Supplementary Material

Supplement to "Strategy-proofness and efficiency with non-quasi-linear preferences: A characterization of minimum price Walrasian rule"

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In this supplement, we provide the proofs that we omitted from the main paper. In Appendix D, we provide the proof of Fact 4 in Section 3. The proof is the same as Mishra and Talman's (2010), but we provide it for completeness. Fact 5 is already shown by Demange and Gale (1985) and Roth and Sotomayor (1990). For completeness, we also give the proof of Fact 5 in Appendix E.

Appendix D: Proof of Fact 4

The following theorem is used to prove Fact 4.

Hall's theorem (Hall 1935). *Let $N \equiv \{1, \dots, n\}$ and $M \equiv \{1, \dots, m\}$. For each $i \in N$, let $D_i \subseteq M$. Then there is a one-to-one mapping x' from N to M such that for each $i \in N$, $x'(i) \in D_i$ if and only if for each $N' \subseteq N$, $|\bigcup_{i \in N'} D_i| \geq |N'|$.*

Fact 4 (Mishra and Talman 2010). *Let $\mathcal{R} \subseteq \mathcal{R}^E$ and $R \in \mathcal{R}^n$. A price vector p is a Walrasian equilibrium price vector for R if and only if no set is overdemanded and no set is underdemanded at p for R.*

Proof. "*Only if.*" Let $p \in P(R)$. Then there is an allocation $z = (x_i, t_i)_{i \in N}$ satisfying conditions (WE-i) and (WE-ii) in Definition 3. Let $M' \subseteq M$.

We show that M' is not overdemanded at p for R. Let $N' \equiv \{i \in N : D(R_i, p) \subseteq M'\}$. Since for each $i \in N'$, $x_i \in D(R_i, p) \subseteq M'$, and each real object is consumed by at most one agent, $|N'| = |\{x_i : i \in N'\}|$. Since $\{x_i : i \in N'\} \subseteq M'$, $|\{x_i : i \in N'\}| \leq |M'|$. Thus, $|N'| \leq |M'|$.

We show that M' is not underdemanded at p for R. Let $N' \equiv \{i \in N : D(R_i, p) \cap M' \neq \varnothing\}$. Suppose that for each $x \in M'$, $p^x > 0$ and $|N'| < |M'|$. Note that

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$|N'| < |M'|$ implies that there is $x \in M'$ such that for all $i \in N$, $x_i \neq x$. Then condition (WE-ii) implies that $p^x = 0$. This is a contradiction. Thus, $|N'| \geq |M'|$.

"*If.*" Assume that no set is overdemanded and no set is underdemanded at p for R.

Let $Z^* \equiv \{z = (x_i, t_i)_{i \in N} \in Z : \text{for each } i \in N, x_i \in D(R_i, p) \text{ and } t_i = p^{x_i}\}$. First, we show $Z^* \neq \varnothing$. Suppose that there is $N' \subseteq N$ such that for each $i \in N'$, $0 \notin D(R_i, p)$ and $|\{\bigcup_{i \in N'} D(R_i, p)\}| < |N'|$. Then $\{\bigcup_{i \in N'} D(R_i, p)\}$ is overdemanded at p for R. Thus, for each $N' \subseteq N$, if for each $i \in N'$, $0 \notin D(R_i, p)$, then $|\{\bigcup_{i \in N'} D(R_i, p)\}| \geq |N'|$. Then, by Hall's theorem, there is $z' \in Z$ such that for each $i \in N$, if $0 \notin D(R_i, p)$, then $x'_i \in D(R_i, p)$ and $t'_i = p^{x'_i}$. Thus, $Z^* \neq \varnothing$.

By the definition of Z^*, for each $z \in Z^*$, (z, p) satisfies (WE-i). We show that there is $z \in Z^*$ such that (z, p) satisfies (WE-ii). Let $M^+(p) \equiv \{x \in M : p^x > 0\}$. Let

$$z \in \underset{z' \in Z^*}{\arg\max} \left|\{y \in M^+(p) : \text{for some } i \in N, x'_i = y\}\right|, \tag{1}$$

that is, z maximizes over Z^* the number of objects in $M^+(p)$ that are assigned to some agents. Then, by the definition of Z^*, (z, p) satisfies (WE-i).

Let $M^0 \equiv \{y \in M^+(p) : \text{for each } i \in N, x_i \neq y\}$. Note that if $M^0 = \varnothing$, then (z, p) also satisfies (WE-ii). Thus, we show that $M^0 = \varnothing$. By contradiction, suppose that $M^0 \neq \varnothing$.

Let $N^0 \equiv \{i \in N : D(R_i, p) \cap M^0 \neq \varnothing\}$. For each $k = 1, 2, \ldots$, let $M^k \equiv \{y \in M : \text{for some } i \in N^{k-1}, x_i = y\}$ and $N^k \equiv \{i \in N : D(R_i, p) \cap M^k \neq \varnothing\} \setminus \{\bigcup_{k'=0}^{k-1} N^{k'}\}$. We claim by induction that for each $k \geq 0$, $M^k \subseteq M^+(p)$ and $N^k \neq \varnothing$.

Induction argument.

Step 1. By the definition of M^0, $M^0 \subseteq M^+(p)$. Since M^0 is not underdemanded at p for R, $|N^0| \geq |M^0|$. Thus, $M^0 \neq \varnothing$ implies that $N^0 \neq \varnothing$.

Step 2. Let $K \geq 1$. As induction hypothesis, assume that for each $k \leq K-1$, $M^k \subseteq M^+(p)$ and $N^k \neq \varnothing$.

First, we show that $M^K \subseteq M^+(p)$. Suppose that there is $x \in M^K \setminus M^+(p)$. Then $p^x = 0$. By the induction hypothesis, there is a sequence $\{x(s), i(s)\}_{s=1}^K$ such that

$$\begin{aligned}
&x(1) = x, \qquad x_{i(1)} = x(1)\\
&x(2) \in D(R_{i(1)}, p) \cap M^{K-1}, \qquad x_{i(2)} = x(2)\\
&x(3) \in D(R_{i(2)}, p) \cap M^{K-2}, \qquad x_{i(3)} = x(3)\\
&\qquad \vdots\\
&x(K) \in D(R_{i(K-1)}, p) \cap M^1, \qquad x_{i(K)} = x(K).
\end{aligned}$$

Let $x(K+1) \in D(R_{i(K)}, p) \cap M^0$. For each $s \in \{1, 2, \ldots, K\}$, let $z'_{i(s)} \equiv (x_{i(s+1)}, p^{x_{i(s+1)}})$, and for each $j \in N \setminus \{i(s)\}_{s=1}^K$, let $z'_j \equiv z_j$. Then $z' \in Z^*$ and

$$\left|\{y \in M^+(p) : \text{for some } i \in N, x'_i = y\}\right| = \left|\{y \in M^+(p) : \text{for some } i \in N, x_i = y\}\right| + 1.$$

This is a contradiction to (1). Thus, $M^K \subseteq M^+(p)$.

Next, we show that $N^K \neq \varnothing$. By $M^K \subseteq M^+(p)$ and the induction hypothesis, $\bigcup_{k=0}^{K} M^k \subseteq M^+(p)$. Thus, since $\bigcup_{k=0}^{K} M^k$ is not underdemanded at p for R,

$$\left|\bigcup_{k=0}^{K} N^k\right| \geq \left|\bigcup_{k=0}^{K} M^k\right|. \tag{2}$$

By the definition of M^k and N^k, for each $k, k' \in \{0, 1, \dots, K\}$ with $k \neq k'$, $N^k \cap N^{k'} = \varnothing$, which also implies that $M^k \cap M^{k'} = \varnothing$. Thus,

$$\left|\bigcup_{k=0}^{K} N^k\right| = \sum_{k=0}^{K} |N^k| \quad \text{and} \quad \left|\bigcup_{k=0}^{K} M^k\right| = \sum_{k=0}^{K} |M^k|.$$

Then, by (2),

$$\sum_{k=0}^{K-1} |N^k| + |N^K| = \sum_{k=0}^{K} |N^k| \geq \sum_{k=0}^{K} |M^k| = \sum_{k=1}^{K} |M^k| + |M^0|. \tag{3}$$

For each $k \geq 1$, by $M^k \subseteq M^+(p)$, $|M^k| = |N^{k-1}|$. Thus, $\sum_{k=0}^{K-1} |N^k| = \sum_{k=1}^{K} |M^k|$. Then, by (3),

$$|N^K| \geq |M^0|.$$

Thus, by $M^0 \neq \varnothing$, $|N^K| \geq 1$ and so $N^K \neq \varnothing$.

Since $M^+(p)$ is finite, by the above induction argument, for large K, $|\bigcup_{k=0}^{K} M^k| = \sum_{k=0}^{K} |M^k| > |M^+(p)|$. Since $\bigcup_{k=0}^{K} M^k \subseteq M^+(p)$, this is impossible. □

Appendix E: Proof of Fact 5

Let $\mathcal{R} \subseteq \mathcal{R}^E$.

Lemma 15. *Let $i \in N$ and $R_i \in \mathcal{R}$. Let $p, q \in \mathbb{R}_+^m$ and $x, y \in L$ be such that $x \in D(R_i, p)$ and $(y, q^y)\ P_i\ (x, p^x)$. Then $y \in M$ and $q^y < p^y$.*

Proof. Since $(y, q^y)\ P_i\ (x, p^x)$ and $x \in D(R_i, p)$, we have $(y, q^y)\ P_i\ (x, p^x)\ R_i\ \mathbf{0}$. Thus, $y \in M$. Also, by $x \in D(R_i, p)$, $(y, q^y)\ P_i\ (x, p^x)\ R_i\ (y, p^y)$. Thus, $(y, q^y)\ P_i\ (y, p^y)$ implies $q^y < p^y$. □

Given $R, R' \in \mathcal{R}^n$, $(z, p) \in W(R)$, and $(z', p') \in W(R')$, let

$$N^1 \equiv \{i \in N : z_i'\ P_i\ z_i\}, \qquad M^2 \equiv \{x \in M : p^x > p'^x\}$$

$$X^1 \equiv \{x \in L : \text{for some } i \in N^1, x_i = x\}, \quad \text{and} \quad X'^1 \equiv \{x \in L : \text{for some } i \in N^1, x_i' = x\}.$$

Lemma 16 (Decomposition (Demange and Gale 1985)). *Let $R \in \mathcal{R}^n$ and $(z, p) \in W(R)$. Let R' be a d-truncation of R such that for each $i \in N$, $d_i \leq -CV_i(0; z_i)$, and let $(z', p') \in W(R')$. Then $X^1 = X'^1 = M^2$.*

PROOF. First, we show $X'^1 \subseteq M^2$. Let $x \in X'^1$. Then there is $i \in N^1$ such that $x'_i = x$. By $i \in N^1$, $(x'_i, p'^{x'_i})\, P_i\, (x_i, p^{x_i})$. Thus, by $x_i \in D(R_i, p)$ and Lemma 15, $x'_i \in M$ and $p'^{x'_i} < p^{x'_i}$, and so $x = x'_i \in M^2$. Thus, $X'^1 \subseteq M^2$.

Next we show $M^2 \subseteq X^1$. Let $x \in M^2$. Then $x \in M$ and $0 \leq p'^x < p^x$. Thus, by (WE-ii), there is $i \in N$ such that $x_i = x$. By $d_i \leq -CV_i(0; z_i)$ and Lemma 2(ii), $(x'_i, p'^{x'_i})\, P_i\, (x_i, p^{x_i})$. Thus, $i \in N^1$ and so $x = x_i \in X^1$. Thus, $M^2 \subseteq X^1$.

Note that by the definition of X^1 and X'^1, $|X^1| \leq |N^1|$ and $|X'^1| \leq |N^1|$. Since $X'^1 \subseteq M^2 \subseteq M$, each agent in N^1 receives a different object and so $|X'^1| = |N^1| \geq |X^1|$. Since $X'^1 \subseteq M^2 \subseteq X^1$, $|X'^1| \leq |M^2| \leq |X^1|$. Thus, $|X'^1| = |M^2| = |X^1|$. By $|X'^1| = |M^2|$ and $X'^1 \subseteq M^2$, $X'^1 = M^2$. By $|M^2| = |X^1|$ and $M^2 \subseteq X^1$, $M^2 = X^1$. □

LEMMA 17 (Lattice Structure (Demange and Gale 1985)). *Let $R \in \mathcal{R}^n$ and $(z, p) \in W(R)$. Let R' be a d-truncation of R such that for each $i \in N$, $d_i \leq -CV_i(0; z_i)$, and let $(z', p') \in W(R')$. Then (i) $\hat{p} \equiv p \wedge p' \in P(R)$ and (ii) $\bar{p} \equiv p \vee p' \in P(R')$.*[1]

PROOF. Let $N^1 \equiv \{i \in N : z'_i\, P_i\, z_i\}$ and $M^2 \equiv \{x \in M : p^x > p'^x\}$.

(i) Let $\hat{z}$ be defined by setting for each $i \in N^1$, $\hat{z}_i \equiv z'_i$, and for each $i \in N \setminus N^1$, $\hat{z}_i \equiv z_i$. We show that $(\hat{z}, \hat{p}) \in W(R)$.

STEP 1. *We have that $(\hat{z}, \hat{p})$ satisfies (WE-i).*

Let $i \in N$ and $x \in L$. In the following two cases, we show $(\hat{x}_i, \hat{p}^{\hat{x}_i})\, R_i\, (x, \hat{p}^x)$, which implies $\hat{x}_i \in D(R_i, \hat{p})$.

CASE 1. $i \in N^1$. By $\hat{x}_i = x'_i$ and Lemma 16, $\hat{x}_i \in M^2$, and so $\hat{x}_i \in M$ and $p'^{\hat{x}_i} < p^{\hat{x}_i}$. Thus, $\hat{p}^{\hat{x}_i} = p'^{\hat{x}_i}$.

First, assume that $x \in M^2$. Then, by $\hat{p}^x = p'^x$,

$$(\hat{x}_i, \hat{p}^{\hat{x}_i}) = z'_i \underset{x'_i \in D(R'_i, p')}{R'_i} (x, p'^x) = (x, \hat{p}^x).$$

Since R'_i is a d_i-truncation of R_i, $\hat{x}_i \neq 0$ and $x \neq 0$, Remark 1(i) implies $(\hat{x}_i, \hat{p}^{\hat{x}_i})\, R_i\, (x, \hat{p}^x)$.

Next, assume that $x \notin M^2$. Then, by $\hat{p}^x = p^x$,

$$(\hat{x}_i, \hat{p}^{\hat{x}_i}) = z'_i \underset{i \in N^1}{P_i} z_i \underset{x_i \in D(R_i, p)}{R_i} (x, p^x) = (x, \hat{p}^x).$$

CASE 2. $i \notin N^1$. By $\hat{x}_i = x_i$ and Lemma 16, $\hat{x}_i \notin M^2$. Thus, $p^{\hat{x}_i} \leq p'^{\hat{x}_i}$ or $\hat{x}_i = 0$. First, we assume that $x \in M^2$. Then $\hat{p}^x = p'^x$. Note that $i \notin N^1$ implies $(\hat{x}_i, \hat{p}^{\hat{x}_i}) = z_i\, R_i\, z'_i$.

CASE 2.1. $x'_i \neq 0$. By $x'_i \in D(R'_i, p')$, $z'_i\, R'_i\, (x, p'^x) = (x, \hat{p}^x)$. Since R'_i is a d_i-truncation of R_i, $x'_i \neq 0$, and $x \neq 0$, Remark 1(i) implies $z'_i\, R_i\, (x, p'^x)$. Thus,

$$(\hat{x}_i, \hat{p}^{\hat{x}_i}) = z_i\, R_i\, z'_i\, R_i\, (x, p'^x) = (x, \hat{p}^x).$$

[1] Denote $p \wedge p' \equiv (\min\{p^x, p'^x\})_{x \in M}$ and $p \vee p' \equiv (\max\{p^x, p'^x\})_{x \in M}$.

CASE 2.2. $x'_i = 0$. Note $z'_i = \mathbf{0}$. Since $x'_i \in D(R'_i, p')$, $CV'_i(x; \mathbf{0}) \le p'^x$. Thus, if $CV_i(x; \mathbf{0}) \le CV'_i(x; \mathbf{0})$, then $z'_i \, R_i \, (x, p'^x)$, which implies that

$$(\hat{x}_i, \hat{p}^{\hat{x}_i}) = z_i \, R_i \, z'_i \, R_i \, (x, p'^x) = (x, \hat{p}^x).$$

Next, assume that $CV_i(x; \mathbf{0}) > CV'_i(x; \mathbf{0})$. Then, since R'_i is a d_i-truncation of R_i, $d_i > 0$, which implies that $x_i \neq 0$.[2] Then, by $d_i \le -CV_i(0; z_i)$, $CV_i(x; z_i) \le CV'_i(x; \mathbf{0}) \le p'^x$, which implies that $z_i \, R_i \, (x, p'^x)$. Thus,

$$(\hat{x}_i, \hat{p}^{\hat{x}_i}) = z_i \, R_i \, y(x, p'^x) = (x, \hat{p}^x).$$

Next assume that $x \notin M^2$. Then $\hat{p}^x = p^x$. Since $\hat{x}_i = x_i \in D(R_i, p)$,

$$(\hat{x}_i, \hat{p}^{\hat{x}_i}) = z_i \, R_i \, (x, p^x) = (x, \hat{p}^x).$$

STEP 2. *We have that* $(\hat{z}, \hat{p})$ *satisfies (WE-ii).*

Let $x \in M$ be such that $\hat{p}^x > 0$. We show that there is $i \in N$ such that $\hat{x}_i = x$. Since $\hat{p} = p \wedge p'$, $\hat{p}^x > 0$ implies $p^x > 0$ and $p'^x > 0$.

CASE 1. $x \in M^2$. By Lemma 16, there is $i \in N^1$ such that $x'_i = x$. Since $i \in N^1$, by construction of $\hat{z}$, $\hat{x}_i = x'_i$. Thus, $\hat{x}_i = x$.

CASE 2. $x \notin M^2$. By $p^x > 0$, there is $i \in N$ such that $x_i = x$. By Lemma 16, $i \notin N^1$. Thus, $\hat{x}_i = x_i$, and so $\hat{x}_i = x$.

(ii) Let $\bar{z}$ be defined by setting for each $i \in N^1$, $\bar{z}_i \equiv z_i$, and for each $i \in N \setminus N^1$, $\bar{z}_i \equiv z'_i$. We show $(\bar{z}, \bar{p}) \in W(R')$.

STEP 1. *We have that* $(\bar{z}, \bar{p})$ *satisfies (WE-i).*

Let $i \in N$ and $x \in L$. In the following two cases, we show $(\bar{x}_i, \bar{p}^{\bar{x}_i}) \, R'_i \, (x, \bar{p}^x)$, which implies $\bar{x}_i \in D(R'_i, \bar{p})$.

CASE 1. $i \in N^1$. By $\bar{x}_i = x_i$ and Lemma 16, $\bar{x}_i \in M^2$, and so $\bar{x}_i \in M$ and $p'^{\bar{x}_i} < p^{\bar{x}_i}$. Thus, $\bar{p}^{\bar{x}_i} = p^{\bar{x}_i}$. First assume that $x \in M^2$. Since $\bar{x}_i = x_i \in D(R_i, p)$ and $\bar{p}^x = p^x$,

$$(\bar{x}_i, \bar{p}^{\bar{x}_i}) = z_i \, R_i \, (x, p^x) = (x, \bar{p}^x).$$

Since R'_i is a d_i-truncation of R_i, $\bar{x}_i \neq 0$, and $x \neq 0$, Remark 1(i) implies $(\bar{x}_i, \bar{p}^{\bar{x}_i}) \, R'_i \, (x, \bar{p}^x)$.

Next, assume that $x \notin M^2$. Then $p^x \le p'^x$ or $x = 0$.

CASE 1.1. $x \neq 0$. Since $\bar{x}_i = x_i \in D(R_i, p)$ and $\bar{p}^x = p'^x \ge p^x$,

$$(\bar{x}_i, \bar{p}^{\bar{x}_i}) = z_i \, R_i \, y(x, p^x) \, R_i \, (x, \bar{p}^x).$$

Since R'_i is a d_i-truncation of R_i and $\bar{x}_i \neq 0$, $(\bar{x}_i, \bar{p}^{\bar{x}_i}) \, R'_i \, (x, \bar{p}^x)$.

[2] To see this, suppose that $x_i = 0$. Then $d_i \le -CV_i(0; z_i) = 0$, which contradicts $d_i > 0$.

Case 1.2. $x = 0$. Since R'_i is a d_i-truncation of R_i and $d_i \leq -CV_i(0; z_i)$,

$$(\bar{x}_i, \bar{p}^{\bar{x}_i}) = z_i \, R'_i \, \mathbf{0} = (x, \bar{p}^x).$$

Case 2. $i \notin N^1$. By $\bar{x}_i = x'_i$ and Lemma 16, $\bar{x}_i \notin M^2$. Thus, $p^{\bar{x}_i} \leq p'^{\bar{x}_i}$ or $\bar{x}_i = 0$. If $\bar{x}_i = 0$,

$$(\bar{x}_i, \bar{p}^{\bar{x}_i}) = \mathbf{0} = z'_i \underset{x'_i \in D(R'_i, p')}{R'_i} (x, p'^x) \underset{p^{\bar{x}} = \max\{p^x, p'^x\}}{R'_i} (x, \bar{p}^x).$$

Thus, assume that $\bar{x}_i \neq 0$. Then

$$(\bar{x}_i, \bar{p}^{\bar{x}_i}) \underset{p^{\bar{x}_i} \leq p'^{\bar{x}_i} = \bar{p}^{\bar{x}_i}}{=} z'_i \underset{x'_i \in D(R'_i, p')}{R'_i} (x, p'^x) \underset{\bar{p}^x = \max\{p^x, p'^x\}}{R'_i} (x, \bar{p}^x).$$

Step 2. *We have that* $(\bar{z}, \bar{p})$ *satisfies (WE-ii).*

Let $x \in M$ be such that $\bar{p}^x > 0$. We show that there is $i \in N$ such that $\bar{x}_i = x$. Since $\bar{p} = p \vee p'$, $\bar{p}^x > 0$ implies $p^x > 0$ or $p'^x > 0$.

Case 1. $x \in M^2$. By Lemma 16, there is $i \in N^1$ such that $x_i = x$. Since $i \in N^1$, by construction of $\bar{z}$, $\bar{x}_i = x_i$. Thus, $\bar{x}_i = x$.

Case 2. $x \notin M^2$. If $p'^x = 0$, then $p'^x = 0 < p^x$. Thus, $x \in M^2$, which is a contradiction. Thus, $p'^x > 0$. Then there is $i \in N$ such that $x'_i = x$. By Lemma 16, $i \notin N^1$, which implies that $\bar{x}_i = x'_i$. Thus, $\bar{x}_i = x$. □

The following is a corollary of Lemma 17.

Corollary 3. *Let* $R \in \mathcal{R}^n$ *and* $p, p' \in P(R)$. *Then* (i) $p \wedge p' \in P(R)$ *and* (ii) $p \vee p' \in P(R)$.

Fact 5 (Roth and Sotomayor 1990). *Let* $R \in \mathcal{R}^n$ *and let* R' *be a* d*-truncation of* R *such that for each* $i \in N$, $d_i \geq 0$. *Then* $p_{\min}(R') \leq p_{\min}(R)$.

Proof. Let $(z', p') \in W(R')$. Then, for each $i \in N$, since $CV'_i(0; z'_i) \leq 0$ and $d_i \geq 0$, $-d_i \leq 0 \leq -CV'_i(0; z'_i)$. Since R is the $(-d)$-truncation of R', Lemma 17 implies $\hat{p} \equiv p' \wedge p_{\min}(R) \in P(R')$. Thus, since $p_{\min}(R') \leq \hat{p}$, $p_{\min}(R') \leq p_{\min}(R)$. □

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