Transparency and collateral: Central versus bilateral clearing

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This paper studies the optimal clearing arrangement for bilateral financial contracts in which an assessment of counterparty credit risk is crucial for efficiency. The economy is populated by borrowers and lenders. Borrowers are subject to limited commitment and hold private information about the severity of such lack of commitment. Lenders can acquire information, at a cost, about the commitment of their borrowers, which affects the assessment of counterparty risk. Clearing through a central counterparty allows lenders to mutualize counterparty credit risk, but this insurance may weaken incentives to acquire and reveal information. If information acquisition is incentive-compatible, then lenders choose central clearing. If it is not, they may prefer bilateral clearing either to prevent strategic default or to optimize the allocation of costly collateral.

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JEL classification. G10, G14, G20, G23.

1. Introduction

Counterparty credit risk is an important element in financial contracting. It is the risk that a counterparty may become unable or unwilling to settle its contractual obligations when they become due. In financial markets, counterparty risk is managed through clearing, i.e., the process of transmitting, reconciling, and confirming payment orders

or instructions to transfer securities prior to settlement. Clearing can be bilateral, via traders' respective clearing banks, or central, through a central counterparty (CCP). A CCP is an entity that interposes itself between two counterparties through a legal process called novation, becoming the buyer to every seller and the seller to every buyer for a specified set of contracts.[1]

Historically, CCPs emerged to provide insurance against counterparty risk for exchange-based derivatives. In the aftermath of the 2007–2009 financial crisis, the mandatory central clearing of many over-the-counter (OTC) traded derivatives has been at the core of financial reforms both in the United States and in Europe. While the objective was to strengthen the resilience and transparency of the derivatives markets, the consequences of these reforms on the flow of information in financial markets are not well understood. OTC markets are characterized by bilateral trading relationships in which firms specialize in understanding and pricing counterparty risk.[2] Information about the exposure of a counterparty to various risks and about its investment opportunities is often limited to the bilateral relationship, as it may not be verifiable by parties outside the relationship. Hence, transforming the nature of bilateral financial relationships with a clearing mandate can have important implications for the availability of such information.

This paper studies the potential trade-offs between bilateral and central clearing with respect to the management of counterparty risk and market transparency. We develop a model of bilateral financial contracting wherein agents' ability to commit to repay is limited. Moreover, agents hold private information about the severity of their commitment problem, to which we refer as the agent's type. Information about a counterparty's type is *soft* in the sense that it can be verified only by agents within the transaction via costly monitoring. The information obtained by monitoring is not available to a third party such as a clearing institution, which must choose contractual terms appropriately in order to induce truthful reporting about the monitoring activity and its outcome. This assumption captures the idea that soft information is often related to significant synergies across different projects and trades that are observable only to the agents involved in those activities.[3] When the CCP does not provide traders with sufficient incentives to monitor and report truthfully, the counterparty type cannot be part of the contractual terms. As a result, the CCP charges inefficient collateral requirements, followed, possibly, by strategic defaults by some of its members, thus undermining the rationale for the central clearing mandate itself.[4]

As an illustrative example of soft information and how it relates to our assumptions, consider a repurchase agreement (repo): a repo borrower may not have incentives to

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[1]See the glossary of terms used in payments and settlement systems provided by the Bank for International Settlements.

[2]See Krishnamurthy, Nagel, and Orlov (2014) for statistical data on tri-party repo and primary dealer statistics from the Federal Reserve Bank of New York, and Afonso and Lagos (2012) as well as Bech and Atalay (2010) for the Federal Funds market.

[3]See Stein (2002), María Liberti and Petersen (2019), Hauswald and Marquez (2006), and Mian (2003).

[4]Borrowers' self-selection into separating contracts is not feasible without preventing default, as the single-crossing condition is not satisfied.

repay the repo lender by repurchasing the asset previously sold if that asset has a lower spot price, or can be borrowed easily and cheaply, or if purchasing other financial instruments is a more profitable investment. Hence, failure to settle the closing leg of the repo, which occurs relatively frequently, can be a strategic decision.[5] In our model, the *type* of borrower is identified with these alternative financial opportunities: in the repo example, a borrower is a low commitment type if engaged in securities borrowing, or if engaged in trading complex financial instruments whose risk assessment requires industry know-how at a specific point in time and in a specific market.[6] Information about the cost of borrowing assets and their availability, and about certain financial investments by a counterparty, are more readily available to market participants than to a clearing institution. In fact, the vast majority of OTC trades are intermediated by broker-dealers who typically facilitate trading by either swiftly finding a matching counterparty for a client in the market or by trading directly with the client using their own inventory of assets. Information about inventories, about the cost of borrowing assets as well as their availability, and about individual transactions may only be available to a dealer or through a dealer.

In our model, traders value insurance against two dimensions of risk, namely, a counterparty's uncertain income and his commitment type. Private information about a counterparty's type introduces adverse selection, which interacts with the value of insurance in different ways in bilateral and central clearing. We represent bilateral clearing as a restriction on pooling counterparty risk: each trader has only a single counterparty. Thus, counterparty risk is managed only through collateral requirements, which are costly in terms of foregone investment opportunities. Monitoring provides the information necessary to tailor collateral requirements to a counterparty's type, but it is costly in terms of effort. We represent central clearing as a restriction on the set of contracts that the CCP can provide. In addition to collateral requirements, CCPs can rely on loss mutualization to manage counterparty risk.[7] We assume that the CCP commits to fully mutualize losses arising from idiosyncratic shocks to income, and it does not condition payments on future realizations of income shocks and default decisions unless it detects misreporting of a counterparty type.

The loss mutualization intrinsic to central clearing activity interacts with the supply of information about agent types. When the CCP can induce each member to monitor a counterparty and truthfully reveal their type, it can implement separating contracts that make central clearing Pareto superior to bilateral clearing. We call the resulting allocations incentive-feasible.[8]

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[5]For example, a charge that has been introduced for U.S. Treasury, agency debt, and agency mortgage-backed securities settlement fails to facilitate efficient clearing of these markets. See also data on Daily Total U.S. Treasury and Agency Fails from the Depository Trust and Clearing Corporation.

[6]For example, collateralized loan obligations, cryptoassets, and so forth.

[7]This modeling strategy is similar to that of Acharya and Bisin (2014), Biais, Heider, and Hoerova (2016), and Koeppl and Monnet (2010).

[8]Because monitoring and truth-telling are incentive-feasible, the CCP tailors collateral requirements to counterparty types and can implement transfers that make every participant weakly better off.

When incentive-feasible allocations do not exist and monitoring is optimal with bilateral clearing, a trade-off between bilateral and central clearing exists. In these situations, central clearing is associated with either higher average collateral requirements or higher average default rates relative to bilateral clearing. Higher collateral requirements or default rates represent the costs associated with the loss of information about a counterparty's type. Traders prefer bilateral to central clearing when the insurance provided by the CCP against idiosyncratic income shocks is insufficient to compensate for the loss of information about a counterparty's type. Note that this result is not related to the common idea that CCPs may generate moral hazard and increase risk by providing insurance. Rather, the activity of the CCP results in the lack of incentives to acquire and transmit information about counterparties.

The model identifies the characteristics of markets and traders that are optimally associated with bilateral and central clearing. For a region of the parameter space, financial institutions with a high opportunity cost of collateral, such as dealers and hedge funds, prefer to clear their trades bilaterally, whereas institutions with a low opportunity cost of collateral, such as money market funds (MMFs), prefer to rely on CCPs.[9]

Our model formalizes the implications of novation on the structure of CCPs. Through novation, the CCP becomes a counterparty in every trade, and observes *all* contracts traded by institutions for which it performs clearing services in a *specified* financial market. Both *all* and *specified* are important components of this definition: the first implies that, in a specific market, the CCP has information about the network of trades across its members, which may not be available to bilateral counterparties; the second implies that the CCP may lack information about its members if that information is learned within a bilateral relationship. Previous research on CCPs focused on the first component, recognizing the potential benefits of central clearing.[10] We focus here on the second component and characterize the conditions under which central clearing might reduce welfare relative to bilateral clearing.

Novation also transforms the nature of the risk exposure of the two parties in a trade by transferring counterparty risk to the CCP. If a clearing member defaults, the CCP needs to manage the risks associated with the outstanding contracts of the defaulted member by finding a counterparty willing to acquire those positions.[11] When this option is unavailable or unsuccessful, the CCP needs to gather enough resources to perform the obligations associated with the defaulted contracts, which makes the CCP dependent on loss mutualization. This is the focus of our paper. We study how loss mutualization affects monitoring incentives and its interaction with the ability of the CCP to mitigate counterparty risk, which has been the core function of CCPs since their inception.[12]

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[9]That dealers have a larger opportunity cost of collateral than MMFs is reflected in the higher returns they produce. That MMFs have taken up central clearing, wherever possible, is reflected, for example, in the increased rate between 2015 and 2017 of repos cleared at fixed income clearing corporations (FICCs), which have tended to replace reverse repos with the Fed.

[10]See Acharya and Bisin (2014).

[11]See the Clearing Rules and Disclosure Framework of ICE Clear Credit, a CCP that clears credit default swaps, and the Rules and Procedures of National Securities Clearing Corporation, a CCP that clears equities, corporate debt, and exchange-traded funds, among other products.

[12]For an analysis of the process of selling defaulted positions and of the relationship between central clearing and market liquidity, see Oleschak (2019).

The assumption that CCPs commit to mutualize losses arising from idiosyncratic shocks to income is consistent with novation and CCPs' default management procedures.[13] Conditioning CCP payments on the realization of income shocks would impair the liquidity of the instruments whose trading CCPs facilitate. The liquidity of the centrally cleared instrument is essential for effective default management, as it enables CCPs to replace or hedge the positions of a defaulting member and it ensures that they can accurately assess the risk of a settlement fail. Traders observing a robust CCP are more willing to trade, thus improving market liquidity and, as a consequence, the chances for successful default management by the CCP. In this respect, CCPs both help create market liquidity and, simultaneously, rely on it to fulfill their mandate.[14] Our framework abstracts from explicitly modeling the liquidity of the underlying financial products, as we focus on the contractual structure resulting from novation and adverse selection, rather than the relationship between central clearing and market liquidity. Nonetheless, the assumption that the CCP commits to mutualize losses from idiosyncratic shocks is consistent with the importance of liquidity in default management.

The results and assumptions of our model formalize concerns expressed by practitioners and analysts about the effects of mandatory central clearing on the credit risk of the CCP (Gregory (2014)), and conform to the empirical evidence in Bignon and Vuillemey (2020). We assume that the CCP cannot directly monitor ultimate investors: Bignon and Vuillemey (2020) find evidence of this information asymmetry in the failure of the *Caisse de Liquidation des Affaires et Marchandises* (CLAM, a CCP that cleared sugar futures) in Paris in 1974, as "retail investors were unsophisticated and non-diversified, did not have enough liquid financial resources," and CLAM could not "directly monitor ultimate investors."[15] We show the existence of equilibria where lenders do not have incentives to acquire information about their counterparties and/or to pass it on to the CCP. In equilibrium, then, the CCP is unable to charge member-specific margins. Bignon and Vuillemey (2020) show that CLAM kept margins *at a constant level across members*, which *was not sufficient to ensure stable clearing* and ended with the failure of a large CCP member and eventually of the CCP itself.

Our paper relates to previous work on financial market infrastructure. Acharya and Bisin (2014) identify that CCPs play a role in increasing welfare, due to the ability of CCPs to observe all trades in a specific market, when lack of transparency causes a negative externality and results in inefficient defaults. Our analysis complements Acharya and Bisin (2014) with the study of market transparency when valuable information is

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[13]As described by Kroszner (1999), the main function of CCPs since their inception has been to reduce counterparty risk. The first CCPs emerged to serve futures exchanges in nineteenth-century Europe, as credit or nonperformance risk in futures contracts was particularly acute due to the potentially long time between entering the contract and the delivery date. The futures exchanges devised CCPs as a mechanism to "make full payment to the aggrieved party, [...] drawing on an assessment fund to which members of the exchange had contributed." This mechanism is the essence of the risk mutualization that these exchanges embedded in their CCPs to guarantee the performance of trades through novation.

[14]See Edwards (1983).

[15]Bignon and Vuillemey (2020) go even further, theorizing *risk-shifting* behavior on the part of the CCP once it realized it was close to bankruptcy.

contained across assets rather than within a given class of assets cleared by a CCP. Biais et al. (2016) and Koeppl (2013) focus on the interaction between clearing and moral hazard with respect to the role of collateral in insuring counterparties and aligning incentives. In our environment, the loss mutualization provided by a CCP interacts with adverse selection and costly monitoring, and it affects traders' incentives to acquire socially valuable information about their trading partners and to transmit it to the CCP.

Our paper is also related to the literature on payment systems, in particular to Koeppl, Monnet, and Temzelides (2012), who study the efficiency of a clearing and settlement system in an environment with asymmetric information between the clearing institution and traders. Our paper complements Koeppl et al. (2012) in characterizing the endogenous effect of central clearing on transparency and default in financial markets.

The paper is organized as follows: Section 2 describes the model; Sections 3 and 4 describe optimal contracts without and with information acquisition; Section 5 presents comparative statics; Section 6 concludes.

2. The model

The economy lasts for two periods, $t = 1, 2$, and is populated by two types of agents: a unit measure of *lenders* and a unit measure of *borrowers*.[16] There are two goods: a consumption good and a capital good. In the first period, lenders receive an endowment of one unit of capital, whereas borrowers receive an endowment of ω units of consumption good. Both lenders and borrowers can store the consumption good from $t = 1$ to $t = 2$, while the capital good is not storable. Borrowers also have access to an investment technology that transforms capital at time $t = 1$ into consumption at time $t = 2$. The return of this technology depends on the realization of the idiosyncratic state of a borrower, s, which is revealed at $t = 2$: if $s = h$, the technology returns $\theta > 0$; if $s = l$, the technology fails, and returns 0. Let $p = \text{Prob}(s = h)$ and assume that the idiosyncratic state is independent and identically distributed (i.i.d.) across borrowers and is publicly observable, and that the law of large numbers holds. We refer to this random return as a borrower's uncertain income and identify it as one source of counterparty risk.

Borrowers' preferences are defined over consumption in period t, c_t, and are represented by the utility function $U(c_1, c_2) = \alpha c_1 + c_2$, where $\alpha > 1$. Borrowers have limited commitment to repay: a borrower can default on his debts, in which case his payoff is a function of his type λ^i, where $i \in \{L, H\}$ denotes low commitment (low type) or high commitment (high type), and $0 < \lambda^i < 1$. A measure q of borrowers are high types, whereas a measure $1 - q$ are low types, with $\lambda^L < \lambda^H$. The type λ^i is private information of the borrower, but it can be learned by a lender at a cost $\gamma > 0$. The possibility of borrowers' strategic default represents the second source of counterparty risk.

Lenders have preferences defined over consumption at $t = 2$, x_2, and the cost in effort of monitoring, $e \in \{0, 1\}$, which are represented by the utility function $V(x_2, e) = u(x_2) - \gamma \cdot e$. We assume that $u'(x) > 0 > u''(x) \ \forall x \geq 0$ and that $\lim_{x \to 0} u'(x) = +\infty$.

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[16]Our analysis extends to any contract with a component of limited commitment to honor a financial obligation, be it a repayment for a loan (as in a repo or a bond) or the transfer of an asset (as in an option that is exercised by its holder).

Trade is bilateral: a lender and a borrower are randomly matched, and the lender makes a take-it-or-leave-it (TIOLI) offer to the borrower, specifying the clearing arrangement as either bilateral or central.

Feasible contracts differ depending on the choice of clearing arrangement. We model bilateral clearing as a contracting problem between a lender and a borrower, as, in practice, their respective clearing banks simply execute payment orders. We model central clearing as a planning problem in which the CCP, after novation, takes over the financial obligations of the original counterparties. With novation, the CCP replaces the bilateral counterparties and "stands in between buyers and sellers and guarantees the performance of trades…[and]…is legally obliged to perform on the contracts it clears."[17] In practice, this is achieved by allocating any losses that the CCP experiences pro rata among the CCP members. In our model, this is achieved by allowing the CCP to pool idiosyncratic risk, similar to Acharya and Bisin (2014), Koeppl and Monnet (2010), and Biais et al. (2016). This implies that risks are mutualized and that no information about the realization of idiosyncratic shocks to a borrower's income is used to allocate losses. However, a CCP can impose sanctions for violations of its rules or for *prohibited conduct*, a practice known as due diligence.[18] With misconduct, the net payment from the CCP to a member might differ from the payment specified in the contract submitted for central clearing. In our model, the CCP detects lack of due diligence when it can identify whether a lender did not monitor her counterparty or did not report her counterparty's type truthfully. Strategic default of the original borrower may reveal misconduct by a lender. Thus, the CCP can use information about the strategic default of the original counterparty to punish misbehavior by the lender.

3. Optimal contracts without information acquisition

The goal of this section is twofold: (1) to introduce notation and the basic mechanics of our model; (2) to set a benchmark for contracts, to which we will refer in subsequent sections, for cases when information acquisition will not occur in equilibrium.

3.1 *Bilateral clearing without information acquisition*

When clearing is bilateral, lenders commit to a mechanism that specifies a menu of contracts. Without loss of generality, we assume direct revelation mechanisms, that is, a contract is executed after the borrower announces his type.

Formally, a strategy for a borrower is a pair $(m^i, \sigma^i) \in \{\lambda^L, \lambda^H\} \times \{0, 1\}$, where m^i is his reporting strategy and σ^i his default decision. Let Δ be the public history of the borrower's default decision, where $\Delta = 1$ if the borrower defaults ($\sigma^i = 1$) and $\Delta = 0$ if the borrower repays ($\sigma^i = 0$). A mechanism with bilateral clearing is a menu of contracts $(\Sigma^i, c_1^i, c_{2,s}^i, x_{2,s}^{i,\Delta})_{i \in \{L,H\}, s \in \{l,h\}}$, where $\Sigma^i \in \{0, 1\}$ is the lender's default recommendation to

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[17]See Gregory (2014), Section 8.3, and Cox and Steigerwald (2017): "CCPs are best seen as commitment mechanisms that assure the performance of financial contract obligations. How they perform that function sets them apart from other infrastructures, intermediaries and financial institutions."

[18]See the Rules and Procedures of ICE Clear Credit, article 701 and rule 609.

a borrower who reports his type to be λ^i, c_1^i and $c_{2,s}^i$ are the borrower's consumption, and $x_{2,s}^{i,\Delta}$ is the lender's consumption when the borrower's idiosyncratic state is s. A contract is incentive-compatible if a borrower's best strategy (m^i, σ^i) is to report his type truthfully, $m^i = \lambda^i$, and to follow the default/repayment recommendation, $\sigma^i = \Sigma^i$.

After reporting his type and accepting the ensuing contract, a borrower receives the capital, which he invests, and transfers $\omega - c_1^i$ units of consumption good to the lender. We interpret the transfer of $\omega - c_1^i$ by the borrower to the lender at $t = 1$ as collateral, as it denotes the amount of consumption good stored by the lender to be consumed at $t = 2$ by either the lender or the borrower. After the idiosyncratic shock is realized, the borrower's deviation payoff is $(1 - \lambda^i)\theta$ when $s = h$ and zero otherwise. Thus, the optimal mechanism solves the problem

$$(P_0^b) \quad V^{\text{bil},e=0} = \max \sum_{i=L,H} q_i \big[p \{ \Sigma^i u(x_{2h}^{i1}) + (1 - \Sigma^i)u(x_{2h}^{i0}) \} + (1 - p)u(x_{2l}^i) \big]$$

such that $\alpha c_1^i + p\big[\Sigma^i(1 - \lambda^i)\theta + (1 - \Sigma^i)c_{2h}^i\big] + (1 - p)c_{2l}^i \geq \alpha\omega,$ (1)

$$\omega \geq c_1^i \geq 0,$$ (2)

$$x_{2h}^{i0} + c_{2h}^i \leq \omega - c_1^i + \theta,$$ (3)

$$x_{2h}^{i1} \leq \omega - c_1^i,$$ (4)

$$x_{2l}^i + c_{2l}^i \leq \omega - c_1^i,$$ (5)

$$(\lambda^i, \Sigma^i) \in \operatorname*{argmax}_{(\hat{m}, \hat{\sigma})} \{ \alpha c_1^{\hat{m}} + p[\hat{\sigma}(1 - \lambda^i)\theta + (1 - \hat{\sigma})c_{2h}^{\hat{m}}] + (1 - p)c_{2l}^{\hat{m}} \}.$$ (6)

Constraint (1) is borrower i's participation constraint: the borrower can always refuse to trade and consume his endowment ω. Constraint (2) is the $t = 1$ feasibility constraint; (3) and (4) are the $t = 2$ feasibility constraints in states $(s, \Delta) = (h, 0)$ and $(s, \Delta) = (h, 1)$, respectively; (5) is the $t = 2$ feasibility condition when $s = l$. Finally, constraint (6) is the incentive-compatibility constraint for a borrower of type λ^i: the strategy pair (λ^i, Σ^i) is incentive-compatible if there is no other strategy pair $(\hat{m}, \hat{\sigma})$ that yields a higher payoff. Notice that a borrower can deviate by reporting a different type $\hat{m} \neq \lambda^i$, or by choosing a different default strategy $\hat{\sigma} \neq \Sigma^i$, or both.[19]

The next section proves that central clearing always dominates bilateral clearing when no information about borrowers' types can be acquired. Thus, we do not characterize the solution to problem (P_0^b), as it is irrelevant to the focus of this paper.

3.2 Central clearing without information acquisition

With novation, the contract between a borrower and a lender is replaced by a contract between the lender and the CCP, and a contract between the borrower and the CCP. The

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[19]As an example of a financial contract between the lender and the borrower, consider a repurchase agreement (repo): then we can think of the unit of capital transferred by the lender to the borrower at $t = 1$ as the *starting leg* of the repo, and of the payment $x_{2,s}^{i,0}$ by the borrower to the lender at $t = 2$ as the *closing leg* of the repo. See Garbade (2006) for the evolution of repo contracts and Antinolfi, Carapella, Kahn, Martin, Mills, and Nosal (2015) for their microfoundations.

CCP takes the terms of the original contract as given, but can require borrowers to post collateral (i.e., margin) and lenders to contribute to a loss mutualization scheme (i.e., default or guarantee fund). We model novation by assuming that the CCP commits to a mechanism at the beginning of $t = 1$, and that lenders and borrowers negotiate over the contracts in such a mechanism. Each contract specifies actions for the borrower, transfers between borrowers and the CCP, and transfers between lenders and the CCP, while no transfer between the borrower and the lender takes place.

Formally, a strategy for a type-i borrower is a pair $(m^i, \sigma^i) \in \{\lambda^L, \lambda^H\} \times \{0, 1\}$ that specifies a message $m^i \in \{\lambda^L, \lambda^H\}$ in $t = 1$ and a default decision $\sigma^i \in \{0, 1\}$ in $t = 2$. As with bilateral clearing, $\sigma^i = 1$ means that the borrower defaults when the idiosyncratic state is $s = h$. A mechanism with central clearing consists of contracts between the CCP and lenders, $\{X_2^i\}_{i=L,H}$, and between the CCP and borrowers, $\{\Sigma^i, C_1^i, C_{2,s}^i\}_{i=L,H}$, which are executed if the borrower reports his type to be λ^i. As with bilateral clearing, Σ^i denotes the default recommendation of the CCP to a borrower who reports his type to be λ^i, while C_1^i, $C_{2,s}^i$ and X_2^i denote borrower's and lender's consumptions, respectively. Notice that contracts between the CCP and lenders are independent of the history of the original borrowers at time $t = 2$, that is, the CCP does not condition payments on the realization of the idiosyncratic state $s \in \{l, h\}$ or on strategic default, as there is no information acquisition. This assumption represents the commitment of the CCP to honor promises inherited through novation. In other words, if the CCP novates two ex ante identical contracts, it cannot discriminate payments ex post.[20] A mechanism is incentive-compatible if it is in the borrower's best response to truthfully report his type and then follow the default recommendation Σ^i. Thus, the optimal mechanism with central clearing and no monitoring solves

$$(P_0) \quad V^{CCP,e=0} = \max \left\{ \sum_i q_i u(X_2^i) \right\}$$

such that
$$\alpha C_1^i + p\left[\Sigma^i (1 - \lambda^i)\theta + (1 - \Sigma^i)C_{2h}^i\right] + (1 - p)C_{2l}^i \geq \alpha\omega, \tag{7}$$

$$0 \leq C_1^i \leq \omega, \tag{8}$$

$$\sum_i q_i \left\{ X_2^i + p(1 - \Sigma^i)C_{2h}^i + (1 - p)C_{2l}^i \right\}$$

$$\leq \sum_i q_i \left\{ \omega - C_1^i + p(1 - \Sigma^i)\theta \right\}, \tag{9}$$

$$(\lambda^i, \Sigma^i)$$

$$\in \underset{(\hat{m}, \hat{\sigma})}{\text{argmax}} \left\{ \alpha C_1^{\hat{m}} + p\left[\hat{\sigma}(1 - \lambda^i)\theta + (1 - \hat{\sigma})C_{2h}^{\hat{m}}\right] + (1 - p)C_{2l}^{\hat{m}} \right\}. \tag{10}$$

Constraint (7) is borrower i's participation constraint; (8) and (9) are feasibility constraints in $t = 1$ and $t = 2$, respectively. Note that the feasibility constraint in $t = 2$ is defined for the aggregate resources of the CCP in $t = 2$, because the CCP pools borrowers'

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[20]As discussed in Section 1, this is necessary for the CCP to manage default events effectively.

idiosyncratic risks by becoming the buyer to every seller and the seller to every buyer. Constraint (10) is the incentive-compatibility constraint of a borrower who must report his type truthfully, $m^i = \lambda^i$, and then follow the default recommendation, $\sigma^i = \Sigma^i$.[21]

Comparing problems (P_0) and (P_0^b), we can prove the following result.

PROPOSITION 1. *Without information acquisition, central clearing is the optimal clearing arrangement: the solution to (P_0) is superior to the solution to (P_0^b).*

When lenders cannot learn the type of their counterparty, they are no better than the CCP at evaluating the risk that a borrower will strategically default. Thus, the CCP can always replicate the optimal borrower contracts of Section 3.1 and, in addition, insure lenders against the idiosyncratic income risk associated with the original counterparty. Hence, central clearing is always preferred to bilateral clearing.

To simplify the exposition of our results, we introduce the following assumption.[22]

ASSUMPTION 2. *Assume that $\omega > \omega(\lambda^L)$, with $\omega(\lambda) \equiv (1 - \lambda) p\theta/\alpha$.*

By setting a lower bound on borrowers' endowment, Assumption 2 guarantees that borrowers' participation constraints are binding and that borrowers do not earn extra rents with respect to autarky. We can then characterize the solution to problem (P_0).

PROPOSITION 3. *Let Assumption 2 hold. The optimal CCP mechanism satisfies*

$$X_2^i \equiv X_2 = \sum_i q_i \big[\omega - C_1^i + (1 - \Sigma^i)p(\theta - C_{2h}^i) - (1 - p)C_{2l}^i\big], \tag{11}$$

where $C_{2,l}^H = C_{2,l}^L = 0$, and for $\hat{q} \equiv 1/\alpha + (1 - 1/\alpha)\lambda^L/\lambda^H$,

$$C_1^H = C_1^L = \begin{cases} \omega - \omega(\lambda^L) & \text{if } q \le \hat{q}, \\ \omega - \omega(\lambda^H) & \text{if } q > \hat{q}, \end{cases} \qquad C_{2,h}^H = C_{2,h}^L = \begin{cases} (1 - \lambda^L)\theta & \text{if } q \le \hat{q}, \\ (1 - \lambda^H)\theta & \text{if } q > \hat{q}. \end{cases}$$

for $\omega(\lambda)$ defined in Assumption 2. Thus, if $q \le \hat{q}$, no borrower defaults ($\Sigma^L = 0$, $\Sigma^H = 0$); if $q > \hat{q}$, low type borrowers default ($\Sigma^L = 1$, $\Sigma^H = 0$).

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[21]Referring to the example in Section 3.1, if the financial contract is a repurchase agreement, the contract between the lender and the CCP involves a starting leg where the lender transfers her endowment of capital to the CCP at $t = 1$ and a closing leg where the CCP pays X_2^i to the lender. The contract between the borrower and the CCP involves a starting leg where the CCP transfers one unit of capital (received from the lender) to the borrower and the borrower transfers $\omega - C_1^i$ units of good to the CCP as a margin requirement. The closing leg of this contract involves the transfer of $\theta - C_{2,s}^i$ units of consumption good from the borrower to the CCP, of which $\theta - C_{2,s}^i - X_2^i$ are default fund contributions from the borrower. If the borrower's income turns out to be low (i.e., $s = l$), then he makes no payment to the CCP at $t = 2$. The CCP can use resources from the margin requirements and the default fund contributions of borrowers able to pay to settle payments to lenders.

[22]In the working paper version, we relax this assumption and show that our main results hold true. See Antinolfi, Carapella, and Carli (2019).

Because the CCP can fully diversify borrowers' idiosyncratic risks, it only needs to maximize resources available in $t = 2$ to pay lenders. This is achieved by offering the same contract to high and low type borrowers, except for the default recommendation in a region of the parameter space. In particular, it is optimal for the CCP to homogenize its collateral requirements and choose between two classes of contracts: one in which no borrower defaults in $t = 2$, and one in which λ^H borrowers repay in $t = 2$, whereas λ^L borrowers default. In the first scenario, the CCP offers a pooling contract that treats all borrowers as if they were the worst possible type, λ^L. As a result, λ^H borrowers end up posting collateral above what their type would optimally require. In the second scenario, all borrowers post the same collateral, as if they were λ^H types. As a result, λ^L borrowers default.

Collateral requirements play an important role in the decision of the CCP between these two classes of contracts. On the one hand, higher collateral requirements increase the level of resources available at $t = 2$ if they prevent λ^L borrowers from defaulting. On the other hand, higher collateral requirements reduce borrowers' consumption in $t = 1$ and, through borrowers' participation constraint, result in fewer resources available at the CCP in $t = 2$, because collateral is costly. The resolution of this trade-off depends on the cost of collateral, α, and on the measure of λ^L borrowers, $1 - q$. In particular, when the population of λ^L types is relatively large, i.e., $q \leq \hat{q}$, the CCP maximizes resources at $t = 2$ by preventing the default of λ^L borrowers. Thus, all borrowers post enough collateral to satisfy the limited commitment problem of λ^L types, namely $\omega(\lambda^L)$ defined in Assumption 2. If instead the population of λ^L types is relatively small, i.e., $q > \hat{q}$, it is too costly for the CCP to prevent the default of λ^L borrowers by means of higher collateral requirements for all borrowers. Thus, the resources of the CCP are maximized when all borrowers post collateral to satisfy the limited commitment constraint of λ^H types, namely $\omega(\lambda^H)$.

Substituting the results from Proposition 3 into (P_0), the value of central clearing with no information acquisition becomes

$$V^{\text{CCP},e=0} = \begin{cases} u(\omega(\lambda^L) + p\theta\lambda^L) & \text{if } q \leq \hat{q}, \\ u(\omega(\lambda^H) + p\theta\lambda^H q) & \text{if } q > \hat{q}. \end{cases} \tag{12}$$

4. Optimal contracts with information acquisition

In this section, we allow a lender to monitor her borrower and learn his type, which remains private information to the lender and the borrower. As a result, when it designs a contract with monitoring, the CCP needs to account for lenders' incentives to monitor their counterparty and to truthfully report the information they learn.

4.1 Bilateral clearing with information acquisition

When a lender learns the type λ^i of her counterparty, the optimal contract prevents default. Given $i \in \{L, H\}$, a contract with bilateral clearing and information acquisition is a list $(c_1^i, c_{2,s}^i, x_{2,s}^i)_{s \in \{l,h\}}$, where c_1^i and $c_{2,s}^i$ are the borrower's consumption amounts, and

$x_{2,s}^i$ is the lender's consumption. Let V_i denote the value to a lender of a match with a borrower of type λ^i after the lender has paid the cost γ. Then optimal contracts solve

$$(P^i) \quad V_i = \max_{(x_{2,h}^i, x_{2,l}^i, c_1^i, c_{2,h}^i, c_{2,l}^i) \in \Re_+^5} pu(x_{2,h}^i) + (1-p)u(x_{2,l}^i) - \gamma$$

$$\text{such that } \alpha c_1^i + pc_{2,h}^i + (1-p)c_{2,l}^i \geq \alpha\omega, \tag{13}$$

$$\omega \geq c_1^i \geq 0, \tag{14}$$

$$c_{2,h}^i + x_{2,h}^i \leq \omega - c_1^i + \theta, \tag{15}$$

$$c_{2,l}^i + x_{2,l}^i \leq w - c_1^i, \tag{16}$$

$$c_{2,h}^i \geq (1 - \lambda^i)\theta. \tag{17}$$

Constraint (13) is the borrower's participation constraint; (14), (15), and (16) are feasibility constraints in $t = 1$ and in $t = 2$ if $s = h, l$ respectively; and (17) is the borrower's limited commitment constraint: if $s = h$, a borrower can default and consume $(1 - \lambda^i)\theta$.

Note that both constraints (15) and (16) bind at a solution. Solving for $x_{2,h}^i$ and $x_{2,l}^i$, and substituting them in objective function (P^i), we can solve for $(c_1^i, c_{2,h}^i, c_{2,l}^i)$. Because $\alpha > 1$, a lender's expected consumption is largest when the borrower consumes the value of his endowment ω in $t = 1$ and is nothing in $t = 2$. Such a contract satisfies (13) but violates (17), and leaves the lender with no consumption in the second period when $s = l$, as implied by constraint (16). As a result, the lender will always store some of the borrower's endowment as collateral. Collateral plays two roles with bilateral clearing. First, it provides insurance to the lender against the risk of low consumption at $t = 2$ when $s = l$. Second, it provides the borrower with incentives to repay at $t = 2$ by increasing the level of resources available to be allocated for his consumption at that time. Which of these two roles dominates depends on the severity of the borrower's commitment problem. To formally define the severity of the commitment problem, let λ^* be the unique solution to

$$\Psi(\lambda^*) = \frac{\alpha - p}{1 - p} \tag{18}$$

for $\Psi(\lambda) = [u'((1 - \lambda)p\theta/\alpha)]/[u'(\theta - (\alpha - p)(1 - \lambda)\theta/\alpha)]$. Intuitively, λ^* is the smallest value of λ such that the limited commitment constraint, (17), is slack. If $\lambda \leq \lambda^*$, then (17) binds, as the borrower's commitment problem is severe: his deviation payoff is large and his counterparty quality is low. In the rest of the paper, we make the following assumption.

ASSUMPTION 4. $\lambda^L < \lambda^*$, with λ^* defined in (18).

Assumption 4 guarantees that the limited commitment constraint (17) of the λ^L borrowers binds. If this condition did not hold, information about counterparty quality would have no value with bilateral clearing.

LEMMA 5. *Let $\omega(\lambda)$ be defined in Assumption 2 and let λ^* be defined in (18). Let Assumptions 2 and 4 hold. Then optimal contracts with bilateral clearing and monitoring satisfy $c_{2,l}^i = 0$, $x_{2,h}^i = \theta - c_{2,h}^i + \omega - c_1^i$, $x_{2,l}^i = \omega - c_1^i$, $c_{2,h}^L = (1-\lambda^L)\theta$, $c_1^L = \omega - \omega(\lambda^L)$, and*

(i) $c_1^H = \omega - \omega(\lambda^*)$, $c_{2,h}^H = (1-\lambda^*)\theta$ if $\lambda^H \geq \lambda^*$

(ii) $c_1^H = \omega - \omega(\lambda^H)$, $c_{2,h}^H = (1-\lambda^H)\theta$ if $\lambda^H < \lambda^*$.

To understand the intuition behind Lemma 5, recall that the limited commitment constraint of a λ^L borrower binds by Assumption 4. Thus, collateral provides λ^L borrowers with incentives to repay at $t = 2$. The limited commitment constraint of a λ^H borrower, however, may or may not bind. If $\lambda^H > \lambda^*$, which corresponds to case (i) in Lemma 5, the commitment problem of λ^H borrowers is not severe and constraint (17) is slack. In this case, the dominant role of collateral is insurance against state $s = l$. If, instead, $\lambda^H < \lambda^*$, which corresponds to case (ii) in Lemma 5, constraint (17) binds and the dominant role of collateral is to provide incentives to repay.

Lemma 5 also shows that, with bilateral clearing, lenders choose to bear some counterparty risk. Because collateral is costly and is the only tool for managing counterparty risk, lenders' consumption is larger in the state of nature where their counterparty experiences a high-income realization: $x_{2,h}^i > x_{2,l}^i$.

4.2 *Central clearing with information acquisition*

In this section, we study mechanisms with central clearing when lenders can monitor and learn, at a cost, their counterparty type. Because neither lenders' monitoring effort nor the outcome of monitoring is observable, the CCP needs to incentivize lenders to monitor and truthfully report the outcome. Given a CCP mechanism, a strategy for a lender is a pair $(e, m) \in \{0, 1\} \times \{\lambda^H, \lambda^L\}$ of effort e and message m to the CCP. A strategy for a borrower is a default decision $\sigma_s(\lambda, m) : \{\lambda^L, \lambda^H\}^2 \times \{l, h\} \to \{0, 1\}$. For example, $\sigma_h(\lambda^L, \lambda^H)$ is the default decision of a λ^L borrower at the node corresponding to lender's message $m = \lambda^H$, when the idiosyncratic state is $s = h$.

Through novation, the CCP becomes the sole counterparty to the borrower and the lender, and it is legally obliged to honor the contracts it clears, as in Section 3.2. However, while the CCP's payments are not contingent on realizations of idiosyncratic shocks to income, as in Section 3.2, when information acquisition is feasible, the CCP's payments can be contingent on strategic default decisions. Indeed, strategic default by a borrower in the state where his income realization is high reveals either lack of monitoring by a lender or misreporting (her borrower's type). In this case, the CCP can punish the lender, even if this implies modifying the payment agreed upon in the contract, as the CCP unambiguously detects that a lender deviated from the contract.

In practice, CCPs impose sanctions for violation of their rules and procedures, and they assess fines if a member shows "prohibited conduct" or conduct that is inconsistent with "just and equitable principles of trade." Violations of the rules of a CCP include failure to provide information regarding the businesses and operations of the member

and its risk management practices, or failure to report the member's financial or operational conditions. Additionally, members may need to submit information to the CCP "as the Corporation from time to time may reasonably require."[23] In this respect, lenders' costly monitoring captures the idea that members may choose to learn their counterparties' businesses and operations by, for example, carrying out specific trades with specific counterparties, even if such a trade comes at a cost.

Using the same notation as in Section 3.1, let $\Delta \in \{0, 1\}$ be the observed default decision of the lender's original counterparty, where $\Delta = 1$ means that the borrower defaults in equilibrium. A mechanism with central clearing and monitoring consists of contracts between the CCP and lenders, $\{X_2^{m,\Delta}\}$, and contracts between the CCP and borrowers, $\{C_1^m, C_{2,s}^m\}$. Let $w^{i,\Delta} = u(X_2^{m,\Delta})$ be the lender's payoff when she reports $m = \lambda^i$ in $t = 1$ and let the observed default of her original borrower in $t = 2$ be Δ. The optimal CCP mechanism that induces monitoring and results in borrowers' separation solves

$$(P_1) \quad \max\{qw^{H,0} + (1-q)w^{L,0} - \gamma\}$$

$$\text{such that } \alpha C_1^i + pC_{2,h}^i + (1-p)C_{2,l}^i \geq \alpha\omega, \tag{19}$$

$$C_{2,h}^i \geq (1-\lambda^i)\theta, \tag{20}$$

$$\omega - \sum_{i\in\{L,H\}} q_i C_1^i \geq 0, \tag{21}$$

$$qu^{-1}(w^{H,0}) + (1-q)u^{-1}(w^{L,0})$$
$$+ \sum_{i\in\{L,H\}} \{q_i[C_1^i + pC_{2,h}^i + (1-p)C_{2,l}^i]\} \leq \omega + p\theta, \tag{22}$$

$$w^{i,0} \geq \sum_{s\in\{l,h\}} p_s[\sigma_s(\lambda^i, \lambda^{-i})w^{-i,1} + [1-\sigma_s(\lambda^i, \lambda^{-i})]w^{-i,0}], \tag{23}$$

$$-\gamma + \sum_{i\in\{L,H\}} q_i w^{i,0}$$
$$\geq \max_{\hat{i}\in\{L,H\}} \left\{ \sum_{i\in\{L,H\}} q_i \left[\sum_{s\in\{l,h\}} p_s[\sigma_s(\lambda^i, \lambda^{\hat{i}})w^{\hat{i},1} \right. \right.$$
$$\left. \left. + [1-\sigma_s(\lambda^i, \lambda^{\hat{i}})]w^{\hat{i},0}] \right] \right\}, \tag{24}$$

$$\sigma_s(\lambda^i, \lambda^j) = 0 \quad \text{if and only if} \quad C_{2,s}^j \geq (1-\lambda^i)\theta_s. \tag{25}$$

Constraint (19) is borrower i's participation constraint and (20) is his limited commitment constraint. Equations (21) and (22) are feasibility constraints in $t = 1$ and $t = 2$, and (23) and (24) are, respectively, ex post and ex ante incentive-compatibility constraints for lenders. Specifically, when (23) is satisfied, a lender reports truthfully her counterparty's type after monitoring. When constraint (24) is satisfied, a lender prefers,

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[23]See the Rules and Procedures of National Securities Clearing Corporation and the Clearing Rules of ICE Clear Credit.

ex ante, to monitor her borrower (and then report his type truthfully) rather than not to monitor and to report that her counterparty is either a high or a low type. Constraint (25) defines borrowers' default decision and implies that $\sigma_l(\lambda^i, \lambda^j) = 0$ for $i, j \in \{L, H\}$, because borrowers have no incentive to default in state $s = l$.

We solve problem (P_1) in two steps. In the first step, the CCP determines the contracts with borrowers that maximize resources in $t = 2$. Doing so relaxes constraints (22), (23), and (24), while satisfying (19), (20), and (21). In the second step, the CCP chooses contracts with lenders to maximize their payoff, given the available resources. Let $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ map any value of the monitoring cost $\gamma > 0$ to the minimum aggregate resources in $t = 2$, consistent with the existence of a solution to the CCP problem (P_1):

$$\phi(\gamma) = qu^{-1}\left(\frac{\gamma}{pq(1-q)}\right) + (1-q)u^{-1}\left(\gamma\left[\frac{1-p(1-q)}{pq(1-q)}\right]\right).$$

Further, with $\omega(\lambda)$ defined in Assumption 2, let the threshold $\hat{\gamma}$ denote the largest value of γ such that a solution to problem (P_1) exists:

$$\phi(\hat{\gamma}) = \sum_{i\in\{L,H\}} q_i[\lambda^i p\theta + \omega(\lambda^i)]. \tag{26}$$

The following proposition characterizes such a solution.

PROPOSITION 6. *A solution to problem (P_1) exists and is unique if and only if $\gamma \leq \hat{\gamma}$. If $\gamma \leq \hat{\gamma}$, (i) optimal contracts satisfy $C_{2h}^i = (1-\lambda^i)\theta$, $C_{2l}^i = 0$, $C_1^i = \omega - \omega(\lambda^i)$, and $w^{H,0^*} = w^{L,0^*} + \gamma/q$, $w^{L,0^*}$ solving $u^{-1}(w^{L,0^*} + \gamma/q) + (1-q)u^{-1}(w^{L,0^*}) = \sum_{i\in\{L,H\}} q_i[\lambda^i p\theta + \omega(\lambda^i)]\}$; (ii) $\max\{u(X_2^*), \sum_{i\in\{L,H\}}\{q_i w^{i,0^*}\} - \gamma\} \geq \sum_{i\in\{L,H\}}\{q_i[\sum_{s=l,h} p_s u(x_{2,s}^{i^*})]\} - \gamma$, where X_2^* is lenders' consumption in (11) in the optimal contract with CCP clearing and no monitoring of Proposition 3, $w^{i,0^*}$ is lenders' payoff in the solution to (P_1), and $x_{2,s}^{i^*}$ is lenders' consumption in the contract of Lemma 5.*

The existence of a solution to problem (P_1) relies on the tools that the CCP employs to provide lenders with incentives to fulfill contractual obligations. A lender may deviate in two ways. First, she may deviate by saving on monitoring cost and reporting that her counterparty is a λ^L type. To prevent this behavior, the CCP must reward lenders matched with a λ^H type, and the reward must increase with the monitoring cost γ. Second, a lender may deviate by saving on monitoring cost and reporting that her counterparty is a λ^H type. Such an incentive to deviate is increasing in the reward that the CCP provides to lenders matched with a λ^H type. The CCP can prevent both types of deviation only when the monitoring cost is not too large, $\gamma \leq \hat{\gamma}$.

When a solution to problem (P_1) exists, collateral does not provide insurance benefits, because the CCP can fully insure lenders by pooling borrowers' idiosyncratic risk. Hence, collateral simply provides borrowers with incentives to repay. The limited commitment constraint (20) binds for both types of borrowers, and so does the participation constraint (19) because of Assumption 2. Constraints (19) and (20) determine borrowers' consumption in $t = 1$ and $t = 2$. Feasibility and incentive-compatibility constraints

(22) and (24) also bind, determining lender's consumption in $t = 2$. Notice that variation in the payoff provided to lenders as a function of their borrowers' types is necessary to induce monitoring and truthful reporting. The second part of Proposition 6 proves that bilateral clearing is never optimal if a solution to problem (P_1) exists. Intuitively, if information is valuable with bilateral clearing, it is even more valuable with central clearing, which could also pool borrowers' type risk, by which we mean the risk that a lender might need to require her borrower to post a relatively large amount of collateral if his type is λ^L.[24] More precisely, if the cost of monitoring is sufficiently low, the CCP can replicate any payoff with bilateral clearing and monitoring. Moreover, by pooling borrowers' idiosyncratic income shocks, the CCP obtains enough resources to induce lenders to monitor their counterparties and report their type truthfully. Doing so allows the CCP to also partially insure lenders against the risk of a λ^L borrower. As a result, for a low monitoring cost, central clearing improves on bilateral clearing.

For economies that do not satisfy the conditions in Proposition 6, bilateral clearing can be optimal. The following proposition provides a full characterization of this result, which is our key finding.

PROPOSITION 7. *Let Assumptions 2 and 4 hold, let λ^* be defined as in (18), and let $\hat{q}$ be defined as in Proposition 3. If*

$$\hat{q}\big[pu(\min\{\lambda^H\theta + \omega(\lambda^H), \lambda^*\theta + \omega(\lambda^*)\}) + (1-p)u(\max\{\omega(\lambda^H), \omega(\lambda^*)\})\big]$$
$$+ (1-\hat{q})\big[pu(\lambda^L\theta + \omega(\lambda^L)) + (1-p)u(\omega(\lambda^L))\big] > u(\omega(\lambda^L) + \lambda^L p\theta), \qquad (27)$$

there exists an interval $(\underline{q}, \overline{q})$ and a function $\overline{\gamma}(q) : (\underline{q}, \overline{q}) \to \Re_+$ such that bilateral clearing (with monitoring) is the optimal clearing arrangement if and only if $\gamma \in (\hat{\gamma}, \overline{\gamma})$.

The conditions in Proposition 7 are necessary and sufficient for the optimality of bilateral clearing. When $\gamma > \hat{\gamma}$, Proposition 6 implies that no mechanism with central clearing and monitoring exists. Without the information generated by monitoring, the CCP can only offer contracts that require all borrowers to post the same amount of collateral, as described in Section 3.2. Thus, central clearing has the limitation of requiring some borrowers to post either excessive or insufficient collateral relative to what their type would require. Central clearing, however, has the advantage of providing insurance by pooling borrowers' idiosyncratic income shocks, which saves on collateral. When $\gamma < \overline{\gamma}(q)$, such insurance does not compensate lenders for the distortion in the use of collateral due to the lack of information about their counterparty quality. Thus, lenders choose to clear contracts bilaterally and to acquire information about their borrowers.

This result relies on a key property of bilateral clearing that preserves lenders' incentives to monitor, even when these incentives are insufficient with central clearing: lenders' consumption varies both with borrowers' realized income and with their

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[24]A match with a λ^L borrower is relatively expensive for the lender, as it requires a larger amount of collateral to prevent strategic default. Hence, risk averse lenders value ex ante insurance against the risk that they might need to require borrowers to post a relatively large amount of collateral. Because both the income risk and the type risk are idiosyncratic to the borrower, with full information, the CCP could offer lenders ex ante insurance against both risks.

types. The loss mutualization inherent in central clearing, instead, limits the variance in the distribution of lenders' consumption with respect to their borrowers' realized income. This variance is important in preserving monitoring incentives because information about the type of counterparty has a different value in different states of nature. When a borrower's idiosyncratic state is $s = l$, his technology fails. Consequently, neither a high nor a low type of borrower can repay. In this case, information about the quality of a counterparty has no value. When a borrower's idiosyncratic state is $s = h$, instead, information about his type is valuable because different types of borrowers have different temptations to default. It is the variability in the lenders' consumption between the two states that preserves their incentive to monitor. Bilateral clearing maintains this variability in consumption because the insurance against the realization of a borrower's idiosyncratic income is incomplete. CCPs offer contracts with repayments that are not state-contingent, eliminate the variability in lenders' consumption, and fail to induce monitoring if its cost is sufficiently high ($\gamma > \hat{\gamma}$).[25] To better understand this result, we can interpret bilateral clearing with monitoring as a lottery $\mathcal{L}^{\text{bil}} = (pq, p(1-q), (1-p)q, (1-p)(1-q))$ over outcomes $(x_{2,h}^H, x_{2,l}^H, x_{2,h}^L, x_{2,l}^L)$, whereas central clearing results in a degenerate lottery over X_2^{CCP}. The threshold $\overline{\gamma}(q)$ can be rewritten as

$$\overline{\gamma} = u(E_{\mathcal{L}^{\text{bil}}} - RP_{\mathcal{L}^{\text{bil}}}) - u(X_2^{\text{CCP}}), \tag{28}$$

where $E_{\mathcal{L}^{\text{bil}}}$ and $RP_{\mathcal{L}^{\text{bil}}}$ are, respectively, the expected value and the risk premium of the lottery $\mathcal{L}^{\text{bil}}$.[26] When Assumptions 2 and 4 are satisfied, $X_2^{\text{CCP}} < E_{\mathcal{L}^{\text{bil}}}$, and a trade-off between bilateral and central clearing may exist if lenders are not overly risk averse and if the population of borrowers is sufficiently heterogeneous. In fact, the value of insurance provided by the CCP is smaller the less risk averse lenders are, as reflected in (27) and (28). Moreover, the benefits from collateral customization are larger if the population of borrowers contains both high and low types, as reflected in the necessary and sufficient condition $q \in (\underline{q}, \overline{q})$. Finally, the results in Proposition 7 are consistent with central clearing arising endogenously in markets where participants are homogenous in terms of their business type. In the model this is equivalent to q being close to 1 or 0. The first central counterparties originated next to grain and coffee exchanges, where *farmers* and *bakers* traded futures (Kroszner (2006), and Gregory (2014)).

5. Implications for collateral and default

The goal of this section is to illustrate the implications of our model for the collateral policies of each clearing arrangement and for the associated default rates at equilibrium.

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[25]As discussed in the Introduction, our assumption that CCPs cannot condition payments on the future realizations of idiosyncratic income shocks captures the crucial role of liquidity of the centrally cleared instruments for CCPs to effectively manage the risk of members' default. Absent any liquidity concerns, we can show that it is optimal for CCPs to move away from full sharing of idiosyncratic risks (see Antinolfi et al. (2019)).

[26]The risk premium of a lottery is a measure of how many resources, in expectation, an agent is willing to give up to avoid uncertainty: $RP_{\mathcal{L}} = E_{\mathcal{L}} - CE_{\mathcal{L}}$.

We focus on economies where the assumptions of Proposition 7 are satisfied, implying that information acquisition is not incentive-compatible with central clearing and, as a consequence, bilateral clearing might be preferred.

While collateral requirements with bilateral clearing are tailored to borrower types, the CCP must choose a homogeneous collateral policy, trading off the cost of collateral with that of default. In economies with a small fraction of λ^H borrowers ($q \leq \hat{q}$), the CCP demands that λ^H borrowers post more collateral than their type would require. Consequently, average collateral is larger with central clearing than with bilateral clearing, and default does not occur in equilibrium, as with bilateral clearing. In contrast, in economies with a large fraction of λ^H borrowers ($q > \hat{q}$), the CCP saves resources by reducing collateral requirements and allowing λ^L borrowers to default. Thus, average collateral is lower with central clearing than with bilateral clearing, but the equilibrium default rate is larger. Lemma 8 summarizes these results.

LEMMA 8. *Let Assumptions 2 and 4 hold. Let $\gamma > \hat{\gamma}$, with $\hat{\gamma}$ defined in (26). If $q \leq \hat{q}$, average collateral is lower with bilateral clearing, and average defaults are the same under the two clearing arrangements. If $q \geq \hat{q}$, average collateral is larger and average defaults are smaller with bilateral clearing than they are with central clearing.*

We then further investigate the effects of an increase in the cost of collateral, α.

LEMMA 9. *Let Assumption 2 hold and let $\lambda^H < \lambda^*$ for λ^* defined in (18).*

(i) *If $q \leq \hat{q}$ and lenders are not prudent, i.e., $u'''(x) \leq 0$, then $d\overline{\gamma}/d\alpha > 0$ and $d\hat{\gamma}/d\alpha < 0$.*

(ii) *If $q > \hat{q}$, $\gamma > \hat{\gamma}$ for $\hat{\gamma}$ defined in (26), lenders are prudent enough, i.e., (a) $u'''(x) > 0$ and (b) $pu'(\omega(\lambda^H) + \lambda^H \theta) + (1 - p)u'(\omega(\lambda^H)) > u'(\omega(\lambda^L) + \lambda^L p\theta)$, then $d\overline{\gamma}/d\alpha < 0$.*

The results in Lemma 9 hinge on the collateral policy adopted by the CCP, which depends on the relative measure of high and low type borrowers. When $q \leq \hat{q}$, the CCP's collateral policy treats all borrowers as low types. Bilateral clearing, alternatively, features information acquisition, which allows lenders to tailor collateral requirements to the type of their counterparty. Thus, when $q \leq \hat{q}$, bilateral clearing saves on collateral requirements. As a result, *ceteris paribus*, an increase in the cost of collateral strengthens the relative advantage of bilateral clearing. However, this mechanism alone is not sufficient to guarantee that bilateral clearing is preferred for a larger set of economies when α increases. The reason is that, with bilateral clearing, lenders' consumption depends also on their counterparty's income realization. Thus, with bilateral clearing, the effect of an increase in the cost of collateral must be weighted by the marginal utility of consumption at different levels of consumption. The assumption that lenders are not prudent in case (i), i.e., $u''' \leq 0$, is sufficient to guarantee that this second-order effect on lenders' payoffs works in the same direction as the first-order effect on average collateral requirements.

The opposite result holds when $q > \hat{q}$: the optimal contract with central clearing treats all borrowers as λ^H types. Hence, low type borrowers post too little collateral relative to what their type would require and then default. In an economy where central clearing is preferred, the benefit from economizing on collateral compensates lenders for the cost of λ^L borrowers defaulting in equilibrium. In this scenario, an increase in the cost of collateral must strengthen, *ceteris paribus*, this direct effect. As in the previous case, this is not enough to guarantee that central clearing is preferred for a larger set of economies, because an increase in α has a second-order effect on the payoffs in bilateral clearing via uncertain consumptions. The assumptions in part (ii) of Lemma 9 are sufficient for this second-order effect to work in the same direction as the first-order effect on collateral.[27]

Finally, notice that large values of α can be associated with financial institutions such as hedge funds, whose opportunity cost of collateral is higher than, say, that of MMFs.[28] In this respect, and considering economies with relatively few λ^H borrowers ($q \leq \hat{q}$), the results in Lemma 9 are broadly consistent with evidence of hedge funds clearing a substantial share of their trades bilaterally, whereas MMFs are more likely to rely on financial market infrastructure (e.g., General Collateral Finance Repo Service (GCF Repo) and tri-party settlement).[29]

6. Conclusions

This paper characterizes optimal clearing arrangements for financial transactions in a model where insurance is valuable because of uncertain returns to investment and the heterogeneous quality of trading counterparties. The contribution of this analysis is the identification and characterization of a trade-off between clearing bilaterally and the channeling of clearing services through a CCP. This trade-off arises when incentives to monitor bilateral counterparties are incompatible with the risk pooling activity of the CCP. Thus, the consequence of mandatory CCP clearing is a potential loss of information across markets due to decreased incentives to monitor trading partners. This result

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[27]Assumption (b) in part (ii) of Lemma 9 sets a lower bound on the degree of prudence evaluated at the bilateral contract associated with a λ^H type, which is the largest expected payoff a lender can obtain with bilateral clearing:

$$\underbrace{\left[pu'\!\left(\omega(\lambda^H) + \lambda^H \theta\right) + (1-p)u'\!\left(\omega(\lambda^H)\right) \right] - u'\!\left(\omega(\lambda^H) + \lambda^H p\theta\right)}_{>0 \text{ by prudence}}$$

$$> \underbrace{u'\!\left(\omega(\lambda^L) + \lambda^L p\theta\right) - u'\!\left(\omega(\lambda^H) + \lambda^H p\theta\right)}_{>0 \text{ by concavity}}.$$

[28]Under normal circumstances, disregarding events as money market funds *breaking the buck*.

[29]For evidence related to the U.S. repo market, see the Office of Financial Research Brief Paper no. 17-04, *Benefits and Risks of Central Clearing in the Repo Market*. For more details on MMFs and central clearing, see footnote 9 in Section 1. Focusing our discussion on the region of the parameters' space where $q \leq \hat{q}$ is justified by the heterogeneity in business lines and operations of the vast majority of financial institutions, which is related to the alternative financial opportunities of a borrower (i.e., his type). Hence, the majority of financial institutions are represented as λ^L types in our model.

should not lead to the conclusion that CCPs are not useful for sharing risk in markets. It rather highlights the limits inherent in the ability of CCPs to effectively provide such risk-sharing in markets with adverse selection, and the importance of the riskiness of the underlying assets and of the degree of heterogeneity in market participants in determining whether CCPs can perform their risk-sharing function effectively.

Appendix

Proof of Proposition 1

Let $(\Sigma^{i*}, c_1^{i*}, c_{2,s}^{i*}, x_{2,s}^{i,\Delta*})_{i=\{L,H\}}$ be the solution to problem (P_0^b) and let $\{\hat{X}_2^i\}_{i=L,H}$, $\{\hat{\Sigma}^i, \hat{C}_1^i, \hat{C}_{2s}^i\}_{i=L,H}$ be a mechanism with central clearing constructed from the solution to (P_0^b) as $\hat{X}_2^i \equiv \hat{X}_2 = \sum_{i=L,H} q_i[p\{\Sigma^{i*}x_{2h}^{i1*} + (1-\Sigma^{i*})x_{2h}^{i0*}\} + (1-p)x_{2l}^{i*}]$, $\hat{\Sigma}_i = \Sigma^{i*}$, $\hat{C}_1^i = c_1^{i*}$, and $\hat{C}_{2s}^i = c_{2s}^{i*}$. By construction, constraints (7), (8), and (10) are satisfied by (1), (2), and (6), respectively. Constraint (9) is satisfied by (3), (4), and (5). Then the mechanism $\{\hat{X}_2^i\}_{i=L,H}$, $\{\hat{\Sigma}^i, \hat{C}_1^i, \hat{C}_{2s}^i\}_{i=L,H}$ is feasible for problem (P_0), and it must be $V^{\mathrm{CCP},e=0} \geq u(\hat{X}_2)$. Concavity of $u(\cdot)$ implies $u(\hat{X}_2) > V^{\mathrm{bil},e=0}$ and the conclusion follows.

Proof of Proposition 3

By monotonicity and concavity of $u(\cdot)$ and linearity of (9) in X_2^i, constraint (9) binds and $X_2^H = X_2^L$. Hence, substitute $X_2 = p\theta - \sum_{i\in\{L,H\}} q_i[(1-\Sigma^i)pC_{2h}^i + (1-p)C_{2l}^i] + \sum_i q_i\{\omega - C_1^i\}$ in the objective of problem (P_0), and solve for $(\Sigma^i, C_1^i, C_{2,h}^i, C_{2,l}^i)$. Notice that we can ignore contracts that recommend $\Sigma^H = 1$. Indeed, suppose, by contradiction, that the optimal contracts recommend $\Sigma^H = 1$. Then, by (10), it must be that λ^H borrowers prefer the strategy $(\hat{m}, \hat{\sigma}) = (\lambda^H, 1)$ to the strategy $(\hat{m}, \hat{\sigma}) = (\lambda^L, 0)$:

$$\alpha C_1^H + p(1-\lambda^H)\theta + (1-p)C_{2,l}^H \geq \alpha C_1^L + pC_{2,h}^L + (1-p)C_{2,l}^L. \qquad (29)$$

Suppose $\Sigma^L = 0$: then λ^L borrowers prefer the strategy $(\hat{m}, \hat{\sigma}) = (\lambda^L, 0)$ over the strategy $(\hat{m}, \hat{\sigma}) = (\lambda^L, 1)$, and (10) yields $\alpha C_1^L + C_{2,h}^L + (1-p)C_{2,l}^L \geq \alpha C_1^H + p(1-\lambda^L)\theta + (1-p)C_{2,l}^H$, which contradicts (29). Suppose, then, that $\Sigma^L = 1$. Consider then the contracts $(\tilde{\Sigma}^i, \tilde{C}_1^i, \tilde{C}_{2,s}^i)$, where $\tilde{C}_{2,h}^H = (1-\lambda^H)\theta$, $\tilde{C}_{2,s}^i = C_{2,s}^i$ if either $i \neq H$ or $s \neq h$, $\tilde{C}_1^i = C_1$ for $i \in \{L, H\}$, $\tilde{\Sigma}^H = 0$, and $\tilde{\Sigma}^L = 1$. It is easy to check that all constraints in problem (P_0) are satisfied, and the new contract is payoff equivalent to the original (optimal) one. Thus, we can ignore contracts with $\Sigma^H = 1$, and only characterize contracts with $(\Sigma^L, \Sigma^H) = (0, 0)$ and with $(\Sigma^L, \Sigma^H) = (1, 0)$.

CLAIM. The solution to (P_0) with $(\Sigma^L, \Sigma^H) = (0, 0)$ is $C_{2,l}^H = C_{2,l}^L = 0$, $C_1^H = C_1^L = \omega - \omega(\lambda^L)$, and $C_{2h}^H = C_{2,h}^L = (1-\lambda^L)\theta$ for $\omega(\lambda)$ defined in Assumption 2.

PROOF. First, constraint (10) is equivalent to the conditions

$$\min\{C_{2,h}^H, C_{2,h}^L\} \geq (1-\lambda^L)\theta, \qquad (30)$$

$$\alpha C_1^H + pC_{2,h}^H + (1-p)C_{2,l}^H = \alpha C_1^L + pC_{2,h}^L + (1-p)C_{2,l}^L. \qquad (31)$$

Indeed, when $\Sigma^H = \Sigma^L = 0$, constraint (10) for λ^L borrowers becomes

$$C^L_{2,h} \geq (1 - \lambda^L)\theta, \tag{32}$$

$$\alpha C^L_1 + p(1 - \lambda^L)\theta + (1 - p)C^L_{2,l}$$
$$\geq \alpha C^H_1 + p\max\{(1 - \lambda^L)\theta, C^H_{2,h}\} + (1 - p)C^H_{2,l}, \tag{33}$$

whereas for λ^H borrowers, it becomes

$$C^H_{2,h} \geq (1 - \lambda^H)\theta, \tag{34}$$

$$\alpha C^H_1 + pC^H_{2,h} + (1 - p)C^H_{2,l} \geq \alpha C^L_1 + pC^L_{2,h} + (1 - p)C^L_{2,l}. \tag{35}$$

Equations (32) and (34) are equivalent to (30). Combining (33) with (35), we obtain $\alpha C^H_1 + pC^H_{2,h} + (1-p)C^H_{2,l} \geq \alpha C^L_1 + pC^L_{2,h} + (1-p)C^L_{2,l} \geq \alpha C^H_1 + p\max\{(1-\lambda^L)\theta, C^H_{2,h}\} + (1-p)C^H_{2,l} \geq \alpha C^H_1 + pC^H_{2,h} + (1-p)C^H_{2,l}$, which must all hold at equality, and (31) follows.

Second, notice that we can ignore the participation constraint (7) of λ^H borrowers. The conclusion follows from (31) and (7) for λ^L borrowers.

Third, we have that $C^L_{2,l} = C^H_{2,l} = 0$. Suppose, by contradiction, that $C^L_{2,l} > 0$. Then it must be $C^L_1 = \omega$: if not, reduce $C^L_{2,l}$ by ϵ and increase C^L_1 by $(1 - p)\epsilon/\alpha$. The new contract is feasible in (P_0) and delivers a higher value of the objective in (P_0). With $C^L_1 = \omega$, $C^L_{2,l} > 0$, and $C^L_{2,h} \geq (1 - \lambda^L)\theta$, the participation constraint (7) of λ^L borrowers can be ignored. Moreover, $C^H_{2,h} = (1 - \lambda^L)\theta$; otherwise, we could reduce $C^L_{2,l}$ by ϵ and $C^H_{2,h}$ by $(1 - p)\epsilon/p$, satisfy all constraints, and deliver a higher value of the objective. Finally, it should be that $C^H_1 = 0$; otherwise, we could reduce $C^L_{2,l}$ by ϵ, reduce C^H_1 by $(1 - p)\epsilon/\alpha$, and attain a higher value of the objective. Substituting $C^H_1 = C^H_{2,l} = 0$, $C^H_{2,h} = (1 - \lambda^L)\theta$, and $C^L_1 = \omega$, (35) yields $(1 - \lambda^L)\theta = \alpha\omega + pC^L_{2,h} + (1 - p)C^L_{2,l}$, which is violated for $C^L_{2,l} > 0$ and $C^L_{2,h} \geq (1 - \lambda^L)\theta$, reaching a contradiction. By a similar argument, $C^H_{2,l} = 0$.

Fourth, $C^H_{2,h} = C^L_{2,h} = (1 - \lambda^L)\theta$. Indeed, if $C^i_{2,h} > (1 - \lambda^L)\theta$, we can reduce $C^i_{2,h}$ by ϵ, increase C^i_1 by $p\epsilon/\alpha$, and attain a higher value of the objective.

Finally, the participation constraint implies $C^i_1 = \omega - (1 - \lambda^L)p\theta/\alpha$, Assumption 2 implies $\omega - (1 - \lambda^L)p\theta/\alpha > 0$, and (31) implies $C^H_1 = C^L_1$. □

CLAIM. Let $\omega(\lambda)$ be defined in Assumption 2. A solution to problem (P_0) with $(\Sigma^L, \Sigma^H) = (1, 0)$ is such that $C^H_{2,l} = C^L_{2,l} = 0$, $C^H_1 = C^L_1$, $C^L_{2,h} = (1 - \lambda^H)\theta$, and

$$C^H_{2,h} = \begin{cases} (1 - \lambda^H)\theta & \text{if } q \geq \dfrac{1}{\alpha}, \\ (1 - \lambda^L)\theta & \text{if } q < \dfrac{1}{\alpha}, \end{cases} \qquad C^i_1 = \begin{cases} \omega - \omega(\lambda^H) & \text{if } q \geq \dfrac{1}{\alpha}, \\ \omega - \omega(\lambda^L) & \text{if } q < \dfrac{1}{\alpha}. \end{cases}$$

PROOF. First, rewrite constraint (10) as

$$C^H_{2,h} \geq (1 - \lambda^H)\theta, \tag{36}$$

$$\alpha C^H_1 + pC^H_{2,h} + (1 - p)C^H_{2,l}$$

$$\geq \alpha C_1^L + p \max\{(1-\lambda^H)\theta, C_{2,h}^L\} + (1-p)C_{2,l}^L, \tag{37}$$

$$C_{2,h}^L \leq (1-\lambda^L)\theta, \tag{38}$$

$$\alpha C_1^L + p(1-\lambda^L)\theta + (1-p)C_{2,l}^L$$
$$\geq \alpha C_1^H + p \max\{(1-\lambda^L)\theta, C_{2,h}^H\} + (1-p)C_{2,l}^H, \tag{39}$$

where (36)–(37) and (38)–(39) are the incentive-compatibility constraints for λ^H and λ^L borrowers, respectively.

Second, note that we can choose $C_{2,h}^L \in [0, (1-\lambda^H)\theta]$, as it satisfies (38) and relaxes (37). Since $C_{2,h}^L$ does not appear in any other constraint, we can ignore (38). Without loss of generality we choose $C_{2,h}^L = (1-\lambda^H)\theta$.

Third, from (39), we can ignore the participation constraint of λ^L borrowers.

Fourth, $C_{2,h}^H \leq (1-\lambda^L)\theta$. If not, $C_{2,h}^H > (1-\lambda^L)\theta > (1-\lambda^H)\theta$ and we can ignore (36). Moreover, $C_1^H = \omega$: if not, we can reduce $C_{2,h}^H$ by ϵ, increase C_1^H by $p\alpha/\epsilon$, and achieve a higher value of the objective. Since $C_1^H = \omega$, we can ignore the participation constraint of λ^H borrowers. Also, (37) binds or the CCP could reduce $C_{2,h}^H$ without violating any constraint and achieve a higher value of the objective. So (37) becomes

$$\alpha\omega + pC_{2,h}^H + (1-p)C_{2,l}^H = \alpha C_1^L + p(1-\lambda^H)\theta + (1-p)C_{2,l}^L. \tag{40}$$

From (40), we can ignore (39). Therefore, the only relevant constraints are (40), the resource constraint (8) for $i = L$, and nonnegativity of $C_{2,l}^i$. Then, as CCP's revenues are decreasing in C_1^L and $C_{2,l}^L$, it must be that $C_1^L = C_{2,l}^L = 0$. If, by contradiction, $C_1^L > 0$ ($C_{2,l}^L > 0$), we can reduce C_1^L ($C_{2,l}^L$) and $C_{2,h}^H$ to leave (40) unchanged, increasing CCP's revenues. But if $C_1^L = C_{2,l}^L = 0$, then (40) implies $C_{2,h}^H < (1-\lambda^H)\theta$, which contradicts $C_{2,h}^H > (1-\lambda^L)\theta$, proving it must be that $C_{2,h}^H \leq (1-\lambda^L)\theta$.

Fifth, constraint (39) holds with equality: $\alpha C_1^L + (1-p)C_{2,l}^L = \alpha C_1^H + (1-p)C_{2,l}^H$. Suppose not: suppose $\alpha C_1^L + (1-p)C_{2,l}^L > \alpha C_1^H + (1-p)C_{2,l}^H$. Then it should easily be $C_1^L = C_{2,l}^L = 0$, since C_1^L and $C_{2,l}^L$ only enter the right-hand side of (37) (and the left-hand side of (39), which is slack). But then (39) implies $0 > \alpha C_1^H + (1-p)C_{2,l}^H$. Hence, (39) holds with equality.

Sixth, constraint (37) can be ignored, as (39) with equality and (38) imply $\alpha C_1^H + pC_{2,h}^H + (1-p)C_{2,l}^H = \alpha C_1^L + pC_{2,h}^H + (1-p)C_{2,l}^L \geq \alpha C_1^L + p(1-\lambda^H)\theta + (1-p)C_{2,l}^L$.

Seventh, the participation constraint (7) of a λ^H borrower binds. Suppose not. Then $C_{2h}^H = (1-\lambda^H)\theta$ and $C_1^H = C_1^L = C_{2,l}^H = C_{2,l}^L = 0$, which can be a solution only if $\omega < (1-\lambda^H)p\theta/\alpha$, contradicting Assumption 2.

Eighth, $C_{2,l}^H = C_{2,l}^L = 0$. Suppose not: if $C_{2,l}^i > 0$, then it must be $C_1^i = \omega$. Otherwise, we could reduce $C_{2,l}^i$ by ϵ and increase C_1^i by $(1-p)\epsilon/\alpha$, increasing the revenues for the CCP. But then if either $C_1^H = \omega$ or $C_1^L = \omega$, (39) implies that the participation constraint of λ^H borrowers is slack, which contradicts our seventh result.

Ninth, with (39) binding and $C_{2,l}^H = C_{2,l}^L = 0$, we obtain $C_1^H = C_1^L$.

Tenth, $C_1^i = \omega$. Suppose, by contradiction, that $C_1^i = \omega$. Then (39) implies that the participation constraint (7) of λ^H types is slack, which contradicts our seventh result.

Next, with $C_{2,l}^i = 0$, constraint (7) binding for a λ^H type, and constraint (39) binding, we can rewrite $C_1^L = C_1^H = pC_{2,h}^H/\alpha$. Substituting these in problem (P_0), requires us to solve only for $C_{2,h}^H$:

$$\max_{C_{2,h}^H} u\left(qp\theta + pC_{2h}^H\left[\frac{1}{\alpha} - q\right] \right)$$

$$\text{s.t. } \min\left\{ \frac{\alpha\omega}{p}, (1-\lambda^L)\theta \right\} \geq C_{2h}^H \geq (1-\lambda^H)\theta.$$

It is easy to see that the objective is increasing in C_{2h} if and only if $q \leq 1/\alpha$. From Assumption 2, we conclude that

$$C_{2,h}^H = \begin{cases} (1-\lambda^H)\theta & \text{if } q \geq 1/\alpha, \\ (1-\lambda^L)\theta & \text{if } q < 1/\alpha. \end{cases} \qquad \Box$$

CLAIM. *The optimal contract with central clearing.* The optimal contract induces no borrower to default, $\Sigma^H = \Sigma^L = 0$, if $q \leq \hat{q}$, and induces λ^L borrowers to default in equilibrium, $\Sigma^H = 0$, $\Sigma^L = 1$, if $q > \hat{q}$ for $\hat{q} \equiv 1/\alpha + (1-1/\alpha)\lambda^L/\lambda^H$.

The proof follows by comparing the payoffs of the two types of contracts above.

Proof of Lemma 5

Replace $x_{2,h}^i$ and $x_{2,l}^i$ from the binding constraints (15) and (16), and rewrite problem (P^i):

$$(P^i) \quad V_i = \max_{(c_1^i, c_{2,h}^i, c_{2,l}^i) \in \mathfrak{R}_+^3} pu(\theta - c_{2,h}^i + \omega - c_1^i) + (1-p)u(\omega - c_1^i - c_{2,l}^i) - \gamma,$$

subject to
$$\alpha c_1^i + pc_{2,h}^i + (1-p)c_{2,l}^i \geq \alpha\omega, \tag{41}$$

$$\omega \geq c_1^i \geq 0, \tag{42}$$

$$c_{2,h}^i \geq (1-\lambda^i)\theta. \tag{43}$$

Notice first that it must be that $c_1^i < \omega$. Indeed, if $c_1 = \omega$, (43) would be slack and we could slightly decrease c_1^i without violating any constraints and increasing the value of the objective function.

Next, notice that it must be that $x_{2,h}^i \geq x_{2,l}^i$. If not, then $x_{2,h}^i < x_{2,l}^i$ and

$$0 < x_{2,l}^i - x_{2,h}^i = c_{2,h}^i - (\theta + c_{2,l}^i) \leq c_{2,h}^i - (1-\lambda^i)\theta. \tag{44}$$

Then (43) is slack and the lender could reduce $c_{2,h}^i$ by ϵ and increase $c_{2,l}^i$ by $p\epsilon/(1-p)$. All constraints would be satisfied and by concavity of $u(\cdot)$ the lender would increase her expected utility. Hence, $x_{2,h}^i \geq x_{2,l}^i$.

Suppose next that $c_{2,l}^i > 0$. Then it should be that $x_{2,h}^i = x_{2,l}^i$. If not, i.e., if $x_{2,h}^i > x_{2,l}^i$, the lender could increase $c_{2,h}^i$ by ϵ and reduce $c_{2,l}^i$ by $p\epsilon/(1-p)$. All constraints would be

satisfied and, by concavity of $u(\cdot)$, the lender would increase her expected utility. Since $x_{2,h}^i = x_{2,l}^i$, (44) becomes $c_{2,h}^i = c_{2,l}^i + \theta > (1 - \lambda^i)\theta$. But then the lender could reduce $c_{2,h}^i$ and $c_{2,l}^i$ by ϵ, and increase c_1 by ϵ/α. All constraints would be satisfied and the lender expected utility would increase. Thus, it must be $c_{2,l}^i = 0$.

Observe next that the participation constraint (41) should bind. Suppose, by contradiction, that (41) is slack. Then (43) should bind. Indeed, if both (41) and (43) were slack, we could decrease $c_{2,h}^i$ without violating any constraints. Since $\omega > \omega(\lambda^L)$, Assumption 2 implies $c_1^i > 0$. But then, if $c_1^i > 0$ and (41) is slack, we could just decrease c_1^i, which delivers higher utility to the lender. Hence, (41) binds

Since $c_{2,l}^i = 0$ and (41) binds, we have that $c_{2,h}^i = \alpha(\omega - c_1^i)/p$, which, substituted in the objective function and in (43), yields

$$(P^i) \quad V_i = \max_{(c_1^i) \in \Re_+} pu\left(\theta - (\omega - c_1^i)\frac{\alpha - p}{p}\right) + (1 - p)u(\omega - c_1^i) - \gamma,$$

$$c_1^i \le \omega - \frac{(1 - \lambda^i)p\theta}{\alpha}. \tag{45}$$

Ignore constraint (45): the first-order condition for optimality is

$$(\alpha - p)u'\left(\theta - (\omega - c_1^i)\frac{\alpha - p}{p}\right) \le (1 - p)u'(\omega - c_1^i) \tag{46}$$

with equality if $c_1^i > 0$. Notice that the left-hand side is decreasing in c_1^i and the right-hand side is increasing in c_1^i. Suppose first that $c_1^i = 0$: (46) requires $(\alpha - p)u'(\theta - \omega(\alpha - p)/p) \le (1 - p)u'(\omega)$. Since $\omega > (1 - \lambda^L)p\theta/\alpha$, by Assumption 2, $(\alpha - p)u'(\theta - (1 - \lambda^L)(\alpha - p)\theta/\alpha) < (\alpha - p)u'(\theta - \omega(\alpha - p)/p) \le (1 - p)u'(\omega) < (1 - p)u'((1 - \lambda^L)p\theta/\alpha)$, which violates Assumption 4. Then there exists a unique $c_1^{i*} > 0$ that solves (46). Given this c_1^*, the solution to problem (P^i) depends on λ^i: either $c_1^i = c_1^*$ if (45) is satisfied or c_1^i is defined by (45) holding at equality. Notice that (45) is decreasing in c_1^i; thus, there exists a unique λ^* such that $c_1^i = c_1^*$ if $\lambda^i \ge \lambda^*$ and c_1^i is pinned down by (45) holding at equality if $\lambda^i < \lambda^*$. Specifically, λ^* solves (46) for $\omega - c_1^i = (1 - \lambda^*)p\theta/\alpha$: $(\alpha - p)u'(\theta - (1 - \lambda^*)p\theta(\alpha - p)/\alpha) = (1 - p)u'((1 - \lambda^*)p\theta/\alpha)$. Let $\Psi(\lambda) = u'((1 - \lambda)p\theta/\alpha)/u'(\theta - (1 - \lambda)\theta(\alpha - p)/\alpha)$. Notice that $\Psi(0) = 1 < (\alpha - p)/(1 - p)$ and $\Psi'(\lambda) > 0$. Thus, there exists a unique $\lambda^* \in (0, 1)$ such that $\Psi(\lambda^*) = (\alpha - p)/(1 - p)$. Then we conclude that $c_1^i = c_1^*$ if $\lambda^i \ge \lambda^*$ and $c_1^i = \omega - (1 - \lambda^i)p\theta/\alpha$ if $\lambda^i < \lambda^*$, where λ^* is defined by (18). Assumption 4 implies that $c_{2,h}^L = (1 - \lambda^L)\theta$ and $c_1^L = \omega - (1 - \lambda^L)p\theta/\alpha$. Also, if $\lambda^H > \lambda^*$, we have $c_1^H = \omega - \omega(\lambda^*)$ and $c_{2,h}^H = (1 - \lambda^*)\theta$, whereas if $\lambda^H < \lambda^*$, we have $c_1^H = \omega - \omega(\lambda^H)$ and $c_{2,h}^H = (1 - \lambda^H)\theta$.

Proof of Proposition 6

CLAIM 1. Constraint (23) is satisfied by (24).

PROOF. We prove this by contraposition: when (23) is violated, then (24) is violated. Suppose that there exists a $\tilde{i} \in \{L, H\}$ such that (23) is violated: $w^{\tilde{i},0} < \sum_{s \in \{l, h\}} p_s\{\sigma_s(\lambda^{\tilde{i}}, \lambda^{-\tilde{i}})w^{-\tilde{i},1} + [1 - \sigma_s(\lambda^{\tilde{i}}, \lambda^{-\tilde{i}})]w^{-\tilde{i},0}\}$. From the definition of the max operator, the

condition above, the fact that $\sigma_s(\lambda^i, \lambda^i) = 0$, and the fact that $\gamma > 0$, we obtain $\max_{\hat{i} \in \{L, H\}} \{\sum_{i \in \{L, H\}} q_i [\sum_{s \in \{l, h\}} p_s [\sigma_s(\lambda^i, \lambda^{\hat{i}}) w^{\hat{i}, 1} + [1 - \sigma_s(\lambda^i, \lambda^{\hat{i}})] w^{\hat{i}, 0}]]\} \geq \sum_{i \in \{L, H\}} q_i \times \{\sum_{s \in \{l, h\}} p_s [\sigma_s(\lambda^i, \lambda^{-\hat{i}}) w^{-\hat{i}, 1} + [1 - \sigma_s(\lambda^i, \lambda^{-\hat{i}})] w^{-\hat{i}, 0}]\} = q_{-\hat{i}} w^{-\hat{i}, 0} + q_{\hat{i}} \sum_{s \in \{l, h\}} p_s \{\sigma_s(\lambda^{\hat{i}}, \lambda^{-\hat{i}}) w^{-\hat{i}, 1} + [1 - \sigma_s(\lambda^{\hat{i}}, \lambda^{-\hat{i}})] w^{-\hat{i}, 0}\} > q_{-\hat{i}} w^{-\hat{i}, 0} + q_{\hat{i}} w^{\hat{i}, 0} = \sum_{i \in \{L, H\}} q_i w^{i, 0} > -\gamma + \sum_{i \in \{L, H\}} q_i w^{i, 0}$, which proves that (24) is also violated. □

CLAIM 2. Maximum punishment for lack of due diligence is optimal: $w^{i, 1} = 0$.

The proof follows from the right-hand side of (24) being strictly increasing in $w^{i, 1}$.

Hence, we substitute $w^{i, 1} = 0$ and, unless necessary to clarify the dependence on the default decision, in the remainder of the proof, we simplify notation by rewriting $w^i = w^{i, 0}$.

CLAIM 3. Central clearing with information acquisition is preferred to central clearing without information acquisition only if $C_{2, h}^H < (1 - \lambda^L)\theta$.

PROOF. The proof is by contradiction. Let (w^H, w^L), $(C_1^i, C_{2, h}^i, C_{2, l}^i)_{i = L, H}$ be the solution to problem (P_1) and suppose $C_{2h}^H \geq (1 - \lambda^L)\theta$. Consider now the contract with central clearing and no monitoring in problem (P_0) defined as $\hat{X}_2 = q u^{-1}(w^H) + (1 - q) u^{-1}(w^L)$, $\hat{C}_{2, s} = q C_{2, s}^H + (1 - q) C_{2, s}^L$, and $\hat{C}_1 = q C_1^H + (1 - q) C_1^L$, $\Sigma^L = \Sigma^H = 0$. Easily, this contract is incentive-compatible in (10) and satisfies (7)–(9). Concavity of $u(\cdot)$ gives $u(\hat{X}_2) \geq q w^H + (1 - q) w^L > q w^H + (1 - q) w^L - \gamma$ and the conclusion follows. □

CLAIM 4. The solution to (P_1) satisfies $\sigma_h(\lambda^L, \lambda^H) = 1$ and $\sigma_h(\lambda^H, \lambda^L) = 0$.

The conclusion $\sigma_h(\lambda^L, \lambda^H) = 1$ follows from Claim 3 above. Alternatively, the conclusion $\sigma_h(\lambda^L, \lambda^H) = 0$ follows easily from (20).

Ignoring constraint (23) and substituting $\sigma_h(\lambda^H, \lambda^L) = 0$ and $\sigma_h(\lambda^L, \lambda^H) = 1$ in (24) and (25), we can rewrite problem (P_1) as

$$(\hat{P}^{FI}) \quad \max_{(w^i, C_1^i, C_{2, s}^i)} q w^H + (1 - q) w^L - \gamma \tag{47}$$

such that $\alpha C_1^i + p C_{2, h}^i + (1 - p) C_{2, l}^i \geq \alpha \omega,$ (48)

$$C_{2, h}^i \geq (1 - \lambda^i)\theta, \tag{49}$$

$$\omega - \sum_{i \in \{L, H\}} q_i C_1^i \geq 0, \tag{50}$$

$$q u^{-1}(w^H) + (1 - q) u^{-1}(w^L)$$
$$+ \sum_{i \in \{L, H\}} \{q_i [C_1^i + p C_{2, h}^i + (1 - p) C_{2, l}^i]\} \leq \omega + p\theta, \tag{51}$$

$$-\gamma + q w^H + (1 - q) w^L \geq \max\{w^L, [q + (1 - q)(1 - p)] w^H\}. \tag{52}$$

CLAIM 5. In problem $(\hat{P}^{FI})$, we can replace constraint (52) with

$$-\gamma + qw^H + (1-q)w^L \geq w^L, \tag{53}$$

$$w^H \geq \frac{\gamma}{pq(1-q)}; \qquad w^L \geq \frac{q+(1-q)(1-p)}{pq(1-q)}\gamma. \tag{54}$$

PROOF. Suppose not: suppose that if, in problem $(\hat{P}^{FI})$, we replace constraint (52) with constraints (53) and (54), we obtain a different solution. Specifically, let $(w^{i*}, C_1^{i*}, C_{2,s}^{i*})$ be the solution to problem $(\hat{P}^{FI})$ and let $(w^{i**}, C_1^{i**}, C_{2,s}^{i**})$ be the solution to the modified problem, i.e., the one with constraints (53) and (54). Note first that (53) and (54) are necessary conditions for (52): from the definition of the max operator, constraint (52) requires both $-\gamma + qw^H + (1-q)w^L \geq w^L$ and $-\gamma + qw^H + (1-q)w^L \geq [q + (1-q)(1-p)]w^H$, and it is easy to show that if these two conditions hold, then (53) and (54) are also satisfied. Then, since (53) and (54) define a larger set for $(w^H, w^L) \in \Re_+^2$ than constraint (52), it must be that the solution to the modified problem, $(w^{i**}, C_1^{i**}, C_{2,s}^{i**})$, is not feasible under the original problem $(\hat{P}^{FI})$ and that

$$qw^{H**} + (1-q)w^{L**} \geq qw^{H*} + (1-q)w^{L*}. \tag{55}$$

The only way in which the solution to the modified problem is not feasible in $(\hat{P}^{FI})$ is that $-\gamma + qw^{H**} + (1-q)w^{L**} < [q+(1-q)(1-p)]w^{H**}$. Consider then in the original problem $(\hat{P}^{FI})$ the new contracts $(w^{i'}, C_1^{i'}, C_{2,s}^{i'})$ constructed as follows: $C_{2,s}^{i'} = C_{2,s}^{i**}$, $C_1^{i'} = C_1^{i**}$, and $(w^{H'}, w^{L'})$ are the unique values solving the two equations $w^{H'} = w^{L'} + \gamma/q$ and $qu^{-1}(w^{H'}) + (1-q)u^{-1}(w^{L'}) = qu^{-1}(w^{H**}) + (1-q)u^{-1}(w^{L**})$. The new contracts satisfy constraints (48)–(51) by construction. Also, constraint (52) is satisfied by construction: if not, it must be that $w^{L'}/(1-p) - \gamma/[(1-p)(1-q)] < w^{H'} = w^{L'} + \gamma/q$, which can hold only if $w^{L'} < \gamma[q+(1-q)(1-p)]/[pq(1-q)]$ and, therefore, $w^{H'} = w^{L'}/[q+(1-q)(1-p)] < \gamma/[pq(1-q)]$. Since u^{-1} is increasing, we have

$$qu^{-1}(w^{H'}) + (1-q)u^{-1}(w^{L'})$$
$$< qu^{-1}\left(\frac{\gamma}{pq(1-q)}\right) + (1-q)u^{-1}\left(\frac{q+(1-q)(1-p)}{pq(1-q)}\gamma\right). \tag{56}$$

However, by construction of $w^{H'}$ and $w^{L'}$ and (54), we have $qu^{-1}(w^{H'}) + (1-q) \times u^{-1}(w^{L'}) = qu^{-1}(w^{H**}) + (1-q)u^{-1}(w^{L**}) \geq qu^{-1}(\gamma/[pq(1-q)]) + (1-q)u^{-1}(\gamma[q+(1-q)(1-p)]/[pq(1-q)])$, which contradicts (56). Therefore the new contracts satisfy constraint (52) as well. Finally, for $X = qu^{-1}(w^{H**}) + (1-q)u^{-1}(w^{L**})$, we have that $qw^{H'} + (1-q)w^{L'} = q(w^{H**} + \int_{w^{L**}}^{w^{L'}}[-u'([X-(1-q)u^{-1}(s)]/q)(1-q)/[qu'(s)]]ds) + (1-q)(w^{L**} + \int_{w^{L**}}^{w^{L'}} 1ds) = qw^{H**} + (1-q)w^{L**} + (1-q)\int_{w^{L**}}^{w^{L'}}[1-u'([X-(1-q)u^{-1}(s)]/q)/u'(s)]ds > qw^{H**}+(1-q)w^{L**}$, where the last inequality follows from concavity of u together with the fact that $[X-(1-q)u^{-1}(s)]/q > s$ for all $s \in [w^{L**}, w^{L'}]$. But then the new contracts $(w^{i'}, C_1^{i'}, C_{2,s}^{i'})$ satisfy constraints (48)–(52) and from (55) deliver lenders a larger expected utility than the optimal contracts $(w^{i*}, C_1^{i*}, C_{2,s}^{i*})$, which contradicts optimality of $(w^{i*}, C_1^{i*}, C_{2,s}^{i*})$ and concludes the proof. □

CLAIM 6. *If a solution to* $(\hat{P}^{FI})$ *exists, it must be such that* (51) *and* (53) *bind:* (i) $qu^{-1}(w^{H*}) + (1-q)u^{-1}(w^{L*}) = \omega + p\theta - \sum_{i\in\{L,H\}}\{q_i[C_1^{i*} + pC_{2,h}^{i*} + (1-p)C_{2,l}^{i*}]\}$ *and* (ii) $-\gamma + qw^{H*} + (1-q)w^{L*} = w^{L*}$.

PROOF. Suppose a solution to $(\hat{P}^{FI})$ exists, but $qu^{-1}(w^{H*})+(1-q)u^{-1}(w^{L*}) < \omega + p\theta - \sum_{i\in\{L,H\}}\{q_i[C_1^{i*} + pC_{2,h}^{i*} + (1-p)C_{2,l}^{i*}]\}$. It is easy to verify that we could construct new contracts $C_1^{i'} = C_1^{i*}$, $C_{2,s}^{i'} = C_{2,s}^{i*}$, and $w^{H'} = w^{H*} + \epsilon$, $w^{L'} = w^{L*} + \epsilon$ for ϵ small that satisfy all constraints and make lenders attain a higher expected utility.

Suppose next that the solution to $(\hat{P}^{FI})$ is such that $-\gamma + qw^{H*} + (1-q)w^{L*} > w^{L*}$. It is easy to verify that we could construct new contracts with $C_1^{i'} = C_1^{i*}$ and $C_{2,s}^{i'} = C_{2,s}^{i*}$, and $(w^{H'}, w^{L'})$ being a mean-preserving contraction on $u^{-1}(w^{H*})$ and $u^{-1}(w^{L*})$, so that (51) is unaffected, but by convexity of $u^{-1}(\cdot)$, the value of the objective function is strictly higher. □

CLAIM 7. *We have* $C_{2h}^i = (1-\lambda^i)\theta$, $C_{2l}^i = 0$, *and* $C_1^i = \omega - \omega(\lambda^i)$.

PROOF. From Claim 6, the objective function is strictly increasing in second period resources: $\omega + p\theta - \sum_{i\in\{L,H\}}\{q_i[C_1^i + pC_{2,h}^i + (1-p)C_{2,l}^i]\}$. From Assumption 2, the participation constraint (48) should bind. From $\alpha > 1$, we obtain that (49) binds, hence $C_{2,h}^i = (1-\lambda^i)\theta$, and that $C_{2,l}^i = 0$. □

CLAIM 8. *A solution to problem* $(\hat{P}^{FI})$ *exists and is unique if and only if* $\gamma \le \hat{\gamma}$, *for* $\hat{\gamma}$ *defined in* (26). *Then if* $\gamma \le \hat{\gamma}$, $qw^H + (1-q)w^L - \gamma = w^L$ *for* w^L *solving* $qu^{-1}(w^L + \gamma/q) + (1-q)u^{-1}(w^L) = \sum_{i\in\{L,H\}}q_i[\lambda^i p\theta + \omega(\lambda^i)]$.

The conclusion follows from Claim 5, Claim 6, and Claim 7.

CLAIM 9. *If* $\gamma \le \hat{\gamma}$ *for* $\hat{\gamma}$ *is defined in* (26), *then* $\max\{u(X_2^*), \sum_{i\in\{L,H\}}\{q_iw^{i,0*}\}\} \ge \sum_{i\in\{L,H\}}\{q_i[\sum_{s=l,h} p_su(x_{2,s}^{i*})]\}$, *where* $x_{2,s}^{i*}$ *is lenders' consumption in the optimal contract with bilateral clearing and monitoring of Lemma 5, and* X_2^* *is lenders' consumption in* (11) *for the optimal contract with CCP clearing and no monitoring in Proposition 3, and* $w^{i,0*}$ *is lenders' utility for the optimal contract with CCP clearing and monitoring.*

PROOF. Suppose not: suppose that the optimal contract with bilateral clearing and monitoring dominates both the optimal contract with central clearing and monitoring and the optimal contract with CCP clearing and no information acquisition:

$$\max\left\{u(X_2^*), \sum_{i\in\{L,H\}}\{q_iw^{i,0*}\} - \gamma\right\} < \sum_{i\in\{L,H\}}\left\{q_i\left[\sum_{s=l,h} p_su(x_{2,s}^{i*})\right]\right\} - \gamma. \tag{57}$$

Let $(x_{2h}^{i*}, x_{2l}^{i*}, c_1^{i*}, c_{2h}^{i*}, c_{2l}^{i*})$ be the optimal contracts with bilateral clearing and monitoring, and let $(w^{i,0*}, C_{2h}^{i*}, C_{2l}^{i*}, C_1^{i*})$ be the optimal contracts with CCP clearing and monitoring. Define $\hat{w}^H$ and $\hat{w}^L$ as $\hat{w}^H = pu(x_{2h}^{H*}) + (1-p)u(x_{2l}^{H*})$, $\hat{w}^L = pu(x_{2h}^{L*}) + (1-p)u(x_{2l}^{L*})$, and consider, in problem $(\hat{P}^{FI})$, the contracts with CCP clearing and monitoring $(\hat{w}^i, \hat{C}_{2h}^i, \hat{C}_{2l}^i, \hat{C}_1^i)$, where $\hat{w}^H$ and $\hat{w}^L$ are defined above, and $\hat{C}_1^i = c_1^{i*}$, $\hat{C}_{2h}^i = c_{2h}^{i*}$, and $\hat{C}_{2l}^i = c_{2l}^{i*}$.

Step 1. The contract $(\hat{w}^i, \hat{C}_1^i, \hat{C}_{2h}^i, \hat{C}_{2l}^i)$ satisfies (48)–(51).

Equations (48)–(50) are satisfied by construction, and equation (51) is satisfied from concavity of $u(\cdot)$:

$$
\begin{aligned}
qu^{-1}&(\hat{w}^H) + (1-q)u^{-1}(\hat{w}^L) \\
&= qu^{-1}\big(pu(x_{2,h}^{H*}) + (1-p)u(x_{2,l}^{H*})\big) + (1-q)u^{-1}\big(pu(x_{2,h}^{L*}) + (1-p)u(x_{2,l}^{L*})\big) \\
&< q\big[px_{2h}^{H*} + (1-p)x_{2l}^{H*}\big] + (1-q)\big[px_{2h}^{L*} + (1-p)x_{2l}^{L*}\big] \\
&= p\theta + \omega - q\big[\hat{C}_1^H + p\hat{C}_{2h}^H + (1-p)\hat{C}_{2l}^H\big] \\
&\quad - (1-q)\big[\hat{C}_1^L + p\hat{C}_{2h}^L + (1-p)\hat{C}_{2l}^L\big].
\end{aligned}
\tag{58}
$$

Step 2. The contracts $(\hat{w}^i, \hat{C}_1^i, \hat{C}_{2h}^i, \hat{C}_{2l}^i)$ satisfy $q\hat{w}^H + (1-q)\hat{w}^L - \gamma \geq \hat{w}^L$.

Consider, in problem (P_0), the contract $(X_2, C_1^i, C_{2h}^i, C_{2l}^i)$ with $C_1^i = c_1^{L*}$, $C_{2,s}^i = c_{2,s}^{L*}$, and $X_2 = u^{-1}(pu(x_{2h}^{L*}) + (1-p)u(x_{2l}^{L*}))$. Such a contract is feasible in (P_0); therefore, it must be that

$$
u(X_2^*) \geq u(X_2) = pu(x_{2h}^{L*}) + (1-p)u(x_{2l}^{L*}) = \hat{w}^L.
\tag{59}
$$

Moreover, since the contract with bilateral clearing and monitoring is preferred to the contract with CCP clearing and no monitoring,

$$
q\underbrace{\big[pu(x_{2h}^{H*}) + (1-p)u(x_{2l}^{H*})\big]}_{\hat{w}^H} + (1-q)\underbrace{\big[pu(x_{2h}^{L*}) + (1-p)u(x_{2l}^{L*})\big]}_{\hat{w}^L} - \gamma \geq u(X_2^*).
$$

Combining the last expression with (59), we conclude that $\hat{w}^H \geq \hat{w}^L + \gamma/q$.

Step 3. The contracts $(\hat{w}^i, \hat{C}_{2h}^i, \hat{C}_{2l}^i, \hat{C}_1^i)$ must be such that (i) $[q+(1-q)(1-p)]\hat{w}^H > \hat{w}^L$ and (ii) $-\gamma + q\hat{w}^H + (1-q)\hat{w}^L < [q+(1-q)(1-p)]\hat{w}^H$.

If either condition were violated, from Step 2 we could conclude that the contracts $(\hat{w}^i, \hat{C}_{2h}^i, \hat{C}_{2l}^i, \hat{C}_1^i)$ satisfy (48)–(52) in problem $(\hat{P}^{FI})$. Thus, by definition of optimality in problem $(\hat{P}^{FI})$, it must be that $qw^{H,0*} + (1-q)w^{L,0*} \geq q\hat{w}^H + (1-q)\hat{w}^L = q[pu(x_{2h}^{H*}) + (1-p)u(x_{2l}^{H*})] + (1-q)[pu(x_{2h}^{L*}) + (1-p)u(x_{2l}^{L*})$, which contradicts (57).

Step 4. Condition (57) must be violated.

Let $(C_1^{i*}, C_{2,h}^{i*}, C_{2,l}^{i*})$ solve Problem $(\hat{P}^{FI})$. From Claim 7 and (58),

$$
qu^{-1}(\hat{w}^H) + (1-q)u^{-1}(\hat{w}^L) < \sum_{i \in \{L,H\}} q_i\big[\lambda^i p\theta + \omega(\lambda^i)\big]
$$

for $\omega(\lambda)$ defined in Assumption 2. Define then $\delta = \sum_{i\in\{L,H\}} q_i[\lambda^i p\theta + \omega(\lambda^i)] - qu^{-1}(\hat{w}^H) - -(1-q)u^{-1}(\hat{w}^L)$ and define $w^{H'}$ such that

$$u^{-1}(w^{H'}) = u^{-1}(\hat{w}^H) + \frac{\delta}{q}. \tag{60}$$

Since $u^{-1}(\cdot)$ is increasing, $w^{H'} \geq \hat{w}^H$. Define now the operator

$$T(y) = u\left(\frac{qu^{-1}(w^{H'}) + (1-q)u^{-1}(\hat{w}^L) - qu^{-1}\left(\frac{y}{q+(1-p)(1-q)}\right)}{1-q}\right) - y.$$

Notice that $T(y)$ is monotone decreasing in y and that for $y = \bar{y} \equiv [q + (1-p)(1-q)]u([qu^{-1}(w^{H'}) + (1-q)u^{-1}(\hat{w}^L)]/q) > 0$, it is $T(\bar{y}) = u(0) - \bar{y} < 0$. Furthermore, the two conditions $w^{H'} \geq \hat{w}^L + \gamma/q$ and $w^{H'} \geq \hat{w}^L/(1-p) - \gamma/[(1-q)(1-p)]$ imply that $w^{H'} \geq \hat{w}^L/[q + (1-p)(1-q)]$, where the second inequality follows from $w^{H'} \geq \hat{w}^H > \hat{w}^L/(1-p) - \gamma/[(1-q)(1-p)]$, which results from Step 3 and from the definition of $w^{H'} \geq \hat{w}^H$. Then for $y = \hat{w}^L$, it is true that $T(\hat{w}_L) = u([qu^{-1}(w^{H'}) + (1-q)u^{-1}(\hat{w}^L) - qu^{-1}(\hat{w}^L/[q + (1-p)(1-q)])]/(1-q)) - \hat{w}^L \geq u(u^{-1}(\hat{w}^L)) - \hat{w}^L = 0$. By the intermediate value theorem, there must be a $w^{L''} \geq \hat{w}^L$ such that $T(w^{L''}) = 0$. Define then $w^{L''} \in [\hat{w}^L, \bar{y})$ to be the value that satisfies $T(w^{L''}) = 0$, and then define $w^{H''}$ as the solution to $w^{H''} = w^{L''}/[q + (1-p)(1-q)]$. Notice that $w^{H''} \leq w^{H'}$, since $w^{L''} \geq \hat{w}^L$.

Consider then the contract $(w^{H''}, w^{L''}, C_1^{i*}, C_{2h}^{i*}, C_{2l}^{i*})$, where $w^{H''}$ and $w^{L''}$ are defined above and $(C_1^{i*}, C_{2h}^{i*}, C_{2l}^{i*})$ solve problem $(\hat{P}^{FI})$. Notice that this contract satisfies (48)–(51) in problem $(\hat{P}^{FI})$. We want to show that this contract satisfies (52) as well. By construction, $[q + (1-q)(1-p)]w^{H''} = w^{L''}$. Also by construction, via the operator T, $qu^{-1}(w^{H''}) + (1-q)u^{-1}(w^{L''}) = qu^{-1}(w^{H'}) + (1-q)u^{-1}(\hat{w}^L) = \sum_{i\in\{L,H\}} q_i[\lambda^i p\theta + \omega(\lambda^i)]$. Combining the last expression with (26) and because $\gamma \leq \hat{\gamma}$, we obtain $qu^{-1}(w^{H''}) + (1-q)u^{-1}(w^{L''}) \geq qu^{-1}(\gamma/[pq(1-q)]) + (1-q)u^{-1}(\gamma[1-p(1-q)]/[pq(1-q)])$. Notice that the last inequality can be rewritten as $qu^{-1}(w^{L''}/[q+(1-p)(1-q)]) + (1-q)u^{-1}(w^{L''}) \geq qu^{-1}(\ddot{w}^L/[q+(1-p)(1-q)]) + (1-q)u^{-1}(\ddot{w}^L)$ for $\ddot{w}^L = \gamma[1-p(1-q)]/[pq(1-q)]$, which can hold if and only if $w^{L''} \geq \ddot{w}^L$, and, therefore, $w^{H''} = w^{L''}/[q+(1-p)(1-q) \geq \ddot{w}^L/[q+(1-p)(1-q)] \equiv \ddot{w}^H$. Observe also that $\ddot{w}^H = \ddot{w}^L + \gamma/q$. Therefore, for $w^{L''} \geq \ddot{w}^L$ and $w^{H''} \geq \ddot{w}^H$, the following equality holds: $w^{H''} = \ddot{w}^H + (w^{L''} - \ddot{w}^L)/[q + (1-q)(1-p)] = \ddot{w}^L + \gamma/q + (w^{L''} - \ddot{w}^L)/[q + (1-q)(1-p)] = w^{L''} + \gamma/q + (w^{L''} - \ddot{w}^L)\{1/[q + (1-q)(1-p)] - 1\} \geq w^{L''} + \gamma/q$, proving that the contract $(w^{H''}, w^{L''}, C_1^{i*}, C_{2h}^{i*}, C_{2l}^{i*})$ satisfies the constraint (52) as well. Then, by optimality, it must be that $\sum_{i\in\{L,H\}}\{q_i w^{i,0*}\} \geq qw^{H''} + (1-q)w^{L''} = qw^{H''} + (1-q)u([qu^{-1}(w^{H'}) + (1-q)u^{-1}(\hat{w}^L) - qu^{-1}(w^{H''})]/(1-q)) = qw^{H''} + (1-q)u([qu^{-1}(\hat{w}^H) + \delta + (1-q)u^{-1}(\hat{w}^L) - qu^{-1}(w^{H''})]/(1-q)) = qw^{H''} + (1-q)u([\Omega - qu^{-1}(w^{H''})]/(1-q)) = q(w^{H'} - \int_{w^{H''}}^{w^{H'}} ds) + (1-q)\{\hat{w}^L + \int_{w^{H''}}^{w^{H'}}\{u'([\Omega - qu^{-1}(s)]/(1-q))q/](1-q)u'(s)\}ds\} = qw^{H'} + (1-q)\hat{w}^L + q\int_{w^{H''}}^{w^{H'}}\{u'([\Omega - qu^{-1}(s)]/(1-q))/u'(s) - 1\}ds \geq qw^{H'} + (1-q)\hat{w}^L \geq q\hat{w}^H + (1-q)\hat{w}^L = \sum_{i\in\{L,H\}}\{q_i[\sum_{s=l,h} p_s u(x_{2,s}^{i*})]\}$, where $\Omega = \sum_{i\in\{L,H\}} q_i[\lambda^i p\theta + \omega(\lambda^i)]$, the first inequality in the last line follows from the

fact that $[\Omega - qu^{-1}(s)]/(1 - q) < s$ for all $s \in (w^{H''}, w^{H'}]$, and the second inequality in the last line follows from the fact that $w^{H'} \geq \hat{w}^H$, given the definition in (60). But this contradicts (57). □

Proof of Proposition 7

When $\gamma > \hat{\gamma}$, Proposition 6 implies that central clearing with information acquisition is not feasible. Then we need to show that there exists a well defined function $\overline{\gamma}(q)$: $(\underline{q}, \overline{q}) \to \mathfrak{R}_+$ such that bilateral clearing with information acquisition is preferred to central clearing with no information acquisition provided that $\gamma \in (\hat{\gamma}, \overline{\gamma})$.

Let $\omega(\lambda)$ be defined in Assumption 2. From Proposition 3 and Lemma 5, bilateral clearing with information acquisition is preferred to central clearing and no monitoring if and only if $\gamma < \overline{\gamma}(q)$, where $\overline{\gamma}(q) : [0, 1] \to \mathfrak{R}$ is defined as

$$\overline{\gamma}(q) = \sum_{i=L,H} \left\{ q_i \sum_{s=l,h} p_s u(x^i_{2,s}) \right\} - u(X_2) \tag{61}$$

for

$$X_2 = \begin{cases} \omega(\lambda^L) + p\theta\lambda^L & \text{if } q \leq \hat{q}, \\ \omega(\lambda^H) + p\theta q\lambda^H & \text{if } q > \hat{q}, \end{cases} \tag{62}$$

where $\hat{q} = 1/\alpha + (1 - 1/\alpha)\lambda^L/\lambda^H$ and

$$x^H_{2,h} = \min\{\lambda^H \theta + \omega(\lambda^H), \lambda^*\theta + \omega(\lambda^*)\}, \qquad x^H_{2,l} = \max\{\omega(\lambda^H), \omega(\lambda^*)\},$$
$$x^L_{2,h} = \lambda^L \theta + \omega(\lambda^L), \qquad x^L_{2,l} = \omega(\lambda^L). \tag{63}$$

STEP 1. *The function $\overline{\gamma}(q)$ defined in (61) satisfies $\overline{\gamma}(0) < 0$ and $\overline{\gamma}(1) < 0$.*

PROOF. From (61), $\overline{\gamma}(0) = [pu(\lambda^L \theta + \omega(\lambda^L)) + (1 - p))u(\omega(\lambda^L))] - u(\omega(\lambda^L) + p\theta\lambda^L) < 0$, where the inequality comes from concavity of $u(\cdot)$. Also, $\overline{\gamma}(1) = [pu(\min\{\lambda^H \theta + \omega(\lambda^H), \lambda^*\theta + \omega(\lambda^*)\}) + (1 - p))u(\max\{\omega(\lambda^H), \omega(\lambda^*)\})] - u(\omega(\lambda^H) + p\theta\lambda^H) < u(p\min\{\lambda^H \theta + \omega(\lambda^H), \lambda^*\theta + \omega(\lambda^*)\} + (1 - p)\max\{\omega(\lambda^H), \omega(\lambda^*)\}) - u(\omega(\lambda^H) + p\theta\lambda^H) \leq u(p[\lambda^H \theta + \omega(\lambda^H)] + (1 - p)\omega(\lambda^H)) - u(\omega(\lambda^H) + p\theta\lambda^H) = 0$, where the first inequality comes from concavity of $u(\cdot)$ and the second one comes from the definition of λ^*. □

STEP 2. *The function $\overline{\gamma}(q)$ defined in (61) is monotonically increasing for $q \leq \hat{q}$.*

PROOF. Evaluate the derivative of $\overline{\gamma}(q)$ at the allocations defined by (62) and (63) for $q \leq \hat{q}$: $\partial\overline{\gamma}(q)/\partial q = pu(\min\{\lambda^H \theta + \omega(\lambda^H), \lambda^*\theta + \omega(\lambda^*)\}) + (1 - p)u(\max\{\omega(\lambda^H), \omega(\lambda^*)\}) - pu(\lambda^L \theta + \omega(\lambda^L)) + (1 - p)u(\omega(\lambda^L)) > 0$, where the inequality comes from the fact that $x^L_{2,h} = \lambda^L \theta + \omega(\lambda^L)$, $x^L_{2,l} = \omega(\lambda^L)$ is feasible, but not optimal, for the problem of a lender who faces a λ^H borrower. □

STEP 3. The function $\overline{\gamma}(q)$ defined in (61) is convex for $q > \hat{q}$.

PROOF. Evaluate the derivative of $\overline{\gamma}(q)$ at the allocations defined by (62) and (63) for $q > \hat{q}$: $\partial\overline{\gamma}(q)/\partial q = pu(\min\{\lambda^H\theta + \omega(\lambda^H), \lambda^*\theta + \omega(\lambda^*)\}) + (1-p)u(\max\{\omega(\lambda^H), \omega(\lambda^*)\}) - pu(\lambda^L\theta + \omega(\lambda^L)) - (1-p)u(\omega(\lambda^L)) - u'(\omega(\lambda^H) + p\theta q\lambda^H)p\theta\lambda^H$, and then concavity of $u(\cdot)$ implies $\partial^2\overline{\gamma}(q)/\partial q^2 = -u''(\omega(\lambda^H) + p\theta q\lambda^H)[p\theta\lambda^H]^2 > 0$.   □

STEP 4. $\overline{\gamma}(q) > 0$ for some $q \in [0, 1]$ if and only if $\overline{\gamma}(\hat{q}) > 0$. Moreover, if $\overline{\gamma}(\hat{q}) > 0$, then there exist $\underline{q}, \overline{q} \in [0, 1]$, where $\underline{q} < \overline{q}$, such that $\overline{\gamma}(q) > 0$ for $\gamma \in [\underline{q}, \overline{q}]$.

PROOF. Consider the "if" direction: assume that $\overline{\gamma}(\hat{q}) > 0$. Since $\overline{\gamma}(q)$ is strictly increasing for $q < \hat{q}$ and $\overline{q}(0) < 0$, by the intermediate value theorem, there exists a unique $\underline{q} \in (0, 1/\alpha)$ such that $\overline{\gamma}(\underline{q}) = 0$. Also, since $\overline{\gamma}(1) < 0$ and $\overline{\gamma}(q)$ is convex for $q > \hat{q}$, given that $\overline{\gamma}(\hat{q}) > 0$, it must be that for $q > \hat{q}$, the function $\overline{\gamma}(q)$ is initially decreasing, crosses the horizontal axes for a unique $\overline{q}$ where $\overline{\gamma}(\overline{q}) = 0$, and then stays negative. Next, consider the only if direction: suppose that $\overline{\gamma}(q) > 0$ for some $q \in (0, 1)$. Assume, by contradiction, that $\overline{\gamma}(\hat{q}) < 0$. Since the function $\overline{\gamma}(q)$ is strictly increasing for $q < \hat{q}$, then $\overline{\gamma}(q) < 0$ for all $q \le \hat{q}$, and it must be that $\overline{\gamma}(q) > 0$ for some $q > \hat{q}$. But then, since $\overline{\gamma}(\hat{q}) < 0$ and the function $\overline{\gamma}(\hat{q})$ is continuous, there should exist an interval $[q', q''] \subset (\hat{q}, 1]$ such that $\overline{\gamma}(q) > 0$ and $\overline{\gamma}'(q) > 0$ for $q \in [q', q'']$. But then, since the function $\overline{\gamma}(q)$ is convex, it must be that $\overline{\gamma}'(q) > 0$ also for all $q > q''$ and, therefore, $\overline{\gamma}(1) > \overline{\gamma}(q'') > 0$, which is a contradiction.   □

STEP 5. The function $\overline{\gamma}(q) : (\underline{q}, \overline{q}) \to \Re_+$ is well defined if and only if (27) holds.

PROOF. It follows from the previous steps that $\overline{\gamma}(q) < 0$ for all $q \in (0, 1)$ if and only if $\overline{\gamma}(\hat{q}) < 0$.   □

Proof of Lemma 9

(i) The conclusion $d\hat{\gamma}/d\alpha < 0$ comes directly from the definition of $\hat{\gamma}$ in (26). For $d\overline{\gamma}/d\alpha$, use the assumption $\lambda^H < \lambda^*$ and the results of Lemma 5: $x_{2,h}^H = \omega(\lambda^H) + \lambda^H\theta$ and $x_{2,l}^H = \omega(\lambda^H)$. As $q \le \hat{q}$, (12) implies $X_2 = \omega(\lambda^L) + \lambda^L p\theta$. Thus, $d\overline{\gamma}/d\alpha = -q\omega(\lambda^H)\{pu'(\omega(\lambda^H) + \lambda^H\theta) + (1-p)u'(\omega(\lambda^H))\}/\alpha - \omega(\lambda^L)\{(1-q)[pu'(\omega(\lambda^L) + \lambda^L\theta) + (1-p)u'(\omega(\lambda^L))] - u'(\omega(\lambda^L) + \lambda^L p\theta)\}/\alpha \ge -q\omega(\lambda^H)u'(\omega(\lambda^H) + \lambda^H p\theta)/\alpha + q\omega(\lambda^L)u'(\omega(\lambda^L) + \lambda^L p\theta)/\alpha > qu'(\omega(\lambda^L) + \lambda^L p\theta)[\omega(\lambda^L) - \omega(\lambda^H)]/\alpha > 0$, where the first inequality follows from $u'''(\cdot) \le 0$, the second one from $u''(\cdot) < 0$, and the last one from $\lambda^H > \lambda^L$.

(ii) Since $\lambda^H < \lambda^*$, Lemma 5 implies $x_{2,h}^H = \omega(\lambda^H) + \lambda^H\theta$ and $x_{2,l}^H = \omega(\lambda^H)$. With $q > \hat{q}$, (12) implies $X_2 = \omega(\lambda^H) + p\theta q\lambda^H$. Then $d\overline{\gamma}/d\alpha = -\omega(\lambda^H)\{q[pu'(\omega(\lambda^H) + \lambda^H\theta) + (1-p)u'(\omega(\lambda^H))] - u'(\omega(\lambda^H) + p\theta q\lambda^H)\}/\alpha - (1-q)\omega(\lambda^L)[pu'(\omega(\lambda^L) + \lambda^L\theta) + (1-p)u'(\omega(\lambda^L))]/\alpha < -\omega(\lambda^H)\{q[pu'(\omega(\lambda^H) + \lambda^H\theta) + (1-p)u'(\omega(\lambda^H))] - u'(\omega(\lambda^L) + p\theta\lambda^L)\}/\alpha - (1-q)\omega(\lambda^L)\{pu'(\omega(\lambda^L) + \lambda^L\theta) + (1-p)u'(\omega(\lambda^L))\}/\alpha < (1-q)\{\omega(\lambda^H)u'(\omega(\lambda^L) + p\theta\lambda^L) - \omega(\lambda^L)[pu'(\omega(\lambda^L) + \lambda^L\theta) + (1-p)u'(\omega(\lambda^L))]\}/$

Theoretical Economics 17 (2022)

$\alpha \le 0$, where the first inequality comes from $u''(x) < 0$ and $q > \hat{q}$, guarantee-
ing that pooling over λ^H yields more resources than pooling over λ^L (that is,
$\omega(\lambda^H) + p\theta q\lambda^H > \omega(\lambda^L) + p\theta\lambda^L$); the second inequality comes from the assump-
tion $pu'(\omega(\lambda^H) + \lambda^H\theta) + (1-p)u'(\omega(\lambda^H)) > u'(\omega(\lambda^L) + \lambda^L p\theta)$; the third inequal-
ity comes from prudence, i.e., $u''' \ge 0$, and from $\lambda^H > \lambda^L$.

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