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Dynamics of strategic information transmission in social networks

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We develop a dynamic framework of strategic information transmission through cheap talk in a social network. Privately informed agents have different preferences about the action to be implemented by each agent and repeatedly communicate with their neighbors in the network. We first characterize myopic (best response) equilibria as well as fully informative myopic equilibria. Second, we provide a sufficient condition for the existence of a fully informative farsighted (perfect Bayesian) equilibrium. Fully informative myopic and farsighted equilibria essentially take a particular simple form: all communication is truthful along a subnetwork that is a tree. We also consider societies in which both myopic and farsighted agents are present, and we analyze equilibrium welfare. Furthermore, we extend our model to public communication and investigate the implications of our results for the design of institutions. Finally, our analysis reveals that myopic equilibria tend to Pareto dominate farsighted equilibria, in particular if a social planner has designed the network optimally.

Keywords. Cheap talk, information aggregation, learning, social networks, strategic communication.

JEL classification. C72, D82, D83, D85.

1. Introduction

In many economic, political, and social situations, decision-making relies on information agents gather through communication with their social environment. Sharing information allows for better individual decisions, but revealing all may not be the best strategy when interests diverge and individual decisions have spillovers on other agents. We consider this type of context but drop the typical assumption that agents communicate directly with all other agents. If a large number of agents are involved in a decentralized decision process, it is likely that not all decision-makers interact—at least not directly.[1] For example, we can think of an organization where decision-making is decentralized at the division level and these divisions do not share the same preferences about

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[1] Framed differently, we consider information aggregation through communication in social networks but drop the typical assumption that the agents' interests are aligned.

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optimal decisions; possible reasons include local costs of adaptation, career concerns, and priorities. Before making decisions, division leaders interact with those other division leaders they have direct operational relations with, resulting in information being transmitted indirectly throughout the organization.

Our objective in this paper is to develop a tractable dynamic framework of belief formation under payoff externalities to analyze how agents strategically transmit information through cheap talk with their neighbors in a social network. The central research questions are whether such belief dynamics lead to the aggregation of dispersed information and how the information is effectively transmitted through the network. A comprehensive analysis of these questions requires distinguishing different degrees of sophistication of the agents—myopic and farsighted—as well as different modes of communication—private and public.[2]

Our framework is a natural extension of the model of cheap talk by Galeotti et al. (2013) (henceforth GGS) to a dynamic game on a network.[3] There is an unknown *state of the world* and each agent has a *bias* relative to the state that determines the agent's ideal action or bliss point. We can view the profile of biases as a measure for the conflict of interest between the agents: each agent would like the other agents' actions to be as close as possible to her ideal action. Each agent initially receives a *private signal* correlated with the unknown state and can communicate with her direct neighbors in the *social network*. Agents sequentially "wake up" (in a random order) to communicate with their neighbors by sending *messages* about their information. They decide for each piece of information they hold whether to transmit it to their neighbors.[4] Agents update their beliefs about the state based on their private signal and information received from their neighbors via Bayes' rule. Finally, after the repeated information exchange has taken place, each agent takes an action and payoffs realize.[5]

Our analysis proceeds in several steps. In the baseline setup, we consider myopic agents and private communication. Myopic agents play myopic best response and thus ignore that other agents' beliefs may change subsequently due to further information transmission.[6] Agents can differentiate their messages across neighbors by sending them privately to every neighbor; private communication is assumed throughout if not

[2]The distinction between myopic and farsighted behavior is common in the literature on strategic interaction on networks. A widely used example is the myopic pairwise stability concept by Jackson and Wolinsky (1996), which has been extended to farsighted network formation by Dutta et al. (2005), Herings et al. (2009), and Page et al. (2005).

[3]These authors study an extension of the uniform-quadratic version of Crawford and Sobel (1982) to multiple players.

[4]More precisely, information is "tagged" so that agents can share information along with its source. Notably, recent experimental evidence by Mobius et al. (2015) suggests that people make use of tagging to aggregate information.

[5]This implies that there are no time constraints, which is a simplifying assumption that allows us to focus on the effects of payoff externalities. In the context of an organization, we can think about this assumption as that agents start exchanging information early enough so that the information exchange is finished when decisions are to be made.

[6]We could also think of these agents as purposely avoiding the computational burden associated with computing a sequentially rational response. We refer to Bala and Goyal (1998) for a discussion of this assumption.

stated otherwise. Herein, we refer to perfect Bayesian equilibria with myopic best response as *myopic equilibria* and employ an adaptation of the intuitive criterion by Cho and Kreps (1987) to restrict off-equilibrium beliefs.[7] We characterize myopic equilibria as well as *optimal myopic equilibria*, which are optimal from an informational point of view, in a myopic sense: they maximize the transmitted information at each time instant, which makes them prominent for myopic agents. Surprisingly, partial information transmission is ruled out in optimal myopic equilibria. We then show that the ability of a sender to communicate all her information truthfully in an optimal myopic equilibrium (at a given time instant) depends on the number of signals the receiver already holds and the number of signals the sender could potentially transmit: the upper bound on the number of signals the sender would like the receiver to hold is decreasing in the conflict of interest.[8] Furthermore, we show that optimal myopic equilibria are *fully informative* if and only if there exists a subnetwork that is a tree along which the conflicts of interest between neighbors are below some threshold. In this case, all communication is truthful along the tree.

Second, we consider farsighted agents who have perfect foresight. Herein, we refer to perfect Bayesian equilibria as *farsighted equilibria*. We show that there exists a fully informative farsighted equilibrium if there exists a subnetwork that is a tree such that for each agent the absolute differences of her bias and the average bias of all agents reachable through each of her neighbors in the tree are below the same threshold as for myopic equilibria. Again, all communication is shown to be truthful along the tree.

We then combine our findings on myopic and farsighted agents, and study heterogeneous societies in which both myopic and farsighted agents are present. With a slight modification to account for the presence of myopic agents, we show that our result for farsighted agents extends to heterogeneous societies. Notably, heterogeneous societies provide testable implications: we can construct a network and preferences such that certain agents can only transmit all their information if they are myopic (farsighted).

Fourth, we analyze equilibrium welfare. As in GGS, more ex ante expected information transmission (for at least one agent) constitutes a strict Pareto improvement. Hence, an equilibrium is Pareto efficient if and only if it is fully informative. We add that optimal myopic equilibria are *utility-maximizing equilibria*—equilibria that Pareto dominate all other equilibria—if they are fully informative, and otherwise they may not be utility-maximizing.

Next, we consider public communication, that is, agents cannot differentiate their messages across neighbors. Our findings are in line with the existing literature regarding local communication; see, e.g., Farrell and Gibbons (1989) and Goltsman and Pavlov (2011). In particular, we also observe what Farrell and Gibbons (1989) refer to as subversion (truthful communication is possible with some audience in private but impossible in public), which interestingly has further implications in our context. When looking

[7] Broadly speaking, the criterion effectively eliminates the possibility of sowing doubt about information that was already transmitted earlier.

[8] Notice that this result essentially is a generalization of the main result in GGS to agents holding multiple private signals.

beyond local communication, we find that, unlike in static models, subversion may also prevent fully informative equilibria if agents are myopic.

Furthermore, we analyze the implications of our model for the design of institutions. Consider a manager who is planning to move her company to new headquarters. How would she organize the new building and place the different divisions? According to the biography by Isaacson (2011), Steve Jobs, the former Chief Executive Officer (CEO) of Apple and Pixar, designed the Pixar headquarters "to promote encounters and unplanned collaborations." If we think that encounters occur if individuals are linked in the network and collaborations are essentially the exchange of information, then, in our context, the objective of the CEO, or, more generally, a social planner, would be to choose a network that maximizes equilibrium welfare. Our analysis suggests that agents should be placed in increasing order of their biases to create a "line" network, both if agents are myopic and (at least for small conflicts) if they are farsighted. Notably, this implies that *homophily*—the well documented tendency of individuals to connect to others with similar preferences ("love of the same") (see McPherson et al. (2001) for a literature overview)—is beneficial for information transmission under conflicts of interest.

Finally, we consider the complete network to investigate whether allowing for dynamic communication aids or impedes information transmission relative to the static game of GGS. We show that dynamics ease the constraints on conflicts of interest for fully informative equilibria to exist, both with myopic and with farsighted agents. Additionally, a general comparison of myopic and farsighted equilibria reveals that myopic agents tend to reach outcomes that Pareto dominate those of farsighted agents, in particular if a social planner has designed the network optimally.

Our work contributes to the literature on cheap talk as well as to that on learning and information aggregation in social networks. We extend the static framework of cheap talk by GGS to a dynamic game on a network. A closely related contribution is Hagenbach and Koessler (2010), who study a model of multiplayer cheap talk in which agents have incentives to coordinate their actions.[9] Furthermore, Ambrus et al. (2013) investigate a game in which the sender and the receiver can communicate only through a chain of intermediators. They assume that all involved agents are strategic—but different from our approach, only the receiver takes an action—and find that pure strategy equilibrium outcomes are monotonic in the intermediators' biases and do not depend on the order of intermediators.[10]

From the point of view of the literature on learning and information aggregation in social networks, our contribution is that we introduce payoff externalities. Acemoglu et al. (2014) study information aggregation through communication on a network in a closely related model, but consider aligned preferences. In their model, agents can take

[9] We refer to GGS for a detailed discussion of the differences from our statistical framework.

[10] Other related contributions study cheap talk with either multiple senders, e.g., Ambrus and Takahashi (2008), Austen-Smith (1993), Battaglini (2002), Krishna and Morgan (2001), Morgan and Stocken (2008), and Wolinsky (2002), or multiple receivers, e.g., Farrell and Gibbons (1989) and Goltsman and Pavlov (2011). Baumann (2017) studies information transmission between agents in a network and an uninformed principal who wants to extract information from the network.

the decision at any time and payoffs are discounted so that agents prefer earlier decisions. They show that under truthful communication, asymptotic learning—defined as the fraction of agents who take the correct action converging to 1 as the society grows—occurs if most agents are close to "information hubs," which receive and distribute a large amount of information.[11] Other papers that study information aggregation through communication in social networks typically use non-Bayesian rules to update beliefs. These approaches emphasize the complexity of updating beliefs when agents share their ex post beliefs (and not messages on the signals as in our case) and study the pattern of the agents' influence on long-run beliefs; see, e.g., Acemoglu et al. (2010), DeMarzo et al. (2003), and Golub and Jackson (2010).[12] Furthermore, our paper is related to contributions on Bayesian models of observational learning, in which agents learn from observing the actions of their neighbors. Despite aligned interests, information aggregation may fail in these models, as observed actions do not reflect all information an agent has; see, e.g., Acemoglu et al. (2011), Bala and Goyal (1998), Banerjee and Fudenberg (2004), Gale and Kariv (2003), and Mueller-Frank (2013).[13] Another related paper is Hagenbach (2011), who studies information centralization on a network. In her model, agents face a trade-off between supporting fast information centralization and increasing the chances to themselves centralize the information first, which would yield an additional personal reward.

The paper is organized as follows. In Section 2 we introduce the model and notation. Section 3 presents our main results on myopic and farsighted agents. We also derive results on heterogeneous societies and analyze equilibrium welfare. Section 4 considers public communication. The problem of a social planner concerned with the formation of the network is analyzed in Section 5. We compare our framework with the static game of GGS in Section 6. Section 7 concludes. The proofs are presented in the Appendix.

2. Model and notation

We consider a set $N = \{1, 2, \dots, n\}$, with $n \geq 2$, of *agents* or *players* and each agent $i \in N$ has a *bias* $b_i \in \mathbb{R}$. The unknown *state of the world* θ is distributed uniformly on $[0, 1]$. At time $t = 0$, each agent i receives a *private signal* $s_i \in \{0, 1\}$ about the realization of θ, where $s_i = 1$ with probability θ and $s_i = 0$ with probability $1 - \theta$. Signals are independent across agents, conditionally on θ.

Let g^N denote the set of all subsets of N with cardinality 2, called the *complete graph*. The *social network* is given by an undirected graph $g \subseteq g^N$. For simplicity, we denote

[11] Fan et al. (2015) build on this model and investigate learning with finite populations. Another related strand of literature is that on knowledge and consensus, which studies repeated information exchange through truthful private messages. In particular, Parikh and Krasucki (1990) and Krasucki (1996) analyze which conditions on the communication network lead to consensus.

[12] While these contributions assume truthful communication, Buechel et al. (2015) study a model in which agents act strategically in that stated beliefs depend on preferences for conformity. Förster et al. (2013) develop a model in which agents aggregate information anonymously, based on how many agents hold certain beliefs.

[13] Mueller-Frank (2014) studies boundedly rational agents and shows that the presence of at least one Bayesian agent in the network is sufficient for every agent to perfectly aggregate information.

a *link* $\{i, j\} \in g$ by ij. Agent i can exchange information with agent j (and vice versa) if $ij \in g$. We denote the *neighborhood* of agent i by $N_i(g) = \{j \in N | ij \in g\}$. A *path* from agent i to agent j in g is a sequence of distinct agents $i = i_1, i_2, \dots, i_K = j$ such that $i_k i_{k+1} \in g$ for all $k = 1, 2, \dots, K-1$. We say that the graph g is a *tree* if there exists a unique path from i to j in g for all distinct $i, j \in N$. The set of agents $C \subseteq N$ is a *component* of g if there is a path from i to j in g for all distinct $i, j \in C$ and $N_i(g) \subseteq C$ for all $i \in C$. The *set of components* of g is denoted $\mathcal{C}(g)$.

Agents communicate with their neighbors and update their beliefs at discrete time periods. At each time instant $t = 1, 2, \dots,$ one agent i wakes up and communicates with her neighbors. We assume, without loss of generality, that agents wake up with equal probability $1/n$ independent across periods.[14] The agent who is *active* at time instant $t \geq 1$ is denoted $\iota(t)$ and the *history* of active agents before time t by

$$H_t = \big(\iota(t')\big)_{t'=1}^{t-1}.$$

The set of possible histories at time $t \geq 1$ is denoted $\mathcal{H}_t$. If agent $i = \iota(t)$ is active at time instant t, she communicates with each neighbor $j \in N_i(g)$ by sending a *private message* $\hat{m}_{ij,t} \in \{0, 1\}^n$. Without loss of generality, the kth entry of $\hat{m}_{ij,t}$, $\hat{m}_{ij,t,k}$, represents the information i sends to j at time instant t regarding the signal of agent k. In other words, information is "tagged" so that agents can share their information along with its sources. The *information set* of agent i at time $t \geq 0$ is defined as

$$I_{i,t} = \big(s_i, (\hat{m}_{ij,t'})_{\substack{i=\iota(t') \\ ij \in g \\ t' < t}}, (\hat{m}_{ki,t'})_{\substack{k=\iota(t') \\ ki \in g \\ t' < t}}\big)$$

and contains the agent's own private signal and all messages that she has sent and received up to time t. The set of possible information sets of agent i at time $t \geq 0$ is denoted $\mathcal{I}_{i,t}$. A *(pure) communication strategy* for the active agent $i = \iota(t)$ at time instant t regarding each agent $j \in N_i(g)$ is defined as a mapping

$$m_{ij,t} : \mathcal{I}_{i,t} \times \mathcal{H}_t \to \{0, 1\}^n.$$

After the repeated information exchange has taken place, i.e., in the limit $t \to \infty$, each agent i has to choose an *action* $\hat{a}_i \in \mathbb{R}$.[15] Agent i's action depends on the information that she has acquired during the information exchange and the history of communication,

$$a_i : \Big(\lim_{t\to\infty} \mathcal{I}_{i,t}\Big) \times \Big(\lim_{t\to\infty} \mathcal{H}_t\Big) \to \mathbb{R}.$$

Given the state of the world θ, the *payoff* of agent i from the profile of actions $\hat{a} = (\hat{a}_1, \hat{a}_2, \dots, \hat{a}_n)$ is

$$u_i(\hat{a}|\theta) = -\sum_{j \in N} (\theta + b_i - \hat{a}_j)^2.$$

[14]Our results hold as long as each agent wakes up with strictly positive probability that is independent across periods.

[15]Notice that, as there are only finitely many agents and signals, the information exchange terminates after finitely many periods (with probability 1) in the equilibria we consider. Therefore, the outcomes were the same with high probability if the agents chose the action at some time period $T < \infty$ large enough.

Agent i's payoff depends on how close her own action $\hat{a}_i$ and the actions $\hat{a}_j$ of the other agents $j \neq i$ are to her ideal action or bliss point $\theta + b_i$. We denote the *belief* of agent i about her ideal action at time $t \geq 0$ by $x_i(t)$; her equilibrium action hence is given by $a_i = \lim_{t\to\infty} x_i(t)$. The profile of beliefs is denoted $x(t) = (x_1(t), x_2(t), \dots, x_n(t))$. We assume that every aspect of the model except the state of the world and the agents' private signals is common knowledge.

The equilibrium concept we employ for farsighted (myopic) agents is perfect Bayesian Nash equilibrium (with myopic best response). Let $m = (m_1, m_2, \dots, m_n)$, with $m_i = (m_{ij,t})_{j\in N_i(g),t=1,2,\dots}$ denote a *communication strategy profile* for all agents. Notice that $m_{ij,t}$ denotes the strategy of agent i with respect to her neighbor j for the case that she is active at time instant t. We denote the strategy profile of all agents except agent i by $m_{-i} = (m_1, m_2, \dots, m_{i-1}, m_{i+1}, \dots, m_n)$. The strategy profile of agent i at all time instants except t is denoted $m_{i,-t} = (m_{ij,t'})_{j\in N_i(g),t'=1,2,\dots,t-1,t+1,\dots}$. We say that there is a *truthful communication path* from i to j in g (with respect to i's signal s_i) at time t under strategy profile m if there exist distinct agents $i = i_1, i_2, \dots, i_K = j$ and time instants $t_1 < t_2 < \dots < t_{K-1} < t$ such that $i_k = \iota(t_k)$, $i_k i_{k+1} \in g$ and $m_{i_k i_{k+1},t_k,i} = s_i$ for all $k = 1, 2, \dots, K-1$. Furthermore, let

$$I_{i,t}^m = \{s_i\} \cup \{s_j | \exists \text{ truthful communication path from } j \text{ to } i \text{ at time } t \text{ under } m\}$$

denote the *actual information set* of agent i at time t under strategy profile m, i.e., the set consisting of her own signal and the signals that were transmitted to her up to time t under strategy profile m. Notice that for convenience we denote the actual information set as a set (and not as a vector) and omit the dependence on the history of communication for notational clarity.

Let us denote by $\mathbb{E}_m$ the conditional expectation when the agents behave according to the strategy profile m. Under strategy profile m, the information set of agent i and the history of communication induce, by sequential rationality, the following belief about her ideal action $\theta + b_i$:

$$\begin{aligned} x_i(t) &= \operatorname*{argmax}_{x\in[0,1]} \Big(-\mathbb{E}_m \Big[(\theta + b_i - x)^2 + \sum_{j\neq i} \big(\theta + b_i - x_j(t)\big)^2 | I_{i,t}, H_t \Big] \Big) \\ &= \operatorname*{argmax}_{x\in[0,1]} \Big(-\mathbb{E} \Big[(\theta + b_i - x)^2 + \sum_{j\neq i} \big(\theta + b_i - x_j(t)\big)^2 | I_{i,t}^m \Big] \Big) \\ &= \operatorname*{argmax}_{x\in[0,1]} \big(-\mathbb{E}\big[(\theta + b_i - x)^2 | I_{i,t}^m \big] \big) \\ &= \mathbb{E}\big[\theta | I_{i,t}^m\big] + b_i. \end{aligned}$$

The standard Beta-binomial model yields

$$x_i(t) = \mathbb{E}\big[\theta | I_{i,t}^m\big] + b_i = \frac{\sum_{s\in I_{i,t}^m} s + 1}{|I_{i,t}^m| + 2} + b_i. \tag{1}$$

We denote this *information exchange game* by $\Gamma(n, g, b)$ and restrict our attention to pure strategy equilibria. Notice that if $s_k \in I^m_{i,t}$ at time instant t under strategy profile m, the communication strategy of the active agent i with each agent $j \in N_i(g)$ regarding k's signal can take, without loss of generality, one of two forms: she either reveals her information, $m_{ij,t,k} = s_k$, or she employs a babbling strategy, $m_{ij,t,k} \in \{0, 1\}$ independent of s_k. If $s_k \notin I^m_{i,t}$, then i's message is disregarded by j, as it is necessarily uninformative. Hence, we refer to $\widetilde{I}^m_{ij,t} = I^m_{i,t} \setminus I^m_{j,t}$ as the set of *potentially transmitted information* from i to j at time t under strategy profile m, and refer to $I^m_{ij,t} = \{s_k \in \widetilde{I}^m_{ij,t} | m_{ij,t,k} = s_k\}$ as the set of *transmitted information* from i to j at time instant t under strategy profile m (in case $i = \iota(t)$). Finally, we say that a strategy profile m is *fully informative* if $\lim_{t\to\infty} I^m_{i,t} = \{s_1, s_2, \dots, s_n\}$ with probability 1 for all $i \in N$.

3. Main results

In this section, we first present the equilibrium analysis in the baseline setup of myopic agents and private communication. We then extend this analysis to farsighted agents and also derive results for societies in which both myopic and farsighted agents are present. Finally, we analyze equilibrium welfare.

3.1 *Myopic agents*

Consider myopic agents who play myopic best response, that is, ignore that beliefs may change subsequently due to further information transmission. The equilibrium concept we employ is perfect Bayesian equilibrium with myopic best response.

Definition 1 (Myopic equilibrium). (i) A strategy profile m^* is a *myopic equilibrium* of $\Gamma(n, g, b)$ if for all $i \in N$ and all time instants t,

$$m^*_{i,t} \in \underset{m_{i,t}}{\operatorname{argmax}}\, \mathbb{E}_{(m_{i,t}, m^*_{i,-t}, m^*_{-i})}\big[u_i\big(x(t+1)|\theta\big)|I_{i,t}, H_t\big].$$

(ii) A myopic equilibrium m^* is *(myopically) optimal* if for all $i \in N$, time instants t, and $j \in N_i(g)$,

$$\big|I^{m^*}_{ij,t}\big| = \max_{m \in \mathcal{M}^*(m^*, t)} \big|I^m_{ij,t}\big|,$$

where $\mathcal{M}^*(m^*, t) = \{m \text{ myopic equilibrium} | I^m_{k,t} = I^{m^*}_{k,t} \text{ for all } k \in N\}$ and $|\cdot|$ denotes the cardinality of a set.

In a myopic equilibrium, each agent maximizes her expected instantaneous payoff (at the next time instant $t + 1$) whenever she is active, i.e., the payoff she would get if actions were taken immediately after she has sent her messages and agents have updated beliefs accordingly. Furthermore, the equilibrium is (myopically) optimal if at each time instant the active agent transmits as much information as possible in equilibrium. For brevity, we refer to these equilibria as *optimal myopic equilibria*. We study optimal myopic equilibria as they are optimal from an informational point of view, in a myopic sense: they maximize the transmitted information at each time instant, which makes them prominent equilibria for myopic agents.[16]

As information could potentially be transmitted through several paths in the network, a deviation by some agent may be detected in some instances. More precisely, a deviation is detected in this information exchange game if and only if some agent receives inconsistent information regarding some signal (possibly from the same neighbor) even though the respective communication was supposed to be truthful under the equilibrium m^*. We propose the following adaptation of the intuitive criterion by Cho and Kreps (1987) to restrict off-equilibrium beliefs to reasonable beliefs. For any strategy profile m, agent i, signal s_k, and time instant t, let

$$N_i(m, s_k, t) = \{(j, t') | j \in N_i(g), t' \leq t, j = \iota(t'), s_k \in I^m_{j,t'}, m_{ji,t',k} = s_k\}$$

be the collection of all pairs of neighbors and time instants (at time t) such that these neighbors are supposed to have truthfully communicated agent k's signal to i at the respective time instant under strategy profile m. Notice that we omit the dependence on the history of communication for notational clarity. Furthermore, let

$$\begin{aligned} N^*_i(m, s_k, t) = \{(j, t') \in N_i(m, s_k, t) | \exists \mu \in \Delta^*(N_i(m, s_k, t)) :& \\ \mathbb{E}_m[u_j(x(t'+1)|\theta)|\widetilde{I}_{j,t'}, H_{t'}] < \mathbb{E}_{\widetilde{m}}[u_j((\widetilde{x}_i(t'+1), x_{-i}(t'+1))|\theta)|\widetilde{I}_{j,t'}, H_{t'}]& \\ \text{for some realization } \widetilde{I}_{j,t'} \text{ consistent with } I^{\widetilde{m},\mu}_{i,t}\}& \end{aligned}$$

be the subset of pairs (j, t') such that agent j possibly (for some belief μ of i and realization $\widetilde{I}_{j,t'}$ of j's information set) had an incentive to deviate regarding s_k at time t', where $\widetilde{m} = (1 - m_{ji,t',k}, m_{-(ji,t',k)})$ is such that agent j has deviated in the communication with agent i at time t' regarding s_k, $\Delta^*(N_i(m, s_k, t))$ is the set of consistent beliefs over $N_i(m, s_k, t)$ regarding the origin of the deviation, $I^{\widetilde{m},\mu}_{i,t}$ is the actual information set of agent i at time t under $\widetilde{m}$ and μ, and $\widetilde{x}_i(t'+1) = \mathbb{E}[\theta | I^{\widetilde{m},\mu}_{i,t'+1}] + b_i$.[17]

Definition 2 (Intuitive criterion). Consider a myopic equilibrium m^*, any agent $i \in N$, signal s_k, and time instant t. If for any $(j, t') \in N^*_i(m^*, s_k, t)$,

$$\mathbb{E}_{m^*}[u_j(x(t'+1)|\theta)|\widetilde{I}_{j,t'}, H_{t'}] < \mathbb{E}_{\widetilde{m}^*}[u_j((\widetilde{x}_i(t'+1), x_{-i}(t'+1))|\theta)|\widetilde{I}_{j,t'}, H_{t'}]$$

for all $\mu \in \Delta^*(N^*_i(m^*, s_k, t))$ and some realization $\widetilde{I}_{j,t'}$ consistent with $I^{\widetilde{m}^*,\mu}_{i,t}$, then we say that m^* *violates the intuitive criterion*. If there is no such agent i, signal s_k, and time instant t, we say that m^* *survives the intuitive criterion*. We refer to myopic equilibria that survive the intuitive criterion as *intuitive myopic equilibria*.

[16]Compared to, for instance, utility-maximizing equilibria. Coordination on such equilibria would certainly require some farsighted reasoning. We discuss this in more detail in Section 3.4.

[17]Notice that beliefs in $\Delta^*(N_i(m, s_k, t))$ are not necessarily probability distributions. We require beliefs in $\Delta^*(N_i(m, s_k, t))$ to be *consistent* with $I_{i,t}$, i.e., agents who have sent the same messages regarding signal s_k are assigned the same deviation probability. Furthermore, two information sets are *consistent* if they do not contradict each other regarding the realization of some signal.

Definition 2 posits a criterion to restrict off-equilibrium beliefs and select only those equilibria that are still equilibria under this restriction. If an agent detects a deviation on some signal, i.e., one of her neighbors has sent an off-equilibrium message with respect to this signal to her, then the intuitive criterion requires her belief regarding the origin of the deviation to be restricted to those agents who possibly could have benefitted from the deviation. The equilibrium violates the criterion if the deviation still could have been profitable despite the restriction of the receiver's beliefs. In other words, intuitive myopic equilibria are such that off-equilibrium deviations are deterred by reasonable beliefs. Notice that existence is not an issue as there always exists an intuitive equilibrium in which no information is transmitted. In the following lemma, we characterize intuitive myopic equilibria.

Lemma 1. *A strategy profile m^* is an intuitive myopic equilibrium if and only if for all $ij \in g$ and all time instants t such that $I_{ij,t}^{m^*} \neq \varnothing$,*

$$\begin{aligned}|b_i - b_j| \leq \frac{1}{|I_{j,t}^{m^*}| + |\widetilde{I}_{ij,t}^{m^*}| + 2}\Bigg[1/2 - \frac{|\widetilde{I}_{ij,t}^{m^*} \backslash I_{ij,t}^{m^*}|}{|I_{j,t}^{m^*}| + |I_{ij,t}^{m^*}| + 2}\Bigg(|I_{i,t}^{m^*} \cap I_{j,t}^{m^*}| \\ + |I_{ij,t}^{m^*}| + 1/2 + |I_{j,t}^{m^*} \backslash I_{i,t}^{m^*}| \frac{|I_{i,t}^{m^*} \cap I_{j,t}^{m^*}| + |I_{ij,t}^{m^*}| + 1}{|I_{i,t}^{m^*}| + 2}\Bigg)\Bigg].\end{aligned} \tag{2}$$

First of all, Lemma 1 says that the only relevant messages are those that actually convey new information. In particular, the intuitive criterion implies that messages regarding signals that were already transmitted earlier do not matter: as the agent who initially transmits a signal to some agent does not have an incentive to deviate (otherwise it would not be a myopic equilibrium), a later deviation on this signal (that would be profitable for some off-equilibrium belief of the receiver) is attributed correctly and, hence, is not profitable.[18] Interestingly, this means that the intuitive criterion effectively eliminates the possibility of sowing doubt about information that was already transmitted earlier. To illustrate this, suppose that $ij \in g$, $I_{i,t}^{m^*} = I_{j,t}^{m^*} = \{s_i, s_j, s_k\}$ and that all communication is truthful whenever possible under equilibrium m^*. Consider the communication of agent $i = \iota(t)$ with agent j at time t. If it is profitable for some belief of agent j to deviate on signal s_k of agent k, $m'_{ij,t,k} = 1 - s_k$, then this deviation is attributed correctly to agent i by the intuitive criterion. Without the criterion, the receiver could also mistake the deviation for truthful communication or assign a certain probability to this possibility, which could make the deviation beneficial.

Second, whether player i can transmit some set of signals to player j depends on the conflict of interest between the two as well as on the information both agents hold at that point in time and the information that i intends to transmit to j. Consider first the—as we will see later on—most important case in which the sender i transmits all

[18] Essentially, the criterion exploits that myopic agents do not anticipate that an initial deviation becomes off equilibrium eventually and, instead, consider it as on equilibrium.

signals she could possibly transmit, i.e., $I^{m^*}_{ij,t} = \widetilde{I}^{m^*}_{ij,t}$. The right-hand side of (2) simplifies drastically in this case and we get

$$|b_i - b_j| \leq \frac{1}{2(|I^{m^*}_{j,t}| + |I^{m^*}_{ij,t}| + 2)}.$$

To understand the intuition behind this condition, notice that by sequential rationality and the properties of the Beta-binomial model (see (1)), the receiver's updated belief is

$$x_j(t+1) = \frac{\sum_{s \in I^{m^*}_{j,t+1}} s + 1}{|I^{m^*}_{j,t+1}| + 2} + b_j = \frac{\sum_{s \in I^{m^*}_{j,t}} s + \sum_{s \in I^{m^*}_{ij,t}} s + 1}{|I^{m^*}_{j,t}| + |I^{m^*}_{ij,t}| + 2} + b_j,$$

and, furthermore, that deviating on one signal is most profitable.[19] Hence, the most profitable deviation induces an expected absolute change in the receiver's belief of $1/(|I^{m^*}_{j,t}| + |I^{m^*}_{ij,t}| + 2)$, i.e., Lemma 1 says that the sender i can transmit all her (currently held) information to the receiver j in equilibrium if and only if the absolute difference of her bias and the receiver's bias is at most half the expected absolute change in the receiver's belief induced by the most profitable deviation. In particular, as the expected absolute change in the receiver's belief is decreasing in the number of signals the receiver already holds as well as in the number of signals the sender is transmitting, truthful communication becomes more difficult over time as agents accumulate information.

Next we turn to the case where the sender i transmits only a part of the signals she could possibly transmit, i.e., $\varnothing \neq I^{m^*}_{ij,t} \subsetneq \widetilde{I}^{m^*}_{ij,t}$. Observe that the right-hand side of (2) has a unique maximizer with respect to $I^{m^*}_{ij,t}$: $\widetilde{I}^{m^*}_{ij,t}$. That is, it is generally more difficult to transmit only a part of the information that could possibly be transmitted than to transmit all the information. In particular, this implies that the transmission of all information may be feasible in equilibrium, while the transmission of only a part of this information is not. Furthermore, whenever the transmission of all information is not feasible in equilibrium, then neither is the transmission of only a part of it.

The reason is that if the sender only transmits a part of her information, i.e., withholds at least one signal from the receiver, then the transmitted information may induce a change in the receiver's belief that is too large relative to the sender's belief (which takes into account the information withheld) and thus make a deviation profitable. To illustrate this, consider for instance $\widetilde{I}^{m^*}_{ij,t} = \{s, s', s''\}$ with realizations $s = 1$, $s' = 1$, and $s'' = 0$. Transmitting only the signals s and s' (instead of all three signals), $I^{m^*}_{ij,t} = \{s, s'\}$, increases the receiver's belief relative to the sender's belief, which makes a deviation more likely to be profitable if $b_j > b_i$, and the same holds for the complementary realization of the signals ($s = 0$, $s' = 0$, and $s'' = 1$) if $b_j < b_i$.

[19] A deviation on more than one signal either cancels out (e.g., if the sender deviates on two signals $s = 1$ and $s' = 0$), is equivalent to a deviation on one signal (e.g., if the sender deviates on three signals $s = 1$, $s' = 1$, and $s'' = 0$, which is equivalent to a deviation only on $s = 1$), or leads to an expected change in the receiver's belief that is too large to be profitable (e.g., if the sender deviates on two signals $s = 1$ and $s' = 1$).

Moreover, a careful inspection of (2) reveals that surprisingly partial information is generically ruled out in equilibrium: the right-hand side of (2) is at most equal to zero in case of partial information transmission. We summarize our findings in the following proposition.

Proposition 1. *A strategy profile m^* is an intuitive myopic equilibrium if and only if for all $ij \in g$ and all time instants t, either*

(i)

$$I_{ij,t}^{m^*} = \widetilde{I}_{ij,t}^{m^*} \quad \textit{and} \quad |b_i - b_j| \leq \frac{1}{2\left(|I_{j,t}^{m^*}| + |\widetilde{I}_{ij,t}^{m^*}| + 2\right)},$$

(ii) $|I_{ij,t}^{m^*}| = 1 < |\widetilde{I}_{ij,t}^{m^*}| = 2$, $|I_{i,t}^{m^*} \cap I_{j,t}^{m^*}| = 0$, *and* $b_i = b_j$, *or*

(iii) $I_{ij,t}^{m^*} = \varnothing$.

Next we turn to optimal myopic equilibria. We know from the above analysis that it is generally more difficult to transmit only a part of the information that could possibly be transmitted than to transmit all the information. Hence, in optimal intuitive myopic equilibria, either all information that could possibly be transmitted is transmitted or no information is transmitted. Combining these findings with Proposition 1 yields the following result.

Theorem 1. *A strategy profile m^* is an optimal intuitive myopic equilibrium if and only if for all $ij \in g$ and all time instants t,*

$$I_{ij,t}^{m^*} = \widetilde{I}_{ij,t}^{m^*} \quad \textit{if}\ |b_i - b_j| \leq \frac{1}{2\left(|I_{j,t}^{m^*}| + |\widetilde{I}_{ij,t}^{m^*}| + 2\right)} \quad \textit{and} \quad I_{ij,t}^{m^*} = \varnothing \quad \textit{otherwise.}$$

Theorem 1 says that the ability of the sender to communicate her information truthfully in an optimal intuitive myopic equilibrium depends—given the conflict of interest between her and the receiver—on the number of signals the receiver already holds and the number of signals the sender could potentially transmit (and actually does transmit). If interests are aligned, $b_i = b_j$, the sender transmits all her information, as the right-hand side of the inequality in Theorem 1 is strictly positive. However, whenever there is a conflict of interest, $b_i \neq b_j$, there exists a threshold that depends on the size of the conflict of interest,

$$\kappa(b_i, b_j) = \left\lfloor \frac{1}{2|b_i - b_j|} - 2 \right\rfloor,$$

such that the sender transmits all her information if $|I_{j,t}^{m^*}| + |\widetilde{I}_{ij,t}^{m^*}|$ does not exceed the threshold, and otherwise no information transmission takes place. This leads to an interesting finding: an agent may not be able to transmit any information to a particular neighbor if she received too much information before communicating with that neighbor for the first time. The following example illustrates this point.

Example 1. Consider $n = 4$ agents, the "line" network $g = \{12, 23, 34\}$, and biases $b_1 = 1/10$ and $b_2 = b_3 = b_4 = 0$. Notice that agents 2, 3, and 4 have aligned interests, while information transmission is only feasible in equilibrium between agents 1 and 2 if jointly both agents do not hold all signals as $\kappa(b_1, b_2) = 3$. Suppose that the agents play an optimal intuitive myopic equilibrium and consider the following two scenarios of active agents in the first three time periods:

(i) $H_4 = (1, 3, 2)$. Agent 1 transmits her signal to agent 2 in period 1. The next period, agent 3 transmits her signal to her neighbors, agent 2 and agent 4. In period 3, agent 2 transmits agent 3's signal as well as her own signal to agent 1, as they jointly only hold three (different) signals, and she also transmits agent 1's signal as well as her own signal to agent 3. The only agent who does not eventually get all information in this scenario is agent 1, who was excluded ex ante.

(ii) $H_4 = (4, 3, 2)$. In this scenario, agent 4 becomes active first instead of agent 1 and transmits her signal to agent 3, who one period later transmits this signal as well as her own signal to agent 2. Hence, when agent 2 becomes active for the first time in period 3, she already holds all signals except that of agent 1, which precludes the transmission of any information to the latter, and the same holds vice versa for agent 1 when she becomes active. As a consequence, no signal is transmitted to agent 1 and no other agent gets to know her signal in this scenario.

A comparison of the information the agents eventually hold in these scenarios reveals that each agent is better informed in scenario (i) than in scenario (ii). In fact, (i) is the best-case scenario from an informational point of view, while (ii) is the worst-case scenario (given the equilibrium).

Notice that we can use Theorem 1 to derive thresholds on the conflict of interest for the extreme cases where information transmission is not constrained and where it does not take place at all in optimal myopic equilibria. As the right-hand side of the inequality in Theorem 1 is strictly decreasing in $|I^{m^*}_{j,t}| + |\widetilde{I}^{m^*}_{ij,t}|$, information transmission is not constrained if the inequality holds even when the sender i knows all signals that the receiver j does not yet know, that is, $|\widetilde{I}^{m^*}_{ij,t}| = n - |I^{m^*}_{j,t}|$. Alternatively, information transmission does not take place at all if the inequality does not hold, even when both agents only know their private signal, i.e., $|I^{m^*}_{j,t}| = |\widetilde{I}^{m^*}_{ij,t}| = 1$.

Corollary 1. *In an optimal intuitive myopic equilibrium m^*, information transmission between agents i and $j \in N_i(g)$*

(i) is not constrained if $|b_i - b_j| \le 1/(2(n + 2))$ (any signal is transmitted) and

(ii) does not take place at all if $|b_i - b_j| > 1/8$ (no signal is transmitted).

We next proceed with the question of when a fully informative equilibrium exists. The first part of Corollary 1 states that the transmission of information between two agents is not constrained if their conflict does not exceed $1/(2(n+2))$. Hence, the transmission of a particular signal to some agent is guaranteed in optimal intuitive myopic equilibria if there exists a path from the agent whose private signal is concerned to the former agent along which this condition is fulfilled for any pair of subsequent agents. Thus, the transmission of all signals to all agents is guaranteed if there exists a path between any two agents along which the condition holds, that is, along a subnetwork that is a tree. In particular, all communication is truthful along the tree. Additionally, as otherwise at least one signal would not be transmitted to everyone, we get the following characterization of fully informative equilibria.

Theorem 2. *Suppose that m^* is an optimal intuitive myopic equilibrium. Then m^* is fully informative if and only if there exists a tree $\tilde{g} \subseteq g$ such that*

$$|b_i - b_j| \le \frac{1}{2(n+2)} \quad \textit{for all } ij \in \tilde{g}. \tag{3}$$

In particular, in this case $I^{m^*}_{ij,t} = \widetilde{I}^{m^*}_{ij,t}$ for all $ij \in \tilde{g}$ and all time instants t.

Notice that in the static game of GGS, there exists a fully informative equilibrium if and only if inequality (3) holds for all $i, j \in N$; see Section 6 for details. Hence, with myopic players, our framework puts weaker constraints on conflicts of interest compared to GGS for a fully informative equilibrium to exist.

3.2 *Farsighted agents*

Next we study farsighted agents who have perfect foresight and correctly anticipate subsequent changes in beliefs due to further information transmission in later time periods. The equilibrium concept we employ is perfect Bayesian equilibrium.

Definition 3 (Farsighted equilibrium). (i) A strategy profile m^* is a *farsighted equilibrium* of $\Gamma(n, g, b)$ if for all $i \in N$ and all time instants t,

$$\left(m^*_{i,l}\right)^{\infty}_{l=t} \in \operatorname*{argmax}_{(m_{i,l})^{\infty}_{l=t}} \mathbb{E}_{((m^*_{i,l})^{t-1}_{l=1}, (m_{i,l})^{\infty}_{l=t}, m^*_{-i})}\left[u_i(a|\theta)|I_{i,t}, H_t\right].$$

In a farsighted equilibrium, each agent maximizes her expected payoff from the actions the agents eventually choose whenever she is active. Hence, agents take into account the consequences the transmission of their information, both in the current period and in future periods, has on the actions of all agents. This implies that the agents' strategies are interdependent, which makes a characterization of equilibria a hopeless endeavor. Therefore, we concentrate on fully informative equilibria, where interdependent strategies are much less an issue, as all agents eventually hold all information. Notice that again existence is not an issue, as there always exists an equilibrium in which no information is transmitted. We provide a sufficient condition for the existence of a fully informative equilibrium.

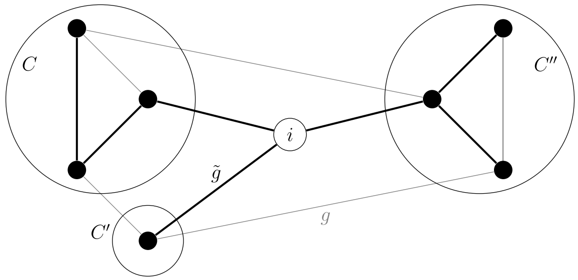

Figure 1. Example for the sufficient condition (4) in Theorem 3. We highlight a possible tree $\tilde{g} \subseteq g$ and the corresponding collection of subsets $\mathcal{C}(\tilde{g}_{-i}) = \{C, C', C''\}$ for some agent i.

Theorem 3. *Suppose that $\tilde{g} \subseteq g$ is a tree such that*

$$\left| b_i - \frac{1}{|C|} \sum_{l \in C} b_l \right| \leq \frac{1}{2(n+2)} \quad \textit{for all } i \in N \textit{ and } C \in \mathcal{C}(\tilde{g}_{-i}). \tag{4}$$

Then there exists a fully informative farsighted equilibrium m^* such that for all $ij \in g$ and all time instants t,

$$I_{ij,t}^{m^*} = \begin{cases} \widetilde{I}_{ij,t}^{m^*} & \textit{if } ij \in \tilde{g}, \\ \varnothing & \textit{if } ij \in g \backslash \tilde{g}. \end{cases}$$

Broadly speaking, Theorem 3 says that there exists a fully informative farsighted equilibrium if we can restrict information transmission to a tree along which all communication is truthful. More precisely, this requires that for each agent the absolute differences of her bias and the average bias of all agents reachable through each of her neighbors in the tree are below the familiar threshold $1/(2(n+2))$. Notably, the sender behaves with respect to each neighbor j in the tree as if j were the only agent in C, where $j \in C \in \mathcal{C}(\tilde{g}_{-i})$, and her bias were $b_C = |C|^{-1} \sum_{l \in C} b_l$. Notice also that there is no need to restrict off-equilibrium beliefs here, as we can assume a babbling strategy with respect to signals that were already transmitted earlier.

To illustrate Theorem 3, consider the network g and the tree $\tilde{g} \subseteq g$ depicted in Figure 1. In this example, agent i can communicate truthfully in equilibrium with all neighbors in the tree if (given all other agents communicate truthfully) the absolute difference of her bias and the average bias of the agents in each of the three sets C, C', and C'' is below $1/(2(n+2))$.

To understand the intuition behind this condition, notice first that we can concentrate on deviations regarding signals that the sender transmits in the current period, as the one-shot-deviation principle by Hendon et al. (1996) applies. Similar to myopic equilibria, the most profitable deviation is to deviate on only one signal. Furthermore, observe that, by sequential rationality and properties of the Beta-binomial model (see (1)), this deviation induces an expected absolute change in the action of each agent who eventually will be affected by this deviation of $1/(n+2)$. Hence, a sender i can transmit all her information to a neighbor $j \in N_i(g)$ in equilibrium if the absolute difference of her bias and the average bias of the agents who would eventually be affected by a deviation is at most half the expected absolute change in the actions of these agents induced by the most profitable deviation. Notice that, similar to myopic players, with farsighted players our framework puts weaker constraints on conflicts of interest compared to GGS for a fully informative equilibrium to exist.

One may ask whether condition (4) is also necessary or at least somehow close to necessity. As it turns out, the construction of a fully informative farsighted equilibrium that does not rely on information being transmitted along a tree is possible even if such an equilibrium does not exist when information transmission is restricted to a tree, but requires a well tailored setup of network and preferences. To illustrate this, we consider the following example.

Example 2. Consider $n = 6$ agents, the circle network $g = \{12, 23, 34, 45, 56, 61\}$, and biases $b_1 = 0$, $b_2 = 1/32 = b_6$, $b_3 = 1/16 = b_5$, and $b_4 = 1/10$. There does not exist a subnetwork that is a tree and fulfills (4): for any tree $\tilde{g} \subseteq g$, either agent 1 or agent 4 does not want to transmit all information, as their conflict of interest is too large. However, there nevertheless exists a fully informative equilibrium m^* that is such that the following statements hold:

(i) Each agent transmits signals s_1 and s_4 to all neighbors.

(ii) Signals s_2 and s_3 are transmitted along the subnetwork $\tilde{g} = g \backslash \{45\}$.

(iii) Signals s_5 and s_6 are transmitted along the subnetwork $\tilde{g} = g \backslash \{34\}$.

For simplicity, we assume that agents babble with respect to some signal whenever the receiver has already received this signal in an earlier time period.[20] Notice that while the signals of agents $i \neq 1, 4$ are still transmitted along subnetworks that are trees on which (4) is fulfilled, the trees now differ across signals. Furthermore, the signals of agents 1 and 4 are transmitted both ways through the network. Thereby, we achieve that a deviation by either agent 1 or agent 4 on her own signal affects either all other agents or half of the other agents in expectation (in case of a deviation with respect to only one of the neighbors), both of which are not profitable. We illustrate this in Figure 2. In fact, agents 1 and 4 face the same incentives when transmitting their own signal under m^* as if they were peripheral agents in a line network and all other agents communicated all signals truthfully.

Finally, we note that condition (4) also becomes necessary for the existence of a fully informative equilibrium as soon as g is a tree.

Corollary 2. *Suppose that g is a tree. Then there exists a fully informative farsighted equilibrium m^* if and only if*

$$\left| b_i - \frac{1}{|C|} \sum_{l \in C} b_l \right| \leq \frac{1}{2(n+2)} \quad \text{for all } i \in N \text{ and } C \in \mathcal{C}(g_{-i}).$$

[20] This buys us that all deviations are on-equilibrium-path deviations.

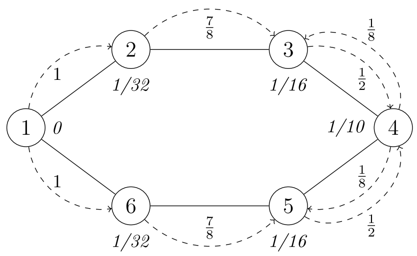

FIGURE 2. Transmission of agent 1's signal in the fully informative equilibrium in Example 2. Italic numbers indicate biases and numbers next to dashed arcs indicate ex ante probabilities of information transmission.

3.3 *Heterogeneous societies*

We now consider heterogeneous societies in which both myopic and farsighted agents are present. Let N_{myc} denote the set of myopic agents and let N_{far} denote the set of farsighted agents, with $N = N_{\text{myc}} \cup N_{\text{far}}$. While farsighted agents have perfect foresight, myopic agents play myopic best responses. We refer to the corresponding equilibrium concept as myopic–farsighted equilibrium.

DEFINITION 4 (Myopic–farsighted equilibrium). A strategy profile m^* is a *myopic–farsighted equilibrium* of $\Gamma(n, g, b)$ if for all time instants t,

(i) $$m^*_{i,t} \in \operatorname*{argmax}_{m_{i,t}} \mathbb{E}_{(m_{i,t}, m^*_{i,-t}, m^*_{-i})}\big[u_i\big(x(t+1)|\theta\big)|I_{i,t}, H_t\big] \quad \text{for } i \in N_{\text{myc}},$$

(ii) $$\big(m^*_{i,l}\big)^{\infty}_{l=t} \in \operatorname*{argmax}_{(m_{i,l})^{\infty}_{l=t}} \mathbb{E}_{((m^*_{i,l})^{t-1}_{l=1}, (m_{i,l})^{\infty}_{l=t}, m^*_{-i})}\big[u_i(a|\theta)|I_{i,t}, H_t\big] \quad \text{for } i \in N_{\text{far}}.$$

In a myopic–farsighted equilibrium, each myopic agent maximizes her expected payoff if actions were taken at the next time instant and each farsighted agent maximizes her expected payoff from the actions the agents eventually choose whenever they are active. Notice that we do not require that the type distribution is common knowledge. In fact, we allow for agents to not have any information on others' types.[21]

Our aim is to combine our findings for myopic and farsighted agents to derive a myopic–farsighted equilibrium. Therefore, we concentrate on fully informative equilibria as we did for farsighted agents. Moreover, we need to restrict information transmission to a tree to apply Theorem 3. Take any tree $\tilde{g} \subseteq g$. Then a farsighted agent $i \in N_{\text{far}}$ can reveal all information to her neighbors in $\tilde{g}$ if condition (4) holds, i.e., if

$$\left| b_i - \frac{1}{|C|}\sum_{l\in C} b_l \right| \leq \frac{1}{2(n+2)} \quad \text{for all } C \in \mathcal{C}(\tilde{g}_{-i}).$$

[21] Common knowledge of the type distribution would be crucial if we wanted to apply the intuitive criterion (in case a deviation is detected). However, as our analysis will concentrate on information transmission along trees, all deviations are on-equilibrium-path deviations.

By Corollary 1, a myopic agent $i \in N_{\text{myc}}$ can reveal all information to her neighbors in $\tilde{g}$ if

$$|b_i - b_j| \le \frac{1}{2(n+2)} \quad \text{for all } j \in N_i(\tilde{g}).$$

The following corollary summarizes our findings.

Corollary 3. *Suppose that there exists a tree $\tilde{g} \subseteq g$ such that*

(i)

$$|b_i - b_j| \le \frac{1}{2(n+2)} \quad \textit{for all } i \in N_{\text{myc}} \textit{ and } j \in N_i(\tilde{g}),$$

(ii)

$$\left| b_i - \frac{1}{|C|} \sum_{l \in C} b_l \right| \le \frac{1}{2(n+2)} \quad \textit{for all } i \in N_{\text{far}} \textit{ and } C \in \mathcal{C}(\tilde{g}_{-i}).$$

Then there exists a fully informative myopic–farsighted equilibrium m^* such that for all $ij \in g$ and all time instants t,

$$I_{ij,t}^{m^*} = \begin{cases} \widetilde{I}_{ij,t}^{m^*} & \textit{if } ij \in \tilde{g}, \\ \varnothing & \textit{if } ij \in g \backslash \tilde{g}. \end{cases}$$

In other words, Corollary 3 says that there exists a fully informative myopic–farsighted equilibrium if we can restrict information transmission to a tree along which all communication is truthful. The latter is feasible in equilibrium if for each myopic agent, the conflict of interest with each of her neighbors in the tree is below $1/(2(n+2))$ (condition (i)), and for each farsighted agent and each of her neighbors in the tree, the absolute difference of her bias and the average bias of that neighbor and agents reachable through the latter in the tree is also below $1/(2(n+2))$ (condition (ii)).

Similarly to the case of only farsighted agents, conditions (i) and (ii) in Corollary 3 also become necessary for the existence of an equilibrium as described as soon as g is a tree.

Corollary 4. *Suppose that g is a tree. Then there exists a fully informative myopic–farsighted equilibrium m^* such that for all $ij \in g$ and all time instants t, $I_{ij,t}^{m^*} = \widetilde{I}_{ij,t}^{m^*}$ if and only if*

(i)

$$|b_i - b_j| \le \frac{1}{2(n+2)} \quad \textit{for all } i \in N_{\text{myc}} \textit{ and } j \in N_i(g),$$

(ii)

$$\left| b_i - \frac{1}{|C|} \sum_{l \in C} b_l \right| \le \frac{1}{2(n+2)} \quad \textit{for all } i \in N_{\text{far}} \textit{ and } C \in \mathcal{C}(g_{-i}).$$

Notice, however, that there is one difference between the case of only farsighted agents (Corollary 3) and Corollary 4. Conditions (i) and (ii) in Corollary 4 are necessary for the existence of a fully informative equilibrium in which each agent immediately reveals all information to her neighbors, but not for the existence of a fully informative equilibrium in general. With heterogeneous types, farsighted agents may in some situations improve the outcome by delaying their information transmission until myopic agents have revealed all information. The following example illustrates this point.

Example 3. Consider $n = 4$ agents, the line network $g = \{12, 23, 34\}$, the type distribution $N_{\text{myc}} = \{2\}$ and $N_{\text{far}} = \{1, 3, 4\}$, and $b_1 = 1/20 = b_4$, $b_2 = 0$, and $b_3 = 1/10$. Then condition (i) in Corollary 4 is violated, as the conflict between agents 2 and 3 is too large. Hence, there is no fully informative equilibrium in which each agent immediately reveals all information to her neighbors. However, there does exist a fully informative equilibrium in which agent 4 does not transmit her signal to agent 3 until agent 2 has transmitted her own signal as well as agent 1's signal to agent 3. The myopic agent 2 is willing to do this if agent 3 does not yet hold the signal of agent 4, as $\kappa(b_2, b_3) = 3$, which agent 4 can guarantee by delaying the transmission of her signal to agent 3.

Notably, a fully informative equilibrium would no longer exist in Example 3 if agent 2 were slightly biased to the left, while such an equilibrium would exist even then if she were farsighted.[22] This shows that studying heterogeneous societies is not only for the sake of generalization, but provides testable implications. We can construct a network and preferences such that a certain agent could only transmit all her information to a particular neighbor if she were myopic (farsighted).

3.4 *Welfare*

Finally, we analyze equilibrium welfare. We provide a straightforward extension of the analysis in GGS.[23] We first derive a condition that allows us to rank equilibria in the Pareto sense depending on the expected number of signals the agents eventually hold. We can write player i's expected utility in equilibrium m^* as

$$\mathbb{E}_{m^*}\big[u_i(a|\theta)\big] = -\sum_{j\in N}\big[(b_j - b_i)^2 + \sigma_j^2(m^*)\big],$$

where $\sigma_j^2(m^*)$ denotes the residual variance of θ that player j expects to have after the information exchange has taken place. Next, standard properties of the Beta-binomial model yield

$$\sigma_j^2(m^*) = \frac{1}{6}\mathbb{E}\left[\frac{1}{\lim\limits_{t\to\infty}|I_{j,t}^{m^*}| + 2}\right].$$

[22]This holds for $b_2 = -\varepsilon$ and $0 < \varepsilon \le 1/120$.

[23]Our analysis differs in that the number of signals each player holds as $t \to \infty$ is not (necessarily) deterministic.

Hence, an equilibrium m^* yields a higher ex ante expected utility to player i than equilibrium m^{**} if and only if it does so to all players $j \in N$. Furthermore, the following result holds (GGS, Theorem 2).

Theorem 4. *Equilibrium m^* Pareto dominates equilibrium m^{**} if and only if*

$$\sum_{j\in N} \mathbb{E}\left[\frac{1}{\lim_{t\to\infty} |I_{j,t}^{m^*}| + 2}\right] < \sum_{j\in N} \mathbb{E}\left[\frac{1}{\lim_{t\to\infty} |I_{j,t}^{m^{**}}| + 2}\right].$$

In particular, Theorem 4 says that an equilibrium Pareto dominates another equilibrium if the expected number of signals the agents get to know during the information exchange is weakly larger for all agents and strictly larger for at least one agent in the former equilibrium. In other words, from an ex ante point of view, more information transmission is better for everyone. Hence, it follows that an equilibrium is Pareto efficient if and only if it is fully informative.

Corollary 5. *Equilibrium m^* is Pareto efficient if and only if m^* is fully informative.*

The next example shows that depending on the network and the agents' preferences, there may be a Pareto efficient myopic equilibrium while there is no such farsighted equilibrium and vice versa.

Example 4. Consider $n = 4$ agents and the line network $g = \{12, 23, 34\}$.

(i) If $b_i = i/15$ for all i, then any optimal intuitive myopic equilibrium is fully informative and, thus, Pareto efficient, while all farsighted equilibria are Pareto dominated.[24]

(ii) If $b_1 = 0 = b_4$ and $b_2 = 1/20 = -b_3$, then there exists a fully informative and, thus, Pareto efficient farsighted equilibrium, while all optimal intuitive myopic equilibria are Pareto dominated.[25]

Finally, we briefly investigate whether optimal myopic equilibria are optimal from a welfare point of view. It follows from the above analysis that we can call an equilibrium *utility-maximizing* if it maximizes the ex ante expected utility of each player. While one may think that farsighted players can coordinate on a utility-maximizing equilibrium, this assumption would seem odd for myopic players. Instead, optimal myopic equilibria could be considered as being prominent in case of myopic players. An interesting question is then whether these equilibria are nevertheless utility-maximizing, i.e., whether the transmission of as much information as possible at each time instant leads to optimal outcomes. It follows from Corollary 5 that this claim is true for fully informative

[24]To see this, notice that $|b_1 - 1/3 \cdot \sum_{j=2}^{4} b_j| = 2/15 > 1/12$, i.e., condition (4) is violated for agent 1, which is also necessary if g is a tree.

[25]To see this, notice that $|b_2 - b_3| = 1/10 > 1/12$, i.e., the condition in Theorem 2 is violated for agents 2 and 3.

equilibria. However, in general, the answer is no and the reason is simply that myopic agents do not take into account the consequences of transmitting information on future information transmission.[26]

Corollary 6. *Consider myopic agents. An optimal intuitive myopic equilibrium is utility-maximizing if it is fully informative, and otherwise it may not be utility-maximizing.*

4. Public communication

In this section, we analyze information transmission with public communication. Agents can no longer discriminate between different neighbors and have to send the same messages to all their neighbors.[27] Accordingly, the information set of agent i at time $t \geq 0$ is

$$I_{i,t} = \big(s_i, (\hat{m}_{i,t'})_{\substack{i=\iota(t') \\ t'<t}}, (\hat{m}_{k,t'})_{\substack{k=\iota(t') \\ ki\in g \\ t'<t}}\big).$$

A communication strategy for the active agent $i = \iota(t)$ regarding her neighborhood $N_i(g)$ at time instant t is a mapping

$$m_{i,t} : \mathcal{I}_{i,t} \times \mathcal{H}_t \to \{0,1\}^n.$$

Furthermore, the set of potentially transmitted information from i to her neighborhood $N_i(g)$ at time t under strategy profile m is denoted $\widetilde{I}^m_{iN_i(g),t} = I^m_{i,t} \backslash (\bigcap_{j\in N_i(g)} I^m_{j,t})$, and the set of transmitted information from i to her neighborhood $N_i(g)$ at time instant t under strategy profile m (in case $i = \iota(t)$) is denoted by $I^m_{iN_i(g),t} = \{s_k \in \widetilde{I}^m_{iN_i(g),t} | m_{i,t,k} = s_k\}$. Notice that if $s \in I^m_{i,t}$, then $s \in \widetilde{I}^m_{iN_i(g),t}$ if and only if $s \notin I^m_{j,t}$ for at least one neighbor $j \in N_i(g)$. Myopic and farsighted equilibria with public communication are defined as follows.

Definition 5 (Equilibria with public communication). (i) A strategy profile m^* is a *myopic equilibrium with public communication* of $\Gamma(n,g,b)$ if for all $i \in N$ and all time instants t,

$$m^*_{i,t} \in \underset{m_{i,t}}{\operatorname{argmax}}\, \mathbb{E}_{(m_{i,t}, m^*_{i,-t}, m^*_{-i})}\big[u_i\big(x(t+1)|\theta\big)|I_{i,t}, H_t\big].$$

(ii) A strategy profile m^* is a *farsighted equilibrium with public communication* of $\Gamma(n,g,b)$ if for all $i \in N$ and all time instants t,

$$\big(m^*_{i,l}\big)^{\infty}_{l=t} \in \underset{(m_{i,l})^{\infty}_{l=t}}{\operatorname{argmax}}\, \mathbb{E}_{((m^*_{i,l})^{t-1}_{l=1}, (m_{i,l})^{\infty}_{l=t}, m^*_{-i})}\big[u_i(a|\theta)|I_{i,t}, H_t\big].$$

[26]For instance, in Example 1 optimal intuitive myopic equilibria are Pareto dominated by an equilibrium in which agent 4 delays her information transmission until all other information transmission has taken place.

[27]More generally, we could also consider partitions of the neighborhoods and assume that agents need to send the same messages to all agents who are in the same partition element. The conditions we derive then need to hold for each partition element instead of for the whole neighborhood.

We first study myopic equilibria. Not surprisingly, whether an agent can communicate truthfully with public communication depends on the conflicts of interest between the sender and her neighbors. Additionally, however, it also depends on the extent to which each neighbor is affected by possible deviations. More precisely, if we take a subset $I_i^d \subseteq I_{iN_i(g),t}^m$ of the signals that the sender $i = \iota(t)$ transmits at time t under strategy profile m and consider a deviation on these signals, then the effect of this deviation on each neighbor $j \in N_i(g)$ certainly depends on the realization of the signals in $I_i^d \backslash I_{j,t}^m$, while it may be independent of the signals in $I_i^d \cap I_{j,t}^m$ (for some choice of off-equilibrium beliefs).[28] Therefore, determining the most profitable deviation becomes a complex issue with public communication, at least if the sender transmits more than one signal.

In the following lemma, we provide a sufficient condition for an equilibrium to exist in which agents either transmit all signals that could possibly be transmitted or none whenever they are active.

Lemma 2. *A strategy profile m^* is a myopic equilibrium with public communication if, for all time instants t and all $i \in N$, $I_{iN_i(g),t}^{m^*} = \widetilde{I}_{iN_i(g),t}^{m^*}$ if*

$$\sum_{j \in N_i(g)} \frac{2 \sum_{s \in I_i^d \backslash I_{j,t}^{m^*}} s - \left|I_i^d \backslash I_{j,t}^{m^*}\right|}{\left|I_{j,t}^{m^*}\right| + \left|\widetilde{I}_{ij,t}^{m^*}\right| + 2}(b_j - b_i) \leq \frac{1}{2} \sum_{j \in N_i(g)} \left(\frac{2 \sum_{s \in I_i^d \backslash I_{j,t}^{m^*}} s - \left|I_i^d \backslash I_{j,t}^{m^*}\right|}{\left|I_{j,t}^{m^*}\right| + \left|\widetilde{I}_{ij,t}^{m^*}\right| + 2}\right)^2$$

for all realizations of $\widetilde{I}_{iN_i(g),t}^{m^*}$ and all $\varnothing \neq I_i^d \subseteq I_{iN_i(g),t}^{m^*}$, and $I_{iN_i(g),t}^{m^*} = \varnothing$ otherwise.

Lemma 2 says that whether player i can transmit the set of signals $I_{iN_i(g),t}^{m^*}$ to her neighbors in equilibrium depends on the conflict of interest between i and each neighbor, on the information agent i and each neighbor hold at that point of time, and on the information that i transmits to her neighbors. The difference with respect to private communication is that, due to complex interdependencies in the expected changes of the receivers' beliefs, we are unable to derive an explicit threshold on the conflicts of interest.

Although this makes Lemma 2 a rather weak result, it nevertheless allows us to compare myopic equilibria with private communication and with public communication. First, when focussing on individual (local) communication, we find that it is generally easier to communicate in public than to communicate with all neighbors in private, in the following sense: whenever an agent can truthfully communicate with all her neighbors under private communication, then, ceteris paribus, she can also do so under public communication.

Theorem 5. *Consider an optimal intuitive myopic equilibrium with private communication, m^*, and a myopic equilibrium with public communication satisfying the conditions of Lemma 2, m^{**}. Furthermore, suppose that at time instant t, $I_{i,t}^{m^*} = I_{i,t}^{m^{**}}$ for all*

[28]The construction of the equilibrium in Lemma 2 below considers off-equilibrium beliefs in the spirit of the intuitive criterion: upon observing a deviation, agents attribute the deviation to the agent who communicated later in time.

$i \in N$. Then, for any $i \in N$,

$$I^{m^{**}}_{iN_i(g),t} = \widetilde{I}^{m^{**}}_{iN_i(g),t} \quad \textit{if } I^{m^*}_{ij,t} = \widetilde{I}^{m^*}_{ij,t} \textit{ for all } j \in N_i(g).$$

The intuition behind Theorem 5 is that public communication *disciplines* the communication with agents with whom the sender has particularly large conflicts of interest. In the worst case, private communication (with all neighbors) and public communication are equally difficult to sustain.[29] This finding is in line with the results obtained in static cheap talk frameworks, e.g., by Farrell and Gibbons (1989) and Goltsman and Pavlov (2011).

We also observe what Farrell and Gibbons (1989) refer to as subversion. When truthful communication is possible with some but not all neighbors in private, then public communication may *subvert* the relationship with the neighbors with whom truthful communication was possible in private and, therefore, make truthful communication impossible. Our contribution is that we look beyond local communication. Interestingly, we find that in our context, subversion may—unlike in static models—also prevent fully informative outcomes.[30] There does not necessarily exist a fully informative myopic equilibrium with public communication whenever this is the case with private communication. The reason is that with private communication, fully informative equilibria may exist despite some large conflicts between neighbors, as direct information transmission is not necessary if there exists a path in the network between them along which conflicts are small enough. With public communication, however, direct information transmission to such neighbors cannot be avoided, which may preclude fully informative myopic equilibria. The following example illustrates this point.

Example 5. Consider $n = 3$ agents, the complete network $g = g^N$, and biases $b_i = i/10$ for $i = 1, 2, 3$.

(i) Private communication. Information transmission is not constrained between agents 1 and 2 as well as between agents 2 and 3, and, hence, optimal intuitive myopic equilibria are fully informative.

(ii) Public communication. Suppose that agent 3 transmits her signal before agent 1 does so. Then agent 2 already holds two signals once agent 1 could transmit her signal, which implies that agent 1 is unable to do so. Hence, there is no fully informative myopic equilibrium with public communication.

Subversion prevents a fully informative equilibrium with public communication; see Figure 3 for an illustration.

Example 5 exploits the fact that the network g is cyclic and, thus, with private communication no direct information transmission between agents 1 and 3 is necessary in

[29] This is the case if the information that could possibly be transmitted to agents with different biases is disjoint.

[30] In static models, this only becomes possible if one allows the sender to make certifiable statements; see, e.g., Koessler (2008).

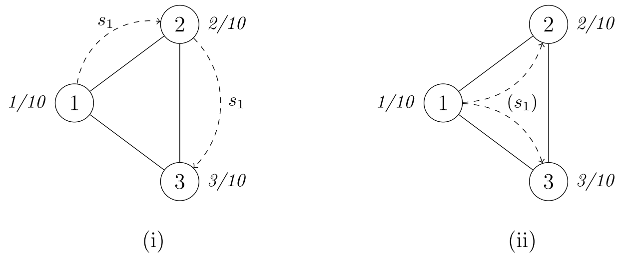

FIGURE 3. (Non-)transmission of agent 1's signal s_1 with private communication (left) and with public communication (right) in Example 5. Parentheses indicate that the signal is only transmitted with certain probability.

fully informative equilibria. In the special case where the network is a tree, this possibility does not exist; in particular, optimal intuitive myopic equilibria with private communication are fully informative if and only if all agents communicate all information truthfully to all their neighbors. In this case, Theorem 5 implies that there exists a fully informative myopic equilibrium with public communication.

COROLLARY 7. *Suppose that g is a tree, and consider an optimal intuitive myopic equilibrium with private communication, m^*, and a myopic equilibrium with public communication satisfying the conditions of Lemma 2, m^{**}. Then m^{**} is fully informative if m^* is fully informative.*

Next, we study farsighted equilibria. Due to the complexity of the problem, we restrict our analysis to trees and show that—similar to myopic equilibria and in line with the existing literature—the conditions for fully informative equilibria with public communication to exist (on trees) are weaker than those for private communication.[31] Corollary 2 tells us that on trees there exists a fully informative farsighted equilibrium with private communication if and only if for each agent and each of her neighbors the absolute difference of her bias and the average bias of that neighbor and agents reachable through the latter is below $1/(2(n+2))$. This implies that for each agent the absolute difference of her bias and the average bias of two or more neighbors and agents reachable through them is also below $1/(2(n+2))$. Hence, it follows that there exists a fully informative farsighted equilibrium with public communication.

PROPOSITION 2. *Suppose that g is a tree. Then there exists a fully informative farsighted equilibrium with public communication if there exists a fully informative farsighted equilibrium with private communication.*

The proof is analogous to that of Theorem 3 and is omitted. Finally, notice that Proposition 2 also holds for myopic-farsighted equilibria.[32]

[31] With public communication, we cannot avoid in general that information flows through different paths in the network, which implies uncertainty (that depends on the network structure and other players' strategies) about whether certain agents would be affected by a deviation.

[32] This is an immediate consequence of Theorem 5 and Proposition 2.

5. Design of institutions

In this section, we analyze the implications of our model for the design of institutions. How should a social planner organize the network to maximize equilibrium welfare? Following the argument in Section 3.4, we assume that myopic agents coordinate on an optimal intuitive myopic equilibrium and farsighted agents coordinate on a utility-maximizing farsighted equilibrium. Furthermore, we restrict our attention to private communication and assume that biases are ordered, $b_1 \leq b_2 \leq \cdots \leq b_n$.

Consider first myopic agents. Denote the optimal intuitive myopic equilibrium that the agents coordinate on in network g by $m^*(g)$. Then the social planner will choose the network g such that $m^*(g)$ maximizes welfare. It follows from Theorem 4 that this means choosing

$$g \in \operatorname*{argmin}_{g'} \sum_{j \in N} \mathbb{E}\left[\frac{1}{\lim\limits_{t\to\infty} |I_{j,t}^{m^*(g')}| + 2}\right]. \tag{5}$$

Theorem 1 tells us that any agent i transmits (weakly) more information to a neighbor $j \in N_i(g)$ in an optimal intuitive myopic equilibrium the smaller is the conflict of interest between them. Hence, one may conjecture that any network g such that $k(k+1) \in g$ for all $k = 1, 2, \ldots, n-1$ satisfies (5). As fully informative equilibria are Pareto efficient (Corollary 5) and, thus, the underlying network necessarily fulfills (5), Theorem 2 tells us that this is indeed true if $b_{k+1} - b_k \leq 1/(2(n+2))$ for all $k = 1, 2, \ldots, n-1$. However, if this condition is not fulfilled, optimal intuitive myopic equilibria are not fully informative any more and, therefore, may not be utility-maximizing (Corollary 6). Some agents may accumulate too many signals in the beginning so that they can only transmit less information to some neighbor than they could if they did not know that many signals; see Example 1 for an illustration of this point. Hence, as the social planner would like to minimize the risk of that happening, the best she can do in this case is to implement the line network with agents placed in increasing order of their biases. The following proposition summarizes our findings.

Proposition 3. *Consider myopic agents. Then*

$$g \in \operatorname*{argmin}_{g'} \sum_{j \in N} \mathbb{E}\left[\frac{1}{\lim\limits_{t\to\infty} |I_{j,t}^{m^*(g')}| + 2}\right]$$

if either

(i) $g = \{k(k+1) | k = 1, 2, \ldots, n-1\}$ *or*

(ii) $g \supseteq \{k(k+1) | k = 1, 2, \ldots, n-1\}$ *and* $b_{k+1} - b_k \leq 1/(2(n+2))$ *for all* $k = 1, 2, \ldots, n-1$.

Let us reconsider Example 1 to illustrate why additional links may lead to worse outcomes from a welfare point of view if conflicts are not sufficiently small.

Example 6. Consider $n = 4$ agents and biases $b_1 = 1/10$ and $b_2 = b_3 = b_4 = 0$. Recall that information transmission is only feasible in equilibrium between agents 1 and 2 if jointly both agents do not hold all signals as $\kappa(b_1, b_2) = 3$. Consider the following two networks:

(i) $g = \{12, 23, 34\}$. In the line network, no information transmission takes place between agents 1 and 2 if the history in the first periods is, e.g., $H_5 = (4, 3, 2, 1)$. Alternatively, if $H'_5 = (3, 4, 2, 1)$, then the outcome is as in a utility-maximizing equilibrium, as agents 1 and 2 jointly hold only three signals when each of them becomes active for the first time.

(ii) $g' = \{12, 23, 24, 34\}$. This is the line network extended by a link between agents 2 and 4. Hence, whereas agent 3 as well as agent 4 is connected to agent 2, both histories $H_5 = (4, 3, 2, 1)$ and $H'_5 = (3, 4, 2, 1)$ result in no information transmission taking place between agents 1 and 2, i.e., the outcome is worse than in the line network from a welfare point of view.

In general, for each history, the outcome is weakly better in g than in g'. Thus, in sum the social planner would strictly prefer g over g'.

Next, we consider farsighted agents. Denote the utility-maximizing farsighted equilibrium on which the agents coordinate in network g by $m^{**}(g)$. Then the social planner will choose a network g such that

$$g \in \operatorname*{argmin}_{g'} \sum_{j \in N} \mathbb{E}\left[\frac{1}{\lim_{t\to\infty} |I_{j,t}^{m^{**}(g')}| + 2}\right]. \tag{6}$$

Whereas farsighted agents coordinate on a utility-maximizing equilibrium, there cannot be too many links in the network as was the case with myopic agents. Hence, when agents are farsighted, a safe choice for the social planner is to implement the complete network. However, at least if links are costly, she may want to implement the network g with the least links that fulfills (6). From this point of view, the best case scenario is the line network, with the natural candidate being the one with agents placed in increasing order of their biases. While we cannot say much in general due to the lack of a characterization of farsighted equilibria, we nevertheless know from Corollary 5 that the existence of a fully informative equilibrium is sufficient for the underlying network (and supersets thereof as agents coordinate on a utility-maximizing equilibrium) to fulfill (6). Hence, applying Theorem 3 to the line network with agents placed in increasing order of their biases yields the following proposition.

Proposition 4. *Consider farsighted agents. Then*

$$g \in \operatorname*{argmin}_{g'} \sum_{j \in N} \mathbb{E}\left[\frac{1}{\lim_{t\to\infty} |I_{j,t}^{m^{**}(g')}| + 2}\right]$$

if either

(i) $g = g^N$ *or*

(ii) $g \supseteq \{k(k+1) | k = 1, 2, \ldots, n-1\}$,

$$b_k \geq \frac{1}{n-k} \sum_{l=k+1}^{n} b_l - \frac{1}{2(n+2)} \quad \textit{for all } k = 1, 2, \ldots, n-1,$$

$$b_k \leq \frac{1}{2(n+2)} + \frac{1}{k-1} \sum_{l=1}^{k-1} b_l \quad \textit{for all } k = 2, 3, \ldots, n.$$

Proposition 4 says that the social planner should implement any superset of the line network with agents placed in increasing order of their biases if, for each agent, the difference of her bias and the average bias of agents with a higher (lower) bias is below $1/(2(n+2))$. Otherwise, we can only say that more links are weakly better and so the complete network is a safe choice for the social planner.[33]

In sum, our analysis suggests that the social planner should implement the line network with agents placed in increasing order of their biases, both if agents are myopic and (at least for small conflicts) if they are farsighted. Notably, our results also imply that when conflicts are small enough such that the welfare-maximizing network yields a fully informative (and, hence, Pareto efficient) equilibrium if agents are farsighted, then the same holds if agents are myopic, but not vice versa.

6. Dynamic versus static communication

An interesting question is whether allowing for dynamic communication aids or impedes information transmission relative to the static game of GGS, where agents communicate simultaneously. Consider the complete network $g = g^N$ and private communication. In this case, players can potentially transmit their signal directly to all other players as in the static game. The difference that remains is that we allow for dynamic transmission of information through the network, but exclude the possibility that all players transmit their information simultaneously. On the one hand, dynamics may aid transmitting information to agents with rather large conflicts—through a chain of intermediaries along which conflicts are small. On the other hand, dynamics may also impede the transmission of information if such a chain of intermediaries does not exist. The reason is that truthful communication becomes more difficult once the sender has received other information that she does not want to transmit further.[34]

However, we establish in the following discussion that dynamics unambiguously aid information transmission when we consider fully informative (and Pareto efficient) equilibria. We first formally introduce the static game studied by GGS. Each agent $i \in N$ simultaneously communicates with each other agent $j \neq i$ by sending a private message $\hat{m}_{ij} \in \{0, 1\}$. The information set of agent i after communication has taken place is given

[33] It remains an open question whether a superset of the line network with agents placed in increasing order of their biases may actually be beneficial in this case.

[34] This holds at least for myopic agents and optimal myopic equilibria; see the discussion in Section 3.1 on partial information transmission for details.

by $I_i = (s_i, (\hat{m}_{ij})_{j \neq i}, (\hat{m}_{ki})_{k \neq i})$. A (pure) communication strategy for agent i regarding each agent $j \neq i$ is defined as a mapping $m_{ij} : \{0, 1\} \to \{0, 1\}$. Let $I_i^m = \{s_i\} \cup \{s_j | m_{ji} = s_j\}$ denote the actual information set of agent i under strategy profile m. After communication has taken place, each agent i has to choose an action $\hat{a}_i \in \mathbb{R}$. The payoff of agent i from the profile of actions $\hat{a} = (\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_n)$ is as in our framework, $u_i(\hat{a}|\theta) = -\sum_{j \in N}(\theta + b_i - \hat{a}_j)^2$. Hence, under strategy profile m, agent i's action is given by $a_i = \mathbb{E}[\theta | I_i^m] + b_i$. A strategy profile m is fully informative if $I_i^m = \{s_1, s_2, \ldots, s_n\}$ for all $i \in N$. Finally, a strategy profile m^{**} is an equilibrium of the static game if for all $i \in N$,

$$m_i^{**} \in \operatorname*{argmax}_{m_i} \mathbb{E}_{(m_i, m_{-i}^{**})}\big[u_i(a|\theta)|I_i\big].$$

Corollary 1 of GGS implies the following result on fully informative equilibria in the static game.

Proposition 5. *There exists a fully informative equilibrium m^{**} of the static game if and only if*

$$|b_i - b_j| \leq \frac{1}{2(n+2)} \quad \textit{for all } i, j \in N. \tag{7}$$

In the static game, fully informative equilibria require the conflict of interest between any two agents not to exceed $1/(2(n+2))$. We show that this condition implies that there also exists a fully informative equilibrium of the dynamic game, with myopic and with farsighted agents (and with both), but not vice versa.

Consider the dynamic game and any tree $\tilde{g} \subseteq g$. Clearly, condition (7) is stronger than condition (3) for myopic agents. Furthermore, consider farsighted agents and any $i \in N$ and $C \in \mathcal{C}(\tilde{g}_{-i})$. Condition (7) implies

$$\left| b_i - \frac{1}{|C|} \sum_{l \in C} b_l \right| \leq \frac{1}{|C|} \sum_{l \in C} |b_i - b_l| \leq \frac{1}{2(n+2)},$$

i.e., condition (7) is also stronger than condition (4) for farsighted agents. Altogether, this yields the following result.

Corollary 8. *Suppose that in the dynamic game, $g = g^N$ and each agent is either myopic or farsighted. Then there exists a fully informative equilibrium m^* of the dynamic game if, but not only if, there exists a fully informative equilibrium m^{**} of the static game.*

The following example illustrates Corollary 8 and shows that dynamics may significantly ease the constraints on conflicts of interest for fully informative equilibria to exist; in particular, if agents are myopic.

Example 7. Suppose that $g = g^N$ in the dynamic game.

(i) Consider a society with n even that consists of two groups N_1 and N_2 of equal size $n/2$ such that $N_1 \cup N_2 = N$. Assume that the groups are homogeneous with respect to preferences, while there is a conflict of interest between them. That is,

$b_i = 0$ for all $i \in N_1$ and $b_i = b > 0$ for all $i \in N_2$. Proposition 5 implies that in the static game, $b \leq 1/(2(n+2))$ is required for a fully informative equilibrium to exist. The same requirement applies in the dynamic game if agents are farsighted. However, the requirement is weaker if agents are myopic. By Proposition 1, we can construct a fully informative equilibrium if $b \leq 1/8$.[35] First, each agent exchanges her signal with one agent of the other group such that afterward each agent holds two signals. This is possible if the conflict between groups does not exceed $1/8$ since myopic agents do not anticipate further information transmission, and implies that each signal is held by some agent in each group. Second, all signals are transmitted to all agents within groups.

(ii) Consider equally distributed biases, $b_i = i \cdot \varepsilon$ for all $i \in N$ and $\varepsilon > 0$. By Proposition 5, a fully informative equilibrium exists in the static game if and only if $\varepsilon \leq 1/(2(n+2)(n-1))$. However, in the dynamic game with each agent being either myopic or farsighted, a fully informative equilibrium exists if $\varepsilon \leq 1/(n(n+2))$, which is roughly twice as large as the threshold in the static game. In this equilibrium, information is transmitted along the line with agents placed in increasing order of their biases, $\tilde{g} = \{k(k+1) | k = 1, 2, \ldots, n-1\}$. Thereby, agents do not need to directly exchange information with those agents with whom they have particularly large conflicts, which relaxes the constraint on the parameter ε.[36]

7. Discussion and conclusion

In this paper, we develop a tractable dynamic framework of belief formation under payoff externalities in a social network. We build on the model of cheap talk by GGS and extend it to a dynamic game on a network. Agents initially receive noisy signals about the state of the world and have different preferences about the actions to be implemented. They repeatedly communicate with their neighbors in the social network by sending cheap talk messages about their information. After the repeated information exchange has taken place, each agent takes an action and payoffs realize.

We characterize (optimal) myopic equilibria as well as fully informative—and Pareto efficient—optimal myopic equilibria and provide a sufficient condition for the existence of a fully informative farsighted equilibrium. Essentially, fully informative equilibria exist if the transmission of all information is possible along a subnetwork that is a tree. These results carry over to heterogeneous societies, which provide testable implications: some agent may only be able to transmit all her information if she is myopic (farsighted).

We also extend our framework to public communication and show that subversion may—unlike in static models—prevent fully informative equilibria if agents are

[35] Notice that the result still holds qualitatively if the groups are not equally large.

[36] The above threshold follows from condition (4) since agent 1 (or n) is the most constrained agent if she is farsighted. Notice also that if all agents are myopic, a fully informative equilibrium exists if $\varepsilon \leq 1/(2(n+2))$, which is $n-1$ times as large as the threshold in the static game.

myopic. Furthermore, we analyze the implications of our model for the design of institutions and find that the social planner should place agents in increasing order of their biases to create a line network, both if agents are myopic and (at least for small conflicts) if they are farsighted. We can interpret these findings in favor of homophily: sufficiently homophilic societies, that is, agents are at least connected to those agents with the most similar preferences, aggregate information efficiently. Finally, we compare our framework with the static game of GGS and show that dynamics ease the constraints on conflicts of interest for fully informative equilibria to exist.

A general comparison of myopic and farsighted equilibria reveals that farsighted agents will typically end up less informed than myopic agents, and this is particularly so if a social planner has designed the network optimally. Interestingly, together with Theorem 4 this implies that myopic agents tend to reach outcomes that Pareto dominate those of farsighted agents. One exception that we have shown is Example 4(ii), where there exists a fully informative and Pareto efficient farsighted equilibrium, but no such myopic equilibrium. The reason is that the agents are not placed in increasing order of their biases in this example, which a social planner could correct and thereby ensure the existence of a Pareto efficient equilibrium regardless of the agents' types. Notably, this finding is in contrast to the literature on network formation, where farsighted agents form the network that Pareto dominates all other networks (if such network exists), while myopic agents may fail to do so; see, e.g., Herings et al. (2009).

Our framework is closely related to Acemoglu et al. (2014), who also study information aggregation through communication on a network. While in their model preferences are aligned, final decisions can be taken at any time and payoffs from decisions are discounted so that agents prefer earlier decisions. We do not allow agents to take the action at any time to focus on the effects of payoff externalities. Doing so would lead to (weakly) less information transmission in our model, with the extent depending on the network architecture.

Furthermore, our model relies on the quadratic loss utility known from Crawford and Sobel (1982) in combination with the Beta-binomial model. We refer to GGS for a detailed discussion of these assumptions and only add one point that is relevant in our context of networks. We assume that all individual decisions are equally important for each agent, but one may also think that agents care more about decisions of those located close to them in the network. For instance, an agent may care more about decisions of her neighbors than about decisions of the neighbors of her neighbors, etc. This assumption would not change our results on myopic agents and would have an ambiguous effect on information transmission for farsighted agents that depends on whether conflicts increase with distance. In particular, farsighted equilibria would become more similar to myopic equilibria. Likewise, we refer to GGS for a discussion of an extension to mixed strategies. Essentially, they find that mixed strategies may sometimes (slightly) improve information transmission.

Finally, our model can be extended to analyze the optimal allocation of decision-making authority. In our model, each agent makes a decision, but in some settings one may want to delegate the decision-making authority to better informed agents. Dewan et al. (2015) investigate this issue in the context of the institutional design of governments based on the model by GGS. They provide a justification for centralized decision-making authority (e.g., by ministers or the prime minister) and cabinet meetings (public communication in our framework). We leave the analysis of this issue in our framework for further research.

Appendix

Proof of Lemma 1. Let m^* be a strategy profile and consider any time instant t, $i = \iota(t)$, and link $ij \in g$. To ease notation, we let $I_1 = I_{i,t}^{m^*} \cap I_{j,t}^{m^*}$ denote the set of common signals between i and j, let $I_2 = I_{j,t}^{m^*} \backslash I_{i,t}^{m^*}$ denote the set of signals of j that i does not know, let $I_3 = I_{ij,t}^{m^*}$ denote the set of signals that i transmits to j, and let $I_4 = \widetilde{I}_{ij,t}^{m^*} \backslash I_{ij,t}^{m^*}$ denote the set of signals that i could, but does not, transmit to j at time instant t under strategy profile m^*. Notice that these four sets are disjoint and, furthermore, $I_{j,t+1}^{m^*} = I_1 \cup I_2 \cup I_3$ and $I_{i,t}^{m^*} = I_1 \cup I_3 \cup I_4$. Let $k_l = |I_l|$ and $k = \sum_{l=1}^4 k_l$.

We only need to consider deviations regarding signals $s \in I_{i,t}^{m^*}$ that i communicates truthfully to j at time instant t under m^*, as messages on other signals are uninformative and, thus, are disregarded by j. Furthermore, the intuitive criterion (Definition 2) implies that among these signals we can disregard deviations regarding signals that j already received at some earlier time period $t' < t$, i.e., signals that are in $I_{j,t}^{m^*}$ as well. To see why, consider such a signal $s \in I_{i,t}^{m^*} \cap I_{j,t}^{m^*}$ and suppose that agent i has an incentive to deviate for some off-equilibrium belief μ of agent j about the origin of the deviation, i.e., $(i, t) \in N_j^*(m^*, s, t)$. As the agent who initially transmitted the signal to agent j (this could have been agent i as well) did not have an incentive to deviate by definition of the equilibrium (with myopic agents, a deviation on a signal that has not been transmitted before is on-equilibrium path from the point of view of the sender), j attributes a detected deviation on signal s to agent i and time t, i.e., $\mu(i, t) = 1$. Given this off-equilibrium belief μ, a deviation by agent i would not alter $x_j(t+1)$ and, thus, would not be profitable, i.e., m^* survives the intuitive criterion for any choice of strategy regarding signals $s \in I_{i,t}^{m^*} \cap I_{j,t}^{m^*}$.

Therefore, we only need to consider deviations regarding signals $s \in I_3$ and write $I_3 = \{s_{r_1}, s_{r_2}, \dots, s_{r_{k_3}}\}$. Consider, without loss of generality, that agent i misreports signals $s_{r_1}, s_{r_2}, \dots, s_{r_d}$, $1 \le d \le k_3$, and write

$$I_3^d = \{1 - s_{r_1}, 1 - s_{r_2}, \dots, 1 - s_{r_d}, s_{r_{d+1}}, s_{r_{d+2}}, \dots, s_{r_{k_3}}\}.$$

Agent i has no incentives to do this deviation if and only if

$$-\int_0^1 \sum_{I_2' \in \{0,1\}^{k_2}} \big(\big(\theta + b_i - \mathbb{E}[\theta | I_1, I_2', I_3] - b_j\big)^2 - \big(\theta + b_i - \mathbb{E}[\theta | I_1, I_2', I_3^d] - b_j\big)^2 \big) f\big(\theta, I_2' | I_1, I_3, I_4\big)\, d\theta \ge 0,$$

where we denote by $f(\cdot)$ the probability density function (pdf) and, with a slight abuse of notation, denote the sets I_2' as vectors. The identity $a^2 - b^2 = (a-b)(a+b)$ and dividing by 2 yields

$$-\int_0^1 \sum_{I_2' \in \{0,1\}^{k_2}} \left(\mathbb{E}\left[\theta | I_1, I_2', I_3\right] - \mathbb{E}\left[\theta | I_1, I_2', I_3^d\right]\right) \cdot \left(\frac{\mathbb{E}\left[\theta | I_1, I_2', I_3\right] + \mathbb{E}\left[\theta | I_1, I_2', I_3^d\right]}{2} + b_j - \theta - b_i\right) f\left(\theta, I_2' | I_1, I_3, I_4\right) d\theta \geq 0.$$

Now define $\Delta(I_2'; d) = \mathbb{E}[\theta | I_1, I_2', I_3] - \mathbb{E}[\theta | I_1, I_2', I_3^d]$. Since $f(\theta, I_2' | I_1, I_3, I_4) = f(\theta | I_1, I_2', I_3, I_4)\mathbb{P}(I_2' | I_1, I_3, I_4)$, it follows that

$$-\sum_{I_2' \in \{0,1\}^{k_2}} \int_0^1 \Delta\left(I_2'; d\right)\left(\frac{\mathbb{E}\left[\theta | I_1, I_2', I_3\right] + \mathbb{E}\left[\theta | I_1, I_2', I_3^d\right]}{2} + b_j - \theta - b_i\right) \cdot f\left(\theta | I_1, I_2', I_3, I_4\right)\mathbb{P}\left(I_2' | I_1, I_3, I_4\right) d\theta \geq 0.$$

Next, as

$$\int_0^1 \theta f\left(\theta | I_1, I_2', I_3, I_4\right) d\theta = \mathbb{E}\left[\theta | I_1, I_2', I_3, I_4\right]$$

and since $\mathbb{E}[\theta | \cdot]$ is independent of θ, we get

$$\begin{aligned} & -\sum_{I_2' \in \{0,1\}^{k_2}} \Delta\left(I_2'; d\right)\left(\frac{\mathbb{E}\left[\theta | I_1, I_2', I_3\right] + \mathbb{E}\left[\theta | I_1, I_2', I_3^d\right]}{2} + b_j - b_i \right. \\ & \quad \left. - \mathbb{E}\left[\theta | I_1, I_2', I_3, I_4\right]\right)\mathbb{P}\left(I_2' | I_1, I_3, I_4\right) \geq 0 \qquad (8) \\ \Leftrightarrow \quad & -\sum_{I_2' \in \{0,1\}^{k_2}} \Delta\left(I_2'; d\right)\left(\frac{\Lambda\left(I_2'; d\right)}{2} + b_j - b_i\right)\mathbb{P}\left(I_2' | I_1, I_3, I_4\right) \geq 0, \end{aligned}$$

where $\Lambda(I_2'; d) = \mathbb{E}[\theta | I_1, I_2', I_3] + \mathbb{E}[\theta | I_1, I_2', I_3^d] - 2\mathbb{E}[\theta | I_1, I_2', I_3, I_4]$. To ease notation, we let $\Sigma_1 = \sum_{s \in I_1} s$, $\Sigma_2' = \sum_{s \in I_2'} s$, $\Sigma_3^1 = \sum_{l=1}^{d} s_{r_l}$, $\Sigma_3^2 = \sum_{l=d+1}^{k_3} s_{r_l}$, and $\Sigma_4 = \sum_{s \in I_4} s$. First, as

$$\sum_{s \in I_3^d} s = \sum_{l=1}^{d}(1 - s_{r_l}) + \sum_{l=d+1}^{k_3} s_{r_l} = d - \Sigma_3^1 + \Sigma_3^2$$

and by (1), we get

$$\begin{aligned} \Delta\left(I_2'; d\right) &= \mathbb{E}\left[\theta | I_1, I_2', I_3\right] - \mathbb{E}\left[\theta | I_1, I_2', I_3^d\right] \\ &= \frac{\Sigma_1 + \Sigma_2' + \Sigma_3^1 + \Sigma_3^2 + 1}{k_1 + k_2 + k_3 + 2} - \frac{\Sigma_1 + \Sigma_2' + d - \Sigma_3^1 + \Sigma_3^2 + 1}{k_1 + k_2 + k_3 + 2} \qquad (9) \\ &= \frac{2\Sigma_3^1 - d}{k - k_4 + 2}. \end{aligned}$$

Second, it follows that

$$\begin{aligned}
\Lambda(I_2'; d) &= \mathbb{E}[\theta | I_1, I_2', I_3] + \mathbb{E}[\theta | I_1, I_2', I_3^d] - 2\mathbb{E}[\theta | I_1, I_2', I_3, I_4] \\
&= \frac{\Sigma_1 + \Sigma_2' + \Sigma_3^1 + \Sigma_3^2 + 1}{k - k_4 + 2} + \frac{\Sigma_1 + \Sigma_2' + d - \Sigma_3^1 + \Sigma_3^2 + 1}{k - k_4 + 2} \\
&\quad - 2\frac{\Sigma_1 + \Sigma_2' + \Sigma_3^1 + \Sigma_3^2 + \Sigma_4 + 1}{k + 2} \\
&= 2\left(\frac{k_4(\Sigma_1 + \Sigma_2' + \Sigma_3^2 + 1) + (k+2)d/2 - (k - k_4 + 2)(\Sigma_3^1 + \Sigma_4)}{(k - k_4 + 2)(k+2)}\right) \\
&= \frac{2}{k+2}\left(\frac{k_4}{k - k_4 + 2}(\Sigma_1 + \Sigma_2' + \Sigma_3^2 + 1 + d/2) + d/2 - \Sigma_3^1 - \Sigma_4\right).
\end{aligned} \tag{10}$$

Substituting (9) and (10) into (8) and using that (9) is independent of I_2' yields

$$\begin{aligned}
& -\frac{2\Sigma_3^1 - d}{k - k_4 + 2} \sum_{I_2' \in \{0,1\}^{k_2}} \Bigg[\frac{1}{k+2}\left(\frac{k_4}{k - k_4 + 2}(\Sigma_1 + \Sigma_2' + \Sigma_3^2 + 1 + d/2) + d/2\right. \\
& \left. - \Sigma_3^1 - \Sigma_4\right) + b_j - b_i\Bigg]\mathbb{P}(I_2' | I_1, I_3, I_4) \geq 0 \\
\Leftrightarrow \quad & -\frac{2\Sigma_3^1 - d}{k - k_4 + 2}\Bigg[\frac{1}{k+2}\left(\frac{k_4}{k - k_4 + 2}(\Sigma_1 + \Sigma_3^2 + 1 + d/2) + d/2 - \Sigma_3^1 - \Sigma_4\right) \\
& + b_j - b_i + \frac{k_4}{(k+2)(k - k_4 + 2)} \sum_{I_2' \in \{0,1\}^{k_2}} \Sigma_2' \mathbb{P}(I_2' | I_1, I_3, I_4)\Bigg] \geq 0.
\end{aligned}$$

As the signals are independent, we have

$$\sum_{I_2' \in \{0,1\}^{k_2}} \Sigma_2' \mathbb{P}(I_2' | I_1, I_3, I_4) = \mathbb{E}[\Sigma_2' | I_1, I_3, I_4] = k_2 \frac{\Sigma_1 + \Sigma_3^1 + \Sigma_3^2 + \Sigma_4 + 1}{k - k_2 + 2}.$$

Hence, it follows that

$$\begin{aligned}
& -\frac{2\Sigma_3^1 - d}{k - k_4 + 2}\Bigg[\frac{1}{k+2}\left(\frac{k_4}{k - k_4 + 2}(\Sigma_1 + \Sigma_3^2 + 1 + d/2) + d/2 - \Sigma_3^1 - \Sigma_4\right) \\
& + b_j - b_i + \frac{k_2 k_4}{(k+2)(k - k_4 + 2)} \frac{\Sigma_1 + \Sigma_3^1 + \Sigma_3^2 + \Sigma_4 + 1}{k - k_2 + 2}\Bigg] \geq 0 \\
\Leftrightarrow \quad & -\frac{2\Sigma_3^1 - d}{k - k_4 + 2}\Bigg[\frac{1}{k+2}\left(\frac{k_4}{k - k_4 + 2}\left(\Sigma_1 + \Sigma_3^2 + 1 + k_2 \frac{\Sigma_1 + \Sigma_3^1 + \Sigma_3^2 + \Sigma_4 + 1}{k - k_2 + 2}\right.\right. \\
& \left.\left. + d/2\right) + d/2 - \Sigma_3^1 - \Sigma_4\right) + b_j - b_i\Bigg] \geq 0 \\
\Leftrightarrow \quad & \Sigma_3^1 = d/2 \vee \Bigg[\Sigma_3^1 > d/2 \wedge -\frac{1}{k+2}\left(\frac{k_4}{k - k_4 + 2}\left(\Sigma_1 + \Sigma_3^2 + 1 + d/2\right.\right.
\end{aligned}$$

$$+ k_2 \frac{\Sigma_1 + \Sigma_3^1 + \Sigma_3^2 + \Sigma_4 + 1}{k - k_2 + 2}\biggr) + d/2 - \Sigma_3^1 - \Sigma_4\biggr) \geq b_j - b_i\biggr]$$

$$\vee \biggl[\Sigma_3^1 < d/2 \wedge \frac{1}{k+2}\biggl(\frac{k_4}{k-k_4+2}\biggl(\Sigma_1 + \Sigma_3^2 + 1 + d/2$$

$$+ k_2 \frac{\Sigma_1 + \Sigma_3^1 + \Sigma_3^2 + \Sigma_4 + 1}{k - k_2 + 2}\biggr) + d/2 - \Sigma_3^1 - \Sigma_4\biggr) \geq b_i - b_j\biggr].$$

Thus, the strategy profile m^* is an equilibrium if and only if the above statement holds for all realizations $\Sigma_1 \leq k_1$, $\Sigma_3^1 \leq d$, $\Sigma_3^2 \leq k_3 - d$, and $\Sigma_4 \leq k_4$, and all $1 \leq d \leq k_3$,[37] which is equivalent to

$$\biggl[\min_{\substack{1\leq d\leq k_3, \Sigma_1\leq k_1, d/2<\Sigma_3^1\leq d,\\ \Sigma_3^2\leq k_3-d, \Sigma_4\leq k_4}} \frac{-1}{k+2}\biggl(\frac{k_4}{k-k_4+2}\biggl(k_2\frac{\Sigma_1 + \Sigma_3^1 + \Sigma_3^2 + \Sigma_4 + 1}{k - k_2 + 2} + \Sigma_1$$

$$+ \Sigma_3^2 + 1 + d/2\biggr) + d/2 - \Sigma_3^1 - \Sigma_4\biggr) \geq b_j - b_i\biggr]$$

$$\wedge \biggl[\min_{\substack{1\leq d\leq k_3, \Sigma_1\leq k_1, \Sigma_3^1<d/2,\\ \Sigma_3^2\leq k_3-d, \Sigma_4\leq k_4}} \frac{1}{k+2}\biggl(\frac{k_4}{k-k_4+2}\biggl(\Sigma_1 + \Sigma_3^2 + 1 + d/2$$

$$+ k_2 \frac{\Sigma_1 + \Sigma_3^1 + \Sigma_3^2 + \Sigma_4 + 1}{k - k_2 + 2}\biggr) + d/2 - \Sigma_3^1 - \Sigma_4\biggr) \geq b_i - b_j\biggr]$$

$$\begin{aligned}
\Leftrightarrow \quad & \biggl[\min_{1\leq d\leq k_3} -\frac{1}{k+2}\biggl(\frac{k_4}{k-k_4+2}\biggl(k_1 + k_3 - d/2 + 1 \\
& + k_2 \frac{k_1 + \lfloor d/2 \rfloor + k_3 - d + 2}{k - k_2 + 2}\biggr) \\
& + d/2 - \lfloor d/2 \rfloor - 1\biggr) \geq b_j - b_i\biggr] \wedge \biggl[\min_{1\leq d\leq k_3} \frac{1}{k+2}\biggl(\frac{k_4}{k-k_4+2}\biggl(1 + d/2 \\
& + k_2 \frac{\lceil d/2 \rceil + k_4}{k - k_2 + 2}\biggr) + d/2 - \lceil d/2 \rceil + 1 - k_4\biggr) \geq b_i - b_j\biggr] \\
\Leftrightarrow \quad & \biggl[-\frac{1}{k+2}\biggl(\frac{k_4}{k-k_4+2}\biggl(k_1 + k_3 + 1/2 + k_2 \frac{k_1 + k_3 + 1}{k - k_2 + 2}\biggr) - 1/2\biggr) \\
& \geq b_j - b_i\biggr] \\
& \wedge \biggl[\frac{1}{k+2}\biggl(\frac{k_4}{k-k_4+2}\biggl(3/2 + k_2 \frac{1 + k_4}{k - k_2 + 2}\biggr) + 1/2 - k_4\biggr) \geq b_i - b_j\biggr] \\
\Leftrightarrow \quad & \frac{1}{k+2}\biggl(1/2 - \frac{k_4}{k-k_4+2}\biggl(k_1 + k_3 + 1/2 + k_2 \frac{k_1 + k_3 + 1}{k - k_2 + 2}\biggr)\biggr) \geq |b_j - b_i|,
\end{aligned} \tag{11}$$

which finishes the proof. □

[37] To ease the exposition, we omit the restriction that all parameters need to be nonnegative.

Proof of Proposition 1. Let m^* be a strategy profile and consider any time instant t, $i=\iota(t)$, and link $ij \in g$. Also recall the notation introduced in the proof of Lemma 1. Consider $k_3 \geq 1$ (otherwise (iii) is satisfied and there is nothing to show). If (i) is satisfied, i.e., $k_4 = 0$, then by (11), agent i has no incentives to deviate if and only if

$$\frac{1}{2(k_1+k_2+k_3+2)} \geq |b_j - b_i|.$$

Second, suppose that (ii) is satisfied, i.e., $k_1 = 0$, $k_3 = 1$, $k_4 = 1$, and $b_i = b_j$. Then the left-hand side of (11) vanishes and, thus, agent i has no incentives to deviate as $b_i = b_j$. Hence, it is left to show that no other case is possible if $k_3 \geq 1$. It is sufficient to show that if (11) holds, $k_3 \geq 1$ but not ($k_1 = 0$, $k_3 = 1$, $k_4 = 1$, $b_i = b_j$), then $k_4 = 0$. We proceed by case distinction:

Case 1. Suppose $k_3 \geq 1$ and $b_i \neq b_j$. We show that (11) does not hold if $k_4 \geq 1$, thereby implying that $k_4 = 0$. Notice that it is sufficient to show that

$$\frac{1}{k+2}\left(1/2 - \frac{k_4}{k-k_4+2}\left(k_1+k_3+1/2+k_2\frac{k_1+k_3+1}{k-k_2+2}\right)\right) \leq 0$$

for all $k_1, k_2 \geq 0$, $k_1+k_2 \geq 1$,[38] $k_3 \geq 1$, and $k_4 \geq 1$, which is equivalent to

$$\min_{\substack{k_1,k_2\geq 0, k_1+k_2\geq 1,\\ k_3\geq 1, k_4\geq 1}} \frac{k_4}{k_1+k_2+k_3+2}\left(k_1+k_3+1/2+k_2\frac{k_1+k_3+1}{k_1+k_3+k_4+2}\right) \geq 1/2.$$

We get

$$\begin{aligned}
&\min_{\substack{k_1,k_2\geq 0, k_1+k_2\geq 1,\\ k_3\geq 1, k_4\geq 1}} \frac{k_4}{k_1+k_2+k_3+2}\left(k_1+k_3+1/2+k_2\frac{k_1+k_3+1}{k_1+k_3+k_4+2}\right)\\
&= \min_{\substack{k_1,k_2\geq 0,\\ k_1+k_2\geq 1, k_3\geq 1}} \frac{1}{k_1+k_2+k_3+2}\left(k_1+k_3+1/2+k_2\frac{k_1+k_3+1}{k_1+k_3+3}\right)\\
&= \min_{k_1\geq 0, k_2\geq 1, k_3\geq 1} \frac{1}{k_1+k_2+k_3+2}\left(k_1+k_3+1/2+k_2\frac{k_1+k_3+1}{k_1+k_3+3}\right)\\
&= 1/2,
\end{aligned}$$

which establishes the claim.

Case 2. Suppose $k_3 \geq 1$ and not $(k_1 = 0, k_3 = 1, k_4 = 1)$. We show that (11) does not hold if $k_4 \geq 1$, thereby implying that $k_4 = 0$. Notice that it is sufficient to show that

$$\frac{1}{k+2}\left(1/2 - \frac{k_4}{k-k_4+2}\left(k_1+k_3+1/2+k_2\frac{k_1+k_3+1}{k-k_2+2}\right)\right) < 0$$

[38] We cannot have $k_1 = k_2 = 0$, as agent j holds at least her own private signal, which she may ($k_1 \geq 1$) or may not ($k_2 \geq 1$) have transmitted to agent i before time t.

for all $k_1, k_2 \geq 0$, $k_1 + k_2 \geq 1$, $k_3 \geq 1$, and $k_4 \geq 1$ such that not $(k_1 = 0, k_3 = 1, k_4 = 1)$. Applying the same arguments as in the first case reveals that the left-hand side attains its maximum 0 only if $(k_1 = 0, k_3 = 1, k_4 = 1)$, which is ruled out and, therefore, finishes the proof. □

Proof of Theorem 2. Suppose that there exists a tree $\tilde{g} \subseteq g$ such that

$$|b_i - b_j| \leq \frac{1}{2(n+2)} \quad \text{for all } ij \in \tilde{g}. \tag{12}$$

We show that m^* is fully informative. Fix any distinct $i, j \in N$ and consider the path $i = i_1, i_2, \ldots, i_k = j$ such that $i_l i_{l+1} \in \tilde{g}$ for all $l = 1, 2, \ldots, k-1$. Next, define the stopping time

$$\tau_{ij} = \min\bigl\{t | \exists t_1 < t_2 < \cdots < t_{k-1} = t : \iota(t_l) = i_l \ \forall l = 1, 2, \ldots, k-1\bigr\}.$$

By Corollary 1, $I_{ij,t}^{m^*} = \widetilde{I}_{ij,t}^{m^*}$ for all $ij \in \tilde{g}$ and all time instants t. Hence, $s_i \in I_{j,t}^{m^*}$ for $t > \tau_{ij}$. Moreover, notice that τ_{ij} is almost surely finite. Define $\tau = \max_{j \in N} \max_{i \neq j} \tau_{ij}$ and notice that $I_{j,t}^{m^*} = \{s_1, s_2, \ldots, s_n\}$ for $t > \tau$ and all $j \in N$. Furthermore, τ is also almost surely finite as a maximum over finitely many almost surely finite stopping times, which finishes this part.

Second, suppose that m^* is fully informative. We show that there exists a tree $\tilde{g} \subseteq g$ such that (12) holds. Suppose to the contrary that there exist distinct $i, j \in N$ such that there does not exist a path $i = i_1, i_2, \ldots, i_k = j$ such that $i_l i_{l+1} \in g$ and

$$|b_{i_l} - b_{i_{l+1}}| \leq \frac{1}{2(n+2)} \quad \text{for all } l = 1, 2, \ldots, k-1.$$

Hence, on any path $i = i_1, i_2, \ldots, i_k = j$ from i to j in g,

$$|b_{i_l} - b_{i_{l+1}}| > \frac{1}{2(n+2)} \quad \text{for some } l \in \{1, 2, \ldots, k-1\}. \tag{13}$$

Next, consider the event that no more information transmission would take place under m^* by agents other than agent i at time instant t such that $i = \iota(t)$ for the first time, which occurs with strictly positive probability. We can assume, without loss of generality, that $I_{i,t}^{m^*} = \{s_1, s_2, \ldots, s_n\}$ and $I_{k,t}^{m^*} = \{s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n\}$ for $k \neq i$. As m^* is a fully informative optimal myopic equilibrium, there exists a path $i = i_1, i_2, \ldots, i_k = j$ such that $i_l i_{l+1} \in g$ and

$$I_{i_l i_{l+1}, t_l}^{m^*} = \{s_i\} \quad \text{for all } l = 1, 2, \ldots, k-1,$$

where $t = t_1 < t_2 < \cdots < t_{k-1}$ are such that $t_l = \min\{t > t_{l-1} | \iota(t) = i_l\}$ for $l = 2, 3, \ldots, k-1$. By Theorem 1, this implies that

$$|b_{i_l} - b_{i_{l+1}}| \leq \frac{1}{2\bigl(|I_{i_{l+1}, t_l}^{m^*}| + |\widetilde{I}_{i_l i_{l+1}, t_l}^{m^*}| + 2\bigr)} = \frac{1}{2(n+2)} \quad \text{for all } l = 1, 2, \ldots, k-1,$$

which is a contradiction to (13) and thus finishes the proof. □

Proof of Theorem 3. Suppose that $\tilde{g} \subseteq g$ is a tree such that

$$\left| b_i - \frac{1}{|C|} \sum_{l \in C} b_l \right| \leq \frac{1}{2(n+2)} \quad \text{for all } i \in N \text{ and } C \in \mathcal{C}(\tilde{g}_{-i})$$

and let the strategy profile m^* be such that

$$I_{ij,t}^{m^*} = \begin{cases} \widetilde{I}_{ij,t}^{m^*} & \text{if } ij \in \tilde{g}, \\ \varnothing & \text{if } ij \in g \backslash \tilde{g} \end{cases} \quad \text{for all } t.$$

Without loss of generality, we restrict our attention to signals that the receiver did not yet receive and assume a babbling strategy regarding all other signals.[39] First, notice that, analogous to the first part of the proof of Theorem 2, m^* is fully informative. Hence, it is left to show that m^* is a farsighted equilibrium.

Take any time instant t, $i = \iota(t)$, and link $ij \in g$. We only need to consider deviations regarding signals $s \in \widetilde{I}_{ij,t}^{m^*}$ that i transmits to j at time instant t under m^*, i.e., signals $s \in I_{ij,t}^{m^*}$, as messages on other signals are uninformative and, thus, are disregarded by j. Hence, consider any link $ij \in \tilde{g}$. We can treat each of these links separately, as $\tilde{g}$ is a tree and, thus, the strategies with respect to different neighbors of agent i are independent. In particular, a deviation on a signal $s \in I_{ij,t}^{m^*}$ eventually affects the beliefs of all agents $l \in C$, where C is such that $j \in C \in \mathcal{C}(\tilde{g}_{-i})$. To ease notation, we let $I_1 = I_{i,t}^{m^*} \cap I_{j,t}^{m^*}$ denote the set of common signals between agents i and j, let $I_2 = \{s_l : l \in C\} \backslash I_{i,t}^{m^*}$ denote the set of signals of agents in C that i does not know, let $I_3 = \{s_l : l \notin C\} \backslash I_{i,t}^{m^*}$ denote the set of signals of agents not in C that i does not know (and, hence, i will eventually transmit them to C under m^*), and let $I_4 = I_{ij,t}^{m^*}$ denote the set of signals i transmits to j at the current communication round (and, hence, indirectly to the other agents in C as well). Notice that these four sets are disjoint and, furthermore, $I_{i,t}^{m^*} = I_1 \cup I_4$ and $\lim_{t \to \infty} I_{l,t}^{m^*} = \bigcup_{p=1}^{4} I_p = \{s_1, s_2, \ldots, s_n\}$ for all $l \in C$. Let $k_p = |I_p|$ such that $n = \sum_{p=1}^{4} k_p$.

As communication is restricted to a tree under m^*, all deviations are on-equilibrium-path deviations. Furthermore, agent i transmits information at most $n - |C|$ different time instants to agent j, and the last time instant is t such that $i = \iota(t)$ and $k_4 \geq 1$ as well as $k_3 = 0$. Hence, we can apply the one-shot-deviation principle of Hendon et al. (1996). In particular, we need to check local deviations at one communication round only, but take into account the possibility that agent i herself deviated in earlier communication rounds. Therefore, denote $I_4 = \{s_{r_1}, s_{r_2}, \ldots, s_{r_{k_4}}\}$ and $I_1 = \{s_{r'_1}, s_{r'_2}, \ldots, s_{r'_{k_1}}\}$, and consider, without loss of generality, that agent i misreports signals $s_{r_1}, s_{r_2}, \ldots, s_{r_d}$ and has misreported signals $s_{r'_1}, s_{r'_2}, \ldots, s_{r'_{d'}}$ in earlier communication rounds, where $d \geq 1$, $d' \geq 0$, and $d + d' \leq n - |C|$. We write

$$I_4^d = \{1 - s_{r'_1}, 1 - s_{r'_2}, \ldots, 1 - s_{r'_d}, s_{r'_{d+1}}, s_{r'_{d+2}}, \ldots, s_{r'_{k_4}}\},$$

$$I_1^{d'} = \{1 - s_{r_1}, 1 - s_{r_2}, \ldots, 1 - s_{r_{d'}}, s_{r_{d'+1}}, s_{r_{d'+2}}, \ldots, s_{r_{k_1}}\}.$$

[39] Alternatively, one could also assume that all communication is truthful and take off-equilibrium beliefs similar to intuitive myopic equilibria.

Agent i has no incentives to do this deviation if and only if

$$-\int_0^1 \sum_{l\in C} \sum_{I_2'\in\{0,1\}^{k_2}, I_3'\in\{0,1\}^{k_3}} \Big(\big(\theta+b_i-\mathbb{E}\big[\theta|I_1,I_2',I_3',I_4\big]-b_l\big)^2 \\ -\big(\theta+b_i-\mathbb{E}\big[\theta|I_1^{d'},I_2',I_3',I_4^d\big]-b_l\big)^2\Big) f\big(\theta,I_2',I_3'|I_1,I_4\big)\,d\theta\geq 0,$$

where we denote by $f(\cdot)$ the pdf and, with a slight abuse of notation, the sets I_2' and I_3' as vectors. Similar manipulations as in the proof of Lemma 1 yield

$$-\sum_{l\in C} \sum_{I_2'\in\{0,1\}^{k_2}, I_3'\in\{0,1\}^{k_3}} \Delta\big(I_2',I_3';d,d'\big)\bigg(-\frac{\Delta\big(I_2',I_3';d,d'\big)}{2}+b_l-b_i\bigg) \\ \cdot\mathbb{P}\big(I_2',I_3'|I_1,I_4\big)\geq 0, \tag{14}$$

where $\Delta(I_2',I_3';d,d')=\mathbb{E}[\theta|I_1,I_2',I_3',I_4]-\mathbb{E}[\theta|I_1^{d'},I_2',I_3',I_4^d]$. To ease notation, we let $\Sigma_1^1=\sum_{l=1}^{d'} s_{r_l'}$, $\Sigma_1^2=\sum_{l=d'+1}^{k_1} s_{r_l'}$, $\Sigma_2'=\sum_{s\in I_2'} s$, $\Sigma_3'=\sum_{s\in I_3'} s$, $\Sigma_4^1=\sum_{l=1}^{d} s_{r_l}$, and $\Sigma_4^2=\sum_{l=d+1}^{k_4} s_{r_l}$. Notice that

$$\sum_{s\in I_1^{d'}} s=\sum_{l=1}^{d'}(1-s_{r_l'})+\sum_{l=d'+1}^{k_1} s_{r_l'}=d'-\Sigma_1^1+\Sigma_1,$$

$$\sum_{s\in I_4^{d}} s=\sum_{l=1}^{d}(1-s_{r_l})+\sum_{l=d+1}^{k_4} s_{r_l}=d-\Sigma_4^1+\Sigma_4^2.$$

Hence, (1) yields

$$\begin{aligned}\Delta\big(I_2',I_3';d,d'\big)&=\mathbb{E}\big[\theta|I_1,I_2',I_3',I_4\big]-\mathbb{E}\big[\theta|I_1^{d'},I_2',I_3',I_4^d\big]\\ &=\frac{\Sigma_1^1+\Sigma_1^2+\Sigma_2'+\Sigma_3'+\Sigma_4^1+\Sigma_4^2+1}{n+2}\\ &\quad-\frac{d'-\Sigma_1^1+\Sigma_1^2+\Sigma_2'+\Sigma_3'+d-\Sigma_4^1+\Sigma_4^2+1}{n+2}\\ &=\frac{2\Sigma_1^1+2\Sigma_4^1-d'-d}{n+2}.\end{aligned} \tag{15}$$

Substituting (15) into (14) and using that (15) is independent of I_2' and I_3' yields

$$-\sum_{l\in C}\frac{2\Sigma_1^1+2\Sigma_4^1-d'-d}{n+2}\bigg(-\frac{2\Sigma_1^1+2\Sigma_4^1-d'-d}{2(n+2)}+b_l-b_i\bigg)\geq 0$$

$$\Leftrightarrow\quad -\frac{2\Sigma_1^1+2\Sigma_4^1-d'-d}{n+2}\bigg(-|C|\frac{2\Sigma_1^1+2\Sigma_4^1-d'-d}{2(n+2)}+\sum_{l\in C} b_l-|C|b_i\bigg)\geq 0$$

$$\Leftrightarrow \quad \Sigma_1^1+\Sigma_4^1=\frac{d'+d}{2} \vee \left[\Sigma_1^1+\Sigma_4^1>\frac{d'+d}{2} \wedge |C|\frac{2\Sigma_1^1+2\Sigma_4^1-d'-d}{2(n+2)} \geq \sum_{l\in C} b_l - |C|b_i\right] \vee \left[\Sigma_1^1+\Sigma_4^1<\frac{d'+d}{2} \wedge -|C|\frac{2\Sigma_1^1+2\Sigma_4^1-d'-d}{2(n+2)} \geq |C|b_i - \sum_{l\in C} b_l\right].$$

Thus, the strategy profile m^* is an equilibrium if and only if the above statement holds for all realizations $\Sigma_1^1 \leq d$ and $\Sigma_4^1 \leq d'$, and all $d \geq 1$ and d' such that $d+d' \leq n-|C|$,[40] which is equivalent to

$$\begin{aligned}
&\left[\min_{\substack{d\geq 1, d+d'\leq n-|C|, \Sigma_4^1\leq d,\\ \Sigma_1^1\leq d', \Sigma_1^1+\Sigma_4^1>(d'+d)/2}} |C|\frac{2\Sigma_1^1+2\Sigma_4^1-d'-d}{2(n+2)} \geq \sum_{l\in C} b_l - |C|b_i\right]\\
&\quad\wedge \left[\min_{\substack{d\geq 1, d+d'\leq n-|C|, \Sigma_4^1\leq d,\\ \Sigma_1^1\leq d', \Sigma_1^1+\Sigma_4^1<(d'+d)/2}} -|C|\frac{2\Sigma_1^1+2\Sigma_4^1-d'-d}{2(n+2)} \geq |C|b_i - \sum_{l\in C} b_l\right]\\
&\Leftrightarrow \quad \left[\min_{d\geq 1, d+d'\leq n-|C|} |C|\frac{2\big(\lfloor(d'+d)/2\rfloor+1\big)-d'-d}{2(n+2)} \geq \sum_{l\in C} b_l - |C|b_i\right]\\
&\qquad\quad \wedge \left[\min_{d\geq 1, d+d'\leq n-|C|} -|C|\frac{2\big(\lceil(d'+d)/2\rceil-1\big)-d'-d}{2(n+2)} \geq |C|b_i - \sum_{l\in C} b_l\right]\\
&\Leftrightarrow \quad \left[\frac{|C|}{2(n+2)} \geq \sum_{l\in C} b_l - |C|b_i\right] \wedge \left[\frac{|C|}{2(n+2)} \geq |C|b_i - \sum_{l\in C} b_l\right]\\
&\Leftrightarrow \quad \frac{1}{2(n+2)} \geq \left|b_i - \frac{1}{|C|}\sum_{l\in C} b_l\right|,
\end{aligned}$$

which holds by assumption and, thus, finishes the proof. □

Proof of Lemma 2. Let m^* denote the candidate strategy profile and consider any time instant t such that $i=\iota(t)$. To ease notation, we let, for each $j \in N_i(g)$, $I_{1j}=I_{i,t}^{m^*} \cap I_{j,t}^{m^*}$ denote the set of common signals of agents i and j, let $I_{2j}=I_{j,t}^{m^*} \setminus I_{i,t}^{m^*}$ denote the set of signals of j that i does not know, let $I_3=I_{iN_i(g),t}^{m^*}$ denote the set of signals that i transmits to her neighbors, and let $I_{3j}=I_{ij,t}^{m^*}$ denote the subset of these signals that she transmits to j under strategy profile m^*. Notice that, for each j, the sets I_{1j}, I_{2j}, and I_{3j} are disjoint, and, furthermore, $I_{j,t+1}^{m^*}=I_{1j}\cup I_{2j}\cup I_{3j}$ and $I_{i,t}^{m^*}=I_{1j}\cup I_{3j}$. Let $k_{lj}=|I_{lj}|$ and $k_j=k_{1j}+k_{2j}+k_{3j}$.

As argued in the proof of Lemma 1, we need to consider only deviations regarding signals that agent i communicates truthfully at time instant t. Furthermore, we consider off-equilibrium beliefs to be such that agents ignore deviations on signals that have already been transmitted to them in earlier periods. Hence, we need to consider

[40] To ease the exposition, we omit that all parameters need to be nonnegative.

only the case $I_3 = \widetilde{I}^{m^*}_{iN_i(g),t}$ and deviations on these signals. In particular, a deviation on $s \in I_3$ affects agent $j \in N_i(g)$ if and only if $s \in I_{3j}$. Let $I_3 = \{s_{r_1}, s_{r_2}, \dots, s_{r_{|I_3|}}\}$ and $I_{3j} = \{s_{j_1}, s_{j_2}, \dots, s_{j_{k_{3j}}}\}$.

Suppose, without loss of generality, that agent i misreports signals $s_{r_1}, s_{r_2}, \dots, s_{r_d}$, $1 \le d \le |I_3|$, and let

$$I_3^d = \{1 - s_{r_1}, 1 - s_{r_2}, \dots, 1 - s_{r_d}, s_{r_{d+1}}, s_{r_{d+2}}, \dots, s_{r_{|I_3|}}\}.$$

Hence, without loss of generality, agent i misreports the subset $s_{j_1}, s_{j_2}, \dots, s_{j_{d_j}}$ to agent j, where $d_j = |\{k \in \{1, 2, \dots, d\} | s_{r_k} \in I_{3j}\}|$ and

$$I_{3j}^d = \{1 - s_{j_1}, 1 - s_{j_2}, \dots, 1 - s_{j_{d_j}}, s_{j_{d_j+1}}, \dots, s_{j_{k_{3j}}}\} \subseteq I_3^d.$$

Agent i has no incentives to do this deviation if and only if

$$-\int_0^1 \sum_{j \in N_i(g)} \sum_{I'_{2j} \in \{0,1\}^{k_{2j}}} \big(\big(\theta + b_i - \mathbb{E}[\theta | I_{1j}, I'_{2j}, I_{3j}] - b_j\big)^2$$

$$- \big(\theta + b_i - \mathbb{E}[\theta | I_{1j}, I'_{2j}, I_{3j}^d] - b_j\big)^2\big) f\big(\theta, I'_{2j} | I_{1j}, I_{3j}\big)\, d\theta \ge 0,$$

where we denote by $f(\cdot)$ the pdf and, with a slight abuse of notation, denote the sets I'_{2j} as vectors. Similar manipulations as in the proof of Lemma 1 yield

$$-\sum_{j \in N_i(g)} \left(\sum_{I'_{2j} \in \{0,1\}^{k_{2j}}} \Delta_j\big(I'_{2j}; d\big) \left(\frac{-\Delta_j\big(I'_{2j}; d\big)}{2} + b_j - b_i \right) \mathbb{P}\big(I'_{2j} | I_{1j}, I_{3j}\big) \right) \ge 0,$$

where, with $\Sigma_{3j}^d = \sum_{l=1}^{d_j} s_{j_l}$,

$$\Delta_j\big(I'_{2j}; d\big) = \mathbb{E}[\theta | I_{1j}, I'_{2j}, I_{3j}] - \mathbb{E}[\theta | I_{1j}, I'_{2j}, I_{3j}^d] = \frac{2\Sigma_{3j}^d - d_j}{k_j + 2}.$$

Using that $\Delta_j(I'_{2j}; d)$ is independent of I'_{2j} yields

$$-\sum_{j \in N_i(g)} \frac{2\Sigma_{3j}^d - d_j}{k_j + 2} \left(-\frac{2\Sigma_{3j}^d - d_j}{2(k_j + 2)} + b_j - b_i \right) \ge 0$$

$$\Leftrightarrow \quad \frac{1}{2} \sum_{j \in N_i(g)} \left(\frac{2\Sigma_{3j}^d - d_j}{k_j + 2} \right)^2 \ge \sum_{j \in N_i(g)} \frac{2\Sigma_{3j}^d - d_j}{k_j + 2} (b_j - b_i),$$

which holds by assumption for all realizations of I_3 and all $I_3^d \subseteq I_3$, $d \ge 1$, and, thus, finishes the proof. □

Proof of Theorem 5. Consider any $i \in N$, time instant t, realization of $\widetilde{I}^{m^{**}}_{iN_i(g),t}$, and $\varnothing \ne I_i^d \subseteq \widetilde{I}^{m^{**}}_{iN_i(g),t}$. Furthermore, suppose that $I^{m^*}_{j,t} = I^{m^{**}}_{j,t}$ for all $j \in N$ and $I^{m^*}_{ij,t} = \widetilde{I}^{m^*}_{ij,t}$ for all $j \in N_i(g)$. We show that then $I^{m^{**}}_{iN_i(g),t} = \widetilde{I}^{m^{**}}_{iN_i(g),t}$.

Fix any $j \in N_i(g)$. As m^* is an optimal intuitive myopic equilibrium with private communication, we have

$$\frac{1}{2\left(\left|I_{j,t}^{m^*}\right|+\left|\widetilde{I}_{ij,t}^{m^*}\right|+2\right)} \geq |b_j - b_i| \geq b_j - b_i. \tag{16}$$

Furthermore, as $2\sum_{s\in I_i^d\setminus I_{j,t}^{m^*}} s - |I_i^d \setminus I_{j,t}^{m^*}|$ is an integer,

$$\left(2\sum_{s\in I_i^d\setminus I_{j,t}^{m^*}} s - \left|I_i^d\setminus I_{j,t}^{m^*}\right|\right)^2 \geq 2\sum_{s\in I_i^d\setminus I_{j,t}^{m^*}} s - \left|I_i^d\setminus I_{j,t}^{m^*}\right|. \tag{17}$$

Together, (16) and (17) yield

$$\frac{1}{2}\left(\frac{2\sum_{s\in I_i^d\setminus I_{j,t}^{m^*}} s - \left|I_i^d\setminus I_{j,t}^{m^*}\right|}{\left|I_{j,t}^{m^*}\right|+\left|\widetilde{I}_{ij,t}^{m^*}\right|+2}\right)^2 \geq \frac{2\sum_{s\in I_i^d\setminus I_{j,t}^{m^*}} s - \left|I_i^d\setminus I_{j,t}^{m^*}\right|}{\left|I_{j,t}^{m^*}\right|+\left|\widetilde{I}_{ij,t}^{m^*}\right|+2}(b_j - b_i).$$

Summing up both sides over all $j \in N_i(g)$ yields the claim by construction of m^{**}. □

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