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Collective household welfare and intra-household inequality

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We investigate the relationship between the individual and household indirect utility functions in the context of a collective household model. Our analysis produces new results that explain how the rule governing the distribution of resources among household members is related to the measurement of household welfare and intra-household inequality. We show that in a collective model of private consumption, income shares are equal to the product of two weights: the Pareto weight and a distribution weight reflecting income effects across individuals. For a weighted Bergsonian representation of household utility and general assumptions about individual preferences, we derive the associated household welfare functions and intra-household inequality measures belonging to a family of entropy indexes. We illustrate our findings with an empirical application that estimates a collective demand system to recover associated individual and household welfare functions along with the measures of intra-household inequality. This is the first application that estimates the Pareto weight and examines its role within a measure of income dispersion among household members.

Keywords. Collective household model, sharing rule, household welfare, intra-household inequality, Theil index.

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1. Introduction

In consumption analysis, the role of aggregation over individuals has been a subject of much interest (Blundell and Stoker 2005, Chiappori and Ekeland 2011, Gorman, 1953, 1961, Jorgenson et al. 1980, 1982, Muellbauer 1975). These studies treat households as individuals by implicitly assuming an egalitarian distribution that equates the levels of well-being among family members. But there is strong evidence that the levels of well-being can differ among family members (Browning and Chiappori 1998, Browning et al. 2014, Chiappori 1988, 1992). In collective household theory, knowledge about individual welfare entails a family welfare function that relates the set of potentially unequal levels of well-being for family members to an aggregate measure for the family as a whole (Sen 1984, p. 378). This "mini social choice problem," as Sen terms it, involves understanding the linkages among individual behavior, household demand, and the aggregation of unequal well-being. However, the relationships between individual and household welfare are complex and remain imperfectly understood. This suggests a need for a refined analysis of the linkages between individual demand behavior and household welfare. The composition of this missing piece of aggregation theory is our major contribution.

In the tradition of collective household models, the efficient allocation of resources within a household is captured by a Bergsonian household utility function involving different welfare weights applied to each individual. Maximizing this Bergsonian utility function subject to a household budget constraint generates centralized household demands that, in a Pareto efficient household, give the same demands obtainable from a decentralized household program. By examining the relationship between centralized and decentralized demands for private goods within a household economy, we show that income shares are equal to the product of two weights: the Pareto weight and a distribution weight reflecting income effects across individuals. We also show how these weights play a role in the evaluation of both household welfare and intra-household inequality for a specific class of indirect utilities.

Chiappori and Meghir (2015) contend that developments in collective theory should extend the analysis to the treatment of public goods and household production and to the measurement of intra-household inequality. Our investigation characterizes how the distribution of resources within the household impacts inequality. While our approach focuses on private consumption goods, note that our results could accommodate public goods and household technologies with minimal adjustments.[1] Our analysis exploits Chiappori and Meghir's (2015) key remark assertion that the sharing rule contains all the information required to implement the measurement of intra-household inequality because it is possible to construct a general inequality index as a function of individual incomes. Our novel contribution shows that for a weighted Bergsonian representation of household utility and general assumptions about individual preferences, the inequality index takes the form of a family of entropy indexes.

We illustrate our findings with an empirical application that estimates a collective demand system to recover associated individual and household welfare functions along

[1]In a household production context involving public goods, our analysis would still apply reliance on shadow prices of utility-yielding goods, prices that can differ among individuals due to individual differences in the household production process.

with the measures of intra-household inequality. This is the first application that estimates the Pareto weight and examines its role within a measure of income dispersion across household members.

The paper is structured as follows. Section 2 sets notation and introduces the basic model of collective consumption. Section 3 describes the role played by individual preferences and associated welfare and distribution weights for different household welfare functions of empirical interest, and derives measures of inequality describing intra-household distribution of resources in a formal way. Section 4 empirically illustrates the theory developed in the paper. Section 5 concludes.

2. The collective model of household consumption

Consider a household composed of K individuals involved in the consumption of goods. The kth individual consumes a bundle of n_k goods $\mathbf{x}_k = (x_{k1}, \dots, x_{kn_k}) \in \mathbb{R}^{n_k}$ with associated prices $\mathbf{p}_k = (p_{k1}, \dots, p_{kn_k}) \in \mathbb{R}^{n_k}_{++}$, $k = 1, \dots, K$. Let $\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_K)$ and $\mathbf{p} = (\mathbf{p}_1, \dots, \mathbf{p}_K)$. In our conceptual framework, the vector of market prices $\mathbf{p}$ is given. The consumer goods $\mathbf{x}_k$ are specific to the kth household member with prices $\mathbf{p}_k$, $k = 1, \dots, K$. This includes the case where some goods are private and assigned to a given household member (e.g., male clothing versus female clothing in a two-person household). This also includes the case of time allocation where the labor of each household member faces a different wage rate. Finally, this covers the case where the household is involved in production activities, using a bundle of goods $\mathbf{z} = (z_1, \dots, z_m)$ to produce the consumer goods $\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_K)$. The goods in $\mathbf{z}$ can be private goods or public goods, purchased by the household at prices $\mathbf{q} = (q_1, \dots, q_m) \in \mathbb{R}^m_{++}$. The household production technology is represented by the feasible set F, where $(\mathbf{x}, \mathbf{z}) \in F$ means that goods $\mathbf{x}$ can be obtained when $\mathbf{z}$ is purchased (Barten 1964, Browning and Chiappori 1998, Browning et al. 2013, Chavas 1989, Ferreira and Perali 1992, Lewbel 1985, Perali 2003). The household production technology can model either the production of "commodities" from which agents derive utility Becker (1965), or the marketable production of farm households (Squire et al. 1986), or the production of quality (Gorman 1980, Lancaster 1966). Assume that the production technology exhibits constant return to scale (CRS). Under efficiency, the household purchases goods z to produce consumer goods $\boldsymbol{x}$ so as to minimize expenditure $E(\mathbf{x}) = \min_z\{\sum_{i=1}^m q_i \cdot z_i : (\mathbf{x}, \mathbf{z}) \in F\}$. Let $\mathbf{p}_k = \partial E/\partial \mathbf{x}_k$ denote the shadow price of $\mathbf{x}_k$, $k = 1, \dots, K$. Under CRS, $E(\mathbf{x})$ is a linear homogenous function of $\mathbf{x}$ and can be written (at least locally) as $E(\mathbf{x}) = \sum_{k=1}^K \mathbf{p}_k \cdot \mathbf{x}_k$, yielding the household budget constraint $\sum_{k=1}^K \mathbf{p}_k \cdot \mathbf{x}_k = y$, where $\mathbf{p}_k \cdot \mathbf{x}_k$ denotes the inner product of the two vectors $\mathbf{p}_k$ and $\mathbf{x}_k$, and $y \in \mathbb{R}_{++}$ is household income. Note that, in this case, the shadow prices $\mathbf{p}_k$ can differ across individuals because of differences in the production process of goods $\mathbf{x}_k$ from $\mathbf{z}$, $k = 1, \dots, K$.

The kth household member has preferences over $\mathbf{x}_k$ represented by a nonsatiated and quasi-concave utility function u_k: $\mathbb{R}^{n_k} \to \mathbb{R}$, $k = 1, \dots, K$. The household utility function is defined as $U(u_1(\mathbf{x}_1), \dots, u_K(\mathbf{x}_K); \mathbf{p}, y)$, where U is a strictly increasing function of $(u_1, \dots, u_K)$ that aggregates individual preferences into household preferences and reflects distribution issues within the household.[2] As discussed in the Introduction, our analysis focuses on the household allocation of market goods privately consumed by family members. Consumption of private goods can be either assigned or nonassigned to a specific member of the household. The family decision process is conducted in a deterministic environment and leads to Pareto-efficient outcomes provided that individual utility functions are well behaved (Menon et al. 2016).

The utility function $U(u_1, \ldots, u_K; \mathbf{p}, y)$ is a generalization of the Samuelson welfare function and represents household preferences across individuals,[3] and may reflect the relative bargaining power of individuals within the household. Throughout the paper, we assume that $u_k(\mathbf{x}_k)$ is a strongly monotone and strictly quasi-concave function of $\mathbf{x}_k$. The household faces a linear budget constraint $\sum_{k=1}^{K} \mathbf{p}_k \cdot \mathbf{x}_k = y$.

Efficient allocations within the household are derived from

$$V(\mathbf{p}, y) = \max_{\mathbf{x}_1, \ldots, \mathbf{x}_K} \left\{ U\big(u_1(\mathbf{x}_1), \ldots, u_K(\mathbf{x}_K); \mathbf{p}, y\big) : \sum_{k=1}^{K} \mathbf{p}_k \mathbf{x}_k = y \right\}, \tag{1}$$

where $V(\mathbf{p}, y)$ is the household indirect utility function. The optimal choices are the household centralized demands $\mathbf{x}^C(\mathbf{p}, y) = (\mathbf{x}_1^C(\mathbf{p}, y), \ldots, \mathbf{x}_K^C(\mathbf{p}, y))$.

Consider a situation where the household income y is allocated among the K household members and where the kth individual receives the monetary amount $\phi_k \in \mathbb{R}_{++}$ subject to the household income constraint $y = \sum_{k=1}^{K} \phi_k$. The efficient household allocation in (1) can then be decomposed into two stages as

$$V(\mathbf{p}, y) = \max_{\phi_1, \ldots, \phi_K} \left\{ \max_{\mathbf{x}_1, \ldots, \mathbf{x}_K} \big\{ U\big(u_1(\mathbf{x}_1), \ldots, u_K(\mathbf{x}_K); \mathbf{p}, y\big) : \mathbf{p}_k \mathbf{x}_k = \phi_k \big\} : \sum_{k=1}^{K} \phi_k = y \right\}. \tag{2}$$

Because the household preference function $U(u_1, \ldots, u_K; \mathbf{p}, y)$ is strictly increasing in each individual utility u_k, (2) can be written equivalently as

$$V(\mathbf{p}, y) = \max_{\phi_1, \ldots, \phi_K} \left\{ U\big(V_1(\mathbf{p}_1, \phi_1), \ldots, V_K(\mathbf{p}_K, \phi_K); \mathbf{p}, y\big) : \sum_{k=1}^{K} \phi_k = y \right\}, \tag{3}$$

$$V_k(\mathbf{p}_k, \phi_k) = \max_{\mathbf{x}_k} \big\{ u_k(\mathbf{x}_k) : \mathbf{p}_k \mathbf{x}_k = \phi_k \big\} \quad \forall k = 1, \ldots, K, \tag{4}$$

where $V_k(\mathbf{p}_k, \phi_k)$ is the indirect utility function for the kth individual, conditional on prices $\mathbf{p}_k$ and on the income allocation. Equation (3) describes the optimal income allocation among all household members with solution denoted by $(\phi_1(\mathbf{p}, y), \ldots, \phi_K(\mathbf{p}, y))$,

[2]One can accept that household preferences for the distribution of resources depend on prices and income because they define the values of outside options of household members or because they can be a measure of member's contribution to the formation of household economic resources. Note that price- and income-dependent utility functions entail that the usual results of consumption theory, in particular Slutsky symmetry, will no longer hold true. For an interesting discussion of the specification of a household welfare function with prices and income, see Apps and Rees (2009).

[3]Household preferences can also be affected by the sociodemographic characteristics of each household member (Barten 1964, Blackorby and Donaldson 1991, Lewbel 1999, 2004, Perali 2003), but we choose not to include them to simplify the notation. While we do not explore the direct role of sociodemographic effects in the present context, we allow for heterogeneity of preferences among household members.

where the income allocated to the kth individual is $\phi_k(\mathbf{p}, y)$, which depends on all prices $\mathbf{p}$ and on household income y. Below, we assume that $U(u_1, \ldots, u_K; \mathbf{p}, y)$ is differentiable in $(u_1, \ldots, u_K)$ and that (3) has an interior solution with $\phi_k(\mathbf{p}, y) > 0$, $k = 1, \ldots, K$. Equation (4) defines the decentralized Marshallian demand for the kth individual, with solution denoted by $\mathbf{x}_k^D(\mathbf{p}_k, \phi_k)$ that depends on $(\mathbf{p}_k, \phi_k)$. Under household efficiency, the optimal income allocation to the kth individual satisfies $\phi_k(\mathbf{p}, y) = \mathbf{p}_k \mathbf{x}_k^C(\mathbf{p}, y)$, and the maximization problem in (1) is equivalent to (3) and (4).

A household decides to allocate income y among its K members as described in (3). As expected, this allocation is related to the household's preference ordering for the utility of its members.

Proposition 1. *Under differentiability of the household utility function $U(u_1(x_1), \ldots, u_K(x_K); p, y)$ and at the optimum, an efficient household allocation for the kth member satisfies*

$$\frac{\lambda(\mathbf{p}, y)}{\lambda_k(\mathbf{p}_k, \phi_k)} = \frac{\partial U(u_1, \ldots, u_K; \mathbf{p}, y)}{\partial u_k}, \tag{5}$$

where $\lambda(\mathbf{p}, y)$ and $\lambda_k(\mathbf{p}_k, \phi_k)$ are the optimizing value of the Lagrange multipliers of the household and individual budget constraints, respectively.

Proof. Consider the constrained maximization problem in (3). The associated Lagrangian is $\mathcal{L} = U(V_1(\mathbf{p}_1, \phi_1), \ldots, V_K(\mathbf{p}_K, \phi_K); \mathbf{p}, y) + \lambda[y - \sum_{k=1}^K \phi_k]$, where λ is the Lagrange multiplier of the budget constraint $y = \sum_{k=1}^K \phi_k$. The first-order necessary condition for an interior solution with respect to ϕ_k is

$$\partial\mathcal{L}/\partial\phi_k = 0 \quad \Rightarrow \quad \big[\partial U(V_1, \ldots, V_K; \mathbf{p}, y)/\partial V_k\big]\big[\partial V_k(\mathbf{p}_k, \phi_k)/\partial\phi_k\big] - \lambda = 0. \tag{6}$$

Under household efficiency, note that (4) implies that $V_k(\mathbf{p}_k, \phi_k) = u_k(\mathbf{x}_k^D(\mathbf{p}_k, \phi_k))$, and at the optimum, applying the envelope theorem to (4) gives $\partial V_k(\mathbf{p}_k, \phi_k)/\partial\phi_k = \lambda_k(\mathbf{p}_k, \phi_k)$. Combining these results with (6) yields

$$\frac{\lambda(\mathbf{p}, y)}{\lambda_k(\mathbf{p}_k, \phi_k)} = \frac{\partial U(u_1, \ldots, u_K; \mathbf{p}, y)}{\partial u_k},$$

which gives (5). □

3. Household welfare evaluation and intra-household inequality

There has been much interest in linking welfare at the individual level with the aggregate level Gorman (1953), Muellbauer (1975). This section analyzes the issue in the context of the household, with a focus on the evaluation and measurement of household welfare. While the household utility function $U(u_1, \ldots, u_K; \mathbf{p}, y)$ reflects preferences with respect to distribution within the household, it does not make such preferences explicit. As shown below, in general, the sharing rule identifies the role played by $\partial U(u_1, \ldots, u_K; \mathbf{p}, y)/\partial u_k$ as a welfare weight. This corresponds to the intuition of close relationships between welfare weights and income distribution within the household.

To analyze distribution issues, it is convenient to focus on a specific representation of the household utility function. In this regard, consider the weighted Bergsonian specification in the tradition of the collective household literature

$$U(u_1, \ldots, u_K; \mathbf{p}, y) = \sum_{k=1}^{K} \mu_k(\mathbf{p}, y) u_k(\mathbf{x}_k), \tag{7}$$

where $\mu_k(\mathbf{p}, y) \in (0, 1)$ is a welfare weight[4] reflecting the relative contribution of the kth individual to household welfare. This can be a function of market prices, income, and distribution factors, that is, exogenous variables that affect the Pareto welfare weight but do not affect either individual preferences or the budget constraint. In the theoretical sections, we omit the distribution factor notation for convenience. Further, without loss of generality, these welfare weights are normalized to satisfy $\mu_k(\mathbf{p}, y) \in (0, 1)$ and $\sum_{k=1}^{K} \mu_k(\mathbf{p}, y) = 1$. From Proposition 1, specification (7) gives the following corollary.

Corollary 1. Under (7), for all $k = 1, \ldots, K$, (5) reduces to

$$\frac{\lambda(\mathbf{p}, y)}{\lambda_k(\mathbf{p}_k, \phi_k)} = \mu_k(\mathbf{p}, y). \tag{8}$$

Equation (8) explicitly shows the relationships between the welfare weights $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_K)$ and the Lagrange multipliers. When $0 < \mu_k < 1$, it follows that $\lambda < \lambda_k$. For the kth member, the value of one extra unit of income when living with her/his family is worth less than in the situation where she/he has control over her/his own income. As shown below, (8) provides useful insights into the economic and welfare analysis of household behavior. Note that expression (8) is the cardinal representation of the ordinal object (5) presented in Proposition 1. The reader may refer to Proposition 2 in Browning et al. (2013) to find an alternative way to derive the Pareto weight. Further, from (8) it follows that

$$\mu_k(\mathbf{p}, y)/\mu_r(\mathbf{p}, y) = \lambda_r(\mathbf{p}_r, \phi_r)/\lambda_k(\mathbf{p}_k, \phi_k)$$

for all $k, r = 1, \ldots, K$, $k \neq r$. In a way consistent with Browning et al. (2013), this states that, under household efficiency, the relative welfare weights $\mu_k(\mathbf{p}, y)/\mu_r(\mathbf{p}, y)$ are equal to the relative decentralized marginal utilities $\lambda_r(\mathbf{p}_r, \phi_r)/\lambda_k(\mathbf{p}_k, \phi_k)$.

For any given economic situation $(\mathbf{p}, y)$, the Bergsonian specification (7) can apply to general household preferences $U(u_1, \ldots, u_K; \mathbf{p}, y)$ as long as $\partial U(u_1, \ldots, u_K; \mathbf{p}, y)/\partial u_k = \mu_k(\mathbf{p}, y)$, which means that, conditional on given $(\mathbf{p}, y)$, it is valid to analyze the behavioral and welfare implications of alternative Bergsonian welfare weights $(\mu_1, \ldots, \mu_K)$. This proves particularly convenient for our analysis: the presence of explicit welfare weights $(\mu_1, \ldots, \mu_K)$ in (7) can shed useful light on their role in consumption decisions and the welfare analysis of distribution issues within the household. We use this property extensively in this section.

[4]The weight μ_k can be interpreted as the Lagrange multiplier of the Pareto optimal problem where agent k maximizes her/his own utility while ensuring that the utility of agents r is greater than or equal to a predetermined utility level.

But what happens when economic conditions $(\mathbf{p}, y)$ change? Then, under household efficiency, $V(\mathbf{p}, y)$ would change and so would $\partial U(u_1, \dots, u_K; \mathbf{p}, y)/\partial u_k$. In this context, the Bergsonian utility function (7) would remain valid under general household preferences $U(u_1, \dots, u_K; \mathbf{p}, y)$ but only on the condition that we allow the welfare weights $(\mu_1, \dots, \mu_K)$ to adjust to changing economic conditions. In other words, the Bergsonian approach would continue to hold provided that the welfare weights $(\mu_1, \dots, \mu_K)$ are treated as endogenous to satisfy $\mu_k(\mathbf{p}, y) = \partial U(u_1, \dots, u_K; \mathbf{p}, y)/\partial u_k$ at the optimum for all $(\mathbf{p}, y)$. To simplify the notation, the analysis presented below treats the dependency of the welfare weights $(\mu_1, \dots, \mu_K)$ on $(\mathbf{p}, y)$ as implicit.

The income sharing rule ϕ_k/y, that is, the proportion of household income allocated to the kth individual, has a specific correspondence with the Bergsonian welfare weights.

Proposition 2. *Under the Bergsonian representation of household preferences (7), (3) implies the income sharing rule for the kth individual,*

$$\frac{\phi_k}{y} = \mu_k(\mathbf{p}, y) w_k, \tag{9}$$

where $w_k = [\partial V_k(\mathbf{p}_k, \phi_k)/\partial \ln(\phi_k)]/[\partial V(\mathbf{p}, y)/\partial \ln(y)]$ is a weight satisfying

$$\sum_{k=1}^{K} \mu_k(\mathbf{p}, y) w_k = 1.$$

Proof. Let $y > 0$ and $\phi_k > 0$. For a given μ_k, the relationship in (5) can be alternatively written as $[\partial V(\mathbf{p}, y)/\partial \ln(y)][1/y] = [\partial U(u_1, \dots, u_K; \mathbf{p}, y)/\partial u_k][\partial V_k(\mathbf{p}_k, \phi_k)/\partial \ln(\phi_k)] \times [1/\phi_k]$. This gives (9). □

The relationship in (9) shows that the proportion of household income ϕ_k/y received by the kth individual is equal to the product of two weights: the welfare weight $\mu_k(\mathbf{p}, y)$ and the distribution weight w_k, capturing marginal effects of intra-household income distribution. For the kth individual, w_k is the ratio between the marginal individual utility due to a change in individual income $\partial V_k(\mathbf{p}_k, \phi_k)/\partial \ln(\phi_k)$ and the marginal household utility due to a change in household income $\partial V(\mathbf{p}, y)/\partial \ln(y)$ for a given μ_k.[5] This can also be interpreted as an elasticity describing how the curvature of the indirect utility function of the household changes as the distribution of income varies across household members (Menon et al. 2016). The result (9) extends the existing literature by showing how the Pareto weight μ_k relates to the income sharing rule ϕ_k/y. It identifies the role played by the distribution weight w_k. As shown below, this structure reveals fundamental relationships in the measurement of intra-household inequality that could not have been characterized otherwise.

We now investigate the welfare implications of these general results by examining in detail how household welfare varies with the distribution of income within the household. We also examine the linkages between individual preferences and income sharing

[5]As shown by Browning and Chiappori (1998), the dual representation of the collective program is conditional on a given level of household utility and Pareto weights.

rules under alternative individual preferences. We start the analysis with the indirect utility specification for the kth individual,

$$V_k(\mathbf{p}_k, \phi_k) = f_k\left(\frac{g_k(\phi_k, \mathbf{p}_k)}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k) g_k(\phi_k, \mathbf{p}_k)}\right), \tag{10}$$

where f_k is a strictly increasing function, and the functions $B_k(\mathbf{p}_k)$, $C_k(\mathbf{p}_k)$ and $g_k(\phi_k, \mathbf{p}_k)$ are chosen so that $V_k(\mathbf{p}_k, \phi_k)$ is homogeneous of degree 0 in $(\phi_k, \mathbf{p}_k)$. The utility specification (10) is very flexible and includes as special cases many models commonly found in the literature. Following Lewbel (1989b, 1990, 1991, 1995), the preference specification (10) belongs to the class of rank-3 demand systems that can exhibit nonlinear Engel curves. For example, when $g_k(\phi_k, \mathbf{p}_k) = \phi_k - A_k(\mathbf{p}_k)$, (10) gives the quadratic expenditure system (QES) proposed by Howard et al. (1979), where demands are quadratic functions of income.[6] Alternatively, when $g_k(\phi_k, \mathbf{p}_k) = \ln(\phi_k) - \ln(A_k(\mathbf{p}_k))$, (10) gives the quadratic almost ideal demand system (QAIDS) proposed by Banks et al. (1997) where budget shares are quadratic functions of the logarithm of income.

The general implications of individual preferences (10) are now presented.

Proposition 3. *Under the Bergsonian household utility (7) and individual preferences (10), the household welfare is*

$$V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k f_k\left(\frac{g_k(\mu_k w_k y, \mathbf{p}_k)}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k) g_k(\mu_k w_k y, \mathbf{p}_k)}\right). \tag{11}$$

Proof. We start with the relationship $V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k V_k(\phi_k, \mathbf{p}_k)$. Using (9) and (10) gives (11). □

The income sharing rule (9) and the indirect household utility function (11) apply under fairly general conditions. As further discussed below, the specification (10) allows for nonlinear Engel curves. This means that the income sharing rule (9) and the household utility function (11) apply in the presence of nonlinear income effects that can vary across individuals. Below, we investigate in more detail the implications of varying assumptions about the functions f_k and g_k as summarized in the synoptic Table 1. Note that cases a and c in the table refer to Section 3.1, whereas cases b and d refer to Section 3.2. The last subsection discusses measures of intra-household inequality.

3.1 *Translated sharing rule*

Consider the specification where $g_k(\phi_k, \mathbf{p}_k) = \phi_k - A_k(\mathbf{p}_k)$ in (10), which corresponds to the QES preference representation (Howard et al. 1979). The indirect utility function for the kth individual takes the form

$$V_k(\mathbf{p}_k, \phi_k) = f_k\left(\frac{\phi_k - A_k(\mathbf{p}_k)}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)(\phi_k - A_k(\mathbf{p}_k))}\right), \tag{12}$$

[6] Another specification of preferences is the trans-log utility function (Christensen et al. 1975, Jorgenson et al. 1982, Lewbel 1989a).

$f_k(\zeta)$	$g_k(\phi_k, \mathbf{p}_k)$		Income Sharing Rule	Household Welfare
ζ	a. $\phi_k - A_k(\mathbf{p}_k)$	(i) $C_k \neq 0$	$\frac{\phi_k - A_k(\mathbf{p}_k)}{y - A(\mathbf{p})} = \mu_k W_k$	$\frac{y - A(\mathbf{p})}{P^*(\mathbf{p}, y, \boldsymbol{\mu})}$
		(ii) $C_k = 0$	$\frac{\phi_k - A_k(\mathbf{p}_k)}{y - A(\mathbf{p})} = \mu_k$	$\frac{y - A(\mathbf{p})}{P_0^*(\mathbf{p}, y, \boldsymbol{\mu})}$
		(iii) $C_k = 0$ and $A_k = 0$	$\frac{\phi_k}{y} = \mu_k$	$\frac{y}{P_0^*(\mathbf{p}, y, \boldsymbol{\mu})}$
	b. $\ln(\phi_k / A_k(\mathbf{p}_k))$	(i) $C_k \neq 0$	$\frac{\phi_k}{y} = \mu_k w_k$	$\sum_k \mu_k \frac{\ln(y) + \ln(\mu_k w_k) - \ln(A_k(\mathbf{p}_k))}{\tilde{P}_k(\mathbf{p}_k, y, \mu_k)}$
		(ii) $C_k = 0$	$\frac{\phi_k}{y} = \mu_k w_k$	$\sum_k \mu_k \frac{\ln(y) + \ln(\mu_k w_k) - \ln(A_k(\mathbf{p}_k))}{B_k(\mathbf{p}_k)}$
		(iii) $C_k = 0$ and $A_k = 1$	$\frac{\phi_k}{y} = \mu_k w_k$	$\sum_k \mu_k \frac{\ln(y) + \ln(\mu_k w_k)}{B_k(\mathbf{p}_k)}$
$\ln(\zeta)$	c. $\phi_k - A_k(\mathbf{p}_k)$	(i) $C_k \neq 0$	$\frac{\phi_k - A_k(\mathbf{p}_k)}{y - A(\mathbf{p})} = \mu_k W_{0k}$	$\ln(\frac{y - A(\mathbf{p})}{P(\mathbf{p}, y, \boldsymbol{\mu})})$
		(ii) $C_k = 0$	$\frac{\phi_k - A_k(\mathbf{p}_k)}{y - A(\mathbf{p})} = \mu_k$	$\ln(\frac{y - A(\mathbf{p})}{P_0(\mathbf{p}, y, \boldsymbol{\mu})})$
		(iii) $C_k = 0$ and $A_k = 0$	$\frac{\phi_k}{y} = \mu_k$	$\ln(\frac{y}{P_0(\mathbf{p}, y, \boldsymbol{\mu})})$
	d. $\ln(\phi_k / A_k(\mathbf{p}_k))$	(i) $C_k \neq 0$	$\frac{\phi_k}{y} = \mu_k w_k$	$\sum_k \mu_k \ln[\frac{\ln(y) + \ln(\mu_k w_k) - \ln(A_k(\mathbf{p}_k))}{\tilde{P}_k(\mathbf{p}_k, y, \mu_k)}]$
		(ii) $C_k = 0$	$\frac{\phi_k}{y} = \mu_k w_k$	$\sum_k \mu_k \ln[\frac{\ln(y) + \ln(\mu_k w_k) - \ln(A_k(\mathbf{p}_k))}{B_k(\mathbf{p}_k)}]$
		(iii) $C_k = 0$ and $A_k = 1$	$\frac{\phi_k}{y} = \mu_k w_k$	$\sum_k \mu_k \ln[\frac{\ln(y) + \ln(\mu_k w_k)}{B_k(\mathbf{p}_k)}]$

Table 1. A synoptic table of the sharing rules and household welfare specifications. Individual indirect utility function $V_k(\mathbf{p}_k, \phi_k) = f_k(\frac{g_k(\phi_k, \mathbf{p}_k)}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k) g_k(\phi_k, \mathbf{p}_k)})$.

where f_k is a strictly increasing function, the functions $A_k(\mathbf{p}_k)$ and $B_k(\mathbf{p}_k)$ are homogeneous of degree 1 in $\mathbf{p}_k$, the function $C_k(\mathbf{p}_k)$ is homogeneous of degree 0 in $\mathbf{p}_k$, and specification (12) allows for flexible price effects (e.g., when $A_k(\mathbf{p}_k)$ is specified to be a quadratic function of $\mathbf{p}_k$). Throughout, we assume that $\phi_k - A_k(\mathbf{p}_k) > 0$. In this context, $\phi_k - A_k(\mathbf{p}_k)$ is a measure of "translated income," where $A_k(\mathbf{p}_k)$ is a price function that can be interpreted as a committed level of subsistence income for the kth individual. Using Roy's identity, the decentralized demand for x_{ki} associated with QES preferences (12) is

$$x_{ki}^D = \frac{\partial A_k(\mathbf{p}_k)}{\partial p_{ki}} + \frac{\partial \big(B_k(\mathbf{p}_k)\big)}{\partial p_{ki}} \big[\phi_k - A_k(\mathbf{p}_k)\big] + \frac{\partial C_k(\mathbf{p}_k)}{\partial p_{ki}} \frac{1}{B_k(\mathbf{p}_k)} \big[\phi_k - A_k(\mathbf{p}_k)\big]^2.$$

Note that QES preferences (12) include as special cases quasi-homothetic preferences when $C_k = 0$. Under quasi-homothetic preferences, individual Engel curves are linear because $\partial x_{ki}^D / \partial \phi_k$ is independent of ϕ_k (Deaton and Muellbauer 1980a, 1980b, Gorman Gorman (1961)). In turn, QES preferences (12) include as special cases homothetic preferences when $C_k = 0$ and $A_k = 0$. In this case, all Engel curves are linear and go through the origin. This indicates that QES preferences (12) offer some flexibility in capturing income effects, including nonlinear Engel curves when $\partial C_k / \partial \mathbf{p}_k \neq 0$. As such, they provide a good basis on which to evaluate the linkages between individual welfare and household welfare.

The implications of individual preferences (12) for income sharing and household welfare are presented next.

Proposition 4. *Under the Bergsonian household utility (7) and individual preferences (12), the income sharing rule satisfies*

$$\frac{\phi_k - A_k(\mathbf{p}_k)}{y - A(\mathbf{p})} = \mu_k W_k, \quad k = 1, \ldots, K, \tag{13}$$

and household welfare is given by

$$V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k f_k\left(\mu_k W_k \frac{y - A(\mathbf{p})}{P_k(\mathbf{p}, y, \mu_k)}\right), \tag{14}$$

where $A(\mathbf{p}) = \sum_{k=1}^{K} A_k(\mathbf{p}_k)$, $W_k = [\partial V_k(\mathbf{p}_k, \phi_k)/\partial \ln(\phi_k - A_k(\mathbf{p}_k))]/[\partial V(\mathbf{p}, y)/\partial \ln(y - A(\mathbf{p}))]$ *is a weight satisfying* $\sum_{k=1}^{K} \mu_k W_k = 1$, *and* $P_k(\mathbf{p}, y, \mu_k) = B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\mu_k W_k(y - A(\mathbf{p}))$, $k = 1, \ldots, K$.

Proof. Starting from the relationship $V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k V_k(\phi_k, \mathbf{p}_k)$ and using (12), we have

$$V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k f_k\big[\big(\phi_k - A_k(\mathbf{p}_k)\big)/\big(B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\big(\phi_k - A_k(\mathbf{p}_k)\big)\big)\big]. \tag{15}$$

Let $A(\mathbf{p}) = \sum_{k=1}^{K} A_k(\mathbf{p}_k)$. Under the Bergsonian household utility (7), (9) can alternatively be written as

$$\frac{\phi_k - A_k(\mathbf{p}_k)}{y - A(\mathbf{p})} = \mu_k\left(\frac{\partial V_k(\mathbf{p}_k, \phi_k)}{\partial \ln(\phi_k - A_k(\mathbf{p}_k))}\right)\Big/\left(\frac{\partial V(\mathbf{p}, y)}{\partial \ln\big(y - A(\mathbf{p})\big)}\right) = \mu_k W_k, \tag{16}$$

where $W_k = [\partial V_k(\mathbf{p}_k, \phi_k)/\partial \ln(\phi_k - A_k(\mathbf{p}_k))]/[\partial V(\mathbf{p}, y)/\partial \ln(y - A(\mathbf{p}))]$ is a weight satisfying $\sum_{k=1}^{K} \mu_k W_k = 1$. This gives (13). Substituting (13) into (15) gives the desired result. □

Proposition 4 applies for any increasing function f_k in (12) and (14). The sharing rule in (13) states that the proportion of "translated income" received by the kth individual $[\phi_k - A_k(\mathbf{p}_k)/y - A(\mathbf{p})]$ is equal to the product of two weights: the Bergsonian welfare weight μ_k and the weight W_k that reflects income effects under optimal income distribution within the household. This can be important when marginal income effects are observed to vary with income levels. In this context, the term W_k in (13) captures the effects of preference heterogeneity across individuals within the household. This term also enters the household utility (14). This raises the following question: Are there situations where these heterogeneity effects vanish? In other words, are there scenarios where $W_k = 1$ in (13) and (14)? Our analysis identifies such conditions below.

In a first step, we examine the special case where $f_k(\zeta) = \ln(\zeta)$ in (12), which is summarized in panel c of Table 1.

Proposition 5. *Under the Bergsonian household utility (7) and individual preferences (12), let* $f_k(\zeta) = \ln(\zeta)$. *Then the income sharing rule is*

$$\frac{\phi_k - A_k(\mathbf{p}_k)}{y - A(\mathbf{p})} = \mu_k W_{0k}, \quad k = 1, \ldots, K, \tag{17}$$

and the household welfare function is

$$V(\mathbf{p}, y) = \ln\left(\frac{y - A(\mathbf{p})}{P(\mathbf{p}, y, \boldsymbol{\mu})}\right), \tag{18}$$

where $A(\mathbf{p}) = \sum_{k=1}^{K} A_k(\mathbf{p}_k)$, $P(\mathbf{p}, y, \boldsymbol{\mu})$ *is a household price index satisfying*

$$\ln P(\mathbf{p}, y, \boldsymbol{\mu}) = \sum_{k=1}^{K} \mu_k \ln\bigl(B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\mu_k W_{0k}\bigl(y - A(\mathbf{p})\bigr)\bigr) - \sum_{k=1}^{K} \mu_k \ln(\mu_k W_{0k}), \tag{19}$$

and

$$W_{0k} = \left(1 - \frac{C_k\bigl(\phi_k - A_k(\mathbf{p}_k)\bigr)}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\bigl(\phi_k - A_k(\mathbf{p}_k)\bigr)}\right) \Bigg/ \sum_{k=1}^{K} \mu_k \left(1 - \frac{C_k\bigl(\phi_k - A_k(\mathbf{p}_k)\bigr)}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\bigl(\phi_k - A_k(\mathbf{p}_k)\bigr)}\right).$$

Proof. Under individual preferences (12), having $f_k(\zeta) = \ln(\zeta)$ implies that $\partial V_k(\mathbf{p}_k, \phi_k)/\partial \ln(\phi_k - A_k(\mathbf{p}_k)) = 1 - [C_k(\mathbf{p}_k)(\phi_k - A_k(\mathbf{p}_k))]/[B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)(\phi_k - A_k(\mathbf{p}_k))]$. Substituting this result into (16) gives

$$\frac{\phi_k - A_k(\mathbf{p}_k)}{y - A(\mathbf{p})} = \mu_k\left(1 - \frac{C_k(\mathbf{p}_k)\bigl(\phi_k - A_k(\mathbf{p}_k)\bigr)}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\bigl(\phi_k - A_k(\mathbf{p}_k)\bigr)}\right) \Bigg/ \left(\sum_k \frac{\partial V(\mathbf{p}, y)}{\partial \ln\bigl(\phi_k - A_k(\mathbf{p}_k)\bigr)}\right). \tag{20}$$

Summing (20) over all k and using $\sum_{k=1}^{K}[\phi_k - A_k(\mathbf{p}_k)] = y - A(\mathbf{p})$, we obtain $\sum_{k=1}^{K}[\partial V(\mathbf{p}, y)/\partial \ln(\phi_k - A_k(\mathbf{p}_k))] = \sum_{k=1}^{K} \mu_k[1 - [C_k(\mathbf{p}_k)(\phi_k - A_k(\mathbf{p}_k))/(B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)(\phi_k - A_k(\mathbf{p}_k)))]]$. Substituting this result into (13) yields (17). When $f_k(\zeta) = \ln(\zeta)$, it follows that (14) becomes

$$V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k \ln\bigl[\mu_k W_{0k}\bigl(y - A(\mathbf{p})\bigr)/\bigl(B_k(\mathbf{p_k}) + C_k(\mathbf{p}_k)\mu_k W_{0k}\bigl(y - A(\mathbf{p})\bigr)\bigr)\bigr]$$

or, using $\sum_{k=1}^{K} \mu_k = 1$,

$$V(\mathbf{p}, y) = \ln\bigl(y - A(\mathbf{p})\bigr) + \sum_{k=1}^{K} \mu_k \ln(\mu_k W_{0k}) - \sum_{k=1}^{K} \mu_k \ln\bigl(B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\mu_k W_{0k}\bigl(y - A(\mathbf{p})\bigr)\bigr),$$

which gives (18) and (19). □

Proposition 5 gives the income sharing rule (17) and the household indirect utility function (18) when $f_k(\zeta) = \ln(\zeta)$, allowing for departures from quasi-homothetic preferences. Again, the weights W_{0k} affect both the sharing rules and the indirect utility function, reflecting income effects under optimal income sharing. The household utility function $V(\mathbf{p}, y)$ given in (18) is equal to $\ln[(y - A(\mathbf{p}))/P(\mathbf{p}, y, \boldsymbol{\mu})]$. It is a monotonic transformation of the household income. Interestingly, the logarithm of the price index of (19) involves the terms $\sum_{k=1}^{K} \mu_k \ln(B_k(\mathbf{p}_k) + C_k \mu_k W_{0k}(y - A(\mathbf{p})))$ and $-\sum_{k=1}^{K} \mu_k \ln(\mu_k W_{0k})$. The first term is a weighted sum of $\ln(B_k(\mathbf{p}_k) + C_k \mu_k W_{0k}(y - A(\mathbf{p})))$, which reflects the cost of living for the kth individual with μ_k as a weight. The second term captures the effects of the Bergsonian welfare weights μ_k and income distribution weights W_{0k} on the household price index.

While Proposition 5 allows for departures from quasi-homotheticity (when $C_k \neq 0$), it includes as special cases situations where individual preferences exhibit quasi-homotheticity (when $C_k = 0$, $k = 1, \dots, K$). The link with the decentralized demands can readily be applied in this context using our results. Here, we say that individual preferences in (12) are *log-quasi-homothetic* if they exhibit quasi-homotheticity with $C_k = 0$ and if $f_k(\zeta) = \ln(\zeta)$, $k = 1, \dots, K$. Note that $C_k = 0$ for all $k = 1, \dots, K$ implies that $W_{0k} = 1$ for all $k = 1, \dots, K$ in Proposition 5. This generates the following result.

Corollary 2. *Under Bergsonian household utility (7) and individual preferences (12) that satisfy log-quasi-homotheticity, with $C_k = 0$ and $f_k(\zeta) = \ln(\zeta)$, the income sharing rule is*

$$\frac{\phi_k - A_k(\mathbf{p}_k)}{y - A(\mathbf{p})} = \mu_k \tag{21}$$

and the household welfare function is

$$V(\mathbf{p}, y) = \ln\left(\frac{y - A(\mathbf{p})}{P_0(\mathbf{p}, y, \boldsymbol{\mu})}\right), \tag{22}$$

where $A(\mathbf{p}) = \sum_{k=1}^{K} A_k(\mathbf{p}_k)$ and *$P_0(\mathbf{p}, y, \boldsymbol{\mu})$ is a household price index satisfying*

$$\ln\bigl[P_0(\mathbf{p}, y, \boldsymbol{\mu})\bigr] = \sum_{k=1}^{K} \mu_k \ln\bigl(B_k(\mathbf{p}_k)\bigr) - \sum_{k=1}^{K} \mu_k \ln(\mu_k). \tag{23}$$

Corollary 2 reports the income sharing rule (21) and the household welfare function (22) obtained when individual preferences exhibit log-quasi-homotheticity. First, comparing (18) and (22) is instructive. It shows that the evaluation of household welfare is similar with or without quasi-homotheticity, with one notable exception: the price indices P in (19) and P_0 in (23) differ. The price index P in (19) includes the terms involving C_k, while the price index P_0 obtained under quasi-homotheticity does not. This illustrates that non-homothetic preferences (or nonlinear Engel curves) affect the evaluation of household welfare. Second, Corollary 2 shows that the preference heterogeneity weight W_k no longer plays a role under log-quasi-homotheticity (as $W_{0k} = 1$ for all $k = 1, \dots, K$). This establishes that under log-quasi-homotheticity, heterogeneity in income effects across individuals no longer affects income sharing or household welfare. Note that while individual Engel curves are linear under quasi-homotheticity, the results in Corollary 2 still allow Engel curves to have different slopes across individuals.

Under log-quasi-homotheticity, the income sharing rule (21) becomes simpler: the proportion of household "translated income" allocated to the kth individual $(\phi_k - A_k(\mathbf{p}_k))/(y - A(\mathbf{p}))$ is equal to the Bergsonian welfare weight μ_k. This is an intuitive result: increasing the Bergsonian welfare for an individual means a proportional increase in her/his share of translated income. Also, under log-quasi-homotheticity, household welfare (22) takes a simple form: it is a monotonic function of translated income $(y - A(\mathbf{p}))$ deflated by the household price index P_0 in (23). The price index P_0 in (23) shows that the welfare weights $(\mu_1, \dots, \mu_K)$ have two effects on household welfare: as weights on the terms involving $B_k(\mathbf{p}_k)$ and through the term

$$-\sum_{k=1}^{K} \mu_k \ln(\mu_k).$$

This term is the Shannon entropy index. It reflects the impacts of individual welfare weights μ_k on household utility. We investigate the consequences on intra-household inequality in Section 3.3.

Finally, when $C_k = 0$, note that letting $A_k(\mathbf{p}_k) = 0$ moves preferences of the kth individual from being quasi-homothetic (with linear Engel curves) to being homothetic (with linear Engel curves going through the origin). In this context, we say that individual preferences in (12) are *log-homothetic* if they are log-quasi-homothetic with $C_k = 0$ and $f_k(\zeta) = \ln(\zeta)$ and if $A_k = 0$ for all $k = 1, \dots, K$. This gives the following result.

Corollary 3. *Under the Bergsonian household utility (7) and individual preferences (12) satisfying log-homotheticity (with $A_k = 0$, $C_k = 0$ and $f_k(\zeta) = \ln(\zeta)$), the income sharing rule is*

$$\frac{\phi_k}{y} = \mu_k \tag{24}$$

and the household utility function is

$$V(\mathbf{p}, y) = \ln\left(\frac{y}{P_0(\mathbf{p}, y, \boldsymbol{\mu})}\right),$$

where $P_0(\mathbf{p}, y, \boldsymbol{\mu})$ is given in (23).

Corollary 3 shows that under log-homotheticity the income sharing rule given by (24) becomes very simple: the proportion of household income allocated to the kth individual (ϕ_k/y) is equal to the Bergsonian welfare weight μ_k, $k = 1, \dots, K$.[7]

[7] The interested reader may observe that because of the logarithmic transformation, Euler's theorem specializes to $h = x_1^k(\partial u^k/\partial x_1) + x_2^k(\partial u^k/\partial x_2)$, where h is the degree of homogeneity. From equating the first-order conditions of the centralized and decentralized model, h is equal to $\phi^k \lambda^k$. Using $\lambda/\mu^k = \lambda^k$ and the centralized Lagrange multiplier, $\lambda = h/y$, it follows that $\phi^k = \mu^k y$. This derivation shows how the Lagrange multipliers are associated with the sharing rules for log-homothetic preferences.

We now present a simple example that illustrates the results presented in Corollary 3.

Example 1. Consider the case where $k = 1, 2$ and $i = 1, 2$ under individual Cobb–Douglas utility functions $\ln(u_1) = \alpha_1 \ln(x_{11}) + \alpha_2 \ln(x_{12})$ and $\ln(u_2) = \beta_1 \ln(x_{21}) + \beta_2 \ln(x_{22})$, with $\alpha_1 + \alpha_2 = \beta_1 + \beta_2 = 1$. This is a case of log-homothetic individual preferences (as presented in Corollary 3). The associated collective household welfare function is $U = \mu_1(\alpha_1 \ln(x_{11}) + \alpha_2 \ln(x_{12})) + \mu_2(\beta_1 \ln(x_{21}) + \beta_2 \ln(x_{22}))$. From the necessary conditions, the centralized and decentralized demands are, respectively,

$$\begin{cases} x_{1i}^C = \alpha_i \mu_1 y / p_{1i}, \\ x_{2i}^C = \beta_i \mu_2 y / p_{2i}, \\ \lambda = 1/y, \end{cases} \qquad \text{and} \qquad \begin{cases} x_{1i}^D = \alpha_i \phi_1 / p_{1i}, \\ x_{2i}^D = \beta_i \phi_2 / p_{2i}, \\ \lambda_k = 1/\phi_k. \end{cases}$$

Application of the second theorem of welfare economics shows that $x_{ki}^C = x_{ki}^D$, yielding $\phi_k = \mu_k y$. With log-homothetic individual preferences, the level of individual income ϕ_k is equal to the product of household income y and the Bergsonian welfare weight μ_k. In the log-homothetic case, the distribution weights w_k equal 1, because the linearity of individual and household Engel curves presupposes expenditure proportionality and, as a consequence, the ratio between individual and household marginal utility of incomes is indeed income independent. In fact, $w_k = (\lambda_k/\lambda)(\phi_k/y) = (1/\mu_k)(\mu_k) = 1$. ♢

For the sake of completeness, we examine also the translated case corresponding to $g_k(\phi_k, \mathbf{p}_k) = \phi_k - A_k(\mathbf{p}_k)$ and $f_k(\zeta)$ equal to the identity function, summarized in panel a of Table 1.

Proposition 6. *Under Bergsonian household utility (7) and individual preferences (12) such that $f_k(\zeta) = \zeta$ and $g_k(\phi_k, \mathbf{p}_k) = \phi_k - A_k(\mathbf{p}_k)$, the income sharing rule satisfies*

$$\frac{\phi_k - A_k(\mathbf{p}_k)}{y - A(\mathbf{p})} = \mu_k W_k \tag{25}$$

and household welfare is given by

$$V(\mathbf{p}, y) = \frac{y - A(\mathbf{p})}{P^*(\mathbf{p}, y, \boldsymbol{\mu})}, \tag{26}$$

where $1/P^*(\mathbf{p}, y, \boldsymbol{\mu}) = \sum_k \mu_k[\mu_k W_k/(B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\mu_k W_k(y - A(\mathbf{p})))]$.

Proof. Recall that $V_k(\mathbf{p}_k, \phi_k) = [(\phi_k - A_k(\mathbf{p}_k))/(B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)(\phi_k - A_k(\mathbf{p}_k)))]$. Then using (25), we have $V(\mathbf{p}, y) = \sum_k \mu_k V_k = \sum_k \mu_k[\mu_k W_k(y - A(\mathbf{p}))/(B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\mu_k W_k(y - A(\mathbf{p})))]$ and $V(\mathbf{p}, y) = (y - A(\mathbf{p})) \sum_k \mu_k[\mu_k W_k/(B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\mu_k \times W_k(y - A(\mathbf{p})))] = (y - A(\mathbf{p}))/P^*(\mathbf{p}, y, \boldsymbol{\mu})$. □

Note that the indirect utility function depending on $1/P^*(\mathbf{p}, y, \boldsymbol{\mu}) = \sum_k \mu_k[\mu_k W_k/(B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\mu_k W_k(y - A(\mathbf{p})))]$ would be linear in income if μ_k is constant and $C_k = 0$, giving corner solutions for the sharing rule.

3.2 *Deflated sharing rule*

We now consider the specification where $g_k(\phi_k, \mathbf{p}_k) = \ln(\phi_k) - \ln(A_k(\mathbf{p}_k))$, corresponding to QAIDS preferences as reported in panels b and d of Table 1.[8] Following Banks et al. (1997), under a QAIDS model for the kth individual, the indirect utility function (10) takes the form

$$V_k(\mathbf{p}_k, \phi_k) = f_k\left(\frac{\ln(\phi_k) - \ln\bigl(A_k(\mathbf{p}_k)\bigr)}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\bigl(\ln(\phi_k) - \ln\bigl(A_k(\mathbf{p}_k)\bigr)\bigr)}\right), \tag{27}$$

where f_k is a strictly increasing function, the functions $\ln(A_k(\mathbf{p}_k))$ and $B_k(\mathbf{p}_k)$ are each homogeneous of degree 1 in $\mathbf{p}_k$, and the function $C_k(\mathbf{p}_k)$ is homogeneous of degree 0 in $\mathbf{p}_k$. Note that $g_k(\phi_k, \mathbf{p}_k) = \ln(\phi_k) - \ln(A_k(\mathbf{p}_k)) = \ln(\phi_k/A_k(\mathbf{p}_k))$. In this context, $\phi_k/A_k(\mathbf{p}_k)$ measures the "deflated income" and $A_k(\mathbf{p}_k)$ is a price function that can be interpreted as the weight that scales the individual income. The budget share associated with x_{ki} under the QAIDS model (27) is

$$\begin{aligned}\frac{p_{ki}x^D_{ki}}{\phi_k} = {} & \frac{\partial \ln\bigl(A_k(\mathbf{p}_k)\bigr)}{\partial \ln(p_{ki})} + \frac{\partial\bigl(B_k(\mathbf{p}_k)\bigr)}{\partial \ln(p_{ki})}\bigl(\ln(\phi_k) - \ln\bigl(A_k(\mathbf{p}_k)\bigr)\bigr) \\ & + \frac{\partial C_k(\mathbf{p}_k)}{\partial \ln(p_{ki})}\frac{\bigl(\ln(\phi_k) - \ln\bigl(A_k(\mathbf{p}_k)\bigr)\bigr)^2}{B_k(\mathbf{p}_k)}.\end{aligned} \tag{28}$$

The share equation (28) shows that the QAIDS specification (27) allows for flexible price effects (e.g., when $\ln(A_k(\mathbf{p}_k))$ is specified to be a quadratic function of $\ln(\mathbf{p}_k)$) as well as quadratic income effects (when $\partial C_k/\partial \ln(\mathbf{p}_k) \neq 0$). Note that the QAIDS model reduces to the almost ideal demand system (AIDS) proposed by Deaton and Muellbauer (1980a) when $C_k = 0$, where AIDS budget shares are linear functions of the log of income. This indicates that, like the QES specification evaluated in Section 3.1, the QAIDS specification (27) is also a good candidate for evaluating the linkages between individual welfare and household welfare.

The implications of QAIDS individual preferences (27) for income sharing and household welfare are presented next.

Proposition 7. *Under the Bergsonian household utility (7) and QAIDS individual preferences (27), let $f_k(\zeta) = \zeta$. Then the income sharing rule satisfies*

$$\frac{\phi_k}{y} = \mu_k w_k, \quad k = 1, \ldots, K, \tag{29}$$

[8] It is interesting to note that, changing notation, panels b and d give the same functional representations of household welfare as a and c, respectively. Consider the substitutions $\tilde{\phi}_k = \ln(\phi_k)$ and $\tilde{A}_k(\mathbf{p}_k) = \ln A_k(\mathbf{p}_k)$, with $\tilde{y} = \sum_k \ln(\phi_k)$ and $\tilde{A}(\mathbf{p}) = \sum_k \ln A_k(\mathbf{p}_k)$. Rewriting $\ln(\phi_k/A_k(\mathbf{p}_k)) = \ln\phi_k - \ln A_k(\mathbf{p}_k) = \tilde{\phi}_k - \tilde{A}_k(\mathbf{p}_k)$, one obtains $\tilde{\phi}_k/\tilde{y} = \mu_k\tilde{w}_k$, where $\tilde{w}_k = (\partial V_k(\mathbf{p}_k, \phi_k)/\partial \ln(\ln(\phi_k)))/(\partial V(\mathbf{p}, y)/\partial \ln(\ln(y)))$ is a weight satisfying $\sum_{k=1}^K \mu_k\tilde{w}_k = 1$. Note that $\ln y = \sum_k \ln(\phi_k)$ is admissible if $\ln(\phi_k) = (\mu_k\tilde{w}_k)\ln y$, and summing over all k, one gets $\sum_k \ln(\phi_k) = \sum_k \mu_k\tilde{w}_k \ln y = \ln y$.

and household welfare is given by

$$V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k \frac{\ln(y) + \ln(\mu_k w_k) - \ln\bigl(A_k(\mathbf{p}_k)\bigr)}{\tilde{P}_k(\mathbf{p}_k, y, \mu_k)}, \tag{30}$$

where $w_k = (\partial V_k(\mathbf{p}_k, \phi_k)/\partial \ln(\phi_k))/(\partial V(\mathbf{p}, y)/\partial \ln(y))$ *is a weight satisfying* $\sum_{k=1}^{K} \mu_k \times w_k = 1$ *and* $\tilde{P}_k(\mathbf{p}_k, y, \mu_k) = B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)[\ln(y) + \ln(\mu_k w_k) - \ln(A_k(\mathbf{p}_k))]$.

Proof. Under the Bergsonian household utility (7), start with the relationship $V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k V_k(\phi_k, \mathbf{p}_k)$ and (27). When $f_k(\zeta) = \zeta$, we have

$$V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k \frac{\ln(\phi_k) - \ln\bigl(A_k(\mathbf{p}_k)\bigr)}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\bigl(\ln(\phi_k) - \ln\bigl(A_k(\mathbf{p}_k)\bigr)\bigr)}. \tag{31}$$

Equation (29) is obtained from (9). Substituting (29) into (31) gives the desired result. □

The last case that we consider is $f_k(\zeta) = \ln(\zeta)$.

Proposition 8. *Under the Bergsonian household utility (7) and QAIDS individual preferences (27), let* $f_k(\zeta) = \ln(\zeta)$. *Then the income sharing rule satisfies (29) and household welfare is given by*

$$V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k \ln\left[\frac{\ln(y) + \ln(\mu_k w_k) - \ln\bigl(A_k(\mathbf{p}_k)\bigr)}{\tilde{P}_k(\mathbf{p}_k, y, \mu_k)}\right].$$

Proof. It follows from substituting (29) in $V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k V_k(\mathbf{p}_k, \phi_k)$, where $V_k(\mathbf{p}_k, \phi_k) = \ln[(\ln(\phi_k) - \ln(A_k(\mathbf{p}_k)))/(B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)(\ln(\phi_k) - \ln(A_k(\mathbf{p}_k))))]$. □

In this context, the distribution weight w_k in (29) captures the effects of heterogeneous income effects across individuals within the household. This term also enters the household utility (30).

Next, we examine the case of AIDS preferences where $C_k = 0$ in (27). When $f_k(\zeta) = \zeta$ and from Proposition 7, (29) and (30) still apply under the AIDS model, where $w_k = (1/B_k(\mathbf{p}_k))/(\sum_{k=1}^{K} \mu_k/B_k(\mathbf{p}_k))$. This shows that under the AIDS specification, unless $B_k(\mathbf{p}_k)$ is a constant across household members, the distribution weights w_k, reflecting intra-household income effects, play a role and affect both the sharing rule (29) and the household utility (30). In other words, the AIDS model (or its QAIDS generalization) typically implies nonlinear sharing rules that depend on two sets of weights: the Bergsonian weights μ_k and the weights w_k. This property holds also for the household utility function $V(\mathbf{p}, y)$.

3.3 *Intra-household inequality*

A desirable property of social evaluation functions is the possibility to express social welfare in terms of an efficiency and equity component (Lambert 1993). We now show that the collective (indirect) household welfare function derived so far can be "abbreviated" into an efficiency component, as if the household were in a unitary framework, which assumes an equal distribution of resources, and an equity component, which accounts for the dispersion in individual prices and resources across family members. The collective household welfare function is increasing in income and decreasing in the inequality index.

For example, collective household welfare under the QES specification (22) is a monotonic function of household income y deflated by the household price index P_0 as defined in (23):

$$V(\mathbf{p}, y) = \ln(y) - \ln P_0(\mathbf{p}, y, \boldsymbol{\mu}) = \ln(y) - \left(\sum_{k=1}^{K} \mu_k \ln\big(B_k(\mathbf{p}_k)\big) + \sum_{k=1}^{K} \mu_k \ln(\mu_k)\right).$$

Again, the log of price index P_0 is composed of two additive terms: the weighted sum $\sum_{k=1}^{K} \mu_k \ln(B_k(\mathbf{p}_k))$, describing the dispersion in individual prices, and the Shannon entropy index $\sum_{k=1}^{K} \mu_k \ln(\mu_k)$, capturing inequality in the distribution of household resources. Suppose that one is comparing two households with the same level of income. Collective household welfare functions admit that the two households may differ both in the dispersion of individual prices and in the distribution of resources.

Interestingly, for each individual, this term has an inverted U-shape relationship with μ_k: it first increases as μ_k rises from 0 to 0.368, it reaches a maximum when $\mu_k = 0.368$, and then decreases as μ_k rises between 0.368 and 1. Note that, in this case, the Shannon index is closely related to the Theil inequality index. Using (24), the Theil T inequality index (TT) applied to a k-member household can be written as

$$TT \equiv \frac{1}{K}\left(\sum_{k=1}^{K} (\phi_k/\bar{y}) \ln(\phi_k/\bar{y})\right) = \sum_{k=1}^{K} \mu_k \ln(\mu_k) + \ln K,$$

where $\bar{y} = y/K$. The TT index measures the distance from the situation where every member of the household attains the same resource share associated with the ideal condition of maximum disorder. If all the power is concentrated in only one member, then the index gives the value corresponding to maximum order. In this case $TT = \ln K$, because $\sum_{k=1}^{K} \mu_k \ln(\mu_k)$ tends to 0. Being negative, it is a measure of inequality rather than equality. A distribution of resources skewed toward one member of the household implies scarcity for the other members (Jorgenson et al. 1980, 1982). Interestingly, if $\mu_k = 1/K$, then $TT = 0$ as in the unitary model. Note also that in the unitary framework, individual prices are equal to market prices and the collective indirect household welfare function degenerates into the traditional unitary indirect welfare function.

If we now consider the specification of the household indirect utility function in (26),

$$V(\mathbf{p}, y) = \frac{y - A(\mathbf{p})}{P^*(\mathbf{p}, y, \boldsymbol{\mu})},$$

where the price aggregator is $1/P^*(\mathbf{p}, y, \boldsymbol{\mu}) = \sum_k [\mu_k \mu_k W_k/(B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\mu_k W_k(y - A(\mathbf{p})))]$, we realize that we can no longer exploit the additive separability property carried by the logarithmic transformation of z, $f_k(\zeta) = \ln(\zeta)$. This structural feature limits both the theoretical and empirical appeal of this representation of collective household welfare. However, it is interesting to note that in the absence of price variation (e.g., in cross-sectional demand analysis), letting $A(\mathbf{p}) = 0$, $B_k(\mathbf{p}_k) = 1$ and $C_k(\mathbf{p}_k) = 0$ would imply that $1/P^*(\mathbf{p}, y, \boldsymbol{\mu})$ becomes

$$\sum_{k=1}^{K} \mu_k \mu_k W_k,$$

which is a weighted Simpson diversity index. In this case, household inequality would be relatively more influenced by the members with greater weight in the family.

4. Application

For illustrative purposes, we apply our theoretical findings to a deflated model where $g_k(\phi_k, \mathbf{p}_k) = \ln(\phi_k/A_k(\mathbf{p}_k))$. Our analysis of household distribution issues uses observable information about assignable goods as part of our identification strategy. A distinctive feature of our empirical procedure is that we estimate a complete demand system, which is a precondition for deriving both the individual and household welfare functions. Knowledge of these objects is necessary to recover the Pareto weight μ_k and the distribution weight w_k describing intra-household inequality. A detailed description of the empirical procedure is shown in the Appendix.

The individual indirect utility function for deflated QAIDS demand models is $V_k(\mathbf{p}_k, \phi_k) = f_k((\ln(\phi_k) - \ln(A_k(\mathbf{p}_k)))/(B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)(\ln(\phi_k) - \ln(A_k(\mathbf{p}_k)))))$. By further assuming that $f_k(\zeta) = \zeta$ and $C_k = 0$, which corresponds to the case b(ii) in Table 1, we specialize to the AIDS indirect utilities

$$V_k(\mathbf{p}_k, \phi_k) = \frac{\ln(\phi_k) - \ln\big(A_k(\mathbf{p}_k)\big)}{B_k(\mathbf{p}_k)}. \tag{32}$$

The associated household welfare function is

$$V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k \frac{\ln(y) + \ln(\mu_k w_k) - \ln\big(A_k(\mathbf{p}_k)\big)}{B_k(\mathbf{p}_k)} \tag{33}$$

and the income share is

$$\frac{\phi_k}{y} = \mu_k w_k, \tag{34}$$

where μ_k is the welfare weight and $w_k = (\partial V_k(\mathbf{p}_k, \phi_k)/\partial \ln(\phi_k))/(\partial V(\mathbf{p}, y)/\partial \ln(y))$ is the distribution weight. Given the chosen functional form, we can express

$$w_k = \left(\frac{1}{B_k(\mathbf{p}_k)}\right) \Big/ \left(\sum_{k=1}^{K} \frac{\mu_k}{B_k(\mathbf{p}_k)}\right). \tag{35}$$

Note that, in the case of two agents $k = 1, 2$, the ratio of the distribution weights does not depend on the welfare weights $w_1/w_2 = B_2/B_1$.[9] Similarly, using this result, the condition $\sum_{k=1}^{K} \mu_k w_k = 1$, and (34), the ratio of the welfare weights is independent of the distribution weights $\mu_1/\mu_2 = \phi_1 B_1/\phi_2 B_2$. From this expression and recalling that $\sum_{k=1}^{K} \mu_k = 1$, the welfare weights are

$$\mu_1 = \frac{\phi_1 B_1}{\phi_1 B_1 + \phi_2 B_2} \quad \text{and} \quad \mu_2 = \frac{\phi_2 B_2}{\phi_1 B_1 + \phi_2 B_2}.$$

Substituting these expressions in the condition $\sum_{k=1}^{K} \mu_k w_k = 1$ and using the relationship that $w_2 = w_1 B_1/B_2$, we obtain the distribution weights[10]

$$w_1 = \frac{\phi_1 B_1 + \phi_2 B_2}{B_1 y} \quad \text{and} \quad w_2 = \frac{\phi_1 B_1 + \phi_2 B_2}{B_2 y}. \tag{36}$$

Given the welfare and distribution weights, we can evaluate the intra-household inequality associated with the AIDS specification. The household welfare function can now be rewritten as

$$V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k \frac{\ln(y) - \ln\big(A_k(\mathbf{p}_k)\big)}{B_k(\mathbf{p}_k)} + \sum_{k=1}^{K} \frac{1}{B_k(\mathbf{p}_k)}\big(\mu_k \ln(\mu_k w_k)\big), \tag{37}$$

where the first term is the deflated level of household income and the second term is a measure of dispersion in individual welfare levels similar to a Theil inequality index scaled by the inverse of the price index $B_k(\mathbf{p}_k)$. Unlike a Theil index, the distribution weight w_k rescales the Pareto weight μ_k. The measure of dispersion vanishes in the limiting case where ϕ_k approaches y and the distribution weight approaches $1/\mu_k$. The household welfare function then reduces to the level of deflated household income. This limiting case gives the least possible weight to equity considerations.

Our analysis can provide estimates of μ_k and w_k and the measurement of the level of intra-household inequality. This is novel in the applied collective household literature. Our empirical analysis involves estimating a collective system of AIDS budget shares linear in the log of total expenditure, extending the AIDS model to account for the intra-household allocation of resources. The application uses Italian household budget data for the year 2007 on couples without children. The data, model, estimation technique, identification strategy, and results are reported in the Appendix.

Table 2 reports the means, standard deviations, minimum and maximum values for the individual and household welfare levels, and the Pareto and distribution weights along with the income shares. The level of welfare of females is higher than that of males. Alternatively, the Pareto weights are higher for males. On average, the contri-

[9]Rewriting (35) as $w_k B_k(\mathbf{p}_k) = 1/\sum_{k=1}^{K} \mu_k/B_k(\mathbf{p}_k)\ \forall k$ yields $w_1 B_1 = w_2 B_2$.

[10]An alternative representation of (36) is $w_1 = B_2/(\mu_1 B_2 + \mu_2 B_1)$ and $w_2 = B_1/(\mu_1 B_2 + \mu_2 B_1)$.

	Mean	Std. Dev.	Min	Max
Individual welfare levels				
Female	5.100	0.662	3.110	8.365
Male	2.888	0.396	1.843	4.271
Household welfare levels	3.688	0.466	2.566	5.281
Pareto weight				
Female	0.363	0.016	0.307	0.424
Male	0.637	0.016	0.576	0.693
Distribution weight				
Female	1.374	0.065	1.175	1.644
Male	0.789	0.023	0.715	0.872
Income share				
Female	0.498	0.005	0.485	0.532
Male	0.502	0.005	0.468	0.515

Table 2. Welfare levels, Pareto and distribution weights, and income share.

bution to household welfare, corresponding to the product of the Pareto weight with the individual welfare level, is about the same for the wife and the husband. The income shares are on average close to half. Judging from the distance between minimum and maximum values, there is, however, sufficient variation informing about the intrahousehold welfare and distribution of household resources. The income share is decomposed in the Pareto and distribution weight. The distribution weights scale up the Pareto weight of the wife and scale down the Pareto weight of the husband. The relative contribution to the household marginal valuation of money is higher for the female.

Table 3 shows the measure of intra-household inequality corresponding to the second component of the household welfare function (37) and describing the dispersion of individual welfare levels. This term is computed for the whole sample, and for the poor and rich families corresponding to the lower and upper tertile of the income distribution. This index allows comparison to be made of two households with similar levels of household income, but differing in the intra-household distribution of resources. As shown in Table 3, the distribution of resources within less affluent Italian households is more equitable compared to the distribution of resources in rich families and the difference between the two samples is statistically significant with a t-test equal to 17.28. This provides new and useful insights into the welfare implications of intra-household allocations.

5. Conclusions

We model a household in terms of the utility functions of its members using a weighted Bergsonian household utility. In a Pareto efficient household environment with privately consumed goods, we recover the relationships between centralized and decentralized programs where the maximization of the utility of each household mem-

	Mean	Std. Dev.	Min	Max
All sample	−0.336	0.028	−0.434	−0.244
Poor	−0.319	0.023	−0.382	−0.265
Rich	−0.365	0.024	−0.434	−0.284

Table 3. Intra-household inequality.

ber is subject to an income sharing rule, and we investigate the relationship among the sharing rules, household welfare, and intra-household inequality. The linkages established between centralized and decentralized demands provide new and useful information on the economics of intra-household allocations. Our analysis describes the general properties of the income sharing rule as a function of two sets of weights: the Bergsonian welfare weights and distribution weights reflecting income effects across household members. We show that these weights play a role in the evaluation of both household welfare and inequality. The theoretical results provide new insights into intra-household decisions and implications for the distribution of household welfare. Intra-household inequality is described by a family of entropy indexes that are functions of the sharing rule. We illustrate our findings with an empirical application that estimates a collective demand system to recover associated individual and household welfare functions along with the measures of intra-household inequality. This is the first application that estimates the Pareto weight and examines its role within a measure of income dispersion across household members. In our sample, richer couples distribute resources among their members less equally.

Appendix. Estimation of the collective complete demand system

To derive the sharing rule, the welfare and distribution weights, and the measure of intra-household inequality, we estimate a collective complete demand system of the AIDS form that is linear in income. This appendix describes the household data used in the analysis, the model specification, and estimation strategy.

Data

The data set used in this study is the 2007 Household Budget Survey of the Italian Statistics Institute (ISTAT), which collects information on harmonized international classifications of expenditure items of a representative national sample of 24,400 households. For illustration purposes, our analysis focuses on couples without children. The sample size thus reduces to 1,407 households. Consumption expenditures are at the household level except for a set of assignable goods, such as clothing for males and females, used for the identification of the income share.

A limitation of the Italian household budget survey, common to many household budget surveys, is its lack of information on the quantities consumed by each household necessary for the computation of household-specific unit values. We approximate unit values with a method suggested by Lewbel (1989b) and empirically developed by Hoderlein and Mihaleva (2008) and Menon et al. (2017b). This technique captures the spatial and quality variability typical of unit values as deduced from household socioeconomic characteristics. This variability is then added to the aggregate price indices published monthly by national statistics institutes.

We estimate a complete demand system comprising budget shares for five goods: food, housing, transportation and communications, clothing, and other goods. Goods such as housing or transportation can be consumed either publicly or privately. In our estimation, this is not a critical concern because we estimate budget shares aggregated at the household level. Further, identification of the sharing rule relies on the observability of assignable goods privately consumed by household members. We acknowledge that accounting for household public goods (e.g., by estimating a conditional sharing rule; see Chiappori and Meghir (2015)) would have affected the demand estimates and the measures of inequality. In the estimation, we control for geographic location (northwest, northeast, center, south Italy, and islands), working status of spouses, seasonality effects (using a dummy equal to 1 if the family has been interviewed in winter months), home ownership, and the number of cars owned by the spouses. Distribution factors (i.e., exogenous variables not affecting either preferences or the budget constraint) include the age and education ratio of the spouses. Such variables would help improve the robustness of the estimated parameters of the income share.

Table 4 shows the definition of the variables used in the empirical analysis and the descriptive statistics for our sample. The larger budget shares relate to other goods (37.4%) and food (23.5%), while clothing has the smallest budget share (9.9%) divided by about half between spouses. Price variability, as measured by standard deviations, varies across goods and is substantial for all goods. It is relatively large for other goods, housing, and clothing, while it is smaller for transportation and communications and food. The large price variability of our data help identification of price effects: the empirical results show that a large number of estimated price coefficients are statistically significant. In regard to distribution factors, the spouses' education levels and age are similar, while the price of clothing is higher for female clothing than for male clothing. The likelihood of participating in the labor market is higher for the husband (86.5%) than for the wife (64.7%). Less than half of the sample (42.9%) was interviewed from October to February. Finally, the majority of households own the home (71.9%) and have more than one car.

Demand system estimation

Our empirical analysis considers a demand system linear in the log of total expenditure (Deaton and Muellbauer 1980a) extended to a household collective context with two adults. Following the theory presented in Section 3.2 with $C_k = 0$, the kth individual's budget share for good i, ω_{ki}^D in (28), reduces to

$$\omega_{ki}^D = \frac{\partial \ln\big(A_k(\mathbf{p}_k)\big)}{\partial \ln(p_{ki})} + \frac{\partial B_k(\mathbf{p}_k)}{\partial \ln(p_{ki})}\big[\ln(\phi_k) - \ln\big(A_k(\mathbf{p}_k)\big)\big],$$

Variables		Budget Shares Mean	Budget Shares Std. Dev.	Log Prices Mean	Log Prices Std. Dev.
Food		0.235	0.115	1.762	0.350
Housing		0.115	0.056	0.804	0.377
Clothing		0.099	0.039	−1.163	0.833
Transport. & communications		0.177	0.093	−3.337	0.696
Other goods		0.374	0.154	0.799	0.809
Female clothing		0.054	0.031	−0.725	0.774
Male clothing		0.044	0.014	−1.233	0.879
				Mean	Std. Dev.
Distribution factors					
Ageratio	$\frac{\text{wife age}}{\text{wife age} + \text{husband age}}$			0.486	0.033
Eduratio	$\frac{\text{wife education}}{\text{wife education} + \text{husband education}}$			0.488	0.087
Ratiop	$\frac{p_{c_f}}{p_{c_m}}$			1.718	0.469
Household characteristics					
$\log(y)$	Log of total expenditure			7.468	0.501
$\log(y_{DI})$	Log of disposable income			7.783	0.399
R1	Northwest			0.289	0.454
R2	Northeast			0.251	0.434
R3	Center			0.195	0.396
R4	South			0.180	0.384
R5	Islands			0.085	0.279
Tj	= 1 if husband works			0.865	0.342
Ts	= 1 if wife works			0.647	0.478
Winter	= 1 if interviewed in Oct.–Feb.			0.429	0.495
Landlord	= 1 if owner of the house			0.719	0.450
No. of cars	Number of cars			1.478	0.590
No. of observations			1,407		

Table 4. Descriptive statistics.

where $\ln(A_k(\mathbf{p}_k))$ and $B_k(\mathbf{p}_k)$ are differentiable and concave price aggregators with the functional form

$$\ln\bigl(A_k(\mathbf{p}_k)\bigr) = \alpha_{k0} + \sum_i \alpha_{ki} \ln p_{ki} + \frac{1}{2} \sum_i \sum_j \gamma_{kij} \ln p_{ki} \ln p_{kj}$$

and

$$B_k(\mathbf{p}_k) = \prod_i p_{ki}^{\beta_{ki}}.$$

The price aggregators $A_k(\mathbf{p}_k)$ and $B_k(\mathbf{p}_k)$ can be interpreted as individual portions of household subsistence and bliss costs, respectively. It is maintained that both spouses have equal access to those costs as if they faced the same prices for nonassignable goods.

	Food	Housing	Clothing	Trans. & Comm.	Other Goods
Intercept	0.205	0.482	−0.277	−1.205	1.795
	0.120	*0.086*	*0.043*	*0.111*	*0.099*
Price effects					
Food	−0.037	0.040	−0.013	0.020	−0.011
	0.015	*0.011*	*0.008*	*0.026*	*0.035*
Housing		−0.001	0.016	0.095	−0.150
		0.011	*0.006*	*0.021*	*0.023*
Clothing			−0.039	−0.044	0.080
			0.005	*0.009*	*0.013*
Trans. & comm.				−0.461	0.390
				0.047	*0.039*
Other goods					−0.309
					0.051
Income effects					
β_1	−0.004	−0.064	0.177	0.202	−0.312
	0.023	*0.015*	*0.007*	*0.020*	*0.020*
β_2	0.050	−0.064	−0.073	0.231	−0.144
	0.022	*0.014*	*0.007*	*0.019*	*0.020*
Distribution factor effects			Param		S.E.
Ageratio			−0.010		*0.062*
Eduratio			−0.061		*0.025*
Ratiop			0.015		*0.005*
Error correction terms					
Food			−0.152		*0.024*
Housing			0.000		*0.015*
Clothing			−0.086		*0.007*
Trans. & comm.			−0.325		*0.020*
Other goods			0.564		*0.022*

Note: Standard errors are given in italics.

Table 5. Collective AIDS estimation: price, income, and distribution factor parameters.

The individual budget shares ω_{ki}^D can be aggregated to obtain

$$\begin{aligned}\omega_i &= \omega_{1i}^D + \omega_{2i}^D \\ &= \alpha_i + \sum_j \gamma_{ij} \ln p_j + \sum_{k=1}^{2} \beta_{ki}\big(\ln \phi_k - \ln A_k(\mathbf{p}_k)\big),\end{aligned} \tag{53}$$

the budget share for each good i at the household level.

Observed heterogeneity is introduced using a translating household technology $t_i(\mathbf{d})$, which modifies the demand system (53), so that demographic characteristics interact additively with income in a theoretically plausible way Gorman (1976), Lewbel (1985), Perali (2003). Thus, the demographically modified collective share equation ω_i

	Food	Housing	Clothing	Trans. & Comm.	Other Goods
Northwest	−0.034	0.023	−0.017	−0.002	0.031
	0.011	*0.007*	*0.002*	*0.011*	*0.017*
Northeast	−0.032	0.029	−0.025	0.007	0.020
	0.011	*0.007*	*0.002*	*0.011*	*0.017*
Center	−0.019	0.018	−0.016	0.002	0.016
	0.011	*0.006*	*0.002*	*0.011*	*0.017*
South	0.015	0.001	0.001	−0.006	−0.011
	0.010	*0.006*	*0.002*	*0.011*	*0.017*
Tj	−0.034	−0.004	0.003	−0.025	0.059
	0.008	*0.004*	*0.001*	*0.007*	*0.011*
Ts	−0.024	0.003	−0.001	−0.009	0.031
	0.006	*0.003*	*0.001*	*0.005*	*0.008*
Winter	−0.002	0.003	0.005	0.000	−0.007
	0.005	*0.003*	*0.001*	*0.005*	*0.007*
Landlord	0.001	0.006	−0.006	−0.010	0.008
	0.006	*0.003*	*0.001*	*0.005*	*0.008*
No. of cars	−0.036	−0.006	−0.005	0.036	0.011
	0.005	*0.003*	*0.001*	*0.004*	*0.006*

Note: Standard errors are given in italics.

Table 6. Collective AIDS estimation: demographic effects.

in (53) becomes

$$\omega_i = \alpha_i + t_i(\mathbf{d}) + \sum_j \gamma_{ji} \ln p_j + \sum_{k=1}^{2} \beta_{ki}\bigl(\ln \phi_k^* - \ln A_k(\mathbf{p}_k)\bigr) + \varepsilon_i,$$

where ε_i is an error term and $\ln \phi_k^*$ is the log of individual income modified by a translating household technology as

$$\ln \phi_k^* = \ln \phi_k - \sum_i t_i(\mathbf{d}) \ln p_i,$$

the translating demographic functions $t_i(\mathbf{d})$ being specified as $t_i(\mathbf{d}) = \sum_l \tau_{il} \ln d_l$ for $l = 1, \dots, L$. This demand system is very similar to a traditional demand system, except for the specification of the income term, which is expressed at individual levels (and not the household level). Following Chavas et al. (2014), Dunbar et al. (2013), and Menon et al. (2017a), individual incomes are derived by partitioning household incomes with a resource share computed using the available information about the observed expenditure on assignable goods, with nonassignable goods assumed to be consumed in equal proportions by each household member. To obtain the sharing rule ϕ_k, we then anchor the income scaling function $m_k(\boldsymbol{\psi})$ to individual total expenditures y_k, incorporating the identifying information about assignable consumption as $\phi_k = m_k(\boldsymbol{\psi})y_k$. In line with the theory results in Chavas et al. (2014) and the empirical findings of Menon et al. (2012), m_k is independent of total expenditure.

	Food	Housing	Clothing	Trans. & Comm.	Other Goods
			Income elasticities		
Female	0.981	0.388	2.914	2.289	0.094
	0.027	*0.300*	*0.485*	*0.572*	*0.322*
Male	1.242	0.389	0.209	2.469	0.582
	0.356	*0.299*	*0.200*	*0.652*	*0.148*
			Uncompensated price elasticities		
Food	−1.172	0.133	−0.031	0.228	−0.270
	0.249	*0.205*	*0.048*	*0.292*	*0.365*
Housing	0.357	−0.679	−0.014	0.176	−0.230
	0.175	*0.123*	*0.016*	*0.202*	*0.186*
Clothing	−0.115	−0.134	−1.266	0.194	−0.240
	0.044	*0.030*	*0.070*	*0.077*	*0.081*
Trans. & comm.	0.159	−0.155	0.090	−2.219	−0.253
	0.127	*0.112*	*0.051*	*0.515*	*0.215*
Other goods	−0.044	−0.070	0.054	0.316	−0.594
	0.057	*0.036*	*0.024*	*0.135*	*0.163*
			Compensated price elasticities		
Food	−0.914	0.259	0.080	0.424	0.147
	0.271	*0.203*	*0.039*	*0.309*	*0.306*
Housing	0.457	−0.629	0.026	0.244	−0.099
	0.138	*0.107*	*0.034*	*0.161*	*0.333*
Clothing	0.251	0.045	−1.113	0.471	0.357
	0.055	*0.036*	*0.102*	*0.069*	*0.172*
Trans. & comm.	0.720	0.118	0.327	−1.826	0.661
	0.174	*0.057*	*0.134*	*0.553*	*0.416*
Other goods	0.027	−0.035	0.084	0.370	−0.449
	0.079	*0.053*	*0.023*	*0.126*	*0.117*

Note: Elasticities are evaluated at the data means. Standard errors are given in italics.

TABLE 7. Income and price elasticities.

This strategy enables the identification of the parameters of the m_k function and of resource sharing within a Pareto efficient family. The income scaling function m_k is analogous to the price scaling function introduced by Barten (1964) or the income scaling function described in Lewbel (1985, Theorem 8). This function is specified in a parsimonious fashion and the vector $\boldsymbol{\psi}$ includes relative prices of the assignable goods as required by the theory (Menon et al. 2016) and distribution factors.

The estimations are carried out using the full maximum likelihood method. The parameters of the omitted equation, other goods, are recovered from the constraints imposed by demand theory (Banks et al. 1997, Browning et al. 2013). The log of total expenditure is likely to be endogenous because of measurement errors, due either to infrequency of purchases or to recall errors. To adjust for the possibility of endogeneity of the log of total expenditure, we apply the control function method (Blundell and Robin 1999) and regress the log of total expenditure against the exogenous explanatory variables of the demand system and the log of net disposable income acting as the instrument for total expenditure. The residual r of the auxiliary regression is then included in the system of budget shares and, conditional on this residual, the log of total expenditure is exogenous. Thus, the error term ε_i can be decomposed as $\varepsilon_i = \nu_i r + \upsilon_i$, where ν_i is a parameter to be estimated and υ_i is an error term. Testing whether ν_i is significantly different from zero corresponds to testing the exogeneity of the log of total expenditure.

Table 5 reports the estimated parameters for price, income, and distribution factors. These are often statistically significant. The parameters ν_i associated with the control variables are also significantly different from zero, with the exception of the housing share, thus justifying the correction for the endogeneity of total expenditure. Two of the three distribution factors are statistically significant. The income parameters are differentiated by gender and provide information that allows the recovery of individual welfare functions. Table 6 presents the demographic effects. They are also significantly different from zero, documenting that demographic factors affect consumption behavior.

Estimated compensated and uncompensated price elasticities along with the individual income effects are consistent with consumer theory (Table 7). From these estimates, we recover the individual and household welfare functions given in (32) and (33) incorporating demographic heterogeneity, and the associated measures of intra-household inequality (37).

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