Theoretical Economics 18 (2023), 267–301 1555-7561/20230267

Multistage information transmission with voluntary monetary transfers

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We analyze a cheap-talk model in which an informed sender and an uninformed receiver engage in a finite-period communication before the receiver chooses a project. During the communication phase, the sender sends a message in each period, and the receiver then voluntarily pays money for the message. As in the canonical cheap-talk model, all the equilibria are interval partitional; in our setting, however, the set of equilibrium partitions becomes larger. We show that the multistage information transmission with voluntary monetary transfers can improve welfare if the receiver cares more about the decision and the sender cares more about money or if the ex post sender–receiver incentive conflict over the project choice is small. We derive a multistage information elicitation mechanism without commitment that can be more beneficial to the receiver than a broad class of other communication protocols (e.g., mediation and arbitration).

Keywords. Incomplete information, cheap talk, multistage strategic communication, voluntary monetary transfers.

JEL classification. C72, C73, D82, D83.

1. Introduction

A lack of information typically leads to inefficient decisions. Therefore, in many economic situations, decision-makers need to gather relevant information before making their decisions. One canonical way of gathering information is consulting informed experts. For example, chief executive officers (CEOs), politicians, and law enforcement officers gather information from consultation with management consultants, strategic planners, and informants, respectively. These consultants are often paid for providing information. In this regard, contract theory indicates that an appropriately designed contract with information-contingent payments helps the decision-maker to screen the information possessed by the informed expert. However, contractibility does not always

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This study is based on the second chapter of my Ph.D. dissertation at Kobe University. An earlier version (Sadakane (2019), ISER Discussion Paper No. 1006) was awarded the 19th ISER-Moriguchi Prize. I am deeply indebted to Yasuyuki Miyahara for his invaluable support. I also thank Hideo Suehiro, Hideshi Itoh, Hisao Hisamoto, Kohei Kawamura, Shinsuke Kambe, Shintaro Miura, Takashi Shimizu, Takeshi Murooka, and seminar participants at the Contract Theory Workshop, Contract Theory Workshop summer camp, KIER, and ISER for their helpful comments and suggestions. I gratefully acknowledge the financial support from the Grants-in-Aid for Scientific Research (17H06778, 19H01471). I owe special thanks to three anonymous referees and the editor for their careful review and constructive suggestions. Any remaining errors are my own.

Available at https://econtheory.org. https://doi.org/10.3982/TE3501

exist if the information is transmitted through ordinary and informal talk, or equivalently, through "cheap talk." In such situations, the decision-maker cannot commit to information-contingent payments. Hence, it seems that allowing the decision-maker to make "voluntary" payments does not affect information transmission. Nevertheless, the information transmitted via cheap talk is often bought and sold without signing a contract.

This study investigates the incentives to decision-makers to make voluntary payments in order to facilitate cheap-talk communication. To this end, we enrich the canonical cheap-talk model originally provided by Crawford and Sobel (1982) (hereafter, CS). Specifically, we analyze a sender–receiver game in which an informed expert (sender or he) and an uninformed decision-maker (receiver or she) engage in a finite-period communication. During the communication phase, in each period, the sender sends a cheap-talk message to the receiver, who in return makes a voluntary payment to the sender. After the communication phase, the receiver chooses a project. As in the canonical cheap-talk model, all equilibria in our model are interval partitional. However, the set of equilibrium partitions becomes larger. As a result, the receiver can improve her equilibrium payoff.

The key underlying the result is the two-way dependence between the multistage information transmission and the receiver's voluntary payments. In the CS model, the project choice and the underlying asymmetric information are one-dimensional. We assume that the sender is upwardly biased. In other words, the sender's most desirable project is always higher than that of the receiver to a certain degree. Given this, the sender has an incentive to cheat the receiver into choosing a project higher than the receiver's most profitable one. In this case, by paying a higher monetary compensation for messages inducing the lower projects, the receiver can weaken the sender's exaggeration incentive. During the communication phase, the sender gradually conveys information about the state. If the receiver deviates from the payments, then she is punished by the sender babbling thereafter. Since we consider the finite-period communication, there must be multiple CS equilibria in the remaining game after the receiver's last payment. When the receiver perceives that she will not receive additional information, she stops further payments. Therefore, the piece of information transmitted after the final payments serves as "the last hostage." Consequently, the message-contingent payments can be self-enforcing, and the receiver can elicit more information in the early periods.

In Section 4, we demonstrate the benefit of multistage information transmission with voluntary transfer payments. If the receiver cares more about the decision and the sender cares more about money or if the ex post incentive conflict between the sender and receiver over the project choice is small, the multistage information transmission with voluntary monetary transfers can improve welfare relative to the optimal mediation. We also provide a simple upper bound on the receiver's equilibrium payoff and show that if the receiver places greater importance on the project than the sender does, the receiver's equilibrium payoff in the long communication can approximate this upper bound.

To this end, we investigate a class of equilibrium that induces a monotone payment scheme resembling an optimal contract established when the receiver credibly commits to payments. This produces an explicit lower bound on the optimal equilibrium payoff to the receiver. In equilibrium, if the communication phase has a sufficiently large number of periods, the sender will reveal nearly full information in the middle states; however, the sender will convey imprecise information in the high and low states. In other words, the equilibrium will involve almost a separation in the middle states and pooling in the high and low states. Specifically, in the first period, the sender conveys either one of the pooling intervals in the high states or information that the state does not belong to any of them. If the sender reveals that the state is in one of the intervals in the high states, then the receiver will stop the payments and the sender will stop transmitting information. Otherwise, the sender will gradually reveal nearly full information in the middle states by the last period. During this phase, in each period, the receiver will compensate the sender whenever the latter conveys that "the state is even lower." In the last period, the sender will convey one of the pooling intervals in the low states only when he received payments in all prior periods.

The pooling for the high states reflects the tradeoff between the direct benefit and the indirect cost of inducing separation in this region. Since the sender is upwardly biased, the payments for precise information increases the payments for all the lower states. Given this, the receiver can save substantially by inducing pooling for the highest states and reducing payments for the lower states. The more the sender cares about the money, the smaller will be the payments aligned with the sender's incentive. This will reduce the cost-saving benefits and narrow the width of the pooling intervals in the high states. The information about the pooling intervals in the low states serves as the hostage released after the last payment. As CS shows, the smaller the ex post sender–receiver incentive conflict over the project choice, the finer is the pooling partition and the harsher is the punishment by babbling. In other words, we can effectively incentivize the receiver to make payments with fewer hostages (i.e., the total width of pooling intervals in the low states becomes narrower). In addition, a decline in the necessary payments associated with the sender's growing interest in money also decreases the number of hostages needed. As a result of these effects, the more the sender cares about the money and/or the smaller the sender–receiver bias, the smaller are the equilibrium payments and the wider is the middle "nearly full information revelation" range. Hence, the receiver can obtain a higher equilibrium payoff.

Our model is potentially applicable to studying the effective use of informants. The Federal Bureau of Investigation (FBI) states that the "use of informants to assist in the investigation of criminal activity may involve an element of deception,..., or cooperation with persons whose reliability and motivation may be open to question."[1] This statement suggests that informants are often biased and that their information may neither be credible nor certifiable. Alemany (2002) indicates that cooperation agreements between the Drug Enforcement Agency (DEA) and informants are often silent about the informants' compensation. This implies that parties may not always be able to sign a contract containing information-contingent payments. Indeed, there are numerous cases of oral promises made by DEA agents to informants subsequently being broken.[2]

[1]FBI, Frequently Asked Questions, "What is the FBI's policy on the use of informants?" (https://www.fbi.gov/about/faqs/what-is-the-fbis-policy-on-the-use-of-informants).

[2]For details, see Alemany (2002).

We show that the use of multistage information elicitation and voluntary transfer payments can improve information transmission, even in situations lacking contractibility. This can be exemplified through the following scenario. Suppose a police informant asks a police officer advance payment in return for information. Here, the information serves as a hostage that is released after the payment. Moreover, the act of asking for money may bring a piece of information to the police officer. If the criminal the police officer is searching for is an enemy of this informant, he may have an incentive to provide the information for free.

The model can also be applied to discuss the budget allocation in a firm. Consider a firm in which a CEO determines the product quantity the firm must sell and a manager possesses the demand information. Before arriving at a decision, the CEO seeks the manager's advice on the demand condition. If the manager faces competition among divisions and has empire-building motives, he may be overly enthusiastic (upwardly biased from the firm's optimal). In this case, the manager has an incentive to exaggerate the demand for the product. When the manager reports that the product is in low demand, the CEO can provide a supplementary budget to the division to rebuild the business. Allocating such a supplementary budget can ease the enthusiastic manager's exaggeration. However, the CEO may break the verbal promise to provide extra funding. Our mechanism shows that gradual information transmission can help the manager to secure a supplementary budget. It also shows that the CEO can elicit detailed information by allocating the budget based on the manager's report.

Our results have important implications for the theory of organizational economics on the design of communication protocols and organizational structures. The results show that multistage information transmission with voluntary transfer payments can benefit the receiver more than a wide range of other mechanisms without transfer payments (e.g., mediated or noisy communication: Krishna and Morgan (2004), Blume, Board, and Kawamura (2007), Goltsman, Hörner, Pavlov, and Squintani (2009), Ivanov (2010), and Ambrus, Azevedo, and Kamada (2013); and delegation mechanisms without transfer: Dessein (2002), Holmström (1977), Melumad and Shibano (1991), and Alonso and Matouschek (2008)).

Related literature Krishna and Morgan (2008) extend the CS model by allowing the parties to draw up a contract containing message-contingent payments. They show that full information revelation is feasible but not optimal and they characterize the optimal contract. We show that when the communication phase has multiple periods, the receiver can control the sender's incentive through voluntary payments even though there is no contractibility. Using the outcome of the optimal contract as a reference point, we derive an explicit lower bound on the receiver's optimal equilibrium payoff in our model.

Our results are closely related to those of Krishna and Morgan (2004) in that we both show how the receiver's active participation in the communication process improves information transmission. Krishna and Morgan (2004) add a long communication protocol to the CS model.[3] They show that if bilateral (face-to-face) communication between

[3] Aumann and Hart (2003) study a finite simultaneous-move (long conversation) game between two players, where one is better informed than the other. They provide a complete geometrical characterization of

the receiver and sender is possible before the sender sends a message about his private information to the receiver, an equilibrium whose outcome Pareto dominates all the equilibrium outcomes in the CS model exists. The key factor of their result is the existence of multiple equilibria in the remaining game after the sender conveys some information during face-to-face communication. The outcome of this face-to-face communication, which could be random, determines which of these equilibria is played in the future. The randomness over the choice of equilibrium in the continuation game influences information conveyed during face-to-face communication. Therefore, in Krishna and Morgan (2004), the receiver tries to control the sender's incentive by controlling the degree of uncertainty associated with the outcome of the face-to-face communication. Contrastingly, in our model, the receiver tries to control the sender's incentive directly through voluntary transfer payments.

Spence (1973) shows that costly signaling helps the sender convey his private information credibly. Within the framework of the CS model, Austen-Smith and Banks (2000), Kartik (2007), and Karamychev and Visser (2017) show that information transmission can be improved when the sender can send a costly message (money burning or, equivalently, paying money to the receiver) to signal information.[4] In their settings, there can be a fully separating equilibrium that is optimal from the receiver's perspective. However, in this case, the sender's signaling would depend on his sacrifice of incurring all the costs of information transmission. Therefore, if the sender has a worthy outside option, it may be challenging to involve the sender in the project. In contrast, our mechanism allows for raising the sender's utility while facilitating information transmission. Roughly speaking, the sender joins the project because he can sell his information to the receiver. From the receiver's viewpoint, by compensating for the information gradually, she can incentivize the sender to engage in the project and convey detailed information, even without a contract. This observation suggests that in some cases, it may be better for the receiver to generate the signaling structure by herself through voluntary payments rather than to rely on the sender's costly signaling.

Hörner and Skrzypacz (2016) examine a model of gradual persuasion in which the sender is paid and gradually reveals "verifiable" information.[5] They show that the sequential revelation of partially informative signals can increase payments to the sender who is trying to sell his information to the receiver. Similar to our mechanism, the most effective punishment for the receiver's deviations is not conveying information in the future. In their model, however, cheap-talk communication is not helpful. The observability of the signal structure that the sender employs and the verifiability of the realized signal are crucial. In contrast, the present study demonstrates that gradual information

the set of equilibrium payoffs when the state of the world is finite and long communication is possible. In Aumann and Hart (2003), the state space and the players' action space are finite. Therefore, we cannot directly apply the results of Aumann and Hart (2003) to the model in this study.

[4]Relatedly, Kartik, Ottaviani, and Squintani (2007) and Kartik (2009) study amendments to the CS model with other means of costly signals such as lying costs.

[5]Kolotilin and Li (2021) also study a model of dynamic Bayesian persuasion with monetary transfers. Unlike our study, the sender's private information is not persistent in their model, and hence it does not lead to the gradual information elicitation.

transmission can perform well with voluntary transfer payments even when the sender can send only cheap-talk messages.

In all the studies mentioned above, once the communication phase is over, the receiver chooses a project. In other words, the project choice is once and for all. Unlike these studies, Golosov, Skreta, Tsyvinski, Wilson (2014) examine strategic information transmission in a finitely repeated cheap-talk game in which there are multiple rounds of communication and actions. Specifically, in each period, the sender sends a message and the receiver chooses a project.[6] Only the sender knows the state of the world, which remains constant throughout the game. They show that the sender can condition his message on the receiver's past actions; in addition, the receiver can choose actions that reward the sender for following a path of messages that eventually leads to the full revelation of information. In contrast to this result, there is no fully revealing equilibrium in our model.

Paper outline The remainder of this paper is organized as follows. Section 2 introduces the model. Section 3 provides the general properties of the equilibria. Section 4 demonstrates the benefits of multistage information transmission with voluntary monetary transfers. Section 5 provides some concluding remarks.

2. Model

There are two players, a sender (he) and a receiver (she). The receiver has the authority to choose a project $y \in Y \equiv \mathbb{R}$, but the outcome produced by project y depends on the sender's private information, θ, which is distributed uniformly on $\Theta \equiv [0, 1]$. Players engage in T-period communication, followed by an action stage in which the receiver chooses y. The timing in each of the T rounds of communication is as follows. First, the sender sends a costless and unverifiable message to the receiver. Second, after receiving a message from the sender, the receiver voluntarily pays money to the sender. Let $w_t \in \mathbb{R}_+$ be the amount the receiver pays at the second stage in period t. We denote by $\mathbf{w}$ a sequence of payments the receiver made, $\mathbf{w} \equiv (w_1, \dots, w_T) \in \mathbb{R}_+^T$.

The receiver's payoff function is

$$U^R(y, \theta, \mathbf{w}) = -(y - \theta)^2 - \sum_{t=1}^{T} w_t,$$

and the sender's payoff function is

$$U^S(y, \theta, \mathbf{w}) = -s(y - \theta - b)^2 + \sum_{t=1}^{T} w_t,$$

where s and b are positive constants. The term $\sum_{t=1}^{T} w_t$ represents the total amount of payments. Here, $-(y - \theta)^2$ and $-s(y - \theta - b)^2$ denote utilities from project y for the

[6] Margaria and Smolin (2018) examine an infinitely repeated cheap-talk game in which the senders with state-independent payoffs communicate to a single receiver. They show that if players are sufficiently patient, any feasible and individually rational payoff can be approximated.

receiver and the sender, respectively. For any given θ, the sender's preferred project is $y = \theta + b$, while the receiver's preferred project is $y = \theta$. Therefore, the parameter $b > 0$ represents "bias," which measures how much the sender's interest regarding the project choice differs from that of the receiver. The constant $s > 0$ measures the weight that the sender places on his payoff from the project choice, relative to the payments he receives. The weight that the receiver places on her payoff from the project choice is normalized as one. Hence, $s < 1$ ($s > 1$) implies that the receiver places greater (less) importance on the project choice than the sender does.

The timing of game is summarized as follows:

(i) Before the game starts, nature randomly draws a state θ according to a uniform distribution on Θ, and the sender observes θ privately.

(ii) Players engage in T-period communication. Each period t consists of two stages.

- At the first stage in period t, the sender sends a message m_t to the receiver.
- At the second stage in period t, the receiver voluntarily pays w_t to the sender.

(iii) After T-period communication, the receiver chooses a project y and the game ends.

Hereafter, $\Gamma(b, s, T)$ denotes this T-period communication game.

2.1 *History and strategies*

A public history h_j^t is a sequence of players' past actions realized until the beginning of the stage $j \in \{1, 2\}$ in period t:

$$h_j^t \equiv \begin{cases} (m_1, w_1, \dots, m_{t-1}, w_{t-1}) & \text{if } j = 1, \\ (m_1, w_1, \dots, m_{t-1}, w_{t-1}, m_t) & \text{if } j = 2. \end{cases}$$

A public history h^{T+1} is a sequence of players' past actions realized in T-period communication:

$$h^{T+1} \equiv (m_1, w_1, \dots, m_T, w_T).$$

Let H_j^t and H^{T+1} be the set of h_j^t and h^{T+1}, respectively. We assume that H_1^1 is a singleton set $\{\phi\}$. Let $h_\theta^t = (\theta, h_1^t) \in \Theta \times H_1^t \equiv H_\Theta^t$ be the sender's private history at stage 1 in period t.

A strategy for the sender is a collection of message rules $\sigma = \{\sigma^t\}_{t=1}^T$, where $\sigma^t : H_\Theta^t \to M$ specifies the message in period t as a function of the sender's private history. A strategy for the receiver is a collection $\rho = \{\rho^t\}_{t=1}^{T+1}$, where (i) $\rho^t : H_2^t \to \mathbb{R}_+$ specifies a payment amount in period $t \leq T$ and (ii) $\rho^{T+1} : H^{T+1} \to Y$ specifies a project the receiver chooses after T-period communication. A belief system $f = \{f^t\}_{t=1}^{T+1}$, where $f^t : H_2^t \to \Delta\Theta$ and $f^{T+1} : H^{T+1} \to \Delta\Theta$, specifies the receiver's belief as a function of the public history to date.

We analyze the (pure strategy) perfect Bayesian equilibria[7]—both players' strategies must maximize their expected payoffs after all histories, and the system of beliefs f must be consistent with the conditional probability derived from $((\sigma, \rho), f)$ and the prior probability distribution. Hereafter, we call a perfect Bayesian equilibrium simply *equilibrium*.

2.2 *An example*

This subsection presents a simple example illustrating the mechanism of the multistage information transmission with voluntary monetary transfers. Suppose that $b = 1/12$ and $s = 0.06$. In this case, there are two equilibria in the CS model. One is the uninformative equilibrium (the babbling equilibrium), and the other is a partially informative equilibrium in which the sender conveys one of two intervals $\{[0, 1/3], [1/3, 1]\}$ to which the state belongs. After receiving a message conveying one of the intervals, the receiver chooses a project in the middle of the interval—$y = 1/6$ or $2/3$—to maximize her expected payoff. Since the sender is upwardly biased, the right-side interval must be wide enough for the sender to be indifferent at the boundary between intervals. Intuitively, by compensating for the information associated with the lower projects, the receiver can decrease the sender's incentive of exaggeration. We will show that such message-contingent payments can be self-enforcing if the sender gradually conveys the information.

To this end, we consider the following information elicitation. First, the sender conveys one interval in $\{[0, 5/6], [5/6, 1]\}$. If the highest interval is chosen, then the receiver will choose $y = 11/12$; otherwise, the receiver will pay $w = 0.0084375$ and again ask advice. The sender conveys one interval in $\{[0, 1/4], [1/4, 5/6]\}$ if and only if the receiver pays w for the information in the first period. The partition $\{[0, 1/4], [1/4, 5/6]\}$ coincides with one of the CS equilibria in the case where the state space is $[0, 5/6]$. After receiving additional information in the second period, the receiver optimally chooses $y = 1/8$ or $y = 13/24$. The sender of type $\theta = 5/6$ is indifferent between the project $y = 13/24$ associated with the payment w and project $y = 11/12$ without payment. Therefore, the sender is willing to convey the interval truthfully.

The key property is that the information conveyed in the second period serves as a "hostage" that is released after the payment. In other words, "no payment" is punished using "no additional information." From the receiver's viewpoint, given the information that $\theta \in [0, 5/6]$, the expected gain by eliciting additional information is $0.0364 > w$. Therefore, the receiver has an incentive to pay w in the previous period. The receiver's payment strategy involves "a high payment for the low states" properties. As noted earlier, this payment scheme prevents the sender's exaggeration to a certain extent.

3. Partition equilibria

In this section, we provide the properties of equilibria. First, we briefly note the relationship between the equilibria in the CS model and those in our model. Consider a strategy

[7]We use the typical extension of the perfect Bayesian equilibria for the infinite state and action spaces. An equilibrium that is essentially equivalent to one of the CS equilibria always exists. Hence, we do not prove the existence theorem. For more details, see Fact 1.

profile such that the sender sends an informative message only in the first period, and the receiver does not pay anything to the sender at any payment stage. If the sender's and receiver's strategies regarding m_1 and project choice, respectively, are the same as an equilibrium in the CS model, then this strategy profile will constitute an equilibrium in $\Gamma(b, s, T)$. This result immediately yields the following Fact 1.

Fact 1. *Any equilibrium outcome in the CS model can be achieved under an equilibrium in $\Gamma(b, s, T)$.*

3.1 *Sender incentive-compatible decision and payment rule*

Now, we introduce some notation and terminologies. A sequence of actions players choose on the path of a given strategy profile is recursively determined on the basis of the realized state θ; $m_1 = \sigma^1(\theta)$, $w_1 = \rho^1(\sigma^1(\theta))$, $m_1 = \sigma^2(\theta, \sigma^1(\theta), \rho^1(\sigma^1(\theta)))$, and so on. Given an equilibrium ξ, let $y_\xi(\theta)$ be the project the receiver chooses on the path when the state is θ. Let $\omega_\xi(\theta)$ be the total payment received by the sender of type θ. Suppose that the given pair ξ constitutes an equilibrium. Then the sender has no incentive to act as if the state is different from the realized state. Therefore, for any $\theta, \theta' \in \Theta$,

$$-s\big(y_\xi(\theta) - \theta - b\big)^2 + \omega_\xi(\theta) \geq -s\big(y_\xi(\theta') - \theta - b\big)^2 + \omega_\xi(\theta'), \tag{1}$$

$$-s\big(y_\xi(\theta') - \theta' - b\big)^2 + \omega_\xi(\theta') \geq -s\big(y_\xi(\theta) - \theta' - b\big)^2 + \omega_\xi(\theta). \tag{2}$$

A pair of decision and payment rule, $(y_\xi(\cdot), \omega_\xi(\cdot))$, is *sender incentive compatible* if Conditions (1) and (2) hold. The incentive compatibility is necessary for ξ to be an equilibrium.

The first result provides the properties of the sender incentive-compatible decision and payment rule $y_\xi(\cdot)$ and $\omega_\xi(\cdot)$.

Lemma 1. *If a pair of $y_\xi(\cdot)$ and $\omega_\xi(\cdot)$ is sender incentive compatible, $y_\xi(\cdot)$ is nondecreasing in θ. Moreover, if $y_\xi(\underline{\theta}) = y_\xi(\overline{\theta})$ for $\underline{\theta} < \overline{\theta}$, then $y_\xi(\underline{\theta}) = y_\xi(\theta) = y_\xi(\overline{\theta})$ and $\omega_\xi(\underline{\theta}) = \omega_\xi(\theta) = \omega_\xi(\overline{\theta})$ for $\theta \in [\underline{\theta}, \overline{\theta}]$.*

Proof. The inequalities (1) and (2) can be simplified into

$$(\theta + b)\big(y_\xi(\theta) - y_\xi(\theta')\big) \geq \big(\theta' + b\big)\big(y_\xi(\theta) - y_\xi(\theta')\big).$$

Hence, we obtain $y_\xi(\theta) \geq y_\xi(\theta')$ for $\theta \geq \theta'$. We immediately obtain that if $y_\xi(\underline{\theta}) = y_\xi(\overline{\theta})$ for $\underline{\theta} < \overline{\theta}$, then $y_\xi(\underline{\theta}) = y_\xi(\theta) = y_\xi(\overline{\theta})$ for $\theta \in [\underline{\theta}, \overline{\theta}]$.

Next, we suppose that $y_\xi(\underline{\theta}) = y_\xi(\overline{\theta}) = y$ for $\underline{\theta} < \overline{\theta}$. Then the sender's incentive compatibility implies that

$$-s(y - \underline{\theta} - b)^2 + \omega_\xi(\underline{\theta}) \geq -s(y - \underline{\theta} - b)^2 + \omega_\xi(\overline{\theta}),$$

$$-s(y - \overline{\theta} - b)^2 + \omega_\xi(\overline{\theta}) \geq -s(y - \overline{\theta} - b)^2 + \omega_\xi(\underline{\theta}).$$

Obviously, $\omega_\xi(\underline{\theta}) = \omega_\xi(\theta) = \omega_\xi(\overline{\theta})$ for $\theta \in [\underline{\theta}, \overline{\theta}]$. This completes the proof. □

The sender's incentive-compatibility condition yields the monotonicity of the project choice $y_\xi(\cdot)$, and the monotonicity of $y_\xi(\cdot)$ implies that if two different types induce the same project, all the types between these two types induce the same project. Moreover, if different types induce the same project, the amount of the payments must also be kept the same to prevent their incentive to imitate the other types.

Since the sender's incentive compatibility is necessary for the given ξ to be an equilibrium, Lemma 1 implies that in any equilibrium, the state space is partitioned into intervals on the equilibrium path. We fix an equilibrium ξ, and let $\mathcal{I}_\xi$ be the interval partition of the state space induced by ξ. If an interval $[a_{i-1}, a_i]$ belongs to $\mathcal{I}_\xi$, we obtain $(y_\xi(\theta), \omega_\xi(\theta)) = (y_\xi(\theta'), \omega_\xi(\theta'))$ for $\theta, \theta' \in [a_{i-1}, a_i]$; and $y_\xi(\theta) \neq y_\xi(\theta'')$ for $\theta \in [a_{i-1}, a_i]$ and $\theta'' \notin [a_{i-1}, a_i]$. In other words, all $\theta \in [a_{i-1}, a_i] \in \mathcal{I}_\xi$ induce the same public history h_j^t and h^{T+1}. Moreover, if θ and θ' belong to different intervals (i.e., if $\theta \in [a_{i-1}, a_i]$ and $\theta \in [a_{i'-1}, a_{i'}]$ for $[a_{i-1}, a_i] \neq [a_{i'-1}, a_{i'}]$), they induce different public histories on the equilibrium path. In equilibrium, the sender conveys only the interval to which the state belongs. Since we assume that θ is drawn from the uniform distribution on $[0, 1]$, if $\theta \in [a_{i-1}, a_i] \in \mathcal{I}_\xi$, the receiver's posterior belief induced in equilibrium is the uniform distribution on $[a_{i-1}, a_i]$. Given this belief, the receiver chooses $y_\xi(\theta) = y_i = (a_{i-1} + a_i)/2$ to maximize her conditional expected utility from the project choice.

3.2 *"Information as a hostage" effect*

We assume that there is no contractibility in the model. Therefore, in any equilibrium, if the receiver makes a positive payment at the second stage in the current period, she must receive additional information in the future communication round. In other words, the sender must be able to punish the receiver's deviation by not conveying information in the future.

Lemma 2. *Fix an equilibrium ξ that induces an equilibrium partition $\mathcal{I}_\xi$ and suppose that the receiver makes a positive payment on the equilibrium path. Then there exists a public history $\hat{h}_2^t$ on the equilibrium path such that the receiver makes a positive payment at $\hat{h}_2^t$ and no payment thereafter. Moreover, the support of the receiver's belief at $\hat{h}_2^t$ must contain at least two nondegenerate intervals $[a_{i-1}, a_i]$ and $[a_{i'-1}, a_{i'}]$ in $\mathcal{I}_\xi$ such that $a_{i-1} < a_i$ and $a_{i'-1} < a_{i'}$.*

Proof. The first statement is trivial. Suppose that the support of the receiver's belief at $\hat{h}_2^t$, $\operatorname{supp} f^t(\cdot|\hat{h}_2^t)$, contains only one interval in $\mathcal{I}_\xi$. Obviously, the receiver has no incentive to pay money at $\hat{h}_2^t$. Hence, $\operatorname{supp} f^t(\cdot|\hat{h}_2^t)$ must contain at least two intervals in $\mathcal{I}_\xi$.

Next, we confirm that $\operatorname{supp} f^t(\cdot|\hat{h}_2^t)$ must not contain two degenerated intervals $[\underline{a}, \underline{a}]$ and $[\overline{a}, \overline{a}]$. Since the sender's incentive compatibility implies that $y_\xi(\underline{a}) = \underline{a}$ and $y_\xi(\overline{a}) = \overline{a}$, we obtain $\omega_\xi(\underline{a}) - \omega_\xi(\overline{a}) = 2sb(\overline{a} - \underline{a}) > 0$. Therefore, if one of $\{[\underline{a}, \underline{a}], [\overline{a}, \overline{a}]\}$ is in $\operatorname{supp} f^t(\cdot|\hat{h}_2^t)$, then the other is not. Therefore, $\operatorname{supp} f^t(\cdot|\hat{h}_2^t)$ must contain at least one nondegenerate interval $[a_{i-1}, a_i]$.

If there is no nondegenerate interval in $\operatorname{supp} f^t(\cdot|\hat{h}_2^t)$, except for $[a_{i-1}, a_i]$, then the receiver has no incentive to pay money at $\hat{h}_2^t$. This completes the proof. $\square$

The intuition behind Lemma 2 is simple. Since the communication round has only finite periods, roughly speaking, the information released after the last payment serves as a hostage.

3.3 *Characterization of the set of equilibrium partitions*

Proposition 1 shows that all the equilibria in $\Gamma(b, s, T)$ are finite partition equilibria, as is the case in the CS model.

Proposition 1. *In any equilibrium, the equilibrium partition consists of a finite number of pooling intervals.*

The proof is in Appendix A. In the proof, we show that a pair of strategy profile and belief system ξ inducing an interval partition $\mathcal{I}_\xi$ with infinitely many elements is incompatible with Lemma 2. Specifically, if ξ induces an interval partition $\mathcal{I}_\xi$ with infinite elements and the receiver optimally chooses a project given interval $[a_{i-1}, a_i] \in \mathcal{I}_\xi$ (i.e., $y_\xi(\theta) = (a_{i-1} + a_i)/2$ for $\theta \in [a_{i-1}, a_i]$), and if the payment rule $\omega_\xi(\cdot)$ satisfies the sender's incentive compatibility, then the receiver reaches a history at which she makes a positive payment, although she receives no information in the future after this payment.

We next discuss the relationship between the equilibrium partition $\mathcal{I}_\xi$ and the payment rule $\omega_\xi(\cdot)$. Suppose that the given equilibrium ξ induces an equilibrium partition $\mathcal{I}_\xi = \{[a_{i-1}, a_i]\}_{i\in\{1,\dots,N\}}$, where $a_{i-1} < a_i < a_{i+1}$. Consider two adjacent intervals $[a_{i-1}, a_i]$ and $[a_i, a_{i+1}]$ in $\mathcal{I}_\xi$.

Given that the state is uniformly distributed on $[a_{i-1}, a_i]$, the receiver chooses $(a_{i-1} + a_i)/2$ to maximize her conditional expected utility from the project. To satisfy the sender's incentive-compatibility condition, the sender on the boundary $\theta = a_i$ must be indifferent between $y_i = (a_{i+1} + a_i)/2$ and $y_{i-1} = (a_i + a_{i-1})/2$.[8]

Let $\omega_i = \omega_\xi(\theta)$ denote the total payment the sender receives by revealing $\theta \in [a_{i-1}, a_i]$. The sender's indifferent condition at $\theta = a_i$ is

$$-s\left(\frac{a_{i-1}+a_i}{2} - a_i - b\right)^2 + \omega_i = -s\left(\frac{a_i+a_{i+1}}{2} - a_i - b\right)^2 + \omega_{i+1}.$$

Hence, the difference in the total payment is

$$\omega_i - \omega_{i+1} = \frac{s}{4}(x_i + x_{i+1})(x_i - x_{i+1} + 4b), \tag{3}$$

where $x_i = a_i - a_{i-1}$ and $x_{i+1} = a_{i+1} - a_i$.

Condition (3) provides the intuition behind the effect of payments on the sender's incentive. Figure 1 and 2 illustrate the payoff from project y_i and y_{i+1} received by the sender type θ, respectively. If the length of the right interval is strictly less than that of

[8]Note that the players' payoff functions satisfy the single crossing property: $(\partial^2/\partial y \partial\theta)(-(y-\theta-b)^2) > 0$ for any b. This condition ensures that the best value of y, from a fully informed sender's standpoint, is strictly increasing in θ. Therefore, the sender's indifferent condition at the boundaries of adjacent intervals corresponds to the sender's incentive-compatibility condition.

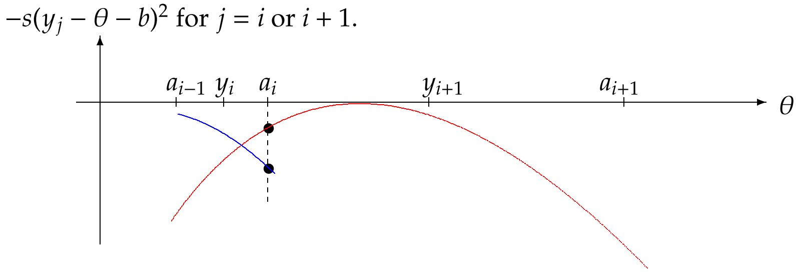

FIGURE 1. $x_{i+1} < x_i + 4b$.

the left one added by $4b$, that is, $x_{i+1} < x_i + 4b$, then the sender type $\theta < a_i$ must receive more money on the equilibrium path. If $x_{i+1} > x_i + 4b$, the payment for the sender type $\theta > a_i$ must be higher than that for $\theta < a_i$. If the sender receives the same payments between two adjacent intervals, it must satisfy that $x_{i+1} = x_i + 4b$, which corresponds to the sender's incentive condition in the CS model. The above result implies that by paying more money for the messages inducing the left interval, the receiver can weaken the sender's exaggeration incentive:

Now, we consider an N-step partition of the state space: $\{[a_{i-1}, a_i]\}_{i\in\{1,\dots,N\}}$ where $0 = a_0 < a_1 < \cdots < a_{N-1} < a_N = 1$. By Condition (3), given $\{[a_{i-1}, a_i]\}_{i\in\{1,\dots,N\}}$, the sender incentive-compatible payment rule $\{\omega_i\}_{i\in\{1,\dots,N\}}$ satisfies that

$$\omega_i = \omega_N + s\left[b\left\{2\left(1 - \sum_{\ell=1}^{i} x_\ell\right) - x_N + x_i\right\} - \frac{x_N^2}{4} + \frac{x_i^2}{4}\right], \tag{4}$$

where $x_i = a_i - a_{i-1}$ and $\sum_{i=1}^N x_i = 1$. The second term is uniquely determined given a partition $\mathcal{I} \equiv \{[a_{i-1}, a_i]\}_{i\in\{1,\dots,N\}}$. Therefore, we denote ω_i by $\omega_N + s \cdot \beta(\mathcal{I}, i)$.

The following lemma provides a simple necessary condition for the equilibrium partition and the payment for the highest interval.

LEMMA 3. *In any equilibrium inducing an equilibrium partition $\mathcal{I}_\xi$, the total payment for the highest interval satisfies that*

$$\omega_N \geq \max\left\{-\min_{i\in\{1,\dots,N\}} s \cdot \beta(\mathcal{I}_\xi, i), 0\right\}.$$

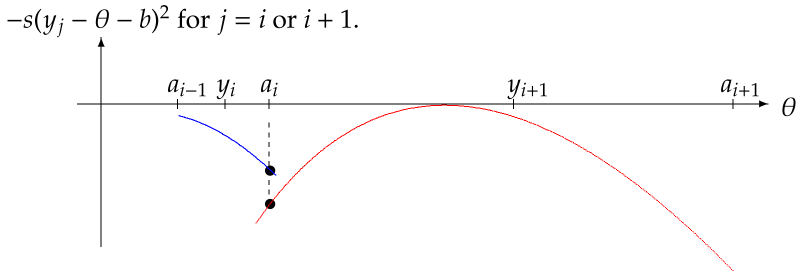

FIGURE 2. $x_{i+1} > x_i + 4b$.

Moreover, if the receiver makes a positive payment on the equilibrium path, that is, $\omega_i = \omega_N + s \cdot \beta(\mathcal{I}, i) > 0$ *for some* $i \in \{1, \dots, N\}$, *the equilibrium partition* $\mathcal{I}_\xi$ *satisfies that*

$$\#\Big\{\arg\max_{i\in\{1,\dots,N\}} \beta(\mathcal{I}_\xi, i)\Big\} \geq 2. \tag{5}$$

Proof. Since ω_i must not be less than zero for all $i \in \{1, \dots, N\}$, we obtain the first inequality. If $\#\{\arg\max_{i\in\{1,\dots,N\}} \beta(\mathcal{I}_\xi, i)\} = 1$, there is an interval $[a_{i-1}, a_i] \in \mathcal{I}_\xi$ such that $\omega_i > \omega_j$ for all $j \in \{1, \dots, N\} \setminus \{i\}$. Then, if the true state belongs to $[a_{i-1}, a_i]$, the given ξ induces a public history $\hat{h}_2^t$ such that the receiver makes a positive payment at $\hat{h}_2^t$ and $\operatorname{supp} f(\cdot|\hat{h}_2^t) = [a_{i-1}, a_i]$. This result contradicts Lemma 2. □

The first inequality originates from the model restriction that the receiver cannot make a negative payment. The second inequality corresponds to Lemma 2.

Finally, we provide a sufficient condition for a partition $\{[a_{i-1}, a_i]\}_{i\in\{1,\dots,N\}}$ to be achieved in equilibrium.

Proposition 2. *Fix a partition* $\mathcal{I} = \{[a_{i-1}, a_i]\}_{i\in\{1,\dots,N\}}$, *where* $0 = a_0 < a_1 < \cdots < a_N = 1$ *and* $N \geq 2$. *If* $\beta(\mathcal{I}, i)$ *satisfies that*

$$\#\Big\{\arg\max_{i\in\{1,\dots,N\}} \beta(\mathcal{I}, i)\Big\} \geq 2$$

and $T > N - \#\{\arg\max_{i\in\{1,\dots,N\}} \beta(\mathcal{I}, i)\}$, *then there exists* $s^* > 0$ *such that for all* $s \in (0, s^*]$, *there is an equilibrium of* $\Gamma(b, s, T)$ *with the equilibrium partition* $\mathcal{I}$.

The proof is in Appendix B. In the proof, we construct an equilibrium in which information is transmitted in the following steps. Fix an N-step partition $\mathcal{I} = \{[a_{i-1}, a_i]\}_{i\in\{1,\dots,N\}}$ and the sender incentive-compatible payment rule $\omega_i = \omega_N + s \cdot \beta(\mathcal{I}, i)$ for $i \in \{1, \dots, N\}$. Suppose that $\#\{\arg\max_{i\in\{1,\dots,N\}} \beta(\mathcal{I}, i)\} \geq 2$. Let I^H denote the set $\{\arg\max_{i\in\{1,\dots,N\}} \beta(\mathcal{I}, i)\}$. In other words, for all $i^H \in I^H \subset \{1, \dots, N\}$, the interval $[a_{i^H-1}, a_{i^H}]$ induces the highest total payment ω_{i^H}. Let us sort the elements of $\{\omega_i\}_{i\in\{1,\dots,N\}}$ in ascending order and denote ω_{i^t} as the tth-smallest payment; $\omega_{i^1} \leq \omega_{i^2} \leq \cdots \leq \omega_{i^t} \leq \cdots \leq \omega_{i^H}$. In equilibrium, the sender gradually and successively conveys information in the order of intervals endowed with the smaller payments.

Specifically, in the first period, if the sender conveys that the true state belongs to the interval $[a_{i^1-1}, a_{i^1}]$ endowed with the smallest payment ω_{i^1}, then the receiver will neither pay money nor obtain additional information in the future. We can take $\omega_{i^1} = 0$ without loss of generality. Otherwise, the receiver will pay a certain amount of money to the sender.[9] After this payment, in the second period, the sender conveys whether the true state belongs to the interval $[a_{i^2-1}, a_{i^2}]$ associated with the second-smallest payment ω_{i^2}. If the receiver learns that the true state belongs to this interval, she will neither pay money nor obtain additional information in the future. Otherwise, the receiver will pay money to the sender, who will convey additional information in the next period.[10]

[9]This payment takes the value of zero if $\omega_{i^1} = \omega_{i^2}$.

[10]Similar to the above, this payment takes the value of zero if $\omega_{i^2} = \omega_{i^3}$.

This information elicitation is repeated in the communication phase until the sender completes revealing the interval to which the state belongs. It takes $N - \#\{I^H\}$ periods at most.

After the receiver makes the last payment, that is, the total amount of the past payments reaches $\omega_N + s \cdot \beta(\mathcal{I}, i^H)$, the sender conveys one interval in $\{[a_{i^H-1}, a_{i^H}]\}_{i^H \in I^H}$. If the receiver deviates from the payments, the sender conveys no information thereafter. Thus, the receiver makes a payment in the current period to avoid the babbling in the future. After the communication, the receiver chooses her best project based on the acquired information. The logic underlying this outcome is similar to that in Benoît and Krishna (1985). The dependence of the selection of the future communication equilibrium on the receiver's past payments constructs punishments for the receiver's deviation of decreasing the payment amount.

Once we fix the partition, the changes in the value of s will never affect the value of information. However, $s \cdot \beta(\mathcal{I}, i)$ will decline with a decline in s, given an increase in the effect of the message-contingent payment on the sender's incentive. Therefore, if the receiver places greater importance on the project choice than the sender does, the punishment by babbling is effective.

Proposition 2 provides a tractable way to identify whether an outcome of information elicitation (i.e., the partition of the state space) can be attained in an equilibrium. Since, in equilibrium, the sender chooses a pair of the interval and the total payments endowed with it on the equilibrium path, the receiver designs a payment rule such that the sender is left without an incentive to act as if the state is in a different interval from the true one. Therefore, the sender incentive-compatible payment rule is determined as a function of the partition. By examining the shape of this function, we can determine whether the given partition can be attained and how long it takes to elicit all the information about this partition.

In the above equilibrium construction, the sender conveys intervals one by one in each period. However, if two or more intervals are endowed with the same payment, we can shorten the processing time by allowing the sender to reveal information about these intervals simultaneously. This modification does not change the players' incentives at any history. The simplest example is the case where the partition is the same as one of the CS equilibria. In this case, since the receiver does not have to control the sender's incentive by paying money, she can elicit information in the first period.

Next, we will find a simple upper bound of the receiver's equilibrium payoff. Lemma 2 implies that the information elicitation through voluntary payments works only when there are informative equilibria without transfers. Hence, if $1 < 4b$, only babbling equilibrium without transfer exists. In this case, the receiver obtains the babbling-equilibrium payoff, that is, $-1/12$. Contrastingly, if $4b < 1$, the set of equilibrium partitions can enlarge. Recall that if the receiver makes a positive payment on the equilibrium path, at least two intervals in $\mathcal{I}_\xi$ induce the same total payment. Let $[a_{i-1}, a_i]$ and $[a_{j-1}, a_j]$ be intervals such that $\omega_i = \omega_j = \omega_N + \max_i s \cdot \beta(\mathcal{I}, i)$ and $a_i \leq a_{j-1}$. Condition (3) yields that

$$\omega_i - \omega_j = +2sb(a_{j-1} - a_i) + \frac{s}{4}(x_i + x_j)(x_i - x_j + 4b) = 0,$$

where $x_i = a_i - a_{i-1}$ and $x_j = a_j - a_{j-1}$. Since $a_i \le a_{j-1}$, we obtain $x_j > x_i + 4b$. To summarize, if $4b < 1$, in any equilibrium, at least one interval must have a length greater than $4b$.

The receiver's equilibrium payoff is given by

$$-\sum_{i=1}^{N}\left[\int_{a_{j-1}}^{a_i}\left(\frac{a_{i-1}+a_i}{2}-\theta\right)^2 d\theta + (a_i - a_{i-1})\omega_i\right].$$

Therefore, the receiver's equilibrium payoff cannot exceed

$$\overline{U}(b) \equiv -\int_{a_{j-1}}^{a_{j-1}+4b}\left(\frac{a_{j-1}+a_{j-1}+4b}{2}-\theta\right)^2 d\theta = -\frac{16b^3}{3}.$$

Remark 1. In any equilibrium ξ, if $4b < 1$, the receiver's equilibrium payoff is less than $\overline{U}(b)$. Otherwise, it is $-1/12$.

As discussed already, in order to not violate the condition in Lemma 2, if the receiver makes positive payments on the equilibrium path, there must be at least one interval whose length is greater than $4b$. The upper bound $\overline{U}(b)$ originates from the information loss caused by this interval.

4. Benefit of multistage information transmission with voluntary monetary transfers

This section examines when the multistage information transmission with voluntary payments can be beneficial to the receiver. We show that the multistage information transmission with voluntary monetary transfers can improve welfare relative to the canonical communication protocols (e.g., mediation and arbitration) if the receiver cares more about the decision and the sender cares more about money or if the ex post sender–receiver incentive conflict over the project choice is small.

To this end, we derive an explicit lower bound on the optimal equilibrium payoff to the receiver. Specifically, we refer to the optimal commitment contract characterized by Krishna and Morgan (2008) to construct a particular partition and payment rule meeting the necessary condition for the equilibrium (Lemma 3). Subsequently, we derive the conditions under which the constructed partition is achieved by an equilibrium, providing a lower bound on the equilibrium payoff to the receiver. There may exist a better equilibrium,[11] but at least one equilibrium with the derived partition exists; it can also dominate the optimal mediation and arbitration.

With the uniform-quadratic assumption, if the receiver is better off with the payments than the best equilibrium without payments, the sender is also better off with

[11]We conjecture that the receiver's optimal equilibrium always induces the monotonically decreasing payment rule as in our equilibrium. Recall that by paying more money for the messages that induce the left interval, the receiver can weaken the sender's exaggeration incentive. Hence, the conjecture has a strong intuition; however, a general argument is elusive due to the complexity of the relationship between the gradual information transmission and the receiver's incentive for payments.

the payments. However, the converse is not always true due to the payments from the receiver to the sender. Hence, we discuss the benefit of the multistage information elicitation with voluntary payments from the receiver's viewpoint.

As discussed in the previous section, the set of equilibrium partitions can enlarge if and only if $b < 1/4$. Therefore, we will assume hereafter $b < 1/4$.

4.1 *The optimal commitment contract*

Section 4.1 characterizes the optimal commitment contract. If the receiver can commit to message-contingent payments, she can receive more benefit than in the case without commitment. This is because the receiver can credibly promise a payment without future punishments by the sender. Krishna and Morgan (2008) characterized the optimal commitment contract in such a case.[12]

Under the optimal commitment contract, the receiver pays only for eliciting precise information and never for imprecise information. Specifically, in the low states, the receiver pays the sender to reveal the state fully and, subsequently, chooses her ideal project; in the high states, the receiver does not pay the sender at all. Consequently, the sender conveys what he knows imprecisely in the high states.

Proposition 3 (Krishna and Morgan (2008)). *An optimal commitment contract involves (i) positive payments and separation over an interval* $[0, \overline{a}]$, *and (ii) no payments and a division of* $[\overline{a}, 1]$ *into a finite number of pooling intervals.*

The optimal commitment contract induces the following (uncountably infinite) partition of the type space:

$$\big\{\{\{\theta\}\}_{\theta\in[0,\overline{a}]}, [\overline{a}, \tilde{a}_1], \dots, [\tilde{a}_{K-1}, 1]\big\}, \tag{6}$$

where

$$\overline{a} = \frac{1+2s}{1+3s} - \frac{1}{1+3s}\sqrt{4s^2 + \frac{1}{3}\big\{3s - 2b(1+3s)K(K-1)\big\}\big\{2b(1+3s)K(K+1) - 3s\big\}},$$

and K *is the unique integer such that*

$$\frac{3s}{2(1+3s)K(K+1)} \le b < \frac{3s}{2(1+3s)K(K-1)}. \tag{7}$$

This proposition is a combination of Propositions 4–7 in Krishna and Morgan (2008). [13] The set of pooling intervals $\{[\overline{a}, \tilde{a}_1], \dots, [\tilde{a}_{K-1}, 1]\}$ is a K-step CS partition of the interval $[\overline{a}, 1]$. Therefore, $\tilde{a}_{k+1} - \tilde{a}_k = \tilde{a}_{k-1} - \tilde{a}_k + 4b$ for $k \in \{1, \dots, K-1\}$. Note that $\tilde{a}_0 = \overline{a}$

[12] In the optimal contract characterized in Krishna and Morgan (2008), the receiver can commit to transfers but retain the decision-making authority. They refer to this (imperfect) commitment contract as the optimal compensation contract. However, to avoid confusion, we simply call it *the optimal commitment contract* in this study.

[13] See Appendix C.

and $\tilde{a}_K = 1$. The payment rule for the message $m = \theta$ satisfies that

$$\omega^c(\theta) = \begin{cases} 0 & \text{for } \theta > \overline{a}, \\ \frac{s}{4}(\tilde{a}_1 - \overline{a})\{4b - (\tilde{a}_1 - \overline{a})\} & \text{for } \theta = \overline{a}, \\ \omega^c(\overline{a}) + 2sb(\overline{a} - \theta) & \text{for } \theta < \overline{a}. \end{cases}$$

To understand the property of no payment for the imprecise information, consider a small change in the contract such that the receiver induces full revelation in $[0, \overline{a}]$ but makes a small payment for the states in $[\overline{a}, \tilde{a}_1]$. On the one hand, this change will distort the pooling intervals in the high states, thereby leading to more information revelation. On the other hand, to maintain the sender's incentive compatibility, a payment for $[\overline{a}, \tilde{a}_1]$ raises the payments for $[0, \overline{a}]$ also. Proposition 3 says that the increased cost of aligning incentives in the low states always outweighs the information gains by distorting the pooling intervals in the high states. That is, by inducing pooling intervals in the high states and giving up a certain amount of information, the receiver can realize substantial savings through the global reduction in payments for the low states. The smaller s is, the smaller the payments to align the sender's incentive is. Then the above cost-saving benefits become smaller. Consequently, the separation interval becomes wider: $\lim_{s\to 0} \overline{a}(s) = 1$.

4.2 *An explicit lower bound on the receiver's optimal equilibrium payoff*

Any equilibrium in $\Gamma(b, s, T)$ cannot approximate the outcome of the optimal commitment contract, regardless of how high is T. Recall that the states for which the receiver makes the highest payments must belong to one of the hostage intervals. Since $\omega^c(\cdot)$ has a single peak at $\theta = 0$ and is continuously decreasing in θ over $[0, \overline{a}]$, and $\omega^c(\theta) = 0$ for $\theta > \overline{a}$, any strategy profile whose outcome resembles the payment and partition under the optimal commitment contract violates Lemma 3.

The above problem is attributed to the nonexistence of the information released after the last payment. Therefore, we alter the partition (6) by adding "hostage" intervals to the left end:

$$\big\{[0, a_1], \dots, [a_{L-1}, a_L], \big\{\{\theta\}\big\}_{\theta\in[a_L,\overline{a}]}, [\overline{a}, \tilde{a}_1], \dots, [\tilde{a}_{K-1}, 1]\big\}, \tag{8}$$

where $a_1 \le 4b$ and $a_{l+1} - a_l = a_l - a_{l-1} + 4b$ for $l \in \{1, \dots, L-1\}$. Note that the left-side pooling intervals coincide with the maximum size (L-step) CS equilibrium partition of the interval $[0, a_L]$. Hence, we have $a_l - a_{l-1} = a_1 + 4b(l-1)$ and $a_l = la_1 + 2bl(l-1)$ for $l \in \{1, \dots, L\}$.

The payment rule is

$$\omega^*(\theta) = \begin{cases} \omega^c(\theta) & \text{for } \theta > a_L, \\ \omega^c(a_L) + \frac{s}{4}(a_L - a_{L-1})(a_L - a_{L-1} + 4b) & \text{for } \theta \le a_L. \end{cases} \tag{9}$$

The payment $\omega^*(\cdot)$ makes the sender indifferent at all boundaries between adjacent intervals. The payment is highest for the leftmost L-step partition. Therefore, the receiver elicits information regarding these intervals in the last period; given this, the information on these intervals serves as a hostage, as we desired.

The partition (8) and the payment rule (9) are well-defined and the sender incentive compatible if and only if $a_L < \overline{a}$. We now identify how small b should be, given $L \geq 2$.

Lemma 4. *Fix $L \geq 2$. Then there exists $b_L^* > 0$ such that if $b \in (0, b_L^*)$, then $a_L = La_1 + 2bL(L-1) < \overline{a}$ for any $a_1 \in [0, 4b]$.*

Proof. Given $b < 1/4$, Condition (7) uniquely determines the number of pooling intervals in high states, $K = K(b)$:

$$K(b) = \left\lceil -\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{6s}{(1+3s)b}} \right\rceil, \tag{10}$$

where $\lceil z \rceil$ is the smallest integer greater than or equal to z. If $b \geq 3s/[4(1+3s)]$, we obtain $K(b) = 1$. Then the boundary $\overline{a}$ is no less than $[(1+2s) - \sqrt{4s^2+s}]/(1+3s)$. If $b < 3s/[4(1+3s)]$, we obtain $K(b) \geq 2$. The boundary $\overline{a}$ is no less than $[(1+2s) - s\sqrt{(4\{K(b)\}^2-1)/(\{K(b)\}^2-1)}]/(1+3s)$. Since $a_1 \in [0, 4b]$, we obtain $a_L = L_s a_1 + 2bL(L-1) \leq 2bL(L+1)$. Therefore, the desired inequality is

$$L(L+1) < \frac{A(b)}{2b} \quad \Rightarrow \quad a_L < \overline{a}, \tag{11}$$

where

$$A(b) = \begin{cases} \dfrac{1+2s}{1+3s} - \dfrac{1}{1+3s}\sqrt{4s^2+s} & \text{for } b \geq 3s/[4(1+3s), \\ \dfrac{1+2s}{1+3s} - \dfrac{s}{1+3s}\sqrt{\dfrac{4\{K(b)\}^2-1}{\{K(b)\}^2-1}} & \text{for } b < 3s/[4(1+3s). \end{cases}$$

Note that $A(b)$ is decreasing in $b \in (0, 1/4)$ and converges to $1/(1+3s)$ as b goes to zero. Therefore, $A(b)/b$ goes to infinity as b approaches zero. Therefore, for any $L > 1$, there is b_L^* such that for all $b \in (0, b_L^*)$ and $a_1 \in [0, 4b]$, we obtain $a_L = La_1 + 2bL(L-1) < \overline{a}$. $\square$

Intuitively, b_L^* becomes smaller as L increases. Given parameters, we always have $2bL(L-1) \leq a_L$ and $\overline{a} < 1/(1+3s)$. Hence, the inequality $a_L < \overline{a}$ becomes harder to satisfy as L increases.

Next, we compute the difference between the receiver's payoff under the optimal commitment contract and that under the modified partition and payment rule. Fix L and suppose that $b < b_L^*$, that is, $a_L < \overline{a}$. Let $E^c(b, s)$ and $E^*(a_1, L, b, s)$ be the receiver's expected payoff under the optimal commitment contract and that under the modified partition and payment rule, respectively. Then we obtain

$$E^c(b, s) = -\frac{\sum_{k=1}^{K} \tilde{x}_k^3}{12} - \int_0^1 \omega^c(\theta)\, d\theta,$$

where $\tilde{x}_k = \tilde{a}_k - \tilde{a}_{k-1}$ for $k \in \{1, \ldots, K\}$.

$$E^*(a_1, L, b, s) = -\frac{\sum_{l=1}^{L} x_l^3}{12} - \frac{\sum_{k=1}^{K} \tilde{x}_k^3}{12} - \int_0^1 \omega^*(\theta)\, d\theta,$$

where $a_1 \in [0, 4b]$ and $x_l = a_l - a_{l-1}$ for $l \in \{1, \ldots, L\}$. Therefore, we obtain

$$E^*(a_1, L, b, s) = E^c(b, s) - \eta(a_1, L, b, s),$$

where $a_1 \in [0, 4b]$ and

$$\begin{aligned}\eta(a_1, L, b, s) = {} & \frac{\sum_{l=1}^{L}\big\{a_1 + 4b(l-1)\big\}^3}{12} - sb\big\{La_1 + 2bL(L-1)\big\}^2 \\ & + \frac{s\big\{La_1 + 2bL(L-1)\big\}}{4}\big\{a_1 + 4b(L-1)\big\}(a_1 + 4bL).\end{aligned}$$

The first term in $\eta(\cdot)$ reflects the loss caused by adding the hostage partition and locally giving up a certain amount of information in the low states. The second term reflects a reduction in payments by giving up the perfect screening in $[0, a_L]$. The third term reflects an increase in payments for $[0, a_L]$ by adjusting the sender's incentive at a_L. A straightforward calculation yields that the third term is higher than the second term. In other words, the expected payments increase, relative to the optimal commitment contract. Therefore, an addition of the hostage partition exerts the following negative effects on the receiver's payoff: coarser partition and higher payments. Note that $\eta(\cdot)$ is increasing in $a_1 \in [0, 4b]$ and $L \in \mathbb{N}$, but it does not depend on K and $\overline{a}$. Moreover, $\eta(4b, L, b, s) = \eta(0, L+1, b, s)$ because two hostage partitions, $\{[0, a_1], \ldots, [a_{L-1}, a_L]\}$ and $\{[0, a_1'], \ldots, [a_L', a_{L+1}']\}$, coincide when $a_L = a_{L+1}'$, $a_1 = 4b$, and $a_1' = 0$. Therefore, the receiver wants to take a_1 and L as small as possible.

Remark 2. $E^*(\cdot, L, b, s)$ is increasing in $a_1 \in [0, 4b]$; $E^*(a_1, L-1, b, s) > E^*(0, L, b, s)$ for any $a_1 \in [0, 4b)$; and $E^*(4b, L-1, b, s) = E^*(0, L, b, s)$.

We now confirm that given the hostage partition $\{[0, a_1], \ldots, [a_{L-1}, a_L]\}$, the modified partition (8) optimally separates the subspace $[a_L, 1]$. Given $a_L < 1$, a commitment contract optimally separating $[a_L, 1]$ involves the following: (i) positive payments and separation over an interval $[a_L, a^*]$ for $a^* \in (a_L, 1)$ and (ii) and no payments and a division of $[a^*, 1]$ into a size K^* of pooling intervals.[14] Since $\eta(a_1, L, b, s)$ does not depend on K and $\overline{a}$, as long as a_L is less than $\overline{a}$, the problem of finding the optimal K^* and a^* that maximizes the receiver's expected payoff, given $\{[0, a_1], \ldots, [a_{L-1}, a_L]\}$, is equivalent to the problem of finding K and $\overline{a}$ in the optimal commitment contract. Consequently, we

[14]We can directly apply the proofs of Propositions 4–6 in Krishna and Morgan (2008).

obtain $K^* = K$ and $a^* = \overline{a}$. Hence, to evaluate the benefit of the multistage communication with voluntary payments, we seek an equilibrium approximating the partition (8) and payment rule (9).

If we have an equilibrium in $\Gamma(b, s, T)$ that approximates the partition (8) and payment rule (9), $E^*(a_1, L, b, s)$ will derive an explicit lower bound on the optimal equilibrium payoff to the receiver. Hence, we consider a strategy profile $\xi^*_{a_1,L,T}$ in which the state information is conveyed, in order, from the rightmost interval over time. In the first period, the sender conveys one of the intervals in $\{[0, \overline{a}], [\overline{a}, \tilde{a}_1], \dots, [\tilde{a}_{K-1}, 1]\}$. If the state is in $[a_L, \overline{a}]$, the sender gradually conveys almost the full information in (sufficiently high) $T-2$ periods. The interval $[a_L, \overline{a}]$ is divided into a size $(T-2)$ partition $\{[a_{L-1+i}, a_{L+i}]\}_{i\in\{1,T-2\}}$ at evenly spaced lengths $a_{L+i} - a_{L-1+i} = \varepsilon \equiv (\overline{a} - a_L)/(T-2)$. In each period, the sender conveys the rightmost interval in the remaining subpartition as long as the receiver pays a certain amount of money in the previous period. In the last period, the sender conveys one of the intervals in $\{[0, a_1], \dots, [a_{L-1}, a_L]\}$. If the receiver deviates from the payments, then she is punished by the sender babbling thereafter.

The given strategy profile $\xi^*_{a_1,L,T}$ induces an interval partition $\{[a_{j-1}, a_j]\}_{j\in\{1,\dots,L+K+T-2\}}$:

$$\Big\{\underbrace{\big\{[a_{j-1}, a_j]\big\}_{j\in\{1,\dots,L\}}}_{=\{[0,a_1],\dots,[a_{L-1},a_L]\}}, \underbrace{\big\{[a_{j-1}, a_j]\big\}_{j\in\{L+1,\dots,L+T-2\}}}_{\approx\{\{\theta\}\}_{\theta\in[a_L,\overline{a}]}},$$
$$\underbrace{\big\{[a_{j-1}, a_j]\big\}_{j\in\{L+T-1,\dots,L+K+T-2\}}}_{=\{[\overline{a},\tilde{a}_1],\dots,[\tilde{a}_{K-1},1]\}}\Big\}. \tag{12}$$

The total payment $\omega^*_j = \omega_{\xi^*_{a_1,L,T}}(\theta)$ the sender receives by revealing $\theta \in [a_{j-1}, a_j]$ must satisfy

$$\omega^*_j = \begin{cases} 0 & \text{for } j \in \{L+T-1, \dots, L+K+T-2\}, \\ W_{L+T-2} + 2sb\big(\overline{a} - \{a_L + \varepsilon(j-L)\}\big) & \\ & \text{for } j \in \{L+1, \dots, L+T-2\}, \\ \omega^*_{L+1} + W_L & \text{for } j \le L, \end{cases} \tag{13}$$

where $W_{L+T-2} = s(\varepsilon + \tilde{a}_1 - \overline{a})\{\varepsilon + 4b - (\tilde{a}_1 - \overline{a})\}/4$ and $W_L = s(a_1 + 4b(L-1) + \varepsilon)(a_1 + 4bL - \varepsilon)/4$. Since we suppose that $b < b^*_L$, the partition (12) and the payment rule (13) are well- defined and sender incentive compatible. Due to its construction, $\omega_{\xi^*_{a_1,L,T}}(\cdot)$ uniformly converges to $\omega^*(\cdot)$ as T approaches infinity. Also, the receiver's expected payoff under the strategy profile $\xi^*_{a_1,L,T}$, denoted by $E^R(a_1, L, b, s, T)$, converges to $E^*(a_1, L, b, s)$ as T approaches infinity.

Remark 3. For every $\varepsilon > 0$, there exists a horizon T^R such that $|E^R(a_1, L, b, s, T) - E^*(a_1, L, b, s)| < \varepsilon$ whenever $T \ge T^R$.

Now, we seek how small L we can take. As discussed in Remarks 2 and 3, if T is sufficiently high and $\xi^*_{a_1,L,T}$ constitutes an equilibrium, the receiver prefers L to be as small as possible. Obviously, we cannot take $L = 1$. Let L_s be the minimum integer L such that $[(L-1)L(L+1)(L+2)]/[3\{4L(L+1)+1\}] > s$. Proposition 4 shows that there exists $\hat{a} \in (0, 4b)$ such that for $a_1 \in (\hat{a}, 4b]$, the given strategy profile $\xi^*_{a_1, L_s, T}$ can constitute an equilibrium in $\Gamma(b, s, T)$ whenever T is sufficiently high.

Proposition 4. *Fix $L = L_s$ and $b \in (0, b^*_{L_s})$. Then there exists $\hat{a} \in [0, 4b)$ such that for $a_1 \in (\hat{a}, 4b]$, there is a horizon T^* for which the strategy profile $\xi^*_{a_1, L_s, T}$ constitutes an equilibrium in $\Gamma(b, s, T)$ whenever $T \geq T^*$.*

Proof. Fix $L = L_s$ and $b \in (0, b^*_{L_s})$. Then $a_{L_s} = L_s a_1 + 2bL_s(L_s - 1) < \overline{a}$, that is, the partition (12) and the payment rule (13) are well-defined and sender incentive compatible. Therefore, we have only to ensure the receiver's incentive compatibility for voluntary payments.

In period $t \in \{1, \dots, T-1\}$, the receiver pays $\delta^*_t = \omega^*_{L_s+T-1-t} - \omega^*_{L_s+T-1-(t-1)}$ if the sender conveys that $\theta \in [0, a_{L_s+T-1-t}]$. The receiver is willing to pay δ^*_t if and only if

$$\begin{aligned} &D(t|T) \\ &= \underbrace{-\frac{\sum_{j=1}^{L_s+T-1-t} x_j^3}{12a_{L_s+T-1-t}} - \frac{\sum_{j=1}^{L_s+T-1-t} x_j\left(\omega^*_j - \omega^*_{L_s+T-1-t}\right)}{a_{L_s+T-1-t}}}_{\text{The expected future gain and payments}} - \underbrace{\left\{-\frac{a^2_{L_s+T-1-t}}{12}\right\}}_{\text{The payoff from deviation}} \\ &\geq \underbrace{\delta^*_t}_{\text{The payment in period t}}, \end{aligned} \tag{14}$$

where $x_j = a_j - a_{j-1}$ and $t \in \{1, \dots, T-1\}$.

For $t = T-1$, this inequality is simplified into

$$\begin{aligned} D(T-1|T) &= \frac{-\sum_{j=1}^{L_s}\left\{a_1 + 4b(j-1)\right\}^3 + \left\{L_s a_1 + 2bL_s(L_s-1)\right\}^3}{12\left\{L_s a_1 + 2bL_s(L_s-1)\right\}} \geq \delta^*_{T-1} \\ \Leftrightarrow \quad \zeta(a_1, L_s, s) &= \left\{(L_s-1)(L_s+1) - 3s\right\}a_1^2 \\ &\quad + 4b\left\{(L_s-1)^2(L_s+1) - 3s(2L_s-1)\right\}a_1 \\ &\quad + 4b^2L_s(L_s-1)\left\{(L_s+1)(L_s-2) - 12s\right\} \\ &\geq 3s\varepsilon(4b - \varepsilon), \end{aligned} \tag{15}$$

where $\varepsilon = \frac{\overline{a} - a_{L_s}}{T-2}$. Note that $\zeta(4b, L_s, s) > 0$ is strictly positive if and only if $(L_s-1)(L_s+2)/12 > s$. Recall that L_s is the minimum integer L such that $[(L-1)L(L+1)(L+2)]/[3\{4L(L+1)+1\}] > s$. The inequality $[(L_s-1)L_s(L_s+1)(L_s+2)]/[3\{4L_s(L_s+1)+$

$1\}] > s$ implies that $(L_s - 1)(L_s + 2)/12 > s$. Due to the continuity of $\zeta(\cdot, L_s, s)$, there is $\hat{a}_{T-1} \in [0, 4b)$ such that for $a_1 \in (\hat{a}_{T-1}, 4b]$, we obtain $\zeta(a_1, L_s, s) > 0$.[15]

Since the right-hand side converges to zero when T approaches infinity, the inequality (15) holds for any $a_1 \in (\hat{a}_{T-1}, 4b]$ whenever T is sufficiently high. Precisely, for $a_1 \in (\hat{a}_{T-1}, 4b]$, there exists T^*_{T-1} such that if $T \geq T^*_{T-1}$, we obtain

$$\zeta(a_1, L_s, s) \geq 3s\frac{\overline{a} - a_{L_s}}{T-2}\left(4b - \frac{\overline{a} - a_{L_s}}{T-2}\right).$$

Next, we consider the period $t \in \{1, T-2\}$. Fix $\theta \in (a_{L_s}, a_{L_s+T-3})$. Let $t_{\theta,T}$ denote the period such that $\theta \in (a_{L_s+T-2-t}, a_{L_s+T-1-t}]$.

The inequality (14) can be simplified into

$$\begin{aligned}
D(t_{\theta,T}|T) = {} & \frac{-\displaystyle\sum_{j=1}^{L_s}\big\{a_1 + 4b(j-1)\big\}^3 - (T-1-t_{\theta,T})\varepsilon^3 + \{a_{L_s+T-1-t_{\theta,T}}\}^3}{12a_{L_s+T-1-t_{\theta,T}}} \\
& - \frac{\displaystyle\sum_{i=1}^{T-1-t_{\theta,T}} 2sb\varepsilon^2 i}{a_{L_s+T-1-t_{\theta,T}}} \\
& - \frac{a_{L_s}s}{a_{L_s+T-1-t_{\theta,T}}}\left[\frac{1}{4}\big\{a_1 + 4b(L_s-1) + \varepsilon\big\}(a_1 + 4bL_s - \varepsilon)\right. \\
& \left. + 2b(a_{L_s+T-1-t_{\theta,T}} - a_{L_s})\right] \\
\geq {} & \delta^*_{t_{\theta,T}} = 2sb\varepsilon,
\end{aligned}$$

where $\varepsilon = \frac{\overline{a}-a_{L_s}}{T-2}$.

For $t_{\theta,T} \geq 2$, the payment $\delta^*_{t_{\theta,T}}$ converges to zero as T approaches infinity. The limit as T approaches infinity of the left-hand side is

$$\begin{aligned}
D(\theta|a_1) = {} & \frac{-\displaystyle\sum_{j=1}^{L_s}\big\{a_1 + 4b(j-1)\big\}^3 + \theta^3}{12\theta} - \frac{\displaystyle\int_0^{\theta - a_{L_s}} 2sbz\,dz}{\theta} \\
& - \frac{a_{L_s}s}{\theta}\left[\frac{1}{4}\big\{a_1 + 4b(L_s-1)\big\}(a_1 + 4bL_s) + 2b(\theta - a_{L_s})\right].
\end{aligned}$$

[15]Note that $\zeta(0, L, s) > 0$ if and only if $(L-2)(L+1)/12 > s$ given L. The definition of L_s implies that $[(L_s-1)L_s(L_s+1)(L_s+2)]/[3\{4L_s(L_s+1)+1\}] > s \geq [(L_s-2)(L_s-1)L_s(L_s+1)]/[3\{4L_s(L_s-1)+1\}]$. Since, moreover, $[(L_s-1)L_s(L_s+1)(L_s+2)]/[3\{4L_s(L_s+1)+1\}] > (L_s-2)(L_s+1)/12 > [(L_s-2)(L_s-1)L_s(L_s+1)]/[3\{4L_s(L_s-1)+1\}]$, if we have $(L_s-2)(L_s+1)/12 > s \geq [(L_s-2)(L_s-1)L_s(L_s+1)]/[3\{4L_s(L_s-1)+1\}]$, then we can take $\hat{a}_{T-1} = 0$.

Let x denote $\theta - a_{L_s}$. Then we obtain

$$\begin{aligned} D(a_{L_s}+x|a_1) &= \frac{a_{L_s}\zeta(a_1, L_s, s)}{12(a_{L_s}+x)} \\ &\quad + \left[\frac{L_s a_1^2 + 4b\{L_s(L_s-1)-2s\}a_1 + 4b^2(L_s-1)\{L_s(L_s-1)-4s\}}{4(a_{L_s}+x)}\right]Lx \\ &\quad + \left[\frac{L_s a_1 + 2b\{L_s(L_s-1)-2s\}}{4(a_{L_s}+x)}\right]x^2 + \frac{x^3}{12(a_{L_s}+x)}. \end{aligned}$$

By the definition of L_s, the second, third, and fourth terms in $D(a_{L_s}+x|a_1)$ are strictly positive, and $D(a_{L_s}+x|a_1) > \zeta(a_1, L_s, s)/12$. Since $\zeta(a_1, L_s, s)/12 > 0$ for $a_1 \in (\hat{a}_{T-1}, 4b]$, we have $D(a_{L_s}+x|4b) > \zeta(a_1, L_s, s)/12 > 0$.

For all $\theta \in (a_L, \overline{a})$, $|D(\theta|a_1) - D(t_{\theta,T}|T)|$ and $\delta^*_{t_{\theta,T}}$ converge to zero as T approaches infinity. Therefore, for $a_1 \in (\hat{a}_{T-1}, 4b]$, there exists T^*_θ such that if $T \geq T^*_\theta$, we obtain

$$D(t_{\theta,T}|T) \geq \zeta(a_1, L_s, s)/12 \geq \delta^*_{t_{\theta,T}} \quad \text{for every } t_{\theta,T} \in \{2, \dots, T-2\}. \tag{16}$$

Finally, we ensure the receiver's incentive to pay δ^*_1 in period 1. In this case, if $D(1|T) \geq \delta^*_1 = s(\varepsilon + \tilde{a}_1 - \overline{a})\{\varepsilon + 4b - (\tilde{a}_1 - \overline{a})\}/4$, paying δ^*_1 is optimal for the receiver. Note that $D(\overline{a}|a_1) > \zeta(a_1, L_s, s)/12$ and that $|D(\overline{a}|a_1) - D(1|T)|$ and $|s(\tilde{a}_1 - \overline{a})\{4b - (\tilde{a}_1 - \overline{a})\}/4 - \delta^*_1|$ converge to zero as T approaches infinity. Moreover, $sb^2 \geq s(\tilde{a}_1 - \overline{a})\{4b - (\tilde{a}_1 - \overline{a})\}/4$. Note that we have

$$\frac{\zeta(4b, L_s, s)}{12} > sb^2 \quad \Leftrightarrow \quad \frac{(L_s-1)L_s(L_s+1)(L_s+2)}{3\{4L_s(L_s+1)+1\}} > s. \tag{17}$$

Hence, we obtain $\zeta(4b, L_s, s)/12 > s(\tilde{a}_1 - \overline{a})\{4b - (\tilde{a}_1 - \overline{a})\}/4$. Recall that L_s is the minimum integer that satisfies (17). Due to the continuity of $\zeta(\cdot, L_s, s)$, there is $\hat{a} \in (\hat{a}_{T-1}, 4b)$ such that $\zeta(a_1, L_s, s)/12 > sb^2$ for every $a_1 \in (\hat{a}, 4b]$. In other words, we obtain $D(\overline{a}|a_1) > s(\tilde{a}_1 - \overline{a})\{4b - (\tilde{a}_1 - \overline{a})\}/4$ for $a_1 \in (\hat{a}, 4b]$. Therefore, there exists T^*_1 such that if $T \geq T^*_1$, we obtain

$$D(1|T) \geq \delta^*_1. \tag{18}$$

To summarize, for a given $a_1 \in (\hat{a}, 4b]$, if $T \geq T^* \equiv \max\{T^*_{T-1}, T^*_\theta, T^*_1\}$, the receiver has an incentive to make voluntary payments as defined in $\xi^*_{a_1, L_s, T}$. This completes the proof. □

In what follows, we discuss the conditions in Proposition 4. The number of the hostage intervals, L_s, originates from the receiver's incentive to make the first and last payments, δ^*_1 and δ^*_{T-1}. Under the given equilibrium inducing near separation in the interval $[a_{L_s}, \overline{a}]$, once information elicitation begins after the first payment, the revelation regarding the middle states takes place in the form of equal-sized information elicitation. The receiver makes equal-sized payments $(2sb\varepsilon)$ to the sender, while the value of additional information is monotonically decreasing as the game proceeds, until the last payment. Hence, when T is sufficiently high, the receiver is willing to pay δ^*_t in period

$t \in \{2, \ldots, T-2\}$ whenever she is willing to pay δ^*_{T-1} in period $T-1$. Given this, we have only to confirm whether the receiver is willing to make the first and last payments, δ^*_1 and δ^*_{T-1}. The inequality $[(L_s - 1)L_s(L_s + 1)(L_s + 2)]/[3\{4L_s(L_s + 1) + 1\}] > s$ identifies a sufficient number of the elements of $\{[0, a_1], \ldots, [a_{L_s-1}, a_{L_s}]\}$ for the receiver's first payments to be optimal, and it also ensures the receiver's incentive for the last payment. The lower the s, the smaller is L_s. This reflects that the payment amount required to adjust the sender's incentives declines with a decline in s. In other words, an increase in s implies that it is difficult to satisfy the receiver's incentive at the first and last payments.

As discussed, a sufficient length of the communication round, T^*, is determined so that the receiver's incentive conditions in each period aggregate to the inequality $[(L_s - 1)L_s(L_s + 1)(L_s + 2)]/[3\{4L_s(L_s + 1) + 1\}] > s$, which corresponds to the receiver's incentive condition to make the first and last payments. In other words, T^* must be sufficiently high so that the receiver is willing to pay δ^*_t for $t \in \{2, \ldots, T-2\}$ whenever she is willing to pay δ^*_{T-1}. Finding the explicit value of T^* is elusive. However, in period $t \in \{2, \ldots, T-2\}$ where the receiver is supposed to make an equal-sized payment, the benefit from obtaining additional information must be higher than this payment. The following sufficient condition for the receiver's incentive compatibility in period $t \in \{2, \ldots, T-2\}$ (from condition (16)) provides us with an intuition behind how changes in L_s and T will influence the receiver's incentive:

$$D(t|T) \geq \frac{1}{12}\zeta(a_1, L_s, s) \geq \delta^*_t = 2sb\frac{\overline{a} - a_{L_s}}{T-2}.$$

The left-hand side, $D(t|T)$, denotes the benefit from obtaining additional information by paying δ^*_t. We have shown that if T is sufficiently high, $D(t|T)$ will be higher than $\zeta(a_1, L_s, s)/12$. Note that $\zeta(a_1, L_s, s)/12$ comprises the value of the hostage partition $\{[0, a_1], \ldots, [a_{L_s-1}, a_{L_s}]\}$, and it is increasing in L_s. The right-hand side of this inequality is the receiver's equal-sized payment during periods in $\{2, \ldots, T-2\}$. The higher the T, the finer is the partition $\{[a_{j-1}, a_j]\}_{j\in\{L_s+1,\ldots,L_s+T-2\}}$ and, therefore, the smaller is payment δ^*_t. Therefore, the higher the L and/or T, the more likely is that this inequality will be satisfied.

We next discuss the effect of s on the lowest number of periods needed to maintain the equilibrium. On the one hand, the equilibrium payments is linearly increasing in s and goes to zero as s becomes smaller. On the other hand, the information gain from a given hostage partition $\{[0, a_1], \ldots, [a_{L_s-1}, a_{L_s}]\}$ is independent of s. Therefore, if s is small enough, we can take $T^* = 3$; that is, an equilibrium $\xi^*_{a_1, L_s, 3}$ exists.[16] However, obviously, $\xi^*_{a_1, L_s, 3}$ cannot nearly separate the middle interval $[a_{L_s}, \overline{a}]$; thus, the receiver's equilibrium payoff cannot approximate $E^*(a_1, L_s, b, s)$.

Recall that given L_s, the information value of the partition $\{[0, a_1], \ldots, [a_{L-1}, a_L]\}$ is increasing in $a_1 \in [0, 4b]$. The hostage partition with L_s-elements can ensure the receiver's incentive for the payment when a_1 is close to $4b$; however, when a_1 is close to zero, it may not ensure an incentive. Note that if we fix $L' > L_s$ and $b < b^*_{L'}$, for any $a'_1 \in [0, 4b]$, the given profile $\xi^*_{a'_1, L', T}$ can constitute an equilibrium whenever T is sufficiently

[16] $T = 3$ is the shortest periods with which the partition (12) is well-defined.

high. The receiver's equilibrium payoff under $\xi^*_{a'_1,L',T}$ approaches $E^*(a'_1, L', b, s)$. However, based on Remark 2, the receiver's equilibrium payoff under $\xi^*_{a_1,L_s,T}$ for $a_1 \in [\hat{a}, 4b]$, is always higher than that under $\xi^*_{a'_1,L',T}$ for $a'_1 \in (0, 4b]$ and $L' > L_s$.

Proposition 5 shows that the receiver's equilibrium payoff under $\xi^*_{a_1,L_s,T}$ can approximate $E^*(\hat{a}, L_s, b, s)$ when T is sufficiently high.

Proposition 5. *Fix $L = L_s$, and $b \in (0, b^*_{L_s})$. Then, for every $\varepsilon > 0$, there exists a horizon $\tilde{T}$ for which the receiver's optimal equilibrium payoff is higher than $E^*(\hat{a}, L_s, b, s) - \varepsilon$ whenever $T \geq \tilde{T}$.*

Proof. Proposition 4 shows that for $a_1 \in (\hat{a}, 4b]$, the strategy profile $\xi^*_{a_1,L_s,T}$ constitutes an equilibrium for sufficiently high T. Moreover, based on Remark 3, the receiver's equilibrium payoff $E^R(a_1, L_s, b, s, T)$ converges to $E^*(a_1, L_s, b, s)$ as T approaches infinity, that is, $|E^*(a_1, L_s, b, s) - E^R(a_1, L_s, b, s, T)| < \varepsilon/2$ for sufficiently high T. Recall that $E^*(a_1, L_s, b, s)$ is continuously decreasing in $a_1 \in [\hat{a}, 4b]$. This implies that there exists $a_1(\varepsilon) \in (\hat{a}, 4b]$ such that if $a_1 \in (\hat{a}, a_1(\varepsilon))$, we obtain $E^*(\hat{a}, L_s, b, s) - E^*(a_1, L_s, b, s) < \varepsilon/2$. Fix $a_1 \in (\hat{a}, a_1(\varepsilon))$. Then there is a horizon $\tilde{T}$ for which the receiver's equilibrium payoff $E^R(a_1, L_s, b, s, T)$ is higher than $E^*(\hat{a}, L_s, b, s) - \varepsilon$ whenever $T \geq \tilde{T}$. □

Remark 2 implies that $E^*(\hat{a}, L_s, b, s) > E^*(4b, L_s, b, s)$. Hence, $E^*(4b, L_s, b, s)$ is an explicit lower bound on the receiver's optimal equilibrium payoff in $\Gamma(b, s, T)$ when T is sufficiently high. Next, we confirm that $E^*(4b, L_s, b, s)$ can provide a nontrivial lower bound when b is sufficiently small. The following proposition describes the circumstances under which we find an equilibrium in $\Gamma(b, s, T)$ that Pareto dominates all the mediated equilibria; given this, it dominates the best equilibrium of the CS model.

Proposition 6. *For every $s > 0$, there exists $\tilde{b}(s) > 0$ such that for $b \in (0, \tilde{b}(s))$, the lower bound $E^*(4b, L_s, b, s)$ exceeds the payoff that the receiver can obtain in any mediated equilibrium:*

$$E^*(4b, L_s, b, s) > -\frac{1}{3}b(1-b).$$

Proof. Goltsman et al. (2009) show that the receiver's expected payoff under the optimal mediation is given by $-b(1-b)/3$. Since $\eta(4b, L_s, b, s) = 4(1+3s)L_s^2(L_s+1)^2b^3/3$ and $2bK(K-1) < 3s/(1+3s) \leq 2bK(K+1)$, we obtain

$$\begin{aligned}
&E^*(4b, L_s, b, s) \\
&\quad > -sb + s4b^2K(K-1) - \frac{4(1+3s)b^3}{3}\left\{K^2(K-1)^2 + L_s^2(L_s+1)^2\right\} \\
&\quad > -\frac{s}{1+3s}b - 4s\left(\sqrt{1+\frac{6s}{(1+3s)b}} - 1\right)b^2 - \frac{4(1+3s)L_s^2(L_s+1)^2}{3}b^3.
\end{aligned}$$

Since $s/(1+3s) < 1/3$, there exists $\tilde{b}(s) \in (0, b^*_{L_s}]$ such that for $b \in (0, \tilde{b}(s))$,

$$-\frac{s}{1+3s}b - 4s\left(\sqrt{1+\frac{6s}{(1+3s)b}} - 1\right)b^2 - \frac{4(1+3s)L_s^2(L_s+1)^2}{3}b^3 > -\frac{1}{3}b(1-b).$$

This completes the proof. □

Intuitively, $\tilde{b}(s)$ is decreasing in s and converges to zero as s approaches infinity: $\lim_{s\to\infty} \tilde{b}(s) = 0$ and $\lim_{s\to 0} \tilde{b}(s) \approx 2/25$. Here, using $E^*(4b, L_s, b, s)$ as a lower bound, we evaluated the receiver's optimal equilibrium payoff. Since the total width of the hostage partition is $a_1 + (a_1 + 4b) = 12b$ when $a_1 = 4b$ and $L_s = 2$, $\tilde{b}(s)$ in Proposition 6 cannot exceed $1/12$. However, $\hat{a}$ declines with a decline in s. Thus, we will show that for any $b < 1/4$, we can find a tighter bound if s is sufficiently small.

To this end, considering an ideal case in which s is sufficiently small, we discuss the limitation of our information-elicitation mechanism. Proposition 7 shows that when T is sufficiently high and s is sufficiently small, the simple upper bound $\overline{U}(b)$ provided in Remark 1 can be approximated by the receiver's equilibrium payoff under $\xi^*_{a_1, L_s, T}$.

Proposition 7. *Fix $b \in (0, 1/4)$. For every $\varepsilon > 0$, there exists $\overline{s} > 0$ and $\overline{T}$ such that if $s < \overline{s}$ and $T \geq \overline{T}$, the receiver can obtain a higher ex ante expected payoff than $\overline{U}(b) - \varepsilon$.*

Proof. If $s < 8/25$, we obtain $L_s = 2$. Further, Condition (7) ensures that given $b \in (0, 1/4)$, we obtain $K = 1$ when s is sufficiently small. Therefore, for a fixed $b \in (0, 1/4)$, there exists $\tilde{s} > 0$ such that $L_s = 2$ and $K = 1$ for $s < \tilde{s}$. In what follows, we suppose that $s < \tilde{s}$.

Given $b \in (0, 1/4)$, we must have $a_{L_s} = 2a_1 + 4b < \overline{a}$ so that the partition (8) and payment rule (9) are well-defined. We first ensure that $\hat{a} < \overline{a}/2 - 2b$ and $\overline{U}(b) - E^*(\hat{a}, 2, b, s) < \varepsilon/3$ for sufficiently small s. Recall that $E^*(\hat{a}, 2, b, s)$ is given by

$$E^*(\hat{a}, 2, b, s) = -\frac{\hat{a}^3 + (\hat{a}+4b)^3}{12} - \frac{\sum_{k=1}^{K} \tilde{x}_k^3}{12} - \int_0^1 \omega^*(\theta)\, d\theta,$$

where $\tilde{x}_k = \tilde{a}_k - \tilde{a}_{k-1}$ for $k \in \{1, \dots, K\}$ and $\tilde{a}_0 = \overline{a}$. Recall that $K = 1$, $\overline{a}$ converges to one, and $\omega^*(\theta)$ converges to zero as s approaches zero. Hence, the second and third terms in $E^*(\hat{a}, 2, b, s)$ converge to zero as s approaches zero. Moreover, the first term converges to $\overline{U}(b)$ as $\hat{a}$ approaches zero. Therefore, we have only to ensure that $\hat{a}$ converges to zero as s approaches zero. Due to the definition of $\hat{a}$ in the proof of Proposition 4, if $\zeta(a_1', 2, s) > sb^2$ for some $a_1' \in (0, 4b]$, then $\hat{a} < a_1'$. Note that $\zeta(a_1, 2, s)$ is continuous in s and $\zeta(a_1, 2, 0) > 0$ for $a_1 > 0$. Thus, how small a_1 we choose, we obtain $\zeta(a_1, 2, s) > sb^2$ for sufficiently small s. This implies that $\hat{a}$ converges to zero as s approaches zero. Hence, we can find $\overline{s} \in (0, \tilde{s}]$ such that $\hat{a} < \overline{a}/2 - 2b$ and $\overline{U}(b) - E^*(\hat{a}, 2, b, s) < \varepsilon/3$ for $s < \overline{s}$.

Suppose that $s < \overline{s}$, then we have $\hat{a} < \overline{a}/2 - 2b$. Moreover, for any $a_1 \in (\hat{a}, \overline{a}/2 - 2b)$ and $\varepsilon > 0$, $\xi^*_{a_1, 2, T}$ constitutes an equilibrium and the equilibrium payoff $E^R(a_1, 2, b, s, T)$

can be higher than $E^*(a_1, 2, b, s) - \varepsilon/3$ when T is sufficiently high. Furthermore, there exists $a_1(\varepsilon) \in (\hat{a}, \overline{a}/2 - 2b)$ such that $|E^*(\hat{a}, 2, b, s) - E^*(a_1, 2, b, s)| < \varepsilon/3$ for every $a_1 \in (\hat{a}, a_1(\varepsilon))$. Fix $a_1 \in (\hat{a}, a_1(\varepsilon))$. Then there is a horizon $\overline{T}$ for which $\xi^*_{a_1,2,T}$ constitutes an equilibrium and the receiver's equilibrium payoff $E^R(a_1, 2, b, s, T)$ is higher than $E^*(a_1, 2, b, s) - \varepsilon/3 > E^*(\hat{a}, 2, b, s) - 2\varepsilon/3 > \overline{U}(b) - \varepsilon$ whenever $T > \overline{T}$. □

The intuition behind Proposition 7 is straightforward. We constructed an equilibrium involving 2-step hostage partition $\{[0, a_1], [a_1, 2a_1 + 4b]\}$ and (near) separation in $[2a_1 + 4b, 1]$ with equal-sized payments $(2sb\varepsilon)$. Precisely, $\delta_1^* = s(1 - \overline{a})\{4b - (1 - \overline{a})\}/4$, $\delta_t^* = 2sb\varepsilon$ for $t < T - 1$, and $\delta_{T-1}^* = s(a_1 + 4b + \varepsilon)(a_1 + 8b - \varepsilon)/4$, where $\overline{a} \approx 1$ and $\varepsilon = (1 - 2a_1 - 4b)/(T - 1)$. The payment rule is linearly increasing in s and converges to zero as s approaches zero. This implies that given $a_1 < 1/2 - 2b$, the receiver's payoff converges to $-\{a_1^3 + (2a_1 + 4b)^3\}/12$ as s approaches zero. As shown in the proof of Proposition 4, if we can choose an arbitrarily high T and the revelation takes place as a limit of equal-sized information elicitation, we will have only to ensure the receiver's incentive so that her first and last payment holds. The information gain from the hostage partition, $\{(2a_1 + 4b)^2\}/12 - \{a_1^3 + (a_1 + 4b)^3\}/\{12(2a_1 + 4b)\}$, does not depend on s, whereas δ_1^* and δ_{T-1}^* converges to zero as s approaches zero. Therefore, regardless of how small a_1 we choose, there exists $\overline{s} > 0$ and $\overline{T}$ such that if $s < \overline{s}$ and $T \geq \overline{T}$, we can construct an equilibrium inducing 2-step hostage intervals $\{[0, a_1], [a_1, 2a_1 + 4b]\}$ and (nearly) separation in $[2a_1 + 4b, 1]$. As a result, if s are sufficiently small, and T is sufficiently high, there is an equilibrium in which the receiver's equilibrium payoff is higher than $\overline{U}(b) - \varepsilon$.

It is known that the mechanisms without transfer (i.e., mediation and arbitration) help the receiver elicit information. Under these mechanisms, the sender's incentives are controlled by properly designed information-contingent communication (mediation) or decision (arbitration) rules. Therefore, the sender's emphasis on the project choice relative to the receiver does not matter under these mechanisms. However, if the project is crucial for the receiver but not for the sender, the receiver should exploit this difference to improve her decision-making. Our mechanism provides a clear intuition about how the receiver utilizes this difference without any commitment. Interestingly, Proposition 6 shows that, if the bias is sufficiently small, an equilibrium Pareto dominating all the mediated equilibria will exist, even when s is relatively high. Moreover, Proposition 7 shows that if s is sufficiently small, the receiver can obtain a higher expected payoff than that under the optimal arbitration, although she cannot commit to predetermined decision rules.[17]

5. Concluding remarks

This study analyzed a cheap-talk game in which an informed sender and an uninformed receiver engage in a finite-period communication before the receiver's decision-making. During the communication phase, the sender sends multiple (cheap-talk) messages and

[17] Goltsman et al. (2009) show that the receiver's ex ante expected payoff under the optimal arbitration and the optimal mediation are $-b^2(1 - 4b/3)$ and $-b(1 - b)/3$, respectively.

the receiver monetarily compensates the sender whenever she receives a message. We showed that the dependence of future information on past payments creates an incentive for the receiver to pay money. This result ensures that the receiver makes message-contingent payments, to a certain extent, even when there is no contractibility. Thus, this model improves the information transmission, relative to the CS model.

We focused on the multistage *unilateral* communication. Intuitively, the punishment by the sender babbling can motivate the receiver's payment incentive even when the players engage in more general communication protocols such as multistage *bilateral* communication. However, this leads to the question of whether and how much the players' welfare can be improved with such general communication protocols. Considering such a model remains for further research.

Appendix A: Proof of Proposition 1

If there exists an interval $[\underline{a}_F, \overline{a}_F] \subset [0, 1]$ such that $\{\theta\} \in \mathcal{I}_\xi$ for all $\theta \in [\underline{a}_F, \overline{a}_F]$, we refer to this interval as the *separation interval.*

First, we show that in any equilibrium, the equilibrium partition $\mathcal{I}_\xi$ does not contain a nondegenerate separation interval. Suppose that the equilibrium partition $\mathcal{I}_\xi$ contains at least one nondegenerate separation interval $[\underline{a}_F, \overline{a}_F]$ such that $\underline{a}_F < \overline{a}_F$.

We denote the union of separation intervals by Θ_F. For the sender's truth telling to be satisfied on Θ_F, the payment rule $\omega_\xi(\cdot)$ must satisfy that for any $\underline{\theta}, \overline{\theta} \in \Theta_F$, if $\underline{\theta} < \overline{\theta}$, then $\omega_\xi(\underline{\theta}) = \omega_\xi(\overline{\theta}) + 2sb(\overline{\theta} - \underline{\theta})$. Therefore, $\omega_\xi(\cdot)$ is strictly decreasing on Θ_F and $\omega_\xi(\theta) > 0$ for a.e. $\theta \in \Theta_F$.

Given a strategy profile, each $\theta' \in \Theta$ induces a sequence of players' actions $(m'_1, w'_1, \dots, m'_T, w'_T)$ according to ξ. Fix $\theta' \in \Theta_F$. Then there exists $t(\theta') < T$ such that $w'_{t(\theta')} > 0$ and $w'_t = 0$ for any $t > t(\theta')$.[18] Moreover, $\sum_{t=1}^{t(\theta')} w'_t = \omega_\xi(\theta')$ holds. Since the length of the communication round is a finite periods $T < +\infty$ and Θ_F contains non-degenerate interval $[\underline{a}_F, \overline{a}_F]$, we can take a period $\hat{t} < T$ such that $\{\theta \in \Theta_F : t(\theta) = \hat{t}\}$ has a positive measure (on the probability space on which the prior probability distribution is defined) and that $\{\theta \in \Theta_F : t(\theta) = t\}$ has measure zero for $t > \hat{t}$.

We denote $\{\theta \in \Theta_F : t(\theta) = \hat{t}\}$ by $\Theta_F^{\hat{t}}$. Let $h_2^{\hat{t}}(\theta)$ denote the history at the second stage in period $\hat{t}$ induced by ξ and $\theta \in \Theta_F^{\hat{t}}$. For almost every $\theta \in \Theta_F$, on the equilibrium path induced by θ, the receiver never makes a positive payment after the period $\hat{t}$. Since $\omega_\xi(\theta)$ is strictly decreasing on Θ_F, it must be satisfied that $h_2^{\hat{t}}(\theta) \neq h_2^{\hat{t}}(\theta')$ for $\theta, \theta' \in \Theta_F^{\hat{t}}$. Recall that $\Theta_F \setminus \{\Theta_F^t\}_{t \leq \hat{t}}$ has measure zero. Hence, it must be satisfied that for (a.e.) $\theta, \theta' \in \Theta_F$, if $\theta \neq \theta'$, $\operatorname{supp}\{f^{\hat{t}}(\cdot|h_2^{\hat{t}}(\theta))\} \cap \operatorname{supp}\{f^{\hat{t}}(\cdot|h_2^{\hat{t}}(\theta'))\} = \emptyset$.

The subpartition $\mathcal{I}_\xi \setminus \{\theta\}_{\theta \in \Theta_F}$ has at most countably infinite numbers of pooling intervals. Therefore, for (a.e.) $\theta \in \Theta_F^{\hat{t}}$, the support of the receiver's belief at $h_2^{\hat{t}}(\theta)$ is $\{\theta\}$ although the receiver makes a positive payment at this history. This contradicts Lemma 2.

Next, suppose that the equilibrium partition has infinitely many pooling intervals. Let $\{[a_{i-1}, a_i]\}_{i \in I}$ denote the equilibrium partition $\mathcal{I}_\xi$. In this case, $\{[a_{i-1}, a_i]\}_{i \in I}$

[18] Since the receiver cannot obtain additional information about θ after the second stage in the final period of communication round, she has no incentive to choose $w^T > 0$. Therefore, w^T must be equal to 0 in any equilibrium.

can be divided into two subpartitions $\{[a_{i_+-1}, a_{i_+}]\}_{i_+\in I_+}$ and $\{[a_{i_--1}, a_{i_-}]\}_{i_-\in I_-}$, where $a_{i_+} - a_{i_+-1} \geq 4b$ for $i_+ \in I_+$, and $a_{i_-} - a_{i_--1} < 4b$ for $i_- \in I_-$. Moreover, $\{[a_{i_+-1}, a_{i_+}]\}_{i_+\in I_+}$ consists of a finite number of intervals, whereas $\{[a_{i_--1}, a_{i_-}]\}_{i_-\in I_-}$ has infinitely many pooling intervals. Let ω_i denote $\omega_\xi(\theta)$ for $\theta \in [a_{i-1}, a_i] \in \{[a_{i-1}, a_i]\}_{i\in I}$.

From the sender incentive-compatibility conditions, for any $i'_-, i''_- \in I_-$ such that $a_{i'_-} \leq a_{i''_--1}$, we obtain

$$\omega_{i'_-} - \omega_{i''_-} = 2sb\chi_- + \frac{s}{4}(x_{i'_-} + x_{i''_-})(x_{i'_-} - x_{i''_-} + 4b) > 0, \tag{19}$$

where $\chi_- = a_{i''_--1} - a_{i'_-}$, $x_{i'_-} = a_{i'_-} - a_{i'_--1}$, and $x_{i''_-} = a_{i''_-} - a_{i''_--1}$. Therefore, the payment rule ω_ξ is decreasing over $\{[a_{i_--1}, a_{i_-}]\}_{i_-\in I_-}$.

Given ξ, each $\theta' \in \Theta$ induces a sequence of players actions $(m'_1, w'_1, \dots, m'_T, w'_T)$. If $\omega(\theta') > 0$ for $\theta' \in \Theta$, there exists $t(\theta') < T$ such that $w'_{t(\theta')} > 0$ and $w'_t = 0$ for any $t > t(\theta')$. Moreover, $\sum_{t=1}^{t(\theta')} w'_t = \omega_\xi(\theta')$ holds.

Define Θ^t_- as $\{[a_{i-1}, a_i] \in \{[a_{i_--1}, a_{i_-}]\}_{i_-\in I_-} : t(\theta) = t \text{ for } \theta \in [a_{i-1}, a_i]\}$. Since $\{[a_{i_--1}, a_{i_-}]\}_{i_-\in I_-}$ contains countably infinite intervals, there exists at least one $t < T$ such that Θ^t_- also contains countably infinite intervals. Let $\hat{t}$ be the maximum period in which $\Theta^{\hat{t}}_-$ contains countably infinite intervals. Let $h^{\hat{t}}_2(\theta)$ denote the history at the second stage in period $\hat{t}$ induced by ξ and $\theta \in \Theta^{\hat{t}}_-$.

Fix $\theta \in [a_{i_--1}, a_{i_-}] \in \Theta^{\hat{t}}_-$. At history $h^{\hat{t}}_2(\theta)$, the support of the receiver's belief does not contain intervals in $\{\bigcup_{t\leq\hat{t}} \Theta^t_-\} \setminus [a_{i_--1}, a_{i_-}]$.[19] Since, moreover, $\#\{\{[a_{i_--1}, a_{i_-}]\}_{i_-\in I_-} \setminus \{\Theta^t_-\}_{t\leq\hat{t}}\} < \infty$, we obtain $\#\{\{[a_{i-1}, a_i]\}_{i\in I} \setminus \{\Theta^t_-\}_{t\leq\hat{t}}\} < \infty$.

Since $\Theta^{\hat{t}}_-$ contains infinite number of intervals and $\#\{\{[a_{i-1}, a_i]\}_{i\in I} \setminus \{\Theta^t_-\}_{t\leq\hat{t}}\} < \infty$, there are infinitely many intervals in $\Theta^{\hat{t}}_-$ such that $\theta \in [a_{i_--1}, a_{i_-}]$ and ξ induce a history $h^{\hat{t}}_2(\theta)$ at which the receiver makes a positive payment, although $\text{supp}\{f^{\hat{t}}(\cdot|h^{\hat{t}}_2(\theta))\} = [a_{i_--1}, a_{i_-}]$. This result contradicts Lemma 2. Therefore, the equilibrium partition consists of a finite number of pooling intervals.

Appendix B: Proof of Proposition 2

Consider a strategy profile and belief system described below. In the first period, the sender conveys whether the state belongs to the interval $[a_{i^1-1}, a_{i^1}]$.[20] After receiving the message that means $\theta \in [a_{i^1-1}, a_{i^1}]$, the receiver will neither pay money nor obtain additional information in the future. Otherwise, the receiver pays $\omega_{i^2} - \omega_{i^1} = s \cdot \beta(\mathcal{I}, i^2) - s \cdot \beta(\mathcal{I}, i^1)$ to the sender. After this payment, in the second period, the sender conveys whether the true state belongs to $[a_{i^2-1}, a_{i^2}]$. In period t where the receiver believes that $\theta \in \Theta \setminus \bigcup_{\hat{t}\leq t-1}\{[a_{i^{\hat{t}}-1}, a_{i^{\hat{t}}}]\}$, the sender conveys whether $\theta \in [a_{i^t-1}, a_{i^t}]$, and the

[19] Take two intervals $[a_{i_--1}, a_{i_-}]$ and $[a_{i'_--1}, a_{i'_-}]$ in $\{\bigcup_{t\leq\hat{t}} \Theta^t_-\} \subset \{[a_{i_--1}, a_{i_-}]\}_{i_-\in I_-}$. Recall that if θ belongs to an interval in $\{\bigcup_{t\leq\hat{t}} \Theta^t_-\} \setminus [a_{i'_--1}, a_{i'_-}]$, the receiver never pays positive amount of money after the period $\hat{t}$. If $\theta \in [a_{i_--1}, a_{i_-}]$ and $\theta' \in [a_{i'_--1}, a_{i'_-}]$ induce $h^{\hat{t}_2(\theta)} = h^{\hat{t}_2(\theta')}$, we obtain $\omega(\theta) = \omega(\theta')$. This outcome is inconsistent with Condition (19). Hence, θ and θ' induce different histories at the second stage in period $\hat{t}$.

[20] Consider the case in which two or more intervals induce the smallest payment (e.g., $\omega_{i^1} = \omega_{j^1} = 0$). In this case, if the state belongs to one of them, the sender conveys to which interval the true state belongs. Otherwise, he conveys that the state does not belong to each of them.

receiver pays $\omega_{i^{t+1}} - \omega_{i^t} = s\{\beta(\mathcal{I}, i^{t+1}) - \beta(\mathcal{I}, i^t)\}$ only when she receives the message that means $\theta \notin [a_{i^t-1}, a_{i^t}]$.

This information elicitation is repeated in the communication phase until the sender completes revealing the interval to which the state belongs. We denote by t^* the last period in which the sender conveys information on the equilibrium path. Note that $t^* \leq N - \#\{I^H\} + 1$.

Let $\beta_t(\mathcal{I})$ denote $\beta(\mathcal{I}, i^t)$ for $t \leq t^*$. Note that $\beta_{t^*}(\mathcal{I}) = \beta(\mathcal{I}, i^H)$. If the receiver deviates from payments, the sender conveys no information thereafter. It means that all the sender types send the same message. We suppose that if the sender deviates to some off-path message, then the receiver assigns her belief to one of the beliefs induced by the on-path messages and acts as if she receives that on-path message.[21] Given the belief f^T, the receiver chooses a project to maximize $E[-(y-\theta)^2|f^T]$. Therefore, the decision chosen on the equilibrium path is $y_i = (a_{i-1} + a_i)/2$ for $i \in \{1, \dots, N\}$.

Since the pair of $\{\omega_i\}_{i\in\{1,\dots,N\}}$ and $\{y_i\}_{i\in\{1,\dots,N\}}$ associated with the given partition $\{[a_{i-1}, a_i]\}_{i\in\{1,\dots,N\}}$ is sender incentive compatible, the sender has no incentive to deviate from the given message strategy on the equilibrium path.[22] Hence, we have only to ensure the optimality of the receiver's payment strategy.

Consider a history h_2^t where the receiver makes a positive payment $\rho^t(h_2^t) = s\{\beta_{t+1}(\mathcal{I}) - \beta_t(\mathcal{I})\} > 0$. Note that $f^t(\cdot|h_2^t)$ is a uniform distribution on $\Theta_t \equiv \Theta \setminus \bigcup_{\hat{t}\leq t}\{[a_{i^{\hat{t}}-1}, a_{i^{\hat{t}}}]\}$. Let $\mathrm{U}(X)$ denote the uniform distribution on the set X. The receiver's continuation payoff by paying $\rho^t(h_2^t)$ is

$$\underbrace{E\Big[E\big[-(y_{i^{\tilde{t}}}-\theta)^2\big|\mathrm{U}\big([a_{i^{\tilde{t}}-1}, a_{i^{\tilde{t}}}]\big)\big] - s\big\{\beta_{\tilde{t}}(\mathcal{I}) - \beta_{t+1}(\mathcal{I})\big\}\Big|\mathrm{U}(\Theta_t)\Big]}_{\text{The expected future gain and payments}} - \underbrace{s\big\{\beta_{t+1}(\mathcal{I}) - \beta_t(\mathcal{I})\big\}}_{\text{The payment in period } t}$$

$$= -E\Big[\mathrm{Var}\big(\theta\big|\mathrm{U}\big([a_{i^{\tilde{t}}-1}, a_{i^{\tilde{t}}}]\big)\big)\Big|\mathrm{U}(\Theta_t)\Big] - s\Big\{E\big[\beta_{\tilde{t}}(\mathcal{I})\big|\mathrm{U}(\Theta_t)\big] - \beta_t(\mathcal{I})\Big\},$$

where $E[\beta_{\tilde{t}}(\mathcal{I})|\mathrm{U}(\Theta_t)]$ is the conditional expected total payment associated with intervals, which the sender conveys in the later periods, $\tilde{t} \in \{t+1, \dots, t^*\}$. Since we sorted $\beta_t(\mathcal{I})$ in ascending order, we obtain $E[\beta_{\tilde{t}}(\mathcal{I})|\mathrm{U}(\Theta_t)] - \beta_t(\mathcal{I}) > 0$.

By decreasing the payment amount, the receiver obtains at most

$$E\big[-(\tilde{y}-\theta)^2\big|\mathrm{U}(\Theta_t)\big],$$

where $\tilde{y}$ is the optimal value maximizing this conditional expectation. We obtain

$$-E\Big[\mathrm{Var}\big(\theta\big|\mathrm{U}\big([a_{i^{\tilde{t}}-1}, a_{i^{\tilde{t}}}]\big)\big)\Big|\mathrm{U}(\Theta_t)\Big] = E\Big[E\big[-(y_{i^{\tilde{t}}}-\theta)^2\big|\mathrm{U}\big([a_{i^{\tilde{t}}-1}, a_{i^{\tilde{t}}}]\big)\big]\Big|\mathrm{U}(\Theta_t)\Big]$$

[21] By allowing the sender to randomize his message, we can exclude the possibility that the receiver observes an off-path message at the on-path histories. For example, consider the sender's strategy in the first period as follows. If the state is in $[a_{i^1-1}, a_{i^1}]$, the sender randomizes his message uniformly on $M_1 \subset M$. Otherwise, the sender randomizes his message uniformly on $M \setminus M_1$. The receiver's belief induced by the above message strategy is the same as that under the given pure message strategy. However, allowing the sender's randomization complicates the discussion in the previous section, although it does not afford new insights into the equilibrium characterization. Therefore, we focus on the class of the sender's pure message strategies.

[22] Under the given construction of off-path beliefs, the receiver has no incentive to send off-path messages at any history of h_θ^t.

$$> E\big[-(\tilde{y}-\theta)^2 | \mathrm{U}(\Theta_t)\big].$$

Therefore, if $\rho^t(h_2^t) > 0$, then paying $\rho^t(h_2^t)$ is optimal for the receiver if and only if

$$s \le s^*(t, \mathcal{I}) = \frac{-E\big[\mathrm{Var}\big(\theta | \mathrm{U}\big([a_{i\tilde{t}-1}, a_{i\tilde{t}}]\big)\big) | \mathrm{U}(\Theta_t)\big] - E\big[-(\tilde{y}-\theta)^2 | \mathrm{U}(\Theta_t)\big]}{E\big[\beta_{\tilde{t}}(\mathcal{I}) | \mathrm{U}(\Theta_t)\big] - \beta_t(\mathcal{I})} > 0.$$

Note that we define $s^*(t, \mathcal{I}) = +\infty$ for h_2^t such that $\rho^t(h_2^t) = s\{\beta_{t+1}(\mathcal{I}) - \beta_t(\mathcal{I})\} = 0$. Let $s^*(\mathcal{I})$ be the minimum value of $s^*(t, \mathcal{I})$ in $t \le t^*$. If $s \le s^*(\mathcal{I})$, the prescribed strategy profile and belief system constitute an equilibrium. This completes the proof of Proposition 2.

Appendix C: The optimal commitment contract

Proposition 3 corresponds to Propositions 4–7 in Krishna and Morgan (2008). The sender's preference differentiates the model between Krishna and Morgan (2008) and the present study. Krishna and Morgan (2008) mainly consider the case in which $s = 1$; however, all the proofs can be generalized easily.

Proposition 8 (Proposition 4 in Krishna and Morgan (2008)). *Full revelation commitment contracts are never optimal.*

Proof. Krishna and Morgan (2008) show this result in the general model. Therefore, the statement is true in our model. □

Proposition 9 (Proposition 5 in Krishna and Morgan (2008)). *An optimal commitment contract involves separation in low states and pooling in high states.*

Proof. Suppose that there is a pooling interval $[\underline{a}, a_i]$ and separation interval $[a_i, \overline{a}]$. In the interval $[a_i, \overline{a}]$, the contract must satisfy $\omega^c(\theta) = 2sb(\overline{a} - \theta) + \omega^c(\overline{a})$.

The indifference condition at a is

$$-s\left(\frac{\underline{a} + a_i}{2} - a_i - b\right)^2 + \omega_i^c = -sb^2 + 2sb(\overline{a} - a_i) + \omega^c(\overline{a}).$$

Note that $\omega_i^c > 0$. At $\underline{a}$, the sender must be indifferent between some project y together with some transfer ω_y^c, and the project $(\underline{a} + a)/2$ together with ω_i^c. Therefore, we obtain

$$\omega_y^c = s\big(\underline{a}^2 + 2\overline{a}b + y^2 - 2y\underline{a} - 2yb\big) + \omega^c(\overline{a})$$

Note that ω_y^c does not depend on a_i. Therefore, the receiver's expected payoff in these intervals is

$$\begin{aligned} V &= \int_{\underline{a}}^{a_i} \left(-\left(\frac{\underline{a} + a_i}{2} - \theta\right)^2 - \omega_i^c\right) d\theta - \int_{a_i}^{\overline{a}} \big[2sb(\overline{a} - \theta) + \omega^c(\overline{a})\big]\, d\theta \\ &= -\frac{1 + 3s}{12}(a_i - \underline{a})^3 + sb\big(2\overline{a}\underline{a} - \overline{a}^2 - \underline{a}^2\big). \end{aligned}$$

Obviously, $dV/da_i < 0$. Since $\omega_i^c > 0$, a small change in a_i is feasible. □

Lemma 5 (Lemma 5 in Krishna and Morgan (2008)). *Suppose that a contract induces calls for revelation on* $[0, a]$ *and pooling with no payment on* $[a, 1]$. *Such a contract is feasible (i.e., all payments are nonnegative) if and only if the no-contract equilibrium (the CS equilibrium) that subdivides* $[a, 1]$ *into the maximum number of pooling intervals is played.*

Proof. The proof is straightforward and, therefore, omitted. For details, refer to Lemma 5 in Appendix C in Krishna and Morgan (2008). □

Proposition 10 (Proposition 6 in Krishna and Morgan (2008)). *In an optimal commitment contract, the receiver never pays for the pooling interval(s).*

Proof. Proposition 9 implies that an optimal contract must have separation on $[0, \overline{a}]$ for some $\overline{a} \geq 0$ and pooling intervals in $[\overline{a}, 1]$. Let n^* be the number of pooling intervals. Suppose that the total expected transfer in this contract is B^*. Any optimal contract must maximize the receiver's expected payoff from the project choice among all the contracts in which the expected expenditure is B^*.

We will show that (i) given a budget B, if a "low states separation and high states pooling" contract maximizes the receiver's expected payoff, it must satisfy the "no payment for pooling" property; and (ii) an optimal contract must be a solution to such a problem. The proof of the second statement is straightforward from Krishna and Morgan (2008). Here, we will show that the optimization problem we have to solve to prove the first statement is equivalent to that solved by Krishna and Morgan (2008).

Fix $n \geq \max\{n^*, N(b)\}$ and B, where $N(b)$ is the maximum number of partition elements of $[0, 1]$ with no transfer and the budget B is arbitrary.

Let $[0, a_0]$ be the separation interval and $\{[a_0, a_1], \ldots, [a_{n-1}, a_n]\}$ be pooling intervals. Let ω_i^c be the payment for interval $[a_{i-1}, a_i]$. The sender's incentive compatibility implies that[23]

$$\omega_i^c = \frac{s}{4}(a_i - a_{i-1})^2 - s(a_i + a_{i-1})b - \frac{s}{4}(1 - a_{n-1})^2 + s(1 + a_{n-1})b + \omega_n^c.$$

The payment for $\theta \in [0, a_0]$ is

$$\omega^c(\theta) = 2sb(a_0 - \theta) + \omega^c(a_0),$$

where

$$\omega_0^c = -2sba_0 - \frac{s}{4}(1 - a_{n-1})^2 + s(1 + a_{n-1})b + \omega_n^c.$$

Given B, an optimal contract is the solution to the following problem:

$$\max_{(a_0, a_1, \ldots, a_{n-1}, \omega_n^c)} -\frac{1}{12}\sum_{i=1}^{n}(a_i - a_{i-1})^3$$

[23] See Condition (4) in Section 3.

subject to

$$\underbrace{sba_0^2 + \omega_0^c a_0 + \sum_{i=1}^{n} \omega_i^c (a_i - a_{i-1}) \leq B}_{\text{The expected payments cannot exceed the budget B.}} \quad ,$$

$$\omega_i^c \geq 0 \text{ for } i \in \{0, 1, \dots, n-1\},$$

where ω_i^c satisfies the prescribed conditions.

The Lagrangian is given by

$$L = -\frac{1}{12} \sum_{i=1}^{n} (a_i - a_{i-1})^3 + \lambda \left(B - sba_0^2 + \omega_0^c a_0 - \sum_{i=1}^{n} \omega_i^c (a_i - a_{i-1}) \right) + \sum_{i=0}^{n-1} \mu_i \omega_i^c,$$

where λ and μ_i are multipliers. The first-order necessary conditions are

$$\begin{aligned}
\frac{\partial L}{\partial a_0} &= \frac{1+3\lambda s}{4} (a_1 - a_0)^2 - 2\mu_0 sb - \frac{1}{2}\mu_1 s(a_1 - a_0 + 2b) = 0, \\
\frac{\partial L}{\partial a_i} &= \frac{1+3\lambda s}{4} \left\{ (a_{i+1} - a_i)^2 - (a_i - a_{i-1})^2 \right\} + \frac{1}{2}\mu_i s(a_i - a_{i-1} - 2b) \\
&\quad - \frac{1}{2}\mu_{i+1} s(a_{i+1} - a_i + 2b) = 0 \text{ for } i \in \{1, 2, \dots, n-2\}, \\
\frac{\partial L}{\partial a_{n-1}} &= \frac{1+3\lambda s}{4} \left\{ (1 - a_{n-1})^2 - (a_{n-1} - a_{n-2})^2 \right\} - \frac{\lambda s}{2}(1 - a_{n-1} + 2b) \\
&\quad + \frac{1}{2}(1 - z_{n-1} + 2b) \left(\sum_{n=0}^{n-2} s\mu_i \right) + \frac{1}{2}\mu_{n-1} s(1 - a_{n-2}) = 0, \\
\frac{\partial L}{\partial \omega_n^c} &= -\lambda + \sum_{n=0}^{n-1} \mu_i = 0.
\end{aligned}$$

By replacing multipliers with $\tilde{\lambda} = \lambda s$ and $\tilde{\mu}_i = s\mu_i$, we obtain the equivalent problem solved by Krishna and Morgan (2008) (Problem 1 in Appendix D). □

Following the same routine in the proof of Proposition 7 in Krishna and Morgan (2008), we can verify that the optimal cutoff $\overline{a}$ is given by

$$\overline{a} = \frac{1+2s}{1+3s} - \frac{1}{1+3s} \sqrt{4s^2 + \frac{1}{3} \left\{ 3s - 2b(1+3s)K(K-1) \right\} \left\{ 2b(1+3s)K(K+1) - 3s \right\}},$$

where

$$\frac{3s}{2(1+3s)K(K+1)} \leq b < \frac{3s}{2(1+3s)K(K-1)}.$$

References

Alemany, Joaquin J. (2002), "United States contracts with informants: An illusory promise." *U. Miami Inter-Am. L. Rev.*, 33, 251. [269]

Alonso, Ricardo and Niko Matouschek (2008), "Optimal delegation." *The Review of Economic Studies*, 75, 259–293. [270]

Ambrus, Attila, Eduardo M. Azevedo, and Yuichiro Kamada (2013), "Hierarchical cheap talk." *Theoretical Economics*, 8, 233–261. [270]

Aumann, Robert J. and Sergiu Hart (2003), "Long cheap talk." *Econometrica*, 71, 1619–1660. [270, 271]

Austen-Smith, David and Jeffrey S. Banks (2000), "Cheap talk and burned money." *Journal of Economic Theory*, 91, 1–16. [271]

Benoît, Jean-Pierre and Vijay Krishna (1985), "Finitely repeated games." *Econometrica*, 53, 905–922. [280]

Blume, Andreas, Oliver J. Board, and Kohei Kawamura (2007), "Noisy talk." *Theoretical Economics*, 2, 395–440. [270]

Crawford, Vincent P. and Joel Sobel (1982), "Strategic information transmission." Econometrica: Journal of the Econometric Society, 1431–1451. [268]

Dessein, Wouter (2002), "Authority and communication in organizations." *The Review of Economic Studies*, 69, 811–838. [270]

Golosov, Mikhail, Vasiliki Skreta, Aleh Tsyvinski, and Andrea Wilson (2014), "Dynamic strategic information transmission." Journal of Economic Theory, 304–341. [272]

Goltsman, Maria, Johannes Hörner, Gregory Pavlov, and Francesco Squintani (2009), "Mediation, arbitration and negotiation." *Journal of Economic Theory*, 144, 1397–1420. [270, 291, 293]

Holmström, Bengt Robert (1977), *On Incentives and Control in Organizations*. Ph.D. thesis, Stanford University. [270]

Hörner, Johannes and Andrzej Skrzypacz (2016), "Selling information." *Journal of Political Economy*, 124, 1515–1562. [271]

Ivanov, Maxim (2010), "Communication via a strategic mediator." *Journal of Economic Theory*, 145, 869–884. [270]

Karamychev, Vladimir and Bauke Visser (2017), "Optimal signaling with cheap talk and money burning." *International Journal of Game Theory*, 46, 813–850. [271]

Kartik, Navin (2007), "A note on cheap talk and burned money." *Journal of Economic Theory*, 136, 749–758. [271]

Kartik, Navin (2009), "Strategic communication with lying costs." *The Review of Economic Studies*, 76, 1359–1395. [271]

Kartik, Navin, Marco Ottaviani, and Francesco Squintani (2007), “Credulity, lies, and costly talk.” *Journal of Economic theory*, 134, 93–116. [271]

Kolotilin, Anton and Hongyi Li (2021), “Relational communication.” *Theoretical Economics*, 16, 1391–1430. [271]

Krishna, Vijay and John Morgan (2004), “The art of conversation: Eliciting information from experts through multi-stage communication.” *Journal of Economic theory*, 117, 147–179. [270, 271]

Krishna, Vijay and John Morgan (2008), “Contracting for information under imperfect commitment.” *The RAND Journal of Economics*, 39, 905–925. [270, 281, 282, 285, 297, 298, 299]

Margaria, Chiara and Alex Smolin (2018), “Dynamic communication with biased senders.” *Games and Economic Behavior*, 110, 330–339. [272]

Melumad, Nahum D. and Toshiyuki Shibano (1991), “Communication in settings with no transfers.” *The RAND Journal of Economics*, 22, 173–198. [270]

Sadakane, Hitoshi (2019), “Multistage information transmission with voluntary monetary transfer.” SSRN 2998440, ISER Discussion Paper No. 1006. [267]

Spence, Michael (1973), “Job market signaling.” *The Quarterly Journal of Economics*, 87, 355–374. [271]

Co-editor Simon Board handled this manuscript.

Manuscript received 31 October, 2018; final version accepted 20 March, 2022; available online 28 March, 2022.