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Monotone contracts

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We develop a framework for deriving dynamic monotonicity results in long-term stochastic contracting problems with symmetric information. Specifically, we construct a notion of *concave separable activity* that encompasses many prevalent contractual components (e.g., wage, effort, level of production, etc.). We then provide a tight condition under which such activities manifest a form of seniority in *every* contracting problem in which they are present: any change that occurs in the level of the activity over time favors the agent. Our work unifies and significantly generalizes many existing results and can also be used to establish monotonicity results in other settings of interest.

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1. Introduction

Interactions between a principal and an agent often take place in complex and dynamic environments: seasonality and random shocks affect demands, workers accumulate skills, and business opportunities arrive and disappear at random. Contracts are used to specify the obligations of each party and, in particular, how these obligations should respond to changes in the contracting environment. Faithfully describing realistic contracting environments and deriving optimal contracts therein is eminently difficult. A standard approach is to fully characterize the optimal contracts in a "stylized" contracting problem that captures the essential features of the original setting. This approach has been used to derive valuable insights into qualitative features of real-life phenomena in a wide array of economic settings. For example, Milton and Holmström (1982), Holmström (1983), and Postel-Vinay and Robin (2002a,b) study competitive labor markets and derive a downward wage rigidity property; Krueger and Uhlig (2006) study competitive insurance markets and show that changes in the terms of insurance contracts always favor the insured; Albuquerque and Hopenhayn (2004) study

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entrepreneur financing and find that an entrepreneur's access to capital increases over time; Fudenberg and Rayo (2019) and Bird and Frug (2021) study effort dynamics and show that a worker's effort decreases over time; and Forand and Zápal (2020) study dynamic project selection and find that project selection criteria change in the agent's favor as time goes by.

In this paper, we take an alternative approach to studying the qualitative features of desirable contracts that circumvents the need to fully characterize optimal contracts. We develop a conceptual framework and use it to establish a general dynamic monotonicity result that unifies and remarkably generalizes most of the monotonicity results developed in the above-mentioned papers.[1] Furthermore, the framework we develop is significantly more general not only with respect to the potential complexity of the environment in which the interaction occurs, but also in terms of the structure of the contractual components encompassed by our result. Thus, in addition to offering a generalization and unification of several seemingly unrelated results in the existing literature, our framework paves the way for deriving related results for new, more intricate, contractual components in richer settings.

The key restrictions we impose are that information is symmetric and that only the principal has commitment power. We model a contracting problem as a stochastic game—in each period, the players play a randomly drawn stage game, observe its outcome, and collect payoffs—in which the principal commits to a long-term strategy and the agent re-optimizes his play at every history. As the calendar time, previous stage games, and players' past moves may affect the games the players will play in the future, the class of contracting problems we consider is fairly general. It accommodates a wide variety of settings, including, but not limited to, settings where the agent's cost of effort depends on past events, there is seasonality in demand, there is uncertainty about the principal's ability to provide compensation in the future, there are long-term (or storable) investment opportunities, or there are research and development processes that may change future production methods and costs.

The main notion we develop is that of *activity*. Broadly speaking, an activity is a recurring component of the interaction for which the players have monotone and opposite preferences. Examples include worker's daily effort, monthly wage, production volume, level of authority of a bureaucrat or a unit in an organization, financing to an entrepreneur, quality of supplied products, and more. Some activities are unilaterally controlled by one of the players (e.g., worker's effort) while others are jointly controlled by both players (e.g., a situation where output depends on the agent's effort as well as the amount of resources provided by the principal). Our analysis will show that the class of jointly controlled activities gives rise to a strategic aspect that is absent in the case of the unilaterally controlled activities.[2]

Two characteristics of activities will play an important role in our analysis: concavity and separability. An activity is concave if the principal's activity-related payoff is a

[1]The only exceptions are Albuquerque and Hopenhayn (2004) and Fudenberg and Rayo (2019). In Section 7, we discuss the connection between their monotonicity results and our result.

[2]The activities in the above papers, with the exception of Forand and Zápal (2020), are unilaterally controlled activities.

strictly concave function of the agent's activity-related payoff. An activity is separable if changes *within* the activity do not affect the distribution of games that the players will play in the future. Note that the separability requirement allows events unrelated to the activity to affect the availability of the activity in the future. Hence, situations where the availability of an activity is endogenous and/or path-dependent fall within the scope of our analysis.

Our main result identifies a property of concave separable activities that guarantees that, in optimum, and irrespective of the exact details of the contracting problem, the level of the activity changes over time only in the direction that favors the agent (Theorem 1). Furthermore, our result is tight in the sense that under mild technical requirements, for every concave separable activity that fails to satisfy the property, there exist contracting problems in which, as time goes by, the level of the activity changes in the opposite direction (Proposition 1).

The essence of the mechanism behind our result can be described as "incentive-constrained smoothing." Intuitively, consider an incentive-compatible contract and, for each concave separable activity, consider the joint play induced by the contract in all components of the interaction except for that specific activity. Mechanically, this play can be thought of as imposing incentive-compatibility constraints on how the designated activity can be played over time. The concavity of the activity implies that smoothing out fluctuations in the activity-play over time is profitable. However, in general, such smoothing may destabilize incentive compatibility by creating new (within-activity) deviation opportunities. In this light, an additional contribution of this paper is in showing how a relatively standard intertemporal smoothing argument can be extended to much richer models of dynamic contracting and, in particular, in identifying tight limits imposed by short-run activity-specific strategic incentives.

The rest of the paper is organized as follows. In Sections 2 and 3, we define the contracting environment and develop the notion of activity. Section 4 reports the main result of the paper (Theorem 1). Section 5 offers a (partial) converse of Theorem 1, and Section 6 is devoted to some robustness results. We review the related literature in Section 7 and offer concluding remarks in Section 8. All proofs are relegated to Appendix A.

2. Contracting environment

We consider dynamic interactions between a principal and an agent that can be represented as follows. In each period $t \in \{1, 2, \dots, T\}$, where $T \leq \infty$, the players play a randomly drawn (strategic-form) stage game $G(t)$, observe the outcome of $G(t)$, and receive payoffs. The game in period t is drawn from a commonly known history-dependent distribution $f(h_t)$, where h_t lists the realized (stage) games in all previous periods $(G(1), \dots, G(t-1))$ and the players' actions in those games. We impose the following measurability constraint on $f(\cdot)$: for any h_t, $s > 0$, and strategy profiles in periods $t, \dots, t+s-1$, the distribution of the periodic game in period $t+s$ is well defined. We refer to a stochastic process $f(\cdot)$ as a *contracting problem*.

We assume that the only asymmetry between the players is in their ability to commit. While the principal enjoys full commitment power, the agent cannot commit to a course of play. Thus, it is convenient to think of the principal's problem at the beginning of the interaction in terms of committing to a contract that the agent would find optimal to follow at any history. Formally, a *contract* specifies, for every finite history h_t, an action profile in every stage game that can be realized at[3] h_t. A contract is *incentive compatible* if, for every pair $(h_t, G(t))$, the agent's continuation strategy (from period t onward) specified by the contract is a best response to the principal's continuation strategy specified by the contract. We assume that the players maximize (discounted) expected utility and use the same positive discount factor δ. Hence, the principal's objective is to select an incentive-compatible contract that maximizes his expected discounted value at time zero.

3. Concave separable activities

In this section, we develop the main concept of the paper, which can be used to draw economically relevant conclusions in complex or even partially specified contracting problems (we illustrate some applications in Section 4.2). We start by defining an activity and presenting some examples, after which we define a notion of separability with respect to a contracting problem.

3.1 *Activities*

We denote a strategic-form game between a principal and an agent by $G = \langle S_p, S_a; u_p, u_a\rangle$, where S_i and $u_i : S_p \times S_a \to \mathbb{R}$ are, respectively, the action space and the von Neumann–Morgenstern (vNM) utility function of player $i \in \{a, p\}$.

Definition 1 (Activity). An activity is a pair (G, Σ), where $G = \langle S_p, S_a; u_p, u_a\rangle$ and $\Sigma \subseteq S_p \times S_a$ such that there exist a real-valued nondegenerate interval L and a bijection $\eta : L \to \Sigma$ for which the functions $u_p \circ \eta : L \to \mathbb{R}$ and $u_a \circ \eta : L \to \mathbb{R}$ are continuous and strictly monotone in opposite directions.

To define when an activity is available in a given period, we first define the addition operator $\oplus$ for games. Roughly speaking, the game $G^1 \oplus G^2$ is played when the players play the games G^1 and G^2 simultaneously "side by side" and their payoffs are added.

Definition 2 (The $\oplus$ Operator). Given a pair of strategic-form games between a principal and an agent, $G^1 = \langle S^1_p, S^1_a; u^1_p, u^1_a\rangle$ and $G^2 = \langle S^2_p, S^2_a; u^2_p, u^2_a\rangle$. The strategic-form game $G^1 \oplus G^2$ is defined as $\langle S_p, S_a; u_p, u_a\rangle$, where, for $i \in \{p, a\}$, $S_i := S^1_i \times S^2_i$, and, for all action profiles $((s^1_p, s^2_p), (s^1_a, s^2_a)) \in S_p \times S_a$, $u_i : S_p \times S_a \to \mathbb{R}$ satisfies $u_i((s^1_p, s^2_p), (s^1_a, s^2_a)) = u^1_i(s^1_p, s^1_a) + u^2_i(s^2_p, s^2_a)$.

Using this operator, we can now state the following definition.

[3]We assume that the contract is deterministic in the sense that it assigns to each history and realized game a deterministic action profile. Nevertheless, since the action space in $G(t)$ can be uncountable, our framework can accommodate "mixing" by the principal by considering $G(t)$ to be a game where the principal's strategies are lotteries over his actions in a more basic strategic-form game.

Definition 3 (Activity in a Contracting Problem). The activity (G, Σ) is available in period t if the realized game in t, $G(t)$, can be written as $G' \oplus G$ for some game G'.

It is useful to emphasize certain aspects of the above definitions. In essence, an activity is a part of the interaction over which the players have opposite preferences, and that can be measured in terms of linearly ordered levels (the interval L) and adjusted continuously. Note that the definition of activity is agnostic about the choice of L. A particularly useful candidate, especially in general arguments, is $L = u_a(\Sigma)$, i.e., measure the activity in terms of the agent's activity-related payoff; however, when specific applications are considered, alternative units (e.g., production volume) may be natural and convenient. Furthermore, note that the above definition of an activity is silent about the player's preference over outcomes of G outside of Σ. Thus, our results will apply only to the play of action profiles in Σ.

Additionally, defining when an activity is part of an interaction via the operator $\oplus$ imposes important restrictions on the relation between the activity and the rest of the interaction in periods when the activity is available. First, the players' payoffs in the activity game G are additively separable from other payoffs obtained in the same period. Second, the action space in periods when the activity is available must have a cross-product structure. This rules out the possibility that activity-related actions impose restrictions on the players' possible actions outside of G and vice versa.

Since there is a bijection between Σ and an interval L, and the players' preferences are strictly monotone in opposite directions on L, an activity can be thought of as a means of transferring utility between the players. Given an activity (G, Σ), let $U_p(u)$ denote the principal's payoff from the (unique) action profile in Σ for which the agent's payoff is u. Formally, $U_p : u_a(\Sigma) \to \mathbb{R}$ is defined as

$$U_p(u) = u_p \circ (u_a|_\Sigma)^{-1}(u),$$

where $(u_a|_\Sigma)$ is u_a restricted to the domain Σ.

Definition 4 (Concavity). An activity (G, Σ) is concave if $U_p(\cdot)$ is strictly concave.

In other words, an activity is concave if the principal's marginal loss due to an increase in the agent's utility from the activity is strictly increasing.[4]

3.2 *Examples of activities*

In this section, we illustrate how certain components of different economic interactions can be formulated as activities. The usefulness of this stems from the fact that typically identifying an activity within an interaction is much easier than solving for the optimal contract therein, and our analysis in the following sections will allow us to draw important and general qualitative conclusions (e.g., downward wage rigidity) that do not depend on the exact details of the interaction. We provide two examples. The first is the wage paid by the principal, and the second is the agent's labor provision as part of a jointly controlled production process.

[4]In Corollary 1 below, we will provide a weaker version of our result for the case where $U_p(\cdot)$ is weakly concave.

Wage Suppose that when $w \geq 0$ is the agent's wage (in a given period), the principal's and agent's payoffs from wage are $-w$ and $g(w)$, respectively, for some increasing function $g(\cdot)$. The formulation of wage as an activity is $(G_{\text{wage}}, \Sigma_{\text{wage}})$, where

$$G_{\text{wage}} = \langle S_p = \mathbb{R}_+, S_a = \{a_\emptyset\}; u_p(w, a_\emptyset) = -w, u_a(w, a_\emptyset) = g(w)\rangle,$$

and Σ_{wage}, in this case, is equal to the set of all possible outcomes of G_{wage}. The interpretation of the players' action spaces in the activity game G_{wage} is that the principal unilaterally controls the wage paid to the agent. If the function $g(\cdot)$ is strictly concave, then this activity is concave.[5] A crucial aspect of the activity, which is necessary for our main result, is that the activity-related payoff—e.g., utility from wage—is independent of other actions—e.g., effort—in that period. In Section 4.3, we illustrate an example where such dependence occurs and show how partial conclusions, similar to those in our main result, can be inferred in such cases.

Next, we illustrate a jointly controlled activity. An important feature of such activities is that Σ is typically a proper subset of the action space of the activity game. We adapt an idea that appeared in Albuquerque and Hopenhayn (2004).

Labor in joint production Consider a production process where, first, the principal provides capital k, after which the agent can either supply labor l or reallocate the capital provided by the principal for his private benefit. Moreover, suppose that the output is given by $z(l, k) = \min\{l, k\}$ and that the value of output z for the principal is $\pi(z)$. In addition, suppose that the principal's marginal cost of capital, the agent's marginal cost of labor, and the agent's marginal utility from capital used for private benefit are all 1. This activity can be represented as (G_z, Σ_z), where

$$G_z = \langle S_p = \mathbb{R}_+, S_a = \mathbb{R}_+ \cup \{\text{steal}\}; u_p(k, s_a), u_a(k, s_a)\rangle,$$

where

$$u_p(k, s_a) = \begin{cases} -k & \text{if } s_a = \text{steal} \\ \pi\big(z(s_a, k)\big) - k & \text{if } s_a \neq \text{steal}, \end{cases} \qquad u_a(k, s_a) = \begin{cases} k & \text{if } s_a = \text{steal} \\ -s_a & \text{if } s_a \neq \text{steal}, \end{cases}$$

and

$$\Sigma_z = \{(s_p, s_a) \in \mathbb{R}_+^2 : s_p = s_a\}.$$

Note that, in this example, there are two types of action profiles that do not belong to Σ_z: those where the agent steals the capital and those where the input bundle is inefficient ($k \neq l$). The activity is concave if $\pi(\cdot)$ is strictly concave.

[5]This activity is an example from a class of activities that are unilaterally controlled by one of the players. A natural activity that is unilaterally controlled by the agent is his effort. A possible representation of this activity is $G_{\text{effort}} = \langle S_p = \{p_\emptyset\}, S_a = \mathbb{R}_+; u_p(p_\emptyset, e) = \pi(e), u_a(p_\emptyset, e) = -c(e)\rangle$, where $\pi(\cdot)$ and $c(\cdot)$ are, respectively, the principal's profit and the agent's cost of effort, and Σ_{effort} is again the set of all possible outcomes of G_{effort}.

3.3 *Separability*

Up until now, we have only considered the relation between an activity and other parts of the interaction within a stage game. Next, we define a notion of separability that places a dynamic restriction on the contracting problem. Intuitively, this notion imposes that changes in the activity level do not affect the distribution of periodic games in the future. Whether or not a given activity satisfies this notion depends on the context of the contracting problem. For example, in many cases (e.g., low-level employees), it is reasonable to assume that changes in the agent's effort will have no impact on his prospects in the labor market. However, in other cases (e.g., chief executive officers), the firm's profitability—which is impacted by the agent's effort—acts as a publicly observed signal about the agent's skill (i.e., "career concerns" à la Holmström (1999)), and so changing the agent's effort can impact his prospects in the labor market. Thus, while in the former case the agent's effort may satisfy the dynamic separability constraint, in the latter case it will not.

Definition 5 (Separability With Respect to a Contracting Problem). An activity (G, Σ) is separable with respect to $f(\cdot)$ if, for any pair of same-length histories h_t, $\hat{h}_t$ along which the sequence of realized games $\{G(\tau)\}_{\tau=1}^{t-1}$ is identical, the statement below holds:

If there is $s < t$ such that

(i) $G(s) = G' \oplus G$ for some G' and

(ii) the sequence of outcomes of $(G(1), \dots, G(s-1), G', G(s+1), \dots, G(t-1))$ is identical under h_t and $\hat{h}_t$,

then $f(h_t) = f(\hat{h}_t)$.

Notice that our notion of separability is inherently asymmetric. It only requires that changes in the action profile in the activity game G not affect the stochastic process according to which future games are drawn. By contrast, changes that affect the future distribution of games through actions outside the activity game G are entirely legitimate. Thus, this notion of separability does not rule out situations where the availability of activities is endogenously controlled by the players.

The object of interest in this paper is concave activities that are separable with respect to the contracting problem under consideration. We refer to such activities as *concave separable activities*.

4. Monotonicity of concave separable activities

We first illustrate the workings of the main result with a simple example. Consider a four-period contracting problem in which the agent exerts effort in periods 1 and 3, and the principal compensates him in periods 2 and 4. In the present illustration, we will mainly focus on the compensation component of the interaction. Therefore, we will simplify the part related to effort as much as possible by assuming that the principal's and agent's possible actions in periods $t \in \{1, 3\}$ are, respectively, $S_p(t) = \{p_\emptyset\}$

and $S_a(t) = \{0, x_t\}$, where $x_t > 0$ measures the agent's cost of effort required in period t. Moreover, we restrict attention to cases where it is strictly optimal for the principal to incentivize positive effort in both $t = 1$ and $t = 3$, and assume that players do not discount the future.

Assume first that compensation is provided via $(G_{\text{wage}}, \Sigma_{\text{wage}})$ (the periodic wage activity that is specified in the first example in Section 3.2). If $(G_{\text{wage}}, \Sigma_{\text{wage}})$ is concave, it is immediate that the cheapest way to compensate the agent for his total effort $(x_1 + x_3)$ is to pay him a wage worth $(x_1 + x_3)/2$ utils in each of periods 2 and 4. When $x_1 \geq x_3$, this form of compensation satisfies all incentive-compatibility constraints and is thus uniquely optimal. If, on the other hand, $x_1 < x_3$, this form of compensation is not incentive compatible in period[6] 3. To restore incentive compatibility, some of the compensation must be postponed from period 2 to 4, which would lead to an increasing compensation over time. A decreasing compensation plan (where the wage paid in period 4 is strictly lower than that paid in period 2), however, is suboptimal for all x_1 and x_3.

An alternative way to frame the argument (which will make transparent the role of Property 1 that we define below) is as follows. Suppose that a decreasing compensation plan is proposed. Reducing the compensation in period 2 by a small amount and increasing it in period 4 so that the agent's total utility from wage remains the same decreases the overall cost of compensation (this is a basic smoothing argument). If the original decreasing compensation plan was part of an incentive-compatible contract, then, a fortiori, so is the modified compensation plan, because deferring compensation only relaxes some of the incentive-compatibility constraints of the forward-looking agent.

The above argument relies on the implicit assumption that changing the level of compensation in a given period does not create new deviation opportunities for the agent. Assume now that compensation in our example is provided via a more complex concave activity for which the agent's deviation payoff does vary with the level of compensation. To fix ideas, suppose that our agent is a civil servant who is compensated by being granted a higher level of authority. To keep the illustration concise, we will assume that, given any level of authority, the civil servant decides whether or not to *abuse* his authority, and that abusing authority provides him with the highest payoff (within the activity game) for every authority level granted by the principal. Denote this activity by $(G_{\text{authority}}, \Sigma_{\text{authority}})$, where

$$G_{\text{authority}} = \left\langle S_p = [\underline{y}, \overline{y}], S_a = \{\text{use}, \text{abuse}\}; u_p(s_p, s_a), u_a(s_p, s_a)\right\rangle$$

and

$$\Sigma_{\text{authority}} = [\underline{y}, \overline{y}] \times \{\text{use}\}.$$

As before, start with a decreasing and incentive-compatible compensation plan (now via $(G_{\text{authority}}, \Sigma_{\text{authority}})$) and consider a smoothing modification that reduces the

[6]Recall that compensation is nonnegative and so the only threat available for the principal from period 3 onward is to provide a compensation of zero in period 4. Since $x_3 > x_1$, the compensation for the average effort is insufficient to incentivize the agent to exert the necessary effort in period 3.

principal's cost of compensation while keeping the agent's total utility from compensation fixed. What is now unclear is whether this modification results in an incentive-compatible contract. To understand when this is indeed the case, we now analyze the agent's considerations in periods 2 and 4.

The smoothing modification decreases the civil servant's periodic payoff from following the contract in period 2. However, by construction (in particular, since period 2 is the first period involved in the modification), the civil servant's *continuation payoff* from following the contract does not change in period 2. Hence, to guarantee that the smoothing modification did not create opportunities for profitable deviations in that period, it must be the case that the civil servant's payoff from abusing authority in period 2 did not increase. A sufficient condition for this is that $u_a(y, \text{abuse}) \geq u_a(y', \text{abuse})$ for any y, y' such that $u_a(y, \text{use}) > u_a(y', \text{use})$.

On the other hand, since period 4 is the last period of the smoothing modification, the civil servant's periodic payoff and his continuation payoff in period 4 increase by the same amount. Hence, to guarantee that the smoothing modification did not generate opportunities for profitable deviations in period 4, the change in the civil servant's payoff from abusing authority in period 4 must be bounded from above by the increase in his payoff from following the contract in that period. A sufficient condition for this is that $u_a(y', \text{abuse}) - u_a(y, \text{abuse}) \leq u_a(y', \text{use}) - u_a(y, \text{use})$ for any y, y' such that $u_a(y', \text{use}) > u_a(y, \text{use})$.

The above illustration contains a number of special features. One important such feature is that the distribution of periodic games did not depend on the players' past actions; that is, all activities were separable with respect to the contracting problem. If an activity is not separable with respect to a contracting problem, then by choosing the activity level, the principal attempts not only to maximize his activity-related payoff, but also to improve the distribution of future stage games. Clearly, in certain interactions, the latter motive may be the dominating one. For example, if the principal is evaluated for promotion based on the output on the last day of each month, he has an incentive to require exceptionally high effort on that day. Thus, separability of an activity with respect to the contracting problem is essential in order to obtain a general monotonicity result for that activity.

4.1 *Main result*

To state the key condition of our main result, we define the following activity-specific functions.[7] Given an activity (G, Σ), the function $\bar{U}_a : \Sigma \to \mathbb{R}$ maps every action profile $\sigma \in \Sigma$ to the agent's highest payoff in G provided that the principal's action is[8] σ_p (where σ_p is the principal's part of the action profile σ):

$$\bar{U}_a(\sigma) = \sup_{s_a \in S_a} u_a(\sigma_p, s_a).$$

[7]To simplify notation, we do not explicitly add (G, Σ) as an argument of these functions, but leave this dependence implicit.

[8]To ease notation, we define the mapping $\bar{U}_a$ on action profiles in Σ rather than on the principal's actions consistent with these profiles.

Next, given a pair of distinct action profiles $\sigma^1, \sigma^2 \in \Sigma$, we define the function

$$\phi(\sigma^1, \sigma^2) = \frac{\bar{U}_a(\sigma^1) - \bar{U}_a(\sigma^2)}{u_a(\sigma^1) - u_a(\sigma^2)}.$$

Note that since $\phi(\cdot, \cdot)$ is defined only for distinct action profiles and by the definition of activity $u_a(\sigma^1) \neq u_a(\sigma^2)$ whenever $\sigma^1 \neq \sigma^2$, the function $\phi(\cdot, \cdot)$ is well defined.

Property 1. The activity (G, Σ) satisfies Property 1 if $\phi(\sigma^1, \sigma^2) \in [0, 1]$ for every pair of distinct action profiles $\sigma^1, \sigma^2 \in \Sigma$.

Property 1 restricts the extent to which the agent's incentives to deviate (within the activity-game G) may vary between action profiles in Σ. For any (distinct) profiles $\sigma^1, \sigma^2 \in \Sigma$, two magnitudes need to be compared: (i) the difference between the agent's payoffs under σ^1 and σ^2, and (ii) the difference between the agent's maximal attainable payoffs when only the principal plays in accordance with σ^1 and σ^2. Property 1 holds if, for any $\sigma^1, \sigma^2 \in \Sigma$, (i) and (ii) do not have opposing signs, and the absolute value of (i) is at least as large as the absolute value of (ii).

In many cases, it is easy to verify that Property 1 holds. For instance, if an activity is unilaterally controlled by the principal (i.e., the agent's action space in the activity-game is a singleton), then the numerator and denominator of $\phi(\cdot, \cdot)$ are always identical. Hence, Property 1 holds at the upper bound, $\phi(\sigma^1, \sigma^2) \equiv 1$. On the other hand, if an activity is unilaterally controlled by the agent (i.e., the principal's action space in the activity-game is a singleton), then the two terms in the numerator of $\phi(\cdot, \cdot)$ are the same, and, therefore, for such activities, Property 1 holds at the lower bound, $\phi(\sigma^1, \sigma^2) \equiv 0$. The following lemma summarizes the above discussion.

Lemma 1. *Unilaterally controlled activities satisfy Property 1.*

It follows that the activity of periodic wage (first example described in Section 3.2) satisfies Property 1. By contrast, the joint-production example (second example in Section 3.2) does not satisfy Property 1: in order to increase production, both players need to provide more inputs. Since providing labor is costly to the agent, it follows that an increase in the intended production *decreases* the agent's payoff. On the other hand, "more capital on the table" *increases* the agent's deviation payoff. As the numerator and denominator of $\phi(\cdot, \cdot)$ are of opposing signs, Property 1 does not hold.

The activities in most of the papers we mentioned earlier are unilaterally controlled and, hence, Property 1 readily holds. Below, we show that Property 1 draws the exact limits to the standard smoothing arguments, imposed by short-term activity-specific incentives. Notably, a condition similar to our Property 1 appeared as part of Assumption A.3 in Ray (2002), who considers an abstract repeated-game setting with partial commitment.[9]

[9] Ray's assumption also imposes continuity of payoffs from other contractual components. Such continuity is crucial for Ray's construction, which relies not on the curvature of payoffs but rather on the possibility of marginally modifying several contractual components to enable the principal to appropriate surplus. We compare our model and results to Ray (2002) in Section 7.

Given an activity (G, Σ), we refer to a player's payoff from action profiles in Σ as his *activity-related payoff*.

Definition 6 (Nondecreasing Agent's Activity Payoff). Fix a contracting problem $f(\cdot)$, an incentive-compatible contract therein, and an activity (G, Σ). The agent's activity-related payoff is nondecreasing over time if there is zero probability of observing a history in which there exist two periods $t < s$ such that (G, Σ) is available in both periods, the action profiles played in G in these periods are both members of Σ, and $u_a(\sigma_t) > u_a(\sigma_s)$.

We are now ready to state the main result of the paper.

Theorem 1. *Let (G, Σ) be a concave activity that is separable with respect to $f(\cdot)$. If (G, Σ) satisfies Property 1, then, under any optimal contract, the agent's activity-related payoff is nondecreasing over time.*

4.2 *Implications and applications of Theorem 1*

Theorem 1 unifies and significantly generalizes many classic as well as more recent results in the literature. A substantial strand of literature has shown that an employee's wage (at a given workplace) rises over time when there are fluctuations in the value of his outside option. See, for example, Milton and Holmström (1982), Holmström (1983), and Postel-Vinay and Robin (2002a,b). To derive their results, these papers specify a full-blown model of the labor market that embeds fluctuations in the worker's outside option, and then use that specific structure to obtain the downward rigidity of wage directly. This standard approach is inherently limited as it derives the result only for the particular specification under consideration.

By contrast, our result establishes the downward rigidity of wage in any contracting problem where wage constitutes a concave separable activity, as is the case in all the aforementioned papers. Similarly, our result establishes the upward rigidity of effort in any contracting problem where effort constitutes a concave separable activity. Thus, our result suggests a general insight into seniority-based dynamics in stochastic contracting problems: a worker's effort can only decrease over time while his wage can only increase.

In addition, our approach draws connections between seemingly unrelated monotonicity results that have been derived in the literature. For example, in addition to the body of literature on wage dynamics mentioned above, our result generalizes monotonicity results regarding the dynamics of insurance contracts (Marcet and Marimon (1992), Krueger and Uhlig (2006)), dynamic project selection (Forand and Zápal (2020)), and effort dynamics (Bird and Frug (2021)). The objects of interest in each of these papers are concave separable activities that satisfy Property 1, and, hence, Theorem 1 delivers the qualitative monotonicity results derived directly in all of these papers.[10]

[10]In Forand and Zápal (2020) there are multiple projects that can be thought of as weakly concave separable activities. Their result follows from two corollaries of Theorem 1 that we establish in Section 4.3.

In Appendix B we establish the connection between Theorem 1 and some of the aforementioned monotonicity results in a more formal manner. In particular, we first construct the exact mapping between the models suggested in those papers and our general framework. Then we show that the objects of interest in those papers can be represented as concave separable activities that satisfy Property 1 and, hence, by Theorem 1, must exhibit a monotone dynamics under an optimal contract.

Theorem 1 can also be used to establish related monotonicity results in other settings of interest. Examples of possible applications of Theorem 1 include the following situations.

Power allocation in organizations It is well known that within large organizations, incentives are often provided via the reallocation of power rather than via monetary transfers (e.g., Cyert and March, 1963; Aghion and Tirole, 1997; Li, Matouschek, and Powell, 2017). “Excess power,” i.e., the power a division manager has beyond what is required for him to perform his job, can sometimes be represented as a concave separable activity.[11] Our result shows that the evolution of a division manager's power is inherently related to his potential benefit from abusing his power.

Consider, for example, a setting in which a division manager is occasionally required to exert an extreme amount of effort to deal with shocks in the firm's business environment. If the potential for “abuse” of power is low, then increasing the manager's power should have a small impact on his incentive to deviate and Property 1 is likely to hold. In this case, Theorem 1 implies that, regardless of the exact details of the contracting problem, if the manager's power in the organization increases after such a shock, then the increase will be permanent. If, on the other hand, the potential for abuse of power is high, then after a shock, the manager's power in the organization may increase temporarily and then decrease back to its former level.

Quality provision over time In a dynamic interaction between a supplier (the principal) and a client (the agent), the quality of the supplied goods may be an important component of the contractual terms. In some cases, quality provision constitutes a concave separable activity.[12] In such cases, as quality is unilaterally controlled by the supplier, it will satisfy Property 1 (Lemma 1). Hence, by Theorem 1, quality can only increase over time, regardless of the details of the contracting problem.

Foreign investments Consider a setting of foreign direct investment or entrepreneur financing (à la Thomas and Worrall (1994) and Albuquerque and Hopenhayn (2004)) where, in some periods, a lender (the principal) finances production by an entrepreneur (the agent) who may default on his debts. In the above papers, the amount of funding in each period is not an activity, as the chosen level of funding restricts the entrepreneur's actions in the rest of the interaction.[13] However, in alternative specifications of such

[11] We used an example of such an activity to provide the intuition for Theorem 1.

[12] For example, if the supplier has access to a competitive market where he can sell the goods he does not sell to the client, and the market's marginal valuation for quality is higher than the client's.

[13] Albuquerque and Hopenhayn (2004) derive a monotonicity result that relies on this restriction. We discuss their result in detail in Section 7.

financing problems, the (periodic) funding may constitute a concave separable activity. If the entrepreneur's payoff from defaulting is such that Property 1 holds, then by Theorem 1, the level of funding can only increase over time regardless of the exact specification of the financing problem.

4.3 *Two corollaries of Theorem 1*

In this section, we offer two corollaries of Theorem 1 that further extend the set of economic settings where results analogous to Theorem 1 hold.[14] First, recall the role of the concavity assumption in our characterization: smoothing the agent's activity-related payoff is strictly profitable for the principal. If an activity is only weakly concave (i.e., $U_p(u_a)$ is only weakly concave), then smoothing the agent's activity-related payoff does not decrease the principal's profit. Hence, we can establish the following corollary.[15]

Corollary 1. *Let (G, Σ) be a weakly concave activity that is separable with respect to $f(\cdot)$. If (G, Σ) satisfies Property 1 and an optimal contract exists, then there exists an optimal contract in which the agent's activity-related payoff is nondecreasing over time.*

Recall that a (stage) game can represent the principal randomizing over a finite set of alternatives, and observe that an activity based on such a game is weakly concave. Corollary 1 is particularly important for this class of activities.

Second, in a contracting problem with *multiple* activities (e.g., a problem where an agent exerts effort in return for wage), qualitative properties of the *joint* dynamics can be inferred. In particular, consider a contracting problem that contains two concave separable activities that satisfy Property 1. In an optimal contract, the marginal cost of increasing the agent's activity-related payoff via one activity today must be no greater than the marginal cost of doing so via the other activity in the future. In other words, observing the level of a *single* activity in a given period establishes a bound on the level of *every* activity in the contracting problem in the future.

Corollary 2. *Let $(\tilde{G}, \tilde{\Sigma})$ and $(\hat{G}, \hat{\Sigma})$ be two concave activities that satisfy Property 1 and are separable with respect to $f(\cdot)$, and consider a history where $(\tilde{G}, \tilde{\Sigma})$ is available in period s and $(\hat{G}, \hat{\Sigma})$ is available in period $t > s$. If the selected action profiles in $\tilde{G}$ and $\hat{G}$ in those periods are, respectively, $\tilde{\sigma} \in \tilde{\Sigma}$ and $\hat{\sigma} \in \hat{\Sigma}$, then*[16] $\tilde{U}_p'(\tilde{u}_a(\tilde{\sigma})) \leq \hat{U}_p'(\hat{u}_a(\hat{\sigma}))$.

In addition to linking the dynamics of genuinely distinct activities, Corollary 2 enables us to draw useful inferences about components of an interaction that resemble

[14] The proofs of these corollaries are analogous to the proof of Theorem 1 and are, therefore, omitted.

[15] The agent's activity-related payoff from a concave separable activity is nondecreasing over time under every optimal contract, and so Theorem 1 can be phrased in a way that holds vacuously if an optimal contract does not exist. By contrast, the agent's activity-related payoff from a weakly concave separable activity may not be nondecreasing over time under some optimal contracts, and so the corollary cannot be phrased in a way that holds vacuously if an optimal contract does not exist.

[16] We denote payoff functions associated with $(\tilde{G}, \tilde{\Sigma})$ by a tilde, and those associated with $(\hat{G}, \hat{\Sigma})$ by a hat. Moreover, recall that for a concave activity, we denote by $U_p'(\cdot)$ the right-hand side derivative of $U_p(\cdot)$.

activities, but do not satisfy additive separability in their payoffs or do not have the cross-product structure of the strategy space. For instance, consider an interaction where the agent performs different types of tasks and his utility from wage in a given period depends on the task performed in that period. In particular, suppose that there are two possible tasks: a *regular* task that is always available and requires an effort of $e = 1$, and an *opportunity* task that arrives occasionally and, if implemented, demands the agent's full attention—i.e., it replaces the regular task—and requires an effort of $e = 2$. In each period, the principal chooses wage $w \in \mathbb{R}_+$ and the agent's utility from wage is $\sqrt{w}/e$.

The game played in "regular periods," i.e., when only the regular task is available, is

$$G_{\text{reg}} = \langle S_p = \mathbb{R}_+, S_a = \{1\}; u_p(w, 1) = \pi(1) - w, u_a(w, 1) = \sqrt{w}\rangle,$$

where $\pi(1)$ denotes the principal's profit from the regular task. The game G_{reg}, together with Σ being the set of all its outcomes, forms the activity of wage in regular periods. In "opportunity periods," the periodic game is

$$G_{\text{opp}} = \left\langle S_p = \mathbb{R}_+, S_a = \{1, 2\}; u_p(w, e) = \pi(e) - w, u_a(w, e) = \frac{\sqrt{w}}{e}\right\rangle,$$

where $\pi(2)$ denotes the principal's profit from the opportunity task. Note that this game reflects both the selection among mutually exclusive tasks and the choice of wage. Since the agent's utility from wage is task-dependent, we need two "different activities" to formally represent wage in opportunity periods. In addition, since tasks are mutually exclusive, while the set Σ varies with the selected task, in order to satisfy the cross-product structure, the activity game for these two activities is identical and given by G_{opp}. This, in turn, implies that even the wage payments in periods when the regular task is performed require two distinct activities since $G_{\text{opp}} \neq G_{\text{reg}}$. Hence, to fully describe the dynamics of wage, formally, we need to consider three wage-related activities. By Theorem 1, the wage paid via each of these activities separately will never decrease over time. Corollary 2 complements the analysis and links the different activities. In particular, while wage need not be monotone over time, it can only decrease between a period in which the opportunity task is performed and a period in which the regular task is performed.

5. Tightness of Theorem 1

Our characterization in Theorem 1 is tight in the sense that under mild technical conditions, the level of an activity that does not satisfy Property 1 changes over time in the principal's favor in some contracting problems. To illustrate this point, we first consider variants of the joint-production activities from the second example in Section 3.2, after which we provide a formal (partial) converse to Theorem 1.

Recall that Property 1 holds if $0 \leq \phi(\sigma^1, \sigma^2)$ and $\phi(\sigma^1, \sigma^2) \leq 1$ for all $\sigma^1, \sigma^2 \in \Sigma$. Therefore, to demonstrate that the dynamics of an activity that does not satisfy Property 1 may be inconsistent with the dynamics implied by Theorem 1, we present two counterexamples. In particular, for the case where $\phi(\cdot, \cdot) < 0$, we consider a production technology under which capital and labor are complements, whereas for the case where $\phi(\cdot,\cdot) > 1$, we consider a production technology where capital and labor are (strong) substitutes. In each case, we construct a counterexample by an appropriate choice of compensation and production opportunities. For simplicity, we assume that the players do not discount the future.

Pay at the end For the case where Property 1 is violated because $\phi(\cdot,\cdot) < 0$, consider a slightly modified version of our joint-production activity where the production function is $z(k,l) = \min\{l, k/2\}$, where, as before, l is the agent's labor input and k is the capital provided by the principal. Moreover, assume that there are production opportunities in periods 1 and 2. To further simplify the example, suppose that the principal has very limited discretion on how to provide compensation to the agent: the principal can only decide whether or not to pay the agent a compensation of 1 at the end of the interaction (period 3).

Optimal production requires that if the agent provides l units of labor, then the principal provides $k(l) = 2l$ units of capital. Hence, we set $\Sigma = \{(k,l) \in \mathbb{R}^2_+ : k = 2l\}$. Let $\hat{\sigma} = (2\hat{l}, \hat{l})$ and $\tilde{\sigma} = (2\tilde{l}, \tilde{l})$ for $\hat{l} \neq \tilde{l}$. Under our assumptions that both the agent's marginal cost of labor and his marginal utility from reallocating capital are 1, we obtain

$$\phi(\hat{\sigma}, \tilde{\sigma}) = \frac{2\hat{l} - 2\tilde{l}}{-\hat{l} - (-\tilde{l})} = -2.$$

As the main focus of the present illustration is on the agent's incentives, we prefer not to provide a specific $\pi(\cdot)$ that makes the activity concave, but simply assume that the principal seeks to maximize aggregate production, and that, given a fixed level of aggregate production, he seeks to minimize the variance in the production level between periods. Due to the minimum production technology, this is equivalent to maximizing the agent's *aggregate* labor $l_1 + l_2$ (where l_t denotes the agent's labor in period $t \in \{1, 2\}$), where, between pairs (l_1, l_2) that add up to the same total, the principal prefers the one with the minimal difference between l_1 and l_2. Under these assumptions, the optimal contract can be identified by solving the linear programming problem

$$\begin{aligned} &\max\{l_1 + l_2\} \text{ such that} \\ \mathrm{IC}_1:\quad & 1 - l_1 - l_2 \geq 2 \cdot l_1 \\ \mathrm{IC}_2:\quad & 1 - l_2 \geq 2 \cdot l_2, \end{aligned}$$

where IC_t is the agent's incentive-compatibility constraint in period t. The *unique* solution to this linear programming problem is $(l_1 = \frac{2}{9}, l_2 = \frac{1}{3})$. That is, the optimal contract induces an *increasing* labor schedule, and, hence, the agent's activity-related payoff decreases over time in contrast to the dynamics implied by Theorem 1.

To understand the intuition for this counterexample, observe that the agent's continuation utility after he provides labor in period 2 is 1 (since only the compensation in period 3 is left), whereas his continuation utility after he provides labor in period 1 is $1 - l_2$. Accordingly, the threat of *losing* the continuation utility is greater in period 2 than in period 1. This, in turn, implies that the agent's gain from deviating in the activity-game ($U(k(l_t), l_t) - u_a(k(l_t), l_t) = k(l_t) + l_t = 3l_t$) under an incentive-compatible contract can be greater in period 2 than in period 1. Since this gain is increasing in labor, the maximal amount of labor the agent can be asked to provide is higher in period 2 than in period 1.

Carrot and stick For the the case where Property 1 is violated because $\phi(\cdot,\cdot) > 1$, consider the production function $z(l, k) = l + k/2$ and suppose that there is an "output target" of 1 that must be fulfilled in the periods in which the activity is available. Due to the output target, to make this activity concave, we assume that the principal's cost of providing capital is given by an increasing and strictly convex function $c(\cdot)$. Under this production technology, the principal's optimal capital input as a function of the agent's labor input is $k(l) = 2 \cdot (1 - l)$. This activity is given by

$$G_z = \langle S_p = \mathbb{R}_+, S_a = \mathbb{R}_+ \cup \{\text{steal}\}; u_p(k, s_a), u_a(k, s_a)\rangle,$$

where

$$u_p(k, s_a) = \begin{cases} \pi(1) - c(k) & \text{if } s_a \geq 1 - k \\ -c(k) & \text{else ,} \end{cases} \qquad u_a(k, s_a) = \begin{cases} k & \text{if } s_a = \text{steal} \\ -s_a & \text{if } s_a \neq \text{steal,} \end{cases}$$

and

$$\Sigma = \{(k, l) \in \mathbb{R}_+^2 : k = 2 \cdot (1 - l)\}.$$

Note that for any distinct $(l, k), (l', k') \in \Sigma$,

$$\phi\big((l, k), (l', k')\big) = \frac{k' - k}{(-l') - (-l)} = \frac{2(1 - l') - 2(1 - l)}{l - l'} = 2.$$

The specification of the contracting problem is as follows. In period 1, the agent chooses whether to opt out—which secures him a payoff of 0—or to participate. If the agent participates, then there are production opportunities in periods 2 and 4, and the principal's compensation opportunities are as follows: he can provide a compensation of 1 at the end of the interaction (period 5), and in period 3, he can either offer a compensation of $\frac{1}{2}$ or impose a fine of 1. The agent's incentive-compatibility constraints are

$$\begin{aligned} \mathrm{IC}_1: \quad & \frac{1}{2} + 1 - l_2 - l_4 \geq 0 \\ \mathrm{IC}_2: \quad & \frac{1}{2} + 1 - l_2 - l_4 \geq 2(1 - l_2) - 1 \\ \mathrm{IC}_4: \quad & 1 - l_4 \geq 2(1 - l_4). \end{aligned}$$

Note that IC_4 is equivalent to $l_4 \geq 1$, while the assumed production target and technology give $l_4 \leq 1$. Thus, a contract is incentive compatible only if $l_4 = 1$. Moreover, the incentive-compatibility constraints in periods 1 and 2 evaluated at $l_4 = 1$ jointly imply that the agent's labor input in period 2 must equal $\frac{1}{2}$. Thus, the *only* incentive-compatible contract has the agent's labor increase from $l_2 = \frac{1}{2}$ to $l_4 = 1$.

Intuitively, in this counterexample, the principal's ability to punish the agent for deviating in period 2 is greater than his ability to punish the agent for deviating in period 4. Accordingly, the agent's gain from deviating in the activity-game ($U(k(l_t), l_t) - u_a(k(l_t), l_t) = k(l_t) + l_t = 2 - l_t$) can be greater in period 2 than in period 4. Since this gain is decreasing in labor, the principal may have to require more labor in period 4 than in period 2. Indeed, due to the choice of compensation opportunities, the principal must require a full unit of labor in period 4, but cannot require that amount of labor in both periods, and so the agent's labor increases over time. Hence, the agent's activity-related payoff decreases over time under the optimal contract, in contrast to the dynamics implied by Theorem 1.

To establish a converse result to Theorem 1 beyond the above examples, we need to address two relatively technical points. First, in both counterexamples, Σ is chosen in such a way that it is suboptimal for the principal to specify an action profile outside of Σ in the activity-game G. In general, as the definition of an activity is agnostic about the choice of Σ, it may be the case that the principal will select action profiles outside of Σ under an optimal contract. For example, consider the joint-production activity-game G_z, where $z = \min\{l, k/2\}$, that is paired with the *inefficient* set of input bundles $\Sigma'_z = \{(k, l) \in \mathbb{R}^2_+ : k = 2l + 1\}$. Even though (G_z, Σ'_z) is a well defined activity, any contract in which the principal selects an action profile in Σ'_z is worse than some contract in which he assigns capital efficiently. Hence, an action profile in Σ'_z will never be played under an optimal contract in any contracting problem. To circumvent such problems, in our converse result we restrict attention to activities that satisfy the following efficiency notion.

Definition 7 (Strictly Pareto-Efficient Activity). An activity (G, Σ) is strictly Pareto-efficient if the payoffs associated with every action profile in Σ are on the Pareto frontier of the convex hull of the payoff set of G, and are not a convex combination of payoffs associated with action profiles outside of Σ.

Second, Property 1 stipulates that a "small change" in the agent's activity-related payoff does not have a large impact on his deviation payoff. Thus, a class of activities that obviously fail to satisfy Property 1 are those for which the agent's deviation payoff is discontinuous with respect to his activity-related payoff. To bypass the need to use a solution concept that is suitable for such discontinuous contracting problems (e.g., ϵ-optimality), we restrict attention to activities for which the function $\bar{u}_a : u_a(\Sigma) \to \mathbb{R}$, defined as

$$\bar{u}_a(u_a) = \bar{U}_a\big(\eta(u_a)\big),$$

where $\eta : u_a(\Sigma) \to \Sigma$ is a bijection that satisfies the requirements in the definition of the activity, is continuous.[17] In our converse result we fully construct optimal contracts. To simplify their derivation, we further assume that $\bar{u}_a(\cdot)$ is differentiable.

[17] An alternative converse to Theorem 1 that does not impose continuity on $\bar{u}_a(\cdot)$ and uses ϵ-optimality as a solution concept is available upon request.

Proposition 1. *Let (G, Σ) be a strictly Pareto-efficient concave activity for which $\bar{u}_a(\cdot)$ is differentiable and Property 1 does not hold. There exists a contracting problem $f(\cdot)$ with respect to which (G, Σ) is separable such that the agent's activity-related payoff decreases over time under the optimal contract.*

6. Robustness

A natural question that arises is whether the dynamics of activities that "almost" satisfy Property 1 can admit arbitrary decreases in the agent's activity-related payoff or if any such decreases will be "small." It turns out that violations of the upper and lower bounds of Property 1 have an asymmetric impact on the possible dynamics of a concave separable activity. In particular, given a sequence of activities that are identical on Σ and for which the *infimum* of $\phi(\cdot, \cdot)$ converges to 0, the size of the maximal decrease in the agent's activity-related payoff converges to 0 as well. On the other hand, if the *supremum* of $\phi(\cdot, \cdot)$ exceeds 1 even slightly, then there exist contracting problems in which a large decrease in the agent's activity-related payoff is observed.

To illustrate the intuition for the first result alluded to above, consider, for example, a parametrized family of activities, $\{(G^c, \Sigma)\}_{c\in\mathbb{R}_{++}}$, such that, within Σ, all of the activities are identical and satisfy $u_a(\Sigma) = [0, 1]$ and $U_p(u) = -u^2/2$ (recall that $U_p(u)$ is the principal's payoff from the unique action profile in Σ from which the agent's payoff is u), whereas the agent's payoffs in G^c outside of Σ are such that $\bar{U}_a^c(\sigma) = 1 + c - cu_a(\sigma)$ (where $\bar{U}_a^c(\sigma)$ is the agent's highest payoff in G^c when the principal plays σ_p). This specification is convenient because for every such (G^c, Σ), $\phi(\sigma, \sigma') = -c$ for any pair of distinct action profiles[18] $\sigma, \sigma' \in \Sigma$.

Consider a contracting problem with no discounting with respect to which (G^c, Σ) is separable and in which it is available in periods 1 and 2. Moreover, assume that $u_1 > u_2$ under an optimal contract, where u_t is the agent's (G^c, Σ)-related payoff in period t. By a standard smoothing argument, decreasing the agent's activity-related payoff in period 1 by a small ϵ and increasing it in period 2 by the same ϵ is profitable for the principal. Therefore, our assumption that $u_1 > u_2$ is part of an optimal contract implies that the aforementioned modification is not incentive compatible. Since $\phi(\cdot, \cdot) \leq 1$, increasing the agent's activity-related payoff in period 2 cannot violate the incentive-compatibility constraint in that period. Therefore, the above modification must violate the incentive-compatibility constraint in period 1.

Reducing the agent's activity-related payoff by ϵ in period 1 has two effects. First, it reduces the agent's utility in that period (on the path of play) by ϵ; second, it *increases* the agent's payoff from deviating in that period by ϵc. Thus, increasing the agent's activity-related payoff in period 2 by $(1 + c)\epsilon$ restores incentive compatibility. The principal's marginal profit from such a modification is $-U_p'(u_1) + U_p'(u_2)(1 + c)$. Since $U_p'(u_a) = -u_a$ and $u_t \leq 1$, the marginal profit from this modification is positive if $u_1 - u_2 > c$. Hence, $u_1 > u_2$ can be consistent with optimality only if $u_1 - u_2 \leq c$ (i.e., the bound on the decrease in agent's utility from period 1 to period 2 vanishes with c).

[18] For this specificaiton $\phi(\sigma, \sigma') = \frac{\bar{U}_a(\sigma) - \bar{U}_a(\sigma')}{u_a(\sigma) - u_a(\sigma')} = \frac{(1+c-cu_a(\sigma)) - (1+c-cu_a(\sigma'))}{u_a(\sigma) - u_a(\sigma')} = -c$.

While the above example has a very specific structure, the main part of the argument—that smoothing a decrease in the agent's activity-related payoff when $\phi(\cdot,\cdot) \leq 1$ can violate the incentive-compatibility constraint only in the *earlier* period—is general. This observation plays a fundamental role in our robustness result. In particular, it implies that if smoothing the agent's activity-related payoff between two periods violates incentive compatibility, then the principal can always restore incentive compatibility by increasing the agent's activity-related payoff in the later period. Hence, if $\phi(\cdot,\cdot)$ is bounded from below (and does not exceed 1), we can place an upper bound on the size of a decrease in the agent's activity-related payoff by comparing the marginal gain from smoothing the agent's activity-related payoff and the marginal cost of increasing his aggregate (discounted) activity-related payoff.

Proposition 2. *Consider a set of concave activities $\{(G^c, \Sigma)\}_{c\in\mathbb{R}_{++}}$ for which Σ, $(u_a|_\Sigma)$, and $(u_p|_\Sigma)$ are identical, $u_a(\Sigma)$ is compact, and (G^c, Σ) are such that $\phi(\cdot,\cdot) \subseteq [-c, 1]$. There exists a set of positive numbers $\{M^c\}_{c\in\mathbb{R}_+}$ for which $\lim_{c\to 0} M^c = 0$, such that the agent's (G^c, Σ)-related payoff does not decrease over time by more than M^c in any contracting problem with regard to which (G^c, Σ) is separable.*

On the other hand, if $\phi(\cdot,\cdot) > 1$, then smoothing a decrease in the agent's activity-related payoff can violate the incentive-compatibility constraint only in the *later* period. Consequently, smoothing a decrease in the agent's activity-related payoff may require the principal to increase the agent's continuation utility in the nonactivity part of the contracting problem. However, as some contracting problems do not contain such compensation opportunities (or providing additional compensation is prohibitively costly), a large decrease in the agent's activity-related payoff can be observed if $\phi(\cdot,\cdot) > 1$.

Proposition 3. *Let (G, Σ) be a strictly Pareto-efficient concave activity for which $\bar{u}_a(\cdot)$ is differentiable, $u_a(\Sigma)$ is compact, and $\phi(\cdot,\cdot)$ is bounded from below by $1 + c$ for some $c > 0$. There exists a contracting problem with respect to which (G, Σ) is separable, in which, under the optimal contract, the agent's activity-related payoff decreases by* $\max_{\sigma,\sigma'\in\Sigma}\{u_a(\sigma) - u_a(\sigma')\}$.

7. Literature review

The monotonicity result we derive embeds many results that have been mentioned throughout the paper. Milton and Holmström (1982), Holmström (1983), and Postel-Vinay and Robin (2002a,b) analyze labor markets and establish that an employee's wage does not decrease over time. Marcet and Marimon (1992) and Krueger and Uhlig (2006) study the dynamics of insurance contracts and show that the transfer received by the insured does not decrease over time. Forand and Zápal (2020) show that project selection criteria shift in the agent's favor as time goes by, and Bird and Frug (2021) show that while wage increases over time, the effort on similar tasks decreases over time.

Clearly, there are many dynamic monotonicity results that arise due to mechanisms other than the incentive-constrained smoothing mechanism analyzed in the present paper. Such results are prevalent in the literature on wage dynamics, which predicts the deferral of compensation for a multitude of reasons.[19] For example, Becker (1962) and Parsons (1972) emphasize the effect of relationship-specific investment, Freeman and Medoff (1984) highlight the role of labor unions and collective bargaining, and Salop and Salop (1976), Lazear (1981), Carmichael (1983), and Akerlof and Katz (1989) illustrate the impact of asymmetric information.

As in our setting, Albuquerque and Hopenhayn (2004) and Fudenberg and Rayo (2019) assume symmetric information and full commitment on the part of the principal, and obtain a dynamic monotonicity result. However, the results in these papers do not follow from Theorem 1. Albuquerque and Hopenhayn (2004) consider a model of entrepreneur financing where the entrepreneur's profit from production in a given period, which depends on the capital he receives, is an upper bound on his repayment to the lender in that period. Thus, the cross-product structure of available actions does not hold and so (periodic) financing is not an activity. Moreover, the mechanism that generates the monotonicity result in Albuquerque and Hopenhayn (2004) is different from our "incentive-constrained smoothing" mechanism and so their result does not require concavity of the payoff functions but only quasi-concavity. At the start of the interaction in their model, the entrepreneur owes a large debt to the lender and, hence, has a low continuation utility. Since the entrepreneur's deviation payoff in a given period is increasing in the size of the loan he received in that period, the entrepreneur's continuation utility limits the size of the loan he can receive. As time goes by, the entrepreneur repays the initial debt and his continuation utility increases, and, hence, it becomes incentive compatible for him to receive larger and more efficient loans. Fudenberg and Rayo (2019) show that an apprentice's unskilled effort decreases over time. However, the unskilled effort in their model is not an activity: first, the sum of the apprentice's skilled and unskilled effort must be less than 1 (which violates the cross-product structure of available actions), and second, the apprentice's cost of effort is a function of his aggregate effort level (which violates separability of payoffs).

Our dynamic monotonicity result relies on the assumptions that there is symmetric information and that the principal has full commitment power. Earlier work on stochastic environments has shown that if one (or both) of these assumptions is relaxed, optimal outcomes may necessitate non-montone dynamics. For example, Möbius (2001), Hauser and Hopenhayn (2008), and Samuelson and Stacchetti (2017) show this in the context of "trading favors," and Li, Matouschek, and Powell (2017), Bird and Frug (2019), and Lipnowski and Ramos (2020) show this in the context of dynamic project selection.

Another related paper is Ray (2002), who studies a model where the principal has partial commitment power in a repeated (and constant across periods) interaction. Ray shows that regardless of the exact details of the interaction, if it is not possible to support unconstrained efficient agreements in all periods, then there is a form of transition in the agent's favor as time goes by. Specifically, Ray shows that the periodic contract converges over time to the agent's preferred contract.[20] The driving force behind Ray's

[19] See Prendergast (1999), Edmans and Gabaix (2009), and Pavan (2017) for a review of this literature.

[20] Thomas and Worrall (2018) generalize Ray's results to a setting where both agents can take an action, and neither of them can commit. Moreover, they establish that in their setting, efficient (relational) contracts may exhibit qualitative properties that cannot occur in Ray's setting.

result is that compensating the agent in a future period for his current actions increases his liability (should he deviate) in the intermediate periods. In an environment where unconstrained Pareto efficiency is precluded by limited liability, this enables the principal to increase his payoff by offering agreements that are nearer to the Pareto frontier in those intermediate periods.[21] By contrast, we do not study the evolution of the agent's payoffs over time (which, in fact, can exhibit any dynamics due to the fluctuations in the environment), but rather show that certain components of the contract shift monotonically in the agent's favor due to an activity-specific incentive-constrained smoothing motive.

8. Concluding remarks

Persistent asymmetric information The methodology developed in this paper can be extended beyond symmetric information environments and used to derive monotonicity results in contracting problems endowed with certain types of asymmetric information. Consider, for example, a model in which the agent works on stochastically arriving tasks in which the agent's productivity of effort is strictly concave and his cost of effort is linear. However, assume that the marginal cost of effort is the agent's private information and is constant over time. In such a model, there is a screening element in the principal's problem that is absent in this paper. Since the arrival of tasks is stochastic and the agent can stop working at any time, finding the optimal intertemporal allocation of information rents is a complex problem. Nevertheless, we now briefly explain how our methodology can be used to show that the optimal effort schedule for every type of agent is non-increasing over time.

In principle, in this screening problem there are two possible reasons for offering a contract with an increasing effort schedule to a certain type of agent: either to maximize the principal's profit from that agent type or to reduce the cost of providing information rents to other agent types. From Theorem 1, we know that an increasing effort schedule does not maximize the principal's profit from his interaction with any single agent type. Hence, an increasing effort schedule can only be offered to a certain agent type if it reduces the cost of providing information rents to other agent types.

Incentive compatibility in this model requires that, given a "menu of contracts," every agent type (weakly) prefers accepting the contract intended for him (and adhering to its terms indefinitely) to selecting a contract intended for another type, adhering to its terms until an arbitrary point of time in the interaction and then disregarding it. Notice that smoothing a decrease in one agent-type's effort schedule (weakly) reduces every agent-type's payoff from selecting that contract and adhering to its terms until any given point in time. Hence, smoothing a decrease in the effort schedule of one agent type (weakly) reduces the cost of providing information rents to other agent types. Thus, in optimum, the principal will offer contracts with a non-increasing effort schedule to all agent types.

[21] Lazear (1981), among others, shows that similar mechanisms are relevant also in dynamic contracting problems with full commitment.

Unconnected support for activity levels Our definition of an activity requires that the set of possible activity levels be a real-valued interval. This assumption, which may seem like a mere simplification, is, in fact, necessary for our main result. Consider a contracting problem with an infinite horizon and a discount factor of $\delta = \frac{1}{2}$. Moreover, assume that in each of the first two periods the agent can exert an effort of $e \in \{0, 1, 2\}$, and that in every period the principal can provide compensation worth 2 utils to the agent.

Requiring high effort ($e = 2$) in period 1 and low effort ($e = 1$) in period 2 is not incentive compatible: the agent's discounted cost of effort is $2 + \delta = \frac{5}{2}$, whereas his maximal discounted utility from compensation from period 2 onward is $\delta \cdot 2/(1 - \delta) = 2$. Thus, requiring high effort in both periods is also not a viable option. However, requiring low effort in period 1 and high effort in period 2 is incentive compatible: in both periods the agent's discounted cost of effort is 2, which is exactly his discounted utility from future wages if the principal provides compensation from period 2 onward. Therefore, under the optimal contract, the agent's effort increases over time, even though Property 1 holds and effort is separable with respect to the contracting problem.

Appendix A: Proofs

We use the following notation in the proofs. We denote a generic periodic game by $\tilde{G}$, a generic action profile therein by $s(\tilde{G})$, and a generic finite history by $\tilde{h}$. Let $\tilde{\omega} = (\tilde{h}, \tilde{G})$ denote the history and the game that has been realized following that history. We refer to $\tilde{\omega}$ as the state.

Proof of Theorem 1. For a generic contract X and state $\tilde{\omega}$, denote by $X(\tilde{\omega})$ the action profile suggested by X at $\tilde{\omega}$. For any $\tilde{\omega}$ at which (G, Σ) is available and the action profile that is played in G under X is an element of Σ, we sometimes replace $s(\tilde{G})$ with $(\sigma(\tilde{\omega}), s^{-G}(\tilde{\omega}))$, where $\sigma(\tilde{\omega})$ is the action profile that is played in G at $\tilde{\omega}$.

Consider an incentive-compatible contract C and suppose that there exists a state ω_t for which there exist $\Delta > 0$, $p > 0$, and a set $\Omega_{t'}$ of states of length $t' > t$ that are consistent with ω_t such that (i) (G, Σ) is available at ω_t and at every $\omega_{t'} \in \Omega_{t'}$, and the action profile in G, specified by C, in those states is an element of Σ, (ii) $u_a(\sigma(\omega_t)) - u_a(\sigma(\omega_{t'})) \geq \Delta$ for every $\omega_{t'} \in \Omega_{t'}$, and (iii) $\Pr(\Omega_{t'} | \omega_t, C) = p$.

Next we show that the continuation of C at ω_t is suboptimal by modifying the activity-play relative to C. Such modifications do not alter the distribution of periodic games, since (G, Σ) is separable with respect to $f(\cdot)$.

We begin by defining a continuation contract at ω_t that smooths out the decrease in the agent's payoff from (G, Σ) under C. Fix an $\epsilon > 0$ for which $\epsilon + \frac{\epsilon}{p\delta^{(t'-t)}} < \Delta$ and define $\hat{C}$ by making the following modifications to the activity-play under C.

First, given the action profile suggested by C at ω_t, $(\sigma(\omega_t), s^{-G}(\omega_t))$, define

$$\hat{C}(\omega_t) = \big(\hat{\sigma}(\omega_t), s^{-G}(\omega_t)\big),$$

where $\hat{\sigma}(\omega_t)$ is the action profile in Σ for which $u_a(\hat{\sigma}(\omega_t)) = u_a(\sigma(\omega_t)) - \epsilon$. This action profile exists since, by the choice of ϵ, it provides the agent with an activity-related payoff that is between two feasible activity-related payoffs, and by the definition of an activity, the set of possible activity-related payoffs is an interval.

In the subsequent steps, we denote a generic sequence of periodic games and their play between periods t_1 and t_2 by $\Xi_{t_1}^{t_2} = \{G(\tau), s(G(\tau))\}_{\tau=t_1}^{t_2}$.

Second, we modify the contract so that the first change does not alter the path of play in the periods up to t'. Formally, for $\tau \in \{t+1, \dots, t'-1\}$ and any $(\Xi_{t+1}^{\tau-1}, G(\tau))$ that are consistent with some $\omega_{t'} \in \Omega_{t'}$, define

$$\hat{C}\big(\omega_t, \big(\hat{\sigma}(\omega_t), s^{-G}(\omega_t)\big), \Xi_{t+1}^{\tau-1}, G(\tau)\big) = C\big(\omega_t, \big(\sigma(\omega_t), s^{-G}(\omega_t)\big), \Xi_{t+1}^{\tau-1}, G(\tau)\big).$$

Third, we decrease the agent's payoff from (G, Σ) in period t'. For any $\omega_{t'} = (\omega_t, (\sigma(\omega_t), s^{-G}(\omega_t)), \Xi_{t+1}^{t'-1}, G(t')) \in \Omega_{t'}$ and action profile suggested by C at $\omega_{t'}$, $(\sigma(\omega_{t'}), s^{-G}(\omega_{t'}))$, define

$$\hat{C}\big(\omega_t, \big(\hat{\sigma}(\omega_t), s^{-G}(\omega_t)\big), \Xi_{t+1}^{t'-1}, G\big(t'\big)\big) = \big(\hat{\sigma}(\omega_{t'}), s^{-G}(\omega_{t'})\big),$$

where $\hat{\sigma}(\omega_{t'})$ is the strategy profile in Σ for which $u_a(\hat{\sigma}(\omega_{t'})) = u_a(\sigma(\omega_{t'})) + \frac{\epsilon}{p\delta^{t'-t}}$. This profile exists for the same reason described above.

Finally, we modify the contract so that the previous changes do not alter the path of play after t'. Formally, for any $\tau > t'$, and any $\Xi_{t+1}^{t'-1}$ that is consistent with some $\omega_{t'} \in \Omega_{t'}$ and $(\Xi_{t'+1}^{\tau-1}, G(\tau))$ that are consistent with C, define

$$\begin{aligned}&\hat{C}\big(\omega_t, \big(\hat{\sigma}(\omega_t), s^{-G}(\omega_t)\big), \Xi_{t+1}^{t'-1}, G\big(t'\big), \big(\hat{\sigma}(\omega_{t'}), s^{-G}(\omega_{t'})\big), \Xi_{t'+1}^{\tau-1}, G(\tau)\big)\\ &\quad = C\big(\omega_t, \big(\sigma(\omega_t), s^{-G}(\omega_t)\big), \Xi_{t+1}^{t'-1}, G\big(t'\big), \big(\sigma(\omega_{t'}), s^{-G}(\omega_{t'})\big), \Xi_{t'+1}^{\tau-1}, G(\tau)\big).\end{aligned}$$

Lemma A.1. *$\hat{C}$ is incentive compatible.*

Proof. For all states ω_s such that $s \geq t'$ and $\omega_s \notin \Omega_{t'}$, $\hat{C}$ is identical to C and so $\hat{C}$ is incentive compatible at such states.

At $\omega_{t'} \in \Omega_{t'}$, the agent's continuation utility from following the contract increases by $u_a(\hat{\sigma}(\omega_{t'})) - u_a(\sigma(\omega_{t'}))$ while his deviation payoff increases by $\bar{U}_a(\hat{\sigma}(\omega_{t'})) - \bar{U}_a(\sigma(\omega_{t'}))$. By Property 1, $\frac{\bar{U}_a(\hat{\sigma}(\omega_{t'})) - \bar{U}_a(\sigma(\omega_{t'}))}{u_a(\hat{\sigma}(\omega_{t'})) - u_a(\sigma(\omega_{t'}))} \leq 1$. Since $u_a(\hat{\sigma}(\omega_{t'})) - u_a(\sigma(\omega_{t'})) > 0$, it follows that $\bar{U}_a(\hat{\sigma}(\omega_{t'})) - \bar{U}_a(\sigma(\omega_{t'})) \leq u_a(\hat{\sigma}(\omega_{t'})) - u_a(\sigma(\omega_{t'}))$ and so $\hat{C}$ is incentive compatible at $\omega_{t'}$. Since $\hat{C}$ and C are identical at all states (strictly) between periods t and t', it follows that $\hat{C}$ is incentive compatible at all such states.

At ω_t, by the construction of $\hat{C}$, if the agent does not deviate, then the expected discounted increase in his payoff in period t' equals the decrease in his payoff at ω_t. By Property 1, $0 \leq \frac{\bar{U}_a(\hat{\sigma}(\omega_t)) - \bar{U}_a(\sigma(\omega_t))}{u_a(\hat{\sigma}(\omega_t)) - u_a(\sigma(\omega_t))}$. Since $u_a(\hat{\sigma}(\omega_t)) < u_a(\sigma(\omega_t))$, it follows that $\bar{U}_a(\hat{\sigma}(\omega_t)) \leq \bar{U}_a(\sigma(\omega_t))$. Thus, $\hat{C}$ is incentive compatible at ω_t. □

Lemma A.2. *The principal's continuation payoff at ω_t under $\hat{C}$ is greater than his continuation payoff under C.*

Proof. Since the activity-related payoff is additively separable from any other payoff in the contracting problem, conditional on ω_t, the contract $\hat{C}$ outperforms the contract C if

$$U_p\big(u_a(\sigma(\omega_t))-\epsilon\big)+p\delta^{t'-t}\int U_p\bigg(u_a(\sigma(\omega_{t'}))+\frac{\epsilon}{p\delta^{t'-t}}\bigg)\,d\mu$$
$$> U_p\big(u_a(\sigma(\omega_t))\big)+p\delta^{t'-t}\int U_p\big(u_a(\sigma(\omega_{t'}))\big)\,d\mu,$$

where μ denotes the distribution of states in $\Omega_{t'}$ induced by the contract C, conditional on ω_t. We establish that this inequality holds via the following smoothing argument. Since $U_p(\cdot)$ is concave, it has left- and right-hand side derivatives, which we denote, respectively, by $\partial_- U_p(\cdot)$ and $\partial_+ U_p(\cdot)$,

$$\begin{aligned}
&U_p\big(u_a(\sigma(\omega_t))-\epsilon\big)+p\delta^{t'-t}\int U_p\bigg(u_a(\sigma(\omega_{t'}))+\frac{\epsilon}{p\delta^{t'-t}}\bigg)\,d\mu\\
&> U_p\big(u_a(\sigma(\omega_t))\big)-\epsilon\cdot\partial_+ U_p\big(u_a(\sigma(\omega_t))-\epsilon\big)\\
&\quad + p\delta^{t'-t}\int\bigg(U_p\big(u_a(\sigma(\omega_{t'}))\big)+\frac{\epsilon}{p\delta^{t'-t}}\cdot\partial_- U_p\bigg(u_a(\sigma(\omega_t))-\Delta+\frac{\epsilon}{p\delta^{t'-t}}\bigg)\bigg)\,d\mu\\
&= U_p\big(u_a(\sigma(\omega_t))\big)+p\delta^{t'-t}\int U_p\big(u_a(\sigma(\omega_{t'}))\big)\,d\mu\\
&\quad -\epsilon\bigg(\partial_+ U_p\big(u_a(\sigma(\omega_t))-\epsilon\big)-\partial_- U_p\bigg(u_a(\sigma(\omega_t))-\Delta+\frac{\epsilon}{p\delta^{t'-t}}\bigg)\bigg)\\
&> U_p\big(u_a(\sigma(\omega_t))\big)+p\delta^{t'-t}\int U_p\big(u_a(\sigma(\omega_{t'}))\big)\,d\mu,
\end{aligned}$$

where the first inequality follows from the fact that $U_p(\cdot)$ is decreasing and strictly concave, and the second inequality follows from the same fact and the choice of ϵ. □

If the agent's activity-related payoff is not nondecreasing over time under C, then there exists a set of length-t states with positive measure, Ω_t, at which (G,Σ) is available and the action profile in G (specified by C) in those states is an element of Σ, such that for each $\omega_t\in\Omega_t$, there exists a set $\Omega_{t'}(\omega_t)$ that satisfies the following properties: (i) $\Omega_{t'}(\omega_t)$ is a set of states of length t', where $t'>t$, that are consistent with ω_t; (ii) (G,Σ) is available at each $\omega_{t'}\in\Omega_{t'}(\omega_t)$, and the action profile in G (specified by C) in those states is an element of Σ; (iii) there exists $\Delta>0$ such that $u_a(\sigma(\omega_t))-u_a(\sigma(\omega_{t'}))\geq\Delta$ for every $\omega_{t'}\in\Omega_{t'}(\omega_t)$; and (iv) $\Pr(\Omega_{t'}(\omega_t)|\omega_t)=p$ for some $p>0$. To obtain a contract that is better than C, at every $\omega_t\in\Omega_t$, perform the modification described above. This is feasible as each modification is performed on the continuation contract from a distinct state, and thus these modifications are mutually exclusive. Moreover, these modifications do not violate incentive compatibility at states ω_s, where $s<t$, as, by construction, the modification performed at ω_t does not change the agent's continuation utility at that state. □

Proof of Proposition 1. To establish this result, we construct a counterexample with two parameters, $u', u'' \in u_a(\Sigma)$, and select appropriate parameter values for each violation of Property 1. The counterexample is an interaction where, in period 1, the agent chooses whether to enter the interaction or quit. In period 2, the players play the activity-game G. In period 3, the agent can either quit the interaction or continue, and the principal can either continue with the interaction by giving the agent a payoff of $-u'$ or quit by giving the agent a payoff of $-\bar{u}_a(u')$. In period 4, the players play the activity-game G, and in period 5 the principal can either capitalize on the interaction by giving the agent a payoff of $-u''$ and receive a (large) payoff himself, or quit the interaction by giving the agent a payoff of $-\bar{u}_a(u'')$.

Formally, the counterexample is constructed from the games

$$G_1 = \langle S_p = \{p_\emptyset\}, S_a = \{Q, E\}; u_p(\cdot,\cdot) \equiv u_a(\cdot,\cdot) \equiv 0\rangle$$

$$G_3 = \left\langle S_p = S_a = \{Q, C\}; u_p(\cdot,\cdot) \equiv 0, u_a(s_p, s_a) = \begin{cases} -u' & \text{if } s_p = C \\ -\bar{u}_a(u') & \text{if } s_p = Q \end{cases} \right\rangle$$

$$G_5 = \left\langle S_p = \{Q, C\}, S_a = \{a_\emptyset\}; u_p(s_p, a_\emptyset) = \begin{cases} m & \text{if } s_p = C \\ 0 & \text{if } s_p = Q, \end{cases} \right.$$

$$\left. u_a(s_p, a_\emptyset) = \begin{cases} -u'' & \text{if } s_p = C \\ -\bar{u}_a\big(u''\big) & \text{if } s_p = Q \end{cases} \right\rangle$$

$$G_n = \langle S_p = \{p_\emptyset\}, S_a = \{a_\emptyset\}; u_p(\cdot,\cdot) \equiv u_a(\cdot,\cdot) \equiv 0\rangle,$$

where m is a (sufficiently) large positive number. The contracting problem is given by $f(\cdot) = G_n$ if either player played Q in the past. Otherwise $f(h_t) = G_t$ for $t \in \{1, 3, 5\}$ and $f(h_t) = G$ for $t \in \{2, 4\}$. Finally, we assume that $\delta = 1$.

First, we restrict attention to contracts in which whenever (G, Σ) is available, the specified action profile is an element of Σ. Hence, we can denote by σ_t the action profile that should be played in period $t \in \{2, 4\}$. Under an optimal contract the principal must incentivize the agent to enter and then incentivize him to play his action in the action profile σ_t without quitting the interaction. Note that in this counterexample, it is without loss of generality to assume that after the agent deviates, the principal quits. Thus, the incentive-compatibility constraints are

$$\begin{aligned}
\text{IC}_1: &\quad u_a(\sigma_2) - u' + u_a(\sigma_4) - u'' \geq 0 \\
\text{IC}_2: &\quad u_a(\sigma_2) - u' + u_a(\sigma_4) - u'' \geq \bar{U}_a(\sigma_2) - \bar{u}_a\big(u'\big) \\
\text{IC}_3: &\quad -u' + u_a(\sigma_4) - u'' \geq -u' \\
\text{IC}_4: &\quad u_a(\sigma_4) - u'' \geq \bar{U}_a(\sigma_4) - \bar{u}_a\big(u''\big).
\end{aligned}$$

We now show that there exist $u' > u''$ such that under the unique optimal contract, $u_a(\sigma_2) = u'$ and $u_a(\sigma_4) = u''$. Note that for such a contract, all incentive-compatibility constraints are binding. Since $U_p(\cdot)$ is a strictly concave and decreasing function, this implies that a contract can be both incentive compatible and more profitable than the one suggested above only if $u_a(\sigma_2), u_a(\sigma_4) \in (u'', u')$.

Case 1: $\phi(\sigma'', \sigma') = -c$ for some distinct $\sigma'', \sigma' \in \Sigma$ and $c > 0$. Due to the differentiability of $\bar{u}_a(\cdot)$, there exists $u^* \in u_a(\Sigma)$ for which $\frac{d\bar{u}_a}{du}(u^*) \leq -c$. Moreover, as any discontinuity of the function $\frac{d\bar{u}_a}{du}(\cdot)$ is an essential discontinuity, there exists a nondegenerate interval to the left or to the right of u^* on which $\frac{d\bar{u}_a}{du}(\cdot) \leq -\frac{c}{2}$. Let J be one such interval.

The concavity of $U_p(\cdot)$ implies that we can choose $u'' < u' \in J$ such that for any $\epsilon \in (0, u' - u'')$ it holds that $U_p(u') + U_p(u'') > U_p(u' - \epsilon) + U_p(u'' + \epsilon(1 + \frac{c}{2}))$ (we prove that such values exist in Lemma A.3 that is established below). In order for σ_2, σ_4 to satisfy IC_1, where $u_a(\sigma_2), u_a(\sigma_4) \in (u'', u')$ and $u_a(\sigma_2) = u' - \tilde{\epsilon}$, it must be that $u_a(\sigma_4) \geq u'' + \tilde{\epsilon}(1 + \frac{c}{2})$. However, by the choice of u'' and u', for any such σ_2 and σ_4, it holds that $U_p(u_a(\sigma_2)) + U_p(u_a(\sigma_4)) < U_p(u') + U_p(u'')$.

Case 2: $\phi(\sigma'', \sigma') = 1 + c$ for some distinct $\sigma'', \sigma' \in \Sigma$ and $c > 0$. In this case, by an analogous argument to the one used in Case 1, there exists an interval $J \subset u_a(\Sigma)$ on which $\frac{d\bar{u}_a}{du}(\cdot) \geq 1 + \frac{c}{2}$. Set u'' and u' to be the endpoints of J. Note that the action profile σ_4 for which $u_a(\sigma_4) = u''$ is the only one in Σ for which the agent's activity-related payoff is an element of J and IC_4 holds.

To complete the proof, we must show that the restriction we imposed on the contracting space (by which the action profile in even periods is an element of Σ) is without loss of generality. To see this, note that under an optimal contract, neither player quits the interaction along the path of play. Moreover, the agent must receive a nonnegative utility from any incentive-compatible contract (otherwise he will play Q in period 1). Since the agent's utility in the contract defined above is zero, it follows that if this contract were suboptimal in the unrestricted contracting space, the players' payoffs in the above contract in period 2 or 4 would be Pareto-dominated by the average of two action profiles in G (outside of Σ). However, this contradicts the assumption that (G, Σ) is strictly Pareto efficient. □

Lemma A.3. *Consider a concave activity (G, Σ). For any $k > 0$ and $u_1 \in \mathrm{int}(u_a(\Sigma))$, there exists $\tilde{u}_2 \in u_a(\Sigma)$ such that $\tilde{u}_2 < \tilde{u}_1$ and, for any $\epsilon > 0$,*

$$U_p(\tilde{u}_1) + U_p(\tilde{u}_2) > U_p(\tilde{u}_1 - \epsilon) + U_p\big(\tilde{u}_2 + \epsilon(1 + k)\big). \tag{1}$$

Proof. As $U_p(\cdot)$ is concave, it is absolutely continuous on any compact interval that is a subset of $u_a(\Sigma)$, and, hence, differentiable a.e., and (1) can be written as

$$\int_{\tilde{u}_1 - \epsilon}^{\tilde{u}_1} U_p'(u)\, du > \int_{\tilde{u}_2}^{\tilde{u}_2 + \epsilon(1+k)} U_p'(u)\, du. \tag{2}$$

As $U_p(\cdot)$ is concave, the left-hand side of (2) is bounded from below by $\epsilon U_p'(\tilde{u}_1)$, and the right-hand side of (2) is bounded from above by $(1 + k)\epsilon U_p'(\tilde{u}_2)$. Select $\tilde{u}_2 < \tilde{u}_1$ at which $U_p(\cdot)$ is differentiable and $U_p'(\tilde{u}_1) > (1 + k)U_p'(\tilde{u}_2)$. □

Proof of Proposition 2. Consider the activity (G^c, Σ) and a contracting problem $f(\cdot)$ with respect to which that activity is separable. Suppose that under a given incentive-compatible contract there exist ω_t, $\Delta > 0$, $p > 0$, and a set of states $\Omega_{t'}$ of length $t' > t$ that are consistent with ω_t such that (i) (G, Σ^c) is available at ω_t and at every $\omega_{t'} \in \Omega_{t'}$, and the action profile in G in those states is an element of Σ, (ii) $u_a(\sigma(\omega_t)) - u_a(\sigma(\omega_{t'})) \geq \Delta$ for every $\omega_{t'} \in \Omega_{t'}$, and (iii) $\Pr(\Omega_{t'}|\omega_t) = p$.

Consider modifications of the continuation contract at ω_t that are analogous to the modification of $\hat{C}$ in the proof of Theorem 1, with the exception that at period t' the agent's activity-related payoff is increased by $\alpha\epsilon$ for some $\alpha > \frac{1}{p\delta^{t'-t}}$ (as opposed to by exactly $\frac{\epsilon}{p\delta^{t'-t}}$). Recall that such changes are always feasible for a sufficiently small ϵ (since the agent's activity-related payoff is between two feasible values) and do not impact the distribution of future games (since (G^c, Σ) is separable). Furthermore, as $\alpha > \frac{1}{p\delta^{t'-t}}$ and $\phi(\cdot, \cdot) \leq 1$, such changes (weakly) relax the incentive-compatibility constraints at all states apart from ω_t.

For any $\alpha < \frac{1}{p\delta^{t'-t}} \frac{\partial_- U_p(u_a(\sigma(\omega_t)))}{\partial_+ U_p(u_a(\sigma(\omega_{t'})))}$, where $\partial_- U_p(\cdot)$ and $\partial_+ U_p(\cdot)$ are, respectively, the left- and right-hand side derivatives of $U_p(\cdot)$, the above modification is profitable for sufficiently small ϵ. Moreover, as $\phi(\cdot, \cdot) \geq -c$, the above modification is incentive compatible at ω_t if $\alpha \geq \frac{1+c}{p\delta^{t'-t}}$. Thus, if $\frac{\partial_- U_p(u_a(\sigma(\omega_{t'}))+\Delta)}{\partial_+ U_p(u_a(\sigma(\omega_{t'})))} > 1 + c$, there exists a modification that is both profitable and incentive compatible.

Define

$$X_c = \left\{ x : x > 0 \text{ and } \exists u \in u_a(\Sigma) \text{ s.t. } \frac{\partial_- U_p(u + x)}{\partial_+ U_p(u)} \leq 1 + c \right\}.$$

It follows, that if $\Delta > \sup\{X_c\}$ there exists a modification of the contract under consideration that is both profitable and incentive compatible.

Since $U_p(\cdot)$ is concave, it is differentiable a.e., and, hence, the set X_c is nonempty for any $c > 0$. As $u_a(\Sigma)$ is compact, $\sup\{X_c\}$ is finite and so we can set $M^c = \sup\{X_c\}$. Finally, note that M^c decreases when c decreases, and that since $U_p(\cdot)$ is strictly concave, $\lim_{c \to 0} M^c = 0$. □

Proof of Proposition 3. The proof of this result uses the same counterexample used in the second part of the proof of Proposition 1. To construct a decrease of size $\max_{\sigma, \sigma' \in \Sigma}(u_a(\sigma) - u_a(\sigma'))$ in the agent's activity-related payoff under the unique optimal contract, use that counterexample and set the endpoints of J to be the payoffs that support the maximum. □

Appendix B: Monotonicity of concave separable activities in selected papers

In this appendix, we apply Theorem 1 to obtain the monotonicity results of selected papers.

Milton and Holmström (1982)

Consider the model presented in Milton and Holmström (1982) for a specific level of education e. The contracting problem in that paper has the following components:[22]

A parametrized quitting game

$$G_q^m = \left\langle S_p = \{p_\emptyset\}, S_a = \{Q, C\}; u_p(p_\emptyset, s_a) \equiv 0, u_a(p_\emptyset, s_a) = \begin{cases} \dfrac{m}{1-\delta} & \text{if } s_a = Q \\ 0 & \text{if } s_a = C \end{cases} \right\rangle,$$

where m is the belief about the agent's ability and Q (C) represents the agent's choice to quit (continue with) his current employer.

A parametrized profit game (that signals the agent's ability)

$$G_\pi^y = \left\langle S_p = \{p_\emptyset\}, S_a = \{a_\emptyset\}; , u_p(p_\emptyset, a_\emptyset) = \frac{y}{\delta^2}, u_a(p_\emptyset, a_\emptyset) = 0 \right\rangle,$$

where y is the output generated by the agent.

A null game

$$G_N = \langle S_p = \{p_\emptyset\}, S_a = \{a_\emptyset\}; u_p(p_\emptyset, a_\emptyset) = u_a(p_\emptyset, a_\emptyset) = 0 \rangle.$$

A game that represents the activity of wage,

$$G_{\text{wage}} = \left\langle S_p = \mathbb{R}_+, S_a = \{a_\emptyset\}; u_p(w, a_\emptyset) = -\frac{w}{\delta}, u_a(w, a_\emptyset) = \frac{U(w)}{\delta} \right\rangle,$$

where w is the agent's wage and $U(w)$ is his strictly concave vNM utility function from wage.

The function $f(\cdot)$ is given as $G(1) = G_q^{m_1}$, where m_1 is the prior expectation over the agent's ability. If the agent chooses Q, then in all future periods $G(t) = G_N$; otherwise, $G(2) = G_{\text{wage}}$ and $G(3) = G_\pi^{y_1}$, where y_1 is drawn from a normal distribution with mean m_1 and precision $p_1 = \frac{\hat{p}_1}{\hat{p}_1+1}$, where $\hat{p}_1$ is the prior precision of the distribution over the agent's ability (h_1 in their paper). The construction of $f(\cdot)$ proceeds in an iterative manner. If, in a history h_t, there exists a period in which the agent chose Q in a quitting game, then $G(t) = G_N$. Otherwise, if $t = 3(n-1)+1$ (for some positive integer n), then $G(t) = G_q^{m_n}$, where m_n is drawn according to a normal distribution with expectation $\frac{p_{n-1}m_{n-1}+y_{n-1}}{p_{n-1}+1}$ and precision $p_n = p_{n-1}+1$; if $t = 3(n-1)+2$, then $G(t) = G_{\text{wage}}$; and if $t = 3n$, then $G(t) = G_\pi^{y_n}$, where y_n is drawn according to a normal distribution with expectation m_n and precision $\frac{1+p_n p_{n-1}}{p_n p_{n-1}}$.

In this contracting problem, $(G_{\text{wage}}, \Sigma_{\text{wage}})$ (where $\Sigma_{\text{wage}} = \mathbb{R}_+ \times a_\emptyset$) is a concave separable activity. Moreover, it satisfies Property 1 as it is unilaterally controlled by the principal. Hence, Theorem 1 implies that the wage of an agent who has not quit does not decrease over time.

[22]The interaction in each period of Milton and Holmström (1982) (as well as some of the other papers we refer to in this appendix) consists of distinct subperiods. Hence, to embed their model in our framework, we must map each period in their model into multiple periods in our framework, and scale payoffs to adjust for discounting.

Holmström (1983)

We consider the multiperiod version of this paper and focus on the contracting problem between the firm and a single worker it has hired. The contracting problem in this paper has the following components:

A parametrized quitting game

$$G_q^s = \left\langle S_p = S_a = \{Q, C\}; \right.$$

$$\left. u_p(s_p, s_a) = \begin{cases} \Pi(s) & \text{if } s_a = s_p = C \\ 0 & \text{otherwise,} \end{cases} \quad {}_a(s_p, s_a) = \begin{cases} 0 & \text{if } s_a = s_p = C \\ V(s) & \text{otherwise} \end{cases} \right\rangle,$$

where Q represents the agent's choice to quit or the firm's decision to fire him, s is the state, and $V(s)$ and $\Pi(s)$ are the agent's value of quitting and the firm's value of employing the worker in state s, respectively.

A null game

$$G_N = \langle S_p = \{p_\emptyset\}, S_a = \{a_\emptyset\}; u_p(p_\emptyset, a_\emptyset) = u_a(p_\emptyset, a_\emptyset) = 0 \rangle.$$

A game that represents the activity of wage,

$$G_{\text{wage}} = \left\langle S_p = \mathbb{R}_+, S_a = \{a_\emptyset\}; u_p(w, a_\emptyset) = -\frac{w}{\delta}, u_a(w, a_\emptyset) = \frac{U(w)}{\delta} \right\rangle,$$

where w is the agent's wage, and $U(w)$ is his strictly concave vNM utility function from wage.

The function $f(\cdot)$ is given as $G(1) = G_q^{s_1}$, where s_1 is the initial state, if either player chooses Q, then in all future periods $G(t) = G_N$; otherwise, $G(2) = G_{\text{wage}}$ and $G(3) = G_q^{s_3}$, where s_3 is the realized state. The construction of $f(\cdot)$ proceeds in an iterative manner. If, in a history h_t, there exists a period in which a player chose Q in G_q^s, then $G(t) = G_n$. Otherwise, in an even period, $G(t) = G_{\text{wage}}$, and in an odd period, $G(t) = G_q^{s_t}$.

The pair $(G_{\text{wage}}, \Sigma_{\text{wage}})$ is a concave activity (for $\Sigma_{\text{wage}} = \mathbb{R}_+ \times a_\emptyset$) that satisfies Property 1. Moreover, it is separable with respect to $f(\cdot)$ if the evolution of s_t is exogenous (as in Holmström (1983)). Therefore, Theorem 1 implies that the wage of an agent who has not quit or been fired does not decrease over time, regardless of the evolution of his productivity and outside options. Note that in Holmström (1983) the evolution of these two objects is interconnected; however, this is not needed to obtain this result.

Marcet and Marimon (1992)

We consider the contracting problem analyzed in Section 4 of this paper, where there is symmetric information and the investor (principal) has full commitment power. Moreover, Marcet and Marimon study socially efficient outcomes; hence, in general, the manager's (who is the agent in their model) consumption is not necessarily an activity, as both players may prefer to increase his consumption. To circumvent this problem, we assume that the weight the planner assigns to the manager's utility is 0 ($\lambda = 0$). The contracting problem in this paper consists of the following components:

A parametrized investment game

$$G^I_{\theta,k} = \left\langle S_p = S_a = \mathbb{R}^+; u_p(c,i) = \frac{h(k) - c - i}{\delta}, u_a(c,i) = \frac{u(c)}{\delta} \right\rangle,$$

where k is the capital stock, i is the investment in capital, c is the manager's consumption and $u(c)$ is his strictly concave utility function from consumption, $h(k)$ is the production function (denoted by $f(k)$ in their paper), and θ is an investment shock.

A parametrized (contract) breaching game

$$G^B_{\theta,k} = \left\langle S_p = \{p_\emptyset\}, S_a = \cup\{A, B\}; u_p(p_\emptyset, s_a) = 0, u_a(p_\emptyset, s_a) = \begin{cases} 0 & \text{if } s_a = A \\ V^a(k,\theta) & \text{if } s_a = B \end{cases} \right\rangle,$$

where A (B) represents the agent's choice to adhere (breach) the contract, and $v^a(k,\theta)$ is his value from an autarkic regime with an initial state of (k,θ).

A null game

$$G_N = \langle S_p = \{p_\emptyset\}, S_a = \{a_\emptyset\}; u_p(p_\emptyset, a_\emptyset) = u_a(p_\emptyset, a_\emptyset) = 0\rangle.$$

The function of $f(\cdot)$ is given as $G(1) = G^B_{\theta_1,k_1}$ for some initial state (θ_1, k_1). If the agent chooses B in period 1, then $G(t) = G_N$ for all $t \geq 2$. Otherwise, $G(2) = G^I_{\theta_1,k_1}$ and $G(3) = G^B_{\theta_2,k_2}$, where θ_2 follows from θ_1 via an autoregressive (AR1) process, and $k_2 = dk_1 + g(i_1; \theta_1)$ for some $d \in [0,1]$ and capital accumulation function $g(\cdot;\cdot)$. The construction of $f(\cdot)$ follows in an iterative manner. If the agent has played B in the past, then $G(t) = G_N$. Otherwise, in a period where $t = 2n-1$ (for some integer n), the players play the game $G(t) = G^B_{\theta_n,k_n}$, where θ_n follows an AR1 process and $k_n = dk_{n-1} + g(i_{2n-2}; \theta_n)$, and in period $t+1 = 2n$, the players play the game $G(t+1) = G^I_{\theta_n,k_n}$.

As long as the agent has adhered to the contract, the concave activity (G, Σ) given by

$$\langle S_p = \mathbb{R}_+, S_a = \{a_\emptyset\}; u_p(c, a_\emptyset) = -w, u_a(c, a_\emptyset) = u(c)\rangle$$

and $\Sigma = \mathbb{R}_+ \times a_\emptyset$ is available in every odd period. Moreover, this activity is separable with respect to the contracting problem, and since it is unilaterally controlled by the principal, it satisfies Property 1. Hence, Theorem 1 implies that the manager's consumption is nondecreasing over time under an optimal contract.

Forand and Zápal (2020)

The contracting problem in this paper consists of a finite family of games parametrized by $(v_a, v_p) \in \mathbb{R}^2$ (representing different projects) that arrive according to an exogenous distribution,

$$G^{v_a,v_p} = \langle S_p = [0,1], S_a = \{0,1\}; u_p(s_p, s_a) = s_a s_p v_p, u_a(s_p, s_a) = s_a s_p v_a\rangle,$$

where v_a and v_p are, respectively, the agent's and principal's payoffs from implementing the project, s_a represents the agent's consent to implement the project, and s_p represents the probability that they implement it.[23]

For any G^{v_a,v_p} such that $v_a v_p < 0$, $(G^{v_a,v_p}, \Sigma^{v_a,v_p})$ is a weakly concave separable activity for $\Sigma^{v_a,v_p} = \{(s_p, s_a) : s_a = 1\}$. Since the authors assume that the set of projects is finite and any deviation provides the agent with a payoff of 0, an optimal contract exists. Hence, Corollary 1 implies that there exists an optimal contract under which the agent's utility from every type of project is nondecreasing over time. Moreover, Corollary 2 establishes that there is a threshold such that a project is implemented if and only if the marginal cost of providing 1 util to the agent is below that threshold.

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