

Informed intermediaries

PAULA ONUCHIC

Department of Economics, New York University

I develop a theory of intermediation in a market where agents meet bilaterally to trade and buyers cannot commit to payments. Some agents observe the past trading history of traders in the market. These informed agents can secure trades by punishing traders who previously defaulted. The punishing strategy affects equilibrium prices and determines which trades are hindered by the risk of default. Intermediation is a robust equilibrium feature, generated by asymmetric punishing strategies that yield informed agents either more effective opportunities to trade or the ability to extract more surplus in trades.

KEYWORDS. Intermediation, limited commitment, bilateral trade, trade network.

JEL CLASSIFICATION. D83, D85.

1. INTRODUCTION

In most decentralized markets, a buyer and seller meet and negotiate a price for a good, expecting that the seller will deliver the good and the buyer will pay the agreed-upon price. If either party fails to honor the terms, they face consequences from a legal system. In some markets, however, agents cannot rely on an exogenous authority to guarantee that contracts are honored. For obvious reasons, in markets for stolen goods or corruption markets, agents who fail to honor their debts cannot be prosecuted through formal means. Even in markets that are not illegal, legal fees can be prohibitively high or writing certain contracts may be infeasible. In extreme cases, an effective state or justice system may not exist.

In a variety of empirical contexts, such markets are documented to have a hierarchical trading structure, where some central agents often trade goods not for their own use, but rather to profit from intermediating trades between other market participants. For example, [Schneider \(2005\)](#) interviews 50 “prolific burglars” and finds their most common method for disposal of burgled or shoplifted goods is selling them to fences, who then resell the goods to final consumers for a higher price. [Della Porta and Vanucci \(2016\)](#) also document the presence of brokers in corruption networks in Italy, and make a case for the importance of mafias as enforcers in the Italian market for corrupt

Paula Onuchic: p.onuchic@nyu.edu

I am grateful for the advice and guidance I received from Ricardo Lagos and Debraj Ray, and am indebted to Joshua Weiss and Samuel Kapon for long discussions and helpful comments. I also thank Paulo Arvate, three anonymous referees, the participants of the Search Theory Workshop at NYU, the 2018 Summer Workshop on Money, Banking, Payments and Finance, and the 2019 Summer School of the Econometric Society. This research was supported by NSF Grant SES-1629370 to Debraj Ray.

exchange. Additionally, many papers document the hierarchical network structure of trade in different financial markets.¹

In this paper, I propose a model where agents trade bilaterally and payments are not enforced by an outside authority. Rather, agents honor terms of trade because they wish to maintain a reputation of being trustworthy. In their 1990 paper, Milgrom, North, and Weingast write, “A good reputation can be an effective bond for honest behavior in a community of traders if members of the community know how others have behaved in the past—even if any particular pair of traders meets only infrequently.” Much in the spirit of the quote, a share of traders in the model are *informed* and observe others’ past history of trade. Informed agents emerge as tacit “police” who secure transactions.

Consistent with empirical observations, I show in the model that a robust feature of equilibria is a hierarchical trade network in which informed agents are central and often intermediate trades between other market participants.

The model is a variation of the over-the-counter market in Duffie, Garleanu, and Pedersen (DGP, (2005)). A mass of agents meet bilaterally and continuously trade due to differences in their valuation for an asset. Unlike in DGP, buyers have no exogenous ability to commit to payments: Sellers first transfer the asset; only then do buyers decide whether to make the agreed-upon payment. If a buyer does not pay, the seller has no other recourse. Some traders are informed and observe a record of all past meetings. Informed agents can secure trades by refusing to trade with past defaulters.

When a potential seller meets a potential buyer, she makes a take-it-or-leave-it price offer to the buyer. Despite this protocol, the seller does not necessarily extract all of the buyer’s trade surplus, because she must choose a price that induces the buyer not to default. Aware of the punishment a defaulting buyer is subject to, the seller proposes the highest price that induces no default. For example, if a buyer is punished with a long period of exclusion from trade, he would not default even if the proposed price is high, and the seller indeed offers a high price. If, otherwise, a buyer is only lightly punished for defaulting, he can only be trusted to pay a low price. Through this channel, the strategy that informed agents use to punish defaulters shapes the terms of trade in equilibrium.

In Proposition 1, I propose a first class of equilibria that can be sustained if agents are sufficiently patient and enough informed agents exist: all-trade equilibria, where the limited commitment friction is completely overcome and no trades are hindered by the risk of default. In all-trade equilibria, informed agents who default are punished lightly, but defaulting uninformed agents are punished harshly. As such, informed agents are able to buy assets at a cheaper price and sell them at a higher price than uninformed agents do.

All-trade equilibria are efficient, because all trades in which the buyer values the asset more highly than the seller take place, despite the limited commitment friction. Moreover, *intermediation* trades also happen: informed agents trade with uninformed agents not for the consumption value of the asset, but rather for its future trade value. Because informed agents buy assets at a cheaper price and sell them at a higher price than uninformed agents, they profit from trading and re trading the asset.

¹See, for example, Ashcraft and Duffie (2007), Bech and Atalay (2010), and Afonso and Lagos (2015) for the market for federal funds.

Proposition 2 proves the existence of a second equilibrium class: core–periphery equilibria. These equilibria are supported by a punishing strategy whereby uninformed buyers who default against uninformed sellers face no consequence. As such, uninformed traders are peripheral agents who do not trade with each other, but rather trade only with informed agents who form the core of the network. Because uninformed agents are constrained by the limited commitment friction, core–periphery equilibria are not efficient.

In core–periphery equilibria, informed agents also act as intermediaries, precisely because they form the core of the trade network and have more trade opportunities than uninformed agents. As such, uninformed agents sell assets to informed agents at a discount because finding a final buyer on their own would take longer, and they buy assets for a higher price for a similar motive. This price spread makes intermediation profitable to core traders.

Given the equilibrium multiplicity, Section 5 proposes a refinement: I look for equilibria that maximize informed agents' values. Because all equilibria rely on informed agents coordinating on punishing strategies to ensure trade, it is reasonable to expect informed agents' preferred equilibria. The main results show that using punishing strategies that yield them better terms of trade (Proposition 5) and create a core–periphery trade network by preventing trade between uninformed agents (Proposition 4) can benefit informed agents.

Both of these channels that benefit informed agents also generate motives for equilibrium intermediation, and we can conclude that intermediation is connected to rewarding informed agents who “monitor” trades in the market. Proposition 5 shows informed agents can receive a higher value without sacrificing efficiency, through more favorable terms of trade. However, a consequence of Proposition 4 is that sacrificing efficiency by preventing trades between uninformed agents is even better from the point of view of informed agents.

1.1 *Related literature*

This paper is related to recent literature on intermediation in over-the-counter markets. Initial models feature exogenously given middlemen who facilitate trade (e.g., Duffie et al. (2005), Lagos and Rocheteau (2009)). Many papers since tackle the question of the endogenous emergence of some agents as intermediaries. The papers that are closest to mine are Farboodi, Jarosch, and Shimer (2020) and Farboodi, Jarosch, Menzio, and Wiriadinata (2019).² The former finds that agents with faster meeting rates, and, hence, more opportunities to trade, become intermediaries, whereas the latter shows that intermediation arises purely due to differences in bargaining power across agents. I propose an alternative theory, which relies on agents' limited commitment to future payments. This new theory can be seen as a microfoundation for the two previous explanations, because the punishing strategies that informed agents in my model use lead to some agents endogenously having better trade opportunities or surplus extraction.

²See also Afonso and Lagos (2015), Chang and Zhang (2019), and Bethune, Sultanum, and Trachter (2020).

Babus and Hu (2017) study a repeated game played in a network and also show a link between intermediation and trade when agents cannot commit to payments.³ In their model, without intermediation, trade fails as the number of agents becomes large, and with intermediation, trade can be sustained regardless of the size of the market. The first main difference between their model and mine is that I study the limit case with a continuum of agents. Second, intermediation emerges dynamically in my model, when agents trade for future resale purposes. In their framework, dynamic intermediation is not possible, because assets are not carried between periods. Finally, my paper highlights the effect of punishing strategies on equilibrium features.

My paper also relates to the sizable literature on limited commitment in search-theoretic models of liquidity. Generally, the main difference between those models and mine is that the assets agents trade in my environment are long-lived and might be traded for speculative motives, rather than a commodity that is consumed before the end of each period. This distinction is key for linking the limited commitment friction to intermediation.⁴

Finally, I contribute to the literature linking intermediation and trade efficiency. Previous work found that intermediation facilitates trade by minimizing transaction costs (Townsend (1978)), minimizing search frictions (Rubinstein and Wolinsky (1987), Duffie et al. (2005)), or reducing monitoring costs (Diamond (1984)). In terms of the monitoring motivation for intermediation, my paper is closely related to Diamond (1984) and the subsequent literature. In Diamond's model, intermediation increases efficiency by lowering the total cost of monitoring. In my model, intermediation can be welfare-improving by providing incentives for informed agents to join the market and then monitor trades.

2. MODEL

I study an economy where time is continuous and the horizon is infinite, and future utility flows are discounted by all agents at rate $r > 0$. A unit measure of agents bilaterally meet to trade an indivisible asset with supply fixed at $1/2$. In these meetings, a numeraire good is used as the medium of exchange.

Preferences. At any time, each agent in the market either holds an asset or does not. Agents cannot accumulate assets or sell assets they do not yet have. An agent with idiosyncratic valuation $v \in \{L, H\}$ receives flow value δ_v when holding an asset, where $\delta_H > \delta_L \geq 0$. Agents' valuations and asset holdings are observable to other market participants.

Asset holdings are subject to shocks: At a Poisson rate $\eta > 0$, an agent who is holding an asset loses it and, at that same rate, an agent who is not holding an asset receives one.

³Fainmasser (2019) is another paper that considers the link between intermediation and cooperation in repeated games in networks.

⁴My paper is close to Cavalcanti and Wallace (1999), where agents are either monitored, having all their previous trades recorded, or not monitored. Monitored agents can commit to future payments since they can be punished for defaults. In my paper, agents are heterogeneous in their access to a public record rather than in their being recorded or not. Other related papers are Carapella and Williamson (2015) and Bethume, Hu, and Rocheteau (2018).

The difference in flow payoffs between high- and low-valuation agents, as well as the asset-holding shocks, implies agents wish to continuously trade and retrade the assets.⁵

Bilateral Trade and Limited Commitment. At rate $\lambda > 0$, an agent meets another randomly selected agent. If one of the agents in the meeting has an asset and the other does not, they have an opportunity to trade.

The agent who holds an asset (the potential seller) makes a take-it-or-leave-it price offer, in terms of the numeraire good, to the agent who does not have an asset (the potential buyer). If the offer is accepted, the asset is transferred from seller to buyer. After this transfer takes place, the buyer chooses to pay the agreed-upon price or to default.⁶

Despite the seller making a take-it-or-leave-it price offer, she does not necessarily extract all of the buyer's surplus, because she must also make sure the buyer chooses not to default at the proposed price. The seller-optimal price offer is such that the buyer is made exactly indifferent between defaulting and not defaulting.

Information and Trigger Punishing Strategies. A measure ϕ of agents is *informed* (I) and upon meeting a potential trading partner, observe their past trading history, i.e., the identity of past trading partners and whether they defaulted. The other $1 - \phi$ agents are *uninformed* (U) and do not observe said history. Agents' information types are independent of their valuation and are observable to other market participants.

Informed agents use their knowledge of past trading history to punish defaulting agents. To that end, they use *trigger punishing strategies*: when a buyer defaults, a punishment regime is triggered with some probability. In that case, the buyer is forever excluded from trade with informed agents.⁷ All informed agents use the same anonymous⁸ trigger strategy.

Formally, the set of trading histories plus the realization of a public randomization device is partitioned into elements that induce informed agents to trade (*unflagged* histories) and elements that induce informed agents not to trade (*flagged* histories).

To fully describe the punishing strategy, I also need to assign punishments to informed agents who fail to punish when called upon to do so, that is, informed agents who trade with flagged agents. Imposing that informed agents who trade with flagged agents become flagged is sufficient. If a flagged agent wants to buy from an informed agent, she has no incentive to repay and, hence, the informed agent will not engage in this sale. If a flagged agent wants to sell to an informed agent, the informed agent already gets punished from engaging in this trade and, hence, has no incentive to pay. Anticipating that possibility of default, the flagged agent does not sell.

⁵In most of the literature stemming from DGP, this trading motive is achieved through shocks in agents' valuations for the asset, rather than to the asset holdings as in my model. In the absence of limited commitment, these modeling choices are equivalent.

⁶The buyer has limited commitment and can choose to default, but the model could seamlessly be flipped to the case where the buyer first pays and then the seller chooses whether to transfer the good.

⁷Other non-trigger strategies also support the equilibria I find, and for some parameter values, other strategies might support equilibria that are not supported by the trigger class. However, the harshest trigger strategy, whereby a buyer is punished with probability 1 after default, is also the harshest punishment informed agents can inflict on defaulters across all the possible, even non-trigger, strategies.

⁸An informed agent who meets two agents with the same trading history chooses the same action in both meetings.

3. EQUILIBRIUM DEFINITION

Let V_{va}^i be the value to an unflagged agent of information type $i \in \{I, U\}$, valuation $v \in \{H, L\}$, and asset holding $a \in \{0, 1\}$. Accordingly, let \tilde{V}_{va}^i be the value of that same agent when he is in the flagged regime. Sixteen potential trades occur between unflagged agents, and (i_s, v_s, i_b, v_b) denotes a trade between a seller of type $(i_s, v_s) \in \{I, U\} \times \{H, L\}$ and a buyer of type $(i_b, v_b) \in \{I, U\} \times \{H, L\}$. The surplus in this meeting is given by $V_{v_s 0}^{i_s} - V_{v_s 1}^{i_s} + V_{v_b 1}^{i_b} - V_{v_b 0}^{i_b}$, the value to the buyer of acquiring an asset minus the value to the seller of losing an asset.

Punishing Strategy and Price Determination. Define the trigger punishing strategy $\tau : (\{I, U\} \times \{H, L\})^2 \rightarrow [0, 1]$, where $\tau(i_s, v_s, i_b, v_b)$ is the probability that the buyer becomes flagged after defaulting on trade (i_s, v_s, i_b, v_b) . In meeting (i_s, v_s, i_b, v_b) , the buyer chooses to pay the seller if the seller's price offer p satisfies

$$V_{v_b 1}^{i_b} - p \geq (1 - \tau(i_s, v_s, i_b, v_b))V_{v_b 1}^{i_b} + \tau(i_s, v_s, i_b, v_b)\tilde{V}_{v_b 1}^{i_b}. \quad (1)$$

The left-hand side of condition (1) shows the value to the buyer of paying the price p requested by the seller: the buyer leaves the meeting with the asset and without triggering any punishment, and pays price p . The right-hand side show the value of defaulting: with probability $(1 - \tau(i_s, v_s, i_b, v_b))$, the buyer leaves the meeting with the asset and does not trigger punishment, and with probability $\tau(i_s, v_s, i_b, v_b)$, the buyer leaves the meeting with the asset but triggers the punishment regime.

When set to equality, condition (1) determines the highest price the seller can charge while guaranteeing the buyer does not default. Thus, the seller's optimal take-it-or-leave-it price offer in meeting (i_s, v_s, i_b, v_b) is

$$p(i_s, v_s, i_b, v_b) = \tau(i_s, v_s, i_b, v_b)(V_{v_b 1}^{i_b} - \tilde{V}_{v_b 1}^{i_b}). \quad (2)$$

If participating in the market is valuable, $V_{v_b 1}^{i_b}$ is larger than $\tilde{V}_{v_b 1}^{i_b}$ and the seller can charge a positive price and guarantee the buyer does not default. Equation (2) shows that a harsher punishment for defaulting (larger τ) raises the maximum price the seller can charge, because it decreases the buyer's deviation value. In the other limit, in the absence of punishment for defaulting ($\tau = 0$), the price is zero.

It is convenient to define the buyer and seller surplus shares in each meeting.

Let $\beta : (\{I, U\} \times \{H, L\})^2 \rightarrow \mathbb{R}$, where $\beta(i_s, v_s, i_b, v_b)$ is the proportion of the surplus that remains with the seller in meeting (i_s, v_s, i_b, v_b) :

$$\beta(i_s, v_s, i_b, v_b) = \frac{V_{v_s 0}^{i_s} - V_{v_s 1}^{i_s} + p(i_s, v_s, i_b, v_b)}{V_{v_s 0}^{i_s} - V_{v_s 1}^{i_s} + V_{v_b 1}^{i_b} - V_{v_b 0}^{i_b}}. \quad (3)$$

Equilibrium Trades. Let $\mathcal{I} : (\{I, U\} \times \{H, L\})^2 \rightarrow \{0, 1\}$ be such that $\mathcal{I}(i_s, v_s, i_b, v_b) = 1$ indicates trade (i_s, v_s, i_b, v_b) takes place in equilibrium, and let $\mathcal{I}(i_s, v_s, i_b, v_b) = 0$ indicate it does not. A trade is mutually beneficial if both the seller and buyer retain a positive surplus. In equilibrium, only mutually beneficial trades take place. Additionally, all

strictly beneficial trades must take place:

$$\begin{aligned} \mathcal{I}(i_s, v_s, i_b, v_b) &= 1 \\ \Rightarrow \beta(i_s, v_s, i_b, v_b) &\in [0, 1] \quad \text{and} \quad V_{v_s 0}^{i_s} - V_{v_s 1}^{i_s} + V_{v_b 1}^{i_b} - V_{v_b 0}^{i_b} \geq 0, \end{aligned} \tag{4}$$

$$\begin{aligned} \beta(i_s, v_s, i_b, v_b) &\in (0, 1) \quad \text{and} \quad V_{v_s 0}^{i_s} - V_{v_s 1}^{i_s} + V_{v_b 1}^{i_b} - V_{v_b 0}^{i_b} > 0 \\ \Rightarrow \mathcal{I}(i_s, v_s, i_b, v_b) &= 1. \end{aligned} \tag{5}$$

Value Functions. Given the surplus sharing rules defined above, we can write the unflagged and flagged agents' value functions. To that end, also let $\{\mu_{va}^i\}$ denote the stationary distribution of unflagged agents across valuations and asset holdings. I focus on stationary equilibria with no default on path, and so I refrain from adding notation for the stationary measure of flagged agents, which must be zero.

Value functions consist of the flow value received if holding an asset, the value due to asset-holding shocks, and flows from trade. For unflagged and flagged agents, respectively, they are

$$\begin{aligned} rV_{v0}^i &= \eta(V_{v1}^i - V_{v0}^i) \\ &+ \lambda \sum_{v_s \in \{L, H\}} \mu_{v_s 1}^I \mathcal{I}(I, v_s, i, v) (1 - \beta(I, v_s, i, v)) (V_{v1}^i - V_{v0}^i + V_{v_s 0}^I - V_{v_s 1}^I) \\ &+ \lambda \sum_{v_s \in \{L, H\}} \mu_{v_s 1}^U \mathcal{I}(U, v_s, i, v) (1 - \beta(U, v_s, i, v)) (V_{v1}^i - V_{v0}^i + V_{v_s 0}^U - V_{v_s 1}^U), \end{aligned} \tag{6}$$

$$\begin{aligned} rV_{v1}^i &= \delta_v + \eta(V_{v0}^i - V_{v1}^i) \\ &+ \lambda \sum_{v_b \in \{L, H\}} \mu_{v_b 0}^I \mathcal{I}(i, v, I, v_b) \beta(i, v, I, v_b) (V_{v_b 1}^I - V_{v_b 0}^I + V_{v0}^i - V_{v1}^i) \\ &+ \lambda \sum_{v_b \in \{L, H\}} \mu_{v_b 0}^U \mathcal{I}(i, v, U, v_b) \beta(i, v, U, v_b) (V_{v_b 1}^U - V_{v_b 0}^U + V_{v0}^i - V_{v1}^i), \end{aligned} \tag{7}$$

$$r\tilde{V}_{v0}^i = \eta(\tilde{V}_{v1}^i - \tilde{V}_{v0}^i) + \lambda \sum_{v_s \in \{L, H\}} \mu_{v_s 1}^U \mathcal{I}(U, v_s, i, v) (\tilde{V}_{v1}^i - \tilde{V}_{v0}^i), \tag{8}$$

$$\begin{aligned} r\tilde{V}_{v1}^i &= \delta_v + \eta(\tilde{V}_{v0}^i - \tilde{V}_{v1}^i) \\ &+ \lambda \sum_{v_b \in \{L, H\}} \mu_{v_b 0}^U \mathcal{I}(i, v, U, v_b) \beta(i, v, U, v_b) (V_{v_b 1}^U - V_{v_b 0}^U + \tilde{V}_{v0}^i - \tilde{V}_{v1}^i). \end{aligned} \tag{9}$$

Agents in flagged agents do not trade with informed agents; thus (8) and (9) do not account for value of meeting with informed agents. Flagged agents can still trade with uninformed agents, who are not able to observe their past defaults. Moreover, because the flagged agent is already in the punishment regime and cannot be punished further, flagged buyers always default, which is also accounted for in (8). Because I work with equilibria with no default on path, these trades never take place. However, they still affect the value of the deviation.

Stationary Distribution. The distribution of agents across types must satisfy the adding-up constraints

$$\sum_{v \in \{H, L\}} \sum_{a \in \{0, 1\}} \mu_{va}^I = \phi, \quad (10)$$

$$\sum_{v \in \{H, L\}} \sum_{a \in \{0, 1\}} \mu_{va}^U = 1 - \phi, \quad (11)$$

$$\sum_{i \in \{I, U\}} \sum_{v \in \{H, L\}} \mu_{v1}^i = \frac{1}{2}. \quad (12)$$

Finally, in any stationary equilibrium, the stationary distribution is such that the inflow to each state is equal to the outflow. For any $v \in H, L$ and $i \in \{I, U\}$,

$$\begin{aligned} & \mu_{v1}^i \left[\eta + \sum_{v_b \in \{H, L\}} \sum_{i_b \in \{I, U\}} \mu_{v_b 0}^{i_b} \mathcal{I}(i, v, i_b, v_b) \right] \\ &= \mu_{v0}^i \left[\eta + \sum_{v_s \in \{H, L\}} \sum_{i_s \in \{I, U\}} \mu_{v_s 1}^{i_s} \mathcal{I}(i_s, v_s, i, v) \right] \end{aligned} \quad (13)$$

must hold.

Symmetry. I focus on symmetric equilibria, where an agent's equilibrium trading behavior depends only on her information type and on whether their asset holdings are well aligned with their valuation.⁹ Agents' portfolios are misaligned if they have high valuation but no asset or if they hold an asset but have low valuation; their portfolios are well aligned otherwise. Symmetry requires a misaligned seller to have a trading pattern that mirrors that of a misaligned buyer of the same information type. Formally,

$$\mathcal{I}(i_s, v_s, i_b, v_b) = \mathcal{I}(i_b, \sim v_b, i_s, \sim v_s), \quad (14)$$

$$\beta(i_s, v_s, i_b, v_b) = 1 - \beta(i_b, \sim v_b, i_s, \sim v_s). \quad (15)$$

Equilibrium Definition. A stationary equilibrium with no default is a set of unflagged value functions $\{V_{va}^i\}$, flagged value functions $\{\tilde{V}_{va}^i\}$, trade indicator \mathcal{I} , seller surplus shares β , punishment strategy τ , and stationary distribution $\{\mu_{va}^i\}$ such that (3)–(15) are satisfied.

4. ALL-TRADE AND CORE-PERIPHERY EQUILIBRIA

I propose two classes of equilibria. In the first class, all-trade equilibria, the limited commitment friction is completely overcome and no positive-surplus trades are hindered by the threat of default. In the second class, core-periphery equilibria, the threat of default prevents uninformed agents from directly trading with each other. In these equilibria, uninformed agents are *peripheral* traders, only trading with the informed *core* traders. Core traders trade among themselves and also with peripheral traders.

⁹Farboodi et al. (2020) similarly restrict attention to the set of symmetric equilibria in their model.

In the language of Italian markets for corrupt exchange, as in Della Porta and Vanucci (2016), “the mafia” can be seen as the group of informed agents who have detailed knowledge of the behavior of individuals in the corrupt community. In all-trade equilibria, the mafia punishes individuals who fail to honor their debt to someone, regardless of whether the cheated creditor belongs to the mafia. In core–periphery equilibria, the mafia turns a blind eye to individuals who default on creditors who are not mafia participants, but punish anyone who cheats on a mafia participant.

The two classes of equilibria illustrate that two types of punishing strategy asymmetries generate intermediation: surplus-sharing asymmetries (present in both all-trade and core–periphery equilibria) and asymmetries in effective trade opportunities (present in core–periphery equilibria). In Section 5, I show that both channels can be used by informed agents to increase their equilibrium values.

4.1 All-trade equilibria

Equations (2)–(5) show a mapping between the punishing strategy τ and the surplus-sharing rule β and trade indicator \mathcal{I} . One possible approach to find equilibria is to start with a punishing strategy τ and find the equilibrium it induces. I take the reverse approach: First, I conjecture potential equilibrium trade indicator \mathcal{I} and surplus-sharing rule β and then search for a punishing strategy τ that supports the conjectures.

To build the class of all-trade equilibria, I start with the conjecture that all positive-surplus trades take place. Formally, \mathcal{I} is such that

$$V_{v_s0}^{i_s} - V_{v_s1}^{i_s} + V_{v_b1}^{i_b} - V_{v_b0}^{i_b} > 0 \quad \Rightarrow \quad \mathcal{I}(i_s, v_s, i_b, v_b) = 1. \quad (16)$$

I also conjecture a surplus-sharing rule where, when informed and uninformed agents trade, the informed agent keeps at least half of the surplus. In all other trades, the surplus is split equally between the buyer and seller.

Formally, for some $\beta^I \in [0.5, 1)$, β is given by¹⁰

$$\begin{aligned} \beta(I, v_s, U, v_b) &= \beta^I \geq \frac{1}{2}, & \beta(U, v_s, I, v_b) &= 1 - \beta^I \leq \frac{1}{2}, \\ \beta(I, v_s, I, v_b) &= \frac{1}{2}, & \beta(U, v_s, U, v_b) &= \frac{1}{2}. \end{aligned} \quad (17)$$

Notice again that the seller makes take-it-or-leave-it price offers in all meetings, but he is subject to the limited commitment constraint and must make a price offer that induces the buyer not to default. Consequently, equilibria exist in which the seller does not keep all the surplus, as in the conjecture above.

I show later in this section that the proposed β conjecture is supported by a punishing strategy τ that punishes uninformed defaulters very harshly, with a high trigger probability of exclusion from the market, but informed defaulters only lightly. Knowing the punishment for an uninformed defaulter is harsh, an informed seller can make a

¹⁰The reverse equilibria, where the uninformed agents keep at least half of the surplus ($\beta^I < 0.5$), also exist and are discussed later in the paper.

high price offer and extract a large share of the surplus. Conversely, informed defaulters are punished lightly, so an uninformed seller must demand a low price from an informed buyer so as to avoid default. As such, an informed buyer keeps a large share of the surplus of the trade to himself.

In Proposition 1, I show that if the measure of informed agents is large enough and agents are sufficiently patient, some punishing strategy τ supports an equilibrium consistent with conjectures (16) and (17).

PROPOSITION 1. *For each $\beta^I \in [0.5, 1)$, an all-trade equilibrium satisfying (16) and (17) exists if ϕ is sufficiently large ($\phi \rightarrow 1$) and r is sufficiently small ($r \rightarrow 0$).*

Conversely, for each $r, \phi \in (0, 1)$, a $\bar{\beta} \in (0.5, 1)$ exists such that if $\beta^I > \bar{\beta}$, an all-trade equilibrium does not exist.

All-trade equilibria are efficient and feature intermediation by informed agents.

The proof of Proposition 1, which I describe in the rest of this section, is provided in the Appendix. In the Appendix, I also characterize the unflagged and flagged values, and the stationary distribution in the all-trade equilibria.

Equilibrium Values and Trades. I start by substituting the \mathcal{I} and β conjectures in (16) and (17) into the equations that define agents' value functions both in the unflagged and the flagged regimes, and the stationary distribution in (6)–(13).

The first observation is that the implied stationary distribution is symmetric: the measure of uninformed (informed, respectively) agents with high valuation but without an asset is equal to that of uninformed (informed) agents with low valuation and with an asset. I define $\mu^U \equiv \mu_{H0}^U = \mu_{L1}^U$ and $\mu^I \equiv \mu_{H0}^I = \mu_{L1}^I$, the measures of agents whp have misaligned portfolios.

Now define S_v^i to be the value of holding an asset to agent of information type i and valuation type v , so that $S_v^i \equiv V_{v1}^i - V_{v0}^i$. With some algebra, I show the starting conjectures imply

$$\begin{aligned} S_H^i &= \left[\frac{1}{2(r+2\eta)} \right] (\delta_H + \delta_L) + \alpha^i (\delta_H - \delta_L), \\ S_L^i &= \left[\frac{1}{2(r+2\eta)} \right] (\delta_H + \delta_L) - \alpha^i (\delta_H - \delta_L), \end{aligned}$$

where

$$\alpha^I = \frac{1}{2(r+2\eta + \lambda(2\mu^U \beta^I + \mu^I))} \quad (18)$$

and

$$\alpha^U = \left[\frac{r+2\eta + \lambda\beta^I(2\mu^U + \mu^I) + \lambda(1-\beta^I)(\phi/2 - \mu^I)}{r+2\eta + \lambda(\mu^U + \phi/2(1-\beta^I))} \right] \alpha^I. \quad (19)$$

Because $\beta^I \geq 0.5$, it is easy to check that the term multiplying α^I in (19) is weakly larger than 1, so that $\alpha^U \geq \alpha^I$. Thus agents' values are ordered $S_H^U \geq S_H^I > S_L^I \geq S_L^U$, with all inequalities being strict if β^I is strictly larger than 0.5.

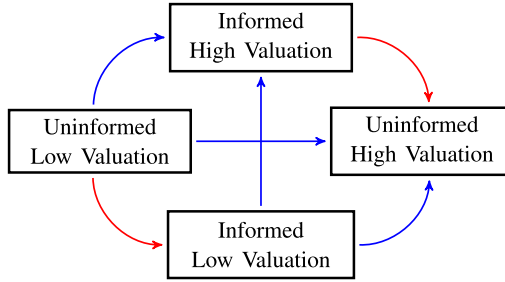


FIGURE 1. Trade pattern supported in the all-trade equilibria. The upper left, middle, and lower right arrows indicate portfolio balancing trades. The upper right and lower left arrows indicate intermediation trades.

The constants α^I and α^U are effective discount rates that determine the wedge between S_H^i and S_L^i . This wedge stems from the difference in the flow value of holding an asset for agents with high or low valuation. If α^i is higher, this difference is larger. Note that α^i is higher whenever agents get fewer opportunities to trade due to more frictions (lower λ). The intuition is that when an agent with high valuation gets few opportunities to trade, it is valuable to hold onto an asset. Otherwise, if opportunities to trade are abundant, the value of having an asset in hand is not as large, because the opportunity to buy one presents itself often.

With these values at hand, we can describe the set of trades with positive surplus in this conjectured equilibrium. These trades are pictured in Figure 1. The first type is *portfolio-balancing trades*, where assets move from sellers with low valuation to buyers with high valuation. The surplus in these trades is positive, given by $S_H^{i_b} - S_L^{i_s} > 0$, where i_b and i_s are the information types of the buyer and seller, respectively. These trades are portfolio-balancing because both agents enter the meeting with misaligned portfolios and leave it with aligned portfolios.

The second type of positive-surplus trades is *intermediation trades*, where the buyer and seller have the same valuation. When an informed buyer with low valuation meets an uninformed seller with low valuation, this trade has positive surplus given by $S_L^U - S_L^I \geq 0$. Similarly, when an uninformed buyer with high valuation meets an informed seller with high valuation, this trade also has positive surplus, given by $S_H^I - S_H^U \geq 0$.

In these trades, informed agents act as intermediaries: they trade not for the consumption value of the asset, but rather for its future trade value. Because informed agents keep more than half of the surplus in all trades with uninformed agents, they buy assets at a lower price than they expect to sell them for in a future encounter. As such, they wish to buy assets even if they have low valuation. Conversely, they sell assets even in the high-valuation state, because they charge a higher price than they expect to buy a new asset for in a future trade.

Note that the ability to extract more than half of the surplus is not inherent to informed agents, but rather is an endogenous feature of the conjectured equilibrium.

All-Trade Equilibrium with $\beta^I = 0.5$. When $\beta^I = 0.5$, surplus is shared evenly between the buyer and seller in all meetings, even between informed and uninformed

agents. In that case, we have $S_H^U = S_H^I > S_L^I = S_L^U$, and the surplus of intermediation trades is equal to 0, so that agents are indifferent between trading and not trading in intermediation meetings. This finding illustrates that in all-trade equilibria, the motive for intermediation stems from the asymmetry in surplus sharing between informed and uninformed agents.

Verifying Equilibrium Existence. So far, I have assumed that some punishing strategy τ supports the all-trade equilibrium conjectures. With the equilibrium values calculated above, I can now verify when such a τ exists. Effectively, I need to verify that for some τ , the seller's optimal take-it-or-leave-it price offer in each positive-surplus meeting exactly implements the conjectured surplus-sharing rule in (17).

For each positive-surplus meeting (i_s, v_s, i_b, v_b) , $\tau(i_s, v_s, i_b, v_b) \in [0, 1]$ must exist such that

$$p(i_s, v_s, i_b, v_b) = \tau(i_s, v_s, i_b, v_b)(V_{v_b1}^{i_b} - \tilde{V}_{v_b1}^{i_b}), \quad (20)$$

where

$$p(i_s, v_s, i_b, v_b) = V_{v_s1}^{i_s} - V_{v_s0}^{i_s} + \beta(i_s, v_s, i_b, v_b)[(V_{v_s1}^{i_s} - V_{v_s0}^{i_s}) - (V_{v_b1}^{i_b} - V_{v_b0}^{i_b})].$$

To build such a punishing strategy, I need to verify that the loss in value from being excluded from the market is larger than the price in each meeting: $V_{v_b1}^{i_b} - \tilde{V}_{v_b1}^{i_b} \geq p(i_s, v_s, i_b, v_b)$. In this case, some $\tau(i_s, v_s, i_b, v_b) \in [0, 1]$ exists for which (20) is exactly satisfied. In the [Appendix](#), I show that for each value of $\beta^I \in (0.5, 1)$, if r is sufficiently small and ϕ is sufficiently large, the loss in value from being excluded from trade with informed agents is large enough that this condition holds.

Moreover, I show that for any given values of r and ϕ , some $\bar{\beta}$ strictly below 1 exists such that the all-trade equilibrium cannot be sustained if $\beta^I > \bar{\beta}$. This upper bound exists because if β^I is too large, the value to uninformed agents from trading with informed agents is very low. In that case, losing access to those trades is not a sufficiently strong punishment to ensure uninformed buyers do not default.

In Figure 2, I display parameter regions where the proposed equilibrium holds. In the top left panel, I fix $\beta^I = 0.7$ and plot out the region in the (ϕ, r) space. In the bottom left panel, I fix $\phi = 0.7$ and show that, for higher β^I values, agents need to be more patient for the all-trade equilibrium to be sustained.

In the top and bottom right panels of Figure 2, I fix the value of r and display the punishing strategy τ as a function of ϕ and β^I . The black lines are the probabilities of becoming flagged when uninformed buyers default, and the gray lines correspond to the probability of flagging informed buyers who default in each of the three trades where informed agents are buyers. Notice that uninformed agents are always punished more harshly for defaults: the black lines are always above the gray ones. This finding is consistent with uninformed agents being charged higher prices in trades and retaining a smaller share of surplus.

Efficiency. Allocative efficiency is attained when trades reallocate assets from low- to high-valuation agents. A measure of efficiency is the proportion of misaligned agents

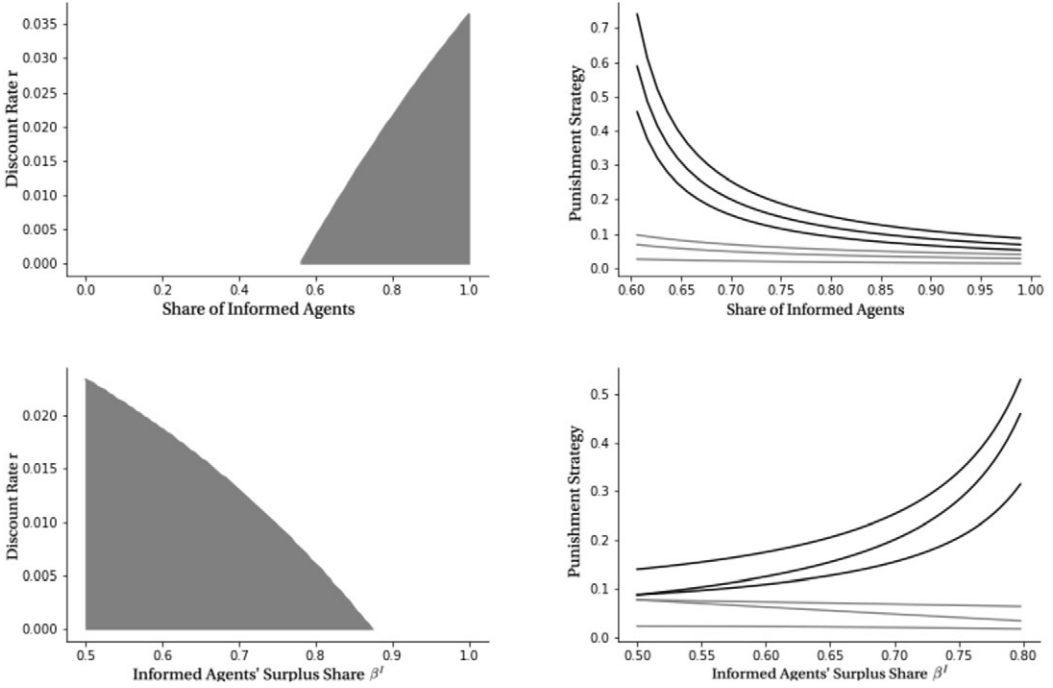


FIGURE 2. In the left panels, the shaded gray area represents the parameter region where the all-trade equilibrium exists. Other parameters are fixed at $\lambda = 2$, $\eta = 0.2$, $\delta_H = 1$, and $\delta_L = 0$. For the top left panel, $\beta^I = 0.7$; for the bottom left panel, $\phi = 0.7$. In the top right panel, I fix the discount factor $r = 0.003$ and $\beta^I = 0.7$, and show, for each ϕ , the punishment strategy τ . The top three τ lines (in black) correspond to punishment in trades in which uninformed agents are buyers subject to the punishment and the bottom three τ lines (in gray) correspond to punishments to informed buyers. Accordingly, the bottom right panel displays τ as a function of β^I when $r = 0.003$ and $\phi = 0.7$.

in the economy in the stationary distribution, namely, $\mu^I + \mu^U$. In the absence of misaligned agents, all assets are successfully allocated to high-valuation agents. In all-trade equilibria, constrained efficiency is met: the measure of misaligned agents is minimized, subject to the meeting technology and the asset-holding shocks. The reason is that every portfolio-balancing meeting results in trade, so that assets efficiently flow from low- to high-valuation agents.

The total measure of misaligned agents is

$$(\mu^U + \mu^I) = \left[-\frac{\eta}{\lambda} + \sqrt{\left(\frac{\eta}{\lambda}\right)^2 + \frac{1}{2} \frac{\eta}{\lambda}} \right].$$

If meeting frictions vanish, that is, $\lambda \rightarrow +\infty$, the proportion of misaligned agents goes to 0. Instead, when η is much larger than λ , $\mu^U + \mu^I$ converges to $1/4$, which is equivalent to the asset being randomly assigned across agents in the market.

4.2 Core–periphery equilibria

I now build a class of equilibria (core–periphery equilibria) in which uninformed agents do not trade with each other. Trades between uninformed agents are hindered by the risk of default, because informed agents do not punish uninformed buyers who default on uninformed sellers.

As before, I start with a conjecture of trade indicator \mathcal{I} and surplus-sharing rule β . Positive-surplus trades take place if and only if they involve at least one informed agent:

$$\mathcal{I}(i_s, v_s, i_b, v_b) = 1 \Leftrightarrow (i_s, i_b) \neq (U, U) \quad \text{and} \quad V_{v_s 0}^{i_s} - V_{v_s 1}^{i_s} + V_{v_b 1}^{i_b} - V_{v_b 0}^{i_b} > 0. \quad (21)$$

Informed agents keep at least half of the surplus when trading with uninformed agents: for some $\beta^I \in [0.5, 1)$,

$$\begin{aligned} \beta(I, v_s, U, v_b) &= \beta^I \geq \frac{1}{2}, & \beta(U, v_s, I, v_b) &= 1 - \beta^I \leq \frac{1}{2}, \\ \beta(I, v_s, I, v_b) &= \frac{1}{2}. \end{aligned} \quad (22)$$

Proposition 2 shows that a core–periphery equilibrium can be sustained by some punishing strategy if the fraction of informed agents is sufficiently high and agents are sufficiently patient.

PROPOSITION 2. *For each $\beta^I \in [0.5, 1)$, a core–periphery equilibrium satisfying (21) and (22) exists if ϕ is sufficiently large ($\phi \rightarrow 1$) and r is sufficiently small ($r \rightarrow 0$).*

Conversely, for each $r, \phi \in (0, 1)$, a $\bar{\beta} \in (0.5, 1)$ exists such that if $\beta^I > \bar{\beta}$, a core–periphery equilibrium does not exist.

Core–periphery equilibria are not efficient and feature intermediation by informed agents.

A detailed proof of Proposition 2 is provided in the [Appendix](http://econtheory.org/supp/4072/supplment.pdf) and continues in the Supplemental Material (available in a supplementary file on the journal website, <http://econtheory.org/supp/4072/supplment.pdf>), where I also characterize the values and stationary distribution in the core–periphery equilibria. The proof follows the same steps as the proof of Proposition 1.

Equilibrium Values and Trades. First, I substitute the \mathcal{I} and β conjectures in (21) and (22) into the equations defining agents' value functions in both the unflagged and flagged regimes, and the stationary distribution in (6)–(13).

The implied stationary distribution is symmetric, as in the all-trade equilibrium. Let $\hat{\mu}^U$ and $\hat{\mu}^I$ be the measures of uninformed and informed agents with misaligned portfolios in the core–periphery equilibrium, respectively.

With some algebra, I show that the value of holding an asset to an agent of information type i and valuation type v is

$$\begin{aligned} S_H^i &= \left[\frac{1}{2(r + 2\eta)} \right] (\delta_H + \delta_L) + \hat{\alpha}^i (\delta_H - \delta_L), \\ S_L^i &= \left[\frac{1}{2(r + 2\eta)} \right] (\delta_H + \delta_L) - \hat{\alpha}^i (\delta_H - \delta_L), \end{aligned}$$

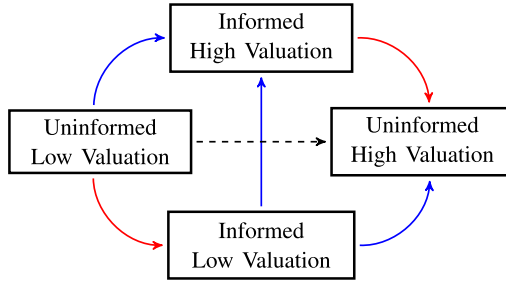


FIGURE 3. Trade pattern supported in core–periphery equilibria. The upper left, lower right, and vertical arrows indicate portfolio-balancing trades. The upper right and lower left arrows indicate intermediation trades. The dashed arrow indicates a trade that does not occur in this pattern, due to limited commitment.

where

$$\hat{\alpha}^I = \frac{1}{2(r + 2\eta + \lambda(2\hat{\mu}^U \beta^I + \hat{\mu}^I))} \tag{23}$$

and

$$\hat{\alpha}^U = \left[\frac{r + 2\eta + \lambda\beta^I(2\hat{\mu}^U + \hat{\mu}^I) + \lambda(1 - \beta^I)\left(\frac{\phi}{2} - \hat{\mu}^I\right)}{r + 2\eta + \lambda\frac{\phi}{2}(1 - \beta^I)} \right] \hat{\alpha}^I. \tag{24}$$

Because $\beta^I \geq 0.5$, $\alpha^U > \alpha^I$ and agents’ values are ordered $S_H^U > S_H^I > S_L^I > S_L^U$.

With these values, and the conjecture that positive-surplus trades take place if and only if they involve at least one informed trader, we can verify the trading pattern in core–periphery equilibrium, which is depicted in Figure 3. As in all-trade equilibria, informed agents engage in portfolio-balancing trades with both informed and uninformed trading partners. However, portfolio-balancing trades between two uninformed agents do not happen.

Intermediation trades also occur in equilibrium, where informed sellers with high valuation sell to uninformed buyers with high valuation, and informed buyers with low valuation buy assets from uninformed buyers with low valuation.

This equilibrium thus embeds a core–periphery trade network, whereby informed agents are core traders, who trade among themselves, as well as with the peripheral uninformed agents. Uninformed agents form the network periphery and do not trade with each other directly, but rather have their trades endogenously intermediated by informed agents in the network’s core.

In all-trade equilibria, intermediation stems from the surplus-sharing rule that benefits informed agents. Although this channel is still present in core–periphery equilibria, an additional motive for intermediation exists: informed agents at the network’s core effectively have more trade opportunities and trade at a faster rate than uninformed agents in the periphery.

The difference in opportunities to trade is not inherent to the agents, but rather a feature of the equilibrium conjecture that uninformed agents do not trade with each other even if the trade has positive surplus. This conjecture is sustained in equilibrium by a punishing strategy that does not punish uninformed buyers who default on uninformed sellers. As such, the threat of default prevents trades between peripheral uninformed agents.

Core–Periphery Equilibrium with $\beta^I = 0.5$. When $\beta^I = 0.5$, surplus is shared evenly between the buyer and seller in all meetings that result in trade, even between informed and uninformed agents. In that case, we *still* have $S_H^U > S_H^I > S_L^I > S_L^U$. To verify this ordering, substitute $\beta^I = 0.5$ into (23) and (24) to find

$$\hat{\alpha}^I = \frac{1}{2(r + 2\eta + \lambda(\hat{\mu}^U + \hat{\mu}^I))} \quad \text{and} \quad \hat{\alpha}^U = \left[\frac{r + 2\eta + \lambda\hat{\mu}^U + \frac{\lambda\phi}{4}}{r + 2\eta + \frac{\lambda\phi}{4}} \right] \hat{\alpha}^I,$$

so that $\hat{\alpha}^U > \hat{\alpha}^I$ even if $\beta^I = 0.5$. This observation illustrates that in core–periphery equilibria, intermediation does not stem solely from the asymmetry in surplus sharing between informed and uninformed agents. Rather, the core–periphery structure of the trading network itself yields a strictly positive value to intermediation trades.

Verifying Equilibrium Existence. To verify that a τ that supports the core–periphery equilibrium conjectures exists, I follow the same steps as with all-trade equilibria in Section 4.1, with one caveat, which I now explain.

To ensure that no trade takes place between uninformed agents, even when trades have positive surplus, informed agents must set no punishment for default in trades between two uninformed agents. Formally, set $\tau(i_s, v_s, i_b, v_b) = 0$ whenever $(i_s, i_b) = (U, U)$. With this (non-) punishment in place, an uninformed seller foresees that an uninformed buyer will default on any price offer and, thus, refuses to sell.

For all the trades (i_s, v_s, i_b, v_b) that are conjectured to take place, I show that if r is sufficiently small and ϕ is sufficiently large, a probability of punishment $\tau(i_s, v_s, i_b, v_b)$ exists that exactly implements the conjectured surplus-sharing rule.

Efficiency. Constrained efficiency is not achieved in core–periphery equilibria. Because meetings between uninformed agents do not result in trade, some opportunities to transfer assets from low- to high-valuation agents are missed. Consequently, the proportion of misaligned agents is higher than in all-trade equilibria,¹¹ and the total flow payoff to the economy is not as large as it could be.

4.3 Other equilibria

The complete equilibrium set is not exhausted by all-trade and core–periphery equilibria.

Reverse-Intermediation Equilibria. In the equilibria proposed in Sections 4.1 and 4.2, informed agents are assigned higher shares of the surplus in meetings as well as

¹¹This fact is verified in the Appendix.

more opportunities to securely trade. As discussed, both channels lead to intermediation trades whereby informed agents act as the intermediaries.

For each of the equilibria proposed, a reverse equilibrium exists where *uninformed* agents are the intermediaries. For example, “reverse” all-trade equilibria exist where $\beta^I < 0.5$, in which case uninformed agents keep a higher share of trade surplus than informed agents. In that case, reverse intermediation takes place. Equally, “reverse” core–periphery equilibria exist where $\beta^I < 0.5$ and informed agents are peripheral and do not trade with each other due to the risk of default. Again, reverse intermediation takes place.

Such equilibria are not intuitive, as they rely on informed agents coordinating on uninformed agents’ larger shares of the trade surplus than that of informed agents. Indeed, in Section 5, I show that reverse-intermediation equilibria are not selected if informed agents can coordinate to maximize their own value.

Other. Equilibrium trading is determined by the ordering of agents’ values ($S_H^I, S_H^U, S_L^I, S_L^U$) as well as whether, when agents meet, the trades are secured by the punishing strategy. Because agents with high valuation have a higher flow value from holding the asset, in any equilibrium $S_H^i > S_L^j$ for $i, j \in \{I, U\}$. Moreover, the symmetry requirements in (14) and (15) imply $(S_H^I - S_H^U) = -(S_L^I - S_L^U)$.

Thus, in any equilibrium, either $S_H^U \geq S_H^I > S_L^I \geq S_L^U$ or $S_H^I > S_H^U > S_L^U > S_L^I$. Although the whole equilibrium set is not exhausted by the equilibria described so far, they do illustrate these two possibilities. They also illustrate that intermediation trades happen whenever $S_H^I \neq S_H^U$ and $S_L^I \neq S_L^U$, and that these differences can be generated either due to the equilibrium surplus sharing or due to the set of secured trade opportunities in an equilibrium.

5. MAXIMIZING VALUE TO INFORMED AGENTS

The equilibria shown in the last section demonstrate the channels that generate intermediation, determined by the punishing strategy. I now refine the set of equilibria by characterizing equilibria that informed agents prefer. This requirement is natural because informed agents are the ones who coordinate on punishing strategies that sustain trade in equilibrium.

Define the objects V^I and V^U , the value of being informed and uninformed, respectively:

$$V^I \equiv \frac{V_{H0}^I + V_{L0}^I}{2}, \tag{25}$$

$$V^U \equiv \frac{V_{H0}^U + V_{L0}^U}{2}. \tag{26}$$

The weighting in V^I and V^U reflects that half the agents have high valuation and half have low valuation, regardless of their information type. The definition of these values also considers agents who enter the market without holding an asset. It already accounts for the value of holding the asset, because these agents over time are hit by asset-holding shocks and also trade assets. The results in this section equally hold if we

look at the opposite case, where the value accounts for an agent who enters the market holding an asset.

Proposition 3, shows that although equilibria where uninformed agents act as intermediaries exist, they are not the equilibria that informed agents prefer. This result allows us to rule out reverse-intermediation equilibria, as in Section 4.3, and asserts that intermediation is typically performed by informed agents (as in all-trade and core-periphery equilibria).

PROPOSITION 3. *If there exists an equilibrium featuring intermediation by uninformed agents, then another equilibrium exists, yielding a higher V^I , in which uninformed agents are not intermediaries.*

The next result states that, to the informed agent, core-periphery equilibria are preferable to all-trade equilibria: preventing uninformed agents from trading with each other is beneficial to informed agents. One implication of Proposition 4 is, thus, that informed agents select inefficient equilibria to maximize their own value.

PROPOSITION 4. *For any $\beta^I \in [0.5, 1)$, the core-periphery equilibrium with surplus share β^I yields a higher V^I than the all-trade equilibrium with that same surplus share β^I .*

All-trade and core-periphery equilibria differ in that, in the latter, uninformed agents do not trade with each other. When trade between uninformed agents is shut down, they have fewer trade opportunities and are more eager to trade in meetings with informed agents. Therefore, the surplus in such meetings increases and informed agents can extract higher value from them.

Finally, Proposition 5 states that the value to informed agents is higher when informed agents keep a larger share of surplus in trades with uninformed agents.

PROPOSITION 5. *The value to informed agents (V^I) is increasing in β^I in both all-trade and core-periphery equilibria.*

The proofs of Propositions 3, 4, and 5 are provided in the [Appendix](#).

5.1 Numerical exercise

Propositions 4 and 5 allow us to compare informed agents' values across different equilibria. However, they do not determine which equilibrium maximizes informed agents' values among the equilibria that can be supported for a given set of parameters. In this numerical exercise, I compare all-trade and core-periphery equilibria that can be supported across different parameterizations.

Figure 4 displays three main results from numerical simulations.¹²

¹²The qualitative features of Figure 4 described in Numerical Results 1, 2, and 3 are robust to other parameter specifications.

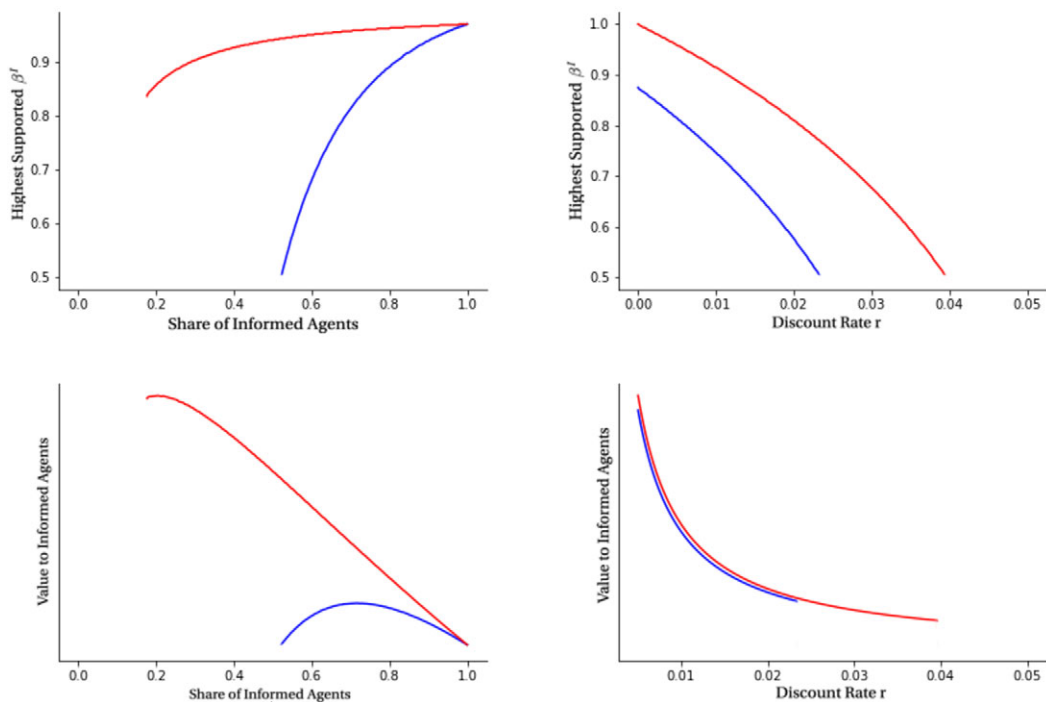


FIGURE 4. In the top two panels, I plot the highest β^I achievable in all-trade (lower lines) and core-periphery (higher lines) equilibria as a function of the share of informed agents ϕ (left panel, keeping $r = 0.005$) and discount rate r (right panel, keeping $\phi = 0.7$). In the bottom two panels, I plot the highest V^I achievable in all-trade (again, lower lines) and core-periphery (again, higher lines) equilibria as a function of the share of informed agents ϕ and discount rate r . Other parameters are $\lambda = 2$, $\eta = 0.2$, $\delta_H = 1$, and $\delta_L = 0$.

NUMERICAL RESULT 1. If an all-trade equilibrium with surplus share β^I exists, then a core-periphery equilibrium with surplus share of at least β^I also exists.

This result, along with Proposition 4, implies that when an all-trade equilibrium exists, it is dominated—in terms of value to informed agents—by a core-periphery equilibrium that also exists. In Figure 4, I confirm that the highest value to informed agents achieved by an all-trade equilibrium is smaller than the highest value achieved by a core-periphery equilibrium.

NUMERICAL RESULT 2. Informed agents’ value is higher under the best available core-periphery equilibrium than under the best available all-trade equilibrium.

Finally, I state a couple of comparative static results. First, the upper bound on β^I mentioned in Propositions 1 and 2 is higher when agents are more patient and more informed agents exist. Second, informed agents’ value is highest at some interior value of ϕ .

NUMERICAL RESULT 3. In both all-trade and core–periphery equilibria

- (i) the highest supported β^I is increasing in ϕ and decreasing in r
- (ii) informed agents' value is maximized at an interior $\phi \in (0, 1)$.

One way to interpret the measure of informed agents ϕ is as the underlying technology, where $\phi = 1$ is the frictionless benchmark where all agents access the record-keeping technology. In that case, Numerical Result 3 shows that informed agents attain the highest value in the presence of nonzero friction. In a first region, where ϕ values are very low, no equilibrium exists. In a second region, where equilibria can be sustained, increasing ϕ leads to two effects: first, the highest β^I that can be sustained is higher, which increases V^I ; second, the share of uninformed agents decreases, so the measure of agents from whom informed agents can extract high surplus shares or intermediate trades is lower, which decreases V^I .

6. CONCLUSION

I developed a dynamic model where agents meet bilaterally to trade and buyers cannot commit to payments. This limited commitment friction is present in many applications, such as markets for stolen goods or markets for corrupt exchange. A robust feature of equilibria in the model, also empirically observed in these markets, is the presence of intermediation, where some central agents trade goods not for their own use, but rather to profit from future trade value.

In the model, equilibria with trade are supported by informed agents who punish traders who do not honor payments. In illegal markets, the presence of well connected (informed) groups is indeed important to ensure the “good” behavior of market participants. For example, Della Porta and Vanucci (2016) argue that the mafia, whose business leans on detailed knowledge of the behavior of individuals in a community, are important enforcers in the Italian market for corrupt exchange.

In Section 3, I proposed two main classes of equilibria, which are supported by informed agents' threats to punish defaulting agents. All-trade equilibria are efficient and all agents are able to trade despite the limited commitment friction, and core–periphery equilibria are inefficient because peripheral uninformed agents are unable to trade among themselves. One important result, stated in Section 5, is that these latter inefficient equilibria are robust, because they yield higher value to informed agents than the former, efficient, equilibria.

Another feature of core–periphery equilibria is that informed agents do not use information about trades between uninformed agents. Consequently, they are also robust to variations in the information technology: if informed agents have access only to information about trades involving at least one informed agent, these equilibria are still supported. On the contrary, equilibria in which uninformed agents trade with each other require informed agents to have access to all the information available.

APPENDIX

A.1 *Proof of Proposition 1 (all-trade equilibria)*

I build the all-trade equilibria through a big guess and verify.

Guesses.

Surplus sharing:

$$\beta(I, v_s, U, v_b) = \beta^I \geq \frac{1}{2}, \quad \beta(U, v_s, I, v_b) = 1 - \beta^I \leq \frac{1}{2},$$

$$\beta(I, v_s, I, v_b) = \frac{1}{2}, \quad \beta(U, v_s, U, v_b) = \frac{1}{2}.$$

Trading pattern: $\mathcal{I}(i_s, v_s, i_b, v_b) = 1 \Leftrightarrow v_s = L$ and $v_b = H$.

Stationary Distribution. Given the guesses for \mathcal{I} , the inflow equal to outflow equations for the stationary distribution become, for $i \in \{I, U\}$,

$$\mu_{L1}^U(\eta + \lambda(\mu_{H0}^I + \mu_{H0}^U + \mu_{L0}^I)) = \eta\mu_{L0}^U, \quad (27)$$

$$\mu_{H0}^U(\eta + \lambda(\mu_{L1}^I + \mu_{L1}^U + \mu_{H1}^I)) = \eta\mu_{H1}^U, \quad (28)$$

$$\mu_{L1}^I(\eta + \lambda(\mu_{H0}^U + \mu_{H0}^I)) = (\eta + \lambda\mu_{L1}^U)\mu_{L0}^I, \quad (29)$$

$$\mu_{H0}^I(\eta + \lambda(\mu_{L1}^U + \mu_{L1}^I)) = \mu_{H1}^I(\eta + \lambda\mu_{H0}^U). \quad (30)$$

In (27), substitute $\mu_{L0}^U = \frac{1-\phi}{2} - \mu_{L1}^U$ (which is true because half of uninformed agents have low valuation). Similarly, in (28), substitute $\mu_{H1}^U = \frac{1-\phi}{2} - \mu_{H0}^U$. Then combine the two equations to get

$$\begin{aligned} \mu_{L1}^U(2\eta + \lambda(\mu_{H0}^U + \mu_{H0}^I)) + \lambda\mu_{L0}^I\mu_{L1}^U &= \mu_{H0}^U(2\eta + \lambda(\mu_{L1}^U + \mu_{L1}^I)) + \lambda\mu_{H1}^I\mu_{H0}^U \\ &= \frac{\eta(1-\phi)}{2}. \end{aligned} \quad (31)$$

In (29), substitute $\mu_{L0}^I = \frac{\phi}{2} - \mu_{L1}^I$ (as before, this is true because half of informed agents have low valuation). Similarly, in (30), substitute $\mu_{H1}^I = \frac{\phi}{2} - \mu_{H0}^I$. Then combine the two equations to get

$$\begin{aligned} \mu_{L1}^I(2\eta + \lambda(\mu_{H0}^U + \mu_{H0}^I)) - \lambda\mu_{L0}^I\mu_{L1}^U &= \mu_{H0}^I(2\eta + \lambda(\mu_{L1}^U + \mu_{L1}^I)) - \lambda\mu_{H1}^I\mu_{H0}^U \\ &= \frac{\eta\phi}{2}. \end{aligned} \quad (32)$$

Adding up (31) and (32) yields

$$(\mu_{L1}^I + \mu_{L1}^U)(2\eta + \lambda(\mu_{H0}^U + \mu_{H0}^I)) = (\mu_{H0}^I + \mu_{H0}^U)(2\eta + \lambda(\mu_{L1}^U + \mu_{L1}^I)) = \frac{\eta}{2} \quad (33)$$

$$\Rightarrow (\mu_{L1}^I + \mu_{L1}^U) = (\mu_{H0}^I + \mu_{H0}^U). \quad (34)$$

Now substitute (33) back into (34) to get

$$\begin{aligned} \lambda(\mu_{L1}^I + \mu_{L1}^U)^2 + 2\eta(\mu_{L1}^I + \mu_{L1}^U) - \frac{\eta}{2} &= 0 \\ \Rightarrow (\mu_{L1}^I + \mu_{L1}^U) &= (\mu_{H0}^I + \mu_{H0}^U) = -\frac{\eta}{\lambda} + \sqrt{\left(\frac{\eta}{\lambda}\right)^2 + \frac{1}{2}\frac{\eta}{\lambda}}. \end{aligned} \quad (35)$$

Plug (35) back into (31) and (32) to get $\mu_{H0}^U = \mu_{L1}^U =: \mu^U$ and $\mu_{H0}^I = \mu_{L1}^I =: \mu^I$, and, finally,

$$\mu^U = -\left(\frac{\eta}{\lambda} + \frac{\phi}{4}\right) + \sqrt{\left(\frac{\eta}{\lambda}\right)^2 + \frac{1}{2}\frac{\eta}{\lambda} + \left(\frac{\phi}{4}\right)^2},$$

$$\mu^I = -\frac{\eta}{\lambda} - \mu^U + \sqrt{\left(\frac{\eta}{\lambda}\right)^2 + \frac{1}{2}\frac{\eta}{\lambda}}.$$

Unflagged Values. Taking into account the guesses for \mathcal{I} and β , unflagged values are given by the system

$$\begin{aligned} rV_{H0}^I &= \eta(V_{H1}^I - V_{H0}^I) + \lambda\mu_{L1}^I \frac{V_{H1}^I - V_{H0}^I + V_{L0}^I - V_{L1}^I}{2} + \lambda\mu_{L1}^U \beta^I [V_{H1}^I - V_{H0}^I + V_{L0}^U - V_{L1}^U], \\ rV_{H1}^I &= \delta_H + \eta(V_{H0}^I - V_{H1}^I) + \lambda\mu_{H0}^U \beta^I [V_{H0}^I - V_{H1}^I + V_{H1}^U - V_{H0}^U], \\ rV_{L1}^I &= \delta_L + \eta(V_{L0}^I - V_{L1}^I) + \lambda\mu_{H0}^I \frac{V_{L0}^I - V_{L1}^I + V_{H1}^I - V_{H0}^I}{2} \\ &\quad + \lambda\mu_{H0}^U \beta^I [V_{L0}^I - V_{L1}^I + V_{H1}^U - V_{H0}^U], \\ rV_{L0}^I &= \eta(V_{L1}^I - V_{L0}^I) + \lambda\mu_{L1}^U \beta^I [V_{L1}^I - V_{L0}^I + V_{L0}^U - V_{L1}^U], \\ rV_{H0}^U &= \eta(V_{H1}^U - V_{H0}^U) + \lambda\mu_{L1}^I (1 - \beta^I) [V_{H1}^U - V_{H0}^U + V_{L0}^I - V_{L1}^I] \\ &\quad + \lambda\mu_{H1}^I (1 - \beta^I) [V_{H1}^U - V_{H0}^U + V_{H0}^I - V_{H1}^I] + \lambda\mu_{L1}^U \frac{V_{H1}^U - V_{H0}^U + V_{L0}^U - V_{L1}^U}{2}, \\ rV_{H1}^U &= \delta_H + \eta(V_{H0}^U - V_{H1}^U), \\ rV_{L1}^U &= \delta_L + \eta(V_{L0}^U - V_{L1}^U) + \lambda\mu_{H0}^I (1 - \beta^I) [V_{L0}^U - V_{L1}^U + V_{H1}^I - V_{H0}^I] \\ &\quad + \lambda\mu_{L0}^I (1 - \beta^I) [V_{L0}^U - V_{L1}^U + V_{L1}^I - V_{L0}^I] + \lambda\mu_{H0}^U \frac{V_{H1}^U - V_{H0}^U + V_{L0}^U - V_{L1}^U}{2}, \\ rV_{L0}^U &= \eta(V_{L1}^U - V_{L0}^U). \end{aligned}$$

Using the result from the stationary distribution and writing the system in terms of the values of holding an asset, we get

$$\begin{aligned} rS_H^I &= \delta_H - 2\eta S_H^I + \lambda\mu^U \beta^I (S_H^U - S_H^I) + \lambda\mu^I \frac{(S_L^I - S_H^I)}{2} + \lambda\mu^U \beta^I (S_L^U - S_H^I), \\ rS_L^I &= \delta_L - 2\eta S_L^I + \lambda\mu^U \beta^I (S_L^U - S_L^I) + \lambda\mu^I \frac{(S_H^I - S_L^I)}{2} + \lambda\mu^U \beta^I (S_H^U - S_L^I), \\ rS_H^U &= \delta_H - 2\eta S_H^U + \lambda\mu^I (1 - \beta^I) (S_L^I - S_H^U) \\ &\quad + \lambda \frac{\phi - 2\mu^I}{2} (1 - \beta^I) (S_H^I - S_H^U) + \lambda\mu^U \frac{(S_L^U - S_H^U)}{2}, \\ rS_L^U &= \delta_L - 2\eta S_L^U + \lambda\mu^I (1 - \beta^I) (S_H^I - S_L^U) \end{aligned}$$

$$+ \lambda \frac{\phi - 2\mu^I}{2} (1 - \beta^I) (S_L^I - S_L^U) + \lambda \mu^U \frac{(S_L^U - S_H^U)}{2}.$$

Add up the first two and the last two to get

$$(r + 2\eta)(S_H^I + S_L^I) = \delta_H + \delta_L + 2\lambda\mu^U \beta^I (S_H^U + S_L^U) - 2\lambda\mu^U \beta^I (S_H^I + S_L^I),$$

$$(r + 2\eta)(S_H^U + S_L^U) = \delta_H + \delta_L + \frac{\lambda\phi}{2} (1 - \beta^I) (S_H^I + S_L^I) - \frac{\lambda\phi}{2} (1 - \beta^I) (S_H^U + S_L^U).$$

These imply $(S_H^I + S_L^I) = (S_H^U + S_L^U) = \frac{\delta_H + \delta_L}{r + 2\eta}$. Now, from the original system, subtract the second equation from the first and the fourth from the third to find

$$(r + 2\eta)(S_H^I - S_L^I) = \delta_H - \delta_L - \lambda(2\mu^U \beta^I + \mu^I)(S_H^I - S_L^I),$$

$$(r + 2\eta)(S_H^U - S_L^U) = \delta_H - \delta_L - \lambda \frac{\phi}{2} (1 - \beta^I) (S_H^U - S_L^U) + \lambda \left(\frac{\phi}{2} - \mu^I \right) (1 - \beta^I) (S_H^I - S_L^I) - \lambda \mu^U (S_H^U - S_L^U).$$

Rearrange these to get the expressions

$$(S_H^I - S_L^I) = \alpha^I (\delta_H - \delta_L),$$

$$(S_H^U - S_L^U) = \alpha^U (\delta_H - \delta_L), \tag{36}$$

where

$$\alpha^I = \frac{1}{2(r + 2\eta + \lambda(2\mu^U \beta^I + \mu^I))},$$

$$\alpha^U = \left[\frac{r + 2\eta + \lambda\beta^I(2\mu^U + \mu^I) + \lambda(1 - \beta^I)(\phi/2 - \mu^I)}{r + 2\eta + \lambda(\mu^U + \phi/2(1 - \beta^I))} \right] \alpha^I,$$

which finally implies

$$S_H^i = \left[\frac{1}{2(r + 2\eta)} \right] (\delta_H + \delta_L) + \frac{\alpha^i}{2} (\delta_H - \delta_L),$$

$$S_L^i = \left[\frac{1}{2(r + 2\eta)} \right] (\delta_H + \delta_L) - \frac{\alpha^i}{2} (\delta_H - \delta_L).$$

Punishing Strategy. To conclude that an all-trade equilibrium exists, all that is left to show is that there exists a punishing strategy τ under which the conjectured β satisfies (3).

For the trades for which $\mathcal{I}(i_s, v_s, i_b, v_b) = 0$, which have nonpositive surplus, as found above, we can set any punishment level. For instance, let $\tau(i_s, v_s, i_b, v_b) = 0$ if $\mathcal{I}(i_s, v_s, i_b, v_b) = 0$.

Now define $D_{va}^i = V_{va}^i - \tilde{V}_{va}^i$. The other conditions that need to be satisfied by τ so as to support the conjectured β are

$$\tau(I, L, I, H)D_{H1}^I = \frac{S_H^I + S_L^I}{2} \tau(U, L, I, L)D_{L1}^I = \beta^I S_L^U + (1 - \beta^I) S_L^I,$$

$$\begin{aligned}\tau(U, L, I, H)D_{H1}^I &= \beta^I S_L^U + (1 - \beta^I) S_H^I \tau(I, H, U, H)D_{H1}^U \\ &= \beta^I S_H^U + (1 - \beta^I) S_H^I,\end{aligned}\quad (37)$$

$$\tau(I, L, U, H)D_{H1}^U = \beta^I S_H^U + (1 - \beta^I) S_L^I \tau(U, L, U, H)D_{H1}^U = \frac{S_H^U + S_L^U}{2}.$$

First, I solve for the values D_{H1}^I , D_{L1}^I , and D_{H1}^U . The system that defines $\{D_{va}^I\}$ is

$$\begin{aligned}rD_{H1}^I &= (\eta + \lambda\mu^U)(D_{H0}^I - D_{H1}^I), \\ rD_{L1}^I &= (\eta + \lambda\mu^U)(D_{L0}^I - D_{L1}^I) + \lambda\mu^I \frac{S_H^I - S_L^I}{2}, \\ rD_{L0}^I &= (\eta + \lambda\mu^U)(D_{L1}^I - D_{L0}^I) - \lambda\mu^U (\beta^I S_L^U + (1 - \beta^I) S_L^I), \\ rD_{H0}^I &= (\eta + \lambda\mu^U)(D_{H1}^I - D_{H0}^I) + \lambda\mu^I \frac{S_H^I - S_L^I}{2} - \lambda\mu^U (\beta^I S_H^I + (1 - \beta^I) S_L^U).\end{aligned}$$

Combining the first line with the fourth and the second line with the third yields

$$\begin{aligned}r(D_{H1}^I - D_{H0}^I) &= -2(\eta + \lambda\mu^U)(D_{H1}^I - D_{H0}^I) - \lambda\mu^I \frac{S_H^I - S_L^I}{2} + \lambda\mu^U (\beta^I S_H^I + (1 - \beta^I) S_L^U), \\ r(D_{L1}^I - D_{L0}^I) &= -2(\eta + \lambda\mu^U)(D_{L1}^I - D_{L0}^I) + \lambda\mu^I \frac{S_H^I - S_L^I}{2} + \lambda\mu^U (\beta^I S_L^U + (1 - \beta^I) S_L^I).\end{aligned}$$

Solve to find

$$\begin{aligned}(D_{H1}^I - D_{H0}^I) &= -\frac{\lambda\mu^I}{(r + 2\eta + 2\lambda\mu^U)} \frac{S_H^I - S_L^I}{2} + \frac{\lambda\mu^U}{(r + 2\eta + 2\lambda\mu^U)} (\beta^I S_H^I + (1 - \beta^I) S_L^U), \\ (D_{L1}^I - D_{L0}^I) &= \frac{\lambda\mu^I}{(r + 2\eta + 2\lambda\mu^U)} \frac{S_H^I - S_L^I}{2} + \frac{\lambda\mu^U}{(r + 2\eta + 2\lambda\mu^U)} (\beta^I S_L^U + (1 - \beta^I) S_L^I).\end{aligned}$$

Plug this back into the original system to get

$$D_{H1}^I = \frac{\eta + \lambda\mu^U}{r(r + 2\eta + 2\lambda\mu^U)} \left[\lambda\mu^I \frac{S_H^I - S_L^I}{2} - \lambda\mu^U (\beta^I S_H^I + (1 - \beta^I) S_L^U) \right], \quad (38)$$

$$\begin{aligned}D_{L1}^I &= \frac{r + \eta + \lambda\mu^U}{r(r + 2\eta + 2\lambda\mu^U)} \lambda\mu^I \frac{S_H^I - S_L^I}{2} \\ &\quad - \frac{\lambda\mu^U (\eta + \lambda\mu^U)}{r(r + 2\eta + 2\lambda\mu^U)} (\beta^I S_L^U + (1 - \beta^I) S_L^I).\end{aligned}\quad (39)$$

The system that defines $\{D_{va}^U\}$ is

$$rD_{H1}^U = \eta(D_{H0}^U - D_{H1}^U),$$

$$rD_{H0}^U = (\eta + \lambda\mu^U)(D_{H1}^U - D_{H0}^U)$$

$$\begin{aligned}
 & + \lambda\mu^I(1 - \beta^I)(S_H^U - S_L^I) + \lambda(\phi/2 - \mu^I)(1 - \beta^I)(S_H^U - S_H^I) - \lambda\mu^U \frac{S_H^U + S_L^U}{2}, \\
 rD_{L1}^U & = (\eta + \lambda\mu^U)(D_{L0}^U - D_{L1}^U) + \lambda\mu^I(1 - \beta^I)(S_H^I - S_L^U) \\
 & + \lambda(\phi/2 - \mu^I)(1 - \beta^I)(S_L^I - S_L^U), \\
 rD_{L0}^U & = \eta(D_{L1}^U - D_{L0}^U).
 \end{aligned}$$

Subtract the second line from the first to get

$$\begin{aligned}
 r(D_{H1}^U - D_{H0}^U) & = -(2\eta + \lambda\mu^U)(D_{H1}^U - D_{H0}^U) - \lambda\mu^I(1 - \beta^I)(S_H^U - S_L^I) \\
 & - \lambda(\phi/2 - \mu^I)(1 - \beta^I)(S_H^U - S_H^I) + \lambda\mu^U \frac{S_H^U + S_L^U}{2} \\
 \Rightarrow (D_{H1}^U - D_{H0}^U) & = -\frac{\lambda\mu^I}{r + 2\eta + \lambda\mu^U}(1 - \beta^I)(S_H^U - S_L^I) \\
 & - \frac{\lambda(\phi/2 - \mu^I)}{r + 2\eta + \lambda\mu^U}(1 - \beta^I)(S_H^U - S_H^I) + \frac{\lambda\mu^U}{r + 2\eta + \lambda\mu^U} \frac{S_H^U + S_L^U}{2}.
 \end{aligned}$$

Finally, substitute this back into the original system to get

$$\begin{aligned}
 D_{H1}^U & = \frac{\eta\lambda\mu^I(1 - \beta^I)}{r(r + 2\eta + \lambda\mu^U)}(S_H^U - S_L^I) + \frac{\eta\lambda(\phi/2 - \mu^I)(1 - \beta^I)}{r(r + 2\eta + \lambda\mu^U)}(S_H^U - S_H^I) \\
 & - \frac{\eta\lambda\mu^U}{r(r + 2\eta + \lambda\mu^U)} \frac{S_H^U + S_L^U}{2}.
 \end{aligned} \tag{40}$$

With these expressions in hand, we can check whether the inequalities

$$\begin{aligned}
 D_{H1}^I(0) & \geq \max\left\{\frac{S_H^I + S_L^I}{2}, \beta^I S_L^U + (1 - \beta^I)S_H^I\right\}, \\
 D_{L1}^I(0) & \geq \beta^I S_L^U + (1 - \beta^I)S_L^I, \\
 D_{H1}^U(0) & \geq \max\left\{\beta^I S_H^U + (1 - \beta^I)S_H^I, \beta^I S_H^U + (1 - \beta^I)S_L^I, \frac{S_H^U + S_L^U}{2}\right\}
 \end{aligned}$$

are satisfied. It is easy to check that these conditions hold as $\phi \rightarrow 1$ (i.e., $\mu^I > 0, \mu^U \rightarrow 0$) and $r \rightarrow 0$: when ϕ is large, the left-hand side of all three inequalities is positive, and when r is small, the left-hand side grows unboundedly, whereas the right-hand side is bounded.

Under these parameter conditions, I can conclude that $\tau(I, L, I, H)$, $\tau(U, L, I, L)$, $\tau(U, L, I, H)$, $\tau(I, H, U, H)$, $\tau(I, L, U, H)$, and $\tau(I, L, I, H)$ exist such that expressions (37) are satisfied, as desired.

Moreover, notice that for each $\phi \in (0, 1)$, there is some $\bar{\beta}$ such that if $\beta^I > \bar{\beta}$, then D_{H1}^U (given in (40)) is negative, and so $D_{H1}^U(0) < \max\{\beta^I S_H^U + (1 - \beta^I)S_H^I, \beta^I S_H^U + (1 - \beta^I)S_L^I, \frac{S_H^U + S_L^U}{2}\}$. In that case, no τ exists that can support the equilibrium conjecture.

This proves the second part of the proposition, namely, that some $\bar{\beta}$ exists such that if $\beta^I > \bar{\beta}$, an all-trade equilibrium does not exist.

A.2 Proof of Proposition 2 (core–periphery equilibria)

Once again, we build this equilibrium with a big guess and verify. The steps to solving for the equilibrium objects, which I report here, are similar to those in all-trade equilibria. They can be found in the Supplemental Material.

Guesses.

Surplus sharing:

$$\beta(I, v_s, U, v_b) = \beta^I \geq \frac{1}{2}, \quad \beta(U, v_s, I, v_b) = 1 - \beta^I \leq \frac{1}{2}$$

$$\beta(I, v_s, I, v_b) = \frac{1}{2}.$$

Efficient trading if and only if an informed agent is involved:

$$\mathcal{I}(i_s, v_s, i_b, v_b) = 1 \Leftrightarrow (i_s, i_b) \neq (U, U) \text{ and } V_{v_s 0}^{i_s} - V_{v_s 1}^{i_s} + V_{v_b 1}^{i_b} - V_{v_b 0}^{i_b} > 0.$$

Stationary Distribution. We have

$$\mu_{L1}^U = \mu_{H0}^U = \hat{\mu}^U = \frac{1 - \phi}{4 + \frac{\eta}{\lambda}}$$

$$\mu_{H0}^I = \mu_{L1}^I = \hat{\mu}^I = -\frac{\eta + \lambda \hat{\mu}^U}{\lambda} + \sqrt{\left(\frac{\eta + \lambda \hat{\mu}^U}{\lambda}\right)^2 + \frac{\phi}{2} \frac{\eta + \lambda \hat{\mu}^U}{\lambda}}.$$

Unflagged Values. We have

$$S_H^i = \left[\frac{1}{2(r + 2\eta)} \right] (\delta_H + \delta_L) + \frac{\alpha^i}{2} (\delta_H - \delta_L),$$

$$S_L^i = \left[\frac{1}{2(r + 2\eta)} \right] (\delta_H + \delta_L) - \frac{\alpha^i}{2} (\delta_H - \delta_L),$$

where $\hat{\alpha}^I = \frac{1}{2(r + 2\eta + \lambda(2\hat{\mu}^U \beta^I + \hat{\mu}^I))}$ and $\hat{\alpha}^U = \left[\frac{r + 2\eta + \lambda \hat{\mu}^U + \frac{\lambda \phi}{4}}{r + 2\eta + \frac{\lambda \phi}{4}} \right] \hat{\alpha}^I$.

This confirms that the conjectured \mathcal{I} and β indeed satisfy (4) and (5).

Punishing Strategy. To support the core–periphery equilibrium, I need to find a τ that supports the conjectures. Once again, for the trades for which $\mathcal{I}(i_s, v_s, i_b, v_b) = 0$, this is trivial, and we can set $\tau(i_s, v_s, i_b, v_b) = 0$ when $\mathcal{I}(i_s, v_s, i_b, v_b) = 0$. For all other trades, we need τ to satisfy

$$\tau(I, L, I, H) D_{H1}^I = \frac{S_H^I + S_L^I}{2} \tau(U, L, I, L) D_{L1}^I = \beta^I S_L^U + (1 - \beta^I) S_L^I,$$

$$\tau(U, L, I, H) D_{H1}^I = \beta^I S_L^U + (1 - \beta^I) S_H^I,$$

$$\tau(I, H, U, H) D_{H1}^U = \beta^I S_H^U + (1 - \beta^I) S_H^I, \tag{41}$$

$$\tau(I, L, U, H)D_{H1}^U = \beta^I S_H^U + (1 - \beta^I)S_L^I.$$

We can use the same steps as in the proof of Proposition 1 to find

$$D_{H1}^I = \frac{\eta + \lambda \hat{\mu}^U}{r(r + 2\eta + 2\lambda \hat{\mu}^U)} \left[\lambda \hat{\mu}^I \frac{S_H^I - S_L^I}{2} - \lambda \hat{\mu}^U ((1 - \beta^I)S_H^I + \beta^I S_L^U) \right],$$

$$D_{L1}^I = \frac{r + \eta + \lambda \hat{\mu}^U}{r(r + 2\eta + 2\lambda \hat{\mu}^U)} \lambda \hat{\mu}^I \frac{S_H^I - S_L^I}{2} - \frac{\eta + \lambda \hat{\mu}^U}{r(r + 2\eta + 2\lambda \hat{\mu}^U)} \lambda \hat{\mu}^U ((1 - \beta^I)S_L^U + \beta^I S_L^I),$$

$$D_{H1}^U = \frac{\eta \lambda \hat{\mu}^I}{r(r + 2\eta)} (1 - \beta^I)(S_H^U - S_L^I) + \frac{\eta \lambda (\phi/2 - \hat{\mu}^I)}{r(r + 2\eta)} (1 - \beta^I)(S_H^U - S_H^I).$$

With these expressions in hand, we can check whether the inequalities

$$D_{H1}^I(0) \geq \max \left\{ \frac{S_H^I + S_L^I}{2}, \beta^I S_L^U + (1 - \beta^I)S_H^I \right\},$$

$$D_{L1}^I(0) \geq \beta^I S_L^U + (1 - \beta^I)S_L^I,$$

$$D_{H1}^U(0) \geq \max \{ \beta^I S_H^U + (1 - \beta^I)S_H^I, \beta^I S_H^U + (1 - \beta^I)S_L^I \}$$

are satisfied. It is easy to check that these conditions hold as $\phi \rightarrow 1$ (i.e., $\mu^I > 0, \mu^U \rightarrow 0$) and $r \rightarrow 0$: when ϕ is large, the left-hand side of all three inequalities is positive, and when r is small, the left-hand side grows unboundedly, whereas the right-hand side is bounded.

Under these parameter conditions, I can conclude that there exist $\tau(I, L, I, H), \tau(U, L, I, L), \tau(U, L, I, H), \tau(I, H, U, H), \tau(I, L, U, H),$ and $\tau(I, L, I, H)$ such that (41) is satisfied.

Moreover, notice that, for each $\phi \in (0, 1)$, some $\bar{\beta}$ exists such that if $\beta^I > \bar{\beta}$, then $D_{H1}^U(0) < \max \{ \beta^I S_H^U + (1 - \beta^I)S_H^I, \beta^I S_H^U + (1 - \beta^I)S_L^I \}$. In that case, there is no τ that supports the equilibrium conjecture. This proves the second part of the proposition, namely, that there exists some $\bar{\beta}$ such that if $\beta^I > \bar{\beta}$, a core-periphery equilibrium does not exist.

Showing that there are more misaligned agents than in the symmetric equilibrium.

In all-trade equilibria, the inflow-outflow equations can be written as

$$\mu^U (\eta + \lambda(\mu^U + \phi/2)) = \left(\frac{1 - \phi}{2} - \mu^U \right) \eta,$$

$$\mu^I (\eta + \lambda(\mu^U + \mu^I)) = \left(\frac{\phi}{2} - \mu^I \right) \eta$$

$$\Rightarrow \lambda(\mu^U + \mu^I)^2 + 2\eta(\mu^U + \mu^I) = \eta/2. \tag{42}$$

In core-periphery equilibria, the inflow-outflow equations can be written as

$$\hat{\mu}^U (\eta + \lambda\phi/2) = \left(\frac{1 - \phi}{2} - \hat{\mu}^U \right) \eta,$$

$$\begin{aligned} \hat{\mu}^I(\eta + \lambda(\hat{\mu}^U + \hat{\mu}^I)) &= \left(\frac{\phi}{2} - \hat{\mu}^I\right)(\eta + \lambda\hat{\mu}^U) \\ \Rightarrow \lambda(\hat{\mu}^U + \hat{\mu}^I)^2 + 2\eta(\hat{\mu}^U + \hat{\mu}^I) &= \eta/2 + \lambda(\hat{\mu}^U)^2. \end{aligned} \quad (43)$$

To satisfy (42) and (43), it must be that $\hat{\mu}^U + \hat{\mu}^I > \mu^U + \mu^I$.

A.3 Proof of Proposition 3

Suppose an equilibrium exists where uninformed agents act as intermediaries. In such an equilibrium, it must hold that $S_H^I \geq S_H^U > S_L^U \geq S_L^I$. If these inequalities are not strict, agents are indifferent between engaging in intermediation trades or not. In that case, another equilibrium exists where no intermediation trades take place and V^I and V^U stay the same, which confirms the statement in the proposition.

Now suppose instead that the inequalities are strict: $S_H^I > S_H^U > S_L^U > S_L^I$. This ordering implies that in that equilibrium, no intermediation by informed agents occurs. The values to informed agents with aligned portfolios then are given by

$$\begin{aligned} V_{L0}^I &= \frac{\eta S_L^I}{r}, \\ V_{H1}^I &= \frac{\delta_H - \eta S_H^I}{r}. \end{aligned}$$

The values to uninformed agents with aligned portfolios are given by

$$\begin{aligned} V_{L0}^U &= \frac{\eta S_L^U + \lambda \mu_{L1}^I (1 - \beta(I, L, U, L))(S_L^U - S_L^I)}{r} \geq \frac{\eta S_L^U}{r}, \\ V_{H1}^U &= \frac{\delta_H - \eta S_H^U + \lambda \mu_{H0}^I \beta(U, H, I, H)(S_H^I - S_H^U)}{r} \geq \frac{\delta_H - \eta S_H^U}{r}. \end{aligned}$$

Rewriting (25) and (26) gives

$$\begin{aligned} V^I &= \frac{1}{2}(V_{H1}^I + V_{L0}^I) - \frac{1}{2}S_H^I = \frac{\delta_H}{2r} - \frac{\eta}{2r}(S_H^I - S_L^I) - \frac{1}{2}S_H^I, \\ V^U &= \frac{1}{2}(V_{H1}^U + V_{L0}^U) - \frac{1}{2}S_H^U \geq \frac{\delta_H}{2r} - \frac{\eta}{2r}(S_H^U - S_L^U) - \frac{1}{2}S_H^U. \end{aligned}$$

Because $S_H^I \geq S_H^U$ and $S_L^U \geq S_L^I$, it must be that $V^U \geq V^I$.

Therefore, we can construct a new equilibrium, which mirrors the equilibrium we started out with, by flipping the labels U and I . This new equilibrium yields a higher value to informed agents, equal to the value to uninformed agents in the original equilibrium. Moreover, in the new equilibrium, no intermediation by uninformed agents occurs.

A.4 Proof of Proposition 4

I proceed in three steps.

Step 1. Let μ^I and μ^U be the measures of misaligned agents in the all-trade equilibria, and let $\hat{\mu}^I$ and $\hat{\mu}^U$ be the measures of misaligned agents in the core–periphery equilibria.

In the proof of Proposition 2, I show $\mu^I + \mu^U < \hat{\mu}^I + \hat{\mu}^U$ (fact (i)).

Similarly manipulating the inflow–outflow equations, we can also establish two more facts: (ii) $\mu^U < \hat{\mu}^U$ and (iii) $\mu^I/\mu^U > \hat{\mu}^I/\hat{\mu}^U$.

Step 2. Take α^I and α^U as given by (18) and (19) in the all-trade equilibria, and take $\hat{\alpha}^I$ and $\hat{\alpha}^U$ as given by (23) and (24) in the core–periphery equilibria.

Then facts (i) and (ii) in Step 1 imply $\alpha^I > \hat{\alpha}^I$.

Moreover, we can show $\mu^U(\alpha^U - \alpha^I) < \hat{\mu}^U(\hat{\alpha}^U - \hat{\alpha}^I)$. To show this, we can first write

$$\mu^U(\alpha^U - \alpha^I) = \frac{\lambda(2\beta^I - 1)(\mu^U + \mu^I)}{r + 2\eta + \lambda(\mu^U + \phi/2(1 - \beta^I))} \frac{\mu^U}{r + 2\eta + \lambda(2\mu^U\beta^I + \mu^I)}, \quad (44)$$

$$\hat{\mu}^U(\hat{\alpha}^U - \hat{\alpha}^I) = \frac{\lambda(2\beta^I - 1)(\hat{\mu}^U + \hat{\mu}^I) + \lambda\hat{\mu}^U}{r + 2\eta + \lambda\phi/2(1 - \beta^I)} \frac{\hat{\mu}^U}{r + 2\eta + \lambda(2\hat{\mu}^U\beta^I + \hat{\mu}^I)}. \quad (45)$$

Facts (i) and (ii) in Step 1 imply that the first term on the right-hand side of (45) is larger than the first term on the right-hand side of (44), and facts (ii) and (iii) imply that the second term on the right-hand side of (45) is larger than the second term on the right-hand side of (44).

Step 3. Using the trading pattern in all-trade equilibria, we can rewrite (25) as

$$\begin{aligned} V^I &= \frac{1}{2}(V_{H1}^I + V_{L0}^I) - \frac{1}{2}S_H^I \\ &= \frac{\delta_H}{2r} - \frac{\eta}{2r}(S_H^I - S_L^I) - \frac{1}{2}S_H^I + \frac{\lambda\mu^U\beta^I}{2r}[(S_H^U - S_H^I) + (S_L^I - S_L^U)] \\ &= \frac{\delta_H}{2r} - \frac{\eta}{2r}\alpha^I(\delta_H - \delta_L) - \frac{1}{2}\left[\frac{1}{2(r + 2\eta)}(\delta_H + \delta_L) + \alpha^I(\delta_H - \delta_L)\right] \\ &\quad + \frac{\lambda\beta^I}{r}\mu^U(\alpha^U - \alpha^I)(\delta_H - \delta_L). \end{aligned} \quad (46)$$

Likewise for core–periphery equilibria, we write (25) as

$$\begin{aligned} V^I &= \frac{1}{2}(V_{H1}^I + V_{L0}^I) - \frac{1}{2}S_H^I \\ &= \frac{\delta_H}{2r} - \frac{\eta}{2r}(S_H^I - S_L^I) - \frac{1}{2}S_H^I + \frac{\lambda\hat{\mu}^U\beta^I}{2r}[(S_H^U - S_H^I) + (S_L^I - S_L^U)] \\ &= \frac{\delta_H}{2r} - \frac{\eta}{2r}\hat{\alpha}^I(\delta_H - \delta_L) - \frac{1}{2}\left[\frac{1}{2(r + 2\eta)}(\delta_H + \delta_L) + \hat{\alpha}^I(\delta_H - \delta_L)\right] \\ &\quad + \frac{\lambda\beta^I}{r}\hat{\mu}^U(\hat{\alpha}^U - \hat{\alpha}^I)(\delta_H - \delta_L). \end{aligned} \quad (47)$$

Using these expressions and the inequalities found in Steps 1 and 2, we can straightforwardly check that the proposition holds.

A.5 Proof of Proposition 5

It is immediate to see that α^I and $\hat{\alpha}^I$ are decreasing in β^I .

Now rewrite (44) as

$$\alpha^U - \alpha^I = \frac{\lambda(\mu^U + \mu^I)}{r + 2\eta + \lambda(\mu^U + \phi/2(1 - \beta^I))} \frac{(2\beta^I - 1)}{(r + 2\eta + \lambda(2\mu^U\beta^I + \mu^I))}.$$

The first term on the right-hand side is increasing in β^I . The second term is increasing in β^I as well, because $\beta^I/(2\beta^I - 1)$ is decreasing in β^I .

Thus, $(\alpha^U - \alpha^I)$ is increasing in β^I . We can similarly check that $(\hat{\alpha}^U - \hat{\alpha}^I)$ is increasing in β^I as well.

Now use this fact and (46) and (47) to verify that the proposition holds.

REFERENCES

- Afonso, Gara and Ricardo Lagos (2015), "Trade dynamics in the market for federal funds." *Econometrica*, 83, 263–313. [58, 59]
- Ashcraft, Adam and Darrell Duffie (2007), "Systemic illiquidity in the federal funds market." *American Economic Review*, 97, 221–225. [58]
- Babus, Ana and Tai-Wei Hu (2017), "Endogenous intermediation in over-the-counter markets." *Journal of Financial Economics*, 125, 200–215. [60]
- Bech, Morten and Enghin Atalay (2010), "The topology of the federal funds market." *Physica A: Statistical Mechanics and its Applications*, 389, 5223–5246. [58]
- Bethune, Zach, Tai-Wei Hu, and Guillaume Rocheteau (2018), "Indeterminacy in credit economies." *Journal of Economic Theory*, 175, 556–584. [60]
- Bethune, Zach, Bruno Sultanum, and Nicholas Trachter (2020), "An information-based theory of financial intermediation." Working paper. [59]
- Carapella, Francesca and Steve Williamson (2015), "Credit markets, limited commitment, and government debt." *Review of Economic Studies*, 82, 963–990. [60]
- Cavalcanti, Ricardo and Neil Wallace "Inside and outside money as alternative media of exchange." *Journal of Money, Credit and Banking*, 31, 443–457. [60]
- Chang, Briana and Shengxing Zhang (2019), "Endogenous market making and network formation." Working paper. [59]
- Della Porta, Donatella and Alberto Vannucci (2016), "The hidden order of corruption: An institutional approach." Routledge. [57, 65, 76]
- Diamond, Douglas (1984), "Financial intermediation and delegated monitoring." *Review of Economic Studies*, 51, 393–414. [60]
- Duffie, Darrell, Nicolae Garleanu, and Lasse Pedersen (2005), "Over-the-counter markets." *Econometrica*, 73, 1815–1847. [58, 59, 60]

- Fainmesser, Itay (2019), “Exclusive intermediation in unobservable networks.” *Games and Economic Behavior*, 113, 533–548. [60]
- Farboodi, Maryam, Gregor Jarosch, Guido Menzio, and Ursula Wiriadinata (2019), “Intermediation as rent extraction.” Working paper. [59]
- Farboodi, Maryam, Gregor Jarosch, and Robert Shimer (2020), “The emergence of market structure.” Working paper. [59, 64]
- Lagos, Ricardo and Guillaume Rocheteau (2009), “Liquidity in asset markets with search frictions.” *Econometrica*, 77, 403–426. [59]
- Milgrom, Paul, Douglass North, and Barry Weingast (1990), “The role of institutions in the revival of trade: The law merchant, private judges, and the champagne fairs.” *Economics & Politics*, 2, 1–23. [58]
- Rubinstein, Ariel and Asher Wolinsky (1987), “Middlemen.” *Quarterly Journal of Economics*, 102, 581–593. [60]
- Schneider, Jacqueline (2005), “Stolen-goods markets: Methods of disposal 1.” *British Journal of Criminology*, 45, 129–140. [57]
- Townsend, Robert (1978), “Intermediation with costly bilateral exchange.” *Review of Economic Studies*, 45, 417–425. [60]

Co-editor Florian Scheuer handled this manuscript.

Manuscript received 11 December, 2019; final version accepted 8 March, 2021; available online 17 March, 2021.