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Transparency and price formation

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We study the role that price transparency plays in determining the efficiency and surplus division in a sequential bargaining model of price formation with asymmetric information. Under natural assumptions on type distributions and for any discount factor, we show that the unobservability of past negotiations leads to lower prices and faster trading. Unobservability, therefore, enhances the "Coasian effect" by fostering efficiency and diverting more of the surplus to the player who possesses private information. In addition, we show that the equilibrium is unique and is in pure strategies in the nontransparent regime; this stands in sharp contrast to the existing literature and allows for a better understanding of the Coasian effect and price observability.

Keywords. Coase conjecture, bargaining, durable goods monopoly, incomplete information, price formation, transparency.

JEL classification. C61, C73, C78.

1. Introduction

Sequential bargaining is not only a workhorse in analyzing bilateral interactions, with applications ranging from dispute resolution to labor contracting, but also a model of price formation and surplus division, which are of fundamental importance in economic theory. In dynamic trading environments, the details of information structures matter, because potential informational spillovers across players and over time introduce various channels through which incomplete information influences the price formation process. In bargaining games, a particular variation in the information structure is whether price offers are made publicly or in private. The goal of this paper is to investigate the effect of transparency in this sense on the price formation process in an important class of bargaining environments: Coasian bargaining.

In our model, there is an impatient buyer and an infinite sequence of sellers. The first seller makes an offer to the buyer, which the buyer either accepts or rejects. If the

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buyer accepts the seller's offer, the game ends. If the buyer rejects the seller's offer, the buyer moves to the next seller, who makes the buyer an offer. The game continues in the same fashion: if seller t's offer is rejected, the buyer moves to seller $t+1$. The gains from trade are commonly known, but the buyer has private information about his willingness to pay.

Formally, we amend the classic Coasian bargaining model with a sequence of sellers instead of one long-run seller. This model allows us to compare two different configurations of information flow among the short-run sellers. As in a standard Coasian model, with appropriate adjustments, our analysis remains valid when the roles and asymmetric information of the buyer and seller are switched. Several real-world markets—housing transactions, certain labor markets, corporate acquisitions, and over-the-counter derivatives trading—exhibit characteristics of this model with either a long-run informed buyer or a long-run informed seller. The observability of past rejected offers varies across these markets: among the examples listed, tender offers for corporate acquisitions are most often made publicly and are observed even when rejected, even though covert offers are not rare either. In housing markets and labor markets, offers are often covert, while rejected offers made public are not uncommon either. In over-the-counter markets as well, transactions are largely opaque and quotes are unobservable to subsequent traders (see, e.g., Zhu 2012).[1] Granted, all these markets have distinct characteristics and the market outcomes are the result of a complicated interaction of various institutional details. Our model, which is admittedly stark, captures one mechanism via which observability of past offers may impact the outcomes and, therefore, we believe that our results contribute to the understanding of such markets.

We identify the role of price observability in determining the surplus distribution (measured by the equilibrium prices) and the efficiency of trade (measured by the amount of delay before trade takes place). More specifically, we compare the equilibrium price sequences and expected delay under two opposing specifications: one where past prices are observable to subsequent sellers (*transparent regime*) and one where they are not (*nontransparent regime*). Under natural restrictions on the distribution of the buyer's valuations, we find that prices are uniformly lower in the nontransparent regime than in the transparent regime for each given discount factor of the long-run buyer. Moreover, even though an agreement is eventually reached in either regime, under stronger restrictions on the type distribution, we show that the expected delay is larger in the transparent regime. In the flip side of the model, when an informed long-run seller sequentially meets buyers who make offers, the transparent regime leads to *lower* prices and *longer* expected delay.

All of our results are obtained for the arbitrary discount factors of the long-run buyer and not just for the case where the buyer is sufficiently patient. This feature makes our analysis of the effect of price transparency robust to market frictions, which, beyond its

[1] Indeed, even though the precise number of past quotes is unlikely to be known, the amount of time that traders spent searching for a particular deal might be observable. The same is true when an unemployed worker searches for jobs. Therefore, a deterministic arrival of players in our Coasian model is a plausible description of these markets.

theoretical interest, is valuable in understanding interactions in real markets where frictions cannot be ignored. Indeed, for the frictionless limit, the outcomes of both regimes are degenerate: trade is efficient and the informed player captures all the surplus. This is consistent with the classic Coase conjecture. From this perspective, our results imply that, away from the frictionless limit, a lack of transparency enhances the “Coasian effect” by fostering efficiency and diverting more of the surplus to the informed player.[2] Moreover, the comparison with the Coase conjecture implies that market frictions amplify the effect of transparency.

In an infinite horizon bargaining game without parametric assumptions or closed-form solutions, comparing equilibrium price paths in two different extensive forms is a rather challenging task. The observation that allowed us to make progress in this task comes from elementary demand theory: the comparison of equilibrium prices in two markets boils down to the comparison of demand elasticities in these markets. We identify conditions ensuring an appropriate demand elasticity ranking for our two dynamic bargaining environments. Our appeal to demand theory not only resolves the analytical difficulties, but also highlights the economic forces at play in dynamic bargaining problems.

To gain some intuition, first recall the well known “skimming property” from Fudenberg et al. (1985): regardless of the regime and in any equilibrium, a price is accepted by the buyer if and only if his valuation is above an associated cutoff. This property allows one to interpret the buyer’s decisions as defining an endogenous demand curve that each seller faces in equilibrium, where the probability of trade at each price is interpreted as the quantity sold. Gul et al. (1986) point out that, with this interpretation, when both parties are long-run, their bargaining problem can equivalently be viewed as the problem of a durable goods monopolist lacking the power to commit to a price. In contrast to Gul et al. (1986), where a single durable goods monopolist competes with his future selves, in our model, a sequence of sellers compete with each other over time. Each seller faces a *residual market* characterized by a demand curve endogenously determined by the equilibrium strategies of all past and future sellers.

With this interpretation at hand, the main exercise is to compare the demand curves faced by each seller in both regimes. First consider a hypothetical price change by seller 1. Transparency forces seller 2, who enters the game only when there is no trade in the first period, to respond with a price change in the same direction. In contrast, in the nontransparent regime, seller 2 cannot react to such a price change. In equilibrium, the buyer fully anticipates the reaction of seller 2 and, hence, is less sensitive to the price

[2]We believe this result may have something to add to the discussion of the design of certain markets. Since transparency affects both the surplus distribution and the speed of trading, the answer to the design question necessarily depends on the details of the markets and the designer’s objectives. If the market designer cares more about faster trading when there is common knowledge of gains from trade, then nontransparency should be preferred. For instance, in over-the-counter (OTC) markets, a trader often gets involved in many transactions and he is the more informed party on some and the less informed party on others. In this sense, perhaps the surplus distribution over different transactions might be averaged out for a given trader. In this case, maximizing the speed of transaction could be a plausible mechanism design objective. If this were the case, our results would give support to using a dark pool as opposed to an open-order book.

change by seller 1 in the transparent regime. That is, the demand curve faced by seller 1 in the transparent regime is *steeper* than that in the nontransparent regime. However, the ranking of the slopes of the demand curves does not translate directly into the ranking of elasticities or the ranking of profit-maximizing prices. Indeed, since a seller's demand curve is determined jointly by the strategic choices of all previous and future sellers, the relative positions of the two demand curves corresponding to the same seller in either regime is a priori unclear. Therefore, the above simple intuition is not enough given the subtleties of our problem. We show that a relatively benign regularity condition on the buyer's type distribution—increasing hazard rate—pins down the relative positions of the demand curves and, therefore, allows an unambiguous comparison of prices in the two regimes.

We next explore which regime leads to a larger delay in trade. Delay in the context of the bargaining model is related to the quantity traded in the analogous dynamic monopoly model with larger quantities corresponding to smaller delay. Typically, a less elastic demand curve implies a higher price, but, as is well understood in demand theory, elasticities alone do not determine the ranking of quantities. One needs to uncover additional details about the demand functions, which are endogenous equilibrium objects in our model. In spite of this, we are able to show that transparency entails more delay if the buyer's type distribution is concave.

Our contribution is not limited to the comparison of the two regimes. Even though the transparent regime in isolation is the focus of the Coasian bargaining and durable goods monopoly literature, the equilibrium characterization of the nontransparent regime is novel to this paper. We are able to show that the equilibrium outcome in the nontransparent regime is unique and is necessarily in pure strategies under the minimal assumption of increasing virtual valuation. This result, which does not require any genericity assumption, is in sharp contrast to the classic results of Coase bargaining where offers are publicly observable, in which case randomization can happen in the first period and may be necessary off the equilibrium path (see Fudenberg et al. 1985 and Ausubel et al. 2002). Gul et al. (1986) conjectured that a pure strategy equilibrium can be obtained for the no-gap case with continuous distributions, and they argued that this is one of the properties for an equilibrium to be "a salient predictor of market behavior." Their conjecture remains open. The pure strategy property in our model is also surprising in view of the results on dynamic markets for lemons with unobservable offers where randomization is a generic property (see Hörner and Vieille 2009 and Fuchs et al. 2014). This equilibrium property allows for a characterization of the role of transparency that is not possible elsewhere.

Related literature

The role of observability has been investigated in different environments. Bagwell (1995) studies the connection between commitment power and observability and shows that the first-mover's advantage can be eliminated if its action is not perfectly observed. Rubinstein and Wolinsky (1990) study random matching and bargaining. In their complete information environment, observability enlarges the equilibrium set by a folk theorem argument that is not at work in the presence of incomplete information. Swinkels

(1999) analyzes a dynamic Spencian signaling model and obtains pooling equilibrium under private offers, while Nöldeke and van Damme (1990) previously obtained the Riley outcome in the case of public offers.

Related to our study of the more standard Coasian bargaining with independent valuations is the strand of literature that studies bargaining with interdependent values; see Evans (1989), Vincent (1989), and Deneckere and Liang (2006). Our closest precursor is the work of Hörner and Vieille (2009), who study an interdependent-value model with a single long-run seller and a sequence of short-run buyers, and find that inefficiencies take different forms in the two opposing information structures. They show that in the hidden-offer case, multiple equilibria exist—all in mixed strategies—and an inefficient delay occurs even as the discount factor goes to 1, while in the public-offer case, remarkably, an inefficient impasse ensues beyond the first period. The question about the impact of price transparency on surplus division and the timing of trade for general discount factors is not addressed, however, and no clear-cut comparison of the two regimes in terms of price paths and the long-run player's welfare is obtained. It is noteworthy that our independent-value model is not a limiting case of Hörner and Vieille's model, and, hence, the qualitative divergence of results in terms of equilibrium structure, efficiency, and price comparisons is not completely surprising.[3] Beyond Hörner and Vieille's (2009) initial exploration, the role of transparency has been extended to other settings. Kim (2012) presents a random matching model in which efficiency of trade may not be monotonic in the search friction. Bergemann and Hörner (2010) study the role of transparency on auction outcomes.

In our model, efficiency is always obtained when the discounting friction vanishes. We emphasize that bilateral sequential bargaining, rather than other centralized mechanisms, is an appropriate model for thin markets in which trading opportunities do not arise frequently and, hence, discounting frictions are nonnegligible. Accordingly we focus on a comparison of price dynamics, surplus division, and the timing of trade in the two regimes that is robust to all discounting frictions, and this task requires new methods. We obtain an unambiguous comparison and show that the informed player has a clear-cut preference over nontransparent market information structure.

Several bargaining models that feature discounting as a source of search friction are similar in structure to our model. Fudenberg et al. (1987) consider bargaining games where a seller can decide whether to switch to a new buyer or continue to bargain with an incumbent buyer. They show that a take-it-or-leave-it offer endogenously emerges as an equilibrium outcome. However, there are also other equilibria.

The paper is organized as follows. Section 2 introduces the formal model. Section 3 considers a two-period example to demonstrate the forces driving our results. Section 4 establishes the existence and the uniqueness of equilibrium for the nontransparent regime. Sections 5 and 6 present our results concerning the comparison of prices

[3]The results of Hörner and Vieille (2009) rely crucially on the assumption that buyer–seller types are sufficiently interdependent and the discount factor is sufficiently large. Indeed, as the interdependence vanishes (i.e., the values of the uninformed short-run players become a constant), the lower bound required for the discount factor converges to 1, implying that the limiting case is not a well defined Coasian bargaining game.

and speed of trade across two regimes, and Section 7 concludes. All omitted proofs are relegated to the Appendix. Additional material is available in a supplementary file on the journal website, http://econtheory.org/supp/1566/supplement.pdf.

2. Model

A buyer bargains with a sequence of sellers. In each period $t = 1, 2, \dots$, a new seller enters the game. We refer to the seller at period t as *seller* t. Each seller has one unit to sell for which his reservation value is normalized to 0. The buyer has demand for one unit.

The buyer discounts future payoffs at rate $\delta \in (0, 1)$ and has private information about his valuation, v, which we refer to as his *type*. The prior cumulative distribution of buyer types is F, which has support $[\underline{v}, \bar{v}]$, with $\bar{v} > \underline{v} > 0$, and admits density f. We assume that there exists a constant $m > 0$ such that $1/m < f(v) < m$ for any $v \in [\underline{v}, \bar{v}]$.[4] Throughout, we assume that F has increasing "virtual valuation"; that is,

$$v - \frac{1 - F(v)}{f(v)} \quad \text{is increasing.}$$

This assumption is standard in the mechanism design literature. Bulow and Roberts (1989) point out that this assumption is equivalent to the monotonicity of the marginal revenue of a monopolist seller facing an inverse demand curve $1 - F$.

The bargaining within each period t is as follows. Seller t proposes a price p_t to the buyer. The buyer may choose to accept or reject this offer. If the price is accepted, the transaction takes place at this price and the bargaining game ends; the buyer obtains a payoff of $\delta^{t-1}(v - p_t)$, while the seller t obtains a payoff of p_t. If the price is turned down, seller t leaves the market and the game proceeds to period $t + 1$.

We refer to the information structure in which past rejected offers are observable to subsequent sellers as the *transparent regime* and the structure in which these offers are unobservable as the *nontransparent regime*.

We consider the perfect Bayesian equilibria of the two specifications of the bargaining game.[5] The first thing to notice is that in both regimes, the "skimming property" is satisfied. That is, after any history, on or off the equilibrium path, if a price offer p is accepted by a type v, then it is also accepted by all types $v' > v$. This allows us to cast

[4]That is, we focus on the so-called gap case; see Section 7 for additional discussion. We emphasize that the assumption that f is bounded below by a strictly positive number is important for the gap case. For instance, if $F(v) = (v - \underline{v})^2/(\bar{v} - \underline{v})^2$, then the model behaves like a no-gap case even though $\underline{v} > 0$.

[5]A perfect Bayesian equilibrium for finite games with observed actions is defined in Fudenberg and Tirole (1991, p. 333). The extension of this concept to our bargaining games is natural. A perfect Bayesian equilibrium in either regime specifies history-dependent sequences of the sellers' price offers, the acceptance and rejection decisions of all buyer types, and the sellers' beliefs about the buyer's types such that the strategies are best responses given the beliefs. The beliefs are updated at each observable history from the strategies by Bayes' rule whenever possible. After a seller's deviation in the transparent regime, all future sellers' beliefs are consistent with the buyer's strategies upon this deviation; in the nontransparent regime, an off-path event occurs only when the buyer remains on the market beyond the last period in which trade happens with a positive probability on the equilibrium path. The off-path belief here turns out to be irrelevant. See also Hörner and Vieille (2009, p. 33) for a related discussion.

the problem of each seller choosing a price as a problem of each seller choosing a cutoff type k to trade with (or a probability of trade).[6]

In the equivalent dynamic monopoly interpretation of Gul et al. (1986), a sequence of sellers face an inverse demand function $1-F$. Each seller has unlimited supplies and can serve a fraction of the market at some transaction price. The game is prolonged not because all previous prices are rejected (in each period some prices can be offered and accepted), but because the market is not fully penetrated.

3. An example

We first consider a two-period version of our model. For further simplicity, we assume that buyer types are uniformly distributed with support $[0, 1]$.[7]

By the skimming property, for each on- or off-equilibrium-path history, there exists k_1 such that seller 2 believes that buyer types higher than k_1 trade with seller 1 and the remaining types are $[0, k_1]$. Since the second period is the final period, regardless of the regime, a remaining buyer type k accepts seller 2's offer p_2 if p_2 is below k. Therefore, seller 2's problem in either regime can be cast as choosing $p_2 = k$ to solve

$$\max_k k(k_1 - k).$$

Then, regardless of the regime, when the remaining types are $[0, k_1]$, seller 2 charges a price $p_2 = \frac{1}{2}k_1$ and trades with buyer types $[\frac{1}{2}k_1, k_1]$.

The two regimes differ in the formation of beliefs off the equilibrium path: whereas an off-path price of seller 1 in the nontransparent regime does not affect the belief of seller 2, who cannot observe this deviation, it does do so in the transparent regime. To be more specific, in the nontransparent regime, seller 2 believes that the highest remaining buyer type is a *fixed constant* k_1^*, even when the actual cutoff of seller 1 is different. Therefore, seller 2's price in the second period is a *fixed constant* equal to $\frac{1}{2}k_1^*$. If seller 1 wishes to sell to types $[k, 1]$, the highest price he can charge is

$$p_1(k) = (1-\delta)k + \tfrac{1}{2}\delta k_1^*. \tag{1}$$

This is the price that makes the marginal type k indifferent between buying at a price $p_1(k)$ now and waiting until the second period for the constant price $\frac{1}{2}k_1^*$. Hence, seller 1 in the nontransparent regime solves the problem

$$\max_k (1-k)p_1(k). \tag{2}$$

[6] See, for example, Fudenberg and Tirole (1991, p. 406). The proof for the skimming property does not rely on the assumption of observability; the crucial elements are price posting by the seller and single-unit demand by the buyer. In the nontransparent regime, it can be shown that the buyer uses a reservation price strategy, which is stronger than the skimming property.

[7] Even though our general model assumes $\underline{v} > 0$, assuming $\underline{v} = 0$ greatly simplifies the computation in the example. The intuition highlighted in this example applies to the nontrivial gap case where trade takes more than one period to complete.

	Period 1 cutoff	Period 2 cutoff	Period 1 price	Period 2 price
Nontransparent	$\frac{1}{2}(1-\frac{\delta}{4-3\delta})$	$\frac{1}{4}(1-\frac{\delta}{4-3\delta})$	$(\frac{1}{2}-\frac{1}{4}\delta)(1-\frac{\delta}{4-3\delta})$	$\frac{1}{4}(1-\frac{\delta}{4-3\delta})$
Transparent	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}-\frac{1}{4}\delta$	$\frac{1}{4}$

TABLE 1. Comparison of the two regimes.

	Period 1 cutoff	Period 2 cutoff	Period 1 price	Period 2 price
Nontransparent	0	0	0	0
Transparent	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

TABLE 2. Comparison of the two regimes as $\delta \to 1$.

It follows that

$$k_1^* = \frac{1}{2}\left(1 - \frac{1}{2}\frac{\delta}{1-\delta}k_1^*\right).$$

Hence,

$$k_1^* = \frac{1}{2}\left(1 - \frac{\delta}{4-3\delta}\right), \qquad p_1^* = \left(\frac{1}{2} - \frac{1}{4}\delta\right)\left(1 - \frac{\delta}{4-3\delta}\right), \qquad p_2^* = \frac{1}{4}\left(1 - \frac{\delta}{4-3\delta}\right).$$

In contrast, in the transparent regime, if seller 1 sells to buyer types $[k, 1]$ for any k, seller 2 will correctly anticipate the remaining types to be $[0, k]$ and charge a price $\frac{1}{2}k$ accordingly. Moreover, seller 2's response of setting price $\frac{1}{2}k$ to seller 1's deviation is fully anticipated by the buyer, implying that the highest price that seller 1 can charge and sell to buyer types $[k, 1]$ is

$$p_1(k) = (1-\delta)k + \tfrac{1}{2}\delta k = k\left(1 - \tfrac{1}{2}\delta\right). \tag{3}$$

Now, seller 1's problem is given by (2), where $p_1(k)$ is specified by (3). Simple algebra shows that

$$k_1^* = \tfrac{1}{2}, \qquad p_1^* = \tfrac{1}{2}\left(1 - \tfrac{1}{2}\delta\right), \qquad p_2^* = \tfrac{1}{4}.$$

In this example, the demand curves faced by seller 1 are linear (where the quantity sold is $1-k$). The demand curve of the nontransparent regime, (1), is flatter than the demand curve of the transparent regime, (3). We summarize our finding in Table 1.

The contrast becomes more apparent if we take $\delta \to 1$, as shown in Table 2.

This example illustrates the following qualitative results which we generalize later. The prices are uniformly higher in the *transparent* regime and, hence, the lack of transparency diverts more surplus to the informed long-run buyer. In addition, the expected delay in trade, that is, the expected value of $1 - \delta^{\tau(k)}$, where $\tau(k)$ is the period in which type k trades, is higher in the *nontransparent* regime and, hence, the lack of transparency fosters efficiency.

4. Equilibrium

The analysis of the transparent regime is known from Fudenberg et al. (1985). Their model has two bargainers with unequal discount factors and thus includes our model as a special case where the sequence of short-run sellers can effectively be thought of as one seller with a discount factor equal to 0. For completeness, we include the relevant results as Theorem 0.

Theorem 0. *The equilibrium of the transparent regime exists and the equilibrium outcome is generically unique. There exists* $0 < T < \infty$ *such that trade takes place with probability* 1 *within* T *periods.*

It is worth emphasizing that even though in this regime the equilibrium need not be in pure strategies, *on the equilibrium path* randomization can occur only in the first period. However, randomization is necessary off the equilibrium path for the gap case.

We now turn to the analysis of the nontransparent regime, which is novel to this paper. We establish that there is a unique equilibrium that is in pure strategies. This property is quite convenient for our later analysis.

Theorem 1. *Fix any* $\delta \in (0, 1)$. *The equilibrium in the nontransparent regime exists and is unique. In addition, there exists* $0 < T < \infty$, *such that all buyer types trade with probability* 1 *within* T *periods and all players use pure strategies at or before period* T.

This result is in contrast to existing results in two strands of the literature. On the one hand, in Coasian bargaining models, uniqueness is established under a genericity condition and randomization is required off the equilibrium path. Gul et al. (1986) argued that pure strategy is one of the properties for an equilibrium to be "a salient predictor of market behavior."[8] On the other hand, in bargaining models with interdependent values, typically no pure strategy equilibrium exists; see, for example, Hörner and Vieille (2009).[9]

The unobservability of the price history entails two competing effects that are absent in the transparent regime. On the one hand, the skimming property implies only that the posterior beliefs are *distributions over truncations of the prior*—instead of simple truncations of the prior, as would be the case if the history were observable. This is simply because the outcomes of potential randomizations by previous sellers are not observable (except trivially in the first period when there is no prior randomization). On the other hand, if the posterior beliefs were indeed simple truncations of the prior, our assumption of increasing virtual valuation (or, equivalently, the decreasing marginal

[8]In the no-gap case, pure strategy equilibrium is obtained in special cases that allow for closed-form solutions; see, for example, Stokey (1981) and Sobel and Takahashi (1983).

[9]When seller $T+1$ is approached by the buyer, which is off the equilibrium path, the seller's belief can be arbitrary in a perfect Bayesian equilibrium. For instance, the belief can have $\{\underline{v}, \bar{v}\}$ as the support and, hence, lead to the randomization of seller $T+1$. The potential multiple off-path plays in period $T+1$, when the game ends in period T in equilibrium, does not affect the equilibrium outcome. The Coasian bargaining literature does not consider this kind of multiplicity of off-path play.

revenue property) would imply that each seller has a unique optimal pure strategy in the nontransparent regime, since in this regime the "inverse demand curve" faced by the analogous monopolist is simply a linear transformation of $1 - F$. This is in contrast to the transparent regime, where the demand faced by each seller must take into account the reaction of subsequent sellers, which depends on the details of F. Therefore, the crux of our proofs for pure strategy and uniqueness is to show that the posterior beliefs, even when price history is not observable, are necessarily simple truncations of the prior.

The strategy we use to prove Theorem 1 is to successively narrow down the possible supports of mixed strategies that can be used by sellers. The details of the proof in Appendix A are rather complicated and tedious. Here, we offer a brief outline to explain the basic idea. Fix an equilibrium and let T be the last period in which trade occurs with a positive probability in that equilibrium.[10] As mentioned above, by the skimming property, we can identify seller t's offer p_t in period t with the *marginal* buyer type k_t, i.e., the lowest type that will accept the price p_t. Since seller t can play mixed strategies, the marginal types can be random as well. Let K_t denote the support of marginal types in seller t's randomization. In the fixed equilibrium of the nontransparent regime, K_t depends only on the calendar time t, not on the realizations of previous price offers. Write $\bar{k}_t = \sup K_t$ as the supremum of the support of seller t's randomization. Our goal is to show that for each $0 < t \leq T$, $K_t = \{\bar{k}_t\}$ and thus to establish that all equilibrium strategies at or before T must be pure. The next lemma is the key step toward establishing this result and makes critical use of the increasing virtual valuation assumption. For the proof, see Appendix A.2.

Lemma 1. *For any* $\tau = 1, \ldots, T-1$, $(\bigcup_{t=1}^{\tau} K_t) \cap [\bar{k}_{\tau+1}, \bar{k}_1] = \{\bar{k}_1, \bar{k}_2, \ldots, \bar{k}_\tau\}$.

Figure 1 illustrates the content of this lemma for $\tau = 1$. The lemma establishes that, for any $t < T$, all but the highest cutoff types in the support of seller t's randomization are smaller than the supremum of the support of seller T's randomization, $\bar{k}_T$. Then, since all trade must take place in T periods and, hence, $\bar{k}_T = \underline{v}$, there are no cutoffs other than $\bar{k}_t$ in the support of seller t's randomization, which completes the argument for the pure strategy.

With the pure strategy property at hand, we know that the posterior type distribution must always be a truncation of the prior distribution F. Then the seller who faces the highest remaining type k solves the profit-maximization problem

$$\max_{k'} (F(k) - F(k'))[(1-\delta)k' + \delta p].$$

The assumption of increasing virtual valuation guarantees that whenever the continuation equilibrium price p is less than k, there is a unique solution k' to the seller's problem. In contrast, in the transparent regime, since the continuation price p depends on today's choice k', the uniqueness of the solution to the seller's profit-maximization problem is not guaranteed. This distinction allows us to establish the uniqueness of equilibrium in the nontransparent regime, unlike in the transparent regime.

[10]The number of periods it takes for the game to end depends on δ, and it grows unboundedly as $\delta \to 1$.

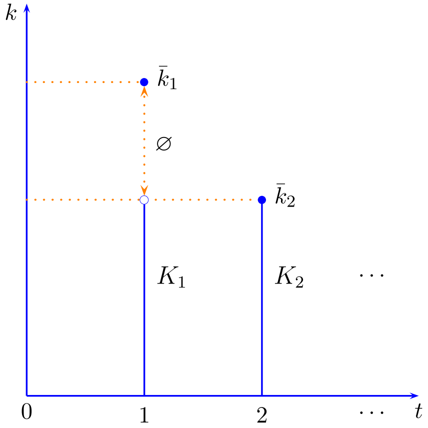

Figure 1. The notation K_t is the support of seller t's randomization. Lemma 1 implies that K_1 does not intersect with $[\bar{k}_2, \bar{k}_1)$ and, hence, the support of seller 1's randomization is narrowed down.

5. Price comparison

In this section, we make the stronger assumption that F exhibits an increasing hazard rate; that is, $f(v)/(1-F(v))$ is nondecreasing over $[\underline{v}, \bar{v}]$. This assumption is introduced into the bargaining setup by Ausubel and Deneckere (1993), and we uncover a connection between this assumption and demand theory. Under this assumption, we establish that *any* realized price sequence of *any* equilibrium in the transparent regime is uniformly above that in the nontransparent regime.

Let $i = \text{TR}, \text{NTR}$ indicate the transparent and nontransparent regimes, respectively. We let $\{p_t^i\}$ represent a *realized* equilibrium price sequence in regime i and let T^i be the last period in which trade takes place with positive probability along this equilibrium path. We adopt the convention that $p_t^i = \underline{v}$ for $t > T^i$. Thus the price comparison is well defined even if $T^{\text{TR}} \neq T^{\text{NTR}}$.

Theorem 2. *Fix any $\delta \in (0, 1)$. Suppose that F exhibits an increasing hazard rate. Let $\{p_t^{\text{TR}}\}$ be any realization of the price sequence of any equilibrium in the transparent regime, and let $\{p_t^{\text{NTR}}\}$ be the unique equilibrium price sequence in the nontransparent regime. Then $p_t^{\text{TR}} \geq p_t^{\text{NTR}}$ for all t.*

Theorem 2 establishes a uniform ranking of equilibrium prices over all periods for any two equilibria in the two regimes. Since equilibrium posteriors differ across regimes, equilibria, and periods, it is convenient to prove the following stronger form of Theorem 2 that establishes the ranking even when the type distributions in the two regimes are *different* truncations of F as long as the support in the transparent regime is larger.

Proposition 1. *Fix any $\bar{v} \geq \bar{k}^{\text{TR}} \geq \bar{k}^{\text{NTR}} \geq \underline{v}$. Suppose the buyer's type distribution is a truncation of F with support $[\underline{v}, \bar{k}^i]$ in regime $i = \text{TR}, \text{NTR}$. Suppose F exhibits an increasing hazard rate on $[\underline{v}, \bar{v}]$. Let $\{p_t^{\text{TR}}\}$ be any realization of the price sequence of any* *equilibrium in the transparent regime, and let* $\{p_t^{\mathrm{NTR}}\}$ *be the unique equilibrium price sequence in the nontransparent regime, under the respective truncated distributions.*[11] *Then* $p_t^{\mathrm{TR}} \geq p_t^{\mathrm{NTR}}$ *for all* t.

Theorem 2 is an immediate corollary of Proposition 1 with $\bar{k}^{\mathrm{TR}} = \bar{k}^{\mathrm{NTR}} = \bar{v}$. The complete proof of Proposition 1 is relegated to Appendix B. The form of Proposition 1, as opposed to Theorem 2, facilitates the induction argument: the continuation games in both regimes have truncated posteriors and trade in the continuation equilibria finishes one period faster than in the original equilibria of the original game. Our induction argument is not straightforward. Here, we first lay out the main steps of our proof. Then we further flesh out the steps where our economic intuition plays a key role and where the increasing hazard rate assumption is used.

Proposition 1 is vacuously true if the truncations of F and the continuation equilibria are such that in either regime all trade takes place within the first period, in which case the transaction prices are identically $\underline{v}$, the lowest buyer type. If the truncations in the two regimes and the equilibrium price paths are such that it takes more than one period to complete trade in either regime, it would suffice to show that the first-period equilibrium prices can be ranked as desired, *and* the posteriors after the first period can be ranked. In this case, we could invoke Proposition 1 for the continuation game with the continuation equilibrium path and the truncated posteriors. The first-period equilibrium prices are determined by the continuation equilibrium prices. Hence a ranking of first-period prices requires a ranking of the second-period prices. However, we cannot establish the ranking of posteriors; instead, we utilize induction in an argument by contradiction.

Formally, we do induction on $\max\{T^{\mathrm{TR}}, T^{\mathrm{NTR}}\}$, where T^i is the number of periods it takes for all buyer types to trade for an arbitrary equilibrium price path in regime i given that the posterior is a truncation of F with support $[\underline{v}, \bar{k}^i]$. We have suppressed the dependence of T^i on the quantifiers to save on notation. Suppose for the purpose of induction that the claim in Proposition 1 is true whenever the posterior truncations and continuation equilibrium price paths are such that $\max\{T^{\mathrm{TR}}, T^{\mathrm{NTR}}\} = 1, \ldots, \tau$. We want to show that the claim is true if the posterior truncations and continuation equilibrium price paths are such that $\max\{T^{\mathrm{TR}}, T^{\mathrm{NTR}}\} = \tau + 1$. Our proof in Appendix B is split into three main parts: (i) We show that *under the induction hypothesis*, the second-period price of the nontransparent regime is lower than any second-period equilibrium price of the transparent regime (Lemma 14 in Appendix B); (ii) then we show that, *under the induction hypothesis* and using (i), the first-period price in the transparent regime must be higher than the equilibrium price of the nontransparent regime (Lemma 15 in Appendix B); finally, we complete the proof by showing that (iii) the prices in the later periods must also be ranked as claimed. We prove step (iii) by way of contradiction: if the price ranking in later periods violates the desired ranking, it must be that the ranking of posterior truncations does not satisfy the condition of *the induction hypothesis*; we then use equilibrium conditions to argue that the supposed price ranking and the posterior

[11] By Lemma 2, a truncation of F on $[\underline{v}, k^{\mathrm{NTR}}]$ exhibits monotone marginal revenue. Hence Theorem 1 implies that there is a unique equilibrium in pure strategies.

ranking cannot be compatible. Since most of the economic intuition we developed is utilized in step (ii) (Lemma 15), we highlight it in this section.

To establish (ii), that is, to show that $p_1^{\mathrm{TR}} \geq p_1^{\mathrm{NTR}}$, the intuition is discerned by comparing the "demand curves" that seller 1 in either regime faces. Our proof amounts to showing that the demand curve in the nontransparent regime is *more elastic* than that in the transparent regime at p_1^{TR}, the equilibrium price offer of the transparent regime. It follows from the elementary monopoly-pricing theory that the first-period profit-maximizing price in the nontransparent regime is *lower* than p_1^{TR}.

We start by defining the relevant elasticities. To do this, let $p_2^{\mathrm{TR}}(p)$ be the equilibrium second-period price in the transparent regime, as a function of the first period-price p.[12,13] Then, for any first-period price p, the cutoff buyer types who purchase in the first period in the two regimes, $k_1^{\mathrm{TR}}(p)$ and $k_1^{\mathrm{NTR}}(p)$, are determined by the indifference conditions

$$p = (1-\delta)k_1^{\mathrm{TR}}(p) + \delta p_2^{\mathrm{TR}}(p)$$

and

$$p = (1-\delta)k_1^{\mathrm{NTR}}(p) + \delta p_2^{\mathrm{NTR}},$$

which can be rearranged, respectively, as

$$k_1^{\mathrm{TR}}(p) = \frac{p - \delta p_2^{\mathrm{TR}}(p)}{1-\delta} \quad \text{and} \quad k_1^{\mathrm{NTR}}(p) = \frac{p - \delta p_2^{\mathrm{NTR}}}{1-\delta}.$$

As explained earlier, seller 1's problem in either regime can be thought of as the problem of a monopolist who faces a demand curve that is shaped by the strategies of the subsequent sellers, by identifying the probability of trade $1 - F(k_1^i(p))$ as the quantity sold at price p. The key, then, is to compare the following two demand curves faced by seller 1 in the two regimes $i = \mathrm{NR}, \mathrm{TNR}$:

$$Q^i(p) = 1 - F(k_1^i(p)).$$

Letting p_1^i represent the first-period equilibrium price in regime i, and assuming *for the purpose of contradiction* that $p_1^{\mathrm{TR}} < p_1^{\mathrm{NTR}}$, we shall show that

$$\frac{F(k^{\mathrm{TR}}(p_1^{\mathrm{NTR}})) - F(k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}))}{1 - F(k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}))} \leq \frac{F(k^{\mathrm{NTR}}(p_1^{\mathrm{NTR}})) - F(k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}}))}{1 - F(k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}}))}. \tag{4}$$

In terms of the demand curves $Q^i(p)$, this inequality says that the percentage decline in the quantity sold in response to a price increase from p_1^{TR} to p_1^{NTR} is smaller in the transparent regime than in the nontransparent regime.

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[12]Note that first-period price (and the rejection decision of the buyer) is the only history observable to seller 2. Therefore, writing his price choice as solely a function of first-period price is without loss of generality.

[13]To give a clean intuition, in this section, we present the argument assuming that all sellers make pure strategy price offers after every history. As discussed earlier, this is not generally true in an equilibrium of the transparent regime. Even though it is known that the second period equilibrium choice is necessarily pure, randomization may be used after a first-period offer p, which is off-equilibrium. Appendix B deals with the general case.

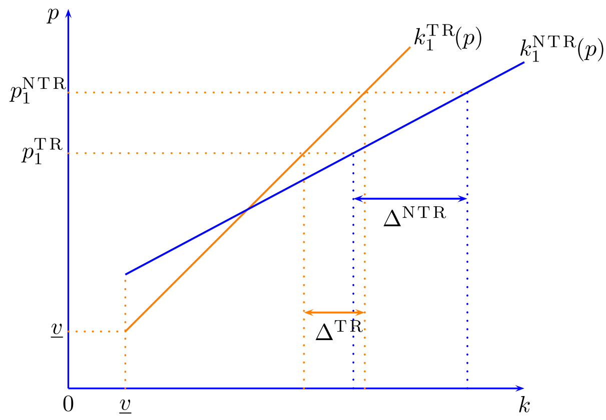

FIGURE 2. The role of the increasing hazard rate assumption.

It is convenient to refer to Figure 2 to describe the argument. The figure depicts representative curves indicating the cutoff types that will purchase at each given price that seller 1 can charge in either regime, that is, $k_1^{\mathrm{NTR}}(p)$ and $k_1^{\mathrm{TR}}(p)$. As indicated in the figure, let Δ^i, $i = \mathrm{TR}, \mathrm{NTR}$, be the "size" of the interval of types switching from buying to not buying in the first period of regime i, in response to a price change from p_1^{TR} to p_1^{NTR}. That is, $\Delta^i \equiv k_1^i(p_1^{\mathrm{NTR}}) - k_1^i(p_1^{\mathrm{TR}})$.

Two main steps of our argument are to show, as depicted in Figure 2, that $k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}) \leq k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}})$ and that $\Delta^{\mathrm{NTR}} \geq \Delta^{\mathrm{TR}}$. That is, the monopolist corresponding to the transparent regime sells *more* at price p_1^{TR} than the monopolist corresponding to the nontransparent regime and loses a *smaller* interval of types for switching to the higher price p_1^{TR}.

The latter step is intuitive and is the exact consequence of the economic forces we have been emphasizing: any price change by seller 1 in the transparent regime elicits a response by seller 2 in the form of a price change in the same direction, which is anticipated by the buyer. In particular, a price increase in the transparent regime implies higher prices in ensuing periods, so that, following such an increase, a smaller range of types switch from buying to not buying when compared with the case of the nontransparent regime, where a price change cannot be matched by subsequent sellers. It is worth noting that this step does not make use of the induction hypothesis, but is a pure consequence of the economic forces at play. The former step is more involved and is proven under the induction hypothesis.[14]

[14]The argument for this result uses the induction hypothesis as well as the economic intuition behind the ranking of the elasticities. First, the induction hypothesis implies that the second-period price can be higher in the *nontransparent* regime only if seller 2 of that regime has a more "optimistic" belief; that is, if the cutoff type purchasing in the first period is *higher* in the nontransparent regime. In other words, seller 1 sells *more* in the transparent regime while charging the *lower price* p_1^{TR}. This leads to a contradiction because switching to the *smaller* quantity choice of seller 1 of the nontransparent regime leads to a *larger* absolute increase in prices (due to the economic forces), which also translates into a larger percentage increase in price since the p_1^{TR} is smaller. Therefore, if seller 1 of the nontransparent regime (weakly) prefers the smaller quantity, then seller 1 of the transparent regime should strictly prefer it. For a formal argument, see Appendix B.

Notice that the two results, $\Delta^{\mathrm{NTR}} \geq \Delta^{\mathrm{TR}}$ and $k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}) \leq k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}})$, would immediately imply (4) if, say, F were the uniform distribution. However, for an arbitrary F, the ranking of the numerators in (4) is not clear, simply because the relevant intervals of discouraged types typically do not share end points and, therefore, a smaller-sized interval (Δ^{TR}) can pack a larger measure under F than can the larger interval (Δ^{NTR}). This would imply that the price increase to p_1^{NTR} would reduce the quantity sold by seller 1 in the transparent regime by a larger absolute amount. Then the ranking of the percentage changes would be ambiguous. This ambiguity is resolved for F satisfying the *increasing hazard rate property*. Now, the increasing hazard rate property is precisely that the quantity $(F(k+\Delta)-F(k))/(1-F(k))$ is increasing in k (see Lemma 11 in Appendix B). This quantity is also increasing in Δ by the monotonicity of F. Therefore, (4) follows from the two observations (i) $\Delta^{\mathrm{NTR}} \geq \Delta^{\mathrm{TR}}$ and (ii) $k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}) \leq k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}})$ for F satisfying the increasing hazard rate property.

To summarize, (4) means that the percentage change in quantity in response to a given price change (from p_1^{TR} to p_1^{NTR}) is larger in the nontransparent regime; i.e., at this range of prices, the demand curve faced by the monopolist in the nontransparent regime is more elastic. Nevertheless, this monopolist strictly prefers the higher price p_1^{NTR} to the lower price p_1^{TR}, since p_1^{NTR} is the unique solution to his profit-maximizing problem. There, the monopolist in the transparent regime, facing a less elastic demand, should also have the same preference over these two prices, which contradicts the optimality of p_1^{TR}. This establishes that $p_1^{\mathrm{TR}} \geq p_1^{\mathrm{NTR}}$.

Since the price paths in the two regimes that the long-run buyer faces are uniformly ranked, it follows immediately that the buyer has a clear-cut preference over the two regimes.

Corollary 1. *Fix any $\delta \in (0,1)$. Suppose that F exhibits an increasing hazard rate. Then the long-run buyer is better off in the nontransparent regime.*

6. Expected delay

As is well understood in the Coasian bargaining literature, as the buyer becomes extremely patient, the outcome becomes efficient. However, the literature so far has had little to say about the delay and efficiency when δ is bounded away from 1. Instead, the literature has focused on the limiting case of $\delta \to 1$ to study "real delay" in various environments. Studying the equilibrium outcomes in the limiting case is not only conceptually important for our understanding of commitment power but also facilitates definite conclusions, such as the limiting efficiency result for the gap case. Yet, to understand fully the applications in real-market environments, it is necessary to consider discount factors that are bounded away from 1. This section is concerned with the question of which regime leads to a longer delay in trade for *any* buyer discount factor $\delta \in (0,1)$.

Under our interpretation, which identifies the probability of sale with the quantity sold by a residual monopolist, the expected delay in sale is smaller if the sales are more "front-loaded," that is, if earlier sellers cover a larger share of the market. In the example of Section 3, a comparison of the two regimes in this dimension was immediate from

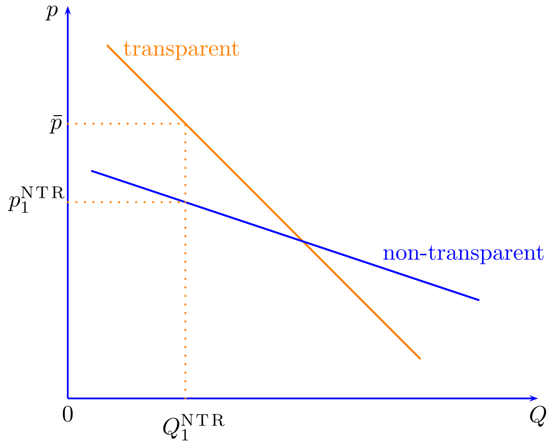

FIGURE 3. Representative demand curves of seller 1 for the infinite horizon model. Note that the two demand curves intersect at a price lower than the first-period equilibrium price of the unobservable regime. This is because when the horizon is longer than two periods, the game starting in the second period onward is no longer identical across the two regimes.

the fact that the first seller in the nontransparent regime serves a larger share of the market (targets a smaller cutoff type). In the general model, however, the comparison of quantities sold by seller 1—or subsequent sellers, for that matter—in the two regimes is not possible and should not be expected. This is because the demand curves faced by seller 1 in either regime typically are related to each other in the manner shown in Figure 3. Therefore, even though the ranking of the elasticities is possible, it implies only that seller 1 in the transparent regime chooses a price above p_1^{NTR}, and not necessarily above $\bar{p}$. Intuitively, when the game has a longer horizon, since the future prices are also expected to be lower in the nontransparent regime, the buyer may have a stronger incentive to wait, reducing the incentives of the seller to increase prices. Nevertheless, we are able to establish the result for the general model under the additional assumption that the buyer's type distribution is concave.

To formally state our result, let $\{k_t^{\mathrm{TR}}\}$ be a *realization* of an equilibrium cutoff sequence in the transparent regime and let $\{k_t^{\mathrm{NTR}}\}$ be the unique equilibrium cutoff sequence in the nontransparent regime with the convention that $k_0^i = \bar{v}$ and $k_t^i = \underline{v}$ for $t > T^i$, where T^i is the last period such that trade takes place with positive probability along the realized path. Given these sequences, for each type $v < \bar{v}$, and for either $i = \mathrm{NTR}, \mathrm{TR}$, there is a unique t such that $k_{t-1}^i > v \geq k_t^i$. Let $\tau^i(v)$ represent this t.

Then a measure of the delay that type v experiences is $1 - \delta^{\tau^i(v)-1}$, which is the portion of the payoff lost due to the delay in reaching an agreement. Therefore, the expected delay in regime i is

$$\int_{\underline{v}}^{\bar{v}} (1 - \delta^{\tau^i(v)-1})\, dF(v) = 1 - \int_{\underline{v}}^{\bar{v}} \delta^{\tau^i(v)-1}\, dF(v).$$

Notice that, ex ante, the probability that the trade will take place at period t is $F(k^i_{t-1}) - F(k^i_t)$. Therefore, the above expectation can alternatively be expressed as

$$1 - \sum_{t=1}^{\infty} \delta^{t-1}(F(k^i_{t-1}) - F(k^i_t)),$$

which simplifies to

$$(1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} F(k^i_t). \tag{5}$$

Proposition 2. *Fix any $\delta \in (0,1)$. Assume that $p^{\mathrm{TR}}_t \geq p^{\mathrm{NTR}}_t$ for any t, where $\{p^{\mathrm{TR}}_t\}$ is a realization of the equilibrium price sequence of any equilibrium in the transparent regime and $\{p^{\mathrm{NTR}}_t\}$ is the unique equilibrium sequence of prices in the nontransparent regime. Then, for a concave F, the expected delay in the transparent regime is larger than the expected delay in the nontransparent regime.*

The complete proof is presented in Appendix C. To gain some intuition into how the ranking of prices helps and what role the concavity of the type distribution F plays, note that for each i, we have[15]

$$p^{\mathrm{TR}}_t = (1-\delta)\sum_{l=t}^{\infty} \delta^{l-t} k^{\mathrm{TR}}_l \geq (1-\delta)\sum_{l=t}^{\infty} \delta^{l-t} k^{\mathrm{NTR}}_l = p^{\mathrm{NTR}}_t. \tag{6}$$

In words, the discounted sum of the tails of the cutoff sequence from period t on (which is equal to the price in that period) is larger for the transparent regime than for the nontransparent regime. It is clear that when F is applied to each k^i_t to obtain the expression for the expected delay in (5), this ranking need not be preserved. Proposition 2 shows that this ranking is preserved when F is concave. The intuition can most easily be gleaned from the following thought experiment: suppose that in each regime, trade takes place in the second period at the latest. Then (6) implies that

$$(1-\delta)k^{\mathrm{NTR}}_1 + \delta k^{\mathrm{NTR}}_2 \leq (1-\delta)k^{\mathrm{TR}}_1 + \delta k^{\mathrm{TR}}_2 \quad \text{and} \quad k^{\mathrm{NTR}}_2 \leq k^{\mathrm{TR}}_2.$$

This means that one of the following two rankings must hold:

(i) $k^{\mathrm{NTR}}_1 \leq k^{\mathrm{TR}}_1$ and $k^{\mathrm{NTR}}_2 \leq k^{\mathrm{TR}}_2$.

(ii) $k^{\mathrm{NTR}}_1 > k^{\mathrm{TR}}_1 \geq k^{\mathrm{TR}}_2 \geq k^{\mathrm{NTR}}_2$.

[15] Since a pure strategy is played on the equilibrium path after period 1 (Fudenberg et al. 1985), for any $t \geq 1$, the indifference condition of the cutoff buyer type k_t is given by

$$p^i_t = (1-\delta)k^i_t + \delta p^i_{t+1},$$

where p^i_{t+1} is deterministic. Iterating this indifference condition, we obtain that prices are discounted sums of the cutoff types. Indeed, this is the only equilibrium condition that is invoked in the proof. Proposition 2 holds for all ranked sequences of prices for which the corresponding sequences of cutoff types are decreasing.

If the ranking in (i) obtains, (5) follows immediately from the monotonicity of F without referring to concavity. Under the ranking in (ii), the cutoffs in the transparent regime are "less spread out"—as well as, on average, higher—than those in the nontransparent regime, which implies that, evaluated under a concave and increasing function F, their expectation is larger.

To see how this intuition is generalized to longer horizons, consider an alternative interpretation of our model as a bargaining game with a stochastic deadline: suppose that δ, instead of representing the discount factor of the buyer, represents the probability with which the bargaining ends before the next period, conditional on the fact that it has not yet ended. It is well understood that such a game is strategically equivalent to the game we have analyzed so far. Then "the smallest buyer type that gets to trade" in either regime is a random variable assigning probability δ^{t-1} to k_t^i. If the realization of this random variable is k, then the realized probability (or quantity) of sale is $1 - F(k)$. Under this interpretation, 1 minus the expression in (5) is the expected quantity sold or, equivalently, the expected probability of sale in either regime. What (6) allows us to show is that the random variable determining the smallest buyer type that trades in the transparent regime *second-order stochastically dominates* its nontransparent regime counterpart. That is why the expectation of this random variable evaluated at concave F is larger in the *transparent* regime, and so there is a lower expected probability of trade or, equivalently, a higher expected delay, when past rejected prices are observable.

7. Concluding remarks

To conclude, we emphasize several aspects of our model that we feel deserve further elaboration.

We can compare our model with classical models of oligopoly. In particular, the bargaining model where previous prices are observable to future sellers is reminiscent of the "price leadership" in oligopoly, as later sellers observe and react to choices that earlier sellers have "committed to." The nontransparent regime, alternatively, is suggestive of "Bertrand competition," because each seller makes a price choice without observing the choices of any other seller. Obviously, the analogy requires that the products of various sellers be vertically differentiated, since in either regime, for a given price, each type of buyer prefers to buy from an earlier seller rather than wait for a later seller—because of discounting. However the analogy is superficial. The extensive form of our sequential bargaining game implies that later sellers face an endogenously determined residual market, while in the standard oligopoly literature, there is a single demand curve common to all sellers. The implication of this is that a given seller in the oligopoly model, by lowering his price, can steal buyers from higher quality (earlier) sellers as well as lower quality (later) sellers, whereas in the dynamic bargaining model, the buyers of higher quality (earlier) sellers are locked in, so that a deviation to a lower price can lure buyers only from lower quality (later) sellers. This discrepancy creates very distinct incentives to deviate and leads to different equilibrium outcomes. A second implication of this distinction is that, while in our model, quantity competition is equivalent to price competition, this is not true in an oligopoly model. Therefore, even though the forces we uncover that lead to the comparison of the two regimes in our model are also present in models of oligopoly, they are obscured by other mechanisms and, therefore, are not highlighted in that literature. In fact, to our knowledge, the industrial organization literature has not studied analogous questions to those we asked in this paper. In addition, most papers look at simple models with closed-form solutions. Our result might indeed shed light on the oligopoly literature.

Another "competitive" benchmark with which we can contrast the performance of the two regimes is one where there is "within-period competition" between the short-run sellers; that is, the buyer meets two (or more) sellers each period and price is determined by competitive bidding. In this case, it is easy to see that the prices will immediately be zero in either regime. Therefore, our results suggest that the nontransparent regime leads to outcomes that are "closer" to this benchmark. Rather than studying within-period competitive bidding, which would more reasonably describe a "thick" market, our purpose in studying one-to-one bargaining is precisely to unlock the strategic aspect of price formation in a thin market.

We consider take-it-or-leave-it offers by the uninformed players. Introducing different bargaining protocols would be an interesting exercise. If the informed buyer makes all the offers, in both regimes an equilibrium is one where the buyer always offers the price 0 and all sellers accept it. In this equilibrium, the buyer gets all the surplus. This is intuitive. In the complete information of this game, when the long-run buyer is making offers, in the unique equilibrium he would always offer 0 regardless of his valuation. If, in addition, the buyer's type is his private information, he cannot do worse than in the case of complete information when his type is known. This observation is made by Ausubel and Deneckere (1989b, Theorem 4). Indeed, they show that this is the unique equilibrium. This suggests that in our context, the uninformed player must have some bargaining power for the impact of transparency to be present. If the bargaining protocol is such that both parties can make offers, then the game becomes complicated due to the signaling effect. The literature makes strong refinement assumptions. Ausubel et al. (2002, Theorem 7) show that Coase conjecture holds in alternating offer bargaining games under a refinement they call "assuredly perfect equilibrium." This refinement requires that when an off-equilibrium action is observed, the players believe that it is more likely to come from low buyer types, giving a strong structure to the off-path beliefs. We believe the same logic works in our transparent regime. Similarly, two-sided uncertainty will also give rise to signaling issues; see, for example, Cramton (1984), Chatterjee and Samuelson (1987), and Abreu and Gul (2000).

Our bargaining model corresponds to the "gap" case—well known in the bargaining literature. In fact, the Coase conjecture fails in the no-gap case with short-run sellers: we can employ the insights of Ausubel and Deneckere (1989a) to construct multiple nonstationary equilibria, where the long-run buyer builds a "reputation" for having a low willingness to pay. Nevertheless, the gap case nicely captures the insights of imperfect competition by silencing the reputation effect.

Some interesting questions within our exact model remain unresolved as well. In particular, we do not consider the ranking—across the two regimes—of ex ante sums of expected payoffs or of cutoff types that trade each period. If we were to be able to establish the ranking of cutoffs, the ranking of the discounted sum of payoffs would immediately follow. Yet, this is a challenging exercise: To obtain a ranking of the cutoffs and, hence, the sum of payoffs, one needs to be able to reason about the magnitude of price differences/ratios across regimes—and not only their ordinal rankings. As the price path is an endogenous object in an infinite horizon model, this becomes rather involved and our current proof strategies do not work. In the supplementary material, we present our results on the special cases of power function distributions and two-period models for general type distributions. In these two cases, we are able to obtain unambiguous rankings of cutoffs and, therefore, efficiency. In addition, we show in these two cases that we can drop the concavity assumption in Proposition 2. Moreover, in the two-period case, we are able to dispense with the increasing hazard rate condition for Theorem 2.[16]

Appendix A: Proof of Theorem 1

We first prove that any equilibrium must be in pure strategies (Lemma 1). We then apply this property to prove equilibrium existence and uniqueness.

By virtue of the skimming property (Fudenberg and Tirole 1991, p. 407) and the fact that the only observable history in the nontransparent regime is rejection, in any equilibrium we can identify seller t's price p_t with the infimum buyer type who accepts p_t. For any fixed equilibrium, since the seller in period t can potentially randomize, let K_t be the support of the cutoffs in period t in this equilibrium and write

$$K = \bigcup_{t=1}^{T} K_t.$$

For each t, define

$$\bar{k}_t := \sup K_t \quad \text{and} \quad \underline{k}_t := \inf K_t.$$

Hence, after period t, the largest possible interval of remaining types is $[\underline{v}, \bar{k}_t)$, while the smallest such interval is $[\underline{v}, \underline{k}_t)$. Define

$$\bar{k}'_t := \sup K_t \setminus \{\bar{k}_t\}.$$

By convention, $\sup \varnothing = -\infty$. Therefore, $[\underline{v}, \bar{k}'_t)$ is the second largest possible interval after a (potential) seller randomization in period t. It is possible a priori that $\bar{k}'_t = \bar{k}_t$. *All these variables just defined depend on the fixed equilibrium. We suppress the dependence for notational convenience.*

A.1 *Preliminary results*

In our dynamic environment, the distribution of types varies over time. We first make the observation that a lower truncation of F inherits the monotone marginal revenue (virtual valuation) property from F.

[16]Note that power function distributions satisfy the monotone hazard rate assumption.

Lemma 2. *Assume that $k-(1-F(k))/f(k)$ is strictly increasing. Then $k-(\alpha-F(k))/f(k)$ is strictly increasing in k whenever $F(k)<\alpha\le 1$.*

Proof. Consider $k'<k$ and $F(k)<\alpha$. We want to show

$$k-\frac{\alpha-F(k)}{f(k)}>k'-\frac{\alpha-F(k')}{f(k')}. \tag{7}$$

Define

$$L(\alpha)=k-k'+\frac{F(k)-F(k')}{f(k')}-(\alpha-F(k))\left(\frac{1}{f(k)}-\frac{1}{f(k')}\right).$$

Inequality (7) is equivalent to $L(\alpha)>0$. Notice that $L(1)>0$ since F has increasing marginal revenue (virtual valuation). For $\alpha<1$, we have two cases to consider: if $1/f(k)-1/f(k')\le 0$, then $L(\alpha)>0$ follows immediately by the definition of $L(\alpha)$; if $1/f(k)-1/f(k')>0$, then $L(\alpha)$ is decreasing in α and, hence, $L(\alpha)>L(1)>0$. □

By standard arguments, in any equilibrium, $\underline{k}_t\ge \underline{v}$ for any $t$and the game ends in finite time with a price equal to $\underline{v}$. This is formalized in Lemma 3.

Lemma 3. *In any equilibrium of the nontransparent regime, there exists $0<T<\infty$ such that trade takes place with probability 1 within T periods.*

Proof. We proceed in the following steps.

Step 1. A seller never makes a price offer below $\underline{v}$. The argument is standard: all buyer types will accept a price of $(1-\delta)\underline{v}$ immediately, which is better than waiting for a price of 0 next period; but then $(1-\delta^2)\underline{v}$ will be accepted for sure because the best price in the next period is bounded below by $(1-\delta)\underline{v}$; iterating this argument shows that a seller will never make a price offer below $(1-\delta^n)\underline{v}$ for any n, and the claim follows.

Step 2. Suppose to the contrary that there is an equilibrium in which there is some positive measure of types that never trade for some history. Then in this equilibrium, $\bar{k}_t>\underline{v}$ for all $t>0$. In this case $\{\bar{k}_t\}$ is a decreasing and, hence, convergent sequence: if, however, $\bar{k}_t<\bar{k}_{t+1}$ for some t, then seller $t+1$ makes a profit of 0 by making an offer close to $\bar{k}_{t+1}$; but he can make a strictly positive profit by offering $\underline{v}$ according to the previous step. Consequently, $|\bar{k}_t-\bar{k}_{t+1}|\to 0$. Thus, the profit of seller t converges to 0 as $t\to\infty$. Moreover, it must be that $\bar{k}_t\downarrow \underline{v}$. To see this, suppose that $\lim_{t\to\infty}\bar{k}_t=k^*>\underline{v}$. Then seller t will get a profit close to 0 if t is large enough, but any seller can deviate to charge a price $\underline{v}$, which, by the previous claim, guarantees a strictly positive profit $(F(k^*)-F(\underline{v}))\underline{v}$, a contradiction.

Step 3. Now from Step 2, for each $\varepsilon>0$, there exists t such that $\underline{v}<\bar{k}_t<\underline{v}+\varepsilon$. Then we claim that there exists ε such that for any $k\in(\underline{v},\underline{v}+\varepsilon)$ and any $k'\in(\underline{v},k]$,

$$(F(k)-F(k'))k'<F(k)\underline{v}. \tag{8}$$

To see this, note that the left-hand side is differentiable in k' and its derivative is $-f(k')k' + F(k) - F(k')$. Now

$$\begin{aligned}-f(k')k' + F(k) - F(k') &< -\frac{1}{m}\underline{v} + F(k) - F(k') \\ &< -\frac{1}{m}\underline{v} + F(\underline{v}+\varepsilon) - F(\underline{v}) \\ &< -\frac{1}{m}\underline{v} + m\varepsilon.\end{aligned}$$

Hence, when $\varepsilon < \underline{v}/m^2$, then $-f(k')k' + F(k) - F(k') < 0$, and (8) follows immediately.

Step 4. Notice that the left-hand side of (8) is the highest possible payoff a seller can obtain when facing buyer types $[\underline{v}, k]$ if he wants to sell to the types $[k', k]$ (it assumes that a price equal to k' will be accepted by all types above k'), while the right-hand side of (8), by Step 1, is the seller's exact payoff from making a price offer $\underline{v}$. Therefore, (8) implies that if $\bar{k}_t < \underline{v} + \underline{v}/m^2$, it is an ex post strictly dominant strategy for seller $t+1$ to make a price offer equal to $\underline{v}$ for each realization of $k_t \in (\underline{v}, \bar{k}_t]$. Therefore, $k_{t+1} = \underline{v}$ is an ex ante strictly dominant strategy for seller t as long as $\underline{v} < \bar{k}_t < \underline{v} + \underline{v}/m^2$. This contradicts the supposition that $\bar{k}_{t+1} > \underline{v}$ for each t. □

We next argue that the upper bound of the support of a seller's potential randomization in the fixed equilibrium is strictly decreasing over the periods during which trade takes place with positive probability.

Lemma 4. *In any equilibrium in which T is the last period in which trade takes place with a positive probability, we have $\bar{v} > \bar{k}_t > \bar{k}_{t+1}$ for any $t < T$.*

Proof. If $\bar{k}_t \le \bar{k}_{t+1}$, then seller $t+1$ gets 0 profit. He can get positive profit by charging $\underline{v}$. Moreover, if $\bar{k}_t = \bar{v}$, then seller t can charge $\underline{v}$ and get a strictly higher profit. □

A.2 *Pure strategy: Proof of Lemma 1*

Lemma 5. *In any equilibrium, $K_1 \cap [\bar{k}_2, \bar{k}_1] = \{\bar{k}_1\}$.*

Proof. To prove this claim, note that by the definition of $\bar{k}_2$, the buyer type $k \in [\bar{k}_2, \bar{k}_1]$ is guaranteed to trade at or before period 2. Therefore, by choosing a marginal type $k \in [\bar{k}_2, \bar{k}_1]$, seller 1 would sell with probability $1 - F(k)$. The price $p_1(k)$ is such that the marginal type k, who will buy for sure next period, is indifferent between buying now or waiting,

$$k - p_1(k) = \delta(k - E[p_2]),$$

where $E[p_2]$ is the expected price in period 2 (seller 2 could potentially randomize). Hence, $p_1(k) = (1-\delta)k + \delta E[p_2]$ and, therefore, seller 1's problem is

$$\max_k (1 - F(k))[(1-\delta)k + \delta E[p_2]].$$

The first-order derivative of the objective function can be calculated to be

$$-f(k)\left[(1-\delta)\left(k-\frac{1-F(k)}{f(k)}\right)+\delta E[p_2]\right]. \tag{9}$$

Since $k-(1-F(k))/f(k)$ is strictly increasing by assumption, (9) is strictly increasing over the interval $[\bar{k}_2, \bar{k}_1]$. Now, since $\bar{k}_1$ maximizes seller 1's profit (or types arbitrarily close to $\bar{k}_1$ if $\bar{k}_1 = \sup K_1$ is not achieved by any $k \in K_1$) and $\bar{k}_1$ is in the interior of $[\underline{v}, \bar{v}]$, it must be that (9) is 0 at $k = \bar{k}_1$. Moreover, since (9) is strictly increasing, it must be negative for any $k \in [\bar{k}_2, \bar{k}_1)$. Hence, no $k \in [\bar{k}_2, \bar{k}_1)$ is optimal. Therefore, $K_1 \cap [\bar{k}_2, \bar{k}_1] = \{\bar{k}_1\}$. □

Lemma 5 does not imply that seller 1 must play a pure strategy. It does not rule out the case that K_1 contains points that are not in $[\bar{k}_2, \bar{k}_1]$, that is, the case $K_1 \setminus [\bar{k}_2, \bar{k}_1] \neq \varnothing$. However, we are able to successively narrow down K_1. This is done in Lemma 1 of the main text, which is repeated as follows.

Lemma 1. *Fix any equilibrium in which the game ends for sure at T. For any $\tau = 1, \dots, T-1$, $(\bigcup_{t=1}^{\tau} K_t) \cap [\bar{k}_{\tau+1}, \bar{k}_1] = \{\bar{k}_1, \bar{k}_2, \dots, \bar{k}_\tau\}$.*

Proof. The proof is by induction. We proceed in the following steps.

Step 1. First note that $K_1 \cap [\bar{k}_2, \bar{k}_1] = \{\bar{k}_1\}$. This is what we proved in Lemma 5. This step shows that $\bar{k}_1$ is an isolated point in K_1.

Step 2. Next we argue that for $1 \le \tau + 2 \le T$, if

$$\left(\bigcup_{t=1}^{\tau} K_t\right) \cap [\bar{k}_{\tau+1}, \bar{k}_1] = \{\bar{k}_1, \bar{k}_2, \dots, \bar{k}_\tau\}, \tag{10}$$

then

$$\left(\bigcup_{t=1}^{\tau+1} K_t\right) \cap [\bar{k}_{\tau+2}, \bar{k}_1] = \{\bar{k}_1, \bar{k}_2, \dots, \bar{k}_{\tau+1}\}.$$

In words, we want to show inductively that $\bar{k}_t$ is an isolated point in the support of seller t's cutoffs and no seller will ever set a cutoff in the interval $(\bar{k}_t, \bar{k}_{t+1})$. The induction step is illustrated in Figure 4.

From the induction hypothesis, $(\bigcup_{t=1}^{\tau} K_t) \cap [\bar{k}_{\tau+1}, \bar{k}_1] = \{\bar{k}_1, \bar{k}_2, \dots, \bar{k}_\tau\}$. Recall that $\bar{k}'_t = \sup K_t \setminus \{\bar{k}_t\}$. Take the *smallest* t^* such that $\bar{k}'_{t^*} = \sup\{\bar{k}'_t \mid t = 1, \dots, \tau+1\}$. That is, $\bar{k}'_{t^*}$ is the highest among the "second highest equilibrium cutoffs" in periods up to $\tau+1$. By the induction hypothesis, for any $t \le \tau$,

$$\bar{k}'_{t^*} \le \bar{k}_{\tau+1} < \bar{k}_t. \tag{11}$$

If $K_t \setminus \{\bar{k}_t\} = \varnothing$ for all $t = 1, \dots, \tau+1$, then the proof is complete already. Suppose that this is not the case. If $\bar{k}'_{t^*} < \bar{k}_{\tau+2}$, then the induction is complete as well. Now suppose that $\bar{k}'_{t^*} \ge \bar{k}_{\tau+2}$.

Step 3. We establish the following claims.

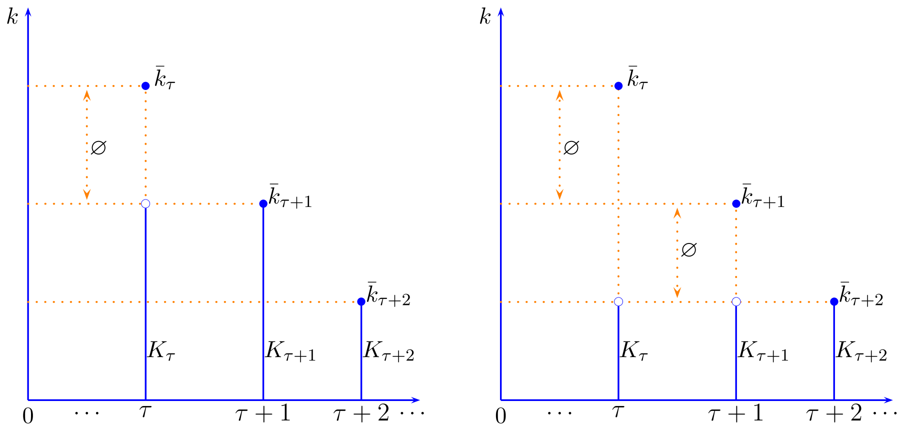

FIGURE 4. The left panel depicts the induction hypothesis; the right panel depicts the induction step.

CLAIM A.1. *There exists $\varepsilon > 0$ such that $(\bar{k}'_{t^*}, \bar{k}'_{t^*} + \varepsilon) \cap K = \varnothing$. That is, there is no (future or past) cutoff immediately above $\bar{k}'_{t^*}$. Hence, in any period t after any history, buyer types $[\bar{k}'_{t^*}, \bar{k}'_{t^*} + \varepsilon)$ must be either entirely in the support of the posterior or entirely outside of the support of the posterior.*

PROOF. By (11), we have either $\bar{k}'_{t^*} = \bar{k}_{\tau+1}$ or $\bar{k}'_{t^*} < \bar{k}_{\tau+1}$. By (11), in the former case we have

$$\bar{k}_{\tau+2} < \bar{k}'_{t^*} = \bar{k}_{\tau+1};$$

in the latter case, we have

$$\bar{k}_{\tau+2} \le \bar{k}'_{t^*} < \bar{k}_{\tau+1}.$$

It follows immediately from Lemma 4 that there is no offer (buyer cutoff) within $(\bar{k}'_{t^*}, \bar{k}'_{t^*} + \varepsilon)$ in all periods $t = 1, \dots, T$. ◁

CLAIM A.2. *There exists $\varepsilon > 0$ such that $(\bar{k}_{t^*} - \varepsilon, \bar{k}_{t^*} + \varepsilon) \cap K = \{\bar{k}_{t^*}\}$. That is, there is no (future or past) cutoff in an ε-neighborhood of $\bar{k}_{t^*}$.*

PROOF. If $t^* \le \tau$, the claim follows from the induction hypothesis (10). If $t^* = \tau + 1$, the problem arises only when $\bar{k}_{\tau+1} = \bar{k}'_{\tau+1}$ because then $\bar{k}_{\tau+1}$ is not an isolated point. This means that there exists $k^n_{\tau+1} \uparrow \bar{k}_{\tau+1}$. Then it must be that there exists $\bar{t} < \tau + 1$ with equilibrium cutoffs $k^n_{\bar{t}} \in K_{\bar{t}}$ such that $k^n_{\bar{t}} \uparrow \bar{k}_{\tau+1}$; otherwise, by the same line of arguments in Step 1 that establishes Lemma 5, seller $\tau + 1$ will not offer both $k^n_{\tau+1}$ and $\bar{k}_{\tau+1}$. But then we must have $\bar{k}'_{\bar{t}} = \bar{k}'_{t^*}$. Since $\bar{t} < \tau + 1 = t^*$, this contradicts the definition of t^*. ◁

CLAIM A.3. $\bar{k}'_{t^*} \neq \bar{k}_{t^*}$. *In addition, $(\bar{k}'_{t^*}, \bar{k}_{t^*}) \cap K_t = \varnothing$ for all $t \le t^*$. That is, $(\bar{k}'_{t^*}, \bar{k}_{t^*})$ includes no past cutoffs.*

Proof. First note that $\bar{k}'_{t^*} \neq \bar{k}_{t^*}$ by Claim A.2 above. The remaining part of Claim A.3 follows from the induction hypothesis. ◁

Claim A.4. *We have $(\bar{k}'_{t^*}, \bar{k}_{t^*-1}) \cap K_t = \varnothing$ for all $t < t^*$. That is, at the beginning of period t^*, buyer types $(\bar{k}'_{t^*}, \bar{k}_{t^*-1})$ are either entirely in the support of the posterior or entirely outside of the support of the posterior.*

The proof follows from the induction hypothesis (10) and Claim A.3.

From now on, we shall consider seller t^*'s optimization problem. For any buyer type $k \geq \underline{v}$, let $\tau(k)$ be the (random) period at which type k ends up trading if he does not trade at or before time t^*. The distribution of $\tau(k)$ for each k is determined by the equilibrium strategies of sellers $t > t^*$.

Step 4. To target a cutoff type k, seller t^* must choose a price $p(k)$ satisfying the indifference condition

$$k - p(k) = \sum_{t=t^*+1}^{\infty} \Pr(\tau(k) = t)\delta^{t-t^*} E[k - p_t(k_t) \mid k_t \leq k, k_t \in K_t]. \tag{12}$$

Using the fact that type $k \geq \underline{v}$ must eventually trade in the future, the right-hand side of the above expression can be rewritten as

$$kE[\delta^{\tau(k)-t^*}] - \sum_{t=t^*+1}^{\infty} \Pr(\tau(k) = t)\delta^{t-t^*} E[p_t(k_t) \mid k_t \leq k, k_t \in K_t].$$

Define

$$\mathbf{p}(k) := \sum_{t=t^*+1}^{\infty} \Pr(\tau(k) = t)\delta^{t-t^*} E[p_t(k_t) \mid k_t \leq k, k_t \in K_t]$$

and

$$\mathbf{d}(k) := E[\delta^{\tau(k)-t^*}].$$

Hence $\mathbf{p}(k)$ and $\mathbf{d}(k)$ are type k's expected *discounted* trading price and *discounted* trading probability from period $t^* + 1$ onward, conditional on this type not having traded at or before t^*. Hence, (12) can be rewritten as

$$k - p(k) = k\mathbf{d}(k) - \mathbf{p}(k).$$

Therefore, the cutoff price for type k at period t^* can be written as $p(k) = k(1 - \mathbf{d}(k)) + \mathbf{p}(k)$.

Step 5. We now consider seller t^*'s objective function. If seller t^* targets a cutoff type $k \in [\bar{k}'_{t^*}, \bar{k}_{t^*-1})$, the trading probability can be written in the form of

$$\beta(\alpha - F(k))$$

for some positive number $\alpha \in (F(\bar{k}_{t^*-1}), 1]$ and $\beta < 1$. This follows from Claim A.4 in Step 3: either all buyer types in the interval $(\bar{k}'_{t^*}, \bar{k}_{t^*-1})$ have traded before t^* or none of them has traded before t^*. Therefore, seller t^*'s payoff by choosing a cutoff $k \in [\bar{k}'_{t^*}, \bar{k}_{t^*-1})$ is

$$R(k) = \beta(\alpha - F(k))\big[k(1 - \mathbf{d}(k)) + \mathbf{p}(k)\big]. \tag{13}$$

Moreover, by Claim A.2 in Step 3, if types $(\bar{k}_{t^*} - \varepsilon, \bar{k}_{t^*} + \varepsilon)$ do not trade at period t^*, they will trade together in the future. Thus we have

$$\mathbf{d}(k) \equiv \mathbf{d}(\bar{k}_{t^*}) \quad \text{and} \quad \mathbf{p}(k) \equiv \mathbf{p}(\bar{k}_{t^*}) \quad \text{for all } k \in (\bar{k}_{t^*} - \varepsilon, \bar{k}_{t^*} + \varepsilon).$$

By Claim A.1 in Step 3, we have

$$\mathbf{d}(k) \equiv \mathbf{d}(\bar{k}'_{t^*}) \quad \text{and} \quad \mathbf{p}(k) \equiv \mathbf{p}(\bar{k}'_{t^*}) \quad \text{for all } k \in [\bar{k}'_{t^*}, \bar{k}'_{t^*} + \varepsilon).$$

In sum, seller t^*'s payoff (13) as a function of k is such that

$$\begin{aligned} R(k) &= \beta(\alpha - F(k))\big[k(1 - \mathbf{d}(\bar{k}_{t^*})) + \mathbf{p}(\bar{k}_{t^*})\big] \quad \text{if } k \in (\bar{k}_{t^*} - \varepsilon, \bar{k}_{t^*} + \varepsilon) \\ R(k) &= \beta(\alpha - F(k))\big[k(1 - \mathbf{d}(\bar{k}'_{t^*})) + \mathbf{p}(\bar{k}'_{t^*})\big] \quad \text{if } k \in [\bar{k}'_{t^*}, \bar{k}'_{t^*} + \varepsilon). \end{aligned}$$

Step 6. Now consider the derivative of seller t^*'s payoff with respect to k. In the case that $k \in (\bar{k}_{t^*} - \varepsilon, \bar{k}_{t^*} + \varepsilon)$,

$$R'(k) = -f(k)\left[(1 - \mathbf{d}(\bar{k}_{t^*}))\left(k - \frac{\alpha - F(k)}{f(k)}\right) + \mathbf{p}(\bar{k}_{t^*})\right].$$

Hence, we have

$$R'(k) = 0. \tag{14}$$

In the case that $k \in [\bar{k}'_{t^*}, \bar{k}'_{t^*} + \varepsilon)$, $R(k)$ is right-differentiable in k. As a result, for $k \in (\bar{k}'_{t^*}, \bar{k}'_{t^*} + \varepsilon)$,

$$R'(k) = -f(k)\left[(1 - \mathbf{d}(\bar{k}'_{t^*}))\left(k - \frac{\alpha - F(k)}{f(k)}\right) + \mathbf{p}(\bar{k}'_{t^*})\right].$$

Let $R'(\bar{k}'_{t^*})$ denote the left derivative of $R(k)$ at $k = \bar{k}'_{t^*}$. Then

$$\begin{aligned} R'(\bar{k}'_{t^*}) &= (1 - \mathbf{d}(\bar{k}'_{t^*}))\left(\bar{k}'_{t^*} - \frac{\alpha - F(\bar{k}'_{t^*})}{f(\bar{k}'_{t^*})}\right) + \mathbf{p}(\bar{k}'_{t^*}) \\ &= (1 - \mathbf{d}(\bar{k}'_{t^*}))\left[\left(\bar{k}'_{t^*} - \frac{\alpha - F(\bar{k}'_{t^*})}{f(\bar{k}'_{t^*})}\right) + \frac{\mathbf{p}(\bar{k}'_{t^*})}{1 - \mathbf{d}(\bar{k}'_{t^*})}\right] \\ &< (1 - \mathbf{d}(\bar{k}'_{t^*}))\left[\left(\bar{k}_{t^*} - \frac{\alpha - F(\bar{k}_{t^*})}{f(\bar{k}_{t^*})}\right) + \frac{\mathbf{p}(\bar{k}_{t^*})}{1 - \mathbf{d}(\bar{k}_{t^*})}\right] \\ &= 0 \quad \text{(by (14))}. \end{aligned}$$

The inequality in the previous display follows from (i) the fact that both $\mathbf{p}(k)$ and $\mathbf{d}(k)$ are increasing in k, and (ii) the fact that

$$\bar{k}_{t^*} - \frac{\alpha - F(\bar{k}_{t^*})}{f(\bar{k}_{t^*})} > \bar{k}'_{t^*} - \frac{\alpha - F(\bar{k}'_{t^*})}{f(\bar{k}'_{t^*})}.$$

Fact (i) follows from the definition of $\mathbf{p}(k)$ and $\mathbf{d}(k)$. Fact (ii) follows from Lemma 2.

Step 7. Now we have established that $R'(\bar{k}'_{t^*}) < 0$. Hence $\bar{k}'_{t^*} \notin K_{t^*}$. In particular, there exists $\eta > 0$ such that

$$R\left(\bar{k}'_{t^*} + \tfrac{1}{2}\varepsilon\right) > R(\bar{k}'_{t^*}) + \eta. \tag{15}$$

Now, since $\bar{k}'_{t^*} \notin K_{t^*}$, there exists a sequence $k^n_{t^*} \in K_{t^*}$ such that $k^n_{t^*} \uparrow \bar{k}'_{t^*}$. By the skimming property, a price acceptable to $k^n_{t^*}$ is also acceptable to $\bar{k}'_{t^*}$. As $k^n_{t^*}$ becomes arbitrarily close to $\bar{k}'_{t^*}$, the probability of sale from targeting $k^n_{t^*}$ becomes arbitrarily close to the probability of sale from targeting $\bar{k}'_{t^*}$ and, hence, for n large enough,

$$R(\bar{k}'_{t^*}) > R(k^n_{t^*}) - \tfrac{1}{2}\eta. \tag{16}$$

It follows from (15) and (16) that for large n,

$$R\left(\bar{k}'_{t^*} + \tfrac{1}{2}\varepsilon\right) > R(k^n_{t^*}) + \tfrac{1}{2}\eta.$$

Hence, $k^n_{t^*}$ cannot be optimal, a contradiction. This establishes that either $K_t \setminus \{\bar{k}_t\} = \varnothing$ for all $t = 1, \ldots, \tau + 1$ or, otherwise, $\bar{k}'_{t^*} < \bar{k}_{\tau+2}$. The induction is therefore complete. □

A.3 *Existence and uniqueness*

Define the function

$$k(b, p) := \arg\max_{k \geq p} [F(b) - F(k)][(1 - \delta)k + \delta p]. \tag{17}$$

The solution to the maximization problem exists by the continuity of F and is unique by the assumption that F satisfies increasing virtual valuation. Therefore, $k(b, p)$ is a well defined function. Intuitively, $k(b, p)$ is the current period marginal type that maximizes the seller's revenue given that the next period price is p and the highest type that has not traded so far is b.

Lemma 6. *(i) The type $k(b, p)$ is continuous. (ii) If $k(b, p) > p$, then $k(b, p)$ is strictly increasing in b and strictly decreasing in p.*

Proof. Part (i) follows from the maximum theorem. Part (ii) says that when the next period price is higher, the current seller chooses a lower cutoff (sells to more buyer types) and when the highest remaining type b is higher, he chooses a higher cutoff (sells to fewer types). The first-order condition of (17) is given by

$$k-\frac{F(b)-F(k)}{f(k)}=-\frac{\delta}{1-\delta}p$$

and the claim follows from Lemma 2. □

Define two sequences $b_0, b_1, \dots$ and $p_0, p_1, \dots$ inductively, as

$$\begin{aligned} b_0 &= p_0 = \underline{v} \\ b_s &= \sup\{b : k(b, p_{s-1}) = b_{s-1}\} \\ p_s &= (1-\delta)b_s + \delta p_{s-1}. \end{aligned} \tag{18}$$

Intuitively, we have reversed the timeline for the purpose of backward induction, where $s \geq 0$ indicates the number of periods that remain before the game ends (see Fudenberg et al. 1985). When the game is over, that is, when $s = 0$, the largest remaining type is $b_0 = \underline{v}$ and the price that leads to $b_0 = \underline{v}$ (in the previous period) is $p_0 = \underline{v}$. Now b_1 is the largest type such that the game will finish this period if the remaining set of types is $[\underline{v}, b_1]$. Then p_1 is the price that leads to b_1 (in the previous period). Hence, if the remaining set of types is $[\underline{v}, b_s]$, the game will finish in s periods (including the current period), and p_s is the price that leads to the marginal type b_s (in the previous period).

Given (b_{s-1}, p_{s-1}) and $b_{s-1} \geq p_{s-1}$, then b_s (and, therefore, p_s) is uniquely defined. However, we cannot remove "sup" in (18) because if $b_{s-1} = p_{s-1}$, then any $b \in (b_{s-1}, b_s]$ satisfies (17). The following lemma is immediate by definition.

Lemma 7. *The set* $\{b : k(b, p_{s-1}) = b_{s-1}\}$ *is a singleton whenever* $b_{s-1} > p_{s-1}$.

Lemma 8 (Existence). *For each initial belief* $b \in (b_s, b_{s+1}]$, *there is an equilibrium that ends in exactly* $s+1$ *periods.*

Proof. By the definition of b_1, there is an equilibrium in which the game ends in one period if the initial belief is in $(b_0, b_1]$. This establishes the claim for $s = 0$. Now we construct an equilibrium for $s > 1$. For each $b \in (b_0, b_1]$, set $\beta_0(b) = b$ and $\pi_0(b) = \underline{v}$, and for $n = 1, \dots, s$, define $\pi_n, \beta_n : (b_0, b_1] \to \mathbb{R}$ inductively by

$$\pi_n(b) = (1-\delta)\beta_{n-1}(b) + \delta\pi_{n-1}(b) \quad \text{and} \quad k(\beta_n(b), \pi_{n-1}(b)) = \beta_{n-1}(b).$$

Now we claim that β_n, π_n satisfy the following properties:

Property 1. $\beta_n(b)$ is strictly increasing and continuous.

Property 2. $\beta_n(b) > \pi_{n-1}(b)$.

Property 3. $\pi_n(b)$ is continuous and weakly increasing.

Property 4. $\beta_n(b_0) = b_n$ and $\beta_n(b_1) = b_{n+1}$.

First, we argue that Properties 1–4 hold for $n = 1$. Since $b > b_0 = \pi_0(b)$, by Lemma 6, $\beta_1(b)$ is strictly increasing and continuous, establishing Property 1. Property 4 follows by definition of the sequence $b_0, b_1, \dots$. Property 2 follows because $\beta_1(b)$ is strictly increasing, $b > b_0$, and $\beta_1(b_0) = b_1 \geq b_0 = \pi_0(b)$.

Now assume that Properties 1–4 hold for $n = 1, \dots, s-1$. We claim that they hold for $n = s$. Note that $\beta_s(b)$ is continuous by the induction hypothesis Property 3 and the continuity of $k(\cdot, \cdot)$. Again, by Lemma 6 and the induction hypothesis Property 3, $\beta_s(b)$ is strictly increasing. There Property 1 is confirmed. Property 3 is immediate by definition of $\pi_s(b)$. By the definition of β_s,

$$k(\beta_s(b_0), \pi_{s-1}(b_0)) = \beta_{s-1}(b_0) = b_{s-1}$$

$$k(\beta_s(b_1), \pi_{s-1}(b_0)) = \beta_{s-1}(b_1) = b_s.$$

By the definition of b_s and the uniqueness of β_s, it is immediate that

$$\beta_s(b_0) = b_s$$

$$\beta_s(b_1) = b_{s+1}.$$

Therefore, Property 4 is confirmed. Notice that

$$\beta_s(b) > \beta_{s-1}(b) > \pi_s(b).$$

Property 2 for $n = s$ follows immediately.

So far, we have shown that β_n is a one-to-one and onto map from the interval $(b_0, b_1]$ into $(b_n, b_{n+1}]$. This implies that for any initial belief $b \in (b_s, b_{s+1}]$, there exists a unique $b^* \equiv \beta_s^{-1}(b) \in (b_0, b_1]$. Moreover, it is easy to see that the sequence $\{\beta_{s-1}(b^*), \dots, \beta_0(b^*), \underline{v}\}$ forms a sequence of equilibrium cutoffs for the game starting with belief b. This establishes the claim. □

Next, we show that the equilibrium constructed above is the unique equilibrium.

Lemma 9 (Uniqueness). *For all initial beliefs $b \in (b_s, b_{s+1}]$, in any equilibrium, trade must be completed in exactly $s+1$ periods. Moreover, there is a unique equilibrium.*

Proof. We proceed in the following steps.

Step 1. We first show that this is true for $s = 0$, that is, there are no equilibria that last more than one period for $b \in (b_0, b_1]$ and there is a unique equilibrium in which the first seller charges $\underline{v}$.

First, there exists b^* very small such that the game ends in one shot. Then consider $b \in (b^*, b^* + \varepsilon]$, where ε is such that for any $v \in [b^*, b_1]$,

$$(F(b) - F(b - \varepsilon))b < F(b)\underline{v}$$

or

$$\left(1 - \frac{F(b-\varepsilon)}{F(b)}\right)b < \underline{v}.$$

If, in equilibrium, seller 1 chooses a cutoff in $(b^*, b^* + \varepsilon)$, then it contradicts the choice of ε. If, in equilibrium, seller 1 chooses a cutoff in $(\underline{v}, b^*]$, then the game ends in two periods. Then from (17), seller 1's problem is

$$k(b, \underline{v}) = \arg\max_{k \geq \underline{v}} [F(b) - F(k)][(1-\delta)k + \delta\underline{v}].$$

Since the game ends in two periods, we know that $k(b, \underline{v}) > \underline{v}$. Therefore, by the monotonicity established in Lemma 6, $k(b, \underline{v})$ is strictly increasing in b. Therefore, for $b \leq b_1$, we have $k(b, \underline{v}) \leq k(b_1, \underline{v}) = \underline{v}$, which contradicts the assumption that $k(b, \underline{v}) > \underline{v}$. That is, the game must end in one period if $b \in (b^*, b^* + \varepsilon]$.

Now consider $v \in (v^* + \varepsilon, v^* + 2\varepsilon]$. In equilibrium, the period 2 cutoff cannot be in $(\underline{v} + \varepsilon, v^* + 2\varepsilon]$ because of the choice of ε. If the period 2 cutoff is in $(\underline{v}, v^* + \varepsilon]$, then the game ends in two periods. But we can apply the previous argument again to derive a contradiction. The proof for this step is completed by induction.

Suppose for the purpose of induction that the lemma is true for $s = 1, 2, \ldots, N-1$ and consider $b \in (b_N, b_{N+1}]$.

Step 2. We first show that the game ends in exactly $N+1$ periods. We consider two cases.

Case 1. Suppose that there exists an equilibrium that lasts longer than $N+1$ periods. Then it must be the case that in this equilibrium, the first seller with initial belief b chooses a cutoff $\tilde{k}_1 > b_N$, because, by the induction hypothesis, for any smaller cutoff, the game ends in exactly N additional periods. Similarly, the second seller with initial belief $\tilde{k}_1$ chooses a cutoff level $\tilde{k}_2 > b_{N-1}$ (this does not exclude the case where $\tilde{k}_2 > b_N$) and the sth seller with initial belief $\tilde{k}_{s-1}$ chooses $\tilde{k}_s > b_{N-s+1}$ so that the game lasts more than $N+1$ periods. Therefore, the price that the second seller with initial belief $\tilde{k}_1$ charges is strictly greater than

$$(1-\delta)(b_{N-1} + \delta b_{N-2} + \cdots + \delta^{N-2} b_1) + \delta^{N-1}\underline{v}.$$

On the other hand, we also know, by Lemma 8, that there is an equilibrium that lasts exactly $N+1$ periods. Let k_s^* be the cutoff sequence of that equilibrium. Notice that $k_s^* \leq b_{N+1-s}$. Therefore, the price charged by the second period seller with initial belief k_1^* is at most

$$(1-\delta)(b_{N-1} + \delta b_{N-2} + \cdots + \delta^{N-2} b_1) + \delta^{N-1}\underline{v}.$$

But then the first seller in the candidate equilibrium chooses a higher cutoff than the first seller in the equilibrium of Lemma 8, even though both of these sellers have the same initial belief and the second period price is less in the latter equilibrium. Since the optimal cutoff is decreasing in the continuation price, this is a contradiction by Lemma 6.

Case 2. Now, suppose that there is an equilibrium that lasts N periods or less.

Suppose, first, that the cutoff $\hat{k}_1$ that the first seller with belief b chooses in equilibrium is less than b_{N-1}. Thereafter, there is a unique continuation equilibrium in which for all s, the cutoff chosen by the sth seller with belief $\hat{k}_{s-1}$ is at most b_{N-s}. Therefore, the price charged by the second seller is at most

$$(1-\delta)(b_{N-2}+\delta b_{N-3}+\cdots+\delta^{N-2}b_1)+\delta^{N-1}\underline{v}.$$

On the other hand, the cutoff k_s^* chosen by seller s in the equilibrium of Lemma 8 is strictly greater than b_{N-s}, and, therefore, the price is strictly above

$$(1-\delta)(b_{N-2}+\delta b_{N-3}+\cdots+\delta^{N-2}b_1)+\delta^{N-1}\underline{v}.$$

But this is a contradiction since $k(b,p)$ is decreasing in p.

Now suppose that $\hat{k}_1 > b_N$. Let s be the first period when the cutoff $\hat{k}_s \le b_N$. Then it must be that $\hat{k}_s \le b_{N-s}$, because for any $k \in (b_{N-s}, b_N]$, there is a unique continuation equilibrium that lasts at least $N-s+1$ periods. Now, consider the equilibrium constructed in Lemma 8, starting from initial belief $\beta_{N-s}(\hat{k}_s)$. Then, by construction, the seller with this belief chooses $\hat{k}_s$. Moreover, by the induction hypothesis, the continuation of this equilibrium coincides with the continuation of the other equilibrium where $\hat{k}_{s-1}$ chooses $\hat{k}_s$. This implies that the next period price is the same in both equilibria. Call this price p. Note that $\hat{k}_{s-1} > b_N > \beta_{N-s}(\hat{k}_s)$. But this is a contradiction since $k(\hat{k}_{s-1}, p) > k(\beta_{N-s}(\hat{k}_s), p)$.

Step 3. Step 2 establishes that for $b \in (b_N, b_{N+1}]$, all equilibria last exactly $N+1$ periods. We show that the equilibrium is unique, which is the one we constructed in Lemma 8.

Suppose, by contradiction, that there is another equilibrium (in addition to the one constructed in the proof of Lemma 8) that lasts exactly $N+1$ periods. Let k^* be the first period cutoff of the equilibrium constructed in the proof of Lemma 8. Then $k^* \le b_N$. Let k' be the first cutoff of the other equilibrium. Then it must be that $k^* \neq k'$ because there is a unique N period equilibrium following cutoff k^* by the induction hypothesis.

Now, if $k' \le b_N$, it must be that $k' > b_{N-1}$, since otherwise the equilibrium lasts at most $N-1$ periods. Suppose, without loss of generality (w.l.o.g.), that $k^* > k'$. Then it must be the case that the second-period price in equilibrium of Lemma 8 is higher than the second-period price following k'. This is because, in the unique continuation equilibrium, all cutoffs are increasing in the initial belief, since the functions $\beta_s(\cdot)$ are increasing; and because, after each of these cutoffs, the equilibrium lasts exactly N additional periods. But this leads to a contradiction since $k(b,p)$ is decreasing in p.

Now suppose $k^* > b_N$. We shall use an argument similar to the one used to establish that all equilibria last at least $N+1$ periods. Let s be the first period when the cutoff $\hat{k}_s \le b_N$. Then it must be that $b_{N-s} \le \hat{k}_s \le b_{N-s+1}$, because that is the only way that the equilibrium will have $N-s+1$ additional periods. Now consider the equilibrium constructed in Claim A.1, starting from initial belief $\beta_{N-s}(\hat{k}_s)$. Then, by construction, the seller with this belief chooses $\hat{k}_s$. Moreover, by the induction hypothesis, the continuation of this equilibrium coincides with the continuation of the other equilibrium where $\hat{k}_{s-1}$ chooses $\hat{k}_s$. This implies that the next period price is the same in both equilibria. Call this price p. Note that $\hat{k}_{s-1} > b_N > \beta_{N-s}(\hat{k}_s)$. But this is a contradiction since $k(\hat{k}_{s-1}, p) > k(\beta_{N-s}(\hat{k}_s), p)$. □

Appendix B: Proof of Proposition 1

B.1 *Preliminary results*

Before presenting our induction argument, we present some preliminary results that facilitate the ensuing discussion. The first set of results are technical and they do not rely on equilibrium conditions.

B.1.1 *Technical results* The first result establishes a strong form of monotonicity for the solution of the profit-maximization problem of a seller facing a truncation of F. In particular, the profit-maximizing price of any seller is nondecreasing in the highest type $\bar{k}$ that he believes to be remaining, regardless of the continuation play. The solution to this maximization problem may not be unique (in the transparent regime). Therefore, it is necessary to make precise the notion of monotonicity for the set of profit-maximizing prices. The appropriate definition in this context is as follows.

Definition 1. Consider two sets $X, Y \subset \mathbb{R}$. We say that the set X is greater than the set Y if and only if for all $x \in X$ and $y \in Y$, $x \geq y$.

This set order is stronger than the *strong set order* (see Topkis 1998), which allows for nonsingleton intersections of two sets. We shall use the set order defined in Definition 1 when we refer to monotonicity of sets.

Remark 1. Unlike in the nontransparent regime in which there is a unique equilibrium in pure strategies, the equilibrium in the transparent regime may involve mixed strategies (especially off the equilibrium path) and there may be multiple equilibria. The multiplicity of the equilibria is not an issue as we fix an arbitrary equilibrium in the transparent regime and compare it with the unique pure strategy equilibrium in the nontransparent regime. To deal with mixed strategies, we consider sets of prices and introduce a strong notion of monotonicity of sets. In addition, since the current period cutoff buyer type might face a random price by delaying trade to the next period, we need to consider the expectation of this random price to study this cutoff buyer's incentives.

In our game, when a short-run seller faces buyer types $[\underline{v}, \bar{k}]$, he chooses a price p to maximize $[F(\bar{k}) - F(k^i(p))]p$, where $k^i(p)$ is the cutoff buyer type in regime i. It turns out that regardless of the properties of $k^i(p)$, we can show that the set of optimal prices $\arg\max_p [F(\bar{k}) - F(k^i(p))]p$ is nondecreasing in $\bar{k}$ in the sense of Definition 1.

Lemma 10. *For any real-valued function* $k(p)$, $\arg\max_p [F(\bar{k}) - F(k(p))]p$ *is nondecreasing in* $\bar{k}$ *in the sense of Definition 1.*

Proof. Notice that the objective function has increasing differences in $\bar{k}$ and p. It follows from Topkis' theorem that the solution set is nondecreasing in $\bar{k}$ in the strong set order (Topkis 1998). We strengthen this conclusion below. Suppose to the contrary that the ordering in Definition 1 does not hold. That is, there exist $p \in \arg\max[F(\bar{k}) - F(k(p))]p$ and $p' \in \arg\max[F(\bar{k}') - F(k(p))]p$, $\bar{k} < \bar{k}'$ but $p > p'$. Then it follows from Topkis' theorem that p and p' are maximizers for both objective functions:

$$\big[F(\bar{k}) - F(k(p))\big]p = \big[F(\bar{k}) - F(k(p'))\big]p'$$
$$\big[F(\bar{k}') - F(k(p))\big]p = \big[F(\bar{k}') - F(k(p'))\big]p'.$$

Subtracting the second equation from the first, we have

$$[F(\bar{k}) - F(\bar{k}')]p = [F(\bar{k}) - F(\bar{k}')]p'.$$

It follows immediately that $p = p'$, a contradiction. □

Next, we establish a direct implication of the increasing hazard rate property for F. Specifically, we show that a truncation of F inherits the hazard rate property from F. We present the version that is actually used in our proof.

Lemma 11. *Suppose that $v < v + \Delta \le k$. Then if F has increasing hazard rate, that is, $f(v)/(1 - F(v))$ is nondecreasing, then $(F(v+\Delta) - F(v))/(F(k) - F(v))$ is nondecreasing in v.*

Proof. First note that

$$\frac{f(v)}{F(k) - F(v)}$$

is nondecreasing in v. To see this, simply note that

$$\begin{aligned}\frac{f(v)}{F(k) - F(v)} &= \frac{f(v)}{1 - F(v)} \cdot \frac{1 - F(v)}{F(k) - F(v)} \\ &= \frac{f(v)}{1 - F(v)}\left(\frac{1 - F(k)}{F(k) - F(v)} + 1\right)\end{aligned}$$

and both terms are increasing in v. Now note that

$$\frac{F(v+\Delta) - F(v)}{F(k) - F(v)} = 1 - \frac{F(k) - F(v+\Delta)}{F(k) - F(v)}.$$

Thus,

$$\begin{aligned}\frac{\partial}{\partial v}\left(\frac{F(v+\Delta) - F(v)}{F(k) - F(v)}\right) &= \frac{f(v+\Delta)}{F(k) - F(v)} - \frac{F(k) - F(v+\Delta)}{(F(k) - F(v))^2} f(v) \\ &= \frac{F(k) - F(v+\Delta)}{F(k) - F(v)}\left(\frac{f(v+\Delta)}{F(k) - F(v+\Delta)} - \frac{f(v)}{F(k) - F(v)}\right) \\ &\ge 0.\end{aligned}$$

This completes the proof. □

B.1.2 *Preliminary equilibrium characteristics* Next we establish two intuitive, but not immediate, properties of equilibria in the transparent regime. The first result establishes that for seller 1, the probability of sale decreases in his price offer. The second establishes that a higher price in the first period leads to higher expected continuation prices. To make these statements precise, it is necessary to introduce additional notation.

Notice that under the transparent regime, the second-period price (on or off the equilibrium path) can depend only on the first-period price, as this is the only observable history. Following an off-equilibrium first-period price, seller 2 may play a mixed strategy. Let $\hat{p}_2^{\mathrm{TR}}(p)$ be the expected second-period price if the first-period seller in the transparent regime chooses p. For any price p that is accepted with a positive probability in equilibrium, there exists a unique k that satisfies the indifference condition:

$$k - p = \delta(k - \hat{p}_2^{\mathrm{TR}}(p)).$$

That is, the buyer type k is indifferent between buying at p in this period versus waiting for a (random) price with an expectation of $\hat{p}_2^{\mathrm{TR}}(p)$ in the next period. This indifference condition is well defined because, in equilibrium, if k is the highest type in the next period, all prices in the support of the seller's equilibrium strategy must induce the acceptance of k; that is, all of these prices are lower than k. We reformulate the indifference condition as follows for future reference:

$$p = (1-\delta)k + \delta\hat{p}_2^{\mathrm{TR}}(p).$$

Denote the unique cutoff type defined by this indifference condition by

$$k_1^{\mathrm{TR}}(p) = \frac{p - \delta\hat{p}_2^{\mathrm{TR}}(p)}{1-\delta}.$$

For future reference, let $k_1^{\mathrm{NTR}}(p)$ be the cutoff type defined by the indifference condition of the nontransparent regime,

$$p = (1-\delta)k_1^{\mathrm{NTR}}(p) + \delta p_2^{\mathrm{NTR}},$$

where p_2^{NTR} is the unique equilibrium price in period 2 (in pure strategy).

Lemma 12. *We have that $k_1^{\mathrm{TR}}(p)$ is nondecreasing in p.*

Proof. Take $p > p'$ and suppose that $k_1^{\mathrm{TR}}(p) < k_1^{\mathrm{TR}}(p')$. Then, by Lemma 10,

$$p = (1-\delta)k^{\mathrm{TR}}(p) + \delta\hat{p}_2(p) \leq (1-\delta)k^{\mathrm{TR}}(p') + \delta\hat{p}_2(p') = p',$$

a contradiction. □

The next lemma shows that in the transparent regime, a deviation by the first seller to a higher price weakly increases the expected price in the second period.

Lemma 13. *Let p_1^{TR} be any price in the support of seller 1's strategy. If $p_1^{\mathrm{TR}} < p$, then the expected price in period 2 satisfies $\hat{p}_2^{\mathrm{TR}}(p_1^{\mathrm{TR}}) \leq \hat{p}_2^{\mathrm{TR}}(p)$.*

Proof. Take $p > p_1^{\mathrm{TR}}$. Then Lemma 12 implies that $k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}) \le k_1^{\mathrm{TR}}(p)$. Moreover, $k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}) = k_1^{\mathrm{TR}}(p)$ contradicts the optimality of p_1^{TR}. Therefore, $k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}) < k_1^{\mathrm{TR}}(p)$. Then the claim follows from Lemma 10. □

Fudenberg et al. (1985) prove that the *equilibrium* price in the second period is a pure strategy, that is, $\hat{p}_2^{\mathrm{TR}}(p_1^{\mathrm{TR}}) = p_2^{\mathrm{TR}}$. Our proof above does not utilize this result.

B.2 *The induction proof of Proposition 1*

Recall that T^i is the last period during which trade takes place with a positive probability in regime i on the given equilibrium path. If $\max\{T^{\mathrm{TR}}, T^{\mathrm{NTR}}\} = 1$, then the prices in the two regimes must equal $\underline{v}$ and the claim in Proposition 1 is vacuously satisfied.

Induction hypothesis. Fix a distribution function F that exhibits an increasing hazard rate. For *any* $\bar{v} \ge \bar{k}^{\mathrm{TR}} \ge \bar{k}^{\mathrm{NTR}} \ge \underline{v}$ and *any* realization of the price sequence $\{p^i\}$ of *any* equilibrium in regime $i = \mathrm{TR}, \mathrm{NTR}$, where the buyer's type distribution is a truncation of F with support $[\underline{v}, \bar{k}^i]$, assume $p_t^{\mathrm{TR}} \ge p_t^{\mathrm{NTR}}$ for all t whenever $\max\{T^{\mathrm{TR}}, T^{\mathrm{NTR}}\} \le \tau$, where $\tau \ge 1$.

Remark 2. We shall show that if $\bar{k}^{\mathrm{TR}} \ge \bar{k}^{\mathrm{NTR}} \ge \underline{v}$ is such that $\max\{T^{\mathrm{TR}}, T^{\mathrm{NTR}}\} = \tau + 1$ in some equilibria of the two regimes, the prices can be ranked. We do this in three steps: (i) We show that under the induction hypothesis, the second-period price of the nontransparent regime is smaller than any possible second-period price of the transparent regime. (ii) Then we show that, under the induction hypothesis and using (i), the first-period price in the transparent regime must be larger than the equilibrium price of the nontransparent regime. (iii) Finally, we complete the proof by showing that the prices in the later periods must also be ranked as claimed. Our discussion in the main text refers to step (ii).

We start with (i) mentioned in Remark 2.

Lemma 14. *Fix any $\bar{k}^{\mathrm{TR}} \ge \bar{k}^{\mathrm{NTR}} \ge \underline{v}$. Fix any equilibrium in the transparent regime and let p_1^{TR} be any price in the support of seller 1's equilibrium strategy. Suppose that* $\max\{T^{\mathrm{TR}}, T^{\mathrm{NTR}}\} = \tau + 1$. *Then $\hat{p}_2^{\mathrm{TR}}(p_1^{\mathrm{TR}}) \ge p_2^{\mathrm{NTR}}$. That is, the expected second-period price in the transparent regime following any equilibrium path history is no less than the second-period price in the nontransparent regime.*

Proof. We abuse notation slightly by writing k_1^i as $k_1^i(p_1^i)$. That is, k_1^i is the *equilibrium* marginal type that purchases at the *given* realized equilibrium price p_1^i in period 1 in regime i.

Suppose to the contrary that the claim is false: $\hat{p}_2^{\mathrm{TR}}(p_1^{\mathrm{TR}}) < p_2^{\mathrm{NTR}}$. Then we claim that the highest buyer type at the beginning of period 2 following p_1^i must satisfy $k_1^{\mathrm{TR}} < k_1^{\mathrm{NTR}}$. To see this, first note that from the second period onward, all trade takes place in at most τ periods in the continuation equilibrium in either regime. If, however, $k_1^{\mathrm{TR}} \ge k_1^{\mathrm{NTR}}$, it follows from the induction hypothesis (with $\bar{k}^{\mathrm{TR}} = k_1^{\mathrm{TR}}$ and $\bar{k}^{\mathrm{NTR}} = k_1^{\mathrm{NTR}}$ on the continuation game) that $\hat{p}_2^{\mathrm{TR}}(p_1^{\mathrm{TR}}) \geq p_2^{\mathrm{NTR}}$, a contradiction.

For each $k \in [\underline{v}, \bar{k}^i]$, define

$$p_1^i(k) := \sup\{p : k_1^i(p) \leq k\}.$$

That is, $p_1^i(k)$ is the highest price that seller 1 can charge so that buyer type k buys in period 1. Since p_1^{NTR} is seller 1's unique optimal price in the nontransparent regime, the following inequality must hold:

$$(F(\bar{k}^{\mathrm{NTR}}) - F(k_1^{\mathrm{TR}}))p_1^{\mathrm{NTR}}(k_1^{\mathrm{TR}}) < (F(\bar{k}^{\mathrm{NTR}}) - F(k_1^{\mathrm{NTR}}))p_1^{\mathrm{NTR}}(k_1^{\mathrm{NTR}}). \tag{19}$$

The left-hand side of (19) is seller 1's profit in regime NTR if he targets a cutoff type k_1^{TR} with a price $p_1^{\mathrm{NTR}}(k_1^{\mathrm{TR}})$; the right-hand side is seller 1's profit by following the unique pure strategy equilibrium, that is, targeting equilibrium cutoff type k_1^{NTR} with the equilibrium price $p_1^{\mathrm{TR}} = p_1^{\mathrm{NTR}}(k_1^{\mathrm{NTR}})$.[17] By the previous claim, $k_1^{\mathrm{TR}} < k_1^{\mathrm{NTR}}$.

Inequality (19) can be rewritten as

$$\frac{F(\bar{k}^{\mathrm{NTR}}) - F(k_1^{\mathrm{TR}})}{F(\bar{k}^{\mathrm{NTR}}) - F(k_1^{\mathrm{NTR}})} < \frac{p_1^{\mathrm{NTR}}(k_1^{\mathrm{NTR}})}{p_1^{\mathrm{NTR}}(k_1^{\mathrm{TR}})}$$

and further as

$$\frac{F(k_1^{\mathrm{NTR}}) - F(k_1^{\mathrm{TR}})}{F(\bar{k}^{\mathrm{NTR}}) - F(k_1^{\mathrm{NTR}})} < \frac{p_1^{\mathrm{NTR}}(k_1^{\mathrm{NTR}}) - p_1^{\mathrm{NTR}}(k_1^{\mathrm{TR}})}{p_1^{\mathrm{NTR}}(k_1^{\mathrm{TR}})} \tag{20}$$

by substracting 1 from each side.[18]

We now argue that in regime TR, increasing the cutoff from k_1^{TR} to k_1^{NTR} strictly increases the payoff of seller 1. This will lead to the desired contradiction. The idea is to show that in the transparent regime, this change of cutoff types leads to a smaller percentage decrease in trading probability (the left-hand side of (20)) but is accompanied by an even larger percentage increase in price than in the nontransparent regime (the right-hand side of (20)).

We first compare the percentage changes in trading probability in the two regimes. Since $\bar{k}^{\mathrm{TR}} \geq \bar{k}^{\mathrm{NTR}}$ by assumption, and $k_1^{\mathrm{TR}} < k_1^{\mathrm{NTR}}$ by a previous claim, it is immediate that

$$\frac{F(k_1^{\mathrm{NTR}}) - F(k_1^{\mathrm{TR}})}{F(\bar{k}^{\mathrm{TR}}) - F(k_1^{\mathrm{NTR}})} \leq \frac{F(k_1^{\mathrm{NTR}}) - F(k_1^{\mathrm{TR}})}{F(\bar{k}^{\mathrm{NTR}}) - F(k_1^{\mathrm{NTR}})}. \tag{21}$$

[17] To avoid introducing further notation, we slightly abuse notation.

[18] In words, (20) has the following interpretation: seller 1 in regime NTR faces a set of buyer types $[\underline{v}, \bar{k}^i]$; if seller 1's targeted type increases from k_1^{TR} to k_1^{NTR}, the trading probability decreases by a percentage factor of $(F(k_1^{\mathrm{NTR}}) - F(k_1^{\mathrm{TR}}))/(F(\bar{k}^{\mathrm{NTR}}) - F(k_1^{\mathrm{NTR}}))$, but this is accompanied by a larger percentage increase in price of $(p_1^{\mathrm{NTR}}(k_1^{\mathrm{NTR}}) - p_1^{\mathrm{NTR}}(k_1^{\mathrm{TR}}))/p_1^{\mathrm{NTR}}(k_1^{\mathrm{TR}})$. Therefore, increasing the cutoff type from k_1^{TR} to k_1^{NTR} is desirable (recall that k_1^{NTR} is the equilibrium cutoff level).

We now compare the percentage changes in price in the two regimes. Note that

$$\begin{aligned} p_1^{\mathrm{TR}}(k_1^{\mathrm{TR}}) &= (1-\delta)k_1^{\mathrm{TR}} + \delta \hat{p}_2(p_1^{\mathrm{TR}}) \\ &< (1-\delta)k_1^{\mathrm{TR}} + \delta p_2^{\mathrm{NTR}} \\ &= p_1^{\mathrm{NTR}}(k_1^{\mathrm{TR}}), \end{aligned} \tag{22}$$

where inequality (22) follows from the supposition that $\hat{p}_2(p_1^{\mathrm{TR}}) < p_2^{\mathrm{NTR}}$.

Note also that since $k_1^{\mathrm{TR}} < k_1^{\mathrm{NTR}}$, it follows from Lemma 10 that $p_1^{\mathrm{TR}}(k_1^{\mathrm{TR}}) \leq p_1^{\mathrm{TR}}(k_1^{\mathrm{NTR}})$ and, hence, by Lemma 13,

$$\hat{p}_2^{\mathrm{TR}}(p_1^{\mathrm{TR}}(k_1^{\mathrm{TR}})) \leq \hat{p}_2^{\mathrm{TR}}(p_1^{\mathrm{TR}}(k_1^{\mathrm{NTR}})). \tag{23}$$

Therefore,

$$\begin{aligned} & p_1^{\mathrm{NTR}}(k_1^{\mathrm{NTR}}) - p_1^{\mathrm{NTR}}(k_1^{\mathrm{TR}}) \\ &\quad = [(1-\delta)k_1^{\mathrm{NTR}} + \delta p_2^{\mathrm{NTR}}] - [(1-\delta)k_1^{\mathrm{TR}} + \delta \hat{p}_2(p_1^{\mathrm{TR}})] \\ &\quad = (1-\delta)(k_1^{\mathrm{NTR}} - k_1^{\mathrm{TR}}) \\ &\quad \leq (1-\delta)(k_1^{\mathrm{NTR}} - k_1^{\mathrm{TR}}) + \delta\big(\hat{p}_2^{\mathrm{TR}}(p_1^{\mathrm{TR}}(k_1^{\mathrm{NTR}})) - \hat{p}_2^{\mathrm{TR}}(p_1^{\mathrm{TR}}(k_1^{\mathrm{TR}}))\big) \\ &\quad = \big[(1-\delta)k_1^{\mathrm{NTR}} + \delta \hat{p}_2^{\mathrm{TR}}(p_1^{\mathrm{TR}}(k_1^{\mathrm{NTR}}))\big] - \big[(1-\delta)k_1^{\mathrm{TR}} + \delta \hat{p}_2^{\mathrm{TR}}(p_1^{\mathrm{TR}}(k_1^{\mathrm{TR}}))\big] \\ &\quad = p_1^{\mathrm{TR}}(k_1^{\mathrm{NTR}}) - p_1^{\mathrm{TR}}(k_1^{\mathrm{TR}}), \end{aligned} \tag{24}$$

where inequality (24) follows from (23).

It then follows from (22) and (24) that

$$\frac{p_1^{\mathrm{NTR}}(k_1^{\mathrm{NTR}}) - p_1^{\mathrm{NTR}}(k_1^{\mathrm{TR}})}{p_1^{\mathrm{NTR}}(k_1^{\mathrm{TR}})} < \frac{p_1^{\mathrm{TR}}(k_1^{\mathrm{NTR}}) - p_1^{\mathrm{TR}}(k_1^{\mathrm{TR}})}{p_1^{\mathrm{TR}}(k_1^{\mathrm{TR}})}. \tag{25}$$

Combining (20), (21), and (25), we have

$$\frac{F(k_1^{\mathrm{NTR}}) - F(k_1^{\mathrm{TR}})}{F(\bar{k}^{\mathrm{TR}}) - F(k_1^{\mathrm{NTR}})} < \frac{p_1^{\mathrm{TR}}(k_1^{\mathrm{NTR}}) - p_1^{\mathrm{TR}}(k_1^{\mathrm{TR}})}{p_1^{\mathrm{TR}}(k_1^{\mathrm{TR}})}. \tag{26}$$

This says that, in regime TR, increasing the cutoff from k_1^{TR} to k_1^{NTR} leads to a smaller percentage change in trading probability than in price.

Applying the same argument between (19) and (20), we can rewrite (26) as

$$(F(\bar{k}^{\mathrm{TR}}) - F(k_1^{\mathrm{TR}}))p_1^{\mathrm{TR}}(k_1^{\mathrm{TR}}) < (F(\bar{k}^{\mathrm{TR}}) - F(k_1^{\mathrm{NTR}}))p_1^{\mathrm{TR}}(k_1^{\mathrm{NTR}}).$$

This inequality says that in regime TR, seller 1 can be strictly better off by targeting the cutoff type k_1^{NTR} rather than the equilibrium cutoff type k_1^{TR}, a contradiction. □

The next lemma establishes (ii) mentioned in Remark 2. That is, it shows that the first-period equilibrium price of the transparent regime is larger than the first-period equilibrium price in the nontransparent regime.

Lemma 15. *Fix any* $\bar{k}^{\mathrm{TR}} \geq \bar{k}^{\mathrm{NTR}} \geq \underline{v}$. *Fix any equilibrium in the transparent regime and let* p_1^{TR} *be any realized first-period equilibrium price. Let* p_1^{NTR} *be the unique first-period equilibrium of the nontransparent regime. Suppose* $\max\{T^{\mathrm{TR}}, T^{\mathrm{NTR}}\} = \tau + 1$. *Then* $p_1^{\mathrm{TR}} \geq p_1^{\mathrm{NTR}}$.

Proof. For a contradiction, suppose that there exists p_1^{TR} in the support of seller 1's equilibrium strategy in the transparent regime such that $p_1^{\mathrm{TR}} < p_1^{\mathrm{NTR}}$. Since seller 1 in the nontransparent regime has a unique optimal strategy, we have, as in (19),

$$\big(F(\bar{k}^{\mathrm{NTR}}) - F(k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}}))\big)p_1^{\mathrm{TR}} < \big(F(\bar{k}^{\mathrm{NTR}}) - F(k_1^{\mathrm{NTR}}(p_1^{\mathrm{NTR}}))\big)p_1^{\mathrm{NTR}},$$

which can be rewritten as

$$\frac{F(k_1^{\mathrm{NTR}}(p_1^{\mathrm{NTR}})) - F(k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}}))}{F(\bar{k}^{\mathrm{NTR}}) - F(k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}}))} < \frac{p_1^{\mathrm{NTR}} - p_1^{\mathrm{TR}}}{p_1^{\mathrm{NTR}}}. \tag{27}$$

Now we compare

$$\frac{F(k_1^{\mathrm{NTR}}(p_1^{\mathrm{NTR}})) - F(k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}}))}{F(\bar{k}^{\mathrm{NTR}}) - F(k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}}))} = \frac{F(k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}}) + \Delta^{\mathrm{NTR}}) - F(k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}}))}{F(\bar{k}^{\mathrm{NTR}}) - F(k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}}))}$$

to

$$\frac{F(k_1^{\mathrm{TR}}(p_1^{\mathrm{NTR}})) - F(k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}))}{1 - F(k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}))} = \frac{F(k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}) + \Delta^{\mathrm{TR}}) - F(k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}))}{1 - F(k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}))},$$

where

$$\Delta^i \equiv k_1^i(p_1^{\mathrm{NTR}}) - k_1^i(p_1^{\mathrm{TR}}).$$

We make the following two claims.

Claim B.1. *We have* $k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}}) \geq k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}})$.

Proof. If p_1^{TR} is offered by seller 1 in regime NTR, then $k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}})$ is the cutoff buyer type; the indifference condition for $k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}})$ is

$$p_1^{\mathrm{TR}} = (1-\delta)k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}}) + \delta p_2^{\mathrm{NTR}}. \tag{28}$$

If p_1^{TR} is offered by seller 1 in regime TR, then $k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}})$ is the cutoff buyer type; the indifference condition for $k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}})$ is

$$p_1^{\mathrm{TR}} = (1-\delta)k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}) + \delta \hat{p}_2^{\mathrm{TR}}(p_1^{\mathrm{TR}}). \tag{29}$$

By Lemma 14, $p_2^{\mathrm{NTR}} \leq \hat{p}_2^{\mathrm{TR}}(p_1^{\mathrm{TR}})$. Taken together, equations (28) and (29) imply that $k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}}) \geq k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}})$. ◁

Claim B.2. *We have* $\Delta^{\mathrm{NTR}} \geq \Delta^{\mathrm{TR}}$.

Proof. The indifference condition for the cutoff type $k_1^{\mathrm{NTR}}(p_1^{\mathrm{NTR}})$ in regime NTR is given by

$$p_1^{\mathrm{NTR}} = (1-\delta)k_1^{\mathrm{NTR}}(p_1^{\mathrm{NTR}}) + \delta p_2^{\mathrm{NTR}}. \tag{30}$$

The indifference condition for the cutoff type $k_1^{\mathrm{TR}}(p_1^{\mathrm{NTR}})$ in regime TR is given by

$$p_1^{\mathrm{NTR}} = (1-\delta)k_1^{\mathrm{TR}}(p_1^{\mathrm{NTR}}) + \delta \hat{p}_2^{\mathrm{TR}}(p_1^{\mathrm{NTR}}). \tag{31}$$

Therefore,

$$\begin{aligned} k_1^{\mathrm{NTR}}(p_1^{\mathrm{NTR}}) - k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}}) &= \frac{p_1^{\mathrm{NTR}} - p_1^{\mathrm{TR}}}{1-\delta} \\ &\geq \frac{p_1^{\mathrm{NTR}} - p_1^{\mathrm{TR}} - (\hat{p}_2^{\mathrm{TR}}(p_1^{\mathrm{NTR}}) - \hat{p}_2^{\mathrm{TR}}(p_1^{\mathrm{TR}}))}{1-\delta} \\ &= k_1^{\mathrm{TR}}(p_1^{\mathrm{NTR}}) - k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}), \end{aligned}$$

where the first line follows from (28) and (30), the second line follows from Lemma 13 and the supposition that $p_1^{\mathrm{TR}} < p_1^{\mathrm{NTR}}$, and the third line follows from (29) and (31). This establishes the observation. ◁

With these two claims, we are ready to prove the lemma. Note that

$$\begin{aligned} \frac{F(k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}}) + \Delta^{\mathrm{NTR}}) - F(k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}}))}{F(\bar{k}^{\mathrm{NTR}}) - F(k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}}))} &\geq \frac{F(k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}}) + \Delta^{\mathrm{TR}}) - F(k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}}))}{F(\bar{k}^{\mathrm{NTR}}) - F(k_1^{\mathrm{NTR}}(p_1^{\mathrm{TR}}))} \\ &\geq \frac{F(k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}) + \Delta^{\mathrm{TR}}) - F(k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}))}{F(\bar{k}^{\mathrm{NTR}}) - F(k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}))} \\ &\geq \frac{F(k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}) + \Delta^{\mathrm{TR}}) - F(k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}))}{F(\bar{k}^{\mathrm{TR}}) - F(k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}))}, \end{aligned}$$

where the first inequality follows from Claim B.2, the second inequality follows from Claim B.1 and Lemma 11, and the third inequality follows from the assumption that $\bar{k}^{\mathrm{TR}} \geq \bar{k}^{\mathrm{NTR}}$. Combining this with (27), we get

$$\frac{F(k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}) + \Delta^{\mathrm{TR}}) - F(k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}))}{F(\bar{k}^{\mathrm{TR}}) - F(k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}))} < \frac{p_1^{\mathrm{NTR}} - p_1^{\mathrm{TR}}}{p_1^{\mathrm{NTR}}},$$

which, after substituting in Δ^{TR}, can be rewritten as

$$\big(F(\bar{k}^{\mathrm{TR}}) - F(k_1^{\mathrm{TR}}(p_1^{\mathrm{NTR}}))\big)p_1^{\mathrm{NTR}} > p_1^{\mathrm{TR}}\big(F(\bar{k}^{\mathrm{TR}}) - F(k_1^{\mathrm{TR}}(p_1^{\mathrm{TR}}))\big).$$

This says that in regime TR, p_1^{NTR} gives seller 1 a larger profit than the equilibrium price p_1^{TR}, a contradiction. □

We now complete the induction proof of Proposition 1. This is step (iii) mentioned in Remark 2.

Proof of Proposition 1. We have already shown that $p_1^{\mathrm{TR}} \geq p_1^{\mathrm{NTR}}$. Suppose to the contrary that for some $s \leq \tau + 1$, we have $p_t^{\mathrm{TR}} \geq p_t^{\mathrm{NTR}}$ for all $t < s$, but $p_s^{\mathrm{TR}} < p_s^{\mathrm{NTR}}$. By the induction hypothesis, this is only possible if $k_{s-1}^{\mathrm{TR}}(p_{s-1}^{\mathrm{TR}}) < k_{s-1}^{\mathrm{NTR}}(p_{s-1}^{\mathrm{NTR}})$. But then the indifference condition of buyer type $k_{s-1}^{\mathrm{TR}}(p_{s-1}^{\mathrm{TR}})$ in period $s-1$ in regime TR is

$$p_{s-1}^{\mathrm{TR}} = (1-\delta)k_{s-1}^{\mathrm{TR}}(p_{s-1}^{\mathrm{TR}}) + \delta p_s^{\mathrm{TR}},$$

and the indifference condition of buyer type $k_{s-1}^{\mathrm{NTR}}(p_{s-1}^{\mathrm{NTR}})$ in period $s-1$ in regime NTR is

$$p_{s-1}^{\mathrm{NTR}} = (1-\delta)k_{s-1}^{\mathrm{NTR}}(p_{s-1}^{\mathrm{NTR}}) + \delta p_s^{\mathrm{NTR}}.$$

Since $k_{s-1}^{\mathrm{TR}}(p_{s-1}^{\mathrm{TR}}) < k_{s-1}^{\mathrm{NTR}}(p_{s-1}^{\mathrm{NTR}})$ and $p_s^{\mathrm{TR}} < p_s^{\mathrm{NTR}}$, we have, from the above two indifference conditions, that $p_{s-1}^{\mathrm{TR}} < p_{s-1}^{\mathrm{NTR}}$, a contradiction. □

Appendix C: Proof of Proposition 2

Let $\{k_t^i\}$ be a realization of an equilibrium cutoff sequence in any equilibrium in regime i, with the convention that $k_t^i = \underline{v}$ for $t > T^i$, where T^i is the latest period in which trade takes place with a positive probability. Define a random variable, x^i, that takes values in $\{k_t^i\}$, with a cumulative distribution G^i defined as

$$G^i(k) = \Pr(x^i \leq k) = \delta^{\tau^i(k)},$$

where $\tau^i(k)$ is the unique number that satisfies $k \in [k_{\tau^i(k)}^i, k_{\tau^i(k)-1}^i)$.

In words, the support of x^i is the equilibrium cutoff in regime i. The marginal types trading at time t or earlier in each regime have a total probability of δ^{t-1} under the relevant random variable.

Lemma 16. *The variable x^{TR} second-order stochastically dominates x^{NTR}. That is,*

$$\forall k: \int_{\underline{v}}^{k} \Pr(x^{\mathrm{TR}} \leq x)\, dx \leq \int_{\underline{v}}^{k} \Pr(x^{\mathrm{NTR}} \leq x)\, dx.$$

Proof.

$$\int_{\underline{v}}^{k} \Pr(x^{\mathrm{TR}} \leq x)\, dx = \sum_{t=\tau^{\mathrm{TR}}(k)}^{\infty} (1-\delta)\delta^{t-1}(k - k_t^{\mathrm{TR}})$$

and

$$\int_{\underline{v}}^{k} \Pr(x^{\mathrm{NTR}} \leq \tilde{k})\, d\tilde{k} = \sum_{t=\tau^{\mathrm{NTR}}(k)}^{\infty} (1-\delta)\delta^{t-1}(k - k_t^{\mathrm{NTR}}).$$

First assume that $\tau^{\mathrm{TR}}(k) \geq \tau^{\mathrm{NTR}}(k)$. Then

$$\begin{aligned}
&\sum_{t=\tau^{\mathrm{NTR}}(k)}^{\infty} (1-\delta)\delta^{t-1}(k-k_t^{\mathrm{NTR}}) - \sum_{t=\tau^{\mathrm{TR}}(k)}^{\infty} (1-\delta)\delta^{t-1}(k-k_t^{\mathrm{TR}}) \\
&\qquad = \sum_{t=\tau^{\mathrm{TR}}(k)}^{\infty} (1-\delta)\delta^{t-1}(k_t^{\mathrm{TR}}-k_t^{\mathrm{NTR}}) + \sum_{t=\tau^{\mathrm{NTR}}(k)}^{\tau^{\mathrm{TR}}(k)-1} (1-\delta)\delta^{t-1}(k-k_t^{\mathrm{NTR}}) \\
&\qquad \geq \sum_{t=\tau^{\mathrm{TR}}(k)}^{\infty} (1-\delta)\delta^{t-1}(k_t^{\mathrm{TR}}-k_t^{\mathrm{NTR}}) \\
&\qquad \geq 0,
\end{aligned}$$

where the first inequality is due to the fact that $k \geq k_t^{\mathrm{NTR}}$ for all $t \geq \tau^{\mathrm{NTR}}(k)$, and the last inequality is due to the price ranking. Now assume that $\tau^{\mathrm{TR}}(k) < \tau^{\mathrm{NTR}}(k)$. Then

$$\begin{aligned}
&\sum_{t=\tau^{\mathrm{NTR}}(k)}^{\infty} (1-\delta)\delta^{t-1}(k-k_t^{\mathrm{NTR}}) - \sum_{t=\tau^{\mathrm{TR}}(k)}^{\infty} (1-\delta)\delta^{t-1}(k-k_t^{\mathrm{TR}}) \\
&\qquad = \sum_{t=\tau^{\mathrm{NTR}}(k)}^{\infty} (1-\delta)\delta^{t-1}(k_t^{\mathrm{TR}}-k_t^{\mathrm{NTR}}) - \sum_{t=\tau^{\mathrm{TR}}(k)}^{\tau^{\mathrm{NTR}}(k)-1} (1-\delta)\delta^{t-1}(k-k_t^{\mathrm{TR}}) \\
&\qquad \geq \sum_{t=\tau^{\mathrm{NTR}}(k)}^{\infty} (1-\delta)\delta^{t-1}(k_t^{\mathrm{TR}}-k_t^{\mathrm{NTR}}) - \sum_{t=\tau^{\mathrm{TR}}(k)}^{\tau^{\mathrm{NTR}}(k)-1} (1-\delta)\delta^{t-1}(k_t^{\mathrm{NTR}}-k_t^{\mathrm{TR}}) \\
&\qquad = \sum_{t=\tau^{\mathrm{TR}}(k)}^{\infty} (1-\delta)\delta^{t-1}(k_t^{\mathrm{TR}}-k_t^{\mathrm{NTR}}) \\
&\qquad \geq 0,
\end{aligned}$$

where the first inequality follows because, by the definition of $\tau^{\mathrm{NTR}}(k)$ for $t < \tau^{\mathrm{NTR}}(k)$, we have $k \leq k_t^{\mathrm{NTR}}$, and the last inequality follows from the price ranking. □

The proof of Theorem 2 immediately follows from the following lemma.

Lemma 17. *If F is concave, then*

$$\sum_{t=1}^{\infty} \delta^{t-1} F(k_t^{\mathrm{TR}}) \geq \sum_{t=1}^{\infty} \delta^{t-1} F(k_t^{\mathrm{NTR}}). \tag{32}$$

Proof. Notice that for the random variables defined above,

$$\Pr(x^{\mathrm{TR}} = k_t^{\mathrm{TR}}) = \Pr(x^{\mathrm{NTR}} = k_t^{\mathrm{NTR}}) = \delta^{t-1} - \delta^t = (1-\delta)\delta^{t-1}.$$

Then the left- and right-hand sides of (32) are the expectation of $F(x^{\mathrm{TR}})/(1-\delta)$ and the expectation of $F(x^{\mathrm{NTR}})/(1-\delta)$, respectively. Then the claim follows by the second-order stochastic dominance. □

References

Abreu, Dilip and Faruk Gul (2000), "Bargaining and reputation." *Econometrica*, 68, 85–117. [359]

Ausubel, Lawrence M., Peter Cramton, and Raymond J. Deneckere (2002), "Bargaining with incomplete information." In *Handbook of Game Theory With Economic Applications*, Vol. 3 (Robert J. Aumann and Sergiu Hart, eds.), 1897–1946, North-Holland, Amsterdam. [344, 359]

Ausubel, Lawrence M. and Raymond J. Deneckere (1989a), "Reputation in bargaining and durable goods monopoly." *Econometrica*, 57, 511–531. [359]

Ausubel, Lawrence M. and Raymond J. Deneckere (1989b), "A direct mechanism characterization of sequential bargaining with one-sided incomplete information." *Journal of Economic Theory*, 48, 18–46. [359]

Ausubel, Lawrence M. and Raymond J. Deneckere (1993), "Efficient sequential bargaining." *Review of Economic Studies*, 60, 435–461. [351]

Bagwell, Kyle (1995), "Commitment and observability in games." *Games and Economic Behavior*, 8, 271–280. [344]

Bergemann, Dirk and Johannes Hörner (2010), "Should auctions be transparent?" Discussion Paper 1764, Cowles Foundation. [345]

Bulow, Jeremy and John Roberts (1989), "The simple economics of optimal auctions." *Journal of Political Economy*, 97, 1060–1090. [346]

Chatterjee, Kalyan and Larry Samuelson (1987), "Bargaining with two-sided incomplete information: An infinite horizon model with alternating offers." *Review of Economic Studies*, 54, 175–192. [359]

Cramton, Peter C. (1984), "Bargaining with incomplete information: An infinite-horizon model with two-sided uncertainty." *Review of Economic Studies*, 51, 579–593. [359]

Deneckere, Raymond and Meng-Yu Liang (2006), "Bargaining with interdependent values." *Econometrica*, 74, 1309–1364. [345]

Evans, Robert (1989), "Sequential bargaining with correlated values." *Review of Economic Studies*, 56, 499–510. [345]

Fuchs, William, Aniko Öry, and Andrzej Skrzypacz (2014), "Transparency and distressed sales under asymmetric information." Working paper, Stanford University. [344]

Fudenberg, Drew, David K. Levine, and Jean Tirole (1985), "Infinite-horizon models of bargaining with one-sided incomplete information." In *Game-Theoretic Models of Bargaining* (Alvin E. Roth, ed.), 73–98, Cambridge University Press, Cambridge. [343, 344, 349, 357, 368, 375]

Fudenberg, Drew, David K. Levine, and Jean Tirole (1987), "Incomplete information bargaining with outside opportunities." *Quarterly Journal of Economics*, 102, 37–50. [345]

Fudenberg, Drew and Jean Tirole (1991), *Game Theory*. MIT Press, Cambridge, Massachusetts. [346, 347, 360]

Gul, Faruk, Hugo Sonnenschein, and Robert Wilson (1986), "Foundations of dynamic monopoly and the Coase conjecture." *Journal of Economic Theory*, 39, 155–190. [343, 344, 347, 349]

Hörner, Johannes and Nicolas Vieille (2009), "Public vs. private offers in the market for lemons." *Econometrica*, 77, 29–69. [344, 345, 346, 349]

Kim, Kyungmin (2012), "Information about sellers' past behavior in the market for lemons." Working paper, University of Iowa. [345]

Nöldeke, Georg and Eric van Damme (1990), "Signalling in a dynamic labour market." *Review of Economic Studies*, 57, 1–23. [345]

Rubinstein, Ariel and Asher Wolinsky (1990), "Decentralized trading, strategic behaviour and the Walrasian outcome." *Review of Economic Studies*, 57, 63–78. [344]

Sobel, Joel and Ichiro Takahashi (1983), "A multistage model of bargaining." *Review of Economic Studies*, 50, 411–426. [349]

Stokey, Nancy L. (1981), "Rational expectations and durable goods pricing." *Bell Journal of Economics*, 12, 112–128. [349]

Swinkels, Jeroen M. (1999), "Education signalling with preemptive offers." *Review of Economic Studies*, 66, 949–970. [344, 345]

Topkis, Donald M. (1998), *Supermodularity and Complementarity*. Princeton University Press, Princeton, New Jersey. [372, 373]

Vincent, Daniel R. (1989), "Bargaining with common values." *Journal of Economic Theory*, 48, 47–62. [345]

Zhu, Haoxiang (2012), "Finding a good price in opaque over-the-counter markets." *Review of Financial Studies*, 25, 1255–1285. [342]

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