Theoretical Economics 17 (2022), 331–369 1555-7561/20220331

Optimal organ allocation policy under blood-type barriers with the donor-priority rule

Jaehong Kim
The Wang Yanan Institute for Studies in Economics (WISE) and Department of Economics, School of Economics, Xiamen University

Mengling Li
Department of Economics, School of Economics, Xiamen University and The Wang Yanan Institute for Studies in Economics (WISE), Xiamen University

Shortages in organs for transplantation have resulted in a renewed interest in designing incentive policies to promote organ supply. The donor-priority rule, which grants priority for transplantation based on deceased organ donor registration status, has proven to be effective in both theory and practice. This study investigates the implications of the donor-priority rule for optimal deceased organ allocation policy design under a general formulation of blood-type barriers. We find that for any blood typing and organ matching technology, reserving type X organs for only type X patients maximizes the aggregate donation rate under regular distributions, which also ensures equity in organ sharing. Moreover, this is the unique optimal allocation policy if and only if the directed compatibility graph that corresponds to a given organ matching technology is acyclic.

Keywords. Market design, organ donation, priority rule, blood-type compatibility, equity.

JEL classification. D47, D64, D78, H42, I11.

1. Introduction

Most of the world is facing increasing shortages in the supply of human organs for transplantation. On average, 20 patients in the U.S. die each day while waiting for an organ transplant. Deceased donors represent the most common source of transplant organs in the U.S. and much of the rest of the world. Each deceased donor can provide multiple organs and potentially save 8 lives.[1] In an effort to address chronic organ shortages, a wide

Jaehong Kim: jaehongkim@xmu.edu.cn
Mengling Li: menglingli@xmu.edu.cn

We are grateful to the two anonymous referees for their constructive suggestions and comments, which have improved the paper in many ways. We thank the participants of the 2019 Society for the Advancement of Economic Theory (SAET) Conference for their helpful comments. This research is supported by the National Natural Science Foundation of China (grant 71703132, grant 72173103) and the Ministry of Education of China (grant 17YJC790074). All errors are our own. Mengling Li is the correpsonding author.

[1] Data were obtained from the U.S. Health Resources & Services Administration (HRSA) at https://www.organdonor.gov/statistics-stories/statistics.html

 https://doi.org/10.3982/TE3969

range of policy initiatives have been proposed and implemented to improve the availability of organs for transplantation (Bernstein (2016)). Among these incentive policies, the donor-priority rule, which grants priority for organ transplantation based on the organ donor registration status, has generated increasing interest among economists (Kessler and Roth (2012)). In practice, Singapore adopted the donor-priority rule in 1987 (Iyer (1987)), and this priority rule was introduced in Israel in 2008 (Lavee et al. (2010)), in Chile in 2013 (Zuniga-Fajuri (2015)) and in China in 2018.[2] Meanwhile, the donor-priority rule and its variants have been constantly proposed in other countries, including the U.S. (Chan (2020)), the U.K. (Gray (2013)) and Canada (Burkell et al. (2013)). In this study, we investigate the optimal design of organ allocation policies among different blood-type groups in terms of promoting the aggregate donation rate under the donor-priority rule.[3]

Blood-type compatibility is a major medical requirement for most successful organ transplantations. Among the 36 human blood group systems recognized by the International Society of Blood Transfusion (Storry et al. (2016)), the ABO classification with the four blood-type groups of O, A, B, and AB is the most standard and widely discussed in the economics literature (Roth et al. (2004, 2005, 2007)). The ABO classification defines one of the most important biological barriers in organ transplantations from donors to patients:[4] O donors are blood-type compatible with patients of all four blood types, A donors are compatible with A and AB patients, B donors are compatible with B and AB patients, and AB donors are compatible with patients of only type AB.[5]

In practice, two types of deceased organ allocation policies naturally arise from the ABO compatibility requirement in organ transplantations. The first policy type, which is referred to as the ABO-identical allocation policy in the medical literature, reserves type X organs for only type X patients. This policy is implemented for deceased donor kidney allocations in the U.S. and for most of the organ types in Israel. By contrast, under the class of ABO-compatible allocation policies, organs can be offered to any compatible patient. There is potentially a continuum of the ABO-compatible policies that are characterized by the number of organs transferred to compatible blood-type groups. This type of policy is the current practice in the U.S. for organs with greater medical urgency (other than kidneys), for all organ types in China and for the Eurotransplant Kidney Allocation System (Glander et al. (2010)).

[2]Please refer to https://www.codac.org.cn/cstatute/transcplantationdocments/201601223/699067.htm for more details of the Chinese policy.

[3]Although a difference exists in practice between registering for deceased donation in advance and being an available donor at death, we refer to both as "donating" in this paper for convenience.

[4]There is another type of compatibility requirement, known as tissue-type compatibility, for the transplantation of organs such as kidneys and hearts. Our model framework is sufficiently general to incorporate tissue-type compatibility, as discussed in Section 4.3.

[5]Although recent advances in desensitization and immunosuppressive protocols have made blood-type (and tissue-type) incompatibility much less of a constraint in transplantations associated with incompatible *living* donor allograft recipients (see Andersson and Kratz (2020), Heo et al. (2021) and the references in these papers), to the best of our knowledge, these advances have been almost exclusively practiced among living donor transplantations instead of deceased donor transplantations.

In this paper, we attempt to provide an economic rationale for the choice of ABO-identical or ABO-compatible policies to promote deceased donor registration and increase the organ supply, which could influence practical organ allocation policies and donation decisions. In practice, the considerations of these allocation policies vary depending on the organ type, transplantation network and country (Park et al. (2013), Lai and Roberts (2016)). The current choices of different ABO allocation policies are largely driven by disparities in the waiting times to transplantation among different blood groups and the post-transplantation survival rate (Jawitz et al. (2013), Cai et al. (2015)). Although the medical literature on whether ABO-compatible transplants have comparable outcomes to ABO-identical transplants remains controversial (Aladag et al. (2006), Koukoutsis et al. (2007), Bergenfeldt et al. (2015), Taghavi et al. (2014)), the economic consequences of such allocation policies in combination with the donor-priority rule have not been clearly established. These organ allocation policies may affect not only the relative organ availability to each blood type but also the incentives to contribute to the organ supply.

Our model setup is sufficiently general to account for both existing technologies and future advances in blood typing and organ matching from two perspectives. First, we consider a finite and possibly large number of blood types, which can incorporate any of the current 36 human blood group systems and the more detailed blood subtyping technologies. It is important to model a wide range of blood types because new allocation policies are usually based on medical advances in blood grouping technologies (Sönmez et al. (2018)). For instance, under the standard ABO classification, blood type A can be further classified into A1 and A2, which have different immunological properties (Nelson et al. (2002)). This advance in subtyping technology formed the basis of a policy reform in 2014 in the allocation of deceased-donor kidneys in the U.S. Second, our model allows for a wide class of organ matching functions, which can account for the current biological barriers and future medical possibilities of organ-specific matching technology. This generalization ensures that our results are robust to future advances in organ matching technology. For example, an improved technology that dates to the 1980s allows a certain fraction of blood type A kidneys, which are referred to as subtype A2 kidneys, to be transplanted to medically qualified blood type B and O patients (Brynger et al. (1983)). Therefore, the ABO-identical and ABO-compatible allocation policies based on the ABO classification serve merely as particular examples in our general model framework. In accordance with the terminologies in the medical literature, we use the prefix "XYZ" to represent allocation policies under a general blood-type classification and refer to them as the XYZ-identical and XYZ-compatible allocation policies.

For a wide class of donation cost (regular type) distributions, we find that for any blood-typing and organ matching technology, the XYZ-identical deceased organ allocation policy, which reserves type X organs for only type X patients, maximizes the aggregate donation incentives under the donor-priority rule. This result remains robust after considering several extensions of our general framework. The XYZ-identical policy has a desirable byproduct of equalizing access to organs among different blood-type

groups, which is consistent with the equity objective stated in the organ transplantation policies of many countries. For instance, the primary goals of the OPTN in the U.S. are "to increase and ensure the equity of organ sharing in the national system of organ allocation" and "to increase the supply of donated organs available for transplantation" (Duda (2005)). Meanwhile, a patient's blood type is identified as one of the top three contributors to the disparities in access to deceased donor kidney and liver transplants (Stewart et al. (2018)). Such equity considerations among blood-type groups have also been widely discussed and promoted in both the economics and medical literature, for instance, Bertsimas et al. (2013), Lai and Roberts (2016) and Sönmez et al. (2020). Therefore, our findings indicate that the XYZ-identical allocation policy can simultaneously achieve both objectives of increasing the organ supply and ensuring equity across blood-type groups.

In the general organ donation and allocation framework, we establish that equal access among all the blood-type groups is essential to achieve maximal aggregate donation incentives under any given organ matching technology. The main reason is that unequal access to deceased donations leads to distortions in the donation incentives across blood-type groups, which can compromise the aggregate donation rate. Furthermore, we show that the XYZ-identical allocation policy is the unique optimal policy in terms of maximizing the aggregate donation incentive if and only if the directed compatibility graph that corresponds to a given organ matching technology is acyclic. The condition of acyclicity essentially implies that "autarky" is the only way to eliminate incentive distortions since it is impossible to achieve a balanced "trade" of organs across groups if there is no directed cycle in the compatibility graph. A direct implication of this result is that under the standard ABO organ matching technology, it is strictly suboptimal to allocate organs to patients of nonidentical types in terms of promoting the organ supply.

By calibrating our model with U.S. heart donation and allocation data, we conduct numerical simulations to document that the potential improvement in the aggregate donation rate with the ABO-identical organ allocation policy is considerable. After introducing the donor-priority rule, the ABO-identical policy achieves a further increase of more than 10% in the aggregate donation rate relative to the improvement from the donor-priority rule itself, which is robust with respect to several specifications of cost type distributions. Under the de facto ABO-compatible policy implied from recent U.S. heart transplantation data, we observe significantly imbalanced incentives among the four blood-type groups, with pairwise differences in the group donation rates that reach over 19 percentage points when the donation cost is relatively small.

The remainder of this paper proceeds as follows. In the rest of this section, we briefly review the related literature and emphasize our main contributions. Section 2 starts with a simple example and then develops the general organ donation and allocation model. We analyze the optimal organ allocation policy in Section 3 and discuss its extensions in Section 4. In Section 5, we perform numerical simulations to explore the size of the improvement with the optimal policy. Section 6 concludes. All proofs are presented in the Appendix.

1.1 *Related literature and contribution*

This paper is most closely related to the emerging literature on incentivizing deceased organ donations via different organ allocation rules and, in particular, the donor-priority rule, which provides priority on organ waiting lists to individuals who previously registered as organ donors. The pioneering work by Kessler and Roth (2012) shows both experimentally and theoretically that the donor-priority rule significantly outperforms an alternative policy that does not utilize such a priority allocation in terms of donation rates. This seminal work was later extended with several variants of the donor-priority rule in laboratory experiments by Li et al. (2013), Kessler and Roth (2014), and Herr and Normann (2016) and in theory by Kim et al. (2021) and Dai et al. (2020), among many others. Empirically, Stoler et al. (2017) document the positive impacts of the donor-priority rule in Israel on the donor registration rate. However, none of these previous studies considers how the donor-priority rule interacts with blood-type barriers and consequently affects the donation incentives among different blood-type groups. Our paper contributes to the theory of deceased organ allocations and donations by formally investigating the implications of the donor-priority rule for the allocation policies that naturally arise from the biological barriers to organ transplantations.

Our model considers one of the most important and extensively studied medical barriers to deceased organ transplantation, namely, blood-type compatibility, in a unified deceased organ donation and allocation framework. The explicit consideration of blood-type compatibility in organ transplantation is widely featured in the kidney exchange literature as one of the most successful applications of matching theory initiated by Roth et al. (2004, 2005, 2007), who study the matching and exchange mechanisms among blood-type incompatible living donor-patient pairs to improve the efficiency and welfare of organ markets. These seminal works have played important roles in designing kidney exchange policies in the U.S. and Europe. However, these papers and subsequent studies have mostly considered the standard ABO blood-type classification. Sönmez et al. (2018), as one of the few exceptions, consider technological advances in blood typing and study the economic consequences of a recent reform by the United Network for Organ Sharing (UNOS) in 2014 that prioritizes subtype A2 deceased-donor kidneys for blood type B patients. In our paper, we consider more general blood typing and organ matching technologies that apply to existing and future medical advances, and our findings are robust in terms of these technologies.

The optimal organ allocation policy in our framework not only maximizes the number of organ transplants by directly promoting the total organ supply but also guarantees equity in organ sharing. Equity in access to organ transplantation has emerged as an important concern in the kidney exchange literature (Sönmez et al. (2020)). In the medical literature, Lai and Roberts (2016) investigate the impact of ABO-nonidentical liver transplantation on waitlist disparities by blood types. More broadly, our study also contributes to the operations research literature on optimal organ allocation policies, for instance, Ruth et al. (1985), Su and Zenios (2006), Kong et al. (2010), and Bertsimas et al. (2013), among many others. While most of these studies have considered an exogenous supply of deceased organs, we explicitly analyze how organ allocation policies

among different blood-type groups endogenously determine the organ supply by blood type, which in turn can affect the optimal design of organ allocation policies.

Finally, the design of a default organ registration status is also an important policy dimension to incentivize organ donations that has received much attention. Although many previous studies have provided direct evidence that an "opt-out" system, where everyone is presumed to be a registered donor unless an individual actively indicates otherwise, generates higher registration rates compared with an "opt-in" system (Johnson and Goldstein (2003), Abadie and Gay (2006)), there are critical arguments against shifting from an opt-in to an opt-out policy (Fabre et al. (2010)). In particular, a recent study by Glazier and Mone (2019) has raised concerns that switching to an opt-out system may have unintended consequences that make this policy less effective at increasing final donation rates under the current U.S. gift law. In addition, recent empirical studies by Sharif (2018) and Arshad et al. (2019) have offered opposing evidence that an opt-out system will not automatically lead to increased organ donation.

2. Model

This section first discusses a simplified example to present the essentials of our model and the intuition behind the analysis and then formally defines the general framework.

2.1 *Illustrative example*

There are two groups of agents with blood types A and O, with a continuum of agents in each, and each group has a measure of $1/2$. In the first stage, each agent decides whether to register for organ donation. The net cost of committing to donation is c, which is uniformly distributed on $[-\gamma, 1-\gamma]$, with cumulative distribution function (CDF) $F(c) = \gamma + c$ and $\gamma \in (0, 1)$. In the second stage, each agent may be brain dead with probability $\beta \in (0, 1)$ or may encounter organ failure with probability $\theta \in (0, 1)$. After brain death, a registered donor donates one organ, which is to be allocated among those who need one, with a higher priority granted to registered donors. We focus on a scenario with organ shortage, that is, $\beta < \theta$, such that among those with organ failure, a patient may receive an organ donation only if he is a previously registered donor. The payoff from receiving an organ donation is normalized to 1.

Under the AO-identical policy, type-O (A) organs are allocated to only type-O (A) patients, although type-O donors are also blood-type compatible with type-A patients. Let c_i with $i \in \{A, O\}$ denote the cutoff cost such that only an agent of blood type i with a donation cost lower than c_i will commit to donation. The total supply of type i organs is $\beta(c_i + \gamma)/2$, and the effective demand for type i organs by registered donors is $\theta(c_i + \gamma)/2$. Therefore, the probability for patients who are registered donors to obtain organ allocations is $p_i \equiv \beta/\theta$. In equilibrium, the cutoff costs should satisfy $c_i = \theta p_i$ with $i \in \{A, O\}$, which results in $c_A^* = c_O^* = \beta$. The aggregate donation rate under the AO-identical policy is therefore $[F(c_A^*) + F(c_O^*)]/2 = \beta + \gamma$.

We next consider the class of AO-compatible policies, which allow the transfer of organs from group O to A (but not from A to O due to blood-type incompatibility given current organ matching technology). Let $k/2 > 0$ denote the number of organs transferred

from group O to A under a given AO-compatible policy. Then the probabilities for registered donors to receive organ allocations become $p_A = [\beta(c_A + \gamma) + k]/[\theta(c_A + \gamma)]$ and $p_O = [\beta(c_O + \gamma) - k]/[\theta(c_O + \gamma)]$. If $k > 0$, then $p_A > \beta/\theta > p_O$; therefore, $c_A > \beta > c_O$. This indicates that when exporting organs from group O donors to group A patients, agents in group A obtain a higher transplantation rate and are thus more incentivized to donate, while agents in group O become less incentivized than their counterparts under the AO-identical policy. Accordingly, unequal access to the organ transplantations between the two groups distorts the donation incentives.

Based on the equilibrium conditions $\hat{c}_A = \theta \hat{p}_A$ and $\hat{c}_O = \theta \hat{p}_O$, we can derive that $\hat{c}_A + \hat{c}_O$ is strictly decreasing in k, and so is the equilibrium aggregate donation rate $[F(\hat{c}_A) + F(\hat{c}_O)]/2$. Therefore, the class of AO-compatible policies reduces the number of organ donations and transplantations, i.e., $[F(\hat{c}_A) + F(\hat{c}_O)]/2 < [F(c_A^*) + F(c_O^*)]/2$ for any $k > 0$. This is mainly because transferring $k/2 > 0$ organs from group O to group A decreases the transplantation rate of group O by $\Delta_{p_O} = k/[\theta(c_O + \gamma)]$, while it increases the transplantation rate of group A by only $\Delta_{p_A} = k/[\theta(c_A + \gamma)] < \Delta_{p_O}$, since $c_A > c_O$. Specifically, any given AO-compatible policy disincentivizes donations in group O more than it incentivizes donations in group A and, therefore, results in a decrease in the aggregate donation rate compared with the AO-identical policy.

This simple example illustrates the optimality of the AO-identical policy in incentivizing deceased donor registrations by considering two equally populated blood-type groups and a simplified version of the standard organ matching technology. It is also a unique optimal policy in this simplified context. We next present a general model framework that incorporates not only a large number of blood types with any possible population distributions but also a wide class of organ matching functions.

2.2 *General framework*

Based on Kessler and Roth (2012), we consider a parsimonious model of organ donation and allocation that includes a unit mass of agents. The organ donation decisions of the agents are modeled as a simple two-period game. In the first period, agents simultaneously decide whether to register for deceased organ donation. Although there are psychological and logistical costs associated with deceased donor registration, it is also an altruistic act that brings expected utility gains by improving someone's life and reducing suffering. Therefore, we consider the net costs as the psychological and logistical costs of registration minus the expected utility of registering. The private net costs of registering (or equivalently, donating) c are independently and identically distributed (i.i.d.) with a smooth CDF $F(c)$ and probability density function (PDF) $f(c) = F'(c) > 0$. The net donation cost is modeled as heterogeneous since each agent perceives the organ donation decision differently. Some agents may feel a great psychological burden when thinking about death at the time of making this decision, whereas other agents could be altruistic and enjoy a rewarding feeling that overrides the psychological and logistical costs associated with donation. Therefore, the net cost of donating may even be negative. To account for the presence of such altruistic agents, we consider $F(0) > 0$ such that a group of donors always exists regardless of the incentive rules. For the simplicity of presentation, we refer to c as the donation cost in the rest of this paper.

Our main analysis assumes that $1/F(c)$ is strictly convex. This property is implied by regularity, ρ-concavity, and satisfied by many distributions that are widely used in the literature (Ewerhart (2013)). The first related concept is the notion of regularity, which is critical for several most remarkable results in the mechanism design literature, for instance, Myerson (1981) and Myerson and Satterthwaite (1983). $1/F(c)$ is strictly convex if and only if the virtual cost $r(c) := c + F(c)/f(c)$ is strictly increasing in c. The second related concept pertains to a general notion of concavity, known as ρ-concavity, with the index $\rho \in [-\infty, \infty]$ measuring the degree of concavity (Caplin and Nalebuff (1991a, 1991b)). Any ρ-concave distribution F with $\rho > -1$ has a strictly convex $1/F$.

In the second period, agents encounter health shocks and receive payoffs. We consider two mutually exclusive shocks, namely, brain death (due to a nonorgan-related disease or accident) and organ failure. Each agent experiences brain death with probability $\beta \in (0, 1)$ and organ failure with probability $\theta \in (0, 1)$ and remains healthy with probability $1 - \beta - \theta \in (0, 1)$. When a registered donor encounters brain death, he contributes α organs to the pool of deceased organs, which are allocated among agents with organ failure via a waiting list mechanism. Each agent with organ failure, also referred to as a patient, needs to obtain an organ for transplantation from the pool of deceased organs. If such a patient receives an organ allocation, he gains a normalized payoff of V, which represents the value of receiving an organ with an improved life expectancy and quality of life. Otherwise, if an agent in need of an organ does not receive one, his payoff from organ allocation is normalized to zero. Without loss of generality, we consider $\alpha = 1$ and normalize $V = 1$ throughout the theoretical analysis for expositional simplicity, which are relaxed in Section 5 for simulation analysis.

The available organs are distributed based on a waiting list mechanism. Although in practice, waiting lists are administered separately for different organ types depending on many factors, such as medical emergency, immunological match, and geographical distance, our main analysis abstracts away from these nonincentive-related details to focus on the aspects that are more likely to influence the incentive to register for deceased donation. More specifically, we consider a waiting list mechanism that features three key elements. First, the biological barriers among different blood-type groups are determined by organ matching technology. Second, a class of allocation policies describes the discretion in transplants between blood-type-compatible but nonidentical donors and recipients. Third, the donor-priority rule grants a higher priority to registered donors than to nondonors in receiving a transplant.

2.2.1 *Matching technology* A major factor for the successful transplantation of organs is blood-type compatibility. We consider a general classification of blood types that can incorporate any of the current human blood group systems and future developments in blood subtyping technologies. Let $\mathbf{B} = \{b_1, b_2, \ldots, b_m\}$ denote a finite and possibly large set of blood types, with $n_i \in (0, 1)$ as the measure of blood type i agents such that $\sum_{i=1}^{m} n_i = 1$. We use b to denote a generic blood group in $\mathbf{B}$.

Under the general blood-type classification, we consider a large class of matching functions $\mathbf{M}$ to describe the blood-type compatibility barriers to organ transplantations,

which can incorporate any existing or future organ matching technology, as follows:

$$\mathbf{M} := \big\{\mu : \mathbf{B} \times \mathbf{B} \to \{0, 1\} \mid \mu(b_i, b_i) = 1, \text{ for any } b_i \in \mathbf{B}\big\},$$

where $\mu(b_i, b_j)$ defines the medical feasibility of transplanting organs of blood type b_j donors to blood type b_i patients. Equivalently, $\mathbf{M}$ is a set of $m \times m$ matrices whose diagonal entries are one, and the off-diagonal entries are either zero or one. For notational simplicity, we denote $\mu_{ij} := \mu(b_i, b_j)$. Note that the only restriction in this class of matching functions is compatibility within the same blood group, which is a minimal requirement in practice.

A standard matching technology among this large class is ABO blood-type compatibility, which is most widely discussed and modeled in the kidney exchange literature (Roth et al. (2004, 2005, 2007)). It can be described by four blood types $\mathbf{B} = \{\text{O, A, B, AB}\}$ and the standard ABO organ matching function $\tilde{\mu}$ as follows:

$$\tilde{\mu}(\text{AB}, b) = 1 \quad \text{for any } b \in \{\text{O, A, B, AB}\}; \qquad \tilde{\mu}(\text{A}, b) = \begin{cases} 1 & \text{if } b \in \{\text{O, A}\}, \\ 0 & \text{otherwise;} \end{cases}$$

$$\tilde{\mu}(\text{B}, b) = \begin{cases} 1 & \text{if } b \in \{\text{O, B}\}, \\ 0 & \text{otherwise;} \end{cases} \qquad \tilde{\mu}(\text{O}, b) = \begin{cases} 1 & \text{if } b = \text{O}, \\ 0 & \text{otherwise.} \end{cases}$$

The ABO-compatible transplantation is a conventional matching technology, and ABO incompatibility has long been considered to be a contraindication to organ transplantation in practice. Nevertheless, over the past 25 years, ABO-incompatible transplantation has increasingly been performed to overcome donor shortages in the case of adult kidney (for living donors) and liver transplantation and pediatric heart transplantation, and the outcomes have steadily improved (Morath et al. (2017), Yu et al. (2017)). Therefore, with future medical advances, an ideal organ matching technology would be full compatibility, which could make the transplantations between any donor-patient pair medically feasible. The full compatible matching function μ^* can be represented as $\mu^*_{ij} = 1$ for any $i, j \in \{1, \ldots, m\}$, i.e., μ^* is a matrix of ones.

2.2.2 *Allocation policy* An allocation policy is represented by an $m \times m$ matrix $\Lambda := \{\lambda_{ij}\}_{\{i,j=1,\ldots,m\}}$, where $\lambda_{ij} \in [0, 1]$ denotes the proportion of type b_j donors (organs) allocated to type b_i patients and $\sum_{i=1}^{m} \lambda_{ij} = 1$ for any $j \in \{1, \ldots, m\}$. For any given organ matching technology $\mu \in \mathbf{M}$ (by nature or technology), an optimal allocation policy must satisfy

$$\sum_{i=1}^{m} \lambda_{ij}\mu_{ij} = 1, \quad \forall j = 1, \ldots, m.$$

These constraints guarantee the full utilization, i.e., nonwastefulness, of organs, which immediately implies that if $\mu_{ij} = 0$, then we must have $\lambda_{ij} = 0$ in an optimal policy.

A typical example of an organ allocation policy is the XYZ-identical allocation policy, which reserves type b_i organs for type b_i patients, even if the existing organ matching technology allows transplantations of type b_i organs to patients of another type, i.e.,

$\mu_{ij} = 1$ for some $j \neq i$. The XYZ-identical allocation policy can be represented as an identity matrix with $\lambda^*_{ii} = 1$ and $\lambda^*_{ij} = 0$ for any $i, j \in \{1, \ldots, m\}$ and $i \neq j$, i.e., $\Lambda^* \equiv I$. Under the ABO blood-type classification, the XYZ-identical policy largely corresponds to the ABO-identical allocation policy applied in practice for deceased kidney transplants in the U.S. One of the main arguments for adopting this policy is that the ABO classification leads to an immunological asymmetry in the sense that blood type O patients are especially disadvantaged by having less access to the organ supply pool than patients with the other three blood types (A, B, and AB). The ABO-identical allocation policy for deceased-donor transplants thus aims to mitigate the resulting disadvantage to type O patients by reserving O kidneys for O patients.

By contrast, the class of XYZ-compatible allocation policies are flexible in the sense that organs can be offered to any compatible patients by a proportion of λ_{ij} with $i \neq j$ under the existing organ matching technology. We refer to any allocation policy with $\lambda_{ij} > 0$ for some $i \neq j$, i.e., a nonidentity allocation matrix, as an XYZ-compatible policy. Note that this requirement is imposed merely to exclude the XYZ-identical policy from the class of XYZ-compatible policies, which can facilitate our following discussions. Under the ABO blood-type classification, the corresponding ABO-compatible allocation policies are more common in the U.S. for organs with greater medical urgency. The Eurotransplant Kidney Allocation System adopts the ABO-compatible scheme, but it results in a substantial drain of O kidneys with longer waiting times and worse outcomes for blood type O patients on organ waitlists (Glander et al. (2010)).

2.2.3 *Donor-priority rule* With the donor-priority rule, registered donors are strictly prioritized over nondonors to receive organ allocations, which is conditioned on the same blood-type group. The assumption of strictly higher priority for donors is relaxed in Section 4.1. To capture the shortage of the organ supply in practice, we assume that the demand rate of organs is larger than the supply rate, i.e., $\alpha\beta = \beta \leq \theta$, which is consistent with the calibrated parameter values in Section 5.1. A similar assumption is adopted in Sönmez et al. (2020). Accordingly, our main analysis focuses on the case in which the organ supply is insufficient to satisfy the demand of donors for each blood-type group. Let $p_i \in [0, 1]$ denote the survival rate of blood type i patients (i.e., agents with organ failure) who have registered for donation, i.e., the probability of receiving an organ allocation in the case of organ failure. For the following discussions, we refer to p_i as the survival rate for simplicity. We denote the vector of survival rates as $P := (p_1, \ldots, p_m) \in [0, 1]^m$, which measures the exclusivity of the donor-priority queue.

2.2.4 *Donation decision and market equilibrium* Given any blood-type classification $\mathbf{B}$ and any associated organ matching technology $\mu \in \mathbf{M}$, a waiting list mechanism, which distributes organs procured from deceased donors to patients with organ failure, can be defined by the allocation policies among different blood-type groups Λ and the survival rates under the donor-priority rule P.

In making the donation decision, an agent needs to weigh his private cost of donation over the marginal benefit of donation, which is measured by the probability of being in need of an organ, the value for receiving an organ and his incremental chance

of receiving a transplant. The donation decision for type i agents is therefore characterized by a threshold donation cost c_i such that $c_i = \theta p_i$. A type i agent with donation cost c would register for donation if and only if $c \leq \theta p_i$, i.e., his cost of donation is no more than his marginal benefit of donating. In this sense, an agent with donation cost c_i is known as a threshold agent. We use $\delta_i = F(c_i) = F(\theta p_i)$ to denote the donation rate among type i agents. Therefore, $\delta_i n_i = F(\theta p_i) n_i$ is the measure of blood type i donors in the population.

To define the organ market equilibrium, we first need to specify the organ supply available to each blood-type group and the effective organ demand by each blood-type group. The organ supply made available to type i agents under an allocation policy Λ is measured by $\beta \sum_{j=1}^{m} \delta_j n_j \lambda_{ij} \mu_{ij}$, which accounts for the brain death rate in the population β, the measure of each type of donors $\delta_j n_j$, the allocation policy from type j donors to type i patients λ_{ij}, and the matching technology μ_{ij}. The effective organ demand of type i agents (donors) is $\theta \delta_i n_i p_i$, which is determined by the probability of organ failure θ, the proportion of type i donors δ_i, the measure of type i agents n_i in the population and the survival rate of type i patients p_i. Therefore, we have m market clearing conditions in the equilibrium

$$\theta \delta_i n_i p_i = \beta \sum_{j=1}^{m} \delta_j n_j \lambda_{ij} \mu_{ij}, \quad \forall i = 1, \ldots, m.$$

3. Analysis of the optimal policy

Given any blood-type classification $\mathbf{B}$ and organ matching technology μ, we consider the social planner's objective of maximizing the aggregate donation rate by choosing the allocation policy Λ and the survival rates of patients with organ failure P as

$$\max_{\Lambda \in [0,1]^{m \times m}, P \in [0,1]^m} \sum_{i=1}^{m} F(\theta p_i) n_i \tag{1a}$$

$$\text{subject to} \quad \theta F(\theta p_i) n_i p_i - \beta \sum_{j=1}^{m} F(\theta p_j) n_j \lambda_{ij} \mu_{ij} = 0, \quad \forall i = 1, \ldots, m; \tag{1b}$$

$$\sum_{i=1}^{m} \lambda_{ij} \mu_{ij} - 1 = 0, \quad \forall j = 1, \ldots, m. \tag{1c}$$

where constraints (1b) are the *market clearing* conditions for each blood type, and equations (1c) represent the *full utilization* constraints on the allocation policies among m blood-type groups conditioned on the given matching technology.

3.1 *Optimal allocation policy*

To derive the optimal organ allocation policy, we start by considering a simplified optimization problem with fully compatible matching technology, i.e., $\mu = \mu^*$. With $\mu^*_{ij} = 1$

for all $i, j \in \{1, \ldots, m\}$, the optimization problem defined by (1a)–(1c) becomes

$$\max_{\Lambda \in [0,1]^{m\times m}, P\in[0,1]^m} \sum_{i=1}^{m} F(\theta p_i) n_i \tag{2a}$$

$$\text{subject to} \quad \theta F(\theta p_i) n_i p_i - \beta \sum_{j=1}^{m} F(\theta p_j) n_j \lambda_{ij} = 0, \quad \forall i = 1, \ldots, m; \tag{2b}$$

$$\sum_{i=1}^{m} \lambda_{ij} - 1 = 0, \quad \forall j = 1, \ldots, m. \tag{2c}$$

By adding the m constraints in (2b), we can derive a more simplified optimization problem that concerns only the choice of P as follows:

$$\max_{P\in[0,1]^m} \sum_{i=1}^{m} F(\theta p_i) n_i \tag{3a}$$

$$\text{subject to} \quad \sum_{i=1}^{m} \theta F(\theta p_i) n_i p_i - \beta \sum_{j=1}^{m} F(\theta p_j) n_j = 0. \tag{3b}$$

It is clear from (3b) that the problem of maximizing the aggregate donation rate in (3a)–(3b) is equivalent to maximizing the aggregate organ transplantations. Furthermore, we can show that this simplified problem in (3a)–(3b) is equivalent to the problem defined by (2a)–(2c) by the following lemma.

LEMMA 1. *Given any $P \in [0,1]^m$ that satisfies equation (3b), we can always find $\Lambda \in [0,1]^{m\times m}$ that satisfies constraints (2b) and (2c).*

Since the optimization problem in (3a)–(3b) with $\mu = \mu^*$ pertains only to the choice of P, we first explore the survival rates for patients with organ failure that result from an optimal allocation policy, without yet specifying the corresponding optimal allocation matrix Λ. For the simplified problem defined by (3a)–(3b), the following proposition states that the unique optimal solution is to equalize the survival rates after organ failure among all blood-type groups with $P = P^* = (p^*, \ldots, p^*)$, where $p^* := \beta/\theta$.

PROPOSITION 1. *If $\mu = \mu^*$, we must have $P = P^*$ in an optimal policy.*

Proposition 1 suggests that under the ideal matching technology of full compatibility μ^*, an optimal allocation policy to maximize the aggregate donation incentives (equivalently, organ transplantations) should always provide equal access to organ transplants among all blood-type groups, regardless of the population size of each group n_i, i.e., $P = P^*$ uniquely maximizes the aggregate donation under μ^*. It follows immediately from the equilibrium donation decisions that the threshold agent in each blood-type group has the same donation cost $c_i = c^* = \theta p^* = \beta$ and that the donation incentive among each group is balanced with $\delta_i = \delta^* = F(\theta p^*) = F(\beta)$. By equalizing the marginal

cost of contribution in the organ procurement process, this optimal policy simultaneously equalizes the marginal benefit of allocation in organ sharing among each group, which results in balanced donation incentives across the blood-type groups.

To provide intuition for this result, consider otherwise that an optimal allocation policy under μ^* leads to unequal access to organs across the blood-type groups. It immediately follows that the corresponding allocation policy must be XYZ-compatible. This is because under the XYZ-identical policy with $\Lambda = I$, each blood-type group essentially operates as an "autarky" in a separate and identical submarket (except for possible differences in market size), without any interactions among them. Therefore, the survival rate within each group must be uniformly determined by the market clearing condition $\theta\delta_i n_i p_i = \beta\delta_i n_i$ such that $p_i = p^* = \beta/\theta$. Under an XYZ-compatible policy, unequal access to donated organs implies that at least one group is a "net importer" of donated organs in the sense of receiving (or "importing") more donations from the other $m-1$ groups than the total amount that it has transferred (or "exported") to the other groups. Meanwhile, since there is no external source of organ supply, at least one group is a "net exporter" of organs in the market. Net importers have higher donation incentives than net exporters since net importers enjoy a relatively higher marginal benefit in receiving organ allocations with a larger survival rate after organ failure. As a result, unequal access to organs distorts the donation incentives across the blood-type groups.

Now let us consider the incentive effects of redistributing a marginal amount of the organs enjoyed by a net importing group b_i to a net exporting group b_e. Note that it is always feasible to do this because of the fully compatible matching technology currently under consideration. With strictly convex $1/F(c)$, such a redistribution incentivizes the donations in group b_e to a larger extent on the margin than it decreases the donation incentives of group b_i. This is because organs are relatively more scarce for group b_e, and a marginal increment in the benefits of receiving organ allocations stimulates more type-b_e agents to donate when $1/F(c)$ is strictly convex. Therefore, eliminating the trade imbalances in organs among the different blood-type groups can mitigate the incentive distortions and achieve the optimal aggregate donation rate.

More generally, the following theorem states that for any given organ matching technology $\mu \in \mathbf{M}$, equal access among all the blood-type groups is a necessary and sufficient condition to achieve maximal aggregate donation incentives. We establish this result in two parts. The first step is to observe that the optimization problem with $\mu = \mu^*$ in (3a)–(3b) is in fact a relaxed problem of the original optimization problem with a general matching technology $\mu \in \mathbf{M}$ defined by (1a)–(1c). This is because adding the m market clearing constraints in (1b) and applying the m full utilization constraints in (1c) result in (3b). This indicates that for any given μ, if an allocation policy results in equal access with $P = P^*$ such that the donation rate achieves the upper bound of $\delta^* = F(\beta)$, it must be an optimal allocation policy under the given matching technology. In the second step, we show that under any matching technology, an allocation policy that results in unequal access with $P \neq P^*$ must be suboptimal. This is established by first transforming the original optimization problem with $\mu \in \mathbf{M}$ to the optimization problem with $\mu = \mu^*$ and then applying the result of Proposition 1.

THEOREM 1 (Equal access in organ sharing). *For any given organ matching technology $\mu \in \mathbf{M}$, if an allocation policy results in $P = P^*$, it must be an optimal policy. In contrast, any allocation policy that results in $P \neq P^*$ is suboptimal.*

The intuition of Theorem 1 similarly follows from the discussions of Proposition 1 in that unequal access in organ sharing leads to incentive distortions among the blood-type groups and, therefore, undermines the aggregate donation rate. Based on the previous discussions, with an arbitrary matching technology, the redistribution of organs from a net importing group b_i to a net exporting group b_e requires either compatibility between at least one type of the organs currently consumed by group b_i and type-b_e patients or, in an indirect way, that there is a path to transfer organs through several compatible intermediary groups that terminates at group b_e. The indirect method of redistribution is always feasible by simply reversing the pairwise organ transfers between groups with $\lambda_{ij} > 0$ in the corresponding allocation matrix.

An alternative way of understanding the intuition for equal access in an optimal policy under any given matching technology is to observe that the incentive to donate for each group is largely determined by the incremental chance of receiving a transplant from being a nondonor to a registered donor. When each group has identical survival rates, transferring an organ from group b_j to a compatible group b_i results in a marginal increase in $n_i p_i$ by $1/(\theta\delta_i)$ and a marginal decrease in $n_j p_j$ by $1/(\theta\delta_j)$, according to the market clearing conditions of these two groups. Since group b_i now has a higher donation incentive with more access to organs, i.e., $\delta_i > \delta_j$, such a transfer decreases the amount of organ donations in group b_j more than it incentivizes donations in group b_i with $1/(\theta\delta_i) < 1/(\theta\delta_j)$, provided that $1/F(c)$ is strictly convex. Therefore, moving from equal access to unequal access not only has distributional effects but, more importantly, leads to incentive distortions and undermines the aggregate donation rate.

We now turn to the optimal design of specific allocation policies. For any general matching technology $\mu \in \mathbf{M}$, our next main result establishes that the XYZ-identical allocation policy with $\Lambda^* = I$ maximizes the aggregate incentives to donate under the donor-priority rule, which is general in terms of the classifications and population distributions of blood types and the organ-specific matching technologies.

THEOREM 2 (Optimality of the XYZ-identical policy). *For any $\mu \in \mathbf{M}$, the allocation policy of $\lambda^*_{ii} = 1$ and $\lambda^*_{ij} = 0$ for $i \neq j$ (i.e., $\Lambda^* = I$), which results in $P = P^*$, is optimal.*

This result indicates that regardless of the organ matching technology, it is always optimal to offer blood-type b_i organs only to type b_i patients while simultaneously ensuring equal access to organs among all blood-type groups, as in Theorem 1. The proof of this result relies on a relaxation of the original optimization problem in (1a)–(1c) and, subsequently, the application of Proposition 1. To understand the intuition behind the optimality of the XYZ-identical policy, we note that with each group staying in autarky, this equalizes the access to organs and balances the donation incentives across the different blood-type groups. In contrast, allowing organ transfers between compatible but nonidentical blood-type donor-patient pairs may result in an incentive distortion

among these groups, as a standard matching technology naturally creates immunological asymmetries between different blood types. For instance, with the ABO matching function $\tilde{\mu}$, type O patients have inherent disadvantages because they are not compatible with any other type of organ and, consequently, have less access to deceased donor organs than the other three types of patients, while type AB patients are in a more favorable situation. Such disparity in access to transplants has motivated many interesting studies in the living donor-patient kidney exchange literature (Sönmez et al. (2018, 2020)). In particular, with $\mu = \tilde{\mu}$, our result in Theorem 2 is parallel to the finding in Sönmez et al. (2020) that the ABO-identical exchange policy is optimal in the sense of maximizing the measure of transplants to the donor-recipient pairs.[6]

3.2 *Uniqueness of the optimal policy*

Although Theorem 2 suggests that the XYZ-identical policy maximizes the aggregate donation incentive, it may not be a unique allocation policy to achieve this. Intuitively, any allocation policy that results in a balanced trade of organs among the blood-type groups ensures equal access in organ sharing and can therefore be optimal according to Theorem 1. To further explore the general set of optimal allocation policies, we next investigate the implications of bilateral matching feasibility on the optimal transfers among the different blood-type groups. Let us first consider group b_j from a supplier's or donor's perspective. If the type-b_j organs (such as type AB under the standard ABO classification) are not compatible with any other type-b_i (where $i \neq j$) patients, an optimal policy should not export any type-b_j organs because of the full utilization constraint for group b_j. Meanwhile, group b_j should not import any organs from another group due to the equal access condition in optimal policies. We next consider group b_i as a receiver in the market. A symmetric argument applies from the perspective of recipients or patients as follows. If type-b_i patients (such as type O under the standard ABO classification) are not compatible with any type-b_j (where $j \neq i$) organs under a given matching technology, it would be wasteful to transfer any type-b_j organs to group b_i based on the full utilization constraint. Meanwhile, it is also suboptimal to export any type-b_i organs to any other group because of the equal access property in an optimal allocation policy based on Theorem 1. These arguments lead to the following lemma.

LEMMA 2. *Consider any fixed j or any fixed i. If $\mu_{ij} = 0$ for all $i \neq j$, an optimal allocation policy should satisfy $\lambda_{ii} = 1$ and $\lambda_{ij} = \lambda_{ji} = 0$ for any $i \neq j$.*

An important implication of Lemma 2 is that the suboptimality of the XYZ-compatible policies is closely related to the configuration of μ, i.e., the given matching technology. The next theorem establishes the necessary and sufficient condition for the XYZ-identical policy to be the unique optimal policy. For notational simplicity and clarity, we denote, for instance, $\mu(i_1, i_\ell) := \mu_{i_1 i_\ell} := \mu(b_{i_1}, b_{i_\ell})$, in the following.

[6]In the kidney exchange literature with living donors, the ABO-identical exchange policy matches an arriving incompatible pair (with a blood-type b_i recipient and a blood-type b_j donor) with a mutually compatible pair of its reciprocal type (with a blood-type b_j recipient and a blood-type b_i donor).

THEOREM 3 (Uniqueness of optimal policy). *If* $\mu(i_1, i_\ell)\mu(i_\ell, i_{\ell-1})\cdots\mu(i_3, i_2)\mu(i_2, i_1) = 0$ *for any* $i_1, i_2, \ldots, i_\ell$ *and* $\ell \geq 2$*, the optimal allocation policy is unique with* $\Lambda^* = I$ *and* $P = P^*$*. Otherwise, a continuum of optimal policies exists with* $\Lambda \neq I$ *and* $P = P^*$*.*

When $\ell = 2$, the condition in Theorem 3 corresponds to $\mu(i_1, i_2)\mu(i_2, i_1) = 0$ for any two groups i_1 and i_2, which means that, for any pair of blood-type groups, at least one group must be incompatible with the other. More generally, to better understand the condition for the uniqueness of the optimal policy, we note that given any blood-type classification $\mathbf{B}$ and the corresponding matching technology μ, we can construct a directed compatibility graph $\mathbf{G} = (\mathbf{B}, \mathbf{E})$ by encoding each blood-type group $b_i \in \mathbf{B}$ as a vertex and adding a directed edge $e = (b_j, b_i) \in \mathbf{E}$ from b_j to b_i if organs of type b_j are compatible with patients of type b_i, i.e., $\mu_{ij} = 1$. Note that G permits self-loops with edges that connect a vertex to itself. Following the standard definition in graph theory, the adjacency matrix of $\mathbf{G}$ is a square $m \times m$ matrix $A_{\mathbf{G}} = \{a_{ij}\}$ such that $a_{ij} = 1$ when there is a directed edge from vertex b_i to vertex b_j and $a_{ij} = 0$ otherwise. Therefore, the compatibility matrix representation of a matching technology is essentially the transpose of the adjacency matrix representation of the corresponding directed compatibility graph, i.e., $\mu = A_G^T$. It follows that the condition of $\mu(i_1, i_\ell)\mu(i_\ell, i_{\ell-1})\cdots\mu(i_3, i_2)\mu(i_2, i_1) = 0$ for any $i_1, i_2, \ldots, i_\ell$ and $\ell \geq 2$ is equivalent to the requirement that the compatibility graph G does not contain any directed cycles (not accounting for the self-loops from b_i to b_i), i.e., G is a directed acyclic graph (which permits self-loops).

The uniqueness of an optimal allocation policy critically depends on the configuration of the directed compatibility graph that corresponds to a given organ matching technology. The first part of Theorem 3 states that when the directed compatibility graph is acyclic, the XYZ-identical policy is the unique optimal allocation policy, which also results in equal access to organ transplants with $P = P^*$. The second part establishes that if there exists at least one directed cycle in the compatibility graph, we have a continuum of XYZ-compatible policies that are optimal and can achieve the same donation rate as the XYZ-identical policy while ensuring equal access to organs. The intuition is as follows. To maximize the aggregate donation incentives, an allocation policy must result in equal access to organ transplants as established in Theorem 1. The key to guaranteeing an identical survival rate across groups is to have a balanced trade of organs under the class of XYZ-compatible policies or simply have no trade at all as in the XYZ-identical policy. When the compatibility graph is acyclic, it is impossible to achieve a balanced trade with XYZ-compatible policies. This is because starting at any blood-type group b_i, which exports some of the type-b_i organs to other groups, there is no consistently directed sequence of edges to allow for continuous flows of organs that eventually loops back to group b_i again. That is, group b_i cannot import as much as it exports, which makes it a net exporter in the market, and thus results in incentive distortions. Therefore, when the directed compatibility graph that corresponds to a given organ matching technology is acyclic, the only feasible way to avoid a trade surplus or trade deficit in organs is to keep each group in a state of autarky, i.e., to restrict organ imports and exports by reserving type X organs for only type X patients such that $\Lambda^* = I$, which is the XYZ-identical policy. In contrast, if there is a directed cycle in the compatibility graph, it

is possible to realize a fairly even reciprocal trade pattern among the blood-type groups along the cycle, where each group can balance the overall trade by offsetting a trade deficit (with one group) with a trade surplus (with another group). Accordingly, a continuum of optimal allocation policies achieves a balanced trade of organs and equal access to organ transplants when the compatibility graph G contains directed cycles.

These arguments also have direct implications for the optimal way to allocate each specific type of organs. When the condition of acyclicity in Theorem 3 is violated, the continuum of optimal policies actually applies to only the groups that are in the compatible cycles, whereas the remaining groups that are not a part of any directed trade cycle should remain in autarky. More specifically, if blood-type group b_i cannot form any directed cycle with the other groups, it is optimal for this group to refrain from trade, i.e., to reserve type b_i organs for type b_i patients only such that $\lambda_{ii} = 1$ and $\lambda_{ij} = \lambda_{ji} = 0$ for any $j \neq i$. Otherwise, group b_i can engage in a balanced trade along at least one directed compatible cycle with the other groups, which thus creates a continuum of optimal allocation policies for group b_i with $\lambda_{ii} < 1$. This argument leads to Corollary 1, which is a generalization of Lemma 2. When $\ell = 1$, the condition in Corollary 1 corresponds to $\mu(i, j_1)\mu(j_1, i) = 0$ for any blood-type group $j_1 \neq i$.

Corollary 1. *For any fixed blood-type group i, if $\mu(i, j_\ell) \cdots \mu(j_2, j_1)\mu(j_1, i) = 0$ for any $\ell \geq 1$ and $j_k \neq i$, where $k = 1, 2, \ldots \ell$, the optimal allocation policy for group i is unique with $\lambda_{ii} = 1$ and $\lambda_{ij} = \lambda_{ji} = 0$ for any $j \neq i$. Otherwise, a continuum of optimal allocation policies exists for group i with $\lambda_{ii} < 1$.*

Theorem 3 immediately suggests the suboptimality of the ABO-compatible policies, i.e., the ABO-identical policy is the unique optimal allocation policy. Intuitively, with the asymmetry and acyclicity in the ABO blood-type compatibility matrix, the O group is especially disadvantaged because it can only serve as an exporter while never serving as an importer in the market for organ transplants. This intuition is consistent with the well-documented empirical evidence that the ABO-compatible allocation policy in the Eurotransplant Kidney Allocation System results in a substantial drain of O kidneys and longer waiting times, higher death rates, and an accumulation of blood type O patients on the waiting list for kidneys, which will further aggravate the so-called "blood group O problem" in the future (Glander et al. (2010)). The next corollary formalizes and generalizes this result to any general blood-type classifications.

Corollary 2. *If $\mu = \tilde{\mu}$, or more generally, if μ is a triangular matrix, there is a unique optimal policy with $\Lambda^* = I$ and $P = P^*$.*

In general, if μ can be represented in the form of an $m \times m$ triangular matrix (by rearranging the rows and columns that correspond to each blood-type group, if necessary), the second part of Corollary 2 states that the XYZ-identical policy is the unique optimal allocation policy. This result intuitively follows from a sequence of iterative eliminations of groups that can never participate in cross-group trades. Without loss of generality, we consider a lower triangular compatibility matrix with the rows and columns representing blood-type groups $b_1, b_2, \ldots, b_m$ in sequence. In the first step, we observe that group

b_1 cannot receive any other type of organs since $\mu(b_1, b_j) = 0$ for all $j > 1$. In an optimal policy, equal access to organ transplants requires that organs donated by group b_1 should not be exported to any other group since group b_1 cannot import any organ due to incompatibility. Since group b_1 can never be involved in any trade, we can remove it from the market, and thus eliminate the first row and the first column of the compatibility matrix μ. In the second step, we consider group b_2 in the reduced compatibility matrix among the remaining $m-1$ groups $b_2, b_3, \ldots, b_m$, which is an $(m-1) \times (m-1)$ lower triangular matrix. In this reduced market, type b_2 patients cannot receive any other type of organs since $\mu(b_2, b_j) = 0$ for all $j > 2$. By a similar argument, group b_2 should also not be involved in any trade in an optimal policy, and thus can be removed from the market. By repeating these steps, each group is iteratively eliminated from the market for possible organ trade across groups. Therefore, this sequence of procedures rules out all the XYZ-compatible policies and results in a unique optimal allocation policy, which is the XYZ-identical policy.

Finally, we note that our main results rely on a common condition of a regular cost-type distribution, i.e., $1/F(c)$ is strictly convex. In our model, the cost of donation captures the disincentives involved in deceased donor registration in practice, which may encompass both the psychological costs and financial costs (Hawley et al. (2018)). In practice, the psychological costs are likely to play a more important role and are often difficult to measure (Kessler and Roth (2012), Dai et al. (2020)). Although it is a challenging task to directly measure the exact cost distribution, many empirical studies have explored and identified various factors, including the sociodemographic characteristics, attitudes, beliefs and subjective norms, that can influence the intention of donation through surveys (Baughn et al. (2006), López et al. (2018)). Based on these empirical studies, the regularity condition, which is satisfied under many common distributions, is a reasonable approximation of the underlying cost distribution. If regularity fails to hold, i.e., if $1/F(c)$ is not strictly convex, then we have the following negative result on the suboptimality of $P = P^*$. For the proof, we simply construct a counterexample with a cost distribution such that $1/F(c)$ is not convex at β.

Proposition 2. *If $1/F(c)$ is not strictly convex, then $P = P^*$ may be suboptimal, i.e., an XYZ-compatible policy may strictly dominate the XYZ-identical policy.*

4. Extension and discussion

Our main analysis, although general in terms of the blood-type group classifications and organ matching technology, has made a few simplifying assumptions. In particular, to capture as directly as possible the key feature of potential incentive distortions due to asymmetries in matching technology, we have abstracted away from additional medical considerations in practical organ transplantations. This section further discusses several extensions and alternative model specifications, which are largely motivated by the details in practical allocation policies, to explore the robustness of our main results.

4.1 *Relative priority*

The prioritization criteria for patients on waiting lists are complex, which vary across organ types and depend on many characteristics of patients and organs beyond the registered donor status and blood type (Bertsimas et al. (2013)), which tend to make the relative priority between donors and nondonors less stark than that in our model. For instance, the deceased kidney allocation policy in Israel conforms with a point system that ranks patients according to individual allocation scores, which take into account the patient's deceased donor registration status, age, waiting period, genetic compatibility, and level of antibodies (Lavee et al. (2010)).

To account for these details, we further examine a scenario with only a relatively higher priority for donors instead of the absolute priority in the main analysis. Specifically, we consider p_i as the transplantation rate of blood type i patients who were registered for donation and $(1-t)p_i$ as the transplantation rate of blood type i patients who are not registered donors. The relative priority between donors and nondonors is measured by $t \in (0, 1]$, with a higher t indicating a starker difference and higher relative priority for the donors. Note that the marginal increment in the probability of getting a transplantation by registering to become a donor is tp_i, and hence decreasing t reduces the incentive to donate. If t was considered to be a policy parameter, it is trivial that $t^* = 1$ would be the optimal policy, and the analysis degenerates to our previous discussions. Therefore, we consider $t \in (0, 1]$ as exogenously given in the following analysis, which may incorporate other practical factors in determining organ allocation priorities, such as geographic distances (Kong et al. (2010)).

Individuals in each blood-type group compare the expected utility gain of the donors from receiving organ transplantation θp_i and the expected utility gain of the nondonors $\theta(1-t)p_i$ in making donation decisions. Therefore, a type i individual with donation cost c will register for donation if and only if $c \le \theta t p_i$. The market clearing condition becomes $\theta(1-t+t\delta_i)n_i p_i = \beta \sum_{j=1}^{m} \delta_j n_j \lambda_{ij} \mu_{ij}$ for each group $i \in \{1, \dots, m\}$. The following proposition states the optimal allocation policy after introducing the relative priority between donors and nondonors.

Proposition 3. *If* $1/(F(c) + \frac{1-t}{t})$ *is strictly convex in* c*, then given any* $\mu \in \mathbf{M}$ *and* $t \in (0, 1]$*, the allocation policy* $\Lambda^* = I$*, with survival rates* $P = \bar{P} := (\bar{p})_{\{i=1,\dots,m\}}$*, is optimal, where* $\bar{p} \in (0, 1)$ *is a solution to* $(1-t)\theta\bar{p} = (\beta - \theta t\bar{p})F(\theta t\bar{p})$.

With relative priority between donors and nondonors, the XYZ-identical allocation policy is still optimal to maximize the aggregate donation rate, regardless of the extent of priority provided to donors. This optimal policy also has a nice byproduct of equalizing the access to organ transplants with $P = \bar{P}$ among the different blood-type groups, similar to our previous analysis. In particular, when $t = 1$, the optimal policy sets $\bar{p} = p^*$, which coincides with the absolute priority case in Theorem 2, and when $t \to 0$, we have $\bar{p} \to F(0)p^* < p^*$, which is the case without donor priority. This further suggests that the access to organ transplantation measured by p_i unanimously improves with the XYZ-identical allocation policy under the donor-priority rule.

4.2 *Degree of severity*

Organ allocation policies are designed to address multiple objectives in practice, such as the equality across the blood-type groups and regions (Kong et al. (2010), Bertsimas et al. (2013)) and an efficient match of organs to patients of different degrees of severity (Assfalg et al. (2016), Kratz (2019)). Indeed, the degree of severity of the patient's conditions plays an important role in determining the waitlist for certain types of deceased organ transplantations, including livers, lungs, and hearts (Colvin-Adams et al. (2012)). For instance, in Israel, the waiting list for heart transplants takes medical urgency as the first consideration in the allocation decision, and within the less urgent group, registered donors are prioritized at the top of the candidacy list (Lavee et al. (2010), Berzon (2008)).

To accommodate the feature of medical urgency, we consider two types of organ shocks, namely, severe (S) and mild (M), which result in severe and mild patients. The probability of a severe organ failure shock is $\theta_S \in (0, 1)$, while a mild organ failure shock occurs with a probability of $\theta_M \in (0, 1)$. These two shocks are mutually exclusive, and $\theta = \theta_S + \theta_M \in (0, 1)$ is the probability of organ failure shock. Agents only know the distributions of these two types of organ shocks but are unaware of which type of organ shock will be realized at the time of their donation decisions. We focus on a scenario with insufficient organ supply for severe patients by assuming $\beta \leq \theta_S$. If a patient (severe or mild) in need of organ transplantation receives an allocation, he gains a utility of $V = 1$, as in our previous setup. To distinguish between severe and mild patients, we assume that in the case of not receiving an organ transplant, a severe patient's payoff from organ allocation is zero, while a mild patient obtains $1 - x$ with $x \in (0, 1)$, where x represents the cost of additional medical aids to maintain a functioning life.[7]

To account for both the donor priority and the degree of severity in an allocation policy, we consider the probabilities for patients who are registered donors to receive an organ allocation as follows. Let p_i denote the transplantation rate of blood type i severe patients and τp_i denote the transplantation rate of blood type i mild patients. In this setup, $\tau \in [0, 1]$ is an additional policy parameter that determines the extent of priority granted to severe patients, and a lower τ indicates more priority for severe patients.[8] In making donation decisions, agents weigh the expected utility gain for donors in organ allocations, which is $\theta_S p_i + \theta_M[\tau p_i + (1 - \tau p_i)(1 - x)]$, and the expected utility gain for nondonors in organ allocations, which is $\theta_M(1 - x)$. Therefore, a type i individual with donation cost c registers for donation if and only if $c \leq (\theta_S + \tau x \theta_M) p_i$. The market clearing condition for each blood-type group $i \in \{1, \ldots, m\}$ becomes $(\theta_S + \tau\theta_M)\delta_i n_i p_i = \beta \sum_{j=1}^{m} \delta_j n_j \lambda_{ij} \mu_{ij}$. The following proposition states that it is optimal to provide absolute priority to severe patients with $\tau^* = 0$ and to equalize the access to organ transplants with $\tilde{p} = \beta/\theta_S \in (0, 1]$.

[7] Alternatively, one can interpret x as a reduced formulation for severity or urgency, and in this case, severe patients will be dead with a probability of one without transplantation, while mild patients will be dead with a probability of $x \in (0, 1)$ without transplantation.

[8] In the kidney exchange literature, the prioritization for patients with severe conditions is modeled by Dickerson and Sandholm (2014) and Kratz (2019), with patients sorted into priority groups based on the severity of their conditions.

PROPOSITION 4. *For any $\mu \in \mathbf{M}$, the allocation policy $\Lambda^* = I$ with survival rates $P = \tilde{P} := (\tilde{p})_{\{i=1,\dots,m\}}$ and $\tau^* = 0$ is optimal.*

This result again confirms that the XYZ-identical allocation policy with equal access to organ transplantations is optimal in terms of maximizing donation incentives by considering different degrees of severity. In addition, this result suggests that considering an efficient match of organs to patients with different severities of conditions also increases the donation rate. All else being equal, it is optimal to provide severe patients an absolutely higher priority over mild patients with $\tau^* = 0$ in incentivizing deceased donor registrations. This is consistent with the practice in Israel where candidates with urgent conditions are given priority for heart, lung, and liver transplantations (Lavee et al. (2010)).

4.3 *Alternative model specifications*

Excess supply. Although medical urgency plays an important role in certain types of organ transplantations, the number of patients in the most urgent group is typically small compared with the potential deceased organ supply. For instance, at any given moment, there are generally fewer than 50 candidates listed nationwide in the U.S. with the most urgent designation waiting for liver transplantations. To feature an excess supply of organs for severe patients, we further extend the analysis in Section 4.2 by considering $\beta > \theta_S$. In this case, among the registered donors, severe patients can always receive a transplant, and the transplantation rate for type i mild patients is denoted by p_i. The expected utility gain from the organ transplantation of the donors now becomes $\theta_S + \theta_M[p_i + (1 - p_i)(1 - x)]$, while the expected utility of the nondonors is $\theta_M(1 - x)$. Therefore, a type i individual with donation cost c will register for donation if and only if $c \leq \theta_S + x\theta_M p_i$. It follows that the market clearing condition for each blood-type group i is $(\theta_S + \theta_M p_i)\delta_i n_i = \beta \sum_{j=1}^{m} \delta_j n_j \lambda_{ij} \mu_{ij}$. Similar to the previous discussions, we can establish that it remains optimal to distribute organs equally across different blood types with identical transplantation rates $\hat{p} = (\beta - \theta_S)/\theta_M \in (0, 1]$, and the XYZ-identical allocation policy maximizes the aggregate donation rate.

Heterogeneous health shocks. Empirical evidence suggests that organ failure shock may disproportionately affect different blood-type groups. For instance, it is well documented that African Americans are two to four times more likely to have kidney failure than white Americans because of differences in clinical, socioeconomic, or genetic risk factors (Collins et al. (2014), Carnethon et al. (2017)). Given that nearly 70% of African Americans are blood types O or B, the pervasive racial disparity in organ-related disease naturally translates to different risks of organ failure among blood types. To take this fact into account, our model can introduce heterogeneous organ failure rates across blood types with θ_i. More generally, we can allow for further heterogeneity in brain death rates with $\beta_i < \theta_i$ for $i \in \{1, 2, \dots, m\}$. The previous analysis similarly applies, and the XYZ-identical allocation policy remains optimal in terms of incentivizing deceased donor registrations.

Equity across tissue type/donor service area. The OPTN report on equity in access has identified the blood type, donor service area, and tissue type as the top three main

contributors to disparities in access to deceased donor kidney transplants (Stewart et al. (2018)). Although our main analysis focuses on the blood-type groups, it can also be easily extended to incorporate these additional characteristics, provided that the agents are aware of their individual attributes at the time of donation decisions. More specifically, each individual's type can be denoted by a multidimensional vector that specifies his blood type, donor service area, tissue type, and other attributes that factor into the practical organ allocation process. As long as the type space is discrete, the matching technology defined in Section 2.2.1 is sufficiently general to encompass the feasibility of organ allocations across all of these types, which can account for current and future medical technologies or legal constraints. All of our main results directly translate to this extended setting. Alternatively, if the individual tissue type is unknown at the time of the donation decision, a similar analysis as that in Section 4.2 can be applied to account for tissue-type compatibility by considering severe patients as the highly sensitized type and τ as a measure for the relative priority for such disadvantaged patients. The result in Proposition 4 suggests prioritizing such disadvantaged patients within the same blood-type group, which is consistent with practical deceased organ allocation polices (Bertsimas et al. (2013), Stewart et al. (2018)). In addition, as noted by Sönmez et al. (2020), the primary method of reducing inequity in access for highly sensitized patients is to enlarge the organ pool size. From this perspective, by maximizing the aggregate donation incentives, the XYZ-identical policy also indirectly contributes to the objective of equitable organ allocation across tissue types.

5. Simulations

This section investigates the extent of the donation rate improvement by adopting the optimal ABO-identical allocation policy. For this purpose, we conduct counterfactual simulations by calibrating our model with U.S. heart donation and transplantation data. Note that deceased donation is the only source of supply, and the ABO-compatible allocation policy is currently implemented for heart transplantations in the U.S.

5.1 *Simulation setup*

Table 1 reports the calibrated parameters of our model with the extension of relative priority. The blood-type distribution in the population follows from Sönmez et al. (2020), which is calculated based on the blood-type distribution for different ethnicities and fractions of each ethnicity in the U.S. population. Since our theoretical framework models deceased donor registration behavior, we focus on a subsample of deceased donors authorized through state registries in the calibration, which accounted for approximately 49% of all deceased donors in 2018.[9] The average number of hearts supplied by each donor ($\alpha = 0.318$) is measured by the number of heart transplants performed over the number of deceased donors in 2018, as reported by the OPTN. According to the organ donation statistics provided by the HRSA, only 3 in 1000 people die in a manner

[9]The other deceased donations were authorized by family or next of kin consent. Data obtained from https://www.organdonor.gov/statistics-stories/statistics.html.

TABLE 1. Parameter calibration for numerical simulation.

Parameter	Definition	Value
n_O	Fraction of blood-type O in the population	0.456
n_A	Fraction of blood-type A in the population	0.378
n_B	Fraction of blood-type B in the population	0.126
n_{AB}	Fraction of blood-type AB in the population	0.040
α	Average number of hearts supplied by each deceased donor	0.318
β	Probability of dying in a way that allows for organ donation	0.003
θ	Probability of being in need of a heart transplant	0.001
t	Relative priority between donors and nondonors	0.966
$F(0)$	Default donation rate as the eligible designated donor rate	0.380
V	Value for receiving a heart transplant	485,000
$\tilde{\Lambda}$	De-facto ABO-compatible allocation matrix used in practice	Calculated*

*The ABO-compatible allocation matrix Λ is calculated based on the type-specific number of donors and number of heart transplants performed according to the OPTN data, which represents the de-facto ABO-compatible policy currently used for the heart transplantation practice in the U.S. The detailed methods of the calculations are discussed in Section 5.1.

that allows for organ donation. Therefore, the probability of brain death is measured as $\beta = 0.003$ in the numerical calculations.

For the default donation rate without any donor-priority incentive as in the current U.S. policy, $F(0)$ is measured by the eligible designated donor rate, which is 38% and obtained from the 2018 Donate Life America Annual Report. This measure extends beyond the number of registered donors in the general adult population to measure the rate of donor registration among the individuals who are in the population of likely donors, which is consistent with the full utilization of donated organs in our model setting. It follows that we can calculate the probability of being in need of a heart transplant in the population as $\theta = \beta F(0)W/D$, where W is the number of patients added to the waitlist for heart transplants and D represents the number of deceased donors. To measure the value of receiving a heart transplant, we estimate the underlying exponential distribution of patient survival among adult heart transplant recipients from 2010 to 2012 based on Colvin et al. (2019). The results suggest that the expected post-transplant life expectancy is approximately 9.7 years. Following Dai et al. (2020), we assume that the economic value per quality-adjusted life-year to be \$50,000 (Diamond and Kaul (2009)), which provides an estimate of $V = \$485{,}000$ as the value of receiving a heart transplant.

To uncover the current ABO-compatible allocation policy matrix $\tilde{\Lambda}$ for heart transplantation, we first note that with the standard ABO organ matching technology $\tilde{\mu}$, the full utilization constraints in an ABO-compatible allocation policy require $\tilde{\lambda}_{O,j} = 0$ for $j \in \{A, B, AB\}$, $\tilde{\lambda}_{A,j} = 0$ for $j \in \{B, AB\}$, $\tilde{\lambda}_{B,j} = 0$ for $j \in \{A, AB\}$, and $\tilde{\lambda}_{AB,AB} = 1$. According to the definition of an allocation policy matrix, each column must add up to one, i.e., $\sum_{i\in\{O,A,B\}} \tilde{\lambda}_{ij} = 1$ for $j \in \{O, A, B\}$, which results in 3 feasibility constraints for $\tilde{\Lambda}$. In addition, based on the type-specific number of donors D_i and the number of heart transplants performed R_i in the OPTN data, we have 4 more market clearing constraints, i.e., $\sum_{j\in\{O,A,B,AB\}} \tilde{\lambda}_{ij}\alpha\beta D_j = R_i$, for $i \in \{O, A, B, AB\}$. With a total number of 7 equations and 8 parameters to recover, this leaves one degree of freedom. Without loss of generality, we set $\tilde{\lambda}_{A,O}$ as the free parameter and derive its range as $\tilde{\lambda}_{A,O} \in [0.061, 0.092]$. In

the numerical simulations, we consider the intermediate value of $\tilde{\lambda}_{A,O} = 0.076$, which defines the de facto ABO-compatible allocation matrix *endogenously implied* from the observed US data as follows:

$$\tilde{\Lambda} \equiv \begin{pmatrix} \tilde{\lambda}_{O,O} & 0 & 0 & 0 \\ \tilde{\lambda}_{A,O} & \tilde{\lambda}_{A,A} & 0 & 0 \\ \tilde{\lambda}_{B,O} & 0 & \tilde{\lambda}_{B,B} & 0 \\ \tilde{\lambda}_{AB,O} & \tilde{\lambda}_{AB,A} & \tilde{\lambda}_{AB,B} & 1 \end{pmatrix} = \begin{pmatrix} 0.797 & 0 & 0 & 0 \\ 0.076 & 0.977 & 0 & 0 \\ 0.086 & 0 & 0.998 & 0 \\ 0.041 & 0.023 & 0.002 & 1 \end{pmatrix}.$$

Due to the lack of reliable empirical data, we consider a wide range of donation cost distributions in the numerical simulations and obtain comparable results. In the following discussions, we focus on the set of simulation results based on normal distributions with a mean that varies from \$600 to \$200 while calibrating the standard deviation to match the donor registration rate before the introduction of the donor-priority rule, i.e., $F(0) = 0.38$. Existing studies suggest that introducing the donor-priority rule can lead to an approximately 13% increase in organ availability in the U.S. (ScienceDaily (2019)). Therefore, we calibrate the level of relative priority between donors and nondonors as $t = 0.966$ such that the donation rate under the ABO-identical policy is 51%, with an intermediate level of average donation cost of $\bar{c} = 400$. Based on these calibrated parameters, we solve for the equilibrium in each set of the simulations and compare the respective donation rates under the donor-priority rule with the de facto ABO-compatible allocation policy $\tilde{\Lambda}$ and with the ABO-identical allocation policy Λ^*.

5.2 *Simulation results*

Table 2 summarizes the simulation results, with the rows corresponding to different average donation costs. The row with $\bar{c} = 400$ is the baseline used for calibrating the degree of relative priority, while the remaining rows serve as robustness checks of our simulation results. In addition, these results provide insights into the interactive effects of allocation policy design and other initiatives to reduce donation costs.

We observe substantial increases in the aggregate donation rates shown in columns (2) and (3) compared with the current donation rate without any donor-priority incentive, which is 38%. The donor-priority rule can realize an increase in the donation rate of approximately 8 percentage points even when the average cost of donation is relatively high, with $\bar{c} = 600$. Comparisons between columns (2) and (3) suggest a considerably higher donation rate under the ABO-identical allocation policy than under the de facto ABO-compatible allocation policy $\tilde{\Lambda}$ implied from the U.S. data. For all the distributions examined, the relative increment in the donation rate shown in column (4) is more than 10%, which is measured as the improvement by moving from the de-facto ABO-compatible policy to the ABO-identical policy (with the donor-priority rule) over the improvement by introducing the donor-priority rule (under the de facto ABO-compatible policy). Further comparisons across distributions suggest that the superior performance of the ABO-identical policy is even more stark when the average donation cost becomes smaller. In the case of $\bar{c} = 200$, implementing the donor-priority rule while keeping the current ABO-compatible allocation policy increases the aggregate donation

TABLE 2. The equilibrium donation rates under different allocation policies (%).

Mean Cost (1)	ABO-Compatible (2)	ABO-Identical (3)	Relative Increment (4)	Donation Rate by Groups under $\tilde{\Lambda}$			
				O (5)	A (6)	B (7)	AB (8)
600	45.72	46.56	10.87	44.77	47.13	45.78	42.97
500	47.28	48.33	11.31	46.17	49.01	47.22	43.70
400	49.61	51.00	11.96	48.29	51.84	49.30	44.63
300	53.46	55.47	12.99	51.84	56.57	52.46	45.75
200	60.84	64.23	14.82	58.94	65.74	57.54	46.63

Note: This table summarizes the aggregate donation rates in columns (2)–(3) and the type-specific donation rates under the de facto ABO-compatible policy $\tilde{\Lambda}$ in columns (5)–(8). The relative increment in column (4) measures the increase in the donation rate while moving from the ABO-compatible policy to the ABO-identical policy (with the donor-priority rule) over the improvement by introducing the donor-priority rule (under the de facto ABO-compatible policy).

rate from 38 to 60.84%. By adopting the ABO-identical policy, we can achieve a further increase in the donation rate of 3.39 percentage points, which accounts for approximately 14.82% of the relative increment. These results further imply that the efficacy of the donor-priority rule combined with the ABO-identical allocation policy can be enhanced by complementary initiatives that aim to reduce the psychological and logistical costs of deceased organ donation.

In Columns (5)–(8) of Table 2, we further examine how the relative donation incentive varies across the different blood-type groups under the current ABO-compatible allocation policy. Although each group unanimously has a higher incentive to donate after introducing the donor-priority rule than the default rate of 38%, the improvement is not equally shared among the four groups. We observe significantly unbalanced incentives among the four blood-type groups, with the highest increase for group A and the lowest increase for group AB. The individuals in group B are relatively less incentivized than the individuals in group A but more incentivized than those in group O. Compared with the aggregate donation rate under the ABO-identical allocation policy in Column (3), group A is more incentivized, while groups O, B, and AB are all less incentivized under the ABO-compatible policy $\tilde{\Lambda}$. The blood-type O agents export part of the supply by O donors to the other three groups with $\tilde{\lambda}_{A,O}, \tilde{\lambda}_{B,O}, \tilde{\lambda}_{AB,O} > 0$, which decreases their marginal benefits in becoming organ donors and results in a smaller donation rate. Although groups A, B, and AB are all net importers of organs, only group A is more incentivized to donate than its counterpart under the ABO-identical policy. This is because the current ABO-compatible policy $\tilde{\Lambda}$, as endogenously implied from the observed transplantation statistics, provides an excess supply for the donors in these three groups. As a result, a considerable proportion of nondonors become "free riders." The marginal benefit of donor registration is then determined as one minus the probability of nondonors to receive transplants. With a moderate excess supply, more donation is incentivized for group A. However, if the excess supply is considerably large, the free-riding effect dominates, making the marginal benefit of donation less attractive and resulting in a relatively small donation rate, which is the case for groups B and AB.

By implementing the optimal ABO-identical allocation policy, the incentive among each group becomes balanced since each group is autarkic and there is no excess supply.

Overall, the pairwise difference in the group donation rates under $\tilde{\Lambda}$ increases when the mean cost of donation becomes smaller. In particular, the difference in the donation rates between groups A and AB is approximately 4 percentage points when $\bar{c}$ is high at 600, while it reaches over 19 percentage points with $\bar{c} = 200$. One implication is that the ABO-compatible allocation policy creates more distortions in the incentives among different blood-type groups when the cost of donation is relatively lower. As a result, the policy initiatives and efforts to reduce donation costs should also be complemented by the ABO-identical allocation policy.

6. Concluding remarks

From the perspective of practical market design, it is crucial to understand the incremental changes associated with a new policy initiative and, more importantly, how it interacts with pre-existing rules and arrangements (Roth (2018)). Our results provide a better understanding about how the interaction between the donor-priority rule and matching technology barriers affects donation incentives, which could possibly influence practical organ allocation policies and decisions. More broadly, our model framework can also be generalized to the donation and allocation problem of other scarce public resources that are privately provided ex ante and rationed through waiting list mechanisms ex post, such as the family replacement program in blood donations (Lacetera et al. (2013), Sun et al. (2016)) and the recent proposal for incentivizing plasma donations from recovered COVID-19 patients (Kominers et al. (2020)). How these resources are allocated and distributed can moderate their scarcity through changes in both donation incentives and allocation equity.

Appendix: Proofs

Proof of Lemma 1

Fix any $P \in [0,1]^m$ that satisfies (3b). First, we choose any λ_{1j}, $j = 1, \dots m$ that satisfies (2b) and (2c). Note that this is always possible because otherwise, we have

$$0 < \theta F(\theta p_1) n_1 p_1 - \beta \sum_{j=1}^{m} F(\theta p_j) n_j \leq \sum_{i=1}^{m} \theta F(\theta p_i) n_i p_i - \beta \sum_{j=1}^{m} F(\theta p_j) n_j,$$

which violates the condition (3b). Next, we recursively choose any $\lambda_{2j}, \lambda_{3j}, \dots$ for $j = 1, \dots, m$ that satisfies (2b) and (2c). This is always possible since otherwise, let $i_0 \leq m$ be the smallest index such that we cannot choose $\lambda_{i_0 j}$, which satisfies (2b) and (2c). Then we have

$$0 < \sum_{i=1}^{i_0} \theta F(\theta p_i) n_i p_i - \beta \sum_{j=1}^{m} F(\theta p_j) n_j \leq \sum_{i=1}^{m} \theta F(\theta p_i) n_i p_i - \beta \sum_{j=1}^{m} F(\theta p_j) n_j,$$

which violates (3b). Therefore, we can construct $\Lambda \in [0,1]^{m \times m}$ that satisfies (2b) and (2c).

Proof of Proposition 1

We first focus on the interior solutions and later show that boundary solutions cannot be optimal for the problem defined by (3a)–(3b). To find the critical points, we derive the (interior) first-order conditions as

$$H_i := \theta f(\theta p_i) n_i + \theta f(\theta p_m) n_m \frac{\partial p_m}{\partial p_i} = 0, \quad i = 1, \dots, m-1, \tag{4}$$

$$\text{where } \frac{\partial p_m}{\partial p_i} = -\frac{n_i}{n_m} \frac{F(\theta p_i) - (\beta - \theta p_i) f(\theta p_i)}{F(\theta p_m) - (\beta - \theta p_m) f(\theta p_m)}. \tag{5}$$

That is, any critical point satisfies the following system of equations:

$$\frac{f(\theta p_i)}{F(\theta p_i) - (\beta - \theta p_i) f(\theta p_i)} = \frac{f(\theta p_m)}{F(\theta p_m) - (\beta - \theta p_m) f(\theta p_m)}, \quad \forall i \neq m; \tag{6}$$

$$\sum_{i=1}^{m} \theta F(\theta p_i) n_i p_i - \beta \sum_{j=1}^{m} F(\theta p_j) n_j = 0. \tag{7}$$

Since $1/F(c)$ is strictly convex, we have $(1/F)'' = (2f^2F - f'F^2)/F^4 > 0$, which gives $2f^2 - f'F > 0$. We can use this inequality to show that

$$\Omega := \frac{f(\theta p)}{F(\theta p) - (\beta - \theta p) f(\theta p)} \tag{8}$$

is strictly decreasing in p by considering its first-order derivative with respect to p as

$$\frac{\partial \Omega}{\partial p} = \frac{\theta \left[f'(\theta p) F(\theta p) - 2 f^2(\theta p) \right]}{\left[F(\theta p) - (\beta - \theta p) f(\theta p) \right]^2} < 0. \tag{9}$$

It immediately follows that to have equations (6) hold, we need to have $p_1 = p_2 = \cdots = p_m$. Then, by equation (7), we have

$$\sum_{i=1}^{m} \theta F(\theta p_i) n_i p_i - \beta \sum_{j=1}^{m} F(\theta p_j) n_j = \theta F(\theta p_i) p_i - \beta F(\theta p_i) = 0,$$

which gives $p_i = p^* = \beta/\theta < 1$ for all i, since $F(\theta p^*) \neq 0$. Thus, $P = P^*$ is the unique critical point for (3a)–(3b).

To further show that the second-order conditions are satisfied at $P = P^*$, we need to show that the Hessian matrix is negative definite. By differentiating H_i in (4) and $\partial p_m / \partial p_i$ in (5), we obtain

$$H_{ii} := \frac{\partial H_i}{\partial p_i} = \theta^2 f'(\theta p^*) n_i + \theta^2 f'(\theta p^*) n_m \left(\frac{\partial p_m}{\partial p_i} \right)^2 + \theta f(\theta p^*) n_m \frac{\partial^2 p_m}{\partial p_i^2};$$

$$H_{ij} := \frac{\partial H_i}{\partial p_j} = \theta^2 f'(\theta p^*) n_m \left(\frac{\partial p_m}{\partial p_i} \right) \left(\frac{\partial p_m}{\partial p_j} \right) + \theta f(\theta p^*) n_m \frac{\partial^2 p_m}{\partial p_i \partial p_j}, \quad \forall j \neq i, m;$$

$$\frac{\partial^2 p_m}{\partial p_i^2} = -\frac{2 n_i (n_i + n_m) \theta}{n_m^2} \frac{f(\theta p^*)}{F(\theta p^*)}; \qquad \frac{\partial^2 p_m}{\partial p_i \partial p_j} = -\frac{2 n_i n_j \theta}{n_m^2} \frac{f(\theta p^*)}{F(\theta p^*)}, \quad \forall j \neq i, m.$$

Together with $\partial p_m/\partial p_i = -n_i/n_m$ at $P = P^*$, we can derive that the Hessian matrix H at $P = P^*$ has entries H_{ij}, where

$$H_{ii} = n_i(n_i + n_m)\frac{\theta^2}{n_m}\left[f'(\theta p^*) - \frac{2f^2(\theta p^*)}{F(\theta p^*)}\right];$$

$$H_{ij} = n_i n_j \frac{\theta^2}{n_m}\left[f'(\theta p^*) - \frac{2f^2(\theta p^*)}{F(\theta p^*)}\right].$$

Note that for any nonzero $\mathrm{x} = (x_1, x_2, \ldots, x_{m-1}) \in \mathbb{R}^{m-1}$,

$$\mathrm{x}H\mathrm{x}^T = \frac{\theta^2}{n_m}\left[f'(\theta p^*) - \frac{2f^2(\theta p^*)}{F(\theta p^*)}\right]\left[\left(\sum_{i=1}^{m-1} n_i x_i\right)^2 + n_m\left(\sum_{i=1}^{m-1} n_i x_i^2\right)\right] < 0.$$

The inequality comes from the strict convexity of $1/F$. Therefore, the Hessian matrix H is negative definite at $P = P^*$. As a result, the local maximum is achieved at $P = P^*$.

Next, we show that boundary solutions are not possible. Consider $p_j = 1$ for some j. Since we have established in (8) and (9) that strict convexity of $1/F$ implies that $f(\theta p)/[F(\theta p) - (\beta - \theta p)f(\theta p)]$ is strictly decreasing in p, with $p_j = 1 > p_m$, we have

$$\frac{f(\theta p_j)}{F(\theta p_j) - (\beta - \theta p_j)f(\theta p_j)} < \frac{f(\theta p_m)}{F(\theta p_m) - (\beta - \theta p_m)f(\theta p_m)}.$$

By rearranging this inequality and substituting (5), it then follows that

$$H_j = \theta f(\theta p_j)n_j + \theta f(\theta p_m)n_m\frac{\partial p_m}{\partial p_j} < 0.$$

Therefore, it is strictly better to decrease p_j. Similarly, when $p_j = 0$, we can establish

$$H_j = \theta f(\theta p_j)n_j + \theta f(\theta p_m)n_m\frac{\partial p_m}{\partial p_j} > 0,$$

and thus, it is strictly better to increase p_j. Therefore, P^* is the global maximizer for (3a)–(3b), and we must have $P = P^*$ in an optimal policy when $\mu = \mu^*$.

Proof of Theorem 1

We first establish the optimality of $P = P^*$ for any given $\mu \in \mathbf{M}$. Let Δ_μ^* denote an optimal donation rate under any given μ, i.e.,

$$\Delta_\mu^* := \max_{\Lambda \in [0,1]^{m\times m}, P\in[0,1]^m} \sum_{i=1}^m F(\theta p_i)n_i \tag{10a}$$

$$\text{subject to} \quad \theta F(\theta p_i)n_i p_i - \beta\sum_{j=1}^m F(\theta p_j)n_j\lambda_{ij}\mu_{ij} = 0, \quad \forall i = 1, \ldots, m; \tag{10b}$$

$$\sum_{i=1}^m \lambda_{ij}\mu_{ij} - 1 = 0, \quad \forall j = 1, \ldots, m. \tag{10c}$$

We note that the optimization problem with $\mu = \mu^*$ in (3a)–(3b) (or equivalently, (2a)–(2c)) is a relaxation of (1a)–(1c), because adding (1b) and applying (1c) give

$$\sum_{i=1}^{m} \theta F(\theta p_i) n_i p_i - \beta \sum_{j=1}^{m} \left[F(\theta p_j) n_j \sum_{i=1}^{m} \lambda_{ij} \mu_{ij} \right] = \sum_{i=1}^{m} \theta F(\theta p_i) n_i p_i - \beta \sum_{j=1}^{m} F(\theta p_j) n_j = 0,$$

which is equivalent to (3b). According to Proposition 1, when $\mu = \mu^*$, the maximum donation rate in (3a)–(3b) is $F(\beta)$. It then follows that we must have $\Delta_\mu^* \le F(\beta)$. Furthermore, it is clear that $\Delta_\mu^* = F(\beta)$ can be achieved at $P = P^*$. Therefore, for any given μ, if an allocation policy results in equal access with $P = P^*$ such that the donation rate achieves the upper bound of $F(\beta)$, it must be an optimal policy. This proves the first part of the theorem.

Next, to show the suboptimality of $P \neq P^*$ for any given μ, we prove it by contradiction. Suppose that an allocation policy denoted by $\tilde{\Lambda} := (\tilde{\lambda}_{ij})_{\{i,j=1,\dots,m\}}$ with $P = \tilde{P} := (\tilde{p}_i)_{\{i=1,\dots,m\}} \neq P^*$ is optimal for a given μ, i.e.,

$$\sum_{i=1}^{m} F(\theta \tilde{p}_i) n_i = \sum_{i=1}^{m} F\big(\theta p^*\big) n_i = F(\beta), \tag{10a$'$}$$

$$\text{where } \theta F(\theta \tilde{p}_i) n_i \tilde{p}_i - \beta \sum_{j=1}^{m} F(\theta \tilde{p}_j) n_j \tilde{\lambda}_{ij} \mu_{ij} = 0, \quad \forall i = 1, \dots, m, \tag{10b$'$}$$

$$\sum_{i=1}^{m} \tilde{\lambda}_{ij} \mu_{ij} - 1 = 0, \quad \forall j = 1, \dots, m. \tag{10c$'$}$$

By the full utilization constraints, it is clear that if $\mu_{ij} = 0$ (where $i \neq j$), then $\tilde{\lambda}_{ij} = 0$. It follows that by replacing $\mu_{ij} = 0$ with $\mu'_{ij} = 1$ in (10b′)–(10c′) and setting the corresponding $\tilde{\lambda}_{ij} = 0$, we obtain an equivalent set of constraints since $\tilde{\lambda}_{ij} \mu_{ij} = \tilde{\lambda}_{ij} \mu'_{ij} = 0$. Therefore, we can replace μ by μ^* while restricting $\tilde{\lambda}_{ij} = 0$ if $\mu_{ij} = 0$ in (10b′)–(10c′), yielding the following equivalent set of conditions:

$$\theta F(\theta \tilde{p}_i) n_i \tilde{p}_i - \beta \sum_{j=1}^{m} F(\theta \tilde{p}_j) n_j \tilde{\lambda}_{ij} \mu^*_{ij} = 0, \quad \forall i = 1, \dots, m,$$

$$\sum_{i=1}^{m} \tilde{\lambda}_{ij} \mu^*_{ij} - 1 = 0, \quad \forall j = 1, \dots, m.$$

By Lemma 1, these two conditions are equivalent to the conditions by adding the m market clearing conditions. As a result, we have

$$\sum_{i=1}^{m} F(\theta \tilde{p}_i) n_i = \sum_{i=1}^{m} F\big(\theta p^*\big) n_i = F(\beta),$$

$$\text{where } \sum_{i=1}^{m} \theta F(\theta \tilde{p}_i) n_i \tilde{p}_i - \beta \sum_{j=1}^{m} F(\theta \tilde{p}_j) n_j = \sum_{i=1}^{m} \theta F\big(\theta p^*\big) n_i p^* - \beta \sum_{j=1}^{m} F\big(\theta p^*\big) n_j = 0.$$

This suggests that $\tilde{P}$ is an optimal solution for (3a)–(3b). This contradicts Proposition 1 in that $P = P^*$ uniquely maximizes the donation rate if $\mu = \mu^*$.

Proof of Theorem 2

Note that the simplified problem in (3a)–(3b) with $\mu = \mu^*$ gives a relaxed optimization problem of (1a)–(1c), which is established in the proof of Theorem 1. It is easy to check that the optimal solution for (3a)–(3b), i.e., $p_i = p^* = \beta/\theta$, and $\lambda_{ii} = 1$, $\lambda_{ij} = 0$ for $i \neq j$ together define a feasible solution for (1a)–(1c). Accordingly, it is also an optimal solution for the original optimization problem in (1a)–(1c).

Proof of Lemma 2

Consider a fixed j with $\mu_{ij} = 0$ for all $i \neq j$. If $\lambda_{ij} > 0$ for some $i \neq j$, then $\sum_{i=1}^{m} \lambda_{ij}\mu_{ij} = \lambda_{jj} < 1$ since $\sum_{i=1}^{m} \lambda_{ij} = 1$. This violates the full utilization constraint for type-j organs. Thus, we must have $\lambda_{ij} = 0$ for all $i \neq j$ in an optimal allocation policy, and it immediately follows that $\lambda_{jj} = 1$. If $\lambda_{ji} > 0$ for some $i \neq j$, we must have either of the following:

$$\sum_{k=1}^{m} \lambda_{ki}\mu_{ki} = \sum_{k\neq j} \lambda_{ki}\mu_{ki} \leq \sum_{k\neq j} \lambda_{ki} < 1, \quad \text{if } \mu_{ji} = 0,$$

$$p_j = \frac{\beta\delta_j n_j + \beta \sum_{i\neq j} \delta_i n_i \lambda_{ji}\mu_{ji}}{\theta\delta_j n_j} \geq p^* + \frac{\beta\delta_i n_i \lambda_{ji}\mu_{ji}}{\theta\delta_j n_j} > p^*, \quad \text{if } \mu_{ji} = 1.$$

That is, it either violates the full utilization constraint for type-i organs or the optimality condition of $P = P^*$ in Theorem 1. Therefore, we must have $\lambda_{ji} = 0$ for $i \neq j$ in an optimal allocation policy. This proves the first part of the lemma.

Next, consider a fixed i with $\mu_{ij} = 0$ for all $j \neq i$. If $\lambda_{ij} > 0$ for some $j \neq i$, we must have

$$\sum_{k=1}^{m} \lambda_{kj}\mu_{kj} = \sum_{k\neq i} \lambda_{kj}\mu_{kj} \leq \sum_{k\neq i} \lambda_{kj} < 1.$$

This violates the full utilization constraint for type-j organs. Accordingly, we must have $\lambda_{ij} = 0$ if $j \neq i$ in an optimal policy. If $\lambda_{ji} > 0$ for some $j \neq i$, then $\lambda_{ii} < 1$ since $\sum_{j=1}^{m} \lambda_{ji} = 1$, and we must have $p_i < p^*$, as

$$p_i = \frac{\beta\delta_i n_i \lambda_{ii} + \beta \sum_{j\neq i} \delta_j n_j \lambda_{ij}\mu_{ij}}{\theta\delta_i n_i} = \frac{\beta\delta_i n_i \lambda_{ii}}{\theta\delta_i n_i} < \frac{\beta}{\theta} = p^*.$$

This violates the optimality condition $P = P^*$ in Theorem 1. Thus, in an optimal allocation policy, we must have $\lambda_{ji} = 0$ if $j \neq i$, and it immediately follows that $\lambda_{ii} = 1$. This completes the proof.

Proofs of Theorem 3

To prove the first part of Theorem 3, suppose that for any sequence of distinct $i_1, i_2, \dots, i_\ell$ with $\ell \geq 2$, the given matching technology μ satisfies $\mu(i_1, i_\ell)\mu(i_\ell, i_{\ell-1}) \cdots \mu(i_3, i_2)\mu(i_2, i_1) = 0$, and Λ is an optimal policy with $\lambda_{j_2 j_1} > 0$ for some $j_2 \neq j_1$. It immediately follows that $\mu(j_2, j_1) = 1$ since otherwise, the full utilization constraint of type-j_1 organs is violated. Additionally, by the assumption of $\mu(j_1, j_2)\mu(j_2, j_1) = 0$, we must have $\mu(j_1, j_2) = 0$.

If $\lambda_{kj_2} = 0$ for all $k \neq j_2$, we have $\lambda_{j_2 j_2} = 1$ since $\sum_{k=1}^m \lambda_{kj_2} = 1$ and, therefore,

$$\begin{aligned} p_{j_2} &= \frac{\beta\delta_{j_2} n_{j_2} \lambda_{j_2 j_2} + \beta \sum_{k \neq j_2} \delta_k n_k \lambda_{j_2 k} \mu(j_2, k)}{\theta \delta_{j_2} n_{j_2}} \\ &\geq \frac{\beta\delta_{j_2} n_{j_2} + \beta\delta_{j_1} n_{j_1} \lambda_{j_2 j_1} \mu(j_2, j_1)}{\theta \delta_{j_2} n_{j_2}} > \frac{\beta}{\theta} = p^*, \end{aligned}$$

which violates the optimality condition of $P = P^*$ in Theorem 1. Similarly, if $\lambda_{j_1 k} = 0$ for all $k \neq j_1$, we have

$$p_{j_1} = \frac{\beta\delta_{j_1} n_{j_1} \lambda_{j_1 j_1} + \beta \sum_{k \neq j_1} \delta_k n_k \lambda_{j_1 k} \mu(j_1, k)}{\theta \delta_{j_1} n_{j_1}} = \frac{\beta\delta_{j_1} n_{j_1} \lambda_{j_1 j_1}}{\theta \delta_{j_1} n_{j_1}} < \frac{\beta}{\theta} = p^*,$$

where the inequality follows from $\lambda_{j_1 j_1} < 1$ since $\lambda_{j_2 j_1} > 0$ and $\sum_{k=1}^m \lambda_{kj_1} = 1$. This again violates the optimality condition of $P = P^*$ in Theorem 1. Accordingly, we must have $\lambda_{j_3 j_2} > 0$ and $\lambda_{j_1 j_\ell} > 0$ for some $j_3 \neq j_2$ and $j_\ell \neq j_1$. It immediately follows that $\mu(j_3, j_2) = 1$ and $\mu(j_1, j_\ell) = 1$ by the full utilization constraints of type-j_2 or type-j_ℓ organs, which also implies $j_3 \neq j_1$ and $j_\ell \neq j_2$ since $\mu(j_1, j_2) = 0$. Moreover, according to our assumption, we must have $j_3 \neq j_\ell$ since otherwise, we have $\mu(j_1, j_3)\mu(j_3, j_2)\mu(j_2, j_1) = 1$. Thus $\{j_1, j_2, j_3, j_\ell\}$ are all distinct from one another.

With similar arguments as above, we must have $\lambda_{j_4 j_3} > 0$ and $\lambda_{j_\ell j_{\ell-1}} > 0$ for some $j_4 \notin \{j_1, j_2, j_3, j_{\ell-1}, j_\ell\}$ and $j_{\ell-1} \notin \{j_1, j_2, j_3, j_4, j_\ell\}$. Otherwise, we have $P \neq P^*$, which violates the optimality condition of $P = P^*$ in Theorem 1. It immediately follows that $\mu(j_4, j_3) = 1$ and $\mu(j_\ell, j_{\ell-1}) = 1$ according to the full utilization constraints of type-j_3 or type-$j_{\ell-1}$ organs. Moreover, according to our assumption, we must have $j_4 \neq j_{\ell-1}$ since otherwise, we have $\mu(j_1, j_\ell)\mu(j_\ell, j_4)\mu(j_4, j_3)\mu(j_3, j_2)\ \mu(j_2, j_1) = 1$. Therefore, $\{j_1, j_2, j_3, j_4, j_{\ell-1}, j_\ell\}$ are all distinct from one another. By repeating this process, we find that the optimal Λ requires more than m distinct blood types, which contradicts the fact that only m blood types exist. This completes the proof for the first part.

Next, suppose that there exists a sequence of distinct $i_1, i_2, \dots, i_\ell$ with $\ell \geq 2$ such that $\mu(i_1, i_\ell)\mu(i_\ell, i_{\ell-1}) \cdots \mu(i_3, i_2)\mu(i_2, i_1) = 1$. Without loss of generality, we can assume that $\min\{n_{i_1}, n_{i_2}, \dots, n_{i_\ell}\} = n_{i_1}$. We construct an allocation policy $\Lambda \neq I$ as follows. Pick any $\lambda_{i_2 i_1} = x \in [0, 1]$ and let

$$\lambda_{i_{k+1} i_k} = \frac{n_{i_1}}{n_{i_k}} x, \qquad \lambda_{i_k i_k} = 1 - \lambda_{i_{k+1} i_k} \quad \text{for any } k = 1, 2, \dots, \ell - 1,$$

$$\lambda_{i_1 i_\ell} = \frac{n_{i_1}}{n_{i_\ell}} x, \qquad \lambda_{i_\ell i_\ell} = 1 - \lambda_{i_1 i_\ell}.$$

In addition, let $\lambda_{ii} = 1$ for any $i \notin \{i_1, i_2, \dots, i_\ell\}$. We can check that the market clearing conditions and the full utilization constraints are all satisfied under this policy Λ with $P = P^*$. By Theorem 1, Λ is optimal, which, however, is not the XYZ-identical policy. Moreover, by varying the size of $x \in [0, 1]$, we have a continuum of optimal allocation policies that achieve $P = P^*$ and donation rate $F(\beta)$, which completes the proof.

Proof of Corollary 1

The first part can be proved by contradiction. Suppose that the stated condition is satisfied and that there is an optimal allocation policy for group i with $\lambda_{ii} < 1$, $\lambda_{ji} > 0$ or $\lambda_{ij} > 0$ for some $j \neq i$. Similar to the arguments in the first half of the proof of Theorem 3, we find that either case would require more than m distinct blood types, which contradicts the fact that there are only m blood types. To prove the second part, when $\mu(i, j_\ell) \cdots \mu(j_2, j_1)\mu(j_1, i) = 1$ for some $\ell \geq 1$, $j_k \neq i$, we can construct a continuum of optimal allocation policies for group i by transferring a positive measure of organs along the directed compatible cycle $i \to j_1 \to j_2 \to \cdots \to j_\ell \to i$ such that $\lambda_{ii} < 1$ in a similar manner as the second half of the proof of Theorem 3.

Proof of Corollary 2

The proof is immediate from Theorem 3 by noticing that the ABO-compatibility matrix $\tilde{\mu}$ is a 4×4 triangular matrix, and any triangular compatibility matrix satisfies the condition of acyclicity.

Proof of Proposition 2

We consider the following example with the full compatibility matching technology $\mu = \mu^*$. There are two blood types with an equal share of the population, i.e., $m = 2$ and $n_1 = n_2 = 1/2$. The optimal policy is determined by (3a)–(3b). Given any $0 < a < b < 1$, let F, q_1, and q_2 satisfy the following:

$$F(c) \begin{cases} = a & \text{for } c = \theta q_1 := \beta \dfrac{a}{a+b}, \\ \in \left(a, \dfrac{a+b}{2}\right) & \text{for } c = \theta p^* = \beta, \\ = b & \text{for } c = \theta q_2 := \beta\left(1 + \dfrac{a}{a+b}\right). \end{cases}$$

Basically, we construct F such that $1/F$ is not convex at θp^*. Here, we find that

$$\theta F(\theta q_1) n_1 q_1 + \theta F(\theta q_2) n_2 q_2 = \beta F(\theta q_1) n_1 + \beta F(\theta q_2) n_2,$$

$$F\big(\theta p^*\big) n_1 + F\big(\theta p^*\big) n_2 < \frac{a+b}{2} = F(\theta q_1) n_1 + F(\theta q_2) n_2,$$

where the first equation shows that $p_1 = q_1$, $p_2 = q_2$ satisfies the market clearing condition (3b), and the second inequality indicates that the aggregate donation rate is higher under $p_1 = q_1$, $p_2 = q_2$ than under $p_1 = p_2 = p^*$.

Proof of Proposition 3

With the relative priority, the social planner's optimization problem becomes

$$\max_{\Lambda\in[0,1]^{m\times m}, P\in[0,1]^m} \sum_{i=1}^{m} F(\theta t p_i) n_i, \tag{11a}$$

$$\text{subject to} \quad \theta\big[1 - t + tF(\theta t p_i)\big] n_i p_i = \beta \sum_{j=1}^{m} F(\theta t p_j) n_j \lambda_{ij} \mu_{ij}, \quad \forall i = 1, \dots, m; \tag{11b}$$

$$\sum_{i=1}^{m} \lambda_{ij} \mu_{ij} - 1 = 0, \quad \forall j = 1, \dots, m; \tag{11c}$$

Similar to the previous analysis, we start by considering a simplified optimization problem for (11a)–(11c) with $\mu = \mu^*$ as follows:

$$\max_{P\in[0,1]^m} \sum_{i=1}^{m} F(\theta t p_i) n_i, \tag{11a$'$}$$

$$\text{subject to} \quad \sum_{i=1}^{m} \theta\big[1 - t + tF(\theta t p_i)\big] n_i p_i - \beta \sum_{j=1}^{m} F(\theta t p_j) n_j = 0. \tag{11b$'$}$$

We first focus on the interior solutions and later show that boundary solutions cannot be optimal. The critical points are determined by the (interior) first-order conditions as

$$\theta t f(\theta t p_i) n_i + \theta t f(\theta t p_m) n_m \frac{\partial p_m}{\partial p_i} = 0, \quad \forall i \neq m,$$

$$\text{where } \frac{\partial p_m}{\partial p_i} = -\frac{n_i}{n_m} \frac{1 - t + tF(\theta t p_i) - (\beta - \theta t p_i) t f(\theta t p_i)}{1 - t + tF(\theta t p_m) - (\beta - \theta t p_m) t f(\theta t p_m)}.$$

Note that if $1/[F + (1-t)/t]$ is strictly convex, then $f(\theta t p)/[1 - t + tF(\theta t p) - (\beta - \theta t p) t f(\theta t p)]$ is strictly decreasing in p. Therefore, for the first-order conditions to hold, we need to have $p_1 = p_2 = \cdots = p_m \equiv \bar{p}$. The market clearing condition becomes $\theta[1 - t + tF(\theta t \bar{p})]\bar{p} - \beta F(\theta t \bar{p}) = 0$, which implies that $\bar{p}$ should satisfy $(1-t)\theta\bar{p} = (\beta - \theta t \bar{p}) F(\theta t \bar{p})$.

Let $\bar{P} := (\bar{p})_{\{i=1,\dots,m\}}$. We next check the second-order conditions at $P = \bar{P}$. For simpler notation, let

$$N(\bar{p}) := 2 t f(\theta t \bar{p}) - (\beta - \theta t \bar{p}) t f'(\theta t \bar{p}),$$

$$D(\bar{p}) := 1 - t + tF(\theta t \bar{p}) - (\beta - \theta t \bar{p}) t f(\theta t \bar{p}).$$

We obtain the following derivatives at $P=\bar{P}$:

$$\frac{\partial p_m}{\partial p_i}=-\frac{n_i}{n_m},\qquad \frac{\partial^2 p_m}{\partial p_i^2}=-\frac{n_i(n_i+n_m)}{n_m^2}\frac{\theta t N(\bar{p})}{D(\bar{p})},\qquad \frac{\partial^2 p_m}{\partial p_i\partial p_j}=-\frac{n_i n_j}{n_m^2}\frac{\theta t N(\bar{p})}{D(\bar{p})}.$$

Thus the Hessian at $P=\bar{P}$ is

$$H_{ii}=n_i(n_i+n_m)\frac{(\theta t)^2}{n_m}\left[f'(\theta t\bar{p})-\frac{f(\theta t\bar{p})N(\bar{p})}{D(\bar{p})}\right],$$
$$H_{ij}=n_i n_j\frac{(\theta t)^2}{n_m}\left[f'(\theta t\bar{p})-\frac{f(\theta t\bar{p})N(\bar{p})}{D(\bar{p})}\right],\quad \text{for } i\neq j,\, m.$$

By strict convexity of $1/[F+(1-t)/t]$, the Hessian is negative definite at $P=\bar{P}$. We can similarly check that boundary solutions are not possible. Accordingly, the global maximum is achieved at $P=\bar{P}$.

Next, since the optimization problem in (11a′)–(11b′) is a relaxed problem of (11a)–(11c) and $P=\bar{P}$ with $\Lambda^*=I$ is a feasible solution for (11a)–(11c), it provides an optimal solution for the original optimization problem. This completes the proof.

Proof of Proposition 4

We denote $\tilde{\alpha}:=\theta/(\theta_S+\tau\theta_M)$ and $\tilde{V}:=(\theta_S+\tau x\theta_M)/\theta$. The social planner's objective under different degrees of severity can be formulated as

$$\max_{\Lambda\in[0,1]^{m\times m},P\in[0,1]^m,\tau\in[0,1]}\sum_{i=1}^{m}F(\tilde{V}\theta p_i)n_i \tag{12a}$$

$$\text{subject to}\quad \theta F(\tilde{V}\theta p_i)n_i p_i=\tilde{\alpha}\beta\sum_{j=1}^{m}F(\tilde{V}\theta p_j)n_j\lambda_{ij}\mu_{ij},\quad \forall i=1,\dots,m; \tag{12b}$$

$$\sum_{i=1}^{m}\lambda_{ij}\mu_{ij}-1=0,\quad \forall j=1,\dots,m; \tag{12c}$$

We first note that if treating τ as a free parameter, the optimization problem defined by (12a)–(12c) is equivalent to the original problem specified in (1a)–(1c) through a change of notation (by considering the general scenario that $\alpha\neq 1$ and $V\neq 1$ in the original problem). Therefore, by Proposition 1, the optimal solution is $\lambda_{ii}^*=1$ and $\lambda_{ij}^*=0$ for $i\neq j$, with survival rates $p_1=p_2=\cdots=p_m=\tilde{\alpha}\beta/\theta=\beta/(\theta_S+\tau\theta_M)$. Next, if we consider $\tau\in[0,1]$ as a decision variable, the aggregate donation rate becomes

$$\sum_{i=1}^{m}\delta_i n_i=\sum_{i=1}^{m}F(c_i)n_i=F(\tilde{V}\tilde{\alpha}\beta)=F\left(\frac{\beta(\theta_S+\tau x\theta_M)}{\theta_S+\tau\theta_M}\right),$$

which is strictly decreasing in τ. Accordingly, the optimal solution is to set $\tau^*=0$, which gives $p_1=p_2=\cdots=p_m=\beta/\theta_S$. This completes the proof.

References

Abadie, Alberto and Sebastien Gay (2006), "The impact of presumed consent legislation on cadaveric organ donation: A cross-country study." *Journal of Health Economics*, 25, 599–620. [336]

Aladag, Murat, Ahmet Gurakar, Cemalettin Camci, Ye Yong, Harlan Wright, Bakr Nour, and Anthony Sebastian (2006), "Compatible ABO mismatch and liver transplantation: A single center's experience." *Experimental and Clinical Transplantation*, 4, 467–469. [333]

Andersson, Tommy and Jorgen Kratz (2020), "Kidney exchange over the blood group barrier." *The Review of Economic Studies*, 87, 1091–1133. [332]

Arshad, Adam, Benjamin Anderson, and Adnan Sharif (2019), "Comparison of organ donation and transplantation rates between opt-out and opt-in systems." *Kidney International*, 95, 1453–1460. [336]

Assfalg, Volker, Norbert Hüser, Marieke van Meel et al. (2016), "High-urgency kidney transplantation in the Eurotransplant kidney allocation system: Success or waste of organs? The Eurotransplant 15-year all-centre survey." *Nephrology Dialysis Transplantation*, 31, 1515–1522. [350]

Baughn, Daniel, James R. Rodrigue, and Danielle L. Cornell (2006), "Intention to register as organ donors: A survey of adolescents." *Progress in Transplantation*, 16, 260–267. [348]

Bergenfeldt, Henrik, Peter Höglund, Bodil Andersson, Göran Rådegran, Mattias Ohlsson, and Johan Nilsson (2015), "ABO-identical blood group matching has no survival benefit for AB heart transplant recipients." *The Annals of Thoracic Surgery*, 99, 762–768. [333]

Bernstein, Lenny (2016), "Private sector act to reduce organ transplant waiting list." *Washington Post*. [332]

Bertsimas, Dimitris, Vivek F. Farias, and Nikolaos Trichakis (2013), "Fairness, efficiency, and flexibility in organ allocation for kidney transplantation." *Operations Research*, 61, 73–87. [334, 335, 349, 350, 352]

Berzon, Corinne (2008), "Israel's 2008 organ transplant law: Continued ethical challenges to the priority points model." *Israel Journal of Health Policy Research*, 7, 11. [350]

Brynger, H., L. Rydberg, B. Samuelsson, I. Blohmé, A. Lindholm, and L. Sandberg (1983), "Renal transplantation across a blood group barrier—'A2' kidneys to 'O' recipients." In *Proceedings of the European Dialysis and Transplant Association*, volume 19, 427–431. [333]

Burkell, Jacquelyn A., Jennifer A. Chandler, and Sam D. Shemie (2013), "Attitudes toward reciprocity systems for organ donation and allocation for transplantation." *Journal of Health Politics, Policy and Law*, 38, 957–986. [332]

Cai, Junchao, Xin Qing, Guosheng Wu, Matthew Everly, Elaine Cheng, and Paul Terasaki (2015), "Is ABO-compatible but non-identical intestinal transplant comparable to ABO-identical transplant? An analysis of the UNOS registry." *SOJ Immunology*, 3, 1–9. [333]

Caplin, Andrew and Barry Nalebuff (1991a), "Aggregation and imperfect competition: On the existence of equilibrium." *Econometrica*, 59, 25–59. [338]

Caplin, Andrew and Barry Nalebuff (1991b), "Aggregation and social choice: A mean voter theorem." *Econometrica*, 59, 1–23. [338]

Carnethon, Mercedes R., Jia Pu, George Howard et al. (2017), "Cardiovascular health in African Americans: A scientific statement from the American Heart Association." *Circulation*, 136, 393–423. [351]

Chan, Alex (2020) "US organ donation policy." *JAMA*, 323, 278–279. [332]

Collins, Allan J., Robert N. Foley, Blanche Chavers et al. (2014), "US renal data system 2013 annual data report." *American Journal of Kidney Diseases*, 63, A7. [351]

Colvin, M., J. M. Smith, N. Hadley et al. (2019), "OPTN/SRTR 2017 annual data report: Heart." *American Journal of Transplantation*, 19, 323–403. [353]

Colvin-Adams, M., M. Valapour, M. Hertz et al. (2012), "Lung and heart allocation in the United States." *American Journal of Transplantation*, 12, 3213–3234. [350]

Dai, Tinglong, Ronghuo Zheng, and Katia Sycara (2020), "Jumping the line, charitably: Analysis and remedy of donor-priority rule." *Management Science*, 66, 622–641. [335, 348, 353]

Diamond, George A. and Sanjay Kaul (2009), "Cost, effectiveness, and cost-effectiveness." *Circulation: Cardiovascular Quality and Outcomes*, 2, 49–54. [353]

Dickerson, John and Tuomas Sandholm (2014), "Balancing efficiency and fairness in dynamic kidney exchange." In *Proceedings of the MAIHA Workshop at AAAI-14*. [350]

Duda, Lara (2005), "National organ allocation policy: The final rule." *Virtual Mentor*, 7, 604–607. [334]

Ewerhart, Christian (2013), "Regular type distributions in mechanism design and ρ-concavity." *Economic Theory*, 53, 591–603. [338]

Fabre, John, Paul Murphy, and Rafael Matesanz (2010), "Presumed consent: A distraction in the quest for increasing rates of organ donation." *BMJ*, 341. [336]

Glander, Petra, Klemens Budde, Danilo Schmidt, T. Florian Fuller, Markus Giessing, Hans-Hellmut Neumayer, and Lutz Liefeldt (2010), "The 'blood group O problem' in kidney transplantation—time to change?" *Nephrology Dialysis Transplantation*, 25, 1998–2004. [332, 340, 347]

Glazier, Alexandra and Thomas Mone (2019), "Success of opt-in organ donation policy in the United States." *JAMA*. [336]

Gray, Richard (2013), "Registered organ donors could be given priority for transplants." *The Telegraph*. [332]

Hawley, Zackary, Danyang Li, Kurt Schnier, and Nicole Turgeon (2018), "Can we increase organ donation by reducing the disincentives? An experimental analysis." *Economics & Human Biology*, 29, 128–137. [348]

Heo, Eun Jeong, Sunghoon Hong, and Youngsub Chun (2021), "Kidney exchange with immunosuppressants." *Economic Theory*, 72, 1–19. [332]

Herr, Annika and Hans-Theo Normann (2016), "Organ donation in the lab: Preferences and votes on the priority rule." *Journal of Economic Behavior & Organization*, 131, 139–149. [335]

Iyer, T. K. K. (1987), "Kidneys for transplant—"opting out" law in Singapore." *Forensic Science International*, 35, 131–140. [332]

Jawitz, Oliver K., Nicole G. Jawitz, David D. Yuh, and Pramod Bonde (2013), "Impact of ABO compatibility on outcomes after heart transplantation in a national cohort during the past decade." *The Journal of Thoracic and Cardiovascular Surgery*, 146, 1239–1246. [333]

Johnson, Eric J. and Daniel Goldstein (2003), "Do defaults save lives?" *Science*, 302, 1338–1339. [336]

Kessler, Judd B. and Alvin E. Roth (2012), "Organ allocation policy and the decision to donate." *American Economic Review*, 102, 2018–2047. [332, 335, 337, 348]

Kessler, Judd B. and Alvin E. Roth (2014), "Loopholes undermine donation: An experiment motivated by an organ donation priority loophole in Israel." *Journal of Public Economics*, 114, 19–28. [335]

Kim, Jaehong, Mengling Li, and Menghan Xu (2021), "Organ donation with vouchers." *Journal of Economic Theory*, 191, 105159. [335]

Kominers, Scott Duke, Parag A. Pathak, Tayfun Sönmez, and M. Utku Ünver (2020), "Paying it backward and forward: Expanding access to convalescent plasma therapy through market design." NBER Working Paper No. 27143. [356]

Kong, Nan, Andrew J. Schaefer, Brady Hunsaker, and Mark S. Roberts (2010), "Maximizing the efficiency of the U.S. liver allocation system through region design." *Management Science*, 56, 2111–2122. [335, 349, 350]

Koukoutsis, Ilias, Riccardo Bellagamba, Appou Tamijmarane et al. (2007), "Outcomes after identical and compatible orthotopic liver transplantation for fulminant hepatic failure: A single center experience in UK." *Transplant International*, 20, 659–665. [333]

Kratz, Jorgen (2019), "Triage in kidney exchange." Discussion Papers 19/04, University of York. [350]

Lacetera, Nicola, Mario Macis, and Robert Slonim (2013), "Economic rewards to motivate blood donations." *Science*, 340, 927–928. [356]

Lai, Jennifer C. and John P. Roberts (2016), "ABO-nonidentical liver transplantation in the United States." *American Journal of Transplantation*, 16, 2430–2436. [333, 334, 335]

Lavee, Jacob, Tamar Ashkenazi, Gabriel Gurman, and David Steinberg (2010), "A new law for allocation of donor organs in Israel." *The Lancet*, 375, 1131–1133. [332, 349, 350, 351]

Li, Danyang, Zackary Hawley, and Kurt Schnier (2013), "Increasing organ donation via changes in the default choice or allocation rule." *Journal of Health Economics*, 32, 1117–1129. [335]

López, Jorge S., Maria Soria-Oliver, Begoña Aramayona, Rubén García-Sánchez, José M. Martínez, and María J. Martín (2018), "An integrated psychosocial model of relatives' decision about deceased organ donation: Joining pieces of the puzzle." *Frontiers in Psychology*, 9. [348]

Morath, Christian, Martin Zeier, Bernd Döhler, Gerhard Opelz, and Caner Süsal (2017), "ABO-incompatible kidney transplantation." *Frontiers in Immunology*, 8, 234. [339]

Myerson, Roger B. (1981), "Optimal auction design." *Mathematics of Operations Research*, 6, 58–73. [338]

Myerson, Roger B. and Mark A. Satterthwaite (1983), "Efficient mechanisms for bilateral trading." *Journal of Economic Theory*, 29, 265–281. [338]

Nelson, Paul W., Charles F. Shield, Nicolas A. Muruve, Daniel Murillo, Bradley A. Warady, Mark I. Aeder, and Christopher F. Bryan (2002), "Increased access to transplantation for blood group B cadaveric waiting list candidates by using A2 kidneys: Time for a new national system?" *American Journal of Transplantation*, 2, 94–99. [333]

Park, M. S., K. W. Lee, T. You et al. (2013), "The "ABO cross-transplantation problem" in liver transplantation in Korea." *Transplantation Proceedings*, 45, 2878–2879. [333]

Roth, Alvin E. (2018), "Marketplaces, markets, and market design." *American Economic Review*, 108, 1609–1658. [356]

Roth, Alvin E., Tayfun Sönmez, and M. Utku Ünver (2004), "Kidney exchange." *The Quarterly Journal of Economics*, 119, 457–488. [332, 335, 339]

Roth, Alvin E., Tayfun Sönmez, and M. Utku Ünver (2005), "A kidney exchange clearinghouse in New England." *American Economic Review*, 95, 376–380. [332, 335, 339]

Roth, Alvin E., Tayfun Sönmez, and M. Utku Ünver (2007), "Efficient kidney exchange: Coincidence of wants in markets with compatibility-based preferences." *American Economic Review*, 97, 828–851. [332, 335, 339]

Ruth, R. Jean, Leon Wyszewianski, and Gary Herline (1985), "Kidney transplantation: A simulation model for examining demand and supply." *Management Science*, 31, 515–526. [335]

ScienceDaily (2019), "New research offers solution to reduce organ shortage crisis: Two-part method could increase organ availability by 13% and add 18 years to patient life expectancy." Institute for Operations Research and the Management Sciences. [354]

Sharif, Adnan (2018), "Presumed consent will not automatically lead to increased organ donation." *Kidney International*, 94, 249–251. [336]

Sönmez, Tayfun, M. Utku Ünver, and M. Bumin Yenmez (2020), "Incentivized kidney exchange." *American Economic Review*, 110, 2198–2224. [334, 335, 340, 345, 352]

Sönmez, Tayfun, M. Utku Ünver, and Özgür Yilmaz (2018), "How (not) to integrate blood subtyping technology to kidney exchange." *Journal of Economic Theory*, 176, 193–231. [333, 335, 345]

Stewart, D. E., A. R. Wilk, A. E. Toll et al. (2018), "Measuring and monitoring equity in access to deceased donor kidney transplantation." *American Journal of Transplantation*, 18, 1924–1935. [334, 352]

Stoler, Avraham, Judd B. Kessler, Tamar Ashkenazi, Alvin E. Roth, and Jacob Lavee (2017), "Incentivizing organ donor registrations with organ allocation priority." *Health Economics*, 26, 500–510. [335]

Storry, Jill R., Frederik Banch Clausen, Lilian Castilho et al. (2016), "International society of blood transfusion working party on red cell immunogenetics and terminology: Report of the Seoul and London meetings." *ISBT Science Series*, 11, 118–122. [332]

Su, Xuanming and Stefanos A. Zenios (2006), "Recipient choice can address the efficiency-equity trade-off in kidney transplantation: A mechanism design model." *Management Science*, 52, 1647–1660. [335]

Sun, Tianshu, Susan Feng Lu, and Ginger Zhe Jin (2016), "Solving shortage in a priceless market: Insights from blood donation." *Journal of Health Economics*, 48, 149–165. [356]

Taghavi, Sharven, Senthil N. Jayarajan, Yuka Furuya et al. (2014), "Single-lung transplantation with ABO-compatible donors results in excellent outcomes." *The Journal of Heart and Lung Transplantation*, 33, 822–828. [333]

Yu, Ji Hyun, Byung Ha Chung, Chul Woo Yang et al. (2017), "Impact of ABO incompatible kidney transplantation on living donor transplantation." *PLOS ONE*, 12, 1–17. [339]

Zuniga-Fajuri, Alejandra (2015), "Increasing organ donation by presumed consent and allocation priority: Chile." *Bulletin of the World Health Organization*, 93, 199–202. [332]

Co-editor Federico Echenique handled this manuscript.

Manuscript received 10 October, 2019; final version accepted 30 December, 2020; available online 7 January, 2021.