where firms are concerned about their reputation is observationally equivalent to one where default is low and insensitive to movements in fundamentals most of the time, but under some conditions (in the example when normalized fundamentals go below values around -2), aggregate default rates experience sudden jumps without noticeable changes in fundamentals.

In the second panel of Figure 7, I allow the reputation distribution to evolve over time, starting from a uniform distribution in the initial period. I assume that entrants replace failing firms with an exogenous prior reputation $\phi_0 = 0.5$. As this distribution evolves, it biases toward good reputations, which makes aggregate default even less sensitive to movements in fundamentals: aggregate default fluctuates less most of the time, given the same movements in fundamentals. This can be seen by comparing the light dotted line with the darker solid line. Still, when fundamentals drop low enough, unnoticeable shocks trigger a sudden clustering of risk-taking and aggregate default. Intuitively, the market becomes more populated by firms with intermediate and good reputations, whose behavior reacts similarly to fundamental fluctuations. The evolution of the reputational distribution is shown in the Appendix.

Again, this exercise provides hints about the main determinants of clustering in risk-taking. First, a reputational distribution that is more biased toward intermediate and large reputation levels makes aggregate default low and less sensitive to changes in fundamentals. As discussed above, this is captured in the second panel of Figure 7.

[18] I am aggregating only strategic firms, whose behavior changes with fundamentals. If all of them choose safe projects, aggregate default is 10% (since $p_s = 0.9$). If all of them take risks, aggregate default is 30% (since $p_r = 0.7$). Risky firms always have a default probability of 30%.

What determines the bias of the reputational distribution toward high reputation levels? Two parameters are critical. On the one hand, it is the fraction of strategic firms that enter the market to replace dying firms, ϕ_0, which pins down the initial reputation of entrants. Since in expectation the reputation of strategic firms increases, the distribution will be more biased toward high reputation levels if ϕ_0 is large.[19] On the other hand, it is the rate at which risky firms leave the market in relation to strategic firms. Risky firms leave the market at a rate p_r, while strategic firms leave the market at a lower rate, which is a convex combination between p_r and p_s that depends on fundamentals (in the numerical example this rate fluctuates over time and is captured by the aggregate default probability in the second panel of Figure 7). Then, when shocks that induce strategic firms to choose risky projects are not very likely, risky firms disappear faster in relation to strategic firms, biasing the reputation distribution toward high reputation levels at a faster rate.

Finally, what is the effect of the variance of the fundamental distribution? Naturally, this critically depends on the assumptions about the shape of the distribution. In this exercise with a normal distribution, a lower variance does not change the cutoffs in the first panel of Figure 5, but does reduce the ex ante probability of risk-taking for all reputations, increasing the gap between the two lines in the second panel of Figure 5. Lower ex ante probabilities of risk-taking generate lower interest rates and higher continuation values for every reputation level in Figure 6. This affects feedback into reducing cutoffs, reinforcing the previous effects.

Conditional on guaranteeing the conditions for uniqueness and convergence, a lower variance of fundamentals induces more reputational discipline, and a lower sensitiveness of risk-taking and defaults to fundamentals, which translates into more fragility when fundamentals weaken enough, with sporadic large changes in response to unnoticeable shocks.

4. Summary and implications

Firms' concerns about their reputation reduce excessive risk-taking. This positive effect of reputation is widely accepted on both formal and informal grounds. Here I have studied the effects of reputation concerns from an aggregate perspective, when incentives to take risks vary with both aggregate conditions and the actions of other firms. My main finding is that reputation concerns may have negative as well as positive aggregate effects. These concerns are, in fact, fragile and may suddenly disappear, leading to large changes in aggregate risk- taking as well known and reputable firms shift their behavior in response to small and not obvious changes in fundamentals.

In my model, reputations can eventually be constructed, destroyed, and managed. However, this desirable feature comes with a cost in terms of equilibria multiplicity. To overcome this problem, I have interpreted the reputation model extended with fundamentals as a nonstandard dynamic global game in which strategic complementarities

[19]This initial reputation is endogenized by Atkeson et al. (2012).

arise endogenously from reputation formation. This allows me to select a unique equilibrium that is robust to perturbations in information about fundamentals, which become a coordination device for risk-taking.

Since this paper provides a tractable model of the collapse of reputation incentives, a natural extension for future research is to study more specifically the role of reputation in magnifying the recent financial crisis. First, it is widely agreed that financial institutions, even those with high reputation, took excessive risk before the crisis, exactly as the housing bubble was expanding and risky mortgages became tempting.

Second, there is a widespread view that deregulation of financial markets during the years leading to the financial crisis was responsible for excessive risk-taking. As documented by Keeley (1990), the main changes in regulation since the 1980's led to a decline in bank charter values through reductions in entry restrictions, easier conditions for competition, and interest rate ceilings. This is also consistent with my model when I interpret fundamentals as the level of competition faced by financial institutions, which reduces profits, the long-term gains from maintaining reputation, and then discipline.

Finally, Plantin (2009) shows that liquidity spreads respond to default risk highly nonlinearly in the presence of learning by holding securities, providing a new interpretation for the magnitude and momentum of the recent subprime crisis. This paper also highlights the importance of coordination failures in explaining sudden financial breaks down. Unlike Plantin, my work focuses on the role of reputation in generating such coordination failures and can then be useful to study avenues and conditions under which aggregate confidence based on reputation can flourish and collapse over time.

Even though these extensions are purely suggestive, further research on the interactions of aggregate fundamentals and short-term payoffs can shed light on the specific role of reputation in magnifying crises such as the recent one. For example, versions of this model can be used to understand "shadow banking," a system sustained by confidence and reputation incentives (Gorton and Metrick 2009, Gorton 2010). Indeed, this model was recently applied by Ordoñez (2012) and Chari et al. (2012) to study special purpose vehicles and secondary loan markets, respectively.

Appendix

A.1 *Proof of Proposition 1*

First we prove two lemmas that describe single crossing properties to identify a unique cutoff in the set of fundamentals (θ) and in the set of beliefs ($\widehat{x}$).

Lemma 3 (Fundamental single crossing). *For every reputation level $\phi \in (0,1)$, fix a $\widehat{x} \in [0,1]$ for all θ. There exists a unique $\theta^* \in [\underline{\theta}(\phi), \overline{\theta}(\phi)]$ such that $\Delta(\phi, \theta|\widehat{x}) < 0$ for $\theta < \theta^*$, $\Delta(\phi, \theta|\widehat{x}) = 0$ for $\theta = \theta^*$, and $\Delta(\phi, \theta|\widehat{x}) > 0$ for $\theta > \theta^*$. Furthermore, θ^* is increasing in $\widehat{x}$. For $\phi = \{0,1\}$, $\theta^* = \underline{\theta}(\phi) = \overline{\theta}(\phi)$ for any $\widehat{x}$.*

Proof. For $\phi \in (0,1)$, by Assumption 2 and since $\partial\Delta(\phi, \theta|\widehat{x})/(\partial\theta) > 0$, there is a unique θ^* such that $\Delta(\phi, \theta^*|\widehat{x}) = 0$. Since $\widehat{x} \in [0,1]$, $\theta^* \in [\underline{\theta}(\phi), \overline{\theta}(\phi)]$. Since $\partial\Delta(\phi, \theta|\widehat{x})/(\partial\widehat{x}) \leq 0$, θ^* is increasing in $\widehat{x}$ (this is so if $\widehat{x}' > \widehat{x}$, $\Delta(\phi, \theta^{*'}|\widehat{x}') = 0$ at $\theta^{*'} > \theta^*$). Finally, for $\phi \in \{0,1\}$, $\widehat{x}$ is irrelevant for updating. Then $\theta^* = \underline{\theta}(\phi) = \overline{\theta}(\phi)$ for any $\widehat{x}$. □

Lemma 4 (Belief single crossing). *For every reputation level* $\phi \in (0,1)$*, fix a* $\theta \in [\underline{\theta}(\phi), \overline{\theta}(\phi)]$*. There exists a unique* $\widehat{x}^* \in [0,1]$ *such that* $\Delta(\phi, \theta|\widehat{x}) > 0$ *for* $\widehat{x} < \widehat{x}^*$*,* $\Delta(\phi, \theta|\widehat{x}) = 0$ *for* $\widehat{x} = \widehat{x}^*$*, and* $\Delta(\phi, \theta|\widehat{x}) < 0$ *for* $\widehat{x} > \widehat{x}^*$*. Furthermore,* $\widehat{x}^*$ *is increasing in* θ*. For* $\phi = \{0,1\}$*, any* $\widehat{x} \in [0,1]$ *delivers* $\Delta(\phi, \theta|\widehat{x}) = 0$*.*

Proof. Fix a $\theta \in [\underline{\theta}(\phi), \overline{\theta}(\phi)]$. For $\phi \in (0,1)$, $\partial\Delta(\phi, \theta|\widehat{x})/(\partial\widehat{x}) < 0$, there is a unique $\widehat{x}^*$ such that $\Delta(\phi, \theta|\widehat{x}^*) = 0$. Since $\partial\Delta(\phi, \theta|\widehat{x})/(\partial\theta) > 0$, $\widehat{x}^*$ is increasing in θ (this is so if $\theta' > \theta$, $\Delta(\phi, \theta'|\widehat{x}') = 0$ at $\widehat{x}' > \widehat{x}$). Finally, for $\phi \in \{0,1\}$, $\widehat{x}$ is irrelevant for updating; hence it delivers a $\Delta(\phi, \theta|\widehat{x}) = 0$ for $\theta = \underline{\theta}(\phi) = \overline{\theta}(\phi)$. □

The proposition follows directly from Lemmas 3 and 4. A cutoff $k(\phi)$ is an equilibrium strategy only if it is a best response for any realization of the fundamental θ. Take a cutoff $k(\phi) = \theta^*(\phi)$ such that $\theta^*(\phi) \in (\underline{\theta}(\phi), \overline{\theta}(\phi))$. Such a $\theta^*(\phi)$ is guaranteed by Assumption 2. From cutoff strategies, we know that $x(\phi, \theta) = 0$ for all $\theta > \theta^*(\phi)$ and $x(\phi, \theta) = 1$ for all $\theta < \theta^*(\phi)$. From Lemma 4, at $\theta^*(\phi)$, indifference occurs at some $0 \le x^*(\phi, \theta^*) \le 1$.

The cutoff $\theta^*(\phi)$ is an equilibrium because, for all $\theta > \theta^*(\phi)$, $\Delta(\phi, \theta|\theta^*) > 0$, and hence it is optimal for the firm to choose safe actions (i.e., $x(\phi, \theta) = 0$). Similarly, for all $\theta < \theta^*(\phi)$, $\Delta(\phi, \theta|\theta^*) < 0$, and hence it is optimal for the firm to take risks (i.e., $x(\phi, \theta) = 1$). Finally, since the conditions for equilibrium are both $\Delta(\phi, \theta^*|\widehat{x} = 0) > 0$ and $\Delta(\phi, \theta^*|\widehat{x} = 1) < 0$, an arbitrarily close fundamental $\theta^* + \varepsilon$ (with $\varepsilon \to 0$) is also an equilibrium cutoff.

A.2 *Proof of Proposition 3*

Before proving the result, I formally define the equilibrium in a given period, with endogenous interest rates and for any arbitrary set of expected continuation values. Since we are analyzing the repeated game, I explicitly denote variables for each period with a subscript t. To save notation, I just refer to generic ϕ and ϕ', since the proposition applies to each $\phi \in [0,1]$ at each period t.

Definition 2. A *single period* Markov perfect equilibrium in cutoff strategies consists of a (symmetric) strategy for the firms $k_t(\phi) = \theta_t^*(\phi) : [0,1] \to \mathbb{R}$, interest rates $R_t(\phi|k_t)$, and posteriors $\phi'(\phi, \widehat{x}_t) : [0,1] \times [0,1] \to [0,1]$, for each $\phi \in [0,1]$, such that, for a given set of expected continuation values $\Upsilon' = \{\mathbf{V}(\phi'\}_{\phi'=0}^1$, the following statements hold.

- The equality $k_t(\phi) = \theta_t^*(\phi)$ defines the $x_t^*(\phi, \theta_t) \in \arg\max_{x_t \in [0,1]} \widetilde{V}(\phi, \theta_t, x_t|k_t, \widehat{x}_t)$ for all θ_t.
- Lenders charge $R_t(\phi|k_t)$ to obtain the risk-free rate $\overline{R}$ in expectation, where $k_t(\phi) = \theta_t^*(\phi)$.
- The term $\phi'(\phi, \widehat{x}_t)$ is obtained using Bayes' rule (1), where $\widehat{x}_t(\phi, \theta_t) = x_t^*(\phi, \theta_t)$ for all θ_t is the updating rule that lenders must use if their beliefs are to be correct (that is, consistent with equilibrium strategies of a continuum of firms ϕ_t, which determine aggregate default $D_t(\phi, x_t^*)$).

The equilibrium in a game in which firms live for a finite time T and $\mathbf{V}_{T+1}(\phi)=0$, for all ϕ, is defined as follows.

Definition 3. A *finite-horizon* Markov perfect equilibrium in cutoff strategies consists of a (symmetric) strategy for the firms $\{k_t(\phi)=\theta_t^*(\phi)\}_{t=0}^T$, interest rates $\{R_t(\phi|k_t)\}_{t=0}^T$, posteriors $\phi'(\phi,\widehat{x}_t):[0,1]\times[0,1]\to[0,1]$, and firms' expected continuation values $\{\mathbf{V}_t(\phi)\}_{t=0}^T$, for each ϕ, such that the folowing statements hold.

- A single period Markov perfect equilibrium exists in each $t\in\{0,1,\ldots,T\}$.
- The equality

$$\mathbf{V}_t(\phi)=\int_{-\infty}^{\theta_t^*(\phi)}\widetilde{V}_t(\phi,\theta_t,x_t=1|\theta_t^*,\widehat{x}_t=1)\,d\mathcal{N}(\theta_t)+\int_{\theta_t^*(\phi)}^{\infty}\widetilde{V}_t(\phi,\theta_t,x_t=0|\theta_t^*,\widehat{x}_t=0)\,d\mathcal{N}(\theta_t)$$

  exists.
- At each period t, $\theta_t^*(\phi)$ is determined by the corresponding $\mathbf{V}_{t+1}(\phi')$.

This definition is based on perfect information about the fundamental θ. As in the simple model, in the case of firms that observe a signal about fundamentals, cutoffs are defined for each firm over signals $k_t(z_t^*(\phi))$ and continuation values are computed using expectations over signals for each potential fundamental θ. In what follows, I prove the uniqueness result with highly precise signals ($\sigma\to0$) and show how continuation values are constructed.

Proof of Proposition 3. In a first part, I prove uniqueness in a single period t for arbitrary distributions and a fixed set of continuation values. In particular, I assume $z_i=\theta+\sigma\epsilon_i$, where $\epsilon_i\sim F$, with density f, mean 0, c.d.f. F increasing over a connected set, and a monotone likelihood ratio property, that is, for all $a>b$, $(f(a-\theta))/(f(b-\theta))$ is increasing in θ. Note that conditional on θ, signals $z_i\sim F((z_i-\theta)/\sigma)$. Fundamentals are distributed with mean μ, a density $v(\theta)$, and c.d.f. $\mathcal{V}(\theta)$ strictly increasing over a connected set that includes the dominance regions.

For this part, to reduce notation, I do not introduce explicitly the time subscript t. I proceed in four steps. First, I derive the posterior density and distribution of θ given a signal z. Second, I prove that there is a unique signal $z^*(\phi)$ that makes a strategic firm ϕ indifferent in expectation between taking risk or not. Third, I show that, for any σ, using $z^*(\phi)$ is a best response when the prior about θ follows a uniform distribution on the real line and lenders believe $z^*(\phi)$ is the equilibrium cutoff. Finally, I show that, as $\sigma\to0$, the best response in a game with any prior distribution of θ uniformly converges to following the unique cutoff $z^*(\phi)$ when other firms follow the same cutoff $z^*(\phi)$.

In a second part, I prove uniqueness in the fully dynamic game, showing that value functions are recursively well defined and that the dynamic game can be solved as a series of static games. In this part, I denote explicitly time periods t.

• *First part of the proposition: Uniqueness in a single period for arbitrary distributions*

Step 1: Distributions of fundamentals conditional on signals

LEMMA 5. *The posterior density $f_{\theta|z}$ and distribution $F_{\theta|z}$ of θ given a signal z are given by*

$$f_{\theta|z}(\eta|z) = \frac{v(\eta)f((z-\eta)/\sigma)}{\int_{-\infty}^{\infty} v(\theta)f((z-\theta)/\sigma)\,d\theta} \tag{10}$$

$$F_{\theta|z}(\eta|z) = \frac{\int_{-\infty}^{\eta} v(\theta)f((z-\theta)/\sigma)\,d\theta}{\int_{-\infty}^{\infty} v(\theta)f((z-\theta)/\sigma)\,d\theta} = \frac{\int_{(z-\eta)/\sigma}^{\infty} v(z-\sigma u)f(u)\,du}{\int_{-\infty}^{\infty} v(z-\sigma u)f(u)\,du}. \tag{11}$$

PROOF. By Bayes' rule,

$$f_{\theta|z}(\theta|z) = \frac{v(\theta)f_{z|\theta}(z|\theta)}{f_z(z)}, \tag{12}$$

where f_z and $f_{z|\theta}$ are the densities of z and $z|\theta$, respectively. Since z is the sum of θ and $\sigma\epsilon$, its density is given by the convolution of their densities, i.e., v and $f_{\sigma\epsilon}$. Considering that $F_{\sigma\epsilon}(\eta) = F(\eta/\sigma)$, $f_{\sigma\epsilon}(\eta) = f(\eta/\sigma)/\sigma$, then f_z can be defined as

$$f_z(z) = \sigma^{-1}\int_{-\infty}^{\infty} v(\theta)f\left(\frac{z-\theta}{\sigma}\right)d\theta. \tag{13}$$

We can obtain the distribution of the observed signal z after observing a fundamental θ:

$$F_{z|\theta}(\eta|\theta) = \Pr(z \le \eta|\theta) = F\left(\frac{\eta-\theta}{\sigma}\right)$$

$$f_{z|\theta}(\eta|\theta) = \frac{dF_{z|\theta}(\eta|\theta)}{dz} = \sigma^{-1}f\left(\frac{\eta-\theta}{\sigma}\right).$$

Plugging (14) and (13) into (12), we obtain (10). The posterior distribution is obtained by integrating over the density,

$$F_{\theta|z}(\eta|z) = \int_{-\infty}^{\eta} f_{\theta|z}(\theta|z)\,d\theta = \frac{\int_{-\infty}^{\eta} v(\theta)f((z-\theta)/\sigma)\,d\theta}{\int_{-\infty}^{\infty} v(\theta)f((z-\theta)/\sigma)\,d\theta},$$

and the expression in (11) follows from variable transformation $u = (z-\theta)/\sigma$. ◁

Step 2: Unique equilibrium cutoff $z^*(\phi)$.

LEMMA 6. *There is a unique cutoff signal for each reputation ϕ such that $\Delta(\phi, z|z^*) = 0$ for $z = z^*(\phi)$, $\Delta(\phi, z|z^*) > 0$ for $z > z^*(\phi)$, and $\Delta(\phi, z|z^*) < 0$ for $z < z^*(\phi)$, where $\Delta(\phi, z|z^*)$ is defined by (3) when $k_z(\phi) = z^*(\phi)$, and Δ represents the expected differential gains from choosing safe actions for a firm ϕ that observes z when other firms follow a cutoff $z^*(\phi)$.*

This cutoff $z^*(\phi)$ is obtained using Laplacian beliefs over the probability that the firm plays risky when the fundamental is θ, where $\widehat{x} = F((z^* - \theta)/\sigma)$, such that

$$\Delta(\phi, z^*|z^*) = \int_0^1 [\Delta(\phi, \theta(\widehat{x})|\widehat{x})|z^*]\, d\widehat{x} = 0. \tag{14}$$

Proof. When fundamentals θ are not observed directly, differential gains Δ turn into *expected* differential gains conditional on the signal. When the firm observes a signal z and believes other firms ϕ use a cutoff $k_z(\phi)$, expected gains from choosing safe actions are as defined in (3):

$$\Delta(\phi, z|k_z) = E_{\theta|z}(\Delta(\phi, \theta|\widehat{x})|k_z).$$

Introducing noise in the observation of fundamentals pins down the expected fraction of strategic risk-takers $\widehat{x}$ as a function of cutoff beliefs $k_z(\phi)$ and observed fundamentals θ:

$$\widehat{x} = F\left(\frac{k_z(\phi) - \theta}{\sigma}\right). \tag{15}$$

Developing the expectation from (3), explicitly denoting that $\widehat{x}$ is a function of θ, gives

$$\Delta(\phi, z|k_z) = \int_{-\infty}^{\infty} [\Delta(\phi, \theta|\widehat{x}(\theta))|k_z]\, dF_{\theta|z}(\theta|z).$$

Note that $\theta = k_z(\phi) - \sigma F^{-1}(\widehat{x})$. From (11), define

$$\Psi(\widehat{x}|z, k_z) = F_{\theta|z}(k_z - \sigma F^{-1}(\widehat{x})|z, k_z) = \frac{\int_{(z-k_z)/\sigma + F^{-1}(\widehat{x})}^{\infty} v(z - \sigma u) f(u)\, du}{\int_{-\infty}^{\infty} v(z - \sigma u) f(u)\, du}.$$

Changing variables from θ to $\widehat{x}$ yields

$$\Delta(\phi, z|k_z) = \int_0^1 [\Delta(\phi, \theta(\widehat{x})|\widehat{x})|k_z]\, d\Psi(\widehat{x}|z, k_z).$$

Laplacian beliefs arise from

$$\Psi(\widehat{x}|z, k_z) = \Pr(\theta < k_z - \sigma F^{-1}(\widehat{x})|z) = F\left[\frac{z - k_z}{\sigma} + F^{-1}(\widehat{x})\right].$$

In equilibrium, $k_z(\phi) = z^*(\phi)$. Evaluating the expectation at $z = z^*(\phi)$, $\Psi(\widehat{x}|z^*, z^*) = \widehat{x}$ gives

$$\Delta(\phi, z^*|z^*) = \int_0^1 [\Delta(\phi, \theta(\widehat{x})|\widehat{x})|z^*]\, d\widehat{x} = 0.$$

By Lemmas 3 and 4, we know there is a unique solution $z^*(\phi)$ to this equation. ◁

Step 3: Best response with uniform priors over fundamentals

Now we need to verify that a firm ϕ playing risky if $z < z^*(\phi)$ and safe if $z > z^*(\phi)$ indeed constitutes an equilibrium. Signals z allow firms to have an idea not only about the fundamental, but also about the signals other firms have observed. Following Toxvaerd (2008), I first assume θ is drawn from a uniform distribution on the real line, hence is an improper distribution with infinite probability mass. This assumption allows us to normalize the prior distribution assuming $v(\theta) = 1$, simplifying the density to $f_{\theta|z}(\theta|z) = \sigma^{-1} f((z-\theta)/\sigma)$ and the distribution to $F_{\theta|z}(\theta|z) = F((z-\theta)/\sigma)$. I denote by $\widetilde{\Delta}(\phi, z|k_z)$ the expected differential gains from safe projects for the special case in which the prior of fundamentals is uniform,

$$\widetilde{\Delta}(\phi, z|k_z) = \int_{-\infty}^{\infty} \left[\Delta\left(\phi, \theta \middle| F\left(\frac{k_z - \theta}{\sigma}\right)\right) \middle| k_z\right] \sigma^{-1} f\left(\frac{z-\theta}{\sigma}\right) d\theta.$$

Changing variables that introduce $m = (\theta - k_z(\phi))/\sigma$,

$$\widetilde{\Delta}(\phi, z|k_z) = \int_{-\infty}^{\infty} [\Delta(\phi, \theta|F(-m))|k_z] \sigma^{-1} f\left(\frac{z-k_z}{\sigma} - m\right) d\theta.$$

We can rewrite this more conveniently by defining $\widehat{\Delta}$,

$$\widetilde{\Delta}(\phi, z|k_z) = \widehat{\Delta}(\phi, z, z'|k_z) = \int_{-\infty}^{\infty} B(z', m|k_z) D(z, m|k_z)\, dm,$$

and renaming θ as z', to write the expressions in terms of m, where $B(z', m|k_z) = \Delta(\phi, z', F(-m)|k_z)$ and $D(z, m|k_z) = \sigma^{-1} f((z-k_z)/\sigma - m)$. As shown in Athey (2002), because of the monotone likelihood property, $\widehat{\Delta}(\phi, z, z'|k_z)$ inherits the single crossing property of $\Delta(\phi, \theta|k_z)$. This means there exists a $z^*(\phi, k_z, z')$ such that $\widehat{\Delta}(\phi, z, z'|k_z) > 0$ if $z > z^*(\phi, k_z, z')$ and $\widehat{\Delta}(\phi, z, z'|k_z) < 0$ if $z < z^*(\phi, k_z, z')$. Assuming $z < z'$ and $\widehat{\Delta}(\phi, z, z|k_z) = 0$,

$$\widehat{\Delta}(\phi, z', z'|k_z) \geq \widehat{\Delta}(\phi, z, z'|k_z) \geq \widehat{\Delta}(\phi, z, z|k_z) = 0 \quad (\text{strictly } > \text{ for } \phi \in (0,1)).$$

The first inequality comes from the state monotonicity and the second comes from the single crossing property. A symmetric argument holds for $z > z'$. Hence, there exists a best response $\chi : \mathbb{R} \to \mathbb{R}$ such that

$$\begin{aligned}
\widetilde{\Delta}(\phi, z|k_z) > 0 &\quad \text{if } z > \chi(k_z)\\
\widetilde{\Delta}(\phi, z|k_z) = 0 &\quad \text{if } z = \chi(k_z)\\
\widetilde{\Delta}(\phi, z|k_z) < 0 &\quad \text{if } z < \chi(k_z).
\end{aligned}$$

There exists a unique $z^*(\phi)$ that solves

$$\widetilde{\Delta}(\phi, z^*|z^*) = \int_0^1 \widetilde{\Delta}(\phi, \theta(\widehat{x}), \widehat{x}|z^*)\, d\widehat{x} = 0.$$

Hence, $\chi(z^*(\phi)) = z^*(\phi)$, showing that there is a unique equilibrium in cutoff strategies for each ϕ such that

$$x^*(\phi, z) = \begin{cases} 0 & \text{if } z > z^*(\phi) \\ 1 & \text{if } z < z^*(\phi). \end{cases} \qquad \triangleleft$$

Step 4: Best response with general priors over fundamentals

Lemma 7. *The term $\Delta(\phi, z|k_z) \to \widetilde{\Delta}(\phi, z|k_z)$ uniformly, when $k_z(\phi) = z - \sigma\xi$, as $\sigma \to 0$.*

Proof. First, $\Delta(\phi, z|z - \sigma\xi) \to \widetilde{\Delta}(\phi, z|z - \sigma\xi)$ continuously as $\sigma \to 0$, that is,

$$\Psi(\widehat{x}|z, z - \sigma\xi) = \frac{\int_{\xi + F^{-1}(\widehat{x})}^{\infty} v(z - \sigma u) f(u)\, du}{\int_{-\infty}^{\infty} v(z - \sigma u) f(u)\, du} \to 1 - F(\xi + F^{-1}(\widehat{x})) \equiv \widetilde{\Psi}(\widehat{x}|z, z - \sigma\xi).$$

As in Toxvaerd (2008), I show convergence with respect to the uniform convergence norm, which implies uniform convergence. Uniformity ensures that the equivalence between the games with the two different assumptions about the prior distributions is not the result of a discontinuity at $\sigma = 0$.

Pick $\underline{z}(\phi) < \underline{\theta}(\phi)$ and $\overline{z}(\phi) > \overline{\theta}(\phi)$, and restrict attention to the compact sets $Z \equiv [\underline{z}(\phi), \overline{z}(\phi)]$ and $Z_\sigma \equiv [\underline{z}(\phi) - \sigma\xi, \overline{z}(\phi) + \sigma\xi]$. Hence, $\Delta(\phi, z|k_z)$ maps into a compact set.

Define the uniform convergence norm as

$$\|\Delta(\phi)\| \equiv \sup_{z, k_z}\{|\Delta(\phi, z|k_z)|\}.$$

We can show continuity with respect to the Euclidean metric. Fix z' and $k'_z(\phi)$ such that

$$\begin{aligned} \forall \epsilon_1 > 0, \exists \delta_1 |z - z'| < \delta_1 \quad &\Rightarrow \quad |\Delta(\phi, z|k_z) - \widetilde{\Delta}(\phi, z'|k_z)| < \epsilon_1, \forall k_z \\ \forall \epsilon_2 > 0, \exists \delta_2 |k_z - k'_z| < \delta_2 \quad &\Rightarrow \quad |\Delta(\phi, z|k_z) - \widetilde{\Delta}(\phi, z|k'_z)| < \epsilon_2, \forall z. \end{aligned}$$

This implies

$$\sqrt{(z - z')^2 + (k_z - k'_z)^2} < \sqrt{\delta_1^2 + \delta_2^2}.$$

By the triangle inequality,

$$\begin{aligned} |\Delta(\phi, z|k_z) - \Delta(\phi, z'|k'_z)| &= |\Delta(\phi, z|k_z) - \Delta(\phi, z'|k_z) + \Delta(\phi, z'|k_z) - \Delta(\phi, z'|k'_z)| \\ &\leq |\Delta(\phi, z|k_z) - \Delta(\phi, z'|k'_z)| + |\Delta(\phi, z'|k_z) - \Delta(\phi, z'|k'_z)| \\ &\leq \epsilon_1 + \epsilon_2. \end{aligned}$$

Hence, $\Delta(\phi, z|k_z)$ belongs to the space of continuous functions on $Z \times \widehat{Z}$.

Uniform convergence is equivalent to

$$\|\Delta(\phi) - \widetilde{\Delta}(\phi)\| = \sup_{z, k_z}\{\Delta(\phi, z|k_z) - \widetilde{\Delta}(\phi, z|k_z)\} \to 0$$

with respect to the uniform convergence norm, as $\sigma \to 0$, after substituting for the functions and taking limits. ◁

• *Second part of the proposition: Uniqueness in the fully dynamic game*

In the last period T, the cutoff $z_T^*(\phi)$ is unique (under the condition in (7)) since $\Delta_T(\phi, z_T|k_{z,T})$ is well defined and $\mathbf{V}_{T+1}(\phi) = 0$ for all ϕ; thus here reputation concerns do not generate multiplicity. Once $z_T^*(\phi)$ is determined, the equilibrium interest rate at T for each ϕ is

$$R_T(\phi|z_T^*) = \frac{\overline{R}}{\Pr(c)_T} = \frac{\overline{R}}{(1-\phi)p_r + \phi[p_r\mathcal{V}(z_T^*) + p_s(1-\mathcal{V}(z_T^*))]}.$$

Then we can define expected continuation values in T for each reputation level ϕ. For signals $z_T < z_T^*(\phi)$, firms take risks, and for signals $z_T > z_T^*(\phi)$, firms choose safe actions. As $\sigma \to 0$, errors coming from misscoordination go to zero and, in the limit, expected profits in the last period T are independent of σ:

$$\mathbf{V}_T(\phi) = \int_{-\infty}^{z_T^*(\phi)} p_r[K - R_T(\phi|z_T^*)]v(\theta_T)\,d\theta_T + \int_{z_T^*(\phi)}^{\infty} p_s[K + \theta_T - R_T(\phi|z_T^*)]v(\theta_T)\,d\theta_T.$$

Since equilibrium thresholds are well defined and unique in period T, continuation values $\mathbf{V}_T(\phi)$ are also well defined and unique for all ϕ.

Now consider the decision of a firm ϕ in the next to last period $T-1$. The problem is essentially static, since continuation values $\mathbf{V}_T(\phi)$ are well defined and unique for all ϕ. Furthermore, since fundamentals are independent and identically distributed (i.i.d.) over time, θ_{T-1} is irrelevant in forecasting θ_T, and then value functions are independent of θ. Then $\Delta_T(\phi, z_{T-1}|k_{z,T-1})$ is also well defined for all ϕ, thus leading to a unique equilibrium cutoff $z_{T-1}^*(\phi)$ (following Proposition 2), and a unique and well defined $\mathbf{V}_{T-1}(\phi)$ for all ϕ.

By straightforward inductive reasoning, we know that as $\sigma \to 0$, a unique sequence of cutoffs $\{z_t^*(\phi)\}_{t=0}^T$ and a unique sequence of expected continuation values exist for each reputation level ϕ in each period t, characterized by (9). □

A.3 *Conditions for Proposition 4*

In this section, we discuss the conditions for $\mathbf{V}_t(\phi) \to \overline{\mathbf{V}}(\phi)$ as $T \to \infty$ (i.e., by backward induction, continuation values converge to a fixed point for all ϕ and periods t far enough from T). These fixed points are the bounded limits required to show that there is an infinite-horizon equilibrium that is a unique limit of the finite-horizon Markov perfect equilibrium.[20]

[20] I have not yet examined the broader issue of what other equilibria there might be in the infinite-horizon game.

In short, the condition for convergence is that the variance of fundamentals is large enough. I prove this by steps. First, I discuss the case without reputation formation as a benchmark in which reputation levels do not interact and I obtain sufficient conditions for convergence. Then I introduce reputation formation and show that those conditions are also sufficient.

Step 1: No reputation formation

This is an artificial and expositionally convenient case in which a firm is born with a given reputation ϕ and cannot change it (because age cannot be observed, for example). First assume that safe projects deliver higher expected continuation values. That is, if commitment were feasible, firms would choose to take safe projects rather than risky projects, regardless of their reputation. This assumption makes sense in our context, since the focus is on the case in which safe projects are almost always the efficient behavior.

We prove this part by generalizing any payoff $\Pi_r(\theta)$ and $\Pi_s(\theta)$ such that $\partial\Pi_r(\theta)/(\partial\theta) < \partial\Pi_s(\theta)/(\partial\theta)$. From Proposition 3 and without reputation formation (i.e., $\mathbf{V}_{t+1}(\phi') = \mathbf{V}_{t+1}(\phi)$),

$$\begin{aligned}\mathbf{V}_t(\phi) = \mathcal{V}(z_t^*(\phi))[\beta p_r \mathbf{V}_{t+1}(\phi) - p_r R_t(\phi|z_t^*)] + \int_{-\infty}^{z_t^*(\phi)} p_r \Pi_r(\theta_t) v(\theta_t)\, d\theta_t \\ + \big(1 - \mathcal{V}(z_t^*(\phi))\big)[\beta p_s \mathbf{V}_{t+1}(\phi) - p_s R_t(\phi|z_t^*)] + \int_{z_t^*(\phi)}^{\infty} p_s \Pi_s(\theta_t) v(\theta_t)\, d\theta_t.\end{aligned}$$

Applying the envelope theorem yields

$$\begin{aligned}\frac{\partial \mathbf{V}_t(\phi)}{\partial \mathbf{V}_{t+1}(\phi)} = \mathcal{V}(z_t^*(\phi))\beta p_r + \big(1 - \mathcal{V}(z_t^*(\phi))\big)\beta p_s \\ - \frac{\partial R_t(\phi|z_t^*)}{\partial z_t^*} \frac{\partial z_t^*}{\partial \mathbf{V}_{t+1}(\phi)} \big[p_s - \mathcal{V}(z_t^*(\phi))(p_s - p_r)\big].\end{aligned}$$

The cutoff z_t^* is determined by $p_s \Pi_s(z_t^*) - p_r \Pi_r(z_t^*) - (p_s - p_r) R_t(\phi|z_t^*) = -\beta(p_s - p_r)\mathbf{V}_{t+1}(\phi)$, since there is no reputation formation. Taking derivatives with respect to $\mathbf{V}_{t+1}(\phi)$ gives

$$\frac{\partial z_t^*}{\partial \mathbf{V}_{t+1}(\phi)} = -\frac{\beta(p_s - p_r)}{[p_s \partial\Pi_s/(\partial z_t^*) - p_r \partial\Pi_r/(\partial z_t^*)]} < 0.$$

Also

$$\frac{\partial R_t(\phi|z_t^*)}{\partial z_t^*} = \frac{\phi \overline{R}(p_s - p_r)}{\Pr(c)^2} v(z_t^*) > 0.$$

Recall that $\partial \mathbf{V}_t(\phi)/(\partial \mathbf{V}_{t+1}(\phi)) > 0$ and $\mathbf{V}_t(\phi) > 0$ when $\mathbf{V}_{t+1}(\phi) = 0$. Hence, convergence to a fixed point $\overline{\mathbf{V}}(\phi)$ happens if $\partial \mathbf{V}_t(\phi)/(\partial \mathbf{V}_{t+1}(\phi)) < 1$. It is clear that this is the case for $\phi = 0$ (since $\partial R_t(\phi|z_t^*)/(\partial z_t^*) = 0$). At the other extreme, when $\phi = 1$, imposing the worst combination of parameters to fulfill the requirement ($\mathcal{V}(z_t^*(\phi)) = 0$ and $\Pr(c) = p_r$) and considering all fundamentals θ, the sufficient condition for convergence is

$$v(\theta) < \frac{1-\beta p_s}{\beta p_s} \frac{p_r^2[p_s\,\partial\Pi_s/(\partial\theta) - p_r\,\partial\Pi_r/(\partial\theta)]}{\overline{R}(p_s-p_r)^2} \quad \text{for all } \theta \in \mathbb{R}. \tag{16}$$

In the case of linear payoffs and normal distributions studied in the text, $\partial\Pi_s/(\partial\theta) = 1$, $\partial\Pi_r/(\partial\theta) = 0$, and $v(\theta) \le \sqrt{\alpha/(2\pi)}$ for all θ. Hence, the sufficient condition is $\alpha/(2\pi) < [p_s p_r^2/(\overline{R}(p_s-p_r)^2)]^2[(1-\beta p_s)/(\beta p_s)]^2$ for all $\theta \in \mathbb{R}$.

In words, the variance of fundamentals should be large enough (or the density low enough) to have convergence in continuation values for all reputation levels when reputation cannot be modified. Recall that this is a really stringent sufficient condition, since the worst combination of parameters we use is not jointly consistent. For example, if $\phi = 1$ and $\mathcal{V}(z^*(\phi)) = 0$, then $\Pr(c)$ is not p_r but p_s, hence convergence conditions are effectively more relaxed.

Step 2: Reputation formation

Assume the sufficient condition expressed in (16) is met. Then there is a unique $\overline{\mathbf{V}}(\phi)$ for all ϕ such that, considering reputation formation,

$$\overline{\mathbf{V}}(\phi) = \frac{\beta p_s - \mathcal{V}(z^*(\phi))\beta p_s}{1-\mathcal{V}(z^*(\phi))\beta p_r}\overline{\mathbf{V}}(\phi') - \frac{1-\mathcal{V}(z^*(\phi))p_s + \mathcal{V}(z^*(\phi))p_r}{1-\mathcal{V}(z^*(\phi))\beta p_s} R(\phi|z^*)$$
$$+ \frac{1}{1-\mathcal{V}(z^*(\phi))\beta p_r}\left[\int_{-\infty}^{z^*(\phi)} p_r\Pi_r(\theta)v(\theta)\,d\theta + \int_{z^*(\phi)}^{\infty} p_s\Pi_s(\theta)v(\theta)\,d\theta\right].$$

Taking derivatives to consider a greater continuation value than taking safe actions, in terms of a higher reputation, is

$$\frac{\partial\overline{\mathbf{V}}(\phi)}{\partial\overline{\mathbf{V}}(\phi')} = \frac{\beta p_s - \mathcal{V}(z^*(\phi))\beta p_s}{1-\mathcal{V}(z^*(\phi))\beta p_r} - \frac{\partial R(\phi|z^*)}{\partial z^*}\frac{\partial z^*}{\partial\overline{\mathbf{V}}(\phi')}\big[\mathcal{V}(z^*(\phi))p_r + \big(1-\mathcal{V}(z^*(\phi))\big)p_s\big].$$

It is straightforward to see that $\partial\overline{\mathbf{V}}(\phi)/(\partial\overline{\mathbf{V}}(\phi')) > 0$. It is also possible to check monotonicity of continuation values, since $\partial\overline{\mathbf{V}}(\phi)/(\partial\overline{\mathbf{V}}(\phi')) < 1$ when the sufficient condition expressed in (16) is fulfilled. With and without reputation formation, extreme continuation values, $\overline{\mathbf{V}}(0)$ and $\overline{\mathbf{V}}(1)$, are the same. Since reputation generates a convex combination between unique values in a compact set, the resulting continuation values $\overline{\mathbf{V}}(\phi)$ with reputation formation are also unique.

A.4 *Proof of Proposition 5*

Proof. As a first step, assume convergence has been achieved (Proposition 4). From Proposition 3, for a general $\Pi_s(\theta)$ and $\Pi_r(\theta)$, $z^*(\phi)$ is determined by solving

$$\int_0^1 \Delta(\phi, z^*|z^*, \widehat{x})\,d\widehat{x} = p_s\Pi_s(z^*) - p_r\Pi_r(z^*) + (p_s-p_r)\left[\beta\int_0^1 \mathbf{V}(\phi'(\phi,\widehat{x}))\,d\widehat{x} - R(\phi|z^*)\right]$$
$$= 0.$$

Taking derivatives with respect to ϕ,

$$\frac{dz^*(\phi)}{d\phi} = -\int_0^1 \frac{\partial\Delta(\phi, z^*|z^*, \widehat{x})}{\partial\phi}\, d\widehat{x} \Big/ \left(\frac{\partial\Delta(\phi, z^*)}{\partial z^*}\right), \tag{17}$$

where

$$\frac{\partial\Delta(\phi, z^*|z^*, \widehat{x})}{\partial\phi} = (p_s - p_r)\left[\beta \frac{\partial \mathbf{V}(\phi')}{\partial\phi'} \frac{\partial\phi'}{\partial\phi}\bigg|_{\widehat{x}} - \frac{\partial R(\phi|z^*)}{\partial\phi}\right]$$

and

$$\frac{\partial\Delta(\phi, z^*|z^*, \widehat{x})}{\partial z^*} = p_s\frac{\partial\Pi_s}{\partial z^*} - p_r\frac{\partial\Pi_r}{\partial z^*} - (p_s - p_r)\frac{\partial R(\phi|z^*)}{\partial z^*} \quad \text{for all } \widehat{x}.$$

From (5),

$$\frac{dR(\phi|z^*)}{d\phi} = \frac{\partial R}{\partial\phi} + \frac{\partial R}{\partial z^*}\frac{dz^*}{d\phi}, \tag{18}$$

where

$$\frac{\partial R}{\partial\phi} = -\frac{\overline{R}(p_s - p_r)(1 - \mathcal{V}(z^*))}{\Pr(c)^2} < 0 \quad \text{and} \quad \frac{\partial R}{\partial z^*} = \frac{\overline{R}(p_s - p_r)\phi v(z^*)}{\Pr(c)^2} > 0 \quad \text{for all } \phi.$$

Finally, from (9), using the envelope theorem,

$$\frac{d\overline{\mathbf{V}}(\phi)}{d\phi} = \frac{\partial\overline{\mathbf{V}}}{\partial\phi} + \frac{\partial\overline{\mathbf{V}}}{\partial z^*}\frac{dz^*}{d\phi}, \tag{19}$$

where

$$\frac{\partial\overline{\mathbf{V}}}{\partial\phi} = \beta\left[\mathcal{V}(z^*)p_r\frac{\partial\overline{\mathbf{V}}(\phi)}{\partial\phi} + (1-\mathcal{V}(z^*))p_s\frac{\partial\overline{\mathbf{V}}(\phi')}{\partial\phi'}\frac{\partial\phi'}{\partial\phi}\bigg|_{\widehat{x}=0}\right] - (p_s - \mathcal{V}(z^*)(p_s - p_r))\frac{\partial R(\phi)}{\partial\phi}$$

and

$$\frac{\partial\overline{\mathbf{V}}}{\partial z^*} = \beta v(z^*)\left[p_s\left(\int_0^1 \overline{\mathbf{V}}(\phi'|\widehat{x})\, d\widehat{x} - \overline{\mathbf{V}}(\phi'|\widehat{x}=0)\, d\widehat{x}\right) - p_r\left(\int_0^1 \overline{\mathbf{V}}(\phi'|\widehat{x})\, d\widehat{x} - \overline{\mathbf{V}}(\phi)\, d\widehat{x}\right)\right].$$

We are interested in the sign of the derivatives (17)–(19). To determine them, I solve backward from the last period T.

At period T, $\partial\Delta_T(\phi, z_T^*|z_T^*, \widehat{x})/(\partial\phi) > 0$ for all $\widehat{x}$ (since $\mathbf{V}_{T+1} = 0$ for all ϕ) and $\partial\Delta_T(\phi, z_T^*)/(\partial z_T^*) > 0$ (from (7)). Hence, $dz_T^*/(d\phi) < 0$. From (18), $dR_T(\phi)/(d\phi) < 0$. Finally, from (19) (since $\mathbf{V}_{T+1} = 0$ for all ϕ), $d\mathbf{V}_T(\phi)/(d\phi) > 0$.

At period $T-1$, we additionally have the effects coming from $\mathbf{V}_T$. From Bayesian learning, $(\partial\phi'/(\partial\phi))|_{\widehat{x}} = p_r[\widehat{x}p_r + (1-\widehat{x})p_s]/(p_r + (p_s - p_r)(1-\widehat{x})\phi) > 0$ for all $\widehat{x}$ and all ϕ. From results at T, $\int_0^1 (\partial\mathbf{V}_T(\phi')/(\partial\phi'))(\phi'/\phi)|_{\widehat{x}}\, d\widehat{x} > 0$. Hence, $\int_0^1 (\partial\Delta_{T-1}(\phi, z_{T-1}^*|\widehat{x})/(\partial\phi))\, d\widehat{x} > 0$ and $dz_{T-1}^*/(d\phi) < 0$. From (18), $dR_{T-1}(\phi)/(d\phi) < 0$. Finally, it follows that $d\mathbf{V}_{T-1}(\phi)/(d\phi) > 0$ from (19) and from the fact that $\partial\mathbf{V}_T(\phi)/(\partial\phi) > 0$ for all ϕ and that $\int_0^1 \mathbf{V}_T(\phi'|\widehat{x})\, d\widehat{x}$ can be written as a convex combination between $\mathbf{V}_T(\phi)$ and $\mathbf{V}_T(\phi'|\widehat{x}=0)$.

Solving backward until convergence, $d\mathcal{V}(z^*(\phi))/(d\phi) < 0$ $(dz^*(\phi)/(d\phi) < 0)$, $dR(\phi)/(d\phi) < 0$, and $d\overline{\mathbf{V}}(\phi)/(d\phi) > 0$ for all $\phi \in [0, 1]$. □

A.5 *Proof of Lemma 2*

Proof. Differentiating (17) with respect to ϕ, we get

$$\frac{d^2 z^*}{d\phi^2} = -\frac{1}{\partial\Delta/(\partial z^*)}\left[\frac{\partial^2\Delta}{\partial\phi^2} + 2\frac{\partial^2\Delta}{\partial\phi\,\partial z^*}\frac{dz^*}{d\phi} + \frac{\partial^2\Delta}{\partial z^{*2}}\left(\frac{dz^*}{d\phi}\right)^2\right]. \tag{20}$$

In what follows, I go back to a linear relation between payoffs and fundamentals, so the shape of cutoffs is not just an artifice of the shape of payoffs. The components of (20) are

$$\frac{\partial^2\Delta}{\partial\phi^2} = (p_s - p_r)\left[\beta\int_0^1\left(\frac{\partial\overline{\mathbf{V}}}{\partial\phi'}\frac{\partial^2\phi'}{\partial\phi^2}\bigg|_{\widehat{x}} + \frac{\partial^2\overline{\mathbf{V}}}{\partial\phi'^2}\frac{\partial\phi'}{\partial\phi}^2\bigg|_{\widehat{x}}\right)d\widehat{x} - \frac{\partial^2 R}{\partial\phi^2}\right] \tag{21}$$

$$\frac{\partial^2\Delta}{\partial\phi\,\partial z^*} = -(p_s - p_r)\frac{\partial^2 R}{\partial\phi\,\partial z^*}$$

and

$$\frac{\partial^2\Delta}{\partial z^{*2}} = -(p_s - p_r)\frac{\partial^2 R}{\partial z^{*2}}.$$

From (19),

$$\begin{aligned}\frac{\partial^2\overline{\mathbf{V}}}{\partial\phi^2}\bigg|_{\widehat{x}} &= \mathcal{V}(z^*)p_r\left[\beta\frac{\partial^2\overline{\mathbf{V}}(\phi)}{\partial\phi^2} - \frac{\partial^2 R(\phi|z^*)}{\partial\phi^2}\right]\\ &\quad + (1-\mathcal{V}(z^*))p_s\left[\beta\left(\frac{\partial\overline{\mathbf{V}}}{\partial\phi'}\frac{\partial^2\phi'}{\partial\phi^2}\bigg|_{\widehat{x}} + \frac{\partial^2\overline{\mathbf{V}}}{\partial\phi'^2}\frac{\partial\phi'}{\partial\phi}^2\bigg|_{\widehat{x}}\right) - \frac{\partial^2 R(\phi|z^*)}{\partial\phi^2}\right],\end{aligned} \tag{22}$$

where

$$\frac{\partial^2 R(\phi|z^*)}{\partial\phi^2} = \frac{2\overline{R}(p_s - p_r)^2(1-\mathcal{V}(z^*))^2}{\Pr(c)^3} > 0.$$

I proceed in two steps. First, as a benchmark, I solve backward from T when reputation cannot be updated. Then I show how reputation formation convexifies the schedule of cutoffs.

Step 1: No reputation formation. Assume a firm is born with a given reputation ϕ and cannot change it. I call the cutoffs in this case $\widetilde{z}^*(\phi)$. In this case, beliefs $\widehat{x}$ do not play any role, $\partial\phi'/(\partial\phi) = 1$, and $\partial^2\phi'/(\partial\phi^2) = 0$ for all ϕ. Hence, (21) and (22) can be rewritten as

$$\frac{\partial^2\Delta_t}{\partial\phi^2} = (p_s - p_r)\left[\beta\frac{\partial^2\mathbf{V}_{t+1}}{\partial\phi^2} - \frac{\partial^2 R_t}{\partial\phi^2}\right]$$

and

$$\frac{\partial^2 \mathbf{V}_t}{\partial \phi^2} = \big(\mathcal{V}(\widetilde{z}_t^*) p_r + (1-\mathcal{V}(\widetilde{z}_t^*)) p_s\big)\bigg[\beta \frac{\partial^2 \mathbf{V}_{t+1}(\phi)}{\partial \phi^2} - \frac{\partial^2 R_t(\phi|z^*)}{\partial \phi^2}\bigg].$$

At period T, since $\mathbf{V}_{T+1} = 0$ for all ϕ, then $\partial^2 \Delta_T/(\partial\phi^2) < 0$ and $\partial^2 \mathbf{V}_T/(\partial\phi^2) < 0$. However, these signs do not guarantee that (20) is positive. The sufficient condition for $d^2\widetilde{z}_T^*/(d\phi^2) > 0$ is $|d\widetilde{z}_T^*/(d\phi)| > (v(\widetilde{z}_T^*)/\phi)[p_r - (p_s - p_r)(1-\mathcal{V}(\widetilde{z}_T^*))\phi]/[\Pr(c)v'(\widetilde{z}_T^*) + 2(p_s - p_r)v^2(\widetilde{z}_T^*)]$, which is easier to be fulfilled with a large variance of fundamentals and for high values of ϕ. This condition requires some algebra that is available on request.

At period, $T-1$, $\partial^2\Delta_{T-1}/(\partial\phi^2) < \partial^2\Delta_T/(\partial\phi^2) < 0$ and $\partial^2\mathbf{V}_{T-1}/(\partial\phi^2) < \partial^2\mathbf{V}_T/(\partial\phi^2) < 0$. This means $d^2\widetilde{z}_{T-1}^*/(d\phi^2) > 0$ for a higher range of ϕ values. The same analysis hold until convergence. In this case, $\partial^2\overline{\mathbf{V}}/(\partial\phi^2) = -(\psi(\widetilde{z}^*)/(1-\beta\psi(\widetilde{z}^*)))\,\partial^2 R/(\partial\phi^2)$ and $\partial^2\Delta/(\partial\phi^2) = -(p_s - p_r)/(1-\beta\psi(\widetilde{z}^*))\,\partial^2 R/(\partial\phi^2)$, with $\psi(\widetilde{z}^*) = \mathcal{V}(\widetilde{z}^*)p_r + (1-\mathcal{V}(\widetilde{z}^*))p_s$. Without reputation concerns, it may be that $d^2\widetilde{z}^*/(d\phi^2) > 0$ for all ϕ, but this is less likely at lower reputation levels.

Step 2: Reputation formation. Consider now the full model with reputation formation. This leads to convexity by combining continuation values of different reputation levels. We consider again (21) and (22).

At period T, as in Step 1, $\partial^2\Delta_T/(\partial\phi^2) < 0$, $\partial^2\mathbf{V}_T/(\partial\phi^2) < 0$, and $d^2 z_T^*/(d\phi^2) = d^2\widetilde{z}_T^*/(d\phi^2)$.

At period $T-1$, since $\partial\phi'/(\partial\phi) = p_r(p_s(1-\widehat{x}) + p_r\widehat{x})/[p_r + (p_s - p_r)(1-\widehat{x})\phi]^2 > 0$ and $\partial^2\phi'/(\partial\phi^2) = -2p_r(p_s(1-\widehat{x}) + p_r\widehat{x})(p_s - p_r)(1-\widehat{x})/[p_r + (p_s - p_r)(1-\widehat{x})\phi]^3 < 0$ for all $\widehat{x} \in [0,1]$, then $\partial^2\Delta_{T-1}/(\partial\phi^2) < \partial^2\Delta_T/(\partial\phi^2) < 0$ and $\partial^2\mathbf{V}_{T-1}/(\partial\phi^2) < \partial^2\mathbf{V}_T/(\partial\phi^2) < 0$, exactly as in Step 1. Furthermore, $\int_0^1 (\partial\mathbf{V}_T/(\partial\phi'))(\partial^2\phi'/(\partial\phi^2))|_{\widehat{x}}\,d\widehat{x} < 0$ and $\int_0^1 (\partial^2\mathbf{V}_T/(\partial\phi'^2))(\partial\phi'/(\partial\phi))^2|_{\widehat{x}}\,d\widehat{x} < \partial^2\mathbf{V}_T/(\partial\phi^2)$, which means $\partial^2\Delta_{T-1}/(\partial\phi^2)$ and $\partial^2\mathbf{V}_{T-1}/(\partial\phi^2)$ are lower than their counterparts without reputation concerns, derived in Step 1. This implies that $d^2 z_{T-1}^*/(d\phi^2) > d^2\widetilde{z}_{T-1}^*/(d\phi^2)$ for all ϕ.

Solving backward until convergence, reputation formation introduces pressure for concavity of continuation values, and hence the convexity of the schedule of cutoffs and interest rates at all reputation levels, leading to $d^2 z^*/(d\phi) > d^2\widetilde{z}^*/(d\phi)$ for all ϕ.

Even more importantly, as reputation formation becomes easier (i.e., signals are more precise), for $p_r/p_s \to 0$, $(\partial\phi'/(\partial\phi))|_{\phi=0} \to \infty$ and $(\partial^2\phi'/(\partial\phi^2))|_{\phi=0} \to \infty$, hence $d^2 z_{T-1}^*/(d\phi^2) > 0$ for all ϕ (since it always convexifies the schedule of cutoffs for low reputation levels, which are the levels of reputation where convexity is more difficult to obtain without reputation formation). Hence, for any reputation ϕ, there is always a $\overline{(p_r/p_s)}(\phi) \in (0,1]$ such that $d^2 z_{T-1}^*/(d\phi^2) = 0$. Furthermore, from the condition in Step 1, $\overline{(p_r/p_s)}(\phi)$ is weakly increasing in ϕ. □

A.6 *Numerical exercise*

The parameters in the numerical exercise are $K = 1.5$, $\mu = -0.4$, $\alpha = 25$, $p_s = 0.9$, $p_r = 0.7$, $\beta = 0.95$, and $\overline{R} = 1$. These parameters have been chosen to fulfill four conditions.

- Risk-taking is almost never efficient. It happens for normalized fundamentals below $\theta_E = -4$, which occurs ex ante with a probability of only 0.001%.
- Pledgable short-term cash flows are higher than equilibrium interest rates ($K > 1/p_r$). Hence, firms can always pay back debts if they continue.
- Without reputation concerns, interest rates are not convex in ϕ, so I can show the forces of reputation concerns in convexifying them (Lemma 2).
- Conditions for uniqueness from interest rates (8) and convergence of continuation values to a fixed point (from Proposition 4) are fulfilled.

Finally, I introduce an additional set of signals, which are correlated to projects and observable if the firm continues. Assume, for example, a firm "grows" or "produces a new idea" (generates a "good signal") more likely if the project was safe ($q_s = 0.8 > q_r = 0.4$). I make this extension for two reasons. First, it shows the flexibility of the model. Second, it breaks down the perfect correlation between reputation and age that arises when the only positive signal about the firm's type is continuation.

A.6.1 *Computational procedure*

- Set a large grid of $\phi \in [0, 1]$.
- Solve the full information (FI) environment (efficiency).
  - Guess a $\mathbf{V}_{\text{FI},0} = 0$.
  - Obtain $\theta^*_{\text{FI},0}$ from $\Delta(\theta)_{\text{FI}} = p_s\Pi_s(\theta) - p_r\Pi_r(\theta) + \beta(p_s - p_r)\mathbf{V}_{\text{FI},0} = 0$.
  - Obtain
$$\mathbf{V}_{\text{FI},1} = \frac{\int_{-\infty}^{\theta^*_{\text{FI},0}}[p_r\Pi_r(\theta) - \overline{R}]v(\theta)\,d\theta + \int_{\theta^*_{\text{FI},0}}^{\infty}[p_s\Pi_s(\theta) - \overline{R}]v(\theta)\,d\theta}{1 - \beta(p_s - \mathcal{V}(\theta^*_{\text{FI},0})(p_s - p_r))}.$$
  - Use $\mathbf{V}_{\text{FI},1}$ as the new guess and iterate until $\mathbf{V}_{\text{FI},i} - \mathbf{V}_{\text{FI},i-1} < \varepsilon$.
- Solve the environment without (wo) reputation formation.
  - Guess $\mathbf{V}(\phi)_{\text{wo},0} = 0$ and $\theta^*(\phi)_0 = \theta^*_{\text{FI}}$ for all ϕ.
  - Obtain $\theta^*(\phi)_1$ from $\Delta(\phi, \theta^*(\phi)_1) = 0$, where
$$\Delta(\phi, \theta) = p_s\Pi_s(\theta) - p_r\Pi_r(\theta) + (p_s - p_r)\big[\beta\mathbf{V}(\phi)_{\text{wo},0} - R(\phi|\theta^*(\phi)_0)\big].$$
  - For each ϕ, obtain
$$\mathbf{V}(\phi)_{\text{wo},1} = \frac{\int_{-\infty}^{\theta^*(\phi)_1} p_r[\Pi_r(\theta) - R(\phi|\theta^*(\phi)_1)]v(\theta)\,d\theta + \int_{\theta^*(\phi)_1}^{\infty} p_s[\Pi_s(\theta) - R(\phi|\theta^*(\phi)_1)]v(\theta)\,d\theta}{[1 - \beta(p_r + \mathcal{V}(\theta^*(\phi)_1)(p_s - p_r))]}.$$
  - Use $\mathbf{V}(\phi)_{\text{wo},1}$ and $\theta^*(\phi)_1$ as new guesses and iterate until $\mathbf{V}(\phi)_{\text{wo},i} - \mathbf{V}(\phi)_{\text{wo},i-1} < \varepsilon_1$ and $\theta^*(\phi)_i - \theta^*(\phi)_{i-1} < \varepsilon_2$ for all ϕ.

- Solve the environment with reputation formation.
  - Guess a $\mathbf{V}(\phi)_0 = 0$ and $z^*(\phi)_0 = \theta^*(\phi)$ for all ϕ.
  - Using $\mathbf{V}(\phi)_0$, for each belief $\widehat{x} \in [0,1]$ from a large grid of size N_x, obtain

$$\Delta(\phi, z, \widehat{x}|z^*(\phi)_0)_0 = E_z[p_s\Pi_s(\theta) - p_r\Pi_r(\theta)] + (p_s - p_r)[\beta\mathbf{V}(\phi'|\widehat{x})_0 - R(\phi|z^*(\phi)_0)].$$

    Recall that for $\sigma \to 0$, this expression can be well approximated by

$$\Delta(\phi, z, \widehat{x}|z^*(\phi)_0)_0 = p_s\Pi_s(z) - p_r\Pi_r(z) + (p_s - p_r)[\beta\mathbf{V}(\phi'|\widehat{x})_0 - R(\phi|z^*(\phi)_0)].$$

  - Solve for $z^*(\phi)_1$ from $\sum \Delta(\phi, z, \widehat{x}|z^*(\phi)_0)_0/N_x = 0$.
  - For all $\theta < (>) z^*(\phi)_1$, $x(\phi, \theta)_1 = 1\ (= 0)$,
    * $R(\phi|z^*(\phi)_1)$ follows from $z^*(\phi)_1$
    * ϕ' follows from $x(\phi, \theta)_1$.
  - Obtain $\mathbf{V}(\phi)_1$ as

$$\mathbf{V}(\phi)_1 = \int_{-\infty}^{z^*(\phi)_1} p_r\big[\Pi_r(\theta) - R(\phi|z^*(\phi)_1) + \beta\mathbf{V}(\phi)_0\big]v(\theta)\,d\theta + \int_{z^*(\phi)_1}^{\infty} p_s\big[\Pi_s(\theta) - R(\phi|z^*(\phi)_1) + \beta\mathbf{V}(\phi')_0\big]v(\theta)\,d\theta.$$

  - Use $\mathbf{V}(\phi)_1$ and $z^*(\phi)_1$ as new guesses and iterate until $\mathbf{V}(\phi)_i - \mathbf{V}(\phi)_{i-1} < \varepsilon_1$ and $z^*(\phi)_i - z^*(\phi)_{i-1} < \varepsilon_2$ for all ϕ.

A.6.2 *Additional results* To obtain the stationary expected distribution, I assume that for every firm that disappears, there is a new one that enters the market. Among those firms, 50% are strategic, which implies that every new firm enters with a fresh reputation $\phi_0 = 0.5$. Figure 8 shows the stationary expected distribution of reputation in the market and the evolution of firms with reputation 0.01, 0.99 and the assigned prior 0.5, as a fraction of the total of firms. The fraction of firms with poor reputation tends to disappear, while the fraction of firms with good reputation grows over time toward a stationary distribution. However, this evolution is not monotonic. When a spike of risk-taking occurs, good firms die at a higher rate than in normal times and are replaced by new firms with an intermediate reputation of $\phi_0 = 0.5$. Hence, in those periods of high risk-taking, there is a decline in the average quality of firms in the market. This is relevant because it suggests that a bad enough shock in fundamentals not only magnifies the crisis, but also makes it persistent.

Finally, Figure 9 shows aggregate net returns to lenders.[21] When fundamentals weaken enough, we see that returns decline catastrophically, since most firms, regardless of their reputations, take risks. Since lenders charge low rates to good reputation

[21] First I obtained individual net returns for each reputation level (computed by the lending rate charged to ϕ multiplied by the true probability of no default minus the risk-free rate). Then I calculated the weighted sum of individual returns to obtain aggregate net returns.

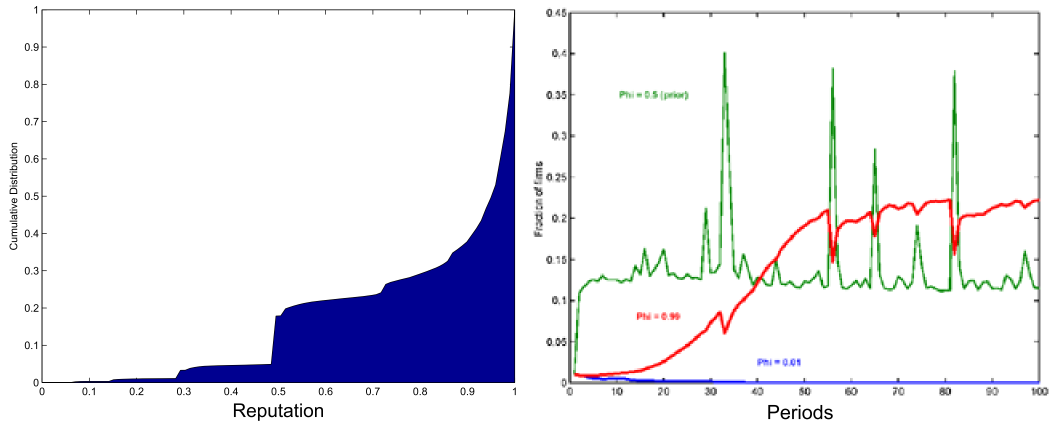

Figure 8. Stationary reputation distribution and initial evolution of certain reputations.

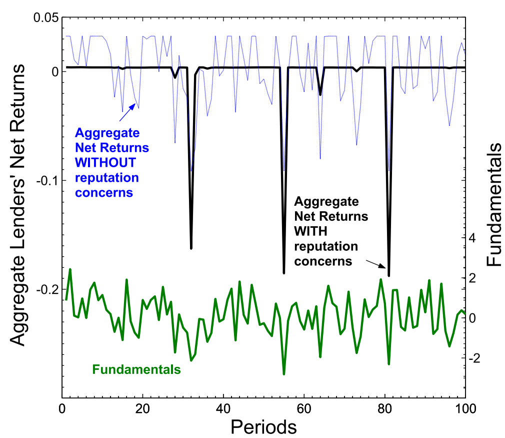

Figure 9. Simulated aggregate net returns to lenders.

firms, sudden losses are large. With reputation concerns, lending rates are lower and more stable, while lenders losses are greater when they rarely occur. Reputation concerns reduce the frequency of crises, but they magnify lenders' losses when crises do occur.

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