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Nonexclusive competition under adverse selection

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A seller of a divisible good faces several identical buyers. The quality of the good may be low or high, and is the seller's private information. The seller has strictly convex preferences that satisfy a single-crossing property. Buyers compete by posting menus of nonexclusive contracts, so that the seller can simultaneously and privately trade with several buyers. We provide a necessary and sufficient condition for the existence of a pure-strategy equilibrium. Aggregate equilibrium trades are unique. Any traded contract must yield zero profit. If a quality is actually traded, then it is efficiently traded. Depending on parameters, both qualities may be traded, or only one of them, or the market may break down to a no-trade equilibrium.

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1. Introduction

The recent financial crisis has spectacularly recalled that the liquidity of financial markets cannot be taken for granted, even for markets that usually attract many traders and on which exchanged volumes tend to be very high. For instance, Adrian and Shin (2010) document that the issuance of asset-backed securities declined from over 300 billion dollars in 2007 to only a few billion in 2009. Similarly, Brunnermeier (2009) emphasizes the severe liquidity dry-up of the interbank market over the 2007–2009 period, when many banks chose to keep their liquidity idle instead of lending it even at short maturities. It is tempting to associate these difficulties with asymmetries in the allocation of information among traders. Indeed, during the crisis, one of the banks' main concerns was the unknown exposure to risk of their counterparties.[1] Moreover, structured financial products such as mortgage-backed securities, collateralized debt obligations, and credit default swaps often involve many different underlying assets, and their designers are likely to hold private information about their quality; this creates an adverse selection problem that reduces liquidity provision.[2] Finally, most of these securities are traded outside of organized exchanges on over-the-counter markets, with poor information on the trading volumes or on the net positions of traders. Hence agents are able to interact secretly with multiple partners, at the expense of information release. These two features, adverse selection and nonexclusivity, are at the heart of the present paper.

Theoretical studies of adverse selection in competitive environments have mainly been developed in the context of two alternative paradigms. Akerlof (1970) studies an economy where privately informed sellers and uninformed buyers act as price takers. All trades are assumed to take place at the same price. Competitive equilibria typically exist, but feature a form of market failure: because the market-clearing price must be equal to the average quality of the goods offered by the sellers, the highest qualities are generally not traded in equilibrium. It seems, therefore, natural to investigate whether such a drastic outcome can be avoided by allowing buyers to screen goods of different qualities. In this spirit, Rothschild and Stiglitz (1976) consider a strategic model in which buyers offer to trade different quantities at different unit prices, thereby allowing sellers to credibly communicate their private information. They show that low-quality sellers trade efficiently, while high-quality sellers end up trading a suboptimal, but nonzero quantity. For instance, on insurance markets, high-risk agents are fully insured, while low-risk agents obtain only partial coverage; no pure-strategy equilibrium exists if the proportion of low-risk agents is too high.

The present paper revisits these classical approaches by relaxing the assumption of exclusive competition, which states that each seller is allowed to trade with at most one buyer. This assumption plays a central role in Rothschild and Stiglitz's (1976) model, and is also satisfied in the simplest versions of Akerlof's (1970) model, in which sellers can trade only one or zero unit of an indivisible good. However, situations where sellers can simultaneously and secretly trade with several buyers naturally arise on many

[1] See, among others, Taylor and Williams (2009) and Philippon and Skreta (2012).

[2] See Gorton (2009). There is also some evidence that lending standards and the intensity of screening have been progressively deteriorating with the expansion of the securitization industry in the pre-2007 years. See, for instance, Keys et al. (2010) and Demyanyk and Van Hemert (2011).

markets—one may even say that nonexclusivity is the rule rather than the exception. In addition to the contexts we have already mentioned, well known examples include the European banking industry, the U.S. credit card market, and the life insurance and annuity markets of several Organization for Economic Cooperation and Development (OECD) countries.[3]

Our aim is to study the impact of adverse selection in markets with such nonexclusive trading relationships. To do so, we allow for nonexclusive trading in a generalized version of Rothschild and Stiglitz's (1976) model. This exercise is interesting per se: as we shall see, the reasonings that lead to the characterization of equilibria are quite different from those put forward by these authors. The results are also different: the equilibria we construct typically feature linear pricing, possibly with a bid–ask spread, and trading is efficient whenever it takes place. Alternatively, pure-strategy equilibria may fail to exist, as in Rothschild and Stiglitz (1976), and some types may be excluded from trade, as in Akerlof (1970). It might even be that the only equilibrium involves no trade.

Our analysis builds on the following simple model of trade. There is a finite number of buyers, who compete for a divisible good offered by a seller.[4] The seller is privately informed of the quality of the good, which may be low or high. The seller's preferences are strictly convex, but otherwise arbitrary, provided they satisfy a single-crossing property. Buyers compete by simultaneously posting menus of contracts, where a contract specifies both a quantity and a transfer. After observing the menus offered and taking into account her private information, or type, the seller chooses which contracts to trade. Our model encompasses pure-trade, insurance, and credit environments as special cases.[5]

In this context, we fully characterize the seller's aggregate trades in any pure-strategy equilibrium. The contribution of this paper is twofold. First, we provide a necessary and sufficient condition for such an equilibrium to exist. This condition can be stated as follows: Let v be the average quality of the good. Then a pure-strategy equilibrium exists if and only if, at the no-trade point, the low-quality type would be willing to *sell* a small quantity of the good at price v, whereas the high-quality type would be willing to *buy* a small quantity of the good at price v. Second, we show that there exists a unique aggregate equilibrium allocation. Each buyer earns zero profit in equilibrium. If the willingness to trade at the no-trade point varies enough across types, equilibria are first-best efficient: the low-quality type *sells* the efficient quantity, while the high-quality type

[3]Detragiache et al. (2000) and Ongena and Smith (2000) document that multiple banking relationships have become very widespread in Europe. Rysman (2007) provides recent evidence of multi-homing in the U.S. credit card industry. Cawley and Philipson (1999) and Finkelstein and Poterba (2004) report similar findings for the U.S. life insurance market and the U.K. annuity market. The structure of annuity markets is of particular interest because some legislations explicitly rule out the possibility of designing exclusive contracts: for instance, on September 1, 2002, the U.K. Financial Services Authority ruled in favor of the consumers' right to purchase annuities from suppliers other than their current pension provider (Open Market Option).

[4]We argue in Section 5 that our results extend to the case of multiple sellers, provided contracting is bilateral and private.

[5]The labels *seller* and *buyers* are only used for expositional purposes. On financial markets, one may sell as well as buy assets. This translates in our model to allowing for negative as well as positive quantities. We argue in Section 5 that our results extend to the case where only nonnegative quantities can be traded.

buys the efficient quantity. By contrast, if the two types have similar willingness to trade at the no-trade point, there is no trade in equilibrium. Finally, in intermediate cases, one type of seller trades efficiently, while the other type does not trade at all.

These results suggest that under nonexclusivity, the seller may only signal her type through the sign of the quantity she proposes to trade with a buyer. This is, however, a very rough signalling device, which is only effective when one type acts as a seller, while the other type acts as a buyer. As a consequence, there is no equilibrium in which both types trade nonzero quantities on the same side of the market. In the context of insurance markets, for instance, this rules out situations in which both the low-risk and the high-risk agents purchase a basic policy at a medium price, with the high-risk agent purchasing on top of this a supplementary policy at a higher price. The general message is thus that nonexclusive competition exacerbates the adverse selection problem: if the first-best outcome cannot be achieved, a nonzero level of trade for one type can be sustained in equilibrium only if the other type is left out of the market. In particular, no cross-subsidization between types takes place in equilibrium. That is, each buyer earns zero profit on any contract he trades in equilibrium. To establish this result, we exhibit a class of deviations that make it possible for at least one buyer to keep trading with the type with which he would hypothetically make a profit, while minimizing the loss he would make with the other type by exploiting the equilibrium offers of his rivals. Overall, our analysis shows that a partial or complete market breakdown may arise under nonexclusive competition when buyers compete in arbitrary menu offers, with very few restrictions on the set of instruments available to them.

Related literature

The implications of nonexclusive competition have been extensively studied in moral-hazard contexts. Following the seminal contributions of Hellwig (1983) and Arnott and Stiglitz (1993), many recent works emphasize that in financial markets where agents can make noncontractible effort decisions, the impossibility of enforcing exclusive contracts can induce positive profits for financial intermediaries and a reduction in trades. Positive profits arise in equilibrium because none of the intermediaries can profitably deviate without inducing the agents to trade several contracts and select inefficient levels of effort.[6] The present paper rules out moral-hazard effects and argues that nonexclusive competition under adverse selection drives intermediaries' profits to zero.

Pauly (1974), Jaynes (1978), and Hellwig (1988) pioneered the analysis of nonexclusive competition under adverse selection. Pauly (1974) suggests that Akerlof-like outcomes can be supported in equilibrium when buyers are restricted to offer linear price schedules. Jaynes (1978) points out that the separating equilibrium characterized by Rothschild and Stiglitz (1976) is vulnerable to entry by an insurance company proposing additional trades that can be concealed from its competitors. He further argues that the nonexistence problem identified by Rothschild and Stiglitz (1976) can be overcome if insurance companies can share the information they have about the agents' trades.

[6]See, for instance, Parlour and Rajan (2001), Bisin and Guaitoli (2004), and Attar and Chassagnon (2009) for applications to credit and insurance markets.

Hellwig (1988) discusses the relevant extensive form for the interfirm communication game.

Biais et al. (2000) study a model of nonexclusive competition among uninformed market-makers who supply liquidity to an informed insider whose preferences are quasilinear, and quadratic in the quantities she trades. Although our model encompasses this specification of preferences, we develop our analysis in a two-type framework, whereas Biais et al. (2000) consider a continuum of types. Despite the similarities between the two setups, their results stand in stark contrast to ours. Indeed, restricting attention to equilibria where market-makers post convex menus of contracts, they argue that nonexclusivity leads to a Cournot-like equilibrium outcome, in which each market-maker earns a positive profit. This is very different from our Bertrand-like equilibrium outcome, in which each traded contract yields zero profit. We postpone until Section 5.3 a more detailed comparison between these contrasting sets of results.

Attar et al. (2011) consider a situation where a seller is endowed with one unit of a good, the quality of which she privately knows. The good is divisible, so that the seller may trade any quantity of it with any of the buyers, as long as she does not trade more than her endowment in the aggregate. Both the buyers' and the seller's preferences are linear in quantities and transfers. It is shown that pure-strategy equilibria always exist and that the corresponding aggregate allocations are generically unique. Depending on whether quality is low or high, and on the probability with which quality is high, the seller may either trade her whole endowment or abstain from trading altogether. Buyers earn zero profit in any equilibrium. These results offer a fully strategic foundation for Akerlof's (1970) classic study of the market for lemons, based on nonexclusive competition. Besides equilibrium existence, a key difference with our setting is that equilibria in Attar et al. (2011) may exhibit nontrivial pooling and, hence, cross-subsidies across types. This reflects the notion that trades are subject to an aggregate capacity constraint. By contrast, the present paper considers a situation where the seller's trades are unrestricted, as in a financial market where agents can take arbitrary positions. Another feature of our model is that we consider general preferences for the seller, provided that they are strictly convex and satisfy a single-crossing property. Thus the range of applications of the present paper is different than in Attar et al. (2011).

In contemporaneous work, Ales and Maziero (2011) study nonexclusive competition in an insurance context similar to that analyzed by Rothschild and Stiglitz (1976). Relying on free-entry arguments, they argue that only the high-risk agent can obtain a positive coverage in equilibrium. This is consistent with the results derived in the present paper; however, a distinguishing feature of our analysis is that it is fully strategic and avoids free-entry arguments. Our results are also more general in that we do not rely on a particular parametric representation of the seller's preferences, which allows us to uncover the common logical structure of a broad class of models.[7]

This paper also contributes to the common-agency literature that analyzes situations where several principals compete through mechanisms to influence the decisions

[7] For instance, a special feature of the insurance model is that efficiency requires that both types of agents be fully insured, whereas our analysis covers situations where efficiency requires that different types of sellers trade different quantities.

of a common agent. In our bilateral-contracting setting, the trades between the seller and the buyers are not public, and the seller may choose to trade with any subset of buyers. Moreover, in line with our focus on competitive environments, the profit of each buyer depends only on the trade he makes with the seller, not on the other trades his competitors may make with her. In the terminology of common agency, our model is thus a *private* and *delegated* common-agency game with *no direct externalities* between principals.[8] In contrast to most of the common-agency literature, our analysis yields a unique prediction for aggregate equilibrium trades and equilibrium payoffs. In our view, this uniqueness result is tied to three key ingredients of our model. First, there are no direct externalities between principals.[9] Second, each buyer's profit is linear in the contract he trades; whereas if some convexity were introduced in the buyers' preferences, then multiple equilibrium outcomes would arise even in a complete-information version of our model.[10] Finally, each type of the seller cares only about the aggregate quantity she sells to the buyers and the aggregate transfer she receives in return, whereas if the buyers' offers were not perfectly substitutable from the seller's viewpoint, then one would again expect multiple equilibrium outcomes to arise even under complete information.[11] Observe that these three assumptions are natural in a broad range of situations, including financial and insurance markets.

Finally, it should be stressed that our uniqueness result obtains despite the fact that very few restrictions are imposed on the set of instruments available to the buyers, who are basically free to propose arbitrary menus of contracts. In this respect, our results contrast with the literature on supply-function equilibria, which considers oligopolistic industries where firms compete in supply schedules instead of simple price or quantity offers. Wilson (1979) and Grossman (1981) are the first to observe that this additional degree of freedom may significantly expand the set of equilibrium outcomes. Klemperer and Meyer (1989) and Kyle (1989) suggest that the introduction of some uncertainty, either in the form of imperfect information over market demand or in the form of noise traders, may limit the multiplicity of equilibria. Vives (2011) develops these intuitions in a general setting where rational traders interact in the presence of idiosyncratic shocks;

[8]The distinction between *delegated* common-agency games, in which the agent can trade with any subset of principals, and *intrinsic* common-agency games, in which the agent must either trade with all principals or with none of them, was introduced by Bernheim and Whinston (1986). Martimort (2006) formulates the distinction between *public-agency* settings, in which each principal's transfer can be made contingent on all the agent's decisions, and *private-agency* settings, in which the transfer made by each principal is only contingent on the trades that the agent makes with him. Finally, the role of *direct externalities* between principals has been emphasized by Martimort and Stole (2003) and Peters (2003).

[9]Direct externalities between principals typically lead to multiple equilibrium outcomes even in complete-information environments, as shown by Martimort and Stole (2003) and Segal and Whinston (2003).

[10]This setting is analyzed by Chiesa and Denicolò (2009), who show that although the aggregate quantity traded in equilibrium always coincides with the first-best quantity, equilibrium transfers and payoffs are not uniquely determined.

[11]Examples in this direction are provided by d'Aspremont and Dos Santos Ferreira (2010), who provide a strategic analysis of competition between firms selling differentiated goods to a representative consumer under complete information, both in the cases of intrinsic and delegated agency.

he shows that there exists a unique symmetric equilibrium in which supply functions are linear.

The paper is organized as follows. Section 2 describes the model. Section 3 characterizes pure-strategy equilibria. Section 4 derives necessary and sufficient conditions under which such equilibria exist. Section 5 discusses extensions of our analysis, imposing nonnegative trades or allowing for multiple sellers and more than two types. Section 6 concludes.

2. The model

Our model features a seller who can simultaneously trade with several identical buyers. To simplify the general description and the analysis of the model, in most of the paper, and unless otherwise mentioned, we impose no restriction on the sign of the quantities traded by the seller or, for that matter, on the sign of the transfers she receives in return. The labels *seller* and *buyers*, although useful, are, therefore, to a large extent conventional. In some of the applications presented in Section 2.4, however, it is more natural to impose that quantities traded be nonnegative. As explained in Section 5.1, our analysis and results extend to these cases as well, with minor modifications. Which assumption is more appropriate should be clear from the context.

2.1 *The seller*

The seller is privately informed of her preferences. She may be of two types, L or H, with positive probabilities m_L and m_H such that $m_L + m_H = 1$. Subscripts i and j are used to index these types, with the convention that $i \neq j$. Each type cares only about the aggregate quantity Q she sells to the buyers and the aggregate transfer T she receives in return. Type i's preferences over aggregate quantity-transfer bundles (Q, T) are represented by a utility function u_i defined over $\mathbb{R}^2$. For each i, we assume that u_i is continuously differentiable, with $\partial u_i/\partial T > 0$, and that u_i is strictly quasiconcave. Hence, type i's marginal rate of substitution of the good for money

$$\tau_i \equiv -\frac{\partial u_i/\partial Q}{\partial u_i/\partial T}$$

is everywhere well defined and strictly increasing along her indifference curves. Note that $\tau_i(Q, T)$ can be interpreted as type i's marginal cost of supplying a higher quantity, given that she already trades (Q, T). We impose no restriction on the sign of $\tau_i(Q, T)$. The following assumption is key to our results.

Assumption SC. *For each* (Q, T), $\tau_H(Q, T) > \tau_L(Q, T)$.

Assumption SC expresses a strict single-crossing property: type H is less eager to sell a higher quantity than type L is. As a result, in the (Q, T) plane, a type-H indifference curve crosses a type-L indifference curve only once, from below.

2.2 *The buyers*

There are $n \geq 2$ identical buyers. There are no direct externalities between them: each buyer cares only about the quantity q he purchases from the seller and the transfer t he makes in return. Each buyer's preferences over individual quantity-transfer bundles (q, t) are represented by a linear profit function: if a buyer receives from type i a quantity q and makes a transfer t in return, he earns a profit $v_i q - t$. We impose no restriction on the sign of v_i. The following assumption will be maintained throughout the analysis.

Assumption CV. *We have* $v_H > v_L$.

We let $v \equiv m_L v_L + m_H v_H$ be the average quality of the good, so that $v_H > v > v_L$. Assumption CV reflects common values: the seller's type has a direct impact on the buyers' profits. Together with Assumption SC, Assumption CV captures a fundamental trade-off of our model: type H provides a more valuable good to the buyers than type L, but at a higher marginal cost. These assumptions are natural if we interpret the seller's type as the quality of the good she offers. Together, they create a tension that will be exploited later on: Assumption SC leads type H to offer less of the good, but Assumption CV induces buyers to demand more of the good offered by type H, if only they could observe quality.

2.3 *The nonexclusive trading game*

Trading is nonexclusive in that no buyer can control and a fortiori contract on the trades that the seller makes with other buyers. The timing of events is as follows. First, buyers compete in menus of contracts for the good offered by the seller.[12] Next, the seller can simultaneously trade with several buyers. Accordingly, the extensive form is as follows.

1. Each buyer k proposes a menu of contracts, that is, a set $C^k \subset \mathbb{R}^2$ of quantity-transfer bundles that contains at least the no-trade contract $(0, 0)$.[13]
2. After privately learning her type, the seller selects one contract from each of the menus C^k offered by the buyers.

A pure strategy for type i is a function that maps each menu profile $(C^1, \ldots, C^n)$ into a vector of contracts $((q^1, t^1), \ldots, (q^n, t^n)) \in C^1 \times \cdots \times C^n$. To ensure that type i's utility-maximization problem

$$\max\left\{u_i\left(\sum_k q^k, \sum_k t^k\right) : (q^k, t^k) \in C^k \text{ for each } k\right\}$$

always has a solution, we require the buyers' menus C^k to be compact sets. This allows us to use perfect Bayesian equilibrium as our equilibrium concept. Throughout the paper, we focus on pure-strategy equilibria.

[12]As shown by Peters (2001) and Martimort and Stole (2002), there is no need to consider more general mechanisms in this multiple-principal single-agent setting.

[13]This requirement allows one to deal with participation in a simple way. It reflects the fact that the seller cannot be forced to trade with any particular buyer.

2.4 *Applications*

The following examples illustrate the range of our model.

Pure trade In the pure-trade model, the seller has quasilinear preferences

$$u_i(Q,T)=T-c_i(Q),$$

where the cost function c_i is continuously differentiable and strictly convex. Then $\tau_i(Q,T)=c_i'(Q)$. Assumption SC requires that $c_H'(Q)>c_L'(Q)$ for all Q. For instance, in line with Biais et al. (2000), one may consider a quadratic cost function $c_i(Q)=\theta_iQ+\gamma Q^2$ for some positive constant γ. Assumption SC then reduces to $\theta_H>\theta_L$. Note that Biais et al. (2000) moreover assume that the first-best quantities are implementable, a situation sometimes called *responsiveness* in the literature (Caillaud et al. 1988). In our two-type specification, this amounts to assuming that $v_H-\theta_H<v_L-\theta_L$. Our analysis does not rely on this assumption. Finally, in the financial market-microstructure interpretation of Biais et al. (2000) model, it is natural to assume that the seller can take long or short positions in the financial asset she trades with the buyers. By contrast, if the seller produces a physical good, it is more natural to assume that only nonnegative quantities of it can be traded.

Insurance In the insurance model, an agent can sell a risk to several insurance companies. As in Rothschild and Stiglitz (1976), the agent faces a binomial risk on her wealth, which can take two values W_G and W_B, with probabilities π_i and $1-\pi_i$ that define her type. Here W_G-W_B is the positive monetary loss that the agent incurs in the bad state. A contract specifies a reimbursement r to be paid in the bad state and an insurance premium p. Let R be the sum of the reimbursements and let P be the sum of the insurance premia. We assume that the agent's preferences have an expected utility representation

$$\pi_iu(W_G-P)+(1-\pi_i)u(W_B-P+R)$$

for some von Neumann–Morgenstern utility function u that is assumed to be continuously differentiable, strictly increasing, and strictly concave. An insurance company's profit from trading the contract (r,p) with type i is $p-(1-\pi_i)r$, which can be written as v_iq-t if we set $v_i\equiv-(1-\pi_i)$, $q\equiv r$, and $t\equiv -p$, so that $Q=R$ and $T=-P$. Hence, the agent purchases for a transfer $-T$, a reimbursement Q in the bad state. Note that reimbursements must remain nonnegative if negative insurance is ruled out. The agent's expected utility writes as

$$u_i(Q,T)=\pi_iu(W_G+T)+(1-\pi_i)u(W_B+T+Q).$$

Then

$$\tau_i(Q,T)=-\frac{(1-\pi_i)u'(W_B+T+Q)}{\pi_iu'(W_G+T)+(1-\pi_i)u'(W_B+T+Q)},$$

so that Assumption SC requires that type H has a lower probability of incurring a loss, $\pi_H>\pi_L$. Given our parametrization, this implies that $v_H>v_L$, so that Assumption CV is satisfied. Therefore, our model encompasses the nonexclusive version of Rothschild and Stiglitz's (1976) model considered by Ales and Maziero (2011). Note that we could also allow for nonexpected utility in the modelling of the agent's preferences. Thus, for instance, we can consider state-dependent utilities, as in Cook and Graham (1977), or rank-dependent utilities, as in Quiggin (1982).

Credit In the credit model, a borrower raises nonnegative amounts of capital from several investors to fund a variable-size project. In the default state, the project generates a zero cash flow and the borrower defaults. In the no-default state, the project generates a positive cash flow and the borrower does not default. The type of the borrower affects both the probability of the no-default state, π_i, and the cash flow in that state, $f_i(B)$, given aggregate borrowed capital B. We assume that for each i, the function f_i is continuously differentiable, strictly increasing, and strictly concave, with $f_i(0)=0$. As in Stiglitz and Weiss (1981), we restrict our analysis to standard debt contracts. Thus a contract is a borrowed capital/promised repayment pair (b, p). Let P be the aggregate promised repayment. We assume that the borrower's preferences are represented by

$$\pi_i[f_i(B)-P].$$

An investor's expected payoff from trading the contract (b, p) with type i is $\pi_i p - b$, which can be written as $v_i q - t$ if we set $v_i \equiv \pi_i$, $q \equiv p$, and $t \equiv b$, so that $Q = P$ and $T = B$. Thus the borrower raises T against a promise of repaying Q, and her expected utility writes as

$$u_i(Q, T) = \pi_i[f_i(T)-Q].$$

Then $\tau_i(Q, T) = 1/f_i'(T)$. Assumptions SC and CV are satisfied if $f_H'(T) < f_L'(T)$ for all T and $\pi_H > \pi_L$. Intuitively, type H is less likely to default, but her investment project has lower returns than type L's, so that her marginal cost of repaying her debts in the no-default state is higher than type L's.

3. Equilibrium characterization

An equilibrium allocation specifies individual trades (q_i^k, t_i^k) between each type i and each buyer k, and corresponding aggregate trades $(Q_i, T_i) = (\sum_k q_i^k, \sum_k t_i^k)$. In this section, we characterize these equilibrium trades, assuming that an equilibrium exists, and we provide a simple necessary condition for the existence of an equilibrium.

3.1 *Pivoting*

In line with Rothschild and Stiglitz (1976), we examine well chosen deviations by a buyer, and use the fact that in equilibrium, deviations cannot be profitable. A key difference, however, is that in Rothschild and Stiglitz (1976), competition is exclusive, whereas in our setting, competition is nonexclusive.

Under exclusive competition, what matters from the viewpoint of any given buyer k is simply the maximum utility level U_i^{-k} that each type i can get by trading with some other buyer. A deviation by buyer k targeted at type i is then a contract (q_i^k, t_i^k) that gives type i a strictly higher utility, $u_i(q_i^k, t_i^k) > U_i^{-k}$. Type j may be attracted or not by this contract; in each case, one can compute the deviating buyer's profit.

In contrast, under nonexclusive competition, all the contracts offered by the other buyers matter from the viewpoint of buyer k. Suppose indeed that the seller can trade (Q^{-k}, T^{-k}) with buyers other than k. Then buyer k can use this as an opportunity to build more attractive deviations. For instance, to attract type i, buyer k can propose the contract $(Q_i - Q^{-k}, T_i - T^{-k} + \varepsilon)$ for some positive number ε: combined with (Q^{-k}, T^{-k}), this contract gives type i a strictly higher utility than her aggregate equilibrium trade (Q_i, T_i). In that case, we say that buyer k *pivots on* (Q^{-k}, T^{-k}) *to attract type* i. Type j may be attracted or not by this contract; in each case, one can provide a condition on profits that ensures that the deviation is not profitable.

The key difference between exclusive and nonexclusive competition is thus that in the latter case, each buyer k faces a single seller whose type is unknown, but whose preferences are defined by an *indirect* utility function, rather than by the primitive utility function u_i as in the former case. Formally, type i's indirect utility from trading a contract (q, t) with buyer k is given by

$$z_i^{-k}(q,t) = \max\left\{u_i\left(q + \sum_{l \neq k} q^l, t + \sum_{l \neq k} t^l\right) : (q^l, t^l) \in C^l \text{ for each } l \neq k\right\},$$

so that in equilibrium $U_i \equiv u_i(Q_i, T_i) = z_i^{-k}(q_i^k, t_i^k)$ for all i and k. Notice that $z_i^{-k}(q,t)$ is strictly increasing in t. Moreover, because u_i is continuous and the menus C^l, $l \neq k$, are compact, it follows from Berge's maximum theorem that z_i^{-k} is continuous.[14]

What makes the analysis difficult is that the functions z_i^{-k} are endogenous, because they depend on the menus offered by the buyers other than k, on which we impose no restriction besides compactness. As a result, there is no a priori guarantee that the functions z_i^{-k} are well behaved, which prevents us from using mechanism-design techniques to determine each buyer's best response to the other buyers' menus. Instead, we rely only on pivoting arguments to fully characterize aggregate equilibrium trades and individual equilibrium payoffs, as in Attar et al. (2011).

Remark. The idea of determining each principal's equilibrium behavior by considering his interaction with an agent endowed with an indirect utility function that incorporates the optimal choices she makes with the other principals is a standard device in the common-agency literature.[15] In private-agency settings, this methodology has been applied to games of complete information (Chiesa and Denicolò 2009, d'Aspremont

[14]This differs from Attar et al. (2011), where the presence of a capacity constraint may induce discontinuities in the seller's indirect utility function. Note that the function z_i^{-k} is independent of buyer k's menu offer. Therefore, saying that it is continuous does not commit us in any way to restricting buyer k's menu offer. Indeed, we will only use deviations that consist of at most two nontrivial contracts.

[15]A similar approach has been followed in the literature on supply-function equilibria, in which each supplier's equilibrium behavior is determined by taking into account the residual demand he faces given the supply functions offered by his competitors (see Wilson 1979, Grossman 1981, Klemperer and Meyer 1989, Kyle 1989, and Vives 2011).

and Dos Santos Ferreira 2010), as well as to games of incomplete information (Biais et al. 2000, Martimort and Stole 2003, 2009, Calzolari 2004, Laffont and Pouyet 2004, or Khalil et al. 2007). Although this approach has been used to derive a full characterization of equilibrium payoffs under complete information, the analysis of incomplete-information environments typically involves additional restrictions. Indeed, attention is usually restricted to equilibria in which the screening problem faced by each principal is regular enough, which amounts to considering well behaved z_i^{-k} functions that are concave in quantities and satisfy a single-crossing property.[16] A distinguishing feature of our analysis is that we provide a full characterization of aggregate equilibrium trades and individual equilibrium payoffs by exploiting only the continuity of the z_i^{-k} functions and the fact that each of them is strictly increasing in transfers.

Denote type-by-type individual profits by $b_i^k \equiv v_i q_i^k - t_i^k$ and denote expected individual profits by $b^k \equiv m_L b_L^k + m_H b_H^k$. The following lemma encapsulates our pivoting technique.

Lemma 1. *In equilibrium, for all q and t, if the seller can trade $(Q_i - q, T_i - t)$ with buyers other than k, then*

$$v_i q - t > b_i^k \quad \textit{implies} \quad vq - t \leq b^k. \tag{1}$$

The intuition for this result is as follows. If the seller can trade $(Q_i - q, T_i - t)$ with buyers other than k, then buyer k can pivot on this aggregate trade to attract type i, while still offering the contract (q_j^k, t_j^k). If the contract (q, t) allows buyer k to increase the profit he earns with type i, then it must be that type j also selects it instead of (q_j^k, t_j^k) following buyer k's deviation. Moreover, this contract cannot increase buyer k's average profit if traded by both types; otherwise, we would have constructed a profitable deviation.

We are now ready to use our pivoting technique to gain insights into the structure of aggregate equilibrium trades. Because each type cares only about her aggregate trade, and because buyers care only about their individual trades and have identical linear profit functions, in equilibrium aggregate trades and aggregate profits can be computed as if both types were trading (Q_j, T_j), with type i additionally trading $(Q_i - Q_j, T_i - T_j)$. What can be said about this additional trade? The first information comes from Assumption SC, which, along with $\partial u_i/\partial T > 0$, implies that $Q_L - Q_H$ is nonnegative. More interestingly, Lemma 1 allows us to show that in the aggregate, buyers cannot make a profit by trading $(Q_i - Q_j, T_i - T_j)$ with type i. Formally, denote by $S_i \equiv v_i(Q_i - Q_j) - (T_i - T_j)$ the corresponding aggregate profit. Then the following result obtains.

Proposition 1. *In any equilibrium, $S_i \leq 0$ for each i.*

[16] See Martimort and Stole (2009) for a general exposition of this methodology and for a detailed analysis of the conditions that need to be imposed on the agent's preferences and on the corresponding virtual surplus function to guarantee the regularity of each principal's program.

Proof. Choose i and k, and set $q \equiv q_j^k + Q_i - Q_j$ and $t \equiv t_j^k + T_i - T_j$. Then the seller can trade $(Q_i - q, T_i - t) = (\sum_{l \neq k} q_j^l, \sum_{l \neq k} t_j^l)$ with buyers other than k. One has

$$\begin{aligned} v_i q - t - b_i^k &= v_i(q_j^k + Q_i - Q_j) - (t_j^k + T_i - T_j) - b_i^k \\ &= v_i(Q_i - Q_j) - (T_i - T_j) - [v_i(q_i^k - q_j^k) - (t_i^k - t_j^k)] \\ &= S_i - s_i^k, \end{aligned}$$

where $s_i^k \equiv v_i(q_i^k - q_j^k) - (t_i^k - t_j^k)$ and

$$\begin{aligned} v_j q - t - b_j^k &= v_j(q_j^k + Q_i - Q_j) - (t_j^k + T_i - T_j) - b_j^k \\ &= -[v_j(Q_j - Q_i) - (T_j - T_i)] \\ &= -S_j. \end{aligned}$$

Therefore, according to (1),

$$S_i > s_i^k \quad \text{implies} \quad m_i(S_i - s_i^k) \leq m_j S_j. \tag{2}$$

Suppose, to the contrary, that $S_i > 0$. Because $S_i = \sum_k s_i^k$ by construction, one must have $S_i > s_i^k$ for some k. From (2), we obtain that $S_j > 0$ and, thus, that $S_i + S_j > 0$. As $S_i + S_j = (v_i - v_j)(Q_i - Q_j)$ and $v_H > v_L$, this implies that $Q_L < Q_H$, a contradiction. Hence the result. Note for future reference that because $S_j \leq 0$, it actually follows from (2) that $S_i \leq s_i^k$ for all i and k. □

The intuition for Proposition 1 can easily be understood in the context of a free-entry equilibrium. Indeed, under free entry, the seller can trade (Q_j, T_j) with the existing buyers, so that an entrant can pivot on (Q_j, T_j) to attract type i. That is, an entrant could simply propose to buy a quantity $Q_i - Q_j$ in exchange for a transfer slightly above $T_i - T_j$. This contract would certainly attract type i; in addition, if it also attracts type j, this would be good news for the entrant, because $v_j(Q_i - Q_j) \geq v_i(Q_i - Q_j)$ as $v_H > v_L$ and $Q_L \geq Q_H$. In a free-entry equilibrium, it must, therefore, be that $v_i(Q_i - Q_j) \leq T_i - T_j$. Proposition 1 shows that the same result holds when the number of buyers is fixed, although the argument is more involved. Indeed, it is then unclear that if S_i were positive, then *each* buyer would have a profitable deviation. For instance, if a buyer earning a positive profit with type j in equilibrium decides to propose an attractive deviation to type i as in the above free-entry deviation, he might incur a net loss with type j if she chose to trade a different contract than in equilibrium. This loss might, in turn, offset the gains from attracting type i, making the deviation unprofitable. Rather, the proof of Proposition 1 amounts to showing that in these circumstances, *at least one* buyer must have a profitable deviation.

As simple as it is, this result is powerful enough to rule out equilibrium outcomes that have been emphasized in the literature. Consider first the separating equilibrium of Rothschild and Stiglitz's (1976) exclusive-competition model of insurance provision. In this equilibrium, insurance companies earn zero profit and no cross-subsidization takes place. Using the parametrization of Section 2.4, this means that the equilibrium contract (Q_i, T_i) of each type i lies on the line with negative slope $v_i = -(1 - \pi_i)$ going through the origin. Moreover, the high-risk agent, that is, in our parametrization, type L, is indifferent between the contracts (Q_L, T_L) and (Q_H, T_H). Hence, as $Q_L > Q_H > 0$, the line connecting these two contracts has a negative slope that is strictly lower than v_L. That is, $T_L - T_H < v_L(Q_L - Q_H)$, in contradiction to Proposition 1. Therefore, the Rothschild and Stiglitz (1976) equilibrium allocation is not robust to nonexclusive competition.

Another class of candidate equilibria that has been considered in the literature, especially in the case of insurance, is equilibria with linear prices in which different types trade nonzero quantities on the same side of the market. For instance, Pauly (1974) explicitly restricts insurance companies to post linear price schedules. Similarly, in the context of annuities, Sheshinski (2008) makes the assumption that each type of annuity is traded at a common price available to all potential agents that is equal to the average longevity of the buyers of this type of annuity, weighted by the purchased equilibrium amounts. Finally, Chiappori (2000) argues that under nonexclusivity, agents can linearize any nonlinear schedule by trading small contracts with different insurance companies; as a result, standard linear pricing ensues. This argument, however, presumes that such small contracts are offered, which need not be the case as the supply of contracts is endogenous. More strikingly, we now show that such equilibria are ruled out by Proposition 1. To see this, suppose that there exists an equilibrium in which each buyer stands ready to buy any quantity at a unit price p and that $Q_L > Q_H > 0$ in this equilibrium. Because the expected aggregate profit $B \equiv \sum_k b^k$ must be nonnegative, one must have $v > p$. Moreover, according to Proposition 1 and the definition of S_L, one must have $p \geq v_L$. Hence buyers make profits when trading with type H and cannot make profits when trading with type L. Now, any buyer k can attempt to reap the aggregate profit on type H: to do so, he may simply deviate by offering a contract $(Q_H, T_H + \varepsilon_H)$ for some positive number ε_H. This contract certainly attracts type H. Because $p \geq v_L$, at worst it also attracts type L and, therefore, one must have $b^k \geq (v - p)Q_H$ by letting ε_H go to zero. Summing these inequalities over k yields

$$B \geq n(v - p)Q_H. \tag{3}$$

Because one can compute the aggregate profit as if both types were trading (Q_H, T_H), with type L additionally trading $(Q_L - Q_H, T_L - T_H)$, one has

$$B = vQ_H - T_H + m_L S_L = (v - p)Q_H + m_L S_L. \tag{4}$$

Merging (3) and (4) yields $m_L S_L \geq (n-1)(v - p)Q_H$. Because $n \geq 2$, $v > p$, and $S_L \leq 0$ by Proposition 1, one must thus have $Q_H \leq 0$, a contradiction. Hence there is no equilibrium with linear prices in which both types actively trade on the same side of the market.

3.2 *The zero-profit result*

In any Bertrand-like setting, the usual argument consists in making buyers compete for any profit that may result from serving the whole demand. This also applies to our setting, although the logic is different. Specifically, the following zero-profit result obtains.

Proposition 2. *In any equilibrium, $B=0$, so that $b^k=0$ for each k.*

Proof. Denote type-by-type aggregate profits by $B_i \equiv \sum_k b_i^k$ and recall that the expected aggregate profit is denoted by B. We first prove that for each j and k,

$$B_j > b_j^k \quad \text{implies} \quad B - b^k \leq m_i S_i. \tag{5}$$

Indeed, if $B_j > b_j^k$, buyer k can deviate by proposing a menu that consists of the no-trade contract, and of the contracts $c_i^k = (q_i^k, t_i^k + \varepsilon_i)$ and $c_j^k = (Q_j, T_j + \varepsilon_j)$, for some positive numbers ε_i and ε_j. Because $U_j \geq z_j^{-k}(q_i^k, t_i^k)$ and the function z_j^{-k} is continuous, it is possible, given the value of ε_j, to choose ε_i small enough so that type j trades c_j^k following buyer k's deviation. Turning now to type i, observe that she must trade either c_i^k or c_j^k following buyer k's deviation: indeed, because ε_i is positive, type i strictly prefers c_i^k to any contract she could have traded with buyer k before the deviation. If type i selects c_i^k, then buyer k's profit from this deviation is $m_i(b_i^k - \varepsilon_i) + m_j(B_j - \varepsilon_j)$, which, because $B_j > b_j^k$ by assumption, is strictly higher than b^k when ε_i and ε_j are small enough, a contradiction. Therefore, type i must select c_j^k following buyer k's deviation and for this deviation not to be profitable, one must have $vQ_j - T_j - \varepsilon_j \leq b^k$. In line with (4), this can be rewritten as $B - m_i S_i - \varepsilon_j \leq b^k$, from which (5) follows by letting ε_j go to zero.

Now, if $B > 0$, then $B > b^k$ for some k. Because $S_i \leq 0$ and $S_j \leq 0$ by Proposition 1, it follows from (5) that $B_i \leq b_i^k$ and $B_j \leq b_j^k$ for each k. Averaging over types yields $B \leq b^k$ for each k, a contradiction. Hence the result. □

The intuition for Proposition 2 can easily be understood in the context of a free-entry equilibrium. Indeed, suppose, for instance, that the aggregate profit from trading with type j is positive, $B_j > 0$. Then an entrant could propose to buy Q_j in exchange for a transfer slightly above T_j. This contract would certainly attract type j, which would benefit the entrant; in equilibrium, it must, therefore, be that this trade also attracts type i and that $vQ_j - T_j \leq 0$. Now recall that the aggregate profit can be written as $B = vQ_j - T_j + m_i S_i$. Our first result in Proposition 1 is that $S_i \leq 0$ and we just argued that $vQ_j - T_j \leq 0$ when $B_j > 0$. Hence the aggregate profit must be zero. Proposition 2 shows that the same result holds when the number of buyers is fixed, which is not a priori obvious. In line with the proof of Proposition 1, the proof of Proposition 2 amounts to showing that if B is positive, then at least one buyer must have a profitable deviation.

Remark. An inspection of the proofs of Propositions 1 and 2 reveals that these results require only weak assumptions on feasible trades, namely that if the quantities q and q' are tradable, then so are the quantities $q+q'$ and $q-q'$. Hence we allow for negative and positive trades, but we may, for instance, have integer constraints on quantities. Finally, we did use in Lemma 1 the fact that the functions u_i and, thus, the functions z_i^{-k} are continuous with respect to transfers, but, for instance, we did not use the fact that the seller's preferences are convex.

3.3 *Pooling versus separating equilibria*

We say that an equilibrium is *pooling* if both types of the seller trade the same aggregate quantity, $Q_L = Q_H$, and that it is *separating* if they trade different aggregate quantities, $Q_L > Q_H$. We now investigate the basic price structure of these two kinds of candidate equilibria.

Lemma 2. *The following statements hold.*

- *In any pooling equilibrium,* $T_L = vQ_L = T_H = vQ_H$.
- *In any separating equilibrium, the following cases occur.*
  - *(i) If* $Q_L > 0 > Q_H$, *then* $T_L = v_L Q_L$ *and* $T_H = v_H Q_H$.
  - *(ii) If* $Q_L > Q_H \geq 0$, *then* $T_H = vQ_H$ *and* $T_L - T_H = v_L(Q_L - Q_H)$.
  - *(iii) If* $0 \geq Q_L > Q_H$, *then* $T_L = vQ_L$ *and* $T_H - T_L = v_H(Q_H - Q_L)$.

The first statement of Lemma 2 is an immediate consequence of the zero-profit result. Otherwise, the equilibrium is separating and the three cases may, in principle, arise. In case (i), type L *sells* a positive quantity Q_L, while type H *buys* a positive quantity $|Q_H|$. There are no cross-subsidies in equilibrium, as each type i trades at the fair price v_i. In case (ii), everything happens as if, in the aggregate, both types were selling a quantity Q_H at the fair price v, with type L selling an additional quantity $Q_L - Q_H$ at the fair price v_L. Two scenarios are conceivable. If $Q_H > 0$, there are cross-subsidies in equilibrium, with $B_L < 0 < B_H$. In that case, the structure of aggregate equilibrium trades is similar to that obtained by Jaynes (1978) and Hellwig (1988) in a nonexclusive version of Rothschild and Stiglitz's (1976) model, where insurance companies can share information about their clients. It is also reminiscent of the equilibrium of the limit-order book analyzed by Glosten (1994). Further results in Section 3.4 rule out this scenario and, more generally, any equilibrium in which both types trade nonzero quantities on the same side of the market. Alternatively, if $Q_H = 0$, the structure of aggregate equilibrium trades is similar to that which prevails in a two-type version of Akerlof's (1970) model when adverse selection is severe. Finally, case (iii) is the mirror image of case (ii).

3.4 *The no-cross-subsidization result*

In this section, we prove that our nonexclusive competition game has no equilibrium with cross-subsidies. We first establish that the aggregate profit earned on each type must be zero in equilibrium. As discussed below, this drastically reduces the set of candidate equilibria. We then refine this result by showing that any traded contract must actually yield zero profit in equilibrium.

The first step of the analysis consists of showing that if buyers make profits in the aggregate when trading with type j, then type j must trade inefficiently in equilibrium. Specifically, her marginal rate of substitution at her aggregate equilibrium trade is not equal to the quality of the good she sells, but rather to the *average* quality of the good.

Lemma 3. *If in equilibrium $B_j > 0$, then $\tau_j(Q_j, T_j) = v$.*

The intuition for Lemma 3 is as follows. If $\tau_j(Q_j, T_j)$ were different from v, then any buyer could propose a contract in the neighborhood of (Q_j, T_j) that would attract type j, thereby generating a positive profit close to B_j, and that would generate a small positive profit even if it were traded by both types. This, however, is impossible according to the zero-profit result.

Remark. Consider a candidate equilibrium in which $B_j > 0$ and let k be such that $b_j^k > 0$. Then, for any such buyer k, type i could get her equilibrium utility $U_i = z_i^{-k}(q_i^k, t_i^k)$ by trading (q_j^k, t_j^k) instead of (q_i^k, t_i^k) with buyer k. That is,

$$U_i = z_i^{-k}(q_i^k, t_i^k) = z_i^{-k}(q_j^k, t_j^k), \tag{6}$$

which can be interpreted as a binding incentive compatibility constraint, taking into account the nonexclusivity of trades. (Note the difference with the exclusive competition case, in which incentive constraints bear only on aggregate quantities, $U_i = u_i(Q_i, T_i) \geq u_i(Q_j, T_j)$.) The argument goes as follows. Suppose that $z_i^{-k}(q_i^k, t_i^k) > z_i^{-k}(q_j^k, t_j^k)$. Then buyer k could deviate by proposing a menu that consists of the no-trade contract and of the contracts $c_i^k = (q_i^k, t_i^k - \varepsilon_i)$ and $c_j^k = (q_j^k, t_j^k + \varepsilon_j)$ for some positive numbers ε_i and ε_j such that $m_i\varepsilon_i > m_j\varepsilon_j$. Clearly, type j selects c_j^k following buyer k's deviation. Turning now to type i, observe that because $z_i^{-k}(q_i^k, t_i^k) > z_i^{-k}(q_j^k, t_j^k)$ and the function z_i^{-k} is continuous, she is better off selecting c_i^k rather than c_j^k following buyer k's deviation as long as ε_i and ε_j are small enough. If she decides to trade c_i^k, then buyer k makes a positive profit $m_i\varepsilon_i - m_j\varepsilon_j$. Thus type i must not trade with buyer k following his deviation and for this deviation not to be profitable, one must have $v_j q_j^k - t_j^k - \varepsilon_j \leq 0$. Letting ε_j go to zero, we get $b_j^k \leq 0$. By contraposition, (6) must hold as soon as $b_j^k > 0$.

The second step of the analysis consists of showing that if buyers make profits in the aggregate when trading with type j, then the aggregate trade made by type j in equilibrium must remain available if any buyer withdraws his menu offer. In our oligopsony model, this rules out Cournot-like outcomes in which the buyers share the market in such a way that each of them needs to provide type j with her aggregate equilibrium trade, as is the case in the equilibrium described in Biais et al. (2000). This is more in the spirit of Bertrand competition, where cross-subsidies are harder to sustain.

Lemma 4. *In equilibrium, if $B_j > 0$, then for each k, the seller can trade (Q_j, T_j) with buyers other than k.*

The proof of Lemma 4 proceeds as follows. First, we show that if B_j is positive, then the equilibrium utility of type j must remain available following any buyer's deviation; the reason for this is that otherwise, a buyer could deviate and reap the aggregate profit on type j. As a result, for any buyer k, there exists an aggregate trade (Q^{-k}, T^{-k}) with buyers other than k that allows buyer j to achieve the same level of utility as in equilibrium, $u_j(Q^{-k}, T^{-k}) = U_j$. From the strict quasiconcavity of u_i and Lemma 3, we obtain that if $Q^{-k} \neq Q_j$, then $T^{-k} > vQ^{-k}$. We finally show that this would allow buyer k to profitably deviate by pivoting on (Q^{-k}, T^{-k}).

We are now ready to state and prove the main result of this section.

Proposition 3. *In any equilibrium, $B_j = 0$ for each j.*

Proof. Suppose, to the contrary, that $B_j > 0$ for some j. Then any buyer k such that $b_j^k > 0$ can deviate by proposing a menu that consists of the no-trade contract and of the contracts $c_i^k = (Q_i - Q_j + \delta_i, v_i(Q_i - Q_j) + \varepsilon_i)$ and $c_j^k = (q_j^k, t_j^k + \varepsilon_j)$ for some numbers δ_i, ε_i, and ε_j. Choose δ_i and ε_i such that $\tau_i(Q_i, T_i)\delta_i < \varepsilon_i$. This ensures that when δ_i and ε_i are small enough, type i can strictly increase her utility by trading c_i^k with buyer k and trading (Q_j, T_j) with buyers other than k, thereby trading $(Q_i + \delta_i, T_i + \varepsilon_i)$ in the aggregate; according to Lemma 4, this is feasible as $B_j > 0$. Because $U_i \geq z_i^{-k}(q_j^k, t_j^k)$ and the function z_i^{-k} is continuous, it is possible, given the values of δ_i and ε_i, to choose ε_j positive and small enough so that type i trades c_i^k following buyer k's deviation. Turning now to type j, observe that she must trade either c_i^k or c_j^k following buyer k's deviation: indeed, because ε_j is positive, type j strictly prefers c_j^k to any contract she could have traded with buyer k before the deviation. If type j selects c_j^k, then buyer k's profit from this deviation is $m_i(v_i\delta_i - \varepsilon_i) + m_j(v_j q_j^k - t_j^k - \varepsilon_j)$, which, because $v_j q_j^k - t_j^k = b_j^k > 0$ by assumption, is positive when δ_i, ε_i, and ε_j are small enough, in contradiction to the zero-profit result. Therefore, type j must select c_i^k following buyer k's deviation and for this deviation not to be profitable, one must have

$$v(Q_i - Q_j + \delta_i) - v_i(Q_i - Q_j) - \varepsilon_i \leq 0. \tag{7}$$

Now, recall that as a consequence of Assumption SC, $(v - v_i)(Q_i - Q_j) \geq 0$. Therefore, letting δ_i and ε_i go to zero in (7), we get $Q_i = Q_j$ and, hence, the equilibrium must be pooling. Using the equality $Q_i = Q_j$ to simplify (7), we obtain that for any small enough δ_i and ε_i such that $\tau_i(Q_i, T_i)\delta_i < \varepsilon_i$, one has $v\delta_i \leq \varepsilon_i$. As δ_i can be positive or negative, it follows that $\tau_i(Q_i, T_i) = v$. However, according to Lemma 3, one also has $\tau_j(Q_j, T_j) = v$ as $B_j > 0$. Because $(Q_i, T_i) = (Q_j, T_j)$, this contradicts Assumption SC. Hence the result. □

Along with Lemma 2, this no-cross-subsidization result leads to the conclusion that one must have $Q_H \leq 0 \leq Q_L$ in any equilibrium. This excludes two types of equilibrium outcomes that have been emphasized in the literature: first, pooling outcomes such as the one described in Attar et al. (2011), in which both types trade the same nonzero quantity at a price equal to the average quality of the good; second, separating outcomes such as the one described by Jaynes (1978), Hellwig (1988), and Glosten

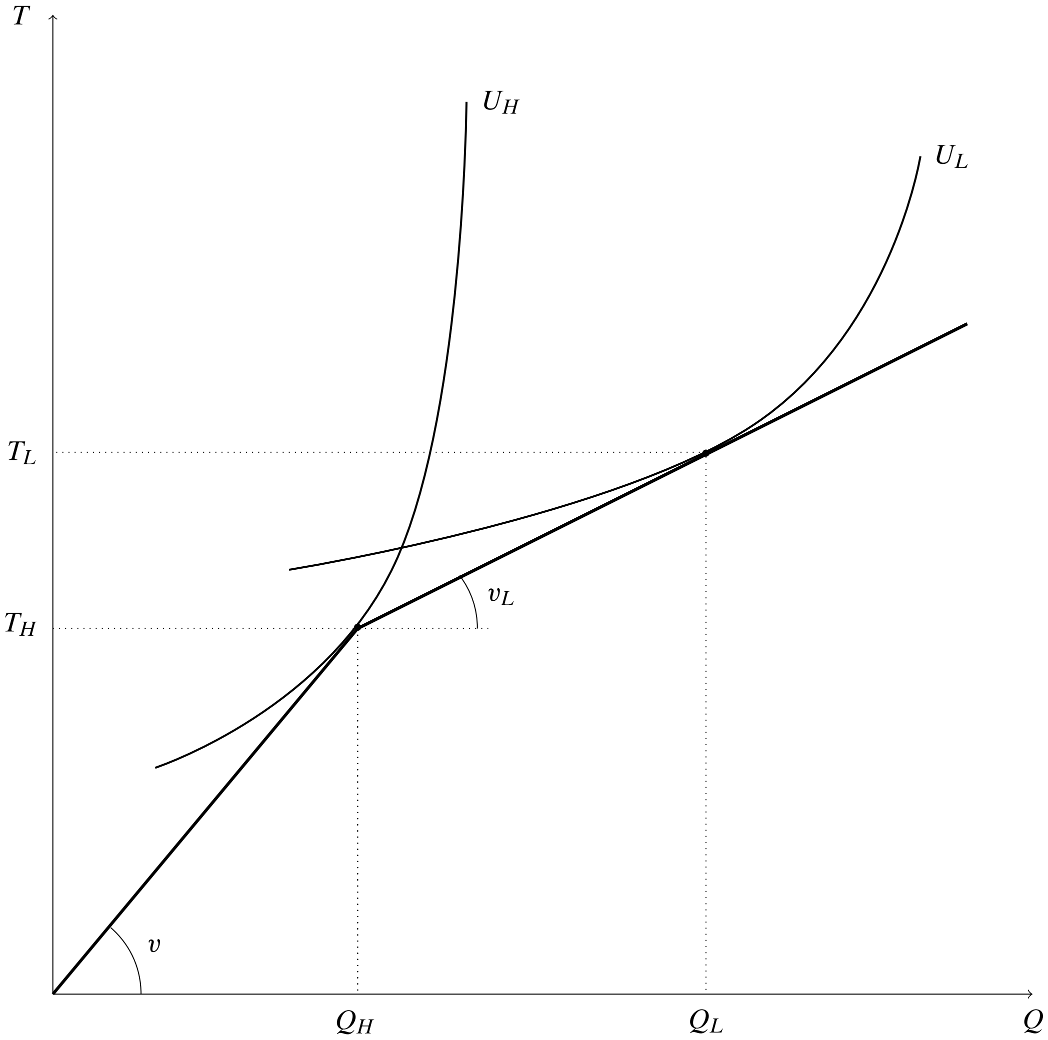

FIGURE 1. Depiction of a candidate Jaynes–Hellwig–Glosten equilibrium with $Q_L > Q_H > 0$.

(1994), and illustrated in Figure 1. If one leaves aside the case in which both types trade nonzero quantities on opposite sides of the market, the remaining possibilities for equilibrium outcomes are either that there is no trade in the aggregate or that only one type actively trades at a fair price in the aggregate.

To illustrate the logic of the no-cross-subsidization result, consider a candidate separating equilibrium with positive quantities $Q_L > Q_H > 0$, as illustrated in Figure 1. The basic price structure of such an equilibrium is delineated in Lemma 2(ii).

Let k be a buyer whose profit b_H^k from trading with type H is positive. According to Lemma 4, the aggregate trade (Q_H, T_H) remains available if buyer k removes his menu offer. He can thus attempt to pivot on (Q_H, T_H) to attract type L, which amounts to offering a contract $c_L^k = (Q_L - Q_H, T_L - T_H + \varepsilon_L)$ for some positive number ε_L. When ε_L is small enough, the loss for buyer k from trading c_L^k with type L is negligible, as the slope of the line segment that connects (Q_H, T_H) and (Q_L, T_L) is the fair price v_L. For buyer k's deviation to be profitable, he must make a profit when trading with type H. To do so, he can offer an additional contract $c_H^k = (q_H^k, t_H^k + \varepsilon_H)$ for some positive number ε_H. Because (q_H^k, t_H^k) was available for trade in equilibrium, c_L^k is more attractive than c_H^k for type L as long as ε_L is large enough relative to ε_H. Now, if type H trades c_H^k, the deviation is profitable, because when ε_H is small enough, c_H^k yields a profit close to $b_H^k > 0$ when traded by type H, whereas the loss from trading c_L^k with type L is negligible. If type H trades c_L^k instead, the deviation is still profitable, because c_L^k yields a positive profit when traded by both types. This shows that there exists no separating equilibrium with positive quantities. The reasoning for a pooling equilibrium is slightly more involved, but reaches the same conclusion.

Remark. The proof of Proposition 3 shows that cross-subsidies are not sustainable in equilibrium because it would otherwise be possible for some buyer to neutralize the type on which he makes a loss by proposing that she mimic the behavior of the other type when facing the other buyers. A key feature of this deviation is that it is performed by a buyer who is actively and profitably trading with one type in equilibrium.[17] Moreover, it is crucial for the argument that this buyer deviates to a menu that includes two nontrivial contracts targeted at the two types of sellers. Observe that this class of deviations was not considered in the early contributions of Jaynes (1978) and Glosten (1994). Jaynes (1978), who studies strategic competition between insurance providers under nonexclusivity, indeed restricts firms to the use of simple insurance policies. That is, each firm can propose at most one contract that is different from the no-trade contract.[18] As a consequence, an incumbent firm cannot profitably deviate by simultaneously making a loss when trading with the high-risk agent and compensating for this loss when trading with the low-risk agent. Glosten (1994) characterizes an aggregate price–quantity schedule that is robust to entry. In our setting, this schedule would be as depicted in Figure 1. By contrast, we do not take the aggregate price–quantity schedule as given, but we derive it from the individual menus offered by the buyers.

So far, we have focused on the aggregate equilibrium implications of our model. We now briefly sketch a few implications for individual equilibrium trades. The following result shows that each traded contract yields zero profit and that aggregate and individual equilibrium trades have the same sign.

[17] It is unclear that an entrant would be able to upset the above candidate equilibrium. One might think that an entrant could successfully attempt to nearly reap the aggregate profit on type H, say by proposing a contract of the form $(Q_H, T_H + \varepsilon_H)$, while making limited losses on type L by proposing a contract of the form $(Q_L - Q_H, T_L - T_H + \varepsilon_L)$ as above. Yet this would overlook the fact that by proposing such a contract to type H, the entrant would globally modify the structure of available trades, unlike the local deviation $(q_H^k, t_H^k + \varepsilon_H)$ we used in the proof of Proposition 3. As a result, type L might well be attracted by the contract $(Q_H, T_H + \varepsilon_H)$ because she may find it profitable to trade this contract along with some contracts offered by the incumbents, thereby upsetting the attempt at a successful entry.

[18] This assumption is maintained in the reformulation of Jaynes (1978) proposed by Hellwig (1988).

PROPOSITION 4. *In any equilibrium,* $b_j^k = 0$ *and* $q_L^k \geq 0 \geq q_H^k$ *for all* j *and* k.

Proposition 4 reinforces the basic insight of our model, according to which, in equilibrium, the seller can signal her type only through the sign of the quantities she trades. It follows that if a type does not trade in the aggregate, then she does not trade at all. Hence a pooling equilibrium, when it exists, is actually a no-trade equilibrium.

3.5 *Aggregate equilibrium trades*

In this section, we fully characterize the candidate aggregate equilibrium trades and we provide necessary conditions for the existence of an equilibrium. Given the price structure of equilibria delineated in Section 3.3 and the no-cross-subsidization result established in Section 3.4, all that remains to be done is to give restrictions on each type's equilibrium marginal rate of substitution. Two cases need to be distinguished, according to whether a type's aggregate trade is zero in equilibrium.

Our first result is that if type j does not trade in the aggregate, then her equilibrium marginal rate of substitution must lie between v and v_j. This is why an equilibrium may fail to exist for some parameter values.

LEMMA 5. *In equilibrium, if* $Q_j = 0$, *then* $v_j - \tau_j(0,0)$ *and* $\tau_j(0,0) - v$ *have the same sign.*

The intuition for Lemma 5 is as follows. Suppose, for instance, that $Q_H = 0$. If $v_H > \tau_H(0,0)$, then any buyer could attract type H by proposing a contract that offers to buy a small positive quantity at a unit price lower than v_H. For this deviation not to be profitable, type L must also trade this contract, and one must have $\tau_H(0,0) \geq v$, so that the deviator makes a loss when both types trade this contract. The same reasoning applies if $v_H < \tau_H(0,0)$, by considering a contract that offers to sell a small positive quantity at a unit price higher than v_H. The case $Q_L = 0$ can be handled in a symmetric way.

Our second result is that if type i trades a nonzero quantity in the aggregate, then she must trade efficiently in equilibrium.

LEMMA 6. *In equilibrium, if* $Q_i \neq 0$, *then* $\tau_i(Q_i, T_i) = v_i$.

The intuition for Lemma 6 is as follows. Suppose, for instance, that $Q_L > 0$. As cross-subsidization cannot occur in equilibrium, $T_L = v_L Q_L$. If type L were trading inefficiently in equilibrium, that is, if $\tau_L(Q_L, T_L) \neq v_L$, then there would exist a contract that offers to buy a positive quantity at a unit price lower than v_L, and that would give type L a strictly higher utility than (Q_L, T_L). Any of the buyers could profitably attract type L by proposing this contract, which would be even more profitable for the deviating buyer if traded by type H. Hence type L must trade efficiently in equilibrium. The case $Q_H < 0$ can be handled in a symmetric way.

To state our characterization result, it is necessary to define first-best quantities. The following assumption ensures that these quantities are well defined.

Assumption FB. *For each i, there exists Q_i^* such that $\tau_i(Q_i^*, v_i Q_i^*) = v_i$.*

Assumption FB states that Q_i^* is the efficient quantity for type i to trade at a unit price v_i that gives an aggregate zero profit for the buyers. An important consequence of the strict quasiconcavity of u_i is that $Q_i^* \geq 0$ if and only if $\tau_i(0,0) \leq v_i$, and that $Q_i^* = 0$ if and only if $\tau_i(0,0) = v_i$. In the pure-trade model, Q_i^* is defined by $c_i'(Q_i^*) = v_i$. In the insurance model, because of the agent's risk aversion, efficiency requires full insurance for each agent i, so that $Q_i^* = W_G - W_B$. The credit model is special in that the constraint that quantities must remain nonnegative may be binding. Efficiency requires that the net present value of the project, $\pi_i f_i(T) - T$, be maximized; if this leads to a positive and finite investment, the promised repayment Q_i^* that makes the investors just break even satisfies $\pi_i f_i'(\pi_i Q_i^*) = 1$. In contrast, if $\pi_i f_i'(0) \leq 1$, borrower i's investment project has a nonpositive net present value and it is efficient not to invest in her project.

We can now state our main characterization result.

Theorem 1. *If an equilibrium exists, then $\tau_L(0,0) \leq v \leq \tau_H(0,0)$. Moreover, the following statements hold.*

- *If $v_L \leq \tau_L(0,0) \leq v \leq \tau_H(0,0) \leq v_H$, all equilibria are pooling, with $Q_L = Q_H = 0$.*
- *Otherwise, all equilibria are separating and the following cases occur.*

  (i) If $\tau_L(0,0) < v_L < v < v_H < \tau_H(0,0)$, then $Q_L = Q_L^* > 0$ and $Q_H = Q_H^* < 0$.

  (ii) If $\tau_L(0,0) < v_L < v \leq \tau_H(0,0) \leq v_H$, then $Q_L = Q_L^* > 0$ and $Q_H = 0$.

  (iii) If $v_L \leq \tau_L(0,0) \leq v < v_H < \tau_H(0,0)$, then $Q_L = 0$ and $Q_H = Q_H^* < 0$.

The first message of Theorem 1 is negative: the nonexclusive competition game need not have an equilibrium. A necessary condition for an equilibrium to exist is that at a price equal to the average quality v, type L would like to *sell* some of the good, whereas type H would like to *buy* some of it. In the pure-trade model, no equilibrium exists if $c_L'(0) > v$ or $c_H'(0) < v$, that is, if the low-cost seller L is not eager enough to sell or if the high-cost seller H is too eager to sell. In the insurance model, no equilibrium exists if $[\pi_H/(1-\pi_H)]u'(W_G)/u'(W_B) < \pi/(1-\pi)$, where $\pi \equiv m_L \pi_L + m_H \pi_H$, that is, if the low-risk agent H is too eager to buy insurance.[19] In the credit model, no equilibrium exists if $\pi f_H'(0) > 1$, where again $\pi \equiv m_L \pi_L + m_H \pi_H$, that is, if the low-default-risk borrower H is too eager to invest.[20] Overall, Theorem 1 reinforces the insight of the no-cross-subsidization result: an equilibrium exists only if the adverse selection problem is severe enough so that both types' incentives to trade are not too closely aligned. On a more positive note, we show in Theorem 2 that the necessary condition $\tau_L(0,0) \leq v \leq \tau_H(0,0)$ also turns out to be sufficient for the existence of an equilibrium.

[19]This result is also obtained in Ales and Maziero (2011), assuming free entry. The second existence condition $\tau_L(0,0) \leq v$ or, equivalently, $[\pi_L/(1-\pi_L)]u'(W_G)/u'(W_B) \leq \pi/(1-\pi)$, is automatically satisfied in the insurance model as $\pi > \pi_L$ and $u'(W_B) > u'(W_G)$.

[20]The second existence condition $\tau_L(0,0) \leq v$ or, equivalently, $\pi f_L'(0) \geq 1$, is irrelevant in the credit model because the borrower cannot raise negative amounts of capital; see Section 5.1 and the Appendix.

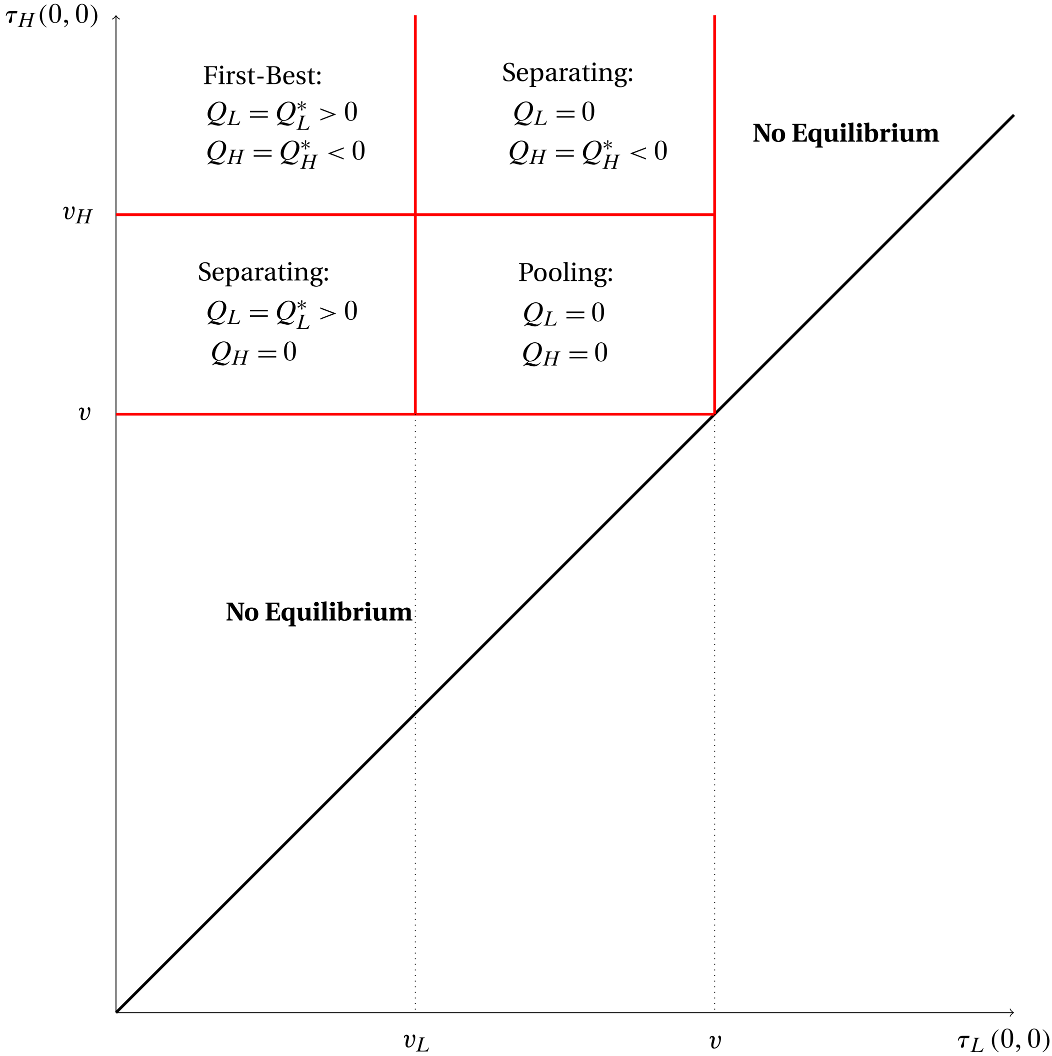

FIGURE 2. Depiction of the structure of equilibrium aggregate trades as a function of $\tau_L(0,0)$ and $\tau_H(0,0) > \tau_L(0,0)$ for fixed parameters v_L, v_H, and v.

Thus Theorem 1 provides a complete description of the structure of possible aggregate equilibrium outcomes, which is summarized in Figure 2.

The second message of Theorem 1 is that pooling additionally requires $v_L \leq \tau_L(0,0)$ and $v_H \geq \tau_H(0,0)$; by the no-cross-subsidization result, we already know that a pooling equilibrium involves no trade for both types. The conditions $v_L \leq \tau_L(0,0)$ and $v_H \geq \tau_H(0,0)$ together imply that $Q_L^* \leq 0 \leq Q_H^*$. When one of these inequalities is strict, the first-best quantities are not implementable. Thus pooling requires a strong form of nonresponsiveness: namely, in the first-best scenario, type L would like to buy and type H would like to sell. This cannot arise in the insurance model, for in that case $Q_L^* = Q_H^* = W_G - W_B$. Therefore, the insurance model admits no pooling equilibrium. In the pure-trade model, a pooling equilibrium requires that $c_L'(0) \geq v_L$ and $c_H'(0) \leq v_H$.[21] In the credit model, a pooling equilibrium requires that $\pi_L f_L'(0) \leq 1$ or, equivalently, that the investment project of the high-default-risk borrower L has nonpositive net present value.[22]

The third message of Theorem 1 is that in a separating equilibrium, at least one of the types trades efficiently. In case (i), the preferences of types L and H are sufficiently far apart from each other, in the sense that $Q_L^* > 0 > Q_H^*$: in the first-best scenario, type L would like to sell and type H would like to buy—a strong form of responsiveness. In that case, both types end up trading their first-best quantities in equilibrium. Clearly, neither the insurance model nor the credit model admits an equilibrium of this kind. In the pure-trade model, a first-best equilibrium exists if $c_L'(0) < v_L$ and $c_H'(0) > v_H$. In case (ii), both Q_L^* and Q_H^* are nonnegative: in the first-best scenario, both types would like to sell. The seller's preferences may or may not satisfy responsiveness. The unique candidate equilibrium outcome is then that seller L trades efficiently, while seller H does not trade at all. This is the situation that prevails in the insurance model when an equilibrium exists: in that case, the high-risk agent L obtains full insurance at an actuarially fair price, while the low-risk agent H purchases no insurance. In the pure-trade model, this type of equilibrium exists only if $c_L'(0) < v_L$ and $c_H'(0) \leq v_H$. In the credit model, this type of equilibrium exists only if $\pi_L f_L'(0) > 1$, that is, if the investment project of the high-default-risk borrower L has positive net present value.[23] Finally, case (iii) is symmetric to case (ii), exchanging the roles of types L and H. Note that in any separating equilibrium, each type strictly prefers her aggregate equilibrium trade to that of the other type. This contrasts with the predictions of models of exclusive competition under adverse selection, such as Rothschild and Stiglitz's model (1976), in which the high-risk agent L is indifferent between her equilibrium contract and that of the low-risk agent H.

Remark. It is interesting to compare the conclusions of Theorem 1 with those reached by Attar et al. (2011) in a nonexclusive version of Akerlof's (1970) market for lemons. Compared to the present setup, the two distinguishing features of their model is that the seller has linear preferences, $u_i(Q, T) = T - \theta_i Q$, and makes choices under an aggregate capacity constraint, $Q \leq 1$. Observe that in this context, type i's marginal rate of substitution is constant and equal to θ_i up to capacity. In a two-type version of their model in which there are potential gains from trade for each type, that is, $v_L > \theta_L$ and $v_H > \theta_H$, Attar et al. (2011) show that the nonexclusive competition game always admits an equilibrium, that the buyers earn zero profits, and that the aggregate equilibrium allocation is generically unique. If $\theta_H > v$, the equilibrium is similar to the separating equilibrium found in case (ii) of Theorem 1: type L trades efficiently, $Q_L = 1$ and $T_L = v_L$, while type H does not trade at all, $Q_H = T_H = 0$. In contrast, if $\theta_H < v$, the situation is markedly

[21] This is, for instance, the case in the Biais et al. (2000) setting if $\theta_L \geq v_L$ and $\theta_H \leq v_H$. It should, however, be noted that they explicitly rule out this parameter configuration.

[22] The second pooling condition $\tau_H(0,0) \leq v_H$ or, equivalently, $\pi_H f_H'(0) \geq 1$, is irrelevant in the credit model because the borrower cannot raise negative amounts of capital; see Section 5.1 and the Appendix.

[23] Again, the condition $\tau_H(0,0) \leq v_H$ is irrelevant in the credit model.

different from that described in Theorem 1. First, an equilibrium exists, whereas, in the analogous situation where $\tau_H(0,0) < v$, no equilibrium exists in our model. Second, any equilibrium is pooling and efficient, that is, $Q_L = Q_H = 1$ and $T_L = T_H = v$, whereas cross-subsidies and, therefore, nontrivial pooling equilibria are ruled out in our model. The key difference between the two setups that explains these discrepancies is that in the present paper, we do not require the seller's choices to satisfy an aggregate capacity constraint. This implies that some deviations that are crucial for our characterization result are not available in Attar et al. (2011). A case in point is the no cross-subsidization result: key to the proof of Proposition 3 is the possibility for a deviator who makes a profit when trading with type j to pivot on (Q_j, T_j) to attract type i, while preserving the profit he makes with type j. However, for the argument to go through, there must be no restrictions on the quantities traded in such deviations; in particular, it is crucial that the deviator be able to induce type i to consume more than Q_i in the aggregate.[24] This, however, is precisely what is impossible to do in the presence of a capacity constraint when both types trade up to capacity, as in the pooling equilibrium described in Attar et al. (2011).

4. Equilibrium existence

To establish the existence of an equilibrium, we impose the following technical assumption on preferences.

Assumption T. *There exist $\underline{Q}_H$ and $\overline{Q}_L$ such that*

$$\tau_H(Q,T) < v_H \quad \text{if } Q < \underline{Q}_H, \quad \text{and} \quad \tau_L(Q,T) > v_L \quad \text{if } Q > \overline{Q}_L,$$

uniformly in T.

Assumption T ensures that equilibrium menus can be constructed as compact sets of contracts and does not affect in any way our previous results. It should be emphasized that the restrictions it imposes on preferences are rather mild. In the pure-trade model, because of the quasilinearity of preferences, Assumption T follows from Assumption FB, and one can take $\underline{Q}_H = Q_H^*$ and $\overline{Q}_L = Q_L^*$. In the insurance model, Assumption T follows from the agent's risk aversion, and one can take $\underline{Q}_H = \overline{Q}_L = W_G - W_B = Q_H^* = Q_L^*$. In the credit model, Assumption T needs to be slightly modified, because quantities traded remain nonnegative and, more importantly, because $\tau_i(Q,T) = 1/f_i'(T)$ depends only on T. It is, however, easy to check from the proof of Theorem 2 that on this side of the market, we need only to require Assumption T to hold for aggregate trades (Q,T) such

[24]Formally, it follows from the proof of Proposition 3 that if $B_H > 0$ in a pooling equilibrium where each type trades a positive aggregate quantity Q, then for any small enough additional trade $(\delta_L, \varepsilon_L)$ such that $\tau_L(Q,T)\delta_L < \varepsilon_L$ and that would thus attract type L, one must have $v\delta_L \le \varepsilon_L$. If there are no restrictions on δ_L, this implies that $\tau_L(Q,T) = v$, from which a contradiction can be derived using Lemma 3. Yet if, for some reason, only nonpositive δ_L were admissible, say, because the seller could not trade more than Q in the aggregate, then one could conclude only that $\tau_L(Q,T) \le v$, from which no contradiction would follow.

that $T \geq v_L Q$. In the credit model, this amounts to assuming that $1 > \pi_L f_L'(\pi_L Q)$ for Q large enough, which is automatically satisfied if Assumption FB holds for $i = L$.

We can now state our existence result.

Theorem 2. *An equilibrium exists if and only if $\tau_L(0,0) \leq v \leq \tau_H(0,0)$. Moreover, there exists $\overline{Q} > 0 > \underline{Q}$ such that any equilibrium can be supported by at least two buyers posting the same tariff*

$$t(q) \equiv \min\{v_L q, v_H q\}, \quad \underline{Q} \leq q \leq \overline{Q},$$

while the other buyers stay inactive.

Theorem 2 shows that the necessary condition for the existence of an equilibrium given in Theorem 1 is also sufficient. These two results together provide a complete description of the aggregate equilibrium outcomes of our game. As for individual strategies, the tariffs chosen here to support equilibria entail linear pricing for both positive and negative quantities, with a kink at zero that one may interpret as a bid–ask spread. Another noteworthy feature of these strategies is that in no case can a buyer make a loss. Hence, even if these strategies involve contracts that are not traded in equilibrium, these latent contracts cannot turn out to be costly for the buyers. The number of active buyers is indifferent.

The lower and upper bounds $\underline{Q}$ and $\overline{Q}$ were introduced only to make sure that the corresponding menus of contracts are compact, to be consistent with the assumptions of our main characterization result. Yet the intuition of our existence result is easier to grasp when one eliminates these bounds. Suppose that some buyer were to deviate, for instance, in the hope of making profits from trading with type H. Because his competitors cannot make losses, this implies that following the deviation, the aggregate trade $(\hat{Q}_H, \hat{T}_H)$ chosen by type H should verify $v_H \hat{Q}_H > \hat{T}_H$. As the trade $(\hat{Q}_H, t(\hat{Q}_H))$ is available anyway, we get $\hat{T}_H \geq t(\hat{Q}_H)$, which implies $\hat{Q}_H > 0$. Because we have $\tau_H(0,0) \geq v$ by assumption, we also get that the final transfer $\hat{T}_H$ cannot be less than $v\hat{Q}_H$.

Similarly define $(\hat{Q}_L, \hat{T}_L)$ as the aggregate trade of type L following the deviation. Type L could trade as type H does and additionally sell a quantity $\hat{Q}_L - \hat{Q}_H$ in exchange for a transfer $t(\hat{Q}_L - \hat{Q}_H)$. By the single-crossing property, $\hat{Q}_L \geq \hat{Q}_H$. Hence type L can end up selling an aggregate quantity $\hat{Q}_L$ in exchange for a transfer $\hat{T}_H + v_L(\hat{Q}_L - \hat{Q}_H)$. As she chooses to trade $(\hat{Q}_L, \hat{T}_L)$ instead, this shows that $\hat{T}_L \geq \hat{T}_H + v_L(\hat{Q}_L - \hat{Q}_H)$. But we already know that $\hat{T}_H \geq v\hat{Q}_H$. In line with (4), we obtain that aggregate profits cannot be positive. As the deviator's competitors cannot make losses, the deviation cannot be profitable.

The fact that buyers cannot make losses should not be interpreted as an extreme aversion to the hazard of trading under adverse selection. Indeed, recall from Proposition 4 that in equilibrium, the seller credibly signals her information by the sign of the trade she proposes to make with each buyer. The buyers then become perfectly informed of the seller's type and Bertrand competition pushes prices down to their willingness-to-pay. Hence buyers cannot make losses, but they do not make any profits either. The fact that only two active buyers are needed to sustain an equilibrium confirms the Bertrand-like nature of nonexclusive competition in our setting.

Finally, we made no attempt to minimize the size of equilibrium menus. The proof of Theorem 2 provides such an implementation in the efficient case (i) of Theorem 1, for which it is sufficient that at least two buyers propose the efficient trades $(Q_L^*, v_L Q_L^*)$ and $(Q_H^*, v_H Q_H^*)$, but for the other more complex cases, we get only partial results. The question of minimum implementation thus remains open.

5. Extensions

5.1 *Nonnegative trades*

In some situations, a natural constraint on feasible trades is that quantities that are traded remain nonnegative: think, for example, of a producer who is selling a product of unknown quality, of a household that is buying insurance coverage when negative insurance is ruled out, or of a borrower who is seeking credit. It turns out that our results directly extend to this case, with obvious modifications, and the Appendix lists the minor changes that are needed in the proofs. In words, an equilibrium exists if and only if $\tau_H(0,0) \geq v$. Indeed, the condition $\tau_L(0,0) \leq v$ becomes irrelevant, because if $Q_L = 0$ and $\tau_L(0,0) > v$, the option of offering to type L to trade a negative quantity is no longer available. The characterization of aggregate equilibrium trades is then as follows. If $\tau_L(0,0) < v_L$, then, in any equilibrium, type L trades the efficient quantity Q_L^*, while type H does not trade at all. In contrast, if $\tau_L(0,0) \geq v_L$, then, in any equilibrium, neither type L nor H trades. Note that in this case, it is efficient for type L not to trade, but not necessarily so for type H, as, for instance, in the credit model when $\pi_L f_L'(0) \leq 1$ but $\pi_H f_H'(0) > 1$. Finally, any equilibrium outcome can be implemented by having all buyers ready to buy any quantity up to a large upper bound at a constant unit price v_L.

5.2 *Multiple sellers*

Another noteworthy extension to be considered is the case of multiple sellers. So suppose that n buyers now face m sellers indexed by $l = 1, \ldots, m$. Our results extend to this new game under the following assumptions. First, each buyer is able to identify each seller. Second, communication remains private and bilateral: a buyer cannot observe what other buyers propose and what transactions each seller concludes with other buyers. Consequently, a buyer k can only propose to each seller l to choose a contract in a menu C_l^k, a contract being, as above, a quantity-transfer bundle.[25] Third, the profit of a buyer remains additive, and equal to the sum of the profits he obtains with each seller. Under these assumptions, it is easily understood that each interaction between a given seller and the buyers can be studied in isolation.[26]

Hence, choose a seller l and consider the collection of menus $(C_l^1, \ldots, C_l^n)$ that are offered to her. Suppose there exists a buyer k for whom a profitable deviation $\hat{C}_l^k \neq C_l^k$

[25]Indeed, Han (2006, Theorem 1) has established that when attention is restricted to pure-strategy perfect Bayesian equilibria, any profile of equilibrium payoffs that can be more generally supported by arbitrary private bilateral communication mechanisms can also be supported by letting each principal independently offer a menu of contracts to each agent.

[26]The argument presented here parallels the one exposed more formally in Attar et al. (2011, Lemma 2).

exists in the single-seller game associated with seller l. In the multiple-seller game, this means that buyer k could deviate by offering the menus $(C_1^k, \ldots, \hat{C}_l^k, \ldots, C_m^k)$ instead of the menus $(C_1^k, \ldots, C_l^k, \ldots, C_m^k)$. As other sellers and buyers can neither notice nor react to this deviation, this would alter only the interaction with seller l and, thus, would constitute a profitable deviation for buyer k in the multiple-seller game. Conversely, consider the multiple-seller game and choose a buyer k who is offering menus $(C_1^k, \ldots, C_m^k)$. If there is a profitable deviation $(\hat{C}_1^k, \ldots, \hat{C}_m^k)$ for buyer k, then there must exist a seller l for whom offering $\hat{C}_l^k$ instead of C_l^k increases buyer k's profit. Therefore, such a change constitutes a profitable deviation for buyer k in the single-seller game played with seller l.

Together, these two arguments show that an equilibrium exists in the multiple-seller game if and only if an equilibrium exists in each of the m single-seller games, and that any aggregate equilibrium outcome in the multiple-seller game must be such that each seller ends up with the unique aggregate equilibrium allocation characterized in Theorem 1. Thus, our results extend to the case of multiple sellers. Note finally that these arguments also hold for heterogeneous sellers, whose types may be correlated, as is plausibly the case in financial markets.

5.3 *Beyond two types*

In our model, the seller's type can take only two values. The crucial simplification this assumption affords us is the ability to fully control the behavior of each type following some buyer's deviation. This property notably simplifies our pivoting technique, as developed in Lemma 1 and then used in Proposition 1, Lemma 4, and Proposition 4. Indeed, we often use two-contract deviations for buyer k such that one of these contracts attracts type j and increases the profit buyer k makes with her, whereas the other contract is close to the contract buyer k would trade with type i in equilibrium. Then we reason as follows: if type i were to trade the latter contract, buyer k would have a profitable deviation; hence type i must trade the same contract as type j. This, in turn, allows us to infer some information about the structure of equilibrium trades. Observe that because there are only two alternatives to consider, this kind of argument does not require the seller's indirect utility functions z_i^{-k} to be well behaved.

Beyond two types, it becomes hard, if not intractable, to control the behavior of each type following such a deviation. A first attempt to address this problem and, therefore, to extend our analysis to an arbitrary number of types, consists of focusing on equilibria in which the seller's indirect utility functions z_i^{-k} satisfy some additional properties. In related work, we extend our model to an arbitrary but finite number of types, concentrating on pure-strategy equilibria in which buyers offer concave quantity-transfer schedules or, equivalently, convex menus (Attar et al. 2013). It should be noted that the same restriction is made by Biais et al. (2000), which allows us to draw a clear comparison between our results and theirs.

A key implication of the assumption that buyers offer convex menus in equilibrium is that the indirect utility functions z_i^{-k} now satisfy a single-crossing property. This remarkably simplifies the analysis of buyers' deviations, as a given contract may attract only an "interval" of types. The results in Attar et al. (2013) then partially generalize the insights of the present paper. First, equilibria in convex menus—when they exist—involve large inefficiencies: the only type who may actually trade in equilibrium is the one who is the most eager to sell.[27] Next, when she trades, this type does so at a fair price, involving zero profits for the buyers. Finally, equilibrium existence conditions are increasingly difficult to meet when the number of types becomes large.

This Bertrand-like outcome contrasts with the Cournot-like outcome highlighted by Biais et al. (2000) in a version of the same model in which there is a continuum of types. As noted in Attar et al. (2011), a general insight of our analysis thus seems to be that the properties of equilibria crucially depend on the cardinality of the set of seller's types. A possible explanation for this discrepancy can be provided along the following lines. When the number of types is finite, any equilibrium in convex menus has the property that for any buyer k, at least one type is indifferent between two contracts in the menu she offers. If buyer k deviates to an alternative menu that entails a slight perturbation of these two contracts, he can effectively induce a significant change in the seller's behavior as well as a discontinuity in his own profit; hence, the Bertrand-like outcome. In contrast, when the seller's type varies continuously, it might well be that the seller is never indifferent between two contracts offered by any buyer k. Indeed, in the equilibrium constructed by Biais et al. (2000), the seller has a unique best response given the menus offered by the buyers. This implies, in particular, that the contract traded by type θ with any buyer k varies continuously with θ. Then consider again a slight perturbation of the contracts traded by buyer k with types in some small interval (θ_0, θ_1). Such an alternative offer may induce each type in (θ_0, θ_1) to trade contracts that are different from her equilibrium contract. By the single-crossing property, though, each of these types will choose to trade a contract close to her equilibrium contract. Given the continuity of his profit function, the corresponding change in buyer k's profit will be marginal; hence, the Cournot-like outcome.

To the best of our knowledge, this discontinuity between models with discrete type sets and models with continuous type sets is novel in the screening literature. It is hard to think of an a priori argument that would rule in favor of either assumption. In the context of our model, discrete type sets allow one to dispense with the regularity assumptions on distributions and valuations that play an important role in Biais et al. (2000) model.

6. Conclusion

In this paper, we analyzed the impact of adverse selection on markets where competition is nonexclusive. We fully characterized aggregate equilibrium trades, which are uniquely determined, and we provided a necessary and sufficient condition for the existence of a pure-strategy equilibrium. Our results show that under nonexclusivity, market breakdown may arise in a competitive environment where buyers compete through arbitrary menu offers: specifically, whenever the first-best outcome cannot be achieved, equilibria—when they exist—involve no trade for at least one type of seller.

[27] We restrict attention to the case where quantities sold must remain nonnegative.

These predictions contrast with those that obtain under exclusive competition, namely, that one type of seller trades efficiently, while the other type signals the quality of the good she offers by trading a suboptimal, but nonzero, quantity of this good. When competition is nonexclusive, each buyer's inability to control the seller's trades with his opponents creates additional deviation opportunities. This makes screening more costly and implies that the seller either trades efficiently or does not trade at all.

Our results may explain why some markets are underdeveloped. For instance, theory predicts that individual should find it in their best interest to annuitize a large part of their lifetime savings (Yaari 1965), yet in practice the demand for annuities remains low. Although several demand-side explanations, such as bequest motives, have been proposed to solve this puzzle, our analysis points at an alternative supply-side explanation based on nonexclusivity and adverse selection. As mentioned in the Introduction, nonexclusivity is a common feature of annuity markets. Adverse selection may arise because individuals have private information about their survival prospects. In this context, our analysis predicts that market participation should be limited to individuals who have the best survival prospects and have more to gain from purchasing annuities. This severely limits the size of the market unless participation is made mandatory. A similar argument may be put forward to explain the thinness of the long-term-care insurance market.

So far, there have been few investigations of the welfare implications of adverse selection in markets where competition is nonexclusive. A natural development of our analysis would be to study the decision problem faced by a planner seeking to implement an efficient allocation, subject to informational constraints and to the constraint that exclusivity be nonenforceable. It is unclear that such a planner can improve on the market allocation characterized in this paper. If he could, this would provide new theoretical insights in favor of welfare-based regulatory interventions; in particular, in the context of financial or insurance markets.

Appendix

Proof of Lemma 1. Let i, k, q, and t be as in the assumption of the lemma, and suppose that $v_i q - t > b_i^k$. Buyer k can deviate by proposing a menu consisting of the no-trade contract and of the contracts $c_i^k = (q, t + \varepsilon_i)$ and $c_j^k = (q_j^k, t_j^k + \varepsilon_j)$ for some positive numbers ε_i and ε_j. Given the assumption of the lemma, by trading c_i^k with buyer k and $(Q_i - q, T_i - t)$ with buyers other than k, type i gets utility $u_i(Q_i, T_i + \varepsilon_i) > U_i$. In equilibrium one has $U_i \geq z_i^{-k}(q_j^k, t_j^k)$ and the function z_i^{-k} is continuous. Thus, $u_i(Q_i, T_i + \varepsilon_i) > z_i^{-k}(q_j^k, t_j^k + \varepsilon_j)$ for any small enough ε_j, so that, for any such ε_j, type i must select c_i^k following buyer k's deviation. Consider now type j's behavior. By trading c_j^k, type j can get utility $u_j(Q_j, T_j + \varepsilon_j) > U_j$, so that she must select either c_i^k or c_j^k following buyer k's deviation. If type j selects c_j^k, then, by deviating, buyer k earns a profit

$$m_i(v_i q - t - \varepsilon_i) + m_j(v_j q_j^k - t_j^k - \varepsilon_j) = m_i(v_i q - t) + m_j b_j^k - (m_i \varepsilon_i + m_j \varepsilon_j).$$

However, from the assumption that $v_i q - t > b_i^k$, this is strictly higher than b^k when ε_i and ε_j are small enough, a contradiction. Hence type j must select c_i^k following buyer k's deviation. In equilibrium, this deviation cannot be profitable, so that $vq - t - \varepsilon_i \leq b^k$. Letting ε_i go to zero yields the desired implication. The result follows. □

Proof of Lemma 2. In the case of a pooling equilibrium, the conclusion follows immediately from the zero-profit result. Consider next a separating equilibrium and let us start with case (ii): $Q_L > Q_H \geq 0$. We know from Proposition 1 that $S_L \leq 0$. Suppose that $S_L < 0$. From (5) and the zero-profit result, we get $B_H \leq b_H^k$ for each k, which implies that $B_H \leq 0$. Now, notice from (4) that

$$B = vQ_H - T_H + m_L S_L = B_H + m_L[S_L - (v_H - v_L)Q_H].$$

Because $B_H \leq 0$, $S_L < 0$ and $Q_H \geq 0$, we obtain that $B < 0$, a contradiction. Therefore, it must be that $S_L = 0$, so that $T_L - T_H = v_L(Q_L - Q_H)$. This implies that $B = vQ_H - T_H$, so that $T_H = vQ_H$ as $B = 0$. The result follows. Case (iii) follows in a similar manner, exchanging the roles of L and H. Finally, consider case (i): $Q_L > 0 > Q_H$. As above, $B = B_H + m_L[S_L - (v_H - v_L)Q_H] = 0$. Suppose that $B_H > 0$ and thus $B_H > b_H^k$ for some k. Again, from (5), this implies that $S_L = 0$ and thus that $B_H - m_L(v_H - v_L)Q_H = B = 0$. Because $v_H > v_L$ and $B_H > 0$, one must have $Q_H > 0$, a contradiction. Hence $B_H \leq 0$. Symmetrically, using that $B = B_L + m_H[S_H - (v_L - v_H)Q_L] = 0$, we get $B_L \leq 0$. Thus $B_L = B_H = 0$ as $B = 0$, and hence $T_L = v_L Q_L$ and $T_H = v_H Q_H$. The result follows. □

Proof of Lemma 3. If $B_j > 0$, then one must have $T_j = vQ_j$ by Lemma 2. Any buyer k can deviate by proposing a menu consisting of the no-trade contract and of the contract $c_j^k = (Q_j + \delta_j, T_j + \varepsilon_j)$ for some numbers δ_j and ε_j. Suppose, to the contrary, that $\tau_j(Q_j, T_j) \neq v$. Then one can choose δ_j and ε_j such that $\tau_j(Q_j, T_j)\delta_j < \varepsilon_j < v\delta_j$. When δ_j and ε_j are small enough, the first inequality guarantees that type j can strictly increase her utility by trading c_j^k with buyer k. If type i trades c_j^k, then buyer k's profit from this deviation is $v(Q_j + \delta_j) - (T_j + \varepsilon_j) = v\delta_j - \varepsilon_j > 0$, in contradiction to the zero-profit result. Therefore, type i must not trade with buyer k and for this deviation not to be profitable, one must have $m_j[v_j(Q_j + \delta_j) - (T_j + \varepsilon_j)] = m_j(B_j + v_j\delta_j - \varepsilon_j) \leq 0$. Letting δ_j and ε_j go to zero yields $B_j \leq 0$, a contradiction. The result follows. □

Proof of Lemma 4. Suppose first that $U_j > z_j^{-k}(0,0)$ for some k. Then buyer k can deviate by proposing a menu consisting of the no-trade contract and of the contract $c_j^k = (Q_j, T_j - \varepsilon_j)$ for some positive number ε_j. When ε_j is small enough, one has $u_j(Q_j, T_j - \varepsilon_j) > z_j^{-k}(0,0)$, so that type j trades the contract c_j^k following buyer k's deviation. If type i does not trade the contract c_j^k, buyer k's profit from this deviation is $m_j(v_j Q_j - T_j + \varepsilon_j) = m_j(B_j + \varepsilon_j) > 0$, in contradiction to the zero-profit result. If type i trades the contract c_j^k, then, because $T_j = vQ_j$ by Lemma 2, buyer k's profit from this deviation is $vQ_j - T_j + \varepsilon_j = \varepsilon_j > 0$, again in contradiction to the zero-profit result. As in any case $U_j \geq z_j^{-k}(0,0)$, it must be that $U_j = z_j^{-k}(0,0)$ for each k. It follows that for any buyer k, there exists an aggregate trade (Q^{-k}, T^{-k}) with buyers other than k such that $u_j(Q^{-k}, T^{-k}) = U_j$.

Suppose now that $Q^{-k} \neq Q_j$. Then, from the strict quasiconcavity of u_i and Lemma 3, one must have $T^{-k} > vQ^{-k}$. We now examine two deviations for buyer k that pivot on (Q^{-k}, T^{-k}). First, define (q_1, t_1) such that $(q_1, t_1) + (Q^{-k}, T^{-k}) = (Q_j, T_j)$. Then the seller can trade $(Q_j - q_1, T_j - t_1)$ with buyers other than k. Moreover, using the fact that $T_j = vQ_j$ by Lemma 2 and that $T^{-k} > vQ^{-k}$, we get

$$vq_1 - t_1 = v(Q_j - Q^{-k}) - (T_j - T^{-k}) = T^{-k} - vQ^{-k} > 0.$$

Therefore, by Lemma 1, one must have $v_j q_1 - t_1 \leq b_j^k$, that is, again using $T_j = vQ_j$, $T^{-k} - v_j Q^{-k} + (v_j - v)Q_j \leq b_j^k$. As $T^{-k} > vQ^{-k}$, this implies that

$$(v_j - v)(Q_j - Q^{-k}) < b_j^k. \tag{8}$$

Second, define (q_2, t_2) such that $(q_2, t_2) + (Q^{-k}, T^{-k}) = (Q_i, T_i)$. Then the seller can trade $(Q_i - q_2, T_i - t_2)$ with buyers other than k. Moreover, using the fact that $S_i = 0$ and $T_j = vQ_j$ by Lemma 2, that $T^{-k} > vQ^{-k}$, and that $(v - v_i)(Q_i - Q_j) \geq 0$ by Assumption SC, we get

$$\begin{aligned}
vq_2 - t_2 &= v(Q_i - Q^{-k}) - (T_i - T^{-k}) \\
&= T^{-k} - vQ^{-k} + vQ_i - [T_j + v_i(Q_i - Q_j) - S_i] \\
&= T^{-k} - vQ^{-k} + (v - v_i)(Q_i - Q_j) \\
&> 0.
\end{aligned}$$

Therefore, by Lemma 1, one must have $v_i q_2 - t_2 \leq b_i^k$, that is, using again $S_i = 0$ and $T_j = vQ_j$, $T^{-k} - v_i Q^{-k} + (v_i - v)Q_j \leq b_i^k$. As $T^{-k} > vQ^{-k}$, this implies that

$$(v_i - v)(Q_j - Q^{-k}) < b_i^k. \tag{9}$$

Because $v = m_i v_i + m_j v_j$ and $m_i b_i^k + m_j b_j^k = 0$ by the zero-profit result, averaging (8) and (9) yields $0 < 0$, a contradiction. Therefore, one must have $Q^{-k} = Q_j$ and, thus, $T^{-k} = T_j$ as $u_j(Q^{-k}, T^{-k}) = U_j = u_j(Q_j, T_j)$. The result follows. □

Proof of Proposition 4. We first prove that $b_j^k = 0$ for all j and k. Suppose, to the contrary, that $b_j^k > 0$ for some j and k. We first show that $S_i = S_j = 0$. To prove that $S_i = 0$, observe that by the no-cross-subsidization result, one has $b_j^l < 0 = B_j$ for some $l \neq k$. From (5), this implies that $m_i S_i \geq B - b^l$. Because $B - b^l = 0$ by the zero-profit result and because $S_i \leq 0$ by Proposition 1, it follows that $S_i = 0$. To prove that $S_j = 0$, observe that if $b_j^k > 0$, then $b_i^k < 0 = B_i$ by the zero-profit result and the no cross-subsidization result. Arguing as for S_i, it follows that $S_j = 0$. Hence $S_i = S_j = 0$, as claimed. As $S_i + S_j = (v_i - v_j)(Q_i - Q_j)$, one must have $Q_i = Q_j$, and the equilibrium is pooling, with $(Q_i, T_i) = (Q_j, T_j) = (0, 0)$. Now, because $b_j^k > 0$ and because $(Q_j, T_j) = (0, 0)$ can obviously be traded with buyers other than k, one can show as in the proof of Proposition 3 that $\tau_i(0, 0) = v$. Finally, consider buyer l as above. As $b_j^l < 0$, one has $b_i^l > 0$ by the zero-profit result. Because $(Q_i, T_i) = (0, 0)$ can obviously be traded with buyers other than l, it follows along the same lines that $\tau_j(0,0)=v$ as well, which contradicts Assumption SC. Hence the result.

We next prove that $q_L^k \geq 0 \geq q_H^k$ for each k. Because $v_H > v_L$ and

$$s_i^k = v_i(q_i^k - q_j^k) - (t_i^k - t_j^k) = b_i^k - b_j^k - (v_i - v_j)q_j^k = (v_j - v_i)q_j^k$$

as $b_i^k = b_j^k = 0$, we need to show only that $s_i^k \leq 0$ for all i and k. Choose i, k, and $l \neq k$, and set $q \equiv q_i^k + q_i^l - q_j^k$ and $t \equiv t_i^k + t_i^l - t_j^k$. Then the seller can trade $(Q_i - q, T_i - t) = (q_j^k + \sum_{m\neq k,l} q_i^m, t_j^k + \sum_{m \neq k,l} t_i^m)$ with buyers other than l. We can thus apply Lemma 1. One has

$$v_i q - t - b_i^l = v_i(q_i^k + q_i^l - q_j^k) - (t_i^k + t_i^l - t_j^k) - b_i^l = s_i^k$$

and

$$v_j q - t - b_j^l = v_j(q_i^k + q_i^l - q_j^k) - (t_i^k + t_i^l - t_j^k) - b_j^l = -(s_j^k + s_j^l).$$

Therefore, according to (1),

$$s_i^k > 0 \quad \text{implies} \quad m_i s_i^k \leq m_j(s_j^k + s_j^l). \tag{10}$$

Now, suppose, to the contrary, that $s_i^k > 0$ for some i and k. Then, by (10),

$$m_i s_i^k \leq m_j(s_j^k + s_j^l) \tag{11}$$

for each $l \neq k$. Summing on $l \neq k$ yields

$$(n-1)m_i s_i^k \leq m_j[S_j + (n-2)s_j^k].$$

From Proposition 1, we know that $S_j \leq 0$. Hence, if $s_i^k > 0$, one must also have $s_j^k > 0$. Exchanging the roles of i and j in (10) yields

$$m_j s_j^k \leq m_i(s_i^k + s_i^l) \tag{12}$$

for each $l \neq k$. Combining (11) and (12) leads to $m_i s_i^k \leq m_j s_j^l + m_i(s_i^k + s_i^l)$ or, equivalently, $m_i s_i^l + m_j s_j^l \geq 0$ for each $l \neq k$. Note that we also have $m_i s_i^k + m_j s_j^k > 0$, as both s_i^k and s_j^k are positive. Summing all these inequalities yields $m_i S_i + m_j S_j > 0$, in contradiction to Proposition 1. Hence the result. □

Proof of Lemma 5. Suppose that $Q_j = 0$. If $\tau_j(0,0) = v_j$, the result is immediate. Suppose then that $\tau_j(0,0) \neq v_j$. Any buyer k can deviate by proposing a menu consisting of the no-trade contract and of the contract $c_j^k = (\delta_j, \varepsilon_j)$ for some numbers δ_j and ε_j. Choose δ_j and ε_j such that $\tau_j(0,0)\delta_j < \varepsilon_j < v_j \delta_j$. This ensures that when δ_j and ε_j are small enough, type j can strictly increase her utility by trading c_j^k with buyer k and that buyer k thereby makes a positive profit with type j. Therefore, type i must also trade c_j^k following buyer k's deviation and for this deviation not to be profitable, one must have

$\varepsilon_j \geq v\delta_j$. Thus we have shown that for any small enough δ_j and ε_j, $\tau_j(0,0)\delta_j < \varepsilon_j < v_j\delta_j$ implies that $\varepsilon_j \geq v\delta_j$, which is equivalent to the statement of the lemma. The result follows. □

Proof of Lemma 6. By the no-cross-subsidization result, if $Q_i \neq 0$, the equilibrium must be separating. Moreover, from Lemma 2, one must have $T_i = v_i Q_i$. Suppose, to the contrary, that $\tau_i(Q_i, T_i) \neq v_i$. Then any buyer k can deviate by proposing a menu consisting of the no-trade contract and of the contract $c_i^k = (q_i, t_i)$ for some numbers q_i and t_i. As $\tau_i(Q_i, T_i) \neq v_i$, it follows from the strict quasiconcavity of u_i that one can choose (q_i, t_i) close to (Q_i, T_i) such that $U_i < u_i(q_i, t_i)$ and $t_i < v_i q_i$, where q_i is positive if $i = L$ and negative if $i = H$. The first inequality guarantees that type i trades c_i^k following buyer k's deviation. As $v_i q_i > t_i$, type j must also trade c_i^k following buyer k's deviation and one must have $t_i \geq v q_i$, for, otherwise, this deviation would be profitable. Overall, we have shown that $v_i q_i > v q_i$. Because q_i is positive if $i = L$ and negative if $i = H$, and because $v_H > v > v_L$, we obtain a contradiction in both cases. The result follows. □

Proof of Theorem 1. Suppose first that a pooling equilibrium exists. Then, according to the no-cross-subsidization result, $Q_L = Q_H = 0$. Lemma 5 then implies that

$$v_L \leq \tau_L(0,0) \leq v \leq \tau_H(0,0) \leq v_H. \tag{13}$$

Suppose next that a separating equilibrium exists. Then, according again to the no-cross-subsidization result, only three scenarios are possible.

(i) In the first case, $Q_H < 0 < Q_L$. Then, by Lemma 2, $T_L = v_L Q_L$ and $T_H = v_H Q_H$. Moreover, by Lemma 6, $\tau_L(Q_L, T_L) = v_L$ and $\tau_L(Q_H, T_H) = v_H$. As a result, $Q_L = Q_L^*$ and $Q_H = Q_H^*$, so that $Q_H^* < 0 < Q_L^*$. The strict quasiconcavity of u_i then implies that

$$\tau_L(0,0) < v_L \quad \text{and} \quad \tau_H(0,0) > v_H. \tag{14}$$

(ii) In the second case, $Q_H = 0 < Q_L$. Then, by Lemma 5, $v \leq \tau_H(0,0) \leq v_H$. Moreover, by Lemma 2, $T_L = v_L Q_L$. Finally, by Lemma 6, $\tau_L(Q_L, T_L) = v_L$. As a result, $Q_L = Q_L^*$, so that $Q_L^* > 0$. The strict quasiconcavity of u_i then implies that

$$\tau_L(0,0) < v_L \quad \text{and} \quad v \leq \tau_H(0,0) \leq v_H. \tag{15}$$

(iii) In the third case, $Q_H < 0 = Q_L$. Then, by Lemma 5, $v_L \leq \tau_L(0,0) \leq v$. Moreover, by Lemma 2, $T_H = v_H Q_H$. Finally, by Lemma 6, $\tau_H(Q_H, T_H) = v_H$. As a result, $Q_H = Q_H^*$, so that $Q_H^* < 0$. The strict quasiconcavity of u_i then implies that

$$v_L \leq \tau_L(0,0) \leq v \quad \text{and} \quad \tau_H(0,0) > v_H. \tag{16}$$

To conclude the proof, observe that from (13)–(16), an equilibrium exists only if $\tau_L(0,0) \leq v \leq \tau_H(0,0)$. As conditions (13)–(16) are mutually exclusive, the characterization of the candidate aggregate equilibrium trades is complete. Hence the result. □

Proof of Theorem 2. Choose an integer m, $2 \leq m \leq n$, and fix $\underline{Q}$ and $\overline{Q}$ such that $\underline{Q} < \min\{0, \underline{Q}_H\}/(m-1)$ and $\overline{Q} > \max\{0, \overline{Q}_L\}/(m-1)$. Suppose that m buyers post the tariff

t defined as in the theorem, while the other buyers stay inactive and propose only the no-trade contract. Consider any buyer. In the aggregate, his competitors post the tariff

$$T^{-}(Q^{-}) \equiv \min\{v_L Q^{-}, v_H Q^{-}\}, \quad \underline{Q}_1 \le Q^{-} \le \overline{Q}_1,$$

where Q^{-} refers to the aggregate quantity they trade. Here $\underline{Q}_1$ is either $m\underline{Q}$ or $(m-1)\underline{Q}$, and thus is no greater than $\underline{Q}_H$, and similarly for $\overline{Q}_1$, which cannot be smaller than $\overline{Q}_L$. Note also that if the efficient trade Q_H^* is negative, then $\underline{Q}_1 \le Q_H^* \le \overline{Q}_1$; the symmetrical statement applies for Q_L^*.

Suppose that our buyer deviates and ends up trading (q_L, t_L) with type L and (q_H, t_H) with type H. For his deviation to be profitable, he must make a positive profit with at least one type, say type H (the proof for type L is symmetrical). Hence $v_H q_H > t_H$. Define $Q_i^{-} \in [\underline{Q}_1, \overline{Q}_1]$ as the quantity traded by type i with the deviator's competitors following his deviation. Also define $\hat{Q}_i$ as the total quantity traded by type i, so that $\hat{Q}_i = q_i + Q_i^{-}$, and define $\hat{T}_i$ as the total transfer obtained by type i, so that $\hat{T}_i = t_i + T^{-}(Q_i^{-})$. The tariff T^{-} is such that the deviator's competitors cannot make losses following the deviation. Therefore, as $v_H q_H > t_H$, one must have $v_H \hat{Q}_H > \hat{T}_H$. Because the no-trade contract is available, we get

$$u_H(\hat{Q}_H, v_H \hat{Q}_H) > u_H(\hat{Q}_H, \hat{T}_H) \ge u_H(0, 0). \tag{17}$$

If $\hat{Q}_H < 0$, then (17) implies that $\tau_H(0,0) > v_H$, so that $Q_H^* < 0$. By construction of the tariff T^{-}, type H can then trade $(Q_H^*, v_H Q_H^*)$ with the deviator's competitors, thereby getting utility $u_H(Q_H^*, v_H Q_H^*) = \max_Q\{u_H(Q, v_H Q)\} > u_H(\hat{Q}_H, \hat{T}_H)$ by (17), a contradiction. As the case $\hat{Q}_H = 0$ is trivially ruled out by (17), it must be that $\hat{Q}_H > 0$. From (17), we now get $\tau_H(0,0) < v_H$, for, otherwise, type H would be strictly better off not trading at all than trading $(\hat{Q}_H, v_H \hat{Q}_H)$.[28] Because $\tau_H(0,0) \ge v$ by assumption, from (17) again we get $\hat{T}_H \ge v\hat{Q}_H$, for, otherwise, type H would be strictly better off not trading at all than trading $(\hat{Q}_H, \hat{T}_H)$. Finally, notice that $T^{-}(Q^{-}) \le vQ^{-}$ for all $Q^{-} \in [\underline{Q}_1, \overline{Q}_1]$. Thus $v\hat{Q}_H = vq_H + vQ_H^{-} \le \hat{T}_H = t_H + T^{-}(Q_H^{-}) < v_H q_H + vQ_H^{-}$ and hence $q_H > 0$.

Type L may also choose to trade (q_H, t_H) with the deviator. He would then have to choose some Q^{-} to maximize $u_L(q_H + Q^{-}, t_H + T^{-}(Q^{-}))$, subject to $\underline{Q}_1 \le Q^{-} \le \overline{Q}_1$. Notice first from the definition of $\overline{Q}_L$ that the constraint $Q^{-} \le \overline{Q}_1$ does not play any role: indeed $q_H > 0$, so that when Q^{-} reaches its upper bound $\overline{Q}_1$, the total quantity traded $q_H + Q^{-}$ is higher than $\overline{Q}_L$ and, therefore, type L's marginal rate of substitution is higher than v_L by Assumption T. We can thus eliminate the constraint $Q^{-} \le \overline{Q}_1$, taking care of extending the tariff T^{-} beyond $\overline{Q}_1$ by setting $T^{-}(Q^{-}) \equiv v_L Q^{-}$ for all $Q^{-} > \overline{Q}_1$. Now $\hat{Q}_L - q_H$ satisfies the remaining constraint $\underline{Q}_1 \le Q^{-}$; indeed, thanks to Assumption SC, we have $\hat{Q}_L \ge \hat{Q}_H$, so that $\hat{Q}_L - q_H \ge Q_H^{-} \ge \underline{Q}_1$. We thus have shown that type L can get at least utility $u_L(\hat{Q}_L, t_H + T^{-}(\hat{Q}_L - q_H))$. Observe that the transfer in this

[28]Note that the condition $\tau_H(0,0) < v_H$ excludes the efficient case (i) of Theorem 1. In that simple case, an inspection of the above lines reveals that we have used only the fact that $(Q_H^*, v_H Q_H^*)$ is offered by the deviator's competitors. We have thus shown that in the efficient case (i) of Theorem 1, any equilibrium can be sustained by having at least two buyers posting the two trades $(Q_H^*, v_H Q_H^*)$ and $(Q_L^*, v_L Q_L^*)$.

expression can be rewritten as $\hat{T}_H + T^-(\hat{Q}_L - q_H) - T^-(Q_H^-)$, which is no less than $\hat{T}_H + v_L(\hat{Q}_L - \hat{Q}_H)$ by concavity of T^-. Because type L is supposed to end up with utility $u_L(\hat{Q}_L, \hat{T}_L)$ following the deviation, it follows that $\hat{T}_L \geq \hat{T}_H + v_L(\hat{Q}_L - \hat{Q}_H)$. Moreover, as shown above, $\hat{T}_H \geq v\hat{Q}_H$. Therefore, the aggregate profit, which may as usual be written as $v\hat{Q}_H - \hat{T}_H + m_L[v_L(\hat{Q}_L - \hat{Q}_H) - (\hat{T}_L - \hat{T}_H)]$, is at most zero. Because the tariff T^- is such that the deviator's competitors cannot make losses, the deviation cannot be profitable. Hence the result. □

When only nonnegative quantities can be sold

A careful rereading of the proofs leads to the following changes.

Lemma 1 is still valid, but only when q is nonnegative. Proposition 1 now allows only to conclude that $S_L \leq 0$. Because $S_L = B_L - B_H + (v_H - v_L)Q_H$ and $Q_H \geq 0$ by assumption, a useful consequence of Proposition 1 is that $B_H \geq B_L$.

Proposition 2 still holds. Indeed (5) still holds, as its derivation involves only nonnegative trades. Once (5) is proven, one has to include the following argument. Suppose $B > 0$. From the above remark that $B_H \geq B_L$, it must be that $B_H > 0$ and there exists k such that $B_H > b_H^k$. From (5) applied to k at $(i, j) = (L, H)$, we get $m_L S_L \geq B - b^k = \sum_{l \neq k} b^l \geq 0$. As $S_L \leq 0$ by Proposition 1, we get $S_L = 0$. But by the single-crossing property, $S_L + S_H = (v_L - v_H)(Q_L - Q_H) \leq 0$, so we now know that both S_L and S_H are nonpositive. One can then use the last lines of the proof of Proposition 2 to conclude that profits must be zero.

Lemma 2 still holds. Only the pooling case and case (ii) remain.

Lemma 3 and its proof are unchanged. Indeed, notice that if $B_j > 0$, then one must have $Q_j > 0$, so that deviations that involve a small change in this quantity are feasible.

Lemma 4 still holds, but the proof has to be adapted somewhat. Suppose $B_j > 0$. From the zero-profit result and the above remark that $B_H \geq B_L$, it must be that $j = H$. As $B_H > 0$, there exists k such that $B_H > b_H^k$ and, thus, we can apply (5) to k at $(i, j) = (L, H)$ to get $S_L = 0$. The first step of the proof shows without changes that there exists an aggregate trade (Q^{-k}, T^{-k}) with buyers other than k such that $u_H(Q^{-k}, T^{-k}) = U_H$. If $Q^{-k} < Q_H$, then the two deviations used in the rest of the proof are feasible, as both q_1 and q_2 are positive. If $Q_H < Q^{-k} < Q_L$, then only deviation (q_2, t_2) is feasible (recall that $j = H$), so that (9) holds. Therefore, $b_L^k > 0$. Because $B_L \leq 0$, this implies that there exists $l \neq k$ such that $B_L > b_L^l$. We can then apply (5) to l at $(i, j) = (H, L)$ to get $S_H = 0$. Because $S_L = S_H = 0$ and $S_L + S_H = (v_L - v_H)(Q_L - Q_H)$, we get $Q_L = Q_H$, in contradiction to our assumption that $Q_H < Q^{-k} < Q_L$. Finally, it cannot be that $Q^{-k} \geq Q_L$, for, otherwise, type L would strictly prefer (Q^{-k}, T^{-k}) to (Q_L, T_L) by the single-crossing property.

Proposition 3 still holds. Recall that the assumption $B_j > 0$ implies that $j = H$ and, thus, the proof of Proposition 3 needs no change.

At this point, we know from Lemma 2 and Proposition 3 that $Q_H = T_H = 0$ in any equilibrium, so that necessarily $q_H^k = t_H^k = 0$ for each k and that $T_L = v_L Q_L$. Deriving the other results is then easy and requires only minor adaptations to the proofs. The only important difference concerns Lemma 5 and, as a result, the statement of the necessary condition for the existence of an equilibrium in Theorem 1. Indeed, although the reasoning in Lemma 5 remains correct, we must take into account the restriction $\delta_j \geq 0$. When $j = L$, we get that $\tau_L(0,0)\delta_L < \varepsilon_L < v_L\delta_L$ implies that $\varepsilon_L \geq v\delta_L$ and, thus, $v_L\delta_L > \varepsilon_L \geq v\delta_L$, which is impossible if $\delta_L \geq 0$. By contraposition, we can thus conclude only that $\tau_L(0,0) \geq v_L$ if $Q_L = 0$. As a result, $\tau_L(0,0) \leq v$ is no longer a necessary condition for the existence of an equilibrium. Indeed, one may have $Q_L = 0$ and yet $\tau_L(0,0) > v > v_L$. When $j = H$, in contrast, the conclusion of Lemma 5 remains correct, from which it follows that $\tau_H(0,0) \geq v$ is a necessary and indeed sufficient condition for the existence of an equilibrium.

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