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Rational bubbles and middlemen

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This paper develops a model of rational bubbles where trade of an asset takes place through a chain of middlemen. We show that there exists a unique and robust equilibrium, and a bubble can occur due to information frictions in bilateral and decentralized markets. Under reasonable assumptions, the equilibrium price is increasing and accelerating during bubbles although the fundamental value is constant over time. Bubbles may be detrimental to the economy, but any announcement from the central bank has no effect on welfare with risk neutral agents. Middlemen are the source of financial fragility.

Keywords. Rational bubbles, middlemen, higher-order uncertainty, asymmetric information, flippers.

JEL classification. D82, D83, D84, G12, G14.

1. Introduction

As was emphasized in an article "*How Tales of 'Flippers' Led to a Housing Bubble*" by Shiller (2017), the bubble preceding the Great Recession and financial crisis of 2007 to 2009 is tightly linked to flipping in certain housing markets. In fact, this applies to bubbles in many other markets as well. Flipping in decentralized markets often occurs through chains of middlemen who are specialized in purchasing an asset at a low price and quickly reselling it at a higher price. The objective of our paper is to explore a tractable equilibrium framework to understand the theoretical connection between the occurrence of bubbles and middlemen.[1]

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[1]Bubbles that occur via a chain are common not only in housing markets (see Bayer, Geissler, Mangum, and Roberts (2020)), but also, for example, in a market for securities, where the chain starts with an originator of loans (typically, a bank), the agents in the middle of the chain are investment and commercial banks, and the final user is an individual investor.

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In this paper, we construct a tractable finite-period model of rational bubbles caused by a chain of middlemen. We show the following.

THEOREM. *There is a unique equilibrium. A bubble occurs with positive probability and once it occurs, it bursts for sure. The equilibrium is robust to perturbations of parameters.*

Of course, a standard backward induction argument implies that if the value of an asset were common knowledge, then a bubble would never occur. However, we show that with lack of common knowledge, a bubble occurs in a unique and robust equilibrium.

In our model, middlemen contact each other individually and negotiate the terms of the trade bilaterally. Prior to the trade, the final user may or may not know the consumption value of an asset. Importantly, because of rumors about the final user, downstream middlemen may or may not know whether the final user knows the consumption value of the asset, and upstream middlemen may or may not know whether the downstream middlemen know whether the final user knows the consumption value of the asset. This opens room for a bubble—an upstream middleman acquires an asset, knowing it is overpriced, in hopes of finding a downstream middleman, who also knows it is overpriced but believes that he can sell the asset to someone else. In other words, each middleman cares less about the fundamental value of the asset, and more about how much the other agents value it.

The general property that a higher-order uncertainty associated with flippers causes bubbles captures many historical episodes. Notably, Kindleberger and Aliber (2005) says "The bubble involves the purchase of an asset, usually real estate or a security, not because of the rate of return on the investment but in anticipation that the asset or security can be sold to someone else at an even higher price; the term 'the greater fool' has been used to suggest the last buyer was always counting on finding someone else to whom the stock or the condo apartment or the baseball cards could be sold."[2]

Under reasonable assumptions, we show that the equilibrium price increases during bubbles even when the fundamental value is constant over time. This is because each middleman always faces the risk that his downstream middleman rejects the asset, and prices are determined in such a way that the risk is compensated. The price also tends to accelerate because the probability that a middleman can sell the asset decreases over time. So, middlemen who trade in later periods are exposed to bigger risks. This equilibrium price trajectory explains, for instance, the housing bubble during 2003–2005 where over the course of market fluctuations, the price tends to increase, possibly accelerating, until it is interrupted by a sudden markdown or even a crash. Similar price trajectories are observed in many other bubbles, not only in the modern bubbles, for example, the dot-com bubble or the Japanese asset bubble, but also in the classical Dutch tulip bubble (see Figure 1), etc. We show it as a unique and robust equilibrium—we claim not only that it can occur as an equilibrium, but also that it must be this way.

[2]There is a literature of "greater fool models" initiated by Allen, Morris, and Postlewaite (1993). While both their models and ours share some common features—in particular, higher-order uncertainty causes bubbles— there are important differences. We will detail them in Section 3.3.

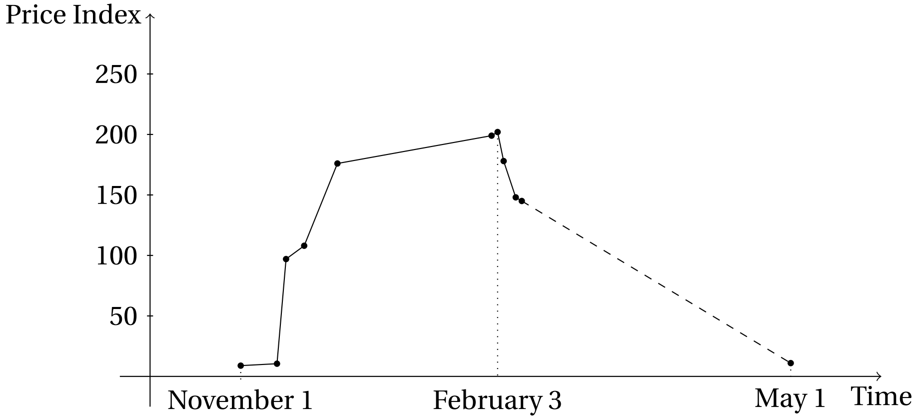

FIGURE 1. Tulip prices over the 1636–1637 period. Source: Thompson (2007).

We focus attention on a bubble, which occurs in a decentralized market with bilateral trades. In particular, the market is assumed to be opaque in the sense that the prices at which individual trades occur are not publicly observable. This is a feature of over-the-counter markets, where middlemen contact each other individually and negotiate the terms of trade bilaterally. Indeed, this opaqueness assumption plays the key role for the robustness of our bubble equilibrium. That is, we show that a bubble can occur even when prices are publicly observable but such an equilibrium is not robust to a small perturbation of parameters (see the discussion section of our main result in Section 3.3). The opaqueness assumption echos Duffie's (2012) sentiment that "The financial crisis of 2007–2009 brought significant concerns and regulatory action regarding the role of OTC markets, particularly from the viewpoint of financial instability. OTC markets for derivatives, collateralized debt obligations, and repurchase agreements played particularly important roles in the crisis and in subsequent legislation."

Thanks to the features of our model—rationality, common prior, uniqueness, and robustness—we are able to offer the following characterizations of our equilibrium. First, like in the traditional view, bubbles facilitate trade in our model. However, we show that bubbles are not necessarily welfare improving. We characterize exactly when a bubble is detrimental: it is welfare reducing (improving) if there are losses (gains) from trade between middlemen.[3]

Second, "irrational exuberance" is the phrase used by Alan Greenspan in a 1996 speech, "*The Challenge of Central Banking in a Democratic Society*." The speech was given during the dot-com bubble. The Greenspan's comment was broadcasted by dedicated financial TV channels around the world live, and its meaning was widely discussed by financial journalists. The Tokyo market moved down sharply after his speech, with other markets following. Inspired by the recent progress in the information design literature (see Kamenica and Gentzkow 2011), we analyze the welfare consequence of

[3]Of course, even if there are losses from trade between middlemen, there are (potential) gains from trade between middlemen and the final user.

such bubble bursting policies à la Conlon (2015) as an information design problem. We assume that the information designer (the central bank), who aims at maximizing total expected utility, is able to send out a public message about the value of the asset. Such public announcements create common knowledge, and burst a bubble. The information designer can burst the bubble even if it does not have superior information technology to the other market participants. Surprisingly, however, given risk neutral agents, it cannot influence ex ante welfare even if it has superior information technology. This neutrality result breaks down if agents are risk averse, and it depends on how we introduce risk aversion whether the bubble bursting policy is beneficial or detrimental.

Finally, based on comparative statics exercises we analyze how market structure influences financial fragility. We find that the wider the rumors are spread, the less likely a bubble occurs and the shorter the expected duration of the bubble is, but the larger its size is. The recent regulation on Credit Default Swap (CDS) markets (e.g., the Dodd–Frank Act) aims at preventing bubbles by facilitating information sharing among investors. Our result predicts a side-effect—while a larger amount of information sharing reduces the probability and length of a bubble, it may increase its size. Also, we find that with a larger number of middlemen, the ex ante probability that a bubble occurs and the expected duration of a bubble can be all magnified—*middlemen are the source of financial fragility*.

Related literature

Allen, Morris, and Postlewaite (1993) construct a model of bubbles with information asymmetry, relaxing the common-knowledge assumption. This line of models is often referred to the "greater fool theory" approach and substantially simplified in a sequence of papers by Conlon (2004, 2015), Liu and Conlon (2018) and Liu, White, and Conlon (2020). These papers and the current paper both point the crucial role of higher-order uncertainty in the occurrence of bubbles. Our innovation is to establish *both* the robustness *and* the uniqueness of equilibrium, which is obtained by explicitly highlighting the role of middlemen in an opaque market. We are able to pursue relatively easily the welfare analysis, policy analysis, and comparative statics based on standard tools. Further, we identify an intuitive necessary and sufficient condition for which price increases, and bubbles are detrimental to welfare.

One crucial difference is that, in these greater fool models, buyers trade either with "bad" sellers—sellers who know that they are selling worthless objects—or "good" sellers. For the occurrence of bubbles, the two types of sellers must behave in the same way. Since this occurs only at knife-edge parameter values, their bubble equilibria are not robust to small perturbations of parameter values.[4] In contrast, in our model, there are no good or bad sellers because buyers are better informed than sellers. The exception is

[4]See, for example, Conlon (2015) and Liu and Conlon (2018) for a discussion of this matter. They argue that with a continuum of states, the bubble equilibrium is robust.

the middleman next to the final user. However, since the final user cannot observe past prices in the opaque market, the final user does not learn whether the middleman is a good or bad seller from behavior. Thus, our bubble equilibrium is robust.[5]

There are papers that model bubbles as a timing game—a game where each trader wants to sell an overpriced asset quicker than its rivals. Abreu and Brunnermeier (2003) show that overpricing can occur in such a model along with an exogenously increasing price path. Our modeling approach differs from them. Moreover, they do not argue whether the equilibrium is unique or not.[6]

Matsushima (2013) and (2020) also model bubbles as a timing game. These papers obtain uniqueness, but unlike the current paper, they assume existence of irrational agents who ride a bubble no matter what. A rational agent also rides the bubble, pretending to be irrational. In our model, uniqueness is obtained in a simpler setting without irrational agents, where only a standard backward induction argument is needed. Further, in Matsushima (2013) and (2020), a bubble occurs in a mixed-strategy equilibrium (a pure-strategy equilibrium does not exist in his models), while in our simpler setting, a bubble occurs in a pure-strategy equilibrium. This is advantageous because we obtain a sharper characterization.

Finally, we also contribute to the literature of middlemen initiated by Rubinstein and Wolinsky (1987). We find a novel motive of middlemen—middlemen care less about the quality of the asset but more about what others know about it once the common-knowledge assumption is relaxed. Our environment is akin to Wright and Wong (2014), who develop a model of intermediation chains. They show that a bubble can occur with symmetric information only when there is an infinite number of middlemen. In contrast, we offer a finite-period model with asymmetric information. Moreover, unlike in Wright and Wong (2014), middlemen in our model can be active even when the asset does not have a positive fundamental value. We study the role of intermediaries for bubbles in finite intermediation chains and show that they are essential for bubbles. Though it is highly stylized, our model delivers novel insights—the role of higher-order uncertainty in the trade involving middlemen—into the occurrence and burst of rational bubbles.

The structure of the paper

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 shows the existence, the uniqueness, and the robustness of equilibrium. In Section 4,

[5]Other existing models in this literature emphasize risk-sharing (Allen, Morris, and Postlewaite (1993), Liu and Conlon (2018)) or intertemporal consumption-smoothing (Liu, White, and Conlon (2020)) as the motive for trade. In contrast, the motive for trade in our model is neither risk-sharing nor consumption-smoothing, but is based on middlemen's incentive to flipping.

[6]In a timing game model of bubble, Doblas-Madrid (2012) also generates a bubble in which a price increases, and his model, gains from trade are rooted in a liquidity shock. In his model, the price increases due to the growth in fundamentals, whereas in our model, the price increases even when fundamentals do not grow. Moreover, like Abreu and Brunnermeier (2003), Doblas-Madrid (2012) also does not obtain uniqueness. See also Araujo and Doblas-Madrid (2019).

we study how the equilibrium price changes over time. Section 5 derives the welfare implications, and in Section 6, we discuss the implications of policies that affect agents' beliefs. Section 7 examines how market structure affects the occurrence and duration of a bubble. Section 8 concludes. Appendix A explores an alternative setup where individual prices are publicly observable. In Appendix B, we show that when agents are risk-averse, policies that affect agents' beliefs have welfare consequences. Finally, in Appendix C we provide a parametric example.

2. The model

In this section, we describe the economic environment in the first subsection and the knowledge of agents in the second subsection.

2.1 *The environment*

There are N agents $A_1, A_2, \dots, A_N$ where $2 < N < \infty$. They are spatially separated in the following fashion: A_n can meet, and hence, trade with A_{n-1} and A_{n+1} but with no one else. Therefore, trade between A_{n-1} and A_{n+1} must go through A_n. We assume that trade is sequential, A_n and A_{n+1} trade in period n, and A_n exits the economy after trading with A_{n+1}. Hence, time is discrete and continues for $N-1$ periods.

There are two objects in this economy. One is an indivisible asset x in fixed supply, and the other is a divisible good y that every agent can produce at unit cost. Only A_1 is endowed with x, that is, A_1 is the initial owner of x. He can try to trade it to A_2 in exchange for some amount of y, say y_1. We assume the consumption value of x for A_1 is zero. More generally, if A_n acquires x from A_{n-1}, he can try to trade it to A_{n+1} for y_n, which generates a payoff

$$U(y_n) = \kappa y_n$$

where $\kappa > 0$ is the value of A_{n+1}'s good to A_n. The consumption value of x for A_n is 0 for each $n < N$. The value of x for the final user A_N is $v > 0$ with some probability and 0 with the remaining probability. The key here is that the final user may have a larger consumption value than the other agents.[7] For simplicity, agents do not discount utilities between any two periods. Middlemen $A_2, A_3, \dots, A_{N-1}$ are a necessary part of the process of getting x from the initial owner A_1 to the final user A_N.

For our purpose of studying the occurrence of bubbles in the presence of middlemen, we keep the determination of terms of trade as simple as possible. We therefore employ a generalized Nash bargaining to determine the terms of trade $(y_n)_{n=1}^{N-1}$, where

[7] Middlemen could have a positive consumption value as well. More precisely, even if the consumption value of x for A_n is $f \geq 0$ for each $n < N$, and the value for A_N is $f + v$ with some probability and f with the remaining probability, all of our results hold as long as v is sufficiently large, f is sufficiently small, or κ is not too small. Just for simplicity, we assume $f = 0$.

agents' utilities are zero if they disagree to trade.[8] In trade between A_n and A_{n+1}, let θ be the bargaining power of A_n with $0 < \theta < 1$.[9]

The chain structure of trade can be justified by the presence of search frictions that only allow trade between two adjacent agents.[10] Alternatively, it can be motivated by a setup where good y produced by A_{n+1} differs from good y by A_n, and only A_n can enjoy good y produced by A_{n+1}. This interpretation would be relevant especially when $\kappa > 1$, in which case homogeneous-good interpretation might allow each agent to produce and consume good y for himself as much as he wants.

We assume that the final user cannot observe trade among the other agents. The Appendix A explores an alternative setup where individual prices are publicly observable. With a transparent market that we study in the Appendix, bubbles are shown to occur in equilibrium in a knife-edge case. With an opaque market that we study in the main text, bubbles are shown to occur in equilibrium for any parameter values.

2.2 *Knowledge*

In this section, we describe the knowledge of agents. It is reminiscent of Rubinstein's (1989) email game and meant to capture the essence of a rational bubble in the simplest possible way. Following the literature, we say that we define a rational bubble as follows.

Definition 1. *A rational bubble occurs* if all agents know that the asset x is worthless, but it is traded for a positive amount of the good y.

There are two necessary ingredients for a rational bubble. First, there should be at least one agent who may not know the consumption value—otherwise, the value of the asset is commonly known and the asset can never be overpriced. Second, there should be at least some situation where all agents, including the agent in question, know the value of the asset—given our definition of a rational bubble, everyone should know it is overpriced.

To capture these ingredients, we shall assume that all parameters describing utilities, costs, etc., are common knowledge, *except* for the consumption value of asset x. We also assume that prior to trade, all agents except for A_N observe the consumption value of x for A_N.[11] This setup can be applied to, for example, housing or antique markets. In such

[8]Formally, we consider a game form where a fictitious third party suggests the exchange ratio following the Nash bargaining solution, and then each agent either accepts or rejects the trade. The trade occurs only when both accept (to get rid of an equilibrium where an agent rejects because the other rejects, we assume that agents move sequentially). While the agents have private information, the terms of trade that the third party proposes do not depend on it. Our result is, however, robust to other bargaining protocols.

[9]When $\theta = 0$, it will turn out that the terms of trade are zero, and hence, we assume $\theta > 0$. When $\theta = 1$, agents are indifferent on whether they buy the asset x or not. Thus, to ensure the uniqueness of equilibrium, we assume $\theta < 1$. However, if we assume that agents buy whenever they are indifferent, all the results hold even with $\theta = 1$.

[10]Hollifield, Neklyudov, and Spatt (2017) and Li and Schürhoff (2019) document the empirical evidence of intermediation chains often observed in OTC financial markets.

[11]Even when we consider a more general setting where $A_1, \ldots, A_{N-1}$ do not know the consumption value at some state, by definition of the rational bubble, we would end up considering a state where all

a market, middlemen are experts who know the fundamental value of the asset, and the final user is not an expert (e.g., imagine a temporal consumer like a mover into a new city or an amateur collector of artistic paintings).

If the value of the asset is zero, A_N receives a signal with some probability. Otherwise, A_N does not receive any signals. Thus, if he receives a signal, then A_N is sure that the consumption value is zero, and in this event, every other agent also knows that x is worthless. Moreover, if A_N receives a signal, he (nonstrategically) sends a signal (email in the terminology of Rubinstein (1989)) to A_{N-1}. The signal reaches A_{N-1} with some probability but is lost with the remaining probability. Thus, if A_{N-1} receives a signal, he is sure that A_N knows that the consumption value is zero. Similarly, if A_{N-1} receives a signal from A_N, he (nonstrategically) sends a signal to A_{N-2}. The signal reaches A_{N-2} with some probability but is lost with the remaining probability. This process continues until a signal is lost between some two agents or the initial owner A_1 receives a signal. In words, the signal (rumor) that A_N knows that x is worthless spreads from A_N to A_1, but it is subject to loss between any two agents.[12] We assume that information does not flow from A_{n-1} to A_n, and in particular, A_{N-1} cannot credibly reveal the true value of the asset to A_N. We finally assume all these signals occur *prior to trade*.

To describe the above situation formally, we introduce $N+2$ states of the world. The consumption value of x for A_N is $v>0$ at state ω_v and 0 at the other states. When the state is ω_ϕ, no agent receives a signal although x is worthless. For each $n=1,\dots,N$, state ω_n corresponds to the case where all the agents $A_N, A_{N-1}, \dots, A_n$ receive signals, while the others do not. Hence, the set of states is

$$\Omega=\{\omega_v, \omega_\phi, \omega_N, \omega_{N-1}, \dots, \omega_1\}.$$

Let μ be the common prior distribution over Ω. We assume it has full-support on Ω.

We represent agents' knowledge by partitions of Ω. Agent A_N's partition is

$$\mathcal{P}_N=\big\{\{\omega_v, \omega_\phi\}, \{\omega_N, \omega_{N-1}, \dots, \omega_1\}\big\}.$$

The first element, $\{\omega_v, \omega_\phi\}$, corresponds to the case where A_N does not receive a signal, and hence, does not know whether x is worthless. The second element, $\{\omega_N, \omega_{N-1}, \dots, \omega_1\}$, corresponds to the case where A_N receives a signal and knows that x is worthless. For each $n<N$, agent A_n's partition is

$$\mathcal{P}_n=\big\{\{\omega_v\}, \{\omega_\phi, \omega_N, \omega_{N-1}, \dots, \omega_{n+1}\}, \{\omega_n, \omega_{n-1}, \dots, \omega_1\}\big\}.$$

agents know the consumption value. Hence, it is sufficient to focus on the model where $A_1, \dots, A_{N-1}$ always know the consumption value. Also, the agent who may not know the consumption value does not need to be A_N, that is, the final user, and could be one of the middlemen. Our formulation, however, is able to generate a bubble in the cleanest way in an intermediation chain setting.

[12]For simplicity, we say that A_n "nonstrategically" sends a signal to A_{n-1} when A_n receives a signal. However, A_n does not actually care whether A_{n-1} receives the signal. It will turn out that, if A_{n-1} receives the signal, the only effect on A_n is that A_n does not receive an offer from A_{n-1}, but it is always optimal for A_n to reject the offer given that A_n has already received the signal. Hence, A_n does not care whether he sends a signal to A_{n-1}, and might therefore use a mixed strategy, or even just let the information randomly leak out.

The first element, $\{\omega_v\}$, corresponds to the case where the consumption value of x for A_N is v. The second element, $\{\omega_\phi, \omega_N, \omega_{N-1}, \dots, \omega_{n+1}\}$, corresponds to the case where the consumption value is zero, but A_n does not receive a signal. The third element, $\{\omega_n, \omega_{n-1}, \dots, \omega_1\}$, corresponds to the case where the consumption value is zero and A_n receives a signal ($A_{n-1}, \dots, A_1$ may also receive signals). An agent can distinguish any two states if those states belong to a different element of his partition, but cannot otherwise.

Assume $\omega \neq \omega_v$. Then, if A_N does not receive a signal, the posterior probability that the consumption value of x for A_N is v is $\mu(\omega_v)/[\mu(\omega_v) + \mu(\omega_\phi)]$, and hence, the expected value is

$$v_e = \frac{\mu(\omega_v)}{\mu(\omega_v) + \mu(\omega_\phi)} v > 0. \tag{1}$$

It will be useful to calculate the probability ψ_n that A_{n+1} does not receive a signal conditional on the event that A_n does not receive a signal. This will turn out to be the probability that A_n can sell the asset x to A_{n+1}, given that A_n has not received a signal. This probability is as follows: for $N-1$,

$$\psi_{N-1} = \frac{\mu(\omega_\phi)}{\mu(\omega_\phi) + \mu(\omega_N)}$$

and for each of $n = 1, \dots, N-2$,

$$\psi_n = \frac{\mu(\omega_\phi) + \mu(\omega_N) + \cdots + \mu(\omega_{n+2})}{\mu(\omega_\phi) + \mu(\omega_N) + \cdots + \mu(\omega_{n+2}) + \mu(\omega_{n+1})}.$$

Note that $0 < \psi_n < 1$ for each $n = 1, \dots, N-1$ because $\mu(\omega) > 0$ for each $\omega \in \Omega$.

3. Equilibrium

In this section, we derive an equilibrium of the economy,[13] and argue it is unique. In the first subsection, we display the equilibrium and in the second subsection, we provide a proof. In the third subsection, we discuss our main result.

3.1 *Life of a bubble*

Define a sequence $(\hat{y}_n)_{n=1}^{N-1}$ as follows: for $N-1$,

$$\hat{y}_{N-1} = \theta v_e$$

and for each $n = 1, \dots, N-2$,

$$\hat{y}_n = \theta \kappa \psi_{n+1} \hat{y}_{n+1}. \tag{2}$$

Similarly, define a sequence $(\hat{y}^v_n)_{n=1}^{N-1}$ as follows: for $N-1$,

$$\hat{y}^v_{N-1} = \theta v_e$$

[13]Our equilibrium concept is sequential equilibrium with the game form defined in footnote 8.

and for each $n = 1, \dots, N-2$,

$$\hat{y}_n^v = \theta\kappa\hat{y}_{n+1}^v.$$

Note that $\hat{y}_n > 0$ and $\hat{y}_n^v > 0$ for each $n = 1, \dots, N-1$ because $\theta > 0$, $v_e > 0$, $\kappa > 0$, and $\psi_{n+1} > 0$ for each $n = 1, \dots, N-2$. Here, $\hat{y}_n$ and $\hat{y}_n^v$ will be the prices A_n receives when $\omega \in \{\omega_\phi, \omega_N, \dots, \omega_{n+2}\}$ and $\omega = \omega_v$, respectively, as shown in the following lemma.

Lemma 1. *The following two statements hold:*

(i) Assume $\omega \neq \omega_v$. In equilibrium, if agent A_{n+1} receives a signal, agent A_{n+1} does not trade with agent A_n, or $y_n = 0$; if agent A_{n+1} does not receive a signal, agent A_{n+1} trades with agent A_n and obtains x in exchange for $y_n = \hat{y}_n$. Moreover, this outcome is unique.

(ii) Assume $\omega = \omega_v$. In equilibrium, A_{n+1} always trades with A_n and obtains x in exchange for $y_n = \hat{y}_n^v$. Moreover, this outcome is unique.

Recall that we assumed that the past prices cannot be observed by the final user. Thus, agent A_N cannot learn the state from whether the terms of trade are $\hat{y}_n$ or $\hat{y}_n^v$.

If $\omega \in \{\omega_1, \omega_2\}$, a bubble does not occur because A_2 receives a signal and does not trade with A_1. If $\omega = \omega_\phi$, trade takes place, but this is simply because A_N does not know that x is worthless. If $\omega \in \{\omega_N, \omega_{N-1}, \dots, \omega_3\}$, Lemma 1 describes the life of a bubble.

To see this, suppose the economy is at ω_{n^*} where $n^* > 2$. Then agents $A_N, \dots, A_{n^*}$ receive signals, while the others do not. Given this realization, every agent knows that x is worthless, and hence, the fundamental value of x is zero. Yet, the asset x is exchanged for a positive amount of the good y for $n^* - 2$ periods, until it reaches agent A_{n^*-1}. In this sense, a bubble is occurring. Obviously, if the fact that the fundamental value of x is zero were common knowledge, then x would not be traded. At period $n^* - 1$, agent A_{n^*} refuses to buy the asset from A_{n^*-1}, and the bubble bursts. We summarize this result as follows.

Theorem 1. *For any parameter values, the equilibrium exists and is unique. In the equilibrium, a bubble occurs when $\omega \in \{\omega_N, \omega_{N-1}, \dots, \omega_3\}$. Moreover, the bubble bursts for sure.*

Since the equilibrium exists for any parameter values, obviously it is robust to small perturbations of parameters. The theorem clarifies the role of middlemen for the occurrence of bubbles. To see this, consider the case without middlemen (i.e., $N = 2$).[14] The set of states in this case is $\Omega = \{\omega_v, \omega_\phi, \omega_2, \omega_1\}$. There is no state at which a bubble occurs, since A_2 only buys in $\{\omega_v, \omega_\phi\}$, where he thinks the asset might be valuable. This gives the following corollary.

Corollary 1. *A bubble does not occur when there is no middlemen in our setup.*

In Section 7, by means of examples, we further examine how middlemen affect the occurrence and duration of bubbles.

[14]The environment extends to this case in an obvious way.

3.2 *Proof of Lemma 1*

Proof is by backward induction. Here, we focus only on the case where $\omega \neq \omega_v$, and a similar proof holds for the other case.

Trade between A_{N-1} and A_N: If A_N receives a signal, A_N knows that x is worthless, and hence, A_N does not trade with A_{N-1}, so $y_{N-1} = 0$.

If A_N does not receive a signal, his expected value of x is v_e. Then A_N and A_{N-1} negotiate the terms of trade:

$$\max_{y_{N-1}} (\kappa y_{N-1})^{\theta}(v_e - y_{N-1})^{1-\theta}$$

subject to incentive constraints: $\kappa y_{N-1} \geq 0$ and $v_e - y_{N-1} \geq 0$. Note that, if they do not agree to trade, they do not obtain any utility. Recall that the consumption value of x for A_{N-1} is commonly known to be zero, and as a result A_{N-1}'s outside option in bargaining does not depend on the state. The solution is

$$\hat{y}_{N-1} = \theta v_e.$$

Hence, A_N obtains x in exchange for $\hat{y}_{N-1}$.

Trade between A_{N-2} and A_{N-1}: If A_{N-1} receives a signal, A_{N-1} knows that A_N receives a signal. Then, as we have shown above, A_{N-1} knows that A_N will not trade with A_{N-1}, and hence, A_{N-1} does not trade with A_{N-2}, so $y_{N-2} = 0$.

If A_{N-1} does not receive a signal, there are exactly two possibilities:

1. both A_{N-1} and A_N do not receive signals; and
2. A_N receives a signal, but A_{N-1} does not.

In the first case, A_N trades with A_{N-1}. In the second case, however, A_N does not trade with A_{N-1}. The first case occurs with probability ψ_{N-1} given that A_{N-1} does not receive a signal, and the second case occurs with the remaining probability. Hence, A_{N-1}'s expected utility of obtaining x is $\psi_{N-1}\kappa\hat{y}_{N-1}$. Then A_{N-1} and A_{N-2} negotiate the terms of trade:

$$\max_{y_{N-2}} (\kappa y_{N-2})^{\theta}(\psi_{N-1}\kappa\hat{y}_{N-1} - y_{N-2})^{1-\theta}$$

subject to incentive constraints: $\kappa y_{N-2} \geq 0$ and $\psi_{N-1}\kappa\hat{y}_{N-1} - y_{N-2} \geq 0$. The solution is

$$\hat{y}_{N-2} = \theta\kappa\psi_{N-1}\hat{y}_{N-1}.$$

Therefore, A_{N-1} obtains x in exchange for $\hat{y}_{N-2}$.

Induction hypothesis: Suppose that:

1. if A_{n+1} receives a signal, A_{n+1} does not trade with A_n, so $y_n = 0$;
2. if A_{n+1} does not receive a signal, A_{n+1} trades with A_n and obtains x in exchange for $y_n = \hat{y}_n$.

Then, we will show that:

1. if A_n receives a signal, A_n does not trade with A_{n-1}, or $y_{n-1} = 0$;
2. if A_n does not receive a signal, A_n trades with A_{n-1} and obtains x in exchange for $y_{n-1} = \hat{y}_{n-1}$.

Trade between A_{n-1} and A_n: If A_n receives a signal, A_n knows that A_{n+1} receives a signal. Then, by the induction hypothesis, A_n knows that A_{n+1} will not trade with A_n, and hence, A_n does not trade with A_{n-1}, so $y_{n-1} = 0$.

If A_n does not receive a signal, there are exactly two possibilities:

1. both A_n and A_{n+1} do not receive signals; and
2. A_{n+1} receives a signal, but A_n does not.

In the first case, by the induction hypothesis, A_{n+1} trades with A_n and obtains x in exchange for $\hat{y}_n$. In the second case, however, A_{n+1} does not trade with A_n, again by the induction hypothesis. The first case occurs with probability ψ_n, given that A_n does not receive a signal, and the second case occurs with the remaining probability. Hence, A_n's expected utility of obtaining x is $\psi_n \kappa \hat{y}_n$. Then A_{n-1} and A_n negotiate the terms of trade:

$$\max_{y_{n-1}} (\kappa y_{n-1})^{\theta} (\psi_n \kappa \hat{y}_n - y_{n-1})^{1-\theta}$$

subject to incentive constraints: $\kappa y_{n-1} \geq 0$ and $\psi_n \kappa \hat{y}_n - y_{n-1} \geq 0$. The solution is

$$\hat{y}_{n-1} = \theta \kappa \psi_n \hat{y}_n.$$

Therefore, A_n obtains x in exchange for $\hat{y}_{n-1}$.

Uniqueness In each trade, both agents obtain positive expected utilities because $0 < \theta < 1$. Moreover, in those states where A_{n+1} is hypothesized not to trade with A_n, A_{n+1} has strict incentives to refuse the trade, so chooses $y_n = 0$. Hence, there is no indifference among choices of each agent, which implies the uniqueness of equilibrium.[15]

Individual rationality From the equilibrium prices, we can see that the interim expected utilities are positive for agents who do not receive signals and zero for the other agents. Hence, every agent has incentives to participate in the economy at the interim stage—after the state is determined but before trade takes place.[16] This further implies that the ex ante—before the state is determined—utility of each agent is positive, and thus every agent has incentives to participate in the economy at the ex ante stage as

[15]Note that conditional upon A_N chooses an action, it is strictly dominant to trade (not to trade) when he does not get (gets) a signal. Given this, again conditional upon A_{N-1} chooses an action, it is strictly dominant to trade (not to trade) when he does not get (gets) a signal, and this is true for all A_n. Thus, in each round we are only eliminating *conditionally dominated* strategies (Shimoji and Watson (1998)).

[16]When the state is ω_v, agents $A_1, \ldots, A_{N-1}$ know that the consumption value of the asset x for the final user A_N is $v > 0$, and the final user A_N does not know this since he never receives a signal. Thus, each trade takes place between each pair of two adjacent agents, and each agent enjoys positive expected utility at the interim stage.

well. Therefore, before trade takes place, it is optimal for each agent to participate in the economy.

3.3 *Discussion of Theorem 1*

Opaqueness In our result, the opaqueness assumption—the final user cannot observe the transacted prices of the other pairs—plays a key role. The prices at state ω_v and at the other states differ, because at state ω_v middlemen know that the final user will buy the asset for sure. However, the final user cannot observe the prices and so cannot update his belief on the state. To understand it more deeply, consider the case where prices are publicly observable. Then it is easy to see that Lemma 1 (and hence, Theorem 1) does not hold, because the final user A_N can learn the state from prices—if asset x is traded at price $\hat{y}_n$ (resp. $\hat{y}_n^v$), then the state must be ω_ϕ (resp., ω_v).

Does a bubble occur with publicly observable prices in our model? For it to be possible, we must have the same price movements in states ω_v and ω_ϕ. One way to achieve the same price movements in these states is to modify the current setup and add new states where middlemen also do not know the true valuation. Even with this trick, the final user might learn the difference between ω_v and ω_ϕ because in general trades among middlemen depend on states. Hence, a bubble can occur only as a knife-edge case and we lose robustness. See Appendix A for the details.

Relationship to "greater fool" models of bubbles Unlike in greater fool models of bubbles (Allen, Morris, and Postlewaite (1993)), in our model there are neither "bad" sellers—sellers who know that they are selling worthless objects—nor "good" sellers in the trade among middlemen. Related and more importantly, traders are in fact selling to other traders who know (weakly) more than themselves. This is actually opposite to the existing greater fool models where sellers are better informed than buyers are.[17]

Intermediation chain Wright and Wong (2014) provide a model of intermediation bubbles and show that bubbles occur if there are an infinite number of middlemen and the potential gains from trade in terms of y is strictly positive. In this vein, our innovation is not only in relaxing the assumption of common knowledge and showing that bubbles can occur with a finite number of middlemen, but also in showing that there is a bubble even when the potential gains from trade in terms of y are zero or negative. In Section 5, we will see that this leads us to different welfare implications from theirs.

4. Price changes

We are ready to investigate the properties of the equilibrium. In this section, we study how the equilibrium price changes over time. We show that $\hat{y}_n$, the price that A_{n+1} has to pay to obtain the asset x, is not only *increasing* but also *accelerating* in n during a bubble under reasonable assumptions. We emphasize that in our model the fundamental value is constant over time. To this end, we show the following technical lemma.

[17] Strictly speaking, the information partition of agent A_{n+1} is not finer than A_n. Still in terms of relevant information, A_{n+1} is (weakly) more informed than A_n.

Lemma 2. *If $\mu(\omega_2) \leq \cdots \leq \mu(\omega_N)$, the probability ψ_n is decreasing in n, that is,*

$$\psi_{n+1} < \psi_n.$$

Proof. For $n = 1, \ldots, N-3$, letting $\mathcal{M} = \mu(\omega_\phi) + \mu(\omega_N) + \cdots + \mu(\omega_{n+3})$,

$$\psi_{n+1} - \psi_n = \frac{\mathcal{M}\big[\mu(\omega_{n+1}) - \mu(\omega_{n+2})\big] - \big[\mu(\omega_{n+2})\big]^2}{\big[\mathcal{M} + \mu(\omega_{n+2})\big]\big[\mathcal{M} + \mu(\omega_{n+2}) + \mu(\omega_{n+1})\big]} < 0$$

where the last inequality holds since we have $\mu(\omega_{n+1}) \leq \mu(\omega_{n+2})$ for each $n = 1, \ldots, N-3$ and $\mu(\omega) > 0$ for each $\omega \in \Omega$ by assumption. A similar argument holds for the case between $N-1$ and $N-2$. □

We now obtain the following result on price changes.

Proposition 1. *If $\theta\kappa \leq 1$, then $\hat{y}_n$ is increasing in n, that is,*

$$\hat{y}_{n+1} - \hat{y}_n > 0.$$

Moreover, if $\mu(\omega_2) \leq \cdots \leq \mu(\omega_N)$, it is accelerating in n, that is,

$$\frac{\hat{y}_{n+2} - \hat{y}_{n+1}}{\hat{y}_{n+1}} > \frac{\hat{y}_{n+1} - \hat{y}_n}{\hat{y}_n}.$$

Proof. To see that $\hat{y}_n$ is increasing, we obtain

$$\hat{y}_{n+1} - \hat{y}_n = \hat{y}_{n+1} - \theta\kappa\psi_{n+1}\hat{y}_{n+1} > 0.$$

To see that $\hat{y}_n$ is accelerating, we obtain

$$\frac{\hat{y}_{n+2} - \hat{y}_{n+1}}{\hat{y}_{n+1}} - \frac{\hat{y}_{n+1} - \hat{y}_n}{\hat{y}_n} = \frac{1}{\theta\kappa\psi_{n+2}} - \frac{1}{\theta\kappa\psi_{n+1}} > 0$$

where the inequality holds because we have $\psi_{n+2} < \psi_{n+1}$ by Lemma 2. □

The fact that $\hat{y}_n$ is increasing follows because each middleman must be compensated for the risk that he may not be able to sell the asset x with positive probability, $1 - \psi_n$. Thus, during a bubble, middlemen "flip"—agent A_{n+1} buys the asset x at the price $\hat{y}_n$ in hopes of reselling it at the price $\hat{y}_{n+1} > \hat{y}_n$. The fact that $\hat{y}_n$ is accelerating follows because the probability that one can resell, ψ_n, is decreasing over time. In other words, flippers who trade in later periods are exposed to a bigger risk, and the prices are determined in such a way that they are compensated for this increasing risk. Note that the growth rate of the price, $(\hat{y}_{n+1} - \hat{y}_n)/\hat{y}_n$, is increasing, not just the price differences $\hat{y}_{n+1} - \hat{y}_n$. In other words, the price differences are growing more quickly than the price itself is growing.

For price increase, $\kappa \leq 1$ is sufficient. For price acceleration, we impose an additional assumption, which is that states where more agents receive signals are realized with smaller probabilities.

The price trajectory shown in Proposition 1 features many observed bubbles, not only in the modern bubbles, for example, the dot-com bubble[18] or the Japanese asset bubble or the housing bubble before the last financial crisis (see, e.g., Figure 1 in Bayer et al. (2020), p. 5219), but also in the classical Dutch tulip bubble, etc.

5. Welfare

Now, we derive welfare implications. Our welfare criterion is utilitarian, that is, welfare is the sum of all agents' utilities. Note that agents have incentives to participate in the economy, and so the welfare analysis is meaningful.

Observe that, in trade between A_n and A_{n+1}, the gains from trade are

$$\kappa y_n - y_n = (\kappa - 1) y_n$$

because we normalized the production cost of y to be 1. Thus, when the economy is at ω_{n^*} with $n^* > 2$, ex post welfare is

$$(\kappa - 1) \sum_{n=1}^{n^*-2} \hat{y}_n.$$

This is because a bubble continues for $n^* - 2$ periods, after which the bubble bursts and A_{n^*-1} does not succeed in selling to A_{n^*}. The terms of trade are $y_n = \hat{y}_n$ during a bubble and $y_n = 0$ after period $n^* - 2$. We obtain the following result on welfare.

Proposition 2. *Bubbles are detrimental to the economy if and only if* $\kappa < 1$.

As in most models of bubbles, a bubble facilitates trade in our theory. However, unlike most greater-fool models,[19] trade can be detrimental. Our innovation is not only to show that bubbles are not necessarily welfare improving but also to characterize exactly when a bubble is detrimental.

There are efficiency losses from trading the good y if and only if $\kappa < 1$. For example, if there are some transaction costs that sellers must pay when they sell the asset, the ex post welfare is negative.[20]

Proposition 2 should be interpreted with caution, because it is only about the welfare implication of bubbles, and not about ex ante welfare.[21] When analyzing the welfare effect of a policy concerning bubbles (as we do in the next section), we will have to consider not only its effect on the states of bubbles but also on the other states.

[18] See the NASDAQ Composite Index.

[19] Most greater fool bubbles involve sellers acting in bad faith, that is, knowingly deceiving ex ante uninformed buyers who ex post wish they had not bought. In our model, this occurs only in the trade with the final user.

[20] Grossman and Yanagawa (1993), Miao and Wang (2014), and Guerrón-Quintana, Hirano, and Jinnai (2020) show that bubbles may reduce welfare in models of endogenous growth.

[21] The ex post welfare in ω_ϕ is $(\kappa - 1)\sum_{n=1}^{N-1} \hat{y}_n$ and that in ω_v is $v + (\kappa - 1)\sum_{n=1}^{N-1} \hat{y}_n^v$.

6. Bubble bursting policy

In his speech, "*The Challenge of Central Banking in a Democratic Society*," Alan Greenspan pointed out the possibility of overpriced assets. In this section, we examine welfare implications of such policies—policies that (intend to) affect agents' beliefs on the quality of the asset by releasing information about it. The finding is somewhat surprising. *Any* of these public policies have *no* welfare consequence.

For this purpose, we employ the information design approach (Kamenica and Gentzkow (2011), henceforth KG). A public *experimentation* $\sigma : \Omega \to \Delta(M)$ sends to the agents a randomized message, where M denotes the message space. The probability that a message $m \in M$ is sent when the state is $\omega \in \Omega$ is denoted by $\sigma_\omega(m)$. Following the literature, we assume that the information designer (central bank) chooses and commits to the experimentation σ before the state is realized, and its choice of rules is observable to agents. Motivated by the fact that Greenspan's comment was taken seriously and spread instantly among all market participants, we restrict our attention to *public* experimentation, where the realized message m is publicly observable.

We assume first agents receive message m, and update their belief to $\mu_m \in \Delta(\Omega)$. Because message m is commonly observable, the updated belief μ_m is commonly known. Next, agents get private signals, as in Section 2. Thus, effectively, after message m is received, the situation is the same as the game described in Section 2, but with the updated "prior" distribution μ_m.[22]

Of course, Bayes plausibility (see KG) holds:

$$\mathbb{E}_{\sigma(m)}[\mu_m] = \mu \tag{3}$$

where $\mu \in \Delta(\Omega)$ is the prior.

One example of a public experimentation is full disclosure, where $M = \Omega$ and $\sigma_{\omega_n}(\omega_{n'}) = \delta_{n,n'}$ with $\delta_{n,n'}$ is the Kronecker delta function. Another example is a *bubble bursting policy* proposed by Conlon (2015) and further examined by Holt (2019).[23]

Example 1. [Bubble bursting policy] The central bank announces whether the state belongs to $\{\omega_N, \dots, \omega_1\}$ or $\{\omega_v, \omega_\phi\}$. That is, $M = \{m_0, m_+\}$ and m_0 (resp., m_+) denotes $\omega \in \{\omega_N, \dots, \omega_1\}$ (resp., $\omega \in \{\omega_v, \omega_\phi\}$) with probability 1. Thus, the probability that each message is sent is $\sigma_\omega(m_0) = 1$ for $\omega \in \{\omega_N, \dots, \omega_1\}$ and $\sigma_\omega(m_+) = 1$ for $\omega \in \{\omega_v, \omega_\phi\}$.

Note that messages m_0 and m_+ induce posteriors

$$\mu_0(\omega) = \begin{cases} \dfrac{\mu(\omega_n)}{\mu(\omega_N) + \cdots + \mu(\omega_1)} & \text{if } \omega = \omega_n \in \{\omega_N, \dots, \omega_1\} \\ 0 & \text{otherwise} \end{cases}$$

[22] Note that we have assumed full support in Section 2. While this may not be the case here, it is routine to extend the model.

[23] Asako and Ueda (2014) use another approach, based on Abreu and Brunnermeier (2003), and examine whether a public announcement can burst a bubble but without welfare analysis.

and

$$\mu_+(\omega) = \begin{cases} 0 & \text{if } \omega = \omega_n \in \{\omega_N, \dots, \omega_1\} \\ \dfrac{\mu(\omega_v)}{\mu(\omega_v) + \mu(\omega_\phi)} & \text{if } \omega = \omega_v \\ \dfrac{\mu(\omega_\phi)}{\mu(\omega_v) + \mu(\omega_\phi)} & \text{if } \omega = \omega_\phi \end{cases},$$

respectively. It is routine to check Bayes plausibility.

This policy is bubble bursting in the sense of Definition 1—we say that a bubble occurs if the asset x is traded despite all agents know it is worthless. At states $\omega_1, \dots, \omega_N$, asset x is not traded, and hence, a bubble does not occur. At state ω_ϕ, asset x is still overpriced, but given that agent A_N does not know this, we do not call this a bubble. At state ω_v, asset x is traded but has a positive value $v > 0$. □ ◊

This experiment is of particular interest, as it does not require the central bank to have a superior information technology to that of agents. Indeed, this policy is implementable if the knowledge of the central bank is the same as that of agent A_N.

Now we examine welfare consequences of such public announcements. Our welfare criterion in this section is—unlike the previous section—ex ante. That is, the information designer also takes into account the states at which a bubble does not occur.

We now prove the following neutrality result.

Proposition 3. *Announcement through any public experimentation does not change the ex ante payoff of any agent.*

This proposition is a consequence of the fact that the ex ante expected prices are not affected by the public experiments, which will be shown below.

Proof. First, using $\hat{y}^v_{N-1} = \hat{y}_{N-1} = \theta v_e$, the ex ante expected price paid by A_N is

$$\mu(\omega_v)\hat{y}^v_{N-1} + \mu(\omega_\phi)\hat{y}_{N-1} = \big[\mu(\omega_v) + \mu(\omega_\phi)\big]\theta v_e = \theta\mu(\omega_v)v.$$

Hence, it is independent of any policy. Here, if y_n is the (stochastic) price in period n,

$$y_n = \theta\kappa\mathbb{E}_n y_{n+1}.$$

Therefore, by the law of iterated expectations, announcement through any public experimentation does not change the expected prices paid by any agents. □

This result means that the central bank cannot affect any agent's ex ante welfare by releasing information. For example, consider the full-disclosure policy where the central bank always knows the consumption value of the asset x. When the state is $\omega \neq \omega_v$, the central bank announces that x is valueless, and thus trade does not take place. When $\kappa < 1$ (when $\kappa > 1$, the opposite happens), trade of good y is detrimental to the economy. In this case, the policy improves interim welfare in these states because it prevents trade of good y. Of course, this policy has a side effect. When the state is ω_v, the central bank announces the information that the consumption value of x for A_N is $v > 0$, and hence, all agents including A_N know that the consumption value is $v > 0$. Thus, the trade of good y is facilitated by the policy in this state because the expected consumption value of x for A_N increases. These effects are exactly canceled out.

The result in Section 5—that a bubble is detrimental when $\kappa < 1$—in contrast, focuses only on welfare in states where bubbles occur. Put differently, when $\kappa < 1$ and $\omega \in \{\omega_N, \dots, \omega_3\}$, a benevolent information designer is happy with his announcement policy. In contrast, when $\omega = \omega_v$, by the policy, the final user learns that the consumption value of the asset x is $v > 0$ and as a result the amounts of good y traded increase. In this state, the benevolent policy maker suffers from ex post regrets that it should not have implemented such a policy.

The result depends on the assumption that the utility function of y is linear, or agents are risk neutral. However, it may serve as a benchmark result in information design for models of bubbles, as the revenue equivalence theorem does in auction theory. In Appendix B, we will provide two examples in which agents are risk averse and public announcements are not neutral—in one example welfare is enhanced and in the other it is hurt.[24]

7. Market structure

In this section, we study the effect of market structure on bubbles. In particular, we investigate the effect of the number of middlemen and the extent to which rumors are spread on the probability, duration and size of bubbles. Below, we specify a parametric form of the prior distribution but ensure the assumptions that we have imposed in the previous sections.

Toward this purpose, we specify the prior μ as follows.[25] Consider a situation where the signal to A_N is lost with probability ε_F, and moreover, between any two adjacent agents, each signal is lost with a common probability, ε_M. In other words, the final user A_N knows that the consumption value of x for A_N is zero with probability $1 - \varepsilon_F$, and the rumor that A_N knows the fact reaches A_{N-1} with probability $1 - \varepsilon_M$, and so on and so forth. The parameter ε_M can be thought as a measure of the extent to which rumors spread—the bigger ε_M is, the more slowly rumors spread.

In this case, we have

$$\mu(\omega_\phi) = \big[1 - \mu(\omega_v)\big]\varepsilon_F$$

and, for each $n = 2, \dots, N$,

$$\mu(\omega_n) = \big[1 - \mu(\omega_v)\big](1 - \varepsilon_F)(1 - \varepsilon_M)^{N-n}\varepsilon_M$$

while for $n = 1$,

$$\mu(\omega_1) = \big[1 - \mu(\omega_v)\big](1 - \varepsilon_F)(1 - \varepsilon_M)^{N-1}.$$

[24] Holt (2019) also considers risk-averse agents and shows that in his model, a bubble bursting policy tends to be detrimental as well.

[25] In Appendix C, we consider another specification.

We have $\mu(\omega_\phi) = [1 - \mu(\omega_v)]\varepsilon_F$ because the final user A_N does not receive any signals with probability ε_F, given that the asset is worthless. For each $n = 2, \dots, N$, we have $\mu(\omega_n) = [1 - \mu(\omega_v)](1 - \varepsilon_F)(1 - \varepsilon_M)^{N-n}\varepsilon_M$ because the signal is not lost between any two adjacent agents until A_n receives it, but it is lost between A_n and A_{n-1}. We have $\mu(\omega_1) = [1 - \mu(\omega_v)](1 - \varepsilon_F)(1 - \varepsilon_M)^{N-1}$ because the signal is not lost between any two adjacent agents. Assume $0 < \mu(\omega_v) < 1$, $0 < \varepsilon_F < 1$, and $0 < \varepsilon_M < 1$. Then it is obvious that $\mu(\omega) > 0$ for each $\omega \in \Omega$ and $\mu(\omega_2) \leq \cdots \leq \mu(\omega_N)$. The probability ψ_n is

$$\psi_n = \frac{\varepsilon_F + (1 - \varepsilon_F)\big[1 - (1 - \varepsilon_M)^{N-n-1}\big]}{\varepsilon_F + (1 - \varepsilon_F)\big[1 - (1 - \varepsilon_M)^{N-n}\big]}$$

and the price $\hat{y}_n$ is

$$\hat{y}_n = \frac{\varepsilon_F}{\varepsilon_F + (1 - \varepsilon_F)\big[1 - (1 - \varepsilon_M)^{N-n-1}\big]}\theta^{N-n}\kappa^{N-n-1}v_e$$

where

$$v_e = \frac{\mu(\omega_v)}{\mu(\omega_v) + \big[1 - \mu(\omega_v)\big]\varepsilon_F}v.$$

The price $\hat{y}_n$ is decreasing in ε_M. See Figure 2 for the graph of $\hat{y}_n$.

A bubble occurs when the state belongs to $\{\omega_N, \omega_{N-1}, \dots, \omega_3\}$, and hence, the probability that a bubble occurs is

$$1 - \big[\mu(\omega_v) + \mu(\omega_\phi) + \mu(\omega_2) + \mu(\omega_1)\big] = \big[1 - \mu(\omega_v)\big](1 - \varepsilon_F)\big[1 - (1 - \varepsilon_M)^{N-2}\big].$$

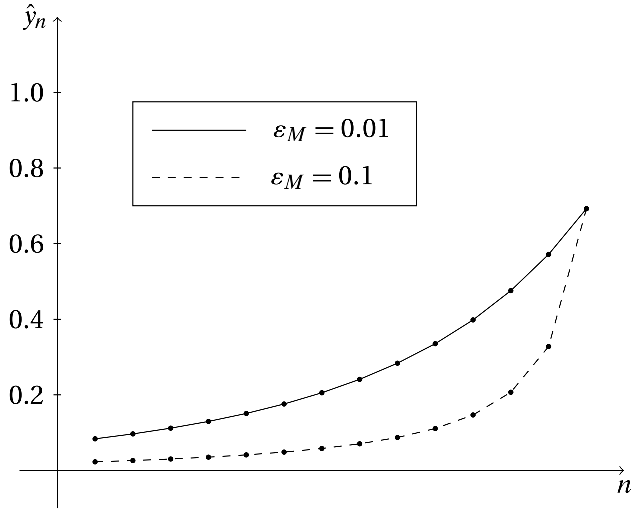

Figure 2. Graph of $\hat{y}_n$ ($N = 15$, $\theta = 0.9$, $\kappa = 1$, $v = 2$, $\mu(\omega_v) = 1/17$, and $\varepsilon_F = 0.1$). Prices are lower in each state with a higher value of ε_M (a less transparent market).

The probability is increasing in ε_M. Focusing on the states where the final user knows that x is worthless, the expected duration of the bubble is

$$\sum_{n=3}^{N} \frac{\mu(\omega_n)(n-2)}{1-[\mu(\omega_v)+\mu(\omega_\phi)]} = N-2-\frac{(1-\varepsilon_M)[1-(1-\varepsilon_M)^{N-2}]}{\varepsilon_M}.$$

The expected duration is increasing in ε_M. We then define *the expected duration ratio* as the expected duration divided by the number of periods. The difference in the expected duration ratio is

$$\begin{aligned}&\frac{N-1}{N}-\frac{(1-\varepsilon_M)[1-(1-\varepsilon_M)^{N-1}]}{N\varepsilon_M}-\left\{\frac{N-2}{N-1}-\frac{(1-\varepsilon_M)[1-(1-\varepsilon_M)^{N-2}]}{(N-1)\varepsilon_M}\right\}\\ &=\frac{1}{N(N-1)\varepsilon_M}\{1-(1-\varepsilon_M)^{N-1}[1+(N-1)\varepsilon_M]\}.\end{aligned}$$

It is routine to show that the right-hand side is positive for all $\varepsilon_M \in (0,1)$. That is, the expected duration ratio is increasing in the number of agents.

The following two propositions summarize the above analysis.

Proposition 4. *The wider the rumors are spread, the less likely a bubble occurs and the shorter the expected duration ratio of the bubble is, but the larger the size of the bubble is.*

Proposition 5. *The larger the number of middlemen, the higher the ex ante probability of a bubble to occur, and the longer the expected duration ratio of the bubble.*

The Dodd–Frank Act aims at improving the ability of market participants to monitor and understand the risks in these markets. Hence, our prediction of Proposition 4 about its effect is not only to reduce the probability and the length of a bubble, but also to increase the size of it (if it occurs at all).

Proposition 5 is consistent with the empirical evidence by Bayer et al. (2020) who report that a housing bubble is accompanied by a large increase in the number of flippers. Further, it is interesting to mention that the Japanese asset/housing bubble from 1986 to 1991 was believed to be caused structurally through bank deregulation. It drastically moved large corporate customers away from bank borrowing toward other financing and as a result increased flipping. More generally, Kindleberger and Aliber (2005) argue that bubbles are often triggered by financial innovations (e.g., futures, acceptance loans, or securitization) or deregulation (opening new business opportunities), which most likely enhances entry of middlemen.

8. Conclusion

In this paper, we construct a simple tractable model of a bubble. This tractability of the model will allow one to investigate some other interesting properties of bubbles, which we have not explored.

For example, our model has an indivisible asset and a divisible good. By replacing the divisible good with divisible money, we may be able to study the relationships between bubbles and monetary policy more explicitly. Also, it would be interesting to endogenize middlemen's entry. The model then may be able to generate a tighter prediction on how regulations on entry of middlemen, which is an important and practical policy target in many markets, can influence the occurrence and welfare consequence of bubble.

Another interesting issue is the relationship between market opaqueness and robustness—in models of bubbles where the motivation of trade is risk-sharing (Allen, Morris, and Postlewaite (1993), Liu and Conlon (2018)) or intertemporal consumption-smoothing (Liu, White, and Conlon (2020)), would equilibrium be robust when prices are not observable? We leave them for future research.

Appendix A: Publicly observable prices

We have assumed that the final user cannot observe prices in past trades among middlemen. In this Appendix, we modify the setup in the main text and show that a bubble can occur in equilibrium but within knife-edge parameters.

To have a bubble with publicly observable prices, we must have the same price movements in states ω_v and ω_ϕ because, otherwise, the price pattern reveals information to the final user. One way to achieve the same price movements in these states is to add new states $\omega^+_{N-1}, \omega^+_{N-2}, \dots, \omega^+_1$. For each $n = 1, \dots, N-1$, at state ω^+_n, the consumption value of x for the final user A_N is zero, and for each $m \leq n$, agent A_m does not know whether the consumption value is zero, while for each $m > n$, agent A_m knows that the consumption value is zero. The set of the states is

$$\Omega^+ = \{\omega_v, \omega_\phi, \omega_N, \dots, \omega_1, \omega^+_{N-1}, \dots, \omega^+_1\}.$$

Let μ^+ be the common prior distribution over Ω^+, and assume that $\mu^+(\omega) > 0$ for each $\omega \in \Omega^+$.

Now, agent A_N's partition is

$$\mathcal{P}^+_N = \{\{\omega_v, \omega_\phi\}, \{\omega_N, \dots, \omega_1, \omega^+_{N-1}, \dots, \omega^+_1\}\}.$$

The first element, $\{\omega_v, \omega_\phi\}$, corresponds to the case where A_N does not know whether x is worthless. The second element, $\{\omega_N, \dots, \omega_1, \omega^+_{N-1}, \dots, \omega^+_1\}$, corresponds to the case where A_N knows that x is worthless. For each $n < N$, agent A_n's partition is

$$\mathcal{P}^+_n = \{\{\omega_v, \omega^+_{N-1}, \dots, \omega^+_n\}, \{\omega_\phi, \omega_N, \dots, \omega_{n+1}\}, \{\omega_n, \dots, \omega_1, \omega^+_{n-1}, \dots, \omega^+_1\}\}.$$

The first element, $\{\omega_v, \omega^+_{N-1}, \dots, \omega^+_n\}$, corresponds to the case where A_n does not know whether x is worthless. The second element, $\{\omega_\phi, \omega_N, \dots, \omega_{n+1}\}$, corresponds to the case where A_n knows that x is worthless, but A_n does not know whether A_{n+1} knows that ... A_N knows that x is worthless. The third element, $\{\omega_n, \dots, \omega_1, \omega^+_{n-1}, \dots, \omega^+_1\}$, corresponds to the case where A_n knows that x is worthless and that A_{n+1} knows that ... A_N knows that x is worthless. Note that middlemen may not know the consumption value of x for A_N when it is $v > 0$.

Analogously to the definitions of v_e and ψ_n in the previous sections, define

$$v_e^+ = \frac{\mu^+(\omega_v)}{\mu^+(\omega_v) + \mu^+(\omega_\phi)} v$$

and for $N-1$,

$$\psi_{N-1}^+ = \frac{\mu^+(\omega_\phi)}{\mu^+(\omega_\phi) + \mu^+(\omega_N)}$$

while for each $n = 1, \dots, N-2$, define

$$\psi_n^+ = \frac{\mu^+(\omega_\phi) + \mu^+(\omega_N) + \cdots + \mu^+(\omega_{n+2})}{\mu^+(\omega_\phi) + \mu^+(\omega_N) + \cdots + \mu^+(\omega_{n+2}) + \mu^+(\omega_{n+1})}.$$

Moreover, define a sequence $(\hat{y}_n^+)_{n=1}^{N-1}$ as follows: for $N-1$,

$$\hat{y}_{N-1}^+ = \theta v_e^+$$

and for each $n = 1, \dots, N-2$,

$$\hat{y}_n^+ = \theta \kappa \psi_{n+1}^+ \hat{y}_{n+1}^+.$$

The following lemma shows we can find parameters under which the equilibrium prices are the same across ω_v and ω_ϕ.[26]

Lemma 3. *Assume that for $N-1$,*

$$\psi_{N-1}^+ = \frac{\mu^+(\omega_v)}{\mu^+(\omega_v) + \mu^+(\omega_{N-1}^+)}$$

and for each $n = 2, \dots, N-2$,

$$\psi_n^+ = \frac{\mu^+(\omega_v) + \mu^+(\omega_{N-1}^+) + \cdots + \mu^+(\omega_{n+1}^+)}{\mu^+(\omega_v) + \mu^+(\omega_{N-1}^+) + \cdots + \mu^+(\omega_{n+1}^+) + \mu^+(\omega_n^+)}.$$

Then, for each $\omega \in \Omega^+$, the following is an equilibrium. Moreover, the outcome is unique.

(i) If $\omega \in \Omega^+ \setminus \{\omega_v, \omega_\phi\}$, agent A_N does not trade with agent A_{N-1}, so $y_{N-1} = 0$; otherwise, agent A_N trades with agent A_{N-1} and obtains x in exchange for $y_{N-1} = \hat{y}_{N-1}^+$.

(ii) If $\omega \in \Omega^+ \setminus \{\omega_v, \omega_\phi, \omega_N, \dots, \omega_{n+2}, \omega_{N-1}^+, \dots, \omega_{n+1}^+\}$, agent A_{n+1} does not trade with agent A_n, so $y_n = 0$; otherwise, agent A_{n+1} trades with agent A_n and obtains x in exchange for $y_n = \hat{y}_n^+$.

A price coincidence in each period is necessary to prevent the final user from distinguishing a "good" state (ω_v) from a "bad" state (ω_ϕ). Note that requiring prices to match in states ω_v and ω_ϕ is rather stringent and it holds true only for certain knife-edge parameters. In other words, a bubble equilibrium is not robust to parameter changes.

[26] For instance, a uniform distribution satisfies the requirement in Lemma 3.

THEOREM 2. *Consider an extended setup with additional states. For some knife-edge parameter values, an equilibrium exists and is unique, and a bubble occurs when* $\omega \in \{\omega_N, \omega_{N-1}, \dots, \omega_3\}$. *Moreover, the bubble bursts for sure.*

APPENDIX B: RISK AVERSION

We obtained a welfare neutrality result for bubble bursting policies when $U(y) = \kappa y$. Here, we consider two examples in which some of agents are risk averse. In both examples, we compare the case where the information designer fully reveals the state ω versus the case where it reveals no information. Of course, once ω is fully revealed, there is no room for a bubble to occur, and asset x is traded only if it has a real value. As for the welfare criterion, we again consider the sum of agents' utilities. In one example, full disclosure is welfare improving, while in the other it is welfare reducing.

Example 1: Full disclosure is welfare reducing

Suppose the utility function is concave in y:

$$U(y) = \kappa y^{\alpha}$$

where $\alpha \in (0, 1]$ is a constant. More precisely, agent A_N's utility is $v - y_{N-1}$ if good x has value and he produces y_{N-1} of good y. If good x does not have value, then it is $-y_{N-1}$. For $n = 2, \dots, N-1$, agent A_n's utility is $U(y_n) - y_{n-1}$ if he produces y_{n-1} and consumes y_n of good y.

First, consider the case where the information designer reveals no information. Define a sequence $(\hat{y}_n)_{n=1}^{N-1}$ as follows (with $\alpha = 1$, this corresponds to the same variable in the main text): for $N-1$,

$$\hat{y}_{N-1} = \frac{\theta\alpha}{1-\theta(1-\alpha)} v_e$$

and for each $n = 1, \dots, N-2$,

$$\hat{y}_n = \frac{\theta\alpha}{1-\theta(1-\alpha)} \kappa\psi_{n+1}\hat{y}_{n+1}^{\alpha}.$$

Similarly, define a sequence $(\hat{y}_n^v)_{n=1}^{N-1}$ as follows: for $N-1$,

$$\hat{y}_{N-1}^v = \frac{\theta\alpha}{1-\theta(1-\alpha)} v_e$$

and for each $n = 1, \dots, N-2$,

$$\hat{y}_n^v = \frac{\theta\alpha}{1-\theta(1-\alpha)} \kappa\left(\hat{y}_{n+1}^v\right)^{\alpha}.$$

As in Lemma 1, the characterization of equilibrium is as follows:

1. Assume $\omega \neq \omega_v$. In equilibrium, if agent A_{n+1} receives a signal, agent A_{n+1} does not trade with agent A_n, so $y_n = 0$; if agent A_{n+1} does not receive a signal, agent

A_{n+1} trades with agent A_n and obtains x in exchange for $y_n = \hat{y}_n$. Moreover, this outcome is unique.

2. Assume $\omega = \omega_v$. In equilibrium, A_{n+1} always trades with A_n and obtains x in exchange for $y_n = \hat{y}_n^v$. Moreover, this outcome is unique.

When $\omega \in \{\omega_v, \omega_\phi\}$, trade takes place each period. When $\omega = \omega_n$, trade takes place until period $n-2$ if $n \geq 3$ and trade never takes place if $n \leq 2$. When $\omega = \omega_v$, the sum of utilities is

$$v - \hat{y}_{N-1}^v + U(\hat{y}_{N-1}^v) - \hat{y}_{N-2}^v + \cdots - \hat{y}_1^v + U(\hat{y}_1^v) = v + \sum_{n=1}^{N-1} \left[\kappa(\hat{y}_n^v)^\alpha - \hat{y}_n^v\right].$$

When $\omega = \omega_\phi$, it is

$$-\hat{y}_{N-1} + U(\hat{y}_{N-1}) - \hat{y}_{N-2} + \cdots - \hat{y}_1 + U(\hat{y}_1) = \sum_{n=1}^{N-1} \left(\kappa \hat{y}_n^\alpha - \hat{y}_n\right),$$

and when $\omega = \omega_{n^*}$ with $n^* \geq 3$, it is

$$-\hat{y}_{n^*-2} + U(\hat{y}_{n^*-2}) - \hat{y}_{n^*-3} + \cdots - \hat{y}_1 + U(\hat{y}_1) = \sum_{n=1}^{n^*-2} \left(\kappa \hat{y}_n^\alpha - \hat{y}_n\right).$$

The ex ante welfare is therefore

$$W = \mu(\omega_v)\left\{v + \sum_{n=1}^{N-1} \left[\kappa(\hat{y}_n^v)^\alpha - \hat{y}_n^v\right]\right\} + \mu(\omega_\phi) \sum_{n=1}^{N-1} \left(\kappa \hat{y}_n^\alpha - \hat{y}_n\right) + \sum_{n^*=3}^{N} \mu(\omega_{n^*}) \sum_{n=1}^{n^*-2} \left(\kappa \hat{y}_n^\alpha - \hat{y}_n\right).$$

Next, consider the full-disclosure policy. In this case, the consumption value of x for A_N is common knowledge, and trade takes place only at ω_v. Asset x is traded in exchange for $(y_n^d)_{n=1}^{N-1}$ of good y in period n (the superscript d denotes full disclosure) where

$$y_{N-1}^d = \frac{\theta\alpha}{1-\theta(1-\alpha)} v$$

and for each $n = 1, \ldots, N-2$,

$$y_n^d = \frac{\theta\alpha}{1-\theta(1-\alpha)} \kappa\left(y_{n+1}^d\right)^\alpha.$$

The ex ante welfare is

$$W^d = \mu(\omega_v)\left\{v + \sum_{n=1}^{N-1} \left[\kappa(y_n^d)^\alpha - y_n^d\right]\right\}.$$

We consider the following numerical example.

Claim 1. *Suppose μ is a uniform distribution with $N = 3$, $\theta = 0.9$, $\kappa = 1$, $v = 2$, and $\alpha = 0.9$. Then we obtain $W = 0.4231 > 0.3682 = W^d$. That is, the full-disclosure policy is welfare reducing.*

In this case, a bubble is welfare improving because it allows risk-averse middlemen to share risks—the Hirshleifer (1971) effect—and so the bubble bursting policy is detrimental. A similar result is also provided by Holt (2019) with a different setup. He constructs a model similar to Conlon (2015), where the motive for trade is risk sharing, and studies bubble bursting policies. He shows that such policies prevent risk sharing, and hence, may be detrimental.

Example 2: Full disclosure is welfare improving

Next suppose agent A_N's utility is $u(v - y_{N-1})$ when $\omega = \omega_v$ and $u(-y_{N-1})$ otherwise, where u is strictly increasing and concave, and moreover satisfies $u(0) = 0$. For $n = 2, \dots, N-1$, agent A_n's utility is $\kappa y_n - y_{n-1}$. Agent 1's utility is κy_1. In words, the initial owner and middlemen are still risk neutral, but the final user is risk averse. For simplicity, we assume $\kappa = 1$—that is, the social gains from trading good y is zero—and $\theta = 1$ in what follows—that is, in the trade between A_n and A_{n+1}, A_n makes a take-it-or-leave-it offer—but by continuity, the result also holds for $\kappa \neq 1$ or $\theta < 1$ if they are sufficiently close to 1.

First, consider the case where the information is fully revealed and the consumption value of x for A_N is common knowledge. Since $\kappa = 1$, there are no gains from trade in terms of y. In trade between A_N and A_{N-1}, the terms of trade are determined by $u(v - y^d_{N-1}) = 0$, or $y^d_{N-1} = v$. Hence, the ex ante welfare under the full-disclosure policy is

$$W^d = \mu(\omega_v)v.$$

Of course, with take-it-or-leave-it offer bargaining, only the initial owner of the asset enjoys the surplus and the other agents' ex ante welfare is 0.

Next, consider the case where the information designer reveals no information. Since A_{N-1} makes a take-it-or-leave-it offer to A_N, in trade between A_N and A_{N-1}, the terms of trade are determined by

$$\frac{\mu(\omega_v)}{\mu(\omega_v) + \mu(\omega_\phi)} u(v - \hat{y}_{N-1}) + \frac{\mu(\omega_\phi)}{\mu(\omega_v) + \mu(\omega_\phi)} u(-\hat{y}_{N-1}) = 0.$$

Let $\hat{y}_{N-1}$ be the quantity satisfying the above equation. Then the ex ante welfare is

$$W = \mu(\omega_v)\big[u(v - \hat{y}_{N-1}) + \hat{y}_{N-1}\big] + \mu(\omega_\phi)\big[u(-\hat{y}_{N-1}) + \hat{y}_{N-1}\big],$$

and thus,

$$\frac{W}{\mu(\omega_v) + \mu(\omega_\phi)} = \hat{y}_{N-1}.$$

Again, only the initial owner of the asset enjoys the surplus and the other agents' ex ante welfare is 0.

By definition of v_e (see (1)), we have

$$\frac{\mu(\omega_v)}{\mu(\omega_v) + \mu(\omega_\phi)}(v - v_e) + \frac{\mu(\omega_\phi)}{\mu(\omega_v) + \mu(\omega_\phi)}(-v_e) = 0.$$

Since u is strictly concave, this equation implies

$$\frac{\mu(\omega_v)}{\mu(\omega_v)+\mu(\omega_\phi)}u(v-v_e)+\frac{\mu(\omega_\phi)}{\mu(\omega_v)+\mu(\omega_\phi)}u(-v_e)<0.$$

Hence, we obtain

$$\hat{y}_{N-1}<v_e.$$

Therefore,

$$W=\big[\mu(\omega_v)+\mu(\omega_\phi)\big]\hat{y}_{N-1}<\big[\mu(\omega_v)+\mu(\omega_\phi)\big]v_e=W^d.$$

Therefore, we established the following.

Claim 2. *The full-disclosure policy is welfare improving.*

Note that this time, it is the final user, not middlemen, who is risk averse. Hence, the full-disclosure policy, which removes the risk, gives the final user the consumption value of good x equal to v for sure whenever he decides to buy it. This is welfare improving because without the policy, the expected consumption value for the risk-averse final user is strictly less than v.

Appendix C: Uniform distribution

We consider an example where the distribution μ is uniform, that is, for each $\omega\in\Omega$,

$$\mu(\omega)=\frac{1}{N+2}.$$

We derive comparative statics results for the number of middlemen similar to Proposition 5 in Section 7.

It is obvious that $\mu(\omega)>0$ for each $\omega\in\Omega$ and $\mu(\omega_2)\leq\cdots\leq\mu(\omega_N)$. The probability ψ_n is

$$\psi_n=\frac{N-n}{N-n+1}$$

and the price $\hat{y}_n$ is

$$\hat{y}_n=\frac{1}{N-n}\theta^{N-n}\kappa^{N-n-1}v_e$$

where $v_e=v/2$. See Figure 3 for the graph of $\hat{y}_n$.

A bubble occurs when the state belongs to $\{\omega_N,\omega_{N-1},\dots,\omega_3\}$, and hence, the probability that a bubble occurs is

$$1-\big[\mu(\omega_v)+\mu(\omega_\phi)+\mu(\omega_2)+\mu(\omega_1)\big]=1-\frac{4}{N+2}.$$

This probability is increasing in N and converges to one as $N\to\infty$. In words, a bubble occurs with an *arbitrarily* high probability. Focusing on the states where the final user

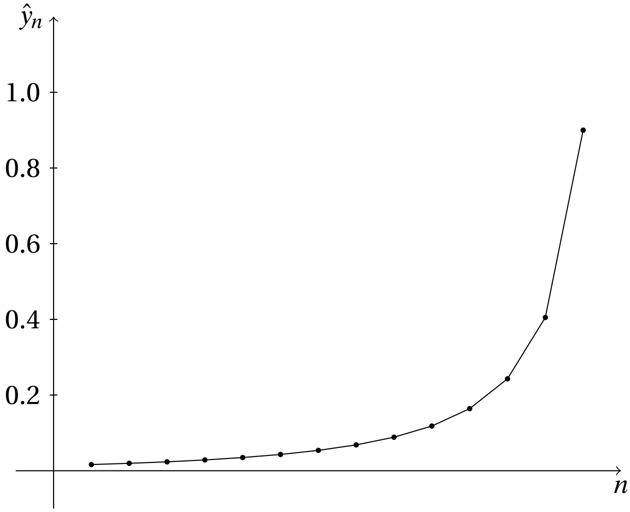

FIGURE 3. Graph of $\hat{y}_n$ for uniform distribution ($N = 15$, $\theta = 0.9$, $\kappa = 1$, and $v = 2$).

knows that x is worthless, the expected duration of the bubble is

$$\sum_{n=3}^{N} \frac{\mu(\omega_n)(n-2)}{1-\left[\mu(\omega_v)+\mu(\omega_\phi)\right]} = \frac{(N-2)(N-1)}{2N}.$$

The expected duration and expected duration ratio are increasing in N. The results, which are analogous to those in Section 7, are summarized as follows.

PROPOSITION 6. *The larger the number of middlemen, the higher the ex ante probability of a bubble to occur, and the longer the expected duration ratio of the bubble.*

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