Supplement to "Condorcet meets Ellsberg"

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Andrew Ellis
Department of Economics, London School of Economics and Political Science

Appendix SA contains the two extensions mentioned in the main text. Appendix SB provides the details of the algebra omitted from the proofs.

Appendix SA: Extensions

SA.1 *Directly updating on the pivotal event*

In this subsection, I assume that each voter directly updates her set of priors conditional on being pivotal. While this assumption is without loss of generality for SEU voters, it implies different behavior for MEU voters. Theorem SA.1 extends Theorem 1 to this setting.

I consider games as in Section 3 except that I restrict attention to $R_a = \{r_a\}$ and $R_b = \{r_b\}$. Instead of updating only on her signal, each voter also updates conditional on being pivotal. Her set of posteriors after observing signal t equals the set $\hat{\Pi}_t$, where $\hat{\pi}_t \in \hat{\Pi}_t$ if and only if there exists $\pi \in \Pi$ so that $\hat{\pi}_t(\cdot) = \pi(\cdot|t_i = t, piv, \sigma_{-i})$. In particular, the marginal probability of a conditional on $t_i = t$ and being pivotal is in the range $[\underline{q}_t(\sigma), \bar{q}_t(\sigma)]$ where

$$\underline{q}_t(\sigma) = \frac{\underline{p}_t \Pr(piv|a, \sigma_{-i})}{\underline{p}_t \Pr(piv|a, \sigma_{-i}) + (1 - \underline{p}_t) \Pr(piv|b, \sigma_{-i})}$$

and

$$\bar{q}_t(\sigma) = \frac{\bar{p}_t \Pr(piv|a, \sigma_{-i})}{\bar{p}_t \Pr(piv|a, \sigma_{-i}) + (1 - \bar{p}_t) \Pr(piv|b, \sigma_{-i})}.$$

After observing signal t, voter i chooses her strategy to maximize

$$\hat{V}_t(\hat{\sigma}, \sigma_{-i}) = \min_{q \in [\underline{q}_t(\sigma), \bar{q}_t(\sigma)]} q\hat{\sigma}(A) + (1 - q)\hat{\sigma}(B).$$

When $\bar{p} > \underline{p}$, $\hat{V}_t(\cdot, \sigma_{-i})$ is an affine transformation of $V_t(\cdot, \sigma_{-i})$ if and only if $\theta_a(\sigma) = \theta_b(\sigma)$; in general, they have different maximizers. Consequently, the best response to σ_{-i} according $V_t(\cdot)$ may differ from that according $\hat{V}_t(\cdot)$. Any collection $\hat{\Gamma} = (I, [\underline{p}, \bar{p}], T, r_a, r_b)$ defines an *ambiguous pivotal voting game* if players maximize $\hat{V}_t(\cdot)$ rather than

Andrew Ellis: a.ellis@lse.ac.uk

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$V_t(\cdot)$. An equilibrium for such a game is as in the main text, except $V_t(\hat\sigma, \sigma_{-i})$ is replaced by $\hat V_t(\hat\sigma, \sigma_{-i})$. Theorem 1 holds as stated when considering ambiguous pivotal voting games rather than ambiguous voting games.

THEOREM SA.1. *Any symmetric equilibrium to an ambiguous pivotal voting game where voters lack confidence does not have correct expected winners.*

PROOF. For the sake of contradiction, suppose that σ is a symmetric equilibrium where $\tau_A(\sigma|a) > \frac{1}{2}$ and $\tau_B(\sigma|a) > \frac{1}{2}$. Note that $\hat V_t(\hat\sigma; \sigma_{-i}) = W_t(\hat\sigma(A); \sigma_{-i})$ where

$$W_t(s; \sigma_{-i}) = \min_{p\in[\underline{p}_t, \bar p_t]} \frac{p \Pr(piv|a, \sigma_{-i})s + (1-p)\Pr(piv|b, \sigma_{-i})(1-s)}{p\Pr(piv|a, \sigma_{-i}) + (1-p)\Pr(piv|b, \sigma_{-i})}.$$

The superdifferential of W_t equals

$$\partial W_t(s; \sigma_{-i}) = \begin{cases} \{\frac{\underline{p}_t \Pr(piv|a,\sigma_{-i}) - (1-\underline{p}_t)\Pr(piv|b,\sigma_{-i})}{\underline{p}_t \Pr(piv|a,\sigma_{-i}) + (1-\underline{p}_t)\Pr(piv|b,\sigma_{-i})}\} & \text{if } s > \frac{1}{2} \\ \{\frac{p \Pr(piv|a,\sigma_{-i}) - (1-p)\Pr(piv|b,\sigma_{-i})}{p \Pr(piv|a,\sigma_{-i}) + (1-p)\Pr(piv|b,\sigma_{-i})} : p \in [\underline{p}_t, \bar p_t]\} & \text{if } s = \frac{1}{2} \\ \{\frac{\bar p_t \Pr(piv|a,\sigma_{-i}) - (1-\bar p_t)\Pr(piv|b,\sigma_{-i})}{\bar p_t \Pr(piv|a,\sigma_{-i}) + (1-\bar p_t)\Pr(piv|b,\sigma_{-i})}\} & \text{if } s < \frac{1}{2}. \end{cases}$$

This implies that $s = 0$ is the only optimum of W_t if and only if

$$\frac{\bar p_t}{1-\bar p_t} < \frac{\Pr(piv|b, \sigma_{-i})}{\Pr(piv|a, \sigma_{-i})},$$

that any $s \in [0, \frac{1}{2}]$ is an optimum of W_t if

$$\frac{\bar p_t}{1-\bar p_t} = \frac{\Pr(piv|b, \sigma_{-i})}{\Pr(piv|a, \sigma_{-i})},$$

that $s = \frac{1}{2}$ is the only optimum of W_t if and only if

$$\frac{\underline{p}_t}{1-\underline{p}_t} < \frac{\Pr(piv|b, \sigma_{-i})}{\Pr(piv|a, \sigma_{-i})} < \frac{\bar p_t}{1-\bar p_t},$$

that any $s \in [\frac{1}{2}, 1]$ is an optimum of W_t if

$$\frac{\underline{p}_t}{1-\underline{p}_t} = \frac{\Pr(piv|b, \sigma_{-i})}{\Pr(piv|a, \sigma_{-i})},$$

and that $s = 1$ is the only optimum of W_t if and only if

$$\frac{\underline{p}_t}{1-\underline{p}_t} > \frac{\Pr(piv|b, \sigma_{-i})}{\Pr(piv|a, \sigma_{-i})}.$$

Label $\bigcap_{t\in T}[\underline{p}_t, \bar p_t] = [\underline{p}_1, \bar p_2]$, noting that $\underline{p}_t \le \underline{p}_1$ and $\bar p_t \ge \bar p_2$ for all $t \in T$. By symmetry, all voters perceive the same strategies of others, so fix any i and consider $\beta = \Pr(piv|b, \sigma_{-i})/\Pr(piv|a, \sigma_{-i})$. If $\beta < \bar p_2/(1-\bar p_2)$, then for all $t \in T$, $\beta < \bar p_t/(1-\bar p_t)$, so $\sigma(t)(A) \geq \frac{1}{2}$ by the above. Conclude that $\tau_B(\sigma|b) \leq \frac{1}{2}$, a contradiction. Finally, if $\beta > \underline{p}_1/(1-\underline{p}_1)$, then for all $t \in T$, $\beta > \underline{p}_t/(1-\underline{p}_t)$, so $\sigma(t)(A) \leq \frac{1}{2}$ by the above. Conclude that $\tau_A(\sigma|a) \leq \frac{1}{2}$, a contradiction. These cases are mutually exhaustive and both result in a contradiction. Therefore, it is impossible that $\tau_A(\sigma|a) > \frac{1}{2}$ and $\tau_B(\sigma|b) > \frac{1}{2}$, completing the proof. □

SA.2 *Ambiguity about likelihoods*

Voters assign a marginal probability to a between $\underline{p}$ and $\bar{p}$, where $0 < \underline{p} \leq \bar{p} < 1$. Conditional on state s, the signal that voter i observes is distributed according to one of the distributions in the set R_s, where each R_s is a closed, convex, nonempty set of probability distributions over T. Formally, $\pi \in \Pi$ if and only if there exists a $p \in [\underline{p}, \bar{p}]$ and an r_s in the convex hull of $\{\bigotimes_{i\in I} r_{s,i} : r_{s,i} \in R_s \; \forall i \in I\}$ for each $s \in S$ so that

$$\pi(a, t) = p r_a(t) \quad \text{and} \quad \pi(b, t) = (1-p) r_b(t)$$

for all $(s, t) \in \Omega$.

Voters form a set of posteriors by updating each measure in Π according to Bayes rule. Denoting the vector of signals seen by other voters as t_{-i}, Bayes rule gives that π_t is an extreme point of $\Pi(\cdot|t_i = t)$ if and only if there exists $p \in \{\underline{p}_t, \bar{p}_t\}$ as well as an $r_{a,i} \in R_a$ and an $r_{b,i} \in R_b$ for every $i \in I$ so that

$$\pi_t(a, t, t_{-i}) = p \prod_{j\neq i} r_{a,j}(t_j) \quad \text{and} \quad \pi_t(b, t, t_{-i}) = (1-p) \prod_{j\neq i} r_{b,j}(t_j)$$

for every $(s, t) \in \Omega$, where

$$\bar{p}_t = \frac{\bar{r}_{a,t}\bar{p}}{\bar{r}_{a,t}\bar{p} + \underline{r}_{b,t}(1-\bar{p})}$$

and

$$\underline{p}_t = \frac{\underline{r}_{a,t}\underline{p}}{\underline{r}_{a,t}\underline{p} + \bar{r}_{b,t}(1-\underline{p})}$$

for $\bar{r}_{a,t} = \max_{r_a \in R_a} r_a(t)$, $\underline{r}_{b,t} = \min_{r_b \in R_b} r_b(t)$, $\underline{r}_{a,t} = \min_{r_a \in R_a} r_a(t)$, and $\bar{r}_{b,t} = \max_{r_b \in R_b} r_b(t)$. Say that $(r^\sigma_{a,i}, r^\sigma_{b,i})_{i\in I}$ is a *minimizing likelihood* for a strategy profile σ if for every $t \in T$, there is a minimizer of $V_t(\sigma_i, \sigma_{-i})$, π'_t, that has the form $\pi'_t(a, t, t_{-i}) = p \prod_{j\neq i} r^\sigma_{a,j}(t_j)$ and $\pi'_t(b, t, t_{-i}) = (1-p) \prod_{j\neq i} r^\sigma_{b,j}(t_j)$ for some $p \in (0, 1)$.

Lemma SA.2. *If σ is a strategy profile, then there exists a minimizing likelihood $(r^\sigma_{a,i}, r^\sigma_{b,i})_{i\in I}$ for σ. If σ is symmetric, then this likelihood can be taken to be symmetric, i.e., $r^\sigma_{a,i} = r^\sigma_{a,j} = r^\sigma_a$ and $r^\sigma_{b,i} = r^\sigma_{b,j} = r^\sigma_b$ for every $i, j \in I$.*

Proof. Fix an arbitrary voter i, a signal $\tau \in T$, and another voter $j \in I \setminus \{i\}$. For an arbitrary likelihood $(r_{a,k})_{k\in I\setminus\{i\}}$, voter i's interim utility conditional on state a and signal τ

can be written as

$$\begin{aligned}\mathbb{E}[u_i|a,\sigma,\tau] = f(\sigma_{-i,-j},(r_{a,k})_{k\in I\setminus\{i,j\}}) + \sum_{t\in T} r_{a,j}(t)\sigma_j(t)(A) \\ \times \big[\sigma_i(\tau)(A)\Pr(n-1 \text{ voters in } I\setminus\{i,j\} \text{ vote for } A|(r_{a,k})_{k\in I\setminus\{i,j\}},\sigma_{-i-j}) \\ + \sigma_i(\tau)(B)\Pr(n \text{ voters in } I\setminus\{i,j\} \text{ vote for } A|(r_{a,k})_{k\in I\setminus\{i,j\}},\sigma_{-i-j})\big],\end{aligned}$$

where $f(\cdot)$ is the probability that $n+1$ voters in $I\setminus\{i,j\}$ vote for A, given their strategies and the distribution of signals. Therefore, for all $\tau\in T$, any $r_{a,j}$ that minimizes $\sum_{t\in T} r_{a,j}(t)\sigma_j(t)(A)$ minimizes $\mathbb{E}[u_i|a,\sigma,\tau]$ regardless of the other likelihoods. Similar rewriting is possible for b, resulting in $r_{b,j}$ minimizing $\sum_{t\in T} r_{b,j}(t)\sigma_j(t)(B)$.

For each j, pick an arbitrary

$$r^{\sigma}_{a,j} \in \arg\min_{r_a\in R_a} \sum_{t\in T} r_a(t)\sigma_j(t)(A)$$

and an arbitrary

$$r^{\sigma}_{b,j} \in \arg\min_{r_b\in R_b} \sum_{t\in T} r_b(t)\sigma_j(t)(B).$$

Given the above observation, the collection $(r^{\sigma}_{a,k}, r^{\sigma}_{b,k})_{k\in I\setminus\{i\}}$ is a minimizing likelihood. If σ is symmetric, then it is without loss to take $r^{\sigma}_{a,j}=r^{\sigma}_{a,k}$ and $r^{\sigma}_{b,j}=r^{\sigma}_{b,k}$ for all $j,k\in I$ since $\arg\min_{r_a\in R_a}\sum_{t\in T} r_a(t)\sigma_j(t)(A) = \arg\min_{r_a\in R_a}\sum_{t\in T} r_a(t)\sigma_k(t)(A)$ (and similarly for b), completing the proof. □

Theorem SA.3. *If voters lack confidence and σ is a symmetric equilibrium, then for any distribution of signals r^*_a, r^*_b, expected winners are not correct.*

Proof. Apply Lemma SA.2. Follow the arguments of Theorem 1 in the main text using the minimizing collection of likelihoods rather than r_a, r_b to establish that $\sigma(t)(A) > \frac{1}{2}$ implies $\sigma(t')(A) \geq \frac{1}{2}$ for all $t'\in T$. Conclude that for any distribution r^*_a, r^*_b, $\tau_A(\sigma|a) > \frac{1}{2}$ implies $\tau_A(\sigma|b) > \frac{1}{2}$, where τ uses r^*_a, r^*_b. □

Appendix SB: Algebra

SB.1 *Algebra for the proof of Theorem 1*

Let $f(m,p,n) = \binom{n}{m}p^m(1-p)^{n-m}$, noting that

$$\begin{aligned}\frac{\partial f}{\partial p} &= \binom{n}{m}[mp^{m-1}(1-p)^{n-m} - (n-m)p^m(1-p)^{n-m-1}] \\ &= n\binom{n-1}{m-1}[p^{m-1}(1-p)^{n-m} - p^m(1-p)^{n-m-1}] \\ &= n[f(m-1,p,n-1) - f(m,p,n-1)]\end{aligned}$$

if $1 < m < n$ and that $\partial f(n, p, n)/\partial p = np^{n-1} = nf(n-1, p, n-1)$. Write

$$\theta_a(\sigma) = \sum_{m=n+1}^{2n} f(m, \tau_A(\sigma|a), 2n)$$

and conclude that

$$\begin{aligned}
\left.\frac{\partial\theta_a}{\partial\tau_A(\sigma|a)}\right|_{\tau_A(\sigma|a)=p} &= 2n\left\{\sum_{m=n+1}^{2n-1}[f(m-1, p, 2n-1) - f(m, p, 2n-1)]\right. \\
&\qquad\qquad\qquad\qquad\qquad\qquad \left. + f(2n-1, p, 2n-1)\right\} \\
&= 2n\big\{[f(n, p, 2n-1) - f(2n-1, p, 2n-1)] + f(2n-1, p, 2n-1)\big\} \\
&= (2n)f(n, p, 2n-1).
\end{aligned}$$

Since $\rho_s(\sigma) = \binom{2n}{n}\tau_c(\sigma|s)^n(1-\tau_c(\sigma|s))^n$, it follows that

$$\begin{aligned}
\left.\frac{\partial p_s}{\partial\tau_c(\sigma|s)}\right|_{\tau_c(\sigma|s)=t} &= 2n[f(n-1; t, 2n-1) - f(n; t, 2n-1)] \\
&= 2n\binom{2n-1}{n-1}[p^{n-1}(1-p)^n - p^n(1-p)^{n-1}] \\
&= (1-2p)2n\binom{2n-1}{n-1}p^{n-1}(1-p)^{n-1}.
\end{aligned}$$

Therefore, the $\rho_s(\sigma)$ decreases as the vote share of the candidate with the most votes in state s increase. Combining yields

$$\begin{aligned}
\left.\frac{\partial[2\theta_a + \rho_a]}{\partial\tau_A(\sigma|a)}\right|_{\tau_A(\sigma|a)=p} &= (2n)[2f(n, p, 2n-1) + f(n-1; t, 2n-1) - f(n; t, 2n-1)] \\
&= (2n)[2f(n, p, 2n-1) + f(n-1; t, 2n-1)] > 0.
\end{aligned}$$

SB.2 *Algebra for the proof of Theorem 3*

Define $f_o(t, m) = \binom{2m+1}{m}t^m(1-t)^{m+1} + \binom{2m+1}{m+1}t^{m+1}(1-t)^m$ and $f_e(t, m) = \binom{2m}{m}t^m(1-t)^m$. Write

$$\rho_{A,s} + \rho_{B,s} = \sum_{i=0}^{n} f(2i; \tau_\varnothing, 2n)f_e(\tau_A^*, i) + \sum_{i=0}^{n-1} f(2i+1; \tau_\varnothing, 2n)\frac{1}{2}f_o(t, i).$$

Since $\binom{2m+1}{m} = \binom{2m+1}{m+1}$, $f_o(t, m)$ can be rewritten as $f_o(t, m) = \binom{2m+1}{m}t^m(1-t)^m$.

To see that $\rho_{A,s} + \rho_{B,s}$ decreases in $|\tau_A^* - \frac{1}{2}|$, note that $\frac{\partial f_e}{\partial t} = (1-2t)m^2\binom{2m}{m}t^{m-1} \times (1-t)^{m-1}$ and $\frac{\partial f_o}{\partial t} = (1-2t)m^2\binom{2m+1}{m}t^{m-1}(1-t)^{m-1}$, establishing the result.

I turn now to establishing that it increases in $\tau_\varnothing$. I first show that every term is increasing. To see $f_e(t,m) > \frac{1}{2} f_o(t,m)$,

$$\begin{aligned}\frac{2f_e(t,m)}{f_o(t,m)} &= \frac{2\binom{2m}{m} t^m (1-t)^m}{\binom{2m+1}{m} t^m (1-t)^m} \\ &= \frac{2\frac{(2m)!}{m!m!}}{\frac{(2m+1)!}{m!m+1!}} = \frac{2m+2}{2m+1} > 1,\end{aligned}$$

and to see $\frac{1}{2} f_o(t,m) > f_e(t,m+1)$,

$$\begin{aligned}\frac{f_o(t,m)}{2f_e(t,m+1)} &= \frac{\binom{2m+1}{m} t^m (1-t)^m}{2\binom{2m+2}{m+1} t^{m+1}(1-t)^{m+1}} \\ &= \frac{\frac{(2m+1)!}{m!(m+1)!}}{2\frac{(2m+2)!}{(m+2)!(m+1)!} t(1-t)} = \frac{(m+1)(m+2)}{2(2m+2)t(1-t)} \\ &> \frac{(m+1)(m+2)}{(2m+2)\frac{1}{2}} = m+2 > 1.\end{aligned}$$

To conclude that $\rho_{A,s} + \rho_{B,s}$ increases in $\tau^*_\varnothing$, use the following lemma.

Lemma SB.1. *If $f : [0,1] \to \mathbb{R}$ is a weakly increasing (decreasing) simple function and π first order stochastically dominates π', then $\int f\, d\pi \geq \int f\, d\pi'$ ($\int f\, d\pi \leq \int f\, d\pi'$).*

Proof. Consider a weakly increasing, simple function f. Proceed by induction on the number of distinct values of f, n. If f is constant, then $\int f\, d\pi = \int f\, d\pi'$. Now let $n \geq 2$ and suppose that $\int g\, d\pi \geq \int g\, d\pi'$ whenever g takes $n-1$ distinct values. Suppose f takes n distinct values, with values $x_1, x_2, \dots, x_n$ on the intervals $E_1, \dots, E_n$ (where $z_i \in E_i$ and $z_{i+1} \in E_{i+1}$ implies $z_i < z_{i+1}$). Then let g take values $x_1, x_2, \dots, x_{n-1}$ on the intervals $E_1, \dots, E_{n-1} \cup E_n$, respectively. Then $f = g + (x_n - x_{n-1})\chi_{E_n}$, where $\chi_E(z)$ equals 1 if $z \in E$ and 0 otherwise. By the induction hypothesis, $\int g\, d\pi \geq \int g\, d\pi'$ and

$$\int \chi_{E_n}\, d\pi = \pi(E_n) \geq \pi'(E_n) = \int \chi_{E_n}\, d\pi'$$

by first order stochastic domination (FOSD). Since

$$\begin{aligned}\int f\, d\pi &= \int (g + (x_n - x_{n-1})\chi_{E_n})\, d\pi \\ &= \int g\, d\pi + \int (x_n - x_{n-1})\chi_{(e_{n-1}]}\, d\pi \\ &\geq \int g\, d\pi' + \int (x_n - x_{n-1})\chi_{(e_{n-1}]}\, d\pi' = \int f\, d\pi',\end{aligned}$$

the result holds when f takes n distinct values. Consequently, it holds for all simple functions. Similar arguments establish the other part of the result when f is weakly decreasing rather than weakly increasing. □

To see that utility in a is larger than b if $(1-\underline{p}_t)/\underline{p}_t < (\rho_{Aa}+\rho_{Ba})/(\rho_{Bb}+\rho_{Ab})$ and a t-voter plays (A,α) if and only if $\alpha \le 1+(\theta_a-\theta_b)/(\rho_{A,a}+\rho_{A,b}) = \alpha^*_A$, note that

$$\begin{aligned}(1-\alpha)\rho_{A,a}+\theta_a &\ge \theta_b-(1-\alpha)\rho_{A,b}\\ -\alpha &\ge \frac{\theta_b-\theta_a}{\rho_{A,a}+\rho_{A,b}}-1\\ \alpha &\le 1+\frac{\theta_a-\theta_b}{\rho_{A,a}+\rho_{A,b}}=\alpha^*_A.\end{aligned}$$

To see that utility in a is larger than b if $(1-\bar{p}_t)/\bar{p}_t > (\rho_{Aa}+\rho_{Ba})/(\rho_{Bb}+\rho_{Ab})$ and a t-voter plays (B,α) if and only if $\alpha^*_B = 1+(\theta_b-\theta_a)/(\rho_{B,b}+\rho_{B,a}) \le \alpha$, note that

$$\begin{aligned}-(1-\alpha)\rho_{B,a}+\theta_a &\ge \theta_b+(1-\alpha)\rho_{B,b}\\ \theta_a-\theta_b &\ge (1-\alpha)[\rho_{B,b}+\rho_{B,a}]\\ \frac{\theta_a-\theta_b}{\rho_{B,b}+\rho_{B,a}}-1 &\ge -\alpha\\ \alpha^*_B=1+\frac{\theta_b-\theta_a}{\rho_{B,b}+\rho_{B,a}} &\le \alpha.\end{aligned}$$

To see that $U(p,m)$ increases in p if $p\ge \frac{1}{2}$, note that $F(k;p,m)=(m-k)\binom{m}{k}\times \int_0^{1-p} t^{m-k-1}(1-t)^k\,dt$ so $\frac{\partial F}{\partial k} = -(m-k)\binom{m}{k}(1-p)^{m-k}p^k$. This implies that $1-F(k;p,m)$ increases in p, immediately establishing the result if m is even. Note that $\frac{\partial f(m/2;p,m)}{\partial p} = \binom{m}{m/2}m^2p^{m-1}(1-p)^{m-1}(2p-1)$, which is also positive if $p\ge\frac{1}{2}$.

To see that $U(p,m)$ increases in m if $p>\frac{1}{2}$, consider $m>0$ and even. Then

$$\begin{aligned}U(p,m) &= 1-F\Big(\tfrac{1}{2}m;\,p,m\Big)+\tfrac{1}{2}f\Big(\tfrac{1}{2}m;\,p,m\Big)\\ &< 1-F\Big(\tfrac{1}{2}m;\,p,m\Big)+pf\Big(\tfrac{1}{2}m;\,p,m\Big)\\ &= (1-p)\Big(1-F\Big(\tfrac{1}{2}m;\,p,m\Big)\Big)+p\Big(1-F\Big(\tfrac{1}{2}m-1;\,p,m\Big)\Big)\\ &= 1-F\Big(\tfrac{1}{2}(m-1);\,p,m+1\Big)=U(p,m+1).\end{aligned}$$

Similarly,

$$\begin{aligned}U(p,m) &= 1-F\Big(\tfrac{1}{2}m;\,p,m\Big)+\tfrac{1}{2}f\Big(\tfrac{1}{2}m;\,p,m\Big)\\ &= (1-p)\Big[1-F\Big(\tfrac{1}{2}m;\,p,m-1\Big)\Big]+p\Big[1-F\Big(\tfrac{1}{2}m-1;\,p,m-1\Big)\Big]+\tfrac{1}{2}f\Big(\tfrac{1}{2}m;\,p,m\Big)\\ &> 1-F\Big(\tfrac{1}{2}m;\,p,m-1\Big)=U(p,m-1).\end{aligned}$$

Hence, $U(p,m)$ increases in m if $p>\frac{1}{2}$.

If expected winners are correct, applying Lemma SB.1 gives that $\theta_s(\sigma)$ is increasing in $\tau(\varnothing|s,\sigma)$ and increasing in $\tau(c_s|\sigma,s)/(\tau(A|\sigma,s)+\tau(B|\sigma,s))$.

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