# Supplement to "Transparency and price formation"

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This supplement has three parts. The first part considers the family of power function distributions $F(v) = v^\alpha$ with support $[0, 1]$.[1] The second part considers a two-period model with general distributions. It is shown that cutoffs (and, hence, the sum of discounted payoffs) can be uniformly ranked. In the third part, we discuss the technical difficulties associated with ranking the equilibrium cutoff levels in more general models.

## S.1. The power function distributions $F(v) = v^\alpha$

Sobel and Takahashi (1983) explicitly solve for equilibria of the Coase bargaining model with equal discounting. Fuchs and Skrzypacz (2013) explicitly solve for equilibria in a model with a real-time deadline in the limiting case where the time period shrinks to 0. They consider the interdependent values case. Here we characterize the unique equilibrium for the finite horizon in discrete-time games for both regimes of our model. Taking the limit, we also obtain the unique stationary equilibrium for the infinite horizon game.

Let $T \le +\infty$ be the length of the horizon and, as in the manuscript, let $\{k_t^i\}_{t=1}^T$, $i = \text{TR}, \text{NTR}$, be the cutoff sequence associated with regime $i$.

Proposition S.1. *We have* $k_t^{\text{NTR}} \le k_t^{\text{TR}}$ *for all* $t = 1, \dots, T$.

We prove this result for $T < +\infty$ and $T = +\infty$. We shall suppress the superscripts TR and NTR, whenever there is no confusion.

### S.1.1 *Finite horizon*

We start by deriving equations that describe the equilibrium cutoff sequences in each regime.

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[1] We consider the no-gap case, which greatly simplifies the computation and the analysis, while still involving the same economic mechanisms driving the price/efficiency rankings.


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S.1.1.1 *Transparent regime* The equilibrium can be found using backward induction. Given the remaining buyer types $[0, k_{T-1}]$, seller $T$ in the last period solves

$$\max_{k\in[0,k_{T-1}]} (k_{T-1}^{\alpha} - k^{\alpha})k.$$

The solution is

$$k_T = \left(\frac{1}{1+\alpha}\right)^{1/\alpha} k_{T-1}.$$

Then the price that seller $T$ charges is given by

$$\left(\frac{1}{1+\alpha}\right)^{1/\alpha} k_{T-1}.$$

Therefore, given the remaining buyer types $[0, k_{T-2}]$, seller $T-1$ solves

$$\max_{k\in[0,k_{T-2}]} (k_{T-2}^{\alpha} - k^{\alpha})\left[(1-\delta)k + \delta\left(\frac{1}{1+\alpha}\right)^{1/\alpha} k\right],$$

where $(1-\delta)k + \delta(1/(1+\alpha))^{1/\alpha}k$ is the price that makes buyer type $k$ indifferent. The solution to this problem is

$$k_{T-1} = \left(\frac{1}{1+\alpha}\right)^{1/\alpha} k_{T-2}.$$

Proceeding this way and using $k_0 = 1$, we find that the equilibrium cutoff sequence is given by

$$k_t = \left(\frac{1}{1+\alpha}\right)^{t/\alpha}, \quad t = 1, 2, \dots, T. \tag{S.1}$$

S.1.1.2 *Nontransparent regime* The equilibrium is described by a sequence $k_1, k_2, \dots, k_T$. Given $k_1, \dots, k_T$, define $p_1, \dots, p_T$ recursively as

$$p_T = k_T \quad \text{and} \quad p_t = (1-\delta)k_t + \delta p_{t+1} \quad \text{for } t = 1, \dots, T-1.$$

Then, $k_1, \dots, k_{T-1}$ must satisfy

$$k_t = \arg\max_k (k_{t-1}^{\alpha} - k^{\alpha})((1-\delta)k + \delta p_{t+1})$$

and $k_T$ must satisfy

$$k_T = \arg\max_k (k_{T-1}^{\alpha} - k^{\alpha})k.$$

Therefore, we obtain the first-order difference conditions

$$k_{T-1}^{\alpha} = (\alpha+1)k_T^{\alpha}$$

and for $t = 1, 2, \dots, T-1$,

$$(k_{t-1}^{\alpha} - k_t^{\alpha})(1-\delta) = \alpha k_t^{\alpha-1}((1-\delta)k_t + \delta p_{t+1}).$$

Manipulating these equations leads to the telescopic sum expression

$$\frac{k_{t-1}^{\alpha}}{k_t^{\alpha}} = 1+\alpha\biggl(1+\delta\frac{k_{t+1}}{k_t}\biggl(1+\delta\frac{k_{t+2}}{k_{t+1}}\biggl(1+\delta\frac{k_{t+3}}{k_{t+2}}\biggl(\cdots\biggl(1+\frac{\delta}{1-\delta}\frac{k_T}{k_{T-1}}\biggr)\biggr)\biggr)\biggr)\biggr). \tag{S.2}$$

Then using $k_0 = 1$, a closed-form solution can be obtained.

S.1.1.3 *Comparing the two regimes* Next, we compare the equilibrium cutoff sequences using (S.1) and (S.2).

In the transparent regime, it follows from (S.1) that seller $t$'s cutoff is such that

$$\left(\frac{k_{t-1}^{\mathrm{TR}}}{k_t^{\mathrm{TR}}}\right)^{\alpha} = 1+\alpha.$$

In the nontransparent regime,

$$\left(\frac{k_{t-1}^{\mathrm{NTR}}}{k_t^{\mathrm{NTR}}}\right)^{\alpha} = 1+\alpha\biggl(1+\delta\frac{k_{t+1}^{\mathrm{NTR}}}{k_t^{\mathrm{NTR}}}\biggl(1+\delta\frac{k_{t+2}^{\mathrm{NTR}}}{k_{t+1}^{\mathrm{NTR}}}\biggl(1+\delta\frac{k_{t+3}^{\mathrm{NTR}}}{k_{t+2}^{\mathrm{NTR}}}\biggl(\cdots\biggl(1+\frac{\delta}{1-\delta}\frac{k_T^{\mathrm{NTR}}}{k_{T-1}^{\mathrm{NTR}}}\biggr)\biggr)\biggr)\biggr)\biggr).$$

The coefficient of $\alpha$ on the right-hand side of the latter expression is strictly greater than 1, which implies that in the nontransparent regime, for $t \le T-1$,

$$\left(\frac{k_{t-1}^{\mathrm{NTR}}}{k_t^{\mathrm{NTR}}}\right)^{\alpha} > 1+\alpha.$$

This, together with the fact that in both regimes $k_0 = 1$, implies that $k_t^{\mathrm{NTR}} \le k_{t-1}^{\mathrm{TR}}$ for any $t$. That is, the equilibrium cutoffs are uniformly smaller in the nontransparent regime.

### S.1.2 *Infinite horizon*

For the transparent regime, taking $T \to \infty$ in (S.2), we obtain that

$$\frac{k_{t-1}^{\alpha}}{k_t^{\alpha}} = 1+\alpha\biggl(1+\delta\frac{k_{t+1}}{k_t}\biggl(1+\delta\frac{k_{t+2}}{k_{t+1}}\biggl(1+\delta\frac{k_{t+3}}{k_{t+2}}(\cdots)\biggr)\biggr)\biggr). \tag{S.3}$$

We conjecture, and then verify, that $k_{t+1}/k_t = \gamma$ for some $\gamma \in (0,1)$. Then it follows from (S.3) that

$$\left(\frac{1}{\gamma}\right)^{\alpha} = 1+\alpha\bigl(1+\delta\gamma\bigl(1+\delta\gamma(1+\delta\gamma(\cdots))\bigr)\bigr),$$

which simplifies to

$$1 = \gamma^{\alpha} + \alpha\gamma^{\alpha}\left(\frac{1}{1-\delta\gamma}\right). \tag{S.4}$$

This equation has a solution $\gamma^* \in (0,1)$. To see this, notice that the right-hand side is continuously increasing in $\gamma$, and it equals 0 when $\gamma = 0$ and equals $1 + \alpha/(1-\delta) > 1$ when $\gamma = 1$.[2]

Observe that in the transparent regime, seller $t$'s cutoff (S.1) is independent of the remaining periods. The resulting cutoff is the equilibrium cutoff even for $T = \infty$. Hence, $k_t^{\mathrm{TR}} = (1/(1+\alpha))^{t/\alpha}$.

To show that $k_t^{\mathrm{TR}} > k_t^{\mathrm{NTR}}$, it is enough to show that $(1/(1+\alpha))^{1/\alpha} > \gamma^*$. To this end, note that when $\gamma = (1/(1+\alpha))^{1/\alpha}$, the right-hand side of (S.4) is

$$\left(\frac{1}{1+\alpha}\right) + \alpha\left(\frac{1}{1+\alpha}\right)\left(\frac{1}{1-\delta(1/(1+\alpha))^{1/\alpha}}\right) > \left(\frac{1}{1+\alpha}\right) + \alpha\left(\frac{1}{1+\alpha}\right) = 1.$$

This proves that $(1/(1+\alpha))^{1/\alpha} > \gamma^*$.

## S.2. Two periods and general type distributions

We now consider a general distribution $F$ with support $[\underline{v}, \bar{v}]$ but focus on a two-period model.

Proposition S.2. *Assume $F(\cdot)$ satisfies increasing virtual valuation. If there is a two-period deadline for the bargaining, then $k_t^{\mathrm{NTR}} \leq k_t^{\mathrm{TR}}$ for $t = 1, 2$.*

Proof. In each regime, if $k_1^i$ is the cutoff chosen in the first period in equilibrium, then the second period cutoff, which is unique by the assumption of increasing virtual valuation, satisfies

$$k_2^i = \arg\max_{k \geq \underline{v}} (F(k_1^i) - F(k))k.$$

Let $k_2^{\mathrm{TR}}(k_1)$ be the solution to the above problem for arbitrary $k_1$.

Now, suppose for a contradiction that $k_1^{\mathrm{NTR}} > k_1^{\mathrm{TR}}$.[3] Consider the percentage change in prices that the seller 1 can charge in either regime when he hypothetically switches from targeting $k_1^{\mathrm{NTR}}$ to targeting $k_1^{\mathrm{TR}}$. Notice that such a switch is not desirable for seller 1 of the nontransparent regime, since in the unique equilibrium he chooses $k_1^{\mathrm{NTR}}$. Then it must be that this percentage change (drop) in price is *smaller* in the transparent regime

[2] For instance, if $\alpha = 1$, then we obtain

$$\gamma^* = \frac{\delta + 2 - \sqrt{\delta^2 + 4}}{2\delta} \in (0,1).$$

Hence, in the transparent regime,

$$k_t^{\mathrm{NTR}} = \left(\frac{\delta + 2 - \sqrt{\delta^2 + 4}}{2\delta}\right)^t.$$

[3] Seller 1 could potentially randomize in the transparent regime, and $k_1^{\mathrm{TR}}$ can be taken as any cutoff in the support.

than in the nontransparent regime so that the seller 1 in the nontransparent regime is willing to make this switch. That is,

$$\frac{(1-\delta)(k_1^{\mathrm{NTR}}-k_1^{\mathrm{TR}})}{(1-\delta)k_1^{\mathrm{NTR}}+\delta k_2^{\mathrm{NTR}}} > \frac{(1-\delta)(k_1^{\mathrm{NTR}}-k_1^{\mathrm{TR}})+\delta(k_2^{\mathrm{TR}}(k_1^{\mathrm{NTR}})-k_2^{\mathrm{TR}}(k_1^{\mathrm{TR}}))}{(1-\delta)k_1^{\mathrm{NTR}}+\delta k_2^{\mathrm{TR}}(k_1^{\mathrm{NTR}})}.$$

But notice that $k_2^{\mathrm{TR}}(k_1^{\mathrm{NTR}}) = k_2^{\mathrm{NTR}}$ and $k_2^{\mathrm{TR}}(k_1^{\mathrm{NTR}}) - k_2^{\mathrm{TR}}(k_1^{\mathrm{TR}}) \geq 0$, which implies that this inequality cannot hold, a contradiction. □

## S.3. More general models

In this section, we would like to detail the technical issues we have encountered in ranking the equilibrium cutoff levels in more general models beyond those that we consider in the previous two sections. We hope to shed light on why our current proof strategies do not work more generally.

Ideally, one would be able to establish a result of the following form:

*Under Condition A, $k_t^{\mathrm{TR}} \geq k_t^{\mathrm{NTR}}$ for all $t$, and under Condition B, $k_t^{\mathrm{TR}} \leq k_t^{\mathrm{NTR}}$ for all $t$.*

Such a result, which is akin to comparing the probability of sales or quantities sold up to any point in time, would be sufficient to answer various questions of interest such as comparing the ex ante surplus implied by equilibria in the two regimes. Our paper provides only a ranking of the prices and not of quantities in this manner.

The strategy we use in proving the price ranking, when stripped of the induction arguments and other complications (such as mixed strategies and multiplicity), comes down to determining conditions (in this case, increasing hazard rate) under which, for a given price change, the percentage change in quantity sold is larger in the transparent regime than in the nontransparent regime. In contrast, to use a similar strategy to establish the ranking of quantities, one needs to compare—across regimes—the percentage change in prices for a given change in quantities.

In the former exercise of price ranking, the proof relies on comparing the impact of an *exogenously given* change in prices on the cutoff types (hence quantities) that are *endogenous*. This is possible because the percentage change in quantities can be expressed in terms of the hazard rate of the type distribution. Using this expression, an *ordinal* ranking of the endogenous objects $k_1^i$ is sufficient to rank the percentage changes.

In contrast, to use the analogue of our proof strategy to rank the equilibrium cutoffs, we would have to compare the impact of an exogenous/given change in cutoffs on prices that are the *endogenously* determined objects in this case. To do this, one needs more than the ordinal ranking of the prices under the two regimes. In fact, information about the size of the differences in prices under different regimes and for different quantities is needed. In particular, to use a similar proof strategy, one needs to be able to establish that if $k_1^{\mathrm{TR}} < k_1^{\mathrm{NTR}}$, then

$$\frac{p_1^{\mathrm{NTR}}(k_1^{\mathrm{NTR}})-p_1^{\mathrm{NTR}}(k_1^{\mathrm{TR}})}{p_1^{\mathrm{NTR}}(k_1^{\mathrm{NTR}})} < \frac{p_1^{\mathrm{TR}}(k_1^{\mathrm{NTR}})-p_1^{\mathrm{TR}}(k_1^{\mathrm{TR}})}{p_1^{\mathrm{TR}}(k_1^{\mathrm{NTR}})}$$

or, equivalently,

$$\frac{p_1^{\text{NTR}}(k_1^{\text{NTR}}) - p_1^{\text{NTR}}(k_1^{\text{TR}})}{p_1^{\text{TR}}(k_1^{\text{NTR}}) - p_1^{\text{TR}}(k_1^{\text{TR}})} < \frac{p_1^{\text{NTR}}(k_1^{\text{NTR}})}{p_1^{\text{TR}}(k_1^{\text{NTR}})}$$

or, equivalently,

$$\frac{p_1^{\text{NTR}}(k_1^{\text{NTR}})}{p_1^{\text{TR}}(k_1^{\text{NTR}})} < \frac{p_1^{\text{NTR}}(k_1^{\text{TR}})}{p_1^{\text{TR}}(k_1^{\text{TR}})}.$$

Clearly (under an appropriate induction hypothesis), both sides of the latter inequality are less than 1. In particular, for the ordinal information about the prices to suffice for establishing this inequality, one needs to argue that the ratio of the *endogenous* inverse demand curves is monotone in quantity.[4] Unfortunately, we have not been able to uncover conditions (i.e., a counterpart to the increasing hazard rate condition that turns out to be sufficient for comparing prices) under which this is necessarily true; neither have we been able to construct a different proof strategy.

Failing this, we have looked for examples where $k_t^{\text{TR}} \geq k_t^{\text{NTR}}$ for all $t$ *does not* hold. Finding such an example would be necessary to answer the questions about efficiency ranking and it would inform our understanding of whether the concavity assumption of Theorem 3 can be relaxed. Unfortunately, we have not been able to come up with such an example either.

## References

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[4]This would be similar to the monotone hazard rate assumption.