Theoretical Economics 15 (2020), 625–667 1555-7561/20200625

Multiplier effect and comparative statics in global games of regime change

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This paper provides a general analysis of comparative statics results in global games. I show that the effect of a change in any parameter of a global game model of regime change can be decomposed into a direct effect, which captures the effect of a change in parameters when agents' beliefs are held constant, and a multiplier effect, which captures the role of adjustments in agents' beliefs. I characterize conditions under which the multiplier effect is strong and relate it to the strength of strategic complementarities and the publicity multiplier emphasized in earlier work. Finally, I use the above insights to identify when comparative statics can be deduced from the model's primitives when they do not depend on the information structure and when they coincide with predictions of the complete information model.

Keywords. Global games, comparative statics, multiplier effect, strategic complementarities, publicity multiplier.

JEL classification. D83, D84.

1. Introduction

Global games of regime change are coordination games with incomplete information where agents' payoffs depend on whether the status quo is preserved or abandoned. This class of games was first introduced to the literature by Carlsson and van Damme (1993) and popularized by Morris and Shin (1998, 2003). Since then, global games have been fruitfully used to study economic phenomena that feature coordination motives.[1]

The popularity of global games stems from the fact that global games tend to have a unique equilibrium, which allows one to obtain unambiguous comparative statics results. Indeed, in applications of global games, a significant effort is typically devoted to establishing comparative statics results. However, these results are derived on a case-by-case basis, and there exist few general results that could be invoked to simplify such

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I am thankful to the two referees for the comments and suggestions that helped me improve the paper. I thank participants at the 2018 North American Summer Meeting of the Econometric Society, NYU Alumni Conference, and UBC lunch microseminar for their helpful suggestions. I gratefully acknowledge the financial support provided by the University of British Columbia under the Hampton New Faculty Award.

[1]For example, currency crises have been considered in Morris and Shin (1998), Hellwig et al. (2006), and Angeletos et al. (2006, 2007); debt crises have been addressed in Szkup (2017) and Zabai (2019); political revolts have been treated in Edmond (2013); and business cycles have been dealt with in Schaal and Taschereau-Dumouchel (2015). For applications to banking, see Eisenbach (2017), Goldstein and Pauzner (2005), Rochet and Vives (2004), or Vives (2014).

 https://doi.org/10.3982/TE3074

analysis. Furthermore, there have been few attempts to understand how the presence of an incomplete information structure and heterogeneous beliefs affects comparative statics results.

The goal of this paper is to fill in this gap in the literature and provide a unified analysis of comparative statics results in global games of regime change. To do so, I consider a general global game model with a unique equilibrium that, as usual, is characterized by a regime change threshold θ^* (i.e., the value of fundamentals below which the status quo collapses and above which the status quo prevails). The main result of the paper, upon which all other results in the paper build, is that, following a change in *any parameter* of the model, the change induced in the threshold θ^* can be decomposed into a product of a "direct effect" and a "multiplier effect." The direct effect captures how a change in a parameter of the model affects the regime change threshold when agents' beliefs are held constant. Thus the direct effect captures the fundamental (i.e., "nonbelief") channels through which a change in a parameter of the model affects the equilibrium. The multiplier effect, alternatively, captures the effect of the adjustment in agents' beliefs about the likelihood of a regime change. I show that the multiplier effect is always greater than 1 and that the multiplier effect is the same for all parameters of the model.

The above result has three immediate consequences. First, it indicates that in order to determine the sign of comparative statics results, one can focus on the direct effect and abstract from adjustments in beliefs. Second, it indicates that adjustments in beliefs act like an amplification mechanism that always magnifies the initial effect of a parameter change. Third, since the multiplier effect associated with a change in a parameter is the same for all parameters, to identify which parameters have the strongest effect on the equilibrium it suffices to compare their direct effects. Thus the above decomposition not only clarifies the role of beliefs in the model but also can be used to simplify comparative statics analysis.

In the remainder of the paper, I investigate the properties of the multiplier and direct effects. I first relate the multiplier effect to the strength of strategic complementarities in the model and to the "publicity multiplier" (as introduced in the literature by Morris and Shin 2003). I find that the multiplier effect is large precisely when best-response functions are steep at the equilibrium threshold. I then use this observation to characterize when the multiplier effect is strong. I also show that the publicity multiplier is a special case of the multiplier effect identified above, and that a similar effect is associated with other parameters of the model.

I use the above results to answer three related questions: (i) When can comparative statics results be deduced from the model's primitives? (ii) When are comparative statics results independent of the assumed information structure? (iii) When do predictions of the global game model coincide with predictions based on analysis of the extremal equilibria of the complete information model? I provide a simple condition on the model's primitives under which comparative statics results are robust to changes in the information structure and can be deduced without solving the model. I also provide conditions under which predictions of the global game model and the underlying complete information model coincide.

In the final part of the paper, I show how the results established herein can be used to derive new results, improve understanding of existing results, or extend existing results. First, I extend the results of Sákovics and Steiner (2012) on the design of optimal subsidies in coordination problems. Second, using the model of Bebchuk and Goldstein (2011), I provide conditions under which a small shock to banks' capital can result in a lending freeze.

It should be noted that in this paper I do not interpret global games merely as an equilibrium selection device for an underlying complete information game. Instead, I take a broader view and interpret the global game model as a description of reality. In particular, I treat the information structure as a part of the model environment in the same way that one treats preferences, technology, or endowments. This approach is motivated by the view that most of the choices made by decision makers are taken under incomplete information about fundamentals. This interpretation of global games has been gaining popularity in recent years (see Angeletos and Lian 2016 for an up-to-date overview of this literature). Motivated by this approach, in this paper I analyze the model in which information is noisy (and hence fundamental uncertainty is present) rather than focus on equilibrium in the limit as the noise in the signals vanishes.[2]

Related Literature. This paper contributes to the ever-growing literature on global games. Global games were introduced by Carlsson and van Damme (1993), and extended by Frankel et al. (2003) and Oury (2013).[3] While global games have been extensively studied, there have been few attempts to derive general comparative statics results for global games or to understand the role that heterogeneous beliefs play in those results. The notable exceptions are Iachan and Nenov (2015), who study the effects of changes in the precision of private information on the regime change threshold, and Guimarães and Morris (2007), who compare the predictions of a global game model with those of a complete information framework in the context of a currency crisis model.

The analysis in this paper builds on insights from Cooper and John (1988) and Vives (2014). Cooper and John (1988) were the first to emphasize that models with strategic complementarities tend to feature a multiplier effect, although their analysis was limited to a complete information framework. Vives (2014) stresses the importance of taking into account the strength of strategic complementarities when performing comparative statics analysis in global games. The direct motivation for this work, however, comes from the applied literature and the difficulty of deriving (and interpreting) comparative statics results in complex global game models such as those in Eisenbach (2017), Szkup (2017), or Zabai (2019). Indeed, in Szkup (2017) I applied the results presented in this

[2]This broader view of global games is shared by a number of authors in the global game literature (see Heinemann and Illing 2002, Iachan and Nenov 2015, Inostroza and Pavan 2018, Morris and Shin 2003, or Vives 2014, among many others).

[3]See also Angeletos et al. (2007), Mathevet and Steiner (2013), and Steiner (2008) for analysis of dynamic global games; Hellwig (2002), Morris and Shin (2004), Angeletos and Werning (2006), Hellwig et al. (2006), and Tarashev (2007) for analysis of the impact of exogenous and endogenous public information on equilibrium in global games; Dasgupta (2007) for analysis of an option to delay decisions; Szkup and Trevino (2015) for analysis of information acquisition; Angeletos et al. (2006) and Angeletos and Pavan (2013) for analysis of signalling; or Edmond (2013) for analysis of information manipulation.

paper to analyze the effects of various government policies aimed at preventing self-fulfilling debt crises.

From a broader perspective, this paper is also related to the work on monotone comparative statics and supermodular games (see, for example, Milgrom and Roberts 1990, Topkis 1998, Van Zandt and Vives 2007, Vives 1990, 2005). One of the goals of these papers is to characterize a condition where a change in a parameter leads to a monotone adjustment either in the agent's choice (in a single-agent decision problem) or in the agent's best-response function (in strategic environments). These papers are also helpful in the analysis of global games; however, by relying on specific properties of global games, I am able to derive more detailed results and to uncover properties that are related to the structure of global games in particular.

2. The model

In this section, I describe the model and briefly characterize its unique equilibrium, which will serve as the starting point for the comparative statics analysis performed in the remainder of the paper.

2.1 *Setup*

There is a continuum of players indexed by $i \in I$, where without loss of generality I is normalized to $[0, 1]$. The set of players I is partitioned into a finite set $\mathcal{S}$ of types of players, $\mathcal{S} = \{s_1, \dots, s_N\}$. For every $n \in \mathcal{N} \equiv \{1, \dots, N\}$, s_n contains a continuum of identical players of measure λ_n, with $\Sigma_{n\in\mathcal{N}} \lambda_n = 1$. The type of player i is denoted by $s(i)$. All agents, regardless of their type, have the same action set $A = \{0, 1\}$. I denote agent i's action by a_i, where $a_i = 1$ corresponds to attacking the regime and $a_i = 0$ corresponds to not attacking the regime (i.e., supporting the status quo). Let m denote the proportion of agents choosing to attack the status quo, that is, $m = \Sigma_{n\in\mathcal{N}} (\int_{i\in s_n} a_i \, di)$.

The economy is characterized by a state variable $\theta \in \mathbb{R}$, referred to as the strength of the regime, and by the regime status $\mathcal{R} \in \{0, 1\}$, where $\mathcal{R} = 1$ indicates that there is a regime change and $\mathcal{R} = 0$ means that the status quo is preserved.[4] Initially, the economy is in the status quo. The regime changes, that is, $\mathcal{R} = 1$, if and only if

$$R(\theta, m; \boldsymbol{\psi}) < 0,$$

where $\boldsymbol{\psi}$ is a vector that contains all the parameters of the model. The function R, which I call the regime change function, measures the resilience of the regime and is assumed to be continuously differentiable in all its arguments, with $R_1 > 0$ and $R_2 < 0$. That is, the resilience of the regime increases with θ, the intrinsic strength of the regime, and decreases with m, the proportion of agents who decide to challenge the status quo. Finally,

[4] In Morris and Shin (1998) the status quo is a currency peg, while the alternative regime is a floating exchange rate regime; in Dasgupta (2007) the status quo is unprofitable (or unsuccessful) investment, while the alternative regime is the state where investment is profitable (successful); in Goldstein and Pauzner (2005) the status quo is a bank being solvent, while the alternative state is the bank becoming insolvent, etc.

I assume that for sufficiently small θ, the regime will change even if no agent attacks it, while for sufficiently large θ, the regime will survive even if all agents decide to challenge it. In other words, there exist $\underline{\theta}$ and $\overline{\theta}$ such that

$$R(\underline{\theta}, 0; \boldsymbol{\psi}) = 0 \quad \text{and} \quad R(\overline{\theta}, 1; \boldsymbol{\psi}) = 0,$$

and the regime collapses for all $\theta < \underline{\theta}$ while it survives for all $\theta > \overline{\theta}$ irrespective of the proportion of agents who decide to attack it.

The types of players differ with respect to their payoff functions. Since the action space is binary, it suffices to specify the payoff differential functions rather than the payoff functions themselves. Let $\pi^n(\theta, m; \boldsymbol{\psi})$ denote the payoff gain from choosing $a_i = 1$ rather than $a_i = 0$ for an agent of type s_n (the superscript on the function π denotes the type of the agent). Then

$$\pi^n(\theta, m, \boldsymbol{\psi}) = \begin{cases} H^n(\theta; \boldsymbol{\psi}) & \text{if } \mathcal{R} = 1 \\ L^n(\theta; \boldsymbol{\psi}) & \text{if } \mathcal{R} = 0, \end{cases}$$

where $H^n(\theta; \boldsymbol{\psi}) > 0$ is the payoff differential between attacking the status quo and not attacking it for an agent of type s_n when the regime changesl, and $L^n(\theta; \boldsymbol{\psi}) < 0$ is the corresponding payoff differential when the status quo is preserved. For every $n \in \mathcal{N}$, H^n and L^n are differentiable in all their arguments, bounded, and nonincreasing in θ. As a tie-breaking rule, I assume that agent i attacks the regime if he is indifferent.

The strength of the regime, θ, is distributed uniformly over the real line and is initially unobserved.[5] Agent i of type s_n observes a private signal

$$x_i = \theta + \tau_n^{-1/2} \varepsilon_i,$$

where ε_i is distributed according to an absolutely continuous distribution F_n with mean 0 and a continuously differentiable density f_n. The ε_i are identically distributed across agents of the same type, independent across all agents, and independent of θ. The parameter τ_n measures the precision of signals received by agents of type s_n. The payoff functions and their derivatives are integrable with respect to the measure induced by the cumulative distribution function (CDF) F_n.

As stated above, $\boldsymbol{\psi} = (\psi_1, \ldots, \psi_M) \in \mathbb{R}^M$ is the vector of all the parameters of the model, with ψ_m denoting a specific parameter. The vector $\boldsymbol{\psi}$ includes the parameters of the information structure as well as the parameters that directly affect the regime change function R or the payoff differential functions H^n and L^n, $n \in \mathcal{N}$.[6]

2.2 *The unique equilibrium*

Let $\alpha_i : \mathbb{R} \to \{0, 1\}$ denote agent i's strategy (i.e., the action agent i will take in response to signal x_i). As usual in the literature, I focus on monotone strategies; that is, strategies

[5]The assumption of a uniform improper prior is made for simplicity. See Sections 4.2 and 4.3.2, as well as Appendix E.4 for the analysis featuring a proper prior.

[6]For example, in Morris and Shin (1998) the vector of parameters $\boldsymbol{\psi}$ would include the benefit that the government receives from keeping the exchange rate fixed, the parameters describing the speculators' payoff function, the transaction cost in the currency markets, and the informativeness of signals.

where there is some $x_i^* \in \mathbb{R}$ such that $\alpha_i(x_i) = 1$ if and only if $x_i \leq x_i^*$ and $\alpha_i(x_i) = 0$ if $x_i > x_i^*$. The threshold x_i^* is referred to as the threshold signal, and an equilibrium in which all agents follow monotone strategies is called a monotone equilibrium.

Proposition 1. *There exists a unique equilibrium where the regime changes if and only if $\theta \leq \theta^*$ and where all of the following statements hold.*

(i) All agents of type $s_n \in \mathcal{S}$ use a monotone strategy with threshold x_n^*, where x_n^* is the unique solution to

$$P^n(\theta^*, x_n^*; \boldsymbol{\psi}) \equiv \int_{-\infty}^{\theta^*} H^n(\theta; \boldsymbol{\psi}) f_n(\theta | x_n^*)\, d\theta + \int_{\theta^*}^{\infty} L^n(\theta; \boldsymbol{\psi}) f_n(\theta | x_n^*)\, d\theta = 0. \tag{1}$$

(ii) The regime change threshold θ^* is the unique solution to

$$R\Big(\theta^*, \sum_{n \in \mathcal{N}} \lambda_n F_n\big(\tau_n^{1/2}(x_n^* - \theta^*)\big); \boldsymbol{\psi}\Big) = 0. \tag{2}$$

(iii) In the limit as $\tau_n \to \infty$ for all $n \in \mathcal{N}$, the regime change threshold θ^* is the unique solution to

$$R\bigg(\theta^*, \sum_{n \in \mathcal{N}} \lambda_n \frac{H^n(\theta^*; \boldsymbol{\psi})}{H^n(\theta^*; \boldsymbol{\psi}) - L^n(\theta^*; \boldsymbol{\psi})}; \boldsymbol{\psi}\bigg) = 0.$$

See the Appendix for all proofs.

I now turn my attention to the main focus of the paper, that is, the comparative statics results and the role played by the beliefs of agents in the determination of those results.

3. The multiplier and direct effects

Let $\psi_m \in \boldsymbol{\psi}$ be a parameter of interest, and suppose we are interested in understanding how a change in ψ_m affects the equilibrium thresholds θ^* and x^*. In what follows, I differentiate between "partial" and "total" changes in θ^* and x^* in response to a change in ψ_m. In particular, I denote by $\partial x_n^*/\partial\psi_m$ the effect that a change in ψ_m has on type s_n agents' threshold signal when agents' beliefs about θ^* are held constant. Similarly, I denote by $\partial\theta^*/\partial\psi_m$ the partial effect of a change in ψ_m on the regime change threshold when agents' strategies are held constant (i.e., with $\{x_n^*\}_{n=1}^N$ held constant).[7] Finally, I denote the total effects of a change in ψ_m on the equilibrium thresholds (including the effects through the change in beliefs) by $d\theta^*/d\psi_m$ and $dx_n^*/d\psi_m$. In other words, $d\theta^*/d\psi_m$ and $dx_n^*/d\psi_m$ correspond to the equilibrium effects induced by a change in ψ_m that one would typically compute when performing comparative statics analysis, while $\partial\theta^*/\partial\psi_m$ and $\partial x_n^*/\partial\psi_m$ correspond to the partial effects implied by a change in ψ_m when ignoring the adjustments in endogenous variables.

[7] Formally, $\partial x_n^*/\partial\psi_m$ is computed by applying the implicit function theorem to the payoff indifference condition for type s_n with θ^* treated as an exogenous constant, while $\partial\theta^*/\partial\psi_m$ is computed by applying the implicit function theorem to (2) with $\{x_n^*\}_{n=1}^N$ held constant.

Having introduced the above notation, I now state the main result of the paper, which all of the subsequent analysis is based on. This result states that a change in θ^* induced by a change in any parameter ψ_m of the model can be decomposed into the "direct effect" (denoted by $\mathcal{D}(\psi_m)$) and the "multiplier effect" (denoted by $\mathcal{M}(\psi_m)$).

THEOREM 1. *Fix $\boldsymbol{\psi}$. For any $\psi_m \in \boldsymbol{\psi}$, we have*

$$\frac{d\theta^*}{d\psi_m} = \underbrace{\left[\frac{1}{1-\sum_{n\in\mathcal{N}} \frac{\partial\theta^*}{\partial x_n^*}\frac{\partial x_n^*}{\partial\theta^*}}\right]}_{\textit{multiplier effect}(\mathcal{M}(\psi_m))} \underbrace{\left[\frac{\partial\theta^*}{\partial\psi_m} + \sum_{n\in\mathcal{N}} \frac{\partial\theta^*}{\partial x_n^*}\frac{\partial x_n^*}{\partial\psi_m}\right]}_{\textit{direct effect}(\mathcal{D}(\psi_m))}.$$

Moreover,

(i) $\mathcal{M}(\psi_m) \in (1,\infty)$ if $\tau_n < \infty$ for all $n \in \mathcal{N}$

(ii) for any $\psi_m, \psi_l \in \boldsymbol{\psi}$, we have $\mathcal{M}(\psi_m) = \mathcal{M}(\psi_l)$

(iii) for all $n \in \mathcal{N}$ and any $\psi_m \in \boldsymbol{\psi}$,

$$\lim_{\tau_n\to\infty} \mathcal{M} = \infty \quad \textit{and} \quad \lim_{\tau_n\to\infty} \mathcal{D}(\psi_m) = 0 \quad \textit{with} \quad \lim_{\tau_n\to\infty} \mathcal{M}\mathcal{D}(\psi_m) \in \mathbb{R}.$$

The above decomposition has an intuitive interpretation. The direct effect captures the effect that a change in ψ_m has on θ^* when agents' beliefs about the regime change threshold are held constant. In particular, a change in ψ_m leads to a change in θ^* by affecting the regime change condition (as captured by $\partial\theta^*/\partial\psi_m$) or by affecting the payoff indifference conditions and leading to a change in individual threshold signals while holding agents beliefs about θ^* unchanged (as captured by $(\partial\theta^*/\partial x_n^*)(\partial x_n^*/\partial\psi_m)$). Both of these effects are captured by $\mathcal{D}(\psi_m)$. Thus the direct effect captures the fundamental (i.e., nonbelief) channels through which a change in a parameter affects the equilibrium.

However, following a change in ψ_m, agents' beliefs are not constant. In particular, agents understand that a change in ψ_m leads to a change in θ^*, and hence adjust their beliefs and actions, inducing a further adjustment in θ^*. This leads to another round of adjustments in agents' beliefs, and hence in θ^* and so on. These adjustments are captured by the multiplier effect. Thus the multiplier effect captures the role that adjustments in beliefs play in the change in θ^*.[8]

The second part establishes several properties of the direct and multiplier effects. First, it states that the multiplier effect is always greater than 1, but finite as long as the precision of the information is finite. Second, a change in any component of $\boldsymbol{\psi}$ results in the same multiplier effect. In other words, if ψ_m and ψ_l are two distinct parameters of the

[8]The above discussion suggests that the decomposition of comparative statics stated in Theorem 1 can be obtained by analyzing equilibrium best-response dynamics (see, for example, Vives 2005). Indeed, in Appendix A, I show that the above result can be derived either by using the implicit function theorem or by computing the best-response dynamics. The latter has the advantage of providing an intuitive interpretation of this result.

model, then the difference in the equilibrium effects of changes in ψ_m and ψ_l are fully attributed to the difference in their direct effects. As a result, we can simply denote the multiplier effect by $\mathcal{M}$. Finally, we see that as signals of type s_n agents become infinitely precise, $\mathcal{M}$ tends to infinity and $\mathcal{D}(\psi_m)$ tends to 0, implying that in the limit, all of the adjustments in θ^* are driven by the adjustments in beliefs.

Why do we have that $\lim_{\tau_n\to\infty}\mathcal{D}(\psi_m)=0$? To understand this result, let $m_n^*(\theta)$ denote the proportion of agents of type s_n attacking the regime given θ. As implied by part (iii) of Proposition 1, $m_n^*(\theta)$ converges to a step function with $m_n^*(\theta)=1$ if $\theta<\theta^*$, $m_n^*(\theta)\in(0,1)$ if $\theta=\theta^*$, and $m_n^*(\theta)=0$ if $\theta>\theta^*$. This in turn implies that in the limit, the regime change function $R(\theta^*, m^*(\theta); \boldsymbol{\psi})$ is strictly less than 0 for $\theta<\theta^*$, takes the value 0 at $\theta=\theta^*$, and is strictly greater than 0 for all $\theta>\theta^*$. As such, any potential effect of a small change in ψ_m on θ^*, holding agents' beliefs constant, is always dominated by the discontinuous jump in the proportion of agents of type s_n attacking the regime. As a result, $\lim_{\tau_n\to\infty}\mathcal{D}(\psi_m)=0$.

It is worth stressing that despite its simplicity, Theorem 1, by clarifying the role of agents' beliefs, leads to new insights. First, it tells us that in to establish $\operatorname{sgn}(d\theta^*/d\psi)$, it suffices to determine the sign of the direct effect. Thus for the purpose of obtaining qualitative predictions, one can treat beliefs as a fixed object, which can substantially simplify the analysis. Second, Theorem 1 implies that an adjustment in beliefs acts like an amplification mechanism that always magnifies the initial response of θ^* to a change in ψ_m. Finally, we see that to determine which parameter has the strongest effect on θ^*, it suffices to compare the direct effect induced by each parameter. I state the above observations as a corollary.

Corollary 1. *Consider the effect of a change in ψ_m on the equilibrium.*

(i) The direction of the change in θ^* is determined by the direct effect, that is,

$$\operatorname{sgn}\left(\frac{d\theta^*}{d\psi_m}\right)=\operatorname{sgn}\big(\mathcal{D}(\psi_m)\big).$$

(ii) The adjustment in beliefs always amplifies the initial response of θ^*, that is,

$$\left|\frac{d\theta^*}{d\psi_m}\right|\geq\left|\frac{\partial\theta^*}{\partial\psi_m}\right|,$$

with strict inequality holding whenever $\partial\theta^*/\partial\psi_m\neq 0$.

(iii) Suppose that $\psi_m\in\boldsymbol{\psi}$ is the parameter that leads to the strongest direct effect. Then

$$\frac{d\theta^*}{d\psi_m}\geq\frac{d\theta^*}{d\psi_k}\quad \textit{for all } \psi_k\in\boldsymbol{\psi}, k\neq m$$

The proof follows immediately from Theorem 1 and the above discussion.

The remainder of the paper is devoted to investigating further properties of the multiplier effect (Section 4) and understanding further implications of Theorem 1 for comparative statics analysis (Section 5). In Section 6, I consider applications of these results. In Section 7, I discuss several extensions of Theorem 1.

4. Understanding the multiplier effect

In this section, I explore how the multiplier effect identified above is related to the strength of strategic complementarities and to the publicity multiplier, and when it is strong. This analysis is motivated by the work of Vives (2005, 2014), Morris and Shin (2003, 2004), and Bebchuk and Goldstein (2011).

4.1 *Relation to strategic complementarities*

In a recent paper, Vives (2014) stressed that "the degree of strategic complementarity of investors' actions is the crucial parameter… for policy analysis" and used this insight to show that the effect of financial regulation depends on the strength of strategic complementarities. Since, as shown in Theorem 1, the multiplier effect determines the overall effect of a given parameter change on the equilibrium, this suggests that there is a close connection between the magnitude of the multiplier effect and the degree of strategic complementarity of agents' actions. The main goal of this section is to understand this relationship.

Let $\widehat{x}_n$ be the threshold used by all the agents of type s_n, $n \in \mathcal{N}$. Let $\widehat{\boldsymbol{x}} = (\widehat{x}_1, \dots, \widehat{x}_N)$ be the vector of these thresholds, and denote by $\widehat{\theta}(\widehat{\boldsymbol{x}})$ the implied regime change threshold. Given that all other agents use monotone strategies, the best response of agent i is to use a monotone strategy with a threshold signal $\beta_{s(i)}(\widehat{\boldsymbol{x}})$, where $\beta_{s(i)}(\widehat{\boldsymbol{x}})$ is implicitly defined as the unique solution to agent i's indifference condition

$$\int_{-\infty}^{\widehat{\theta}(\widehat{\boldsymbol{x}})} H^{s(i)}(\theta; \boldsymbol{\psi}) f_{s(i)}\big(\theta | \beta_{s(i)}(\widehat{\boldsymbol{x}})\big)\, d\theta + \int_{\widehat{\theta}(\widehat{\boldsymbol{x}})}^{\infty} L^{s(i)}(\theta; \boldsymbol{\psi}) f_{s(i)}\big(\theta | \beta_{s(i)}(\widehat{\boldsymbol{x}})\big)\, d\theta = 0.$$

To measure the strength of strategic complementarities, one can ask how much $\beta_{s(i)}(\widehat{\boldsymbol{x}})$ increases as all the $\widehat{x}_n$, $n \in \mathcal{N}$, increase by a small amount. This is equivalent to computing the directional derivative of $\beta_{s(i)}(\widehat{\boldsymbol{x}})$ in the direction $\underline{\mathbf{1}} = (1, \dots, 1) \in \mathbb{R}^N$. I denote this directional derivative by $\nabla_{\underline{\mathbf{1}}} \beta_{s(i)}(\widehat{\boldsymbol{x}})$, where

$$\nabla_{\underline{\mathbf{1}}} \beta_{s(i)}(\widehat{\boldsymbol{x}}) = \sum_{n \in \mathcal{N}} \frac{\partial \beta_{s(i)}(\widehat{\boldsymbol{x}})}{\partial(\widehat{x}_n)}.$$

Using this definition, strategic complementarities are stronger when the best-response functions are steeper. However, while natural, the above definition suffers from the problem that in many cases a change in the setup will result in a best-response function becoming steeper at some $\widehat{\boldsymbol{x}}$ but flatter at others.[9]

Proposition 2, stated below, offers a solution to this problem. Specifically, it establishes that the magnitude of the multiplier effect, and hence the total effect of the change in θ^*, is determined by the slope of the best-response function evaluated at $\widehat{\boldsymbol{x}} = \mathbf{x}^*$, where $\mathbf{x}^* = (x_1^*, \dots, x_N^*)$ is the vector of equilibrium signal thresholds. Thus from the compar-

[9] To circumvent this problem, Vives (2014) suggests using the maximal value of the slope of the best-response function as the measure of the strength of strategic complementarities.

ative statics point of view the relevant measure of the strategic complementarities in global games is the slope of the best-response function evaluated at $\widehat{x} = \mathbf{x}^*$, $\nabla_{\underline{\mathbf{1}}}\beta_n(\mathbf{x}^*)$. I refer to this measure as the *equilibrium degree of strategic complementarities* (as it involves computing the slope of the best-response function at the equilibrium signal thresholds).

Proposition 2. *Let β_n denote the best-response function for type s_n, $n \in \mathcal{N}$. Then the following statements hold:*

(i) The multiplier effect is equal to

$$\mathcal{M} = \frac{1}{1 - \sum_{n\in\mathcal{N}} w_n \nabla_{\underline{\mathbf{1}}}\beta_n(\mathbf{x}^*)},$$

where $\mathbf{x}^* = (x_1^*, \dots, x_N^*)$ is the vector of equilibrium signal thresholds and

$$w_n = \frac{\partial\theta^*/\partial x_n^*}{\sum_{l\in\mathcal{N}}\partial\theta^*/\partial x_l^*}$$

measures the relative sensitivity of θ^* to changes in x_n^*, $n \in \mathcal{N}$.

(ii) If $\tau_n < \infty$ for all $n \in \mathcal{N}$, then $\nabla_{\underline{\mathbf{1}}}\beta_n(\mathbf{x}^*) < 1$ and $\mathcal{M} < \infty$. Moreover, if $\tau_n \to \infty$ for all $n \in \mathcal{N}$, then $\nabla_{\underline{\mathbf{1}}}\beta_n(\mathbf{x}^*) \to 1$ and $\mathcal{M} \to \infty$.

This result establishes the link between the equilibrium degree of strategic complementarities in the model and the multiplier effect. It tells us that the multiplier effect is strong precisely when the "equilibrium strategic complementarities" are strong (part (i) of the proposition). This identifies $\nabla_{\underline{\mathbf{1}}}\beta_n(\mathbf{x}^*)$ as the relevant measure of the strength of strategic complementarities in the model. Second, Proposition 2 indicates that the equilibrium strategic complementarities are maximized in the limit as $\tau_n \to \infty$, which explains why the multiplier effect tends to infinity in this case.

One may wonder how, in the limit as information becomes arbitrarily precise, the strength of strategic complementarities in the global game compares with the strength of strategic complementarities in the complete information game. In Appendix B.2, I show that they are equally strong. This observation underscores the important difference between global games and complete information models, namely the presence of "strategic uncertainty" in global games, which is missing in complete information frameworks. Thus while it is true that the strength of strategic complementarities increases with the precision of private signals, so does the strategic uncertainty, which is maximized precisely in the limit as the noise in the signals vanishes (see Morris and Shin 2003). In other words, even though the incentives to coordinate in a global game model are the highest when $\tau_n \to \infty$, agents are unable to coordinate their actions effectively.

4.2 *Publicity multiplier*

Next, I investigate how the publicity multiplier is related to the multiplier effect and revisit the question of when the publicity multiplier is particularly strong.[10] For the purpose of this section, and following Morris and Shin (2003), I assume that all agents are ex ante identical (i.e., of the same type) and share a common prior belief $\theta \sim N(\mu_\theta, \tau_\theta^{-1})$, and that each of them receives a private signal $x_i = \theta + \tau_x^{-1/2}\varepsilon_i$, $\varepsilon_i \sim N(0,1)$, ε_i independent and identically distributed (i.i.d.) across agents, and independent of θ. Here, μ_θ can be interpreted as the public information available to the agents. I also assume that μ_θ affects the equilibrium play only via its impact on agents' posterior beliefs. Otherwise, the setup is unchanged relative to Section 2.

Let θ^* be the unique equilibrium regime change threshold and let x^* be the associated threshold signal.[11] Morris and Shin (2003) define the publicity multiplier as $\mathcal{P} \equiv (dx^*/d\mu_\theta)/(\partial x^*/\partial \mu_\theta)$, where $dx^*/d\mu_\theta$ is the total change in x^* following a change in μ_θ and $\partial x^*/\partial\mu_\theta$ measures the direct effect of a change in μ_θ on agents' decisions. Since μ_θ is just one of the parameters of the model, I can define a similar multiplier effect for any $\psi_m \in \boldsymbol{\psi}$, which I denote by $\mathcal{P}(\psi_m)$. Nevertheless, μ_θ does have a distinct property in the current setup: μ_θ affects only agents' payoff indifference condition (via its effect on agents' posterior beliefs) but has no effect on the regime change condition. Thus below I limit my attention to parameters for which $\partial R/\partial \psi_m = 0$.

Proposition 3. *Let* $\boldsymbol{\psi}^P \equiv \{\psi_m \in \boldsymbol{\psi} | \partial R/\partial\psi_m = 0\}$. *Then*

$$\mathcal{P}(\psi_m) = \mathcal{M}, \quad \textit{implying that} \quad \frac{dx^*}{d\psi_m} = \mathcal{M}\frac{\partial x^*}{\partial \psi_m} \quad \textit{for all } \psi_m \in \boldsymbol{\psi}^P.$$

Since $\mu_\theta \in \boldsymbol{\psi}^P$, the above proposition implies that there is nothing special about the publicity multiplier and that such a multiplier effect applies to any parameter $\psi_m \in \boldsymbol{\psi}^P$. Moreover, we see that the multiplier effect associated with the comparative statics of x^* is the same as that which is associated with changes in θ^*.

The fact that $\mathcal{P}(\psi_m) = \mathcal{M}$ has important consequences. Since $\lim_{\tau\to\infty}\mathcal{M} = \infty$, the above result implies, counterintuitively, that the publicity multiplier is maximized in the limit as $\tau_x \to \infty$ when public information is ignored by the agents. To understand this apparent contradiction, recall that in light of Proposition 2, we have

$$\frac{dx^*}{d\mu_\theta} = \mathcal{M}\frac{\partial x^*}{\partial\mu_\theta} = \frac{1}{1-\beta'(x^*)}\frac{\partial\beta(x^*)}{\partial\mu_\theta},$$

[10]The role played by public information in global games has also been investigated by Hellwig (2002), Bannier and Heinemann (2005), and Metz (2002). See also Morris and Shin (2002), Angeletos and Pavan (2007), and Ui and Yoshizawa (2015) for analysis of public information in closely related quadratic-Gaussian models.

[11]With public information, an equilibrium is unique if and only if $\tau_x^{1/2}/\tau_\theta > (1/\sqrt{2\pi})(\overline{R}_2/\underline{R}_1)$, where $\underline{R}_1$ is the lower bound on $\partial R/\partial\theta$ and $\overline{R}_2$ is the upper bound on $\partial R/\partial m$.

where β denotes agents' best-response function, $\beta'(x^*) \in (0,1)$, and $\lim_{\tau \to \infty} \beta'(x^*) = 1$.[12] Observe that $\partial\beta(x^*)/\partial\mu_\theta = \tau_\theta/(\tau_x + \tau_\theta)$ and that this simple fact provides an explanation for the above result: As τ_x increases, the direct effect of a higher μ_θ tends to 0 since agents attach lower and lower weight to the public information. This effect dominates the increase in strategic complementarities (i.e., the increase in $\beta'(x^*)$); as a result, $dx^*/d\mu_\theta \to 0$ as $\tau_x \to \infty$. Thus Morris and Shin (2003) are correct to point out that μ_θ has the strongest impact on the equilibrium when τ_θ is high, but this is driven by the direct effect rather than the multiplier effect.[13]

4.3 *When is the multiplier effect strong?*

As argued above, the multiplier effect acts as an amplification mechanism, always magnifying the initial effect of changes to parameters. In this section, I investigate when this amplification mechanism is strong. Thus, the results reported below can be used to understand when small shocks to the model have large equilibrium consequences (see also Section 6.2).

4.3.1 *The general model* Determining the conditions under which the multiplier effect is strong boils down to understanding when agents have strong coordination motives. This happens when a change in x_n^* results in a relatively large adjustment in θ^* (i.e., $\partial\theta^*/\partial x_n^*$ is large) and, in turn, the change in θ^* has a relatively large impact on x_n^* (i.e., $\partial x_n^*/\partial\theta^*$ is large).

By inspection of the equilibrium regime change condition, we see that

$$\frac{\partial\theta^*}{\partial x_n^*} \propto \frac{\partial}{\partial m} R\big(\theta^*, m^*(\theta); \boldsymbol{\psi}\big) f_n\left(\frac{x_n^* - \theta^*}{\tau_n^{-1/2}}\right),$$

implying that $\partial\theta^*/\partial x_n^*$ is large when a change in x_n^* results in a large change in the proportion of agents attacking the regime (i.e., $f(\tau_n^{1/2}(x_n^* - \theta^*))$ is high) and the regime is sensitive to such a change (i.e., $\partial R(\theta^*, m^*(\theta); \boldsymbol{\psi})/\partial m$ is high). Similarly, by inspection of the agents' indifference condition, we see that

$$\frac{\partial x_n^*}{\partial\theta^*} \propto \big[H^n\big(\theta^*; \boldsymbol{\psi}\big) - L^n\big(\theta^*; \boldsymbol{\psi}\big)\big] f_n\left(\frac{x_n^* - \theta^*}{\tau_n^{-1/2}}\right),$$

implying that x_n^* is sensitive to changes in θ^* when the payoff difference between successful attack and unsuccessful attack is large at θ^* (large $H^n(\theta^*; \boldsymbol{\psi}) - L^n(\theta^*; \boldsymbol{\psi})$), and when, conditional on observing the threshold signal x_n^*, agents assign a high probability to θ lying in a close neighborhood of θ^*. This is because, in this case, a small change in θ^* results in a large increase in the expected utility difference between attacking and not attacking the regime at the critical signal x_n^*, prompting agents to increase their threshold signals sharply.

[12] Vives (2014) was the first one to derive the above decomposition of $dx^*/d\mu_\theta$.

[13] Following Morris and Shin (2003), in Section 4.2 I restricted my attention to parameters that affect only the payoff indifference condition and considered a setup in which all agents are of the same type. In Appendix B.3, I show that a similar decomposition can be derived if both assumptions are relaxed.

Beyond this broad intuition, little more can be said without imposing additional structure on the model. Thus in what follows I consider a simple setup that is more amenable to analysis.

4.3.2 *The simple model* In this section, I consider a simplified model that is popular in applications (see, for example, Rochet and Vives 2004, Bebchuk and Goldstein 2011, or Morris and Shin 2016) and is a particular case of a model in Vives (2014).

In particular, I consider a setup with only one type of agent, where the agents' payoff functions are constant in θ, that is, $H(\theta) = H > 0$ and $L(\theta) = L < 0$, and where the regime change function is linear in θ and in the proportion of agents that attack the regime, m, that is,

$$R(\theta, m) = \theta - zm,$$

where $z > 0$ is a parameter that captures the sensitivity of the regime to actions of agents. Each agent receives a private signal $x_i = \theta + \tau_x^{-1/2}\varepsilon_i$, $\varepsilon_i \sim N(0, 1)$, with the ε_i independent across agents and independent of θ, and they all share a common prior $\theta \sim N(\mu_\theta, \tau_\theta^{-1/2})$.[14] In what follows, it is convenient to define $\gamma \equiv -L/(H - L)$. The parameter γ measures the relative benefit of a successful attack to the cost of an unsuccessful attack. Note that $\gamma \in (0, 1)$ tends to 0 as $H \to \infty$ or $L \to 0$, and tends to 1 as $H \to 0$ or $L \to -\infty$. In this setup, the multiplier effect is given by

$$\mathcal{M} = \frac{1}{1 - \dfrac{\tau_x + \tau_\theta}{\tau_x} \dfrac{\tau_x^{1/2}\phi\big(\tau_x^{1/2}(x^* - \theta^*)\big)}{1 + \tau_x^{1/2}\phi\big(\tau_x^{1/2}(x^* - \theta^*)\big)}}. \tag{3}$$

Finally, define $\overline{\tau}_\theta(\boldsymbol{\psi})$ as the highest value of τ_θ for which the model has a unique equilibrium for a given $\boldsymbol{\psi}$.[15]

The multiplier effect as a function of γ, z, and μ_θ Let $\boldsymbol{\psi}_{-\boldsymbol{m}}$ denote the vector containing all the model's parameters except ψ_m. The next proposition characterizes how, for a given information structure (i.e., holding τ_x and τ_θ fixed), the multiplier effect varies with the parameters.

Proposition 4. *For a fixed information structure, define a function $g : \mathbb{R}^3 \to \mathbb{R}_+$ by*

$$g(\mu_\theta, z, \gamma) = \left|\mu_\theta - \frac{1}{2}z + \frac{\sqrt{\tau_x + \tau_\theta}}{\tau_\theta}\Phi^{-1}(\gamma)\right|.$$

(i) The multiplier effect is strong when g is low and achieves its maximum strength when $g(\mu_\theta, z, \gamma) = 0$.

(ii) For each $\psi_m \in \{\mu_\theta, z, \gamma\}$, with $\boldsymbol{\psi}_{-\boldsymbol{m}}$ held constant, the multiplier effect is a single peaked function of ψ_m and achieves its maximum value at $\widehat{\psi}_m(\boldsymbol{\psi}_{-\boldsymbol{m}}) \in \mathbb{R}$.

[14]The analysis of this section can be extended to the case of an arbitrary distribution of signals, but only when the prior is uninformative (i.e., a uniform improper prior).

[15]Derivations of $\overline{\tau}_\theta(\boldsymbol{\psi})$ are standard and hence are omitted from the paper.

The above proposition follows from the observations that $\mathcal{M}$ is a decreasing function of $|x^* - \theta^*|$ and that there is a one-to-one mapping between the value of g and the distance between x^* and θ^*. The second part of the proposition states that, holding other parameters constant, the multiplier effect is weak when $\psi_m \in \{\mu_\theta, z, \gamma\}$ takes extreme values. To understand why this is the case, note that when γ is high, the benefit from a successful attack compared to the loss from an unsuccessful attack is large; when z is high, the regime is likely to collapse even if only few agents will attack; and when μ_θ is low, agent i believes that the regime change will occur regardless of the actions of other agents. In all these cases, agents have weak incentives to coordinate their actions, which, in light of Proposition 2, implies that the multiplier effect is weak.

The multiplier effect as a function of τ_x and τ_θ In the next proposition, I characterize how the multiplier effect varies with τ_x and τ_θ.

Proposition 5. *Consider changes in τ_x and τ_θ.*

(i) There exists $\overline{\tau}_x$ such that for all $\tau_x > \overline{\tau}_x$, we have $\partial\mathcal{M}/\partial\tau_x > 0$.

(ii) For each τ_θ, there exist $\mu_L(\tau_\theta), \mu_H(\tau_\theta) \in \mathbb{R}$ with $\mu_L(\tau_\theta) < \mu_H(\tau_\theta)$ such that $\partial\mathcal{M}/\partial\tau_\theta \geq 0$ if and only if $\mu_\theta \in [\mu_L(\tau_\theta), \mu_H(\tau_\theta)]$.

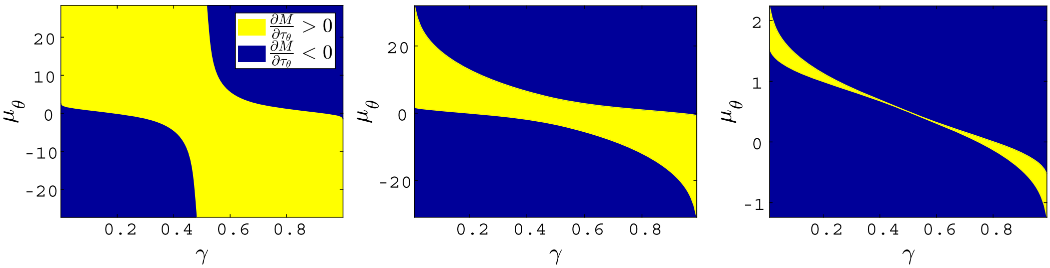
To understand the above proposition, note that a change in τ_θ or τ_x affects $\mathcal{M}$ through two channels that tend to work in opposite directions. First, it affects the sensitivity of x^* to changes in the regime change threshold θ^* ($(\tau_x+\tau_\theta)/\tau_x$ in the denominator of the expression on the right-hand side of (3)). Second, it affects $|x^* - \theta^*|$ and, hence, the sensitivity of the regime's strength to changes in the proportion of agents attacking. An increase in τ_x tends to decrease $\mathcal{M}$ through the first channel but tends to increase it through the second channel. For sufficiently high τ_x, the second effect dominates and, hence, $\partial\mathcal{M}/\partial\tau_x > 0$ for all $\tau_x > \overline{\tau}_x$.

An increase in τ_θ increases $(\tau_x + \tau_\theta)/\tau_x$, which tends to increase $\mathcal{M}$, but also increases $|x^* - \theta^*|$, which tends to decrease $\mathcal{M}$. This negative effect dominates when μ_θ takes extreme values; as a result, $\partial\mathcal{M}/\partial\tau_\theta \geq 0$ if and only if $\mu_\theta \in [\mu_L(\tau_\theta), \mu_H(\tau_\theta)]$. Moreover, this negative effect becomes stronger when τ_θ is high, and thus the region where $d\mathcal{M}/d\tau_\theta > 0$ shrinks rapidly as τ_θ increases (as shown in Figure 1).[16]

5. Implications for comparative statics analysis

In this section, I show how Theorem 1 can shed light on the following important questions about comparative statics results: (i) When are comparative statics predictions "robust" to changes in the information structure? (ii) When can comparative statics results be deduced directly from the model's primitives? (iii) When do comparative statics differ from predictions under complete information based on analysis of the extremal equilibria?

[16]For extreme values of μ_θ, an increase in τ_θ reinforces agents' posterior beliefs about the regime outcome, leading to an increase in $|x^* - \theta^*|$. This effect is stronger when μ_θ is extreme and/or when τ_θ is high, since in these cases a small increase in τ_θ leads to a relatively large change in the agents' posterior beliefs.



Panel A: Small τ_θ ($\tau_\theta \to 0$) Panel B: Medium τ_θ ($\tau_\theta = 1/32\overline{\tau}_\theta$) Panel C: Large τ_θ ($\tau_\theta = \overline{\tau}_\theta$)

FIGURE 1. The effect of an increase in τ_θ on the multiplier effect.

5.1 *Robust predictions*

Theorem 1 implies that to determine whether a change in a parameter of the model increases or decreases θ^*, it suffices to focus on the direct effect. In this section, I go a step further and analyze the direct effect to determine when the comparative statics results do not depend on the assumed information structure and when they can be deduced from the primitives.[17]

The next corollary provide a general, easy-to-check, condition under which $\text{sgn}(d\theta^*/d\psi_m)$ can be determined from the model's primitives and does not depend on the assumed information structure (i.e., neither on the choice of $\{F_n\}_{n\in\mathcal{N}}$ nor on the choice of the precisions $\{\tau_n\}_{n\in\mathcal{N}}$).

COROLLARY 2. *Fix $\boldsymbol{\psi}$. Suppose that for all $\theta \in \mathbb{R}$ and $m \in [0, 1]$, we have*

$$\frac{\partial R(\theta, m; \boldsymbol{\psi})}{\partial \psi_m} \leq (\geq) 0 \quad \text{and} \quad \frac{\partial \pi^n(\theta; \boldsymbol{\psi})}{\partial \psi_m} \geq (\leq) 0, \quad n \in \mathcal{N}. \tag{4}$$

Then $d\theta^*/d\psi_m \geq (\leq) 0$ and $\text{sgn}(d\theta^*/d\psi_m)$ is independent of the assumed information structure.

Condition (4) is intuitive: It states that $d\theta^*/d\psi_m > 0$ if an increase in ψ_m increases the payoffs from attacking the regime and/or decreases the resistance of the regime to attack. It should be stressed that while the hypothesis of Corollary 2 is stringent, it is satisfied in many applications (e.g., in the simple model considered in Section 4.3.2, with the exception of the precision parameters τ_x and τ_θ).

What if the hypothesis of Corollary 2 is not satisfied? In that case, without imposing either additional structure on the model or further conditions on the model's primitives,

[17]There is a close relationship between the conditions under which the sign of comparative statics can be deduced from the model's primitives and those under which their sign does not depend on the assumed information structure. To deduce comparative statics results from the primitives, it must be the case that the effect that a change in ψ has on R and π_n depends on neither θ nor m. Otherwise, we would need to know the equilibrium θ^* and $m^*(\theta^*)$, and these are objects that we can compute only by solving the model. However, changes in the information structure affect precisely θ^* and $m^*(\theta^*)$ and not the model's primitives. It follows that if we can deduce comparative statics from the model's primitives, then these results are "robust" to alternative information structures.

we cannot guarantee that comparative statics results do not depend on the imposed information structure. In the Appendix, I provide two examples of nonrobust predictions. In the first example (Appendix C.1.1), R is increasing in ψ_m for some values of θ and m, but decreasing for others. In that case, a change in the information structure may shift the equilibrium threshold θ^* and the equilibrium proportion of agents that attack the regime, $m^*(\theta^*)$, from the region where $\partial R/\partial\psi_m > 0$ to the region where $\partial R/\partial\psi_m < 0$, or vice versa. In the second example (Appendix C.1.2), a change in ψ_m decreases the payoff functions ($\partial\pi_n/\partial\psi_m < 0$), but also decreases the resilience of the regime ($\partial R/\partial\psi_m$). In this situation, the information structure may determine which effect dominates.

A stronger result can be achieved if we assume that the payoff differential functions are piecewise constant.

Lemma 1. *Suppose that $H^n(\theta; \boldsymbol{\psi}) = H^n > 0$ and $L(\theta; \boldsymbol{\psi}) = L^n < 0$ for all $n \in \mathcal{N}$. Then the model's predictions do not depend on the information structure.*

When the payoffs do not depend on θ, the proportion of agents who attack the regime at θ^* is determined by the payoffs $\{H^n, L^n\}_{n\in\mathcal{N}}$ only (see part (iii) of Proposition 1). In that case, θ^* and, hence, its comparative statics, also do not depend on the information structure. Note that this "robustness" result holds without the need for additional restrictions on the regime change function R that were needed in Corollary 2. However, it does require the strong assumption of piecewise-constant payoffs.

Above, I provided conditions under which predictions are robust to changes in the information structure. However, there are also situations where one may want to know whether adjusting parameter ψ_m always has a larger effect than adjusting parameter ψ_k, irrespective of the information structure (i.e., when the "comparative predictions" are robust).[18]

Corollary 3. *Consider $\psi_m, \psi_k \in \boldsymbol{\psi}$, and suppose that condition (4) in Corollary 2 is satisfied. If for all $\theta \in \mathbb{R}$ and $m \in [0, 1]$, we have*

$$\frac{\partial R(\theta, m; \boldsymbol{\psi})}{\partial\psi_m} < (>) \frac{\partial R(\theta, m; \boldsymbol{\psi})}{\partial\psi_k} \quad \textit{and} \quad \frac{\partial\pi^n}{\partial\psi_m} > (<) \frac{\partial\pi^n}{\partial\psi_k}, \quad n \in \mathcal{N}, \tag{5}$$

then $d\theta^*/d\psi_m > (<)\, d\theta^*/d\psi_k$ irrespective of the information structure.

The corollary states a sufficient condition for comparative predictions to be robust to changes in the information structure. If conditions (4) and (5) are satisfied, then a small change in ψ_m will always lead to a larger adjustment in θ^* than a change in ψ_k. If one or both of those conditions are violated, then, without further assumptions, we cannot guarantee that the ranking of parameters does not vary with the information structure. Unfortunately, condition (5) turns out to be very stringent. For example, as shown below, it is violated in the optimal subsidy design problem in the spirit of Sákovics and Steiner (2012).

[18]For example, consider a government that is deciding whom to subsidize so as to encourage investment (as in Sákovics and Steiner 2012; see also Section 6) and that does not know the information structure faced by the agents. The government might want to know whether the optimal subsidy scheme depends on the underlying information structure.

5.2 *Comparison with the complete information model*

Next, I compare predictions of the global game model with models with complete information. In the complete information version of the setup described in Section 2, all agents observe θ once it has been realized. It is well known that in this case any threshold $\widehat{\theta} \in [\underline{\theta}(\boldsymbol{\psi}), \overline{\theta}(\boldsymbol{\psi})]$ can be supported as an equilibrium threshold. The boundaries $\underline{\theta}(\boldsymbol{\psi})$ and $\overline{\theta}(\boldsymbol{\psi})$ of this "multiplicity region" constitute the smallest and largest equilibrium thresholds, respectively, of the complete information model. The predictions of the complete information model are then often based on the behavior of $\underline{\theta}(\boldsymbol{\psi})$ and $\overline{\theta}(\boldsymbol{\psi})$ in response to changes in $\psi_m \in \boldsymbol{\psi}$.

Let $\boldsymbol{\psi}^R = \{\psi_m \in \boldsymbol{\psi} | \partial R/\partial\psi_m \neq 0\}$ and $\boldsymbol{\psi}^P = \{\psi_m \in \boldsymbol{\psi} | \partial\pi^n/\partial\psi_m \neq 0 \text{ for some } n \in \mathcal{N}\}$, so that $\boldsymbol{\psi}^R$ is the vector of all the parameters that affect the regime change condition and that $\boldsymbol{\psi}^P$ is the vector of all the parameters that affect the payoff functions. Note that a change in ψ_m affects the extremal equilibria if and only if $\psi_m \in \boldsymbol{\psi}^R$. This is because $\underline{\theta}(\boldsymbol{\psi})$ and $\overline{\theta}(\boldsymbol{\psi})$ are defined as solutions to $0 = R(\underline{\theta}(\boldsymbol{\psi}), 0; \boldsymbol{\psi})$ and $0 = R(\overline{\theta}(\boldsymbol{\psi}), 1; \boldsymbol{\psi})$, respectively. Thus in contrast to the global game model, a change in $\psi_m \in \boldsymbol{\psi}^P \backslash \boldsymbol{\psi}^R$ has no effect on the extremal equilibria. This last observation is worth emphasizing, as it constitutes one of the advantages of global game selection over selection mechanisms based on the complete information game.

Corollary 4. *Consider the effect of a change in ψ_m on θ^*, $\underline{\theta}$, and $\overline{\theta}$. Suppose that $\psi_m \in \boldsymbol{\psi}^R$, and that $\partial R/\partial\psi_m > (<)0$ and $\partial\pi^n/\partial\psi_m \leq (\geq)0$ for all θ and m. Then*

$$\operatorname{sgn}\left(\frac{d\theta^*}{d\psi_m}\right) = \operatorname{sgn}\left(\frac{d\underline{\theta}(\boldsymbol{\psi})}{d\psi_m}\right) = \operatorname{sgn}\left(\frac{d\overline{\theta}(\boldsymbol{\psi})}{d\psi_m}\right).$$

Alternatively, if $\psi_m \in \boldsymbol{\psi}^P \backslash \boldsymbol{\psi}^R$, then the predictions of the two models will differ.

6. Applications

In this section, I consider two applications of the results established above. In the first application, I consider how a planner could use subsidies/taxes to coordinate a regime change. This question was first addressed by Sákovics and Steiner (2012). I show that the results developed above allow for a substantially more general analysis. In the second application, inspired by work of Bebchuk and Goldstein (2011), I investigate conditions under which small shocks to banks' balance sheets are substantially amplified by adjustments in beliefs. In both applications, I assume that agents have a proper prior (see Appendix E.4 for a discussion of this extension).

6.1 *Optimal subsidies*

Consider the model of Section 2, and suppose that a social planner wants to use subsidies to ensure that the threshold below which the regime changes is at most $\widehat{\theta}$ and he wants to achieve this in the least costly way.[19] Let $\mathbf{v} = (v_1, \ldots, v_N)$ denote the vector

[19] We can think of the planner encouraging investment (as in Sákovics and Steiner 2012), lending (as in Bebchuk and Goldstein 2011), or foreign direct investment (as in Dasgupta 2007).

of subsidies, with v_n denoting the subsidy granted to agents of type s_n. The planner's problem is then

$$\min_{\{v_n\}_{n=1}^N} \sum_{n\in\mathcal{N}} \lambda_n v_n$$
$$\text{such that } \theta^*(\mathbf{v}) \leq \widehat{\theta} \text{ and } \overline{v} > v_n \geq 0 \quad \text{for all } n \in \mathcal{N},$$

where $\overline{v}$ is the maximum subsidy that can be given to agents (which ensures that the cost of acting against the regime is always nonnegative) and $\theta^*(\mathbf{v})$ is the threshold below which investment is successful when the vector of subsidies is $\mathbf{v}$. Let $\mathcal{D}(v_n; \mathbf{v})$ denote the direct effect of a marginal subsidy to agents in group n when the subsidy vector is $\mathbf{v}$. Moreover, without loss of generality, assume that $\mathcal{D}(v_n; \mathbf{0}) \geq \mathcal{D}(v_{n+1}; \mathbf{0})$ for all $n \in \mathcal{N}$. The following result follows from Theorem 1.

Proposition 6. *Let $\widehat{\theta}$ be the target investment threshold.*

(i) *If for all $n \in \mathcal{N}$ and all feasible $\mathbf{v}$ we have*

$$\frac{\mathcal{D}(v_n; \mathbf{v})}{\lambda_n} > \frac{\mathcal{D}(v_{n+1}; \mathbf{v})}{\lambda_{n+1}},$$

then there exists $n^* \in \mathcal{N}$ such that $v_n^* = \overline{v}$ for all $n < n^*$, $v_n = 0$ for all $n > n^*$, and $v_n \in [0, \overline{v}]$ for $n = n^*$.

(ii) *If for all $n, m \in \mathcal{N}$ and all feasible $\mathbf{v}$ we have*

$$\left.\frac{\mathcal{D}(v_n; \mathbf{v})}{\lambda_n}\right|_{v_n=\overline{v}} < \left.\frac{\mathcal{D}(v_m; \mathbf{v})}{\lambda_m}\right|_{v_m=0},$$

then the following statements hold:

- *The planner subsidizes first the group with the highest $\mathcal{D}(v_n; \mathbf{v})|_{\mathbf{v}=\mathbf{0}}/\lambda_n$.*
- *The planner subsidizes all groups by a positive amount before fully subsidizing any single group.*
- *For all $v_n^*, v_m^* \in (0, \overline{v})$, we have $\mathcal{D}(v_n; \mathbf{v})/\lambda_n = \mathcal{D}(v_m; \mathbf{v})/\lambda_m$.*
- *If, in addition, for all $n \in \mathcal{N}$, $\lim_{v_n\to\overline{v}} \mathcal{D}(v_n; \mathbf{v}) = 0$, then $v_n^* < \overline{v}$ for all $n \in \mathcal{N}$ and $\widehat{\theta} < \theta^*(\overline{v}, \dots, \overline{v})$.*

Proposition 6 characterizes the optimal subsidies in two different scenarios. The first part of the proposition characterizes conditions under which the planner first fully subsidizes agents in group n before subsidizing agents in group $n+1$. This is an extension of the result derived by Sákovics and Steiner (2012) to environments where both the regime change condition and the agents' payoffs are nonlinear and feature a proper prior.[20] It is worth stressing that in environments with a proper prior, the method used

[20] In Sákovics and Steiner (2012), the direct effect of subsidies is always constant and hence their setup satisfies the condition stated in part (i) of Proposition 6.

by Sákovics and Steiner (2012) does not generally apply.[21] The second part of Proposition 6 provides a sufficient condition when such a sharp characterization of subsidies does not hold. When the effect of a full subsidy is low (i.e., $\mathcal{D}(v_n; \mathbf{v})/\lambda_n|_{v_n=\overline{v}}$ is low relative to $\mathcal{D}(v_N; \mathbf{v})/\lambda_N|_{v_N=0}$ for any feasible $\mathbf{v}$), the cost-efficient subsidy scheme tends to partially subsidize several groups at the same time. The subsidies to the targeted groups are chosen so that the direct effects are equalized across all subsidized groups.

Proposition 6 characterizes the optimal subsidies for a given information structure. One may wonder whether the optimal subsidy scheme varies with the underlying information structure. Note that a subsidy v_n has only a direct effect on group n, that is, $x_n^*/\partial v_n < 0$ and $\partial x_m^*/\partial v_n = 0$. It follows that condition (5) in Corollary 3 is violated and, without imposing additional assumptions, we cannot ensure that the optimal subsidy scheme is robust to information changes. Indeed, I next show that in the model of Sákovics and Steiner (2012), the optimal subsidies vary with the information structure.

6.1.1 *Investment subsidies (Sákovics and Steiner 2012)* As an application of Proposition 6, I now revisit the model of Sákovics and Steiner (2012). Their model fits directly into the framework described in Section 2 with $H^n(\theta) = b_n - c_n > 0$, $L^n(\theta) = -c_n < 0$, $R(\theta, m) = 1 - \theta - m$, and $m = \sum_{n=1}^N w_n \lambda_n$, where $\sum_{n=1}^N w_n \lambda_n = 1$.[22] Here, w_n captures the importance (or weight) of group n for the aggregate outcome, b_n is the benefit from successful investment, and c_n is the cost of investment. To demonstrate the importance of accommodating a proper prior, I consider a particular version of their model where agents in group n observe signals $x_i = \theta + \tau_n^{-1/2} \varepsilon_i$, $\varepsilon_i \sim N(0, 1)$, and all agents have the same prior $\theta \sim N(\mu_\theta, \tau_\theta^{-1})$, with $\tau_n^{1/2}/\tau_\theta > 1/\sqrt{2\pi}$, so that the equilibrium is unique.

As shown in Appendix D, in this setup the direct effect of subsidizing group n can be written as $\mathcal{D}(v_n; \mathbf{v}) = \widetilde{\mathcal{D}}(v_n; \mathbf{v})C(\mathbf{v})$, where $C(\mathbf{v})$ is common to all groups of agents. It follows that it is the behavior of $\widetilde{\mathcal{D}}(v_n; \mathbf{v})/\lambda_n$ that drives the design of optimal subsidies.[23] The next result describes some of the properties of $\widetilde{\mathcal{D}}(v_n; \mathbf{v})$.

Lemma 2. *Let $\tau_\theta \in (0, \overline{\tau}_\theta)$ and $\tau_n < \infty$, $n \in \mathcal{N}$. Then for all $n \in \mathcal{N}$, we have*

$$\widetilde{\mathcal{D}}(v_n; \mathbf{v}) < 0 \text{ and } \lim_{v_n \nearrow c_n} \widetilde{\mathcal{D}}(v_n; \mathbf{v}) = 0.$$

Moreover, there exist $\underline{\mu}$ and $\overline{\mu}$, with $\underline{\mu} < \overline{\mu}$, such that the following statements hold:

(i) If $\mu_\theta > \overline{\mu}$, then $\widetilde{\mathcal{D}}(v_n; \mathbf{v})|_{\mathbf{v}=0}$ is decreasing in c_n, τ_n, and w_n but increasing in b_n, and $\partial\widetilde{\mathcal{D}}(v_n; \mathbf{v})/\partial v_n > 0$ for all $v_n \in (0, \overline{v})$.

(ii) If $\mu_\theta < \underline{\mu}$, then $\widetilde{\mathcal{D}}(v_n; \mathbf{v})|_{\mathbf{v}=0}$ is decreasing in b_n, τ_n, and w_n but increasing in c_n, and there exists $\widehat{v}_n$ such that $\partial\widetilde{\mathcal{D}}(v_n; \mathbf{v})/\partial v_n < 0$ if $v_n \in (0, \widehat{v}_n)$ and $\partial\widetilde{\mathcal{D}}(v_n; \mathbf{v})/\partial v_n > 0$ if $v_n \in (\widehat{v}_n, \overline{v})$.

Finally, if $\tau_\theta \to 0$ or $\tau_n \to \infty$, then $\widetilde{\mathcal{D}}(v_n; \mathbf{v})/\lambda_n \to w_n/b_n$.

[21] Sákovics and Steiner (2012) uncover the fundamental property of global games, which states that, under general conditions, the (weighted) average strategic belief is a uniform belief on [0, 1]. Unfortunately, this result does not hold, in general, with a proper nonuniform prior.

[22] To be precise, to map this model into the model described in Section 2, one needs to define the state of the economy as $\widehat{\theta} = 1 - \theta$.

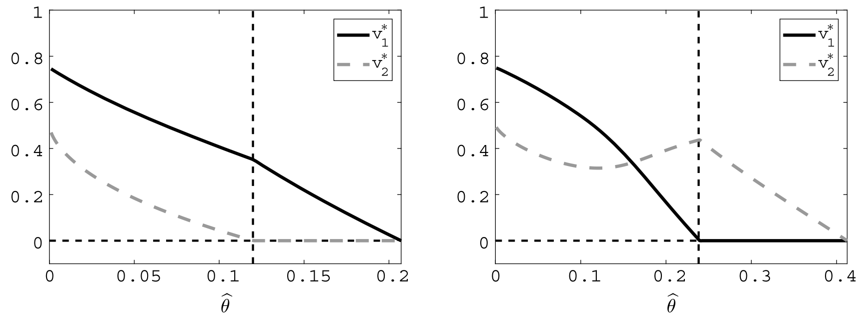
[23] In what follows, I slightly abuse the notation and refer to $\widetilde{\mathcal{D}}(v_n; \mathbf{v})$ as the direct effect.

The first part of Lemma 2 establishes that the direct effect of subsidies is negative, meaning that subsidizing any group decreases θ^*, but this effect tends to 0 as the subsidy to group n approaches its maximal level. It then follows from Proposition 6 that the planner will never choose to fully subsidize any group as long as the target threshold $\widehat{\theta}$ is strictly greater than 0. Instead, if needed, he will partially subsidize several groups.

The second part of Lemma 2 establishes how the direct effect varies with the parameters. If $\mu_\theta > \overline{\mu}$, the effect of initial subsidies is decreasing in c_n, τ_n, and w_n but increasing in b_n. This means that the planner will subsidize first the groups with high costs, weights, and precisions but low benefits. We also see that the effect of subsidies monotonically decreases with v_n, which suggests that subsidies will be monotone in the target threshold $\widehat{\theta}$. When $\mu_\theta < \underline{\mu}$, in contrast, the effect of the initial subsidies is decreasing in b_n, τ_n, and w_n, but increasing in c_n. Thus the planner will subsidize first the groups with high b_n, w_n, and τ_n but low c_n. Moreover, $\widetilde{\mathcal{D}}(v_n; \mathbf{v})$ is nonmonotone in v_n, which opens up the possibility that the subsidies will be nonmonotone in the target threshold. Finally, if $\tau_\theta \to 0$ or $\tau_n \to \infty$, we recover the results of Sákovics and Steiner (2012). In that case, the planner will subsidize first the group with the highest w_n/b_n. Moreover, if needed, he will subsidize that group fully before switching to the next group.

These results are depicted in Figure 2 in the context of the model with two groups of agents that differ in terms of their cost of investment and the precision of their signals. Figure 2 shows that when μ_θ is high (the left panel) the planner chooses to subsidize first the agents with high costs. However, for low enough $\widehat{\theta}$, the planner switches to partially subsidizing both groups. Finally, we see that subsidies are monotone in $\widehat{\theta}$. Alternatively, when μ_θ is low (the right panel), the planner chooses to subsidize first the agents with low costs. Again, for low enough $\widehat{\theta}$, the planner chooses to partially subsidize both groups, but now subsidies are nonmonotone (driven by nonmonotonicity of $\widetilde{\mathcal{D}}(v_n; \mathbf{v})$ in v_n).

The above analysis points to four main differences between the results in the limit as $\tau_n \to \infty$ for all $n \in \mathcal{N}$ and the results away from that limit when $\tau_n < \infty$ for all $n \in \mathcal{N}$. First, since $\lim_{v_n \to c_n} \widetilde{\mathcal{D}}(v_n; \mathbf{v}) = 0$, for a low enough target threshold the planner will tend



(a) High μ_θ (i.e., $\mu_\theta = 0.75$) (b) Low μ_θ (i.e., $\mu_\theta = -0.25$)

Figure 2. Optimal subsidies in the model with two groups of agents. The parameters are $\{b_1 = 2, c_1 = 0.75, w_1 = 1, m_1 = 0.5\}$ for agents in group 1 and $\{b_2 = 2, c_2 = 0.5, w_2 = 1, m_2 = 0.5\}$ for agents in group 2. The information structure is $\{\tau_\theta = 1, \tau_1 = 5, \tau_2 = 3\}$.

to subsidize several groups at the same time. Second, the cost of investment now plays a crucial role in determining who should be subsidized. Third, when μ_θ is low, the planner finds it optimal to subsidize groups with high b_n first, ceteris paribus, a result that is opposite to the one obtained in the limit. Finally, as shown numerically below, the optimal subsidies can be nonmonotone in the target threshold. The above results show that focusing on the limit case is not without loss of generality.

6.2 *Application: Amplification of small shocks*

Using a global game model, Bebchuk and Goldstein (2011) emphasized how a small shock to banks' capital (when amplified by strategic complementarities) can lead to a freeze in lending to the private sector, and they analyzed policies that can help to prevent such an outcome. In this section, I provide conditions under which such an amplification mechanism is likely to be strong.

In their model, there is a continuum $[0, 1]$ of risk-neutral banks, each with a net worth of \$1, that decide whether to invest in a risk-free asset or extend a risky loan. The return on the risk-free investment is equal to $1 + r$. The return on a corporate loan is equal to $1 + R$ (with $R > r$) if the economic fundamentals are strong and a sufficient number of corporations obtain credit, and 0 otherwise. In particular, a corporate loan pays net return R if and only if $\theta + zm > b$, where θ captures the strength of the economy, m is the mass of firms that received funding from the banks, z captures the strength of aggregate investment complementarities in the economy, and b is a threshold level for the loans to be profitable. As usual, banks do not observe θ, but each bank observes a private noisy signal $x_i = \theta + \tau^{-1/2}\varepsilon_i$, $\varepsilon_i \sim N(0, 1)$, with the ε_i independent and identically distributed across agents and independent of θ. This model is a version of the simple model considered in Section 4.3.2 with $L = 1 + r$, $H = R - r$, and $R(\theta, m; \boldsymbol{\psi}) = b - \theta - zm$.

Corollary 5. *For a fixed information structure, define*

$$g^{BG}(\mu_\theta, \alpha, r, R) \equiv \left| \mu_\theta - \frac{1}{2}z + \frac{\sqrt{\tau_x + \tau_\theta}}{\tau_\theta}\Phi^{-1}\left(\frac{1+r}{1+R}\right)\right|.$$

As g^{BG} decreases, the strength of the amplification mechanism increases, achieving its maximum when $g(\mu_\theta, z, r, R) = 0$. Moreover, if the precision of private information is high ($\tau_x > \overline{\tau}_x$), then the strength of the amplification mechanism is increasing in τ_x.

The proof follows from Proposition 4.

Corollary 5 provides potentially important insights for design of prudential macroeconomic policies and financial regulations. First, it stresses that strategic complementarities in lending can be large even if the complementarities at the macroeconomic level are weak (small z). Moreover, it suggests that even if the credit market looks robust (high R or high μ_θ), a small shock can still have a large effect on the provision of credit if g^{BG} takes a low value. Finally, Corollary 5 states that resolving informational asymmetries, as captured by an increase in τ_x, may increase the strength of the amplification mechanism present in the credit market, making the market more vulnerable.

7. Extensions of Theorem 1

As I show in Appendix E, Theorem 1 admits a number of useful extensions. In particular, it generalizes to settings with multiple equilibria, to discrete changes in parameters, and to models with a continuum of actions. Moreover, I show that Theorem 1 can be extended to describe comparative statics of a simultaneous change in several parameters and to environments with a proper prior. In all these extensions the qualitative properties of the direct and multiplier effect are unchanged. This shows that the decomposition derived in this paper is remarkably general and can be used for comparative statics analysis in a wide range of global games models.

8. Conclusions

In this paper, I provided a comprehensive analysis of comparative statics in a global games model of regime change. The central result of the paper is the decomposition of the comparative statics results into the direct effect and the multiplier effect. Despite its simplicity, this decomposition proves to be a surprisingly powerful tool for deriving and interpreting comparative statics results in global games models.

In the remainder of the paper, I analyzed the direct effect and the multiplier effect. In particular, I related the multiplier effect to the strength of strategic complementarities present in the model and to the "publicity multiplier" of Morris and Shin (2003, 2004). Furthermore, I used these insights to characterize conditions under which the multiplier effect is strong, so that a small shock to the model, when amplified by the endogenous adjustments in beliefs, results in large equilibrium adjustments. I then analyzed the direct effect. This analysis allowed me to identify conditions under which (i) comparative statics results can be deduced from the model's primitives, (ii) comparative statics results are independent of the assumed information structure, and (iii) predictions based on the global game coincide with those of the underlying complete information model. Finally, I used these results to study the design of optimal subsidies in coordination problems (see also Sákovics and Steiner 2012) and to understand when small shocks to banks' capital are likely to be amplified by the adjustments in agents' beliefs (see also Bebchuk and Goldstein 2011).

Appendix

This appendix contains the proofs of the results stated in the paper as well as several extensions of Theorem 1. The appendix is divided into five sections. Appendix A contains the proofs of Proposition 1 and Theorem 1. Appendix B contains the proofs of the results stated in Section 4, and Appendices C and D contain the proofs of the results stated in Section 5 and Section 6, respectively. Finally, in Appendix E, I discuss a number of extensions of Theorem 1. Several of the more involved proofs are only sketched here; complete proofs can be found in the working version of this paper, which is available on the author's website.

Appendix A: Proofs for Sections 2 and 3

Proof of Proposition 1. The proof of uniqueness in the case where $\tau_n < \infty$, for all $n \in \mathcal{N}$ is standard and hence is omitted. The proof of the uniqueness result when $\tau_n \to \infty$ for all $n \in \mathcal{N}$ is more technically involved than usual, due to the presence of multiple types of agents. This is because with N types of agents and the indifference conditions, we have to make sure that the whole system converges as $\tau_n \to \infty$. Thus below I prove only the third part of Proposition 1.

Part (iii). Note first that the equilibrium conditions that determine the equilibrium thresholds θ^* and $\{x_n^*\}_{n=1}^N$ are given by (2) and (1), respectively, while agent i's posterior belief density is given by $f_{s(i)}(\tau_{s(i)}^{1/2}(x_i - \theta))$.

It is easy to show that as $\tau_n \to \infty$ for each $n \in \mathcal{N}$, we have $\lim_{\tau_n \to \infty}(x_n^* - \theta^*) = 0$, and that there exists $\overline{\tau}$ such that $\tau_n > \overline{\tau}$ for all $n \in \mathcal{N}$ and the vector of equilibrium thresholds $(\theta^*, x_1^*, \dots, x_N^*) \in [\underline{\theta}, \overline{\theta}] \times [\underline{\theta} - \kappa, \overline{\theta} + \kappa] \times \cdots \times [\underline{\theta} - \kappa, \overline{\theta} + \kappa] \equiv K$, a compact subset of $\mathbb{R}^{N+1}$. Next, let $\boldsymbol{\tau} = (\tau_1, \dots, \tau_N)$ and consider a sequence $\{\boldsymbol{\tau}_k\}_{k=1}^\infty$ such that along this sequence $\tau_n \to \infty$ for each $n \in \mathcal{N}$. Let $(\theta^*(\boldsymbol{\tau}_k), x_1^*(\boldsymbol{\tau}_k), \dots, x_N^*(\boldsymbol{\tau}_k))_{k=1}^\infty$ be the resulting sequence of thresholds. As shown above, for all $\boldsymbol{\tau}_k > \overline{\boldsymbol{\tau}} \equiv (\overline{\tau}, \dots, \overline{\tau})$ we have $(\theta^*(\boldsymbol{\tau}_k), x_1^*(\boldsymbol{\tau}_k), \dots, x_N^*(\boldsymbol{\tau}_k)) \in K$, where K is compact, and thus $(\theta^*(\boldsymbol{\tau}_k), x_1^*(\boldsymbol{\tau}_k), \dots, x_N^*(\boldsymbol{\tau}_k))_{k=1}^\infty$ has a convergent subsequence where each element of this vector converges to a finite limit. Call this subsequence $\{\boldsymbol{\tau}_{k_j}\}_{j=1}^\infty$.

Since $(\theta^*(\boldsymbol{\tau}_k), x_1^*(\boldsymbol{\tau}_k), \dots, x_N^*(\boldsymbol{\tau}_k))$ has to be the solution to the $N+1$ equilibrium conditions for each $\boldsymbol{\tau}_k$, we know that these thresholds satisfy the payoff indifference conditions. The payoff indifference condition for type s_n, after performing the substitution $z = \tau_n^{1/2}(x_n^* - \theta)$, can be written as

$$\int_{-\infty}^{\infty} \Bigg[\mathbf{1}_{\{z \in [\tau_n^{1/2}(x_n^* - \theta^*), \infty)\}} H^n\bigg(x_n^* - \frac{z}{\tau_n^{1/2}}; \boldsymbol{\psi}\bigg) + \mathbf{1}_{\{z \in (-\infty, \tau_n^{1/2}(x_n^* - \theta^*))\}} L^n\bigg(x_n^* - \frac{z}{\tau_n^{1/2}}; \boldsymbol{\psi}\bigg)\Bigg] f_n(z)\, dz = 0,$$

where the integrand is bounded (as both H^n and L^n are bounded). Thus by the bounded convergence theorem, we can pass the limit as $\boldsymbol{\tau}_{k_j} \to \infty$ through the integral. Then the indifference condition converges to

$$\int_{-\infty}^{\infty} \big[\mathbf{1}_{\{z \in [\varsigma_n, \infty)\}} H^n\big(x_n^*; \boldsymbol{\psi}\big) + \mathbf{1}_{\{z \in (-\infty, \varsigma_n)\}} L^n\big(x_n^*; \boldsymbol{\psi}\big)\big] f_n(z)\, dz = 0,$$

where $\varsigma_n \equiv \lim_{\boldsymbol{\tau}_{k_j} \to \infty} \tau_n^{1/2}(x_n^* - \theta^*) \in \mathbb{R}$. Since $H^n(x_n^*; \boldsymbol{\psi})$ and $L^n(x_n^*; \boldsymbol{\psi})$ do not depend on z, we can write the above condition as

$$H^n\big(x_n^*; \boldsymbol{\psi}\big)\big[1 - F_n(\varsigma_n)\big] + L^n\big(x_n^*; \boldsymbol{\psi}\big) F_n(\varsigma_n) = 0.$$

Rearranging, we get

$$F_n(\varsigma_n) = \frac{H^n\big(x_n^*; \boldsymbol{\psi}\big)}{H^n\big(x_n^*; \boldsymbol{\psi}\big) - L^n\big(x_n^*; \boldsymbol{\psi}\big)}.$$

Note that $F_n(s_n)$ is the proportion of agents of type s_n who attack the regime as $\boldsymbol{\tau}_{k_j} \to \infty$. Denote by θ^*_∞ the limit of θ^* as $\boldsymbol{\tau}_{k_j} \to \infty$, and recall that $\lim_{\boldsymbol{\tau}_{k_j} \to \infty}(x^*_n - \theta^*) = 0$. Then θ^*_∞ has to be the unique solution to the regime change condition[24]

$$R\left(\theta^*_\infty, \sum_{n=1}^{N} \lambda_n \frac{H^n(\theta^*_\infty; \boldsymbol{\psi})}{H^n(\theta^*_\infty; \boldsymbol{\psi}) - L^n(\theta^*_\infty; \boldsymbol{\psi})}; \boldsymbol{\psi}\right) = 0.$$

So far I have considered a particular convergent subsequence. However, note that the limit derived above is independent of that convergent subsequence. Therefore, we conclude that $(\theta^*(\boldsymbol{\tau}_n), x^*_1(\boldsymbol{\tau}_n), \ldots, x^*_N(\boldsymbol{\tau}_n))$ converges to the above limit, which completes the proof. □

Proof of Theorem 1.

Derivations using the implicit function theorem. Fix the vector of parameters $\boldsymbol{\psi} \in \mathbb{R}^M$, and let $(\theta^*(\boldsymbol{\psi}), x^*_1(\boldsymbol{\psi}), \ldots, x^*_N(\boldsymbol{\psi}))$ be the associated monotone equilibrium. The equilibrium thresholds have to satisfy the N indifference equations, which can be written succinctly as

$$P^1(\theta^*(\boldsymbol{\psi}), x^*_1(\boldsymbol{\psi}); \boldsymbol{\psi}) = 0, \ldots, P^N(\theta^*(\boldsymbol{\psi}), x^*_N(\boldsymbol{\psi}); \boldsymbol{\psi}) = 0,$$

and the regime change condition, which can be written as

$$R(\theta^*(\boldsymbol{\psi}), x^*_1(\boldsymbol{\psi}), \ldots, x^*_N(\boldsymbol{\psi}); \boldsymbol{\psi}) = 0.$$

The key observation is that the equilibrium conditions written above are identities, as they define implicitly the equilibrium thresholds as a function of $\boldsymbol{\psi}$. Therefore, differentiating the equilibrium conditions with respect to $\psi_m \in \boldsymbol{\psi}$, we obtain

$$\frac{\partial P^1}{\partial \theta^*}\frac{d\theta^*}{d\psi_m} + \frac{\partial P^1}{\partial x^*_1}\frac{dx^*_1}{d\psi_m} + \frac{\partial P^1}{\partial \psi_m} = 0, \ldots, \frac{\partial P^N}{\partial \theta^*}\frac{d\theta^*}{d\psi_m} + \frac{\partial P^N}{\partial x^*_N}\frac{dx^*_N}{d\psi_m} + \frac{\partial P^N}{\partial \psi_m} = 0$$

and

$$\frac{\partial R}{\partial \theta^*}\frac{d\theta^*}{d\psi_m} + \sum_{n=1}^{N} \frac{\partial R}{\partial x^*_n}\frac{dx^*_n}{d\psi_m} + \frac{\partial R}{\partial \psi_m} = 0. \tag{6}$$

Note that using the nth indifference condition $(P^n(\theta^*(\boldsymbol{\psi}), x^*_n(\boldsymbol{\psi}); \boldsymbol{\psi}) = 0)$, we can express $dx^*_n/d\psi_m$ as

$$\frac{dx^*_n}{d\psi_m} = \frac{\partial x^*_n}{\partial \theta^*}\frac{d\theta^*}{d\psi_m} + \frac{\partial x^*_n}{\partial \psi_m},$$

where $\partial x^*_n/\partial \theta^* = -[\partial P^n/\partial \theta^*]/[\partial P^n/\partial x^*_n]$ and $\partial x^*_n/\partial \psi_m = -[\partial P^n/\partial \psi_m]/[\partial P^n/\partial x^*_n]$. Substituting this into (6), we obtain

$$\frac{\partial R}{\partial \theta^*}\frac{d\theta^*}{d\psi_m} + \sum_{n=1}^{N} \frac{\partial R}{\partial x^*_n}\left[\frac{\partial x^*_n}{\partial \theta^*}\frac{d\theta^*}{d\psi_m} + \frac{\partial x^*_n}{\partial \psi_m}\right] + \frac{\partial R}{\partial \psi_m} = 0.$$

[24]Otherwise, there would exist values of $\boldsymbol{\tau}_{k_j}$ such that $\theta^*(\boldsymbol{\tau}_{k_j})$ would violate the regime change condition.

Dividing both sides by $\partial R/\partial\theta^*$, rearranging, and recognizing that $\partial\theta^*/\partial\psi_m = -[\partial R/\partial\psi_m]/[\partial R/\partial\theta^*]$ and $\partial\theta^*/\partial x_n^* = -[\partial R/\partial x_n^*]/[\partial R/\partial\theta^*]$, we obtain

$$\frac{d\theta^*}{d\psi_m} = \frac{1}{1-\sum_{n=1}^{N}\frac{\partial\theta^*}{\partial x_n^*}\frac{\partial x_n^*}{\partial\theta^*}} \times \left[\sum_{n=1}^{N}\frac{\partial\theta^*}{\partial x_n^*}\frac{\partial x_n^*}{\partial\psi_m} + \frac{\partial\theta^*}{\partial\psi_m}\right],$$

as claimed.

Next, I show that the multiplier effect is always greater than 1. To see this, note that $\partial x_n^*/\partial\theta^* \in (0,1)$ since

$$\frac{\partial x_n^*}{\partial\theta^*} = \frac{-H^n(\theta^*;\boldsymbol{\psi})\tau^{1/2}f_n\left(\frac{x_n^*-\theta}{\tau^{-1/2}}\right) + L^n(\theta^*;\boldsymbol{\psi})\tau^{1/2}f_n\left(\frac{x_n^*-\theta}{\tau^{-1/2}}\right)}{-H^n(\theta^*;\boldsymbol{\psi})\tau^{1/2}f_n\left(\frac{x_n^*-\theta}{\tau^{-1/2}}\right) + L^n(\theta^*;\boldsymbol{\psi})\tau^{1/2}f_n\left(\frac{x_n^*-\theta}{\tau^{-1/2}}\right) + \Lambda^n}, \tag{7}$$

where

$$\Lambda^n \equiv \int_{\tau^{1/2}(x_n^*-\theta^*)}^{\infty}\frac{\partial H^n(x_n^*-\tau^{-1/2}z;\boldsymbol{\psi})}{\partial x_n^*}f_n(z)\,dz + \int_{-\infty}^{\tau^{1/2}(x_n^*-\theta^*)}\frac{\partial L^n(x_n^*-\tau^{-1/2}z;\boldsymbol{\psi})}{\partial x_n^*}f_n(z)\,dz < 0.$$

Moreover, from the regime change condition, we have

$$\frac{\partial\theta^*}{\partial x_n^*} = \frac{-R_2\lambda_n\tau_n^{1/2}f_n\left(\frac{x_n^*-\theta^*}{\tau_n^{-1/2}}\right)}{R_1 - R_2\sum_{k=1}^{N}\lambda_k\tau_k^{1/2}f_k\left(\frac{x_k^*-\theta^*}{\tau_k^{-1/2}}\right)} \in (0,1),$$

since $R_1 > 0$ and $R_2 < 0$. Therefore,

$$0 < \sum_{n=1}^{N}\frac{\partial\theta^*}{\partial x_n^*}\frac{\partial x_n^*}{\partial\theta^*} \le \sum_{n=1}^{N}\frac{\partial\theta^*}{\partial x_n^*} = \frac{-R_2\sum_{n=1}^{N}\lambda_n\tau_n^{1/2}f_n\left(\frac{x_n^*-\theta^*}{\tau_n^{-1/2}}\right)}{R_1 - R_2\sum_{k=1}^{N}\lambda_k\tau_k^{1/2}f_k\left(\frac{x_k^*-\theta^*}{\tau_k^{-1/2}}\right)} < 1.$$

This establishes that $\mathcal{M} \in (1,\infty)$ and $\operatorname{sgn}(d\theta^*/d\psi) = \operatorname{sgn}(\mathcal{D}(\psi))$.

Finally, I consider what happens to the multiplier effect as $\tau_n \to \infty$ for some $n \in \mathcal{N}$. In the proof of Proposition 1, I showed that $\tau^{1/2}(x_n^* - \theta^*)$ converges to a finite number as $\tau_n \to \infty$. Therefore, from the expression for $\partial\theta^*/\partial x_n^*$, it is clear that $\lim_{\tau_n\to\infty}\sum_{n\in\mathcal{N}}\partial\theta^*/\partial x_n^* = 1$ and that $\lim_{\tau_n\to\infty}\sum_{n\in\mathcal{N}}\partial\theta^*/\partial x_k^* = 0$. Next consider the expression for $\partial x_n^*/\partial\theta^*$ in (7). Since $\tau_n^{1/2}(x_n^* - \theta^*)$ converges to a finite number as $\tau_n \to \infty$,

$\lim_{\tau_n\to\infty}\Lambda^n$ is finite, and

$$\lim_{\tau_n\to\infty}\left[-H^n(\theta^*;\boldsymbol{\psi})\tau_n^{1/2}f_n\left(\frac{x_n^*-\theta}{\tau^{-1/2}}\right)+L^n(\theta^*;\boldsymbol{\psi})\tau_n^{1/2}f_n\left(\frac{x_n^*-\theta}{\tau_n^{-1/2}}\right)\right]$$
$$=\lim_{\tau_n\to\infty}\tau_n^{1/2}f_n\left(\frac{x_n^*-\theta}{\tau_n^{-1/2}}\right)\underbrace{\left[-H^n(\theta^*;\boldsymbol{\psi})+L^n(\theta^*;\boldsymbol{\psi})\right]}_{<0}=-\infty.$$

It follows that $\lim_{\tau_n\to\infty}\partial x_n^*/\partial\theta^*=1$, implying that $(\partial\theta^*/\partial x_n^*)(\partial x_n^*/\partial\theta^*)\to 1$. Since a change in τ_n has no effect on $\partial x_k^*/\partial\theta^*$, we conclude that $(\partial\theta^*/\partial x_k^*)(\partial x_k^*/\partial\theta^*)\to 0$. From these observations it follows that $\lim_{\tau\to\infty}\mathcal{M}=\infty$. By a similar argument one can show that $\mathcal{D}(\psi)\to 0$ as $\tau_n\to\infty$. □

Derivations using best-response dynamics. I show how one can derive the multiplier and direct effects by considering the best-response dynamics following a change in model parameters.

A small change in ψ_m has two effects on the regime outcome. First, holding agents' beliefs about θ^* constant, for each n it leads to a change in the signal threshold used by type s_n equal to $\partial x_n^*/\partial\psi_m$. This in turn leads to a change in the regime change threshold equal to $(\partial\theta^*/\partial x_n^*)(\partial x_n^*/\partial\psi_m)$. Second, it directly affects the resilience of the status quo, and hence leads to a change in the regime change threshold equal to $\partial\theta^*/\partial\psi_m$. It follows that the change in ψ_m leads to an initial change in θ^* equal to

$$\frac{\partial\theta^*}{\partial\psi_m}+\sum_{n=1}^{N}\frac{\partial\theta^*}{\partial x_n^*}\frac{\partial x_n^*}{\partial\psi_m},$$

which is exactly the direct effect $\mathcal{D}(\psi_m)$ identified above.

The above change in θ^* initiates further adjustments to the new equilibrium. In response to a change in θ^* by $\mathcal{D}(\psi_m)$, each agent adjusts his/her threshold. This adjustment is approximately equal to $(\partial x_n^*/\partial\theta^*)\mathcal{D}(\psi_m)$, that is, the product of sensitivity of x_n^* to changes in θ^* and the actual change of θ^*. This leads to an additional change in θ^* equal to

$$S_1\equiv\sum_{n=1}^{N}\frac{\partial\theta^*}{\partial x_n^*}\frac{\partial x_n^*}{\partial\theta^*}\mathcal{D}(\psi_m),$$

where I am using S_1 to denote the first step of the best-response dynamics initiated by the direct effect. This change in θ^* equal to S_1 induces a further change in agents' beliefs about the regime outcome and hence to a further adjustment. This process continues ad infinitum, with the adjustment of θ^* in the kth round of this process equal to $\mathcal{D}(\psi_m)(\sum_{n=1}^{N}(\partial\theta^*/\partial x_n^*)(\partial x_n^*/\partial\theta^*))^k$. The total change in θ^* is then obtained by adding up the adjustments of θ^* in all rounds (including the initial response), hence (using the convention that $S_0 \equiv \mathcal{D}(\psi_m)$) it is given by

$$\frac{d\theta^*}{d\psi_m} = \sum_{k=0}^{\infty} S_k = \frac{1}{1 - \sum_{n=1}^{N} \frac{\partial\theta^*}{\partial x_n^*} \frac{\partial x_n^*}{\partial\theta^*}} \left[\frac{\partial\theta^*}{\partial\psi_m} + \sum_{n=1}^{N} \frac{\partial\theta^*}{\partial x_n^*} \frac{\partial x_n^*}{\partial\psi_m} \right],$$

where the second equality is valid if $\Sigma_{n=1}^{N}(\partial\theta^*/\partial x_n^*)(\partial x_n^*/\partial\theta^*) < 1$ (which is always the case when there is a unique equilibrium). □

Appendix B: Proofs for Section 4

B.1 *Relation to strategic complementarities*

Proof of Proposition 2. Part (i). Let $\mathbf{x}^* = (x_1^*, \ldots, x_N^*)$, where x_n^* is the threshold used by agents of type s_n and let $\theta^*(\mathbf{x}^*)$ be the implied regime change threshold. Note that

$$\sum_{n=1}^{N} w_n \nabla_{\underline{\mathbf{1}}} \beta_n(\mathbf{x}^*) = \sum_{n=1}^{N} w_n \left(\sum_{k=1}^{N} \frac{\partial\beta_n(\mathbf{x}^*)}{\partial x_k^*} \right) = \sum_{n=1}^{N} w_n \left(\sum_{k=1}^{N} \frac{\partial\beta_n(\mathbf{x}^*)}{\partial\theta^*} \frac{\partial\theta^*(\mathbf{x}^*)}{\partial x_k^*} \right),$$

as x_k^* affects $\beta_n(\mathbf{x}^*)$ only indirectly through its effect on θ^*. Using the definition of w_n, we obtain

$$\begin{aligned} \sum_{n=1}^{N} w_n \left(\sum_{k=1}^{N} \frac{\partial\beta_n(\mathbf{x}^*)}{\partial\theta^*} \frac{\partial\theta^*(\mathbf{x}^*)}{\partial x_k^*} \right) &= \sum_{n=1}^{N} \frac{\frac{\partial\theta^*(\mathbf{x}^*)}{\partial x_n^*}}{\sum_{l=1}^{N} \frac{\partial\theta^*(\mathbf{x}^*)}{\partial x_l^*}} \left(\sum_{k=1}^{N} \frac{\partial\beta_n(\mathbf{x}^*)}{\partial\theta^*} \frac{\partial\theta^*(\mathbf{x}^*)}{\partial x_k^*} \right) \\ &= \sum_{n=1}^{N} \frac{\partial\theta^*(\mathbf{x}^*)}{\partial x_n^*} \frac{\partial x_n^*}{\partial\theta^*} \end{aligned}$$

since in equilibrium $x_n^* = \beta_n(\mathbf{x}^*)$.

Part (ii). Without loss of generality, let us focus on an agent of type s_n whose indifference condition is

$$\int_{-\infty}^{\theta^*(\mathbf{x}^*)} H^n(\theta; \boldsymbol{\psi}) f_n(\theta|\beta_n(\mathbf{x}^*))\, d\theta + \int_{\theta^*(\mathbf{x}^*)}^{\infty} L^n(\theta; \boldsymbol{\psi}) f_n(\theta|\beta_n(\mathbf{x}^*))\, d\theta = 0.$$

In the proof of Theorem 1, I showed that as long as $\tau_n < \infty$, we have $\partial\beta_n(\mathbf{x}^*)/\partial\theta^* < 1$ and $\Sigma_{n\in\mathcal{N}}\partial\theta^*/\partial x_k^* < 1$. It follows that $\nabla_{\underline{\mathbf{1}}}\beta_n(\mathbf{x}^*) = \sum_{k=1}^{N}(\partial\beta_n(\mathbf{x}^*)/\partial\theta^*)(\partial\theta^*(\mathbf{x}^*)/\partial x_k^*) < 1$. I also showed there that $\lim_{\tau_n\to\infty} \partial\beta_n(\mathbf{x}^*)/\partial\theta^* = 1$ and that if $\tau_n = \tau$ for all $n \in \mathcal{N}$, then

$$\lim_{\tau\to\infty} \sum_{k=1}^{N} \frac{\partial\theta^*(\mathbf{x}^*)}{\partial x_k^*} = 1, \quad \text{implying that} \quad \lim_{\tau\to\infty} \nabla_{\mathbf{1}}\beta_n(\mathbf{x}^*) = 1.$$

□

B.2 *Strategic complementarities under the complete information*

For simplicity, I assume that $N = 1$. Consider first the complete information version of the model described in Section 2, that is, $x_i = \theta$. I restrict attention to monotone strategies of the form "attack the status quo if $x_i \leq \widehat{x}_i$ and refrain from attacking the status quo if $x_i > \widehat{x}_i$." Suppose that all agents use the same threshold $\widehat{x}$, and let $\beta_i^{\mathrm{CI}}(\widehat{x})$ denote the optimal threshold of agent i when faced with such a strategy profile. The next lemma characterizes $\beta_i^{\mathrm{CI}}(\widehat{x})$.

Lemma B.1. *When the information structure is complete, the best-response function of agent i is given by*

$$\beta_i^{\mathrm{CI}}(\widehat{x}) = \begin{cases} \underline{\theta} & \text{if } \widehat{x} \leq \underline{\theta} \\ \widehat{x} & \text{if } \widehat{x} \in [\underline{\theta}, \overline{\theta}] \\ \overline{\theta} & \text{if } \widehat{x} > \overline{\theta} \end{cases} \quad \text{and} \quad \frac{\partial \beta_i^{\mathrm{CI}}(\widehat{x})}{\partial \widehat{x}} = \begin{cases} 0 & \text{if } \widehat{x} \leq \underline{\theta} \\ 1 & \text{if } \widehat{x} \in [\underline{\theta}, \overline{\theta}] \\ 0 & \text{if } \widehat{x} > \overline{\theta}. \end{cases}$$

Next consider the global game model with $N = 1$, and denote by τ the common precision level of the private signals. Let $\widehat{x}$ denote the threshold used by all agents, and denote by $\beta_i(\widehat{x})$ the threshold used by agent i in response.

Lemma B.2. *Suppose that $\tau \to \infty$. Then*

$$\lim_{\tau\to\infty} \beta_i(\widehat{x}) = \begin{cases} \underline{\theta} & \text{if } \widehat{x} \leq \underline{\theta} \\ \widehat{x} & \text{if } \widehat{x} \in [\underline{\theta}, \overline{\theta}] \\ \overline{\theta} & \text{if } \widehat{x} > \overline{\theta} \end{cases} \quad \text{and} \quad \lim_{\tau\to\infty} \frac{\partial \beta_i(\widehat{x})}{\partial \widehat{x}} = \begin{cases} 0 & \text{if } \widehat{x} \leq \underline{\theta} \\ 1 & \text{if } \widehat{x} \in [\underline{\theta}, \overline{\theta}] \\ 0 & \text{if } \widehat{x} > \overline{\theta}. \end{cases}$$

Proof. Suppose first that all agents use a threshold $\widehat{x} < \underline{\theta}$. In this case, it is easy to show that $\lim_{\tau\to\infty} \widehat{\theta}(\widehat{x}) = \underline{\theta}$. As a result, it has to be true that $\lim_{\tau\to\infty} \beta_i(\widehat{x}) = \underline{\theta}$.[25] Similarly, if $\widehat{x} > \overline{\theta}$, then $\lim_{\tau\to\infty} \widehat{\theta}(\widehat{x}) = \overline{\theta}$ and hence $\lim_{\tau\to\infty} \beta_i(\widehat{x}) = \overline{\theta}$.

Next consider $\widehat{x} \in [\underline{\theta}, \overline{\theta}]$. First, I establish that if $\widehat{x} \in [\underline{\theta}, \overline{\theta}]$, then $\widehat{\theta}(\widehat{x}) \to \widehat{x}$ as $\tau \to \infty$. To see this, recall that the regime change equilibrium condition is given by $R(\widehat{\theta}, F(\tau^{1/2}(\widehat{x} - \widehat{\theta})); \boldsymbol{\psi}) = 0$. It follows that as $\tau \to \infty$, the left-hand side of this equation converges to $R(\widehat{\theta}, 1; \boldsymbol{\psi}) < 0$ for all $\widehat{\theta} \in [\underline{\theta}, \widehat{x})$ and to $R(\widehat{\theta}, 0; \boldsymbol{\psi}) > 1$ for all $\widehat{\theta} \in [\widehat{x}, \overline{\theta})$. Thus, the only candidate limit is $\widehat{x}$. To see that $\lim_{\tau\to\infty} \widehat{\theta}(\widehat{x}) = \widehat{x}$, one can use the same argument in the proof of Proposition 1, utilizing the observation that $\widehat{\theta}(\widehat{x}; \tau_x) \in [\underline{\theta}, \overline{\theta}]$, which is a compact subset of $\mathbb{R}$.

Next consider $\beta_i(\widehat{x})$ and note that it can be written as $\beta_i(\widehat{\theta}(\widehat{x}))$ since agent i's indifference equation depends on $\widehat{x}$ only indirectly through $\widehat{\theta}$. The indifference equation implies that $\lim_{\tau\to\infty} \beta_i(\widehat{\theta}(\widehat{x})) = \lim_{\tau\to\infty} \widehat{\theta}(\widehat{x}) = \widehat{x}$ for all $\widehat{x} \in [\underline{\theta}, \overline{\theta}]$. It remains to show that $\partial\beta_i(\widehat{x})/\partial\widehat{x} \to 1$ as $\tau \to \infty$ for all $\widehat{x} \in [\underline{\theta}, \overline{\theta}]$. This cannot be concluded directly from the fact that $\lim_{\tau\to\infty} \beta(\widehat{x}) = \widehat{x}$ for all $\widehat{x} \in [\underline{\theta}, \overline{\theta}]$, since, in general, $\lim_{\tau\to\infty} \partial\beta_i(\widehat{x})/\partial\widehat{x} \neq$

[25]To establish that $\lim_{\tau\to\infty} \beta_i(\widehat{x})$ exists, one can follow the same argument that was used in the proof of Proposition 1 (in Appendix A) to show that the equilibrium thresholds converge when the noise in the signals is vanishingly small. In the interest of space, I omit this step.

$\partial[\lim_{\tau\to\infty}\beta_i(\widehat{x})]/\partial\widehat{x}$. However, as argued above, $\beta_i(\widehat{x})=\beta_i(\widehat{\theta}(\widehat{x}))$, and as shown in the proof of Theorem 1, we have $\lim_{\tau\to\infty}\partial\beta_i(\widehat{\theta}(\widehat{x}))/\partial\widehat{\theta}=1$. Furthermore,

$$\lim_{\tau\to\infty}\frac{\partial\widehat{\theta}(\widehat{x})}{\partial\widehat{x}}=\lim_{\tau\to\infty}\frac{-R_2\tau^{1/2}f\left(\dfrac{\widehat{x}-\widehat{\theta}}{\tau_x^{-1/2}}\right)}{R_1-R_2\tau^{1/2}f\left(\dfrac{\widehat{x}-\widehat{\theta}}{\tau_x^{-1/2}}\right)}=1.$$

This establishes the claim. □

B.3 *Publicity multiplier*

I prove a more general result that provides a decomposition of $dx^*/d\psi_m$ into the direct and multiplier effects when there are N distinct types of players and ψ_m may affect the payoff differential functions $\{\pi^n\}_{n=1}^N$ as well as the regime change function. Proposition 3 is then a corollary of this result (with $\partial\theta^*/\partial\psi_m=0$ and $\mathcal{N}\backslash\{n\}=\varnothing$).

Proposition B.1. *Fix $\boldsymbol{\psi}$. The effect that a change in ψ_m has on x_n^* is*

$$\frac{dx_n^*}{d\psi_m}=\mathcal{M}\left[\frac{\partial x_n^*}{\partial\psi_m}+\frac{\partial x_n^*}{\partial\theta^*}\frac{\partial\theta^*}{\partial\psi_m}+\sum_{k\in\mathcal{N}\backslash\{n\}}\frac{\partial x_n^*}{\partial\theta^*}\frac{\partial\theta^*}{\partial x_k^*}\left(\frac{\partial x_k^*}{\partial\psi_m}-\frac{\partial x_n^*}{\partial\psi_m}\right)\right].$$

Proof. From the proof of Theorem 1, we know that

$$\frac{dx_n^*}{d\psi_m}=\frac{\partial x_n^*}{\partial\psi_m}+\frac{\partial x_n^*}{\partial\theta^*}\frac{d\theta^*}{d\psi_m}.$$

Using the fact that $d\theta^*/d\psi_m=\mathcal{M}\times\mathcal{D}(\psi_m)$ and the definition of $\mathcal{D}(\psi_m)$, we get

$$\begin{aligned}\frac{dx_n^*}{d\psi_m}&=\frac{\partial x_n^*}{\partial\psi_m}+\frac{\partial x_n^*}{\partial\theta^*}\mathcal{M}\left[\frac{\partial\theta^*}{\partial\psi_m}+\sum_{k=1}^N\frac{\partial\theta^*}{\partial x_k^*}\frac{\partial x_k^*}{\partial\psi_m}\right]\\&=\frac{\partial x_n^*}{\partial\psi_m}\left[1-\mathcal{M}\frac{\partial x_n^*}{\partial\theta^*}\frac{\partial\theta^*}{\partial x_n^*}\right]+\frac{\partial x_n^*}{\partial\theta^*}\mathcal{M}\left[\frac{\partial\theta^*}{\partial\psi_m}+\sum_{k\neq n}\frac{\partial\theta^*}{\partial x_k^*}\frac{\partial x_k^*}{\partial\psi_m}\right]\\&=\frac{\partial x_n^*}{\partial\psi_m}\left[\frac{1-\sum_{k\neq n}\frac{\partial\theta^*}{\partial x_k^*}\frac{\partial x_k^*}{\partial\theta^*}}{1-\sum_{k=1}^N\frac{\partial\theta^*}{\partial x_k^*}\frac{\partial x_k^*}{\partial\theta^*}}\right]+\frac{\partial x_n^*}{\partial\theta^*}\mathcal{M}\left[\frac{\partial\theta^*}{\partial\psi_m}+\sum_{k\neq n}\frac{\partial\theta^*}{\partial x_k^*}\frac{\partial x_k^*}{\partial\psi_m}\right]\\&=\mathcal{M}\left[\frac{\partial x_n^*}{\partial\psi_m}+\frac{\partial x_n^*}{\partial\theta^*}\frac{\partial\theta^*}{\partial\psi_m}+\frac{\partial x_n^*}{\partial\theta^*}\sum_{k\neq n}\frac{\partial\theta^*}{\partial x_k^*}\left(\frac{\partial x_k^*}{\partial\psi_m}-\frac{\partial x_n^*}{\partial\psi_m}\right)\right].\end{aligned}$$

□

The proof of Proposition 3 follows immediately from Proposition B.1.

The above decomposition is intuitive. The direct effect consists of the effect that a change in ψ_m has on x_n^* through its effect on the regime change condition (captured by $(\partial x_n^*/\partial\theta^*)(\partial\theta^*/\partial\psi_m)$) and through its effect on the payoff indifference condition of a type s_n agent (captured by $\partial x_n^*/\partial\psi_m$). But note that a change in ψ_m also affects the payoff indifference conditions of other types of agents. Since the multiplier effect captures how changes in $\{x_k^*\}_{k=1}^N$, through their effects on θ^*, lead to further changes in x_n^*, we need to take into account the fact that types of agents of types other than s_n may adjust their thresholds by different amounts in response to a change in ψ_m. The last term in the square brackets is an adjustment for this heterogeneity in the initial response of agents to the change in ψ_m.

B.4 *Proofs for Section 4.3*

Lemma B.3. *Consider the setup of Section 4.3.2 and suppose that $\tau_x^{1/2}/\tau_\theta > (1/z)/\sqrt{2\pi}$. Then the equilibrium signal threshold is given by*

$$x^* = \frac{\tau_x+\tau_\theta}{\tau_x}\theta^* - \frac{\tau_\theta}{\tau_x}\mu_\theta - \frac{\sqrt{\tau_x+\tau_\theta}}{\tau_x}\Phi^{-1}(\gamma), \tag{8}$$

the equilibrium regime change threshold θ^* is the unique solution to

$$\theta^* - z\Phi\left(\frac{\tau_\theta}{\tau_x^{1/2}}(\theta^*-\mu_\theta) - \frac{\sqrt{\tau_x+\tau_\theta}}{\tau_x}\Phi^{-1}(\gamma)\right) = 0, \tag{9}$$

and the multiplier effect is given by

$$\mathcal{M} = \frac{1}{1 - \dfrac{\tau_x+\tau_\theta}{\tau_x}\dfrac{z\tau_x^{1/2}\phi(\tau_x^{1/2}(x^*-\theta^*))}{1+z\tau_x^{1/2}\phi(\tau_x^{1/2}(x^*-\theta^*))}}. \tag{10}$$

Proof. Derivations of (8) and (9) are standard. The expression for $\mathcal{M}$ can easily be deduced from these two equations. □

In what follows, I refer to the condition $\tau_x^{1/2}/\tau_\theta > (1/z)(1/\sqrt{2\pi})$ as the uniqueness condition. This condition is maintained for all the results of Section 4.3.2.

Proof of Proposition 4. Note that $\phi(\cdot)$ is a symmetric function that achieves its maximum at 0. Therefore, (10) implies that $\mathcal{M}$ is a decreasing function of $|x^*-\theta^*|$. To establish how changes in parameters affect the distance between x^* and θ^*, suppose that $x^*-\theta^*=a$, where $a\in\mathbb{R}$. From (8), we see that $x^*-\theta^*=a$ if and only if $\theta^*=\mu_\theta+(\tau_x/\tau_\theta)a+(\sqrt{\tau_x+\tau_\theta}/\tau_\theta)\Phi^{-1}(\gamma)$. Next, note that θ^* takes such a value if and only if $\mu_\theta+(\tau_x/\tau_\theta)a+(\sqrt{\tau_x+\tau_\theta}/\tau_\theta)\Phi^{-1}(\gamma)$ is the solution to the regime change condition. Since the regime change condition is given by (9), we know that this happens if and only if

$$\mu_\theta + \frac{\sqrt{\tau_x+\tau_\theta}}{\tau_\theta}\Phi^{-1}(\gamma) = z\Phi(\tau_x^{1/2}a) - \frac{\tau_x}{\tau_\theta}a.$$

Subtracting $(1/2)z$ from both sides, we obtain

$$\mu_\theta - \frac{1}{2}z + \frac{\sqrt{\tau_x + \tau_\theta}}{\tau_\theta}\Phi^{-1}(\gamma) = z\Phi\big(\tau_x^{1/2}a\big) - \frac{1}{2}z - \frac{\tau_x}{\tau_\theta}a. \tag{11}$$

Note that the right-hand side of (11) is equal to 0 at $a = 0$ and that its derivative is given by

$$z\tau_x^{1/2}\phi\big(\tau_x^{1/2}a\big) - \frac{\tau_x}{\tau_\theta} \le z\tau_x^{1/2}\frac{1}{\sqrt{2\pi}} - \frac{\tau_x}{\tau_\theta} = \tau_x^{1/2}\left[z\frac{1}{\sqrt{2\pi}} - \frac{\tau_x^{1/2}}{\tau_\theta}\right] < 0,$$

where the last inequality holds as long as the equilibrium is unique (which is the assumption maintained in Section 4.3). It follows that the right-hand side of (11) is strictly decreasing in a.

From the above observations, it follows that whenever $\mu_\theta - (1/2)z + (\sqrt{\tau_x + \tau_\theta}/\tau_\theta) \times \Phi^{-1}(\gamma) = 0$, we have $a = 0$, meaning that $x^* - \theta^* = 0$, so that the multiplier effect achieves its maximum value (for given values of τ_θ and τ_x). Moreover, by applying the implicit function theorem to (11), it is easy to see that $da/d\mu_\theta < 0$, $da/d\gamma < 0$, and $da/dz > 0$. It follows that a decreases as $\mu_\theta - \frac{1}{2}z + (\sqrt{\tau_x + \tau_\theta}/\tau_\theta)\Phi^{-1}(\gamma)$ increases. Therefore, if we define

$$g(\mu_\theta, \alpha, \gamma) = \left|\mu_\theta - \frac{1}{2}z + \frac{\sqrt{\tau_x + \tau_\theta}}{\tau_\theta}\Phi^{-1}(\gamma)\right|,$$

then a higher value of $g(\mu_\theta, \alpha, \gamma)$ indicates a larger distance between x^* and θ^*, that is, a higher value of $|x^* - \theta^*|$. This proves the result. □

Proof of Proposition 5. The proof of these results can be found in the working paper version of this article (Szkup 2019). Here, I briefly explain the approach used to prove this result.

Understanding how $\mathcal{M}$ varies with τ_x and τ_θ is a challenging task, since a change in either τ_x or τ_θ affects the expression for $\mathcal{M}$ both directly and indirectly (via its effect on x^* and θ^*), and the resulting derivative is a complex object. The approach I take is nevertheless straightforward if tedious. To establish Proposition 5, I compute $\partial\mathcal{M}/\partial\tau_x$ and show that for sufficiently high τ_x we have $\partial\mathcal{M}/\partial\tau_x > 0$. Then I compute and simplify $\partial\mathcal{M}/\partial\tau_\theta$, and show that (for any parameters of the model) (i) the resulting expression is negative as $\mu_\theta \to \pm\infty$, (ii) there exists a nonempty closed interval I of values of μ_θ on which this expression is positive, and (iii) as μ_θ increases, the resulting expression crosses the 0 line from below when $\mu_\theta < \min\{I\}$ and from above when $\mu_\theta > \min\{I\}$. The proposition follows from these observations. □

Appendix C: Proofs for Section 5

Proof of Corollary 2. Note first that, regardless of the assumed information structure, a higher x_i always decreases agent i's incentive to attack the regime, so that $\partial P_{s(i)}(\theta^*, x^*_{s(i)}, \boldsymbol{\psi})/\partial x^*_{s(i)} < 0$. It follows that if $\partial\pi^{s(i)}(\theta; \boldsymbol{\psi})/\partial\psi_m \ge 0$ for all θ, then $\partial x^*_{s(i)}/\partial\psi_m \ge 0$, regardless of $\{F_n\}_{n=1}^N$.

Next consider the direct effect of a change in ψ_m that operates through the regime change function $R(\theta^*, m(\theta^*; \{x_n^*\}_{n=1}^N), \psi)$, where $m(\theta^*, \{x_n^*\}_{n=1}^N) = \sum_{n=1}^N \lambda_n F_n(\tau_n^{1/2}(x_n^* - \theta^*))$. A higher θ^* always decreases the proportion of agents attacking the regime when $\{x_n^*\}_{n=1}^N$ is held constant. Since $R_1 > 0$, it follows that if $\partial R(\theta, m; \boldsymbol{\psi})/\partial \psi_m \leq 0$, then $\partial\theta^*/\partial\psi_m \geq 0$, irrespective of $\{F_n\}_{n\in\mathcal{N}}$.

Finally, we always have $\partial\theta^*/\partial x_n^* > 0$, $n \in \mathcal{N}$, as higher signal thresholds imply a higher proportion of agents attacking the regime for a given θ regardless of $\{F_n\}_{n=1}^N$. Thus we conclude that $\mathcal{D}(\psi_m) \geq 0$, and its sign does not depend on the particular noise structure as characterized by $\{F_n\}_{n=1}^N$. The analogous argument applies when $\partial\pi^{s(i)}/\partial\psi_m \leq 0$ and $\partial R/\partial\psi_m \geq 0$, in which case $\mathcal{D}(\psi_m) \leq 0$. □

Proof of Lemma 1. Suppose that $H^n(\theta) = H^n > 0$ and $L^n(\theta) = L^n < 0$. It is easy to show that in a monotone equilibrium, a player of type s_n uses the threshold x_n^* that solves

$$H^n F_n\left(\frac{x_n^* - \theta^*}{\tau_n^{-1/2}}\right) + L^n\left(1 - F_n\left(\frac{x_n^* - \theta^*}{\tau_n^{-1/2}}\right)\right) = 0.$$

Thus the proportion of agents attacking the regime in equilibrium is given by

$$m^*(\theta^*) = \Sigma_{n=1}^N \lambda_n\bigl(-L^n/\bigl(H^n - L^n\bigr)\bigr) \in (0, 1),$$

which does not depend on the assumed information structure $\{F_n\}_{n=1}^N$. But then it follows that the unique equilibrium threshold θ^*, which is determined by $R(\theta^*, m^*(\theta^*); \boldsymbol{\psi}) = 0$, also does not depend on $\{F_n\}_{n=1}^N$. As such, the comparative statics do not depend on $\{F_n\}_{n=1}^N$. □

Proof of Corollary 3. Recall that $P_n(\theta^*, x_n^*, \boldsymbol{\psi}) = 0$ is the indifference condition for players in group n. Note that if $\partial\pi^n/\partial\psi_k > \partial\pi^n/\partial\psi_m$ for all θ, then $\partial P_n/\partial\psi_k > \partial P_n/\partial\psi_m$, irrespective of F_n and τ_n. Since $\partial P_n/\partial x_n^* < 0$, it follows that $\partial x_n^*/\partial\psi_k > \partial x_n^*/\partial\psi_m$, irrespective of F_n and τ_n. Next suppose that $\partial R/\partial\psi_k < \partial R/\partial\psi_m$ for all θ and m. Since $\partial R/\partial\theta^* > 0$, it follows that irrespective of F_n and τ_n, we have $\partial\theta^*/\partial\psi_k > \partial\theta^*/\partial\psi_m$. Finally, recall that a higher x_n^* always decreases θ^*, that is, $\partial\theta^*/\partial x_n^* = -(\partial R/\partial x_n^*)/(\partial R/\partial\theta^*) < 0$. Combining these observations, we see that $d\theta^*/d\psi_k > d\theta^*/d\psi_m$. □

Proof of Corollary 4. This result follows immediately from the definitions of $\underline{\theta}(\boldsymbol{\psi})$ and $\overline{\theta}(\boldsymbol{\psi})$ and the fact that (as discussed in the proof of Corollary 2) we have $d\theta^*/d\psi_m >$ $(<)0$ when $\partial R/\partial\psi_m > (<)0$ and $\partial\pi^n/\partial\psi_m \leq (\geq)0$ for all θ and m. □

C.1 *Predictions that depend on the information structure*[26]

In this section, I provide two examples to show how the comparative statics depend on the information structure when the hypothesis of Corollary 2 is not satisfied. In both cases, I compare comparative statics predictions derived under two distinct information

[26]Proofs of the results stated in this section can be found in the working paper version of this article (Szkup 2019)

structures. Under the first information structure, agents have the prior belief that $\theta \sim \text{unif}[\mu_\theta - \eta, \mu_\theta + \eta]$, and each of them receives a private signal $x_i = \theta + \varepsilon_i$, with $\varepsilon_i \sim \text{unif}[-\varepsilon, \varepsilon]$.[27] I refer to this information structure as *uniform–uniform*. Under the second information structure, agents have the prior belief that $\theta \sim N(\mu_\theta, \tau_\theta^{-1})$, and each of them receives a private signal $x_i = \theta + \varepsilon_i$, with $\varepsilon_i \sim N(0, \tau_x^{-1/2})$. I refer to this information structure as *normal–normal*.

C.1.1 *Example 1: Nonmonotone regime change function* Consider the following example adapted from Morris and Shin (1998). Suppose that the payoff to speculators from attacking the regime is $1 - t$ if an attack is successful and $-t$ otherwise, while choosing not to attack yields 0. The central bank will keep the peg if $\theta - c(m, L) > 0$ and will abandon it otherwise. Here, θ stands for the benefit of keeping the peg, m is the fraction of speculators who attack the peg, and L is the amount of foreign reserves that the central bank can raise quickly so as to prevent the attack. Finally, $c(m, L)$ is the cost of defending the regime when the size of the attack is m and the foreign reserves are L, with $c_m(m, L) > 0$.

Assume that $c_L(m, L) < 0$ for $m < \overline{m}$, $c_L(m, L) = 0$ if $m = \overline{m}$, and $c_L(m, L) > 0$ if $m > \overline{m}$, so that raising additional liquidity to prevent the attack decreases the cost of defending the peg if and only if m is relatively low. These assumptions capture the idea that when the attack is expected to be small, the central bank can borrow foreign reserves from other foreign central banks cheaply (since the loans are almost risk-free). Alternatively, if the attack is large, then raising additional foreign reserves is costly, as other banks expect that the peg will collapse, in which case they are unlikely to recover their loans. As such, other central banks will charge a high interest rate on those loans.

Let θ^* denote the threshold level of θ such that the peg is abandoned if and only if $\theta < \theta^*$. We then have the following result.

Lemma C.1. *Consider the effects that changes in μ_θ and L have on θ^* (i.e., $d\theta^*/d\mu_\theta$ and $d\theta^*/dL$).*

(i) Under the uniform–uniforminformation structure,

- $d\theta^*/d\mu_\theta = 0$; $d\theta^*/dL < 0$ *if and only if* $1 - t < \overline{m}$.

(ii) Under the normal–normal information structure,

- $d\theta^*/d\mu_\theta < 0$; $d\theta^*/dL < 0$ *if and only if*

$$\mu_\theta + \frac{\sqrt{\tau_x + \tau_\theta}}{\tau_\theta}\Phi^{-1}(t) + \frac{\sqrt{\tau_x}}{\tau_\theta}\Phi^{-1}(\overline{m}) \geq c(\overline{m}, L).$$

Thus we see that when the prior and the noise in the signals have uniform distributions, changes in the mean of the prior have no effect on θ^*, while an increase in L decreases the probability of a currency crisis if and only if the transaction cost t is sufficiently high. Under a Gaussian information structure, when the prior and the noise in

[27] Implicitly, I assume that η is large enough so that $\mu_\theta - \eta < \underline{\theta} - 2\varepsilon$ and $\mu_\theta + \eta > \overline{\theta} + 2\varepsilon$ (i.e., $[\underline{\theta} - 2\varepsilon, \overline{\theta} + 2\varepsilon] \subset (\mu_\theta - \eta, \mu_\theta + \eta)$), which is required for the equilibrium to be unique.

	Repayment	Default
Roll over	D_S	0
Withdraw	1	1

TABLE 1. Short-term Debt holders's payoffs.

the signals have normal distributions, an increase in the mean of the prior always leads to a decrease in the currency crisis threshold. Furthermore, whether an increase in L decreases or increases θ^* depends on all the parameters of the model.

C.1.2 *Example 2:*[28] *counteracting effects of an increase in* ψ_m There is a single firm and a continuum of investors indexed by $i \in [0, 1]$. The firm owns a risky project with a return of θ and total liquidation value 1. The project can be partially liquidated to meet early withdrawals if such a need arises. The firm financed its project by issuing an amount α of short-term debt with face value 1 and an amount $(1-\alpha)$ of long-term debt with face value D_L, with $\alpha \in [0, 1]$. Before the project matures, the short-term debt holders have to decide whether to roll over their debt or withdraw their funds early.

Short-term debt holders who withdraw their funds early get their funds back, that is, they are paid back 1 unit of funds. Short-term debt holders who roll over their loans are promised $D_S > 1$ if the firm's return on the project exceeds its debt obligation. Otherwise, the firm defaults and all the debt holders receive nothing. It follows that short-term debt holders face the payoffs shown in Table 1. Let m denote the fraction of short-term debt holders who withdraw their funds early. The firm repays its debt if and only if $\theta(1-m\alpha) - \alpha(1-m)D_S - (1-\alpha)D_L > 0$. Here, $\theta(1-m\alpha)$ is the return on the scaled-down investment, where a fraction $m\alpha$ of the investment was liquidated to meet early withdrawals. In this setting, "attacking the regime" is associated with withdrawing funds early.

Suppose that the firm would like to avoid early withdrawals and considers increasing D_S to discourage early withdrawals.

LEMMA C.2. *Consider the effect that a change in D_S has on the equilibrium default threshold.*

(i) Under the uniform–uniforminformation structure, we have $d\theta^*/dD_S \geq 0$ if and only if

$$\alpha \geq \min\left\{\frac{D_L - D_S^2}{D_L + 1 - 2D_S}, 0\right\}.$$

(ii) Under the normal–normal information structure, we have $d\theta^*/dD_S < 0$ if and only if $\mu_\theta < \widehat{\mu}_\theta(\boldsymbol{\psi})$, where $\widehat{\mu}_\theta(\boldsymbol{\psi})$ is the unique solution to[29]

$$-\big(1 - m^*\big(\theta^*(\boldsymbol{\psi})\big)\big) - \big(\theta^*(\boldsymbol{\psi}) - D_S\big)\frac{\partial m^*\big(\theta^*(\boldsymbol{\psi})\big)}{\partial D_S} = 0.$$

[28] This is a simplified version of the model in Szkup (2016).

[29] Since $\mu_\theta \in \boldsymbol{\psi}$, a change in μ_θ affects this condition via its impact on $\theta^*(\boldsymbol{\psi})$.

Under the uniform–uniform information structure, whether D_S decreases or increases depends on α, D_L, and D_S, but not on the parameters that affect the information structure. Under the normal–normal information structure, all parameters of the model (including the precision of the private signal, the precision of the prior, and the mean of the prior) matter through their impact on $\widehat{\mu}_\theta$.

Appendix D: Proofs for Section 6

Proof of Proposition 6. The Lagrangian associated with the optimization problem stated in Section 6.1 is given by

$$L = -\sum_{n=1}^{N} \lambda_n v_n + \eta\big[-\widehat{\theta} + \theta^*(v_1, \dots, v_N)\big] + \sum_{n=1}^{N} \xi_n v_n + \sum_{n=1}^{N} \varrho_n[\overline{v} - v_n],$$

where I write the Lagrangian for maximization of $-\Sigma_{n=1}^{N}\lambda_n v_n$ subject to the constraints given in the statement of the problem. Let $d\theta^*/dv_n$ denote the total change in θ^* with respect to v_n. From Theorem 1, we know that $d\theta^*/dv_n = \mathcal{M}\,\mathcal{D}(v_n; \mathbf{v})$. Therefore, the first order conditions (FOCs) can be written as

$$-\lambda_1 + \eta\big[\mathcal{M}\,\mathcal{D}(v_1; \mathbf{v})\big] + \xi_1 - \varrho_1 = 0, \dots, -\lambda_N + \eta\big[\mathcal{M}\,\mathcal{D}(v_N; \mathbf{v})\big] + \xi_N - \varrho_N = 0.$$

Part (i). Let $\mathbf{v}^* = (v_1^*, \dots, v_N^*)$ denote the vector of optimal subsidies, and suppose that $v_n^* < \overline{v}$ and $v_{n+1}^* > 0$. In that case, $\varrho_n = 0$ (as $v_n^* < \overline{v}$) and $\xi_{n+1} = 0$ (as $v_n^* > 0$). It follows that

$$-1 + \eta\left[\mathcal{M}\,\frac{\mathcal{D}(v_n^*; \mathbf{v})}{\lambda_n}\right] + \frac{\xi_n}{\lambda_n} = 0 \quad \text{and} \quad -1 + \eta\left[\mathcal{M}\,\frac{\mathcal{D}(v_{n+1}^*; \mathbf{v})}{\lambda_{n+1}}\right] - \frac{\varrho_{n+1}}{\lambda_{n+1}} = 0.$$

Since $\xi_n, \varrho_{n+1} \geq 0$ and $\eta > 0$, the above equations imply that $\mathcal{D}(v_n; \mathbf{v})/\lambda_n \leq \mathcal{D}(v_{n+1}; \mathbf{v})/\lambda_{n+1}$, which is a contradiction. It follows that the planner always targets the types of agents who have the highest direct effect adjusted by the size of the group.

Part (ii). That the planner subsidizes first the group with the highest $\mathcal{D}(v_n; \mathbf{v})|_{\mathbf{v}=\mathbf{0}}/\lambda_n$ follows by an argument analogous to the one used in part (i). Next suppose that $\exists n, m \in \mathcal{N}$ such that $v_n^* = \overline{v}$ and $v_m^* = 0$. Then $\xi_n = \varrho_m = 0$ and $\varrho_n, \xi_m \geq 0$. From the above FOCs, it follows that $\mathcal{D}(v_n; \mathbf{v})/\lambda_n|_{v_n=\overline{v}} \geq \mathcal{D}(v_m; \mathbf{v})/\lambda_m|_{v_m=0}$, while by assumption we have $\mathcal{D}(v_m; \mathbf{v})/\lambda_m|_{v_m=0} > \mathcal{D}(v_n; \mathbf{v})/\lambda_n|_{v_n=\overline{v}}$, which is a contradiction. Next, suppose that $v_n^*, v_m^* \in (0, \overline{v})$. Then $\xi_n = \varrho_n = 0$, $\xi_m = \varrho_m = 0$, and $\eta > 0$. Thus the above FOCs imply that $\mathcal{D}(v_n^*; \mathbf{v})/\lambda_n = \mathcal{D}(v_m^*; \mathbf{v})/\lambda_m$. Finally, if $\lim_{v_n \to \overline{v}} \mathcal{D}(v_n; \mathbf{v})/\lambda_n = 0$, it is immediate from the above FOCs that $v_n^* = \overline{v}$ can never be optimal. □

Proof of Lemma 2. In equilibrium, agents in group n who received a subsidy v_n invest if and only if $x_i \geq x_n^*$, where

$$x_n^* = \frac{\tau_\theta + \tau_n}{\tau_n}\theta^* - \frac{\tau_\theta}{\tau_n}\mu_\theta + \frac{\sqrt{\tau_n + \tau_\theta}}{\tau_n}\Phi^{-1}\left(\frac{c_n - v_n}{b_n}\right) \quad \text{and}$$

$$\theta^* \text{ solves } \theta^* - \sum_{n=1}^{N} w_n \lambda_n \Phi\left(\frac{x_n^* - \theta^*}{\tau_n^{-1/2}}\right) = 0.$$

It follows that the direct effect of a subsidy v_n to group n is given by

$$\mathcal{D}(v_n;\mathbf{v}) = \frac{\partial\theta^*}{\partial x_n^*}\frac{\partial x_n^*}{\partial v_n}$$
$$= \underbrace{\left(-\frac{1}{1+\sum_{n=1}^{N} w_n\lambda_n\tau_n^{1/2}\phi\left(\frac{x_n^*-\theta^*}{\tau_n^{-1/2}}\right)}\right)}_{C(\mathbf{v})}\underbrace{\frac{\frac{w_n\lambda_n}{b_n}\frac{\sqrt{\tau_n+\tau_\theta}}{\tau_n}\tau_n^{1/2}\phi\left(\frac{x_n^*-\theta^*}{\tau_n^{-1/2}}\right)}{\phi\left(\Phi^{-1}\left(\frac{c_n-v_n}{b_n}\right)\right)}}_{\widetilde{\mathcal{D}}(v_n;\mathbf{v})}.$$

Note that $\widetilde{\mathcal{D}}(v_n;\mathbf{v})$ is decreasing in w_n. Moreover, as $\tau_n\to\infty$ or $\tau_\theta\to 0$ we have $\tau_n^{1/2}(x_n^*-\theta^*)\to\Phi^{-1}((c_n-v_n)/b_n)$ and thus $\widetilde{\mathcal{D}}(v_n;\mathbf{v})/\lambda_n\to w_n/b_n$.

Next I consider how $\widetilde{\mathcal{D}}(v_n;\mathbf{v})|_{v_n=0}$ varies with c_n, b_n, and τ_n to determine which group should be subsidized first. Note that for this purpose I can hold θ^* constant, as all agents face the same θ^*. Consider first $\partial\widetilde{\mathcal{D}}(v_n;\mathbf{v})/\partial c_n$. Computing this derivative, we see that $\operatorname{sgn}(\partial\widetilde{\mathcal{D}}(v_n;\mathbf{v})/\partial c_n)$ is determined by the sign of

$$\left[\frac{\tau_\theta\sqrt{\tau_n+\tau_\theta}}{\tau_n}(\theta^*-\mu_\theta)+\frac{\tau_\theta}{\tau_n}\Phi^{-1}\left(\frac{c_n-v_n}{b_n}\right)\right].$$

When μ_θ is sufficiently small, the expression in the square brackets is positive and hence $\partial\widetilde{\mathcal{D}}(v_n;\mathbf{v})/\partial c_n|_{v_n=0}>0$, while the opposite is true for sufficiently large μ_θ.

Next I consider $\partial\widetilde{\mathcal{D}}(v_n;\mathbf{v})/\partial\tau_n$ and $\partial\widetilde{\mathcal{D}}(v_n;\mathbf{v})/\partial b_n$. It is easy to see that $\operatorname{sgn}(\partial\widetilde{\mathcal{D}}(v_n;\mathbf{v})/\partial\tau_n)$ is determined by the sign of

$$\left[\frac{1}{\sqrt{\tau_n+\tau_\theta}}-\left(\frac{\tau_\theta}{\tau_n^{1/2}}(\theta^*-\mu_\theta)+\sqrt{\frac{\tau_n+\tau_\theta}{\tau_n}}\Phi^{-1}\right.\right.$$
$$\left.\left.\times\left(\frac{c_n-v_n}{b_n}\right)\right)\left(\theta^*-\mu_\theta+\frac{1}{\sqrt{\tau_n+\tau_\theta}}\Phi^{-1}\left(\frac{c_n-v_n}{b_n}\right)\right)\right].$$

The above expression is negative whenever μ_θ is either sufficiently small or sufficiently large; hence in both cases $\partial\widetilde{\mathcal{D}}(v_n;\mathbf{v})/\partial\tau_n|_{v_n=0}>0$. Next, $\operatorname{sgn}(\partial\widetilde{\mathcal{D}}(v_n;\mathbf{v})/\partial b_n)$ is determined by the sign of

$$1-\frac{\left[\frac{\sqrt{\tau_\theta+\tau_n}\tau_\theta}{\tau_n}(\theta^*-\mu_\theta)+\frac{\tau_\theta}{\tau_n}\Phi^{-1}\left(\frac{c_n-v_n}{b_n}\right)\right]\frac{c_n-v_n}{b_n}}{\phi\left(\Phi^{-1}\left(\frac{c_n-v_n}{b_n}\right)\right)},$$

which is positive if μ_θ is sufficiently large, in which case $\partial\widetilde{\mathcal{D}}(v_n;\mathbf{v})/\partial b_n|_{v_n=0}>0$, while the opposite is true when μ_θ is sufficiently small.

Finally, consider how the direct effect of the subsidies varies with their size. Note that $\partial\widetilde{D}(v_n;\mathbf{v})/\partial v_n=-\partial\widetilde{D}(v_n;\mathbf{v})/\partial c_n$. Therefore, if μ_θ is sufficiently large (e.g., $\mu_\theta\geq 1$), then

$$\partial\widetilde{D}(v_n;\mathbf{v})/\partial v_n|_{v_n=0}>0 \quad\Longrightarrow\quad \partial\widetilde{D}(v_n;\mathbf{v})/\partial v_n>0 \quad\text{for all } v_n\in[0,\overline{v}].$$

Alternatively, suppose that $\partial\widetilde{D}(v_n;\mathbf{v})/\partial v_n|_{v_n=0} < 0$ (which happens if μ_θ is sufficiently small). In that case, as $v_n \to c_n$, we have $\Phi^{-1}((c_n - v_n)/b_n) \to -\infty$ and, hence, for sufficiently large v_n, we have $\partial\widetilde{D}(v_n;\mathbf{v})/\partial v_n > 0$. Finally, it is easy to see that, in this case, we have $\partial^2\widetilde{D}(v_n;\mathbf{v})/\partial v_n^2 > 0$ whenever $\partial\widetilde{D}(v_n;\mathbf{v})/\partial v_n = 0$. It follows that $\partial\widetilde{D}(v_n;\mathbf{v})/\partial v_n$ always crosses the 0 line from below. This establishes the claim regarding the behavior of $\partial\widetilde{D}(v_n;\mathbf{v})/\partial v_n$.

Finally, observe that we have a finite number of types of agents and finitely many parameters. Therefore, we can find bounds $\underline{\mu}$ and $\overline{\mu}$ such that if $\mu_\theta > \overline{\mu}$, then μ_θ is large enough to satisfy all the above conditions that require μ_θ to be large, while if $\mu_\theta < \underline{\mu}$, then μ_θ is small enough to satisfy all the above conditions that require μ_θ to be small. □

Appendix E: Extensions of Theorem 1

E.1 *A model with multiple equilibria*

In this section,[30] I discuss how Theorem 1 generalizes to settings that feature multiple monotone equilibria, each characterized by a different regime change threshold θ^*. Thus in what follows, $d\theta^*/d\psi_m$ should be interpreted as a change in a threshold that characterizes a particular equilibrium. To state the main result of this section, I need the following definition of stability of equilibria (see, for example, Vives 2005).

Definition 1 (Stability). A monotone equilibrium characterized by the regime change threshold θ^* is stable if $\exists\varepsilon > 0$ such that for all $\widehat{\theta}_0 \in (\theta^* - \varepsilon, \theta^* + \varepsilon)$ the best-response dynamics initiated at $\widehat{\theta}_0$ converge to θ^*.

As the next proposition shows, Theorem 1 remains valid in an essentially unchanged form as long as we focus on stable equilibria. This is because the derivation of the decomposition of $\partial\theta^*/\partial\psi$ in the proof of Theorem 1 did not utilize the fact that the equilibrium is unique; hence, it remains valid for any equilibrium that does not disappear following a change in ψ_m. Moreover, the necessary and sufficient condition for $\mathcal{M} \in (1, \infty)$ is that $\sum_{n=1}^{N}(\partial\theta^*/\partial x_n^*)(\partial x_n^*/\partial\theta^*) \in (0, 1)$ when evaluated at the initial equilibrium threshold, which is exactly the necessary and sufficient condition for the equilibrium to be stable.

The situation is different when we consider an unstable equilibrium. While the decomposition of comparative statics results into the direct and multiplier effects remains valid, the multiplier effect loses its natural interpretation as capturing the change induced by adjustments in beliefs. This is because when an equilibrium is unstable the best-response dynamics diverge, implying a discrete adjustment in the regime change threshold θ^* regardless of how small the initial change in ψ_m is. As a result, the multiplier effect computed using best-response dynamics is infinite. Alternatively, if we compute the multiplier effect directly using the implicit function theorem as in the proof of Theorem 1, the resulting multiplier effect is negative.[31]

[30] Proofs of the results stated in this section can be found in the working paper version of this article, which is available on the author's website.

[31] The difference between the two approaches stems from the fact that the best-response dynamics describe the change in the equilibrium play following a change in a parameter, while the implicit function

Proposition 7. *Consider any equilibrium regime change threshold θ^*, and let $\mathcal{M}$ and $\mathcal{D}(\psi_m)$ be defined as in Theorem 1. Then $d\theta^*/d\psi_m = \mathcal{M}\mathcal{D}(\psi_m)$. Moreover, the following statements hold:*

(i) *If an equilibrium is stable, then $\mathcal{M} \in (1, \infty)$ and the best-response dynamics following a small change in ψ_m converge.*

(ii) *If an equilibrium is unstable, then $\mathcal{M} < 0$ and the best-response dynamics following a small change in ψ_m diverge.*

E.2 *Discrete changes in parameters*[32]

In this section, I consider discrete changes in parameters. To keep the notation simple, I consider a setup with a single type of agent, where the unique equilibrium is described by thresholds θ_0^* and x_0^*. To derive the decomposition of comparative statics, I compute the best-response dynamics following a change in ψ_m, $\Delta\psi_m$, where $\Delta\psi_m \neq 0$.

I denote by θ_l^* the regime change threshold after l steps of the best-response dynamics, and denote by $\Delta_l\theta^*$ the change in θ^* implied by the lth step of the best-response dynamics; x_l^* and $\Delta_l x^* \equiv x_l^* - x_{l-1}^*$ are defined analogously. I define $\Delta_1 R \equiv R(\theta_1^*, x_1^*, \psi_m + \Delta\psi_m) - R(\theta_0^*, x_0^*, \psi_m)$ as the change in the resilience of the regime following the first round of the best-response dynamics. The change $\Delta_1 R$ can be decomposed as $\Delta_1 R = \Delta_{1,\theta^*}R + \Delta_{1,x^*}R + \Delta_{1,\psi_m}R$, that is, as a sum of changes due to changes in θ^*, x^*, and ψ_m, respectively.[33] Similarly, $\Delta_1 P \equiv P(\theta_0^*, x_1^*, \psi_m + \Delta\psi_m) - P(\theta_0^*, x_0^*, \psi_m)$ is the change in agents' expected payoff difference following the first step of the best-response dynamics, which can be decomposed as $\Delta_1 P = \Delta_{1,x^*}P + \Delta_{1,\psi_m}P$.

Having introduced the required notation, I now state the decomposition of comparative statics for a discrete change in a parameter of the model.

Proposition 8. *Suppose that ψ_m increases by $\Delta\psi_m > 0$ and that $\Delta\theta^* \neq 0$. Then*

$$\frac{\Delta\theta^*}{\Delta\psi_m} = \mathcal{M}\,\mathcal{D}(\psi_m),$$

where

$$\mathcal{D}(\psi_m) = \left[-\frac{\dfrac{\Delta_{1,\psi_m}R}{\Delta\psi_m}}{\dfrac{\Delta_{1,\theta^*}R}{\Delta_1\theta^*}} - \frac{\dfrac{\Delta_{1,x^*}R}{\Delta_1 x^*}}{\dfrac{\Delta_{1,\theta^*}R}{\Delta_1\theta^*}} \frac{\dfrac{\Delta_{1,\psi_m}P}{\Delta\psi_m}}{\dfrac{\Delta_{1,x^*}P}{\Delta_1 x^*}} \right] \in \mathbb{R} \quad \textit{and}$$

$$\mathcal{M} = 1 + \sum_{k=1}^{2} \prod_{l=2}^{k} \frac{\Delta_l\theta^*}{\Delta_l x^*} \frac{\Delta_l x^*}{\Delta_{l-1}\theta^*} \in (1, \infty).$$

theorem characterizes the change in the given equilibrium threshold. These two approaches do not coincide in the case of an unstable equilibrium.

[32] I thank a referee for suggesting this extension.

[33] Formally, $\Delta_{1,\theta^*}R \equiv R(\theta_1^*, x_1^*, \psi_m + \Delta\psi_m) - R(\theta_0^*, x_1^*, \psi_m + \Delta\psi_m)$, $\Delta_{1,x^*}R = R(\theta_0^*, x_1^*, \psi_m + \Delta\psi_m) - R(\theta_0^*, x_0^*, \psi_m + \Delta\psi_m)$, and $\Delta_{1,\psi_m}R = R(\theta_0^*, x_0^*, \psi_m + \Delta\psi_m) - R(\theta_0^*, x_0^*, \psi_m)$. I define $\Delta_{1,x^*}P$ and $\Delta_{1,\psi}P$ in a similar fashion.

Moreover, if θ^* corresponds to a stable equilibrium, then

$$\lim_{\Delta\psi_m \to 0} \frac{\Delta\theta^*}{\Delta\psi_m} = \frac{d\theta^*}{d\psi_m}.$$

E.3 *A model with a continuum of actions*

Let $\mathcal{A}^n = [\underline{a}_n, \overline{a}_n] \subset \mathbb{R}$ be the action set of an agent with type s_n, $n \in \mathcal{N}$. Let $\pi^n(a_i, \theta, \mathcal{R})$ be the payoff function of a player of type s_n when he chooses action a_i, where the strength of the regime is θ and the regime status is $\mathcal{R}$. I assume that $\pi^n(\cdot,\cdot,\cdot)$ is twice continuously differentiable in a_i and θ, and concave in a_i for each θ and $\mathcal{R}$, with $\lim_{a_i \to \underline{a}_n} \pi_1^n(a_i, \theta, \mathcal{R}) > 0$ and $\lim_{a_i \to \overline{a}_n} \pi_1^n(a_i, \theta, \mathcal{R}) < 0$ for all θ and $\mathcal{R}$. These conditions imply that the optimal action exists, is unique, and belongs to the interior of A. In addition, suppose that $\partial\pi^n(a_i, \theta, 1)/\partial a_i - \partial\pi^n(a_i, \theta, 0)/\partial a_i > 0$ and $\partial^2\pi^n(a_i, \theta, \mathcal{R})/\partial\theta\partial a_i < 0$ so that agents prefer a higher action when $\mathcal{R} = 1$ and the optimal action is decreasing in θ. The regime change function is now given by $R(\theta, m; \psi)$, where $m \equiv \int_{i\in[0,1]} a_i \, di$ is the average action in the economy; as before, $R_1 > 0$ and $R_2 < 0$. Finally, denote by $a^*_{s(i)}(i)$ the optimal action of agent i of type $s(i)$.

Proposition 9. *In the model with a continuum of actions, we have*

$$\frac{d\theta^*}{d\psi_m} = \frac{1}{1 - \sum_{n=1}^{N} \int \frac{\partial\theta^*}{\partial a^*_{s(i)}(i)} \frac{\partial a^*_{s(i)}(i)}{\partial\theta^*} \, di} \left[\sum_{n=1}^{N} \int \frac{\partial\theta^*}{\partial a^*_{s(i)}(i)} \frac{\partial a^*_{s(i)}(i)}{\partial\psi} \, di + \frac{\partial\theta^*}{\partial\psi} \right].$$ [34]

E.4 *Proper prior*

It is easy to show that, with appropriate additional assumptions on the information structure that guarantee the uniqueness of equilibrium, all the results extend to the model with a proper prior. In particular, suppose that all agents believe that θ is distributed according to an absolutely continuous distribution G with $\text{supp}(G) = \mathbb{R}$, and a continuously differentiable density g, which is bounded above and strictly positive over $\mathbb{R}$. In this case, one can show that there exists $\overline{\tau}$ such that if for all $n \in \mathcal{N}$ we have $\tau_n \geq \overline{\tau}$, then the equilibrium is unique. Once uniqueness is established, all the results reported in Sections 3–6 can be derived for the model with a proper prior following steps analogous to those in the paper.

E.5 *A simultaneous change in multiple parameters*

Fix $K > 1$ and let $\{\psi_{m_1}, \dots, \psi_{m_K}\} \subset \boldsymbol{\psi}$ be a subset of the parameters of the model. Suppose that we are interested in computing the effect that a simultaneous small change

[34] Here, I slightly abuse the notation when I write $(\partial\theta^*/\partial a^*_n(i))(\partial a^*_n(i)/\partial\theta^*)$. What this term captures is the change in θ^* due to the adjustment in agents' beliefs through the adjustments in their optimal actions $a^*_n(i)$, $n \in \mathcal{N}$.

in $\psi_{m_1}, \dots, \psi_{m_K}$ has on θ^*. To this end, let us write θ^* explicitly as a function of $\psi_{m_1}, \dots, \psi_{m_K}$ and denote by $\nabla_{\underline{\mathbf{1}}}\theta^*(\psi_{m_1}, \dots, \psi_{m_K})$ the directional derivative of θ^* in the direction $\underline{\mathbf{1}} = (1, \dots, 1) \in \mathbb{R}^K$.[35]

Proposition 10. *Let $\{\psi_{m_1}, \dots, \psi_{m_K}\}$ be a subset of the parameters of the model. Then*

$$\nabla_{\underline{\mathbf{1}}}\theta^*(\psi_{m_1}, \dots, \psi_{m_K}) = \mathcal{M} \sum_{k=1}^{K} \mathcal{D}(\psi_{m_k}),$$

where $\mathcal{M}$ and $\mathcal{D}(\psi_{m_k})$ are defined as in Theorem 1.

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[35] The direction $\underline{\mathbf{1}} = (1, \dots, 1)$ implies that all K parameters change at the same rate.

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Co-editor Dilip Mookherjee handled this manuscript.

Manuscript received 19 November, 2017; final version accepted 24 July, 2019; available online 27 September, 2019.