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Selling with evidence

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We study the informed-principal problem in a bilateral asymmetric information trading setting with interdependent values and quasi-linear utilities. The informed seller proposes a mechanism and voluntarily certifies information about the good's characteristics. When the set of certifiable statements is sufficiently rich, we show that there is an ex ante profit-maximizing selling procedure that is an equilibrium of the mechanism proposal game. In contrast to posted price settings, the allocation obtained when product characteristics are commonly known (the unravelling outcome) may not be an equilibrium allocation, even when all buyer types agree on the ranking of product quality. Our analysis relies on the concept of strong Pareto optimal allocation, which was originally introduced by Maskin and Tirole (1990) in private value environments.

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JEL classification. C72, D82.

1. Introduction

Virtually all firms and businesses have more information about their products and services than their customers. In a seminal paper, Akerlof (1970) shows that, under asymmetric information, perceived average quality drives the market price, which leads to market failure because sellers of high quality products are unwilling to sell at such a price. What if sellers can certify their quality? Sellers often offer hard information to their customers in the form of free samples, trial periods, review copies, third party labels, or stamps of approval. Viscusi (1978) argued that when certification is possible, high quality sellers drive the market, because they have the greatest incentive to certify

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and to receive a high price. This market force leads to the well known unravelling of information that, in the absence of other distortions, renders mandatory disclosure rules unnecessary.[1]

What happens when an informed seller cannot only certify, but also employ more sophisticated selling procedures than simply posting a price? This paper answers this question. We consider a privately informed seller (the principal) facing a buyer (the agent) who has private information about his taste. We make no assumptions on how the seller's cost and the buyer's valuation depend on the type profile, capturing scenarios ranging from Hotelling's pure horizontal differentiation model to a pure common value model in which the seller's information is about the quality of the good. The seller, knowing his information, can costlessly provide evidence about product characteristics and can propose any selling procedure: a fixed price, an information fee followed by an acquisition fee, a contingent sales agreement with money back guarantees, or any other sales contract, which we model as a mediated selling mechanism. The mechanism determines the terms of trade as a function of submitted reports as well as evidence. This formulation captures situations in which the amount traded and the price depend, for instance, on whether an asset has a high rating or a good has a certificate of being organic.

Compared to the standard informed-principal model in which the seller has access to soft information only (as in Myerson 1983), the ability of the seller to provide evidence enlarges the set of feasible allocations, because types who cannot offer the same evidence cannot mimic each other. In other words, certifiability relaxes the seller's incentive constraints. At the same time, the ability to certify makes deviations more effective: a high quality seller, for example, can deviate from a selling procedure by providing evidence of his quality and by asking a high price. This force implies a necessary condition for equilibrium interim profits: when the seller can certify his type, he cannot obtain an equilibrium interim profit below the best he can obtain when his type is commonly known, i.e., the *full-information* profit.[2]

The full-information profit vector (the vector of full-information profits for each type of the seller) is actually the *unique* equilibrium profit vector of disclosure games in which the seller's set of certifiable statements is sufficiently rich (Grossman and Hart 1980, Grossman 1981, Milgrom 1981, Koessler and Renault 2012). This is a central insight in the accounting (Verrecchia 2001), industrial organization (Milgrom 2008), and finance (Shin 2003) literatures. The full-information profit vector is typically not ex ante optimal; that is, it does not maximize the expected profit that the seller could achieve if he could commit to a mechanism before knowing his type. Is it still the unique equilibrium profit vector when the seller can propose any selling procedure?

This paper establishes that when the seller's ability to certify product characteristics is sufficiently rich, there is an ex ante optimal selling procedure that is an equilibrium of the mechanism proposal game. Then the seller does not benefit from being able to

[1] An extensive theoretical and empirical literature studies certification and disclosure by firms. See the surveys by Dranove and Jin (2010) and Milgrom (2008) and references therein.

[2] This terminology is used in Maskin and Tirole (1990) and refers to the highest feasible interim profit when the buyer knows the seller's information but the seller does not know the buyer's.

commit to even the best mechanism before knowing his type, and if a highest quality seller type exists, he still does not benefit by deviating from this mechanism. Moreover, we find that the full-information profit vector may *not* be an equilibrium profit vector, even under the conditions making it the unique equilibrium profit vector under price posting.

A key concept in our analysis is the *strong (unconstrained) Pareto optimal* (SPO) profit vector. First introduced by Maskin and Tirole (1990), and generalized by Mylovanov and Tröger (2012), SPO profit vectors have been central in informed-principal games with soft information and private values. They are (roughly) obtained as follows. First, for each buyer's belief, consider all allocations (and the corresponding profit vectors) that satisfy interim incentive and participation constraints for the buyer but not necessarily for the seller (thus the term "unconstrained" in the original definition). Second, take the union of such vectors over all buyer's beliefs and consider the Pareto frontier of this set. Finally, consider an allocation that induces a profit vector on this Pareto frontier that satisfies the buyer's incentive and participation constraints for the prior belief; this is an SPO allocation for the prior. Maskin and Tirole (1990) establish existence for all priors in a private value setting. More importantly, they show that SPO allocations are equilibrium allocations of the mechanism proposal game by establishing that they (i) satisfy the seller's incentive and participation constraints and (ii) are immune to deviations to other mechanisms.[3] Moreover, Maskin and Tirole (1990) observe that SPO profits *coincide* with full-information ones in quasi-linear settings.

We show that SPO allocations also exist in our trading environment with interdependent values. In addition, they are immune to deviations by the seller in the mechanism proposal game. When the seller deviates in this game, he can choose to present directly to the buyer any available evidence and/or propose any alternative mechanism. Hence, once SPO allocations are incentive compatible for the seller—which depends on the certifiability structure—they are equilibrium allocations. In contrast to the private value setting with quasi-linear utilities, SPO allocations typically differ from full-information allocations.

The fact that SPO allocations can be equilibrium allocations in an interdependent value setting can be viewed as somewhat surprising given that one of the main contributions to informed-principal problems under interdependent values, namely Maskin and Tirole (1992), focused on very different candidate equilibrium allocations.[4] The reason is that in such settings, SPO allocations typically fail to be incentive compatible for the seller.

To establish these findings we proceed as follows.

[3]Mylovanov and Tröger (2012) examine a more general private value setting where SPO allocations may fail to exist, which motivates their concept of "strongly neologism-proof" allocations. Wagner et al. (2015) show that a strongly neologism-proof allocation is a perfect Bayesian equilibrium allocation in an informed-principal problem with moral hazard.

[4]In their setup, information is soft and a "worst" type of principal exists, for example, the high cost firm or low productivity worker. They show that if the Rothschild–Stiglitz–Wilson allocation is interim efficient for interior beliefs, any feasible payoff vector that gives each principal type at least as much payoff as the Rothschild–Stiglitz–Wilson allocation can be sustained in an equilibrium.

In Proposition 1 we show that SPO allocations exist for every prior in our interdependent value model. To do so, we follow Maskin and Tirole (1990) and Mylovanov and Tröger (2012) and define a fictitious exchange economy in which traders are the seller's types and the goods are slacks on the ex post incentive and participation constraints of the buyer. Each trader has a zero endowment of each slack and can trade positive or negative amounts of slacks. Beliefs play a role in market clearing, because they capture the proportion of different trader types. The idea of Maskin and Tirole (1990) is to show that for every prior, a Walrasian equilibrium allocation (WEA) exists and is an SPO allocation for that prior. This finding can be extended, even though, with interdependent values, each trader has access to a different set of goods.

When the buyer's utility does not depend on the seller's type (as in Maskin and Tirole 1990, and Mylovanov and Tröger 2012, 2014), each trader (seller type) needs the same slacks for an allocation to satisfy ex post incentive and participation constraints for the buyer. Hence, the resulting WEA is incentive compatible for the seller, because all seller types have the same endowment and are, therefore, able to choose from the same set of allocations. With interdependent values, the buyer's utility depends on the seller's type. Depending on the seller's type, an allocation requires different slacks so as to satisfy the buyer's ex post constraints. Hence, even with the same endowments, the set of allocations each seller's type can choose from depends on his type, and the resulting WEA may not be incentive compatible for the seller. The role of information certification is exactly to restore the seller's incentive constraint at the WEA: if a seller type s wants to mimic type s' (i.e., type s prefers the allocation chosen by type s'), type s' should have a piece of evidence that s does not have. In Section 4.5, we characterize more precisely the conditions on the certifiability structure for the above incentive condition to hold.

In Proposition 2, we show that SPO allocations are immune to arbitrary seller deviations in the mechanism proposal game: for every evidence the seller presents to the buyer and for every mechanism he proposes, there exists a consistent belief for the buyer that makes the deviation unprofitable in the sense that the resulting continuation equilibrium profit is not better than the SPO profit for all seller types. Hence, if an SPO allocation is incentive compatible for the seller, then it is an equilibrium of the mechanism proposal game. In addition, such an equilibrium allocation is ex ante optimal. This is established in Proposition 3, the proof of which leverages the fact that in our model there are transfers. Hence, the mechanism proposal game admits an ex ante optimal equilibrium when the set of certifiable statements about product characteristics is sufficiently rich.

Finally, we provide necessary and sufficient conditions under which ex ante optimal profit vectors coincide with full-information vectors in Proposition 4. This comparison clarifies the value of information for the seller, and relates to the question of whether the "informed-principal problem" is equivalent to a "standard mechanism design problem" without a privately informed designer. In our environment, this equivalence holds under somewhat stringent conditions, even when full certification is possible.

1.0.0.1 *Other related literature* To establish our results, we rely on the general formulation of the informed-principal problem of Myerson (1983) and extend it to allow the information of the principal (the seller), but not the information of the agent (the buyer), to be certifiable. Following the tradition of mechanism design with certifiable information (Green and Laffont 1986, Forges and Koessler 2005, Bull and Watson 2007, Deneckere and Severinov 2008, Strausz 2016), we take the certification structure as exogenous.[5]

The advertising literature assumes the firm is not privately informed when it designs (and commits to) its information disclosure rule.[6] Sun (2011) and Koessler and Renault (2012) study information disclosure by an informed firm at the interim stage, but, unlike this paper, focus on posted prices.

In the informed-principal literature, Mylovanov and Tröger (2014) establish ex ante optimality of equilibrium allocations in a generalized private value setup with transferable utility. A key element of their setup is that the principal's information does not affect the agent's valuation (generalized private values). Mylovanov and Tröger (2014) also provide sufficient conditions that guarantee no equilibrium exists in which the principal's ex ante expected payoff is higher than that corresponding to the full-information payoff vector. Analogous information irrelevance results have been established in different private value settings by Maskin and Tirole (1990), Guofu (1996), Yilankaya (1999), and Skreta (2011).

In Koessler and Skreta (2016), we examine a trading scenario where the seller's type affects the buyer's willingness to pay in an arbitrary way and the seller seeks to maximize revenue. Leveraging the fact that all seller types seek higher revenue, we show that a continuum of Pareto ranked equilibrium profit vectors exists, ranging from the worst case scenario in terms of profits for the seller to a profit vector that is ex ante optimal for the seller. Balestrieri and Izmalkov (2016) consider a symmetric horizontal differentiation trading problem in which the buyer's valuation depends on his type and the seller's privately known location, and they characterize ex ante optimal mechanisms. Balkenborg and Makris (2015) propose an equilibrium refinement for a class of informed-principal problems with common values.

de Clippel and Minelli (2004) study a bargaining problem with bilateral asymmetric information and without transfers, where both parties have verifiable types. They show that an allocation is an equilibrium of the mechanism proposal game if and only if the interim payoffs of the principal and the agent are higher than their interim payoffs at the best allocation for the principal that satisfies ex post participation constraints.[7] As they show (see Example 2 in de Clippel and Minelli 2004), this domination may be strict when utility is not transferable. It would be interesting to study intermediate models in which utilities are not transferable and only the information of the principal is certifiable.

[5]Most mechanism design literature assumes the information structure is exogenous, and the assumption that certification abilities are exogenous is in the same spirit. It captures well that often, in reality, the structure of available certificates is exogenous, that is, takes the form of hygiene letter grades (A, B, C, ...) for restaurants or multi-letter grades (AAA, AA+, BBB, ...) for ratings of financial assets.

[6]See, for example, Johnson and Myatt (2006) and Anderson and Renault (2006). For a comprehensive literature review, see Renault (2015, Section 3).

[7]If the seller's and the buyer's information is perfectly certifiable in our environment, equilibrium allocations are trivial: the seller can extract all surplus in each state.

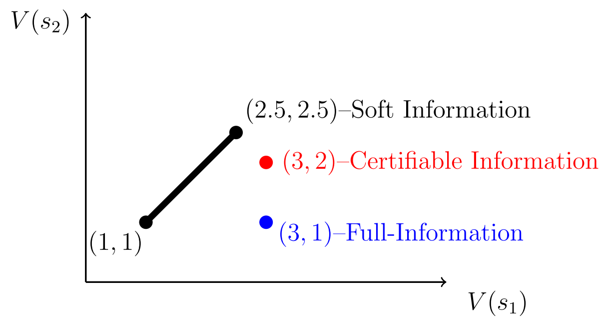

Figure 1. Profit vectors under different benchmarks in the example.

2. Motivating example

Consider a seller who can have two equally likely types, s_1 or s_2, that represent the characteristics of the product he is selling. The seller is facing a buyer whose taste is equally likely to be t_1 or t_2 and whose valuation for the product is described in the matrix

$$u(s,t) = \begin{array}{|c|c|c|} \hline & t_1 & t_2 \\ \hline s_1 & 5 & 3 \\ \hline s_2 & 1 & 2 \\ \hline \end{array}.$$

In this example, the seller's cost is 0 (the seller cares only about revenue). Observe that s_1 is the high quality product, whereas a t_1 consumer values quality more than t_2. When the consumer knows whether the quality is s_1 or s_2, the profit-maximizing selling procedure is for s_1 to ask a price of 3 and for s_2 to ask a price of 1 or 2, resulting in the *full-information* profit vector $(V(s_1), V(s_2)) = (3, 1)$.

Koessler and Renault (2012) establish that when the seller can certify his type and the buyer's valuation function is "pairwise monotonic"—as is the case in this example—unravelling forces make $(V(s_1), V(s_2)) = (3, 1)$ the *unique* equilibrium profit with posted prices. In fact, this result holds for any certifiability structure where s_1 has a piece of evidence not available to s_2.

When the seller cannot certify quality (information is "soft"), but can employ any selling procedure, Koessler and Skreta (2016) show that a continuum of equilibrium interim profit vectors exists; it is described by the line segment along the 45-degree line between the "best safe" and the ex ante optimal profit vectors. The 45-degree line corresponds to incentive compatible profit vectors for the seller: his profit should be the same at s_1 and s_2 because his cost does not depend on his type (as is assumed in Koessler and Skreta 2016, and in this example). The *best safe* profit vector is the highest incentive compatible profit vector the seller can achieve independently of the buyer's belief, so it is $(1, 1)$. An ex ante optimal profit vector maximizes the seller's expected profit before he learns his type, which is 2.5 in this example. Hence, the set of equilibrium profit vectors under soft information is the line segment that connects $(1, 1)$ and $(2.5, 2.5)$ (see Figure 1).

What are the equilibrium profits when the seller's information is certifiable and he can employ any general selling procedure? Clearly, for any certifiability structure where s_1 has a piece of evidence not available to s_2, each seller type can guarantee its full-information profit. So a lower bound on equilibrium profit vectors is the vector $(3, 1)$. Our Proposition 2 establishes that for all such certifiability structures, $(V(s_1), V(s_2)) = (3, 2) > (3, 1)$ is an equilibrium profit vector because it is an SPO profit vector. Observe that this profit vector is ex ante optimal (because the ex ante optimal profit is 2.5), whereas the full-information profit vector is not. Ex ante optimality of SPO profit vectors is shown generally in Proposition 3.

To achieve the profit vector $(3, 2)$, the seller can propose the following simple mechanism. If the buyer accepts the mechanism, he has to pay 3 if the seller presents the evidence s_1 and pay 2 otherwise. The buyer is willing to accept it because he does not know whether he will have to pay 3 or 2, and the expected payment is below the expected valuation for both of his types. This mechanism implements the allocation

$$(p, x)(s, t) = \begin{array}{|c|c|c|} \hline & t_1 & t_2 \\ \hline s_1 & 1, 3 & 1, 3 \\ \hline s_2 & 1, 2 & 1, 2 \\ \hline \end{array},$$

where p is the probability of trade and x is the payment as a function of each type profile (s, t). Another way to implement the profit vector $(3, 2)$ is with a price of 3 and a rebate of 1 if the seller fails to certify s_1.

In this example, the seller benefits from being privately informed, because the buyer's uncertainty about the seller's type relaxes the interim participation constraint (the ex post participation constraint is violated for t_1 when $s = s_2$). More generally, private information can also be beneficial to the seller due to relaxed buyer's incentive constraints (see Section 5.2 in Koessler and Skreta 2016).

3. Setting and definitions

3.1 *The trading problem*

Consider a monopoly seller with one indivisible good facing a single buyer with unit demand. The seller has private information about the *product's characteristics*, denoted by $s \in S$ and also called the *type of the seller*. The buyer has private information about his *taste*, denoted by $t \in T$ and also called the *type of the buyer*. The type space $S \times T$ is finite and types are independently distributed, with full-support probability distributions $\pi^0 \in \Delta(S)$ and $\tau \in \Delta(T)$. The seller's cost for delivering the good is denoted by $v(s, t) \in \mathbb{R}$. The buyer's valuation for the product is denoted by $u(s, t) \in \mathbb{R}$.

An *allocation* is given by $(p, x) : S \times T \to [0, 1] \times \mathbb{R}$, where $p(s, t)$ is the probability of trade and $x(s, t)$ is the transfer from the buyer to the seller. We assume transfers lie in a compact and convex set: $x(s, t) \in [-\mathcal{X}, \mathcal{X}]$ for every s and t, where $\mathcal{X}$ is large.[8]

[8]As in Maskin and Tirole (1990) and Mylovanov and Tröger (2012), we assume for simplicity that transfers are bounded. This assumption ensures that the set of outcomes is compact, which is used in the existence

Both the seller and the buyer are risk neutral. Given an allocation (p, x), the *seller's profit* and the *buyer's utility* are

$$V(s,t) = x(s,t) - p(s,t)v(s,t) \quad \text{and} \quad U(s,t) = p(s,t)u(s,t) - x(s,t).$$

The seller's interim profit is $V(s) \equiv \sum_{t\in T} \tau(t)V(s,t)$. When writing the buyer's interim utility, we keep track of his beliefs because, in the mechanism proposal game, they can potentially differ from the prior. We then let, for every $\pi \in \Delta(S)$, $U_\pi(t) \equiv \sum_{s\in S} \pi(s)U(s,t)$.

3.2 *Certification and mechanisms*

In the standard mechanism design setting without certifiable information, a mechanism specifies a probability of trade and a transfer as a function of (cheap talk) messages sent by both the seller and the buyer. When the seller is able to certify some information by providing evidence about product characteristics, the outcome of a mechanism depends on these standard messages as well as on what has been certified by the seller.

The seller's certification ability is exogenous and is represented by a *certifiability structure* $\mathcal{E} \subseteq 2^S$ that stands for the set of events the seller is able to certify. Let $\mathcal{E}(s) = \{E \in \mathcal{E} : s \in E\}$ be the set of such events when the seller's actual type is $s \in S$.[9] When information is not certifiable, we have $\mathcal{E}(s) = \{S\}$ for every $s \in S$. We assume $S \in \mathcal{E}$, which means the seller always has the option not to certify any information. Following Forges and Koessler (2005), we also assume $\mathcal{E}$ is closed under intersection, which captures the ability of the seller to certify as many events in $\mathcal{E}(s)$ as he wants.[10] A certifiability structure satisfies *own-type certifiability* if $\{s\} \in \mathcal{E}$ for every $s \in S$.

A *mechanism* consists of (finite) sets of cheap talk messages M_S for the seller and M_T for the buyer, and a function

$$\mathcal{M} : \mathcal{E} \times M_S \times M_T \to [0,1] \times [-\mathcal{X}, \mathcal{X}],$$

which specifies a probability of trade and a selling price as a function of the event $E \in \mathcal{E}$ certified by the seller to the mechanism, the cheap talk message $m_S \in M_S$ of the seller, and the cheap talk message $m_T \in M_T$ of the buyer.

A mechanism is played as follows. The seller, when his type is $s \in S$, certifies an event $E \in \mathcal{E}(s)$ and submits a cheap talk message $m_S \in M_S$ to the mechanism. Simultaneously,

proof of Proposition 1. Without bounds on transfers, the set of feasible allocations is unbounded. However, interim transfers and utilities are bounded above and below at any feasible mechanism (because of the interim participation constraints). Mylovanov and Tröger (2014, Lemma 2 in the Appendix) show that large enough bounds on ex post transfers exist so that feasible interim utilities are not affected. The existence of an equilibrium with unbounded transfers is then deduced from the existence of an equilibrium with bounded transfers by considering a sequence of increasing bounds.

[9]Using a certifiability structure is equivalent to using any abstract message correspondence $\mathcal{Z} : S \rightrightarrows Z$ by letting $\mathcal{E}(s) = \{\mathcal{Z}^{-1}(z) : z \in \mathcal{Z}(s)\}$. The set $\mathcal{Z}^{-1}(z)$ is the set of seller types who can send message z, so $\mathcal{Z}^{-1}(z)$ is the event that message z certifies.

[10]This property is called the minimal closure condition in Forges and Koessler (2005) and normality in Bull and Watson (2007).

the buyer decides whether to reject or accept the mechanism and, in the latter case, sends a message $m_T \in M_T$ to the mechanism. The mechanism $\mathcal{M}$ and the reporting and participation strategies implement an allocation (p, x). The default allocation of no trade and no payment arises if the buyer rejects, in which case both players' payoffs are zero.

3.3 *Feasible allocations*

An allocation (p, x) is *feasible* for belief π if it gives positive interim profits to the seller and if there exists a mechanism $\mathcal{M}$, which denotes reporting and participation strategies that implement the allocation (p, x) and form a Bayes–Nash equilibrium given $\mathcal{M}$ and π. Let $E^*(s) = \bigcap_{E \in \mathcal{E}(s)} E$ be the smallest event the seller is able to certify when his actual type is s. The fact that $\mathcal{E}$ is closed under intersection ensures that $E^*(s)$ is certifiable by the seller when his type is s, that is, $E^*(s) \in \mathcal{E}(s)$.

From the certifiability structure $\mathcal{E}$, we uniquely define the *reporting correspondence* of the seller as $R : S \rightrightarrows S$, with

$$R(s) \equiv \{\tilde{s} \in S : E^*(\tilde{s}) \in \mathcal{E}(s)\}.$$

The set $R(s)$ represents all seller types in S that type s is able to mimic when these types certify all the information they can.

The following lemma uses the revelation principle with partially certifiable types (Forges and Koessler 2005) to characterize all feasible allocations in a canonical way. It is similar to the revelation principle without certifiable information: the buyer and the seller each privately make a truthful report about their type t and s, respectively, and, *in addition*, the seller provides maximal evidence by privately certifying $E^*(s)$ to the mechanism.

For a given allocation (p, x), let $V(s' \mid s) \equiv \sum_{t \in T} \tau(t)(x(s', t) - p(s', t)v(s, t))$ be the seller's interim profit when his type is s but he receives the allocation of s', and let $U_\pi(t' \mid t) \equiv \sum_{s \in S} \pi(s)(p(s, t')u(s, t) - x(s, t'))$ be the buyer's interim utility when his actual type is t but he receives the allocation of t'.

Lemma 1. *An allocation (p, x) is feasible for belief π given the certifiability structure $\mathcal{E}$ if and only if the following incentive and participation constraints are satisfied:*

$$V(s) \geq V(s' \mid s) \quad \text{for every } s \in S \text{ and } s' \in R(s) \tag{S-IC}$$

$$V(s) \geq 0 \quad \text{for every } s \in S \tag{S-PC}$$

$$U_\pi(t) \geq U_\pi(t' \mid t) \quad \text{for every } t, t' \in T \tag{B-IC}$$

$$U_\pi(t) \geq 0 \quad \text{for every } t \in T. \tag{B-PC}$$

The proof directly follows the revelation principle with partially certifiable types in Forges and Koessler (2005).[11]

[11] See also Green and Laffont (1986), Bull and Watson (2007), Deneckere and Severinov (2008), or Strausz (2016) for similar versions of the revelation principle.

Note that under own-type certifiability, we have $R(s) = \{s\}$, so (S-IC) is always satisfied. More generally, (S-IC) is satisfied under the following condition: if type s strictly prefers the allocation of type s' (i.e., $V(s' \mid s) > V(s)$), type s' should have evidence that is not available to type s (i.e., $\mathcal{E}(s) \nsubseteq \mathcal{E}(s')$).

3.4 *Ex ante optimal and full-information allocations*

Before proceeding with equilibrium analysis, we define two benchmarks against which we compare equilibria in terms of seller profit.

Definition 1. An allocation (p, x) is *ex ante optimal* if it maximizes the ex ante expected profit $\sum_{s \in S} \pi^0(s) V(s)$ under the *interim* incentive and participation constraints (S-IC), (S-PC), (B-IC), and (B-PC) for $\pi = \pi^0$.

Definition 2. An allocation (p, x) is a *full-information allocation* if for every $s \in S$, it maximizes the interim profit $V(s)$ under the following *ex post* incentive and participation constraints of the buyer:

$$U(s, t) \geq p(s, t')u(s, t) - x(s, t') \quad \text{for every } t, t' \in T$$

$$U(s, t) \geq 0 \quad \text{for every } t \in T.$$

In other words, an ex ante optimal allocation results from a profit-maximizing mechanism the seller chooses before learning his type, whereas a full-information allocation results from profit-maximizing mechanisms the seller chooses when his type is commonly known. The corresponding profits are called the *ex ante optimal profits* and the *full-information profits*.

Note that a full-information allocation does not depend on the certifiability structure and, in general, such an allocation may fail to be feasible because it may not satisfy the seller's incentive constraints. However, it is clearly feasible under own-type certifiability. Note also that if $v(s, t)$ does not depend on t, then when s is known, the seller's reservation value is known and (one of) the full-information allocation(s) is simply a posted price (see Myerson 1981, Riley and Zeckhauser 1983). Finally, note that if a full-information allocation is feasible, it is feasible regardless of the buyer's belief: it is safe according to the terminology of Myerson (1983).

3.5 *Mechanism proposal game*

The timing of the mechanism proposal game is as follows.

Stage 1. Nature selects the seller's type, $s \in S$, according to the probability distribution $\pi^0 \in \Delta(S)$, and selects the buyer's type, $t \in T$, according to the probability distribution $\tau \in \Delta(T)$.

Stage 2. The seller is privately informed about $s \in S$ and the buyer is privately informed about $t \in T$.

Stage 3. The seller certifies an event $F \in \mathcal{E}(s)$ to the buyer and proposes a mechanism $\mathcal{M}$.

Stage 4. The buyer observes the mechanism $\mathcal{M}$ and the event F directly certified to him. The seller and the buyer observe a uniformly distributed public signal in $[0, 1]$,[12] and then play the mechanism $\mathcal{M}$ as described in Section 3.2 (i.e., the buyer decides to reject or to accept and to send a cheap talk message to the mechanism; the seller sends a cheap talk message and certifies an event E to the mechanism).

Notice that we allow the seller to certify information directly to the buyer (in Stage 3), in addition to certifying to the mechanism (in Stage 4, when the mechanism is played). We consider this timing so as to ensure that our analysis is robust to direct information certification, as assumed in more standard disclosure games (e.g., Milgrom 1981, Koessler and Renault 2012). None of our results is affected by this assumption. Indeed, any equilibrium in our setting would also be an equilibrium in the alternative mechanism proposal game in which the seller cannot certify information directly to the buyer.[13]

For every $F \subseteq S$, let $\Delta_F(S) = \{\pi \in \Delta(S) : \pi(s) = 0 \ \forall s \notin F\}$ be the set of beliefs of the buyer with support in F. We adapt the definition of expectational equilibrium of Myerson (1983) to account for the fact that the buyer's belief off the equilibrium path should be consistent with the event F certified to him.

Definition 3. An allocation (p, x) is an *expectational equilibrium* if and only if (i) it is *feasible* for the prior belief π^0 and (ii) no type of seller can benefit from deviating in Stage 3: for every $F \in \mathcal{E}$ and for every mechanism $\tilde{\mathcal{M}}$, there exist a belief $\tilde{\pi} \in \Delta_F(S)$ for the buyer, and reporting and participation strategies that form a continuation Bayes–Nash equilibrium given $\tilde{\mathcal{M}}$ and $\tilde{\pi}$, which induce a profit vector $(\tilde{V}(s))_{s\in S}$, such that $V(s) \geq \tilde{V}(s)$ for every $s \in F$.

By definition, an expectational equilibrium is a perfect Bayesian equilibrium allocation of the mechanism proposal game in which the seller, whatever his type, uses the same mechanism and certifies no information directly to the buyer ($F = S$) along the equilibrium path.[14] Following the logic of the inscrutability principle in Myerson (1983, Section 3), any other perfect Bayesian equilibrium allocation in which information is directly revealed to the buyer would also be a perfect Bayesian equilibrium allocation in which no information is directly revealed to the buyer. Indeed, the seller's information can be incorporated into the mechanism itself by inducing the same allocation and by relaxing the incentive and participation constraints of the buyer.

[12] We consider a public correlation device so that the set of continuation equilibrium profit vectors is convex. We use this property in the proof of Proposition 2.

[13] An equilibrium in this alternative formulation should satisfy condition (ii) in the next definition only for $F = S$.

[14] It is stronger than the weakest version of perfect Bayesian equilibrium because it imposes that all buyer types have the same off-path beliefs after a deviation. None of our results would be affected by considering a weaker equilibrium solution concept.

4. Equilibrium allocations

Compared to the standard setting without certifiable information, certifiability extends the set of feasible allocations because it relaxes the seller's incentive constraints. Certifiability enlarges the set of *potential* equilibrium allocations (the ones that satisfy the expectational equilibrium condition (i)). However, because it does so regardless of the buyer's belief π, it also enlarges the set of continuation equilibrium allocations, so condition (ii) is harder to satisfy. In particular, all profit vectors that arise from continuation equilibria for *some* buyer belief might arise as off-path profit vectors. An upper bound of such profit vectors is the set of SPO profit vectors, which was defined by Maskin and Tirole (1990), and generalized by Mylovanov and Tröger (2012), in private value environments. We establish in Proposition 1 the existence of SPO allocations for all priors in our interdependent value environment. We then show that if an SPO allocation for the prior is feasible (a condition that is always satisfied when the certifiability structure is rich enough), it is an equilibrium (Proposition 2) and is ex ante optimal (Proposition 3).

4.1 *SPO allocations*

Fix the buyer's belief π and consider allocations that satisfy (B-IC) and (B-PC)—the interim incentive and participation constraints of the buyer given π. We refer to such allocations and the associated profit vectors $(V(s))_{s\in S}$ as *π-buyer-feasible*. The Pareto frontier of all such profit vectors when beliefs vary is the set of SPO profit vectors.

Let $\mathcal{V}^B(\pi) \subseteq \mathbb{R}^{|S|}$ be the set of π-buyer-feasible (interim) profit vectors and let $\mathcal{V}^B = \bigcup_{\pi\in\Delta(S)} \mathcal{V}^B(\pi)$ be the set of all buyer-feasible profit vectors as we let beliefs vary.

Definition 4. The set of *SPO profit vectors*, denoted by $\mathcal{V}^{\text{SPO}}$, is the set of buyer-feasible profit vectors $V^* \in \mathcal{V}^B$ such that there is no $\pi \in \Delta(S)$ and no $V \in \mathcal{V}^B(\pi)$ satisfying $V(s) \geq V^*(s)$ for all s, with strict inequality for some s with $\pi(s) > 0$.

That is, $\mathcal{V}^{\text{SPO}}$ is the Pareto frontier of the set of all buyer-feasible profit vectors. Let $\mathcal{V}^{\text{SPO}}(\pi) = \mathcal{V}^{\text{SPO}} \cap \mathcal{V}^B(\pi)$ be the set of *SPO profit vectors for* π. An *SPO allocation* for π is a π-buyer-feasible allocation that results in an SPO profit vector for π.

SPO profit vectors differ dramatically between private and interdependent value environments with quasi-linear utilities: Maskin and Tirole (1990) established that in private value setups with quasi-linear utilities, there is a unique SPO profit vector for all interior beliefs, and it coincides with the full-information profit vector (the seller derives no benefit from information privacy). When values are interdependent, the seller may benefit from information privacy because, in general, SPO profit vectors differ across priors and do not coincide with full-information profit vectors.

In the introductory example, the set of all buyer-feasible profit vectors, $\mathcal{V}^B$, is given by the grey area in Figure 2. Let $\pi(s_1) = \pi$. The SPO profit vectors are the bold segments

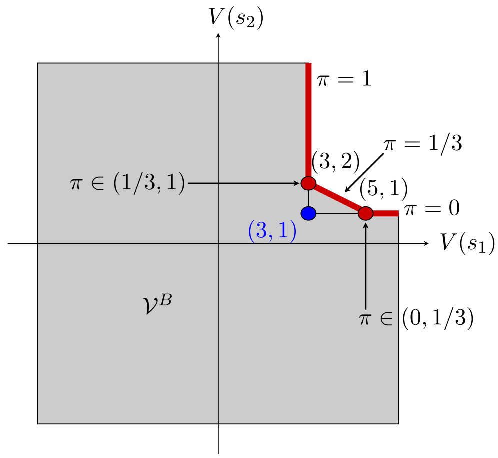

FIGURE 2. Buyer-feasible (grey area), full-information (at $(3, 1)$), and SPO (bold segments) profit vectors in the example.

in the figure and are given by

$$\mathcal{V}^{\mathrm{SPO}}(\pi) = \begin{cases} \{(V_1, 1) : V_1 \geq 5\} & \text{if } \pi = 0, \\ \{(5, 1)\} & \text{if } \pi \in (0, 1/3), \\ \{(V_1, V_2) : (V_1, V_2) = \alpha(5, 1) + (1 - \alpha)(3, 2), \alpha \in [0, 1]\} & \text{if } \pi = 1/3, \\ \{(3, 2)\} & \text{if } \pi \in (1/3, 1), \\ \{(1, V_2) : V_2 \geq 2\} & \text{if } \pi = 1. \end{cases}$$

For example, the SPO profit vector for $\pi \in (1/3, 1)$ can be induced by the allocation

$$(p, x)(s, t) = \begin{array}{|c|c|c|} \hline & t_1 & t_2 \\ \hline s_1 & 1, 3 & 1, 3 \\ \hline s_2 & 1, 2 & 1, 2 \\ \hline \end{array}$$

that we saw in Section 2, and the SPO profit vector for $\pi \in (0, 1/3)$ can be induced by the allocation

$$(p, x)(s, t) = \begin{array}{|c|c|c|} \hline & t_1 & t_2 \\ \hline s_1 & 1, 5 & 1, 5 \\ \hline s_2 & 1, 1 & 1, 1 \\ \hline \end{array}.$$

Interestingly, the full-information profit vector, $(3, 1)$, is never SPO. Also notice that for the prior $\pi^0 = 1/2$, the ex ante optimal expected profit is 2.5. Hence, every profit vector on the segment connecting $(3, 2)$ and $(4, 1)$ is π^0-buyer-feasible and ex ante optimal for $\pi^0 = 1/2$, and yields a higher profit than the full-information profit vector for every seller type. However, only $(3, 2)$ is SPO. In Section 4.7, we show that $(3, 2)$ is actually the unique equilibrium in this example.

4.2 *Existence of SPO allocations*

To show that an SPO allocation exists for every $\pi \in \Delta(S)$, we follow Maskin and Tirole (1990) and Mylovanov and Tröger (2012) and define a fictitious exchange economy in which the seller's types are trading slacks on the buyer's ex post incentive and participation constraints. The belief π plays a role in market clearing, because it captures the proportion of different trader types. The existence of a Walrasian equilibrium in that economy relative to π follows from standard arguments in general equilibrium theory. The only delicate part is that traders' utility functions in that economy are endogenous objects (they correspond to value functions of a program we describe below) and some work is needed to establish that they are continuous so as to get existence. Maskin and Tirole (1990) show that a Walrasian equilibrium allocation relative to π exists and is an SPO allocation for π. With interdependent values, the only difference is that each trader has access to a different set of goods, but this difference does not matter for existence.

Proposition 1. *For every $\pi \in \Delta(S)$, there exists at least one SPO allocation for π.*

For the proof, see the Appendix.

To get a better sense of the proof of Proposition 1, we proceed to describe the fictitious exchange economy. In that economy, a bundle of goods for type s consists of a vector of slacks $(c(s, t), c(s, t, t'))_{t,t'}$, where $c(s, t) \in \mathbb{R}$ is a slack on the ex post participation constraint of buyer type t and $c(s, t, t') \in \mathbb{R}$, $t' \neq t$, are the slacks on the ex post incentive constraints of buyer type t. The endogenous utility function of trader s given c is denoted by $V_I(s \mid c)$ and it corresponds to the indirect profit of type s when slacks $(c(s, t), c(s, t, t'))_{t,t'}$ are present in the ex post incentive and participation constraints of the buyer,

$$V_I(s \mid c) = \max_{x(s,\cdot),\, p(s,\cdot)} \sum_{t \in T} \tau(t)\big(x(s, t) - p(s, t)v(s, t)\big),$$

under the constraints

$$p(s, t)u(s, t) - x(s, t) \geq p\big(s, t'\big)u(s, t) - x\big(s, t'\big) - c\big(s, t, t'\big) \quad \text{for every } t, t' \in T \tag{1}$$

$$p(s, t)u(s, t) - x(s, t) \geq -c(s, t) \quad \text{for every } t \in T. \tag{2}$$

Let $C(s)$ be the (nonempty, closed, and convex) set of slack vectors for which the feasible set of allocations $(x(s, \cdot), p(s, \cdot))$ of this maximization problem is nonempty. When the buyer's valuation, $u(s, t)$, depends on s, the set $C(s)$ also depends on s, which is in contrast to Maskin and Tirole (1990) and Mylovanov and Tröger (2012). In addition, the set of allocations that satisfy the ex post contraints above can differ across s for the same slacks. Hence, different traders with the same slacks have access to different allocations. This point is the key difference between private and interdependent values. This difference does not pose additional challenges for the existence of a Walrasian equilibrium of the fictitious exchange economy, but it matters for the seller's incentive constraint, as we elaborate in Section 4.5.

Given some exogenous prices $\gamma(t)$ and $\gamma(t,t')$ of the slacks $c(s,t)$ and $c(s,t,t')$, trader s's demand correspondence is given by

$$D(s \mid \gamma) \equiv \arg\max_{c \in C(s)} V_I(s \mid c),$$

subject to the budget constraint

$$\sum_{t \in T} \gamma(t) c(s,t) + \sum_{t,t' \in T} \gamma(t,t') c(s,t,t') \leq 0,$$

where we use that the initial endowment of each slack is 0.

A *Walrasian equilibrium* relative to $\pi \in \Delta(S)$ is a vector of nonnegative prices $(\gamma(t), \gamma(t,t'))_{t,t' \in T}$ and slacks $(c(s,t), c(s,t,t'))_{s \in S, t,t' \in T}$ such that demands are optimal, that is, $(c(s,t), c(s,t,t'))_{t,t' \in T} \in D(s \mid \gamma)$ for every $s \in S$, and markets for participation and incentive slacks clear:

$$\sum_{s \in S} \pi(s) c(s,t) \leq 0 \quad \text{for every } t \in T \tag{3}$$

$$\sum_{s \in S} \pi(s) c(s,t,t') \leq 0 \quad \text{for every } t, t' \in T. \tag{4}$$

The last two equations are the "market clearing" conditions, which ensure that a Walrasian equilibrium allocation satisfies the interim incentive and participation constraints of the buyer with belief π. The existence of a Walrasian equilibrium relative to π follows from relatively standard arguments in general equilibrium theory (Lemma 3 in the Appendix) after noting that no trader is satiated, which allows us to show that Walras' law holds for all s.[15]

An allocation (p,x) associated with a Walrasian equilibrium relative to π is called a Walrasian equilibrium allocation relative to π. In Lemma 4 in the Appendix, we show that a Walrasian equilibrium allocation relative to π is an SPO allocation for π. Therefore, we conclude that at least one SPO allocation exists for every π.

4.3 *SPO allocations and equilibrium allocations*

SPO profit vectors may not be feasible, because by definition we ignore the seller's incentive constraints. However, if they are feasible, they are equilibrium allocations because they are immune to deviations. This is established in the next proposition by showing that if $\hat{V}$ is an SPO profit vector, then for any deviation to a mechanism $\mathcal{M}$ and direct

[15] Walras' law is key for existence: Mylovanov and Tröger (2012) show that an SPO allocation may not exist when there is no transfer and a trader may be satiated. To address nonexistence, Mylovanov and Tröger (2012) extend the notion of SPO allocation by defining the notion of strongly neologism-proof allocation, and they show that a strongly neologism-proof allocation exists in generalized private value environments, even if there is no transfer.

certification F to the buyer, a belief $\pi^* \in \Delta_F(S)$ consistent with F and a continuation equilibrium profit vector V^* given π^* exist such that $V^*(s) \leq \hat{V}(s)$ for every $s \in F$.

A particularly easy deviation to understand is the deviation of the seller to full-information disclosure: $F = \{s\}$. In that case, the only consistent belief of the buyer is $\pi^*(s) = 1$, and, therefore, the best profit type s can get is the full-information profit. This deviation cannot be profitable for the seller, because SPO interim profits are never lower than full-information interim profits. To see this, assume to the contrary that $V^*(s)$ is the full-information profit of type s and that $V^*(s) > \hat{V}(s)$. Then the profit vector $\tilde{V}$ such that $\tilde{V}(s) = V^*(s)$ and $\tilde{V}(s') = \hat{V}(s')$ for $s' \neq s$ is π^*-buyer-feasible and dominates $\hat{V}$, which contradicts the fact that $\hat{V}$ is SPO. The proof of the next proposition generalizes the argument for any disclosure $F \subseteq S$.

Proposition 2. *If an SPO allocation is feasible for the prior, then it is an expectational equilibrium allocation of the mechanism proposal game.*

Proof. Let $(\hat{p}, \hat{x})$ be a feasible SPO allocation for the prior belief $\pi^0 \in \Delta(S)$. Let $\hat{V} = (\hat{V}(s))_{s \in S} \in \mathbb{R}^{|S|}$ denote the corresponding profit vector.

To show that $(\hat{p}, \hat{x})$ is an expectational equilibrium, we have to show that for any deviation to a mechanism $\mathcal{M}$ and certification F, there exists an off-path belief $\pi^* \in \Delta_F(S)$ and an equilibrium of $\mathcal{M}$ given this belief that yields an interim profit $V^*(s)$ that is not better than $\hat{V}(s)$ for every $s \in F$. Because the continuation game induced by $\mathcal{M}$ is finite, $\mathcal{M}$ has at least one continuation equilibrium for any off-path belief $\pi \in \Delta_F(S)$. Let $\mathcal{V}(\pi)$ be the convex hull of the set of equilibrium profit vectors of the seller when the off-path belief is π.[16] Let $\mathcal{V} \subseteq \mathbb{R}^{|S|}$ be the convex hull of $\bigcup_{\pi \in \Delta_F(S)} \mathcal{V}(\pi)$.

For every profit vector $V = (V(s))_{s \in S} \in \mathcal{V}$ and belief $\pi \in \Delta_F(S)$, define the correspondence

$$(\pi, V) \mapsto \left(\arg \max_{\tilde{\pi} \in \Delta_F(S)} \sum_{s \in S} \tilde{\pi}(s)\big(V(s) - \hat{V}(s)\big)\right) \times \mathcal{V}(\pi).$$

The cross-product of the belief and the profit sets, $\Delta_F(S) \times \mathcal{V}$, is convex and compact, and the correspondence is upper hemicontinuous and convex-valued, so from Kakutani's fixed point theorem, it has a fixed point $(\pi^*, V^*) \in \Delta_F(S) \times \mathcal{V}$. That is, there exists (π^*, V^*) such that $\pi^* \in \arg\max_{\pi \in \Delta_F(S)} \sum_{s \in S} \pi(s)(V^*(s) - \hat{V}(s))$ and $V^* \in \mathcal{V}(\pi^*)$.

After any deviation to a mechanism $\mathcal{M}$ and certification F, consider such an off-path belief π^* for the buyer and consider the corresponding continuation equilibrium profit vector V^*. Let $I = \{s \in F : V^*(s) > \hat{V}(s)\}$. Assume to the contrary that I is nonempty; then $\pi^*(s) = 0$ for all $s \notin I$ because π^* maximizes $\sum \pi(s)(V^*(s) - \hat{V}(s))$. Because V^* is a continuation equilibrium profit given π^*, it is feasible given π^*, and thus it is π^*-buyer-feasible. Hence, the profit vector $\tilde{V}$ with $\tilde{V}(s) = V^*(s)$ for $s \in I$ and $\tilde{V}(s) = \hat{V}(s)$ for $s \notin I$ is also π^*-buyer-feasible because $\pi^*(s) = 0$ for $s \notin I$. This implies that $\hat{V}$ is not an SPO profit vector because it is dominated by the π^*-buyer-feasible profit vector $\tilde{V}$, a contradiction. Thus, $I = \varnothing$, which means V^* is not profitable compared to $\hat{V}$ for any type $s \in F$. $\square$

[16] In case of multiple equilibria, the random public signal observed by the seller and buyer before they play the mechanism $\mathcal{M}$ selects one.

4.4 *SPO allocations and ex ante optimality*

In the next proposition, we show that if an SPO allocation is feasible, it is ex ante optimal. Mylovanov and Tröger (2014) provide a similar result in a generalized private value environment. To prove this result we show that the ex ante expected profit resulting from an SPO allocation is at least as high as the ex ante expected profit resulting from an ex ante optimal allocation (notice that an ex ante optimal allocation is feasible by definition, whereas an SPO allocation might not satisfy the seller's incentive contraints). Hence, when the SPO allocation is feasible (i.e., satisfies the seller's incentive contraints), the ex ante expected profits coincide.

Proposition 3. *If an SPO allocation is feasible for the prior, then it is ex ante optimal.*

Proof. Let $\mathbf{V}(t) \equiv E_S V(s,t)$ and $\mathbf{V} \equiv E_{T,S} V(s,t)$. To prove the proposition, we show that if a vector of π^0-buyer-feasible profits $(\hat{V}(s))_{s\in S}$ yields a lower ex ante expected profit than the ex ante optimal profit, then it is not an SPO profit vector for π^0. Let (p^*, x^*) be an ex ante optimal allocation, with corresponding $(V^*(s))_{s\in S}$ and $\mathbf{V}^*$. If $(\hat{V}(s))_{s\in S}$ yields lower ex ante profit, we have $\mathbf{V}^* - \hat{\mathbf{V}} > 0$. Let S_1 denote all seller types for which $\hat{V}(s) \geq V^*(s)$, and let S_2 be the complement of S_1. The set S_2 is nonempty because $\hat{V}$ is not ex ante optimal.

Define an allocation $(\tilde{p}, \tilde{x})$ as

$$\begin{aligned}
&\tilde{p}(s,t) = p^*(s,t) \quad \text{for all } s, t\\
&\tilde{x}(s,t) = \hat{V}(s) + p^*(s,t)v(s,t) \quad \text{for } s \in S_1, t \in T\\
&\tilde{x}(s,t) = \hat{V}(s) + p^*(s,t)v(s,t) + \frac{1}{\sum\limits_{s' \in S_2} \pi^0(s')}\big[\mathbf{V}^*(t) - \hat{\mathbf{V}}\big] \quad \text{for } s \in S_2, t \in T.
\end{aligned}$$

Note that

$$\begin{aligned}
&\tilde{V}(s,t) = \hat{V}(s) \quad \text{for } s \in S_1, t \in T\\
&\tilde{V}(s,t) = \hat{V}(s) + \frac{1}{\sum\limits_{s' \in S_2} \pi^0(s')}\big[\mathbf{V}^*(t) - \hat{\mathbf{V}}\big] \quad \text{for } s \in S_2, t \in T.
\end{aligned}$$

The above two equations imply $\tilde{V}(s) = \hat{V}(s)$ for $s \in S_1$ and $\tilde{V}(s) > \hat{V}(s)$ for $s \in S_2$ because $\mathbf{V}^* - \hat{\mathbf{V}} > 0$. The interim payment of the buyer at the allocation $(\tilde{p}, \tilde{x})$ is

$$\begin{aligned}
E_S\big[\tilde{x}(s,t)\big] &= E_S\big[\hat{V}(s) + p^*(s,t)v(s,t)\big] + \big[\mathbf{V}^*(t) - \hat{\mathbf{V}}\big]\\
&= E_S\big[p^*(s,t)v(s,t)\big] + \mathbf{V}^*(t) = E_S\big[x^*(s,t)\big],
\end{aligned}$$

so the resulting allocation $(\tilde{p}, \tilde{x})$ is π^0-buyer-feasible because (p^*, x^*) is. It is also better for all seller types, and strictly better for those in S_2, than $(\hat{V}(s))_s$. Hence, $(\hat{V}(s))_s$ is not an SPO profit vector for π^0. □

Although a feasible SPO allocation for the prior is ex ante optimal, the reverse is not true. In the introductory example, only *one* ex ante optimal vector for the uniform prior is SPO, namely the profit vector $(3, 2)$. For example, $(2.5, 2.5)$ is ex ante optimal but is not SPO.

From Propositions 1, 2, and 3, we immediately get the following corollary:

Corollary 1. *Under own-type certifiability, an expectational equilibrium exists that is ex ante optimal.*

Summing up, given Propositions 2 and 3, we know that an ex ante optimal expectational equilibrium exists whenever the certifiability structure is such that an SPO allocation satisfies the seller's incentive constraints. We explore this further in the next section.

4.5 *When are SPO allocations incentive compatible for the seller?*

SPO allocations for π satisfy (B-IC) and (B-PC) by definition. In addition, as already observed, SPO interim profits are never lower than full-information interim profits, so they trivially satisfy (S-PC). Hence, whether an SPO allocation is feasible depends only on whether or not the seller's incentive constraints (S-IC) are satisfied. Under own-type certifiability, no seller type can mimic another type, so an SPO allocation is trivially always feasible. For other certifiability structures, feasibility depends also on the seller's utility function and the resulting SPO allocation. In the example of Section 2, the feasible allocations with soft information ($\mathcal{E} = \{S\}$) give the same interim profit to both seller types, which is at most 2.5 for the uniform prior (see Figure 1). The intersection of this set with the set of SPO profit vectors is empty (see Figure 2). Hence, when information is not certifiable at all, the SPO allocation in this example is not feasible. However, if the certifiability structure is such that $\{s_1\} \in \mathcal{E}$, then s_2 cannot mimic s_1 ($s_1 \notin R(s_2)$) and the SPO profit vector $(3, 2)$ is feasible.

Under private value as in Maskin and Tirole (1990) or, more generally, when the buyer's valuation does not depend on the seller's type, an SPO allocation is feasible for every certifiability structure (including the case in which information is soft and the seller can only certify the event S). Indeed, in the fictitious exchange economy, the set of allocations each trader s has access to for a given vector of slacks does not depend on s. It follows that the Walrasian equilibrium allocation is incentive compatible for the seller: if type s wants to mimic type s', we can infer he prefers the slacks of type s'. But type s can afford the slacks that type s' is buying, and, moreover, for given slacks, the set of feasible allocations coincide. With interdependent values, the choices of trader s are not necessarily available to s', so we cannot conclude from the Walrasian equilibrium property that (S-IC) is satisfied.

To better understand how feasibility of SPO depends on the richness of the certifiability structure, let us consider some specific classes of trading environments. First, suppose the seller's cost does not depend on his type. In such a trading environment, the profit that type s gets with the allocation of s' is equal to the profit of type

s': $V(s' \mid s) = V(s')$. Hence, an SPO profit vector $(V(s))_{s\in S}$ satisfies (S-IC) if and only if $R(s) \subseteq \{s' \in S : V(s') \le V(s)\}$. That is, if $V(s') > V(s)$, s' should have evidence that is not available to type s. This condition exactly corresponds to the *rich certifiability* assumption in Okuno-Fujiwara et al. (1990, Assumption 1), and is satisfied, for example, under the condition of unilateral distortion discussed in Green and Laffont (1986).

Second, consider trading environments in which the set of seller types S can be partitioned into K elements $\{S_1, \dots, S_K\}$, where each $k = 1, \dots, K$ corresponds to a different set of product characteristics relevant to the buyer: for every $k = 1, \dots, K$, every $s, s' \in S_k$, and every $t \in T$, $u(s,t) = u(s',t)$. The seller's reservation value $v(s,t)$ can depend arbitrarily on s and t. Consider a coarse version of two-way disprovability in Lipman and Seppi (1995): $s \in S_k$, $s' \in S_{k'}$, $k \neq k'$, implies $\mathcal{E}(s') \nsubseteq \mathcal{E}(s)$, that is, $s' \notin R(s)$. For example, this property is satisfied if $s \in S_k$ can certify characteristic k, that is, $S_k \in \mathcal{E}(s)$. This certifiability structure deters any deviation from type $s \in S_k$ to a report of another characteristic $s' \notin S_k$, that is, it deters lies across categories. Lies within a category cannot be profitable either: this follows from the same Walrasian equilibrium argument as in the generalized private value case: type $s \in S_k$ cannot be better off by reporting $s' \in S_k$, because type s can access the same set of allocations as type s' by buying the same slacks as type s'. Hence, under the coarse version of two-way disprovability, an SPO satisfies (S-IC).

Notice that in this last trading environment, even if each set S_k is certifiable, we cannot restrict attention to each S_k separately and consider SPO profit vectors given a restricted type space S_k. Indeed, the SPO profit vectors for types in S_k are not the same when the state space is S as when the state space is S_k (take our leading example with $\{S_1, S_2\} = \{\{s_1\}, \{s_2\}\}$).

4.6 *SPO allocations and full-information allocations*

In our introductory example, the SPO profit vectors do not coincide with the full-information profit vectors (see Figure 2). Hence, when the SPO profit vector is feasible, information is strictly valuable for the seller because there exists an equilibrium allocation (the SPO allocation) that yields higher ex ante and interim profits (remember that SPO profit vectors are always at least as high as the full-information profit vectors). Otherwise, when the SPO profit vector coincides with the full-information profit vector, information is not valuable for the seller. In that case, if the seller can guarantee his full-information profit vector (e.g., when he can certify $F = \{s\}$ to the buyer for every s), the full-information profit vector is the unique equilibrium profit vector whatever the prior. Knowing when SPO profit vectors and full-information profit vectors coincide is, therefore, interesting.

The following proposition, originally obtained in generalized private value environments by Mylovanov and Tröger (2014), provides necessary and sufficient conditions for full-information and SPO profit vectors to coincide. We say that a profit vector is π-*buyer-feasible ex ante optimal* if it maximizes the ex ante expected profit computed with π, $\sum_{s\in S} \pi(s)V(s)$, under the buyer's interim incentive and participation constraints (B-IC) and (B-PC).

Proposition 4. *The full-information profit vector is an SPO profit vector if and only if it is π-buyer-feasible ex ante optimal for all π.*

Proof. The "only if" part follows immediately from the proof of Proposition 3, which shows that an SPO allocation for π must be π-buyer-feasible ex ante optimal. To show the "if" part, assume the full-information profit vector is not an SPO profit vector. Then, by the definition of SPO, there exist π and a vector of π-buyer-feasible profits that dominates (with a strict inequality for some s with $\pi(s) > 0$) the full-information profit vector. This implies the full-information profit vector is not π-buyer-feasible ex ante optimal. □

The full-information allocation is π-buyer-feasible ex ante optimal in several particular cases, for example, when the buyer's valuation depends only on the seller's type ($u(s, t) = u(s, t') = u(s)$ for every t and t') as in Samuelson (1984) and Myerson (1985). Indeed, in this case, the full-information allocation extracts all surplus, so it is necessarily π-buyer-feasible ex ante optimal for all π: trade occurs with probability 1 at price $u(s) = u(s, t)$ when $u(s, t) > v(s, t)$; otherwise no trade occurs. The standard unravelling argument can be applied in this situation as well. Ordering the seller's types according to quality, the highest quality type $\bar{s}$ has an incentive to certify his type so that the buyer accepts price $u(\bar{s})$; then the second highest quality type must separate from the lower types and so on. Hence, the SPO/full-information allocation is feasible if $u(s') > u(s)$ implies that s cannot mimic s' (i.e., $s' \notin R(s)$), as in Milgrom (1981) and Okuno-Fujiwara et al. (1990).

SPO profit vectors also coincide with the full-information profit vectors when the buyer's valuation depends only on his type ($u(s, t) = u(s', t)$ for every s and s'), as in Maskin and Tirole (1990) and Mylovanov and Tröger (2014). Indeed, because we assume quasi-linear utilities, Maskin and Tirole (1990, Proposition 11) and Mylovanov and Tröger (2014, Proposition 8) apply to this case of our environment, and withholding information does not relax the buyer's incentive and participation constraints. Koessler and Skreta (2016) show how to extend this result in a particular class of environments in which the buyer's valuation also depends on the seller's type. In general, however, relaxing the incentive constraint of the buyer by not revealing information is possible, even when the buyer's valuation is strictly increasing in both s and t and utilities are quasi-linear (see Example 2 in Koessler and Skreta 2016).[17]

4.7 *Are all equilibria SPO?*

We have shown that being SPO is a sufficient condition for a profit vector to constitute an equilibrium profit vector of the mechanism proposal game when the seller's ability to certify information is sufficiently rich. We have also shown that it is ex ante optimal.

[17] Another example in which information is valuable for the seller is provided by Mylovanov and Tröger (2014, Section 8) in a more general trading environment with private values. There, withholding information relaxes the buyer's participation constraints. In their example, the buyer's payoff is not monotonic in his type.

If the seller were able to choose a mechanism before learning his type, he would clearly choose an ex ante optimal mechanism. Hence, a feasible SPO allocation has the strong property that it can be optimally chosen at the ex ante stage, and it is immune to deviations by the seller at the interim stage. But are other equilibria of the mechanism proposal game not ex ante optimal?

For the introductory example, we now show that under own-type certifiability, the SPO profit vector is the unique equilibrium profit vector. This uniqueness is in sharp contrast to equilibrium properties under "soft" information (Maskin and Tirole 1992, Koessler and Skreta 2016). Certifiability leads to a unique equilibrium profit vector, which does not belong to the continuum of equilibrium profit vectors under soft information. In this sense, certification increases the "power" of deviations.

To show that $(3, 2)$ is the unique equilibrium profit vector for the uniform prior, we show that for every other feasible vector, small enough $\varepsilon > 0$ exists such that the following deviation dominates this other feasible vector: the seller does not certify information to the buyer directly ($F = S$) and he chooses the mechanism $\tilde{\mathcal{M}} : \mathcal{E} \times M_S \times M_T \to [0, 1] \times \mathbb{R}$, where M_S is a singleton, $M_T = \{\text{left, right}\}$ and

$\tilde{\mathcal{M}} =$

	Left	Right
$\{s_1\}$	$1, 5-\varepsilon$	$1, 3-\varepsilon$
$\{s_2\}$	$1, 1-\varepsilon$	$1, 2-\varepsilon$
$\{s_1, s_2\}$	$1, -10$	$1, -10$

In every continuation Bayes–Nash equilibrium of $\tilde{\mathcal{M}}$ and for every belief $\tilde{\pi}(s_1)$, the buyer never rejects the mechanism and type s always certifies $\{s\}$. The buyer reports "left" if $\tilde{\pi}(s_1) < \frac{1}{3}$, he reports "right" if $\tilde{\pi}(s_1) > \frac{1}{3}$, and he is indifferent between the two reports when $\tilde{\pi}(s_1) = \frac{1}{3}$. Hence, continuation equilibrium profit vectors induced by $\tilde{\mathcal{M}}$ converge to the convex hull of $(3, 2)$ and $(5, 1)$ as $\varepsilon \to 0$, regardless of the buyer's belief. Because only $(3, 2)$ is feasible for the uniform prior in this set, $(3, 2)$ is the unique equilibrium profit vector for the uniform prior.

By varying the buyer's beliefs, the mechanism $\tilde{\mathcal{M}}$ above generates all SPO profit vectors with interior beliefs as $\varepsilon \to 0$ (see Figures 2 and 3). Hence, regardless of the prior belief of the buyer, an equilibrium must be SPO; otherwise, it is dominated by $\tilde{\mathcal{M}}$ for some ε.

The interesting property of $\tilde{\mathcal{M}}$ is that for every belief and no matter what equilibrium is selected given that belief, the equilibrium profit vector converges to the frontier (SPO) as $\varepsilon \to 0$. Ideally, we would like to design games with such properties for any trading scenario covered in our model. Such games would be "canonical deviations," because a mechanism immune to such deviations would be immune to any deviation. We do not know if such canonical (and finite) mechanisms, generating the upper contour set of all

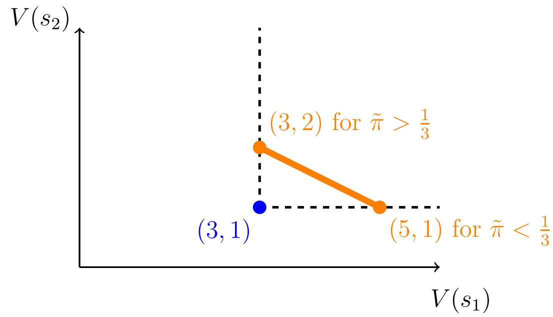

FIGURE 3. Full-information profit vector (at $(3,1)$) and continuation equilibrium profit vectors of $\tilde{\mathcal{M}}$ in the example.

feasible profit vectors as beliefs vary, can be constructed in general. This exploration is left for future research.[18]

5. Conclusion

In this paper, we established that strong Pareto optimal allocations, introduced by Maskin and Tirole (1990) in private value environments, exist in a general bilateral trading model with interdependent values and quasi-linear utilities. We also showed that they are immune to deviations to arbitrary evidence disclosure and mechanism proposal by the seller. A strong Pareto optimal allocation is feasible whenever the set of certifiable statements the seller can make is sufficiently rich. In that case, it is an equilibrium allocation of the informed-principal game. In addition, we showed that this equilibrium allocation is ex ante optimal for the seller. In contrast to private value quasi-linear settings, a strong Pareto optimal allocation typically differs from the full-information allocation.

The earlier papers that predict market unravelling tell us that market forces induce the revelation of all certifiable information, making mandatory disclosure rules unnecessary. Our results highlight that even if product characteristics are fully certifiable, the seller may choose selling procedures that lead to quite different allocations from those obtained when consumers are perfectly informed.

From a theoretical point of view, our analysis suggests that SPO allocations exist and correspond to equilibrium allocations in more general interdependent value environments with certifiable information and transfers, even without quasi-linear utilities.[19] In private value environments without transfers, we know that SPO allocations may fail to exist because some traders are satiated. For such cases, strong neologism-proof allocations introduced by Mylovanov and Tröger (2012) do exist. We conjecture that strong

[18]Maskin and Tirole (1990) get a uniqueness result, but they have to make a sorting assumption and allow infinite mechanisms in an artificial extended game with a third player reporting the buyer's belief to the seller.

[19]For further details, see the Appendix (see footnote 21).

neologism-proof allocations also exist and correspond to equilibrium allocations in general informed-principal problems with interdependent values whenever the certifiability structure is rich enough to guarantee incentive compatibility for the principal.

Appendix: Proof of Proposition 1

We first show that Walras' law holds in the fictitious exchange economy (Lemma 2). Second, we show that a Walrasian equilibrium relative to π exists (Lemma 3). Third, we show that a Walrasian equilibrium profit vector relative to π is an SPO profit vector with prior π (Lemma 4). Therefore, we conclude that at least one SPO profit vector with belief π exists for every π.

Lemma 2. *If* $(c(s,t), c(s,t,t'))_{t,t'} \in D(s \mid \gamma)$, *then* $\sum_{t\in T}\gamma(t)c(s,t) + \sum_{t,t'\in T}\gamma(t,t')c(s,t,t') = 0$.

Proof. The lemma is the standard Walras law, which holds for the same reason as in Maskin and Tirole (1990) and Mylovanov and Tröger (2012) (for nonsatiated types).[20] If, at the optimum for type s, we have $\sum_{t\in T}\gamma(t)c(s,t) + \sum_{t,t'\in T}\gamma(t,t')c(s,t,t') < 0$, then type s can increase the slacks of the participation constraints $c(s,t)$, $t\in T$, by a small constant, independently of t. Therefore, increasing the transfers $x(s,t)$, $t\in T$, by this same constant, independently of t, would increase his indirect interim profit while still satisfying his budget constraint. □

Lemma 3. *For any* $\pi \in \Delta(S)$, *at least one Walrasian equilibrium relative to* π *exists.*

Proof. As in Maskin and Tirole (1990), we show that the indirect interim profit $V_I(s \mid c)$ of each type s in the fictitious economy is continuous and concave, so existence follows by applying the techniques employed in Debreu (1959). We follow below the logic of the proof in Mylovanov and Tröger (2012).

The objective of the maximization problem characterizing $V_I(s \mid c)$ is continuous, and the feasible region defined by (1) and (2) is compact; therefore, from Weierstrass' theorem, the solution value $V_I(s \mid c)$ exists for every $s\in S$ and for $(c(s,t), c(s,t,t'))_{t,t'} \in C(s)$. Because (1) and (2) are linear, the feasible region of the maximization problem is also continuous in the slacks.[21] In addition, the objective is linear. Hence, from the maximum theorem, $V_I(s \mid c)$ is continuous and concave in the slacks.[22]

[20] In Mylovanov and Tröger (2012), this property only applies to traders who are not "satiated," but because we have monetary transfers, at least one participation constraint of the buyer is always binding in our model.

[21] Upper hemicontinuity directly follows from the continuity of the left- and right-hand sides of the inequalities that characterize the feasible region. Lower hemicontinuity follows from the linearity of the system of inequalities; see, for example, Daniilidis et al. (2013, Proposition 3.10) or Dontchev and Rockafellar (2009, Theorem 3C.3).

[22] In Mylovanov and Tröger (2012), the constraints are continuous but not necessarily linear in the allocations. Hence, they provide a more elaborate proof, which uses only the fact that $V_I(s \mid c)$ is upper semicontinuous. Under a mild additional separability condition, they show that the demand correspondence is also upper hemicontinuous and, therefore, Kakutani's fixed point theorem can still be applied.

Because transfers are bounded, we can replace $C(s)$ by a compact subset of slacks $C^*(s) \subset C(s)$. Hence, by the maximum theorem, $D(s \mid \gamma)$ is nonempty, compact-valued, and upper hemicontinuous in γ. It is also convex-valued because $V_I(s \mid c)$ is concave.

Let Δ be the simplex of price vectors, that is, prices such that $\gamma(t), \gamma(t, t') \geq 0$ and $\sum_{t \in T} \gamma(t) + \sum_{t,t'} \gamma(t, t') = 1$. Consider the correspondence $h : \prod_{s \in S} C^*(s) \rightrightarrows \Delta$, where

$$h(c) = \arg\max_{\gamma \in \Delta} \sum_{s \in S} \pi(s) \left(\sum_{t \in T} \gamma(t) c(s, t) + \sum_{t,t' \in T} \gamma(t, t') c(s, t, t') \right).$$

The correspondence h is convex-valued and, by the maximum theorem, it is upper hemicontinuous. Consider the correspondence

$$\left(\prod_{s \in S} C^*(s) \right) \times \Delta \rightrightarrows \left(\prod_{s \in S} C^*(s) \right) \times \Delta$$

$$(c, \gamma) \mapsto \left(\prod_{s \in S} D(s \mid \gamma) \right) \times h(c).$$

By Kakutani's theorem, this correspondence has a fixed point (c^*, γ^*). By construction, we have $(c^*(s, t), c^*(s, t, t'))_{t,t' \in T} \in D(s \mid \gamma^*)$ for every $s \in S$. So to show that (c^*, γ^*) is a Walrasian equilibrium, it remains to show (3) and (4). Assume to the contrary that (3) fails (the same logic applies for (4)), that is,

$$\sum_{s \in S} \pi(s) c^*(s, \tilde{t}) > 0 \quad \text{for some } \tilde{t} \in T.$$

Consider the price vector γ such that $\gamma(\tilde{t}) = 1$ (and 0 for every other slack). This yields

$$\sum_{s \in S} \pi(s) \left(\sum_{t \in T} \gamma(t) c^*(s, t) + \sum_{t,t' \in T} \gamma(t, t') c^*(s, t, t') \right) = \sum_{s \in S} \pi(s) c^*(s, \tilde{t}) > 0.$$

But the budget constraints imply

$$\sum_{s \in S} \pi(s) \left(\sum_{t \in T} \gamma^*(t) c^*(s, t) + \sum_{t,t' \in T} \gamma^*(t, t') c^*(s, t, t') \right) \leq 0,$$

which yields a contradiction with $\gamma^* \in h(c^*)$. □

Lemma 4. *Any Walrasian equilibrium profit vector relative to π is an SPO profit vector with belief π.*

Proof. Let $(\gamma(t), \gamma(t, t'))_{t,t' \in T}$ and $(c(s, t), c(s, t, t'))_{s \in S, t,t' \in T}$ be a Walrasian equilibrium relative to π, with interim profits $V_I(s \mid c)$, $s \in S$. Assume to the contrary that it is not SPO; then $\hat{\pi}$ and a $\hat{\pi}$-buyer-feasible allocation $(\hat{p}, \hat{x})$ exist such that

$$\hat{V}(s) \geq V_I(s \mid c) \tag{5}$$

for every $s \in S$, with a strict inequality for some s with $\hat{\pi}(s) > 0$. Let $(\hat{c}(s,t), \hat{c}(s, t, t'))_{s\in S, t, t' \in T}$ be slack variables associated with $(\hat{p}, \hat{x})$, that is, slacks such that

$$\sum_{s\in S} \hat{\pi}(s)\hat{c}(s,t) \leq 0 \quad \text{and} \quad \sum_{s\in S} \hat{\pi}(s)\hat{c}(s,t,t') \leq 0 \quad \text{for every } t, t' \in T. \tag{6}$$

Because $(\gamma(t), \gamma(t,t'))_{t,t'\in T}$ and $(c(s,t), c(s,t,t'))_{s\in S, t,t'\in T}$ is a Walrasian equilibrium, (5) and Lemma 2 imply

$$\sum_{t\in T} \gamma(t)\hat{c}(s,t) + \sum_{t,t'\in T} \gamma(t,t')\hat{c}(s,t,t') \geq \sum_{t\in T} \gamma(t)c(s,t) + \sum_{t,t'\in T} \gamma(t,t')c(s,t,t') = 0$$

for every s, with a strict inequality whenever $\hat{V}(s) > V_I(s \mid c)$.

Hence,

$$\sum_{s\in S} \hat{\pi}(s)\left[\sum_{t\in T} \gamma(t)\hat{c}(s,t) + \sum_{t,t'\in T} \gamma(t,t')\hat{c}(s,t,t')\right] > 0.$$

Thus, $\sum_{s\in S} \hat{\pi}(s)[\gamma(t)\hat{c}(s,t) + \gamma(t,t')\hat{c}(s,t,t')] > 0$ for some (t,t'), which implies $\sum_{s\in S} \hat{\pi}(s)\hat{c}(s,t) > 0$ or $\sum_{s\in S} \hat{\pi}(s)\hat{c}(s,t,t') > 0$ for some (t,t'), a contradiction to (6). □

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