

# Preventing bank runs

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The work of [Diamond and Dybvig \(1983\)](#) is commonly understood as a theory of bank runs driven by self-fulfilling prophecies. Their contribution may alternatively be interpreted as a theory for *preventing* these bank runs. Absent aggregate risk over liquidity demand, they show that a simple scheme that suspends withdrawals when a target level of bank reserves is reached implements the efficient allocation as the unique equilibrium. Uniqueness implies that there cannot be a bank-run equilibrium. Unfortunately, this scheme cannot implement the efficient allocation when there is aggregate uncertainty over every possible liquidity demand because any realization of liquidity demand may, in this case, be determined by fundamentals instead of psychology. When there is aggregate risk, [Peck and Shell \(2003\)](#) demonstrate that the *constrained* efficient allocation can be implemented by a *direct* mechanism as an equilibrium. They show that the same mechanism can also implement a bank-run equilibrium, which suggests that [Diamond and Dybvig \(1983\)](#) can be understood as a theory of bank runs. The use of direct mechanisms, however, imposes a severe restriction on communications. We propose an indirect mechanism that (i) permits depositors to communicate their beliefs, not just their types, (ii) incentivizes depositors to communicate “rumors” of an impending bank run, and (iii) threatens to suspend payments conditional on what is revealed in these communications. We demonstrate that if commitment is possible, then under some weak parameter restrictions our indirect mechanism uniquely implements an allocation that can be made arbitrarily close to the the constrained efficient allocation as an equilibrium. In other words, our mechanism prevents bank runs.

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## 1. INTRODUCTION

Banking is the business of transforming long-maturity illiquid assets into short-maturity liquid liabilities. The demandable debt issued by commercial banks constitutes the quintessential example of this type of credit arrangement. The use of short-maturity debt to finance long-maturity asset holdings is also prevalent in the shadow-banking sector.<sup>1</sup> Short-maturity debt has long been viewed by economists and regulators as an inherently fragile financial structure—a credit arrangement that is susceptible to runs or rollover risk. The argument is a familiar one. Suppose that depositors expect a run—a wave of early redemptions driven by fear, rather than by liquidity needs. Since the long-maturity assets are illiquid, the value of what can be recouped in a fire sale of these assets will fall short of existing obligations. Because the bank cannot honor its promises in this event, it becomes insolvent. In this manner, the fear of a run can become a self-fulfilling prophecy. If demandable debt is run-prone, then why not tax it or, better yet, legislate it out of existence?<sup>2</sup>

Bryant (1980), however, suggests that the demandability property of bank liabilities is a way to insure against unobservable liquidity risk. In short, banking is an efficient risk-sharing arrangement when assets are illiquid, depositors are risk averse, and liquidity preference is private information. But if this is the case, then the solution to one problem—risk-sharing—seems to open the door to another—a bank run. Indeed, the seminal paper by Diamond and Dybvig (1983) on bank runs demonstrates precisely this possibility: while demandable debt is an efficient risk-sharing arrangement, it is also a source of indeterminacy and financial instability.

The paper by Diamond and Dybvig (1983) is most often viewed as a theory of bank runs. But what is often overlooked is that they also offer a prescription for how to *prevent* bank runs when aggregate uncertainty is absent. (There is no aggregate uncertainty if the fraction of impatient depositors in the economy is known.) The prescription entails embedding bank liabilities with a suspension clause that is triggered when redemption exceeds a specified threshold, i.e., the known fraction of impatient depositors. This simple fix prevents bank runs and implements the efficient outcome.

When aggregate uncertainty is present, Diamond and Dybvig (1983) point out that a full suspension of convertibility, conditional on a threshold level of redemption activity being breached, is not optimal.<sup>3</sup> (There is aggregate risk if the fraction of impatient depositors in the economy is unknown.) In the *absence* of aggregate uncertainty, redemptions that exceed a critical threshold constitutes a signal that a run is occurring. In the presence of aggregate uncertainty, the first-best efficient redemption schedule

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<sup>1</sup>This sector includes, but is not limited to, structured investment vehicles (SIVs), asset-backed commercial paper (ABCP) conduits, money market funds (MMFs), and repurchase agreements (repos).

<sup>2</sup>This is essentially the recommendation recently put forth by Cochrane (2014).

<sup>3</sup>Diamond and Dybvig (1983) do not actually characterize the optimal contract for the case in which aggregate uncertainty is present.

is contingent on a state—the fraction of impatient depositors—that is not observable. But the state of the world cannot be learned or inferred in a way to implement the first-best redemption schedule. As a result, the *constrained* efficient redemption schedule depends on how the state of the world is revealed (through the announcements or redemption behavior of depositors).<sup>4</sup> Notice that it is not possible to confirm whether heavy redemption is driven by fundamentals, i.e., the fraction of impatient depositors is high, or by psychology, i.e., a belief-driven bank run.

Our proposed solution to the bank-run problem under aggregate liquidity risk is based on a simple idea. Consider any model with a bank-run equilibrium. In equilibrium, depositors are assumed to know that a bank run is occurring. This is absolutely necessary to generate the phenomenon of self-fulfilling prophecy. But if depositors are assumed to know what is going on in terms their own collective beliefs, they should be able to communicate such information to a mechanism, and if depositors could somehow be incentivized to report what they know about their beliefs, then the threat of suspension based on such information might prevent the bank run from occurring in the first place. The idea here is to resurrect the [Diamond and Dybvig \(1983\)](#) suspension scheme, where suspension is triggered not by redemption activity, but by “credible rumors of an impending run.”

We provide a reasonable sufficient (but not necessary) condition under which depositors are willing to communicate their beliefs and where the threat of suspension conditional on these communications prevents bank runs. We depart from the direct mechanism approach typically employed in the literature. In a direct mechanism, a depositor in the service queue simply requests to withdraw or not. That is, the depositor communicates only his type: impatient if he wants to withdraw or patient if he does not.

Our indirect mechanism expands the message space to accommodate additional communications. In this way, we permit a depositor to communicate his belief that a run is on. We demonstrate that the threat of suspension conditional on this communication can eliminate the bank-run equilibrium. Our mechanism rewards the depositor for delivering such a message when a run is on. The reward is such that his payoff is higher compared to the payoff associated with concealing his belief that a run is on and making an early withdrawal—that is, misrepresenting his type and running with the other agents. Upon receiving such a message, the mechanism fully suspends all further redemption. The design of our mechanism ensures that a patient agent never has an incentive to run when a run is on or announce that he believes a run is on when it is not. At the end of the day, we are able to construct an indirect mechanism that, subject to a sufficient condition,<sup>5</sup> implements the constrained-efficient allocation in iterated elimination of strictly dominated strategies.<sup>6</sup>

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<sup>4</sup>This property was suggested by Wallace (1988) and later confirmed by [Green and Lin \(2003\)](#).

<sup>5</sup>The sufficient condition is that returns on investment cannot be too low. This sufficient condition is the same condition on returns imposed in the [Diamond and Dybvig \(1983\)](#) paper. In their paper it was imposed to ensure that the incentive-compatibility constraint on the patient agent is slack.

<sup>6</sup>It is also worth mentioning that the principles we propose here to prevent bank runs can be implemented by the private sector without government subsidies, regulations, or implicit guarantees.

*Literature review*

A number of papers have studied bank fragility under optimal arrangements in the [Diamond and Dybvig \(1983\)](#) setting. The standard setting has two important frictions. The first is private information about the depositor's type: patient or impatient. The second concerns the timing and utility of expenditure opportunities for impatient depositors. In particular, impatient depositors are assumed to arrive sequentially with redemption requests that are valued only if they are satisfied on demand. Implicitly, the idea is that some opportunities exist for only a short period of time so that delaying redemption requests (say) for the purpose of ascertaining aggregate withdrawal demand is too costly. The implication of this property is to make sequential service an optimal feature of liquidity providers.<sup>7</sup>

If sequential service is not needed, then it is possible to uniquely implement the efficient allocation—which implies there cannot be a bank-run equilibrium—as shown by [Green and Lin \(2003\)](#). If the private information constraint is relaxed, then it is also possible to uniquely implement the efficient allocation.

Private information and sequential service are thus necessary to generate bank runs under optimal arrangements; however, they are not sufficient. In the absence of aggregate risk, [Diamond and Dybvig \(1983\)](#) demonstrate that the first-best allocation can be uniquely implemented.<sup>8</sup> Unique implementation requires that the redemption schedule has a suspension feature: After a certain fraction of depositors make redemptions in the early period, the bank suspends payments for the remainder of the period. When aggregate risk is present, [Diamond and Dybvig \(1983\)](#) acknowledge that the first-best allocation cannot be implemented. They claim, however, that the institution of *deposit insurance* can be used to uniquely implement the first-best allocation. Their claim appears to be correct insofar as that the government has access to resources outside the banking sector. One merit of the mechanism we propose below is that bank runs can be prevented even without the use of resources outside of the banking sector. To the extent that deposit insurance induces inefficiencies owing to moral hazard ([Cooper and Ross \(2002\)](#)), our mechanism has the added benefit of bypassing these adverse incentive effects.

[Green and Lin \(2003\)](#) are the first to characterize the constrained efficient deposit contract under private information, sequential service, and aggregate uncertainty using a direct mechanism. [Green and Lin \(2003\)](#) depart from [Diamond and Dybvig \(1983\)](#) by assuming that the number of depositors in the economy is discrete and finite; [Diamond and Dybvig \(1983\)](#) assume a continuum of depositors of finite measure. This departure is rather important, as it makes the planner's allocation problem realistic and nontrivial. A continuum of depositors allows the planner to essentially “eliminate” the aggregate risk from the economy. More precisely, if there is a continuum of depositors, then the equilibrium allocation can be made arbitrarily close to the first-best allocation by offering the first  $\epsilon > 0$  depositors one contract and remaining depositors another. The first

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<sup>7</sup>We thank Giuseppe Moscarini and an anonymous referee for pointing out that sequential service is better thought of as a design feature rather than a property of the environment.

<sup>8</sup>The first-best allocation has the feature that all depositors' payments depend only on when they redeem, so, e.g., all depositors who redeem early receive the same payment.

contract is designed to elicit truth-telling as a dominant strategy for the  $\epsilon$  depositors. Hence, the first contract allows the planner to learn the truth state of the world (the fraction of impatient depositors) and the planner can use this information, along with the remaining resources, to implement the first-best allocation for the remaining depositors.<sup>9</sup> Since  $\epsilon$  can be made arbitrarily small, the planner can implement an allocation that is arbitrarily close to the first-best allocation for the given state of the world.

The allocation rule in [Green and Lin \(2003\)](#) allows early withdrawal payments in the sequential service queue to depend on the history of announcements—“impatient,” i.e., “I want to withdraw,” or “patient,” i.e., “I do not want to withdraw”—and aggregate payments to that point. The maximum withdrawal amount faced by an agent in the service queue is lower the larger is the number of preceding withdrawals. This partial suspension scheme is in stark contrast to [Diamond and Dybvig \(1983\)](#), who restrict the maximum withdrawal amount to be insensitive to realized withdrawal demand, so that resources are necessarily exhausted in the event of a run.<sup>10</sup> [Green and Lin \(2003\)](#) demonstrate that the planner can *uniquely* implement the constrained efficient allocation: The optimal bank contract does not admit a bank run.

Since [Green and Lin \(2003\)](#) impose a number of restrictions on their environment, one might wonder if these restrictions are (partly) responsible for their uniqueness result. To check the robustness of their result, [Peck and Shell \(2003\)](#) and [Ennis and Keister \(2009b\)](#) modify the [Green and Lin \(2003\)](#) environment. [Peck and Shell \(2003\)](#) modify the environment in at least two important ways. First, they alter the preferences so that incentive-compatibility (truth-telling) constraints bind at the constrained efficient allocation. Second, they assume that depositors do not know (or are not told) their position in the service queue ([Green and Lin 2003](#) assume that depositors know their position in the queue). If depositors do not know their queue position, then it is not possible to use the backward induction argument of [Green and Lin \(2003\)](#) to eliminate a bank-run equilibrium. [Peck and Shell \(2003\)](#) demonstrate, by example, that the optimal direct mechanism can have a bank-run equilibrium. [Ennis and Keister \(2009b\)](#) modify the [Green–Lin](#) environment by assuming the distribution of depositors types is correlated ([Green and Lin 2003](#) assume independence). They, too, demonstrate that the optimal direct mechanism can have a bank-run equilibrium.

[Green and Lin’s \(2003\)](#) direct mechanism uniquely implements the constrained efficient allocation (it prevents bank-run equilibria) so restricting the mechanism to be direct is without loss of generality regarding unique implementation. In the case of [Peck and Shell \(2003\)](#) and [Ennis and Keister \(2009b\)](#) it is not obvious that restricting attention to direct revelation mechanisms is without loss of generality because their direct mechanisms do not deliver uniqueness results. Perhaps an indirect mechanism is “better” in the sense that it can eliminate the bank-run equilibrium. Indeed, [Cavalcanti and Monteiro \(2016\)](#) examine indirect mechanisms in the [Ennis and Keister \(2009b\)](#) environment

<sup>9</sup>The initial  $\epsilon$  contract, which guarantees truth-telling, cannot be offered to all depositors because it would not satisfy the feasibility constraint. [De Nicolo \(1996\)](#) uses an  $\epsilon, 1 - \epsilon$  contracting setup to “eliminate” aggregate risk in his banking environment.

<sup>10</sup>[Wallace \(1990\)](#) reports that partial suspensions were prevalent in the banking panic of 1907, and that in one form or another must have been a feature of other suspension episodes as well.

and demonstrate that the best allocation can be uniquely implemented in rationalizable strategies. Unfortunately, the backward induction argument embedded in their uniqueness proof will not work in the more general Peck and Shell (2003) environment since depositors do not know their positions in the queue.<sup>11</sup> So it is still an open question if the bank-run equilibrium in Peck and Shell (2003) is a feature of their environment or if it can be prevented by some (possibly indirect) mechanism. This paper provides an answer to this question.

There is a mechanism design literature that studies how indirect mechanisms can help to implement optimal outcomes uniquely. Demski and Sappington (1984) examine a principal–two-agent setting where agents separately make production decisions when their costs are private and correlated. The optimal direct mechanism has two equilibria: a truth-telling equilibrium and a “cheating” equilibrium, where the cheating equilibrium leaves both agents better off and the principal worse off compared to the truth-telling equilibrium. Ma et al. (1988) show how an indirect mechanism can prevent agents from misrepresenting their types—or stop agents from cheating—in the Demski and Sappington (1984) model. Mookherjee and Reichelstein (1990) generalize their approach. Postlewaite and Schmeidler (1986) study a pure exchange economy where agents have private information over endowments and preferences. They produce an example where an indirect mechanism has a unique equilibrium yielding the desirable outcome while the corresponding direct mechanism possesses multiple equilibria. The indirect mechanism they propose extends the set of messages so agents can communicate the strategy profile they believe other agents are using. We use a similar idea in the construction of our indirect mechanism. Unfortunately, none of the aforementioned results can be applied directly to our banking problem because of complications that arise owing to our assumption of sequential service.

The paper is organized as follows. The next section describes the economic environment. Section 3 characterizes the best weakly implementable allocation. In Section 4 we construct an indirect mechanism and provide sufficient conditions for unique implementation of an allocation that is arbitrarily close to the best weakly implementable allocation. In Section 5, we examine numerical examples for which the sufficient conditions are not valid. Some policy remarks are offered in the final section.

## 2. ENVIRONMENT

There are three dates: 0, 1, and 2. The economy is endowed with  $Y > 0$  units of date-1 goods. A constant returns to scale investment technology transforms  $y$  units of date-1 goods into  $yR > y$  units of date-2 goods. There are  $N$  *ex ante* identical agents who turn out to be one of two types:  $t \in T = \{1, 2\}$ . We label a type  $t = 1$  agent “impatient” and a type  $t = 2$  agent “patient.” The number of patient agents in the economy is drawn from the distribution  $\pi = (\pi_0, \dots, \pi_N)$ , where  $\pi_n > 0$ ,  $n \in \mathbb{N} \equiv \{0, 1, \dots, N\}$ , is the probability

<sup>11</sup>One could consider constructing a mechanism that reveals the queue position to depositors and offers the contract given by Cavalcanti and Monteiro (2016). Such an arrangement, however, is not optimal. Providing more information to depositors increases the number of incentive-compatibility constraints that must be satisfied. This reduces the set of implementable allocations, which results in a reduction in welfare.

that there are  $n$  patient agents.<sup>12</sup> The probability density function (pdf)  $\pi$  is the sole source of aggregate uncertainty in the economy. A *queue* is a vector  $t^N = (t_1, \dots, t_N) \in T^N$ , where  $t_k \in T$  is the type of the agent that occupies the  $k$ th position or coordinate in the queue. Let  $P_n = \{t^N \in T^N \mid p(t^N) = n\}$  and  $Q_n(t^N) = \{j \mid t_j = 2 \text{ for } t^N \in P_n\}$ , where  $p(t^N) = \sum_n t_n - N$  denotes the number of patient agents in the queue  $t^N$ ,  $P_n$  is the set of queues with  $n$  patient agents, and  $Q_n(t^N)$  is the set of queue positions of the  $n$  patient agents in queue  $t^N \in P_n$ .<sup>13</sup> The probability of a queue  $t^N \in P_n$  is  $\pi_n / \binom{N}{n}$ , where the binomial coefficient,  $\binom{N}{n}$ , is the number of queues  $t^N \in P_n$ . In other words, all queues with  $n$  patient agents are equally likely. Agents are randomly assigned to a queue position, where the unconditional probability that an agent is assigned to position  $k$  is  $1/N$ . We label an agent assigned to position  $k$  agent  $k$ . The queue realization,  $t^N$ , is observed by no one; not by any of the agents or the planner. Agent  $k$  does not observe his queue position,  $k$ , but does privately observe his type  $t \in T$ . The utility function for an impatient agent is  $U(c^1, c^2; 1) = u(c^1)$  and the utility function of a patient agent is  $U(c^1, c^2; 2) = \rho u(c^1 + c^2)$ , where  $c^1$  is date-1 consumption and  $c^2$  is date-2 consumption. The function  $u$  is increasing, strictly concave, and twice continuously differentiable, and  $\rho > 0$  is a parameter.<sup>14</sup> Agents maximize expected utility.

The timing of events and actions is as follows. At date 0, the planner constructs a mechanism that determines how date-1 and date-2 consumption are allocated among the  $N$  agents. A mechanism consists of a set of announcements,  $M$ , and an allocation rule,  $c = (c^1, c^2)$ , where  $c^1 = (c_1^1, \dots, c_N^1)$  and  $c^2 = (c_1^2, \dots, c_N^2)$ . The planner can commit to the mechanism.<sup>15</sup>

Starting in date 1, agents meet the planner sequentially. Each agent  $k$  makes an announcement  $m_k \in M$ .<sup>16</sup> Only agent  $k$  and the planner can directly observe  $m_k$ . There is a sequential service constraint at date 1, which means the planner allocates date-1 consumption to agent  $k$  based on the announcements of agents  $j \leq k$ ,  $(m^{k-1}, m_k)$ , where  $m^{k-1} = (m_1, \dots, m_{k-1})$ , and each agent  $k$  consumes  $c_k^1(m^{k-1}, m_k)$  at his date-1 meeting with the planner. Date 1 ends after all agents meet the planner.

<sup>12</sup>The full support assumption is not crucial to any result. It is imposed only for simplicity.

<sup>13</sup>We omit the argument of  $Q_n(t^N)$  throughout the paper to reduce notational burden.

<sup>14</sup>These preferences are identical to those in [Diamond and Dybvig \(1983\)](#). In addition, they assume that  $\rho R > 1$  and  $\rho \leq 1$ .

<sup>15</sup>For a discussion of bank fragility in a setting without commitment, see [Ennis and Keister \(2009a\)](#). We conjecture that in an overlapping generations version of the current environment, where the planner lives forever and in each period a new generation of depositors is born, the commitment assumption can be relaxed. Specifically, we conjecture that our main implementation result survives even when the planner cannot commit, as long as he does not discount the future too heavily.

<sup>16</sup>One could imagine that the planner makes announcement  $a_k$  to agent  $k$  before  $k$  makes his announcement. For example, the planner could tell agent  $k$  his queue position, as in [Green and Lin \(2003\)](#), or the set of all messages sent in the previous  $k - 1$  planner-agent meetings, as in [Andolfatto et al. \(2007\)](#), or “nothing”,  $a_k = \emptyset$ , as in [Peck and Shell \(2003\)](#). The optimal mechanism, however, will have the planner announcing nothing. To reduce notation, and without loss of generality, we assume that the planner cannot make announcements to agents, unless otherwise specified. See footnote 19 for a discussion.

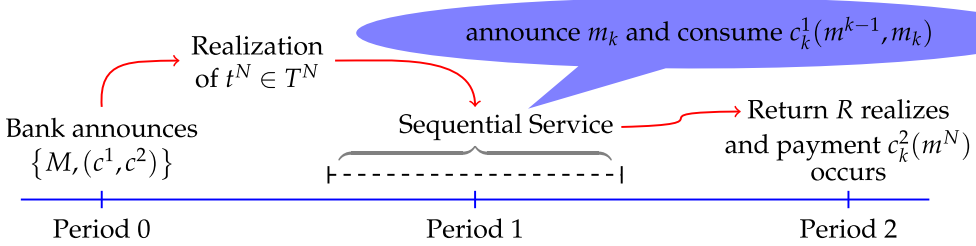


FIGURE 1. Sequence of actions.

While we model sequential service as a constraint on the mechanism, it is probably better thought of as part of a solution that is constrained by deeper aspects of the environment. So, for example, one could think of date 1 as having a continuum of subperiods. Each agent observes his preference type at a particular subperiod of date 1. This sequence of subperiods in which agents observe their preference type is summarized by the queue—those who observe their type first, end up first in the queue. But even though agents observe types in sequence, this itself does not imply a sequential service constraint since nothing in principle prevents the bank from collecting all announcements in sequence and only dispersing payments after all information is collected at the end of date 1. Evidently, an additional assumption is needed to motivate sequential service.

An assumption that seems plausible is that some consumption (or investment) opportunities generate high returns only in a very short window. One must strike while the iron is hot, so to speak. While delay is possible, it greatly diminishes the value of the opportunity. The opportunity to purchase an umbrella after the storm has passed might still have some value, but not as much as before the storm clouds began to gather. Formally, and for simplicity, we model the consumption opportunity at date 1 as completely transitory, in which case sequential service is critical for an efficient risk-sharing arrangement.<sup>17</sup>

In between dates 1 and 2 the planner's resources are augmented by a factor of  $R$ . At date 2, the planner allocates the date-2 consumption good to each agent based on the date-1 announcements, i.e., agent  $k$  receives  $c_k^2(m^N)$ , where  $m^N = (m_1, \dots, m_N) \in M^N$ . Figure 1 depicts the sequence of actions.

### 3. THE BEST WEAKLY IMPLEMENTABLE OUTCOME

An allocation is weakly implementable if it is an equilibrium outcome of a mechanism; it is strongly or uniquely implementable if it is the unique equilibrium outcome of a mechanism. Among the set of weakly implementable allocations, the best weakly implementable allocation provides agents with the highest expected utility. To characterize the best weakly implementable allocation, by the revelation principle, it is without loss of generality to restrict the planner to use a direct revelation mechanism, where

<sup>17</sup>We would like to thank Giuseppe Moscarini and a referee for suggesting this interpretation.



agents only announce  $m_k = t_k \in T = \{1, 2\}$ . It is important to emphasize that the revelation principle is silent on unique implementation; it only allows us to conclude that an equilibrium is weakly implementable.

The welfare—which we measure as the expected utility of an agent before he learns his type—associated with allocation rule  $c$  when agents use strategies  $m_k \in T$  is

$$\sum_{n=0}^N \frac{\pi_n}{\binom{N}{n}} \sum_{t^N \in P_n} \sum_{k=1}^N U[c_k^1(m^{k-1}, m_k), c_k^2(m^N); t_k]. \tag{1}$$

The allocation rule  $c = (c_1, c_2)$  is *feasible* if for all  $m^N \in T^N$ ,

$$\sum_{k=1}^N [Rc_k^1(m^{k-1}, m_k) + c_k^2(m^N)] \leq RY. \tag{2}$$

The best weakly implementable allocation has all agents  $k$  announcing truthfully, i.e.,  $m_k = t_k$ . Allocation rule  $c$  must be incentive compatible in the sense that agent  $k$  has no reason to announce  $m_k \neq t_k$ . Since impatient agents  $k$  only value date-1 consumption, they always announce  $m_k = 1$ .<sup>18</sup> Patient agent  $k$  has no incentive to defect from the strategy  $m_k = 2$ , assuming that all other agents announce truthfully, if

$$\begin{aligned} & \sum_{n=1}^N \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho \{ u[c_k^1(t^{k-1}, 2) + c_k^2(t^N)] \\ & - u[c_k^1(t^{k-1}, 1) + c_k^2(t^{k-1}, 1, t_{k+1}^N)] \} \geq \delta, \end{aligned} \tag{3}$$

where, for any vector  $x^N = (x_1, \dots, x_N)$ ,  $x_i^j$  denotes  $(x_i, \dots, x_j)$ ,  $\delta \geq 0$  is a parameter, and

$$\hat{\pi}_n = \frac{\pi_n / \binom{N}{n}}{\sum_{n=1}^N \pi_n / \binom{N}{n}}$$

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<sup>18</sup>This anticipates the result that the best weakly implementable allocation provides zero date-1 consumption to agents who announce that they are patient, which implies that the incentive-compatibility constraint for impatient agents is always slack.

is the conditional probability that agent  $k$  is in a specific queue with  $n$  patient agents.<sup>19</sup> The  $1/n$  term that appears in (3) reflects that a patient agent has a  $1/n$  chance of occupying each of the patient queue positions in  $Q_n$ .

The best weakly implementable allocation is given by the solution to

$$\max (1) \text{ subject to } (2) \text{ and } (3) \text{ with } \delta = 0, \quad (4)$$

where  $m_k = t_k$  for all  $k \in \mathbb{N}$ . With some abuse of notation, let  $c(\delta) = (c^1(\delta), c^2(\delta))$  denote the solution to problem (4) and let  $W(\delta)$  denote its maximum. Hence, the best weakly implementable allocation is  $c(0) = (c^1(0), c^2(0))$  and  $W(0)$  is the welfare associated with the best weakly implementable allocation.<sup>20</sup>

In our analysis we restrict  $\delta > 0$  to guarantee that the incentive compatibility holds in an open neighborhood of  $c(\delta)$ . The existence of such a neighborhood is necessary for our uniqueness result, but  $\delta > 0$  can be made arbitrarily small. Therefore, we can apply Berge's maximum theorem, which says that  $W(0)$  is approximated by  $W(\delta)$  when  $\delta$  is close to zero. When  $\delta > 0$ , we refer to  $c(\delta)$  as the  $\delta$ -best weakly implementable allocation.

The allocation rule  $c(\delta)$  that solves (4) has the following features: (i) an agent  $k$  who announces  $m_k = 1$  consumes only at date 1, that is,  $c_k^2(\delta)(m_1, \dots, m_{k-1}, 1, m_{k+1}, \dots, m_N) = 0$  for all  $k \in \mathbb{N}$ ; (ii) an agent  $k$  who announces  $m_k = 2$  consumes only at date 2, that is,  $c_k^1(\delta)(m_1, \dots, m_{k-1}, 2) = 0$  for all  $k \in \mathbb{N}$ ; and (iii) all agents  $j$  and  $k$  announcing  $m_j = m_k = 2$  consume identical amounts at date 2, that is,  $c_j^2(\delta)(m^N) = c_k^2(\delta)(m^N)$  for all  $m_j = m_k = 2$ .

Define a bank run as an equilibrium of the direct revelation mechanism  $\{T, c(\delta)\}$  where one or more patient agents misrepresent their type. Both Peck and Shell (2003) and Ennis and Keister (2009b) demonstrate, by way of example, that the direct revelation mechanism  $\{T, c(0)\}$  can have two equilibria: one truth-telling and one bank run.<sup>21</sup> In other words,  $\{T, c(0)\}$  weakly implements  $c(0)$  but does not strongly implement it. In contrast, we show that there is an indirect mechanism that strongly implements  $c(\delta)$

<sup>19</sup>To characterize the best weakly implementable allocation, one wants to choose from the largest possible set of incentive-compatible allocations. This implies the planner should not make any announcements, as noted in footnote 16. In particular, if the planner does not make any announcements, then there is only one incentive-compatibility constraint for all patient agents, (3). If, however, the planner did make an announcement  $a_k$  to agent  $k$ , there will be additional incentive constraints for the agent who received the announcement. For example, suppose that  $a_k = k$  for all  $k$ , i.e., the planner announces to each agent his place in the queue. Then there would be  $N$  incentive-compatibility constraints for patient agents, one for each queue position. Since an appropriately weighted average of these distinct incentive constraints implies (3), the set of incentive-compatible allocations when the planner makes announcements is a subset of the set of incentive-compatible allocations when he does not. By not making any announcements, the planner is able to choose from a larger set of incentive feasible allocations.

<sup>20</sup>The best weakly implementable allocation  $c(0)$  corresponds to the allocation rule derived in Peck and Shell (2003), Appendix B.

<sup>21</sup>The Ennis and Keister (2009b) bank-run example is in Section 4.2 of their paper. There, agents do not know their position in the queue, as in Peck and Shell (2003), and the utility functions of patient and impatient agents are the same as in Green and Lin (2003), who assume that  $\rho = 1$ . Peck and Shell (2003) assume that  $\rho < 1$ .

for any  $\delta$  greater than 0. Because  $\delta$  can be made arbitrarily small, our indirect mechanism can strongly implement an allocation that is arbitrarily close to the best weakly implementable allocation. In other words, our mechanism prevents bank runs at an arbitrarily small welfare cost.

#### 4. AN INDIRECT MECHANISM

Consider an indirect mechanism  $\{\hat{M}, \hat{c}\}$ , where  $\hat{M} = \{1, 2, g\}$  and  $\hat{c}$  is described below. The basic construction of the allocation rule  $\hat{c}$  uses the  $\delta$ -best weakly implementable allocation  $c(\delta)$ , where  $\delta > 0$  is arbitrarily small. If agent  $j$  announces  $\hat{m}_k = 1$ , then

$$\hat{c}_k^1(\hat{m}^{k-1}, 1) = \begin{cases} c_k^1(\delta)(\hat{m}^{k-1}, 1) & \text{if } \forall j < k : \hat{m}_j \in \{1, 2\} \\ 0 & \text{if } \exists j < k : \hat{m}_j = g \end{cases} \quad \text{and} \quad (5)$$

$$\hat{c}_k^2(\hat{m}^{k-1}, 1, \hat{m}_{k+1}^N) = 0.$$

An agent  $k$  announcing  $\hat{m}_k = 1$  receives the date-1 consumption payoff under the direct revelation mechanism  $\{T, c(\delta)\}$  *only if all earlier agents  $j < k$  announce either  $\hat{m}_j = 1$  or  $\hat{m}_j = 2$* ; otherwise he receives zero. The latter implies that there is a suspension of first period payments after an agent  $j < k$  announces  $\hat{m}_j = g$ . The date-2 consumption payoff associated with the announcement  $\hat{m}_k = 1$  is zero, as in the direct revelation mechanism  $\{T, c(\delta)\}$ . If agent  $k$  announces  $\hat{m}_k = g$ , then

$$\hat{c}_k^1(\hat{m}^{k-1}, g) = 0 \quad \text{and} \quad \hat{c}_k^2(\hat{m}^{k-1}, g, \hat{m}_{k+1}^N) = \hat{c}_k^1(\hat{m}^{k-1}, 1) + \epsilon, \quad (6)$$

where  $\epsilon > 0$  is an arbitrarily small number. To keep the presentation simple, we assume throughout this paper that  $\epsilon$  is taken small enough so all results hold. If agent  $k$  announces  $\hat{m}_k = g$ , then he receives a zero payoff at date 1. At date 2, he receives a payoff that is slightly larger than the date-1 payoff he would receive by announcing  $\hat{m}_k = 1$ ; see (6). Hence, announcing  $\hat{m}_k = g$  strictly dominates announcing  $\hat{m}_k = 1$  for any patient agent  $k$ . Note that this is key to our unique implementation result. The mechanism is designed so that announcing  $\hat{m}_k = 1$  is a dominated strategy for patient agents. Finally, if agent  $k$  announces  $\hat{m}_k = 2$ , then

$$\hat{c}_k^1(\hat{m}^{k-1}, 2) = 0 \quad \text{and}$$

$$\hat{c}_k^2(\hat{m}^{k-1}, 2, \hat{m}_{k+1}^N) = \frac{R \left[ Y - \sum_{j=1}^N \hat{c}_j^1(\hat{m}^j) \right] - \sum_{j=1}^N \hat{c}_j^2(\hat{m}^N) \mathbb{1}_{\hat{m}_j=g}}{\hat{p}(\hat{m}^N)}, \quad (7)$$

where  $\hat{p}(\hat{m}^N)$  represents the number of  $\hat{m} = 2$  announcements in the vector  $\hat{m}^N$  and  $\mathbb{1}_{\hat{m}_j=g}$  is an indicator function. If agent  $k$  announces  $\hat{m}_k = 2$ , then he receives a  $1/\hat{p}(\hat{m}^N)$  share of the total output in date 2 that remains after payments to agents  $j$  who announced either  $m_j = 1$  or  $m_j = g$  are made. Since the allocation rule  $\hat{c}$ , given by (5)–(7), depends on  $\delta$  and  $\epsilon$ , we denote it as  $\hat{c}(\delta, \epsilon)$ .

Generally speaking, a patient agent  $j$  who announces  $m_j = 1$  adversely affects the payoffs of truthfully announcing patient agents in two ways. First, the payments to an agent who announces  $m_j = 1$  are made in period 1, which implies that these resources cannot benefit from the investment opportunity,  $R$ , available between dates 1 and 2. Second, if impatient agents have a relatively high marginal utility of consumption compared to patient agents, i.e.,  $\rho$  is small, then, due to risk-sharing considerations, payments to agents who announce  $m_j = 1$  can be quite high, leading to less resources available to the patient agents. Interestingly, the story is a bit different when patient agent  $j$  announces  $\hat{m}_j = g$  and impatient agents have a relatively low marginal utility of consumption compared to patient agents. Following a  $g$  announcement, there is a suspension of date-1 payments and agents who announce  $g$  receive their payments at date 2. Hence, all suspended payments benefit from the investment opportunity that is available between dates 1 and 2, and patient agents who announced truthfully will receive a fraction of the investment return,  $R$ . In addition, if  $\rho$  is relatively large, then the date-2 payment to agent  $j$  (who announced  $\hat{m}_j = g$ ) will be relatively low, which is beneficial for truth-telling patient agents.

Patient agent  $k$  who announces truthfully will benefit from announcement  $m_j = g$  if the allocation rule  $\hat{c}(\delta, \epsilon)$  has the property

$$\hat{c}_k^2(\delta, \epsilon)(\hat{m}^{k-1}, 2, \hat{m}_{k+1}^N) \geq \hat{c}_k^2(\delta, \epsilon)(\hat{i}^{k-1}, 2, \hat{i}_{k+1}^N) = c_k^2(\delta)(\hat{i}^{k-1}, 2, \hat{i}_{k+1}^N), \quad (\text{P1})$$

where  $\hat{i}^i \in T^i$  ( $\hat{i}_i^N \in T_i^N$ ) is a vector of length  $i$  ( $T - i$ ) such that for each  $j \leq i$  ( $i \leq j \leq N$ ),  $\hat{i}_j = 1$  if  $\hat{m}_j = 1$  and  $\hat{i}_j = 2$  if either  $\hat{m}_j = 2$  or  $\hat{m}_j = g$ . In words, vector  $\hat{i}^i$  ( $\hat{i}_i^N$ ) is constructed from the message vector  $\hat{m}^i$  ( $\hat{m}_i^N$ ) by replacing all of the  $g$ s with 2s. The first term in (P1) is the payoff to a truthfully announcing patient agent when some (patient) agents announce  $g$ . The second term is the payoff to patient players when those  $g$  announcements are replaced by 2, which, by construction, also equals the payment from the  $\delta$ -best implementable allocation. If the contract  $\hat{c}(\delta, \epsilon)$  is characterized by property (P1), then, clearly, a truthfully announcing patient agent benefits if some other (patient) agent announces  $g$ . In fact, his payoff will exceed that associated with the  $\delta$ -best weakly implementable allocation,  $c(\delta)$ .

Under what circumstances does the allocation rule  $\hat{c}(\delta, \epsilon)$  have property (P1)? The earlier discussion suggests that truthfully announcing patient agents benefit from an  $m_j = g$  announcement the larger is  $R$  and/or the larger is  $\rho$ . (Recall that the higher is  $\rho$ , the smaller will be the payments to impatient agents.) Our first proposition verifies this intuition.

**PROPOSITION 1.** *If  $\rho R > 1$ , then property (P1) holds.*

For the proof, see the [Appendix](#).

Interestingly, [Diamond and Dybvig \(1983\)](#) assume that  $\rho R > 1$ . In their environment with no aggregate uncertainty, the condition  $\rho R > 1$  ensures that a patient agent has no incentive to announce that he is impatient when all other agents are announcing truthfully. The intuition behind this condition in [Diamond and Dybvig \(1983\)](#) is similar to our earlier discussion: A higher  $\rho$  and/or  $R$  increases a patient agent's payoff relative to an

impatient agent’s payoff and when  $\rho R > 1$ , the relative difference between the payoffs is sufficiently large to ensure that the patient agent has a strict incentive to announce the truth (when all other agents are announcing the truth). Property (P1) and, hence, the condition  $\rho R > 1$ , has a similar incentive-compatibility interpretation for our environment. Intuitively, property (P1) suggests that patient agent  $k$  has no incentive to announce  $\hat{m}_k = g$  when other patient agents  $j$  are announcing *either*  $\hat{m}_j = 2$  or  $\hat{m}_j = g$ . In particular, property (P1),  $\hat{c}_k^2(\delta, \epsilon)(\hat{m}^{k-1}, 2, \hat{m}_{k+1}^N) \geq c_k^2(\delta)(\hat{t}^{k-1}, 2, \hat{t}_{k+1}^N)$ , implies that patient agent  $k$  has no incentive to announce  $\hat{m}_j = 1$  because the  $\delta$ -best weakly implementable contract  $c(\delta)$  is strictly incentive compatible. Since the payoff associated with announcing  $\hat{m}_k = g$  is only slightly higher than announcing  $\hat{m}_k = 1$ , patient agent  $k$  has no incentive to announce  $\hat{m}_k = g$ . Our main proposition demonstrates that this intuition is, in fact, correct.

**PROPOSITION 2.** *If property (P1) holds, then the indirect mechanism  $\{\hat{M}, \hat{c}(\delta, \epsilon)\}$  strongly implements allocation  $c(\delta)$  in rationalizable strategies.*

**PROOF.** The mechanism  $\{\hat{M}, \hat{c}(\delta, \epsilon)\}$  induces a symmetric Bayesian game  $\Gamma = \{T, S\}$ , where  $T = \{1, 2\}$  is the set of types,  $s_t \in \hat{M}$  is the player’s message contingent on his type  $t \in T$ , and  $S = \{(s_1, s_2) \in \hat{M}^2\}$  is the set of pure strategies. We solve the game by iterated elimination of strictly dominated strategies in two rounds.

*Round 1.* Any strategy  $(s_1, s_2) \in S$  with  $s_1 \neq 1$  is strictly dominated by  $(1, s_2)$  since, contingent on being impatient, an agent only derives utility from period 1 consumption. Additionally, any strategy  $(s_1, 1)$  is strictly dominated by  $(s_1, g)$  since, contingent on being patient, agents are indifferent between period 1 or period 2 consumption and announcing  $g$  always gives a total payment that is  $\epsilon$  higher than announcing 1. Let  $S^1 = \{(1, 2), (1, g)\}$  denote the set of strategies that survive the first round of elimination of strictly dominated strategies.

*Round 2.* When strategies are restricted to  $S^1$ , impatient agents announce 1 and patient agents announce either 2 or  $g$ . From property (P1), the lower bound on the expected payoff to a patient player who announces 2 is

$$\sum_{n=1}^N \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho u(c_k^{2*}(\delta)(t^{k-1}, 2, t_{k+1}^N)).$$

Since the payment to agent  $k$  who announces  $m_k = g$  is either  $c_k^{1*}(t^{k-1}, 1) + \epsilon$  or  $\epsilon$ , the expected payoff to a patient player who announces  $g$  is bounded above by

$$\sum_{n=1}^N \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho u(c_k^{1*}(\delta)(t^{k-1}, 1) + \epsilon).$$

Since  $u$  is continuous, there exists an  $\epsilon > 0$  sufficiently small so that

$$\sum_{n=1}^N \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} \{\rho u(c_k^{1*}(\delta)(t^{k-1}, 1) + \epsilon) - \rho u(c_k^{1*}(\delta)(t^{k-1}, 1))\} < \delta.$$

The incentive-compatibility condition (3) can be rewritten as

$$\begin{aligned} & \sum_{n=1}^N \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho u(c_k^{2*}(\delta)(t^{k-1}, 2, t_{k+1}^N)) \\ & \geq \sum_{n=1}^N \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho u(c_k^{1*}(\delta)(t^{k-1}, 1)) + \delta. \end{aligned}$$

Combining the above two inequalities, we get

$$\begin{aligned} & \sum_{n=1}^N \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho u(c_k^{2*}(\delta)(t^{k-1}, 2, t_{k+1}^N)) \\ & > \sum_{n=1}^N \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho u(c_k^{1*}(\delta)(t^{k-1}, 1) + \epsilon). \end{aligned}$$

Therefore, the strategy  $(1, g)$  is strictly dominated by the strategy  $(1, 2)$  in  $S^1$ . Let  $S^2$  be the set of strategies that survive the second round of elimination of strictly dominated strategies. Since  $S^2 = \{(1, 2)\}$  is a singleton, the game is iterated strict dominance solvable. The unique equilibrium strategy is the truth-telling  $s = (1, 2)$ , which implies the same outcome as the truth-telling equilibrium of the direct mechanism  $\{T, c^*(\delta)\}$ .  $\square$

If allocation  $\hat{c}(\delta, \epsilon)$  has property (P1), then the mechanism  $\{\hat{M}, \hat{c}(\delta, \epsilon)\}$  admits only one equilibrium characterized by truth-telling for all agents. Hence, mechanism  $\{\hat{M}, \hat{c}(\delta, \epsilon)\}$  does not allow bank runs. In addition, the allocation delivered by the mechanism,  $\hat{c}(\delta, \epsilon)$ , can be made arbitrarily close to the best weakly implementable allocation  $c(0)$  by choosing  $\delta$  arbitrarily close to zero. Together, Propositions 1 and 2 imply that a sufficient condition for unique implementation is  $\rho R > 1$ . We want to emphasize that conditions stated in Propositions 1 and 2 are only sufficient conditions. Regarding Proposition 1, one can see from the proof that if incentive-compatibility condition (3) does not bind, then the condition  $\rho R > 1$  is *not* necessary. This means that contract  $\hat{c}(\delta, \epsilon)$  can be consistent with property (P1) even if  $\rho R < 1$ . In the subsequent section, we provide an example of this (even when the incentive-compatibility condition (3) binds). Regarding Proposition 2, property (P1) allows us to derive a lower bound on the expected payoff of a patient agent announcing  $m = 2$  and, therefore, to use dominance arguments to demonstrate uniqueness. But neither such a lower bound or dominance arguments are necessary for uniqueness. In the subsequent section we provide an example where contract allocation  $\hat{c}(\delta, \epsilon)$  does not have property (P1) but the indirect mechanism  $\{\hat{M}, \hat{c}(\delta, \epsilon)\}$  uniquely implements  $\hat{c}(\delta, \epsilon)$ .

### 5. SOME EXAMPLES

In this section, we provide some examples that show that the sufficient conditions described in Propositions 1 and 2 are not necessary for unique implementation of the allocation rule  $c^*(\delta)$ . The first example shows that property (P1) can hold when  $\rho R < 1$ .

A second example shows that allocation rule  $c^*(\delta)$  can be uniquely implemented when property (P1) is violated.

Common to all examples are (i)  $R = 1.05$ , (ii)  $Y = 6$ , (iii)  $\rho R < 1$ , (iv)  $\delta = 10^{-10}$ , and (v) that the general structure of preferences is given by<sup>22</sup>

$$u(x) = \frac{(x+1)^{1-\gamma} - 1}{1-\gamma}, \quad \gamma > 1.$$

In the first example,  $N = 2$ ,  $\rho = 0.9$ ,  $\gamma = 1.01$ , and  $(\pi_0, \pi_1, \pi_2) = (0.005, 0.4975, 0.4975)$ . Notice that  $\rho R < 1$ . The best weakly implementable allocation,  $c^*(0)$ , which is obtained by solving (4), has  $c_1^*(1) = 3.1487$  and  $c_2^*(2, 1) = 3.1481$ . The other payments can be derived from the resource constraint (2) holding at equality. It is straight forward to show that the direct mechanism  $\{T, c^*(0)\}$  admits a bank-run equilibrium for this example. For  $\epsilon$  arbitrarily small, property (P1) holds, even though  $\rho R < 1$ . Therefore, although  $\rho R > 1$  is a sufficient condition for property (P1), it is not a necessary one. Since property (P1) is satisfied in this example, Proposition 2 implies that  $\{\hat{M}, \hat{c}(\delta, \epsilon)\}$  uniquely implements allocation  $c^*(\delta)$  for  $\delta$  and  $\epsilon$  small. In this example, constraint (3) binds. This implies that incentive constraints in the Green and Lin (2003) environment—where agents know their queue positions—also bind and that the best implementable allocation from that environment is not equal to  $c^*(0)$ .<sup>23</sup> Hence, the Green and Lin (2003) mechanism is unable to even weakly implement the allocation  $c^*(\delta)$ , where  $\delta$  is arbitrarily small.

The second example replicates the Peck and Shell (2003) example in Appendix B. The only difference between the examples is the specification of preferences. Peck and Shell (2003) assume that  $u(x) = c^{1-\gamma}/(1-\gamma)$ , which implies that  $u(0) = -\infty$ . For these preferences, our mechanism trivially uniquely implements allocation  $c^*(\delta)$ , since patient agent  $k$  will never announce  $m_k = g$  if there is a probability, however small, that some other agent  $j$  will announce  $m_j = g$ . The parameters for our second example are  $N = 2$ ,  $\rho = 0.1$ ,  $\gamma = 2$ , and  $(\pi_0, \pi_1, \pi_2) = (0.25, 0.5, 0.25)$ . Notice that  $\rho R < 1$ . The best weakly implementable allocation,  $c^*(0)$ , is characterized by  $c_1^*(1) = 3.0951$  and  $c_2^*(2, 1) = 3.1994$ . Allocation  $c^*(0)$  features bank runs and a binding incentive constraint (3). It is straightforward to demonstrate that the mechanism  $\{\hat{M}, \hat{c}(\delta, \epsilon)\}$  uniquely implements allocation  $c^*(\delta)$  for  $\delta$  and  $\epsilon$  arbitrarily close to zero. For this example,  $c_1^*(2, 1) + c_2^*(2, 2) > RY$ , which implies that property (P1) is not satisfied for all  $\hat{m}^N \in \hat{M}^N$ . Hence, property (P1) is not necessary for unique implementation. We are not aware of any mechanism in the literature that can implement the best weakly implementable allocations from these two examples. We have experimented with many combinations of model parameters. We are unable to find a set of parameters for which the indirect mechanism  $\{\hat{M}, \hat{c}(\delta, \epsilon)\}$  cannot uniquely implement an allocation that is arbitrarily close to the best weakly implementable allocation. Our search, however, was restricted to  $N \in \{2, 3\}$ . It is, of course, possible that the indirect mechanism  $\{\hat{M}, \hat{c}(\delta, \epsilon)\}$

<sup>22</sup>Notice that  $u(0) = 0$ .

<sup>23</sup>Our environment can be turned into the Green and Lin (2003) environment by allowing the planner to tell agent  $k$  his queue position,  $k$ , before agent  $k$  makes his announcement.

does not uniquely implement the best weakly implementable allocation for some set of parameters when  $\rho R \leq 1$ , but we were unable to construct such an example.

## 6. DISCUSSION

The most common prescription for enhancing the stability of demandable debt is to modify the contract to include a partial suspension clause. For example, [Cochrane \(2014\)](#), suggests that if securities are designed so that debtors have the right to delay payment, suspend convertibility, or pay in part, then it is much harder for a run to develop. [Santos and Neftci \(2003\)](#) recommend the use of extendable debt—which is a suspension in payments—in the sovereign debt market to help mitigate the frequent debt crises that have afflicted emerging economies and, recently, more advanced economies as well. In June 2014, the Securities and Exchange Commission (SEC) announced a set of proposals to help stabilize money market funds (MMFs). One of the key proposals recommends that the MMF board of directors have the discretion to impose penalty redemption fees and redemption gates—or suspension of payments—in times of heavy redemption activity.

The effect of such proposals is to render demandable debt more state-contingent. In this sense, the proposals above are consistent with the properties of the optimal debt contracts described in [Diamond and Dybvig \(1983\)](#), [Green and Lin \(2003\)](#), and [Peck and Shell \(2003\)](#). But given that bank-run equilibria remain a possibility in the latter models, one is led to question whether the use of such measures constitutes only necessary, and not sufficient conditions, for stability.

The key question concerns the issue of precisely *what* information is used to condition the suspension/extension clause. In the [Diamond and Dybvig \(1983\)](#) model without aggregate risk, suspension is triggered when “reserves” reach a predetermined critical level. Evidently, this conditioning factor is sufficient to prevent runs in that environment. Similarly, the partial suspension schedules described in [Green and Lin \(2003\)](#) and [Peck and Shell \(2003\)](#) are triggered by measures of reserve depletion (more precisely, the history of reported types). In reality, the volatility of redemption rates varies across different classes of MMFs. [Schmidt et al. \(2013\)](#), for example, report that MMFs with volatile flow rates prior to the financial crisis of 2008 were more likely to experience runs during the crisis. How are directors of these funds to ascertain whether a spike in redemptions is attributable to fear rather than fundamentals? Our indirect mechanism suggests that information beyond some measure of redemption activity or resource availability is needed to prevent the possibility of a bank run. We need to know *why* depositors are exercising their redemption option. For better or worse, this information is private and must therefore be elicited directly—as in our model—or inferred indirectly—through some other means. Of course, information revelation must be incentive compatible.

Just how realistic is this idea? There is, in fact, historical precedence for the practice of soliciting additional information in periods of heavy redemption activity. For example, banks would sometimes permit limited redemptions to occur for depositors who could demonstrate evidence of impatience, e.g., a need to meet payroll. In footnote



7, [Gorton \(1985\)](#) reports that 19th century clearinghouses would regularly investigate rumors pertaining to the financial health of member banks.

As a practical matter, the spirit of our mechanism could be implemented in several different ways and without any new government regulation. One way would be to permit depositors to pay a small fee for the right to have their funds diverted to a segregated, priority account.<sup>24</sup> Such an action could be interpreted as a communication of an impending run. The priority debt differs from other debt only in the event of failure and the ratio of priority to nonpriority debt outstanding informs the issuer on the degree to which depositors expect the bank to fail. In principle, the suspension clause could be made conditional on this ratio hitting some specified threshold. It does not need to be official as long as there is a mutual understanding that it will be used. And along the lines suggested by our mechanism, if one knows that the bank will suspend before any rumor-induced trouble affects their balance sheet, then depositors know that there will be no reason, in equilibrium, to actually exercise the option of converting their claims to priority debt.

To summarize, current proposals designed to prevent, or at least mitigate, bank runs in demandable debt structures focus on enhancing state contingency, with contingencies dictated by some measure of redemption activity or resource depletion. Our analysis suggests that while state contingency is necessary, it may not be sufficient to prevent bank runs. Suspension clauses should be conditioned on information relating to depositor beliefs about what they perceive to be happening around them. The desired information could be elicited in an incentive-compatible manner through an appropriate modification of the deposit contract—an example of which we described above. The inclusion of such a clause may help to prevent bank runs in debt structures that are presently run-prone.

However, as far as practical application is concerned, policymakers need to understand that, as with other suspension schemes, our proposal requires a high degree of commitment on the part of the banking system or regulatory authority to suspend payments under the stated conditions. In our model, depositors are assumed never to make “mistakes” when reporting what they know to the mechanism. In reality, of course, this may not be the case. Threatening a bank holiday and then following through on the threat because of a mistaken report would be highly undesirable, to say the least. As with any policy conclusion that follows from theory, policymakers must assess the extent to which the assumptions underlying the policy recommendation are reasonably well approximated in reality.

It always appealing to use theory for policy analysis, as we did above, but there is another way to interpret the presented results. Namely, that there is something missing in the [Diamond and Dybvig \(1983\)](#) theory of bank runs. This point was first made by [Green and Lin \(2003\)](#). Anyone who wants to use [Diamond and Dybvig \(1983\)](#) to explain historical episodes of bank runs must provide a consistent theory of why the banks operating during those episodes did not take advantage of contracts capable of preventing runs (as the one we propose here).

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<sup>24</sup>This is effectively what happens in our mechanism when a depositor reports  $m = g$ .

APPENDIX: PROVING PROPOSITION 1

So as to prove Proposition 1, we first establish the following result.

LEMMA 1. *If  $\rho R > 1$ , then  $c_k^1(\delta)(\bar{t}^{k-1}, 1) < c^2(\delta)(\bar{t}^{k-1}, \mathbf{2}^{N-k+1})$  for all  $k \in \mathbb{N}$  and  $\bar{t}^{k-1} \in T^{k-1}$ , where  $\mathbf{2}^n$  denotes the  $n$ -dimensional vector of 2s.*

PROOF. From now on we denote  $c(\delta)$  just by  $c$  to keep the notation short. Since  $c$  solves problem (4), it satisfies the implied first-order conditions. Let  $\lambda_{t^N}$  denote the Lagrange multiplier of the feasibility constraint (2) for each  $t^N \in T^N$  and let  $\mu$  denotes the Lagrange multiplier of the incentive compatibility (3). By simplicity,  $\lambda_{t^N}$  is normalized by  $\pi_{p(t^N)}/\binom{N}{p(t^N)}$ , where  $p(t^N)$  denotes the number of type 2 players in queue  $t^N$ , and  $\mu$  is normalized by  $\bar{\pi} = \sum_{n=1}^N \pi_n/\binom{N}{n}$ . Since  $u'(0) = \infty$ , the constraints  $c^1 \geq 0$  and  $c^2 \geq 0$  are not binding and the respective Lagrange multipliers can be ignored. The first-order conditions of the problem are

$$\begin{aligned}
 [c_k^1(\bar{t}^k)]: \quad & \sum_{n=0}^N \frac{\pi_n}{\binom{N}{n}} \sum_{\substack{t^N \in P_n \\ t^k = \bar{t}^k}} \{u'[c_k^1(\bar{t}^k)] - \lambda_{t^N} R\} \\
 & - \sum_{n=1}^N \frac{\pi_n}{\binom{N}{n}} \sum_{\substack{t^N \in P_n \\ t^k = (\bar{t}^{k-1}, 2)}} \frac{\mu \rho}{p(t^N)} u'[c_k^1(\bar{t}^k)] = 0
 \end{aligned} \tag{8}$$

for all  $k \in \mathbb{N}$  and  $\bar{t}^{k-1} \in T^{k-1}$  such that  $\bar{t}_k = 1$ , and

$$[c^2(t^N)]: \quad \frac{\pi_{p(t^N)}}{\binom{N}{p(t^N)}} \left\{ \rho u'[c^2(t^N)] - \lambda_{t^N} + \frac{\mu \rho}{p(t^N)} u'[c^2(t^N)] \right\} = 0$$

for all  $t^N \in T^N$  such that  $p(t^N) > 0$ . We can solve the above equations for  $\lambda_{t^N}$  and obtain

$$\lambda_{t^N} = \begin{cases} \rho \left( 1 + \frac{\mu}{p(t^N)} \right) u'[c^2(t^N)] & \text{if } p(\bar{t}^N) > 0, \\ \frac{1}{R} u'[c_N^1(t^N)] & \text{if } p(\bar{t}^N) = 0. \end{cases}$$

Note that  $c^2(t^N)$  is not defined if  $t^N = \mathbf{1}^N = (1, 1, \dots, 1)$ : there is no second period payments when every depositor announces to be of impatient type in the first period. To keep the notation short, let us define  $u'[c_N^1(\mathbf{1}^N)] = \rho R u'[c^2(\mathbf{1}^N)]$  and  $1/p(\mathbf{1}^N) = 0$ . Then  $\lambda_{t^N}$  is given by

$$\lambda_{t^N} = \rho \left( 1 + \frac{\mu}{p(t^N)} \right) u'[c^2(t^N)]. \tag{9}$$

After replacing (9) in (8), we obtain that for all  $k \in \mathbb{N}$  and  $\bar{t}^k = (\bar{t}^{k-1}, 1) \in T^{k-1}$ ,

$$\begin{aligned} & \sum_{n=0}^N \frac{\pi_n}{\binom{N}{n}} \sum_{\substack{t^N \in P_n \\ t^k = \bar{t}^k}} u'[c_k^1(\bar{t}^k)] - \sum_{n=1}^N \frac{\pi_n}{\binom{N}{n}} \sum_{\substack{t^N \in P_n \\ t^k = (\bar{t}^{k-1}, 2)}} \frac{\mu\rho}{p(t^N)} u'[c_k^1(\bar{t}^k)] \\ &= \sum_{n=0}^N \frac{\pi_n}{\binom{N}{n}} \sum_{\substack{t^N \in P_n \\ t^k = (\bar{t}^{k-1}, 1)}} R\rho \left(1 + \frac{\mu}{p(t^N)}\right) u'[c^2(t^N)], \end{aligned}$$

which is equivalent to

$$\begin{aligned} & \left\{ \mathbb{P}[t^k = (\bar{t}^{k-1}, 1)] - \sum_{n=1}^N \frac{\pi_n}{\binom{N}{n}} \sum_{\substack{t^N \in P_n \\ t^k = (\bar{t}^{k-1}, 2)}} \frac{\mu\rho}{p(t^N)} \right\} u'[c_k^1(\bar{t}^k)] \\ &= \sum_{n=0}^N \frac{\pi_n}{\binom{N}{n}} \sum_{\substack{t^N \in P_n \\ t^k = (\bar{t}^{k-1}, 1)}} R\rho \left(1 + \frac{\mu}{p(t^N)}\right) u'[c^2(t^N)]. \end{aligned}$$

We can also write the equation in expectations, which yields

$$[1 - \gamma(\bar{t}^{k-1})] u'[c_k^1(\bar{t}^k)] = \mathbb{E}_{t^N | t^k = (\bar{t}^{k-1}, 1)} \left\{ R\rho \left(1 + \frac{\mu}{p(t^N)}\right) u'[c^2(t^N)] \right\}, \quad (10)$$

where  $\gamma(\bar{t}^{k-1}) = \mathbb{P}[t^k = (\bar{t}^{k-1}, 2)] \mathbb{E}_{t^N | t^k = (\bar{t}^{k-1}, 2)} [\mu\rho / p(t^N)] / \mathbb{P}[t^k = (\bar{t}^{k-1}, 1)]$ .

The result is derived from (10). Let us use induction on  $k \in \mathbb{N}$  starting from  $k = N$  and going down until  $k = 1$ .

*Proof for  $k = N$ .* Fix any  $\bar{t}^N = (\bar{t}^{N-1}, 1)$ . From (10) we have that

$$\begin{aligned} & \left[ 1 - \frac{\mathbb{P}[t^N = (\bar{t}^{N-1}, 2)]}{\mathbb{P}[t^N = (\bar{t}^{N-1}, 1)]} \times \frac{\mu\rho}{p(\bar{t}^{N-1}, 2)} \right] u'[c_N^1(\bar{t}^{N-1}, 1)] \\ &= R\rho \left(1 + \frac{\mu}{p(\bar{t}^{N-1}, 1)}\right) u'[c^2(\bar{t}^{N-1}, 1)], \end{aligned}$$

which implies that  $u'[c_N^1(\bar{t}^{N-1}, 1)] > u'[c^2(\bar{t}^{N-1}, 1)]$ . Thus,  $c_N^1(\bar{t}^{N-1}, 1) < c^2(\bar{t}^{N-1}, 1)$ . We know that the resources constraint holds at equality because  $u$  is strictly increasing. Therefore,

$$p(\bar{t}^{N-1}, 2) c^2(\bar{t}^{N-1}, 2) = [p(\bar{t}^N) + 1] c^2(\bar{t}^{N-1}, 2) = p(\bar{t}^N) c^2(\bar{t}^{N-1}, 1) + R c_N^1(\bar{t}^{N-1}, 1).$$

After reorganize the equation above, we have that

$$c^2(\bar{t}^{N-1}, 2) = \frac{p(\bar{t}^{N-1}, 1)}{p(\bar{t}^{N-1}, 1) + 1} c^2(\bar{t}^{N-1}, 1) + \frac{1}{p(\bar{t}^{N-1}, 1) + 1} R c_N^1(\bar{t}^{N-1}, 1) > c_N^1(\bar{t}^{N-1}, 1).$$

Hence, for the case  $k = N$ , we can conclude that  $c_k^1(\bar{t}^{k-1}, 1) < c^2(\bar{t}^{k-1}, 2^{N-k+1})$ .

*Proof for  $k < N$ .* Assume the result holds for all  $j > k$  and  $\bar{t}^j = (\bar{t}^{j-1}, 1) \in T^j$ . That is, for all  $j > k$ , we have  $c_j^1(\bar{t}^{j-1}, 1) < c^2(\bar{t}^{j-1}, \mathbf{2}^{N-j})$ . Let us show that it also holds for  $k$ . Fix some  $\bar{t}^k = (\bar{t}^{k-1}, 1) \in T^{k-1}$ . Then (10) is given by

$$u'[c_k^1(\bar{t}^k)] = \frac{1}{1 - \gamma(\bar{t}^{k-1})} \mathbb{E}_{t^N | t^k = (\bar{t}^{k-1}, 1)} \left\{ R\rho \left( 1 + \frac{\mu}{p(t^N)} \right) u'[c^2(t^N)] \right\}.$$

Note that, for any function  $X : T^N \rightarrow \mathbb{R}$ , the conditional expectation can be decomposed as

$$\begin{aligned} \mathbb{E}_{t^N | t^k = \bar{t}^k} \{X(t^N)\} &= \sum_{j=k+1}^N \mathbb{P}[t^j = (\bar{t}^k, \mathbf{2}^{j-k-1}, 1) | t^k = \bar{t}^k] \mathbb{E}_{t^N | t^j = (\bar{t}^k, \mathbf{2}^{j-k-1}, 1)} \{X(t^N)\} \\ &\quad + \mathbb{P}[t^N = (\bar{t}^k, \mathbf{2}^{N-k}) | t^k = \bar{t}^k] X(\bar{t}^k, \mathbf{2}^{N-k}). \end{aligned}$$

Applying this decomposition to (10), we obtain

$$\begin{aligned} u'[c_k^1(\bar{t}^k)] &= \left\{ \sum_{j=k+1}^N \mathbb{P}[t^j = (\bar{t}^k, \mathbf{2}^{j-k-1}, 1) | t^k = \bar{t}^k] \right. \\ &\quad \times \mathbb{E}_{t^N | t^j = (\bar{t}^k, \mathbf{2}^{j-k-1}, 1)} \left\{ R\rho \left( 1 + \frac{\mu}{p(t^N)} \right) u'[c^2(t^N)] \right\} \\ &\quad \left. + \mathbb{P}[t^N = (\bar{t}^k, \mathbf{2}^{N-k}) | t^k = \bar{t}^k] R\rho \left( 1 + \frac{\mu}{p(\bar{t}^k, \mathbf{2}^{N-k})} \right) u'[c^2(\bar{t}^k, \mathbf{2}^{N-k})] \right\} \\ &\quad \times \frac{1}{1 - \gamma(\bar{t}^{k-1})}. \end{aligned}$$

By (10) we know that

$$[1 - \gamma(\bar{t}^k, \mathbf{2}^{j-k-1})] u'[c_j^1(\bar{t}^k, \mathbf{2}^{j-k-1}, 1)] = \mathbb{E}_{t^N | t^j = (\bar{t}^k, \mathbf{2}^{j-k-1}, 1)} \left\{ R\rho \left( 1 + \frac{\mu}{p(t^N)} \right) u'[c^2(t^N)] \right\}$$

for  $j = k + 1, \dots, N$ . Hence,

$$\begin{aligned} u'[c_k^1(\bar{t}^k)] &= \left\{ \sum_{j=k+1}^N \mathbb{P}[t^j = (\bar{t}^k, \mathbf{2}^{j-k-1}, 1) | t^k = \bar{t}^k] [1 - \gamma(\bar{t}^k, \mathbf{2}^{j-k-1})] u'[c_j^1(\bar{t}^k, \mathbf{2}^{j-k-1}, 1)] \right. \\ &\quad \left. + \mathbb{P}[t^N = (\bar{t}^k, \mathbf{2}^{N-k}) | t^k = \bar{t}^k] R\rho \left( 1 + \frac{\mu}{p(\bar{t}^k, \mathbf{2}^{N-k})} \right) u'[c^2(\bar{t}^k, \mathbf{2}^{N-k})] \right\} \\ &\quad \times \frac{1}{1 - \gamma(\bar{t}^{k-1})}. \end{aligned}$$

By the inductive hypothesis, we know that  $c_j^1(\bar{t}^k, \mathbf{2}^{j-k-1}, 1) < c^2(\bar{t}^k, \mathbf{2}^{N-k})$ , which implies that

$$\begin{aligned}
 u'[c_k^1(\bar{t}^k)] &> \frac{1}{1 - \gamma(\bar{t}^{k-1})} \left\{ \sum_{j=k+1}^N \mathbb{P}[t^j = (\bar{t}^k, \mathbf{2}^{j-k-1}, 1) \mid t^k = \bar{t}^k][1 - \gamma(\bar{t}^k, \mathbf{2}^{j-k-1})] \right. \\
 &\quad \times u'[c^2(\bar{t}^k, \mathbf{2}^{N-k})] \\
 &\quad \left. + \mathbb{P}[t^N = (\bar{t}^k, \mathbf{2}^{N-k}) \mid t^k = \bar{t}^k] R\rho \left( 1 + \frac{\mu}{p(\bar{t}^k, \mathbf{2}^{N-k})} \right) u'[c^2(\bar{t}^k, \mathbf{2}^{N-k})] \right\} \\
 &= \frac{1}{1 - \gamma(\bar{t}^{k-1})} \left\{ \sum_{j=k+1}^N \mathbb{P}[t^j = (\bar{t}^k, \mathbf{2}^{j-k-1}, 1) \mid t^k = \bar{t}^k][1 - \gamma(\bar{t}^k, \mathbf{2}^{j-k-1})] \right. \\
 &\quad \left. + \mathbb{P}[t^N = (\bar{t}^k, \mathbf{2}^{N-k}) \mid t^k = \bar{t}^k] R\rho \left( 1 + \frac{\mu}{p(\bar{t}^k, \mathbf{2}^{N-k})} \right) \right\} u'[c^2(\bar{t}^k, \mathbf{2}^{N-k})] \\
 &= \frac{1}{1 - \gamma(\bar{t}^{k-1})} \left\{ \sum_{j=k+1}^N \mathbb{P}[t^j = (\bar{t}^k, \mathbf{2}^{j-k-1}, 1) \mid t^k = \bar{t}^k] \right. \\
 &\quad - \sum_{j=k+1}^N \mathbb{P}[t^j = (\bar{t}^k, \mathbf{2}^{j-k-1}, 1) \mid t^k = \bar{t}^k] \frac{\mathbb{P}[t^j = (\bar{t}^k, \mathbf{2}^{j-k})]}{\mathbb{P}[t^j = (\bar{t}^k, \mathbf{2}^{j-k-1}, 1)]} \mathbb{E}_{t^N \mid t^j = (\bar{t}^k, \mathbf{2}^{j-k})} \\
 &\quad \times \left[ \frac{\mu\rho}{p(t^N)} \right] \\
 &\quad \left. + \mathbb{P}[t^N = (\bar{t}^k, \mathbf{2}^{N-k}) \mid t^k = \bar{t}^k] R\rho \left( 1 + \frac{\mu}{p(\bar{t}^k, \mathbf{2}^{N-k})} \right) \right\} u'[c^2(\bar{t}^k, \mathbf{2}^{N-k})].
 \end{aligned}$$

After simplifying the above equation, we obtain

$$\begin{aligned}
 u'[c_k^1(\bar{t}^k)] &> \frac{1}{1 - \gamma(\bar{t}^{k-1})} \left\{ 1 - \mathbb{P}[t^N = (\bar{t}^k, \mathbf{2}^{N-k}) \mid t^k = \bar{t}^k] \right. \\
 &\quad - \sum_{j=k+1}^N \frac{\mathbb{P}[t^j = (\bar{t}^k, \mathbf{2}^{j-k})]}{\mathbb{P}[t^k = \bar{t}^k]} \mathbb{E}_{t^N \mid t^j = (\bar{t}^k, \mathbf{2}^{j-k})} \left[ \frac{\mu\rho}{p(t^N)} \right] \\
 &\quad \left. + \mathbb{P}[t^N = (\bar{t}^k, \mathbf{2}^{N-k}) \mid t^k = \bar{t}^k] R\rho \left( 1 + \frac{\mu}{p(\bar{t}^k, \mathbf{2}^{N-k})} \right) \right\} u'[c^2(\bar{t}^k, \mathbf{2}^{N-k})].
 \end{aligned} \tag{11}$$

The fact that the queue position is withdrawn uniformly implies that

$$\mathbb{P}[t^j = (\bar{t}^k, \mathbf{2}^{j-k})] = \mathbb{P}[t^j = (\bar{t}^{k-1}, 1, \mathbf{2}^{j-k})] = \mathbb{P}[t^j = (\bar{t}^{k-1}, \mathbf{2}^{j-k}, 1)]$$

and

$$\begin{aligned} \mathbb{E}_{t^N|t^j=(\bar{t}^k, \mathbf{2}^{j-k})} \left[ \frac{\mu\rho}{p(t^N)} \right] &= \mathbb{E}_{t^N|t^j=(\bar{t}^{k-1}, 1, \mathbf{2}^{j-k})} \left[ \frac{\mu\rho}{p(t^N)} \right] \\ &= \mathbb{E}_{t^N|t^j=(\bar{t}^{k-1}, \mathbf{2}^{j-k}, 1)} \left[ \frac{\mu\rho}{p(t^N)} \right]. \end{aligned}$$

This implies that

$$\begin{aligned} &\sum_{j=k+1}^N \frac{\mathbb{P}[t^j = (\bar{t}^k, \mathbf{2}^{j-k})]}{\mathbb{P}[t^k = \bar{t}^k]} \mathbb{E}_{t^N|t^j=(\bar{t}^k, \mathbf{2}^{j-k})} \left[ \frac{\mu\rho}{p(t^N)} \right] \\ &= \sum_{j=k+1}^N \frac{\mathbb{P}[t^j = (\bar{t}^{k-1}, 1, \mathbf{2}^{j-k})]}{\mathbb{P}[t^k = (\bar{t}^{k-1}, 1)]} \mathbb{E}_{t^N|t^j=(\bar{t}^{k-1}, 1, \mathbf{2}^{j-k})} \left[ \frac{\mu\rho}{p(t^N)} \right] \\ &= \sum_{j=k+1}^N \frac{\mathbb{P}[t^j = (\bar{t}^{k-1}, \mathbf{2}^{j-k}, 1)]}{\mathbb{P}[t^k = (\bar{t}^{k-1}, 1)]} \mathbb{E}_{t^N|t^j=(\bar{t}^{k-1}, \mathbf{2}^{j-k}, 1)} \left[ \frac{\mu\rho}{p(t^N)} \right] \\ &= \sum_{j=k+1}^N \frac{\mathbb{P}[t^j = (\bar{t}^{k-1}, \mathbf{2}^{j-k}, 1)]}{\mathbb{P}[t^k = (\bar{t}^{k-1}, 1)]} \mathbb{E}_{t^N|t^j=(\bar{t}^{k-1}, \mathbf{2}^{j-k}, 1)} \left[ \frac{\mu\rho}{p(t^N)} \right] \\ &\quad + \frac{\mathbb{P}[t^k = (\bar{t}^{k-1}, \mathbf{2}^{N-k})]}{\mathbb{P}[t^k = (\bar{t}^{k-1}, 1)]} \frac{\mu\rho}{p(\bar{t}^{k-1}, \mathbf{2}^{N-k})} \\ &\quad - \frac{\mathbb{P}[t^k = (\bar{t}^{k-1}, \mathbf{2}^{N-k})]}{\mathbb{P}[t^k = (\bar{t}^{k-1}, 1)]} \frac{\mu\rho}{p(\bar{t}^{k-1}, \mathbf{2}^{N-k})} \\ &= \frac{\mathbb{P}[t^k = (\bar{t}^{k-1}, 2)]}{\mathbb{P}[t^k = (\bar{t}^{k-1}, 1)]} \mathbb{E}_{t^N|t^k=(\bar{t}^{k-1}, 2)} \left[ \frac{\mu\rho}{p(t^N)} \right] \\ &\quad - \frac{\mathbb{P}[t^k = (\bar{t}^{k-1}, \mathbf{2}^{N-k})]}{\mathbb{P}[t^k = (\bar{t}^{k-1}, 1)]} \frac{\mu\rho}{p(\bar{t}^{k-1}, \mathbf{2}^{N-k})} \\ &= \gamma(\bar{t}^{k-1}) - \frac{\mathbb{P}[t^k = (\bar{t}^{k-1}, \mathbf{2}^{N-k})]}{\mathbb{P}[t^k = (\bar{t}^{k-1}, 1)]} \frac{\mu\rho}{p(\bar{t}^{k-1}, \mathbf{2}^{N-k})}. \end{aligned} \tag{12}$$

Replacing (12) in inequality (11) and reorganizing the terms in the inequality, we obtain

$$\begin{aligned} u'[c_k^1(\bar{t}^k)] &> \frac{1}{1 - \gamma(\bar{t}^{k-1})} \left\{ 1 - \gamma(\bar{t}^{k-1}) + \frac{\mathbb{P}[t^k = (\bar{t}^{k-1}, \mathbf{2}^{N-k})]}{\mathbb{P}[t^k = (\bar{t}^{k-1}, 1)]} \frac{\mu\rho}{p(\bar{t}^{k-1}, \mathbf{2}^{N-k})} \right. \\ &\quad \left. + \mathbb{P}[t^N = (\bar{t}^k, \mathbf{2}^{N-k}) \mid t^k = \bar{t}^k] R\rho \left( 1 + \frac{\mu}{p(\bar{t}^k, \mathbf{2}^{N-k})} - \frac{1}{R\rho} \right) \right\} \\ &\quad \times u'[c^2(\bar{t}^k, \mathbf{2}^{N-k})]. \end{aligned} \tag{13}$$

Because  $R\rho > 1$ , the inequality (13) implies that

$$u'[c_k^1(\bar{t}^{k-1}, 1)] = u'[c_k^1(\bar{t}^k)] > u'[c^2(\bar{t}^k, \mathbf{2}^{N-k})] = u'[c^2(\bar{t}^{k-1}, 1, \mathbf{2}^{N-k})],$$

and since  $u$  is concave, it implies that  $c_k^1(\bar{t}^{k-1}, 1) < c^2(\bar{t}^{k-1}, 1, \mathbf{2}^{N-k})$ . The resources constraint implies that

$$\begin{aligned} & [p(\bar{t}^{k-1}, 1, \mathbf{2}^{N-k}) + 1]c^2(\bar{t}^{k-1}, \mathbf{2}^{N-k+1}) \\ & = p(\bar{t}^{k-1}, 1, \mathbf{2}^{N-k})c^2(\bar{t}^{k-1}, 1, \mathbf{2}^{N-k}) + Rc_k^1(\bar{t}^{k-1}, 1). \end{aligned}$$

Finally, we can conclude that

$$\begin{aligned} c^2(\bar{t}^{k-1}, \mathbf{2}^{N-k+1}) & = \frac{p(\bar{t}^{k-1}, 1, \mathbf{2}^{N-k})c^2(\bar{t}^{k-1}, 1, \mathbf{2}^{N-k})}{p(\bar{t}^{k-1}, 1, \mathbf{2}^{N-k}) + 1} + \frac{Rc_k^1(\bar{t}^{k-1}, 1)}{p(\bar{t}^{k-1}, 1, \mathbf{2}^{N-k}) + 1} \\ & > c_k^1(\bar{t}^{k-1}, 1). \end{aligned}$$

We have shown that the result holds for  $k = N$  and that if it holds for all  $j \in \{k + 1, \dots, N\}$ , it holds for  $k$ . Therefore, by induction, we can conclude that the result holds for all  $k \in \mathbb{N}$ . □

### *Proof of Proposition 1*

We know that for any vector of announcements  $\hat{m}^N \in \hat{M}^N$ , if either  $\hat{m}^N \in T^N$  or  $\hat{m}_k \neq 2$  for all  $k$ , the result is trivial. Consider a realized vector of announcements  $\hat{m}^N \in \hat{M}^N$ , with  $\hat{m}^N \notin T^N$  and  $\hat{m}_k = 2$ , and let  $j$  be the queue position of the first agent to announce  $g$ . As before,  $\hat{t}^N \in T^N$  denotes the vector  $\hat{m}^N$  and we replace all  $g$ s with 2s. When agent  $j$  announced  $g$  in the first period, payments were suspended; hence, the total resources in the beginning of period 2 are

$$\begin{aligned} R \left[ Y - \sum_{i=1}^N \hat{c}_i^1(\hat{m}^i) \right] & = R \left[ Y - \sum_{i=1}^j \hat{c}_i^1(\hat{t}^i) \right] \\ & = p(\hat{t}^{j-1}, \mathbf{2}^{N-j+1})c_k^2(\hat{t}^{j-1}, \mathbf{2}^{N-j+1}), \end{aligned}$$

where  $p(\hat{t}^{j-1}, \mathbf{2}^{N-j+1})$  is the number of 2s in the vector  $(\hat{t}^{j-1}, \mathbf{2}^{N-j+1})$ . Let  $d(\hat{m}^N)$  denote the number of agents who have announced  $g$  and let  $p(\hat{m}^N)$  denote the number of agents who announced 2. The total payments in the second period to agents who announced  $g$  is given by

$$\sum_{k=1}^N \hat{c}_k^2(\hat{m}^N) \mathbb{1}_{\hat{m}_k=g} = c_j^1(\hat{t}^{j-1}, 1) + d(\hat{m}^N)\epsilon.$$

Hence, payment to agent  $k$  is

$$\begin{aligned} \hat{c}_k^2(\hat{m}^{k-1}, 2, \hat{m}_{k+1}^N) &= \frac{R \left[ Y - \sum_{k=1}^N \hat{c}_k^1(\hat{m}^k) \right] - \sum_{k=1}^N \hat{c}_k^2(\hat{m}^N) \mathbb{1}_{\hat{m}_k=g}}{p(\hat{m}^N)} \\ &= \frac{p(\hat{t}^{j-1}, \mathbf{2}^{N-j+1}) c_k^2(\hat{t}^{j-1}, \mathbf{2}^{N-j+1}) - c_j^1(\hat{t}^{j-1}, 1) - d(\hat{m}^N) \epsilon}{p(\hat{m}^N)}. \end{aligned}$$

By Lemma 1, we know that  $c_j^1(\hat{t}^{j-1}, 1) < c_j^2(\hat{t}^{j-1}, \mathbf{2}^{N-j+1})$ . Thus, by taking  $\epsilon > 0$  small enough, we have that

$$\hat{c}_k^2(\hat{m}^{k-1}, 2, \hat{m}_{k+1}^N) \geq \frac{[p(\hat{t}^{j-1}, \mathbf{2}^{N-j+1}) - 1] c_k^2(\hat{t}^{j-1}, \mathbf{2}^{N-j+1})}{p(\hat{m}^N)}.$$

By construction, we have that

$$\begin{aligned} \hat{c}_k^2(\hat{t}^{k-1}, 2, \hat{t}_{k+1}^N) &= c_k^2(\hat{t}^{k-1}, 2, \hat{t}_{k+1}^N) \\ &= \frac{R \left[ Y - \sum_{i=1}^N \hat{c}_i^1(\hat{t}^i) \right]}{p(\hat{m}^N) + d(\hat{m}^N)} \leq \frac{R \left[ Y - \sum_{i=1}^j \hat{c}_i^1(\hat{t}^i) \right]}{p(\hat{m}^N) + 1} \\ &= \frac{p(\hat{t}^{j-1}, \mathbf{2}^{N-j+1}) c_k^2(\hat{t}^{j-1}, \mathbf{2}^{N-j+1})}{p(\hat{m}^N) + 1}. \end{aligned}$$

Note that

$$\begin{aligned} \frac{p(\hat{t}^{j-1}, \mathbf{2}^{N-j+1}) - 1}{p(\hat{m}^N)} &\geq \frac{p(\hat{t}^{j-1}, \mathbf{2}^{N-j+1})}{p(\hat{m}^N) + 1} \\ \iff p(\hat{t}^{j-1}, \mathbf{2}^{N-j+1}) p(\hat{m}^N) + p(\hat{t}^{j-1}, \mathbf{2}^{N-j+1}) - p(\hat{m}^N) - 1 &\geq p(\hat{t}^{j-1}, \mathbf{2}^{N-j+1}) p(\hat{m}^N) \\ \iff p(\hat{t}^{j-1}, \mathbf{2}^{N-j+1}) &\geq p(\hat{m}^N) + 1. \end{aligned}$$

The last inequality holds because  $p(\hat{t}^{j-1}, \mathbf{2}^{N-j+1}) \geq p(\hat{t}^N) = p(\hat{m}^N) + d(\hat{m}^N)$ . Hence,

$$\begin{aligned} \hat{c}_k^2(\hat{m}^{k-1}, 2, \hat{m}_{k+1}^N) &\geq \frac{[p(\hat{t}^{j-1}, \mathbf{2}^{N-j+1}) - 1] c_k^{*2}(\hat{t}^{j-1}, \mathbf{2}^{N-j+1})}{p(\hat{m}^N)} \\ &\geq \frac{p(\hat{t}^{j-1}, \mathbf{2}^{N-j+1}) c_k^{*2}(\hat{t}^{j-1}, \mathbf{2}^{N-j+1})}{p(\hat{m}^N) + 1} \\ &\geq \hat{c}_k^2(\hat{t}^{k-1}, 2, \hat{t}_{k+1}^N). \end{aligned}$$

This concludes the proof.



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