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Preferences for partial information and ambiguity

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We commonly think of information as an instrument for better decisions, yet evidence suggests that people often decline free information in nonstrategic scenarios. This paper provides a theory for how a dynamically-consistent decision maker can be averse to partial information as a consequence of ambiguity aversion. It introduces a class of recursive preferences on an extended choice domain, which allows the preferences to depend on how information is dynamically revealed and to depart from the standard expected-utility theory. A new notion of ambiguity aversion, called Event Complementarity, exactly characterizes aversion to partial information. Familiar static ambiguity-averse preferences are embedded into the general recursive model, in which conditions for partial information aversion are identified. The findings suggest that Event Complementarity overlaps with yet still differs from the conventional notion of ambiguity aversion.

Keywords. Information avoidance, ambiguity aversion, recursive preferences.

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1. Introduction

In many decision problems, uncertainties are resolved gradually yet decision makers (DMs) appear to be averse to information revealed at intermediate stages. For example, genetic tests can reveal risks of inheritable and often incurable diseases, though many people decline to learn such results.[1] An investor saving for retirement, who could learn about her portfolio's return every month, may instead prefer to review her portfolio less frequently.[2] Anecdotes and experimental studies suggest that avoidance of such freely

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[1] Lerman et al. (1996b) documented that, when asked in an education session, 40% of adults from families of hereditary breast-ovarian cancer refuse BRCA1-genetic test results. Lerman et al. (1996a, 1999) found that 57% of family members at risk for hereditary nonpolyposis colon cancer do not want to learn about genetic test results during a phone interview, and 48% (of more at-risk adults) decline during a structured interview.

[2] Bellemare et al. (2005) found that, when a subject in a portfolio choice experiment is committed to some given portfolio, a higher frequency of feedback information leads to a lower *ex ante* willingness to invest in

available partial information is quite common. Sometimes these information preferences are *intrinsic*, when the DM appears to be directly affected by the information although she takes no action after receiving it. In the same class of problems, DM's also often perceive ambiguity in some uncertain events and fail to form a unique assessment of their probabilities. A DM is averse to the perceived ambiguity if she dislikes betting on such an event with unknown probability relative to a comparable event with a uniquely known probability (Ellsberg 1961). This paper provides a notion of ambiguity aversion, under which a DM will be intrinsically averse to the partial information, even though her preferences are dynamically consistent.[3]

To understand how ambiguity aversion may lead to aversion to partial information, consider a DM who evaluates a fixed act in a dynamic decision problem. She expects to learn some partial information at an earlier stage before the final outcome of the act is revealed. The DM considered here uses a procedure that evaluates the act backward inductively, through the lens of the intermediate partial information. The theory suggests that the anticipated partial information will affect how the DM perceives the uncertainties, if she is also ambiguity averse. To see this, first consider the classic environment with only risk, in which the DM holds a unique belief regarding the uncertainties and updates her belief via the Bayes' rule upon receiving the partial information. In this case, by the law of iterated expectation, receiving partial information does not affect the *ex ante* riskiness of the act compared to the alternative case of receiving no information in the intermediate stage. However, when ambiguity is also present, the DM perceives multiple beliefs as plausible. If the DM updates every plausible belief by the Bayes' rule, the partial information received in the intermediate stage may reduce or enhance the amount of *ex ante* ambiguity relative to no information.[4] The DM, who is averse to this ambiguity, evaluates the fixed act pessimistically by the worst-case scenario and becomes even more pessimistic in light of the partial information. When expecting the partial information, not only does she think of worst-case scenarios when she receives bad news, but she also considers that the bad news is very likely to arrive. In this way, receiving partial information hurts the ambiguity-averse DM by forcing her to perceive the uncertainties more pessimistically than the case of not receiving information. This effect can lead to an intrinsic aversion to partial information.

This paper contributes to the dynamic decision theory by providing a general theory of when and how an ambiguity-averse DM can display intrinsic aversion to partial information according to the story described above. To that end, a general class of recursive

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risky assets. Gneezy et al. (2003) cited the event that in February 1999 Bank Hapoalim, an Israeli mutual fund, announced that it would lower the frequency of performance updates from monthly to quarterly in order to encourage asset holdings among investors.

[3]Caplin and Leahy (2001) and Kőszegi (2003) explained the evidence of intrinsic information avoidance by a model of anticipatory feelings. Eliaz and Spiegler (2006) provided a critique to their approach. My paper explains aversion to partial information by a notion of ambiguity aversion.

[4]It is well known that, when a model accommodates multiple plausible probabilities, the range of conditional probabilities of an event can be larger than the range unconditional of probabilities of the same event, *for every conditioning event* in the information partition (Walley 1991, Seidenfeld and Wasserman 1993). This phenomenon is called "dilation" and clearly demonstrates how partial information can increase ambiguity. Another example of how partial information can increase ambiguity is provided in Section 3.2 in a context of a canonical three-color Ellsberg urn.

preferences over an extended domain of information and acts are axiomatized. For this class of preferences, aversion to partial information can be characterized equivalently in terms of a notion called *Event Complementarity*, which is motivated by Ellsberg's (1961) paradox and explained below. Moreover, conditions for partial information aversion are identified for four widely applied families of static ambiguity-averse preferences, which are naturally embedded to the general recursive preferences model. The findings suggest that Event Complementarity overlaps yet still differs from the conventional notion of ambiguity aversion (Schmeidler 1989).

The primitive of the model is the *ex ante* preference relation $\succcurlyeq$ over the product set of all information partitions and acts. In this way, the *ex ante* preferences over acts depend on the information partition, and for every fixed act the *ex ante* preferences over information partitions can be strict. I provide axioms on $\succcurlyeq$ that characterize a general *partition-dependent recursive utility* representation, which is described as follows. When evaluating a fixed act f, a partition-dependent recursive utility DM proceeds with a folding-back procedure. First, she forms her interim utility of the act f conditional on each event E in the anticipated partition $\pi = \{E_1, \dots, E_n\}$. Then she goes backward to aggregate her interim conditional utilities of the act f for all the events in the partition π to determine her *ex ante* utility of f. In this way, the *ex ante* utility function that represents $\succcurlyeq$ can be dependent on the anticipated partition π. For this class of recursive preferences, a DM is (intrinsically) averse to partial information if she weakly prefers the null information to any partial information partition at all acts.

To characterize aversion to partial information, I propose the notion of Event Complementarity, which is motivated by the violations of Savage's sure-thing principle in the Ellsberg paradox (Ellsberg 1961). To illustrate, consider an act of betting on an unambiguous event F, whose probability is uniquely known (Ghirardato and Marinacci 2002, Epstein and Zhang 2001). When evaluated all at once, this bet is unambiguous. Suppose there is some partition $\{E, E^c\}$ that divides this event into two subsets $E \cap F$ and $E^c \cap F$, which become ambiguous. If the DM's bet on F is separated by the events E and E^c, then its certainty equivalent conditional on E and that conditional on E^c should both reflect the ambiguity embedded in these separated events $E \cap F$ and $E^c \cap F$. Hence, Event Complementarity says that an (Ellsbergian) ambiguity-averse DM will prefer the "whole" bet on F to the two separate bets on $E \cap F$ and $E^c \cap F$, because the latter bets are ambiguous while the former bet is not. In other words, the notion describes the preference for evaluating several events jointly together rather than separately apart, as the joint assessment can potentially create a complementary effect to reduce ambiguity. Under Event Complementary, it is quite intuitive that a DM with a bet on F will dislike receiving the partial information $\{E, E^c\}$.

The main result (Proposition 2) says that, in the general partition-dependent recursive utility model, aversion to partial information is equivalently characterized by Event Complementarity. Since the former notion restricts the dynamic information preferences while the latter is mostly about the static preferences, the observation that they are two sides of the same coin can shed light on other research that tests or applies the ambiguity aversion models in a dynamic environment.

To compare Event Complementarity with the conventional notion of ambiguity aversion (Schmeidler 1989), Section 4 considers several widely applied families of ambiguity-averse preferences (naturally embedded into the partition-dependent recursive utility model), and examines aversion to partial information in each family. The findings are as follows. As Event Complementarity is motivated by violations of Savage's sure-thing principle in the static preferences, preferences that satisfy the sure-thing principle display indifference toward partial information. This is unsurprising and is observed in the popular multiplier preferences family (Hansen and Sargent 2001 and Strzalecki 2011), which is the only family examined in Section 4 that satisfies the sure-thing principle. For other ambiguity-averse preferences, which accommodate violations of the sure-thing principle, there is a close yet delicate link between ambiguity aversion and partial information aversion. For instance, in the widely applied maxmin expected utility (Gilboa and Schmeidler 1989, MEU) family, ambiguity aversion is a sufficient condition for aversion to partial information (Proposition 3). While Event Complementarity is the exact notion of ambiguity aversion that characterizes aversion to partial information, it is not always satisfied for the more general families of variational preferences (Maccheroni et al. 2006a) and second-order belief preferences (Klibanoff et al. 2005), as indicated by Examples 1 and 2. Nevertheless, in both families, ambiguity aversion still implies that partial information aversion holds locally at all acts that are unambiguous (Propositions 4 and 6), reassuring the main intuition. Moreover, in the more general case of variational preferences family, aversion to a given partial information partition at all acts can be equivalently characterized by a simple inequality (Proposition 5), which is a variant of the familiar "no-gain" condition proposed by Maccheroni et al. (2006b). These results suggest that Event Complementarity overlaps with and yet differs from the standard notion of ambiguity aversion proposed by Schmeidler (1989).

The paper proceeds as follows. Section 2 describes the framework. Section 3 axiomatizes the family of partition-dependent recursive utility, and characterizes the equivalence between aversion to partial information and Event Complementarity within this family. Section 4 characterizes aversion to partial information in four popular families of ambiguity-averse preferences. Section 5 discusses the related literature.

2. Preliminaries

Consider an environment with subjective uncertainties, where S is a *finite* set of states of the world. Let s denote a generic state. An event E is a subset of the state space S, and E^c denotes its complement. Let Σ be the collection of all nonempty subsets of S. For all nonempty $E \subseteq S$, let $\Delta(E)$ denote the set of probabilities on E.

The set X describes all consequences. I assume that X is a connected and convex subset of a general topological vector space. An act $f : S \to X$ is a mapping that assigns every state a consequence. Let $\mathcal{F}$ be the set of all such acts that is endowed with the product topology. Note that the set of Anscombe and Aumann (1963) acts is a special case of $\mathcal{F}$. An act f is *constant* if it maps every state to the same consequence x; in this case, f is identified with x. When there is no confusion, for given f and s, sometimes $f(s)$ is used to denote the constant act that gives the corresponding consequence in every state. For $f, g \in \mathcal{F}$ and $E \in \Sigma$, let fEg denote the composed act such that $(fEg)(s) = f(s)$ if $s \in E$ and $(fEg)(s) = g(s)$ if $s \notin E$. For $f, g \in \mathcal{F}$ and $\alpha \in (0, 1)$, $\alpha f + (1 - \alpha)g$ denotes the pointwise mixture of f and g: $(\alpha f + (1 - \alpha)g)(s) = \alpha f(s) + (1 - \alpha)g(s)$ for all s.

The DM faces two-stage resolution of uncertainties, described by the following simple information structures. In stage 0, the DM has no *ex ante* information about the true state. In intermediate stage 1, she receives some partial information that the true state lies in some event $E_i \subseteq S$. Finally, the true state $s \in E_i$ is fully revealed in stage 2. The intermediate stage partial information, denoted $\pi = \{E_1, \dots, E_n\}$, is modeled as a partition of the state space S. Let Π be the set of all such partitions. In particular, $\pi_0 = \{S\}$ denotes the coarsest partition, corresponding to the case when no information is learned in stage 1, and $\pi^* = \{\{s_1\}, \dots, \{s_{|S|}\}\}$ denotes the finest partition, corresponding to the case when all relevant uncertainties are resolved in stage 1. For all π, an act f is π-measurable if it is constant on every event in the partition π. Let $\mathcal{F}_\pi$ be the subset of π-measurable acts in $\mathcal{F}$.

The primitives are the DM's *ex ante* stage-0 preferences over the product space of anticipated information Π and acts $\mathcal{F}$. Denote the preferences and this enriched domain by $\succcurlyeq$ and $\Pi \times \mathcal{F}$. Endow space Π with the discrete topology and space $\Pi \times \mathcal{F}$ with the product topology. If $(\pi, f) \succcurlyeq (\pi', g)$, then the DM prefers act f with anticipated information π to act g with anticipated information π'. In this enriched domain, preferences can be "indexed" in two ways: If $\pi = \pi'$, then $\succcurlyeq$ describes the DM's *ex ante* preferences over acts for fixed interim information π; If $f = g$, then $\succcurlyeq$ describes the DM's *ex ante* preferences over anticipated information at a given act f.[5]

For notational simplicity, hereafter the restriction of $\succcurlyeq$ on $\{\pi\} \times \mathcal{F}$ is sometimes denoted by $\succcurlyeq_\pi$; the static unconditional preference relation is identified with $\succcurlyeq_{\pi_0}$ and denoted by $\succcurlyeq_0$. The preference relation $\succcurlyeq$ satisfies *reduction* if every act can be simply evaluated by its induced mapping from states to consequence, while the courses of this act do not matter; that is, $\succcurlyeq_\pi = \succcurlyeq_{\pi_0}$ for all $\pi \in \Pi$. When the DM's preferences over acts depend on how information is revealed over time, reduction is relaxed.

3. Axiomatic characterization

In this section, I first introduce six axioms on the *ex ante* preferences $\succcurlyeq$ on $\Pi \times \mathcal{F}$, which are necessary and sufficient for $\succcurlyeq$ to admit the partition-dependent recursive utility representation, and then I characterize aversion to partial information for this class of preferences.

Axiom 1 (Weak order). *(i) (Completeness). For all $\pi, \pi' \in \Pi$ and $f, g \in \mathcal{F}$, $(\pi, f) \succcurlyeq (\pi', g)$ or $(\pi, f) \preccurlyeq (\pi', g)$.*

(ii) (Transitivity). For all $\pi, \pi', \pi'' \in \Pi$ and $f, g, h \in \mathcal{F}$, if $(\pi, f) \succcurlyeq (\pi', g)$ and $(\pi', g) \succcurlyeq (\pi'', h)$, then $(\pi, f) \succcurlyeq (\pi'', h)$.

[5] Ahn and Ergin (2010) also consider static preferences over acts indexed by partitions. Their motivation is different, and only partition-measurable acts are considered. Gajdos et al. (2008) used a similar kind of double-indexing, but information is modeled differently.

Axiom 2. *For all* $\pi \in \Pi$, $\succsim_\pi$ *on* $\{\pi\} \times \mathcal{F}$ *satisfies the following.*

(i) *(Continuity). For all* $f \in \mathcal{F}$*, sets* $\{(\pi, g) \in \{\pi\} \times \mathcal{F} : (\pi, g) \succsim (\pi, f)\}$ *and* $\{(\pi, g) \in \{\pi\} \times \mathcal{F} : (\pi, f) \succsim (\pi, g)\}$ *are closed.*

(ii) *(Strong monotonicity). For all* $f, g \in \mathcal{F}$*, if* $(\pi, f(s)) \succsim (\pi, g(s))$ *for all* s*, then* $(\pi, f) \succsim (\pi, g)$*. If in addition one of the preference rankings is strict, then* $(\pi, f) \succ (\pi, g)$.

(iii) *(Constant-act independence). For all* $x, y, z \in X$ *and all* $\alpha \in (0, 1)$,

$$(\pi, x) \succsim (\pi, y) \quad \Leftrightarrow \quad \big(\pi, \alpha x + (1 - \alpha)z\big) \succsim \big(\pi, \alpha y + (1 - \alpha)z\big).$$

(iv) *(Nondegeneracy).* $(\pi, f) \succ (\pi, g)$ *for some* $f, g \in \mathcal{F}$.

Axiom 3 (Stable constant-act preferences). *For all* $\pi, \pi' \in \Pi$ *and* $x, y \in X$, $(\pi, x) \succsim (\pi, y)$ *if and only if* $(\pi', x) \succsim (\pi', y)$*. Moreover,* $(\pi, x) \sim (\pi', x)$ *for all* $\pi, \pi' \in \Pi$ *and* $x \in X$.

The first three axioms are standard. Weak order is a basic assumption of rationality. Continuity is a common technical assumption needed for the existence of a real-valued utility representation. Strong monotonicity says that more is better and improvements in every state "count," where the latter restriction is added to simplify the discussions on conditional preferences elicitation. Constant-act independence imposes the independence axiom on the set of constant acts, which is assumed to be convex and connected. Nondegeneracy rules out the trivial case that all acts are in the same indifference class. Stable constant-act preferences says that the DM's preferences over constant acts are not affected by the anticipated information partitions, and she displays indifference toward information partitions if she is given a constant act.

Axiom 4 (Indifference to redundant information). *For all* π*,* $f \in \mathcal{F}_\pi$*,* $(\pi, f) \sim (\pi^*, f)$.

Axiom 4 is quite innocuous. Intuitively, if an act f is π-measurable, then all outcome uncertainties about f are resolved in stage one after learning which event in π realizes. Thus the additional information in the full information partition π^* relative to that in π should not matter at the π-measurable act f.

Axiom 5 (Anticipated partition separability). *For all* $\pi, \pi' \in \Pi$ *and event* $E \in \pi \cap \pi'$*, and all* $f, g, h, h' \in \mathcal{F}$,

$$(\pi, fEh) \succsim (\pi, gEh) \quad \Leftrightarrow \quad \big(\pi', fEh'\big) \succsim \big(\pi', gEh'\big).$$

Anticipated partition separability says the following. If the DM anticipates an information partition π that contains the event E and compares two acts fEh and gEh that agree with each other on E^c, then two types of modifications will not affect the relative rankings: (i) if one substitutes h, the common component of fEh and gEh on E^c, by another common component h'; and (ii) the anticipated partition π is modified to π', yet changes only occur outside event E.

This axiom facilitates the following elicitation of stage-1 conditional preferences $\{\succsim_E\}_{E\in\Sigma}$ from *ex ante* preferences $\succsim$.

Definition 1 (Conditional preferences elicitation). For all $E \in \Sigma$ and $f, g \in \mathcal{F}$, $f \succsim_E g$ if and only if $(\pi, fEh) \succsim (\pi, gEh)$ for some $h \in \mathcal{F}$ and some π such that $E \in \pi$.

Anticipated partition separability ensures that the elicited conditional preferences $\{\succsim_E\}_{E\in\Sigma}$ are well-defined and satisfy two well-known properties. First, conditional preferences $\{\succsim_E\}_{E\in\Sigma}$ satisfy *consequentialism*, that is, $fEg \sim_E fEh$ for all $f, g, h \in \mathcal{F}$, and all $E \in \Sigma$ (Machina 1989). It says that consequences in states that have been ruled out upon learning the event E should not matter for the conditional preferences on E. Second, $\succsim_\pi$ and $\{\succsim_E\}_{E\in\pi}$ are *dynamically consistent*,that is, for any $\pi \in \Pi$ and $f, g \in \mathcal{F}$, $f \succsim_E g$ for all $E \in \pi$ implies that $f \succsim_\pi g$ (Epstein and Schneider 2003). This property says that an act that is preferred ex-ante should remain preferred as the DM receives additional information.

Lemma 1. *Suppose $\succsim$ satisfies anticipated partition separability and $\{\succsim_E\}_{E\in\Sigma}$ are elicited by Definition 1. Then $\succsim_\pi$ and $\{\succsim_E\}_{E\in\pi}$ satisfy consequentialism and dynamic consistency.*

Observe that anticipated partition separability implies that the *ex ante* preference relation $\succsim_\pi$ is separable with respect to all the events in the partition π. This restriction is reminiscent of Savage's sure-thing principle, which requires that $fEh \succsim_0 gEh$ if and only if $fEh' \succsim_0 gEh'$ for all $E \in \Sigma$ and $f, g, h, h' \in \mathcal{F}$. That is, the sure-thing principle imposes on $\succsim_0$ separability with respect to all the events.

To illustrate the resemblance between anticipated partition separability and the sure-thing principle, consider the classic three-color Ellsberg paradox (Ellsberg 1961). There is an urn that contains 90 balls: among them, 30 balls are red, and 60 balls are either green or yellow, with the exact proportion unknown. The DM places bets on the color of a ball drawn from the urn. Let f_E denote a bet that pays \$1 on event $E \subseteq \{R, G, Y\}$ and \$0 otherwise.

In a static setting, a typical Ellsbergian DM strictly prefers betting on red to betting on green, but strictly prefers betting on the event that the ball is either green or yellow ($\{G, Y\}$), to betting on the event that the ball is either red or yellow ($\{R, Y\}$), that is,

$$f_R \succ_0 f_G \quad \text{and} \quad f_{RY} \prec_0 f_{GY}.$$

The Ellsbergian unconditional preferences are inconsistent with the sure-thing principle.

When anticipating partition $\pi = \{\{R, G\}, \{Y\}\}$, assuming MEU preferences and prior-by-prior updating (see Section 3.2 for details), the DM would again strictly prefer to bet on red versus green after observing "not yellow," that is, $f_{RY} \succ_{RG} f_{GY}$, and, obviously, $f_{GY} \sim_Y f_{RY}$. Then anticipated partition separability would require

$$(\pi, f_R) \succ (\pi, f_G) \quad \text{and} \quad (\pi, f_{RY}) \succ (\pi, f_{GY}).$$

Observe that anticipated partition separability implies that the *ex ante* preference relation $\succsim$ must still display a form of event-separability that is in tension with the usual

Ellsberg intuition, even though this axiom has no bite for the unconditional static preference relation $\succsim_0$.[6] In light of this restriction, anticipated partition separability is the most substantial assumption on $\succsim$ for the representation result.

Axiom 6 (Time neutrality). *For all f, $(\pi^*, f) \sim (\pi_0, f)$.*

Time neutrality says that the DM does not care about the period in which uncertainties are revealed, as long as that happens in a single period. An immediate consequence of this axiom is that the *ex ante* preferences over acts when anticipating the full information and the *ex ante* preferences when anticipating the null information are identical; that is, $\succsim_{\pi^*} = \succsim_{\pi_0}$.

Note that a similar equivalence between ambiguity aversion (in the form of event complementarity) and a preference for full information already holds with only Axioms 1–5 (see Corollary 1 in Section 3.3). Nevertheless, by adding time neutrality, the model abstracts away from intrinsic preferences for the timing of resolution of uncertainties, which can be implied by certain ambiguity-averse preferences (Strzalecki 2013). The focus here is on the sequence of resolution of uncertainties.

As discussed above, Axioms 5 and 6 seem less standard or innocuous than Axioms 1–4. To verify their necessity, Appendix A.9 provides examples of utility representations that violate either one of them but satisfy the other five axioms, and discusses the failure of the main equivalence result (Proposition 2) in each utility representation.

3.1 *Partition-dependent recursive utility representation*

In this subsection, I introduce the recursive utility representation and show that it can be characterized by Axioms 1 to 6.

For a real interval $K \subseteq \mathbb{R}$, let $K^{|S|}$ be the set of $|S|$-dimensional K-ranged vectors, and denote generic vectors by ξ, ϕ. For all $k \in K$, let $\bar{k}$ be the constant vector that equals to k in every dimension. Fix any nonempty $E \in \Sigma$. A composed vector $\xi E \phi$ equals $(\xi E \phi)(s) = \xi(s)$ if $s \in E$, and $(\xi E \phi)(s) = \phi(s)$ if $s \notin E$. For every f and ξ, let f_E and ξ_E be the restrictions of f and ξ on E, respectively. Denote the set of all such restricted acts f_E by $\mathcal{F}_E$. For a functional $I_E : K^{|E|} \mapsto \mathbb{R}$, say that I_E is *monotone* if $\xi_E \geq \phi_E$ implies $I_E(\xi_E) \geq I_E(\phi_E)$; I_E is *strongly monotone* if it is monotone and $\xi_E > \phi_E$ implies $I_E(\xi_E) > I_E(\phi_E)$; I_E is *normalized* if $I_E(\bar{k}) = k$ for all $k \in K$.

Definition 2. Say the preference relation $\succsim$ on $\Pi \times \mathcal{F}$ admits a *partition-dependent recursive utility representation* $(u, (I_E)_{E\in\Sigma}, I_0)$ if there exists (i) a continuous, nonconstant, and affine $u : X \to \mathbb{R}$; (ii) continuous, strongly monotone, and normalized functionals $I_0 : u(X)^{|S|} \mapsto \mathbb{R}$ and $I_E : u(X)^{|E|} \mapsto \mathbb{R}$ for every $E \in \Sigma$ satisfying the following properties. For all $E \in \Sigma$, the conditional preference relation $\succsim_E$ elicited by Definition 1 is represented by a function $V_E : \mathcal{F} \mapsto \mathbb{R}$ where

$$V_E(f) = I_E\big(u(f_E)\big), \tag{1}$$

[6]Suppose $\pi = \pi_0$. Then $E = S$ and so $\pi' = \pi_0$; the equivalence in the axiom holds trivially. Suppose $\pi = \pi^*$. Then E is a singleton and the equivalence in the axiom follows from strong monotonicity.

and the *ex ante* preference relation $\succsim$ is represented by a utility function $V : \Pi \times \mathcal{F} \mapsto \mathbb{R}$ where

$$V(\pi, f) = I_0 \begin{pmatrix} V_{E_1}(f) & \text{if } s \in E_1 \\ \vdots & \vdots \\ V_{E_n}(f) & \text{if } s \in E_n \end{pmatrix} \tag{2}$$

for $\pi = \{E_1, \dots, E_n\}$.

Intuitively, the DM with partition-dependent recursive utility behaves as if she evaluates every partition-act pair (π, f) following a folding-back procedure: by backward induction, she first evaluates for every stage-1 event $E_i \in \pi$ the (elicited) conditional certainty equivalent of f, and then she goes back to stage 0 and evaluates the π-conditional certainty equivalent of f.

Observe that in equation (1), the conditional utility function $V_E(\cdot)$ depends only on the outcomes of act f on the event E. And in the *ex ante* utility representation (equation (2)), $V(\pi, \cdot)$ is separable with respect to events in the partition π.

Proposition 1. *The preference relation $\succsim$ satisfies Axioms 1 to 6 if and only if it admits a partition-dependent recursive utility representation* $(u, (I_E)_{E\in\Sigma}, I_0)$.

Moreover, if both $(u, \{I_E\}_{E\in\Sigma}, I_0)$ and $(u', \{I'_E\}_{E\in\Sigma}, I'_0)$ represent $\succsim$, then there exists $a > 0$ and $b \in \mathbb{R}$ such that $u' = au + b$, $I'_E(\xi_E) = aI_E((\xi_E - \bar{b})/a) + b$ for all $\xi_E \in (u'(X))^{|E|}$, and $I'_0(\xi_E) = aI_0((\xi - \bar{b})/a) + b$ for all $\xi \in (u'(X))^{|S|}$.

Observe that if all conditional probabilities $\{p_E \in \Delta(E)\}_{E\in\Sigma}$ are also uniquely known, that is, all conditional preferences $\{\succsim_E\}_{E\in\Sigma}$ are probabilistically sophisticated à la Machina and Schmeidler (1995), then the representation model in Proposition 1 can be viewed as a special case of Segal (1990). To see this, note that Segal's domain corresponds to the set $\Delta(\Delta X)$. In this model, fix an arbitrary pair of partition and act (π, f). Due to probabilistic sophistication, at every event $E \in \pi$, the pair induces a conditional probability distribution: $\hat{p}_E(B) := p_E(f^{-1}(B) \cap E)$ for all measurable set B in the outcome space X. The pair also induces a probability $\hat{P}$ over these conditional lotteries, given by $\hat{P}(\hat{p}_E) := p_S(E)$, where p_S is the unconditional probability over the state space S. Thus each pair (π, f) induces a compound lottery $\hat{P} \in \Delta(\Delta X)$, and this paper's domain can be mapped to Segal's domain. Yet these two domains are not equivalent under probabilistic sophistication, because the above mentioned mapping is not onto. For a counterexample, let the support of P contain more distinct lotteries than the number of states $|S|$.

Finally, note that the partition-dependent recursive utility model per se is not necessitated by ambiguity. One could imagine that a DM's utility from holding an act is the worst-case scenario in the final period when the state is revealed, that is, her preferences satisfy reduction without regard to how she feels in the interim stage on account of the noninstrumental information. Clearly, this DM does not display any intrinsic information preferences. Hence, independent from ambiguity, some sensitivity to interim variation in utility is necessary to generate intrinsic partial information aversion.

3.2 *A leading example*

To illustrate the intuitive connection between ambiguity aversion and aversion to partial information, recall the three-color Ellsberg urn example mentioned earlier. In the static setting, the typical Ellsbergian DM displays the following preferences:

$$f_R \succ_0 f_G \quad \text{and} \quad f_{RY} \prec_0 f_{GY}.$$

Thus, by adding the same event $\{Y\}$ to both event $\{R\}$ and event $\{G\}$, the DM reverses her preference ranking between them. Obviously, this reflects a preference for betting on events with known probabilities ($p(\{R\}) = \frac{1}{3}$ and $p(\{G, Y\}) = \frac{2}{3}$) to betting on events with unknown probabilities ($p(\{G\}) \in \{0, \frac{1}{90}, \dots, \frac{60}{90}\}$ and $p(\{R, Y\}) \in \{\frac{30}{90}, \frac{31}{90}, \dots, 1\}$), which is the standard interpretation of ambiguity aversion.

To further illustrate the connection to dynamic preferences, assume that the DM (i) is risk-neutral; (ii) has MEU preferences; (iii) conditional on an event E her preferences are updated prior-by-prior (Pires 2002); and (iv) has *ex ante* preferences that are aggregated from conditional preferences by the partition-dependent recursive utility model. Particularly, the DM has vNM utility index over money $u(x) = x$; a set of priors $P = \{(p_R, p_G, p_Y) \in \mathbb{R}^3_+ : p_R = \frac{1}{3}, p_R + p_G + p_Y = 1\}$; and her unconditional atemporal preferences $\succcurlyeq_0$ are represented by

$$V_0(f) = \min_{p \in P} E_p[f].$$

Thus, substituting the payoffs of each bet implies that

$$\frac{2}{3} = V_0(f_{GY}) > V_0(f_G) + V_0(f_Y) = 0,$$
$$\frac{1}{3} = V_0(f_{RY}) = V_0(f_R) + V_0(f_Y) = \frac{1}{3}.$$

The first inequality says that the DM would rather bet on the joint event $\{G, Y\}$ than have two separate bets on the singleton events $\{G\}$ and $\{Y\}$. The intuition is as follows: the singleton events $\{G\}$ and $\{Y\}$ have unknown probabilities (in $\{0, \frac{1}{90}, \dots, \frac{60}{90}\}$) and an Ellsbergian DM dislikes betting on them; when assessed jointly, event $\{G\}$ and event $\{Y\}$ complement each other, and betting on the joint event $\{G, Y\}$, which has a precise probability $\frac{2}{3}$, becomes more attractive. The second equality says the DM is indifferent between betting on the joint event $\{R, Y\}$ and betting separately on the events $\{R\}$ and $\{Y\}$, since there is no complementarity between event $\{R\}$ and event $\{Y\}$. The main point is an MEU DM will never strictly prefer separate bets on two disjoint events to a single bet on their union.

To analyze the information preferences of this Ellsergian DM, suppose her dynamic preferences conforms to the partition-dependent recursive utility model. Consider two intermediate information structures: (i) the null information partition ($\pi_0 = \{\{R, G, Y\}\}$); and (ii) a partial information that reveals whether the ball is yellow or not ($\pi = \{\{R, G\}, \{Y\}\}$). Then the DM's preferences conditional on an event $E \subseteq \{R, G, Y\}$

and her *ex ante* preferences anticipating any partition π are represented by

$$V_E(f) = \min_{p \in P} E_p[f|E],$$

$$V(\pi, f) = \min_{p \in P} \sum_{E \in \pi} p(E) V_E(f).$$

Again, substituting the four bets, priors, and events into the above equations yield $V_{RG}(f_{GY}) = \min_{p \in [0, \frac{2}{3}]} p \cdot 1 = 0$ and $V_{RG}(f_{RY}) = \min_{p \in [\frac{2}{3}, 1]} p \cdot 1 = \frac{1}{3}$; obviously, $V_Y(f_{GY}) = V_Y(f_{RY}) = 1$. Then

$$0 = \min_{p \in [0, \frac{2}{3}]} p \cdot 1 = V(\pi, f_{GY}) < V(\pi_0, f_{GY}) = \frac{2}{3},$$

$$\frac{1}{3} = \min_{p \in [0, \frac{2}{3}]} (1-p) \cdot \frac{1}{3} + p \cdot 1 = V(\pi, f_{RY}) = V(\pi_0, f_{RY}) = \frac{1}{3}.$$

Thus, the DM with MEU preferences also weakly prefers null information π_0 to partial information π, displaying partial information aversion.

3.3 *Ellsbergian behaviors and information preferences*

This subsection formalizes the intuition from the leading Ellsberg example by introducing the relevant notion of ambiguity aversion. The main results (Proposition 2, Corollary 1) show that this notion exactly characterizes an aversion to partial information/a preference for full information in the general partition-dependent recursive utility model.

The classic notion of ambiguity aversion is a preference for hedging (Schmeidler 1989): for all $f, g \in \mathcal{F}$, and all $\alpha \in (0, 1)$, $f \sim_0 g$ implies that $\alpha f + (1-\alpha) g \succsim_0 f \sim_0 g$. In other words, when facing subjective uncertainties, the DM can always weakly benefit from randomization. For example, if the DM in the Ellsberg example can also toss a fair coin, then the mixed act of choosing (the dispreferred) bet f_G if the coin lands head and choosing (the dispreferred) bet f_{RY} if the coin lands tail gives a winning probability one-half, no matter which color the drawn ball is, which gives exactly the same state-by-state outcome as an analogous mixing of the two preferred bets, f_R and f_{GY}. Thus, state-by-state randomization helps the DM hedge against ambiguity. This notion is used in most familiar models of ambiguity-averse preferences considered in Section 4.

Here, I propose a new notion of ambiguity aversion, which is directly motivated by the idea that the DM dislikes betting on an event with unknown probability relative to a comparable event with known probability. Particularly, for any event F, any binary partition $\{E, E^c\}$ divides it into two disjoint subevents: $F_1 = F \cap E$ and $F_2 = F \cap E^c$. While the event F can be unambiguous, its separate components F_1 and F_2 are subject to ambiguity. Hence, an ambiguity-averse DM considered weakly prefers having a single bet on the whole event F rather than having two bets on its separate components F_1 and F_2.

This intuition can be generalized to arbitrary acts to form the notion of *event complementarity*. To illustrate, consider the following "ironing procedure." For any fixed event E, if an act f yields uncertain payoffs on E, then one can "iron out" this variation by replacing the outcomes of the act f on E by its certainty equivalent conditional on E, that is, the constant act x such that $f \sim_E x$. By performing the procedure, any informational complementarity between states in the event E and its complement E^c that is embedded in f is eliminated; while the ironed-out act, denoted xEf, is equivalent to f conditional on the event E and is equal to f on the event E^c. From f to xEf, nothing has changed except for the elimination of informational complementarity of states across E and E^c. Hence, for an ambiguity-averse DM who values the state complementarity, she will find the act f less attractive after being "ironed."

Axiom 7 (π_0-event complementarity). *For all $E \in \Sigma$, $f \in \mathcal{F}$, and $x \in X$, if $f \sim_E x$, then $f \succsim_0 xEf$.*

Note that even though the "ironing procedure" may reduce the total outcome variability of an act, event complementarity implies that the ambiguity aversion embedded in the conditional preference relation $\succsim_E$ precludes the "ironed-out" act from being more attractive than the original act. For instance, consider again the leading Ellsberg example in Section 3.2. Pick the act $g = (2, 0, 1)$, which yields a high prize in red, a low prize in green, and a medium prize in yellow. For the event $E = \{R, G\}$, the "ironed" act xEg replaces payoffs in the states red and green by the conditional certainty equivalent x (which is between 0 and 2) and indeed reduces the total outcome variability of the act. However, the conditional preference relation $\succsim_E$ obtained by prior-by-prior updating is also ambiguity averse, and the E-conditional certainty equivalent of the act g is 0. Hence, the ironed-out act $xEg_{RY} = (0, 0, 1)$ has a lower outcome variability than $g = (2, 0, 1)$, but it is also statewise dominated by g. By monotonicity, $g \succsim_0 xEg$.

Analogously, say that $\succsim$ satisfies *π^*-event complementarity* if the above mentioned procedure is defined with respect to the one-shot preferences under full information $\succsim_{\pi^*}$, that is, for all $E \in \Sigma$, $f \in \mathcal{F}$, and $x \in X$, if $f \sim_E x$, then $f \succsim_{\pi^*} xEf$. The interpretation of its restriction on the one-shot preferences is similar.

The two types of information preferences of interest to this paper can be defined naturally as follows.

Definition 3. The preference relation $\succsim$ exhibits *an (intrinsic) aversion to partial information* if $(\pi_0, f) \succsim (\pi, f)$ for all $f \in \mathcal{F}$ and $\pi \in \Pi$. And the preference relation $\succsim$ exhibits *an (intrinsic) preference for full information* if $(\pi^*, f) \succsim (\pi, f)$ for all $f \in \mathcal{F}$ and $\pi \in \Pi$.

While these two definitions correspond to different patterns of information preferences in the real world, they are equivalent under time neutrality, that is, $(\pi^*, f) \sim (\pi_0, f)$ for all $f \in \mathcal{F}$.

Moreover, *attraction to partial information* can be defined analogously as $(\pi_0, f) \precsim (\pi, f)$ for all $f \in \mathcal{F}$ and $\pi \in \Pi$; if $\succsim$ displays both aversion and attraction to partial information, then $\succsim$ is *neutral to partial information*.

The next result shows that, in the family of partition-dependent recursive preferences, aversion to partial information is equivalent to π_0-event complementarity.

Proposition 2. *Suppose $\succsim$ satisfies Axioms 1–6. Then π_0-event complementarity holds if and only if $\succsim$ exhibits aversion to partial information.*

Proof. Suppose $\succsim$ satisfies π_0-event complementarity. Fix a finite partition $\pi = \{E_1, \ldots, E_n\}$ and an act f. For each $i = 1, \ldots, n$, let $x_i \in X$ be the E_i-conditional certainty equivalent of f; that is,, $x_i \sim_{E_i} f$.[7] Let $f^0 := f, f^1 = x_1 E_1 f^0, f^2 = x_2 E_2 f^1, \ldots, f^n = x_n E_n f_{n-1} = (x_1 E_1 x_2 E_2 \cdots x_{n-1} E_{n-1} x_n) := f_\pi$. Note that f_π is π-measurable. By Definition 1, $x_i \sim_{E_i} f^{i-1}$ for all $i = 1, \ldots, n$. Repeatedly applying event complementarity yields $(\pi_0, f^0) \succsim (\pi_0, f^1) \succsim \cdots \succsim (\pi_0, f^n)$. Furthermore, anticipated partition separability implies that $(\pi, f^0) \sim (\pi, f^1) \sim \cdots \sim (\pi, f^n) = (\pi, f_\pi)$. Putting these results together yield

$$(\pi, f) \sim (\pi, f_\pi) \sim \left(\pi^*, f_\pi\right) \sim (\pi_0, f_\pi) \precsim (\pi_0, f),$$

where the second indifference follows from indifference to redundant information and the third indifference is due to time neutrality. Since this is true for an arbitrary act f and partition π, $\succsim$ exhibits aversion to partial information.

The converse statement is proven by contradiction. Suppose not, so $\succsim$ exhibits aversion to partial information but there exists some π, $E \in \pi$, f, and x such that $f \sim_E x$, but $(\pi_0, xEf) \succ (\pi_0, f)$. Let $n_1, \ldots, n_m$ be labels for states in E^c, that is, $E^c = \{s_{n_1}, \ldots, s_{n_m}\}$. Then consider the finer partition $\pi' = \{E, \{s_{n_1}\}, \ldots, \{s_{n_m}\}\}$. Thus xEf is π'-measurable, and by indifference to redundant information and time neutrality, $(\pi', xEf) \sim (\pi^*, xEf) \sim (\pi_0, xEf)$. By anticipated partition separability, $(\pi', f) \sim (\pi', xEf)$. By transitivity, $(\pi', f) \sim (\pi_0, xEf) \succ (\pi_0, f)$. This violates partial information aversion, a contradiction. □

Obviously, if one defines event substitution as $f \sim_E x \Rightarrow f \precsim_0 xEf$, then a parallel equivalence between attraction to partial information and event substitution also holds.

Proposition 2 suggests that, within the partition-dependent recursive utility model, aversion to partial information and event complementarity are two sides of the same coin. Since the former notion is purely about the dynamics of intrinsic information preferences while the latter mostly restricts on the static preferences over uncertain acts, this exact link can be unexpected and may bring insights to other research that tests or applies the ambiguity-averse preferences in a dynamic environment. Due to this identified link, empirical tests of event complementarity or aversion to partial information can be simplified to testing only one of these two properties (whichever is more convenient); while applications of this model will need to respect the restriction imposed by the link; for instance, the joint assumption of ambiguity aversion and an intrinsic preference for partial information is too strong.

In a similar vein, event complementarity defined with respect to π^* also exactly characterizes a preference for full information, which holds even without the time neutrality axiom.

[7]For all nonempty event E, since $\succsim_E$ inherits strong monotonicity and continuity, there always exists a well-defined conditional certainty equivalent. And all the conditional preferences $\succsim_E$ coincide on X, due to stable constant-act preferences.

Corollary 1. *Suppose $\succcurlyeq$ satisfies Axioms 1–5. Then π^*-event complementarity holds if and only if $\succcurlyeq$ exhibits an intrinsic preference for full information.*

Proof. The proof is analogous to that of Proposition 2. Since Event Complementarity is now defined with respect to $\succcurlyeq_{\pi^*}$, one no longer needs to use Time Neutrality to show $(\pi^*, f_\pi) \sim (\pi_0, f_\pi)$ and $(\pi^*, xEf) \sim (\pi_0, xEf)$. $\square$

This subsection will end with comparisons between the notion of aversion to partial information and two related notions of intrinsic information preferences.

Dillenberger (2010) introduces a notion called preferences for one-shot resolution of uncertainties, which requires that a DM disprefers a two-stage compound lottery relative to its reduced one-shot lottery. While seem similar, aversion to partial information and preferences for one-shot resolution of uncertainties are conceptually different and defined in two distinct domains. In particular, the notion of aversion to partial information compares every partial information partition with the null information; while Dillenberger's notion of preferences for one-shot resolution of uncertainties compares every probabilistic information structure with the null or full information structure. Hence, when restricted to the case with only risk and a unique prior, aversion to partial information is a strictly weaker notion than preferences for one-shot resolution of uncertainties. However, only the domain considered here and the notion of aversion to partial information can accommodate ambiguity and multiple priors. Section 5.2 compares these two papers in further detail.

Grant et al. (1998) and Skiadas (1998) study a notion of intrinsic information aversion, which requires that a less Blackwell-informative partition is preferred to a more Blackwell-informative partition. In contrast, under time neutrality, the notion of aversion of partial information only compares the null information π_0 with some partial information π—it does not impose any restriction on the DM's intrinsic preferences over any two information partitions that are Blackwell-ranked. Hence, in the environment with only risk, Grant et al.'s (1998) intrinsic information aversion is a strictly stronger notion than aversion to partial information.

4. Ambiguity preferences

This section first embeds four well-known ambiguity preferences families to the general partition-dependent recursive model, and then examines the key relation between partial information preferences and ambiguity attitudes in each family.

Within this section, the static preference relation $\succcurlyeq_0$ is assumed to belong to one of the four families of ambiguity-averse preferences: MEU, multiplier preferences, variational preferences, and the second-order belief preferences. Throughout this section, I assume ambiguity aversion à la Schmeidler (1989). Hence, the functional $I_0 : u(X)^S \mapsto \mathbb{R}$ is quasi-concave; that is, $I_0(\alpha u(f) + (1-\alpha)u(g)) \geq \min\{I_0(u(f)), I_0(u(g))\}$.

For each family of ambiguity preferences, assume that the conditional preferences $\{\succcurlyeq_E\}_{E\in\Sigma}$ are updated from $\succcurlyeq_0$ via some intuitive generalized Bayesian updating rule, which satisfies consequentialism and is described below. The key restriction is that the

DM updates all the priors she considers plausible via the Bayes' rule. Under this restriction, updating per se should not bias the conditional preferences toward increasing or decreasing ambiguity. Any implied aversion to partial information must be a consequence of ambiguity aversion.

Each static ambiguity preference relation $\succcurlyeq_0$ is embedded into the (dynamic) partition-dependent recursive preference relation $\succcurlyeq$ as follows. First, preferences under null information and full information are identical to the static preference relation $\succcurlyeq_0$, which is represented by some utility function $V(f) = I_0(u(f))$. Second, a generalized Bayesian updating rule is assumed to map the static preference relation $\succcurlyeq_0$ to the conditional preferences $\{\succcurlyeq_E\}_{E\in\Sigma}$ with utility representation $V_E(f) = I_E(u(f_E))$ for all $E \in \Sigma$. Third, the *ex ante* preferences admit the utility representation $V(\pi, f)$ that aggregates the conditional utilities $\{V_E(f)\}_{E\in\Sigma}$ backward recursively according to Proposition 1. In this case, the *ex ante* preference relation $\succcurlyeq$ is said to be *recursively generated* by the static ambiguity preference relation $\succcurlyeq_0$.

To simplify notation, for each family only the conditional utility functions $\{V_E\}_{E\in\Sigma}$ are displayed. Also, the utility representation for the underlying static preferences is identified with the utility conditional on the event S; that is, $V_0 = I_0 \circ u$ is set to be equal to $V_S = I_S \circ u$, where S is treated as an event.

Finally, in this section I distinguish between *local* and *global* aversions to partial information. The preference relation $\succcurlyeq$ is *locally averse to partial information at some act* f, if $(\pi_0, f) \succcurlyeq (\pi, f)$ for all $\pi \in \Pi$. Say $\succcurlyeq$ is *globally averse to partial information* π if $\succcurlyeq$ is averse to some partial information π at all acts $f \in \mathcal{F}$. Moreover, say $\succcurlyeq$ is *globally averse to partial information* if it is averse to all partial information at all acts.

4.1 *MEU*

For all E, say $\succcurlyeq_E$ belongs to the MEU family (Gilboa and Schmeidler 1989) if it is represented by

$$V_E(f) = \min_{p_E\in P_E} \int_E u(f_E)\, dp_E,$$

where u is the same affine function identified in Proposition 1, and, for some closed and convex set of priors $P \subseteq \Delta(S)$, P_E is the prior-by-prior updated set of posteriors.

Following Epstein and Schneider (2003), for any convex and closed prior set P and any partition π, *the* π*-rectangular hull of* P is

$$\mathrm{rect}_\pi(P) = \left\{ p \in \Delta(S) : p = \sum_{E\in\pi} p^E(\cdot|E)q(E), \text{for all } p^E, q \in P \right\}.$$

Note that $\mathrm{rect}_\pi(P)$ is the largest set of probabilities that induces the same marginal and conditional probabilities with respect to π as the set P. The set P is called π*-rectangular* if $\mathrm{rect}_\pi(P) = P$. The next proposition summarizes the link between MEU preferences and aversion to partial information.

Proposition 3. *Suppose* $\succcurlyeq_0$ *has an MEU representation* (u, P) *and* $\succcurlyeq$ *is recursively generated by* $\succcurlyeq_0$ *with a prior-by-prior updating rule. Then* *(i)* $\succcurlyeq$ *exhibits global aversion to all partial information;*

(ii) $\succcurlyeq$ *is globally neutral to partition* π *if and only if* P *is* π*-rectangular.*

Note that in the two extreme cases when the prior set P is either a singleton or $\Delta(S)$, that is, the DM is either a subjective expected-utility (EU) maximizer or faces full ambiguity, the DM is intrinsically neutral to all information.

Another well-known ambiguity preference family is Choquet EU (Schmeidler 1989). Under ambiguity aversion, Choquet EU becomes a special case of MEU, and hence, Proposition 3 also applies to the Choquet EU model.

4.2 *Multiplier preferences*

For all E, say that $\succcurlyeq_E$ belongs to the multiplier preferences family (Hansen and Sargent 2001, Strzalecki 2011) if it can be represented by

$$V_E(f) = \min_{p_E \in \Delta(E)} \int_E u(f_E)\, dp_E + \theta R(p_E \parallel q_E),$$

where $\theta \in (0, +\infty]$ is a coefficient, $R(p_E \parallel q_E)$ is the relative entropy, and q_E is the Bayesian posterior of some reference belief $q \in \Delta(S)$.

Remark 1. Strzalecki (2011) shows that multiplier preferences satisfy Savage's surething principle, and, hence, $fEx \sim_0 x$ implies $f \sim_0 xEf$. Therefore, $\succcurlyeq_0$ satisfies event neutrality, and, by Proposition 2, the *ex ante* preferences $\succcurlyeq$ display global partial information neutrality.

4.3 *Variational preferences*

For all E, say that $\succcurlyeq_E$ admits a variational preferences representation (Maccheroni et al. 2006a) (u, c_E) if it can be represented by

$$V_E(f) = \min_{p_E \in \Delta(E)} \int_S u(f_E)\, dp_E + c_E(p_E),$$

where $u : X \mapsto \mathbb{R}$ with $u(X) = \mathbb{R}$, and the *conditional cost function* $c_E : \Delta(E) \to [0, +\infty]$ is given by

$$c_E(p_E) = \min_{\{p \in \Delta(S) : p(\cdot|E) = p_E\}} \frac{c(p)}{p(E)}. \tag{3}$$

Here, $c : \Delta(S) \to [0, +\infty]$ is a function that is convex, lower semicontinuous, and grounded, that is, $c(p) = 0$ for some p, and $p(\cdot|E)$ denotes the Bayesian posterior of probability p. The domain of c is $\mathrm{dom}(c) = \{p \in \Delta(S) : c(p) < +\infty\}$, which describes the set of probabilities considered plausible by the DM. The updating rule is given by equation (3), which updates the cost function for all the plausible priors in the domain of c. Li (2015) provided an axiomatic characterization of this updating rule.

For a given cost function c, say *an event* E *is unambiguous* if every probability in $\mathrm{dom}(c)$ assigns the same probability to E. Then, define the set of unambiguous event as $\Sigma^u = \{E \in \Sigma : p(E) = p'(E), \forall p, p' \in \mathrm{dom}(c)\}$. Following Ghirardato and Marinacci (2002), say *an act f is unambiguous* if it is Σ^u-measurable.

Example 1 below indicates that a DM with recursively generated variational preferences may not always display aversion to partial information. Nevertheless, aversion to partial information still holds at acts that are unambiguous, confirming the observation from the lead example (Section 3.2).

Example 1 (Information preferences in variational preferences). Suppose $S = \{s_1, s_2, s_3\}$. Let $X = \mathbb{R}$ and $u(x) = x$. Consider the partition $\pi = \{\{s_1, s_2\}, \{s_3\}\}$ and the event $E = \{s_1, s_2\}$. Generalizing the lead example, let the domain of c be a set $P = \{(\frac{1}{3}, \theta, \frac{2}{3} - \theta) : \theta \in [0, \frac{2}{3}]\}$.

Consider some DM with variational representation (u, c), where the cost function is as follows:

$$c(p) = \begin{cases} 0 & \text{if } p = p_{1/3}, \\ 3\left|\theta - \dfrac{1}{3}\right| & \text{if } p_\theta \in P \backslash \{p_{1/3}\}, \\ +\infty, & \text{otherwise.} \end{cases} \tag{4}$$

Note that c is convex, lower semicontinuous, and grounded.

In this case, Table 1 provides the DM's computed utilities at four acts f_1, f_2, f_3, and f_4 when not anticipating information and anticipating partial information π. Formulas for these computations are provided in Appendix A.5. Clearly, for variational preferences, it is unclear whether a DM is averse or attracted to partial information.[8]

For the cost function given by (4), observe that events $\{s_1\}$, $\{s_2, s_3\}$ are unambiguous while events $\{s_1, s_2\}$ and $\{s_3\}$ are ambiguous. Hence, Table 1 suggests that partial information aversion holds at f_3 and f_4, which are unambiguous acts; while the information preferences pattern is violated at f_1 and f_2, which are unambiguous acts. $\Diamond$

The next proposition confirms that this observation holds in general.

	$V_E(f)$	$V(\pi, f)$	$V(\pi_0, f)$	Information preferences
$f_1 = (9, 0, 9)$	4	5	4	partial information loving
$f_2 = (0, 9, 0)$	3	2	1	partial information loving
$f_3 = (0, 9, 9)$	3	4	6	partial information aversion
$f_4 = (9, 0, 0)$	4	$\frac{7}{3}$	3	partial information aversion

Table 1. *Ex ante* utilities and information preferences for the variational preferences model.

[8]An informal intuition for why ambiguity aversion and partial information loving may coexist in variational preferences is as follows. The variational utility is often interpreted as the DM's payoff from playing a zero-sum game against a malevolent Nature, who picks a probability p that minimizes the DM's expected utility at p plus a transfer $c(p)$. With partial information $\{\{s_1, s_2\}, \{s_3\}\}$, Nature picks probabilities three times, which lowers the DM's *ex ante* utility compared to the no information case, yet it also makes multiple transfers. Partial information loving occurs when the latter effect dominates the former.

Proposition 4. *Suppose $\succsim_0$ has a variational representation (u, c) and $\succsim$ is recursively generated by $\succsim_0$ with the updating rule given by equation (3). Then $\succsim$ exhibits local aversion to partial information at all unambiguous acts.*

A larger set of acts at which local partial information aversion holds is $\hat{\mathcal{F}}^{\mathrm{VP}} = \{f \in \mathcal{F} : M(f) \cap M(x_0) \neq \varnothing\}$, where $M(f) = \arg\min_{p \in \Delta}[\int_S u(f)\, dp + c(p)]$ and x_0 is a constant act such that $u(x_0) = 0$ and $M(x_0) = c^{-1}(0)$. It consists of every act f at which the set of effective minimizing probabilities has nonempty overlap with the set of minimizing probabilities at the constant act x_0. It is straightforward to check that $\hat{\mathcal{F}}^{\mathrm{VP}}$ contains all the unambiguous acts, and, in the MEU special case, $\hat{\mathcal{F}}^{\mathrm{MEU}} = \mathcal{F}$. The proof is analogous to that of Proposition 4 and omitted.

Finally, global aversion to a particular information partition can be characterized by a variant of the familiar "no gain" condition proposed by Maccheroni et al. (2006b). For a given partition π, the "no gain" condition is a recursive equation between the cost functions c and $\{c_E\}_{E \in \pi}$:

$$c(p) = \min_{\{q \in \Delta(S) : q = p \text{ on } \pi\}} \left[\sum_{E \in \pi} p(E) c_E(p_E) + c(q)\right], \quad \forall p \in \Delta(S). \tag{5}$$

Here, p_E is the Bayesian posterior of p at E. This condition generalizes the rectangularity condition in the MEU model to the variational preferences model.

The next proposition generalizes part (2) of Proposition 3 to the variational preferences case.

Proposition 5. *Suppose $\succsim$ is recursively generated by $\succsim_0$ with a variational representation. Then $\succsim$ is globally neutral to partial information π if and only if the "no-gain" condition (5) holds at π.*

Observe that if the "$=$" relation in equation (5) is replaced by "$\geq$", then the resulting inequality is necessary and sufficient for $\succsim$ to exhibit global aversion to partial information π. The proof is analogous to that of Proposition 5, and hence omitted.

4.4 *Second-order belief preferences*

For all E, say that $\succsim_E$ belongs to the second-order belief preferences family (Klibanoff et al. 2005, Seo 2009), if it is represented by

$$V_E(f) = \phi^{-1}\left(\int_{\Delta(S)} \phi\left[\int_E u(f_E)\, dp_E\right] d\mu_E(p)\right),$$

where $u : X \mapsto \mathbb{R}$ is the vNM index, $\phi : u(X) \mapsto \mathbb{R}$ is an increasing function, p_E is the Baysian posterior of probability $p \in \Delta(S)$, and μ_E is the Bayesian posterior of some second-order belief $\mu \in \Delta(\Delta(S))$; that is,

$$\mu_E(p) = \frac{\mu(p) p(E)}{\displaystyle\int_{\Delta(S)} p'(E)\, d\mu(p')}. \tag{6}$$

Note that ϕ captures ambiguity attitude; when ϕ is concave, $\succsim_E$ displays ambiguity aversion.

If ϕ is not linear, say that *an event E is unambiguous* if almost all the beliefs p in the support of the second-order belief μ assign the same probability to E. Let the set of unambiguous events be $\Sigma^u = \{E \in \Sigma : \exists$ some constant $\gamma \in \mathbb{R}$ such that $p(E) = \gamma$ for μ-almost all $p\}$. Say *an act f is unambiguous* if it is measurable with respect to Σ^u (Ghirardato and Marinacci 2002, Klibanoff et al. 2005).

Similar to the variational preferences case, an ambiguity-averse DM with second-order belief preferences may display both partial information aversion and loving, as illustrated by the following example.

Example 2 (Information preferences in second-order belief preferences). Consider $S = \{s_1, s_2, s_3\}$. Suppose there are two plausible probabilities $p_{\frac{1}{3}} = (\frac{1}{3}, \frac{2}{9}, \frac{4}{9})$ and $p_{\frac{2}{3}} = (\frac{1}{3}, \frac{4}{9}, \frac{2}{9})$ that are indexed by $\Theta = \{\frac{1}{3}, \frac{2}{3}\}$, and the second-order belief μ assigns equal weights to these two probabilities. Assume the DM has a vNM utility index $u(x) = x \in \mathbb{R}_{++}$ and an ambiguity index $\phi(y) = \ln(y)$. Consider the partition $\pi = \{\{s_1, s_2\}, \{s_3\}\}$ and the event $E = \{s_1, s_2\}$. By equation (6), the second-order posterior is $\mu_E(p_{\frac{1}{3}}) = \frac{5}{12}$. Table 2 summarizes the computed *ex ante* utility at acts f_1, f_2, f_3, and f_4 when anticipating null information π_0 and partial information π.

Similar to Example 1, for the second-order belief μ given here events $\{s_1\}$, $\{s_2, s_3\}$ are unambiguous while events $\{s_1, s_2\}$ and $\{s_3\}$ are ambiguous. Moreover, partial information aversion holds at the unambiguous acts f_3 and f_4, but fails at the ambiguous acts f_1 and f_2. $\Diamond$

The next proposition formalizes the above observation.

Proposition 6. *Suppose $\succsim_0$ are second-order belief preferences and $\succsim$ is recursively generated by $\succsim_0$ via Bayesian updating of the second-order belief. If $\succsim_0$ is strictly ambiguity averse, then $\succsim$ exhibits local aversion to partial information at all unambiguous acts.*

Alternatively, when ϕ is convex and the DM is ambiguity loving, the *ex ante* preferences $\succsim$ exhibit local partial information loving at all unambiguous acts.

A larger set of acts at which partial information aversion holds is $\hat{\mathcal{F}}^{\text{SOB}} = \{f \in \mathcal{F} : V(\pi_0, f) = \int_{\Delta(S)} \int_S u(f)\, dp\, d\mu(p)\}$. It consists all acts at which $\succsim_0$ displays local smooth ambiguity neutrality (Klibanoff et al. 2005, Definition 4). The proof is analogous to that of Proposition 6 and omitted.

	$V(\pi_0, f)$	$V(\pi, f)$	Information preferences
$f_1 = (0, 1, 0)$	0.314	0.324	partial information loving
$f_2 = (1, 0, 1)$	0.657	0.660	partial information loving
$f_3 = (1, 0, 0)$	0.333	0.324	partial information aversion
$f_4 = (0, 1, 1)$	0.667	0.659	partial information aversion

Table 2. *Ex ante* utilities and information preferences for the second-order belief model.

Hence, while event complementarity is the exact notion that characterizes global aversion to partial information in the general partition-dependent recursive utility model, it only overlaps partially with the traditional notion of ambiguity aversion à la Schmeidler (1989) in the four preference families. If Savage's sure-thing principle holds, any strict partial information preference is ruled out, as shown by the multiplier preferences case. And in the most popular MEU (or Choquet EU) model, ambiguity aversion does imply global partial information aversion. Yet aversion to partial information does not hold everywhere in the more general cases of variational preferences and second-order belief preferences. This suggests that event complementarity, the exact notion of ambiguity aversion that characterizes global aversion to partial information, is not satisfied by all ambiguity models. Nevertheless, for the four ambiguity families considered, (weak) partial information aversion holds locally at all unambiguous acts, which confirms the intuition gained from the leading Ellsberg urn example.

5. Related literature

5.1 *Dynamic ambiguity preferences*

This paper is related to the literature of dynamic decision-making under ambiguity. Siniscalchi (2011) summarizes a "folk theorem" in this literature—a modeler necessarily faces a trade-off among four criteria: (i) dynamic consistency; (ii) consequentialism; (iii) unrestricted (static) ambiguity preferences; and (iv) reduction. The literature have taken various approaches that could be viewed as relaxing one of the four criteria.

Epstein and Schneider (2003) assume reduction and axiomatize recursive preferences over adapted consumption processes where all conditional preferences are MEU, and find that dynamic consistency implies that the prior belief set has to satisfy a "rectangularity" restriction. Later work axiomatizes recursive preferences for other static ambiguity preferences and finds similar restrictions (Maccheroni et al. 2006b, Klibanoff et al. 2009) on static ambiguity preferences.

Siniscalchi (2011) showed that, within a given filtration, dynamic consistency implies Savage's sure-thing principle and Bayesian updating. Together with reduction, dynamic consistency rules out modal Ellsberg preferences and thus ambiguity. To allow for ambiguity, Siniscalchi studies preferences over a richer domain of decision trees, and relaxes dynamic consistency by introducing a weaker axiom called "sophistication." Together with auxiliary axioms, he proposes a general framework in which preferences can be dynamically inconsistent, and the DM addresses these inconsistencies through a Strotz-type solution concept called "consistent planning."

Hanany and Klibanoff (2007, 2009) relaxed consequentialism and characterize weakly dynamically consistent updating rules for ambiguity preferences.

The noted tension between dynamic consistency and ambiguity relies on reduction. However, experimental evidence suggests that reduction is often violated even in environments with objective risk. For example, Halevy (2007) found evidence for nonreduction of compound lotteries and ambiguity aversion, as well as a positive association between the two. In a dynamic portfolio choice experiment, Bellemare et al. (2005) found that when a DM is committed to some *ex ante* portfolio, higher frequency of information feedback leads to lower *ex ante* willingness to invest in risky assets.

This paper takes the approach of relaxing reduction while accommodating the other three criteria. In the general partition-dependent recursive preferences (Proposition 1) and the embedded recursive ambiguity models (Section 4), dynamic consistency and consequentialism are always maintained. Although the set of axioms, especially anticipated partition separability, still requires that the *dynamic* ambiguity preferences are separable with respect to the anticipated partition, they impose no restriction on the *static* ambiguity preferences. For instance, in the special case of MEU preferences, the set of priors representing the *ex ante* preferences must satisfy the well-known "rectangularity" condition with respect to the anticipated information partition. If reduction is assumed, then a temporal act is considered equivalent to its induced mapping from states to consequences, and "rectangularity" is imposed on both the static and the dynamic preferences. This corresponds to the case studied in Epstein and Schneider (2003). Alternatively, the approach taken here drops reduction, and "rectangularity" is imposed only on the dynamic preferences while leaving the static preferences unrestricted.

5.2 *Preferences for temporal resolution of uncertainties*

This paper also relates to the literature that relaxes reduction and studies intrinsic preferences for temporal resolution of uncertainties. This literature is pioneered by Kreps and Porteus (1978), who introduce a domain of objective temporal lotteries to study preferences for the timing of resolution of uncertainties. Epstein and Zin (1989) subsequently applied the Kreps–Porteus preferences to study asset pricing. Grant et al. (1998) linked time preferences to intrinsic information preferences. While these earlier literature directly model information preferences, more recent literature suggest information preferences can emerge endogenously as a consequence of how preferences deviate from the canonical Bayesian EU assumptions. For instance, Palacios-Huerta (1999) showed that disappointment aversion (Gul 1991) could explain preferences for one-shot rather than sequential resolution of uncertainties. Kőszegi and Rabin (2009) found that reference-dependent utility implies preferences for getting information "clumped together rather than apart." Among all, two papers—Strzalecki (2013) and Dillenberger (2010)—are the most relevant to my paper, and hence, in what follows I will provide a detailed comparison with these two papers.

Comparison with Strzalecki (2013) This paper is closely related to Strzalecki (2013). Both papers study the relation between recursive ambiguity preferences and intrinsic preferences for temporal/sequential resolution of uncertainties, and both consider recursive ambiguity preferences in a domain with purely subjective uncertainties. Strzalecki (2013) showed that, even with standard discounting, most models of ambiguity aversion display some preferences with regard to the timing of resolution of uncertainties, with the notable exception of the MEU model. This paper also shares the message that incorporating ambiguity aversion can imply intrinsic attitudes toward temporal/sequential resolution of uncertainties.

The two papers differ in two key aspects.

First, the types of information preferences considered are different. Strzalecki (2013) considers a dynamic model with intermediate prizes, and the state space in each period is the same, with a fixed information structure. Hence, the focus there is a preference for earlier resolution of uncertainties. This paper considers a model with only terminal prizes, and more flexible information structures. Hence, this paper focuses on an aversion to partial resolution of uncertainties, which, if time neutrality is assumed, is orthogonal to the information preferences examined by Strzalecki (2013). More precisely, Strzalecki (2013), adapted to a two-period framework, looks at the state space $\Omega = S \times S$ and a particular information determined by a fixed filtration $\pi = \{\{s\} \times S : s \in S\}$. Any measurable mapping $f : S \mapsto X$ determines two acts $f_1, f_2 : \Omega \mapsto X$, where $f_1(s, s') = f(s)$ for all $s' \in S$ is π measurable and $f_2(s', s) = f(s)$ for all $s' \in S$ is constant on π_0. The notion of a preference for earlier resolution of uncertainties can be translated as $f_1 \succsim_\pi f_2$. By switching the order of s and s' in f_2, a preference for earlier resolution of uncertainties can be considered as the DM prefers π to π_0 at act f_1. In contrast, my paper compares π_0 with arbitrary partitions of the state space.

Second, the role of updating rules and beliefs differs in two papers. In order to focus on the time attitudes, Strzalecki (2013) looked at a setting with identical and indistinguishable (IID) ambiguity in each period—that is, beliefs about uncertainties in the period state space are "constant" over time. In contrast, my paper focuses on the effect of belief updating as new information arrives while, by imposing time neutrality, abstracts away from the time effect. In the simplified two-period model with the fixed filtration $\pi = \{\{s\} \times S : s \in S\}$, for any act $f : S \times S \mapsto X \times X$ adapted to π, Strzalecki (2013) considered the following utility function conditional on event $\{s_1\} \times S$:

$$V_1(s_1, s_2, f) = u(f_1(s_1, s_2) + \beta I\big(u\big(f_2(s_1, s_2)\big)\big),$$

where $\beta \in (0, 1)$ is the discount factor, and I is the *time invariant* (ambiguity) aggregator at the event $\{s_1\} \times S$. My model abstracts away from intermediate prizes and time discounting but allows for a *time varying* (ambiguity) aggregator, that is, the same conditional utility considered by this paper can be expressed as

$$V_1(s_1, s_2, f) = I_{s_1}\big(u\big(f_2(s_1, s_2)\big)\big).$$

Comparison with Dillenberger (2010) Dillenberger (2010) considered a domain of objective compound lotteries proposed by Segal (1990), and studied preferences for one-shot over gradual resolution of uncertainties. In that paper, Dillenberger identified a link between preferences for one-shot resolution of uncertainties and Allais-type behaviors. It shares a few similarities with this paper. At a conceptual level, both papers (and also Palacios-Huerta 1999) rationalized preferences for one-shot over gradual resolution of uncertainties by preference models that deviate from the EU model. At a technical level, both papers use an appropriate framework of recursive preferences with time neutrality, and hence the proof technique in my Proposition 2 bears a similarity to that of Proposition 1 in Dillenberger (2010).

The key difference is that Dillenberger (2010) focused on a domain of objective lotteries with known probabilities, while this paper focuses on a domain of subjective uncertainties where probabilities may or may not be known. This domain difference has two crucial implications. First, the notions of reduction as well as the classes of information preferences considered in the two papers are different. In Dillenberger (2010), a temporal lottery is a two-stage compound lottery, and thus reduction is defined as identifying a compound lottery with the simple lottery that assigns the same total probability to each outcome. In this paper, a temporal act is a mapping from states to outcomes coupled with a two-stage event tree, and a DM can reduce a temporal act to a one-shot act by considering its induced mapping from states to consequences. As a result, aversion to partial information is conceptually different from preferences for one-shot resolution of uncertainties. Second, the key axioms for one-shot preferences $\succsim_0$ in the two papers are quite different. Dillenberger's key axiom, negative certainty independence, says that for all $p, q, \delta_x \in \Delta X$ and $\lambda \in [0, 1]$,

$$p \succsim_0 \delta_x \quad \Rightarrow \quad \lambda p + (1-\lambda)q \succsim_0 \lambda\delta_x + (1-\lambda)q.$$

Intuitively, if a certain outcome x is not better than lottery p, then mixing x with another lottery *by a convex combination*, which eliminates its certainty appeal, will not result in its mixture being more attractive than the corresponding mixture of p. Negative certainty independent preferences are motivated by Allais-type violations of the independence axiom. For a more formal comparison, my key axiom, event complementarity, can be equivalently written as follows (under continuity, monotonicity, and existence of conditional certainty equivalent): for all $f, g \in \mathcal{F}$, $x \in X$, and $E \in \Sigma$,

$$fEx \succsim_0 x \quad \Rightarrow \quad fEg \succsim_0 xEg.^{9}$$

In event complementarity "mixing" a certain outcome x with another act g is by composing them into xEg, and the axiom captures Ellsberg-type violations of Savage's classic sure-thing principle. Moreover, in the static case, Dillenberger's negative certainty independent preferences only overlap with the rank-dependent utility class in the EU family; while the (convex) Choquet EU family, usually considered as the "subjective analogue" of rank-dependent utility family, is a special case of the static preferences satisfying the event complementarity axiom.

Appendix

A.1 *Proof of Lemma 1*

Consequentialism. Let $fEg = f'$ and $fEh = g'$. Then $f'Ef = g'Ef = f$. Therefore, $(\pi, f'Ef) \sim (\pi, g'Ef)$ for $\pi = \{E, E^c\}$. By Definition 1, $f' \sim_E g'$.

Dynamic consistency. Let $\pi = \{E_1, \dots, E_n\}$ and $f \equiv f^0$. Then by Definition 1, $f \succsim_{E_1} g \Rightarrow f \succsim_\pi gE_1f$. Let $gE_1f \equiv f^1$. Then by consequentialism $f \succsim_{E_2} g \Rightarrow f^1 \succsim_{E_2} g$, and by

[9]The stated axiom obviously implies event complementarity. To see the reverse, if $fEx \succsim_0 x$ then by continuity, monotonicity, and existence of conditional certainty equivalent there is some $y \succsim_0 x$ and $fEy \sim_0 y$. Let $h = fEg$, then $hEy = fEy$. By event complementarity, $fEg = h \succsim_0 yEh = yEg \succsim_0 xEg$.

Definition 1 $f^1 \succsim_\pi gE_2 f^1$. Let $gE_2 f^1 \equiv f^2$. Repeat this for all $E_i \in \pi$ $(i = 1, \dots, n)$, let $f^i = gE_i f^{i-1}$ and I have $f^{i-1} \succsim_\pi f^i$. By transitivity, $f = f^0 \succsim_\pi f^n = g$. □

A.2 *Proof of Proposition 1*

First, I verify continuity of $\succsim$ on $\Pi \times \mathcal{F}$.

Lemma 2. *If $\succsim$ satisfies stable constant-act preferences and for every π the preference $\succsim_\pi$ is continuous on $\mathcal{F}$, then $\succsim$ is continuous on $\Pi \times \mathcal{F}$.*

Proof. Fix (π, f). I want to show that sets $U = \{(\pi', g) : (\pi', g) \succsim (\pi, f)\}$ and $L = \{(\pi', g) : (\pi, f) \succsim (\pi', g)\}$ are closed. Let $\{(\pi_n', g_n)\}$ be a convergent sequence in the set U, with limit (π', g). It suffices to show that (π', g) is also in U. Suppose not, then $(\pi, f) \succ (\pi', g')$. Since $\pi_n' \to \pi'$ in the discrete topology on Π, there exists some N such that for all $n > N$, $\pi_n' = \pi'$. Continuity of $\succsim_\pi$ and convexity of X ensure that there exists a constant act x_f such that $(\pi, f) \sim (\pi, x_f) \sim (\pi', x_f)$, where the last statement follows from stable constant-act preferences. If $(\pi', x_f) \sim (\pi, f) \succ (\pi', g)$, then by continuity of $\succsim_{\pi'}$, there exists M $(> N)$ such that for all $n > M$, $(\pi', x_f) \succ (\pi', g_n) = (\pi_n', g_n)$. So $(\pi, f) \succ (\pi_n', g_n)$ for sufficiently large n, a contradiction to the assumption $\{(\pi_n', g_n)\} \subseteq U$. Following similar arguments, I can show set L is closed. □

Second, I prove the sufficiency of Axioms 1 to 6 in Proposition 1.

Constant-act preferences. Fix some arbitrary partition π, then the restriction of $\succsim$ on $\{\pi\} \times X$ is a continuous and independent preference relation on a mixture space X, thus by Herstein and Milnor (1953)'s mixture space theorem it can be represented by an affine function $u : X \mapsto \mathbb{R}$. By stable constant-act preferences, $(\pi, x) \succsim (\pi, y) \Leftrightarrow (\pi', x) \succsim (\pi', y)$ for all $\pi' \in \Pi$, thus u also represents the restriction of $\succsim$ on $\{\pi'\} \times X$. And $u(X) \subseteq \mathbb{R}$ is a real interval since X is connected.

Conditional preferences. Fix arbitrary partition π and event $E \in \pi$. Recall that $f \succsim_E g \Leftrightarrow (\pi, fEh) \succsim (\pi, gEh)$ for some $h \in \mathcal{F}$. The elicited conditional preferences $\succsim_E$ inherits continuity from that of $\succsim_\pi$. By definition of $\succsim_E$ and anticipated partition separability, $\succsim_E$ satisfies consequentialism, that is, $fEh \sim_E fEh'$ for all $f, h, h' \in \mathcal{F}$, and thus without loss of generality I can only look at the restriction of $\succsim_E$ on $\mathcal{F}_E$. Since $\succsim_\pi$ is strongly monotone on $\mathcal{F}$, $\succsim_E$ is strongly monotone on $\mathcal{F}_E$. Finally, $\succsim_E$ agrees with $\succsim_\pi$ on X, and thus can be represented by u. To see this, for all $x, y \in X$, by definition and anticipated partition separability, $x \succsim_E y \Leftrightarrow (\pi, x) \succsim (\pi, yEx)$; by strong monotonicity of $\succsim_\pi$, $(\pi, x) \succsim (\pi, yEx) \Leftrightarrow (\pi, x) \succsim (\pi, y)$.

Let $\succsim_E^*$ be the preference relation on $(u(X))^E$ induced by $\succsim_E$: $\xi_E \succsim_E^* \phi_E$ if and only if $f \succsim_E g$ for some $f, g \in \mathcal{F}$ such that $u(f_E) = \xi_E$ and $u(g_E) = \phi_E$. Then $\succsim_E^*$ is continuous and strongly monotone on $\succsim_E$. Thus there exists a continuous and strongly monotone functional $I_E : (u(X))^E \mapsto \mathbb{R}$ that represents $\succsim_E^*$. By constant-act independence, all $\succsim_E$ and $\succsim_\pi$ agree on X. Hence, it is without loss to normalize them to $I_E(\bar{k}) = k$ for all $k \in u(X)$. Define conditional utility $V_E : \mathcal{F} \mapsto \mathbb{R}$ by $V_E(f) = I_E(u(f_E))$. Then

$$f \succsim_E g \quad \Leftrightarrow \quad u(f_E) \succsim_E^* u(g_E) \quad \Leftrightarrow \quad I_E\big(u(f_E)\big) \geq I_E\big(u(g_E)\big) \quad \Leftrightarrow \quad V_E(f) \geq V_E(g).$$

Moreover, for all $f \in \mathcal{F}$ and $E \in \Sigma$, denote by $x_{f,E}$ the E-conditional certainty equivalent of f; that is, $x_{f,E} \in u^{-1}(I_E(u(f_E)))$. By continuity and strong monotonicity of $\succsim_E$, $x_{f,E}$ exists and is essentially unique; that is, $y \in u^{-1}(I_E(u(f_E))$ implies $y \sim x_{f,E}$.

Ex ante preferences. Fix any $\pi \in \Pi$ and $f \in \mathcal{F}$, I can find conditional certainty equivalent $x_{f,E_i} \sim_{E_i} f$ for all $E_i \in \pi$. Denote $f^0 = f$, $f^1 = x_{f,E_1}E_1 f^0$, $f^2 = x_{f,E_2}E_2 f^1, \dots,$ and $f^n = x_{f,E_n}E_n f^{n-1} = f_\pi$, where

$$f_\pi = \begin{pmatrix} x_{f,E_1} & s \in E_1 \\ x_{f,E_2} & s \in E_2 \\ \vdots & \vdots \\ x_{f,E_n} & s \in E_n \end{pmatrix}$$

is the π-conditional certainty equivalent of f. Then, by Definition 1 and anticipated partition separability, $(\pi, f) = (\pi, f^0) \sim (\pi, f^1) \sim \cdots \sim (\pi, f^n) = (\pi, f_\pi)$. Moreover, $(\pi, f_\pi) \sim (\pi^*, f_\pi) \sim (\pi_0, f_\pi)$, where the first indifference relation is by indifference to redundant information and the second by time neutrality. By transitivity of $\succsim$, $(\pi, f) \sim (\pi_0, f_\pi)$. Thus for all $\pi = \{E_1, \dots, E_n\}$, $\pi' = \{E_1', \dots, E_m'\}$ and $f, g \in \mathcal{F}$, let f_π and $g_{\pi'}$ be the partition conditional certainty equivalents constructed as above. Then

$$\begin{aligned} (\pi, f) \succsim (\pi', g) \quad &\iff \quad (\pi_0, f_\pi) \succsim (\pi_0, g_{\pi'}) \\ &\iff \quad f_\pi \succsim_0 g_{\pi'} \\ &\iff \quad I_0\big(u(f_\pi)\big) \geq I_0\big(u(g_{\pi'})\big) \\ &\iff \quad I_0 \begin{pmatrix} V_{E_1}(f) & E_1 \\ V_{E_2}(f) & E_2 \\ \cdots & \\ V_{E_n}(f) & E_n \end{pmatrix} \geq I_0 \begin{pmatrix} V_{E_1'}(g) & E_1' \\ V_{E_2'}(f) & E_2' \\ \cdots & \\ V_{E_m'}(f) & E_m' \end{pmatrix}. \end{aligned}$$

Third, I verify the necessity of Axioms 1 to 6. Suppose $\succsim$ is represented by $V : \Pi \times \mathcal{F} \mapsto \mathbb{R}$ as stated. The only axiom that is not straightforward to verify is anticipated partition separability. For all partitions $\pi = \{E, F_1, \dots, F_m\}$ and $\pi' = \{E, G_1, \dots, G_n\}$ that both contain E and all acts $f, g, h, h' \in \mathcal{F}$,

$$\begin{aligned} (\pi, fEh) \succsim (\pi, gEh) \quad &\Leftrightarrow \quad I_0 \begin{pmatrix} V_E(f_E) & E \\ V_{F_1}(h_{F_1}) & F_1 \\ \cdots & \\ V_{F_m}(h_{F_m}) & F_m \end{pmatrix} \geq I_0 \begin{pmatrix} V_E(g_E) & E \\ V_{F_1}(h_{F_1}) & F_1 \\ \cdots & \\ V_{F_m}(h_{F_m}) & F_m \end{pmatrix} \\ &\Leftrightarrow \quad V_E(f_E) \geq V_E(g_E) \\ &\Leftrightarrow \quad I_0 \begin{pmatrix} V_E(f_E) & E \\ V_{G_1}(h'_{G_1}) & G_1 \\ \cdots & \\ V_{G_n}(h'_{G_n}) & G_n \end{pmatrix} \geq I_0 \begin{pmatrix} V_E(g_E) & E \\ V_{G_1}(h'_{G_1}) & G_1 \\ \cdots & \\ V_{G_n}(h'_{G_n}) & G_n \end{pmatrix} \\ &\Leftrightarrow \quad \big(\pi', fEh'\big) \succsim \big(\pi', gEh'\big), \end{aligned}$$

where the second and third $\Leftrightarrow$ follow from strong monotonicity of I_0.

Finally, suppose both (u, I_E, I_0) and (u', I'_E, I'_0) represent $\succcurlyeq_E$ and $\succcurlyeq_0$. Since both u and u' are affine representations of $\succcurlyeq_E$ on X, by the mixture space theorem (Herstein and Milnor 1953), $u' = au + b$ for some $a > 0$ and $b \in \mathbb{R}$. For all f, let $x_{f,E}$ be a constant act such that $f \sim_E x_{f,E}$. Then

$$I_E\big(u(f_E)\big) = u(x_{f,E}),$$
$$I'_E\big(u'(f_E)\big) = u'(x_{f,E}).$$

Substituting $u' = au + b$ yields

$$I'_E\big(u'(f_E)\big) = I'_E\big(au(f_E) + \bar{b}\big) = u'(x_{f,E}) = au(x_{f,E}) + b,$$

and thus $I'_E(au(f_E) + \bar{b}) = aI_E(u(f_E)) + b$. Since f is arbitrary, for all $\xi \in (u'(X))^{|E|}$, $I'_E(\xi) = aI_E((\xi - \bar{b})/a) + b$. By similar argument, $I'_0(\xi) = aI_0((\xi - \bar{b})/a) + b$ for all $\xi \in (u'(X))^{|S|}$. $\square$

A.3 *Lemma 3*

Say that the function $I_0 : u(X)^{|S|} \mapsto \mathbb{R}$ satisfies *vertical invariance* if $I_0(\xi + \bar{k}) = I_0(\xi) + k$ for all $\xi, \xi + \bar{k} \in u(X)^S$. We impose an updating axiom for $\succcurlyeq_E$, called *Conditional Certainty Equivalent Consistency (CCEC)*: For all f, x, and E, let $\pi_E = \{E, E^c\}$, then $fEx \sim_0 x \Leftrightarrow f \sim_E x$. It is straightforward to see that under continuity, strong monotonicity, and vertical invariance, there always exists a well-defined conditional certainty equivalent that satisfies CCEC. Hence, under CCEC, event complementarity can be rewritten as a property of $\succcurlyeq_0$ alone: For all E and f, if $fEx \sim_0 x$ for some x, then $f \succcurlyeq_0 xEf$ (Li 2015).

The following lemma provides a useful sufficient condition for $\succcurlyeq_0$-event complementarity.

Lemma 3. *Suppose $\succcurlyeq_0$ is represented by (u, I_0) where I_0 is vertically invariant and $\succcurlyeq_E$ is updated from $\succcurlyeq_0$ by CCEC. If I_0 is superadditive, then $\succcurlyeq_0$ satisfies event complementarity.*

Proof. Fix f, x, E such that $fEx \sim_0 x$. By vertical invariance of I_0,

$$I_0\big(u \circ (xEf)\big) = I_0\big(0E(u \circ f - u \circ x)\big) + u(x).$$

Since $fEx \sim_0 x$, $I_0(u \circ (fEx)) = u(x)$, thus

$$I_0\big(u \circ (xEf)\big) = I_0\big(0E(u \circ f - u \circ x)\big) + I_0\big(u \circ (fEx)\big).$$

Thus $\succcurlyeq_0$ satisfies event complementarity, that is, for all f, E, x such that $fEx \sim_0 x$,

$$\begin{aligned} f \succcurlyeq_0 xEf \quad &\Leftrightarrow \quad I_0(u \circ f) \geq I_0(u \circ xEf) \\ &\Leftrightarrow \quad I_0(u \circ f) \geq I_0(u \circ fEx) + I_0\big(0E(u \circ f - u \circ x)\big). \end{aligned}$$

Note that the last inequality holds whenever I_0 is a supperadditive function: $I_0(\xi + \eta) \geq I_0(\xi) + I_0(\eta)$ for all $\xi, \eta \in u(X)^{|S|}$. $\square$

A.4 *Proof of Proposition 3*

For part (1), note that the MEU functional satisfies vertical invariance and the prior-by-prior updating rule satisfies CCEC (Pires 2002). Hence, by Proposition 2, it suffices to show that $\succsim_0$ satisfies event complementarity. By Lemma 3.3 in Gilboa and Schmeidler (1989), I_0 is superadditive. By Lemma 3, event complementarity follows.

For part (2), if $\succsim_0$ has an MEU representation (u, P) and $\succsim$ is recursively generated by $\succsim_0$, then $\succsim$ can be represented by

$$\begin{aligned} V(\pi, f) &= \min_{p\in P} \sum_{i=1}^{n} \left[\min_{p^i\in P} \int u(f)\, dp^i(\cdot|E_i) \right] p(E_i) \\ &= \min_{p\in P} \min_{p^i\in P} \sum_{i=1}^{n} \left[\int u(f)\, dp^i(\cdot|E_i) \right] p(E_i) \\ &= \min_{p'\in \mathrm{rect}_\pi(P)} \int u(f)\, dp'. \end{aligned}$$

For the "only if" direction, suppose P is not π-rectangular, so there exists $q \in \mathrm{rect}_\pi(P)\backslash P$. Since P is convex and compact, by the strict separating hyperplane theorem, there exists a nonzero and bounded vector $\xi \in u(X)^{|S|}$ such that

$$\int \xi\, dq < \int \xi\, dp, \forall p \in P.$$

Without loss of generality, let $0 \in \mathrm{int}(u(X))$. There exists $f \in \mathcal{F}$ such that $u(f) = \alpha\xi$, for some $\alpha > 0$. Thus without loss of generality, I can replace ξ by $u(f)$ in above inequality. By compactness of P, $\min_{p\in P} \int u(f)\, dp$ attains at some $p^* \in P$, so using above

$$V(\pi, f) = \min_{q'\in \mathrm{rect}_\pi(P)} \int u(f)\, dq' \leq \int u(f)\, dq < \int u(f)\, dp^* = V(\pi_0, f).$$

Thus, $\succsim$ is strictly averse to partition π at f.

For the "if" direction, suppose P is π-rectangular, so $P = \mathrm{rect}_\pi(P)$. Then $V(\pi, f) = V(\pi_0, f)$, for all f, and $\succsim$ is intrinsically neutral to information π. □

A.5 *Calculations for Example 1*

Example 1 uses a variational representation (u, c) where the cost function is given by

$$c(p) = \begin{cases} 0 & \text{if } p = p_{1/3}, \\ 3\left|\theta - \dfrac{1}{3}\right| & \text{if } p_\theta \in P\backslash\{p_{1/3}\}, \\ +\infty, & \text{otherwise.} \end{cases}$$

Observe that c is convex, lower semicontinuous, and grounded.

Consider the conditioning event $E = \{s_1, s_2\}$ and partition $\pi = \{\{s_1, s_2\}, \{s_3\}\}$. Suppose E has occurred, the set of posteriors are

$$P_E = \left\{ \left(\frac{\frac{1}{3}}{\frac{1}{3} + \theta}, \frac{\theta}{\frac{1}{3} + \theta} \right) : \theta \in \left[0, \frac{2}{3}\right] \right\}.$$

And the conditional cost function is

$$c_E(p_E) = \begin{cases} 0, & p_E = p_{\frac{1}{3},E}, \\ \dfrac{3\left|\theta - \frac{1}{3}\right|}{\frac{1}{3} + \theta}, & p \in P_E \setminus \{p_{1/3,E}\}, \\ +\infty, & \text{otherwise.} \end{cases}$$

In this case,

$$V_E(f) = \min_{\theta \in [0, \frac{2}{3}]} \frac{\frac{1}{3}}{\frac{1}{3} + \theta} f(1) + \frac{\theta}{\frac{1}{3} + \theta} f(2) + \frac{3\left|\theta - \frac{1}{3}\right|}{\frac{1}{3} + \theta},$$

$$V(\pi, f) = \min_{\theta \in [0, \frac{2}{3}]} \left(\frac{1}{3} + \theta\right) V_E(f) + \left(\frac{2}{3} - \theta\right) f(3) + 3\left|\theta - \frac{1}{3}\right|,$$

$$V_0(f) = \min_{\theta \in [0, \frac{2}{3}]} \frac{1}{3} f(1) + \theta f(2) + \left(\frac{2}{3} - \theta\right) f(3) + 3\left|\theta - \frac{1}{3}\right|. \tag{7}$$

Substituting the values of the four acts f_1, f_2, f_3, and f_4 to the above equations generates the numbers in Table 1.

A.6 *Proof of Proposition 4*

Suppose an act f is unambiguous. I first show that

$$c^{-1}(0) \subseteq \arg\min_{p \in \Delta} \left[\int_S u(f)\, dp + c(p) \right].$$

If f is unambiguous, then there is some constant γ such that $\int_S u(f)\, dp = \gamma$ for all $p \in \operatorname{dom}(c)$. Take any $p^* \in c^{-1}(0)$, then $c(p^*) \leq c(p)$ for all $p \in \Delta$, and

$$V(f) = \min_{p \in \Delta} \left[\int_S u(f)\, dp + c(p) \right] = \min_{p \in \operatorname{dom}(c)} \left[\int_S u(f)\, dp + c(p) \right] = \int_S u(f)\, dp^* + c(p^*).$$

Hence, $p^* \in \arg\min_{p \in \Delta} [\int_S u(f)\, dp + c(p)]$.

Thus, for any $\pi = \{E_1, \dots, E_n\}$,

$$V(\pi, f) = \min_{p \in \Delta} \sum_{E_i \in \pi} p(E_i) \left[\min_{p_i \in \Delta(E_i)} \int u(f)\, dp_i + c_{E_i}(p_i) \right] + \min_{\{q \in \Delta(S): q = p \text{ on } \pi\}} c(q) \tag{8}$$

$$\leq \sum_{E_i\in\pi} p^*(E_i)\left[\int u(f)\,dp^*(\cdot|E_i)+c_{E_i}\bigl(p^*(\cdot|E_i)\bigr)\right]+\min_{\{q\in\Delta(S):q=p^* \text{ on } \pi\}} c(q) \tag{9}$$

$$= \int_S u(f)\,dp^* \tag{10}$$

$$= \int_S u(f)\,dp^* + c\bigl(p^*\bigr) = V(\pi_0, f),$$

where from (8) to (9) follows from the definition of the min function, and from (9) to (10) follows from

$$0 \leq c_{E_i}\bigl(p^*(\cdot|E_i)\bigr) = \min_{p(\cdot|E_i)=p^*(\cdot|E_i)} \frac{c(p)}{p(E_i)} \leq \frac{c\bigl(p^*\bigr)}{p^*(E_i)} = 0,$$

and

$$0 \leq \min_{\{q\in\Delta(S):q=p^* \text{ on } \pi\}} c(q) \leq c\bigl(p^*\bigr) = 0. \quad \square$$

A.7 *Proof of Proposition 5*

Recall the "no-gain" condition (5)

$$c(p) = \min_{\{q:q(E_i)=p(E_i),\forall E_i\in\pi\}}\left[\sum_{E_i\in\pi} p(E_i)c_{E_i}(p_{E_i}) + c(q)\right], \quad \forall p\in\Delta.$$

Denote the right-hand side by $\tilde{c}(p)$. Note that

$$V(\pi, f) = \min_{p\in\Delta}\left[\sum_{E_i\in\pi} p(E_i)V_{E_i}(f) + c(p)\right]$$

$$= \min_{p\in\Delta}\left[\sum_{E_i\in\pi} p(E_i) \min_{q^i\in\Delta(E_i)}\left(\int_{E_i} u(f)\,dq_{E_i} + c_{E_i}\bigl(q^i\bigr)\right) + c(p)\right]$$

$$= \min_{p\in\Delta}\min_{q^1\in\Delta(E_1)}\cdots\min_{q^n\in\Delta(E_n)}\left[\sum_{i=1}^{n} p(E_i)\int_{E_i} u(f)\,dq^i\right] + \left[\sum_{E_i\in\pi} p(E_i)c_{E_i}\bigl(q^i\bigr) + c(p)\right].$$

Define $q'\in\Delta$ such that its marginal $q' = p$ on π and its posteriors $q'_{E_i} = q^i$ for all $E_i\in\pi$. Then

$$V(\pi, f) = \min_{q'\in\Delta}\min_{\{p\in\Delta:p=q' \text{ on } \pi\}}\left[\sum_{E_i\in\pi} q'(E_i)\int_{E_i} u(f)\,dq'_{E_i}\right] + \left[\sum_{E_i\in\pi} q'(E_i)c_{E_i}\bigl(q'_{E_i}\bigr) + c(p)\right]$$

$$= \min_{q'\in\Delta}\left[\int u(f)\,dq' + \min_{\{p\in\Delta:p=q' \text{ on } \pi\}}\left(\sum_{E_i\in\pi} q'(E_i)c_{E_i}\bigl(q'_{E_i}\bigr) + c(p)\right)\right]$$

$$= \min_{q'\in\Delta}\int u(f)\,dq' + \tilde{c}\bigl(q'\bigr).$$

And $V(\pi_0, f) = \min_{q\in\Delta}\int u(f)\,dq + c(q)$.

If no gain condition (5) holds, then clearly $c=\tilde{c}$ and $V(\pi_0, f) = V(\pi, f)$ for all f. Neutrality to π follows.

Conversely, suppose $c \neq \tilde{c}$. Without loss, assume $c(p) > \tilde{c}(p)$ for some p. By assumption, $(p, \tilde{c}(p))$ is not in cl(epi(c)), i.e., the closure of the epigraph of function c. Since c is convex, epi(c) is convex. By the strict separating hyperplane theorem, there exists some $(\xi, r) \in \mathbb{R}^{|S|+1}$ such that

$$\langle \xi, p\rangle + r\tilde{c}(p) < \langle \xi, p'\rangle + rz, \quad \forall (p', z) \in \mathrm{cl}(\mathrm{epi}(c)). \tag{11}$$

Since $(p, c(p)) \in \mathrm{cl}(\mathrm{epi}(c))$, and thus $r > 0$. Without loss, normalize r to be 1. Then by equation (11),

$$\min_{p'\in\Delta}\langle \xi, p'\rangle + \tilde{c}(p') \leq \langle \xi, p\rangle + \tilde{c}(p) < \min_{p'\in\Delta}\langle \xi, p'\rangle + c(p').$$

By unboundedness, $(u(X)+\mathbb{R}) \subseteq \mathbb{R}$, so one can find some $f \in \mathcal{F}$ such that $u(f) = \xi$. Therefore, $V(\pi_0, f) > V(\pi, f)$ and the DM is strictly averse to π at f. □

A.8 *Proof of Proposition 6*

Fix $\pi = \{E_1, \dots, E_n\}$. Suppose $\succcurlyeq_0$ has the second-order belief representation $(u, \phi; \Theta, \mu)$ and $\succcurlyeq_0$ is ambiguity averse. Then by Proposition 1 in Klibanoff et al. (2005), ϕ is concave.

By definition, if f is unambiguous, then there is some constant c such that $\int_S u(f)\,dp = c$ for all p in the support of μ. Hence,

$$\begin{aligned} V(\pi_0, f) &= \phi^{-1}\int_{\Delta(S)} \phi\left(\int_S u(f)\,dp\right) d\mu(p) = \phi^{-1}\int_{\Delta(S)} \phi(c)\,d\mu(p) \\ &= c = \int_S u(f)\,dp\,d\mu(p). \end{aligned}$$

Then

$$\begin{aligned} V(\pi, f) &= \phi^{-1}\left(\int_\Theta \phi\left[\sum_{i=1}^n p_{\theta'}(E_i)\phi^{-1}\left[\int_\Theta \phi\left(\int u(f)\,dp_{\theta_i}(\cdot|E_i)\right) d\mu_{E_i}(\theta_i)\right]\right] d\mu(\theta')\right) \\ &\leq \phi^{-1}\left(\int_\Theta \phi\left[\sum_{i=1}^n p_{\theta'}(E_i)\left[\int_\Theta \int u(f)\,dp_{\theta_i}(\cdot|E_i)\,d\mu_{E_i}(\theta_i)\right]\right] d\mu(\theta')\right) \\ &= \phi^{-1}\left(\int_\Theta \phi\left[\sum_{i=1}^n p_{\theta'}(E_i)\int_\Theta\int_{E_i} u(f)\,dp_{\theta_i}\frac{d\mu(\theta_i)}{\int p_{\theta''}(E_i)\,d\mu(\theta'')}\right] d\mu(\theta')\right) \\ &= \phi^{-1}\left(\int_\Theta \phi\left[\sum_{i=1}^n \int_\Theta\int_{E_i} u(f)\,dp_{\theta_i}\,d\mu(\theta_i)\frac{p_{\theta'}(E_i)}{\int p_{\theta''}(E_i)\,d\mu(\theta'')}\,d\mu(\theta')\right]\right) \\ &\leq \int_\Theta\left[\sum_{i=1}^n \int_\Theta\int_{E_i} u(f)\,dp_{\theta_i}\,d\mu(\theta_i)\frac{p_{\theta'}(E_i)}{\int p_{\theta''}(E_i)\,d\mu(\theta'')}\,d\mu(\theta')\right] \end{aligned}$$

$$= \sum_{i=1}^{n} \int_{\Theta} \int_{E_i} u(f)\, dp_{\theta_i}\, d\mu(\theta_i) \left(\int_{\Theta} \frac{p_{\theta'}(E_i)}{\int p_{\theta''}(E_i)\, d\mu(\theta'')}\, d\mu(\theta') \right)$$

$$= \int_{\Theta} \int_{S} u(f)\, dp_{\theta}\, d\mu = V(\pi_0, f).$$

The two inequalities follow from the concavity of ϕ and Jensen's inequality. The last equality holds because f is unambiguous. □

A.9 *The necessity of Axioms 5 and 6*

Proposition 1 shows that Axioms 1 to 6 are necessary and sufficient for the existence of a partition-dependent recursive utility representation. For some readers, Axioms 5 and 6 may seem less innocuous than the other four axioms (see discussions in Section 3). This appendix examines the necessity of Axioms 5 and 6, by providing three utility representations that relax either Axiom 5 or Axiom 6 but satisfy the other five axioms. Moreover, the validity of the main result, Proposition 2, is discussed in the context of each representation.

Recall that Axiom 5, anticipated partition separability, implies both dynamic consistency and consequentialism. Below are two recursive utility representations that satisfy all the other axioms but fails Axiom 5 by violating either dynamic consistency or consequentialism.

A representation that violates dynamic consistency Recall the MEU representation with prior-by-prior updating from Section 4.1. Now assume reduction, and the representation becomes

$$V(\pi, f) = \min_{p \in P} \int_{S} u(f)\, dp \quad \text{for all } \pi, f, \tag{12}$$

$$V_E(f) = \min_{p \in P_E} \int_{E} u(f)\, dp_E \quad \text{for all } E \in \Sigma. \tag{13}$$

It is easy to verify that preferences with the above representation given by (12) and (13) satisfy Axioms 1–4 and 6 and consequentialism, but violate dynamically consistency (and hence Axiom 5) at every π such that P is not π-rectangular.

Moreover, note that reduction implies $\succcurlyeq_{\pi} = \succcurlyeq_0$ for all π. Consequently, $\succcurlyeq$ is neutral to partial information. By the proof of Proposition 3, these MEU preferences can satisfy "strict" event complementarity. Thus, the equivalence between event complementarity and aversion to partial information identified in Proposition 2 breaks down.

A representation that violates consequentialism Suppose $\succcurlyeq_0$ has some MEU representation (u, P) as that introduced in Section 4.1, and let P_E be the prior-by-prior updated set of posteriors. For all binary menus $B = \{f, g\}$ and the $\succcurlyeq_0$-optimal act f^* from B, the conditional preferences $\succcurlyeq_{E,f^*,B}$ obtained by the Hanany and Klibanoff's (2007) updating rule are represented by $(u, P_{E,f^*,B})$, where $P_{E,f^*,B} = P_E$ if $f \neq g$ on E^c, and $P_{E,f^*,B}$ is the subset of P_E satisfying $f \succcurlyeq_{E,f^*,B} g$ if $f = g$ on E^c and $f \succcurlyeq_0 g$.[10] Hence, for all π, π', f, g, $(\pi, f) \succcurlyeq (\pi', g)$ if and only if $V(\pi, f; B) \geq V(\pi', g; B)$ for menu $B = \{f, g\}$, where

$$V_E(f; f^*, B) = \min_{p_E \in P_{E,f^*,B}} \int_E u(f)\, dp_E \quad \text{for all } E \in \Sigma, \tag{14}$$

$$V(\pi, f; B) = \min_{p \in P} \sum_{E_i \in \pi} p(E_i) V_{E_i}(f; f^*, B), \tag{15}$$

and $V(\pi', g; B)$ is defined analogously.

Corollary 2. *The utility representation given by (14) and (15) satisfies Axioms 1–4 and 6, but fails Axiom 5.*

Proof. It is obvious that the representation given by (14) and (15) satisfies Axioms 1–4. To see that Axiom 6 holds, note that in its statement only singleton menus like $B = \{f\}$ are considered, and hence, $P_{E,f^*,B} = P_E$ as every belief supports the conditional optimality of f in B. Then the representation becomes the same as that in the main representation, where time neutrality holds.

To see that Axiom 5 is violated, consider again the three-color Ellsberg example. Unconditionally, the set of priors is $P = \{(\frac{1}{3}, \alpha, \frac{2}{3} - \alpha) : \alpha \in [0, \frac{2}{3}]\}$ and the corresponding one-shot MEU preferences imply that $g_1 \succ_0 f_1$ and $f_2 \succ_0 g_2$, where $f_1 = (0, 1, 0)$, $g_1 = (1, 0, 0)$, $f_2 = (0, 1, 1)$, and $g_2 = (1, 0, 1)$. For the nonsingleton event $E = \{R, G\}$ in the partition $\pi = \{\{R, G\}, \{Y\}\}$, the full-Bayesian updated set of posteriors is $P_E = \{(\alpha, 1 - \alpha, 0) : \alpha \in [\frac{1}{3}, 1]\}$. In the menu $B_1 = \{f_1, g_1\}$, to support conditional preference of g_1 over f_1, an additional restriction $P(R) \geq P(G)$ must be imposed on the conditional preferences, and hence, the updated set of posteriors is $P_{E,g_1,B_1} = \{(\alpha, 1 - \alpha, 0) : \alpha \in [\frac{1}{2}, 1]\}$; in the menu $B_2 = \{f_2, g_2\}$, to support the conditional preferences for f_2 over g_2, an additional restriction $P(G) \geq P(R)$ must be imposed, and hence, the updated set of posteriors is $P_{E,f_2,B_2} = \{(\alpha, 1 - \alpha, 0) : \alpha \in [\frac{1}{3}, \frac{1}{2}]\}$. Suppose the *ex ante* preferences have the above representation; then $(\pi, g_1) \succ (\pi, f_1)$ and $(\pi, f_2) \succ (\pi, g_2)$ by dynamic consistency. Moreover, note that $f_2 = f_1 E1$ and $g_2 = g_1 E1$; hence, anticipated partition separability is violated. □

In this class of preferences, event neutrality always holds. To see this, if x is the E-conditional certainty equivalent of f, that is, $f \sim_{E,f,B} xEf$ for $B = \{f, xEf\}$, then there must be $f \sim_0 xEf$. Yet the *ex ante* preferences still displays intrinsic information preferences. Hence, the connection between event complementarity and aversion to partial information breaks down.

[10]That is, when $f = g$ on E^c, $P_{E,f^*,B} = \{p_E \in P_E : \int u(f^*)\, dp_E \geq \min_{q_E \in Q_{E,f^*,B}} \int u(f^*)\, dq_E\}$, where $Q_{E,f^*,B} = \{q_E \in P_E : \int u(f^*)\, dq \geq \max\{\int u(f)\, dq, \int u(g)\, dq\}\}$. See Hanany and Klibanoff's (2007, Proposition 2).

A representation that violates Axiom 6 (time neutrality) Consider a recursive utility representation with discounting:

$$V'_E(f) = \beta I_E\big(u(f_E)\big) \quad \text{for all } E \in \Sigma, \tag{16}$$

$$V'(\pi, f) = \beta I_0 \begin{pmatrix} V'_{E_1}(f) & \text{if } s \in E_1 \\ \vdots & \vdots \\ V'_{E_n}(f) & \text{if } s \in E_n \end{pmatrix}, \tag{17}$$

where u, I_E and I_0 satisfy the same properties as those in the main representation (Proposition 1), and $\beta \in (0, 1)$ is the usual intertemporal discount factor.

By Strzalecki (2013), if the aggregators I_E and I_0 take a general nonlinear form, then, outside the special case of MEU preferences, DM may exhibit a strict intrinsic preference for earlier resolution of uncertainties. As a result, time neutrality fails.

Corollary 3. *The utility representation with discounting given by (16) and (17) satisfies Axioms 1–5 but fails Axiom 6.*

Proof. By a proof similar to that of Proposition 1, it is easy to show that preferences admitting the discounting representation satisfy Axioms 1 to 5.

To see that Axiom 6 (time neutrality) is violated, consider the following counterexample. Consider the variational preferences model from Example 1, now with a discounting representation (16) and (17) and $\beta = \frac{1}{3}$. Then Axiom 6 is violated at act $f = (0, 3, -3)$. To see this, note that by equations (16), (17) and (7),

$$\begin{aligned} V'(\pi_0, f) &= \frac{1}{3}\left(\frac{1}{3} \cdot I_S\big(u(f)\big)\right) = \frac{1}{9} \cdot \min_{\theta \in [0, \frac{2}{3}]} \left\{3\theta - 3\left(\frac{2}{3} - \theta\right) + 3\left|\theta - \frac{1}{3}\right|\right\} \\ &= \frac{1}{9} \cdot \min_{\theta \in [0, \frac{2}{3}]} \left\{4\theta - 2 + 3\left|\theta - \frac{1}{3}\right|\right\} = -\frac{1}{9}, \\ V'\big(\pi^*, f\big) &= \frac{1}{3}\left(I_S\left(\frac{1}{3} \cdot u(f)\right)\right) = \frac{1}{3} \cdot \min_{\theta \in [0, \frac{2}{3}]} \left\{\theta - \left(\frac{2}{3} - \theta\right) + 3\left|\theta - \frac{1}{3}\right|\right\} \\ &= \frac{1}{3} \cdot \min_{\theta \in [0, \frac{2}{3}]} \left\{2\theta - \frac{2}{3} + 3\left|\theta - \frac{1}{3}\right|\right\} = \frac{1}{3} \cdot \left(\frac{2}{3} - \frac{2}{3} + 3 \cdot 0\right) = 0. \end{aligned}$$

Thus, $(\pi^*, f) \succ (\pi_0, f)$. The DM has a strict preference for earlier resolution of uncertainties at f. $\square$

Under the recursive representation with discounting, event complementarity is still equivalent to preferences for full information, as shown by Corollary 1. Yet, due to ambiguity-induced preferences for earlier resolution of uncertainties, there is no clear connection between Event Complementarity and aversion to partial information. Hence, Proposition 2 can fail.

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