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Information and targeted spending

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We present an electoral theory on the public provision of local public goods to an imperfectly informed electorate. We show that electoral incentives lead to greater spending if the electorate is not well informed. A more informed electorate induces candidates to target funds only to specific constituencies, which can reduce aggregate welfare.

Keywords. Local public goods, information, elections, targeted spending.

JEL classification. D72, D82, H41.

1. Introduction

During electoral campaigns, voters pay only limited attention to candidates' policy proposals. In particular, they pay more attention to any proposal that directly affects them and their districts than to other proposals. As a result, voters end up with an information asymmetry: they are better informed about policy proposals targeted to their own district than about proposals targeted to other districts. For instance, U.S. survey data show that voters in Michigan were better informed about the 2008 bailout of the Michigan auto industry than voters in other states; similarly, voters in offshore drilling Louisiana were better informed about offshore drilling proposals; and voters in states bordering Mexico (TX, NM, AZ, and CA) were better informed about border control policies (see Appendix A.1 for details).

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We study how this information asymmetry affects the policy proposals for targeted spending that candidates announce during election campaigns and execute once in office. A literature on government transparency suggests that distortive spending decisions occur because voters are imperfectly informed and that inefficiencies would be eliminated if voters were fully informed.[1] We show that this conjecture does not hold in the context of a central government's targeted spending on local public goods. Given that each local public good project is efficient if its social value exceeds the cost of provision, a government may incur two types of inefficiencies: overspending, by financing projects that are inefficient, or underspending, by not financing efficient projects.[2] We show that a society with a more informed electorate does not resolve these inefficiencies.

We present an electoral theory of local public good provision in a society with multiple districts. Two candidates compete by proposing to provide a local public good to any number of these districts. The benefits of provision—and, hence, the efficiency of provision—can vary across districts. The candidate who wins the election implements her proposal and the cost is paid by common taxation across all districts. Voters observe candidates' proposals for their district, but they only observe proposals for other districts with positive probability π. Parameter π measures the symmetry of voters' information about proposals for their own district and for other districts: if π is low, we say that the information asymmetry is large, and if π is high, we say that the asymmetry is small.

Spending varies with this information asymmetry. If voters' information is very asymmetric ($\pi < \frac{1}{2}$), each voter is likely to vote based solely on what politicians propose for her district. In response, candidates pander to every voter by offering to provide the local public good everywhere. In contrast, if voters' information is more symmetric ($\pi > \frac{1}{2}$), voters are likely to evaluate candidates based on their full slate of proposals, and each voter prefers that fewer projects outside her own district be funded. Candidates respond by targeting spending strategically to a subset of districts to carve out winning majority coalitions, regardless of whether targeting funds to these or any other districts is efficient. In sum, spending in local public goods is driven by information asymmetry, not by efficiency. This finding is consistent with empirical evidence that efficiency criteria play only a limited role in the geographical distribution of government infrastructure investment (Knight (2004), Castells and Solé-Ollé (2005)).

Notably, an increase in voters' information about other districts—which reduces the information asymmetry—can decrease social welfare: if providing a local public good to each district is efficient, we show that this efficient policy is implemented if and only if voters' information is very asymmetric ($\pi < 1/2$), whereas if voters' information is more symmetric, candidates propose a policy of inefficient austerity, providing the local public good to fewer districts.[3]

[1]See Gavazza and Lizzeri (2009), Rogoff and Sibert (1988), Rogoff (1990), Alt and Lassen (2006), and a survey by Eslava (2011).

[2]A given spending policy can also incur both types of inefficiencies. For instance, relative to the welfare maximizing optimum, the U.S. Congress underspends in transportation projects in some districts, while at the same time it overspends in other districts (Knight (2004)).

[3]Note that a fully informed electorate has perfectly symmetric information. Political activists such as the 501(c)(4) nonprofit Ending Spending argue that such an informed electorate would demand and obtain a

We fully characterize the equilibrium for the special case with exactly three districts, allowing local public good provision to be efficient in some districts and inefficient in others. We identify the (large) range of parameters for which an increase in voters' information about other districts leads to an equilibrium reduction in spending and in aggregate welfare.

1.1 *Literature review*

A vast theoretical literature explains targeted redistribution as the equilibrium outcome of an electoral game and argues that candidates aim to buy the votes of a winning majority.[4] This literature studies how political incentives affect redistributive policies under the assumption that voters are fully informed about candidates' proposals.

Downs (1960) suggests that while a well informed electorate would lead to the implementation of the correct policy, alternative policies would be implemented if the electorate is not well informed. Electoral competition with voters who are poorly informed about the state of the world can lead office-motivated politicians to pander, offering the policy that a decisive voter expects to be better for her.[5]

Closer to our work are models in which voters are imperfectly informed about candidates' actions, rather than incompletely informed about candidates' types or about the state of the world: Baron (1994) and Gul and Pesendorfer (2009) assume that some voters are fully informed, while others are uninformed about policy proposals. Glaeser et al. (2005) assume that each voter becomes either informed or uninformed about the policy proposal of each candidate separately, and Boffa et al. (2016) develop a model of political agency in which voters differ in their ability to monitor rent-seeking politicians. Other papers assume that voters may fail to observe politicians' effort (Egorov (2009), Aidt and Shvets (2012)) or preferences (Dhami (2003)).

Closest to us, Gavazza and Lizzeri (2009) study taxation and targeted transfers, and assume that voters may fail to observe the campaign promises of transfers to other voters.[6] In their model, transfers are always inefficient, and absent any informational friction, the equilibrium strategy for candidates is to be completely inactive, and to offer zero taxes and zero transfers to every group. Based on this result, Gavazza and Lizzeri (2009) argue that an increase of transparency in public spending is beneficial. Since in their model the government transfers can do no good, scrutinizing them prevents the government from doing harm. This is, at best, a limited view of government transfers. Our theory of public targeted spending is more general—and more upbeat about the

reduction in wasteful targeted spending. According to its mission statement, "information is the best tool we have to combat the debt crisis and hold politicians accountable." We show that a rational, informed electorate neither demands nor obtains an efficient provision of local public goods.

[4] Ferejohn (1986), Lindbeck and Weibull (1987), Myerson (1993), Dixit and Londregan (1995), Groseclose and Snyder (1996), Banks (2000), Persson and Tabellini (2000), Lizzeri and Persico (2001), Chari et al. (1997), Dal Bó (2007), Roberson (2008), Fernández and Levy (2008), Dekel et al. (2008) and Dekel et al. (2009), Huber and Ting (2013), and Bierbrauer and Boyer (2016).

[5] Heidhues and Lagerlöf (2003), Laslier and der Straeten (2004), Kartik et al. (2013), and Gratton (2014).

[6] See as well subsequent work by Matějka and Tabellini (2017).

role of government—in the sense that we do not assume that targeted spending is necessarily inefficient.

Our advances are twofold. First, we study electoral competition and local public good provision with voters who are imperfectly informed about candidates' policies, an assumption that is more consistent with the empirical evidence about voters' information (Campbell et al. (1980), Bartels (1986), and Alvarez (1997)). Second, by jointly considering efficient and inefficient projects in a unified theory, we are able to better explain the electoral pressures that lead to inefficient targeted spending policies than alternative theories that study only inefficient or only efficient projects in isolation.[7]

2. Model

Overview

We present an electoral competition model in which candidates compete by promising local public good provisions to several electoral districts. There are two candidates A and B, and n voters, one per district. Each candidate chooses a set of districts and proposes to provide a local public good to these districts, with costs covered by general taxation of all districts. A voter learns whether her district is included in each of the proposals and votes for one of the candidates or abstains. The candidate with the most votes wins and implements her proposal.

Players

The set of players is $N \equiv C \cup V$, where $C \equiv \{A, B\}$ is the set of candidates and $V \equiv \{1, \dots, n\}$ is the finite set of voters, with $n \geq 3$.

Candidates' strategies

For each candidate $j \in C$, the set of pure strategies is $S \equiv \{0, 1\}^n$, with arbitrary candidate strategy $s \equiv (s_1, \dots, s_n)$. For each $i \in V$, $s_i = 0$ denotes no provision of the local public good to (the district of) voter i, and $s_i = 1$ denotes provision to this district. Let $\Sigma \equiv \Delta(S)$ denote the set of mixed candidate strategies, let $\sigma \in \Sigma$ denote an arbitrary mixed candidate strategy, and for each $s \in S$, let $\sigma(s)$ denote the probability that σ assigns to s.[8] We say that $\sigma \in \Sigma$ is strictly mixed if $|\{s \in S : \sigma(s) > 0\}| \geq 2$ and that σ is totally mixed if $|\{s \in S : \sigma(s) > 0\}| = |S|$.

For each $j \in C$, let $\sigma^j \in \Sigma$ denote the mixed strategy chosen by candidate j and let $\sigma^C \equiv (\sigma^A, \sigma^B) \in \Sigma^2 \equiv \Sigma \times \Sigma$ denote the candidates' chosen strategy profile.

[7]Our theory relates as well to other analyses of the role of information over political outcomes. Information is typically beneficial (Strömberg (2004), Besley and Burgess (2002)). However, some kinds of information are detrimental in specific contexts: public information may induce agents to disregard useful private signals (Prat (2005) and Morris and Shin (2002)) or reduce voluntary contributions toward the private provision of a public good (Teoh (1997)). We identify a third instance in which more information can be detrimental for voters.

[8]For any finite set X, $\Delta(X) \equiv \{w \in [0, 1]^{|X|} : \sum_{x \in X} w_x = 1\}$ denotes the set of probability distributions over X, where $|X|$ denotes the size of X and w_x denotes the probability attached to element x.

For each $j \in C$ and each $i \in V$, let $p_i^j \in \{0, 1\}$ denote the realization of the proposal of candidate j to district i, and let $p^j \equiv (p_1^j, \dots, p_n^j) \in \{0, 1\}^n$ and $p_i \equiv (p_i^A, p_i^B) \in \{0, 1\}^2$.

Timing and information

First, each candidate $j \in C$ chooses $\sigma^j \in \Sigma$ and pair of proposals $(p^A, p^B) \in (\{0, 1\}^n)^2$ is realized.[9]

Second, Nature determines whether all information is fully revealed. Information is fully revealed with probability $\pi \in (0, 1]$. If information is fully revealed, each voter $i \in V$ observes (p^A, p^B). If information is not fully revealed, each voter i observes only (p_i^A, p_i^B) and remains uninformed about proposals in any district other than her own.

Third, each voter votes for one of the candidates or abstains. The candidate with the most votes wins, with ties broken randomly, the proposal of the winning candidate is implemented, and payoffs accrue.

The assumption that each voter is fully informed about funding in his/her district is for ease of exposition: our theory and results generalize if there is some probability that voters are not informed about any policy proposal.[10]

Voters' strategies

If information is fully revealed, each voter's problem is straightforward: a voter with full information votes for the candidate whose proposal she prefers. For simplicity, we collapse the branch of the game in which information is fully revealed, directly imputing to voters the payoffs that accrue (as detailed below) if each voter plays the undominated strategy of voting for the candidate whose proposal she prefers and abstains if she is indifferent. Denote abstention by $\varnothing$.

If information is not fully revealed, each voter $i \in V$ observes $p_i \in \{(0, 0), (0, 1), (1, 0), (1, 1)\}$.

A pure strategy for any voter is a mapping $z: \{(0, 0), (0, 1), (1, 0), (1, 1)\} \longrightarrow \{A, B, \varnothing\}$ and $z(p_i)$ is the vote cast according to z after observing p_i. Let $Z \equiv \{A, B, \varnothing\}^4$ denote each voter's strategy set and let $z_i \in Z$ be the strategy chosen by voter i.

Payoffs

Candidates are purely office motivated. The payoff for each candidate $j \in C$ is equal to the probability that j wins the election.

Voters care about the local public good in their district and about the total cost of public good provision. The cost of providing the local public good to each district is normalized to 1. Each voter $i \in V$ enjoys a benefit β_i if the local public good is provided

[9] Note that $\forall j \in C$, $\forall s \in S$, $\Pr[p^j = s] = \sigma^j(s)$.

[10] Formally, for any $\pi_{\text{In}} \in (0, 1)$, we could assume instead that voters are informed only about funding in their district with probability $(1 - \pi)\pi_{\text{In}}$ and fully informed with probability $(\pi)\pi_{\text{In}}$, so they are fully uninformed with probability $1 - \pi_{\text{In}}$. Voters who are completely uninformed do not affect the candidates' equilibrium strategies (in equilibrium, these voters abstain).

to district i and receives no benefit from provision to other districts. All districts share the total cost of public good provision equally. Therefore, if candidate $j \in C$ proposes $p^j \in \{0, 1\}^n$ and wins the election, then voter i obtains a payoff

$$\beta_i p_i^j - \frac{1}{n} \sum_{h \in V} p_h^j.$$

We allow for heterogeneity in β_i across districts. We say provision of the local public good to district i is efficient if $\beta_i > 1$ and is inefficient if $\beta_i < 1$. We say a policy p^j is efficient if it provides the local public good to each district in which provision is efficient ($\beta_i > 1 \Longrightarrow p_i^j = 1$) and it does not provide it to any district in which provision is inefficient ($\beta_i < 1 \Longrightarrow p_i^j = 0$). An efficient policy maximizes aggregate welfare. We say that a policy p^j underspends if it does not provide the local public good to a district in which provision would be efficient ($\beta_i > 1$ but $p_i^j = 0$).

We assume that $\beta_i > (n+1)/2n$ for each $i \in V$, which rules out inefficiencies so severe that any majority of districts prefers not to provide the local public good to any district, than to provide it only to districts in this majority.[11] We also assume that a voter i is not indifferent between any proposal that funds the local public good in her district and no provision to any district, i.e., we assume that for any integer $k \in ((n+1)/2, n]$ and any $i \in V$, $\beta_i \neq k/n$.

Beliefs

If information is not revealed, each voter $i \in V$ computes her expected payoff if $j \in \{A, B\}$ wins based on her observation of $p_i^j \in \{0, 1\}$ and on her conjectures about candidate j's play. In an equilibrium in which candidate j plays σ^j, for each $i \in V$, if p_i^j is consistent with σ^j, voter i uses Bayes rule, p_i^j, and σ^j to form expectations over p^j. If p_i^j is not consistent with σ^j, then we assume that voter i uses Bayes rule, p_i^j, and the limit of a sequence of totally mixed strategies $\{\sigma_t^j\}_{t=1}^{\infty}$ that converges to σ^j, to form expectations over p^j.

Solution concept

We assume that candidates are strategic, rational expected utility maximizers. Voters are sequentially rational (Kreps and Wilson (1982)).

To rule out uninteresting equilibria in which no voter is pivotal because all voters vote for the same candidate, we restrict attention to equilibria in which both candidates play the same strategy and in which voters, if strategically indifferent, vote for the candidate whose expected proposal they sincerely prefer, and if they are again indifferent, they abstain.

[11] If the benefit of provision is less than $(n+1)/2n$ in every district, the efficient policy of no provision is a Condorcet winner (it is simple majority preferred to any other proposal) and it can be sustained in equilibrium regardless of whether the electorate is informed.

Even with these restrictions, we face the challenge of multiplicity of equilibria. Following Myerson's (1978) idea of "properness," we resolve this challenge by requiring any voter who observes a deviation by candidate j to form beliefs consistent with the premise that j is infinitely more likely to have chosen a deviation that is less costly for j than a costlier one. We provide a formal definition of the beliefs and the equilibrium notion in Appendix A.3.

3. Results

We first consider a society in which the provision of the local public good to each district is efficient.

We show that the equilibrium outcome is efficient if and only if the electorate is unlikely to become informed. If the electorate is not informed, each district bases its vote on local information alone, which induces universal provision, whereas if the electorate is informed, then voters also condition their vote on provision to other districts, which they oppose, driving down overall spending and with it, efficiency.

Proposition 1. *Assume $\beta_i > 1$ for each $i \in V$. An equilibrium exists.*

If $\pi \in (0, \frac{1}{2})$, the equilibrium is pure, unique, and efficient: it provides the local public good to every district (universal provision).

If $\pi \in (\frac{1}{2}, 1]$, all equilibria are in strictly mixed strategies and, therefore, inefficient, underspending in expectation on local public goods.

The intuition for the equilibrium with universal provision if the electorate is unlikely to be informed ($\pi \in (0, \frac{1}{2})$) is as follows. Consider voter's beliefs such that a voter i who observes a deviation to $p_i^j = 0$, believes that the deviating candidate j has proposed provision to $n-1$ districts, that is, to all districts except i. If information is not revealed, voter i with these beliefs votes against the deviating candidate j; since information is unlikely to be revealed, the deviating candidate is likely to lose, making the deviation unprofitable. These beliefs satisfy the restriction that voters who observe a deviation must believe candidates to have played the deviation most profitable to them, because offering the local public good to $n-1$ districts is indeed the best deviation for each candidate.

In contrast, if the electorate is likely to be informed ($\pi > \frac{1}{2}$), the branches of the game in which information is revealed become determinant. No pure equilibrium holds: if a candidate uses a pure strategy to provide the local public good to k districts, the other candidate can deviate to propose provision to only $k-1$ of these districts, and all voters except the excluded one prefer the deviation, so the deviator wins if information is revealed, which occurs with probability $\pi > \frac{1}{2}$. Since in any equilibrium, both candidates win with equal probability, the deviation is then profitable. If a candidate proposes not to provide the local public good to any district, the other candidate can propose to provide it to $(n+1)/2$ districts if n is odd or to $n/2+1$ if n is even, and the deviator wins if information is revealed.

We provide a more detailed exploration of the relation between information and local public good spending in a society with three districts ($n = 3$). For this special case,

Table 1. Mixed candidate strategies' weights

	$s=(0,0,0)$	$\forall s \in S_1$	$\forall s \in S_2$	$s=(1,1,1)$
$\sigma^\pi(s)$	0	$\frac{2\pi-1}{10\pi-3}$	$\frac{1}{10\pi-3}$	$\frac{4\pi-3}{10\pi-3}$
$\hat{\sigma}^\pi(s)$	$\frac{4\pi-3}{10\pi-3}$	$\frac{1}{10\pi-3}$	$\frac{2\pi-1}{10\pi-3}$	0

we relax the assumption that provision to each district is efficient, allowing provision to be inefficient in an arbitrary subset of districts. We fully characterize the set of equilibria in this environment. We use the following notation to precisely describe the equilibrium strategies. Let β_{med} be the median value of $\{\beta_1, \beta_2, \beta_3\}$. Define the voter strategy $z^* \in Z$ by $z^*((0,0)) = z^*((1,1)) = \varnothing$, $z^*((0,1)) = B$, and $z^*((1,0)) = A$. For $k \in \{1,2\}$, define $S_k \equiv \{s \in S : \sum_{i=1}^n s_i = k\}$; that is, S_k is the set of strategies that provide the local public good to exactly k districts. In addition, for each $\pi \in (0,1]$, define the mixed candidate strategies $\sigma^\pi \in \Sigma$ and $\hat{\sigma}^\pi \in \Sigma$ by the weights in Table 1.

Proposition 2. *Assume $n=3$. An equilibrium exists. In all equilibria, each voter i plays $z_i = z^*$. Let $\hat{\pi} \equiv (11+\sqrt{61})/20$. The equilibrium is unique for any $\pi \in (0,1]$ except $\pi = \frac{1}{2}$ and $\pi = \hat{\pi}$.*

(i) If $\pi \in (0, \frac{1}{2})$, candidates play the pure strategy $(1,1,1)$ (universal provision).

(ii) If $\pi \in (\frac{1}{2}, \frac{3}{4}]$, candidates play the mixed strategy σ with $\sigma(s) = \frac{1}{3}$ for any $s \in S_2$ (provision to two randomly chosen districts).

(iii) For any $\pi \in (\frac{3}{4}, \hat{\pi})$ and for any $\pi \in [\hat{\pi}, 1]$ if $\beta_{\text{med}} > 1$, the equilibrium is unique and candidates play σ^π.

(iv) For any $\pi \in (\hat{\pi}, 1]$, if $\beta_{\text{med}} < 1$, candidates play $\hat{\sigma}^\pi$.

(v) If $\pi = \hat{\pi}$ and $\beta_{\text{med}} < 1$, a strategy profile $((\sigma,\sigma), z_V)$ is an equilibrium if and only if $z_i = z^*$ for each $i \in V$ and $\sigma = \lambda\sigma^\pi + (1-\lambda)\hat{\sigma}^\pi$ for some $\lambda \in [0,1]$.

If the electorate is unlikely to be informed ($\pi \in (0, \frac{1}{2})$) and provision is inefficient in each district ($\beta_i \in (\frac{2}{3}, 1)$ for each $i \in \{1,2,3\}$), then solution concepts that allow great freedom to off-path beliefs, such as perfect Bayesian equilibrium or sequential equilibrium (Kreps and Wilson (1982)), offer an indeterminate prediction: a sequential equilibrium with no provision to any district and another with universal provision both hold, supported by beliefs such that a voter who observes a deviation believes that the deviator has proposed provision to every other district.

In contrast, once we refine voters' off-path beliefs as in our solution concept, then we find that even if provision is inefficient in each district, only the equilibrium with universal provision holds.

In support of this sharper prediction, we note that in a laboratory experiment with $\pi = 0.25$ and $\beta_i = 0.9$, 94% of observed equilibrium play corresponded to the equilibrium with universal provision (Eguia et al. (2014, Section 4.3)).

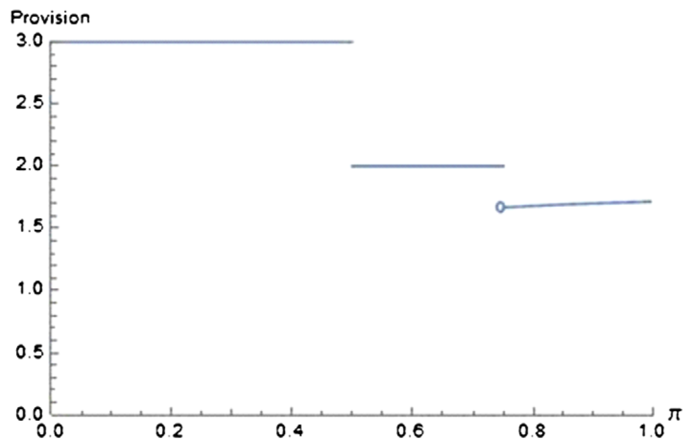

Figure 1. Expected provision if $\beta_{\text{med}} > 1$.

From Proposition 2, we obtain the following corollary on the total expenditure on local public good provision.

Corollary 1. *Assume* $n = 3$. *In the unique equilibrium, the expected number of districts that receive the public good is:*

(i) three if $\pi \in (0, \frac{1}{2})$

(ii) two if $\pi \in (\frac{1}{2}, \frac{3}{4}]$

(iii) in $[\frac{35}{21}, \frac{36}{21}]$ *if* $\pi \in (\frac{3}{4}, \hat{\pi})$ *or* $\pi \in (\hat{\pi}, 1]$ *and* $\beta_{\text{med}} > 1$

(iv) in $[\frac{27}{21}, \frac{30}{21}]$ *and strictly decreasing in* π *if* $\pi \in (\hat{\pi}, 1]$ *and* $\beta_{\text{med}} < 1$.

Figure 1 illustrates the comparative static on aggregate spending as a function of the probability that the electorate becomes informed, for the case in which the median benefit for a district β_{med} is greater than 1. A similar figure obtains if $\beta_{\text{med}} < 1$ (see Figure 2 in Appendix A.2).

If the electorate is not informed, each district bases its vote on local information alone, which induces universal provision, whereas, if the electorate is informed, then voters also condition their vote on provision to other districts, which they oppose regardless of efficiency considerations, driving down overall spending.

However, there are regions over which spending is locally increasing in information: while we can partition the range of the information parameter π into three (if $\beta_{\text{med}} > 1$) or four (if $\beta_{\text{med}} < 1$) intervals such that total spending decreases as π increases from any interval to a higher one, an increase of π within the third interval leads to increased spending.

The intuition for this local increase for the case $\beta_{\text{med}} > 1$ is as follows. For $k \in \{1, 2\}$, let σ^k denote the mixed strategy consisting of provision to k districts, randomizing which ones. For $\pi \in (\frac{1}{2}, \frac{3}{4})$, the equilibrium proposal is σ^2. The payoff from deviating to σ^1 increases in π, because conditional on information being revealed, j playing σ^1

wins against $-j$ playing σ^2 with probability $\frac{2}{3}$ (j loses if information is not revealed). At $\pi = \frac{3}{4}$, there is a discontinuity: the equilibrium proposal σ^2 and the deviation σ^1 yield the same expected payoff. For $\pi > \frac{3}{4}$, the payoff of deviating to σ^1 is higher and the equilibrium breaks down. The new mixed equilibrium for $\pi \in (\frac{3}{4}, 1]$ re-attains equality in the expected payoff from playing σ^1 or σ^2 by introducing a positive weight to playing full provision: $\sigma((1,1,1)) > 0$. If information is revealed, full provision defeats any realization of σ^1, but it is defeated by any realization of σ^2. So playing full provision lowers the payoff of playing σ^1 and increases the payoff of σ^2. The weight of full provision $\sigma((1,1,1))$ required to equate the payoff of σ^1 and σ^2 monotonically increases in π from 0 for $\pi = \frac{3}{4}$ to $\frac{1}{7}$ for $\pi = 1$. As a result, the expected spending in this mixed equilibrium increases monotonically in the interval $\pi \in (\frac{3}{4}, 1]$ from 1.67 to 1.71, while remaining far below the expected spending of 3 in the equilibrium with $\pi \in (0, \frac{1}{2})$ or 2 in the equilibrium with $\pi \in (\frac{1}{2}, \frac{3}{4})$, as shown in Figure 1.

We also obtain the following corollary on welfare. Let $\bar{\beta} = (1/n)\sum_{i=1}^{n} \beta_i$ be the average project benefit.

Corollary 2. *Assume $n = 3$. If $\bar{\beta} > 1$, then for any $\pi_L < \frac{1}{2}$, any $\pi_M \in (\frac{1}{2}, \frac{3}{4}]$, and any $\pi_H \in (\frac{3}{4}, 1]$, aggregate welfare decreases as information π increases from π_L to π_M or from π_M to π_H (if $\bar{\beta} < 1$, this comparative static is reversed).*

For the same reason as in the case of Corollary 1, this monotonicity result does not hold locally within the interval $(\frac{3}{4}, 1]$.

4. Discussion

Our theory relates targeted spending on local public good provision to the magnitude of the asymmetry in voters' information about spending targeted across districts.[12] We predict greater aggregate targeted spending if the information asymmetry is large (π small) than if the information asymmetry is low (π large).

We focus on a specific form of voters' lack of information: voters' imperfect observation of the electoral promises made to other voters. Gavazza and Lizzeri (2009) focus on the same information problem to study government spending under the restriction that case spending targeted to any district is inefficient. They conclude that imperfect observability generates incentives for candidates to offer inefficient targeted spending. Our predictions partially align with Gavazza and Lizzeri's (2009): in the special case that any spending is inefficient, if voters become more informed about spending targeted to other districts, then the inefficiency is reduced. However, we also consider more optimistic scenarios in which provision of local public goods can be efficient. In the extreme case in which all projects under consideration are efficient, efficiency is only achieved if the electorate is not informed about targeted spending in other districts: a reduction in this information asymmetry leads to inefficient underspending, a form of excessive austerity.

[12] Our theory also applies to a society in which voters are divided into interest groups, rather than districts, as in Schipper and Woo (2017) or Boyer et al. (2017).

As uninformed public good consumers, citizens push candidates to provide local public goods in every district, irrespective of efficiency. As informed taxpayers, they push candidates to reduce targeted spending on local public good projects, again, irrespective of the efficiency of these projects.

Appendix

A.1 *Evidence of asymmetric information*

We assume that voters are more informed about proposals for projects in their district than about proposals for projects to be executed in other districts. An implication is that given a proposal to execute a project in a given district, voters in this district are better informed about the project than voters from other districts.

We searched for survey data from 2007 to 2012 about policy proposals to be executed in specific districts or that disproportionately affect specific districts in the United States. We found national surveys that ask factual questions to test respondents' knowledge about three such targeted policy proposals: proposals about offshore drilling, about securing the border, and about the auto bailout.

The 2008 National Annenberg Election Survey (NAES) asked a factual question (question CDd08) on campaign proposals to secure the border to 6,864 subjects, and it asked a factual question (question CFa11) on campaign proposals about offshore drilling to 15,048 subjects. Projects to secure the border directly affect Texas, New Mexico, Arizona, and California. Offshore-drilling policy affects mostly Louisiana.

The Pew Research Center for the People and the Press's Survey on Political Knowledge conducted on March 26–29, 2009, asked a factual question on the auto bailout to 1,003 subjects. The auto bailout program directly affects mostly Michigan. The percentages of correct responses to these questions are as follows.

In each of the three questions, respondents from the state(s) directly affected by the policy were more informed. We checked that this result is not driven by citizens of these six states being generally better informed about all issues: these states perform no better and in fact generally worse than the rest of the United States in answering the two questions that do not directly affect them. Table 3 offers this comparison. Each cell gives the difference in "percentage of correct responses from the state(s) in the row to the question in the column" minus "percentage of correct responses from respondents in the rest of the United States excluding the state(s) in the row."

With the caveats that the 2008 NAES and the 2009 Pew poll are not directly comparable, and the sample size by state for the auto bailout question is small, we can say

Table 2. Percentage of correct answers by local and nonlocal respondents.

On Drilling (NAES)		On the Border (NAES)		On Auto Bailout (Pew)	
Louisiana	66.3	Border states	48.6	Michigan	73.1
Rest of U.S.	57.1	Rest of U.S.	43.9	Rest of U.S.	57.5
Diff.	+9.2	Diff.	+4.7	Diff.	+15.6

Table 3. Difference in percentage of correct responses.

	Drill	Border	Auto
Louisiana	+**9.2**	−1.5	−25.1
Border states	+3.1	+**4.7**	−4.5
Michigan	+3.2	−0.8	+**15.5**

Table 4. Observation count, by question and state.

	Drill		Border		Auto	
	Correct	Total	Correct	Total	Correct	Total
Louisiana	108	163	45	104	6	18
Michigan	316	524	88	200	19	26
Texas	496	837	154	390	39	72
New Mexico	58	94	23	44	7	9
Arizona	124	212	73	127	11	20
California	879	1462	354	681	56	107
U.S. total	8,613	15,048	3,074	6,864	581	1,003

that these states on average outperform the nation by 10 percentage points on knowledge about policies that directly affect them (cells in bold in Table 3) and underperform the nation by 4 percentage points on questions that affect other states (all other cells)—a difference of 14 percentage points.

The raw data on the number of correct responses and total responses to each question are shown in Table 4.

A.2 *Additional figure*

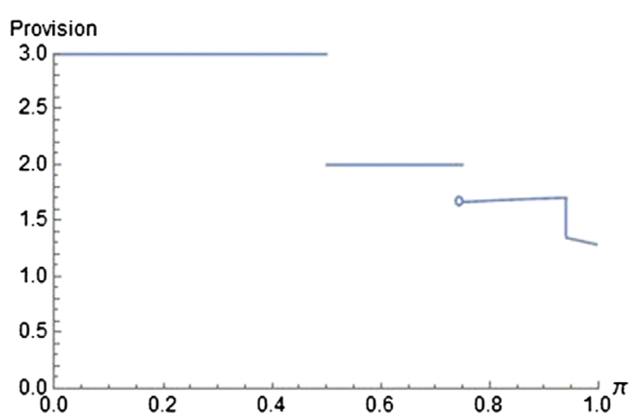

Figure 2. Expected provision if $\beta_{\text{med}} < 1$.

A.3 *Formal notation and definitions*

Notation For each candidate $j \in C$, let $-j$ denote $C \backslash \{j\}$.

Let $z_V \equiv (z_1, \ldots, z_n) \in Z^n$ denote the voters' chosen strategy profile. Let z_V^* be the voter profile in which each voter i uses strategy $z_i = z^*$ (recall that z^* is defined by $z^*((0,0)) = z^*((1,1)) = \varnothing$, $z^*((0,1)) = B$, and $z^*((1,0)) = A$).

Let $\sigma^N \equiv (\sigma^C, z_V) \in \Sigma^2 \times Z^n$ denote the strategy profile chosen by all agents and for each $h \in N$, let $\sigma^{N\backslash\{h\}}$ be the strategy profile of $n-1$ agents constructed by excluding the strategy of agent h from profile σ^N.

For each $j \in C$, for any $\sigma \in \Sigma$, and for any $\sigma^{N\backslash\{j\}} \in \Sigma \times Z^n$, let $u^j(\sigma, \sigma^{N\backslash\{j\}})$ denote j's expected utility given $\sigma^j = \sigma$ and given that all other players play profile $\sigma^{N\backslash\{j\}}$.

Let $s^u \equiv \{1\}^n \in S$ be the candidate strategy of universal provision.

For each $\varepsilon \in (0,1)$, define $\Sigma_\varepsilon \equiv \{\sigma \in \Sigma : \sigma(s) \geq \varepsilon \ \forall s \in S\}$ and $\Sigma_+ \equiv \bigcup_{\varepsilon \in (0,1)} \Sigma_\varepsilon$, so that Σ_+ is the set of all totally mixed strategies.

Beliefs For each candidate $j \in C$ and voter $i \in V$, for any given $\sigma^j \in \Sigma$ and $\{\sigma_t^j\}_{t=1}^\infty \longrightarrow \sigma^j$ such that $\sigma_t^j \in \Sigma_+$ for each $t \in \mathbb{N}$, and for each $s \in S$, let $\omega_i^j(s | p_i^j, \sigma^j, \{\sigma_t^j\}_{t=1}^\infty)$ denote the belief that voter i assigns to $p^j = s$ given that i observes only p_i^j, uses Bayes rule and σ^j to form beliefs about p^j if p_i^j is consistent with σ^j, and uses Bayes rule and $\{\sigma_t^j\}_{t=1}^\infty$ to form beliefs about p^j if p_i^j is not consistent with σ^j. Then, for any $j \in C$ and any $s \in S$ such that $p_i^j = s_i$,

$$\omega_i^j\big(s \big| \big(p_i^j, \sigma^j, \{\sigma_t^j\}_{t=1}^\infty\big)\big) = \begin{cases} \dfrac{\sigma^j(s)}{\displaystyle\sum_{\tilde{s} \in S, \tilde{s}_i = p_i^j} \sigma^j(\tilde{s})} & \text{if } \displaystyle\sum_{\tilde{s} \in S, \tilde{s}_i = p_i^j} \sigma^j(\tilde{s}) > 0 \\ \lim\limits_{t \longrightarrow \infty} \dfrac{\sigma_t^j(s)}{\displaystyle\sum_{\tilde{s} \in S, \tilde{s}_i = p_i^j} \sigma_t^j(\tilde{s}_j)} & \text{otherwise,} \end{cases}$$

and $\omega_i(s | (p_i^j, \sigma^j, \{\sigma_t^j\}_{t=1}^\infty)) = 0$ for any $s \in S$ such that $s_i \neq p_i^j$.

For each candidate $j \in C$, for each voter $i \in V$, for each observed proposal $p_i^j \in \{0,1\}$, and for each $(\sigma^j, \{\sigma_t^j\}_{t=1}^\infty)$ that i uses to form beliefs about p^j, let $u_i(E_i[p^j | (p_i^j, \sigma^j, \{\sigma_t^j\}_{t=1}^\infty)])$ denote the expected value for i of the proposal made by j. Then

$$u_i\big(E_i\big[p^j \big| \big(p_i^j, \sigma^j, \{\sigma_t^j\}_{t=1}^\infty\big)\big]\big) = \beta_i p_i^j - \frac{1}{n} \sum_{s \in S} \bigg(\omega_i^j\big(s \big| \big(p_i^j, \sigma^j, \{\sigma_t^j\}_{t=1}^\infty\big)\big) \sum_{h \in V} s_h \bigg).$$

Additionally, let $EU_i[j | (p_i, \sigma^{N\backslash\{i\}}, \{\sigma_t^C\}_{t=1}^\infty)]$ denote the expected utility for i of voting for j.

Equilibrium concept

Definition 1. For any $\varepsilon \in \mathbb{R}_{++}$, we say that a strategy profile $\sigma^N \equiv (\sigma^C, z_V) \in (\Sigma_+)^2 \times Z^n$ is *ε-proper with respect to C* if it satisfies the following conditions:

(i) For each $j \in C$ and for any $s, \tilde{s} \in S$, if $u^j(s, \sigma^{N\setminus\{j\}}) < u^j(\tilde{s}, \sigma^{N\setminus\{j\}})$, then $\sigma^j(s) < \varepsilon\sigma^j(\tilde{s})$.

(ii) For any $i \in V$, z_i is a best response to $\sigma^{N\setminus\{i\}}$.

Our solution concept is a strategy profile that is the limit of profiles that are ε-proper with respect to C, in which both candidates play the same strategy and in which voters, if strategically indifferent, vote sincerely for the candidate whose expected proposal they prefer. This concept is a variation of Myerson's (1978) notion of an ε-proper strategy profile in which only the set of candidates C is required to use totally mixed strategies, while we allow voters to best respond in a standard fashion. We use the totally mixed candidates' strategy profile to generate voters' beliefs that are consistent with this profile at every voter information set.[13]

Definition 2. We say that a strategy profile $\sigma^N \equiv (\sigma^C, z_V) \in (\Sigma_+)^2 \times Z^n$ is an *equilibrium* $\sigma^A = \sigma^B$ and there exists a pair of convergent sequences $\{\varepsilon_t\}_{t=1}^{\infty} \longrightarrow 0$ and $\{\sigma_t^C\}_{t=1}^{\infty} \longrightarrow \sigma^C$ such that the following statements hold:

(i) For all $t \in \mathbb{N}$, $\varepsilon_t \in \mathbb{R}_{++}$, and $\sigma_t^C \in (\Sigma_+)^2$, (σ_t^C, z_V) is ε_t-*proper* with respect to C.

(ii) For each voter $i \in V$ and candidate $j \in C$, strategy z_i is such that if $EU_i[A|(p_i, \sigma^{N\setminus\{i\}}, \{\sigma_t^C\}_{t=1}^{\infty})] = EU_i[B|(p_i, \sigma^{N\setminus\{i\}}, \{\sigma_t^C\}_{t=1}^{\infty})]$, then

$$
\begin{aligned}
&z_i(p_i) = j \\
&\iff \quad u_i\big(E_i\big[p^j|\big(p_i^j, \sigma^j, \{\sigma_t^j\}_{t=1}^{\infty}\big)\big]\big) > u_i\big(E_i\big[p^{-j}|\big(p_i^{-j}, \sigma^{-j}, \{\sigma_t^{-j}\}_{t=1}^{\infty}\big)\big]\big). \quad (1)
\end{aligned}
$$

Condition (1) above rules out equilibria in which no voter is pivotal because they all vote for the same candidate.

A.4 *Proofs*

Proposition 1. *Assume $\beta_i > 1$ for each $i \in V$. An equilibrium exists.*

If $\pi \in (0, \frac{1}{2})$, the equilibrium is pure, unique, and efficient: it provides the local public good to every district (universal provision).

If $\pi \in (\frac{1}{2}, 1]$, all equilibria are in strictly mixed strategies and, therefore, inefficient: they underspending in expectation on local public goods.

Proof. **Part I.** First we show existence for any $\pi \in (0, 1]$. For each voter $i \in V$, fix $z_i = z^*$. We show below that this is a best response for voter i that satisfies condition (1) in Definition 2.

[13] We do not need to construct a profile of totally mixed voters' strategy profiles because there is no information set in which an agent has observed a voter deviation (voters move last). Candidates anticipate voters will follow their equilibrium strategy, and voters similarly expect other voters to follow their equilibrium strategy.

For each $\varepsilon \in \mathbb{R}_{++}$, define

$$\eta(\varepsilon) \equiv \frac{\varepsilon^{2^n}}{2^n}.$$

Define the correspondence $F_\varepsilon : \Sigma_{\eta(\varepsilon)} \rightrightarrows \Sigma_{\eta(\varepsilon)}$ by $F_\varepsilon(\sigma) = \{\tilde{\sigma} \in \Sigma_{\eta(\varepsilon)} : \tilde{\sigma}(s) \leq \varepsilon\sigma(\tilde{s})$ for any $s, \tilde{s} \in S$ such that $u^A(s, (\tilde{\sigma}, z_V^*)) < u^A(\tilde{s}, \tilde{\sigma}, z_V^*)\}$.

Note that for any $\sigma \in \Sigma_{\eta(\varepsilon)}$, $F_\varepsilon(\sigma) \neq \varnothing$. In particular, for any $\sigma \in \Sigma_{\eta(\varepsilon)}$ and for each $s \in S$, define

$$\lambda(s, \sigma) \equiv \left|\left\{\tilde{s} \in S : u^A\big(s, \big(\sigma, z_V^*\big)\big) < u^A\big(\tilde{s}, \big(\sigma, z_V^*\big)\big)\right\}\right|$$

and

$$\check{\sigma}(s) \equiv \frac{\varepsilon^{\lambda(s,\sigma)}}{\displaystyle\sum_{\tilde{s}\in S} \varepsilon^{\lambda(\tilde{s},\sigma)}}.$$

Then $\check{\sigma} \in F(\sigma)$.

Furthermore, for each $\sigma \in \Sigma_{\eta(\varepsilon)}$, $F_\varepsilon(\sigma)$ is defined by a finite collection of linear weak inequalities, so it is a closed set.

Furthermore, for each $\sigma \in \Sigma_{\eta(\varepsilon)}$, $F_\varepsilon(\sigma)$ is also convex: for any $\sigma \in \Sigma_{\eta(\varepsilon)}$ and for any $\tilde{\sigma}, \sigma' \in F_\varepsilon(\sigma)$ and for any $\lambda \in (0, 1)$, $\bar{\sigma} = \lambda\tilde{\sigma} + (1 - \lambda)\sigma'$ is such that for any $s, \tilde{s} \in S$ such that $u^A(s, (\sigma, z_V^*)) < u^A(\tilde{s}, (\sigma, z_V^*))$,

$$\begin{aligned}\lambda\tilde{\sigma}(s) + (1 - \lambda)\sigma'(s) &\leq \varepsilon\big(\lambda\tilde{\sigma}(\tilde{s}) + (1 - \lambda)\sigma'_A(\tilde{s})\big)\\ \bar{\sigma}(s) &\leq \varepsilon\bar{\sigma}(\tilde{s}).\end{aligned}$$

We next establish that F_ε is upper hemicontinuous**.** Note that for each $\varepsilon \in \mathbb{R}_{++}$, $\Sigma_{\eta(\varepsilon)}$ is compact and $F_\varepsilon(\sigma)$ is closed for any $\sigma \in \Sigma_{\eta(\varepsilon)}$, so F_ε is compact-valued. It suffices then to show that F_ε has a closed graph. For any strategy $\sigma \in \Sigma$ and for any $\delta \in \mathbb{R}_{++}$, let $N_\delta(\sigma) \equiv \{\tilde{\sigma} \in \Sigma : ||\sigma - \tilde{\sigma}|| < \delta\}$ be the open neighborhood of size δ around strategy σ in the standard Euclidean space $\mathbb{R}^{|S|}$. Consider any $\sigma \in \Sigma_{\eta(\varepsilon)}$ and any convergent sequence $\{(\sigma_t, \tilde{\sigma}_t)\}_{t=1}^{\infty} \longrightarrow (\sigma, \tilde{\sigma})$ such that for any $t \in \mathbb{N}$, $\sigma_t \in \Sigma_{\eta(\varepsilon)}$ and $\tilde{\sigma}_t \in F_\varepsilon(\sigma_t)$. We want to establish that $\tilde{\sigma} \in F_\varepsilon(\sigma)$. Suppose not. Then either $\tilde{\sigma} \notin \Sigma_{\eta(\varepsilon)}$ or $\tilde{\sigma} \in \Sigma_{\eta(\varepsilon)}$, and there exist $s, \tilde{s} \in S$ such that $u^A(s, (\sigma, z_V^*)) < u^A(\tilde{s}, (\sigma, z_V^*))$ and $\tilde{\sigma}(s) > \varepsilon\tilde{\sigma}(\tilde{s})$. Suppose first $\tilde{\sigma} \notin \Sigma_{\eta(\varepsilon)}$; then $\exists\bar{\delta} \in \mathbb{R}_{++}$ such that $\hat{\sigma} \notin \Sigma_{\eta(\varepsilon)}$ $\forall\hat{\sigma} \in N_\delta(\tilde{\sigma})$, $\forall\delta \in (0, \bar{\delta})$, which contradicts the statement $\{\tilde{\sigma}_t \in F_\varepsilon(\sigma_t)\ \forall t \in \mathbb{N}$ and $\tilde{\sigma}_t \longrightarrow \tilde{\sigma}\}$.

Suppose instead $\tilde{\sigma} \in \Sigma_{\eta(\varepsilon)}$ and that there exist $s, \tilde{s} \in S$ such that $u^A(s, (\sigma, z_V^*)) < u^A(\tilde{s}, (\sigma, z_V^*))$ and $\tilde{\sigma}(s) > \varepsilon\tilde{\sigma}(\tilde{s})$. Then $\exists\bar{\delta} \in \mathbb{R}_{++}$ such that $\forall\delta \in (0, \bar{\delta})$ and $\forall\hat{\sigma} \in N_\delta(\tilde{\sigma})$, $u^A(s, (\hat{\sigma}, z_V^*)) < u^A(\tilde{s}, (\hat{\sigma}, z_V^*))$ and $\hat{\sigma}(s) > \varepsilon\hat{\sigma}(\tilde{s})$, which again contradicts $\{\tilde{\sigma}_t \in F_\varepsilon(\sigma_t)$ $\forall t \in \mathbb{N}$ and $\tilde{\sigma}_t \longrightarrow \tilde{\sigma}\}$. So $\tilde{\sigma} \in F_\varepsilon(\sigma)$ and F_ε is upper hemicontinuous.

Since $F_\varepsilon(\sigma)$ is a compact and convex set for each $\sigma \in \Sigma_{\eta(\varepsilon)}$, the Cartesian product $F_\varepsilon(\sigma) \times F_\varepsilon(\sigma)$ is also compact and convex. For each $\varepsilon \in \mathbb{R}_{++}$, define $H_\varepsilon : \Sigma_{\eta(\varepsilon)} \rightrightarrows (\Sigma_{\eta(\varepsilon)})^2$ by $H^\varepsilon(\sigma) \equiv F_\varepsilon(\sigma) \times F_\varepsilon(\sigma)$ for each $\sigma \in \Sigma_{\eta(\varepsilon)}$, so H_ε is compact and convex. Further, since F_ε is upper hemicontinuous, the product correspondence $H_\varepsilon = F_\varepsilon \times F_\varepsilon$ is also upper hemicontinuous (Aliprantis and Border (2006, Theorem 17.28)).

Then for each $\varepsilon \in \mathbb{R}_{++}$, H_ε satisfies the conditions of the Kakutani fixed point theorem and it has a fixed point $\sigma_\varepsilon \in \Sigma_{\eta(\varepsilon)}$. Construct the strategy profile $\sigma_\varepsilon^N \equiv ((\sigma_\varepsilon, \sigma_\varepsilon), z_V^*) \in \Sigma^2 \times Z^n$. Note that if players play the profile $\sigma^N = \sigma_\varepsilon^N$, then $\sigma^A = \sigma^B$. Further, $\sigma^A = \sigma_\varepsilon^A$ is a constrained best response to $\sigma_\varepsilon^{N \setminus \{A\}}$, subject to the constraint of $\sigma^A \in \Sigma_{\eta(\varepsilon)}$. Because $\sigma_\varepsilon^A = \sigma_\varepsilon^B$, $z_i((k,k)) = z^*((k,k)) = \varnothing$ for each $k \in \{0,1\}$ is a best response to $\sigma_\varepsilon^{N \setminus \{i\}}$ for each $i \in V$, and because $\beta_i > 1$, $z_i((0,1)) = z^*((0,1)) = B$ and $z_i((1,0)) = z^*((1,0)) = A$ are best responses as well. So $z_i = z^*$ is a best response to $\sigma_\varepsilon^{N \setminus \{i\}}$ for each voter $i \in V$. Furthermore, because each voter $i \in V$ uses a strategy z^*, payoffs for candidate B are the same up to relabeling as for candidate A, and then, since $\sigma_\varepsilon^A = \sigma_\varepsilon^B$, the strategy chosen by A in response to σ_ε^B is also a best response by B to σ_j^A subject to the same constraints for candidate B given by ε that apply to A.

So the fixed point σ_ε^N is a symmetric strategy profile that is ε-proper with respect to $\{A, B\}$.

Then take a sequence $\{\varepsilon_t\}_{t=1}^\infty \longrightarrow 0$ such that $\varepsilon_t \in \mathbb{R}_{++}$ for each $t \in \mathbb{N}$, and take a corresponding sequence of candidates' strategies $\{\sigma_t\}_{t=1}^\infty$ such that each $\sigma_t^N = ((\sigma_t, \sigma_t), z_V^*)$ is ε_t-proper with respect to C. Take a convergent subsequence of $\{\sigma_t\}_{t=1}^\infty$, let σ denote the limit of this convergent subsequence, and let $\{\sigma_t^C\}_{t=1}^\infty \equiv \{(\sigma_t, \sigma_t)\}_{t=1}^\infty$, converging to $\sigma^C \in \Sigma^2$.

We next show that given $(\sigma^C, \{\sigma_t^C\}_{t=1}^\infty)$, $z_V = z_V^*$ satisfies the voting condition (1).

For each voter $i \in V$, because $\sigma^A = \sigma^B$ and $\beta_i > 1$,

$$\begin{aligned} &EU_i\big[A|\big(p_i, \big((\sigma,\sigma), z_V^*\big), \big\{\sigma_t^C\big\}_{t=1}^\infty\big)\big] = EU_i\big[B|\big(p_i, \big((\sigma,\sigma), z_V^*\big), \big\{\sigma_t^C\big\}_{t=1}^\infty\big)\big] \\ &\text{and} \quad u_i\big(E_i\big[p^j|\big(p_i^j, \sigma, \{\sigma_t\}_{t=1}^\infty\big)\big]\big) = u_i\big(E_i[\big[p^{-j}|\big(p_i^{-j}, \sigma, \{\sigma_t\}_{t=1}^\infty\big)\big]\big) \end{aligned}$$

occurs if and only if $p_i \in \{(0,0), (1,1)\}$, so condition (1) requires $z_i(p_i) = \varnothing$ if $p_i \in \{(0,0), (1,1)\}$; $z_i = z^*$ satisfies this restriction. Because $\beta_i > 1$, for $\{j, -j\} = \{A, B\}$,

$$\begin{aligned} &EU_i\big[A|\big(p_i, \big((\sigma,\sigma), z_V^*\big), \big\{\sigma_t^C\big\}_{t=1}^\infty\big)\big] = EU_i\big[B|\big(p_i, \big((\sigma,\sigma), z_V^*\big), \big\{\sigma_t^C\big\}_{t=1}^\infty\big)\big] \\ &\text{and} \quad u_i\big(E_i\big[p^j|\big(p_i^j, \sigma, \{\sigma_t\}_{t=1}^\infty\big)\big]\big) > u_i\big(E_i\big[p^{-j}|\big(p_i^{-j}, \sigma, \{\sigma_t\}_{t=1}^\infty\big)\big]\big) \end{aligned}$$

occurs only if $p_i^j > p_i^{-j}$, and in this case condition (1) requires $z_i(p_i) = j$, which is again satisfied by $z_i = z^*$.

Hence, $\sigma^N = ((\sigma, \sigma), z_V^*)$ is an equilibrium.

Part II. We next prove that $\sigma^N = ((s^u, s^u), z_V^*)$ is an equilibrium for $\pi \in (0, \frac{1}{2})$. Note that given the assumption that $\beta_i > 1$ for each $i \in V$, the efficient policy is universal provision, so $(s^A, s^B) = (s^u, s^u)$ guarantees that the policy outcome is efficient.

Assume $\pi \in (0, \frac{1}{2})$.

We construct a sequence $\{\sigma_t\}_{t=1}^\infty \longrightarrow s^u$ as follows. For each $t \in \mathbb{N}$, let $\varepsilon_t = \frac{1}{2^t}$, and for each $k \in \{0, 1, 2, \dots, n-1\}$ and for each $s \in S_k$, let

$$\sigma_t(s) = \frac{1}{\left(2^t\right)^{n+1-k}}, \tag{2}$$

so $\{\sigma_t(s)\}_{t=1}^\infty \longrightarrow 0$ for any $s \neq s^u$, so $\{\sigma_t\}_{t=1}^\infty \longrightarrow s^u$.

We check that given $\sigma_t^A = \sigma_t^B = \sigma_t$, the weights given by expression (2) along the sequence $\{\sigma_t\}_{t=1}^{\infty}$ satisfy the restriction (i) in Definition 1. We show this for A, and a symmetric argument applies to B. For any sufficiently large $t \in \mathbb{N}$, $u^A(s, (\sigma_t, z_V^*))$ is lexicographic: for any $s, \tilde{s} \in S$, $u^A(s, (\sigma_t, z_V^*)) > u^A(\tilde{s}, (\sigma_t, z_V^*))$ if and only if $\exists l \in \{0, \dots, n\}$ such that

$$u^A\big(s, (\hat{s}, z_V^*)\big) \geq u^A\big(s, (\hat{s}, z_V^*)\big) \quad \forall \hat{s} \in \bigcup_{k=l+1}^{n} S_k$$

$$u^A\big(s, (\hat{s}, z_V^*)\big) > u^A\big(s, (\hat{s}, z_V^*)\big) \quad \text{for any } \hat{s} \in S_l.$$

For any $k \in \{0, \dots, n\}$ and any $\hat{s} \in S_k$, if $(s^A, s^B) = (s, \hat{s})$ and information is not revealed (which occurs with probability $1 - \pi > \frac{1}{2}$), the margin of victory for A is $\sum_{i \in V} s_i - k$, so A wins if $\sum_{i \in V} s_i > k$, ties if $\sum_{i \in V} s_i = k$, and loses if $\sum_{i \in V} s_i < k$. If information is revealed and $\sum_{i \in V} s_i = k$, A ties as well.

Hence, $u^A(s, (\sigma_t, z_V^*)) > u^A(\tilde{s}, (\sigma_t, z_V^*))$ if and only if $\sum_{i \in V} s_i > \sum_{i \in V} \tilde{s}_i$, in which case, according to expression (2), $\sigma_t(s) \geq 2^t \sigma_t(\tilde{s})$, satisfying (i) in Definition 1.

We check that there exists $\bar{t} \in \mathbb{N}$ such that for each $i \in V$, $z_i = z^*$ is a best response for voter i to $((\sigma_t, \sigma_t), z_{V\setminus\{i\}}^*)$ for any $t > \bar{t}$, thus satisfying condition (1). For an arbitrary voter i, given that $\sigma^A = \sigma^B = \sigma_t, \forall t \in \mathbb{N}$, $z_i((0,0)) = z_i((1,1)) = z^*((0,0)) = z^*((1,1)) = \varnothing$ is a best response. In addition, given $p_i = (0,1)$ and given that $p_i^A = 0$ establishes that A has not played the equilibrium strategy s^u, voter i uses $\{\sigma_t\}_{t=1}^{\infty}$ to form beliefs and infers that A has played the strategy that assigns $p_h^A = 1$ for any $h \in V\setminus\{i\}$ with probability 1, whereas $p_i^B = 1$ is consistent with the equilibrium strategy s^u and thus i believes that B has played $s^B = s^u$ with probability 1; hence, $z_i((0,1)) = B$ is a best response. An analogous argument applies to $p_i = (1,0)$.

We complete this part by noting that voters satisfy the voting restriction condition (1). Given that $\sigma^A = s^u = \sigma^B$ and $\{\sigma_t^C\}_{t=1}^{\infty} = \{(\sigma_t, \sigma_t)\}_{t=1}^{\infty}$, for each $i \in V$, if $p_i \in \{(0,0), (1,1)\}$, then

$$EU_i\big[A|\big(p_i, \big((s^u, s^u), z_{V\setminus\{i\}}^*\big), \{\sigma_t^C\}_{t=1}^{\infty}\big)\big] = EU_i\big[B|\big(p_i, \big((s^u, s^u), z_{V\setminus\{i\}}^*\big), \{\sigma_t^C\}_{t=1}^{\infty}\big)\big]$$
$$\text{and} \quad E_i\big[p^A|\big(p_i^A, s^u, \{\sigma_t\}_{t=1}^{\infty}\big)\big] = E_i\big[p^B|\big(p_i^B, s^u, \{\sigma_t\}_{t=1}^{\infty}\big)\big],$$

so condition (1) requires $z_i((0,0)) = z_i((1,1)) = \varnothing$, which is satisfied by $z_i = z^*$. Given $p_i \in \{(0,1), (1,0)\}$, then

$$EU_i\big[A|\big(p_i, \big((s^u, s^u), z_{V\setminus\{i\}}^*\big), \{\sigma_t^C\}_{t=1}^{\infty}\big)\big] \neq EU_i\big[B|\big(p_i, \big((s^u, s^u), z_{V\setminus\{i\}}^*\big), \{\sigma_t^C\}_{t=1}^{\infty}\big)\big]$$

and thus condition (1) is vacuously satisfied.

Part III. We prove uniqueness for $\pi \in (0, \frac{1}{2})$. Assume $\pi \in (0, \frac{1}{2})$. We prove that for any $(\sigma, \tilde{z}_V) \in (\Sigma \times Z^n) \setminus \{(s^u, s^u, z_V^*)\}$, $(\sigma^A, \sigma^B, z_V) = ((\sigma, \sigma), \tilde{z}_V)$ is not an equilibrium.[14]

[14]Note that this claim, together with existence, suffices to establish that $((s^u, s^u), z_V^*)$ is an equilibrium. Part II provides an alternative, constructive proof.

For any $i \in V$, because $\beta_i > 1$, for any $\{\sigma_t^C\}_{t=1}^\infty \longrightarrow (\sigma, \sigma)$ and, given $p_i = (0, 1)$, for $t \in \mathbb{N}$ sufficiently large, we obtain

$$EU_i\big[A|\big(p_i, \big((\sigma, \sigma), \tilde{z}_V\big), \big\{\sigma_t^C\big\}_{t=1}^\infty\big)\big] \leq EU_i\big[B|\big(p_i, \big((\sigma, \sigma), \tilde{z}_V\big), \big\{\sigma_t^C\big\}_{t=1}^\infty\big)\big]$$

and

$$E_i\big[p^A|\big(p_i^A, \sigma, \{\sigma_t\}_{t=1}^\infty\big)\big] < E_i\big[p^B|\big(p_i^B, \sigma, \{\sigma_t\}_{t=1}^\infty\big)\big],$$

and thus, it must be that $\tilde{z}_i((0, 1)) = B$ and, similarly, $\tilde{z}_i((1, 0)) = A$. Further, because $\sigma^A = \sigma^B$, on the equilibrium path it must be $\tilde{z}_i((0, 0)) = \tilde{z}_i((1, 1)) = \varnothing$.

Assume that $\sigma(s^u) \neq 1$. Assume that candidate B deviates to $s^B = s^u$. For any $i \in V$ such that $\sum_{s \in S} \sigma(s)s_i > 0$ (that is, for any voter i who observes $p_i^A = 1$ with strictly positive probability), $p_i = (1, 1)$ is on the equilibrium path and, thus, $\tilde{z}_i((1, 1)) = \varnothing$, and for any $i \in V$, $\tilde{z}_i((0, 1)) = B$ as shown above. Thus, if B deviates to $s^B = s^u$, with probability $1 - \sigma_A(s^u) > 0$, $s^B = s^u$ and $p^A \neq s^u$ (B proposes universal provision and A does not). In this case, with probability $1 - \pi > \frac{1}{2}$, information is not revealed and given that voters play $\tilde{z}_V$, B wins, whereas B may only lose with probability at most $\pi < \frac{1}{2}$. So if A proposes any $p^A \neq s^u$, the deviation is strictly profitable for B. With probability $\sigma_A(s^u)$, $p^A = s^u$ and both candidates tie. So in the aggregate, the deviation is strictly profitable, so $(\sigma^A, \sigma^B, z_V) = ((\sigma, \sigma), \tilde{z}_V)$ is not an equilibrium. Hence, it must be that $\sigma^C = (s^u, s^u)$.

Then, given that the equilibrium candidates' strategy profile is (s^u, s^u) and given that $\beta_i > 1$ for each $i \in V$, and that $\tilde{z}_i((0, 1)) = B$ and $\tilde{z}_i((1, 0)) = A$, the order of possible deviations according to the expected utility that they yield to candidate $j \in C$ is a partial order such that $s^j = s$ is strictly better than $s^j = \tilde{s}$ if and only if $\sum_{i \in V} s_i > \sum_{i \in V} \tilde{s}_i$. That is, if two strategies offer provision to a different number of districts, the one that offers provision to more districts wins; hence, strategies that offer provision to more districts are better and must have greater weights in the sequence $\{\sigma_t^C\}_{t=1}^\infty$. Hence, for each $j \in C$, if voter $i \in V$ observes $p_i^j = 0$, voter i infers that, with probability 1, candidate $j \in C$ has offered provision to $n - 1$ districts. Thus, if voter $i \in V$ observes $p_i = (0, 0)$, then her expectation over the policies of A and B, and her expected utility of voting for either A or B, are the same; hence, i abstains, so $\tilde{z}_i((0, 0)) = \varnothing$ and, thus, $z_i = z^*$ for each $i \in V$.

Part IV. We prove that if $\pi \in (\frac{1}{2}, 1)$, candidates use strictly mixed strategies in any equilibrium. Since the efficient policy is universal provision, if candidates play a strictly mixed strategy, there exists $i \in V$ such that $\Pr[p_i^A = p_i^B = 0] > 0$, which implies that the equilibrium is in expectation inefficient, and the expected number of districts that receive local public good provision is strictly less than n, that is, the equilibrium underspends in expectation.

Assume $\pi \in (\frac{1}{2}, 1)$ and that $((\sigma^A, \sigma^B), z_V) = ((\hat{s}, \hat{s}), \hat{z}_V)$ for some $\hat{s} \in S$ and some $\hat{z}_V \in Z^n$. Define $k \in \{0, 1, \dots, n\}$ by $k = \sum_{i \in V} s_i$. Because in equilibrium both candidates propose the same policy, and voters abstain when indifferent, in equilibrium, both candidates win with equal probability. If $k = 0$ and candidate A deviates to $s^A = s$ for any $s \in S_{(n+1)/2}$ (if n is odd) or any $s \in S_{1+n/2}$ (if n is even), then A wins if information is revealed, which occurs with probability $\pi > \frac{1}{2}$; hence, the deviation is profitable. If $k > 0$ and candidate A deviates to $s^A = s$ for any $s \in S_{k-1}$ such that $|\{i \in V : s_i < \hat{s}_i\}| = 1$, then A

wins if information is revealed, which occurs with probability $\pi > \frac{1}{2}$; hence, the deviation is profitable. □

The proof of Proposition 2 is long. We first prove existence for any $\pi \in (0, 1]$ constructively. Claim 1 establishes that the strategy profiles listed in the statement of Proposition 2 constitute equilibria for the stated values of parameter $\pi \in (0, 1]$. We subsequently prove that no other strategy profile constitutes an equilibrium.

We use the notation $s^0 = (0, 0, 0)$, $s^1 = (1, 0, 0)$, $s^2 = (0, 1, 0)$, $s^3 = (0, 0, 1)$, $s^4 = (1, 1, 0)$, $s^5 = (1, 0, 1)$, and $s^6 = (0, 1, 1)$, and to be consistent with the notation we use for universal provision for an arbitrary n, for $n = 3$, we use $s^u = (1, 1, 1)$. Also note that if $n = 3$, then $S_0 \equiv \{s^0\}$, $S_1 \equiv \{s^1, s^2, s^3\}$, $S_2 \equiv \{s^4, s^5, s^6\}$, and $S_3 \equiv \{s^u\}$.

Claim 1. *Assume $n = 3$. If $\pi \in (0, \frac{1}{2}]$, there exists an equilibrium in which candidates play the pure strategy $(1, 1, 1)$ (full provision) and the voters' strategy profile is z_V^*. For any $\pi \in (\frac{1}{2}, 1]$, there is an equilibrium $((\sigma, \sigma), z_V^*)$ in which σ is*

$\sigma(s^0)$	$\sigma(s)\ \forall s \in S_1$	$\sigma(s)\ \forall s \in S_2$	$\sigma(s^u)$	
0	0	$\frac{1}{3}$	0	*if* $\pi \in (\frac{1}{2}, \frac{3}{4}]$
0	$\frac{2\pi-1}{10\pi-3}$	$\frac{1}{10\pi-3}$	$\frac{4\pi-3}{10\pi-3}$	*if* $\pi \in (\frac{3}{4}, \frac{11+\sqrt{61}}{20}]$ *or* $\pi > \frac{3}{4}$ *and* $\beta_{\text{med}} > 1$
$\frac{4\pi-3}{10\pi-3}$	$\frac{2\pi-1}{10\pi-3}$	$\frac{1}{10\pi-3}$	0	*if* $\pi \in (\frac{11+\sqrt{61}}{20}, 1]$ *and* $\beta_{\text{med}} < 1$

Further, if $\pi = \frac{11+\sqrt{61}}{20}$ and $\beta_{\text{med}} < 1$, for any $\lambda \in [0, 1]$, the strategy profile $((\lambda\sigma^\pi + (1 - \lambda)\hat{\sigma}^\pi, \lambda\sigma^\pi + (1 - \lambda)\hat{\sigma}^\pi), z_V^*)$ is an equilibrium.

Proof. For expositional convenience, without loss of generality, label voters such that $\beta_1 \geq \beta_2 \geq \beta_3$. We partition the parameter range of π into five cases.

Case 1. Assume $\pi \in (0, \frac{1}{2}]$. For any $\bar{V} \subseteq V$, let $z_{\bar{V}}^* \in Z^{|\bar{V}|}$ denote the voters' strategy profile in which $z_i = z^*$ for each $i \in \bar{V}$. We show that if $\pi \in (0, \frac{1}{2}]$, then $((s^A, s^B), z_V) = ((s^u, s^u), z_V^*)$ is an equilibrium. Note that $s^A = s^B$. For each of two subcases, we construct a pair of sequences $\{\varepsilon_t\}_{t=1}^\infty \longrightarrow 0$ and $\{\sigma_t^C\}_{t=1}^\infty \longrightarrow (s^u, s^u)$ such that for each $t \in \mathbb{N}$, $\varepsilon_t \in \mathbb{R}_{++}$, $\sigma_t^N = (\sigma_t^C, z_V^*)$ is ε_t-proper with respect to C. We then note that for each $i \in V$, $z_i = z^*$ satisfies the voting condition (1) given $((s^A, s^B), z_V) = ((s^u, s^u), z_V^*)$ and given $\{\sigma_j^C\}_{t=1}^\infty$; hence, $((s^u, s^u), z_V^*)$ constitutes an equilibrium.

Subcase 1.1: $\beta_2 > 1$. Construct a pair of sequences $(\{\varepsilon_t\}_{t=1}^\infty, \{\sigma_t^N\}_{t=1}^\infty)$ such that $\varepsilon_t = 1/2^t$ for each $t \in \mathbb{N}$; such that for each $t \in \mathbb{N}$, for each $j \in \{A, B\}$, for any $k \in \{0, 1, 2\}$, and for any $s \in S_k$,

$$\sigma_t^j(s) = \frac{1}{2^{t(4-k)}};$$

and such that for each $t \in \mathbb{N}$ and for each $i \in V$, $z_{i,t} = z^*$. Here is the payoff matrix for candidate A as a row player as a function of play by B given that $z_V = z_V^*$. Table 5 shows the payoff matrix for candidate A as a row player as a function of play by B given that $z_V = z_V^*$.

Hence, given that candidate B plays σ_t^B and for $t \in \mathbb{N}$ sufficiently large, $u^A((1, 1, 1), (\sigma_t^B, z_V^*)) > u^A(s, (\sigma_t^B, z_V^*)) > u^A(\tilde{s}, (\sigma_t^B, z_V^*)) > u^A((0, 0, 0), (\sigma_t^B, z_V^*))$ for any $s \in S_2$ and any $\tilde{s} \in S_1$, hence the weights in the sequence $\{\sigma_t^C\}_{t=1}^\infty$ satisfy the ε_t-proper restrictions.

To conclude the proof of this case, we need a further step, which is common to both subcases. Given that $s^A = s^B = s^u$ and $\sigma_t^A = \sigma_t^B$ for each $t \in \mathbb{N}$, $z_i((0,0)) = z_i((1,1)) = z^*((0,0)) = z^*((1,1)) = \varnothing$ is a best response to $(\sigma_t^C, z^*_{V\setminus\{i\}})$ and to $(s^u, s^u, z^*_{V\setminus\{i\}})$ for each $i \in V$ given $p_i \in \{(0,0),(1,1)\}$ for each $t \in \mathbb{N}$. Moreover, given $p_i = (0,1)$,

$$\sum_{s \in S_2} \omega_i^A\big(s|\big(p_i^A, s^u, \{\sigma_t^A\}_{t=1}^\infty\big)\big) = 1, \quad \text{so}$$

$$EU_i\big[A|\big((0,1), \big((s^u, s^u), z^*_{V\setminus\{i\}}\big), \{\sigma_t^C\}_{t=1}^\infty\big)\big] < EU_i\big[B|\big((0,1), \big((s^u, s^u), z^*_{V\setminus\{i\}}\big), \{\sigma_t^C\}_{t=1}^\infty\big)\big],$$

so $z_i((0,1)) = z^*((0,1)) = B$ is a best response, and, by a symmetric argument, $z_i((1,0)) = z^*((1,0)) = A$ is a best response, so $z_i = z^*$ is a best response for each $i \in V$.

Finally, the voting condition (1) is satisfied because

$$EU_i\big[A|\big(p_i, \big((s^u, s^u), z^*_{V\setminus\{i\}}\big), \{\sigma_t^C\}_{t=1}^\infty\big)\big] = EU_i\big[B|\big(p_i, \big((s^u, s^u), z^*_{V\setminus\{i\}}\big), \{\sigma_t^C\}_{t=1}^\infty\big)\big]$$

if and only if $p_i \in \{(0,0),(1,1)\}$, in which case

$$u_i\big(E_i\big[p^A|\big(p_i^A, s^u, \{\sigma_t^A\}_{t=1}^\infty\big)\big]\big) = u_i\big(E_i\big[p^B|\big(p_i^B, s^u, \{\sigma_t^B\}_{t=1}^\infty\big)\big]\big)$$

and, thus, $z_i(p_i) = z^*(p_i) = \varnothing$, as required by condition (1).

Subcase 1.2: $\beta_2 < 1$. Let the pair of sequences $(\{\varepsilon_t\}_{t=1}^\infty, \{\sigma_t^C\}_{t=1}^\infty)$ be such that $\varepsilon_t = 1/2^t$ for each $t \in \mathbb{N}$; such that for each $t \in \mathbb{N}$, for each $j \in \{A, B\}$, $\sigma_t^j((0,0,0)) = 1/2^{3t}$, $\sigma_t^j(s) = 1/2^{4t}$ for any $s \in S_1$ and $\sigma_t^j(s) = 1/2^{2t}$ for any $s \in S_2$; and such that for each $t \in \mathbb{N}$, $z_{V,t} = z_V^*$. The payoff matrix for candidate A as a row player as a function of play by B given that voters play $z_V = z_V^*$ is given in Table 6.

Hence, given that candidate B plays σ_t^B and, for $t \in \mathbb{N}$ sufficiently large,

$$u^A\big((1,1,1), \big(\sigma_t^B, z_V^*\big)\big) > u^A\big(s, \big(\sigma_t^B, z_V^*\big)\big) > u^A\big((0,0,0), \big(\sigma_t^B, z_V^*\big)\big) > u^A\big(\tilde{s}, \big(\sigma_t^B, z_V^*\big)\big)$$

Table 5. Subcase 1.1: $\beta_2 > 1$

$s^A \backslash s^B$	$(0,0,0)$	$\in S_1$	$\in S_2$	$(1,1,1)$
$(0,0,0)$	$\frac{1}{2}$	π	0	0
$\in S_1$	$1-\pi$	$\frac{1}{2}$	$\frac{2\pi}{3}$	0
$\in S_2$	1	$1-\frac{2\pi}{3}$	$\frac{1}{2}$	π
$(1,1,1)$	1	1	$1-\pi$	$\frac{1}{2}$

Table 6. Subcase 1.2: $\beta_2 < 1$

$s^A \backslash s^B$	$(0,0,0)$	$\in S_1$	$\in S_2$	$(1,1,1)$
$(0,0,0)$	$\frac{1}{2}$	π	0	π
$\in S_1$	$1-\pi$	$\frac{1}{2}$	$\frac{2\pi}{3}$	0
$\in S_2$	1	$1-\frac{2\pi}{3}$	$\frac{1}{2}$	π
$(1,1,1)$	$1-\pi$	1	$1-\pi$	$\frac{1}{2}$

Table 7. Subcase 2.1: $\pi \in (\frac{1}{2}, \frac{3}{5}]$

s^A	$u_i(s^A, \sigma^{N\setminus\{A\}})$
$(0, 0, 0)$	0
$\in S_1$	$\frac{2\pi}{3}$
$\in S_2$	$\frac{1}{2}$
$(1, 1, 1)$	$1-\pi$

for any $s \in S_2$ and any $\tilde{s} \in S_1$, hence the weights in the sequence $\{\sigma_t^C\}_{t=1}^\infty$ satisfy the ε_t-proper restrictions.

Further, given that $s^A = s^B = s^u$ and $\sigma_t^A = \sigma_t^B$ for each $t \in \mathbb{N}$, $z_V = z_V^*$ is a best response for voters and satisfies condition (1) for the same arguments as in Subcase 1.1, verbatim.

Case 2. Assume $\pi \in (\frac{1}{2}, \frac{3}{4}]$. Throughout this case, let $\hat{\sigma} \in \Sigma$ refer to the candidate strategy defined by $\hat{\sigma}(s) = \frac{1}{3}\ \forall s \in S_2$ and $\hat{\sigma}(s) = 0\ \forall s \notin S_2$, and let $\hat{\sigma}^N \equiv (\hat{\sigma}, \hat{\sigma}, z_V^*)$. We show that if $\pi \in (\frac{1}{2}, \frac{3}{4}]$, then $\sigma^N = \hat{\sigma}^N$ is an equilibrium. For each of two subcases, we construct a pair of sequences $\{\varepsilon_t\}_{t=1}^\infty \longrightarrow 0$ and $\{\sigma_t^C\}_{t=1}^\infty \longrightarrow (\hat{\sigma}, \hat{\sigma})$ such that for each $t \in \mathbb{N}$, $\varepsilon_t \in \mathbb{R}_{++}$ and (σ_t^C, z_V^*) is ε_t-proper with respect to C. We then note that z_V^* satisfies the voting condition (1) given $\hat{\sigma}^N$ and $\{\sigma_t^C\}_{t=1}^\infty$.

Subcase 2.1: $\pi \in (\frac{1}{2}, \frac{3}{5}]$. The payoff for candidate A as a row player given that B plays $\sigma^B = \hat{\sigma}$ and voters play $z_V = z_V^*$ is given in Table 7.

So for any $t \in \mathbb{N}$ sufficiently large,

$$u^A\big(s, (\sigma_t^B, z_V^*)\big) > u^A\big((1,1,1), (\sigma_t^B, z_V^*)\big) > u^A\big(\tilde{s}, (\sigma_t^B, z_V^*)\big) > u^A\big((0,0,0), (\sigma_t^B, z_V^*)\big)$$

for any $s \in S_2$ and any $\tilde{s} \in S_1$.

Let the pair of sequences $(\{\varepsilon_t\}_{t=1}^\infty, \{\sigma_t^C\}_{t=1}^\infty)$ be such that $\varepsilon_t = 1/2^t$ for each $t \in \mathbb{N}$, and such that for each $t \in \mathbb{N}$, for each $j \in \{A, B\}$, $\sigma_t^j((1,1,1)) = 1/2^{2t}$, $\sigma_t^j(s) = 1/2^{3t}$ for any $s \in S_1$ and $\sigma_t^j((0,0,0)) = 1/2^{4t}$.

Note that the weights in the sequence $\{\sigma_t^C\}_{t=1}^\infty$ satisfy the ε_t-proper restriction for each $j \in C$.

Further, given that $\sigma^A = \sigma^B$ and $\sigma_t^A = \sigma_t^B$ for each $t \in \mathbb{N}$,

$$z_i\big((0,0)\big) = z_i\big((1,1)\big) = z^*\big((0,0)\big) = z^*\big((1,1)\big) = \varnothing$$

is a best response to $(\sigma_t^C, z_{V\setminus\{i\}}^*)$ and to $((\hat{\sigma}, \hat{\sigma}), z_{V\setminus\{i\}}^*)$ for each $i \in V$ given $p_i \in \{(0,0), (1,1)\}$. Moreover, given $p_i = (0,1)$,

$$EU_i\big[A|\big((0,1), \hat{\sigma}^{N\setminus\{i\}}, \{\sigma_t^C\}_{t=1}^\infty\big)\big] < EU_i\big[B|\big((0,1), \hat{\sigma}^{N\setminus\{i\}}, \{\sigma_t^C\}_{t=1}^\infty\big)\big],$$

so $z_i((0,1)) = z^*((0,1)) = B$ is a best response, and, by a symmetric argument, $z_i((1,0)) = z^*((1,0)) = A$ is a best response, so $z_i = z^*$ is a best response for each $i \in V$.

Finally, the voting condition (1) is satisfied because

$$EU_i\big[A|\big(p_i, \hat{\sigma}^{N\setminus\{i\}}, \{\sigma_t^C\}_{t=1}^\infty\big)\big] = EU_i\big[B|\big(p_i, \hat{\sigma}^{N\setminus\{i\}}, \{\sigma_t^C\}_{t=1}^\infty\big)\big]$$

if and only if $p_i \in \{(0,0),(1,1)\}$, in which case

$$u_i\big(E_i\big[p^A|\big(p_i^A, \hat{\sigma}, \{\sigma_t^A\}_{t=1}^{\infty}\big)\big]\big) = u_i\big(E_i\big[p^B|\big(p_i^B, \hat{\sigma}, \{\sigma_t^B\}_{t=1}^{\infty}\big)\big]\big)$$

and, thus, $z_i(p_i) = z^*(p_i) = \varnothing$, as required by condition (1).

Subcase 2.2: $\pi \in (\frac{3}{5}, \frac{3}{4}]$. The payoff for candidate A as a row player given that B plays $\sigma^B = \hat{\sigma}$ and voters play $z_V = z_V^*$ is given in Table 7. So for any $\pi \in (\frac{3}{5}, \frac{3}{4})$ and any $t \in \mathbb{N}$ sufficiently high,

$$u^A\big(s, \big(\sigma_t^B, z_V^*\big)\big) > u^A\big(\tilde{s}, \big(\sigma_t^B, z_V^*\big)\big) > u^A\big((1,1,1), \big(\sigma_t^B, z_V^*\big)\big) > u^A\big((0,0,0), \big(\sigma_t^B, z_V^*\big)\big)$$

for any $s \in S_2$ and any $\tilde{s} \in S_1$. If $\pi = \frac{3}{4}$ and B plays $\sigma^B = \hat{\sigma}$, the payoff for A from playing any $s \in S_2$ or any $\tilde{s} \in S_1$ is $\frac{1}{2}$; we must look at the payoff of A if B plays any strategy.

The payoff for A playing $s^A = s$ for any $s \in S_1$ is $\frac{1}{4}$ if $s^B = (0,0,0)$, $\frac{1}{2}$ if $s^B \in S_1$, and 0 if $s^B \in S_3$; further, since $\sigma_t^B(s) = \sigma_t^B(\tilde{s})$ for any $s, \tilde{s} \in S_2$, it follows that the expected payoff for A from playing $s^A = s$ for any $s \in S_1$ given that $s^B \in S_2$ is $\frac{1}{3}0 + \frac{2}{3}\frac{3}{4} = \frac{1}{2}$. Similarly, the payoff for A playing $s^A = s$ for any $s \in S_2$ is 1 if $s^B = (0,0,0)$, $\frac{1}{2}$ if $s^B \in S_2$, and $\frac{3}{4}$ if $s^B \in S_3$; and since $\sigma_t^B(s) = \sigma_t^B(\tilde{s})$ for any $s, \tilde{s} \in S_1$, it follows that the expected payoff for A from playing $s^A = s$ for any $s \in S_2$ given that $s^B \in S_1$ is $\frac{1}{3}1 + \frac{2}{3}\frac{1}{4} = \frac{1}{2}$.

So again, $u^A(s, (\sigma_t^B, z_V^*)) > u^A(\tilde{s}, (\sigma_t^B, z_V^*))$ for $\pi = \frac{3}{4}$, for any $s \in S_2$ and any $\tilde{s} \in S_1$.

Let the pair of sequences $(\{\varepsilon_t\}_{t=1}^{\infty}, \{\sigma_t^C\}_{t=1}^{\infty})$ be such that $\varepsilon_t = 1/2^t$ for each $t \in \mathbb{N}$, and such that for each $t \in \mathbb{N}$, for each $j \in \{A, B\}$, $\sigma_t^j((1,1,1)) = 1/2^{2t}$, $\sigma_t^j(s) = 1/2^{3t}$ for any $s \in S_1$ and $\sigma_t^j((0,0,0)) = 1/2^{4t}$.

Note that the weights in the sequence $\{\sigma_t^C\}_{t=1}^{\infty}$ satisfy the ε_t-proper restriction for each $j \in C$.

Further, given that $\sigma^A = \sigma^B$ and $\sigma_t^A = \sigma_t^B$ for each $t \in \mathbb{N}$, $z_V = z_V^*$ is a best response for voters and satisfies condition (1) for the same arguments as in previous subcase, verbatim.

Case 3. Assume $\pi \in (\frac{3}{4}, 1]$ and $\beta_2 > 1$. Throughout Cases 3, 4, and 6, let $\sigma^{N,\pi} \equiv ((\sigma^\pi, \sigma^\pi), z_V^*)$. We show that if $\pi \in (\frac{3}{4}, 1]$ and $\beta_2 > 1$, then $\sigma^N = \sigma^{N,\pi}$ is an equilibrium. We construct a pair of sequences $\{\varepsilon_t\}_{t=1}^{\infty} \longrightarrow 0$ and $\{\sigma_t^C\}_{t=1}^{\infty} \longrightarrow (\sigma^\pi, \sigma^\pi)$ such that for each $t \in \mathbb{N}$, $\varepsilon_t \in \mathbb{R}_{++}$ and (σ_t^C, z_V^*) is ε_t-proper with respect to C. We then note that z_V^* satisfies the voting condition (1) given $\sigma^{N,\pi}$ and $\{\sigma_t^C\}_{t=1}^{\infty}$.

The payoff matrix for candidate A as a row player as a function of play by B given that voters play $z_V = z_V^*$ is shown in Table 5. For any $t \in \mathbb{N}$, assume that $\sigma_t^B((0,0,0)) = 1/2^{4t}$, $\sigma_t^B(s) = \delta_{1,t}/3 \in (0,1)\ \forall s \in S_1$, $\sigma_t^B(s^B) = \delta_{2,t}/3 \in (0,1)\ \forall s \in S_2$, and $\sigma_t^B((1,1,1)) = \delta_{3,t}$, and solve the system of equations with $\delta_{1,t}, \delta_{2,t}, \delta_{3,t} \in [0, 1 - 1/2^{4t}]$ as the variables to solve for, such that $u^A(s, (\sigma_t^B, z_V^*)) = u^A(\tilde{s}, (\sigma_t^B, z_V^*))$ for any $s, \tilde{s} \in S \backslash \{(0,0,0)\}$. For each $t \in \mathbb{N}$, let $\delta_{3,t} = 1 - 1/2^{4t} - \delta_{1,t} - \delta_{2,t}$. Then the system of equations is

$$\begin{aligned} &\frac{1}{2^{4t}}(1-\pi) + \delta_{1,t}\left(\frac{1}{2}\right) + \delta_{2,t}\left(\frac{2\pi}{3}\right) \\ &\quad = \frac{1}{2^{4t}}(1) + \delta_{1,t}\left(1 - \frac{2\pi}{3}\right) + \delta_{2,t}\left(\frac{1}{2}\right) + \left(1 - \frac{1}{2^{4t}} - \delta_{1,t} - \delta_{2,t}\right)\pi \end{aligned}$$

$$\frac{1}{2^{4t}}(1-\pi)+\delta_{1,t}\left(\frac{1}{2}\right)+\delta_{2,t}\left(\frac{2\pi}{3}\right)$$
$$=\frac{1}{2^{4t}}(1)+\delta_{1,t}(1)+\delta_{2,t}(1-\pi)+\left(1-\frac{1}{2^{4t}}-\delta_{1,t}-\delta_{2,t}\right)\frac{1}{2}$$

with solutions

$$\delta_{1,t}=\frac{6\pi-3}{10\pi-3}\left(1-\frac{1}{2^{4t}}\right),\qquad \delta_{2,t}=\frac{3}{10\pi-3}+\frac{6\pi-3}{10\pi-3}\frac{1}{2^{4t}}.$$

Let the pair of sequences $(\{\varepsilon_t\}_{t=1}^{\infty},\{\sigma_t^C\}_{t=1}^{\infty})$ be such that $\varepsilon_t=\frac{1}{2^t}$ for each $t\in\mathbb{N}$, and such that for each $t\in\mathbb{N}$, for each $j\in\{A,B\}$, $\sigma_t^j((0,0,0))=1/2^{4t}$,

$$\sigma_t^j(s)=\frac{2\pi-1}{10\pi-3}\left(1-\frac{1}{2^{4t}}\right)\quad\text{for any } s\in S_1$$
$$\sigma_t^j(s)=\frac{1}{10\pi-3}+\frac{2\pi-1}{10\pi-3}\frac{1}{2^{4t}}\quad\text{for any } s\in S_2.$$

Note that $\{\sigma_t^j\}_{t=1}^{\infty}\longrightarrow\bar{\sigma}$ for each $j\in C$, and that for each $t\in\mathbb{N}$, σ_t^j satisfies the weights restriction in the definition of an ε_t-proper equilibrium,

$$u^A\big(s,(\sigma_t^B,z_V^*)\big)=u^A\big(\tilde{s},(\sigma_t^B,z_V^*)\big)>u^A\big((0,0,0),(\sigma_t^B,z_V^*)\big)$$

for any $s,\tilde{s}\in S\backslash\{(0,0,0)\}$ so the only restriction is that

$$\sigma_t^A\big((0,0,0)\big)<\frac{1}{2^t}\sigma_t^A(s)\quad\text{for any } s\in S\backslash\big\{(0,0,0)\big\},$$

which is satisfied.

Further, given that $\sigma^A=\sigma^B$ and $\sigma_t^A=\sigma_t^B$ for any $t\in\mathbb{N}$,

$$z_i\big((0,0)\big)=z_i\big((1,1)\big)=z^*\big((0,0)\big)=z^*\big((1,1)\big)=\varnothing$$

is a best response for each $i\in V$ given $p_i\in\{(0,0),(1,1)\}$. Moreover, for each $i\in V$,

$$EU_i\big[A|\big((0,1),\sigma^{N\backslash\{i\},\pi},\big\{\sigma_t^C\big\}_{t=1}^{\infty}\big)\big]<EU_i\big[B|\big((0,1),\sigma^{N\backslash\{i\},\pi},\big\{\sigma_t^C\big\}_{t=1}^{\infty}\big)\big],$$

so $z_i((0,1))=z^*((0,1))=B$ is a best response, and, by a symmetric argument, $z_i((0,1))=z^*((1,0))=A$ is a best response, so $z_i=z^*$ is a best response for each $i\in V$.

Finally, the voting condition (1) holds because

$$EU_i\big[A|\big(p_i,\sigma^{N\backslash\{i\},\pi},\big\{\sigma_t^C\big\}_{t=1}^{\infty}\big)\big]=EU_i\big[B|\big(p_i,\sigma^{N\backslash\{i\},\pi},\big\{\sigma_t^C\big\}_{t=1}^{\infty}\big)\big]$$

if and only if $p_i\in\{(0,0),(1,1)\}$, in which case

$$u_i\big(E_i\big[p^A|\big(p_i^A,\sigma^\pi,\{\sigma_t^A\}_{t=1}^{\infty}\big)\big]\big)=u_i\big(E_i\big[p^B|\big(p_i^B,\sigma^\pi,\{\sigma_t^B\}_{t=1}^{\infty}\big)\big]\big)$$

and, thus, $z_i(p_i)=z^*(p_i)=\varnothing$, as required by condition (1).

Case 4. Assume $\pi \in (\frac{3}{4}, (11+\sqrt{61})/20]$ and $\beta_2 < 1$. We show that $\sigma^N = \sigma^{N,\pi}$ is an equilibrium. Note that $\sigma^A = \sigma^B$. We construct a pair of sequences $\{\varepsilon_t\}_{t=1}^\infty \longrightarrow 0$ and $\{\sigma_t^C\}_{t=1}^\infty \longrightarrow (\sigma^\pi, \sigma^\pi)$ such that for each $t \in \mathbb{N}$, $\varepsilon_t \in \mathbb{R}_{++}$ and (σ_t^C, z_V^*) is ε_t-proper with respect to C. We then note that z_V^* satisfies the voting condition (1) given $\bar{\sigma}^N$ and $\{\sigma_t^C\}_{t=1}^\infty$.

The payoff matrix for candidate A as a row player as a function of play by B given that voters play $z_V = z_V^*$ is shown in Table 6.

For any $t \in \mathbb{N}$, assume that $\sigma_t^B((0,0,0)) = \frac{1}{2^{4t}}$, $\sigma_t^B(s) = \delta_{1,t}/3 \in (0,1)$ $\forall s \in S_1$, $\sigma_t^B(s^B) = \delta_{2,t}/3 \in (0,1)$ $\forall s \in S_2$, and $\sigma_t^B((1,1,1)) = \delta_{3,t}$, and solve the system of equations with $\delta_{1,t}, \delta_{2,t}, \delta_{3,t} \in [0, 1-1/2^{4t}]$ as the variables to solve for, such that $u^A(s, (\sigma_t^B, z_V^*)) = u^A(\tilde{s}, (\sigma_t^B, z_V^*))$ for any $s, \tilde{s} \in S \backslash \{(0,0,0)\}$. For each $t \in \mathbb{N}$, let $\delta_{3,t} = 1 - 1/2^{4t} - \delta_{1,t} - \delta_{2,t}$. Then the system of equations is

$$
\begin{aligned}
&\frac{1}{2^{4t}}(1-\pi) + \delta_{1,t}\left(\frac{1}{2}\right) + \delta_{2,t}\left(\frac{2\pi}{3}\right) \\
&\quad = \frac{1}{2^{4t}}(1) + \delta_{1,t}\left(1 - \frac{2\pi}{3}\right) + \delta_{2,t}\left(\frac{1}{2}\right) + \left(1 - \frac{1}{2^{4t}} - \delta_{1,t} - \delta_{2,t}\right)\pi \\
&\frac{1}{2^{4t}}(1-\pi) + \delta_{1,t}\left(\frac{1}{2}\right) + \delta_{2,t}\left(\frac{2\pi}{3}\right) \\
&\quad = \frac{1}{2^{4t}}(1-\pi) + \delta_{1,t}(1) + \delta_{2,t}(1-\pi) + \left(1 - \frac{1}{2^{4t}} - \delta_{1,t} - \delta_{2,t}\right)\frac{1}{2},
\end{aligned}
$$

with solutions

$$
\begin{aligned}
\delta_{1,t} &= \frac{6\pi - 3}{10\pi - 3} + \frac{3}{(10\pi - 3)2^{4t}} \\
\delta_{2,t} &= \frac{3}{10\pi - 3} - \frac{3}{(10\pi - 3)2^{4t}}.
\end{aligned}
$$

Note that $\{\sigma_t^j\}_{t=1}^\infty \longrightarrow \sigma^\pi$ for each $j \in C$, and that for each $t \in \mathbb{N}$, σ_t^j satisfies the weights restriction in the definition of an ε_t-proper equilibrium,

$$
u^A\big(s, (\sigma_t^B, z_V^*)\big) = u^A\big(\tilde{s}, (\sigma_t^B, z_V^*)\big) > u^A\big((0,0,0), (\sigma_t^B, z_V^*)\big)
$$

for any $s, \tilde{s} \in S \backslash \{(0,0,0)\}$ and any $\pi \in (\frac{3}{4}, (11+\sqrt{61})/20)$, and analogously for candidate B, so the only restriction is that

$$
\sigma_t^j\big((0,0,0)\big) < \frac{1}{2^t}\sigma_t^j(s)
$$

for any $s \in S \backslash \{(0,0,0)\}$ for any $\pi \in (\frac{3}{4}, (11+\sqrt{61})/20)$, which is satisfied.

The proof of this case concludes following the same steps as in Case 3, verbatim.

Case 5. Assume $\pi \in ((11+\sqrt{61})/20, 1]$ and $\beta_2 < 1$. Throughout Case 5 and Case 6, let $\hat{\sigma}^{N,\pi} \equiv ((\hat{\sigma}^\pi, \hat{\sigma}^\pi), z_V^*)$. We show that if $\pi \in ((11+\sqrt{61})/20, 1]$ and $\beta_2 < 1$, then $\sigma^N = \hat{\sigma}^{N,\pi}$ is an equilibrium. Note that $\sigma^N = \hat{\sigma}^{N,\pi}$ implies $\sigma^A = \sigma^B$. We construct a pair of sequences $\{\varepsilon_t\}_{t=1}^\infty \longrightarrow 0$ and $\{\sigma_t^C\}_{t=1}^\infty \longrightarrow (\hat{\sigma}^\pi, \hat{\sigma}^\pi)$ such that for each $t \in \mathbb{N}$, $\varepsilon_t \in \mathbb{R}_{++}$

and (σ_t^C, z_V^*) is ε_t-proper with respect to C. We then note that $z_V = z_V^*$ satisfies the voting condition (1) given $\hat{\sigma}^{N,\pi}$ and $\{\sigma_t^C\}_{t=1}^{\infty}$.

The payoff matrix for candidate A as a row player as a function of play by B given that voters play $z_V = z_V^*$ is shown in Table 6.

For any $t \in \mathbb{N}$, assume that $\sigma_t^B((0,0,0)) = \delta_{0,t}$, $\sigma_t^B(s) = \delta_{1,t}/3 \in (0,1)$ $\forall s \in S_1$, $\sigma_t^B(s) = \delta_{2,t}/3 \in (0,1)$ $\forall s \in S_2$, and $\sigma_t^B((1,1,1)) = 1/2^{4t}$, and solve the system of equations with $\delta_{0,t}, \delta_{1,t}, \delta_{2,t} \in [0, 1 - 1/2^{4t}]$ as the variables to solve for, such that $u^A(s, (\sigma_t^B, z_V^*)) = u^A(\tilde{s}, (\sigma_t^B, z_V^*))$ for any $s, \tilde{s} \in S \backslash \{(1,1,1)\}$. For each $t \in \mathbb{N}$, let $\delta_{0,t} = 1 - 1/2^{4t} - \delta_{1,t} - \delta_{2,t}$. Then the system of equations is

$$\begin{aligned}
&\left(1 - \frac{1}{2^{4t}} - \delta_{1,t} - \delta_{2,t}\right)\frac{1}{2} + \delta_{1,t}(\pi) + \frac{1}{2^{4t}}(\pi) \\
&\quad = \left(1 - \frac{1}{2^{4t}} - \delta_{1,t} - \delta_{2,t}\right)(1-\pi) + \delta_{1,t}\left(\frac{1}{2}\right) + \delta_{2,t}\left(\frac{2\pi}{3}\right) \\
&\left(1 - \frac{1}{2^{4t}} - \delta_1^t - \delta_2^t\right)\frac{1}{2} + \delta_{1,t}(\pi) \\
&\quad = \left(1 - \frac{1}{2^{4t}} - \delta_1^t - \delta_2^t\right)(1) + \delta_{1,t}\left(1 - \frac{2\pi}{3}\right) + \delta_{2,t}\left(\frac{1}{2}\right)
\end{aligned}$$

with solutions

$$\delta_{1,t} = \frac{3}{10\pi - 3}\left(1 - \frac{1}{2^{4t}}\right) \quad \text{and} \quad \delta_{2,t} = \frac{6\pi - 3}{10\pi - 3} + \frac{3}{(10\pi - 3)2^{4t}}.$$

Let the pair of sequences $(\{\varepsilon_t\}_{t=1}^{\infty}, \{\sigma_t^C\}_{t=1}^{\infty})$ be such that $\varepsilon_t = \frac{1}{2^t}$ for each $t \in \mathbb{N}$, and such that for each $t \in \mathbb{N}$, for each $j \in \{A, B\}$,

$$\begin{aligned}
\sigma_t^j\big((0,0,0)\big) &= \frac{4\pi - 3}{10\pi - 3} \\
\sigma_t^j(s) &= \frac{1}{10\pi - 3}\left(1 - \frac{1}{2^{4t}}\right) \quad \forall s \in S_1 \\
\sigma_t^j(s) &= \frac{2\pi - 1}{10\pi - 3} + \frac{1}{10\pi - 3}\frac{1}{2^{4t}} \quad \forall s \in S_2.
\end{aligned}$$

Note that $\{\sigma_t^j\}_{t=1}^{\infty} \longrightarrow \hat{\sigma}^{\pi}$ for each $j \in C$ and that for each $t \in \mathbb{N}$, σ_t^j satisfies the weights restriction in the definition of an ε_t-proper equilibrium,

$$u^A\big(s, \big(\sigma_t^B, z_V^*\big)\big) = u^A\big(\tilde{s}, \big(\sigma_t^B, z_V^*\big)\big) > u^A\big((1,1,1), \big(\sigma_t^B, z_V^*\big)\big)$$

for any $s, \tilde{s} \in S \backslash \{(1,1,1)\}$, and analogously for candidate B, so the only restriction is that

$$\sigma_t^j\big((1,1,1)\big) < \frac{1}{2^t}\sigma_t^j(s) \quad \forall s \in S \backslash \big\{(1,1,1)\big\},$$

which is satisfied.

Further, given that $\sigma^A = \sigma^B$ and $\sigma_t^A = \sigma_t^B$ for any $t \in \mathbb{N}$,

$$z_i\big((0,0)\big) = z_i\big((1,1)\big) = z^*\big((0,0)\big) = z^*\big((1,1)\big) = \varnothing$$

is a best response for each $i \in V$ given $p_i \in \{(0,0),(1,1)\}$. Moreover, for each $i \in V$,

$$EU_i\big[A|\big((0,1), \hat{\sigma}^{N\setminus\{i\},\pi}, \{\sigma_t^C\}_{t=1}^{\infty}\big)\big] < EU_i\big[B|\big((0,1), \hat{\sigma}^{N\setminus\{i\},\pi}, \{\sigma_t^C\}_{t=1}^{\infty}\big)\big],$$

so $z_i((0,1)) = z^*((0,1)) = B$ is a best response and, by a symmetric argument, $z_i((1,0)) = z^*((1,0)) = A$ is a best response so $z_i = z^*$ is a best response for each $i \in V$.

Finally, the voting condition (1) is satisfied because

$$EU_i\big[A|\big(p_i, \hat{\sigma}^{N\setminus\{i\},\pi}, \{\sigma_t^C\}_{t=1}^{\infty}\big)\big] = EU_i\big[B|\big(p_i, \hat{\sigma}^{N\setminus\{i\},\pi}, \{\sigma_t^C\}_{t=1}^{\infty}\big)\big]$$

if and only if $p_i \in \{(0,0),(1,1)\}$, in which case

$$u_i\big(E_i\big[p^A|\big(p_i^A, \hat{\sigma}^{\pi}, \{\sigma_t^A\}_{t=1}^{\infty}\big)\big]\big) = u_i\big(E_i\big[p^B|\big(p_i^B, \hat{\sigma}^{\pi}, \{\sigma_t^B\}_{t=1}^{\infty}\big)\big]\big)$$

and, thus, $z_i(p_i) = z^*(p_i) = \varnothing$, as required by condition (1).

Case 6. Assume $\pi = (11+\sqrt{61})/20$ and $\beta_2 < 1$. We first want to show that $\hat{\sigma}^{N,\pi}$ defined in Case 5 is also an equilibrium in this special case. The expected utility for candidate j of each pure strategy, given that candidate $-j$ plays an arbitrary $\sigma \in \Sigma$ and voters play $z_V = z_V^*$ is

s^j	$u^j(s^j,(\sigma, z_V^*))$
s^0	$\frac{1}{2}\sigma(s^0) + \pi(\sigma(s^1)+\sigma(s^2)+\sigma(s^3)+\sigma(s^u))$
s^1, s^2 or s^3	$(1-\pi)\sigma(s^0) + \frac{1}{2}(\sigma(s^1)+\sigma(s^2)+\sigma(s^3)) + \pi(\sigma(s^4)+\sigma(s^5))$
s^4, s^5 or s^6	$\sigma(s^0) + (1-\pi)(\sigma(s^1)+\sigma(s^2)) + \sigma(s^3) + \frac{1}{2}(\sigma(s^4)+\sigma(s^5)+\sigma(s^6)) + \pi\sigma(s^u)$
s^u	$(1-\pi)\sigma(s^0)+\sigma(s^1)+\sigma(s^2)+\sigma(s^3)+(1-\pi)(\sigma(s^4)+\sigma(s^5)+\sigma(s^6))+\frac{1}{2}\sigma(s^u)$

Since $\pi = (11+\sqrt{61})/20$,

$$\frac{4\pi-3}{10\pi-3} = \frac{2(-4+\sqrt{61})}{5(5+\sqrt{61})}; \qquad \frac{1}{10\pi-3} = \frac{2}{5+\sqrt{61}}; \quad \text{and} \quad \frac{2\pi-1}{10\pi-3} = \frac{1+\sqrt{61}}{5(5+\sqrt{61})},$$

then

$$u^j\big(s^0,(\hat{\sigma}^{\pi}, z_V^*)\big) = \frac{1}{2}\frac{2(-4+\sqrt{61})}{5(5+\sqrt{61})} + \frac{11+\sqrt{61}}{20}3\frac{2}{5+\sqrt{61}} = \frac{1}{2}$$

$$u^j\big(s^1,(\hat{\sigma}^{\pi}, z_V^*)\big) = \left(1-\frac{11+\sqrt{61}}{20}\right)\frac{2(-4+\sqrt{61})}{5(5+\sqrt{61})} + \frac{1}{2}3\frac{2}{5+\sqrt{61}} + \frac{11+\sqrt{61}}{20}2\frac{1+\sqrt{61}}{5(5+\sqrt{61})}$$
$$= \frac{1}{2}$$

$$u^j\big(s^4,(\hat{\sigma}^{\pi}, z_V^*)\big) = \frac{2(-4+\sqrt{61})}{5(5+\sqrt{61})} + \left(1-\frac{11+\sqrt{61}}{20}\right)2\frac{2}{5+\sqrt{61}} + \frac{2}{5+\sqrt{61}} + \frac{3}{2}\frac{1+\sqrt{61}}{5(5+\sqrt{61})}$$
$$= \frac{1}{2}$$

$$u^j\big(s^u, (\hat{\sigma}^\pi, z_V^*)\big) = \left(1 - \frac{11+\sqrt{61}}{20}\right)\left(\frac{2(-4+\sqrt{61})}{5(5+\sqrt{61})} + 3\frac{1+\sqrt{61}}{5(5+\sqrt{61})}\right) + 3\frac{2}{5+\sqrt{61}} = \frac{1}{2};$$

hence, no candidate deviation is profitable. The condition on voting behavior holds as in Case 5. Hence, $\hat{\sigma}^{N,\pi}$ holds as an equilibrium for $\pi = (11+\sqrt{61})/20$ and $\beta_2 < 1$.

Further, we noted in Case 4 that $\sigma^{N,\pi}$ is an equilibrium; hence, $u^j(s, (\sigma^\pi, z_V^*)) = \frac{1}{2}$ for any $s \in S\backslash\{s^0\}$. Note that

$$\begin{aligned} u^j\big(s^0, (\sigma^\pi, z_V^*)\big) &= \frac{1}{2}\sigma^\pi(s^0) + \pi\big(\sigma^\pi(s^1) + \sigma^\pi(s^2) + \sigma^\pi(s^3) + \sigma^\pi(s^u)\big) \\ &= \frac{11+\sqrt{61}}{20}\left(3\frac{1+\sqrt{61}}{5(5+\sqrt{61})} + \frac{2(-4+\sqrt{61})}{5(5+\sqrt{61})}\right) = \frac{1}{2}. \end{aligned}$$

Therefore, $u^j(s, (\sigma, z_V^*)) = \frac{1}{2}$ for any $s \in S$ and for any $\sigma \in \{\sigma^\pi, \hat{\sigma}^\pi\}$, and, thus, for any $\lambda \in [0,1]$ and for any $j \in \{A, B\}$, there is no profitable deviation for candidate j from $((\lambda\sigma^\pi + (1-\lambda)\hat{\sigma}^\pi, \lambda\sigma^\pi + (1-\lambda)\hat{\sigma}^\pi), z_V^*)$. The voter condition holds as a convex combination of the conditions in Cases 4 and 5, so for any $\lambda \in [0,1]$, the strategy profile $((\lambda\sigma^\pi + (1-\lambda)\hat{\sigma}^\pi, \lambda\sigma^\pi + (1-\lambda)\hat{\sigma}^\pi), z_V^*)$ constitutes an equilibrium for $\pi = (11+\sqrt{61})/20$. □

The proof of uniqueness for

$$\pi \in \left(0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \frac{11+\sqrt{61}}{20}\right) \cup \left(\frac{11+\sqrt{61}}{20}, 1\right]$$

is conceptually simple, but tedious: it proceeds by exhaustively considering and ruling out each other class of strategy profiles. We relegate to a supplementary appendix this proof, together with the proof that for $\pi = (11+\sqrt{61})/20$, the only equilibria that hold are those listed in Proposition 2. This supplementary appendix is available online at https://msu.edu/~eguia/EguiaNicolo18OA.pdf.

References

Aidt, Toke S. and Julia Shvets (2012), "Distributive politics and electoral incentives: Evidence from seven U.S. state legislatures." *American Economic Journal: Economic Policy*, 4, 1–29. [375]

Aliprantis, Charalambos D. and Kim C. Border (2006), *Infinite Dimensional Analysis*. Springer, Berlin. [387]

Alt, James E. and David D. Lassen (2006), "Fiscal transparency, political parties and debt in OECD countries." *European Economic Review*, 50, 1403–1439. [374]

Alvarez, R. Michael (1997), *Information and Elections*. University of Michigan Press. [376]

Banks, Jeffrey S. (2000), "Buying supermajorities in finite legislatures." *American Political Science Review*, 94, 677–681. [375]

Baron, David P. (1994), "Electoral competition with informed and uninformed voters." *American Political Science Review*, 88, 33–47. [375]

Bartels, Larry M. (1986), "Issue voting under uncertainty: An empirical test." *American Journal of Political Science*, 30, 709–728. [376]

Besley, Timothy and Robin Burgess (2002), "The political economy of government responsiveness: Theory and evidence from India." *Quarterly Journal of Economics*, 117, 1415–1451. [376]

Bierbrauer, Felix J. and Pierre C. Boyer (2016), "Efficiency, welfare and political competition." *Quarterly Journal of Economics*, 131, 461–518. [375]

Boffa, Federico, Amedeo Piolatto, and Giacomo A. M. Ponzetto (2016), "Political centralization and government accountability." *Quarterly Journal of Economics*, 131, 381–422. [375]

Boyer, Pierre C., Kai A. Konrad, and Brian Roberson (2017), "Targeted campaign competition, loyal voters, and supermajorities." *Journal of Mathematical Economics*, 71, 49–62. [382]

Campbell, Angus, Philip E. Converse, Warren E. Miller, and Donald E. Stokes (1980), *The American Voter*. Chicago University of Press. [376]

Castells, Antoni and Albert Solé-Ollé (2005), "The regional allocation of infrastructure investment: The role of equity, efficiency and political factors." *European Economic Review*, 49, 1165–1205. [374]

Chari, Varadarajan V., Larry E. Jones, and Ramon Marimon (1997), "The economics of split-ticket voting in representative democracies." *American Economic Review*, 87, 957–976. [375]

Dal Bó, Ernesto (2007), "Bribing voters." *American Journal of Political Science*, 51, 789–803. [375]

Dekel, Eddie, Matthew O. Jackson, and Asher Wolinsky (2008), "Vote buying: General elections." *Journal of Political Economy*, 116, 351–380. [375]

Dekel, Eddie, Matthew O. Jackson, and Asher Wolinsky (2009), "Vote buying: Legislatures and lobbying." *Quarterly Journal of Political Science*, 4, 103–128. [375]

Dhami, Sanjit (2003), "The political economy of redistribution under asymmetric information." *Journal of Public Economics*, 87, 2069–2103. [375]

Dixit, Avinash and John Londregan (1995), "Redistributive politics and economic efficiency." *American Political Science Review*, 89, 856–866. [375]

Downs, Anthony (1960), "Why the government budget is too small in a democracy." *World Politics*, 12, 541–563. [375]

Egorov, Georgy (2009), "Political accountability under special interest politics." Unpublished paper, Harvard University. [375]

Eguia, Jon X., Aniol Llorente-Saguer, Rebecca Morton, and Antonio Nicolò (2014), "Equilibrium selection in sequential games with imperfect information." Max Planck Institute w.p. No. 2014-04. [380]

Eslava, Marcela (2011), "The political economy of fiscal deficits: A survey." *Journal of Economic Surveys*, 25, 645–673. [374]

Ferejohn, John (1986), "Incumbent performance and electoral control." *Public Choice*, 50, 5–25. [375]

Fernández, Raquel and Gilat Levy (2008), "Diversity and redistribution." *Journal of Public Economics*, 92, 925–943. [375]

Gavazza, Alessandro and Alessandro Lizzeri (2009), "Transparency and economic policy." *Review of Economic Studies*, 76, 1023–1048. [374, 375, 382]

Glaeser, Edward L., Giacomo A. M. Ponzetto, and Jesse M. Shapiro (2005), "Strategic extremism: Why republicans and democrats divide on religious values." *Quarterly Journal of Economics*, 120, 1283–1330. [375]

Gratton, Gabriele (2014), "Pandering and electoral competition." *Games and Economic Behavior*, 84, 163–179. [375]

Groseclose, Tim and James M. Snyder Jr. (1996), "Buying supermajorities." *American Political Science Review*, 90, 303–315. [375]

Gul, Faruk and Wolfgang Pesendorfer (2009), "Partisan politics and election failure with ignorant voters." *Journal of Economic Theory*, 144, 146–174. [375]

Heidhues, Paul and Johan Lagerlöf (2003), "Hiding information in electoral competition." *Games and Economic Behavior*, 42, 48–74. [375]

Huber, John D. and Michael M. Ting (2013), "Redistribution, pork and elections." *Journal of the European Economic Association*, 11, 1382–1403. [375]

Kartik, Navin, Francesco Squintani, and Katrin Tinn (2013), "Information revelation and pandering in elections." Unpublished paper, Department of Economics, Columbia University. [375]

Knight, Brian (2004), "Parochial interests and the centralized provision of local public goods: Evidence from congressional voting on transportation projects." *Journal of Public Economics*, 88, 845–866. [374]

Kreps, David M. and Robert Wilson (1982), "Sequential equilibrium." *Econometrica*, 50, 863–894. [378, 380]

Laslier, Jean-François and Karine Van der Straeten (2004), "Electoral competition under imperfect information." *Economic Theory*, 24, 419–446. [375]

Lindbeck, Assar and Jörgen W. Weibull (1987), "Balanced-budget redistribution as the outcome of political competition." *Public Choice*, 52, 273–297. [375]

Lizzeri, Alessandro and Nicola Persico (2001), "The provision of public goods under alternative electoral incentives." *American Economic Review*, 91, 225–239. [375]

Matějka, Filip and Guido Tabellini (2017), "Electoral competition with rationally inattentive voters." Unpublished paper, SSRN 3070204. [375]

McAfee, R. Preston and Marius Schwartz (1994), "Opportunism in multilateral vertical contracting: Nondiscrimination, exclusivity, and uniformity." *American Economic Review*, 84, 210–230.

Morris, Stephen and Hyun Song Shin (2002), "Social value of public information." *American Economic Review*, 92, 1521–1534. [376]

Myerson, Roger B. (1978), "Refinements of the Nash equilibrium concept." *International Journal of Game Theory*, 7, 73–80. [379, 386]

Myerson, Roger B. (1993), "Incentives to cultivate favored minorities under alternative electoral systems." *American Political Science Review*, 87, 856–869. [375]

Persson, Torsten and Guido Tabellini (2000), *Political Economics: Explaining Economic Policy*. MIT Press. [375]

Prat, Andrea (2005), "The wrong kind of transparency." *American Economic Review*, 95, 862–877. [376]

Roberson, Brian (2008), "Pork-barrel politics, targetable policies, and fiscal federalism." *Journal of the European Economic Association*, 6, 819–844. [375]

Rogoff, Kenneth (1990), "Equilibrium political budget cycles." *American Economic Review*, 80, 21–36. [374]

Rogoff, Kenneth and Anne Sibert (1988), "Elections and macroeconomic policy cycles." *Review of Economic Studies*, 55, 1–16. [374]

Schipper, Burkhard C. and Hee Woo (2017), "Political awareness, microtargeting of voters, and negative electoral campaigning." Unpublished paper, SSRN No. 2039122. [382]

Snyder, James M. and David Strömberg (2010), "Press coverage and political accountability." *Journal of Political Economy*, 118, 355–408.

Strömberg, David (2004), "Mass media competition, political competition, and public policy." *Review of Economic Studies*, 71, 265–284. [376]

Teoh, Siew Hong (1997), "Information disclosure and voluntary contributions to public goods." *RAND Journal of Economics*, 28, 385–406. [376]

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