Theoretical Economics 17 (2022), 1529–1557 1555-7561/20221529

Sufficientarianism

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Sufficientarianism is a prominent approach to distributive justice in political philosophy and in policy analyses. However, it is virtually absent from the formal normative economics literature. We analyze sufficientarianism axiomatically in the context of the allocation of 0–1 normalized well-being in society. We present three characterizations of the core sufficientarian criterion, which counts the number of agents who attain a "good enough" level of well-being. The main characterization captures the "hybrid" nature of the criterion, which embodies at the same time a threshold around which the worst off in society is prioritized, and an indifference to equality in other regions. The other two characterizations relate sufficientarianism, respectively, to a liberal principle of noninterference and to a classic neutrality property.

Keywords. Sufficientarianism, distributive justice.

JEL classification. D63, D70.

1. Introduction

Sufficientarianism is a prominent approach to distributive justice in political philosophy (Frankfurt (1987, 2000, 2015)). It is "the doctrine advising the ethical observer to 'maximize the number of people who have enough' in any situation" (Roemer (2004, p. 278)). According to this approach, a concern for equality is philosophically misguided. The social objective should not be to achieve equality in the relevant space (income, well-being, opportunities, and so on). Sufficientarianism grants special status to the threshold, which defines what is "enough." As Casal (2007) has put it, as a theory of distributive justice, sufficientarianism comprises two separate principles: its "negative thesis"

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We thank Marc Fleurbaey, Jordi Massó, Hervé Moulin, John Roemer, Philippe van Parijs, and three anonymous referees for detailed and insightful comments. We also thank audiences at the Universities of Edinburgh, Catania, Murcia, Glasgow, and Colorado State, and participants in the annual meeting of the September Group. J. C. R. Alcantud is grateful to the Junta de Castilla y León and the European Regional Development Fund (Grant CLU-2019-03) for the financial support to the Research Unit of Excellence "Economic Management for Sustainability" (GECOS).

Available at https://econtheory.org. https://doi.org/10.3982/TE4232

is that society should not be concerned with distributive issues above the threshold. Its "positive thesis" is that it is desirable to bring individuals above the threshold.[1]

From an economic perspective, the relevance of sufficientarianism stems from the fact that it is applied explicitly or implicitly in policy contexts ranging from public health, to education, to poverty. The idea that universal access to certain social goods up to a given level should be guaranteed, but it is less pressing to provide additional amounts above the threshold, is rather common in political and public discourse, and in a number of policy arenas. A sufficientarian approach underpins, for example, two-tiered health care systems such as those of Canada and, to an increasing extent, of the UK: "Universal access to basic care is part of what one could call the first tier of a health care system, whereas additional care, provided via a second tier, includes treatments that are only provided to individuals when they opt in to additional insurance" (Fourie (2016, p. 194)). An emphasis on "adequacy" characterizes debates on educational policies in the USA at least since *San Antonio Independent School District v. Rodriguez* (1973), when the Supreme Court effectively sanctioned inequalities in educational funding per pupil by ruling that state-funding formulae for schools based on local taxes were not an unconstitutional violation of the Equal Protection Clause of the Fourteenth Amendment, while acknowledging the importance of securing an "adequate" education for students in all districts (Satz (2007)). In virtually all developed countries, sufficientarianism—rather than, for example, egalitarianism—is the dominant view defining moral obligations in the international realm and concerning humanitarian aid (Satz (2010)). Finally, recent proposals for a universal basic income can also be naturally justified from a sufficientarian perspective (Huseby (2010)).

In spite of its importance and popularity, the theoretical contours of sufficientarianism remain rather vague and undefined. Indeed, one wonders whether the wide appeal of sufficientarianism among both theorists and practitioners holding rather disparate views of distributive justice—including both egalitarians and antiegalitarians—may be partly explained by its embodying seemingly different, if not inconsistent, ethical intuitions.

Sufficientarianism is largely unexplored in normative economics and social choice theory. To the best of our knowledge, it lacks a formal characterization of the type that can be found for most other major approaches to distributive justice, such as egalitarianism and utilitarianism. Benbaji's claim that "as it stands, it does not have a canonical interpretation" (Benbaji (2005, p. 310)) remains valid.

In some recent contributions, Bossert, Kamaga, and Cato (2022, 2021) have applied the axiomatic method to sufficientarian principles. However, their aim is to characterize a class of social welfare orderings within the sufficientarian family and not to explore the foundations of sufficientarianism as a distributive ethic. Therefore, they assume the existence of a unique, exogenously given, normatively relevant threshold,

[1] "The notion of 'having enough' and its ethical significance are by now central to any discussion of the ethics of distribution" (Benbaji (2006, p. 327)). The literature is too vast for a comprehensive set of references. See, for example, Crisp (2003), Temkin (2003), Brown (2005), Casal (2007), Shields (2012), and Vandamme (2017).

which is part of the analytical framework and enters explicitly the formulation of several axioms, which are conceived of as restrictions on *sufficientarian* orderings. More generally, their axiomatic framework is rather different from ours. As their analysis of sufficientarian principles is motivated by an interest in population ethics, they focus on variable-population properties. Further, they impose fairness and efficiency axioms that we derive from more basic principles (see, respectively, Propositions 2 and 3 below).

In this paper, we fill this gap by examining the analytical foundations of sufficientarianism. We provide axiomatic characterizations that dissect its ethical building blocks in a novel way, thus complementing the philosophical analysis, and are a first step in developing a canonical interpretation of the sufficientarian approach.

The plausibility of the sufficientarian view clearly depends on the appropriate interpretation of the threshold that identifies what is "good enough." In turn, this raises the issue of the appropriate variable of normative concern. In his seminal paper, Frankfurt (1987) focuses on income, but this is disputable (Sen (1985)). Bossert et al. (2022, 2021) adopt individual utility as the focus of distributive concern. This is much more satisfactory in that it focuses on what ultimately matters to individuals. However, it also raises complex issues in terms of defining a meaningful, interpersonally comparable utility threshold. Alternatively, one could focus on opportunities in their "chances of success" interpretation (Mariotti and Veneziani (2011, 2018)), as in the sufficientarian approaches proposed by Axelsen and Nielsen (2015) and Nussbaum (1988, 1990). This would lend an objective nature to the alternatives and establish an absolute scale of measure, but one may object—following the literature on luck egalitarianism—that lacking any reference to individual responsibility, chances of success are not an ethically sound way of measuring opportunities.

We do not enter the debate on the appropriate variable of normative concern and analyze sufficientarianism in an abstract framework focusing generically on individual well-being. Given the central role of the threshold in sufficientarian approaches, however, we assume that well-being can be normalized and measured on a 0–1 scale (e.g., as in Karni (1998), Dhillon and Mertens (1999), Segal (2000), and Borgers and Choo (2017) for the case of utilitarianism). We consider criteria that rank profiles of normalized well-being vectors.

Our main characterization of sufficientarianism isolates four key conceptual constituents:

1. The existence of a distinguished threshold profile around which judgments are prioritarian-like, in the sense of giving strict precedence to the worst off in society.

2. A principle asserting that raising an individual t to the maximum level of well-being from a lower level—while leaving the well-being of all other agents unchanged—is an "*absolute individual improvement*" and cannot justify a switch in society's collective ranking against t.

3. A standard *separability* principle: the comparison between two profiles uses as input only the well-being of the individuals who stand to gain or lose in moving from

one profile to the other; and ignores the precise level of well-being of indifferent individuals.

4. A standard requirement of *efficiency* (monotonicity).

We show that these four properties fully characterize sufficientarianism (Theorem 1). The main interest of this way of characterizing sufficientarianism is, in our opinion, that it brings to light the tension between two different ethical stances embedded in the criterion: the focus on lifting the well-being of low well-being individuals around the threshold profile, captured in (1), which tends toward equalizing well-being; and the simultaneous tolerance of major inequalities: as we shall elaborate, the principle in (2) can have extremely inegalitarian consequences.

What is more, we show that these properties imply a key *impartiality* principle (anonymity), asserting that the identities of the agents do not count in the criterion.

The other two characterizations emphasize in different ways the distinctive sufficientarian focus on the individual (captured by the idea of absolute improvements in the main result). Our second characterization (Theorem 2) shows how sufficientarianism satisfies a principle of respect for autonomy with a liberal flavor recently proposed in the literature (Mariotti and Veneziani (2009, 2011, 2013), Lombardi and Veneziani (2016), and Alcantud (2013)). Finally, our third characterization (Theorem 4) uses the classical neutrality axiom of social choice. Evaluating to what extent sufficientarianism incorporates neutrality-like principles will help us understand the informational content of sufficientarian rankings, namely which order-preserving transformations they are invariant to.

An important point to note is that our aim is not to *defend* sufficientarianism as a comprehensive approach in political philosophy and normative economics. Rather, it is to provide a full characterization of the core sufficientarian view so as to clarify its foundations and implications. The use of our axiomatizations can well be negative: if some of the properties in the characterizations we offer are considered unacceptable, then sufficientarianism must be rejected.

2. The framework

2.1 *Preliminaries*

Let $\mathcal{T} = \{1, \dots, T\}$ denote a society of T individuals with $T > 1$. Individual well-being is measured on a $[0, 1]$ scale, where 0 and 1 denote, respectively, the lowest and highest levels of well-being attainable by an individual, and for any agent $t \in \mathcal{T}$, $a_t \in B \equiv [0, 1]$ denotes t's level of well-being or welfare.[2] We are interested in a criterion that guides the allocation of well-being among the T individuals.

A *well-being profile* (or simply a *profile*) is a point $a = (a_1, a_2, \dots, a_T)$ in the box $B^T \equiv [0, 1]^T$. The points $(0, 0, \dots, 0) \in B^T$ and $(1, 1, \dots, 1) \in B^T$ can be thought of as *Hell* and *Heaven*, respectively, and individual t *is in Hell* (resp., *Heaven*) at $a \in B^T$ if $a_t = 0$ (resp.,

[2] None of our results depends on a specific interpretation of the distribuendum, for example, income, wealth, utility, chances of success, which will be generically referred to as well-being.

$a_t = 1$). For any $a \in B^T$, $t \in \mathcal{T}$, and $a'_t \in [0, 1]$, we denote (a'_t, a_{-t}) the profile obtained from a by replacing a_t with a'_t, that is, $(a'_t, a_{-t}) = (a_1, \dots, a_{t-1}, a'_t, a_{t+1}, \dots, a_T)$.

For all $a, b \in B^T$, we write $a \geq b$ to mean $a_t \geq b_t$, for all $t \in \mathcal{T}$; $a > b$ to mean $a \geq b$ and $a \neq b$; and $a \gg b$ to mean $a_t > b_t$, for all $t \in \mathcal{T}$.

A permutation π is a bijective mapping of $\mathcal{T}$ onto itself. For all $a \in B^T$ and all π, $a^\pi \equiv \pi(a) = (a_{\pi(t)})_{t \in \mathcal{T}}$ is a permutation of a.

Given a binary relation $\succcurlyeq$ on a set X and $x, y \in X$, we write $x \succ y$ if and only if $x \succcurlyeq y$ and $y \not\succcurlyeq x$, and $x \sim y$ if and only if $x \succcurlyeq y$ and $y \succcurlyeq x$. We aim to specify desirable properties for a binary relation $\succcurlyeq$ on the box B^T, interpreted as a *social welfare criterion* (or simply a *criterion*).[3]

2.2 *The key properties*

We discuss here four properties of sufficientarianism. These properties are not to be taken as compelling desiderata for a criterion, but just as ethical features of sufficientarianism that we consider relevant. We will show that they are also exhaustive, that is, jointly equivalent to sufficientarianism.

The first two properties are completely standard. One imposes a mild requirement of efficiency.

MONOTONICITY. For all $a, b \in B^T$, $a > b \Rightarrow a \succcurlyeq b$.

The other standard property introduces a form of separability in the criterion by requiring some independence across individuals.

SEPARABILITY. Let $a, b, a', b' \in B^T$ be such that, for some $t \in \mathcal{T}$,

$$a_t = b_t \quad \text{and} \quad a'_t = b'_t,$$
$$a' = (a'_t, a_{-t}),$$
$$b' = (b'_t, b_{-t}).$$

Then $a' \succcurlyeq b'$ whenever $a \succcurlyeq b$.

The logic underlying this axiom is well known and is common to a host of separability axioms in social choice and decision theory. The only information the criterion should use to compare two profiles is the well-being of those individuals who stand to gain or lose by being at one profile rather than the other. The criterion should ignore the exact level of well-being of the individuals who are indifferent among the two profiles. So, if individual t has the same well-being $a_t = b_t$ at profiles a and b and society prefers a

[3] Recall that a relation $\succcurlyeq$ on a set X is said to be: *reflexive* if, for any $x \in X$, $x \succcurlyeq x$; *complete* if, for any $x, y \in X$, $x \neq y$ implies $x \succcurlyeq y$ or $y \succcurlyeq x$; *transitive* if, for any $x, y, z \in X$, $x \succcurlyeq y \succcurlyeq z$ implies $x \succcurlyeq z$. An *ordering* is a reflexive, complete, and transitive binary relation. A social welfare criterion is not necessarily an ordering, but completeness, reflexivity, and transitivity are properties that will be imposed in addition to the axioms below.

to b, it should not change its preference if the levels of well-being of that person change in both profiles to a common new amount $a'_t = b'_t$.

We now introduce two new principles that are more distinctive of sufficientarianism. The first incorporates an explicit emphasis on *levels* of well-being.

Absolute individual improvement. Let $a, b \in B^T$ be such that $a \succ b$ and $a_t < 1$, some $t \in \mathcal{T}$. Then $(1, a_{-t}) \not\prec (b'_t, b_{-t})$ for all $b'_t \in B$.

In words, starting from a situation where a is strictly socially preferred to b, this ranking cannot be reversed if the well-being of an individual t is strictly improved and taken to its maximum level at profile a, no matter how t's well-being changes at b, provided that the well-being of all other agents is unchanged at both profiles. In this sense, taking t to Heaven is an absolute individual improvement that cannot justify a reversal in the collective ranking.

Absolute individual improvement captures the sufficientarian idea that absolute levels of well-being matter, and that they matter for individuals—not because of distributive considerations. As Frankfurt (1997, p. 6) puts it, "what is of genuine moral concern is not formal but substantive. It is whether people have good lives ... The evil lies simply in the unmistakable fact that bad lives are bad." Thus, without any changes in everyone else's well-being, lifting an individual to the highest possible level of well-being, which surely implies a good life no matter how high the threshold that distinguishes a good life, cannot make a profile strictly worse than another profile, when it was initially strictly better. From this perspective, by focusing on changes affecting a single agent, absolute individual improvement captures the fundamentally individualistic nature of the sufficientarian approach. According to Frankfurt (2000, p. 100), for example, "The rights to which a person is entitled do not depend upon any comparison with the rights others possess. The entitlements of each person are based simply upon the relevant characteristics and circumstances of that person. The governing moral requirement is to avoid being arbitrary in the attribution benefits and disadvantages to each individual."

The important feature of the principle is that it is assumed to be valid independently of the possibly harsh distributional consequences. For example, on nonegalitarian views it is typically conceivable that a sacrifice may justifiably be asked of some individual t' for the sake of bringing another individual t to a higher level of well-being. But even nonegalitarians may balk at imposing the same sacrifice on t' just for the sake of bringing t to the very maximum level of well-being if t's gain from this achievement was minuscule. Consider for instance a utilitarian view: suppose that $a_1 + a_2 > b_1 + b_2$ with $a_2 < b_2$, so that a is utilitarian-better than b because the gain of $a_1 - b_1$ for the first individual is larger than the loss of $b_2 - a_2$ for the second individual. But for any $\gamma \in (0, 1)$ for which $\gamma > 1 - (b_2 - a_2)$, the profile $(1, a_2)$ is utilitarian-worse than (γ, b_2), since the gain of $1 - \gamma$ of the first individual does not now compensate for the imposition of the loss $b_2 - a_2$ on the second individual. Yet absolute individual improvement (and sufficientarianism) does justify this loss no matter how high t's level of well-being γ is and, therefore, no matter how little the gain for her by reaching the very top.

The next axiom posits the existence of a distinguished profile around which the criterion is prioritarian.

PRIORITARIAN THRESHOLD. There exists $\beta \in (0, 1)$ such that $(\beta, \beta, \ldots, \beta) \succ b$ for all $b \in B^T$ for which $b_t < \beta$, some $t \in \mathcal{T}$, and $b_i = b_j$ for all $i, j \neq t$.

Prioritarian threshold says that there exists *some* critical profile in which all agents have a common well-being level β that strictly dominates any profile in which one agent has well-being strictly lower than β. Several observations are in order. First, it is intuitive (and consistent with monotonicity) to conceive of the common level of well-being β as being "sufficiently high," hence the terminology "threshold," although this feature is not formally present in the axiom. Second, the "prioritarian" component of the axiom consists in the fact that for $b_i = b_j > \beta$, it states that it is (strictly) not worth decreasing somebody's well-being below β for the sake of increasing everybody else's above it (while for $b_i = b_j \leq \beta$ it simply expresses a mild concern for efficiency). Third, the axiom does not exclude the existence of *multiple* thresholds (as, e.g., in Roemer (2004)): for an extreme example, consider that, in addition to monotonicity, the leximin or maximin criteria satisfy prioritarian threshold for a *continuum* of values β—uniqueness will be implied by the conjunction with absolute individual improvement (Lemma 1). Fourth, by excluding the case $\beta = 1$, and thus preventing the threshold profile from being Heaven, the axiom does not enforce the rather uncontroversial statement that Heaven is better than anything else. Finally, the common critical level of well-being β is restricted to be strictly positive in order to avoid making the property trivial: *any* social ordering vacuously satisfies the axiom when $\beta = 0$ since the condition $b_t < \beta = 0$ cannot be satisfied for $b \in B^T$.

3. THE CORE SUFFICIENTARIAN VIEW

In this section, we define and characterize the core sufficientarian criterion. Let $\alpha \in (0, 1)$ denote an ethically critical threshold identifying a *sufficient*, or satisfactory level of well-being. Then, for all $a \in B^T$, let $P(a, \alpha) = \{t \in \mathcal{T} : a_t \geq \alpha\}$ denote the set of individuals who have (at least) a sufficient level of well-being at profile a, and let $n(a, \alpha)$ be their number—formally, the cardinality of $P(a, \alpha)$. Then, for all $a, b \in B^T$, define the sufficientarian criterion $\succsim_\alpha^s$ on B^T as follows:

$$a \succsim_\alpha^s b \quad \Leftrightarrow \quad n(a, \alpha) \geq n(b, \alpha).$$

Our first result proves that the sufficientarian criterion satisfies the main axioms.

PROPOSITION 1. *The sufficientarian social welfare relation $\succsim_\alpha^s$ on B^T is an ordering, and it satisfies absolute individual improvement, prioritarian threshold, monotonicity, and separability.*

PROOF. It is immediate to see that $\succsim_\alpha^s$ on B^T is an ordering and that it satisfies monotonicity, separability and prioritarian threshold by setting $\beta = \alpha$ (and only for this choice of β). In fact, it satisfies the stronger condition that $n(a, \beta) > n(b, \beta)$ implies $a \succ_\alpha^s b$.

To see that $\succsim_\alpha^s$ on B^T satisfies absolute individual improvement, consider $a, b \in B^T$ such that $a \succ_\alpha^s b$ and $a_t < 1$ for some $t \in \mathcal{T}$. By definition, this implies $n(a, \alpha) > n(b, \alpha)$. Consider $a', b' \in B^T$ such that $a' = (a'_t, a_{-t})$, $b' = (b'_t, b_{-t})$, and $a'_t = 1$. Since $a'_t \geq b'_t$, $n(a', \alpha) \geq n(b', \alpha)$ and so $a' \succsim_\alpha^s b'$, as sought. □

Note that while the sufficientarian criterion is egalitarian at a distinguished level of well-being β, it is not difficult to show that it violates important egalitarian principles, such as the Hammond equity axiom or the Pigou–Dalton condition (Hammond equity says that if $a, b \in B^T$ are such that $a_i < b_i < b_j < a_j$ for some $i, j \in \mathcal{T}$, and $a_k = b_k$ for all $k \in \mathcal{T} \setminus \{i, j\}$, then $b \succcurlyeq a$. The Pigou–Dalton condition states that if $a, b \in B^T$ are such that $b_i = a_i - \delta \geq a_j + \delta = b_j$ for some $i, j \in \mathcal{T}$ and some $\delta > 0$, and $a_k = b_k$ for all $k \in \mathcal{T} \setminus \{i, j\}$, then $b \succcurlyeq a$. To see that the sufficientarian criterion does not incorporate the intuitions behind these properties, let $\alpha = 1/2$ and consider two profiles $a, b \in B^T$ such that $a = (\frac{5}{8}, 0, 1, 1, 1, \ldots, 1)$ and $b = (\frac{3}{8}, \frac{1}{4}, 1, 1, 1, \ldots, 1)$. By definition $a \succ_\alpha^s b$, which violates both axioms).

The main result of the paper is the following.

Theorem 1. *A social welfare ordering $\succcurlyeq$ on B^T satisfies absolute individual improvement, prioritarian threshold, monotonicity, and separability if and only if it is the sufficientarian social welfare ordering $\succcurlyeq_\alpha^s$.*

Before presenting the proof of Theorem 1, we need a series of preliminary results.

3.1 *The ethical threshold, decency, and penury avoidance*

We prove here a fundamental lemma that is useful in the proof of a converse to Proposition 1, but is also interesting in its own right. It establishes the existence of a *unique* ethical threshold β such that profiles in which the well-being of all agents is at least β are strictly better than profiles in which some agents have low well-being.

Lemma 1. *Let $\succcurlyeq$ be an ordering on B^T that satisfies absolute individual improvement, prioritarian threshold, and monotonicity. Then there exists a unique $\beta \in (0, 1)$ such that for all $a, b \in B^T$, $a_t \geq \beta$, all $t \in \mathcal{T}$, and $b_t < \beta$, some $t \in \mathcal{T} \Rightarrow a \succ b$.*

Proof. We first prove that there exists a $\beta \in (0, 1)$ such that for all $a, b \in B^T$, $a_t \geq \beta$, all $t \in \mathcal{T}$, and $b_t < \beta$, some $t \in \mathcal{T} \Rightarrow a \succ b$. Then we prove that such β is unique.

1. Let $\beta \in (0, 1)$ be a parameter value for which prioritarian threshold is satisfied, and consider any $a, b \in B^T$ such that $a_t \geq \beta$, all $t \in \mathcal{T}$, and $b_j < \beta$, some $j \in \mathcal{T}$. By monotonicity, $a \succcurlyeq (\beta, \beta, \ldots, \beta, \beta)$. By prioritarian threshold, $(\beta, \ldots, \beta) \succ (1, \ldots, 1, b_j, 1, \ldots, 1)$. By monotonicity, $(1, \ldots, 1, b_j, 1, \ldots, 1) \succcurlyeq b$. The desired result then follows by transitivity.

2. In order to prove uniqueness, suppose by contradiction that there are $\beta, \beta' \in (0, 1)$, $\beta > \beta'$, such that for all $a, b \in B^T$,

$$n(a, \beta) = T > n(b, \beta) \quad \text{implies} \quad a \succ b, \tag{1}$$

$$n(a, \beta') = T > n(b, \beta') \quad \text{implies} \quad a \succ b. \tag{2}$$

By (2), $(\beta', \ldots, \beta') \succ (\beta, \ldots, \beta, 0)$. By absolute individual improvement, $(\beta', \ldots, \beta', 1) \succcurlyeq (\beta, \ldots, \beta)$, a contradiction with (1). □

In view of this result, when an ordering on B^T satisfies absolute individual improvement, monotonicity, and prioritarian threshold, we will henceforth assume that the latter axiom holds for the unique parameter value α that satisfies the consequent of Lemma 1.

In order to better understand the implications of Lemma 1, it is instructive to consider two properties recently proposed by Roemer (2004, pp. 274 and 277), called *universal decency* and *avoidance of penury*. Let $\beta_1, \beta_2, \beta_3 \in (0, 1)$ represent three prespecified, ethically relevant well-being thresholds such that $\beta_1 \leq \beta_2 \leq \beta_3$. If the inequalities are strict, they can be interpreted, respectively, as the levels of well-being associated with a *life barely worth living*, a *mediocre life*, and a *good or excellent life*.

Universal decency states that a profile such that all individuals flourish is preferable to one in which only some of them enjoy a good or excellent life.

Universal decency. For all $a, b \in B^T$, $a_t \geq \beta_3$ all $t \in \mathcal{T}$, and $b_t < \beta_3$, some $t \in \mathcal{T} \Rightarrow a \succ b$.

Avoidance of penury states that a profile such that all individuals have a decent life is preferable to one in which some of them have a life not worth living.

Avoidance of penury. For all $a, b \in B^T$, $a_t \geq \beta_2$, all $t \in \mathcal{T}$, and $b_t < \beta_1$, some $t \in \mathcal{T} \Rightarrow a \succ b$.

The property in the consequent of Lemma 1 is somehow reminiscent of universal decency and avoidance of penury. However, in our case the property follows from the other, more basic properties. What is more, prioritarian threshold is a merely existential property and, as noted before, it does not impose any restriction on the number of thresholds.

Before proceeding with the general characterization, armed with Lemma 1, it is possible to provide a simple graphical representation of the key steps of the proof of our main result in the case with $T = 2$. This may help gauge the relevance of the various axioms. The uninterested reader may safely skip to the next section for the full proof.

3.1.1 *An illustration with $T = 2$* Consider the following partition of the box B^2:

$$A = \{a \in B^2 : a_1 \geq \alpha, a_2 \geq \alpha\},$$
$$B_1 = \{a \in B^2 : a_1 < \alpha, a_2 \geq \alpha\},$$
$$B_2 = \{a \in B^2 : a_1 \geq \alpha, a_2 < \alpha\},$$
$$C = \{a \in B^2 : a_1 < \alpha, a_2 < \alpha\}.$$

The following subsets will also be useful in the proof below:

$$B_1^F = \{a \in B_1 : a_2 = 1\}, \qquad B_1^{\text{Int}} = B_1 \setminus B_1^F.$$

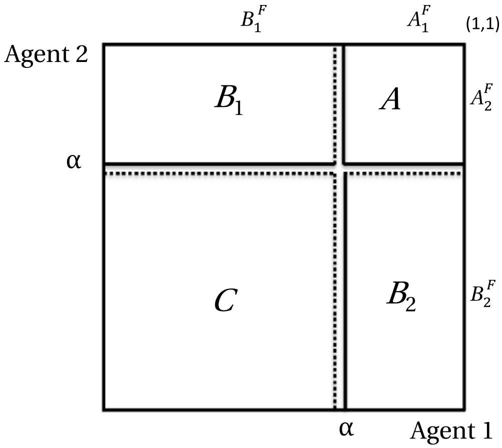

FIGURE 1. The subsets defined in Section 3.1.1.

The set B_1^F is the intersection between B_1 and the frontier of the box, while B_1^{Int} can be loosely interpreted as the "interior" of B_1 in the sense that for all $a \in B_1^{\text{Int}}$, we have $a_t < 1$, $t = 1, 2$. Similarly,

$$B_2^F = \{a \in B_2 : a_1 = 1\}, \qquad B_2^{\text{Int}} = B_2 \setminus B_2^F.$$

Finally, let

$$A_1^F = \{a \in A : a_1 < 1, a_2 = 1\},$$
$$A_2^F = \{a \in A : a_1 = 1, a_2 < 1\},$$
$$A^{\text{Int}} = A \setminus \big(A_1^F \cup A_2^F \cup \{(1, 1)\}\big).$$

The partition can be illustrated in the diagram shown in Figure 1.

Assume that the social welfare ordering satisfies absolute individual improvement, prioritarian threshold, monotonicity, and separability. We know that this implies that Lemma 1 holds and, therefore, $a \in A$, $b \in B^2 \setminus A$, implies $a \succ b$.

First of all, we prove that $B_1^{\text{Int}} \cup B_2^{\text{Int}}$ is contained in one equivalence class. Suppose, by way of contradiction, that $a \in B_1^{\text{Int}}$, $b \in B_2^{\text{Int}}$ but $a \nsim b$. By completeness, either $a \succ b$ or $b \succ a$. Suppose first that $a \succ b$. Then consider $a', b' \in B^2$ such that $a'_2 = 1$, $b'_2 = \alpha$, $a'_1 = a_1$, and $b'_1 = b_1$. By absolute individual improvement, $a' \succcurlyeq b'$. However, noting that $a' \in B_1$ and $b' \in A$, Lemma 1 implies $b' \succ a'$, yielding the desired contradiction. A similar argument—perturbing agent 1's well-being instead—rules out $b \succ a$.

We conclude that for all $a \in B_1^{\text{Int}}$ and $b \in B_2^{\text{Int}}$ it must be $a \sim b$, and transitivity implies that for all $a, b \in B_i^{\text{Int}}$, $i = 1, 2$ it must be $a \sim b$.

Then separability and transitivity immediately imply that any pair of profiles in C, respectively, A^{Int}, B_1^F, B_2^F, A_1^F, and A_2^F, are indifferent.

Finally, by Lemma 1, $(\alpha, \alpha) \succ (1, b_2)$ for any $b_2 < \alpha$. By absolute individual improvement, $(\alpha, 1) \succcurlyeq (1, 1)$. By monotonicity, $(1, 1) \succcurlyeq (\alpha, 1)$ and, therefore, $(1, 1) \sim (\alpha, 1)$. Similarly, $(1, 1) \sim (1, \alpha)$. By transitivity, this implies that any profiles in $A_1^F \cup A_2^F \cup \{(1, 1)\}$

are indifferent. By separability and transitivity, it follows that A is an equivalence class.

The rest of the ranking then follows from separability and transitivity.

3.2 *Impartiality*

We now proceed to establish an auxiliary result, which shows that the distributions where everyone is above the ethical threshold are all equivalent. This will allow us to derive an important property of impartiality.

Lemma 2. *Let $\succcurlyeq$ be an ordering on B^T that satisfies absolute individual improvement, prioritarian threshold, monotonicity, and separability. Then $a \sim b$ for all $a, b \in B^T$ with $n(a, \alpha) = n(b, \alpha) = T$.*

Proof. It suffices to prove that $a \sim (\alpha, \dots, \alpha)$ for all $a \in B^T$ with $n(a, \alpha) = T$ because this yields the desired result by the transitivity of $\sim$.

Fix $a \in B^T$ with $n(a, \alpha) = T$. The fact that $a \succcurlyeq (\alpha, \dots, \alpha)$ follows from monotonicity. In order to prove $(\alpha, \dots, \alpha) \succcurlyeq a$, note, first of all that Lemma 1 implies $(\alpha, \dots, \alpha) \succ (1, \alpha, \dots, \alpha, \alpha/2)$. Therefore, as $\alpha < 1$, by absolute individual improvement, $(\alpha, \dots, \alpha, 1) \succcurlyeq (1, \alpha, \dots, \alpha, 1)$ and by separability, $(\alpha, \dots, \alpha) \succcurlyeq (1, \alpha, \dots, \alpha)$. Similarly, Lemma 1 implies $(\alpha, \alpha, \dots, \alpha) \succ (\alpha/2, 1, \dots, 1)$. Therefore, by absolute individual improvement, $(1, \alpha, \dots, \alpha) \succcurlyeq (1, \dots, 1)$.

Next, by monotonicity, $(1, \dots, 1) \succcurlyeq a$. The desired result then follows from transitivity as $(\alpha, \dots, \alpha) \succcurlyeq (1, \alpha, \dots, \alpha) \succcurlyeq (1, \dots, 1) \succcurlyeq a$. □

Lemma 2 formalizes the "negative thesis" of sufficientarianism, according to which distributive issues cease to be a concern once all agents are above the threshold: "if everyone had enough, it would be of no moral consequence whether some had more than others" (Frankfurt (1987, p. 21)). The normative appeal of Lemma 2 may perhaps derive from its providing a resolution of the tension between equality and freedom. As Arneson (2000, p. 55) puts it, the "negative thesis" addresses "the worry about illiberal restriction of freedom by leaving a wide space of above-threshold matters wherein individual freedom is not constrained by social justice."[4]

Given the previous results, we can now prove that under the conditions of Lemmas 1–2, the social welfare criterion also satisfies a notion of fairness as *impartiality*.

Anonymity. For all $a, b \in B^T$, $a = b^\pi$ for some permutation $\pi \Rightarrow a \sim b$.

Anonymity requires the allocation rule to be insensitive to individual identities.

Proposition 2. *Let $\succcurlyeq$ be an ordering on B^T that satisfies absolute individual improvement, prioritarian threshold, monotonicity, and separability. Then $\succcurlyeq$ satisfies anonymity.*

[4]As noted by an anonymous referee, sufficientarianism may be interpreted precisely as asserting that the role of government is limited, and instead of maximizing social welfare, it should aim to ensure acceptable well-being for as many individuals as possible.

Proof. Let $a, b \in B^T$, and let π be a permutation such that $a = b^\pi$. Because any permutation is a composition of transpositions (i.e., permutations of two elements), and given the transitivity of $\succcurlyeq$, in order to prove $a \sim b$ we just need to assume that π is a transposition. For notational convenience, we consider the case $a = (x, y, a_3, \dots, a_T)$ and $b = (y, x, a_3, \dots, a_T)$. The other transpositions can be dealt with using similar arguments. Without loss of generality, let $x > y$.

Case 1: $x \geq \alpha$. Two subcases arise.

If $y \geq \alpha$, then $a = (x, y, a_3, \dots, a_T) \sim (y, x, a_3, \dots, a_T) = b$ if and only if $(x, y, 1, \dots, 1) \sim (y, x, 1, \dots, 1)$ by separability, and the latter equivalence holds true by Lemma 2.

Assume instead $y < \alpha$.

Suppose, by way of contradiction, that $b \succ a$. Then separability yields $(y, x, 1, \dots, 1) \succ (x, y, 1, \dots, 1)$. If $x < 1$, then by absolute individual improvement, $(y, 1, 1, \dots, 1) \succcurlyeq (x, 1, 1, \dots, 1)$, against Lemma 1. Suppose $x = 1$, that is, $(y, 1, 1, \dots, 1) \succ (1, y, 1, \dots, 1)$. By Lemma 2, $(1, 1, \dots, 1) \sim (1, (1 + \alpha)/2, 1, \dots, 1)$, and separability implies $(y, 1, \dots, 1) \sim (y, (1 + \alpha)/2, 1, \dots, 1)$. By transitivity, $(y, (1 + \alpha)/2, 1, \dots, 1) \succ (1, y, 1, \dots, 1)$ and absolute individual improvement yields $(y, 1, \dots, 1) \succcurlyeq (1, \dots, 1)$, against Lemma 1. A similar argument rules out $a \succ b$.

Case 2: $x < \alpha$.

Let us first prove $(0, z, a_3, \dots, a_T) \sim (0, 0, a_3, \dots, a_T)$ when $z < \alpha$. By contradiction, suppose $(0, z, a_3, \dots, a_T) \succ (0, 0, a_3, \dots, a_T)$, noting that $\succcurlyeq$ is monotonic. By separability, $(\alpha, z, a_3, \dots, a_T) \succ (\alpha, 0, a_3, \dots, a_T)$. We use Case 1 and transitivity to deduce $(\alpha, 0, a_3, \dots, a_T) \sim (0, \alpha, a_3, \dots, a_T)$ and then $(\alpha, z, a_3, \dots, a_T) \succ (0, \alpha, a_3, \dots, a_T)$. By absolute individual improvement, $(1, z, a_3, \dots, a_T) \succcurlyeq (\alpha, \alpha, a_3, \dots, a_T)$. From separability, we get $(1, z, 1, \dots, 1) \succcurlyeq (\alpha, \alpha, 1, \dots, 1)$, in contradiction with Lemma 1.

Similarly, one can prove $(z, 0, a_3, \dots, a_T) \sim (0, 0, a_3, \dots, a_T)$ when $z < \alpha$.

Hence, by transitivity, and noting that $\alpha > x > y$, $(0, y, a_3, \dots, a_T) \sim (0, x, a_3, \dots, a_T)$ and $(x, 0, a_3, \dots, a_T) \sim (y, 0, a_3, \dots, a_T)$. By separability, the former equivalence implies $(x, y, a_3, \dots, a_T) \sim (x, x, a_3, \dots, a_T)$, while the latter equivalence implies $(x, x, a_3, \dots, a_T) \sim (y, x, a_3, \dots, a_T)$. The result then follows from the transitivity of $\sim$. $\square$

Anonymity is often considered to be a fundamental criterion of fairness in normative judgements as it rules out the possibility that agents' identities influence social evaluations. By Proposition 2, we do not need to impose it separately: it follows from our key axioms.

In order to grasp the intuition of the proof, the reader can look at the graphical illustration of the case $T = 2$, showing that $B_1^{\text{Int}} \cup B_2^{\text{Int}}$ is contained in an equivalence class. In fact, the logic of the proof is essentially the same, as $B_1^{\text{Int}}, B_2^{\text{Int}}$ are on opposite sides of the 45-degree line. The key steps of the argument in Proposition 2 are Lemma 1, showing that a unique threshold exists; and Lemma 2, showing that there is "indifference at the top." Then our "perturbational" axioms (notably separability and absolute individual improvement) force the indifference at the top to carry over to profiles that are permutations of one another and have at most two elements below the threshold.

3.3 *Completing the characterization*

We are now ready to establish the main result.

PROOF OF THEOREM 1. Proposition 1 proves necessity.

To prove sufficiency, let $\succcurlyeq$ be an ordering on B^T that satisfies the axioms. By Lemma 1, there is a unique $\alpha \in (0, 1)$ such that for all $a, b \in B^T$, $a_t \geq \alpha$, all $t \in \mathcal{T}$, and $b_t < \alpha$, some $t \in \mathcal{T}$ implies $a \succ b$. We must show that for each $a, b \in B^T$, $a \succcurlyeq b$ if and only if $n(a, \alpha) \geq n(b, \alpha)$.

We show that for all natural numbers h, such that $T \geq h \geq 0$, and for all $a, b \in B^T$,

$$n(a, \alpha) = T - h > n(b, \alpha) \quad \text{implies} \quad a \succ b, \tag{3}$$

and

$$n(a, \alpha) = T - h = n(b, \alpha) \quad \text{implies} \quad a \sim b. \tag{4}$$

We proceed by induction on h.

($h = 0$) Lemma 2 proves (4), while (3) follows from Lemma 1.

(Inductive step) Suppose that (3) and (4) are true for all $0 \leq h \leq k - 1 < T$, and consider $h = k$. We prove first that (4) must hold.

Suppose, by way of contradiction, that there exist $a, b \in B^T$ such that $n(a, \alpha) = T - k = n(b, \alpha)$ but $a \nsim b$. By completeness, suppose that $a \succ b$ without loss of generality.

Suppose $T - k > 0$. Then, noting that $T - k < T$, there are $t, t' \in \mathcal{T}$ such that $a_t \geq \alpha$ and $b_{t'} < \alpha$. By Proposition 2 and transitivity, we assume that $t = t'$ without loss of generality.

If $a_t < 1$, then absolute individual improvement implies $a' = (1, a_{-t}) \succcurlyeq b' = (\alpha, b_{-t})$. This contradicts the induction hypothesis for (3), noting that $n(b', \alpha) = n(b, \alpha) + 1 > n(a, \alpha) = n(a', \alpha)$.

Therefore, suppose that for all $k \in \mathcal{T}$, $a_k \geq \alpha$ implies $a_k = 1$. By Lemma 2 and separability, it follows that $a = (1, a_{-t}) \sim (\alpha, a_{-t}) = a'$ and, therefore, $a' \succ b$ by transitivity. Then absolute individual improvement implies $a = (1, a_{-t}) \succcurlyeq b' = (\alpha, b_{-t})$. This contradicts the induction hypothesis for (3), noting that $n(b', \alpha) = n(b, \alpha) + 1 > n(a, \alpha)$.

Suppose $T - k = 0$.

If there are $t, t' \in \mathcal{T}$ such that $a_t = b_{t'}$, then consider a permutation b^π of b such that $b^\pi_t = b_{t'}$. By Proposition 2 and transitivity, $a \succ b^\pi$. Let $a', b' \in B^T$ be such that $a'_t \geq \alpha$, $b'_t \geq \alpha$, $a'_t = b'_t$, and $a'_j = a_j$, $b'_j = b^\pi_j$, for all $j \neq t$. By separability, $a' \succ b'$, which contradicts the induction hypothesis for (4). Therefore, suppose that there are no $t, t' \in \mathcal{T}$ such that $a_t = b_{t'}$. By monotonicity, $a \succ b$ implies that there is at least some $t \in \mathcal{T}$ such that $a_t > b_t$. Then consider $a' \in B^T$ such that $a_t > a'_t = b_t$ and $a_j = a'_j$ for all $j \neq t$. Noting that $n(a, \alpha) = n(a', \alpha) = n(b, \alpha) = 0$, by the previous argument it must be both $a \sim a'$ and $a' \sim b$, which yields the desired contradiction by transitivity.

Next, we prove that (3) must also hold.

Suppose, by way of contradiction, that there exist $a, b \in B^T$ such that $n(a, \alpha) = T - k > n(b, \alpha)$ but $a \not\succ b$. By completeness, this implies that $b \succcurlyeq a$. Let us fix $t \in \mathcal{T}$ such that $a_t < \alpha$, $b_t < \alpha$ (this is without loss of generality by Proposition 2 and transitivity, and noting that $T - k < T$).

Suppose $b \succ a$. Then by absolute individual improvement, $b' = (1, b_{-t}) \succcurlyeq a' = (\alpha, a_{-t})$, which violates the induction hypothesis for (3).

Suppose $b \sim a$. Consider $a' \in B^T$ such that $a'_t = b_t$ and $a'_j = a_j$ for all $j \neq t$. Because $n(a, \alpha) = T - k = n(a', \alpha)$, by the previous argument, it follows that $a \sim a'$, and so by transitivity $b \sim a'$. Separability then implies $b' = (\alpha, b_{-t}) \sim a'' = (\alpha, a'_{-t})$, however, noting that $n(a'', \alpha) = T - (k - 1) > n(b', \alpha) = n(b, \alpha) + 1$, a contradiction ensues from the induction hypothesis for (3). □

Remark. Theorem 1 continues to hold even if the restrictions $b_t < a_t$ and $b'_t = 1$ are imposed in absolute individual improvement.[5] This is a logical weakening, which leads to greater independence between the axioms. However, we have opted for the current formulation of the result because the weaker version of the axiom seems to be less easily interpretable. Furthermore, the current axiom viewed as a *necessary* condition highlights more directly a strong implication of sufficientarianism.

The proof that the properties in Theorem 1 are independent can be found in Appendix B.1.

4. Alternative characterizations

In this section, we extend our axiomatic approach to dissect sufficientarianism from two alternative perspectives. Specifically, holding fixed separability and prioritarian threshold, we provide characterizations based on key axioms that are different from absolute individual improvement. The first characterization relates sufficientarianism to some elements of liberal thought, while the second harks back to the classical social choice discussion about the informational content of welfare comparisons. In the concluding part of the section, we also offer some remarks on the property of continuity.

4.1 *Liberal roots of sufficientarianism*

Next, we consider a *liberal principle of noninterference* recently proposed by Mariotti and Veneziani (2013).

Noninterference. Let $a, b, a', b' \in B^T$ be such that, for some $t \in \mathcal{T}$,

$$(a_t - a'_t)(b_t - b'_t) > 0,$$
$$a' = (a'_t, a_{-t}), \quad \text{and} \quad b' = (b'_t, b_{-t}).$$

Then $b' \not\succ a'$ whenever $a'_t > b'_t$ and $a \succ b$.

Observe that the first inequality condition in the axiom says that in moving from the unprimed to the primed pair of profiles, individual t either strictly loses at both profiles or strictly gains at both profiles. Then, noninterference says the following: suppose

[5] We are grateful to an anonymous referee for pointing this out to us.

that society strictly prefers a to b and that an individual's well-being changes, for the better or for the worse, while all other agents are unaffected; then society should not reverse its strict preferences in a way that is adverse to the individual whose well-being has changed.

This principle captures the liberal idea of the existence of a sphere of individual autonomy when others are unaffected by a change in someone's circumstances. In particular, society should not use as ethical arguments the reasons behind changes in well-being that concern exclusively one individual, for example, it should not punish further an individual for misfortunes that, because of his negligence, caused harm to him, and him alone (while society remains free to compensate this individual). As argued in Mariotti and Veneziani (2020), the roots of noninterference—and its normative foundations—can be traced back to John Stuart Mill's *Harm Principle*.

Noninterference may be deemed objectionable "as it requires ignoring all information concerning the size of the changes in welfare, and their potentially relevant implications for total utility or for the welfare of the worst off" (Mariotti and Veneziani (2020, p. 572)). Similarly, the restriction to changes in well-being that affect only one individual may be considered arbitrary. The key point to note here is that we are not proposing noninterference as an independently desirable property. Rather, our interest in a liberal axiom falls within our general aim to unpack the normative building blocks of the sufficientarian approach.

From this perspective, one important characteristic of noninterference is its focus on changes in the well-being of a single agent. "The individualistic and nonaggregative nature of noninterference (focusing on changes in the situation of a single agent while keeping everyone else indifferent) aims to capture widely shared liberal views" (Mariotti and Veneziani (2020, p. 572)). As already noted, the individualistic and nonaggregative outlook of classical liberalism is also a key conceptual feature of sufficientarian approaches.

Theorem 2 shows that nonnterference can replace absolute individual improvement and monotonicity in the main characterization.

Theorem 2. *A social welfare ordering $\succcurlyeq$ on B^T satisfies noninterference, prioritarian threshold, and separability if and only if it is the sufficientarian social welfare ordering $\succcurlyeq^s_\alpha$.*

Proof. See Appendix A.1. □

The proof that the properties in Theorem 2 are independent can be found in Appendix B.2.

Theorem 2 highlights an aspect of sufficientarianism that is usually ignored in philosophical debates.[6] What is more, it sheds further light on the relation between sufficientarianism and standard notions of efficiency. To see this, consider the following standard properties.

[6] A partial exception is Crisp (2003) who provides a justification for a sufficientarian view of justice based on a specific interpretation of Mill's concept of the "sentiment of justice."

Weak Pareto. For all $a, b \in B^T$, $a \gg b \Rightarrow a \succ b$.

Strong Pareto. For all $a, b \in B^T$, $a > b \Rightarrow a \succ b$.

Efficiency as weak or strong Pareto is often considered to be a desirable property of social orderings, but it is obvious that the core sufficientarian view is not concerned with full efficiency. For example, its "negative thesis" part asserts precisely the irrelevance of benefits above the critical threshold α.

Consider next the uncontroversial requirement that the social ranking of alternatives does not coincide with the ordering of any individual.

Nondictatorship. For all $t \in \mathcal{T}$, there are $a, b \in B^T$ such that $a_t > b_t$ and $b \succcurlyeq a$.

Mariotti and Veneziani (2013) have proved the following result (adapted to the present context):

Theorem 3. *There exists no ordering on B^T that satisfies noninterference, weak Pareto, and nondictatorship.*

It follows that, since it satisfies noninterference and is not dictatorial, not only is $\succcurlyeq^s_\alpha$ not weakly Pareto optimal, but especially it cannot be extended—by breaking indifferences— to any relation that satisfies weak Pareto (or, a fortiori, strong Pareto) while preserving noninterference and nondictatorship.

4.2 *Neutrality*

In social choice theory, a standard way of capturing consistency in social evaluations is by means of "single-profile" neutrality conditions (see Rubinstein (1984)).

Neutrality. Let $a, b, a', b' \in B^T$, $a \succcurlyeq b$ and $\text{sign}(a_t - b_t) = \text{sign}(a'_t - b'_t)$ for all $t \in \mathcal{T} \Rightarrow a' \succcurlyeq b'$.

Neutrality is often taken to incorporate an emphasis on ordinal judgements with no weight assigned to interpersonal comparisons (for a comprehensive discussion, see Fleurbaey and Mongin (2005)). However, because sufficientarianism *does* allow certain kinds of welfare and level comparisons while being invariant to some ordinal transformation, neutrality needs to be suitably modified.

Although neutrality seemingly incorporates some reasonable properties, in its full force, it has two major shortcomings. First, it is a rather strong requirement, which almost immediately leads to impossibility results (Fleurbaey and Mongin (2005, pp. 386–387)). Second, and related, it has rather unappealing distributive implications. As Samuelson famously argued, neutrality

> "says, 'If it is ethically better to take something (say 1 chocolate or, alternatively, say 50 chocolates) from Person 1 who had all the chocolates in order to give to Person 2 who had none, then it must be ethically preferable to give all the chocolates to Person 2'. One need

> not be a doctrinaire egalitarian to be speechless at this requirement. Is it 'reasonable' to put on an ethical system such a straightjacket? Few will agree that it is." (Samuelson (1977, p. 83))

As Fleurbaey and Mongin (2005, p. 405) noted, "This ingenious parable is virtually all that is needed to deprive [neutrality] from [its] normative appeal as far as distributive applications are concerned ... Incidentally, the parable also illustrates Samuelson's mathematical point that neutrality implies dictatorship 'transparently'."

Both problems arise from the fact that neutrality considers perturbations that affect the well-being of more than one agent. As Samuelson's example shows, in this case imposing a consistency requirement on order preserving perturbations may lead to ethically unattractive implications. However, the property is immune from this line of criticism when the allowable perturbations are restricted to those that concern *only a single individual*.

The following axiom captures the individualistic nature of sufficientarianism while avoiding Samuelson's critique.

Individual neutrality. Let $a, b, a', b' \in B^T$ be such that for some $t \in \mathcal{T}$, $a'_{-t} = a_{-t}$, $b'_{-t} = b_{-t}$ and $a_t > b_t$. Then $a' \succcurlyeq b'$ whenever $a \succ b$ and $a'_t \geq b'_t$.

An interesting—and perhaps surprising—implication of individual neutrality is that if it is assumed, together with the separability and prioritarian threshold axioms, then it is unnecessary to impose monotonicity as a separate requirement.

Proposition 3. *Let $\succcurlyeq$ be an ordering on B^T that satisfies individual neutrality, prioritarian threshold, and separability. Then $\succcurlyeq$ satisfies monotonicity.*

Proof. We first prove the following particular instance: for each $t \in \mathcal{T}$ and $b \in B^T$, $a_t > b_t$ with $a_t \in B$ implies $(a_t, b_{-t}) \succcurlyeq b$.

Fix a $\beta \in (0, 1)$ that verifies prioritarian threshold. For notational convenience, we proceed when $t = 1$, the other cases being identical.

By prioritarian threshold, $(\beta, \dots, \beta) \succ (0, \beta, \dots, \beta)$. By individual neutrality, $(a_1, \beta, \dots, \beta) \succcurlyeq (b_1, \beta, \dots, \beta)$. Now a sequential application of separability implies $(a_1, b_{-1}) \succcurlyeq b$.

Once this property has been established then by transitivity, a routine application to the successive components proves that for all $a, b \in B^T$, $a > b$ implies $a \succcurlyeq b$ because

$$a \succcurlyeq (b_1, a_2, \dots, a_T) \succcurlyeq \cdots \succcurlyeq (b_1, b_2, \dots, b_{T-1}, a_T) \succcurlyeq (b_1, b_2, \dots, b_T) = b. \qquad \square$$

Given Proposition 3, the next result proves that individual neutrality can replace absolute individual improvement and monotonicity in the main characterization.

Theorem 4. *A social welfare ordering $\succcurlyeq$ on B^T satisfies individual neutrality, prioritarian threshold, and separability if and only if it is the sufficientarian social welfare ordering $\succcurlyeq^s_\alpha$.*

Proof. Necessity is trivial.

In order to establish sufficiency, we shall prove that if an ordering $\succcurlyeq$ on B^T satisfies individual neutrality, prioritarian threshold, and separability, then it satisfies absolute individual improvement. To see this, let $a, b \in B^T$ be such that $a \succ b$ and $a_t < 1$, some $t \in \mathcal{T}$.

If $a_t > b_t$, individual neutrality implies $(1, a_{-t}) \succcurlyeq (b'_t, b_{-t})$ for all $b'_t \in B$.

If $a_t = b_t$, separability implies $(1, a_{-t}) \succ (1, b_{-t})$, and then by Proposition 3 and transitivity, $(1, a_{-t}) \succ (b'_t, b_{-t})$ for all $b'_t \in B$.

If $b_t > a_t$, then $(b_t, a_{-t}) \succcurlyeq a \succ b \succcurlyeq (0, b_{-t})$ by Proposition 3 and transitivity. By individual neutrality, $(1, a_{-t}) \succcurlyeq (b'_t, b_{-t})$ for all $b'_t \in B$.

This proves that, under the axioms in the statement, the ordering $\succcurlyeq$ on B^T satisfies absolute individual improvement. Given Proposition 3, the result then follows from Theorem 1. □

The proof that the properties in Theorem 4 are independent can be found in Appendix B.3.

4.3 *Continuity*

Our results also highlight another key aspect of sufficientarianism, namely the discontinuity in ethical judgements at the sufficiency threshold. This emphasis may be disputable, and it may be objected that, intuitively, there is no threshold, which marks a discontinuous, qualitative change in people's lives. Two points can be made in response to this objection.

First, it is not obvious that continuity is a desirable property. Although it is often presented as an innocuous technical condition on social preferences, it rules out some widely used and normatively appealing approaches, such as the lexical version of John Rawls's maximin principle or Amartya Sen's poverty index. (For a discussion of, and counterpoint on, the relevance of continuity properties see, for example, Baigent (2011).) Second, discontinuity may actually express an ethically relevant property: as Shields (2012, p. 108) nicely puts it, "The sufficiency threshold ... seems to mark a shift in the nature of our reasons to benefit people further. This intuitive thought can be formally expressed by what I term the shift thesis." The ethical discontinuity at the threshold is indeed a defining feature of sufficientarianism in all of its variants. Benbaji (2006, p. 332) calls it the "sufficientarian discontinuity," according to which "benefiting a person just below the threshold is much more important than benefiting those who are just above it." From this perspective, the analysis in Section 4.1 can be extended to show precisely the nature of the discontinuity, and loosely speaking, the exact amount of continuity over the entire box B^T that is compatible with sufficientarianism.[7]

[7]In their characterization of a class of sufficientarian orderings, Bossert et al. (2022, 2021) incorporate the ethical discontinuity inherent in the sufficientarian approach by imposing *two* axioms that require continuity to hold above and below the threshold separately, but not over the entire domain.

To be precise, as shown in Appendix A.2, if prioritarian threshold is weakened to include $\beta = 1$, then the three properties in Theorem 2 are not sufficient to obtain the result. An additional axiom is necessary, which incorporates the usual view of a strict preference relation: if two profiles are strictly ranked, then the strict ranking is preserved for profiles that are sufficiently close to the inferior one:

Upper semicontinuity. For all $a \in B^T$, the set $\{b \in B^T : a \succ b\}$ is open in B^T in the Euclidean topology.

5. Concluding remarks

The philosophical analysis of sufficientarianism presents some puzzlingly disparate interpretations. For example, its originator Frankfurt explicitly saw sufficientarianism as an alternative to egalitarianism. Others (e.g., Anderson (1999), Nussbaum (1988, 1990), Satz (2007)) have interpreted it instead as a special form of egalitarianism. The root cause for such discrepancies is that sufficientarianism incorporates just some aspects of several standard principles, without incorporating any of them in its "pure" form. In this paper, we have offered formal characterizations that should help clarify this hybrid aspect of sufficientarianism. The main characterization (Theorem 1) in particular emphasizes, on the one hand, its limited prioritarian features, and on the other hand the possible justification it can provide to harsh inequalities.

We conclude with some comments on possible future directions to extend our work.

Two characteristics of the sufficientarian ordering, among others, have attracted criticism. First, it justifies major losses in well-being for a large number of destitute individuals, for the sake of a small increase in the well-being of one agent, provided this allows her to cross the sufficiency threshold (Roemer (2004)). While sufficientarianism was proposed by Frankfurt precisely in opposition to the idea that equality should always be pursued, this conclusion may seem unpalatable nevertheless, because of the extremeness of its antiegalitarianism. Second, the core sufficientarian view yields very large indifference classes, making it rather insensitive to both efficiency and distributive concerns. *Taken on their own*, neither of these objections seems to seriously undermine sufficientarianism. One may argue, for example, that the sufficientarian view can be interpreted as capturing some widespread intuitions similar to triage: it is not so implausible to opt for a profile, which allows at least one agent to live a decent life, rather than having *everyone* lead a life hardly worth living. Similarly, once everyone thrives, most people would agree that distributive concerns are much less pressing. The problem is that the core sufficientarian view cannot respond to both criticisms *together* (Casal (2007)). A low threshold allows one to respond to the first objection but it makes the second objection more salient. A high threshold has the opposite effect.

One possible answer explored in the literature to the large indifference classes problem is to consider sufficientarianism as part of a more complete distributive theory (Crisp (2003), Benbaji (2005), Brown (2005), Shields (2012), Vandamme (2017)).[8] In

[8]For a discussion, see Casal (2007). Bossert et al.'s (2022, 2021) critical level sufficientarianism can also be interpreted as a hybrid approach.

our framework, this approach could be formalized by means of a refined sufficientarian principle that adds a secondary criterion to the core one. That is, a profile a is judged better than a profile b if the number of people above a normatively relevant threshold is greater at a than at b, or if an equal number of people are above the threshold at a and b, but a is better than b according to a secondary criterion (e.g., utilitarian).

Formally, let $\succcurlyeq^P$ denote an ordering on B^T. Given a threshold level of well-being $\alpha \in (0, 1)$, define a refined sufficientarian criterion $\succcurlyeq_\alpha^{s,P}$ on B^T as follows. For all $a, b \in B^T$,

$$a \succcurlyeq_\alpha^{s,P} b \quad \Leftrightarrow \quad \text{either } n(a, \alpha) > n(b, \alpha), \text{ or } n(a, \alpha) = n(b, \alpha) \text{ and } a \succcurlyeq^P b.$$

(where recall that $n(a, \alpha)$ is the number of individuals who have a sufficient level of well-being at profile a). Observe that $\succcurlyeq_\alpha^{s,P}$ is transitive and complete by the transitivity and completeness of $\succcurlyeq_\alpha^s$ and $\succcurlyeq^P$. It is also known that

$$a \succ_\alpha^{s,P} b \quad \Leftrightarrow \quad \text{either } n(a, \alpha) > n(b, \alpha), \text{ or } n(a, \alpha) = n(b, \alpha) \text{ and } a \succ^P b.$$

A different strategy to respond to the criticisms moved against the core sufficientarian view has been to clearly separate the "positive thesis" and the "negative thesis" by specifying *multiple* different ethical thresholds (Benbaji (2006), Huseby (2010); see also Roemer's (2004) formalization). One can think of the higher threshold as identifying a level of well-being above which agents flourish, while the lower threshold can be set at a level below which an agent will be in a miserable condition. Then, if the former threshold is indeed sufficiently high, it is not implausible to argue that once all agents are above such level distributive concerns are less pressing. Similarly, if the latter threshold is sufficiently low, then one may argue that it is a matter of moral urgency to push as many agents as possible above such a minimum threshold. Formally, let $\alpha, \alpha' \in B$ denote two (ethically determined) distinct thresholds with $1 > \alpha > \alpha' > 0$, identifying, respectively, a *sufficient*, or satisfactory well-being and a *minimum* acceptable well-being level (e.g., the minimum level guaranteeing leading a life worth living). A natural extension of the core sufficientarian view, the *multithreshold sufficientarian criterion*, $\succcurlyeq_{\alpha,\alpha'}^s$, is as follows. For all $a, b \in B^T$,

$$a \succ_{\alpha,\alpha'}^s b \quad \Leftrightarrow \quad \text{either } n(a, \alpha') > n(b, \alpha') \text{ or } n(a, \alpha') = n(b, \alpha') \text{ and } n(a, \alpha) > n(b, \alpha),$$

$$a \sim_{\alpha,\alpha'}^s b \quad \Leftrightarrow \quad n(a, \alpha') = n(b, \alpha') \text{ and } n(a, \alpha) = n(b, \alpha).$$

An analysis of the approaches just sketched seems a promising route to deepen our understanding of sufficientarian ideas.

On a different front, in our treatment, we have not fully exploited the structure of the box of well-being. Given the presence of the extreme profiles Hell and Heaven, "duality" properties analogous to those used in the theory of rationing (e.g., Moulin (2000)) could be defined. This could uncover different ethical aspects of sufficientarianism.

Finally, another interesting development would be to extend the study of sufficientarianism to the context of intergenerational justice (analogously to what has been done for other distributive criteria; for a review, see Asheim (2010)), setting the objective of leaving each generation with a sufficiently high standard of living.

Appendix A: Proof of results in Section 4

A.1 *Proof of Theorem 2*

The basic structure of the proof of Theorem 2 is similar to that of our main characterization. First, we prove that any social welfare ordering satisfying noninterference, prioritarian threshold, and separability satisfies monotonicity.

Proposition 4. *Let $\succcurlyeq$ be an ordering on B^T that satisfies noninterference, prioritarian threshold, and separability. Then $\succcurlyeq$ satisfies monotonicity.*

Proof. We first prove the following particular instance: for each $t \in \mathcal{T}$ and $b \in B^T$, $a_t > b_t$ with $a_t \in B$ implies $(b_1, \dots, b_{t-1}, a_t, b_{t+1}, \dots, b_T) \succcurlyeq b$. Let β be an index that satisfies prioritarian threshold.

We distinguish four cases. For notational convenience, we proceed when $t = 1$, the other cases being identical.

Case 1. If $a_1 = \beta$, then $(a_1, \beta, \dots, \beta) \succ (b_1, \beta, \dots, \beta)$ by prioritarian threshold. A sequential application of separability yields $(a_1, b_2, \dots, b_T) \succ (b_1, b_2, \dots, b_T) = b$.

Case 2. Suppose either $\beta \leq b_1$ or $0 < b_1 < \beta < a_1$. Then $(\beta, \dots, \beta) \succ (0, \beta, \dots, \beta)$ by prioritarian threshold. A sequential application of separability yields $(\beta, b_2, \dots, b_T) \succ (0, b_2, \dots, b_T)$. Noninterference ensures $(a_1, b_2, \dots, b_T) \succcurlyeq (b_1, b_2, \dots, b_T) = b$.

Case 3. If $0 = b_1 < \beta < a_1$, then $(\beta, \dots, \beta) \succ (b_1, \beta, \dots, \beta)$ by prioritarian threshold. A sequential application of separability yields $(\beta, b_2, \dots, b_T) \succ (b_1, b_2, \dots, b_T) = b$. Noninterference yields $(a_1, b_2, \dots, b_T) \succcurlyeq (\beta, b_2, \dots, b_T)$ and the desired result follows from transitivity.

Case 4. If $b_1 < a_1 < \beta$, then consider $\varepsilon > 0$ such that $\beta - \varepsilon > b_1$. By prioritarian threshold, $(\beta, \dots, \beta) \succ (\beta - \varepsilon, \dots, \beta)$, and a sequential application of separability yields $(\beta, b_2, \dots, b_T) \succ (\beta - \varepsilon, b_2, \dots, b_T)$. The desired result then follows from noninterference.

Once this property has been established, by transitivity a routine application to the successive components proves that $a, b \in B^T$, $a > b$ implies $a \succcurlyeq b$ because

$$a \succcurlyeq (b_1, a_2, \dots, a_T) \succcurlyeq \cdots \succcurlyeq (b_1, b_2, \dots, b_{T-1}, a_T) \succcurlyeq (b_1, b_2, \dots, b_T) = b. \qquad \square$$

Given Proposition 4, we prove the existence of a *unique* ethical threshold β such that profiles in which the well-being of all agents is at least β are strictly better than profiles in which some agents have low well-being.

Lemma 3. *Let $\succcurlyeq$ be an ordering on B^T that satisfies noninterference, prioritarian threshold, and separability. Then there is a unique $\beta \in (0, 1)$ such that for all $a, b \in B^T$, $a_t \geq \beta$, all $t \in \mathcal{T}$, and $b_t < \beta$, some $t \in \mathcal{T} \Rightarrow a \succ b$.*

Proof. Given Proposition 4, the demonstration is a straightforward modification of the proof of Lemma 1. □

The next result generalizes the consequent of noninterference to any two profiles in which an agent enjoys the same level of well-being.

Lemma 4. *Let $\succcurlyeq$ be an ordering on B^T that satisfies noninterference, prioritarian threshold, and separability. Then for any $a, b \in B^T$, if $a \succ b$, then $a' \succcurlyeq b'$ for any $a', b' \in B^T$ such that $a'_t = b'_t$ some $t \in \mathcal{T}$ and $a' = (a'_t, a_{-t})$, $b' = (b'_t, b_{-t})$.*

Proof. Let $a, b \in B^T$ be such that $a \succ b$. Consider any $t \in \mathcal{T}$. If $a_t = b_t$, then the result follows from separability. Suppose $a_t \neq b_t$. We argue with $t = 1$, the other possibilities being symmetrical.

1. First of all, we show that for all $a \in B^T$:

$$\text{if } a_1 \geq \alpha, \quad \text{then } \left(a'_1, a_{-1}\right) \sim a \quad \text{for all } a'_1 \in (a_1, 1), \tag{5}$$

$$\text{if } a_1 < \alpha, \quad \text{then } \left(a'_1, a_{-1}\right) \sim a \quad \text{for all } a'_1 \in (a_1, \alpha). \tag{6}$$

To see that (5) holds, fix an arbitrary a'_1 such that $a_1 < a'_1 < 1$. We claim that $(a_1, 1, \dots, 1) \sim (a'_1, 1, \dots, 1)$. By Proposition 4, $(a'_1, 1, \dots, 1) \succcurlyeq (a_1, 1, \dots, 1)$. Suppose, by way of contradiction, that $(a'_1, 1, \dots, 1) \succ (a_1, 1, \dots, 1)$.

Observe that by Lemma 3 we have that $(\alpha, \dots, \alpha) \succ (1, \alpha, \dots, \alpha, \alpha/2)$, and noninterference yields $(\alpha, \dots, \alpha, (1+\alpha)/2) \succcurlyeq (1, \alpha, \dots, \alpha)$. Further, by Lemma 3 $((1+\alpha)/2, \alpha, \dots, \alpha) \succ (\alpha/2, 1, \dots, 1)$, and then because $1 > a'_1 > \alpha > \alpha/2$, noninterference yields $(1, \alpha, \dots, \alpha) \succcurlyeq (a'_1, 1, \dots, 1)$. By transitivity, it follows that $(\alpha, \dots, \alpha, (1+\alpha)/2) \succcurlyeq (a'_1, 1, \dots, 1)$.

But then, transitivity implies $(\alpha, \dots, \alpha, (1+\alpha)/2) \succ (a_1, 1, \dots, 1)$, in contradiction with Proposition 4. Therefore, we conclude that $(a_1, 1, \dots, 1) \sim (a'_1, 1, \dots, 1)$ and the desired result follows from separability.

To see that (6) holds, fix an arbitrary a'_1 such that $a_1 < a'_1 < \alpha$. Proposition 4 implies $(a'_1, a_{-1}) \succcurlyeq a$. Suppose, by way of contradiction, that $(a'_1, a_{-1}) \succ a$. Separability yields $(a'_1, (1+\alpha)/2, \dots, (1+\alpha)/2) \succ (a_1, (1+\alpha)/2 \dots, (1+\alpha)/2)$. Observe that $(\alpha, \dots, \alpha, \alpha/2) \succcurlyeq (a'_1, (1+\alpha)/2, \dots, (1+\alpha)/2)$. [For if $(a'_1, (1+\alpha)/2, \dots, (1+\alpha)/2) \succ (\alpha, \dots, \alpha, \alpha/2)$, then by noninterference, $(a'_1, (1+\alpha)/2, \dots, 1) \succcurlyeq (\alpha, \dots, \alpha, \alpha)$ in contradiction with Lemma 3.] By transitivity, $(\alpha, \dots, \alpha, \alpha/2) \succ (a_1, (1+\alpha)/2 \dots, (1+\alpha)/2$. By noninterference, $(1, \alpha, \dots, \alpha, \alpha/2) \succcurlyeq (\alpha, (1+\alpha)/2 \dots, (1+\alpha)/2)$, in contradiction with Lemma 3.

2. Armed with (5) and (6), we can now prove the result. We distinguish two cases.

Case 1: $a_1 < 1$.

If $b_1 > a_1$, then $b \succcurlyeq (a_1, b_{-1})$ by Proposition 4. Therefore, transitivity implies $a \succ (a_1, b_{-1})$ and the desired result follows from separability.

Suppose $b_1 < a_1$. By (5) and (6), there exists $\tilde{a}_1 \in (a_1, 1)$ such that $a \sim (\tilde{a}_1, a_{-1})$. Then we deduce $(\tilde{a}_1, a_{-1}) \succcurlyeq (a_1, b_{-1})$ by an application of noninterference to $a \succ b$. Transitivity implies $a \succcurlyeq (a_1, b_{-1})$ and separability yields the desired conclusion.

Case 2: $a_1 = 1$.

We proceed in two steps. First, we prove that $a \sim ((1+\alpha)/2, a_{-1})$. By Proposition 4, in order to establish the claim it is sufficient to prove that $((1+\alpha)/2, a_{-1}) \succcurlyeq a$.

Let $c = ((1+\alpha)/2, (1+\alpha)/2, \ldots, (1+\alpha)/2)$. The claim $((1+\alpha)/2, a_{-1}) \succcurlyeq (1, a_{-1}) = a$ is equivalent to $((1+\alpha)/2, c_{-1}) \succcurlyeq (1, c_{-1})$ by separability. Fix an arbitrary $x' \in ((1+\alpha)/2, 1)$. By Lemma 3, $(x', \alpha, (1+\alpha)/2, \ldots, (1+\alpha)/2) \succ (1, 0, (1+\alpha)/2, \ldots, (1+\alpha)/2)$. By noninterference, $(x', x', (1+\alpha)/2, \ldots, (1+\alpha)/2) \succcurlyeq (1, c_{-1})$. The claim then follows from transitivity noting that because $x' \in ((1+\alpha)/2, 1)$, (5) implies $c \sim (x', c_{-1})$ and $(x', c_{-1}) \sim (x', x', (1+\alpha)/2, \ldots, (1+\alpha)/2)$ and the transitivity of $\sim$ ensures $c \sim (x', x', (1+\alpha)/2, \ldots, (1+\alpha)/2)$.

The previous argument proves that $a \sim ((1+\alpha)/2, a_{-1})$. Then Proposition 4 and transitivity imply $(\tilde{a}_1, a_{-1}) \succcurlyeq a$ for all $1 > \tilde{a}_1 > \max(b_1, (1+\alpha)/2)$. Then, by transitivity, it follows that $(\tilde{a}_1, a_{-1}) \succ b$ for all $1 > \tilde{a}_1 > \max(b_1, (1+\alpha)/2)$. Then by noninterference, $a \succcurlyeq (\tilde{a}_1, b_{-1})$. Transitivity ensures $(\tilde{a}_1, a_{-1}) \succcurlyeq (\tilde{a}_1, b_{-1})$ and the desired conclusion follows from separability. □

Next, we show that the profiles where everyone is above the ethical threshold are all equivalent.

Lemma 5. *Let $\succcurlyeq$ be an ordering on B^T that satisfies noninterference, prioritarian threshold, and separability. Then $a \sim b$ for all $a, b \in B^T$ with $n(a, \alpha) = n(b, \alpha) = T$.*

Proof. Given Proposition 4 and Lemmas 3–4, the demonstration is a straightforward modification of the proof of Lemma 2. □

Finally, we prove that under the conditions of Lemmas 3–5, the social welfare ordering satisfies anonymity.

Proposition 5. *Let $\succcurlyeq$ be an ordering on B^T that satisfies noninterference, prioritarian threshold, and separability. Then $\succcurlyeq$ is anonymous.*

Proof. Given Proposition 4 and Lemmas 3–5, the demonstration is a straightforward modification of the proof of Proposition 2. □

We now proceed to prove Theorem 2.

Proof of Theorem 2. *(Necessity)* To see that $\succcurlyeq^s_\alpha$ on B^T satisfies noninterference consider $a, b \in B^T$ such that $a \succ^s_\alpha b$. By definition, this implies $n(a, \alpha) > n(b, \alpha)$. Then consider $a', b' \in B^T$ such that for some $t \in \mathcal{T}$, $(a_t - a'_t)(b_t - b'_t) > 0$, and $a_j = a'_j$, and $b_j = b'_j$ for all $j \neq t$. If $a'_t > b'_t$, then it immediately follows that $n(a', \alpha) \geq n(b', \alpha)$ and so $b' \not\succ^s_\alpha a'$, as sought.

(Sufficiency) Given Propositions 4–5, and Lemmas 3–5, the rest of the proof is a straightforward modification of the proof of Theorem 1. □

A.2 *Upper semicontinuity*

In this section, we provide a formal proof of the claims made in Section 4.3.

First of all, we prove that $\succcurlyeq_\alpha^s$ on B^T satisfies upper semicontinuity. To see this, take any $a \in B^T$ and consider the set $L(a) = \{b \in B^T : a \succ_\alpha^s b\}$. Consider any $b \in L(a)$. By definition $n(a, \alpha) > n(b, \alpha)$. Let $\Delta = \min_{t\in\mathcal{T}\setminus P(b,\alpha)}(\alpha - b_t)$: Δ is well-defined and strictly positive. For any $a, a' \in B^T$, let $d(a, a')$ be the Euclidean distance between a and a'. Then for any $b' \in B^T$ such that $d(b, b') < \Delta$, we observe that when $t \in \mathcal{T} \setminus P(b, \alpha)$, $\alpha - b'_t = \alpha - b_t + b_t - b'_t \geq \Delta + b_t - b'_t \geq \Delta - d(b, b') > 0$. Therefore, we have $n(a, \alpha) > n(b, \alpha) \geq n(b', \alpha)$, as sought.

Next, let prioritarian threshold* denote the weak version of the axiom holding for $\beta \in (0, 1]$. Example 1 shows that separability, noninterference, and prioritarian threshold* do not jointly characterize the sufficientarian ordering.

Example 1. Let $\succcurlyeq_*$ be the ordering on B^2 defined as follows: $a \succcurlyeq_* b$ if and only if $a \in A_i$, $b \in A_j$, and $i \leq j$, where $A_1 = \{(1, 1)\}$, $A_2 = \{(x, 1) : x \in (0, 1)\} \cup \{(1, y) : y \in [0, 1)\}$, $A_3 = \{(0, 1)\} \cup \{(x, y) : x \in (0, 1), y \in [0, 1)\}$, and $A_4 = \{(0, y) : y \in [0, 1)\}$.

The ordering $\succcurlyeq_*$ satisfies separability, noninterference, and prioritarian threshold* (with respect to $\beta = 1$). It does not satisfy upper semicontinuity because $(\frac{1}{n}, 0) \succcurlyeq_* (\frac{1}{2}, 0)$, the sequence $\{(\frac{1}{n}, 0)\}_n$ converges to $(0, 0)$, but $(\frac{1}{2}, 0) \succ_* (0, 0)$. ◊

It is possible to prove, however, that upper semicontinuity is precisely the missing condition. To see this, note first that Proposition 4 continues to hold (only cases 1 and 4 in the proof are relevant). Then we prove the equivalent of Lemma 3.

Lemma 6. *Let $\succcurlyeq$ be an ordering on B^T that satisfies separability, noninterference, prioritarian threshold*, and upper semicontinuity. Then there is a unique $\beta \in (0, 1]$ such that for all $a, b \in B^T$, $a_t \geq \beta$, all $t \in \mathcal{T}$, and $b_t < \beta$, some $t \in \mathcal{T} \Rightarrow a \succ b$.*

Proof. The proof of existence is identical to Lemma 3.

In order to prove uniqueness, we proceed ad absurdum. Assume that there are $\beta, \beta' \in B$, $\beta > \beta' > 0$, such that for all $a, b \in B^T$,

$$n(a, \beta) = T > n(b, \beta) \quad \text{implies} \quad a \succ b, \tag{7}$$

$$n\big(a, \beta'\big) = T > n\big(b, \beta'\big) \quad \text{implies} \quad a \succ b. \tag{8}$$

By (7), $(\beta, \dots, \beta) \succ (\beta, \beta', \dots, \beta')$. Separability assures $(\beta', \beta, \dots, \beta) \succ (\beta', \dots, \beta')$. Let us prove $(\beta', \dots, \beta') \succcurlyeq (\beta', \beta, \dots, \beta)$, which ensures the contradiction.

Using (8), $(\beta', \dots, \beta') \succ (0, \beta, \dots, \beta)$. By noninterference for each $\varepsilon > 0$ such that $\beta' + \varepsilon < 1$, it must be the case that $(\beta' + \varepsilon, \beta', \dots, \beta') \succcurlyeq (\beta', \beta, \dots, \beta)$. Upper semicontinuity then yields the desired $(\beta', \dots, \beta') \succcurlyeq (\beta', \beta, \dots, \beta)$. □

The next result generalizes the consequent of noninterference to any two profiles in which an agent enjoys the same level of well-being.

Lemma 7. *Let $\succsim$ be an ordering on B^T that satisfies separability, noninterference, and upper semicontinuity. Then for any $a, b \in B^T$, if $a \succ b$, then $a' \succsim b'$ for any $a', b' \in B^T$ such that $a'_t = b'_t$ some $t \in \mathcal{T}$ and $a' = (a'_t, a_{-t})$, $b' = (b'_t, b_{-t})$.*

Proof. Consider any $a, b \in B^T$ such that $a \succ b$. Consider any $t \in \mathcal{T}$. If $a_t = b_t$, then the result follows immediately from separability. Therefore, suppose $a_t \neq b_t$. We need to consider three cases.

Case 1. Suppose $a_t > 0$ and $b_t > 0$. Consider any $a^\varepsilon, b' \in B^T$ such that $a_t > a^\varepsilon_t$, $b_t > b'_t$ and $a^\varepsilon_t = b'_t + \varepsilon$, some $\varepsilon > 0$, and $a^\varepsilon_j = a_j$, $b'_j = b_j$ for all $j \neq t$. By noninterference, it follows that $a^\varepsilon \succsim b'$ for any such $\varepsilon > 0$. Therefore, upper semicontinuity yields $a' = (b'_t, a_{-t}) \succsim b'$. The desired result then follows from separability.

Case 2. Suppose either $b_t = 0$ and $1 > a_t$, or $a_t = 0$ and $1 > b_t$. Then a similar argument as in Case 1 can be applied to any $a^\varepsilon, b' \in B^T$ such that $a_t < a^\varepsilon_t$, $b_t < b'_t$ and $a^\varepsilon_t = b'_t + \varepsilon$, some $\varepsilon > 0$, and $a^\varepsilon_j = a_j$, $b'_j = b_j$ for all $j \neq t$.

Case 3. Suppose either $b_t = 0$ and $a_t = 1$, or $a_t = 0$ and $b_t = 1$. In the former case, by upper semicontinuity, there exists $\varepsilon \in (0, 1)$, such that $a \succ b^* = (b_1, \dots, b_{t-1}, \varepsilon, b_{t+1}, \dots, b_T)$. Then the argument in Case 1 can be applied to a, b^*. In the latter case by upper semicontinuity, there exists $\varepsilon \in (0, 1)$, such that $a \succ b^* = (b_1, \dots, b_{t-1}, 1 - \varepsilon, b_{t+1}, \dots, b_T)$. Then the argument in Case 2 can be applied to a, b^*. $\square$

Given Proposition 4 and Lemmas 6–7, the demonstration that the sufficientarian criterion is the only ordering that satisfies separability, noninterference, prioritarian threshold*, and upper semicontinuity is a straightforward modification of the proof of Theorem 2.

Appendix B: Independence of the axioms

B.1 *Independence of axioms used in Theorem 1*

The following examples prove that the properties in Theorem 1 are independent.

Example 2. Let $\succsim^I$ be the trivial ordering given by $a \succsim^I b$ for each $a, b \in B^T$.

The ordering $\succsim^I$ satisfies monotonicity, absolute individual improvement, and separability but not prioritarian threshold (with respect to any β). $\diamondsuit$

Example 3. Let $\alpha \in (0, 1)$ and $\succsim^1_\alpha$ be the ordering on B^T defined by: for each $a, b \in B^T$, $a \succsim^1_\alpha b$ if and only if it is false that $n(b, \alpha) = T > n(a, \alpha)$.

The ordering $\succsim^1_\alpha$ satisfies monotonicity, absolute individual improvement, and prioritarian threshold (with respect to α), but it violates separability: when $T = 2$, $(1, 1) \succ^1_\alpha (1, 0)$ but it is false that $(0, 1) \succ^1_\alpha (0, 0)$. $\diamondsuit$

Example 4. Let $1 > \alpha > \beta > 0$ and $\succsim_{\alpha,\beta}$ be the ordering on B^T defined by: for each $a, b \in B^T$, $a \succsim_{\alpha,\beta} b$ if and only if either $n(a, \alpha) > n(b, \alpha)$ or ($n(a, \alpha) = n(b, \alpha)$ and $n(a, \beta) \geq n(b, \beta)$).

The ordering $\succsim_{\alpha,\beta}$ satisfies monotonicity, separability, and prioritarian threshold (with respect to α), but it violates absolute individual improvement: when $T = 2$, $\alpha = 0.5$, $\beta = 0.3$ one has $(0.6, 0) \succ_{\alpha,\beta} (0.4, 0.4)$ but it is false that $(1, 0) \succsim_{\alpha,\beta} (0.6, 0.4)$. $\diamondsuit$

Example 5. Let $\alpha \in (0, 1)$ and $\succsim^2_\alpha$ be the ordering on B^2 defined as follows: $a \succsim^2_\alpha b$ if and only if $a \in A_i$, $b \in A_j$, and $i \leq j$, where $A_1 = \{(1, 1), (\alpha, \alpha), (\alpha, 1), (1, \alpha)\}$, $A_2 = \{(x, 1) : x \in [0, \alpha) \cup (\alpha, 1)\} \cup \{(1, y) : y \in [0, \alpha) \cup (\alpha, 1)\} \cup \{(x, \alpha) : x \in [0, \alpha) \cup (\alpha, 1)\} \cup \{(\alpha, y) : y \in [0, \alpha) \cup (\alpha, 1)\}$, $A_3 = B^2 \setminus \{A_1 \cup A_2\}$.

The ordering $\succsim^2_\alpha$ satisfies absolute individual improvement, separability and prioritarian threshold (with respect to α), but it violates monotonicity. ◊

B.2 *Independence of axioms used in Theorem 2*

The following examples prove that the properties in Theorem 2 are independent.

Example 6. The ordering $\succsim^I$ defined in Example 2 satisfies noninterference, upper semicontinuity, and separability but not prioritarian threshold (with respect to any β). ◊

Example 7. The ordering $\succsim^1_\alpha$ defined in Example 3 satisfies noninterference, upper semicontinuity, and prioritarian threshold (with respect to α), but it violates separability. ◊

Example 8. The ordering $\succsim_{\alpha,\beta}$ defined in Example 4 satisfies separability, upper semicontinuity, and prioritarian threshold (with respect to α), but it violates noninterference: when $T = 2$, $\alpha = 0.5$, $\beta = 0.3$ one has $(0.6, 0) \succ_{\alpha,\beta} (0.4, 0.4)$ but it is false that $(0.7, 0) \succsim_{\alpha,\beta} (0.6, 0.4)$. ◊

B.3 *Independence of axioms used in Theorem 4*

The following examples prove that the properties in Theorem 4 are independent.

Example 9. The ordering $\succsim^I$ defined in Example 2 satisfies individual neutrality, and separability but not prioritarian threshold (with respect to any β). ◊

Example 10. The ordering $\succsim^1_\alpha$ defined in Example 3 satisfies individual neutrality and prioritarian threshold (with respect to α), but it violates separability. ◊

Example 11. The ordering $\succsim_{\alpha,\beta}$ defined in Example 4 satisfies separability and prioritarian threshold (with respect to α), but it violates individual neutrality: when $T = 2$, $\alpha = 0.5$, $\beta = 0.3$ one has $(0.6, 0) \succ_{\alpha,\beta} (0.4, 0.4)$ but it is false that $(0.7, 0) \succsim_{\alpha,\beta} (0.6, 0.4)$. ◊

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Co-editor Ran Spiegler handled this manuscript.

Manuscript received 31 March, 2020; final version accepted 18 October, 2021; available online 26 October, 2021.