Supplement to "Communication and influence"

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S1. Equilibrium equivalence

In the baseline version of the game, $\Gamma(\mathbf{D}, \mathbf{k}, \mathbf{s})$, agents invest in communication before they learn the value of their local state. In the alternative time line, $\Gamma_\theta(\mathbf{D}, \mathbf{k}, \mathbf{s})$ (mentioned at the end of Section 3), agents choose communication intensities after they observe their local states. In this section, we show that both versions of the game have the same perfect Bayesian equilibrium.

From Theorem 1, we know that the baseline game $\Gamma(\mathbf{D}, \mathbf{k}, \mathbf{s})$ has an equilibrium in linear decision functions

$$a_i = b_{ii}\theta_i + \sum_{j \neq i} b_{ij} y_{ij}.$$

We now show that the alternative version $\Gamma_\theta(\mathbf{D}, \mathbf{k}, \mathbf{s})$ has the exact same equilibrium.

Theorem 12. *The games $\Gamma(\mathbf{D}, \mathbf{k}, \mathbf{s})$ and $\Gamma_\theta(\mathbf{D}, \mathbf{k}, \mathbf{s})$ have the same equilibrium in linear strategies.*

Proof. We are going to show that both games have perfect Bayesian equilibria that satisfy the same four sets of conditions, which in turn correspond to the conditions used in the proof of Theorem 1:

$$D_i b_{ii} = d_{ii} + \sum_{j \neq i} d_{ij} b_{ji} \quad \text{for all } i \tag{19}$$

$$D_i b_{ij} = \frac{r_{ij} p_{ij}}{s_j r_{ij} + s_j p_{ij} + r_{ij} p_{ij}} \sum_{k \neq i} d_{ik} b_{kj} \quad \text{for all } i, j \neq i \tag{20}$$

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$$\frac{\sqrt{d_{ji}}b_{ij}}{k_r} = r_{ij} \quad \text{for all } i, j \neq i \tag{21}$$

$$\frac{\sqrt{D_i}b_{ij}}{k_p} = p_{ij} \quad \text{for all } i, j \neq i. \tag{22}$$

First, note that the second stage is identical in the two versions of the game. Agent i knows that he has chosen $(\tilde{r}_{ji})_{j\neq i}$ and $(\tilde{p}_{ij})_{j\neq i}$ (which may be different from the equilibrium values). He assumes that the other agents have chosen communication intensities according to the equilibrium values and that they will choose actions according to the equilibrium linear strategies. The first-order conditions are, therefore, still given by (8) and (9), which yield (19) and (20).

The only difference in the first stage is that i chooses $(\tilde{r}_{ji})_{j\neq i}$ and $(\tilde{p}_{ij})_{j\neq i}$ after observing θ_i. It is easy to see that the expression for the expected payoff (10) is now

$$\begin{aligned} -E[u_i] = {} & d_{ii}\left((\tilde{b}_{ii}-1)^2\theta_i^2 + \sum_{k\neq i}^{2} \tilde{b}_{ik}^2(\sigma_k + \rho_{ik} + \pi_{ik})\right) \\ & + \sum_{j\neq i} d_{ij}\left(\sum_k (\tilde{b}_{ik} - b_{jk})^2\sigma_k + \sum_{k\neq i}\tilde{b}_{ik}^2(\rho_{ik}+\pi_{ik}) + \sum_{k\neq j} b_{jk}^2(\rho_{jk}+\pi_{jk})\right) \\ & + k_r^2 \sum_{j\neq i}\frac{1}{\rho_{ji}} + k_p^2\sum_{j\neq i}\frac{1}{\pi_{ij}}. \end{aligned}$$

The only difference is that the term $(\tilde{b}_{ii}-1)^2\sigma_i$ is now $(\tilde{b}_{ii}-1)^2\theta_i^2$. But it easy to see that this does not affect the first-order conditions for communication intensities as that term is separate from $(\tilde{r}_{ji})_{j\neq i}$ and $(\tilde{p}_{ij})_{j\neq i}$. Hence the first-order conditions are unchanged, as in (21) and (22). □

This equivalence rests on two assumptions. One is that agents' payoffs are linear-quadratic. While agent i's incentive to coordinate with other agents depends on his local state of the world θ_i, his incentive to reduce the variance of the actions of the other agents does not. But, as the following proof shows, it is only the latter that is affected by unilateral deviations in communication investments. The other assumption is that signals have full support. While the normality assumption may not be essential, it is important that the support of y_{ij} does not depend on communication investment. If it did, a deviation from the equilibrium communication investment could be detectable and costly signaling may be unavoidable in equilibrium.

The question about uniqueness, which we mentioned for game $\Gamma(\mathbf{D}, \mathbf{k}, \mathbf{s})$ and which we discuss in Section S2, is present here as well.

S2. Uniqueness

One may wonder about the importance of the restriction to linear equilibria. Do $\Gamma(\mathbf{D}, \mathbf{k}, \mathbf{s})$ and $\Gamma_\theta(\mathbf{D}, \mathbf{k}, \mathbf{s})$ have equilibria where agents use strategies that are not linear in their signals? A similar question has arisen in other games with quadratic payoff functions, such as Morris and Shin (2007), Angeletos and Pavan (2007, 2009), Dewan and Myatt (2008), and Calvó-Armengol and de Martí Beltran (2009).

Uniqueness in the team-theoretic setting is proven in Marschak and Radner (1972, Theorem 5).

Calvó-Armengol and de Martí Beltran (2009) show that Marschak–Radner's line of proof extends to a strategic setting if the game admits a potential. Unfortunately, this does not apply to the game at hand ($\Gamma(\mathbf{D}, \mathbf{k}, \mathbf{s})$ has a potential only in the special case where $d_{ij} = d_{ji}$ for all pairs ij).

Angeletos and Pavan (2009) prove uniqueness by showing that in their economy the set of equilibria corresponds to the set of efficient allocations. A similar argument is used by Hellwig and Veldkamp (2009).

Dewan and Myatt (2008) prove uniqueness by restricting attention to strategies with nonexplosive higher-order expectations.

For our game, we can prove the following uniqueness result. Consider $\Gamma(\mathbf{D}, \mathbf{k}, \mathbf{s})$ but assume that local states and actions are bounded above and below. Namely, assume that $a_i \in [-\bar{a}, \bar{a}]$ and θ_i is distributed as a truncated normal distribution on $[-k\bar{a}, k\bar{a}]$, where $k < 1$. Call this new game $\Gamma^{\bar{a}}(\mathbf{D}, \mathbf{k}, \mathbf{s})$. We can show that as the bound $\bar{a}$ goes to infinity, the set of equilibria of the game $\Gamma^{\bar{a}}(\mathbf{D}, \mathbf{k}, \mathbf{s})$ contains (at most) one equilibrium and that this equilibrium corresponds to the linear equilibrium that we study here.

Consider the following variation of our game:

- Payoffs are the same as before.
- Local information is bounded: $\theta_i \in [-\bar{\theta}, \bar{\theta}]$, with $\bar{\theta} \in \mathbb{R}$, follows a *truncated* normal distribution[27] with mean 0 and precision s.
- The set of possible actions is bounded. In particular, $a_i \in [-\bar{a}, \bar{a}]$ for all i, where $\bar{a} = c\bar{\theta}$ for some $c \geq 1$. Note that this implies that $[-\bar{\theta}, \bar{\theta}] \subseteq [-\bar{a}, \bar{a}]$.
- Communication reports are defined as in text and, thus, are unbounded: $y_{ij} = \theta_i + \varepsilon_{ij} + \eta_{ij}$ with

$$\varepsilon_{ij} \sim \mathcal{N}(0, r_{ij})$$
$$\eta_{ij} \sim \mathcal{N}(0, p_{ij}).$$

Observe that as $\bar{\theta} \to +\infty$, we converge to our initial specification of the model.

We define the expectation operators $E_i[\cdot] = E[\cdot|\theta_i, \{y_{ij}\}_{j\neq i}]$ for every $i \in \{1, \dots, n\}$.

Lemma 13. *For any action profile* $(a_1, \dots, a_n)$, *we have that* $\omega_{ii}\theta_i + \sum_{j\neq i}\omega_{ij}E_i[a_j] \in [-\bar{a}, \bar{a}]$ *for all* i.

Proof. Just note that $E_i[a_j] \in [-\bar{a}, \bar{a}]$ for all i, j. Since $\theta_i \in [-\bar{\theta}, \bar{\theta}] \subset [-\bar{a}, \bar{a}]$ and $\sum_{i=1}^n \omega_{ij} = 1$, the linear combination $\omega_{ii}\theta_i + \sum_{j\neq i}\omega_{ij}E_i[a_j]$ must be in $[-\bar{a}, \bar{a}]$. □

[27] See, for example, Patel and Read (1996).

LEMMA 14. *The matrix* $\mathbf{\Omega}$ *with off-diagonal entries equal to* ω_{ij} *and diagonal entries equal to* 0 *is a contraction.*

PROOF. The Gerschgorin theorem says that all eigenvalues of a matrix $\mathbf{\Omega}$ are in the union of the sets

$$F_i = \left\{ \lambda \;\middle|\; |\lambda - \omega_{ii}| \le \sum_{j \neq i} |\omega_{ij}| \right\}.$$

In our case, $\omega_{ii} = 0$ and $\sum_{j \neq i} |\omega_{ij}| = 1 - d_{ii}/D_i$, and, hence, all eigenvalues have absolute value smaller than 1. This is the necessary and sufficient condition for $\mathbf{\Omega}$ being a contraction. □

PROPOSITION 15. *Given* $\bar{\theta}$, $\bar{a}$, *and* $(r_{ij}, p_{ij})_{i,j}$, *the game in which agents choose actions* $\{a_i\}_i$ *has a unique equilibrium.*

PROOF. Expected payoffs are

$$-E_i[u_i] = d_{ii}(a_i - \theta_i)^2 + \sum_{j \neq i} d_{ij}(a_i^2 - 2a_i E[a_j] + E[a_j^2]) - k_r^2 \sum_{j \neq i} r_{ji} - k_p^2 \sum_{j \neq i} p_{ij}.$$

Therefore, first-order conditions with respect to actions are

$$-\frac{\partial E_i[u_i]}{\partial a_i} = 2d_{ii}(a_i - \theta_i) + 2\sum_{j \neq i} d_{ij}(a_i - E_i[a_j]) = 0.$$

Given information sets $\{y_i\}_i$, individual actions satisfy Kuhn–Tucker's conditions. Thus, for each $i \in \{1, \dots, n\}$, either

$$a_i = \omega_{ii}\theta_i + \sum_{j \neq i} \omega_{ij} E_i[a_j]$$

or

$$a_i \in \{-\bar{a}, \bar{a}\}.$$

More precisely,

$$BR_i(a_{-i}) = \begin{cases} \omega_{ii}\theta_i + \sum_{j \neq i} \omega_{ij} E_i[a_j] & \text{if } \omega_{ii}\theta_i + \sum_{j \neq i} \omega_{ij} E_i[a_j] \in [-\bar{a}, \bar{a}] \\ \bar{a} & \text{if } \omega_{ii}\theta_i + \sum_{j \neq i} \omega_{ij} E_i[a_j] > \bar{a} \\ -\bar{a} & \text{if } \omega_{ii}\theta_i + \sum_{j \neq i} \omega_{ij} E_i[a_j] < -\bar{a}. \end{cases}$$

We can make use of Lemma 13 to show that, indeed,

$$BR_i(a_{-i}) = \omega_{ii}\theta_i + \sum_{j \neq i} \omega_{ij} E_i[a_j] \quad \text{for all } i.$$

Hence, equilibrium conditions become

$$a_i^* = \omega_{ii}\theta_i + \sum_{j \neq i} \omega_{ij} E_i[a_j^*], \quad i = 1, \dots, n. \tag{23}$$

Nesting these conditions, we get

$$\begin{aligned} a_i^* &= \omega_{ii}\theta_i + \sum_{j\neq i}\omega_{ij}E_i\Big[\omega_{jj}\theta_j + \sum_{k\neq j}\omega_{jk}E_j[a_k^*]\Big] \\ &= \underbrace{\omega_{ii}\theta_i + \sum_{j\neq i}\omega_{ij}\omega_{jj}E_i[\theta_j]}_{\text{expectations on primitives}} + \underbrace{\sum_{j\neq i}\sum_{k\neq j}\omega_{ij}\omega_{jk}E_iE_j[a_k^*]}_{\text{strategic interdependence}}. \end{aligned} \tag{24}$$

The last term in this expression allows for a new level of nestedness that we obtain by plugging (23) into (24):

$$\begin{aligned} a_i^* = &\underbrace{\omega_{ii}\theta_i + \sum_{j\neq i}\omega_{ij}\omega_{jj}E_i[\theta_j] + \sum_{j\neq i}\sum_{k\neq j}\omega_{ij}\omega_{jk}\omega_{kk}E_iE_j[\theta_k]}_{\text{expectations on primitives}} \\ &+ \underbrace{\sum_{j\neq i}\sum_{k\neq j}\sum_{s\neq k}\omega_{ij}\omega_{jk}\omega_{ks}E_iE_jE_k[a_s^*]}_{\text{strategic interdependence}}. \end{aligned}$$

Observe that, again, this last interdependence term allows for adding another level of nestedness and that we can keep repeating this nestedness procedure up to any level. In particular, if we repeat this l times, we obtain the expression

$$\begin{aligned} a_i^* = &\underbrace{\omega_{ii}\theta_i + \sum_{k\neq i}\omega_{ik}\omega_{kk}E_i[\theta_k] + \cdots + \sum_{i_1\neq i}\sum_{i_2\neq i_1}\cdots\sum_{i_l\neq i_{l-1}}\sum_{k\neq i_l}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{kk}E_iE_{i_1}\cdots E_{i_l}[\theta_k]}_{\text{expectations on primitives}} \\ &+ \underbrace{\sum_{i_1\neq i}\sum_{i_2\neq i_1}\cdots\sum_{i_l\neq i_{l-1}}\sum_{k\neq i_l}\sum_{s\neq k}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{ks}E_iE_{i_1}\cdots E_{i_l}E_k[a_s^*]}_{\text{strategic interdependence}}, \end{aligned}$$

where, $i_1, \ldots, i_l$ are indices that run from 1 to n.

We want to show that as $l \to +\infty$, this expression converges and, therefore, that the equilibrium is unique. We are going to show this in two steps:

(i) First, we are going to show that the limit when $l \to +\infty$ of *expectations on primitives* is bounded above and below. This ensures that the expression of expectations on primitives is well defined at the limit.

(ii) Second, we are going to show that the expression of *strategic interdependencies* vanishes when $l \to +\infty$.

The proof of both steps relies on Lemma 14.

To prove (i), first note that all expectations $E_i[\theta_k], E_iE_j[\theta_k], \ldots, E_iE_{i_1}\cdots E_{i_l}[\theta_k]$ are bounded above by $\bar{\theta}$ and bounded below by $-\bar{\theta}$. Then the expression

$$\sum_{k\neq i}\omega_{ik}\omega_{kk}E_i[\theta_k]+\sum_{j\neq i}\sum_{k\neq j}\omega_{ij}\omega_{jk}\omega_{kk}E_iE_j[\theta_k]+\cdots + \sum_{i_1}\cdots\sum_{i_l}\sum_{k\neq i_l}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{kk}E_iE_{i_1}\cdots E_{i_l}[\theta_k]$$

is bounded above by

$$\bar{\theta}\Bigg(\sum_{k\neq i}\omega_{ik}\omega_{kk}+\sum_{j\neq i}\sum_{k\neq j}\omega_{ij}\omega_{jk}\omega_{kk}+\cdots+\sum_{i_1}\cdots\sum_{i_l}\sum_{k\neq i_l}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{kk}\Bigg)$$

and is bounded below by

$$-\bar{\theta}\Bigg(\sum_{k\neq i}\omega_{ik}\omega_{kk}+\sum_{j\neq i}\sum_{k\neq j}\omega_{ij}\omega_{jk}\omega_{kk}+\cdots+\sum_{i_1}\cdots\sum_{i_l}\sum_{k\neq i_l}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{kk}\Bigg).$$

We can apply now the following result: the entry (i,j) of $\mathbf{\Omega}^l$, which we denote $\omega_{ij}^{[l]}$, is equal to $\sum_{i_1}\cdots\sum_{i_{l-1}}\omega_{i,i_1}\omega_{i_1,i_2}\cdots\omega_{i_{l-2},i_{l-1}}\omega_{i_{l-1},j}$. Hence,

$$\sum_{k\neq i}\omega_{ik}\omega_{kk}+\sum_{j\neq i}\sum_{k\neq j}\omega_{ij}\omega_{jk}\omega_{kk}+\cdots+\sum_{i_1}\cdots\sum_{i_l}\sum_{k\neq i_l}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{kk}=\omega_{kk}\sum_{j=1}^{l}\omega_{ik}^{[j]}.$$

The element $\sum_{j=1}^{l}\omega_{ik}^{[j]}$is the (i,k) entry of the matrix $\sum_{1\leq j\leq l}\mathbf{\Omega}^j$. A sufficient condition for the infinite sum $\sum_{j\geq 1}\mathbf{\Omega}^j$ to converge is that $\mathbf{\Omega}$ is a contraction. Thus, by Lemma 14, $\omega_{kk}\sum_{j=1}^{l}\omega_{ik}^{[j]}$ is bounded when $l\to+\infty$ and, hence, the expression of expectations on primitives is bounded too. This proves (i).

To prove (ii), first note that, trivially, $E_iE_{i_1}\cdots E_{i_l}E_k[a_s^*]$ is bounded above by $\bar{a}$ and below by $-\bar{a}$. Hence, the expression

$$\sum_{i_1}\cdots\sum_{i_l}\sum_{k\neq i_l}\sum_{s\neq k}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{ks}E_iE_{i_1}\cdots E_{i_l}E_k[a_s^*]$$

is bounded above by

$$\bar{a}\sum_{i_1}\cdots\sum_{i_l}\sum_{k\neq i_l}\sum_{s\neq k}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{ks}$$

and below by

$$-\bar{a}\sum_{i_1}\cdots\sum_{i_l}\sum_{k\neq i_l}\sum_{s\neq k}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{ks}.$$

Then, since $\sum_{i_1}\cdots\sum_{i_l}\sum_{k\neq i_l}\sum_{s\neq k}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{ks}=\sum_{s\neq k}\omega_{is}^{[l+1]}$ and $\omega_{is}^{[l+1]}\to 0$ when $l\to\infty$ for all $s=1,\dots,n$,[28] we can ensure that $\sum_s \omega_{is}^{[l+1]}\to 0$ when $l\to\infty$. Therefore, the upper and lower bounds of the strategic interdependencies term tend to 0 when $l\to\infty$. This proves (ii). □

Note that this proof does not require normality in our structure of communication reports. Any other information structure would not change the uniqueness result. Of course, it would change the shape of this equilibrium.

Proposition 16. *The unique equilibrium of the game when $\bar{\theta}\to+\infty$ (and, therefore, $\bar{a}\to+\infty$ too) is linear.*

Proof. The previous proposition states that the equilibrium for any given $\bar{\theta}$ and $\bar{a}$ is

$$a_i^* = \lim_{l\to+\infty}\Big\{\omega_{ii}\theta_i+\sum_{k\neq i}\omega_{ik}\omega_{kk}E_i[\theta_k]+\cdots + \sum_{i_1}\cdots\sum_{i_l}\sum_{k\neq i_l}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{kk}E_iE_{i_1}\cdots E_{i_l}[\theta_k]\Big\}.$$

We have to compute explicitly the expectations in the previous expression when $\bar{\theta}\to+\infty$. Observe that when $\bar{\theta}\to+\infty$, all θ_is probability distributions tend to the normal distribution with mean 0 and precision s. Bayesian updating with normal distributions takes a simple linear form. To be more precise, in our setup, since the mean of all prior distributions is equal to 0, we have that

$$E_i[\theta_j]=\alpha_{ij}y_{ij}\quad \text{for all } i\neq j$$
$$E_i[y_{jk}]=\beta_{ijk}y_{ik}\quad \text{for all } k\neq i\neq j\neq k$$

with $\alpha_{ij}\in[0,1]$ and $\beta_{ijk}\in[0,1]$ being constants that depend on the precisions $(r_{ij},p_{ij})_{i,j}$ chosen in the first stage of the game. Observe that this immediately implies that also higher-order expectations $E_iE_{i_1}\cdots E_{i_l}[\theta_k]$ are linear in $\{y_{ij}\}_{j\neq i}$. In particular, $E_iE_{i_1}\cdots E_{i_l}[\theta_k]=\varphi_{ik}^{[l]}y_{ik}$, where $\varphi_{ik}^{[l]}$ is a product of one α (in particular, of $\alpha_{i_l,k}$) and $l-1$ different β's. Note that $\varphi_{ik}^{[l]}\in[0,1]$ for all i,k,l. Therefore,

$$a_i^*=\omega_{ii}\theta_i+\sum_{k\neq i}\omega_{kk}\sum_{l=1}^{+\infty}\varphi_{ik}^{[l]}\omega_{ik}^{[l]}y_{ik}\quad \text{for all } i. \tag{25}$$

To show that this expression is well defined, we proceed as in the proof of Proposition 15. The expression $\sum_{l=1}^{+\infty}\varphi_{ik}^{[l]}\omega_{ik}^{[l]}$ is bounded below by 0 and above by $\sum_{l=1}^{+\infty}\omega_{ik}^{[l]}$. This last infinite sum is the entry (i,k) of the matrix $\sum_{l\geq 1}\mathbf{\Omega}^l$ that is well defined because $\mathbf{\Omega}$ is a contraction. Thus, we conclude that the expression in (25) is well defined for all players and is linear in $(\theta_i,\{y_{ij}\}_{j\neq i})$ for each $i\in\{1,\dots,n\}$. □

[28] This is, precisely, because $\sum_{l\geq 1}\mathbf{\Omega}^l$ converges.

S3. Precluding communication and transfers

In this section, we intend to demonstrate the existence of a strategic effect due to indirect interactions between agents. For example, an agent can inhibit a communication channel by paying some monetary transfer to the agents who would be involved in it. We illustrate this point with a simple three-agent numerical example.

Consider an organization formed by three agents with interaction matrix

$$\mathbf{D} = \begin{pmatrix} 0.3 & 0.3 & 1 \\ 0.3 & 0.3 & 1 \\ 1 & 100 & 1 \end{pmatrix}$$

and such that $s_i = 0.1$ for all i, $k_p = k_r = 0.01$. Agents 1 and 2 occupy an equivalent position inside the organization, and they want primarily to coordinate with agent 3. Instead, agent 3 shows a severe coordination motive with agent 2, compared with any other payoff externality.

When considering unrestricted communication, as we do in the main text, the final utilities of each agent are

$$u_1 = -7.0346, \qquad u_2 = -3.5932, \qquad u_3 = -17.789.$$

If, instead, we consider a setup with inhibited communication in which agents 1 and 2 cannot communicate with each other, some algebra shows that agents' utilities under this communication restriction are

$$u_1 = -11.446, \qquad u_2 = -6.1297, \qquad u_3 = -16.267.$$

Comparing utilities in both cases, one immediately observes that agent 3 benefits from inhibited communication in the communication lines among agents 1 and 2, while the first two agents are worse off. This suggests that there is room in this model to analyze monetary transfers among agents to limit information transmission. Of course, this would raise other strategic considerations, such as the enforcement of the agreements.

S4. Broadcasting

In our baseline game, communication is essentially bilateral. In particular, investments in active communication are link-specific. While this is true in certain circumstances, in other cases there may be economies of scale in active communication. Here we consider the polar opposite, where the investment in active communication that an agent makes affects all his links equally.

Each agent chooses a unique r_i, a common precision for active communication with all other players. This can be understood as an approximation to the analysis of broadcasting. When an agent broadcasts a signal, its quality is the same for all agents. Of course passive communication is still individually chosen. This could be the case of e-mail lists, where the sender is allowed to send a unique message to the organization as a whole, and it is at the discretion of each one of the receivers to attend to it. In our model, when the agent chooses the precision r_i, he determines the possible ambiguity in the message: if the signal is very precise, everybody is going to receive essentially the same common signal; if the signal is very noisy, the receiver needs to exert a high effort to decode this message.

Before proceeding to present and prove the characterization of the equilibrium in the broadcasting case, we need to introduce additional notation. Given a vector $\boldsymbol{\lambda}_{\cdot i}$, let

$$g_{ji}(\boldsymbol{\lambda}_{\cdot i}) = \begin{cases} \omega_{ii} & \text{if } i = j \\ -s_i(\frac{k_p}{\sqrt{D_j}} + \frac{k_r}{\lambda_{ij}}) & \text{otherwise.} \end{cases}$$

With this notation in mind, we can prove a variant of Theorem 1 for the broadcasting case.

Proposition 17. *For any* $(\mathbf{D}, \mathbf{s})$*, if* k_r *and* k_p *are sufficiently low, the game* $\tilde{\Gamma}(\mathbf{D}, \mathbf{k}, \mathbf{s})$ *has a unique pure-strategy equilibrium.*

(i) Decisions are given by

$$\mathbf{b}_{\cdot j} = (\mathbf{I} - \boldsymbol{\Omega})^{-1} \cdot \mathbf{g}_{\cdot j}(\boldsymbol{\lambda}_{\cdot j}) \quad \textit{for all } j,$$

where $\boldsymbol{\lambda}_{\cdot j}$ *is an endogenously determined vector with positive entries that satisfy* $\sum_{k \neq j} d_{jk} b_{kj}^2 = \lambda_{ij}^2 b_{ij}^2$.

(ii) Active communication is

$$r_i = \frac{\lambda_{ij} b_{ij}}{k_r} \quad \textit{for all } j.$$

(iii) Passive communication is

$$p_{ij} = \frac{\sqrt{D_i} b_{ij}}{k_p} \quad \textit{for all } i \neq j.$$

Proof. If agent i chooses a unique ρ_i, the set of first-order conditions is equal to

$$-\frac{1}{2}\frac{\partial E[u_i]}{\partial b_{ii}} = d_{ii}(b_{ii} - 1)\sigma_i + \sum_{k \neq i} d_{ij}(b_{ii} - b_{ji})\sigma_i = 0$$

$$-\frac{1}{2}\frac{\partial E[u_i]}{\partial b_{ij}} = d_{ii} b_{ij}(\sigma_j + \rho_j + \pi_{ij}) + \sum_{k \neq i} d_{ik}((b_{ij} - b_{kj})\sigma_j + b_{ij}\rho_j + b_{ij}\pi_{ij}) = 0$$

$$-\frac{\partial E[u_i]}{\partial \rho_{ij}} = \sum_{j \neq i} d_{ij} b_{ji}^2 + k_r^2 \left(\frac{1}{\rho_i}\right)^2 = 0$$

$$-\frac{\partial E[u_i]}{\partial \pi_{ij}} = D_i b_{ij}^2 + k_p^2 \left(\frac{1}{\pi_{ij}}\right)^2 = 0.$$

This set of first-order conditions is equivalent to

$$D_i b_{ii} = d_{ii} + \sum_{k \neq i} d_{ij} b_{ji}$$

$$D_i b_{ij} = \frac{\sigma_j}{\sigma_j + \rho_j + \pi_{ij}} \sum_{k \neq i} d_{ik} b_{kj} \tag{26}$$

$$\frac{\sqrt{\sum_{k\neq i} d_{ik} b_{ki}^2}}{k_r} = r_i$$

$$\frac{\sqrt{D_i} b_{ij}}{k_p} = p_{ij}. \tag{27}$$

Since

$$\frac{r_j p_{ij}}{s_j r_j + s_j p_{ij} + r_j p_{ij}} = \frac{\sigma_j}{\sigma_j + \rho_j + \pi_{ij}},$$

condition (26) becomes

$$D_i b_{ij} = \frac{r_j p_{ij}}{s_j r_j + s_j p_{ij} + r_j p_{ij}} \sum_{k \neq i} d_{ik} b_{kj}.$$

By permuting i and j in this last expression, we get

$$D_j b_{ji} = \frac{r_i p_{ji}}{s_i r_i + s_i p_{ji} + r_i p_{ji}} \sum_{k \neq i} d_{jk} b_{ki}. \tag{28}$$

Since

$$\frac{\sqrt{\sum_{k\neq i} d_{ik} b_{ki}^2}}{k_r} = r_i,$$

we can define an endogenous value λ_{ji} such that $\sqrt{\sum_{k\neq i} d_{ik} b_{ki}^2} = \lambda_{ji} b_{ji}$ for each $j \neq i$. In particular, it is the unique positive number such that $\sum_{k\neq i} d_{ik} b_{ki}^2 = \lambda_{ji}^2 b_{ji}^2$. Then the first-order condition associated to ρ_i can be rewritten as

$$\frac{\lambda_{ji} b_{ji}}{k_r} = r_i$$

for any $j \neq i$. Plugging this expression and (27) into (28), we get that

$$b_{ji} - \sum_{k \neq i} w_{jk} b_{ki} = -s_i \left(\frac{k_p}{\sqrt{D_j}} + \frac{k_r}{\lambda_{ji}} \right) \quad \text{for all } i$$

or, equivalently, in matrix form,

$$\mathbf{b}_{\cdot i} = (\mathbf{I} - \mathbf{\Omega})^{-1} \cdot \mathbf{g}(\boldsymbol{\lambda}_{\cdot i}). \qquad \square$$

Observe that the main difference in the equilibrium action of the broadcasting case as compared with that of Theorem 1 is the change from the vector $\mathbf{h}$ to the vector $\mathbf{g}(\boldsymbol{\lambda}_{\cdot i})$. The matrix that relates these vectors with the equilibrium actions $\mathbf{b}$ remains the same in both cases.

A natural question that arises with the analysis of this new communication protocol is whether we should expect that agents engage in more active communication than before. The following result gives us an answer in terms of the ratio of passive versus active communication already considered in a previous section.

PROPOSITION 18. *In the symmetric case, in which $d_{ij} = \bar{d}Q$ for all $i \neq j$ and $d_{ii} = (1-(n-1)\bar{d})Q$ for some $Q > 0$, the ratio of passive versus active communication is*

$$\frac{k_p}{k_r}\sqrt{(n-1)\bar{d}}.$$

PROOF. Because of symmetry, for all pairwise different i, j, k we have that $b_{ji} = b_{ki} = b^*$. Therefore,

$$\lambda_{ji} = \sqrt{\sum_{k \neq i} d_{ik}} = \sqrt{(n-1)\bar{d}Q}.$$

This implies that

$$\frac{\sqrt{(n-1)\bar{d}Q}b^*}{k_r} = r_i$$
$$\frac{\sqrt{Q}b^*}{k_p} = p_{ij}.$$

The ratio between active and passive communication in this case is

$$\frac{r_i}{p_{ij}} = \frac{k_p}{k_r}\sqrt{\frac{(n-1)\bar{d}Q}{Q}} = \frac{k_p}{k_r}\sqrt{(n-1)\bar{d}}. \qquad \square$$

Again, when active and passive communication are equally costly, i.e., $k_p = k_r$, the upper bound for this ratio is 1. Observe also, that the ratio in the case of broadcasting does not necessarily decreases when n increases. When $\bar{d} = 1/n$, we obtain that the ratio of active versus passive communication is $\sqrt{(n-1)/n}$. In that case, r_i/p_{ij} tends to 1 when n is large. In clear contrast to the case of pairwise communication, active and passive communication are almost equal when the number of agents is large.

S5. CORNER SOLUTIONS

It is natural to assume that communication intensities cannot be negative. In fact, we assume a small but positive lower bound ξ, to prevent equilibria based on coordination failure. The equilibrium characterization in Theorem 1 rests on the assumption that the communication cost parameters, k_p and k_r, are sufficiently low to guarantee that the lower bound ξ is nonbinding and the equilibrium can be described by unconstrained first-order conditions.

This section allows some or all of the lower-bound constraints to be binding. As one would expect, the solution of the unconstrained case can be extended with Kuhn–Tucker conditions. The resulting set of conditions is more complex than the baseline case, but still tractable—a fact that we illustrate through a numerical example.

We have to include formally in the analysis the inequality constraints

$$p_{ij} \geq \xi \quad \text{for all } i \neq j$$
$$r_{ij} \geq \xi \quad \text{for all } i \neq j.$$

The relevant terms in the Lagrangian that incorporates i's ex ante expected utility and the restrictions are

$$\begin{aligned}
\hat{u}_i &= -D_i \sum_{j\neq i} b_{ij}^2 \pi_{ij} - \sum_{j\neq i} d_{ij} b_{ji}^2 \pi_{ji} \\
&\qquad - k_p^2 \sum_{j\neq i} p_{ij} - k_r^2 \sum_{j\neq i} r_{ji} - \sum_{j\neq i} \lambda_{ij}(\xi - p_{ij}) - \sum_{j\neq i} \mu_{ji}(\xi - r_{ji}) \\
&= -D_i \sum_{j\neq i} b_{ij}^2 \left(\frac{1}{s_j} + \frac{1}{p_{ij}} + \frac{1}{r_{ij}}\right) - \sum_{j\neq i} d_{ij} b_{ji}^2 \left(\frac{1}{s_i} + \frac{1}{p_{ji}} + \frac{1}{r_{ji}}\right) \\
&\qquad - k_p^2 \sum_{j\neq i} p_{ij} - k_r^2 \sum_{j\neq i} r_{ji} - \sum_{j\neq i} \lambda_{ij}(\xi - p_{ij}) - \sum_{j\neq i} \mu_{ji}(\xi - r_{ji}),
\end{aligned}$$

where λ_{ij} and μ_{ij} are the multipliers for the constraint that involve p_{ij} and r_{ij}, respectively. It follows from the Kuhn–Tucker conditions that

$$D_i b_{ij}^2 \frac{1}{\xi^2} \leq k_p^2$$

whenever $p_{ij}^* = \xi$, and

$$d_{ij} b_{ji}^2 \frac{1}{\xi^2} \leq k_r^2$$

whenever $r_{ji}^* = \xi$. Otherwise both inequalities become equalities and we are in the case considered in the main text, where all communication precisions are strictly larger than ξ. Below, we provide sufficient conditions for this latter case.

For each channel, from individual j to individual i, there are four different possibilities depending on whether p_{ij} and/or r_{ij} are greater than or equal to ξ. However, first-order conditions at the second stage of the game determine that, in any case, the system of equations that relates the b's is given by

$$b_{ik} = \frac{p_{ik} r_{ik}}{p_{ik} r_{ik} + s_i p_{ik} + s_i r_{ik}} \sum_{j\neq i} \omega_{ij} b_{jk}.$$

First of all, we are going to check that the relation between the b's that results from the four different combinations of active and passive communication is in all cases linear. We know it for the case where both precisions are strictly larger than ξ.

If $p_{ik} = \xi$ and $r_{ik} > \xi$, we have that

$$b_{ik} = \frac{\xi \frac{\sqrt{d_{ki}}}{k_r} b_{ik}}{\xi \frac{\sqrt{d_{ki}}}{k_r} b_{ik} + s_i \xi + s_i \frac{\sqrt{d_{ki}}}{k_r} b_{ik}} \sum_{j\neq i} \omega_{ij} b_{jk},$$

which after some rearrangement is equal to

$$b_{ik} = -s_i \frac{\xi}{\xi + s_i} \frac{k_r}{\sqrt{d_{ki}}} + \frac{\xi}{\xi + s_i} \sum_{j \neq i} \omega_{ij} b_{jk}.$$

This again provides a linear relation.

When $p_{ik} > \xi$ and $r_{ik} = \xi$, we have that

$$b_{ik} = \frac{\xi \frac{\sqrt{D_i}}{k_p} b_{ik}}{\xi \frac{\sqrt{D_i}}{k_p} b_{ik} + s_i \xi + s_i \frac{\sqrt{D_i}}{k_p} b_{ik}} \sum_{j \neq i} \omega_{ij} b_{jk},$$

which leads to

$$b_{ik} = -s_i \frac{\xi}{\xi + s_i} \frac{k_p}{\sqrt{D_i}} + \frac{\xi}{\xi + s_i} \sum_{j \neq i} \omega_{ij} b_{jk}.$$

Again, we end up with a linear relation between the $b_{\cdot k}$ terms.

Finally, if both p_{ik} and r_{ik} are exactly equal to ξ, then

$$b_{ik} = \frac{\xi}{\xi + 2s_i} \sum_{j \neq i} \omega_{ij} b_{jk},$$

which is clearly linear in the entries $b_{\cdot k}$.

We can gather all these different types of linear relations in a single compact matrix-form expression. Formally, to check for corner equilibria, we have to proceed as follows:

- According to the four different combinations, we can distinguish four types of communication links: bidirectional (where both precisions are strictly larger than ξ), only active (where just the active precision of the communication link is strictly larger than ξ), only passive (where just the passive precision of the communication link is strictly larger than ξ), and mute (where both are equal to ξ).

- After some rearrangement, if necessary and without loss of generality, we can rewrite the system in blocks as

$$\begin{pmatrix} \mathbf{b}_{\cdot k}^{B} \\ \mathbf{b}_{\cdot k}^{OA} \\ \mathbf{b}_{\cdot k}^{OP} \\ \mathbf{b}_{\cdot k}^{M} \end{pmatrix} = \begin{pmatrix} I - \Omega_{B,B} & -\Omega_{B,OA} & -\Omega_{B,OP} & \Omega_{B,M} \\ -\frac{\xi}{\xi+s}\Omega_{OA,B} & I - \frac{\xi}{\xi+s}\Omega_{OA,OA} & -\frac{\xi}{\xi+s}\Omega_{OA,OP} & -\frac{\xi}{\xi+s}\Omega_{OA,M} \\ -\frac{\xi}{\xi+s}\Omega_{OP,B} & I - \frac{\xi}{\xi+s}\Omega_{OP,OA} & I - \frac{\xi}{\xi+s}\Omega_{OP,OP} & -\frac{\xi}{\xi+s}\Omega_{OP,M} \\ -\frac{\xi^2}{\xi^2+2s_i\xi}\Omega_{M,B} & -\frac{\xi^2}{\xi^2+2s_i\xi}\Omega_{M,OA} & -\frac{\xi^2}{\xi^2+2s_i\xi}\Omega_{M,OP} & I - \frac{\xi^2}{\xi^2+2s_i\xi}\Omega_{M,M} \end{pmatrix}^{-1} \cdot \mathbf{h}_{\cdot k},$$

  where individuals are partitioned according to the communication decisions between themselves and individual k. The entries of h_{jk} are the same as in the interior case considered in the main text for the *bidirectional*, $-s_i(\xi/(\xi+s_i))(k_r/\sqrt{d_{ki}})$ if the link is *only active*, $-s_i(\xi/(\xi+s_i))(k_p/\sqrt{D_i})$ if the link is *only passive*, and 0 if ij is *mute*.

- If the solution to the system satisfies the Kuhn–Tucker inequality conditions for the only active, only passive, and mute cases we have presented above, then we have found a corner equilibrium.

We now provide an example of a network that generates a corner equilibrium.

Consider the network

$$1 \leftrightarrow 2 \leftrightarrow 3,$$

where coordination concerns are given by $d_{12} = d_{32} = \beta$, $d_{21} = d_{23} = \gamma$, and $d_{13} = d_{31} = 0$. The adaptation concerns are $d_{11} = d_{22} = d_{33} = 1$. Intuitively, if player 2 has strong interests in coordinating with other players but players 1 and 3 have little interest in coordinating with each other, it seems that there should exist an equilibrium where only player 2 invests in passive communication and nobody else invests in communication, meaning that they fix all their communication decisions to ξ. Let us see.

- With regard to agent 2, consider communication with agent 1 (the case with agent 3 would be symmetric). In this case, agent 2 is passive and agent 3 is mute:

$$\begin{pmatrix} b_{11} \\ b_{21} \\ b_{31} \end{pmatrix} = \begin{pmatrix} 1 & -\frac{\beta}{1+\beta} & 0 \\ -\frac{\xi}{\xi+s}\frac{\gamma}{1+2\gamma} & 1 & -\frac{\xi}{\xi+s}\frac{\gamma}{1+2\gamma} \\ -0 & -\frac{\xi^2}{\xi^2+2s_i\xi}\frac{\beta}{1+\beta} & 1 \end{pmatrix}^{-1} \begin{pmatrix} \frac{1}{1+\beta} \\ -s\frac{\xi}{\xi+s}\frac{k_r}{\sqrt{1+2\gamma}} \\ 0 \end{pmatrix}.$$

If $k_r = k_p = 1$, $\beta = 0.1$, $\gamma = 1$, and $\xi = s = 0.5$,

$$\begin{pmatrix} b_{11} \\ b_{21} \\ b_{31} \end{pmatrix} = \begin{pmatrix} 1 & -\frac{0.1}{1.1} & 0 \\ -\frac{1}{6} & 1 & -\frac{1}{6} \\ -0 & -\frac{1}{3}\frac{0.1}{1.1} & 1 \end{pmatrix}^{-1} \begin{pmatrix} \frac{1}{1.1} \\ -\frac{0.5}{2}\frac{1}{\sqrt{3}} \\ 0 \end{pmatrix} : \begin{pmatrix} 0.90976 \\ 7.3256 \times 10^{-3} \\ 2.2199 \times 10^{-4} \end{pmatrix}.$$

Now for individual 2. In this case, nobody speaks or listens to him. All are mute. The system is very simple:

$$\begin{pmatrix} b_{12} \\ b_{22} \\ b_{32} \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{3}\frac{\beta}{1+\beta} & 0 \\ -\frac{\gamma}{1+2\gamma} & 1 & -\frac{\gamma}{1+2\gamma} \\ -0 & -\frac{1}{3}\frac{\beta}{1+\beta} & 1 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ \frac{1}{1+2\gamma} \\ 0 \end{pmatrix}.$$

In the numerical case considered before, this becomes

$$\begin{pmatrix} b_{12} \\ b_{22} \\ b_{32} \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{3}\frac{0.1}{1.1} & 0 \\ -\frac{1}{3} & 1 & -\frac{1}{3} \\ -0 & -\frac{1}{3}\frac{0.1}{1.1} & 1 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ \frac{1}{3} \\ 0 \end{pmatrix} = \begin{pmatrix} 1.0309 \times 10^{-2} \\ 0.34021 \\ 1.0309 \times 10^{-2} \end{pmatrix}.$$

The case for player 3 is symmetric to player 1.

The inequality Kuhn–Tucker conditions are

$$D_i b_{ij}^2 \frac{1}{\xi^2} \le k_p^2 \quad \text{whenever } p_{ij}^* = \xi$$

$$d_{ij} b_{ji}^2 \frac{1}{\xi^2} \le k_r^2 \quad \text{whenever } r_{ji}^* = \xi.$$

Given the numbers above, these are equivalent to

$$D_i b_{ij}^2 \le 0.25 \quad \text{whenever } p_{ij}^* = \xi$$
$$d_{ij} b_{ji}^2 \le 0.25 \quad \text{whenever } r_{ji}^* = \xi.$$

In particular, these apply to p_{12},

$$1.1(1.0309 \times 10^{-2})^2 \le 0.25$$

(this one holds), and also apply to p_{13},

$$1.1(2.2199 \times 10^{-4})^2 \le 0.25$$

(this one also holds).

It should not hold for agent 2,

$$3(0.90976)^2 > 0.25 \quad (\text{ok}).$$

And it should apply to all active, since we are assuming the only communication larger than ξ is that agent 2 passively communicates with the others:

$$d_{ij} b_{ji}^2 \frac{1}{\xi^2} \le k_r^2.$$

With relations between agents 1 and 3, it is clear because $d_{13} = d_{31} = 0$. Between agents 1 and 2,

$$r_{21}: \quad 0.1(0.90976)^2 \le 0.25 \quad (\text{ok})$$
$$r_{12}: \quad 1(1.0309 \times 10^{-2}) \le 0.25 \quad (\text{ok}).$$

Therefore, all inequalities in the example above work and we get a case with a corner equilibrium.

S6. Adaptation, coordination, and communication

At least since Marschak and Radner (1972), organizational economics has highlighted the trade-off between adaptation and coordination. On the one hand, agents want to adapt to local information. On the other hand, they want to coordinate with the rest of the agents. One of the advantages of the quadratic setup adopted by mostly team-theoretic work is that the relative strength of coordination and adaptation is represented in a simple parametric way and can be used for comparative statics purposes (e.g., Dessein and Santos 2006).

A natural question in our setting is how communication investment varies with the relative importance of adaptation and coordination. As we shall see, the relation is non-monotonic: communication is maximal when the two concerns are balanced.

In our model, adaptation and coordination concerns are captured, respectively, by d_{ii} and d_{ij} (with $i \ne j$). We can thus analyze how different weights on these two concerns affect communication and influence. As we shall see, the relation is nonmonotonic. Define $d'_{ii} = \lambda d_{ii}$ and $d'_{ij} = (2 - \lambda) d_{ij}$ for all $j \ne i$. We make the following observations:

- If $\lambda \to 0$, then $d'_{ii} \to 0$ and $d'_{ij} \to 2d_{ij} \geq 0$, and $d'_{ii}/d'_{ij} \to 0$. Coordination outweighs adaptation.
- If $\lambda = 1$, we have the initial vector of i's interaction terms d.
- If $\lambda \to 2$, we obtain $d'_{ii} = 2d_{ii} > 0$ and $d'_{ij} \to 0$, and $d'_{ii}/d'_{ij} \to +\infty$. Adaptation outweighs coordination.

Proposition 19. *If $\lambda \to 0$ or $\lambda \to 2$, agent i does not engage in active communication and no agent passively communicates with him, i.e., $r_{ji} = p_{ij} = \xi$ for all j.*

Proof. (i) If $\lambda = 0$, then $\omega_{ii} = 0$, and this immediately implies that we hit a boundary equilibrium in which $b_{ji} = 0$ for all j. This implies that agent i is not going to put effort into actively communicating with agent j and that agent j is not going to exert any kind of effort in passive communication to learn about agent i's state of the world.

(ii) If $\lambda \to 2$, the matrix $\mathbf{\Omega}$ tends to $\mathbf{\Omega}'$, where $\mathbf{\Omega}'$ is equal to $\mathbf{\Omega}$ except that row i's entries in $\mathbf{\Omega}'$ are equal to 0. Also

$$h'_{ji} = \begin{cases} w'_{ii} \to 1 & \text{if } i = j \\ -s_i\left(\frac{k_p}{\sqrt{D_j}} + \frac{k_r}{\sqrt{d'_{ij}}}\right) \to -\infty & \text{otherwise.} \end{cases}$$

It is easy to see that the nonnegative matrix $(\mathbf{I} - \mathbf{\Omega}')^{-1}$ satisfies that all entries in row i are also equal to 0, except for $(\mathbf{I} - \mathbf{\Omega}')^{-1}_{ii} = 1$. Hence, following our equilibrium characterization, the elements b'_{ji} would satisfy that, when $\lambda \to 2$, $b'_{ii} \to 1$ and $b'_{ji} \to -\infty$ if $j \neq i$. But this implies that we hit an equilibrium in the boundary that satisfies $b'_{ji} = 0$ for all $j \neq i$. Therefore, again there is neither passive communication by agent j nor active communication by agent i. □

The reasons why communication vanishes when we approach the two extreme situations is different in each case. When coordination motives outweigh the adaptation motive, communication engagement is null because there is a natural focal point that resolves coordination problems: agents, according to prior information, fix their actions to be 0. This trivially resolves coordination and does not affect the decision problem that right now is of negligible magnitude. Local information is unnecessary.

Alternatively, when adaptation outweighs coordination, agents primarily want to resolve their respective local decision problems. The obvious way is to determine their action close to the local information they possess.

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