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Temptation with uncertain normative preference

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We model a decision maker who anticipates being tempted but is also uncertain about what is normatively best. Our model is an extended version of Gul and Pesendorfer's (2001) with three time periods: in the ex ante period, the agent chooses a set of menus; in the interim period, she chooses a menu from this set; in the final period, she chooses from the menu. We posit axioms from the ex ante perspective. Our main axioms on preference state that the agent prefers flexibility in the ex ante period and the option to commit in the interim period. Our representation is a generalization of Dekel et al. (2009) and identifies the agent's multiple normative preferences and multiple temptations. We also characterize the uncertain normative preference analogue to the representation of Stovall (2010). Finally, we characterize the special case where normative preference is not uncertain. This special case allows us to uniquely identify all components of the representations of Dekel et al. (2009) and Stovall (2010).

Keywords. Temptation, uncertain normative preference, interim preference for commitment.

JEL classification. D01, D11.

1. Introduction

We model a decision maker who anticipates being tempted but is also uncertain about what is normatively best. For example, consider an agent who must make a consumption–savings decision. The decision maker knows she will be tempted by higher consumption, but because of an unknown taste shock, she also is uncertain what her optimal consumption level is.

Alternatively, consider the standard example in the temptation literature of someone on a diet. In the morning, the dieter contemplates possible choices for dinner. She wants to make a healthy choice for dinner but is afraid she will be tempted to choose something unhealthy. However, she is also uncertain about what she wants to choose. Perhaps she is uncertain about what is healthiest, possibly because of conflicting information from health studies she has read, or perhaps she simply does not know what she will want that evening. What behavior would someone like this exhibit?

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1.1 *Going beyond menus*

To answer this question, we consider a choice domain that is richer than the one that has become standard in the literature. The temptation literature emanating from Gul and Pesendorfer's (2001) seminal paper has generally studied preference over sets of alternatives, called *menus*. Implicit in this setting are two periods: an ex ante period, where the menu x is chosen, and a final period, where an alternative $\beta \in x$ is chosen. However, this domain is inadequate for our purposes because adaptations of existing temptation models to allow for uncertain normative preference yield no behavioral restrictions, making the models "contentless" in effect.

To see why, note that there is some tension between uncertainty about normative preference on the one hand and temptation on the other hand. Uncertainty about future tastes induces a preference for expanding the choice set to allow for flexibility (Kreps 1979). However, the possibility of future temptation induces a preference for restricting the choice set (i.e., commitment) so as to avoid tempting alternatives (Gul and Pesendorfer 2001, Dekel et al. 2009, Stovall 2010).

The temptation literature has generally avoided this tension and assumed perfect knowledge of normative preference, even while allowing for uncertainty of temptations. For example, Stovall (2010) characterized the representation

$$U(x) = \sum_{i=1}^{I} q_i \Big\{ \max_{\beta \in x} \big[u(\beta) + v_i(\beta) \big] - \max_{\beta \in x} v_i(\beta) \Big\}, \tag{1}$$

where $q_i > 0$ for all i and $\sum_I q_i = 1$. In this representation, the decision maker's temptation preferences (the v functions) vary across state i, while her normative preference (the u function) does not. Thus she is uncertain which temptation will affect her, but she is certain what her normative preference is. When there is no uncertainty (i.e., $I = 1$), then (1) reduces to the representation in Gul and Pesendorfer (2001). Dekel et al. (2009) (henceforth DLR) characterized a similar, but more general, representation that allowed for multiple temptations within each state.

Relaxing the assumption of perfect knowledge of normative preference, one may then wonder exactly what behavior characterizes a decision maker who is both uncertain about normative preference and affected by temptation. Presumably such a decision maker sometimes wants to expand her choice set and sometimes wants to restrict it. Can we say anything about how she wants to expand and restrict her choice set? Are there any restrictions on preference that we can impose that capture these two phenomena? Unfortunately in this setting, there is not.

To see why this is the case, consider a version of (1) but where the normative preference is also uncertain:

$$U(x) = \sum_{i=1}^{I} q_i \Big\{ \max_{\beta \in x} \big[u_i(\beta) + v_i(\beta) \big] - \max_{\beta \in x} v_i(\beta) \Big\}. \tag{2}$$

This is a special case of what is called the *finite additive EU representation*:[1]

$$U(x) = \sum_{k=1}^{K} \max_{\beta \in x} w_k(\beta) - \sum_{j=1}^{J} \max_{\beta \in x} v_j(\beta). \tag{3}$$

DLR give an axiomatic characterization of (3). Their axioms are the standard axioms needed for a representation—completeness, transitivity, continuity—as well as an independence axiom (the underlying alternatives are lotteries) and a finiteness axiom that guarantees that K and J are finite. Hence, (3) is "content-free" in the sense that its underlying behavior is orthogonal to the issues of temptation and uncertainty of future preference.[2]

As we show in Section 2, we can always take (3) and write it in the form of (2), meaning the representation in (2) has no behavioral content beyond that in (3). It is difficult then to interpret (2) as being about uncertain normative preference. After all, the axioms that characterize it suggest no such thing.

Another problem with the interpretation of (2) is that the various u_i functions are not uniquely identified from behavior. This is because there is no unique way to transform (3) into (2), as this transformation entails many arbitrary choices. (See the transformation in Section 2 for details.) Thus a given preference relation may have two different representations in the form of (2), one of which has, say, w in its set of u_i functions while the other does not. Is w a possible normative preference or not? An outside observer cannot know.

1.2 *Preview of results*

However, in situations in which an agent is uncertain about her normative preference, often that uncertainty is resolved before temptation hits. For example, your household spending budget is usually set at home, while your temptation to overspend usually occurs in a store. This suggests a need for a model that has three, not two, periods. Thus we consider preference over the expanded domain of sets of sets of alternatives. To make the exposition less cumbersome, we continue to refer to sets of alternatives as menus, and we call sets of menus *neighborhoods*. We think of a neighborhood as representing a choice problem over three time periods. In the ex ante period, the agent chooses a neighborhood X; in the interim period, she chooses a menu $x \in X$; in the final period, she chooses an alternative $\beta \in x$.

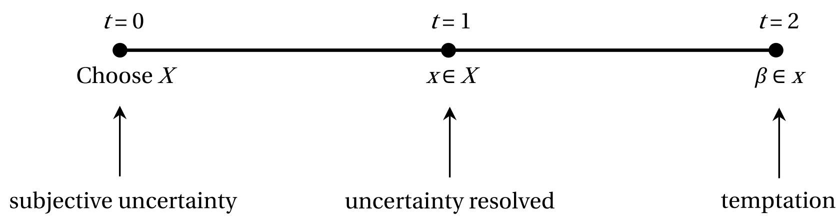
One way to think of our choice domain is that it is enriching the menu environment by adding an interim period to the timeline. In this regard, think of a neighborhood as a set of final outcomes combined with a technology to refine that set in the interim period. For example, for a neighborhood X, let $\hat{x} = \{\beta : \beta \in x \text{ for some } x \in X\}$. That is, $\hat{x}$ is the set of all final outcomes possible under X. Thus when an agent chooses the neighborhood

[1]The finite additive EU representation is a special case of a much broader class of preferences studied by Dekel et al. (2001, 2007).

[2]However, see Noor and Takeoka (2010, 2015) for arguments against Independence in a temptation setting.

X, she is in fact choosing the menu $\hat{x}$. However, X represents more than just $\hat{x}$ since, in the interim, the agent can choose any $x \in X$ and obviously $x \subset \hat{x}$. Thus X encapsulates the ability of the agent to refine $\hat{x}$ in the interim period.

The timeline we envision is the following: When choosing a neighborhood in the ex ante period, the agent knows she will experience temptation in the final period, but not before then. She also faces subjective uncertainty about what her normative preference and temptations will be, but expects that uncertainty to be resolved in the interim period.



We emphasize that the specific way that subjective uncertainty and temptation unfold in this timeline is not inherent to the choice domain of neighborhoods. Rather it is something that comes out of the interpretation of the axioms we impose, which are, in principle, observable.

Our first main axiom, Ex Ante Preference for Flexibility, states simply that larger neighborhoods (by set inclusion) are better. This is motivated by the idea that the decision maker faces subjective uncertainty in the ex ante period. If $X \subset Y$, then Y affords every interim period choice under X and more. If the decision maker expects some of her uncertainty to be resolved in the interim period, then she should prefer the flexibility inherent in Y over X, as the axiom states.

Our other main axiom on preference, Interim Preference for Commitment, states that the agent prefers to have the option to commit in the interim period. This is motivated by the idea that the decision maker expects temptation to be more salient in the final period. Consider two menus x and y. The neighborhood $\{x, y\}$ represents a choice between the alternatives in x and y in the interim period, while the neighborhood $\{x \cup y\}$ represents a choice between the alternatives in x and y in the final period. Thus compared to $\{x \cup y\}$, the neighborhood $\{x, y\}$ provides the option to commit in the interim period. If the decision maker anticipates an alternative in $x \cup y$ to be tempting in the final period, then she would prefer the commitment in $\{x, y\}$ over $\{x \cup y\}$, as the axiom states.

Note, however, that we do not impose a preference for commitment in the ex ante period, only in the interim period. Indeed, such an axiom conflicts with Ex Ante Preference for Flexibility. Thus the decision maker does not expect temptation to be present in the interim period. Similarly, we do not impose a preference for flexibility in the interim period. Such an axiom conflicts with Interim Preference for Commitment. Thus the decision maker does not expect any subjective uncertainty to be resolved in the final period.

Though the separation between when subjective uncertainty is resolved and temptation is present does appear stark, nevertheless some difference in timing is plausible and speaks to a number of real life choices. For example, a lot of personal finance advice is aimed at helping people save more and avoid overspending. One piece of advice to avoid the temptation of overspending is simply to cut up your credit card. Another piece of advice is to freeze your credit card in water.[3] Encased in ice, the credit card cannot be used until after a few hours of thawing, thus separating the decision to use the card from the time the card is actually used. In addition, freezing the card may be preferable to cutting up the card because then you have the option of using it if need be. Simplifying things, let ℓ and h denote low and high level of spending, respectively. Without a credit card, the agent is committed to ℓ. With a credit card, the agent has the option to choose between ℓ and h. However, with the credit card in her wallet, that choice is made at the time of purchase, while the iced credit card means that the choice can be made before the time of purchase. Thus $\{\{\ell, h\}\}$ represents having a credit card in the wallet, while $\{\{\ell\}, \{\ell, h\}\}$ represents having the credit card in ice. An agent who freezes her credit card then reveals $\{\{\ell\}, \{\ell, h\}\} \succ \{\{\ell, h\}\}$, as Interim Preference for Commitment says. Also, $\{\{\ell\}, \{\ell, h\}\} \succ \{\{\ell\}\}$ since the agent did not choose to cut up the card, as Ex Ante Preference for Flexibility says.

These axioms and others are used to characterize representations that are analogues to those in DLR and Stovall (2010), but where the normative preference is uncertain. For example, Theorem 2 characterizes the uncertain normative analogue to (1):

$$\mathcal{U}(X) = \sum_{i=1}^{I} \max_{x \in X} \Big\{ \max_{\beta \in x} \big[u_i(\beta) + v_i(\beta)\big] - \max_{\beta \in x} v_i(\beta) \Big\}.$$

This representation suggests the decision maker has I different possible interim preferences. If state i attains, then the decision maker will choose the menu $x \in X$ that maximizes the value $\max_{\beta \in x}[u_i(\beta) + v_i(\beta)] - \max_{\beta \in x} v_i(\beta)$. This, of course, is the same representation characterized by Gul and Pesendorfer (2001). Thus the decision maker behaves as if she knows she will be a Gul–Pesendorfer decision maker in the interim, but ex ante does not know which type of Gul–Pesendorfer decision maker she will be.

We also consider the special case where the normative preference is not uncertain. This allows us to give alternative characterizations of versions of DLR's and Stovall's representations, but in the neighborhood domain. However, in our framework, these representations are unique. This allows us to identify the decision maker's subjective states and temptation utilities, something that is not done in the menus setting. Nonidentification could be a problem for applications that rely on these functional forms, as results could depend on properties of the particular utility function that have no basis on the properties of the underlying preference.[4]

[3] A quick internet search of "freeze your credit card in water" turns up many examples of people offering this advice. I thank Lars Lefgren for bringing this to my attention.

[4] See also the discussion in DLR concerning identification.

1.3 *Related literature and outline*

Recent work by Ahn and Sarver (2013) suggests an alternative approach to uniquely identifying the representations of DLR and Stovall. Ahn and Sarver consider a two-period model where both ex ante preference over menus and ex post (random) choice from the menu are observed. They ask what joint conditions on ex ante preference and ex post choice imply that the anticipated choice from a menu is the same as the actual choice from the menu. One implication of their result is that with both sets of data (ex ante preference over menus and ex post choice from menus), one is able to uniquely identify the agent's subjective beliefs and state-dependent utilities. Though they do not consider ex ante preferences affected by temptation, Ahn and Sarver's approach suggests that ex post choice data may be useful in identifying the representations of DLR and Stovall. While this may be possible, the present work shows that some identification is possible using just ex ante preference.

Related to this discussion is work by Dekel and Lipman (2012), who discuss a representation that they call a random Strotz representation. One thing they show is that every preference that has Stovall's representation also has a random Strotz representation. Additionally, they show that these representations imply different choices from a menu. Thus these two representations cannot be differentiated by ex ante preference, but they can be by ex post choice. Similar results apply here.

The addition of uncertain normative preference to models of temptation should be important to applications. For example, Amador et al. (2006) study a consumption–savings model where the agent values both commitment and flexibility. Similar to the example given earlier, one of their models uses an agent who receives an uncertain taste shock to her normative preference, but is also uncertain about the strength of temptation to consume rather than save. In contrast to our setting, their model does not have an interim period in which commitment is possible after the resolution of uncertainty. Their central quest is to find the optimal subset of the individual's budget set (i.e., the optimal menu) given the trade-off between commitment and flexibility. In their setting, a minimum savings rule is always optimal.

The domain of preference over neighborhoods is used by others for subjective models of dynamic decision problems. Takeoka (2006) uses this domain to model a decision maker with a subjective decision tree. He imposes Ex Ante Preference for Flexibility as we do. However, his decision maker does not suffer from temptation, but instead anticipates more subjective uncertainty to be resolved in the final period. Thus he also imposes an axiom that is the opposite of Interim Preference for Commitment, which he calls Aversion to Commitment. Kopylov (2009b) uses a similar domain to generalize the work of Gul and Pesendorfer (2001) to multiple periods. Kopylov and Noor (2017) use this domain to model "weak resolve." Their decision maker not only experiences temptation in the final period, but in the interim period as well. Thus they impose an axiom very similar to Interim Preference for Commitment, but in contrast to our model, their decision maker also prefers commitment in the ex ante period.

On a technical note, the proofs of our main theorems rely on the main result from Kopylov (2009a, Theorem 2.1), which is a generalization of DLR's characterization of (3).

Kopylov's setting is general enough to apply to both the menus and the neighborhoods domains, and we exploit this fact in our proofs.

In the next section we introduce the model and the reasons for the expanded domain in more detail. The main axioms and results are presented in Section 3. Section 4 considers the case where normative preference is not uncertain and the identification of the representation that it affords. Proofs of the main theorems and details of our uniqueness results are collected in the Appendix.

2. Background

2.1 *Setup*

We use $\mathbb{N}_0$ to denote the natural numbers with 0. If $I = 0$, then $\{1, \dots, I\}$ is the empty set and statements like "for $i = 1, \dots, I$, we have…" are vacuous. For any set A and binary relation $\succeq$ over A, we say that a function $f : A \to \mathbb{R}$ *represents* $\succeq$ if $f(a) \geq f(b)$ if and only if $a \succeq b$.

Let Δ denote the set of probability distributions over a finite set, and call $\beta \in \Delta$ an *alternative*. Let $\mathcal{M}$ denote the set of closed, nonempty subsets of Δ, and call $x \in \mathcal{M}$ a *menu*. Let $\mathcal{N}$ denote the set of closed, nonempty subsets of $\mathcal{M}$, and call $X \in \mathcal{N}$ a *neighborhood*. Throughout, we use $\alpha, \beta, \dots$ to denote elements of Δ, $x, y, \dots$ to denote elements of $\mathcal{M}$, and $X, Y, \dots$ to denote elements of $\mathcal{N}$.

There are two different ways we can embed $\mathcal{M}$ into $\mathcal{N}$. As these are both important for future discussion, we define them here. So for $x \in \mathcal{M}$, set

$$X^1(x) \equiv \bigcup_{\beta \in x} \{\{\beta\}\}$$

and

$$X^2(x) \equiv \{x\}.$$

For example, if $x = \{\alpha, \beta\}$, then $X^1(x) = \{\{\alpha\}, \{\beta\}\}$ and $X^2(x) = \{\{\alpha, \beta\}\}$. Although both neighborhoods represent a choice between α and β, the difference between the two is that $\{\{\alpha\}, \{\beta\}\}$ forces that choice in the interim period (i.e., $t = 1$) while $\{\{\alpha, \beta\}\}$ forces that choice in the final period (i.e., $t = 2$). Thus the neighborhood $X^1(x)$ is like choosing from the menu x when there is only uncertainty about taste and no temptation, while the neighborhood $X^2(x)$ is like choosing from the menu x when there is both uncertainty about taste and temptation but no interim period to refine x. We slightly abuse notation and let $\mathcal{M}^1$ denote all neighborhoods $X^1(x)$. Similarly, let $\mathcal{M}^2$ denote all neighborhoods $X^2(x)$.

We use the usual metric over Δ. We endow $\mathcal{M}$ with the Hausdorff topology and define the mixture of two menus $x, y \in \mathcal{M}$ as

$$\lambda x + (1 - \lambda) y \equiv \{\lambda \beta + (1 - \lambda)\beta' : \beta \in x, \beta' \in y\}$$

for $\lambda \in [0, 1]$. Similarly, we endow $\mathcal{N}$ with the Hausdorff topology and define the mixture of two neighborhoods $X, Y \in \mathcal{N}$ as

$$\lambda X + (1 - \lambda) Y \equiv \{\lambda x + (1 - \lambda) y : x \in X, y \in Y\}$$

for $\lambda \in [0, 1]$.

Our primitive is a binary relation $\succeq$ over $\mathcal{N}$ that represents the agent's ex ante preference. We do not model choice in the interim or ex post periods explicitly. However, the agent's ex ante preferences are obviously affected by her (subjective) expectations of her future preference and temptations.

2.2 *Inadequacy of the domain* $\mathcal{M}$

As a point of reference, we define Stovall's and DLR's representations. We say that $U : \mathcal{M} \to \mathbb{R}$ is a *simple temptation representation* if

$$U(x) = \sum_{i=1}^{I} q_i \Big\{ \max_{\beta \in x} \big[u(\beta) + v_i(\beta) \big] - \max_{\beta \in x} v_i(\beta) \Big\}, \tag{S}$$

where $I \in \mathbb{N}_0$, $q_i > 0$ for all i, $\sum_I q_i = 1$, and u and each v_i are expected-utility (EU) functions. We say that $U : \mathcal{M} \to \mathbb{R}$ is a *general temptation representation* if

$$U(x) = \sum_{i=1}^{I} q_i \left\{ \max_{\beta \in x} \left[u(\beta) + \sum_{j=1}^{J_i} v_{ij}(\beta) \right] - \sum_{j=1}^{J_i} \max_{\beta \in x} v_{ij}(\beta) \right\}, \tag{G}$$

where $I \in \mathbb{N}_0$, $q_i > 0$ for all i, $\sum_I q_i = 1$, and u and each v_{ij} are EU functions. For ease of future reference, we refer to these as the S and G representations, respectively. Note that the S representation is a special case of the G representation where $J_i = 1$ for every i.

The interpretations of these representations are similar, so consider the S representation. The function u is the agent's commitment preference (i.e., her preference over singleton menus, which are ex ante commitments to a final alternative). Each v_i is interpreted as a temptation, and q_i is the probability the agent assigns to temptation i being realized later. If state i is realized, then the decision maker chooses $\beta \in x$ that maximizes $u + v_i$, and experiences the disutility $\max_{\beta' \in x} v_i(\beta')$, which is the forgone utility of the most tempting alternative in state i. The G representation is similar, only each state i has multiple temptations that might affect the agent.

Consider versions of the S and G representations in which the normative preference varies across states. For the S representation, this looks like

$$U(x) = \sum_{i=1}^{I} q_i \Big\{ \max_{\beta \in x} \big[u_i(\beta) + v_i(\beta) \big] - \max_{\beta \in x} v_i(\beta) \Big\}; \tag{4}$$

for the G representation this looks like

$$U(x) = \sum_{i=1}^{I} q_i \left\{ \max_{\beta \in x} \left[u_i(\beta) + \sum_{j=1}^{J_i} v_{ij}(\beta) \right] - \sum_{j=1}^{J_i} \max_{\beta \in x} v_{ij}(\beta) \right\}. \tag{5}$$

Here the u functions are indexed by i, which captures the idea that the agent is uncertain about her normative preference. As discussed in the Introduction, such representations can be viewed as contentless because they impose no additional restrictions on preference beyond those needed for a finite additive EU representation.

To see this, we first formally define such a representation. We say that $U : \mathcal{M} \to \mathbb{R}$ is a *finite additive EU representation* if

$$U(x) = \sum_{k=1}^{K} \max_{\beta \in x} w_k(\beta) - \sum_{j=1}^{J} \max_{\beta \in x} v_j(\beta), \tag{FA}$$

where $K, J \in \mathbb{N}_0$, and each w_k and v_j is an EU function. For ease of future reference, we refer to this as a FA representation. Note that both the S and G representations are special cases of the FA representation.

Now we show that any FA representation can be written as (4). First, start with a FA representation $U(x) = \sum_K \max_{\beta \in x} w_k(\beta) - \sum_J \max_{\beta \in x} v_j(\beta)$. For every k, choose arbitrary $a_{k1}, a_{k2}, \dots, a_{kJ}$ such that $a_{kj} \geq 0$ for every j and $\sum_J a_{kj} = 1$. Similarly, for every j, choose arbitrary $b_{1j}, b_{2j}, \dots, b_{Kj}$ such that $b_{kj} \geq 0$ for every k and $\sum_K b_{kj} = 1$. Set $I \equiv KJ$ and let $\iota : K \times J \to I$ be any bijection. Choose arbitrary $q_1, q_2, \dots, q_I$ such that $q_i > 0$ for every $i \in I$ and $\sum_I q_i = 1$. Finally, for every i, set $u_i \equiv \frac{a_{kj}}{q_i} w_k - \frac{b_{kj}}{q_i} v_j$ and $\hat{v}_i \equiv \frac{b_{kj}}{q_i} v_j$, where $i = \iota(k, j)$. Then we can rewrite U as

$$U(x) = \sum_{i=1}^{I} q_i \Big\{ \max_{\beta \in x} \big[u_i(\beta) + \hat{v}_i(\beta) \big] - \max_{\beta \in x} \hat{v}_i(\beta) \Big\},$$

which is (4).

Similarly, DLR showed that any FA representation can be written as (5). Hence there is no behavioral distinction (in the domain $\mathcal{M}$) between (4), (5), and the FA representation. In addition, the u_i functions were arbitrarily constructed from the components of the FA representation. Thus it is difficult to interpret the u_i functions as representing the agent's various normative preference as they are not uniquely identified from behavior.

As the results in the next section show, when we expand the choice domain to $\mathcal{N}$, then we are able to distinguish behaviorally between analogues of (4) and (5) and to identify their components uniquely.

3. Uncertain normative preference

We begin with a set of axioms that are modifications of those given in Dekel et al. (2001).[5]

Order. *The preference $\succeq$ is complete and transitive.*

Continuity. *For every Y, the sets $\{X : X \succeq Y\}$ and $\{X : Y \succeq X\}$ are closed.*

Independence. *If $X \succ Y$, then for $Z \in \mathcal{N}$ and $\lambda \in (0, 1]$,*

$$\lambda X + (1 - \lambda) Z \succ \lambda Y + (1 - \lambda) Z.$$

[5] See Dekel et al. (2001) for a discussion of these axioms in the domain $\mathcal{M}$. Also, Kopylov (2009b) discusses them for the domain $\mathcal{N}$.

Following DLR, we also introduce a finiteness axiom.[6] Before we state the axiom, we need some definitions.

Definition 1. We say Y is *critical* for X if $Y \subset X$ and if for all Y' satisfying $Y \subset Y' \subset X$, we have $Y' \sim X$.

We think of a critical subset that is also a proper subset as stripping away irrelevant alternatives from a neighborhood. That is, if Y is critical for X and $x \in X \setminus Y$, then $Y \sim Y \cup \{x\} \sim X \setminus \{x\} \sim X$. Thus we conclude that x is irrelevant to the decision maker in her evaluation of X. Also, note that every neighborhood has a critical subset, namely itself.

Definition 2. We say y is *critical* for $x \in X$ if $y \subset x$ and if for all y' satisfying $y \subset y' \subset x$, we have $(X \setminus \{x\}) \cup \{y'\} \sim X$.

The interpretation of a critical menu is similar to that given above for critical neighborhoods. Also, every menu is critical for itself in any neighborhood.

Finiteness. *There exists $N \in \mathbb{N}$ such that the following statements hold:*

(i) *For every X, there exists Y critical for X, where $|Y| < N$.*

(ii) *For every X and for every $x \in X$, there exists y critical for $x \in X$, where $|y| < N$.*

We refer to the preceding four axioms as the DLR axioms, and assume them throughout.

DLR Axioms. *The preference $\succeq$ satisfies Order, Continuity, Independence, and Finiteness.*

Our next axiom is similar to the Preference for Flexibility axiom introduced by Kreps (1979).

Ex Ante Preference for Flexibility. *If $X \subset Y$, then $Y \succeq X$.*

When an agent is uncertain what her future tastes will be, then she will desire flexibility by preferring larger choice sets. However, as discussed in the Introduction, Ex Ante Preference for Flexibility only imposes this preference for flexibility on neighborhoods and not on the menus that make up the neighborhoods. Thus the agent values flexibility only between the ex ante and interim periods. Flexibility per se is not valued between the interim and final period.

We now introduce our first main axiom concerning temptation. It states that the agent values commitment between the interim and ex post period.

Interim Preference for Commitment. *For every x, y, and X, $\{x, y\} \cup X \succeq \{x \cup y\} \cup X$.*

Because of Ex Ante Preference for Flexibility, the agent does not value commitment in the ex ante period. However, Interim Preference for Commitment says that she does

[6]Finiteness is discussed in DLR and Kopylov (2009a). We note that our axiom is stated slightly differently than either DLR's or Kopylov's axioms. Though our axiom is technically equivalent to Kopylov's (see the proof to Theorem 5), it is stated more in the spirit of DLR's.

want to be able to commit in the interim period since $\{x, y\} \cup X$ provides the option to commit to either x or y in the interim period.

As discussed in the Introduction, Kopylov and Noor (2017) introduce a similar axiom that they call Preference for Earlier Decisions. Their axiom is the special case of Interim Preference for Commitment when $X = \varnothing$, and thus is a less restrictive axiom.[7] Though the focus of their paper is different from ours, their axiom serves a similar purpose in their model. Namely, it implies that temptation is stronger in the final period than in the interim period.

One important aspect of our axioms is that they allow for situations in which a given alternative can alternately be tempting and normatively best. High spending in the frozen credit card example in the Introduction is one example of this. Alternatively, consider the following example.

Example 1. An agent wants to choose the healthiest meal to eat. She is indifferent right now between committing to steak and committing to pasta (i.e., $\{\{s\}\} \sim \{\{p\}\}$) because she does not know which one is healthier for her. If low fat diets are healthier, then she will want to choose pasta. However, if high protein diets are healthier, then she will want to choose steak. She also knows that a study will be published before she has her meal concerning which diet is healthier. Thus she has the preference $\{\{s\}, \{p\}\} \succ \{\{p\}\}$ and $\{\{s\}, \{p\}\} \succ \{\{s\}\}$ because she wants to keep her options open until she knows which diet is healthiest. In addition, she is afraid that no matter which dish is healthiest, she will be tempted by the other. Thus she has the preference $\{\{s\}, \{p\}\} \succ \{\{s, p\}\}$ because the former neighborhood gives her the option to commit after finding out which diet is healthier but before she enters the restaurant and is tempted, while the latter neighborhood does not allow her to commit to the (unknown) healthier option and instead guarantees she will face temptation. ♢

Our first representation takes the following form.

Definition 3. An *uncertain normative preference and general temptation representation* is a function

$$\mathcal{U}(X) = \sum_{i=1}^{I} \max_{x \in X} \left\{ \max_{\beta \in x} \left[u_i(\beta) + \sum_{j=1}^{J_i} v_{ij}(\beta) \right] - \sum_{j=1}^{J_i} \max_{\beta \in x} v_{ij}(\beta) \right\}, \tag{UG}$$

where $I \in \mathbb{N}_0$, $J_i \in \mathbb{N}_0$ for every i, and each u_i and v_{ij} is an EU function.

We refer to the preceding equality as a UG representation.[8] The interpretation of the UG representation is similar to the interpretations given earlier: There are I subjective

[7]Technically, $X = \varnothing$ is not covered by Interim Preference for Commitment, as all neighborhoods are nonempty. However, Interim Preference for Commitment in conjunction with Transitivity implies Preference for Earlier Decisions: For any x and y, alternately set $X = \{x, y\}$ and $X = \{x \cup y\}$, and apply Interim Preference for Commitment. This gives $\{x, y\} \succeq \{x, y, x \cup y\} \succeq \{x \cup y\}$.

[8]Our subsequent representations have a similar naming convention, distinguishing between uncertain (U) and certain (C) normative preference as well as the simple (S) and general (G) temptation representations.

states. In state i, the normative preference is u_i, while the v_{ij}s are the temptations. For a fixed menu $x \in X$, the agent chooses the alternative in x that maximizes $u_i + \sum_{J_i} v_{ij}$ but experiences the disutility $\sum_{J_i} \max_{\beta \in x} v_{ij}(\beta)$, which is the forgone utility from the most tempting alternatives (in state i). For each state i, she chooses a possibly different menu $x \in X$ that maximizes state i's utility and sums across all states to get the total utility for X.

Note that the UG representation is the uncertain normative analogue to the G representation given in Section 2. One key difference is that the UG representation does not have probabilities associated with the states. This is because such probabilities cannot be identified because the normative preferences u_i vary across states. We see in Section 4 that such probabilities can be identified when normative utility is constant across states.

Theorem 1. *The preference $\succeq$ satisfies the DLR axioms, Ex Ante Preference for Flexibility, and Interim Preference for Commitment if and only if $\succeq$ has a UG representation.*

The proof is given in the Appendix. The bulk of the proof is to show that the DLR axioms and Ex Ante Preference for Flexibility characterize a preliminary representation:

$$\mathcal{U}(X) = \sum_{i=1}^{I} \max_{x \in X} \left\{ \sum_{k=1}^{K_i} \max_{\beta \in x} w_{ik}(\beta) - \sum_{j=1}^{J_i} \max_{\beta \in x} v_{ij}(\beta) \right\}.$$

The proof for this preliminary representation, given in Appendix B, relies on the main result from Kopylov (2009a, Theorem 2.1), which is a generalization of DLR's characterization of the FA representation. The primitive in Kopylov's theorem is a binary relation over nonempty compact subsets of a convex compact space. Thus both $\mathcal{M}$ and $\mathcal{N}$ are special cases of Kopylov's setup. The key steps in our proof are to show that Kopylov's axioms are satisfied, first for the ex ante preference $\succeq$ over $\mathcal{N}$ and then for each implied interim preference over $\mathcal{M}$. Once this preliminary representation is established, we use Interim Preference for Commitment to show that $K_i \leq 1$ for every i. From there, straightforward substitutions yield the result.

The various components of a UG representation are uniquely identified from preference. Before we discuss the uniqueness result associated with Theorem 1, we consider a normalization of the UG representation so as to make the statement of the uniqueness result easier. So suppose we have a UG representation in which there was a v_{ij} function that was constant. Then we can remove that function from the representation and end up with a new UG representation that still has the same underlying preference ordering. Now suppose we have a UG representation in which there are functions v_{ij} and $v_{ij'}$ that represent the same ordering over Δ. (Note that i is common for these two functions.) Then v_{ij} and $v_{ij'}$ can be removed from the representation and replaced with the function $\hat{v} = v_{ij} + v_{ij'}$. Again, the resulting representation is still a UG representation and the underlying ordering does not change. Moving a level up, set $U_i(x) \equiv \max_{\beta \in x}[u_i(\beta) + \sum_{J_i} v_{ij}(\beta)] - \sum_{J_i} \max_{\beta \in x} v_{ij}(\beta)$. If there is a U_i function that is constant, then it can be removed from the representation without changing anything.

Finally, if there are functions U_i and $U_{i'}$ that represent the same ordering over $\mathcal{M}$, then they can be removed from the representation and replaced with $\hat{U} = U_i + U_{i'}$.[9] A UG representation that has all constant functions removed and all redundant functions combined in this manner we call *minimal*. Crucially, every UG representation can be made minimal.

Now suppose we have a minimal UG representation. Consider some manipulations of this representation that (i) do not affect the underlying ordering of neighborhoods and (ii) are still minimal: Permuting $\{1, \dots, I\}$, permuting $\{1, \dots, J_i\}$ for any i, multiplying each u_i function and each v_{ij} function by a common positive number, and adding arbitrary constants to any u_i function or v_{ij} function. In fact, our uniqueness result states that these are the only allowed manipulations of a minimal UG representation. Any other change either affects the underlying preference ordering or makes the resulting representation not minimal. A formal statement of this result as well as details for all subsequent uniqueness results are in Appendix C.

One way to see the importance of separating the resolution of uncertainty from the experience of temptation and how that is borne out by the axioms is to note how a UG representation simplifies when restricted to $\mathcal{M}^1$ and $\mathcal{M}^2$, respectively. So for a UG representation $\mathcal{U}$ and $x \in \mathcal{M}$, set $U^1(x) \equiv \mathcal{U}(X^1(x))$. Then it is easy to see that

$$U^1(x) = \sum_{i=1}^{I} \max_{\beta \in x} u_i(\beta),$$

which is Kreps' (1979) representation.[10] This should be as expected since Ex Ante Preference for Flexibility has force on neighborhoods in $\mathcal{M}^1$, while Interim Preference for Commitment does not.

Similarly, set $U^2(x) \equiv \mathcal{U}(X^2(x))$. Then U^2 simplifies to

$$U^2(x) = \sum_{i=1}^{I} \max_{\beta \in x} \left[u_i(\beta) + \sum_{j=1}^{J_i} v_{ij}(\beta) \right] - \sum_{j=1}^{J_i} \max_{\beta \in x} v_{ij}(\beta).$$

This is a special case of (5), which, as discussed in Section 2, is equivalent to the FA representation. Again, this should be as expected, since neither Ex Ante Preference for Flexibility nor Interim Preference Commitment has any force on neighborhoods in $\mathcal{M}^2$.

One problem with the UG representation is that it allows preferences that are arguably not motivated by temptation. Consider the following example.[11]

Example 2. Suppose

$$\big\{\{\alpha\}\big\} \sim \big\{\{\beta\}\big\} \sim \big\{\{\alpha\}, \{\beta\}\big\} \succ \big\{\{\alpha, \beta\}\big\}.$$

[9]It may not be obvious that $\hat{U}$ can be written in the same form as U_i and $U_{i'}$, which is needed for the resulting representation to be a UG representation. However, since U_i and $U_{i'}$ are linear functions, then the fact that they represent the same ordering means that any one, say $U_{i'}$, can be written as a positive affine transformation of the other, U_i. Hence $\hat{U}$ can be written as a positive affine transformation of U_i.

[10]Technically, Kreps' choice domain is simpler, as he does not employ lotteries. However, DLR characterize this exact representation with lotteries using results from Dekel et al. (2001).

[11]Stovall (2010) provides a similar example in the preference-over-menus domain.

Since $\{\{\alpha\}\} \sim \{\{\alpha\}, \{\beta\}\}$, this suggests that there is no state in which the decision maker thinks β is strictly normatively better than α. Similarly, $\{\{\beta\}\} \sim \{\{\alpha\}, \{\beta\}\}$ suggests that there is no state in which the decision maker thinks α is strictly normatively better than β. Hence she thinks α and β are normatively the same across all possible states. However, the strict preference for the option to commit in the interim $\{\{\alpha\}, \{\beta\}\} \succ \{\{\alpha, \beta\}\}$ suggests she expects one to tempt the other. This seems odd, since she thinks α and β are normatively the same across all possible states. ◊

This example is consistent with Interim Preference for Commitment, but not our next axiom.[12]

Interim Choice Consistency. *If* $\{x, x \cup y\} \cup X \succ \{x \cup y\} \cup X$, *then* $\{x, y\} \cup X \succ \{y\} \cup X$.

Consider the preference $\{x, x \cup y\} \cup X \succ \{x \cup y\} \cup X$. This implies that the agent expects to (sometimes) choose x over $x \cup y$ in the interim period. Since $x \cup y$ represents a delay until the final period of the choice between the alternatives in x and the alternatives in y, this preference shows that the agent would rather choose x over y in the interim than in the final period. Now consider the neighborhood $\{x, y\} \cup X$. This neighborhood gives the agent the opportunity to choose directly between x and y in the interim period. Interim Choice Consistency says that the agent should be consistent in her choice of x over y in the interim and have the preference $\{x, y\} \cup X \succ \{y\} \cup X$.

Interim Choice Consistency only makes sense for agents who expect to have only one temptation in the interim. To see why, suppose there were multiple temptations facing the agent after her normative preference was resolved. Suppose menus x and y have the same normatively appealing alternatives, but different tempting alternatives. Then $x \cup y$ would have more tempting alternatives than either x or y. So choosing a normatively appealing alternative from $x \cup y$ may require more self-control than choosing a normatively appealing alternative from just x. Thus in the interim period, this agent may prefer to commit to x over $x \cup y$ (i.e., $\{x, x \cup y\} \succ \{x \cup y\}$) yet be indifferent between x and y (i.e., $\{x, y\} \sim \{y\}$).

The next representation takes the following form.

Definition 4. An *uncertain normative preference and simple temptation representation* is a function

$$\mathcal{U}(X) = \sum_{i=1}^{I} \max_{x \in X} \Big\{ \max_{\beta \in x} \big[u_i(\beta) + v_i(\beta)\big] - \max_{\beta \in x} v_i(\beta) \Big\}, \tag{US}$$

where $I \in \mathbb{N}_0$, and each u_i and v_i is an EU function.

Theorem 2. *The preference $\succeq$ satisfies the DLR axioms, Ex Ante Preference for Flexibility, Interim Preference for Commitment, and Interim Choice Consistency if and only if $\succeq$ is represented by a US representation.*

[12] The example is inconsistent with Interim Choice Consistency, Ex Ante Preference for Flexibility, and Transitivity. Note that Ex Ante Preference for Flexibility implies $\{\{\alpha\}, \{\alpha, \beta\}\} \succeq \{\{\alpha\}\}$. Transitivity then implies $\{\{\alpha\}, \{\alpha, \beta\}\} \succ \{\{\alpha, \beta\}\}$. But then Interim Choice Consistency is violated since $\{\{\alpha\}, \{\beta\}\} \sim \{\{\beta\}\}$.

The proof is given in the Appendix. Also, a straightforward application of the uniqueness result for Theorem 1 means that all the components of the US representation are uniquely identified up to permutations of the state space and common positive affine transformations of the EU functions.

Following the same exercise we did for a UG representation, if we restrict a US representation to $\mathcal{M}^1$ and $\mathcal{M}^2$, then we get the same results. Namely, if $\mathcal{U}$ is a US representation, and if we define U^1 and U^2 as we did earlier, then

$$U^1(x) = \sum_{i=1}^{I} \max_{\beta \in x} u_i(\beta)$$

and

$$U^2(x) = \sum_{i=1}^{I} \max_{\beta \in x} \big[u_i(\beta) + v_i(\beta)\big] - \max_{\beta \in x} v_i(\beta).$$

As discussed in Section 2, U^2 above is equivalent to the FA representation. Thus, when restricted to either $\mathcal{M}^1$ or $\mathcal{M}^2$, the UG and US representations are indistinguishable. This is not the case in the next section when we consider the case of constant normative preference.

4. Constant normative preference

We now focus on the special case when there is no uncertainty about normative preference. As explained in the Introduction, this allows us to give alternative characterizations of analogues of the S and G representations, but where all components of the representations are uniquely identified.

The following example illustrates the importance of uniquely identifying these representations.

Example 3. Suppose there are three final outcomes, and let $w_1 = (2, 2, -4)$, $w_2 = (1, 2, -3)$, $v_1 = (-1, 2, -1)$, and $v_2 = (-2, 2, 0)$ be vectors representing EU functions over Δ. Suppose $\succeq$ is a binary relation over $\mathcal{M}$ with a FA representation

$$U(x) = \sum_{k=1}^{2} \max_{\beta \in x} w_k(\beta) - \sum_{j=1}^{2} \max_{\beta \in x} v_j(\beta).$$

Note that U can be written as two different S representations,

$$U(x) = \frac{1}{2}\Big\{\max_{\beta \in x}\big[u(\beta) + \hat{v}_1(\beta)\big] - \max_{\beta \in x} \hat{v}_1(\beta)\Big\} + \frac{1}{2}\Big\{\max_{\beta \in x}\big[u(\beta) + \hat{v}_2(\beta)\big] - \max_{\beta \in x} \hat{v}_2(\beta)\Big\},$$

where $u = w_1 + w_2 - v_1 - v_2 = (6, 0, -6)$, $\hat{v}_1 = 2v_1$, and $\hat{v}_2 = 2v_2$, and

$$U(x) = \frac{1}{3}\Big\{\max_{\beta \in x}\big[u(\beta) + \bar{v}_1(\beta)\big] - \max_{\beta \in x} \bar{v}_1(\beta)\Big\} + \frac{2}{3}\Big\{\max_{\beta \in x}\big[u(\beta) + \bar{v}_2(\beta)\big] - \max_{\beta \in x} \bar{v}_2(\beta)\Big\},$$

where u is as above, $\bar{v}_1 = 3v_1$, and $\bar{v}_2 = \frac{3}{2}v_2$.

Recall that for the S representation, the interpretation is that $u + v_i$ represents the choice preference in state i, and q_i represents the probability that state i is realized. Hence the first representation suggests that the maximizer of $u + \hat{v}_1 = 2w_1$ is chosen 1/2 of the time, while the second representation suggests that the maximizer of $u + \bar{v}_2 = \frac{3}{2}w_1$ is chosen 2/3 of the time. But since $u + \hat{v}_1$ and $u + \bar{v}_2$ are cardinally equivalent, they represent the same preference over Δ. This means that the two representations suggest different (random) choices from menus even though they represent the same preference over menus. Our results below show that it is possible to behaviorally distinguish between analogues of these two representations in the domain of neighborhoods. We are thus able to separate behaviorally the decision maker's beliefs from her tastes. $\Diamond$

We now consider an axiom that imposes that normative preference be the same across states.

Constant Normative Preference. *If* $\{\{\beta\}\} \succeq \{\{\alpha\}\}$, *then* $\{\{\alpha\}, \{\beta\}\} \sim \{\{\beta\}\}$.

If the agent was not uncertain about her normative preference, then her normative preference would be revealed through her commitment preference (i.e., her preference over the neighborhoods that take the form $\{\{\alpha\}\}$). Thus $\{\{\beta\}\} \succeq \{\{\alpha\}\}$ reveals that the agent thinks β is normatively better than α. Now consider the neighborhood $\{\{\alpha\}, \{\beta\}\}$. Since both $\{\alpha\}$ and $\{\beta\}$ are singleton menus, final consumption is decided in the interim period. Thus temptation is not an issue for the agent when considering $\{\{\alpha\}, \{\beta\}\}$. Therefore, in the interim period, she should choose between α and β according to her normative preference. Since she has already revealed that she thinks β is normatively better than α, then she should choose β over α in the interim period, or $\{\{\alpha\}, \{\beta\}\} \sim \{\{\beta\}\}$.

Constant Normative Preference is obviously necessary for the following representations.

Definition 5. A *constant normative preference and general temptation representation* is a function

$$\mathcal{U}(X) = \sum_{i=1}^{I} q_i \max_{x \in X} \left\{ \max_{\beta \in x} \left[u(\beta) + \sum_{j=1}^{J_i} v_{ij}(\beta) \right] - \sum_{j=1}^{J_i} \max_{\beta \in x} v_{ij}(\beta) \right\}, \tag{CG}$$

where $I \in \mathbb{N}_0$, $J_i \in \mathbb{N}_0$ for every i, $q_i > 0$ for every i, $\sum_I q_i = 1$, and u and each v_{ij} are EU functions.

Definition 6. A *constant normative preference and simple temptation representation* is a function

$$\mathcal{U}(X) = \sum_{i=1}^{I} q_i \max_{x \in X} \left\{ \max_{\beta \in x} \left[u(\beta) + v_i(\beta) \right] - \max_{\beta \in x} v_i(\beta) \right\}, \tag{CS}$$

where $I \in \mathbb{N}_0$, $q_i > 0$ for every i, $\sum q_i = 1$, and u and each v_i are EU functions.

Interestingly, a strict preference for flexibility is still possible at the ex ante stage under the CG and CS representations. One may think that it would not be, since there is no longer any uncertainty about normative preference. However, there is still some taste uncertainty ex ante, as the agent does not know which temptations will affect him. The following example, based off a similar one by DLR, illustrates why uncertainty about temptations lead to an ex ante preference for flexibility.

Example 4. Suppose an agent is on a diet and knows what is normatively best, but is uncertain whether she will have a sugary craving or a salty craving that tempts her. Broccoli (b) is normatively best, while chocolate cake (c) will tempt her if she has a sugary craving and potato chips (p) will tempt her if she has a salty craving. If the temptation uncertainty is resolved in the interim period, then we would expect the agent to have the preference

$$\big\{\{b,c\},\{b,p\}\big\} \succ \big\{\{b,c\}\big\} \quad \text{and} \quad \big\{\{b,c\},\{b,p\}\big\} \succ \big\{\{b,p\}\big\}. \qquad \Diamond$$

With our other axioms, Constant Normative Preference is also sufficient for a CS representation.

Theorem 3. *The preference $\succeq$ satisfies the DLR axioms, Ex Ante Preference for Flexibility, Interim Preference for Commitment, Interim Choice Consistency, and Constant Normative Preference if and only if $\succeq$ is represented by a CS representation.*

However, adding Constant Normative Preference to the list of axioms in Theorem 1 is not sufficient to obtain a CG representation. To see this, note that the representation

$$\mathcal{U}(X) = \sum_{i=1}^{\hat{I}} q_i \max_{x\in X}\left\{\max_{\beta\in x}\left[u(\beta) + \sum_{j=1}^{J_i} v_{ij}(\beta)\right] - \sum_{j=1}^{J_i}\max_{\beta\in x} v_{ij}(\beta)\right\}$$
$$+ \sum_{i=\hat{I}+1}^{I} \max_{x\in X}\left\{\max_{\beta\in x}\left[\sum_{j=1}^{J_i} v_{ij}(\beta)\right] - \sum_{j=1}^{J_i}\max_{\beta\in x} v_{ij}(\beta)\right\}$$

satisfies Constant Normative Preference but it does not, in general, have a CG representation.[13]

So consider the following strengthening of Constant Normative Preference.

Improvement Monotonicity. *If $\{\{\alpha\}\}\cup X \succ X$ and $\{\{\beta\}\} \succ \{\{\alpha\}\}$, then $\{\{\beta\},\{\alpha\}\}\cup X \succ \{\{\alpha\}\}\cup X$.*

Consider the statement $\{\{\alpha\}\}\cup X \succ X$. Since $\{\{\alpha\}\}$ represents commitment to the alternative α, this is saying that commitment to α improves the neighborhood X. If commitment to α improves the neighborhood X, then any commitment strictly better than α must improve the neighborhood $\{\{\alpha\}\}\cup X$.

It is not hard to show that Improvement Monotonicity implies Constant Normative Preference.

[13] Indeed adding Constant Normative Preference to the set of axioms in Theorem 1 characterizes this representation. (This result follows directly from Lemma 6.) Note also that this representation is the analogue to what DLR call a "weak temptation representation."

Lemma 1. *If $\succeq$ satisfies Improvement Monotonicity, Ex Ante Preference for Flexibility, and Continuity, then $\succeq$ satisfies Constant Normative Preference.*

Proof. Suppose $\{\{\beta\}\} \succ \{\{\alpha\}\}$. Then if we also have $\{\{\alpha\}, \{\beta\}\} \succ \{\{\beta\}\}$, Improvement Monotonicity implies $\{\{\alpha\}, \{\beta\}\} \succ \{\{\alpha\}, \{\beta\}\}$ (taking $X = \{\{\beta\}\}$), a contradiction. Hence we must have $\{\{\alpha\}, \{\beta\}\} \preceq \{\{\beta\}\}$. Ex Ante Preference for Flexibility then implies $\{\{\alpha\}, \{\beta\}\} \sim \{\{\beta\}\}$.

Similarly, if $\{\{\alpha\}\} \succ \{\{\beta\}\}$, then we must have $\{\{\alpha\}, \{\beta\}\} \sim \{\{\alpha\}\}$. Continuity guarantees that if $\{\{\alpha\}\} \sim \{\{\beta\}\}$, then we must have $\{\{\alpha\}, \{\beta\}\} \sim \{\{\alpha\}\} \sim \{\{\beta\}\}$. Hence, we have shown that if $\{\{\beta\}\} \succeq \{\{\alpha\}\}$, then $\{\{\alpha\}, \{\beta\}\} \sim \{\{\beta\}\}$. □

Theorem 4. *The preference $\succeq$ satisfies the DLR axioms, Ex Ante Preference for Flexibility, Interim Preference for Commitment, and Improvement Monotonicity if and only if $\succeq$ is represented by a CG representation.*

One interesting aspect of this last theorem is that we are able to obtain a result similar to DLR's but without a technical axiom like their Approximate Improvements Are Chosen (AIC). The intuition behind AIC is complicated and relies on considering the closure of the set of improvements of a menu. (An improvement of a menu is simply an alternative that, when added to the menu, improves that menu.) In our theorem, Improvement Monotonicity plays the same role as AIC. Though our domain is certainly more complicated than that used by DLR, Improvement Monotonicity is arguably more intuitive than AIC.

We end by noting that these uniqueness results depend crucially on the normalization of the representations as well as the specific timing of the model. First, just as in Dekel et al. (2001), our uniqueness results depend on the representations being minimal, meaning all possible redundancies have been removed from the representation. Second, they depend on normalizing the normative utility u across the different states, as the probabilities could not be identified if the magnitude of the normative utilities varied across states.[14] Thus even though the CG and CS representations have state-dependent utility, we are able to identify the subjective probabilities because u is common across the states. Finally, the timing of the model (in which subjective uncertainty is resolved at a different time than the realization of temptation) allows for identification. For example, a model in which the uncertainty about temptation was resolved after the interim period would have similar identification problems as Dekel et al. (2009) and Stovall (2010).

Appendix A: Notation

Throughout the appendices, we identify an EU function with its corresponding vector in Euclidean space consisting of utilities of pure outcomes, e.g., $u(\beta) = u \cdot \beta$. We use $\mathbf{0}$ and $\mathbf{1}$ to represent the vectors of 0s and 1s, respectively. Note that if the vector w satisfies $w \cdot \mathbf{1} = 0$ and if β is in the interior of Δ, then $\beta + \epsilon w \in \Delta$ for small enough ϵ. Also, for $I \in \mathbb{N}_0$, we abuse notation and let I also denote the set $\{1, 2, \dots, I\}$.

[14] Without this normalization, the probabilities could be identified by observing interim choice of menus, similar to Ahn and Sarver's (2013) approach.

Appendix B: A preliminary representation

In this section we introduce a preliminary representation that serves as a foundation for all the other representations. This also allows us to introduce a uniqueness result (see Appendix C) upon which all subsequent uniqueness results are based.

Let $\mathcal{U}:\mathcal{N}\to\mathbb{R}$ be the function

$$\mathcal{U}(X)=\sum_{i\in I}\max_{x\in X}U_i(x), \tag{6}$$

where $I\in\mathbb{N}_0$ and each U_i is a FA representation. Another way to write (6) then is

$$\mathcal{U}(X)=\sum_{i\in I}\max_{x\in X}\left\{\sum_{k\in K_i}\max_{\beta\in x}w_{ik}(\beta)-\sum_{j\in J_i}\max_{\beta\in x}v_{ij}(\beta)\right\},$$

where $K_i, J_i\in\mathbb{N}_0$ for every i, and each w_{ik} and v_{ij} is an EU function.

Theorem 5. *The preference $\succeq$ satisfies the DLR axioms and Ex Ante Preference for Flexibility if and only if $\succeq$ has a representation in the form of (6).*

Proof. Showing that the axioms are necessary is a straightforward exercise and so is omitted.

The proof for sufficiency relies on the main result from Kopylov (2009a, Theorem 2.1). The primitive in Kopylov's theorem is a binary relation over nonempty compact subsets of a convex compact space. Thus both $\mathcal{M}$ and $\mathcal{N}$ are special cases of Kopylov's setup. Since Kopylov's finiteness axiom differs from ours, the key steps are to show that Kopylov's finiteness axiom is satisfied. We do this first for the ex ante preference $\succeq$ over $\mathcal{N}$ and then for each implied interim preference over $\mathcal{M}$.

The following property is defined using Kopylov's setup, i.e., $A_1, A_2, \dots$ are nonempty compact subsets of a convex compact space and $\succeq'$ is a binary relation over such objects.

KF. *There exists N such that for every $N'>N$ and sequence of sets $A_1,\dots,A_{N'}$, there exists $m\in N'$ such that $\bigcup_{n\in N}A_n\sim'\bigcup_{n\in N\setminus\{m\}}A_n$.*

Note that the property KF is not the finiteness axiom used by Kopylov. However, Kopylov shows (Kopylov 2009a, pp. 358–359) that KF is equivalent to his finiteness axiom.

We now begin the sufficiency part of the proof. Let $\succeq$ satisfy the axioms, with N as stated in Finiteness. First we show that $\succeq$ satisfies KF. By way of contradiction, suppose not. Then there exists $X_1,\dots,X_N$ such that $\bigcup_{m\in N}X_m\equiv X_\sigma\nsim X_{-n}\equiv\bigcup_{m\in N\setminus\{n\}}X_m$ for every $n\in N$. By Finiteness, there exists Y critical for X_σ such that $|Y|<N$. But this implies that there exists $n^*\in N$ such that $Y\subset X_{-n^*}$. (If not, then $|Y|\geq N$.) But since Y is critical for X_σ and since $X_{-n^*}\subset X_\sigma$, we have $X_{-n^*}\sim X_\sigma$, a contradiction.

Since $\succeq$ satisfies the DLR axioms and KF, we apply the result from Kopylov (2009a, Theorem 2.1) to obtain the representation for $\succeq$,

$$\mathcal{U}(X)=\sum_{i\in I}\max_{x\in X}U_i(x)-\sum_{j\in J}\max_{x\in X}V_j(x),$$

where $I, J \geq 0$, each U_i and V_j is a continuous linear function from $\mathcal{M}$ to $\mathbb{R}$, and the set $\{U_1, \dots, U_I, V_1, \dots, V_J\}$ is not redundant. Furthermore, since $\succeq$ satisfies Ex Ante Preference for Flexibility, the same result from Kopylov implies that $J = 0$.

For every $i \in I$, let $\succeq_i$ by the binary relation over $\mathcal{M}$ implied by U_i. Since $U_i : \mathcal{M} \to \mathbb{R}$ is a continuous linear function for every i, then $\succeq_i$ satisfies the analogues to Order, Continuity, and Independence.

We show now that for every $i \in I$, $\succeq_i$ satisfies KF for a preference relation over $\mathcal{M}$. So fix $i^* \in I$. By way of contradiction, suppose KF is not satisfied by $\succeq_{i^*}$. Then there exists $x_1, \dots, x_N$ such that $U_{i^*}(x_\sigma) \neq U_{i^*}(x_{-n})$ for every $n \in N$ (where $x_\sigma \equiv \bigcup_{m \in N} x_m$ and $x_{-n} \equiv \bigcup_{m \in N \setminus \{n\}} x_m$ for every n). By Kopylov (2009a, Lemma A.1), there exists $z_1, \dots, z_I$ such that $U_i(z_i) > U_i(z_j)$ for every $i, j \in I$, where $i \neq j$. Hence (by continuity) there exists $\epsilon > 0$ such that

$$U_i\big((1-\epsilon)z_i + \epsilon x\big) > U_i\big((1-\epsilon)z_j + \epsilon x'\big) \tag{7}$$

for every $i, j \in I$, where $i \neq j$, and for every $x, x' \in \{x_\sigma, x_{-1}, \dots, x_{-N}\}$. Set

$$\bar{z}_i \equiv (1-\epsilon)z_i + \epsilon x_\sigma$$

for every $i \in I$. Set $X \equiv \{\bar{z}_i\}_I$. By Finiteness, there exists y critical for $\bar{z}_{i^*} \in X$ such that $|y| < N$. But then there must exist $n^* \in N$ such that $y \subset (1-\epsilon)z_{i^*} + \epsilon x_{-n^*} \equiv y_{n^*}$. (If not, then $|Y| \geq N$.) Note that $y_{n^*} \subset \bar{z}_{i^*}$ since $x_{-n^*} \subset x_\sigma$. Since y is critical for $\bar{z}_{i^*} \in X$, this implies that $(X \setminus \{\bar{z}_{i^*}\}) \cup \{y_{n^*}\} \sim X$, i.e.,

$$\sum_{i \in I} \max_{x \in (X \setminus \{\bar{z}_{i^*}\}) \cup \{y_{n^*}\}} U_i(x) = \sum_{i \in I} \max_{x \in X} U_i(x). \tag{8}$$

Equation (7) implies

$$\bar{z}_i = \arg\max_{x \in X} U_i(x)$$

for every $i \in I$,

$$\bar{z}_i = \arg\max_{x \in (X \setminus \{\bar{z}_{i^*}\}) \cup \{y_{n^*}\}} U_i(x)$$

for every $i \neq i^*$, and

$$y_{n^*} = \arg\max_{x \in (X \setminus \{\bar{z}_{i^*}\}) \cup \{y_{n^*}\}} U_{i^*}(x).$$

Hence, (8) becomes

$$U_{i^*}(y_{n^*}) + \sum_{i \neq i^*} U_i(\bar{z}_i) = \sum_{i \in I} U_i(\bar{z}_i).$$

Subtracting $\sum_{i \neq i^*} U_i(\bar{z}_i)$ from both sides implies that $U_{i^*}(y_{n^*}) = U_{i^*}(\bar{z}_{i^*})$. But the linearity of U_{i^*} implies

$$U_{i^*}(y_{n^*}) = U_{i^*}(\bar{z}_{i^*}),$$

$$U_{i^*}\big((1-\epsilon)z_{i^*} + \epsilon x_{-n^*}\big) = U_{i^*}\big((1-\epsilon)z_{i^*} + \epsilon x_\sigma\big),$$

$$(1-\epsilon)U_{i^*}(z_{i^*})+\epsilon U_{i^*}(x_{-n^*})=(1-\epsilon)U_{i^*}(z_{i^*})+\epsilon U_{i^*}(x_\sigma),$$
$$U_{i^*}(x_{-n^*})=U_{i^*}(x_\sigma).$$

But this contradicts $U_{i^*}(x_\sigma)\neq U_{i^*}(x_{-n^*})$.

We apply the result from Kopylov (2009a, Theorem 2.1) to $\succeq_i$ to get a FA representation for U_i. □

Appendix C: Uniqueness

In this section, we state the uniqueness result associated with Theorem 5. As all of our main representations are special cases of (6), this serves as a basis for their respective uniqueness results.

Let A be any set and let $f, g: A\to\mathbb{R}$ be two real valued functions over A. We say f and g *represent the same ordering over* A if for every $a, b\in A$, $f(a)\geq f(b)$ if and only if $g(a)\geq g(b)$. Let $\{f_i\}_I$ be an indexed family of real valued functions over A. We say that $\{f_i\}_I$ is *redundant* if there exists a constant function in this set or if there exist $i, j\in I$, where $i\neq j$, such that f_i and f_j represent the same ordering over A.

Definition 7. We say that the FA representation $U(x)=\sum_K \max_{\beta\in x} w_k(\beta)-\sum_J \max_{\beta\in x} v_j(\beta)$ is *minimal* if $\{w_k\}_K\cup\{v_j\}_J$ is not redundant.

Definition 8. We say that the representation (6) is *minimal* if $\{U_i\}_I$ is not redundant and each U_i is minimal.

Lemma 2. *If $\succeq$ has a representation in the form of (6), then it has such a representation that is minimal.*

The proof is straightforward and so is omitted.

The following definitions help us to state the uniqueness result. Let f and g be two EU functions. For any $a>0$, we write $f\bowtie_a g$ if there exists $b\in\mathbb{R}$ such that $f=ag+b$. (Thus the standard uniqueness result from expected-utility theory states that f and g represent the same ordering over Δ if and only if there exists $a>0$ such that $f\bowtie_a g$.) More generally, let $\{f_i\}_I$ and $\{g_i\}_I$ be two indexed families of EU functions with the same index set I. For any $a>0$, we abuse notation and write $\{f_i\}_I\bowtie_a\{g_i\}_I$ if there exists a permutation π of I such that $f_i\bowtie_a g_{\pi(i)}$ for every i.

We now state the uniqueness result associated with Theorem 5.

Theorem 6. *If*

$$\mathcal{U}^n(X)=\sum_{i\in I^n}\max_{x\in X}\left\{\sum_{k\in K_i^n}\max_{\beta\in x} w_{ik}^n(\beta)-\sum_{j\in J_i^n}\max_{\beta\in x} v_{ij}^n(\beta)\right\}$$

for $n=1,2$ are both minimal representations of $\succeq$, then the following statements hold:

(i) We have $I^1=I^2$ ($\equiv I$).

(ii) There exist $a > 0$ *and* π *a permutation on* $\{1, \dots, I\}$ *such that for every i,*

(a) $K_i^1 = K_{\pi(i)}^2$ $(\equiv K_i)$

(b) $J_i^1 = J_{\pi(i)}^2$ $(\equiv J_i)$

(c) $\{w_{ik}^1\}_{K_i} \bowtie_a \{w_{\pi(i)k}^2\}_{K_i}$

(d) $\{v_{ij}^1\}_{J_i} \bowtie_a \{v_{\pi(i)j}^2\}_{J_i}$.

The proof of this theorem is omitted as it is a straightforward application of the uniqueness result in Kopylov (2009a, Theorem 2.1).

This result can easily be adapted to our main representations. Take the UG representation for example.

Corollary 1. *If*

$$\mathcal{U}^n(X) = \sum_{i \in I^n} \max_{x \in X} \left\{ \max_{\beta \in x} \left[u_i^n(\beta) + \sum_{j \in J_i^n} v_{ij}^n(\beta) \right] - \sum_{j \in J_i^n} \max_{\beta \in x} v_{ij}^n(\beta) \right\}$$

for $n = 1, 2$ *are both minimal UG representations of* $\succeq$*, then the following statements hold:*

(i) We have $I^1 = I^2$ $(\equiv I)$.

(ii) There exist $a > 0$ *and* π *a permutation on* $\{1, \dots, I\}$ *such that for every i,*

(a) $u_i^1 \bowtie_a u_{\pi(i)}^2$

(b) $J_i^1 = J_{\pi(i)}^2$ $(\equiv J_i)$

(c) $\{v_{ij}^1\}_{J_i} \bowtie_a \{v_{\pi(i)j}^2\}_{J_i}$.

The uniqueness result for the US representation is a special case of Corollary 1. Similarly, the uniqueness result for the CG representation is the following corollary.

Corollary 2. *If*

$$\mathcal{U}^n(X) = \sum_{i \in I^n} q_i^n \max_{x \in X} \left\{ \max_{\beta \in x} \left[u^n(\beta) + \sum_{j \in J_i^n} v_{ij}^n(\beta) \right] - \sum_{j \in J_i^n} \max_{\beta \in x} v_{ij}^n(\beta) \right\}$$

for $n = 1, 2$ *are both minimal CG representations of* $\succeq$ *such that* u^1, u^2 *are not constant, then the following statements hold:*

(i) We have $I^1 = I^2$ $(\equiv I)$.

(ii) There exist $a > 0$ *and* π *a permutation on* $\{1, \dots, I\}$ *such that for every i,*

(a) $u^1 \bowtie_a u^2$

(b) $q_i^1 = q_{\pi(i)}^2$

(c) $J_i^1 = J_{\pi(i)}^2$ $(\equiv J_i)$

(d) $\{v_{ij}^1\}_{J_i} \bowtie_a \{v_{\pi(i)j}^2\}_{J_i}$.

The requirement that u not be constant is needed to identify the probabilities q_i. The uniqueness result for the CS representation is a special case of Corollary 2.

Appendix D: Proofs

D.1 *Proof for Theorem 1*

First we show that Interim Preference for Commitment is necessary. Let x, y, and X be given. Fix i. Set $w_i \equiv u_i + \sum_{j\in J_i} v_j$. Observe that either $\max_{\beta\in x} w_i(\beta) = \max_{\beta\in x\cup y} w_i(\beta)$ or $\max_{\beta\in y} w_i(\beta) = \max_{\beta\in x\cup y} w_i(\beta)$. Note also that for every $j \in J_i$, we have $\max_{\beta\in x} v_j(\beta) \leq \max_{\beta\in x\cup y} v_j(\beta)$ and $\max_{\beta\in y} v_j(\beta) \leq \max_{\beta\in x\cup y} v_j(\beta)$. Hence either

$$\max_{\beta\in x} w_i(\beta) - \sum_{j\in J_i} \max_{\beta\in x} v_j(\beta) \geq \max_{\beta\in x\cup y} w_i(\beta) - \sum_{j\in J_i} \max_{\beta\in x\cup y} v_j(\beta)$$

or

$$\max_{\beta\in y} w_i(\beta) - \sum_{j\in J_i} \max_{\beta\in y} v_j(\beta) \geq \max_{\beta\in x\cup y} w_i(\beta) - \sum_{j\in J_i} \max_{\beta\in x\cup y} v_j(\beta).$$

Thus for every i,

$$\max_{x'\in\{x,y\}\cup X} \left\{ \max_{\beta\in x'} \left[u_i(\beta) + \sum_{j\in J_i} v_j(\beta) \right] - \sum_{j\in J_i} \max_{\beta\in x'} v_j(\beta) \right\}$$
$$\geq \max_{x'\in\{x\cup y\}\cup X} \left\{ \max_{\beta\in x'} \left[u_i(\beta) + \sum_{j\in J_i} v_j(\beta) \right] - \sum_{j\in J_i} \max_{\beta\in x'} v_j(\beta) \right\}.$$

Now we show that the axioms are sufficient. First, the following lemma is useful for proving many of the representation theorems.

Lemma 3. *Let $\{U_i\}_I$ be a nonredundant indexed family of minimal FA representations, where $U_i(x) = \sum_{K_i} \max_{\beta\in x} w_{ik}(\beta) - \sum_{J_i} \max_{\beta\in x} v_{ij}(\beta)$. For $i \in I$ and $m \in K_i \cup J_i$, let $u_{im} = w_{im}$ if $m \in K_i$ and $u_{im} = v_{im}$ if $m \in J_i$. Let $u_{im} \cdot \mathbf{1} = 0$ for every $i \in I$ and every $m \in K_i \cup J_i$.*

Then there exist $x_1, \ldots, x_I$ (in the interior of Δ) such that the following statements hold:

(i) We have $U_i(x_i) > U_i(x_j)$ for every $i \neq j$.

(ii) For any i, for any $m, n \in K_i \cup J_i$ where $m \neq n$, $\arg\max_{\beta\in x_i} u_{im}(\beta)$ is a singleton and $\arg\max_{\beta\in x_i} u_{im}(\beta) \neq \arg\max_{\beta\in x_i} u_{in}(\beta)$.

Proof. Let S denote the set of all w_{ik}s and v_{ij}s normalized to have unit length, i.e.,

$$S \equiv \left\{ s = \frac{u_{im}}{\sqrt{u_{im} \cdot u_{im}}} : i \in I \text{ and } m \in K_i \cup J_i \right\}.$$

Obviously S is finite. Also, for every $s \in S$, $s \cdot \mathbf{1} = 0$ and there exists $m \in K_i \cup J_i$ such that u_{im} and s represent the same ordering over Δ. Thus for every i, we can write

$$U_i(y) = \sum_{s \in S} \max_{\beta \in y} b_{is} s \cdot \beta,$$

where $b_{is} > 0$ if there exists $k \in K_i$ such that w_{ik} and s represent the same ordering, $b_{is} < 0$ if there exists $j \in J_i$ such that v_{jk} and s represent the same ordering, and $b_{is} = 0$ otherwise. (Since U_i is minimal, exactly one of these holds for every s.)

Let x^* denote a sphere in the interior of Δ. For every $s \in S$, set $\beta_s \equiv \arg\max_{\beta \in x^*} s \cdot \beta$. Note that for $s \neq s'$, we have $\beta_s \neq \beta_{s'}$. Set $x \equiv \{\beta_s\}_S$. Hence $U_i(x) = \sum_{s \in S} b_{is} s \cdot \beta_s$. For $a \in \mathbb{R}^S$ and $\epsilon > 0$, set

$$\bar{x}(\epsilon, a) \equiv \{\beta_s + \epsilon a_s s\}_{s \in S}.$$

For fixed a, there exists ϵ_a small enough such that $\beta_s + \epsilon_a a_s s$ is in the interior of Δ and such that $\beta_s + \epsilon_a a_s s = \arg\max_{\beta \in \bar{x}(\epsilon_a, a)} s \cdot \beta$. For every i, set

$$a_i \equiv \left\{ \frac{b_{is}}{\sqrt{\sum_{s' \in S} b_{is'}^2}} \right\}_{s \in S}.$$

Set $\epsilon \equiv \min_i \epsilon_{a_i}$. For every i, set $x_i \equiv \bar{x}(\epsilon, a_i)$. Note that $U_i(x_i) = U_i(x) + \epsilon \sum_{s \in S} a_{is} b_{is}$. Hence $x_i = \arg\max_{i' \in I} U_i(x_{i'})$ since $\{U_i\}_I$ is not redundant and since a_i is the unique solution to the constrained maximization problem: $\max_{\bar{a}} \sum_{s \in S} \bar{a}_s b_{is}$ subject to $\sum_{s \in S} \bar{a}_s^2 = 1$. This proves the first part.

The second part follows from the fact that each U_i is minimal and that $\beta_s + \epsilon a_{is} s = \arg\max_{\beta \in x_i} s \cdot \beta$ for every s. □

Lemma 4. *Let $\succeq$ have a representation in the form of (6) that is minimal. If $\succeq$ also satisfies Interim Preference for Commitment, then $K_i \leq 1$ for every i.*

Proof. Since $\{U_i\}_I$ is not redundant, take $x_1, \ldots, x_I$ from Lemma 3 and set $X \equiv \{x_1, \ldots, x_I\}$. According to Theorem 6, we can assume without loss of generality that $w_{ik} \cdot \mathbf{1} = 0$ for every i and every $k \in K_i$. Fix i^* and by way of contradiction suppose $K_{i^*} > 1$. For any $k \in K_{i^*}$, set $\alpha_k \equiv \arg\max_{\beta \in x_{i^*}} w_{i^*k}(\beta)$. (Lemma 3 guarantees this max is a singleton.) For any $\epsilon > 0$, set $x_k^\epsilon \equiv x_{i^*} \cup \{\alpha_k + \epsilon w_{i^*k}\}$. Take $k, k' \in K_{i^*}$ such that $k \neq k'$. By Lemma 3, $U_i(x_i) > U_i(x_{i^*})$ for every $i \neq i^*$ and $\max_{\beta \in x_{i^*}} v_{i^*j}(\beta) > \max\{v_{i^*j}(\alpha_k), v_{i^*j}(\alpha_{k'})\}$ for every $j \in J_{i^*}$. Hence, there exists $\epsilon > 0$ such that, for every $i \neq i^*$ and $j \in J_{i^*}$,

$$U_i(x_i) > \max\left\{U_i(x_k^\epsilon \cup x_{k'}^\epsilon), U_i(x_k^\epsilon), U_i(x_{k'}^\epsilon)\right\}$$

and

$$\max_{\beta\in x_{i^*}} v_{i^*j}(\beta)=\max_{\beta\in x_k^\epsilon} v_{i^*j}(\beta)=\max_{\beta\in x_{k'}^\epsilon} v_{i^*j}(\beta)$$

hold. This implies

$$U_{i^*}(x_k^\epsilon\cup x_{k'}^\epsilon)>U_{i^*}(x_k^\epsilon),\qquad U_{i^*}(x_{k'}^\epsilon)>U_{i^*}(x_{i^*}).$$

Hence $\mathcal{U}(\{x_k^\epsilon\cup x_{k'}^\epsilon\}\cup X)>\mathcal{U}(\{x_k^\epsilon, x_{k'}^\epsilon\}\cup X)$, violating Interim Preference for Commitment. □

Let $\succeq$ satisfy the stated axioms. By Theorem 5 and Lemma 2, $\succeq$ has a representation $\mathcal{U}$ in the form of (6) that is minimal. By Lemma 4, $K_i\le 1$ for every i. The UG representation follows by setting $u_i=w_i-\sum_{J_i} v_j$ for every i, where $w_i=w_k$ for $k\in K_i$ if $K_i=1$ and $w_i=\mathbf{0}$ if $K_i=0$.

D.2 *Proof for Theorem 2*

First we show that Interim Choice Consistency is necessary. So let x, y, X satisfy $\{x, x\cup y\}\cup X\succ\{x\cup y\}\cup X$. Then there exists $i\in I$ such that

$$\max_{\beta\in x}\bigl[u_i(\beta)+v_i(\beta)\bigr]-\max_{\beta\in x}v_i(\beta)>\max_{\beta\in z}\bigl[u_i(\beta)+v_i(\beta)\bigr]-\max_{\beta\in z}v_i(\beta)$$

for every $z\in\{x\cup y\}\cup X$. Specifically, this holds when $z=x\cup y$. This implies

$$\max_{\beta\in x}v_i(\beta)<\max_{\beta\in x\cup y}v_i(\beta)=\max_{\beta\in y}v_i(\beta).$$

Since

$$\max_{\beta\in x\cup y}\bigl[u_i(\beta)+v_i(\beta)\bigr]\ge\max_{\beta\in y}\bigl[u_i(\beta)+v_i(\beta)\bigr],$$

we have

$$\max_{\beta\in x\cup y}\bigl[u_i(\beta)+v_i(\beta)\bigr]-\max_{\beta\in x\cup y}v_i(\beta)\ge\max_{\beta\in y}\bigl[u_i(\beta)+v_i(\beta)\bigr]-\max_{\beta\in y}v_i(\beta).$$

Hence

$$\max_{\beta\in x}\bigl[u_i(\beta)+v_i(\beta)\bigr]-\max_{\beta\in x}v_i(\beta)>\max_{\beta\in z}\bigl[u_i(\beta)+v_i(\beta)\bigr]-\max_{\beta\in z}v_i(\beta)$$

for every $z\in\{y\}\cup X$, which implies that $\{x, y\}\cup X\succ\{y\}\cup X$.

Now we show the axioms are sufficient. We need the following lemma.

Lemma 5. *Let $\succeq$ have a representation in the form of (6) that is minimal. If $\succeq$ satisfies Interim Choice Consistency, then $J_i\le 1$ for every i.*

Proof. Since $\{U_i\}_I$ is not redundant, take $x_1,\dots,x_I$ from Lemma 3. According to Theorem 6, we can assume without loss of generality that $v_{ij}\cdot\mathbf{1}=0$ for every i and every $j\in J_i$. Fix i^* and by way of contradiction suppose $J_{i^*}>1$. For any $j\in J_{i^*}$, set

$\alpha_j \equiv \arg\max_{\beta\in x_{i^*}} v_{i^*j}(\beta)$. (Lemma 3 guarantees this max is a singleton.) For any $\epsilon > 0$, set $x_j^\epsilon \equiv x_{i^*} \cup \{\alpha_j + \epsilon v_{i^*j}\}$. Set $X_{-i^*} \equiv \{x_1, \dots, x_I\} \setminus \{x_{i^*}\}$. Take $j, j' \in J_{i^*}$ such that $j \neq j'$. By Lemma 3, $U_i(x_i) > U_i(x_{i^*})$ and $U_{i^*}(x_{i^*}) > U_{i^*}(x_i)$ for every $i \neq i^*$, $\max_{\beta\in x_{i^*}} w_{i^*k}(\beta) > w_{i^*k}(\alpha_j)$ for every $k \in K_{i^*}$, $v_{i^*j'}(\alpha_{j'}) > v_{i^*j'}(\alpha_j)$, and $v_{i^*j}(\alpha_j) > v_{i^*j}(\alpha_{j'})$. Hence, there exists $\epsilon > 0$ such that, for every $i \neq i^*$ and $k \in K_{i^*}$,

$$U_i(x_i) > \max\{U_i(x_j^\epsilon \cup x_{j'}^\epsilon), U_i(x_j^\epsilon), U_i(x_{j'}^\epsilon)\},$$

$$U_{i^*}(x_j^\epsilon \cup x_{j'}^\epsilon) > U_{i^*}(x_i),$$

$$\max_{\beta\in x_{i^*}} w_{i^*k}(\beta) = \max_{\beta\in x_j^\epsilon} w_{i^*k}(\beta) = \max_{\beta\in x_{j'}^\epsilon} w_{i^*k}(\beta),$$

$$v_{i^*j'}(\alpha_{j'}) > v_{i^*j'}(\alpha_j + \epsilon v_{i^*j}),$$

and

$$v_{i^*j}(\alpha_j) > v_{i^*j}(\alpha_{j'} + \epsilon v_{i^*j'})$$

hold. Hence $U_{i^*}(x_j^\epsilon) > U_{i^*}(x_j^\epsilon \cup x_{j'}^\epsilon)$ and $U_{i^*}(x_{j'}^\epsilon) > U_{i^*}(x_j^\epsilon \cup x_{j'}^\epsilon)$. Without loss of generality, assume $U_{i^*}(x_j^\epsilon) \geq U_{i^*}(x_{j'}^\epsilon)$. It is easy to verify then that $\mathcal{U}(\{x_{j'}^\epsilon, x_j^\epsilon \cup x_{j'}^\epsilon\} \cup X_{-i^*}) > \mathcal{U}(\{x_j^\epsilon \cup x_{j'}^\epsilon\} \cup X_{-i^*})$ and $\mathcal{U}(\{x_j^\epsilon, x_{j'}^\epsilon\} \cup X_{-i^*}) = \mathcal{U}(\{x_j^\epsilon\} \cup X_{-i^*})$, violating Interim Choice Consistency. □

Let $\succeq$ satisfy the stated axioms. By Theorem 5 and Lemma 2, $\succeq$ has a representation $\mathcal{U}$ in the form of (6) that is minimal. By Lemmas 4 and 5, $K_i \leq 1$ and $J_i \leq 1$ for every i. For every i where $K_i = 1$, set $w_i = w_k$, where $k \in K_i$; otherwise set $w_i = \mathbf{0}$. Similarly, if $J_i = 1$, then set $v_i = v_j$ for $j \in J_i$; otherwise set $v_i = \mathbf{0}$. The US representation follows by setting $u_i \equiv w_i - v_i$.

D.3 *Proof for Theorem 3*

The necessity of Constant Normative Preference is obvious. The sufficiency part relies on the following lemma.

Lemma 6. *Let $\succeq$ have a representation in the form of (6). If $\succeq$ satisfies Constant Normative Preference, then there exists an EU function u such that for every $i \in I$, either $u_i \equiv \sum_{K_i} w_k - \sum_{J_i} v_j$ and u represent the same preference over Δ or u_i is constant.*

Proof. By way of contradiction, assume that there exist $i, i' \in I$ such that u_i and $u_{i'}$ are both nonconstant and represent different preferences over Δ. Then there exist α and α' such that $u_i(\alpha) > u_i(\alpha')$ and $u_{i'}(\alpha') > u_{i'}(\alpha)$. But this implies $\{\{\alpha\}, \{\alpha'\}\} \succ \{\{\alpha\}\}$ and $\{\{\alpha\}, \{\alpha'\}\} \succ \{\{\alpha'\}\}$. So no matter how $\{\{\alpha\}\}$ and $\{\{\alpha'\}\}$ are ranked by $\succeq$, Constant Normative Preference is violated. □

Let $\succeq$ satisfy the stated axioms. By Theorem 2, $\succeq$ has a US representation

$$\mathcal{U}(X) = \sum_{i\in I} \max_{x\in X} \Big\{ \max_{\beta\in x} [u_i(\beta) + v_i(\beta)] - \max_{\beta\in x} v_i(\beta) \Big\}.$$

Since this is a special case of (6), Lemma 2 implies that it is without loss of generality to assume $\mathcal{U}$ is minimal. Here this means that u_i is not constant for every i.

By Lemma 6, there exists u such that for every i, $u_i = q_i u + b_i$ for some $q_i \geq 0$ and $b_i \in \mathbb{R}$. By standard uniqueness results, we can assume without loss of generality that $\sum_I q_i = 1$ and that $b_i = 0$ for every i. Also, the minimality of $\mathcal{U}$ implies that $q_i > 0$ for every i. Thus for every i, set $\hat{v}_i \equiv v_i/q_i$. This gives us the CS representation

$$\mathcal{U}(X) = \sum_{i\in I} q_i \max_{x\in X}\Big\{\max_{\beta\in x}\big[u(\beta) + \hat{v}_i(\beta)\big] - \max_{\beta\in x}\hat{v}_i(\beta)\Big\}.$$

D.4 *Proof for Theorem 4*

First we show the necessity of Improvement Monotonicity. So let $\succeq$ have the CG representation

$$\mathcal{U}(X) = \sum_{i\in I} q_i \max_{x\in X}\left\{\max_{\beta\in x}\left[u(\beta) + \sum_{j\in J_i} v_{ij}(\beta)\right] - \sum_{j\in J_i}\max_{\beta\in x} v_{ij}(\beta)\right\}.$$

Suppose $\{\{\alpha\}\} \cup X \succ X$ and $\{\{\beta\}\} \succ \{\{\alpha\}\}$. Since $\{\{\alpha\}\} \cup X \succ X$, it must be that there exists i such that

$$u(\alpha) > \max_{x\in X}\left\{\max_{\beta\in x}\left[u(\beta) + \sum_{j\in J_i} v_{ij}(\beta)\right] - \sum_{j\in J_i}\max_{\beta\in x} v_{ij}(\beta)\right\}.$$

Since $\{\{\beta\}\} \succ \{\{\alpha\}\}$, we have $u(\beta) > u(\alpha)$. But then we must have

$$u(\beta) > \max_{x\in\{\{\alpha\}\}\cup X}\left\{\max_{\beta\in x}\left[u(\beta) + \sum_{j\in J_i} v_{ij}(\beta)\right] - \sum_{j\in J_i}\max_{\beta\in x} v_{ij}(\beta)\right\}.$$

Hence $\{\{\beta\}, \{\alpha\}\} \cup X \succ \{\{\alpha\}\} \cup X$.

Now we show that the axioms are sufficient. Let $\succeq$ satisfy the stated axioms. By Theorem 1, $\succeq$ has a UG representation

$$\mathcal{U}(X) = \sum_{i\in I} \max_{x\in X}\left\{\max_{\beta\in x}\left[u_i(\beta) + \sum_{j\in J_i} v_{ij}(\beta)\right] - \sum_{j\in J_i}\max_{\beta\in x} v_{ij}(\beta)\right\}.$$

For every i, set

$$V_i(x) \equiv \max_{\beta\in x}\left[u_i(\beta) + \sum_{j\in J_i} v_{ij}(\beta)\right] - \sum_{j\in J_i}\max_{\beta\in x} v_{ij}(\beta)$$

so that $\mathcal{U}(X) = \sum_{i\in I}\max_{x\in X} V_i(x)$. Since this is a special case of (6), Lemma 2 implies that it is without loss of generality to assume $\mathcal{U}$ is minimal. Here this means that $\{v_{ij}\}_{J_i}$ is not redundant for every i and that $\{V_i\}_I$ is not redundant.

By Lemma 1, $\succeq$ satisfies Constant Normative Preference. So by Lemma 6, there exists u such that for every i, we have $u_i = q_i u + b_i$ for some $q_i \geq 0$ and $b_i \in \mathbb{R}$.

Claim 1. *We have $q_i > 0$ for every i.*

Proof. If u is a constant function, then without loss of generality $q_i > 0$ for every i.

So assume that u is not a constant function. Without loss of generality, we can assume that $q_i > 0$ for some i. (If $q_i = 0$ for all i, then we can assume u is constant and choose new q_is.) Hence $\succeq$ is not constant over commitments. Thus there exist α', α'', and α''' such that $\{\{\alpha'\}\} \succ \{\{\alpha''\}\} \succ \{\{\alpha'''\}\}$. Without loss of generality, α', α'', and α''' are all in the interior of Δ.

By standard uniqueness results, we can assume without loss of generality that $u(\alpha') = 0$, $\sum_I q_i = 1$, $b_i = 0$ for every i, and $v_{ij} \cdot \mathbf{1} = 0$ for every i and for every $j \in J_i$. Thus for any β, we have $\mathcal{U}(\{\{\beta\}\}) = u(\beta)$.

Set $I^+ \equiv \{i \in I : q_i > 0\}$ and $I^0 \equiv \{i \in I : q_i = 0\}$. Fix $\epsilon \in (0, -u(\alpha''))$. Set $J \equiv \max_{i \in I} J_i$, $\bar{v} \equiv \max_{i \in I, j \in \bigcup_{i' \in I} J_{i'}} \sqrt{v_{ij} \cdot v_{ij}}$, and $a \equiv \frac{\epsilon}{\bar{v}J}$. Let x^* denote the sphere centered around α' with radius a. (If x^* is not in the interior of Δ, then choose a smaller a.) Thus any β on the boundary of x^* can be written as $\beta = \alpha' + as$, where s is a vector such that $s \cdot \mathbf{1} = 0$ and $s \cdot s = 1$.

Since $\mathcal{U}$ is minimal, apply Lemma 3 to get $x_1, \ldots, x_I$. As is evident from the construction of $x_1, \ldots, x_I$ in Lemma 3, we can assume $\alpha' \in x_i \subset x^*$ for every i. Hence for every i and for every $j \in J_i$, we have

$$\begin{aligned}
\max_{\beta \in x_i} v_{ij}(\beta) &\leq \max_{\beta \in x^*} v_{ij}(\beta) \\
&= v_{ij} \cdot \left(\alpha' + a \frac{v_{ij}}{\sqrt{v_{ij} \cdot v_{ij}}}\right) \\
&= v_{ij} \cdot \alpha' + a \frac{v_{ij} \cdot v_{ij}}{\sqrt{v_{ij} \cdot v_{ij}}} \\
&= v_{ij}(\alpha') + a\sqrt{v_{ij} \cdot v_{ij}}.
\end{aligned}$$

This implies for every i,

$$\begin{aligned}
\sum_{j \in J_i} \max_{\beta \in x_i} v_{ij}(\beta) &\leq \sum_{j \in J_i} \big(v_{ij}(\alpha') + a\sqrt{v_{ij} \cdot v_{ij}}\big) \\
&= \sum_{j \in J_i} v_{ij}(\alpha') + a \sum_{j \in J_i} \sqrt{v_{ij} \cdot v_{ij}} \\
&\leq \sum_{j \in J_i} v_{ij}(\alpha') + a \sum_{j \in J_i} \bar{v} \\
&\leq \sum_{j \in J_i} v_{ij}(\alpha') + a\bar{v}J \\
&= \sum_{j \in J_i} v_{ij}(\alpha') + \epsilon.
\end{aligned}$$

Since $\alpha' \in x_i$, we have for every i,

$$\begin{aligned}
V_i(x_i) &\geq \left[q_i u(\alpha') + \sum_{j \in J_i} v_{ij}(\alpha')\right] - \sum_{j \in J_i} \max_{\beta \in x_i} v_{ij} \cdot \beta \\
&\geq q_i u(\alpha') + \sum_{j \in J_i} v_{ij}(\alpha') - \sum_{j \in J_i} v_{ij}(\alpha') - \epsilon
\end{aligned}$$

$$= -\epsilon$$
$$> u(\alpha'').$$

Note that for every $i \in I^0$, we must have $J_i \geq 2$ (otherwise $\mathcal{U}$ would not be minimal). Recall by Lemma 3, $\arg\max_{\beta \in x_i} v_{ij}(\beta) \neq \arg\max_{\beta \in x_i} v_{ij'}(\beta)$ for every $j, j' \in J_i$, where $j \neq j'$. Hence for every $i \in I^0$, we have

$$V_i(x_i) = \max_{\beta \in x_i} \left[\sum_{j \in J_i} v_{ij} \cdot \beta \right] - \sum_{j \in J_i} \max_{\beta \in x_i} v_{ij} \cdot \beta < 0.$$

Note that for every i, we have $V_i(\{\beta\}) = q_i u(\beta)$. Hence for every $i \in I^+$, we have

$$V_i(x_i) > u(\alpha'') \geq q_i u(\alpha'') = V_i(\{\alpha''\})$$

and

$$V_i(x_i) > u(\alpha''') \geq q_i u(\alpha''') = V_i(\{\alpha'''\}),$$

while for every $i \in I^0$, we have

$$V_i(x_i) < 0 = V_i(\{\alpha''\}) = V_i(\{\alpha'''\}).$$

Let $X = \{x_1, \dots, x_I\}$. All together this implies

$$V_i(\{\alpha''\}) \leq \max_{x \in X \cup \{\{\alpha'''\}\}} V_i(x) \quad \forall i \tag{9}$$

and

$$V_i(\{\alpha'''\}) > \max_{x \in X} V_i(x) \quad \forall i \in I^0. \tag{10}$$

Hence (9) implies $\{\{\alpha'', \alpha'''\}\} \cup X \sim \{\{\alpha'''\}\} \cup X$. Also if I^0 is not empty, then (10) implies $\{\{\alpha'''\}\} \cup X \succ X$. Yet $\{\{\alpha''\}\} \succ \{\{\alpha'''\}\}$ by assumption. Together this violates Improvement Monotonicity. Hence I^0 is empty, so $q_i > 0$ for every i. □

For every i and for every $j \in J_i$, set $\hat{v}_{ij} \equiv v_{ij}/q_i$. Thus we can write $\mathcal{U}$ as

$$\mathcal{U}(X) = \sum_{i \in I} q_i \max_{x \in X} \left\{ \max_{\beta \in x} \left[u(\beta) + \sum_{j \in J_i} \hat{v}_{ij}(\beta) \right] - \sum_{j \in J_i} \max_{\beta \in x} \hat{v}_{ij}(\beta) \right\},$$

which is a CG representation.

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