Theoretical Economics 11 (2016), 455–485 1555-7561/20160455

A model of price discrimination under loss aversion and state-contingent reference points

Juan Carlos Carbajal
School of Economics, University of New South Wales

Jeffrey C. Ely
Department of Economics, Northwestern University

We study optimal price discrimination when a monopolist faces a continuum of consumers with reference-dependent preferences. A consumer's valuation for product quality consists of an intrinsic valuation affected by a private state signal (type) and a gain–loss valuation that depends on deviations of purchased quality from a reference point. Following Kőszegi and Rabin (2006), we consider loss-averse buyers who evaluate gains and losses in terms of changes in the consumption valuation, but in our model each buyer evaluates consumption outcomes relative to his own state-contingent reference quality level. We capture the process by which reference qualities are formed via a reference consumption plan, and use a generalization of the Mirrlees representation of the indirect utility to fully characterize optimal contracts for loss-averse consumers. We find that, depending on the reference plan, optimal price discrimination may exhibit (i) downward distortions beyond the standard downward distortions due to screening, (ii) efficiency gains relative to second-best contracts without loss aversion, and (iii) upward distortions above first-best quality levels without loss aversion. We consider ex ante and ex post consistent contracts in which quality offers by the firm coincide, in expectations or at every state realization, respectively, with the reference quality levels. We find the firm's unique preferred ex ante and ex post consistent contract menu and specify conditions under which, for the second case, it also constitutes the consumers' preferred menu.

Keywords. Price discrimination, product line design, loss aversion, reference-dependent preferences, reference consumption plan, self-confirming contracts.

JEL classification. D03, D44, D81, D82.

Juan Carlos Carbajal: jc.carbajal@unsw.edu.au
Jeffrey C. Ely: jeffely@northwestern.edu
This paper was previously circulated as "Optimal Contracts for Loss-Averse Consumers." We wish to thank a co-editor and the referees for very helpful comments. Thanks also to Jürgen Eichberger, Paul Heidhues, Fabian Herweg, Heiko Karle, Jihong Lee, Konrad Mierendorff, Carlos Pimienta, Ludovic Renou, Antonio Rosato, Klaus M. Schmidt, and seminar participants at ANU, CORE, Maastricht University, Humboldt University–Berlin, Universities of Zurich, Heidelberg, Munich, and New South Wales, NASM–ES 2012, AETW 2012, SITE 2012, and AMES 2013. Early work was conducted while Carbajal visited the University of Bonn; he wishes to thank Benny Moldovanu for his gracious hospitality. Research assistance was provided by Alexandra Bratanova and Kim Nguyen. Financial support from the Australian Research Council (Carbajal) and the National Science Foundation (Ely) is gratefully acknowledged.

DOI: 10.3982/TE1737

1. Introduction

The purpose of this paper is to study monopoly price discrimination in situations where buyers care about comparisons between consumption outcomes and subjective beliefs about these outcomes, which act as reference points. In our model, there is a one-dimensional state parameter θ that affects demand for quality and expectations of future consumption.[1] More precisely, a consumer's utility is determined by his intrinsic quality taste *and* by comparisons between the offered quality and a reference quality level, and *both* are affected by θ. The way we allow the state parameter to determine willingness to pay is standard (for instance, single crossing is satisfied). The interaction between θ and the reference point is captured by a *reference consumption plan*: after observing θ, a buyer anticipates a certain state-contingent reference quality level and experiences gains or losses according to whether his purchased quality exceeds or falls short of his reference point. Reference plans are assumed to be nondecreasing. Thus, a higher intrinsic valuation for quality is associated with a higher anticipated quality outcome.[2] Following Tversky and Kahneman (1991), we assume that consumers are loss-averse. Price discrimination takes reference dependence into account and reflects the interaction between loss aversion and the traditional rent extraction and incentive compatibility trade-off. We are able to derive the optimal contract menu for any monotone reference plan. Our approach enables comparative statics analysis of the offered product line and profits arising from changes in the reference plan, for example, due to targeted advertising.

In Section 3 we analyze the benchmark model under complete information. We show how loss aversion invites upward distortions in the offered quality relative to the loss-neutral case. This occurs, in particular, when buyers enter the market with high reference quality levels: the first-best quality would fall short of the reference level and the loss-averse consumer is willing to pay a premium to reduce the associated loss. The firm exploits this by increasing both quality and price until either these marginal gains are exhausted or the offered quality hits the reference level, shutting down any further gains. In particular, over a wide range of states, profit maximization implies matching reference qualities exactly. A similar logic drives the comparative statics results. If the monopolist is able to inflate consumers' reference levels, the effect would magnify upward distortions and increase profits. While an empirical demonstration of inefficiently high quality offers is challenging, our predictions could be used to indirectly test for loss aversion in the laboratory: all else equal, loss-averse consumers respond to higher reference consumption points differently than loss-neutral consumers.[3]

In Section 4 we turn to the incomplete information case. A contract menu is now feasible if and only if it satisfies the incentive compatibility and participation constraints. Two new effects emerge, compared to the complete information benchmark. First, the

[1] We build on classic monopoly pricing models under asymmetric information developed by Mussa and Rosen (1978) and Maskin and Riley (1984).

[2] Note we allow for all consumers to share a common reference quality level. Alternatively, monotone reference plans restrict how wrong buyers are allowed to be. A consumer with a low signal, and consequently low intrinsic valuation, expects to buy as much quality as, but not more than, a consumer with a high signal.

[3] These predictions are consistent with the findings of Heyman et al. (2004).

marginal profitability of increased quality is reduced due to incentive issues familiar from traditional screening models. Higher quality levels are more attractive to a θ consumer with low willingness to pay, but also to higher type consumers to whom the monopolist was hoping to sell an even higher quality product. The increase in revenues from the θ consumer are offset by information rents ceded to these consumers. Reference dependence modifies this standard trade-off because for any given quality and reference levels, the higher type consumer experiences a larger loss (or smaller gain) than the θ consumer. Thus, it is possible for the firm to increase profits by expanding its product line to both high and low ends of the market, in response to consumers' high expectations. This loss aversion effect can account for three part tariffs and other complex contract schemes that have become increasingly popular among mobile phone operators, Internet providers, and other subscription services,[4] when, for instance, low-type consumers overestimate usage prior to choosing a contract.

The second effect is a novel distortion with no counterpart in a loss-neutral screening model. Consider a monopolist contemplating increasing the quality q offered to a given θ consumer from just below his reference level to just above it. Such a change has a discrete effect on the attractiveness of q to consumers who have higher willingness to pay, but whose reference levels are relatively similar, because they would switch from the loss to the gains domain. As a result, the firm incurs a discrete drop in profits. We quantify this *lump-sum* incentive cost and show its effects on the optimal contract design. It implies an additional downward distortion in quality levels to consumers who would otherwise be offered products that surpass their reference points, making the interaction between reference quality levels and offered qualities quite complex.

The fact that the reference plan determines qualitative features of optimal contracts leads to the question of belief manipulation. This issue is explored in Section 5, where we impose ex ante and ex post consistency requirements on beliefs about future consumption outcomes. A constant reference plan is ex ante consistent if it coincides, in expectations, with actual purchased quality levels. There are many ex ante consistent reference plans. We find that the firm has a unique preferred ex ante consistent contract menu that is generated by the largest ex ante consistent reference plan. A monotone reference plan is ex post consistent if it coincides pointwise (i.e., for every state realization) with actual quality consumption. As in the previous case, there are many possible ex post consistent reference plans. Here we also show that the firm has a unique preferred ex post consistent contract menu. Intuitively, a higher (ex ante) ex post consistent reference plan increases the net total willingness to pay, as it makes the outside option less desirable. In both cases, preferred consistent contracts exclude fewer consumers from the market and have quality levels distorted upward from second-best levels under loss neutrality. While these upward quality distortions improve allocative efficiency for low- and intermediate-type consumers, under ex post consistent contracts buyers with high state parameters end up purchasing overly sophisticated goods.

In practice, firms can manipulate reference points through advertising and other marketing practices.[5] Marketing efforts will be credible as long as they promote optimal

[4] See, for example, Lambrecht and Skiera (2006) and Lambrecht et al. (2007).

[5] Product shows in anticipation of market entrance are standard practice in luxury goods, cars, and consumer electronics industries.

consistent contracts. A higher ex ante or ex post consistent reference plan could improve or hurt consumer's surplus—note that we are comparing how different consistent reference plans affect loss -averse consumers. This is because in either case a higher reference plan implies higher quality offers for (potentially more) active consumers, which translates into more information rents to buyers with positive consumption levels, but it also implies a lower value of the outside option, which increases the net willingness to pay of active consumers.[6] We are able to specify conditions under which the positive information rents effect associated with a higher ex post consistent reference plan dominates the negative participation effect. In this case, the firm's preferred self-confirming contract menu is also the consumer's preferred contract menu.

Section 6 offers some concluding remarks and a review of the related literature. To facilitate exposition, we have gathered all proofs in Section 7.

2. Price discrimination under reference-dependent preferences

Our model builds on Mussa and Rosen (1978) and Maskin and Riley (1984), but in our framework consumers derive utility from consumption *and* from comparisons between consumption and a state-contingent reference point. Following Tversky and Kahneman (1991), we consider loss-averse consumers.

2.1 *The firm*

A profit-maximizing monopolist produces a good of different characteristics captured by the parameter $q \geq 0$. This can be interpreted as either a one-dimensional measure of quality (exclusive features in a luxury product line) or quantity (amount of data offered by a mobile operator). Paying tribute to Mussa and Rosen (1978), we maintain the first interpretation.

The cost of producing one unit of the good with quality q is $c(q) \geq 0$. We assume that the *cost function* $c(\cdot)$ defined on $\mathbb{R}_+$ is (F1) increasing, with $c(0) = 0$, (F2) twice continuously differentiable, and (F3) strongly convex, i.e., there exists $\epsilon > 0$ such that $c''(q) \geq \epsilon$ for all $q > 0$. The firm's problem is to design an optimal menu of posted quality–price pairs for potential buyers with differentiated demands.

2.2 *Consumers*

There is a continuum of consumers with quasi-linear preferences and unit demands for the good offered by the firm. Preference heterogeneity depends on the private type parameter $\theta \in \Theta = [\theta_L, \theta_H]$, where $0 \leq \theta_L < \theta_H < +\infty$. The firm only knows the distribution $F(\cdot)$, with full support on Θ and positive density $f(\cdot)$. We assume that the *inverse hazard rate* $h(\cdot)$ defined by

$$h(\theta) = (1 - F(\theta))/f(\theta)$$

[6]We thank an anonymous referee for pointing out this effect, which was previously omitted from our analysis.

is nonincreasing and continuously differentiable.

The *consumption valuation* is a function $m: \mathbb{R}_+ \times \Theta \to \mathbb{R}$. To avoid any complication arising from the classical screening framework, we impose the following regularity assumptions: (C1) $m(\cdot, \cdot)$ is thrice continuously differentiable; (C2) for all $\theta \in \Theta$, $\theta > \theta_L$, the function $m(\cdot, \theta)$ is strictly increasing and concave, with $m(0, \theta) = 0$, moreover $m(\cdot, \theta_L)$ is everywhere equal to zero; (C3) for every $q \geq 0$, $m(q, \cdot)$ is increasing; (C4) for all $q > 0$ and all $\theta \in \Theta$, $\theta > \theta_L$, single crossing holds, so that $\partial^2 m(q, \theta)/\partial q \, \partial\theta > 0$; (C5) for all $q \geq 0$ and $\theta \in \Theta$,

$$\partial^3 m(q, \theta)/\partial q \, \partial\theta^2 \leq 0 \quad \text{and} \quad \partial^3 m(q, \theta)/\partial q^2 \, \partial\theta \geq 0,$$

and finally (C6) for all $\theta \in \Theta$, $\theta > \theta_L$,

$$\lim_{q \to 0} \left(\frac{\partial m(q, \theta)}{\partial q} - c'(q) \right) > 0 \quad \text{and} \quad \lim_{q \to +\infty} \left(\frac{\partial m(q, \theta)}{\partial q} - c'(q) \right) < 0.$$

This specification includes, among others, Mussa and Rosen's (1978) monopoly model with linear consumption valuation and quadratic costs.[7] It ensures that the quality schedule that maximizes consumption surplus (virtual consumption surplus) is continuously differentiable and increasing on Θ.

2.3 *Loss aversion*

We step outside standard monopoly pricing theory to consider buyers who exhibit reference-dependent preferences for the product attribute. Specifically, a θ consumer derives additional utility from comparing q to a type-contingent reference quality level $r(\theta) \geq 0$. A reference level may reflect (in)correct subjective expectations of future consumption, may be determined by past experiences, or determined by current aspirational considerations. At this stage, it is convenient to study a general reference formation process captured by the *reference consumption plan* $r: \Theta \to \mathbb{R}_+$. We assume that $r(\cdot)$ is (R1) increasing, (R2) continuous and piecewise continuously differentiable, and (R3) admits bounded left and right derivatives everywhere on Θ.

Following Kőszegi and Rabin (2006), comparisons between consumption outcomes and reference points are evaluated in terms of the consumption valuation. Note that we depart from their work in assuming that each buyer assesses q relative to his own state-contingent reference level. The θ consumer has a *gain–loss valuation* given by

$$\mu \times \big(m(q, \theta) - m(r(\theta), \theta)\big),$$

where $\mu = \eta$ if $q > r(\theta)$ and $\mu = \eta\lambda$ if $q \leq r(\theta)$. The parameter $\eta > 0$ is the weight attached to the gain–loss valuation, and $\lambda \geq 1$ is the loss aversion coefficient. Loss neutrality ($\lambda = 1$) is treated as the baseline scenario. The total valuation for the θ consumer is then

$$m(q, \theta) + \mu \times \big(m(q, \theta) - m(r(\theta), \theta)\big).$$

[7] One can accommodate Maskin and Riley's (1984) model by changing (C2) and (F3) to impose strong concavity on $m(\cdot, \theta)$ and convexity on $c(\cdot)$.

A buyer's outside option consists of purchasing a substitute good of minimal quality in a secondary market. For simplicity, we let both quality and price of the substitute good be equal to zero, so that θ consumer's reservation utility equals $-\eta\lambda m(r(\theta),\theta)$. Note this means buying the outside option feels like a loss.[8] His *net total valuation* is then

$$v(q,\theta) = (1+\mu)m(q,\theta) + (\eta\lambda - \mu)m(r(\theta),\theta) \qquad \text{with } \mu = \eta \text{ if } q > r(\theta),\ \mu = \eta\lambda \text{ if } q \le r(\theta). \tag{1}$$

Presented with a contract to buy quality q at price p, a θ consumer chooses to do so as long as his net total utility $v(q,\theta) - p$ is nonnegative. This constitutes the (endogenous) participation constraint.

2.4 *Comment*

We provide the following interpretation of our framework. There is a mass of ex ante identical consumers interacting with the firm on a given time period. Prior to entering the market, consumers share a common reference plan based on (correct or incorrect) subjective beliefs about future consumption outcomes. Each consumer then receives a state signal that affects his willingness to pay and fixes his reference level according to the common reference plan. The firm only knows the distribution of signals. Thus, the firm designs a menu of individually rational and incentive compatible contracts $\{q(\theta), p(\theta)\}_{\theta\in\Theta}$ to maximize expected profits. Once contracts are posted, each buyer evaluates the quality offer of any contract relative to his type-contingent reference level and makes purchasing decisions accordingly.

The consumption valuation, the gain–loss valuation, and the reference plan are common knowledge; in other words, the firm is fully aware of the consumers' behavioral bias. We take a partial equilibrium approach and ignore any budgetary restriction on consumer's behavior. Insofar as the total willingness to pay of loss-averse consumers is influenced by the reference plan, we focus on allocative efficiency alone when discussing welfare implications of loss aversion, considering loss neutrality as the baseline scenario.

3. Price discrimination under complete information

Fix a reference plan $r(\cdot)$. Let $q(\theta)$ and $p(\theta)$ be the quality and price offered to the θ consumer. When types are observable, the firm sets $p(\theta) = v(q(\theta),\theta)$ and earns profits equal to the per-customer *total surplus*

$$\mathrm{TS}(q(\theta),\theta) = (1+\mu(\theta))m(q(\theta),\theta) + (\eta\lambda - \mu(\theta))m(r(\theta),\theta) - c(q(\theta)) \qquad \text{with } \mu(\theta) = \eta \text{ if } q(\theta) > r(\theta),\ \mu(\theta) = \eta\lambda \text{ otherwise.} \tag{2}$$

[8] We are assuming here that the realization of a state, and consequent anticipation of consumption, creates a "pseudo-endowment effect"; see Ariely and Simonson (2003).

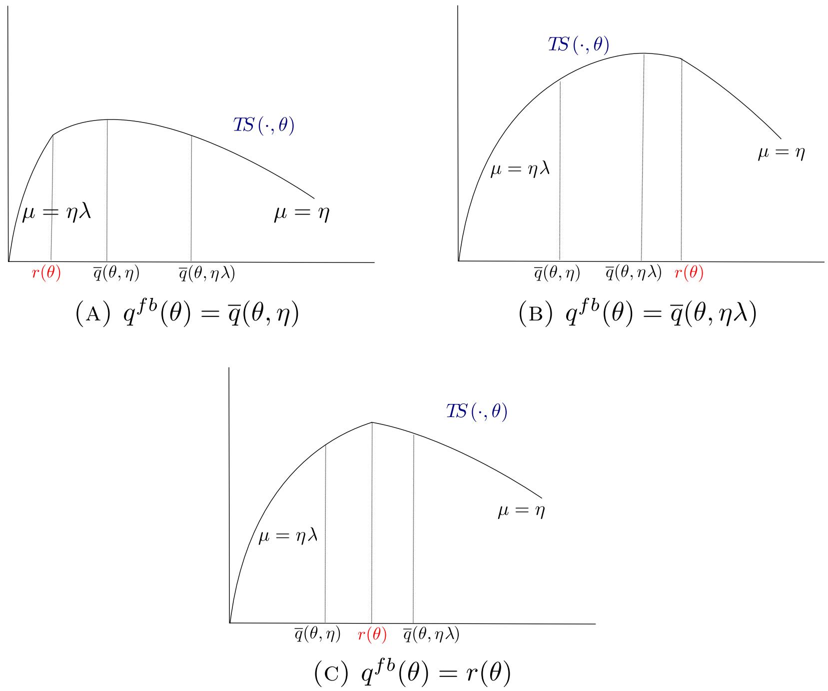


FIGURE 1. The total surplus function $\mathrm{TS}(\cdot, \theta)$.

To find the profit-maximizing quality offer, define for $\theta \in \Theta$ and $\mu \in \{\eta, \eta\lambda\}$,

$$S(q, \theta, \mu) \equiv (1+\mu)m(q, \theta) - c(q)$$

$$\bar{q}(\theta, \mu) \equiv \arg\max_{q \geq 0} S(q, \theta, \mu).$$

Our assumptions imply that $\bar{q}(\cdot, \mu)$ is continuously differentiable and strictly increasing. In particular, since $m(\cdot, \theta)$ is strictly increasing in q for all $\theta > \theta_L$,

$$0 < \bar{q}(\theta, \eta) < \bar{q}(\theta, \eta\lambda) \quad \text{for all } \theta \in \Theta, \theta > \theta_L.$$

When $\lambda = 1$, total surplus in (2) is given by $S(q, \theta, \eta)$. In this case, the first-best monopoly offer is $\bar{q}(\theta, \eta)$, independently of $r(\theta)$. We interpret $\bar{q}(\theta, \eta)$ as the classic efficient (i.e., surplus maximizing) quality level under loss neutrality.

The solution of the firm's problem when $\lambda > 1$ is obtained by noticing that $\mathrm{TS}(q(\theta), \theta)$ coincides with $S(q(\theta), \theta, \eta\lambda)$ when $q(\theta) \leq r(\theta)$ and with a constant-shifted $S(q(\theta), \theta, \eta)$ when $q(\theta) > r(\theta)$. Since $S(\cdot, \theta, \eta)$ has a strictly smaller slope than $S(\cdot, \theta, \eta\lambda)$ for all $\theta > \theta_L$, the total surplus function exhibits a kink at $q = r(\theta)$. The quality level that maximizes profits is determined by the location of the kink relative to the two maximizers $\bar{q}(\theta, \eta)$ and $\bar{q}(\theta, \eta\lambda)$. Figure 1 provides an illustration. If $r(\theta)$ is below $\bar{q}(\theta, \eta)$, profits are increasing at the kink point and the firm chooses $\bar{q}(\theta, \eta)$—Figure 1(A). If $r(\theta)$ is above $\bar{q}(\theta, \eta\lambda)$, profits are decreasing at the kink point and the firm chooses $\bar{q}(\theta, \eta\lambda)$—Figure 1(B). If $r(\theta)$ lies in intermediate ranges, any deviation from the reference level hurts profits—Figure 1(C).

Proposition 1. *For reference plan $r(\cdot)$ and $\lambda > 1$, the complete information optimal contract menu $\{q^{fb}(\theta), p^{fb}(\theta)\}_{\theta\in\Theta}$ is given by*

$$q^{fb}(\theta) = \begin{cases} \bar{q}(\theta, \eta\lambda) & \text{if } r(\theta) > \bar{q}(\theta, \eta\lambda) \\ r(\theta) & \text{if } \bar{q}(\theta, \eta\lambda) \geq r(\theta) \geq \bar{q}(\theta, \eta) \\ \bar{q}(\theta, \eta) & \text{if } \bar{q}(\theta, \eta) > r(\theta) \end{cases}$$

and

$$p^{fb}(\theta) = (1 + \mu(\theta))m(q^{fb}(\theta), \theta) + (\eta\lambda - \mu(\theta))m(r(\theta), \theta)$$

$$\text{with } \mu(\theta) = \eta \text{ if } q^{fb}(\theta) > r(\theta), \mu(\theta) = \eta\lambda \text{ otherwise.}$$

Proposition 1 shows the effects of loss aversion on price discrimination in the absence of screening issues. For a large reference quality level, the optimal quality is in the domain of losses and the firm exploits consumer's loss aversion by increasing its offer from the classic efficient level to $\bar{q}(\theta, \eta\lambda)$. For a low reference level, the firm's optimal quality will be in the domain of gains and therefore coincides with the loss-neutral case. The reference plan entirely determines the shape of first-best quality offers for intermediate ranges. Since the reference consumption plan may in principle be very general, first-best contracts can take various shapes. In particular, a constant reference plan generates *pooling* in the intermediate range of the first-best contracts. To understand these results better we spell out the comparative statics effects of a change in the reference level.

Proposition 2. *The following statements hold under complete information for $\lambda > 1$.*

(i) Optimal quality offers are weakly greater than the loss-neutral efficient levels, and strictly greater when $r(\theta) > \bar{q}(\theta, \eta)$ and $\theta > \theta_L$.

(ii) An increase in the reference level weakly increases the firm's profits. The effect is strict whenever $\bar{q}(\theta, \eta\lambda) > r(\theta)$ and $\theta > \theta_L$.

(iii) An increase in the reference level weakly increases the firm's quality offer; the effect is strict whenever $\bar{q}(\theta, \eta\lambda) > r(\theta) \geq q(\theta, \eta)$ and $\theta > \theta_L$.

The key observation behind Proposition 2 is that a change in the reference level affects how loss-averse consumers evaluate not only the contracted quality, but also the outside option. When $r(\theta) < \bar{q}(\theta, \eta) = q^{fb}(\theta)$, the θ consumer compares his outside option in the loss domain with the firm's offer in the gains domain. An increase in $r(\theta)$ has countervailing effects: it increases the loss associated with the outside option but reduces the gain associated with the contract. The net effect expands the relative attractiveness of the contract: the quality offer is unchanged, but the consumer's willingness to pay, hence the optimal price, increases by

$$(\eta\lambda - \eta)\, \partial m(r(\theta), \theta)/\partial q.$$

As soon as the reference quality exceeds the loss-neutral efficient level, the latter and the outside option are in the loss domain and any further increase in the reference point reduces the value of both equally. However, since total surplus in the loss domain rises more steeply, there are larger surplus gains from quality. The firm captures these direct gains by increasing quality up to the reference level, which raises profits by

$$(1 + \eta\lambda)\, \partial m(r(\theta), \theta)/\partial q - c'(q).$$

Once the reference quality exceeds $\bar{q}(\theta, \eta\lambda)$, all gains from loss aversion have been exhausted and the firm's offered quality and price are unaffected by further increases in reference levels. Note that none of these results holds for $\lambda = 1$. In the loss neutrality case, the optimal contract for the θ consumer consists of quality $\bar{q}(\theta, \eta)$ at price $(1 + \eta)m(\bar{q}(\theta, \eta), \theta)$, independently of $r(\theta)$.

4. Price discrimination under incomplete information

We now study optimal contract design when the realization of the state signal is private information. For the remainder of this section, we fix a reference consumption plan $r(\cdot)$ and derive the optimal contract menu induced by it. In Section 5 we focus on consistent reference plans.

4.1 *The design problem*

Given $r(\cdot)$, the firm's problem is to choose a menu of quality–price schedules $\{q(\theta), p(\theta)\}_{\theta\in\Theta}$ that maximizes expected profits

$$\Pi^{sb} = \int_{\theta_L}^{\theta_H} \big\{p(\theta) - c(q(\theta))\big\}\, dF(\theta)$$

subject to the incentive compatibility constraints

$$v(q(\theta), \theta) - p(\theta) \geq v(q(\hat{\theta}), \theta) - p(\hat{\theta}) \quad \text{for all } \theta, \hat{\theta} \in \Theta \tag{3}$$

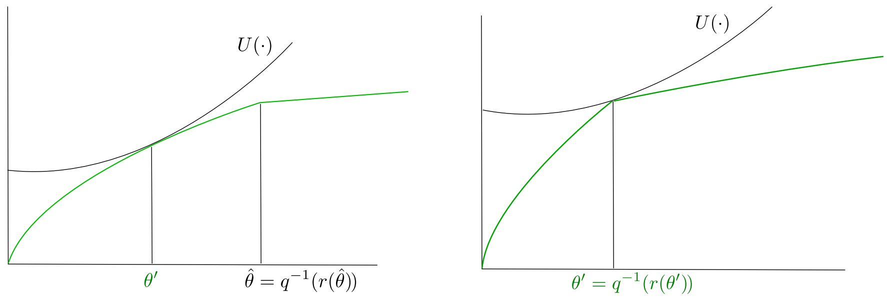
and the individual rationality constraints

$$v(q(\theta), \theta) - p(\theta) \geq 0 \quad \text{for all } \theta \in \Theta.$$

A contract menu $\{q(\theta), p(\theta)\}_{\theta\in\Theta}$ that satisfies both constraints is said to be *incentive feasible*. When there is no risk of confusion, we denote by

$$U(\theta) = v(q(\theta), \theta) - p(\theta)$$

the indirect utility from an incentive feasible contract generated by $r(\cdot)$.



(A) Determinacy (B) Indeterminacy

Figure 2. Standard techniques based on the envelope theorem may fail.

From (1), observe that the value of the gain–loss coefficient μ takes in each side of (3) may differ, as it depends on comparison of $q(\theta)$ with $r(\theta)$ on the left-hand side and comparison of $q(\hat\theta)$ with $r(\theta)$ on the right-hand side. Let $r(\theta) = q(\theta)$ and assume $r(\cdot)$ is strictly increasing around θ. Then μ will change depending on whether $q(\theta)$ is selected by a lower type consumer who experiences a gain relative to his lower reference point, or a higher type consumer who experiences a loss relative to her higher reference point. This sudden variation in the valuation complicates the application of standard contract theoretic techniques, based on the integral representation of incentive compatibility, to characterize incentive feasible contracts. Figure 2 illustrates the source of the problem.

Given an incentive feasible menu, $U(\theta)$ represents the maximum utility the θ consumer can obtain among all of the available options. Therefore, when we consider any particular bundle $(q(\theta'), p(\theta'))$ and plot the mapping

$$v(q(\theta'), \cdot) - p(\theta'),$$

the indirect utility $U(\cdot)$ must lie everywhere above it, and coincide with it at $\theta = \theta'$. When, as in Figure 2(A), $v(q(\theta'), \cdot)$ has partial derivative at θ', this pins down the derivative of the indirect utility. If this is true almost everywhere, then by the envelope theorem these partial derivatives can be integrated to recover $U(\cdot)$. However, when $q(\theta') = r(\theta')$, the mapping $v(q(\theta'), \cdot)$ exhibits a kink at the point $\theta = \theta'$ and this can lead to an indeterminacy, as in Figure 2(B).[9]

Alternatively, $v(q, \cdot)$ has bounded left and right partial derivatives at each $\theta \in \Theta$, denoted, respectively, by

$$\partial v^-(q, \theta)/\partial\theta \quad \text{and} \quad \partial v^+(q, \theta)/\partial\theta.$$

For each $q \geq 0$, we define the correspondence $\varphi(q, \cdot)$ on Θ by

$$\varphi(q, \theta) \equiv \{\delta \in \mathbb{R} \mid \partial v^+(q, \theta)/\partial\theta \leq \delta \leq \partial v^-(q, \theta)/\partial\theta\}.$$

[9]The reference plan is piecewise continuously differentiable, hence we omit discussion of kinks in the valuation due to kinks in $r(\cdot)$. This is inconsequential for the optimal contract derivation.

When $r(\theta)$ does not coincide with q, the partial derivative $\partial v(q,\theta)/\partial\theta$ exists. Hence $\varphi(q,\theta)$ is single-valued and given by

$$\varphi(q,\theta)=\begin{cases}(1+\eta\lambda)\frac{\partial m(q,\theta)}{\partial\theta} & \text{if } q<r(\theta)\\ (1+\eta)\frac{\partial m(q,\theta)}{\partial\theta}+(\eta\lambda-\eta)\frac{d}{d\theta}(m(r(\theta),\theta)) & \text{if } q>r(\theta).\end{cases} \tag{4}$$

When $r(\theta)=q$ and $r(\cdot)$ is strictly increasing at θ, $\varphi(q,\theta)$ is a closed, bounded interval given by (see Section 7 for details)

$$\varphi(q,\theta)=\left[(1+\eta\lambda)\frac{\partial m(q,\theta)}{\partial\theta},(1+\eta)\frac{\partial m(q,\theta)}{\partial\theta}+(\eta\lambda-\eta)\frac{d}{d\theta}(m(r(\theta),\theta))\right]. \tag{5}$$

Because product quality is a choice variable, profit maximization may dictate setting $q(\theta)=r(\theta)$ for a subset of consumers of positive measure—this, for instance, happens in the case of ex post consistent reference plans analyzed in Section 5.2. It follows that in equilibrium $\varphi(q(\theta),\theta)$ may be multivalued and given by (5) on a nonnegligible subset of Θ. We therefore characterize incentive feasible contracts based on an integral monotonicity condition and a generalization of the Mirrlees representation of the indirect utility.[10] Given a (measurable) quality schedule $q:\Theta\to\mathbb{R}_+$, its associated correspondence $\varphi(q(\cdot),\cdot)$ is non-empty-valued, closed-valued, bounded, and measurable. Thus, it admits integrable selections, which we denote by $\delta(q(\cdot),\cdot)$.

Proposition 3. *The menu $\{q(\theta),p(\theta)\}_{\theta\in\Theta}$ with associated indirect utility $U(\cdot)$ is incentive feasible if and only if there exists an integrable selection $\delta(q(\cdot),\cdot)$ of the correspondence $\varphi(q(\cdot),\cdot)$ for which the following conditions are satisfied.*

(a) Integral monotonicity: for all $\theta',\theta''\in\Theta$,

$$v(q(\theta''),\theta'')-v(q(\theta''),\theta')\geq\int_{\theta'}^{\theta''}\delta(q(\tilde\theta),\tilde\theta)\,d\tilde\theta\geq v(q(\theta'),\theta'')-v(q(\theta'),\theta').$$

(b) Generalized Mirrlees representation: for all $\theta\in\Theta$,

$$U(\theta)=U(\theta_L)+\int_{\theta_L}^{\theta}\delta(q(\tilde\theta),\tilde\theta)\,d\tilde\theta.$$

(c) Participation of the θ_L consumer: $U(\theta_L)\geq 0$.

We employ Proposition 3 to reformulate the firm's objective function in terms of a generalized virtual surplus. Ignoring momentarily the restrictions imposed by integral monotonicity, from the participation constraint one has $U(\theta_L)=0$ in equilibrium. The generalized Mirrlees equation yields

$$p(\theta)=v(q(\theta),\theta)-\int_{\theta_L}^{\theta}\delta(q(\tilde\theta),\tilde\theta)\,d\tilde\theta\quad\text{for all }\theta\in\Theta. \tag{6}$$

[10] See Carbajal and Ely (2013) for a general characterization of incentive compatible mechanisms when the valuation function fails to be convex or differentiable in types.

Denote by $\mu(\theta)$ the value of the gain–loss coefficient when the θ consumer selects the bundle $(q(\theta), p(\theta))$. From (5) and (6) it is clear that the firm uses the smallest possible selection, namely

$$\delta(q(\theta),\theta) = (1+\mu(\theta))\frac{\partial m(q(\theta),\theta)}{\partial\theta} + (\eta\lambda - \mu(\theta))\frac{d}{d\theta}(m(r(\theta),\theta)) \quad \text{with } \mu(\theta)=\eta \text{ if } q(\theta) > r(\theta) \text{ and } \mu(\theta)=\eta\lambda \text{ otherwise.} \tag{7}$$

Using (7) in (6), replacing the resulting equation in the expression for expected profits, and integrating by parts, we obtain expected profits in terms of the *virtual consumption valuation* $m^*(q,\theta) = m(q,\theta) - h(\theta)\,\partial m(q,\theta)/\partial\theta$:

$$\Pi^{sb} = \int_{\theta_L}^{\theta_H}\Big\{(1+\mu(\theta))m^*(q(\theta),\theta) + (\eta\lambda-\mu(\theta))m^*(r(\theta),\theta) - c(q(\theta)) - (\eta\lambda-\mu(\theta))h(\theta)\frac{\partial m(r(\theta),\theta)}{\partial q}r'(\theta)\Big\}\,dF(\theta).$$

The first line in the integrand of the above equation is the *virtual total surplus* from the θ consumer and is denoted accordingly by $\mathrm{TS}^*(q(\theta),\theta)$. It expresses the trade-off between marginal and inframarginal revenues that the monopolist faces when increasing the quality allocated to this buyer. The second line, which we denote by $\mathrm{LS}(q(\theta),\theta)$, captures a novel effect in optimal contract design due to loss aversion. Write the firm's objective function as

$$\Pi^{sb} = \int_{\theta_L}^{\theta_H}\big\{\mathrm{TS}^*(q(\theta),\theta) - \mathrm{LS}(q(\theta),\theta)\big\}\,dF(\theta). \tag{8}$$

The next step of the analysis is to understand the trade-offs that stem from the interaction between the two components of the firm's profits.

4.2 *The optimal contract menu*

The monopolist's problem is to find a quality schedule $q(\cdot)$ that maximizes expected profits in (8), subject to the integral monotonicity condition.[11] We solve it in a way that parallels the complete information case to illuminate the new aspects arising from loss aversion. Define for $\theta\in\Theta$ and $\mu\in\{\eta,\eta\lambda\}$,

$$S^*(q,\theta,\mu) \equiv (1+\mu)m^*(q,\theta) - c(q)$$
$$q^*(\theta,\mu) \equiv \arg\max_{q\geq 0} S^*(q,\theta,\mu).$$

Our assumptions ensure that $S^*(\cdot,\theta,\mu)$ is strongly concave. Moreover, $q^*(\cdot,\mu)$ is continuously differentiable—except possibly at a type at which it turns from zero into

[11]Verifying part (a) of Proposition 3 is complicated by the fact that the optimal selection changes depending on whether the quality offer is greater or less than the reference level. We defer this step entirely to Section 7.

positive—and strictly increasing when it attains positive values. Also

$$0 \le q^*(\theta, \eta) \le q^*(\theta, \eta\lambda) \quad \text{for all } \theta \in \Theta,$$

with the last inequality strict for all types for which $q^*(\theta, \eta\lambda)$ is strictly positive.

Analogously to the complete information setting (cf. Figure 1), $\mathrm{TS}^*(q, \theta)$ coincides with $S^*(q, \theta, \eta\lambda)$ when $q \le r(\theta)$ and with an appropriate shift of $S^*(q, \theta, \eta)$ when $q > r(\theta)$. In particular, for a fixed $\theta \in \Theta$, $\mathrm{TS}^*(\cdot, \theta)$ is continuous in q but kinked at $q = r(\theta)$, and achieves its maximum at one of three points

$$q^*(\theta, \eta\lambda), \qquad q^*(\theta, \eta), \quad \text{or} \quad r(\theta),$$

depending on the position of $r(\theta)$ relative to $q^*(\theta, \eta)$ and $q^*(\theta, \eta\lambda)$. Therefore, a maximization based on the surplus component of the firm's objective function develops similarly to the complete information case, with the virtual total surplus accounting for screening-based incentive effects on top of the loss aversion effects.

To gain insights on the second component of the firm's objective, notice that

$$\mathrm{LS}(q, \theta) = \begin{cases} (\eta\lambda - \eta)h(\theta)\frac{\partial m(r(\theta), \theta)}{\partial q} r'(\theta) & \text{if } q > r(\theta) \\ 0 & \text{if } q \le r(\theta). \end{cases} \tag{9}$$

Thus, $\mathrm{LS}(q, \theta)$ represents a *lump-sum cost* incurred whenever the firm's offer exceeds the consumer's reference point. Increasing $q(\theta)$ above $r(\theta)$ moves θ's valuation from the loss domain to the gains domain, and this creates additional costs because the $\hat{\theta}$ consumer, whose reference level $r(\hat{\theta})$ is above $r(\theta)$ but below $q(\theta)$, now views offer $q(\theta)$ as a gain instead of a loss. It follows that the value this consumer attaches to quality offer $q(\theta)$ suffers a discrete change, measured by

$$(\eta\lambda - \eta)m(r(\hat{\theta}), \hat{\theta}).$$

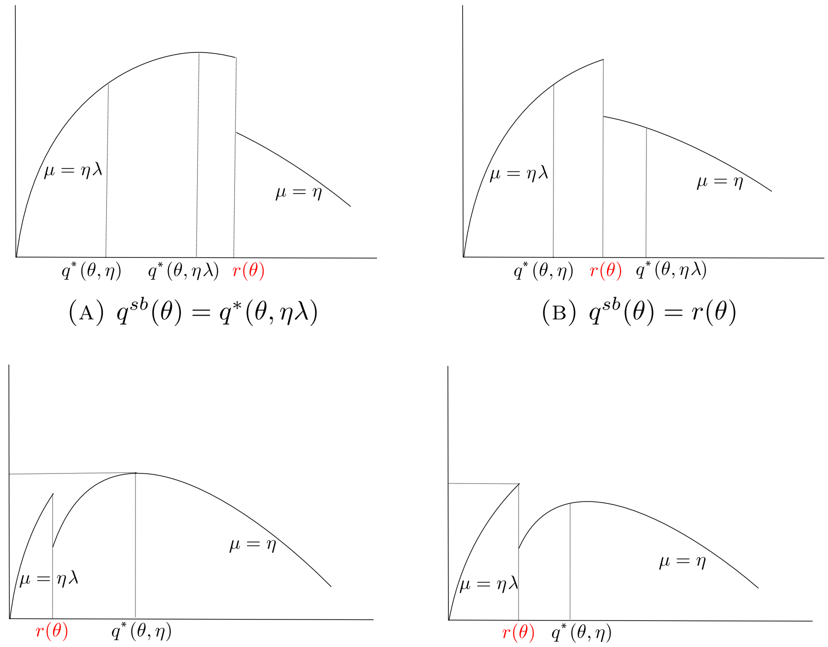
Thus, in equilibrium, $\mathrm{LS}(q(\theta), \theta)$ represents the extra rents ceded to higher type consumers to discourage them from choosing $q(\theta)$, when this offer appears in the gains domain for these consumers.

The combined effect of $\mathrm{TS}^*(q, \theta)$ and $\mathrm{LS}(q, \theta)$ in the objective function implies that there is now, in addition to the kink at $r(\theta)$, a discontinuous jump downward (see Figure 3). We sketch the general solution to the firm's design problem below, leaving formal arguments for Section 7.

Case 1. If $r(\theta) > q^*(\theta, \eta\lambda)$, then the latter constitutes the optimal offer. This is because $q^*(\theta, \eta\lambda)$ is now the unique maximizer of $\mathrm{TS}^*(q, \theta)$ and the lump-sum cost is zero. See Figure 3(A).

Case 2. If $q^*(\theta, \eta\lambda) \ge r(\theta) \ge q^*(\theta, \eta)$, the optimal offer is $r(\theta)$. In this case $\mathrm{TS}^*(q, \theta)$ is strictly decreasing for qualities above the reference point and strictly increasing for qualities below the reference point, and the lump-sum cost is zero. See Figure 3(B).

Case 3. If $q^*(\theta, \eta) > r(\theta)$, the optimal offer is either $q^*(\theta, \eta)$ or $r(\theta)$. The unique maximizer of $\mathrm{TS}^*(q, \theta)$ is $q^*(\theta, \eta)$, but now $\mathrm{LS}(q^*(\theta, \eta), \theta)$ is active. Thus, there is a trade-off between choosing $q^*(\theta, \eta)$ to capture efficiency gains and associated marginal revenues,



(A) $q^{sb}(\theta) = q^*(\theta, \eta\lambda)$ (B) $q^{sb}(\theta) = r(\theta)$

(C) $q^{sb}(\theta) = q^*(\theta, \eta)$ (D) $q^{sb}(\theta) = r(\theta)$

FIGURE 3. The profit-maximizing quality is determined by the position and magnitude of the jump.

or eschewing these and relaxing incentive constraints by offering $r(\theta)$ to ensure that this quality offer is viewed as a loss by higher types. When $r(\theta)$ is below but near $q^*(\theta, \eta)$, efficiency gains are small, hence the firm is more likely to offer $r(\theta)$. For larger differences between $q^*(\theta, \eta)$ and $r(\theta)$, the optimal choice may go the other way. This is illustrated in Figure 3(C–D).

Incentive compatibility implies that the quality schedule must be monotone. Thus, for any subinterval $\Theta_c \subset \Theta$ with $q^*(\theta, \eta) > r(\theta)$, the optimal offer corresponds to one of the following possibilities: either it assigns $q^*(\theta, \eta)$ for all $\theta \in \Theta_c$, or it assigns $r(\theta)$ for all $\theta \in \Theta_c$, or there exists a cutoff $\theta_c \in \Theta_c$ such that the firm offers $r(\theta)$ to each θ consumer below θ_c and offers $q^*(\theta, \eta)$ to each θ consumer above θ_c. Which option is chosen by the firm depends on the details of the model.

PROPOSITION 4. *Fix a reference plan $r(\cdot)$ and $\lambda > 1$. The optimal incentive feasible menu $\{q^{sb}(\theta), p^{sb}(\theta)\}_{\theta\in\Theta}$ is given by*

$$q^{sb}(\theta) = \begin{cases} q^*(\theta, \eta\lambda) & \text{if } r(\theta) > q^*(\theta, \eta\lambda) \\ r(\theta) & \text{if } q^*(\theta, \eta\lambda) \geq r(\theta) \geq q^*(\theta, \eta) \\ r(\theta) & \text{if } q^*(\theta, \eta) > r(\theta) \text{ and } \theta \leq \theta_c \\ q^*(\theta, \eta) & \text{if } q^*(\theta, \eta) > r(\theta) \text{ and } \theta > \theta_c \end{cases} \tag{10}$$

with $\theta_c \in \operatorname{cl}\Theta_c$ *for any subinterval* $\Theta_c \subset \Theta$ *for which* $q^*(\theta, \eta) > r(\theta)$, *and*

$$p^{sb}(\theta) = v(q^{sb}(\theta), \theta) - \int_{\theta_L}^{\theta} \delta(q^{sb}(\tilde{\theta}), \tilde{\theta})\, d\tilde{\theta}, \tag{11}$$

where the optimal selection in the price schedule is as in (7).

We point out that in the above result, there may be finitely many subintervals $\Theta_c \subset \Theta$ for which $q^*(\theta, \eta) > r(\theta)$, and each of these can be partitioned by a cutoff type θ_c as described in (10). See the proof for details.

The specifics of price discrimination under loss aversion exhibit novel elements, compared to the loss-neutral case. High reference plans for low-type consumers generate allocative efficiency gains as optimal offers get closer to the efficient qualities. In particular, there may be an increase in market coverage. High reference plans for high-type consumers, alternatively, generate quality distortions above and beyond the efficient levels. Moreover, it is possible that for a nonnegligible subset of buyers, the optimal quality schedule is determined entirely by the reference plan. This implies that the optimal contract menu may exhibit a degree of complexity—pooling for some mid-range consumers, preceded and followed by separating contracts, discontinuities in the optimal quality schedule—that responds entirely to the reference consumption plan and not to special features of the cost function or the distribution of types.

Corollary 1. *The following statements hold for* $\lambda > 1$ *and* $\theta \in \Theta$.

(i) Downward distortions. *If* $r(\theta)$ *is weakly below* $q^*(\theta, \eta)$, *then so is* $q^{sb}(\theta)$.

(ii) Efficiency gains. *If* $r(\theta)$ *lies between* $q^*(\theta, \eta)$ *and* $\bar{q}(\theta, \eta)$, *then so does* $q^{sb}(\theta)$.

(iii) Upward distortions. *If* $\bar{q}(\theta, \eta) \leq q^*(\theta, \eta\lambda)$ *and* $r(\theta)$ *lies above* $\bar{q}(\theta, \eta)$, *then so does* $q^{sb}(\theta)$.

As in the complete information case, we stress the fact that none of these results is obtained under loss neutrality. When $\lambda = 1$, the optimal quality schedule is $q^{sb}(\cdot) = q^*(\cdot, \eta)$, independently of the reference plan.

4.3 *Application*

We consider the following specification to obtain an explicit solution to the monopolist's problem: States are uniformly distributed on $\Theta = [1, 2]$, consumers have a linear consumption valuation $m(q, \theta) = \theta q$, and the firm's cost function is $c(q) = q^2/2 + q$. The pointwise objective function consists of

$$\begin{aligned} \mathrm{TS}^*(q, \theta) &= (1 + \mu)(2\theta - 2)q - q^2/2 - q + (\eta\lambda - \mu)(2\theta - 2)r(\theta) \\ \mathrm{LS}(q, \theta) &= (\eta\lambda - \mu)(2 - \theta)\theta r'(\theta). \end{aligned}$$

Let $\theta_\mu = (3 + 2\mu)/(2 + 2\mu)$ for $\mu \in \{\eta, \eta\lambda\}$. Observe $\theta_{\eta\lambda} < \theta_\eta$. Readily, one obtains

$$q^*(\theta, \mu) = \begin{cases} 0 & \text{if } 1 \leq \theta \leq \theta_\mu \\ (1 + \mu)(2\theta - 2) - 1 & \text{if } \theta_\mu \leq \theta \leq 2, \end{cases}$$

as the quality level that maximizes $S^*(q, \theta, \mu)$.

To highlight the effects of the reference consumption plan in terms of optimal design, here we consider three different plans. Under the first one, consumers naively believe the firm will offer the (ex ante) expected first-best quality level under loss neutrality: $r_1(\theta) = (1+\eta)3/2 - 1$. Under the second one, consumers anticipate first-best offers: $r_2(\theta) = \bar{q}(\theta, \eta) = (1+\eta)\theta - 1$. From Proposition 4, the optimal quality schedule $q_i^{sb}(\cdot)$ associated with the reference plan $r_i(\cdot)$ is given, respectively, by

$$q_1^{sb}(\theta) = \begin{cases} q^*(\theta, \eta\lambda) & \text{if } 1 \le \theta \le \underline{\theta}_1 \\ r_1(\theta) & \text{if } \underline{\theta}_1 \le \theta \le \bar{\theta}_1 \\ q^*(\theta, \eta) & \text{if } \bar{\theta}_1 \le \theta \le 2, \end{cases}$$

where $\underline{\theta}_1 = (7 + 4\eta\lambda + 3\eta)/(4 + 4\eta\lambda)$, $\bar{\theta}_1 = 7/4$, and

$$q_2^{sb}(\theta) = \begin{cases} q^*(\theta, \eta\lambda) & \text{if } 1 \le \theta \le \theta_2 \\ r_2(\theta) & \text{if } \theta_2 \le \theta \le 2, \end{cases}$$

where $\theta_2 = (2 + 2\eta\lambda)/(1 + 2\eta\lambda - \eta)$.

Consider a third reference plan defined by $r_3(\theta) = r_1(\theta)/2 + r_2(\theta)/2$. An interpretation is that each θ consumer puts equal weight into his reference point being the loss-neutral efficient quality level, given his type, and the average efficient quality. The optimal offer to consumers with reference qualities below $q^*(\cdot, \eta)$ depends on the trade-off between efficiency gains and the lump-sum cost triggered by loss aversion. For such consumers, the difference between profits at $r_3(\theta)$ and $\mu = \eta\lambda$, and profits at $q^*(\theta; \eta)$ and $\mu = \eta$ (cf. (19) in Section 7) is given by

$$\triangle(r_3(\theta), q^*(\theta, \eta)) = \tfrac{1}{2}(1+\eta)(\eta\lambda - \eta)(2-\theta)\theta - \tfrac{1}{8}(1+\eta)^2\Big(3\theta - \tfrac{11}{2}\Big)^2.$$

Note that $r_3(\cdot)$ and $q^*(\cdot, \eta)$ intersect at $\theta = 11/6$. One has that the difference in profits at $\theta = 11/6$ is strictly positive and the difference in profits at $\theta_H = 2$ is strictly negative. Since this profit difference, as a function of types, is continuous and strictly decreasing for all types greater than 11/6, it follows that there exists a unique cutoff $\bar{\theta}_3$ such that $\triangle(r_3(\bar{\theta}_3), q^*(\bar{\theta}_3)) = 0$. For consumers to the left of $\bar{\theta}_3$, the optimal qualities coincide with reference levels and is below quality schedule $q^*(\cdot, \eta)$. The optimal quality schedule under $r_3(\cdot)$ is

$$q_3^{sb}(\theta) = \begin{cases} q^*(\theta, \eta\lambda) & \text{if } 1 \le \theta \le \underline{\theta}_3 \\ r_3(\theta) & \text{if } \underline{\theta}_3 \le \theta \le \bar{\theta}_3 \\ q^*(\theta, \eta) & \text{if } \bar{\theta}_3 \le \theta \le 2, \end{cases}$$

where $\underline{\theta}_3 = (11 + 8\eta\lambda - 3\eta)/(6 + 8\eta\lambda - 2\eta)$. See Figure 4.

Finally, we stress that in contrast with the case of reference independent preferences, none of these menus can be implemented using a single family of linear prices.[12]

[12] Details are available from the authors on request.

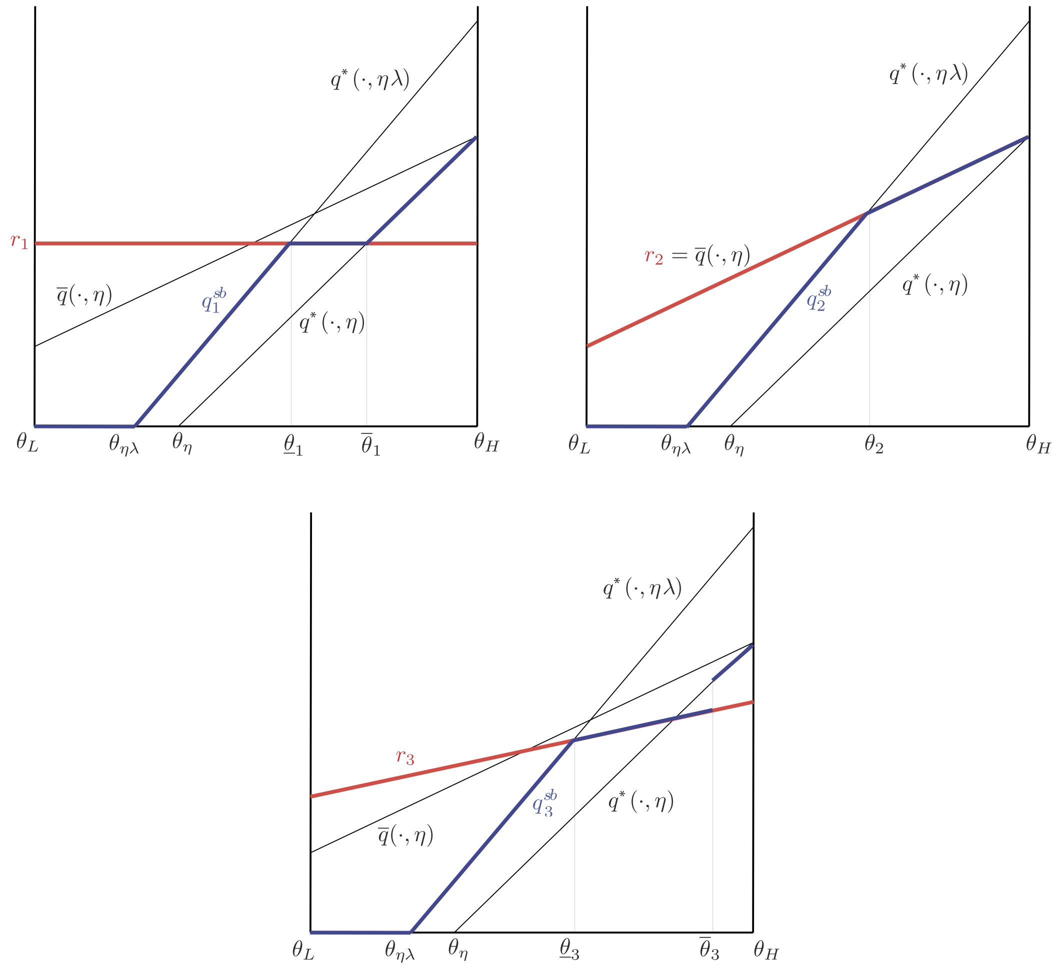


FIGURE 4. The optimal quality schedule q_i^{sb} for reference plan r_i.

5. Consistent reference plans

The analysis of Section 4 allows for differences between optimal offers and reference levels expected by consumers. In this section we focus on correct belief formation, ruling out inconsistencies between expectation-based reference qualities and purchased qualities. We consider ex ante and ex post consistent reference plans and study their effects on profits and contracts. It will be clear from the exposition below that in both cases the lump-sum cost vanishes in equilibrium, which simplifies the construction of optimal contracts. However, ex post consistent reference plans require the generalized envelope techniques previously developed, as in this case quality purchased at every state will be equal to the reference level—hence the correspondence $\varphi(q(\cdot), \cdot)$ is multivalued for a set of types of positive measure.

5.1 *Ex ante consistent reference plans*

When the reference plan $r(\cdot)$ is a constant function, we slightly abuse notation and write $r(\cdot) = r$. From Proposition 4, r generates an optimal menu $\{q^{sb}(\theta), p^{sb}(\theta)\}_{\theta\in\Theta}$ for which

$$q^{sb}(\theta) = \begin{cases} q^*(\theta, \eta\lambda) & \text{if } r > q^*(\theta, \eta\lambda) \\ r & \text{if } q^*(\theta, \eta\lambda) \geq r \geq q^*(\theta, \eta) \\ q^*(\theta, \eta) & \text{if } q^*(\theta, \eta) > r. \end{cases}$$

This follows because the lump-sum cost is zero when all consumers share the same reference point (cf. (9)).

A reference plan r is said to be *ex ante consistent* if it generates an optimal quality schedule $q^{sb}(\cdot)$ that satisfies

$$r = \mathbb{E}[q^{sb}(\theta)]. \tag{12}$$

We claim that the set of ex ante consistent reference plans is nonempty. The null reference plan $r_0 = 0$ generates the optimal schedule $q^*(\cdot, \eta)$. Any constant reference plan $\hat{r} \geq q^*(\theta_H, \eta\lambda)$ induces the optimal schedule $q^*(\cdot, \eta\lambda)$. Thus, without loss of generality we restrict our analysis to constant reference plans lying between 0 and $q^*(\theta_H, \eta\lambda)$.

Fix such a constant reference plan r. Since $q^*(\theta, \eta\lambda) \geq q^*(\theta, \eta)$ for all θ and both $q^*(\cdot, \eta\lambda)$ and $q^*(\cdot, \eta)$ are increasing functions, it is immediate to see that r determines two cutoff types

$$\tau_1(r) \quad \text{and} \quad \tau_2(r), \quad \text{with } \theta_L \leq \tau_1(r) \leq \tau_2(r) \leq \theta_H$$

via the relations

$$q^*(\tau_1(r), \eta\lambda) - r = 0 \quad \text{and} \quad q^*(\tau_2(r), \eta) - r = 0.$$

We can express the optimal quality schedule generated by r, which, to enable us to perform comparative statics, we denote now by $q^{sb}(\cdot; r)$, as

$$q^{sb}(\theta; r) = \begin{cases} q^*(\theta, \eta\lambda) & \text{for } \theta < \tau_1(r) \\ r & \text{for } \tau_1(r) \leq \theta \leq \tau_2(r) \\ q^*(\theta, \eta) & \text{for } \tau_2(r) < \theta. \end{cases}$$

A standard application of the implicit function theorem permits us to deduce the continuity of the mappings $\tau_1(\cdot)$ and $\tau_2(\cdot)$ on the interval $[0, q^*(\theta_H, \eta\lambda)]$. We express the expected quality schedule as a function of r by

$$\mathbb{E}[q^{sb}(\theta; r)] = \int_{\theta_L}^{\tau_1(r)} q^*(\theta, \eta\lambda)\, dF(\theta) + \int_{\tau_1(r)}^{\tau_2(r)} r\, dF(\theta) + \int_{\tau_2(r)}^{\theta_H} q^*(\theta, \eta)\, dF(\theta). \tag{13}$$

It follows that $\mathbb{E}[q^{sb}(\theta; \cdot)]$ is a continuous function of r on the closed interval $[0, q^*(\theta_H, \eta\lambda)]$. Moreover, we have that

$$\mathbb{E}[q^{sb}(\theta; 0)] = \mathbb{E}[q^*(\theta; \eta)] > 0$$

and

$$\mathbb{E}[q^{sb}(\theta; q^*(\theta_H, \eta\lambda))] = \mathbb{E}[q^*(\theta, \eta\lambda)] < q^*(\theta_H, \eta\lambda).$$

By continuity of $\mathbb{E}[q^{sb}(\theta; \cdot)]$, there must exist a fixed point $0 < r < q^*(\theta_H, \eta\lambda)$ that solves (12). This argument shows that the set of ex ante consistent references is nonempty.

Let r be an ex ante consistent reference plan. From (13), we obtain that

$$r = \frac{F(\tau_1(r))}{F(\tau_1(r)) + 1 - F(\tau_2(r))}\mathbb{E}[q^*(\theta; \eta\lambda) \mid \theta \le \tau_1(r)] + \frac{1 - F(\tau_2(r))}{F(\tau_1(r)) + 1 - F(\tau_2(r))}\mathbb{E}[q^*(\theta; \eta) \mid \theta \ge \tau_2(r)].$$

Thus, r can be interpreted as a weighted average between the expected quality schedule that maximizes virtual surplus $S^*(q, \theta, \eta\lambda)$ for types below $\tau_1(r)$ and the expected quality schedule that maximizes $S^*(q, \theta, \eta)$ for types above $\tau_2(r)$. A higher ex ante consistent plan increases the weight assigned to the former, thus generating efficiency gains for the firm, which increase expected profits. Because the set of ex ante consistent plans is compact (see Section 7), we obtain the following proposition.

Proposition 5. *Under complete information and for $\lambda > 1$, the unique preferred ex ante consistent quality schedule for the firm is generated by the largest ex ante consistent reference plan.*

The effect of a higher r on consumer welfare depends on whether or not a higher reference plan leads to changes in optimal offers. For intermediate types it may be possible that a higher reference plan yields higher quality consumption and, when the effects of single crossing are constant throughout consumption levels, this will increase consumer welfare. Things are different for low-type and high-type consumers. In particular, given two ex ante consistent plans $\hat{r}$ and r satisfying $\hat{r} > r$, we have that all consumers with types below $\tau_1(r)$ and above $\tau_2(\hat{r})$ maintain the same purchased quality, namely $q^*(\theta, \eta\lambda)$ in the first case and $q^*(\theta, \eta)$ in the second, under either of the reference plans. However, the utility of these consumers net of the value of the outside option is lower under $\hat{r}$ (see the left-hand side of (14)), so these consumers are worse off.

5.2 *Ex post consistent reference plans*

A reference plan $r(\cdot)$ is said to be *ex post consistent* if the optimal quality schedule it generates satisfies

$$q^{sb}(\theta) = r(\theta) \quad \text{for all } \theta \in \Theta.$$

In this case, buyers correctly anticipate their future consumption outcomes and take those expectations as their reference points. We refer to both an ex post consistent plan and its associated optimal quality schedule by $r(\cdot)$. The set of ex post consistent reference plans is clearly nonempty: from Proposition 4, it contains every plan $\tilde{r}(\cdot)$ for which $q^*(\theta, \eta) \le \tilde{r}(\theta) \le q^*(\theta, \eta\lambda)$ for all $\theta \in \Theta$.

Given this multiplicity, we ask which is the monopolist's preferred ex post consistent reference plan. From (8), given ex post consistent plan $r(\cdot)$, per-customer profits are given by

$$\mathrm{TS}^*(r(\theta),\theta) = S^*(r(\theta),\theta,\eta\lambda) = (1+\eta\lambda)m^*(r(\theta),\theta) - c(r(\theta)).$$

This expression is strictly increasing in $r(\theta)$ for all $0 \leq r(\theta) \leq q^*(\theta,\eta\lambda)$, and attains a unique maximum at $q^*(\theta,\eta\lambda)$. It follows that the reference plan

$$r^*(\cdot) = q^*(\cdot,\eta\lambda)$$

constitutes the unique preferred ex post consistent plan for the monopolist.

A higher ex post consistent reference plan generates two opposite effects on consumer welfare. First, an increase in the reference point (and consequent higher offer) increases the informational rents that the monopolist transfers to active buyers. Second, an increase in the reference point lowers the value of the outside option, which means that active buyers are worse off (in an ex post consistent contract menu, nonactive buyers expect to be excluded from the market). To analyze these countervailing forces, notice that because $\mu(\theta) = \eta\lambda$ holds at every state, the indirect utility for every θ consumer after discounting the value of the outside option is

$$\begin{aligned} U(\theta) - \eta\lambda m(r(\theta),\theta) = \int_{\theta_L}^{\theta} \frac{\partial m(r(\tilde\theta),\tilde\theta)}{\partial\theta}\, d\tilde\theta \\ + \eta\lambda \int_{\theta_L}^{\theta} \left\{ \frac{\partial m(r(\tilde\theta),\tilde\theta)}{\partial\theta} - \frac{\partial m(r(\theta),\tilde\theta)}{\partial\theta} \right\} d\tilde\theta. \end{aligned} \tag{14}$$

See (1) and (7), and condition (b) of Proposition 3, and recall that $m(\cdot,\theta_L)$ is everywhere zero by (C2).

The first integral in the right-hand side of this expression captures the standard informational rents resulting from the screening process. It is positive for consumers buying a positive quality offer, and because of single crossing, its value increases with a higher reference plan. The second integral captures the value of the informational rents vis-à-vis the participation rents that the consumer concedes to the firm to avoid the outside option. Overall, the impact of a higher reference plan depends on the interaction of these two terms. However, when the effects of single crossing are independent of consumption levels, active consumers are also better off with a higher reference plan.

Proposition 6. *The following holds under incomplete information and* $\lambda > 1$.

(i) The unique preferred ex post consistent menu for the firm is $q^*(\cdot,\eta\lambda)$.

(ii) If $\partial^2 m(q,\theta)/\partial q\,\partial\theta$ *is constant in* q *for all* $\theta \in \Theta$, *then the unique preferred ex post consistent menu for consumers is* $q^*(\cdot,\eta\lambda)$.

This result merits some comments. First, with the preferred contract menu, there are allocative efficiency gains for all θ consumers for whom $q^*(\theta,\eta\lambda)$ lies strictly above

$q^*(\theta, \eta)$—the optimal quality offered to loss-neutral consumers—but below $\bar{q}(\theta, \eta)$—the efficient quality for loss-neutral consumers. The θ consumers with $q^*(\theta, \eta\lambda) > \bar{q}(\theta, \eta)$ alternatively end up purchasing excessive quality levels.

Second, notice that the firm exploits consumers' loss aversion in two different, albeit related, ways. A higher reference plan reduces the value of the outside option, thus driving up overall net (virtual) consumer surplus. But also, by offering a quality level equal to the consumer's reference point, the firm takes advantage of the higher marginal willingness to pay for each additional unit of quality, which is captured in the choice of the selection used in the Mirrlees representation of the indirect utility to construct the optimal price schedule in (11).

Third, the fact that consumers also prefer $q^*(\cdot, \eta\lambda)$ is somewhat counterintuitive. A higher reference point diminishes the attractiveness of the outside option, which increases the willingness to pay for quality in the primary market served by the firm (we are ignoring budgetary restrictions on the part of the consumers). Under incomplete information, the firm has to pass some of the extra surplus to consumers in the form of information rents, which are increasing in quality. When the effects of the single crossing conditions are constant, the information rents ceded by the firm to active consumers exceeds the extra participation rents extracted from those consumers when the value of the outside option worsens. Thus, active consumers are better off with a higher ex post consistent reference plan.

6. Concluding remarks

In this paper, we study optimal contract design by a revenue-maximizing monopolist who faces loss-averse consumers. We find that while general insights from standard price discrimination models are present, the reference consumption plan exerts considerable influence on specifics of the optimal contracts. This is due to the appearance of new effects generated by loss aversion under complete and incomplete information. Thus, depending on how potential buyers form their expectations of quality consumption, optimal contract menus may exhibit various distinct features: pooling for intermediate consumers, some discontinuities, efficiency gains, upward distortions from efficiency levels, etc. The expanded range of the optimal contracts is consistent with stylized observations in some industries (e.g., mobile communication, consumer electronics, luxury goods, etc.).

Most of the older empirical literature testing reference-dependent price and quality effects consider memory-based models of the reference point formation process, e.g., Hardie et al. (1993), Briesch et al. (1997). There is however recent evidence of expectation-based reference points in effort provision both in the field, e.g., Crawford and Meng (2011) and Pope and Schweitzer (2011), and in the laboratory, e.g., Abeler et al. (2011) and Gill and Prowse (2012). In our monopoly pricing model with state-contingent reference qualities, there is a multiplicity of expectation-based, consistent reference consumption plans, both in the ex ante and ex post sense, many of which do not rule out marked complexities in optimal contracts. Alternatively, the firm's preferred ex ante and ex post consistent contracts exhibit (allocative) efficiency gains and an increased coverage at the low end of the market, and in the ex post case,exhibit an excess supply of quality compared to the efficient quality levels for the high end of the market.

There are various ways in which the firm may induce consumers to adopt its preferred reference plan, for instance, by announcing salient characteristics of a product line prior to actual market introduction (with no mention of prices). This seems to accord with marketing practices spread across certain industries, where both product announcements and advertising campaigns tend to precede actual market introduction and stress quality attributes over prices. Thus, it is important to understand how, in practice, consumers' (correct) expectations of future consumption are influenced by these marketing campaigns, and by fashion and trend cycles, peer pressure, etc. This is especially important in settings where there are short product cycles due to innovation or in environments of oligopolistic competition where there may be more than one product attribute dimension that can be used as a tool to enter the market. We leave these questions for future research.

Related literature

Our work adds to the literature that investigates how profit-maximizing firms operate in a market where consumers have systematic deviations from traditional preferences; see DellaVigna and Malmendier (2004), Eliaz and Spiegler (2006), Heidhues and Kőszegi (2008) Galperti (2014), and Grubb (2009), among others.

Following Kőszegi and Rabin (2006), we model the gain–loss valuation in terms of differences in the consumption valuation, but we differ in how comparisons take place. In some contexts it is reasonable to assume, as Kőszegi and Rabin (2006) and Heidhues and Kőszegi (2014), that all buyers share an ex ante stochastic reference point and evaluate each realization of stochastic consumption with each realization of the reference point. However, in other situations it is more appropriate to let each buyer assess his quality consumption relative to his state-contingent reference quality level, and this is the approach we follow here. Our work in this respect is closer to Sugden (2003); see also De Giorgi and Post (2011). Recent papers that follow Kőszegi and Rabin's (2006) approach include Rosato (2014), who studies how bait-and-switch tactics manipulate reference points and raise profits even when consumers rationally expect the bait and switch; Hahn et al. (2014), who study nonlinear pricing when consumers form reference points at an ex ante stage before learning their valuation but anticipating the eventual type-dependent consumption; and Eisenhuth (2012), who looks at optimal auctions for bidders with expectation-based reference points; see also Lange and Ratan (2010).

Our work also is related to Orhun (2009), who considers a two-type model in which the reference point of the high-type consumer is influenced by the quality offered to the low-type consumer and vice versa. We consider contract design in response to an arbitrary reference plan, which enables us to study the monopolist's incentives to manipulate reference points, perhaps via advertising. Karle (2014) studies advertising to loss-averse consumers using a model of expectation-based reference point formation à la Kőszegi and Rabin (2006). In his model, advertising creates uncertainty about future consumption and this impacts reference point formation, whereas the logic of our model indicates that the firm will try to drive up each consumer's reference quality level unambiguously.

Throughout this paper, we model reference points in terms of quality levels, departing from recent work in the area such as Herweg and Mierendorff (2013) and Spiegler (2012) that specifies reference points in terms of prices. Despite the evidence supporting the existence of reference price effects on consumer behavior, empirical work from the marketing literature suggests that loss aversion on product quality is at least as important as, if not more important than, loss aversion in prices.[13] This point is also suggested by experimental data reported by Fogel et al. (2004), who confirm the existence of loss aversion for quality in a laboratory setting, and Novemsky and Kahneman (2005), who stress that there is no loss aversion for monetary transactions that are expected to occur and thus are accounted for.

7. Proofs

Proof of Proposition 1. Fix $\theta \in \Theta$ and suppose that $r(\theta) > \bar{q}(\theta, \eta\lambda)$. The unique maximum of the profit function in (2) with $\mu(\theta) = \eta\lambda$ is $\bar{q}(\theta, \eta\lambda)$, which generates profits equal to

$$\mathrm{TS}(\bar{q}(\theta, \eta\lambda), \theta) = (1 + \eta\lambda)m(\bar{q}(\theta, \eta\lambda), \theta) - c(\bar{q}(\theta, \eta\lambda)). \tag{15}$$

Choosing any $q < r(\theta)$ does not change the objective function and strictly reduces profits. Choosing an alternative $\hat{q} \geq r(\theta)$ shifts the objective function in (2) to incorporate $\mu(\theta) = \eta$ instead of $\mu(\theta) = \eta\lambda$. Since $r(\theta) > \bar{q}(\theta, \eta)$, the monopolist would choose a deviation to $\hat{q} = r(\theta)$ with associated profits

$$(1 + \eta\lambda)m(r(\theta), \theta) - c(r(\theta)) \leq \mathrm{TS}(\bar{q}(\theta, \eta\lambda), \theta).$$

Hence, the firm has no profitable deviation.

Showing that the profit maximizing quality is $\bar{q}(\theta, \eta)$ when $r(\theta) < \bar{q}(\theta, \eta)$ is similar and therefore is omitted. In this case, profits are

$$\mathrm{TS}(\bar{q}(\theta, \eta), \theta) = (1 + \eta)m(\bar{q}(\theta, \eta), \theta) + (\eta\lambda - \eta)m(r(\theta), \theta) - c(\bar{q}(\theta, \eta)). \tag{16}$$

Now suppose that $\bar{q}(\theta, \eta) \leq r(\theta) \leq \bar{q}(\theta, \eta\lambda)$. Choosing $\hat{q} > r(\theta)$ yields $\mu(\theta) = \eta$ in (2), and thus we are in the strictly decreasing part of the total surplus. Similarly, choosing $\hat{q} \leq r(\theta)$ yields $\mu(\theta) = \eta\lambda$ in (2), so that we are now in the strictly increasing section of the total surplus. It follows that the profit-maximizing quality is $r(\theta)$, which generates profits equal to

$$\mathrm{TS}(r(\theta), \theta) = (1 + \eta\lambda)m(r(\theta), \theta) - c(r(\theta)). \tag{17}$$

This completes the proof. □

Proof of Proposition 2. (i) Immediate from Proposition 1.

[13]See Hardie et al. (1993), for instance.

(ii), (iii) Assume that $r(\theta) < \bar{q}(\theta, \eta) = q^{fb}(\theta)$. The optimal profits from the θ consumer are given by (16). Increasing the reference level to $r(\theta) < \hat{r}(\theta) \leq \bar{q}(\theta, \eta)$ does not change the offered quality but strictly increases profits, as $m(\cdot, \theta)$ is strictly increasing in its first argument for all $\theta > \theta_L$. When $\bar{q}(\theta, \eta) \leq r(\theta) = q^{fb}(\theta) < \bar{q}(\theta, \eta\lambda)$, optimal profits are given by (17). Thus, an increase in the reference level to $r(\theta) < \hat{r}(\theta) < \bar{q}(\theta, \eta\lambda)$ strictly increases the offer to the new reference point and strictly raises profits, since

$$(1 + \eta\lambda)m(r(\theta), \theta)/\partial q - c'(r(\theta)) > 0.$$

Finally, when $r(\theta) \geq \bar{q}(\theta, \eta\lambda)$, optimal profits are given by (15). Thus, an increase in the reference level alters neither the optimal offer nor profits. □

Derivation of (4) and (5). Fix a quality level $\hat{q} > 0$ and assume that $r(\cdot)$ is strictly increasing around $\hat{\theta} \in \operatorname{int}\Theta$ (the other cases are similar). For any $\theta \in \Theta$, let $\mu(\hat{q}, \theta) \in \{\eta, \eta\lambda\}$ denote the value that μ attains when comparing $\hat{q}$ to $r(\theta)$. The net total valuation as a function of types is given by

$$\begin{aligned} v(\hat{q}, \theta) = (1 + \mu(\hat{q}, \theta))m(\hat{q}, \theta) + (\eta\lambda - \mu(\hat{q}, \theta))m(r(\theta), \theta) \\ \text{where } \mu(\hat{q}, \theta) = \eta \text{ if } \hat{q} > r(\theta) \text{ and } \mu(\hat{q}, \theta) = \eta\lambda \text{ if } \hat{q} \leq r(\theta). \end{aligned} \tag{18}$$

By assumption, $r(\cdot)$ is piecewise continuously differentiable; hence we omit discussion of kinks in the valuation due to kinks in $r(\cdot)$. This will not have any consequence on the derivation of an optimal contract menu. The function $v(\hat{q}, \cdot)$ in (18) has bounded right and left partial derivatives at $\hat{\theta}$ defined, respectively, by

$$\frac{\partial v^+(\hat{q}, \hat{\theta})}{\partial \theta} \equiv \lim_{\theta \downarrow \hat{\theta}} \frac{v(\hat{q}, \theta) - v(\hat{q}, \hat{\theta})}{\theta - \hat{\theta}} \quad \text{and} \quad \frac{\partial v^-(\hat{q}, \hat{\theta})}{\partial \theta} \equiv \lim_{\theta \uparrow \hat{\theta}} \frac{v(\hat{q}, \theta) - v(\hat{q}, \hat{\theta})}{\theta - \hat{\theta}}.$$

Suppose first that $r(\hat{\theta}) > \hat{q}$. Then for all types θ sufficiently close to $\hat{\theta}$, one has $\mu(\hat{q}, \theta) = \mu(\hat{q}, \hat{\theta}) = \eta\lambda$. It follows that

$$\frac{\partial v^+(\hat{q}, \hat{\theta})}{\partial \theta} = \frac{\partial v^-(\hat{q}, \hat{\theta})}{\partial \theta} = (1 + \eta\lambda)\frac{\partial m(\hat{q}, \hat{\theta})}{\partial \theta}.$$

Suppose next that $r(\hat{\theta}) < \hat{q}$. Then for all θ sufficiently close to $\hat{\theta}$, one has $\mu(\hat{q}, \theta) = \mu(\hat{q}, \hat{\theta}) = \eta$. It follows that

$$\frac{\partial v^+(\hat{q}, \hat{\theta})}{\partial \theta} = \frac{\partial v^-(\hat{q}, \hat{\theta})}{\partial \theta} = (1 + \eta)\frac{\partial m(\hat{q}, \hat{\theta})}{\partial \theta} + (\eta\lambda - \eta)\frac{d}{d\theta}(m(r(\hat{\theta}), \hat{\theta})).$$

Finally, suppose that $r(\hat{\theta}) = \hat{q}$. Note that for $\theta' < \hat{\theta} < \theta''$, one has $\mu(\hat{q}, \theta') = \eta$ and $\mu(\hat{q}, \hat{\theta}) = \mu(\hat{q}, \theta'') = \eta\lambda$. We obtain

$$\begin{aligned} \frac{\partial v^+(\hat{q}, \hat{\theta})}{\partial \theta} &= \lim_{\theta \downarrow \hat{\theta}} \frac{1}{\theta - \hat{\theta}} \big\{(1 + \mu(\hat{q}, \theta))m(\hat{q}, \theta) + (\eta\lambda - \mu(\hat{q}, \theta))m(r(\theta), \theta) \\ &\qquad - (1 + \mu(\hat{q}, \hat{\theta}))m(\hat{q}, \hat{\theta}) - (\eta\lambda - \mu(\hat{q}, \hat{\theta}))m(r(\hat{\theta}), \hat{\theta})\big\} \\ &= \lim_{\theta \downarrow \hat{\theta}} (1 + \eta\lambda)\frac{m(\hat{q}, \theta) - m(\hat{q}, \hat{\theta})}{\theta - \hat{\theta}} = (1 + \eta\lambda)\frac{\partial m(\hat{q}, \hat{\theta})}{\partial \theta}, \end{aligned}$$

and similarly

$$\frac{\partial v^-(\hat{q},\hat{\theta})}{\partial\theta} = \lim_{\theta\uparrow\hat{\theta}} \frac{1}{\theta-\hat{\theta}}\{(1+\eta)m(\hat{q},\theta)+(\eta\lambda-\eta)m(r(\theta),\theta) - (1+\eta\lambda)m(\hat{q},\hat{\theta}) \pm \eta m(\hat{q},\hat{\theta})\}$$

$$= \lim_{\theta\uparrow\hat{\theta}}(1+\eta)\frac{m(\hat{q},\theta)-m(\hat{q},\hat{\theta})}{\theta-\hat{\theta}} + \lim_{\theta\uparrow\hat{\theta}}(\eta\lambda-\eta)\frac{m(r(\theta),\theta)-m(r(\hat{\theta}),\hat{\theta})}{\theta-\hat{\theta}}$$

$$= (1+\eta)\frac{\partial m(\hat{q},\hat{\theta})}{\partial\theta} + (\eta\lambda-\eta)\frac{d}{d\theta}\big(m(r(\hat{\theta}),\hat{\theta})\big).$$

Note that

$$\frac{\partial v^+(\hat{q},\hat{\theta})}{\partial\theta} - \frac{\partial v^-(\hat{q},\hat{\theta})}{\partial\theta} = -(\eta\lambda-\eta)\frac{\partial m(r(\hat{\theta}),\hat{\theta})}{\partial q}r'(\hat{\theta}) \le 0.$$

Thus, for the $\hat{\theta}$ consumer, one has that $\varphi(\hat{q},\hat{\theta})$ is equal to the nonempty closed interval

$$\left[(1+\eta\lambda)\frac{\partial m(\hat{q},\hat{\theta})}{\partial\theta}, (1+\eta)\frac{\partial m(\hat{q},\hat{\theta})}{\partial\theta} + (\eta\lambda-\eta)\frac{d}{d\theta}(m(r(\hat{\theta}),\hat{\theta}))\right],$$

which collapses to a single point when $r'(\hat{\theta})=0$. □

Proof of Proposition 3. The equivalence between incentive compatibility of the menu $\{q(\theta), p(\theta)\}_{\theta\in\Theta}$ and parts (a) and (b) follows from Theorem 1 in Carbajal and Ely (2013). Condition (c) clearly holds when contracts are individually rational. Suppose now that (c) is also in place. Using condition (b) we express

$$U(\theta) = U(\theta_L) + \int_{\theta_L}^{\theta}\delta(q(\tilde{\theta}),\tilde{\theta})\,d\tilde{\theta}$$

for any θ consumer. From (4) and (5), any integrable selection is such that $\delta(q(\cdot),\cdot)\ge 0$ everywhere, so $U(\theta)\ge 0$ follows readily. □

Proof of Proposition 4. *Case 1.* Suppose that one has $r(\theta) > q^*(\theta,\eta\lambda)$. The unique maximizer of the integrand in the profit function of (8) with $\mu(\theta)=\eta\lambda$ is $q^*(\theta,\eta\lambda)$. Any deviation to an alternative $q\le r(\theta)$ hurts profits as it decreases virtual total surplus without changing the lump-sum cost, which remains at zero. Now consider a deviation to $\hat{q}\ge r(\theta)$, which switches $\mu(\theta)$ in the objective function from $\eta\lambda$ to η. Since $r(\theta) > q^*(\theta,\eta)$, it follows by the strong concavity of $S^*(\cdot,\theta;\eta)$ that the optimal deviation in this case is $\hat{q}=r(\theta)$. The difference between profits at $r(\theta)$ and $\mu(\theta)=\eta\lambda$, and profits at $r(\theta)$ and $\mu(\theta)=\eta$ is given by

$$(\eta\lambda-\eta)h(\theta)\frac{\partial m(r(\theta),\theta)}{\partial q}r'(\theta)\ge 0.$$

It follows that profits at $q^{sb}(\theta)=q^*(\theta,\eta\lambda)$ and $\mu(\theta)=\eta\lambda$ are strictly greater than profits at $r(\theta)$ and $\mu(\theta)=\eta\lambda$, which in turn are greater than profits at $r(\theta)$ and $\mu(\theta)=\eta$.

Case 2. Suppose that $q^*(\theta, \eta\lambda) \geq r(\theta) \geq q^*(\theta, \eta)$. Consider an alternative quality $\hat{q}$ such that $\hat{q} > r(\theta) > q^*(\theta, \eta)$. The integrand of (8) has $\mu(\theta) = \eta$ for any such $\hat{q}$, so that the lump-sump cost is active. Clearly, profits are strictly decreasing in quality as long as $\hat{q} > r(\theta)$, so there is no upward profitable deviation. One can use a similar argument to show that there is no downward profitable deviation from $r(\theta)$, as any deviation to $\hat{q} < r(\theta)$ has $\mu(\theta) = \eta\lambda$. Therefore $q^{sb}(\theta) = r(\theta)$ is the optimal quality offer.

Case 3. Suppose that $q^*(\theta, \eta) > r(\theta)$. As before, the unique maximizer of the integrand of the profit function in (8) with $\mu(\theta) = \eta$ is $q^*(\theta, \eta)$. Any deviation to a quality level $\hat{q} > r(\theta)$ does not change the value of $\mu(\theta)$ to $\eta\lambda$ and thus will only decrease profits. Among deviations from $q^*(\theta, \eta)$ to quality levels $\hat{q} \leq r(\theta)$ that change the parameter $\mu(\theta)$ to $\eta\lambda$, thus avoiding the lump-sum cost, the one generating the highest profits is $\hat{q} = r(\theta)$. The difference between profits at $r(\theta)$ with associated $\mu(\theta) = \eta\lambda$ and profits at $q^*(\theta, \eta)$ with associated $\mu(\theta) = \eta$ is

$$\begin{aligned}\triangle(r(\theta), q^*(\theta, \eta)) = (\eta\lambda - \eta)h(\theta)\frac{\partial m(r(\theta), \theta)}{\partial q}r'(\theta) \\ - \left\{S^*(q^*(\theta, \eta), \theta, \eta) - S^*(r(\theta), \theta, \eta)\right\}.\end{aligned} \tag{19}$$

The sign of the above expression depends on the difference between gains associated with offering a quality level $r(\theta)$ and avoiding the lump-sum transfer to higher type consumers, and efficiency gains in virtual total surplus at $\mu(\theta) = \eta$ derived from shifting quality from $r(\theta)$ to $q^*(\theta, \eta)$.

Let $\theta'' > \theta'$ be two types for whom $q^*(\theta'', \eta) \geq q^*(\theta', \eta) > r(\theta'') > r(\theta')$. The monopolist either offers to them their respective reference quality levels or $q^*(\theta', \eta)$ and $q^*(\theta'', \eta)$ to each of them, respectively, or it offers to the θ' consumer his reference quality level and the quality level $q^*(\theta'', \eta)$ to the θ'' consumer. The remaining possibility that $q^{sb}(\theta') = q^*(\theta', \eta)$ and $q^{sb}(\theta'') = r(\theta'')$ is not incentive compatible. From Proposition 3, it suffices to show a violation of monotonicity:

$$v(r(\theta''), \theta'') - v(r(\theta''), \theta') < v(q^*(\theta', \eta), \theta'') - v(q^*(\theta', \eta), \theta').$$

One can write the previous inequality as

$$m(r(\theta''), \theta'') - m(r(\theta''), \theta') < m(q^*(\theta', \eta), \theta'') - m(q^*(\theta', \eta), \theta').$$

Since $\theta'' > \theta'$ and $q^*(\theta', \eta) > r(\theta'')$, single crossing implies that this inequality is indeed satisfied.

From the assumptions in Section 2 it follows that $q^*(\cdot, \mu)$ is everywhere continuous and continuously differentiable except possibly at a type where $q^*(\cdot, \mu)$ turns from zero to positive. Therefore the function f_μ defined on Θ by

$$f_\mu(\theta) = r(\theta) - q^*(\theta, \mu)$$

is continuous and piecewise continuously differentiable, with bounded left and right derivatives everywhere on Θ. Let $A \subset \Theta$ be the set of types for which $f_\mu(\theta) = 0$ and $f'_\mu(\theta) \neq 0$. Since f'_μ is continuous, it follows that $\theta \in A$ is an isolated point and thus A is a discrete subset of a compact set; hence it is finite. It follows that there are finitely many subintervals $\Theta_c \subset \Theta$ for which $q^*(\theta, \eta) \geq r(\theta)$. The construction of the optimal quality schedule q^{sb} in (10) follows from these arguments.

It remains to show that the informational constraints, expressed as conditions (a)–(c) of Proposition 3, are in place. One immediately sees from the expression for incentive prices in (11) that both (b) and (c) are in place. To show that condition (a)—integral monotonicity—is satisfied, let $\theta', \theta'' \in \Theta$ be two consumer types such that $\theta' < \theta''$ and suppose $q^{sb}(\theta') < r(\theta') \leq r(\theta'') < q^{sb}(\theta'')$ holds: all remaining cases are similarly proven. By construction of the optimal quality offers, there exists a type θ_c, with $\theta' \leq \theta_c < \theta''$, for which one has $r(\theta') \leq r(\theta_c) = q^{sb}(\theta_c) \leq r(\theta'')$. Moreover, we can choose θ_c so that $q^{sb}(\theta) \leq r(\theta)$ for all $\theta' \leq \theta \leq \theta_c$ and $q^{sb}(\theta) > r(\theta)$ for all $\theta_c < \theta \leq \theta''$.

We first write the valuation differences in a suitable form,

$$\begin{aligned} & v(q^{sb}(\theta''), \theta'') - v(q^{sb}(\theta''), \theta') \\ &\quad = v(q^{sb}(\theta''), \theta'') - v(q^{sb}(\theta''), \theta') \pm v(q^{sb}(\theta''), \theta_c) \\ &\quad \geq v(q^{sb}(\theta''), \theta'') - v(q^{sb}(\theta''), \theta_c) + v(q^{sb}(\theta_c), \theta_c) - v(q^{sb}(\theta_c), \theta'), \end{aligned} \tag{20}$$

where the inequality follows from single crossing. A similar argument yields

$$\begin{aligned} & v(q^{sb}(\theta'), \theta') - v(q^{sb}(\theta'), \theta'') \\ &\quad \geq v(q^{sb}(\theta'), \theta') - v(q^{sb}(\theta'), \theta_c) + v(q^{sb}(\theta_c), \theta_c) - v(q^{sb}(\theta_c), \theta''). \end{aligned} \tag{21}$$

Notice now that

$$\begin{aligned} & v(q^{sb}(\theta''), \theta'') - v(q^{sb}(\theta''), \theta_c) \\ &\quad = \int_{\theta_c}^{\theta''} \left\{ (1+\eta) \frac{\partial m(q^{sb}(\theta''), \tilde{\theta})}{\partial \theta} + (\eta\lambda - \eta) \frac{d}{d\theta}(m(r(\tilde{\theta}), \tilde{\theta})) \right\} d\tilde{\theta} \\ &\quad \geq \int_{\theta_c}^{\theta''} \delta(q^{sb}(\tilde{\theta}), \tilde{\theta})\, d\tilde{\theta}, \end{aligned} \tag{22}$$

where the inequality follows from the monotonicity of q^{sb} and the fact that $\mu(\tilde{\theta}) = \eta$ for all $\theta_c < \tilde{\theta} \leq \theta''$. Furthermore, we can write

$$\begin{aligned} & v(q^{sb}(\theta_c), \theta_c) - v(q^{sb}(\theta_c), \theta') \\ &\quad \geq (1+\eta\lambda) m(q^{sb}(\theta_c), \theta_c) - (1+\eta\lambda) m(q^{sb}(\theta_c), \theta') \\ &\quad = \int_{\theta'}^{\theta_c} (1+\eta\lambda) \frac{\partial m(q^{sb}(\theta_c), \tilde{\theta})}{\partial \theta}\, d\tilde{\theta} \geq \int_{\theta'}^{\theta_c} \delta(q^{sb}(\tilde{\theta}), \tilde{\theta})\, d\tilde{\theta}, \end{aligned} \tag{23}$$

where the second inequality follows from the monotonicity of the optimal quality schedule and the fact that $\mu(\tilde{\theta}) = \eta\lambda$ for all $\theta' \leq \tilde{\theta} \leq \theta_c$. Combining expressions (22) and (23) with (20), we obtain

$$v(q^{sb}(\theta''), \theta'') - v(q^{sb}(\theta''), \theta') \geq \int_{\theta'}^{\theta''} \delta(q^{sb}(\tilde{\theta}), \tilde{\theta})\, d\tilde{\theta},$$

which is the first inequality of the integral monotonicity condition of Proposition 3.

To obtain the second inequality, we write

$$v(q^{sb}(\theta'), \theta_c) - v(q^{sb}(\theta'), \theta') = \int_{\theta'}^{\theta_c} (1+\eta\lambda)\frac{\partial m(q^{sb}(\theta'), \tilde{\theta})}{\partial \theta}\, d\tilde{\theta} \geq \int_{\theta'}^{\theta_c} \delta(q^{sb}(\tilde{\theta}), \tilde{\theta})\, d\tilde{\theta}, \tag{24}$$

and similarly

$$\begin{aligned} v(q^{sb}(\theta_c), \theta'') - v(q^{sb}(\theta_c), \theta_c) &\leq \int_{\theta_c}^{\theta''} \left\{(1+\eta)\frac{\partial m(q^{sb}(\theta_c), \tilde{\theta})}{\partial \theta} + (\eta\lambda - \eta)\frac{d}{d\theta}(m(r(\tilde{\theta}), \tilde{\theta}))\right\} d\tilde{\theta} \qquad (25) \\ &\leq \int_{\theta_c}^{\theta''} \delta(q^{sb}(\tilde{\theta}), \tilde{\theta})\, d\tilde{\theta}. \end{aligned}$$

Combining expressions (24) and (25) with (21) above, we obtain the desired inequality. Thus, integral monotonicity is satisfied. □

Proof of Corollary 1. The properties $q^{sb}(\cdot)$ for $\lambda > 1$ follow readily from (10). □

Proof of Proposition 5. In the main text we showed that the set of ex ante consistent reference plans, or equivalently the set of solutions to the fixed-point (12), is nonempty. Because $\mathbb{E}[q^{sb}(\theta; \cdot)]$ is a continuous function on the compact interval $[0, q^*(\theta_H, \eta\lambda)]$, it follows that the set of ex ante consistent reference plans is also closed. To see this more explicitly, take a sequence of fixed points $\{r_n\}$ and assume that it converges to $\hat{r} \in [0, q^*(\theta_H, \eta\lambda)]$. By continuity, we have that $\mathbb{E}[q^{sb}(\theta; r_n)] \to \mathbb{E}[q^{sb}(\theta; \hat{r})]$. Since $r_n = \mathbb{E}[q^{sb}(\theta; r_n)]$ and $r_n \to \hat{r}$, it follows that $\hat{r} = \mathbb{E}[q^{sb}(\theta; \hat{r})]$, as desired.

Under constant reference plan r, the lump-sum cost is always zero. Thus, using (8), we express expected profits of the firm as a function of r by

$$\begin{aligned} \Pi^{sb}(r) = \int_{\theta_L}^{\tau_1(r)} S^*(q^*(\theta, \eta\lambda), \theta, \eta\lambda)\, dF(\theta) + \int_{\tau_1(r)}^{\tau_2(r)} S^*(r, \theta, \eta\lambda)\, dF(\theta) \\ + \int_{\tau_2(r)}^{\theta_H} \left\{S^*(q^*(\theta, \eta), \theta, \eta) + (\eta\lambda - \eta)m^*(r, \theta)\right\} dF(\theta). \end{aligned}$$

Expected profits are continuous on r, and because the set of ex ante consistent reference plans is a closed subset of $[0, q^*(\theta_H, \eta\lambda)]$, it follows that there exists a preferred ex ante consistent reference plan.

To show that the unique preferred ex ante consistent reference plan for the firm is the largest among all ex ante consistent plans, we argue that profits from every θ consumer are strictly increasing in r.

Case 1. Fix $\theta \in \Theta$ and $\hat{r} > r$ such that $q^{sb}(\theta; \hat{r}) = q^*(\theta, \eta\lambda) > r = q^{sb}(\theta; r)$. In this case, the difference in profits from the θ consumer is given by

$$S^*(q^*(\theta, \eta\lambda), \theta, \eta\lambda) - S^*(r, \theta, \eta\lambda) > 0.$$

Case 2. Fix $\theta, \hat{r} > r$ such that $q^{sb}(\theta; \hat{r}) = q^*(\theta, \eta\lambda) > q^*(\theta, \eta) = q^{sb}(\theta; r) > r$. The profit difference from the θ consumer is given by

$$\begin{aligned}
&S^*(q^*(\theta, \eta\lambda), \theta, \eta\lambda) - S^*(q^*(\theta, \eta), \theta, \eta) - (\eta\lambda - \eta)m^*(r, \theta) \\
&\quad = S^*(q^*(\theta, \eta\lambda), \theta, \eta\lambda) - S^*(q^*(\theta, \eta), \theta, \eta\lambda) + (\eta\lambda - \eta)\big(m^*(q^*(\theta, \eta), \theta) - m^*(r, \theta)\big).
\end{aligned}$$

This difference is strictly positive, so the firm prefers the higher reference plan.

Case 3. Fix $\theta, \hat{r} > r$ such that $q^*(\theta, \eta\lambda) > q^{sb}(\theta; \hat{r}) = \hat{r} > r = q^{sb}(\theta; r)$. In this case the profit difference is given by

$$S^*(\hat{r}, \theta, \eta\lambda) - S^*(r, \theta, \eta\lambda) > 0,$$

as $r < \hat{r} < q^*(\theta, \eta\lambda)$ and we are in the increasing part of the virtual surplus function $S^*(\cdot, \theta, \eta\lambda)$.

Case 4. Fix $\theta, \hat{r} > r$ such that $q^*(\theta, \eta\lambda) > q^{sb}(\theta; \hat{r}) = \hat{r} > q^*(\theta, \eta) = q^{sb}(\theta; r) > r$. Here again we obtain that the profit difference is given by

$$\begin{aligned}
&S^*(\hat{r}, \theta, \eta\lambda) - S^*(q^*(\theta, \eta), \theta, \eta) - (\eta\lambda - \eta)m^*(r, \theta) \\
&\qquad = S^*(\hat{r}, \theta, \eta\lambda) - S^*(q^*(\theta, \eta), \theta, \eta\lambda) + (\eta\lambda - \eta)\big(m^*(q^*(\theta, \eta), \theta) - m^*(r, \theta)\big).
\end{aligned}$$

Since $q^*(\theta, \eta\lambda) > \hat{r} > q^*(\theta, \eta) > r$, this difference is also positive.

These four cases exhaust all relevant comparisons. □

Proof of Proposition 6. (i) Consider expected profits for the firm generated by any ex post consistent reference plan $r(\cdot)$. From (8), when $q^{sb}(\theta) = r(\theta)$ for all θ, one has

$$\Pi^{sb}(r(\cdot)) = \int_{\underline{\theta}}^{\theta_H} S^*(r(\theta), \theta, \eta\lambda) f(\theta)\, d(\theta).$$

For each $\theta \in \Theta$, per-customer profits $S^*(q, \theta, \eta\lambda)$ are strictly increasing in q for all $q < q^*(\theta, \eta\lambda)$, strictly decreasing in q for all $q > q^*(\theta, \eta\lambda)$, and attain a unique maximum at $q = q^*(\theta, \eta\lambda)$.

(ii) For any given q and θ, by single crossing we have $\partial^2 m(q, \theta)/\partial q\, \partial\theta > 0$. Thus, an increase in the reference point for the θ consumer to a new ex post consistent level increases the value of the first integral in the right-hand side of (14). Alternatively, when the function $\partial^2 m(q, \theta)/\partial q\, \partial\theta$ is constant in q for all θ, we obtain that the integrand in the second integral of (14) vanishes. Note also that $m(q, \theta_L) = 0$ for all $q \geq 0$, so this term does not affect our conclusion. □

References

Abeler, Johannes, Armin Falk, Lorenz Goette, and David Huffman (2011), "Reference points and effort provision." *American Economic Review*, 101, 470–492. [475]

Ariely, Dan and Itamar Simonson (2003), "Buying, bidding, playing, or competing? Value assessment and decision dynamics in online auctions." *Journal of Consumer Psychology*, 13, 113–123. [460]

Briesch, Richard A., Lakshman Krishnamurthi, Tridib Mazumdar, and S. P. Raj (1997), "A comparative analysis of reference price models." *Journal of Consumer Research*, 24, 202–214. [475]

Carbajal, Juan Carlos and Jeffrey C. Ely (2013), "Mechanism design without revenue equivalence." *Journal of Economic Theory*, 148, 104–133. [465, 479]

Crawford, Vincent P. and Juanjuan Meng (2011), "New York city cab drivers' labor supply revisited: Reference-dependent preferences with rational-expectations targets for hours and income." *American Economic Review*, 101, 1912–1932. [475]

De Giorgi, Enrico G. and Thierry Post (2011), "Loss aversion with a state-dependent reference point." *Management Science*, 57, 1094–1110. [476]

DellaVigna, Stefano and Ulrike Malmendier (2004), "Contract design and self-control: Theory and evidence." *Quarterly Journal of Economics*, 119, 353–402. [476]

Eisenhuth, Roland (2012), "Reference dependent mechanism design." Mimeo, Northwestern University. [476]

Eliaz, Kfir and Ran Spiegler (2006), "Contracting with diversely naive agents." *Review of Economic Studies*, 73, 689–714. [476]

Fogel, Suzanne, Dan Lovallo, and Carmina Caringal (2004), "Loss aversion for quality in consumer choice." *Australian Journal of Management*, 29, 45–63. [477]

Galperti, Simone (2014), "Optimal screening of time inconsistency." Mimeo, University of California San Diego. [476]

Gill, David and Victoria Prowse (2012), "A structural analysis of disappointment aversion in a real effort competition." *American Economic Review*, 102, 469–503. [475]

Grubb, Michael D. (2009), "Selling to overconfident consumers." *American Economic Review*, 99, 1770–1807. [476]

Hahn, Jong-Hee, Jinwoo Kim, Sang-Hyun Kim, and Jihong Lee (2014), "Price discrimination with loss averse consumers." Mimeo, Seoul National University. [476]

Hardie, Bruce G. S., Eric J. Johnson, and Peter S. Fader (1993), "Modeling loss aversion and reference dependence effects on brand choice." *Marketing Science*, 12, 378–394. [475, 477]

Heidhues, Paul and Botond Kőszegi (2008), "Competition and price variation when consumers are loss averse." *American Economic Review*, 98, 1245–1268. [476]

Heidhues, Paul and Botond Kőszegi (2014), "Regular prices and sales." *Theoretical Economics*, 9, 217–251. [476]

Herweg, Fabian and Konrad Mierendorff (2013), "Uncertain demand, consumer loss aversion, and flat-rate tariffs." *Journal of the European Economic Association*, 11, 399–432. [477]

Heyman, James E., Yesim Orhun, and Dan Ariely (2004), "Auction fever: The effect of opponents and quasi-endowment on product valuations." *Journal of Interactive Marketing*, 18, 7–21. [456]

Karle, Heiko (2014), "Creating attachment through advertising: Loss aversion and prepurchase information." Mimeo, ETH Zurich. [476]

Kőszegi, Botond and Matthew Rabin (2006), "A model of reference-dependent preferences." *Quarterly Journal of Economics*, 121, 1133–1165. [455, 459, 476]

Lambrecht, Anja, Katja Seim, and Bernd Skiera (2007), "Does uncertainty matter? Consumer behavior under three-part tariffs." *Marketing Science*, 26, 698–710. [457]

Lambrecht, Anja and Bernd Skiera (2006), "Paying too much and being happy about it: Existence, causes, and consequences of tariff-choice biases." *Journal of Marketing Research*, 43, 212–223. [457]

Lange, Andreas and Anmol Ratan (2010), "Multi-dimensional reference-dependent preferences in sealed-bid auctions—how (most) laboratory experiments differ from the field." *Games and Economic Behavior*, 68, 634–645. [476]

Maskin, Eric and John Riley (1984), "Monopoly with incomplete information." *Rand Journal of Economics*, 15, 171–196. [456, 458, 459]

Mussa, Michael and Sherwin Rosen (1978), "Monopoly and product quality." *Journal of Economic Theory*, 18, 301–317. [456, 458, 459]

Novemsky, Nathan and Daniel Kahneman (2005), "The boundaries of loss aversion." *Journal of Marketing Research*, 42, 119–128. [477]

Orhun, A. Yesim (2009), "Optimal product line design when consumers exhibit choice set-dependent preferences." *Marketing Science*, 28, 868–886. [476]

Pope, Devin G. and Maurice E. Schweitzer (2011), "Is tiger woods loss averse? Persistent bias in the face of experience, competition, and high stakes." *American Economic Review*, 101, 129–157. [475]

Rosato, Antonio (2014), "Selling substitute goods to loss-averse consumers: Limited availability, bargains, and rip-offs." Mimeo, University of Technology Sydney. [476]

Spiegler, Ran (2012), "Monopoly pricing when consumers are antagonized by unexpected price increases." *Economic Theory*, 51, 695–711. [477]

Sugden, Robert (2003), "Reference-dependent subjective expected utility." *Journal of Economic Theory*, 111, 172–191. [476]

Tversky, Amos and Daniel Kahneman (1991), "Loss aversion in riskless choice: A reference-dependent model." *Quarterly Journal of Economics*, 106, 1039–1061. [456, 458]

Co-editor Faruk Gul handled this manuscript.

Submitted 2013-12-11. Final version accepted 2015-3-13. Available online 2015-3-15.