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The limits of ex post implementation without transfers

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We study ex post implementation in collective decision problems where monetary transfers cannot be used. We find that deterministic ex post implementation is impossible if the underlying environment is neither almost an environment with private values nor almost one with common values. Thus, desirable properties of ex post implementation such as informational robustness become difficult to achieve when preference interdependence and preference heterogeneity are both present in the environment.

Keywords. Ex post implementation, interdependent values, nontransferable utility, mechanism design, collective decision-making, informational robustness.

JEL classification. D71, D82.

1. Introduction

Collective decision-making takes place everywhere, from a committee choosing which job candidates to hire, a congress deciding whether to pass a bill, to a country electing its next president. When designing a decision mechanism for such situations, an important consideration is informational robustness: The mechanism should function effectively for a wide range of information structures, i.e., what agents know and believe about each other's information. Robustness is important because decision mechanisms are often

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institutionalized for repeated use, each time tackling a new problem with a different information structure. Thus, robust, all-purpose mechanisms are best suited for institutions such as committees, legislatures, or elections. Moreover, even in a single decision problem, there is usually uncertainty about the underlying information structure. Thus, narrowly tailored mechanisms may misfire if the actual information structure turns out to be different from what was expected.

One might then ask: Are robust decision mechanisms viable? If monetary transfers are allowed, then the answer can be positive—even if one requires robustness against *all* possible information structures, which, by Bergemann and Morris (2005), amounts to the mechanism in question admitting an *ex post equilibrium*. More specifically, it is known that in interdependent value environments, nontrivial, even efficient, social choice functions can be *ex post incentive compatible (EPIC)*, i.e., implementable in an ex post equilibrium of some mechanism, if private information is one-dimensional.[1] There are limits to ex post implementation with transfers, though, as Jehiel, Meyer-ter-Vehn, Moldovanu, and Zame (2006) show: If private information is continuous and multidimensional, then deterministic EPIC social choice functions must be constant in generic environments.

In many collective decision problems, including the examples mentioned above, monetary transfers cannot be used. One would expect ex post implementation to be further constrained by the absence of transfers, but to what extent? This is the central question we address in this paper. Our main result is as follows: For collective decision problems with a continuous state space, if transfers are not allowed, then deterministic EPIC social choice functions must be constant as long as there is a "small amount" of preference interdependence and preference heterogeneity in the environment, regardless of whether types are one- or multidimensional. If there are only two alternatives, then the conclusion even extends to stochastic social choice functions. Thus, we sharpen the findings of Jehiel et al. (2006) for settings without transfers—we will compare the two papers in more detail after taking a closer look at our result first.

Let us elaborate on the setting. A group of n agents must collectively choose one of finitely many alternatives. Each agent i's private information—her type—is a number or vector θ_i, whereas the collection of everyone's types, $\theta = (\theta_1, \dots, \theta_n)$, constitutes the payoff-relevant *state*. An agent's preferences over the alternatives depend on the state, which includes others' as well as her own information.

The sufficient condition for our impossibility result can be more precisely stated as follows: If in state θ some agents are indifferent between two alternatives (a, b), then among the indifferent agents there exists a certain agent i whose indifference between (a, b) is broken by a slight change in the information of another agent j and, moreover, the preferences of i and j regarding (a, b) do not agree entirely in any arbitrary neighborhood around θ. Thus, locally around θ, there is preference interdependence because

[1]For public goods provision, see Section 5 in Chung and Ely (2003). In auction settings, efficient social choice functions are ex post implementable when preferences satisfy appropriate single-crossing conditions; see Crémer and McLean (1985), Maskin (1992), Dasgupta and Maskin (2000), Jehiel and Moldovanu (2001), Bergemann and Välimäki (2002), Perry and Reny (2002).

the preference of i depends nontrivially on j's information, and there is preference heterogeneity because the preferences of i and j differ.

There are three reasons why we suggest that this sufficient condition requires only a "small amount" of preference interdependence and heterogeneity. First, the condition only imposes restrictions on those "indifference" states where agents are actually indifferent between alternatives. Second, the "magnitude" of preference interdependence and heterogeneity, locally at a state, need not be large. Indeed, the condition is satisfied at θ even if i's preference is barely sensitive to j's information, and their preferences are almost but not entirely identical. Third, for an indifference state and a corresponding pair of alternatives, we merely need two agents, i and j, whose preferences are interdependent and heterogeneous. In other words, two agents are enough to disrupt ex post implementation.

The range of environments where our impossibility theorem applies is not only broad in theory, but also relevant in practice: Decision-relevant information is often dispersed across individuals with diverse intentions and tastes, which formally translates into interdependence and heterogeneity of preferences. In terms of how mechanisms such as voting or deliberation procedures operate in the real world, our result therefore predicts that equilibrium outcomes are likely sensitive to what agents believe about each others' information.

Although, as we have argued above, the sufficient condition for our impossibility result is satisfied in a broad range of environments; there are two prominent types of environments in which it is violated: environments with *private values*, where agents' preferences never depend on the information of others, and environments with *common values*, where agents share the same preferences in every state. It is therefore not surprising that these environments admit nonconstant EPIC social choice functions. In the case of private values, EPIC is known to be equivalent to strategy-proofness. There, dictatorships are strategy-proof, and further nonconstant social choice functions become strategy-proof when the famous Gibbard–Satterthwaite theorem (Gibbard (1973), Satterthwaite (1975)) is circumvented through restrictions on the underlying preference domain.[2] In the case of common values, the social choice function that chooses the common first-best alternative in each state is clearly EPIC. Yet, as we have seen, the possibility of ex post implementation quickly fades as we move away from these two extremes, when both preference interdependence and heterogeneity come into play. In particular, not even dictatorship is EPIC when values are interdependent,[3] and various exceptions to Gibbard–Satterthwaite are killed by even a small amount of preference interdependence.

We already mentioned that our result strengthens the finding of Jehiel et al. (2006) for settings without transfers: while Jehiel et al. (2006) show that deterministic ex post implementation with multidimensional types is generically impossible, even when transfers are available, we show that shutting down transfers further limits the scope of ex

[2] See, e.g., Moulin (1980) and Saporiti (2009).

[3] The reason is that a dictator who decides based on her own information would revise her choice in some states after learning about other agents' information. Also see Jehiel et al. (2006) for disambiguation of the term *dictatorship* in interdependent value environments.

post implementation, especially in environments with one-dimensional types and in important "nongeneric" environments that survive Jehiel et al. (2006), such as those with additively separable preferences.[4]

Both results arise from roughly the same conceptual barrier to ex post implementation, namely that agents who can change the social choice around a given state must have aligned preferences. However, compared to Jehiel et al. (2006), the absence of transfers allows us to expose this barrier more explicitly and translate it into an easily interpretable condition on the underlying preferences. The nature of our result as an impossibility result is then established by the argument that this condition is satisfied in many economically relevant and practically prevalent environments, rather than via a mathematical genericity argument as in Jehiel et al. (2006).

Another difference between the two papers is that Jehiel et al. (2006) only consider a two-agent, two-alternative model. This simple model is sufficient for their goal of establishing generic impossibility, and in principle, the two-by-two setting is also enough to illustrate the key insights of our paper (see Section 2). However, our general analysis with many agents and alternatives covers economically relevant (but in the sense of Jehiel et al. (2006), nongeneric) cases where it is a priori unclear whether some form of ex post implementation becomes possible, e.g., when subsets of agents have aligned preferences over subsets of alternatives.

There are only a few other papers on ex post implementation without transfers. Che, Kim, and Kojima (2015) and Fujinaka and Miyakawa (2020) as well as Pourpouneh, Ramezanian, and Sen (2020) study specific settings, namely object assignment and matching problems, respectively. In these settings, nontrivial ex post implementation is typically possible: Our preference interdependence condition entails allocative externalities, which are typically assumed away in the assignment and matching literature; see Section 5 for a more detailed discussion. The impossibility of ex post implementation can be overcome in the same way when transfers are available: genericity in the sense of Jehiel et al. (2006) also entails allocative externalities. In fact, Bikhchandani (2006) shows by construction that nontrivial ex post implementation is possible in environments with private objects and multidimensional types.

For more general settings, Barberà, Berga, and Moreno (2019, 2022) and Feng and Wu (2020, Section 4.3) also discuss necessary and sufficient conditions for the impossibility of ex post implementation. Unlike us, these papers impose no topological structure on the state space, making their conditions more general yet also more abstract and harder to interpret and verify than our conditions. Indeed, it is precisely because we are working with a continuous state space that we are able to obtain a much sharper result about ex post implementability.

This paper is organized as follows. Section 2 illustrates the main insight in a simple example. Section 3 sets up the general model. Section 4 presents the main result. Section 5 discusses ex post implementation in situations where our result is silent: (1) allowing transfers; (2) matching and assignment problems; (3) discrete state spaces; (4) stochastic social choice with three or more alternatives. All proofs are in the Appendix.

[4]See Section 5.4.2 in Jehiel et al. (2006) for the formal definition.

2. Example

Two agents, 1 and 2, need to make a collective choice from two alternatives, S(afe) and R(isky), e.g., whether or not to pass a law, implement a project, or convict a defendant. The value of S is always 0 to both agents, whereas the value of R depends on an unknown state $\theta = (\theta_1, \theta_2)$, which can take any value from $\Theta = [-1, 1]^2$. Specifically, the value of R to agent $i = 1, 2$ is given by

$$v_i^R(\theta) = \theta_i + \beta\theta_{-i}$$

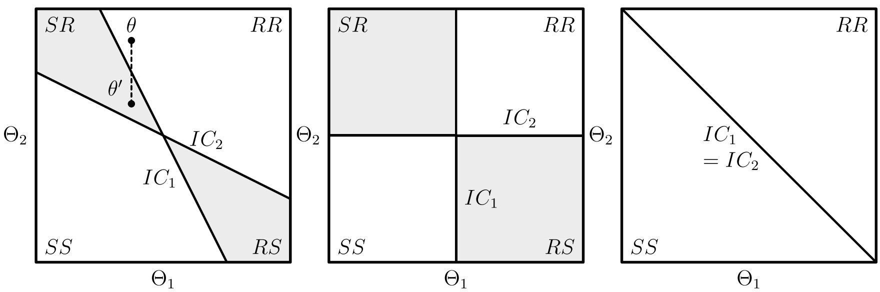
where $\beta \in [0, 1]$.

Agent i can observe θ_i but not θ_{-i}. Thus, each agent only has partial information about the true payoff-relevant state, and β is a parameter that captures the degree to which agent i's valuation depends on the information of the other agent $-i$. Note that when $\beta = 0$, this is a *private value* environment where an agent's preference depends only on her own information, whereas when $\beta = 1$, this is a *common value* environment where the agents preferences are identical. We will return to these special cases in a moment.

We first focus on an intermediate case $\beta = 1/2$. Since each agent's valuation for R is twice as sensitive to her own information as to the other agent's information, the two agents do not always agree on which alternative is better. Indeed, in Figure 1a, which graphically represents the setting, the two agents' *indifference curves* $\mathrm{IC}_i = \{\theta | v_i^R(\theta) = 0\}$, i.e., the respective sets of states where 1 and 2 are indifferent between S and R, partition the state space into four regions, $\{RR, RS, SR, SS\}$, where region XY has the interpretation that within it, agent 1 strictly prefers alternative X and agent 2 strictly prefers alternative Y.

Which deterministic social choice functions $\phi : [-1, 1]^2 \to \{S, R\}$ are EPIC when $\beta = 1/2$? ϕ is EPIC if and only if it is optimal for each agent i to truthfully report her type θ_i to the direct mechanism induced by ϕ *in every state*, given that the other agent also reports truthfully. Obviously, any constant ϕ is EPIC. It turns out that the converse is also true: Any EPIC ϕ must be constant.

Let us briefly sketch the gist of the formal argument. Note that if an agent has the same preference across two states that differ only in her own information, then an EPIC



(a) Interdependent Values (b) Private Values (c) Common Values

Figure 1. An illustration of the example.

social choice function must choose the same alternative in both states. As an example, consider the two states θ and θ' in Figure 1a. These states are aligned vertically (thus differ only in agent 2's information) and are respectively located in RR and SR (thus agent 2 strictly prefers R in both states). If some ϕ chose different alternatives in θ and θ', then agent 2 would be decisive in either state: she could induce the choice of one alternative by reporting her information truthfully, or the choice of the other alternative by misreporting her information to be dimension 2 of the other state. But since she strictly prefers R in both states, she would induce the choice of R in one of the states by misreporting her private information, contradicting EPIC.

Now, any EPIC ϕ must be constant within each of the four regions where both agents' preferences are strict and constant because we could otherwise find two states in the same region that differ only in one agent's information but where different alternatives are chosen, contradicting our previous observation about EPIC.

In addition, ϕ must choose the same alternative across any two adjacent regions because we can always find states such as θ and θ' that link two regions through an agent whose preference is the same. It follows that any EPIC ϕ must choose the same alternative across all four regions.[5]

It is worth noting that the linking argument across regions relies on the existence of the two states (θ, θ') that (1) differ only in one dimension $j \in \{1, 2\}$, and in which (2) agent $i \neq j$ has different ordinal preferences but (3) agent j has the same ordinal preference. Conditions (1) and (2) jointly entail *preference interdependence* between the agents: The change of agent j's information leads to a change in agent i's ordinal preference. Conditions (2) and (3) jointly entail *preference heterogeneity*: The agents' ordinal preferences do not always agree, so that a change in the state may cause a change in one agent's preference but not in the other's. In short, that ϕ is constant relies on the presence of preference interdependence and heterogeneity.

Not surprisingly, there exist nonconstant EPIC ϕ if preference interdependence is absent as in the private value case $\beta = 0$ (Figure 1b) or if preference heterogeneity is absent as in the common value case $\beta = 1$ (Figure 1c) because we cannot find the desired linking states (θ, θ') in either case. For example, the function ϕ that chooses R only in RR is EPIC in both cases.

In contrast, the argument goes through for any $\beta \in (0, 1)$, i.e., when there is at least some preference interdependence and heterogeneity, regardless of how close β is to one of the two exceptional cases. In this sense, if we think of the environments with interdependent values as a spectrum parametrized by $\beta \in [0, 1]$ with private values at one end and common values at the other, then even a slight departure from the two extremes leads to an impossibility of ex post implementation. This insight, as formalized and generalized in Theorem 1, is the main contribution of the paper.

3. Model

A group of agents $N = \{1, \dots, n\}$ must collectively choose an alternative from a finite set A without using monetary transfers. The valuation of agent $i \in N$ for alternative

[5] In this example, it is easy to show that ϕ must then also choose the same alternative on the indifference curves IC_1 and IC_2. In general, one can only show this for the interior of the state space; see the Appendix.

$a \in A$ depends on an underlying *state* $\theta \in \Theta$, where Θ is the set of all possible states. We represent i's valuation for alternative a by a *valuation function* $v_i^a : \Theta \to \mathbb{R}$. In addition, we let $v_i^{ab}(\theta) := v_i^a(\theta) - v_i^b(\theta)$ denote i's *relative valuation function* for a versus another alternative b. Thus, i weakly prefers a over b in state θ if and only if $v_i^{ab}(\theta)$ is nonnegative.

Preference interdependence among the agents is typically modeled by assuming that each agent is only partially informed about the payoff-relevant state θ. Specifically, θ consists of n components, $\theta = (\theta_1, \dots, \theta_n)$, and each agent i only observes θ_i—her *type*. The state space Θ is therefore $\prod_{i \in N} \Theta_i$. We assume $\Theta_i = [-1, 1]^{d_i}$ where $d_i \in \mathbb{N}$ is the dimension of agent i's type, and thus allow for multidimensional types.[6]

Valuation functions are continuously differentiable. Given a relative valuation function v_i^{ab}, let ∇v_i^{ab} denote its gradient, and let $\nabla_{\theta_j} v_i^{ab}$ denote the d_j-dimensional vector of components of ∇v_i^{ab} with respect to the type of agent j. We follow Jehiel et al. (2006) in assuming that an agent's indifference between two alternatives is broken by a slight change in her own information. More precisely,

$$\forall i \in N, \forall \theta \in \Theta, \forall a, b \in A : a \neq b, \quad \left(v_i^{ab}(\theta) = 0 \implies \nabla_{\theta_i} v_i^{ab}(\theta) \neq \mathbf{0}\right).^{7} \tag{RESP}$$

As motivated in the Introduction, we are interested in the ex post implementability of social choice functions. By the revelation principle, we can focus on those that are truthfully ex post implementable in direct mechanisms, or in other words, ex post incentive compatible. Specifically, a (deterministic) *social choice function* $\phi : \Theta \to A$ is *ex post incentive compatible (EPIC)* if truth-telling is an ex post equilibrium of the direct mechanism induced by ϕ, i.e.,

$$\forall i \in N, \forall \theta \in \Theta, \forall \tilde{\theta}_i \in \Theta_i, \quad v_i^{\phi(\theta_i, \theta_{-i})}(\theta) \geq v_i^{\phi(\tilde{\theta}_i, \theta_{-i})}(\theta). \tag{EPIC}$$

Following Jehiel et al. (2006), we say that social choice function ϕ is *trivial* if it is constant on the interior of Θ.

Although we have set up the model in terms of cardinal valuation functions, our findings can be easily transferred to a model where preferences are ordinal. After all, only ordinal preferences matter for ex post incentives when there are no transfers and mechanisms are deterministic. In Section 5, we discuss this alternative specification in more detail.

4. Impossibility of ex post implementation

Let us first formally present the main result and explain it in more detail right after. For a pair of distinct alternatives (a, b), let

$$I^{ab}(\theta) = \left\{ i \in N \middle| v_i^{ab}(\theta) = 0 \right\}$$

[6]Our result still obtains if each Θ_i is a subset of a Euclidean space with connected interior. Moreover, Θ need not be a product state space, provided its interior is connected. Our proof explicitly assumes only these properties of the state space.

[7]This assumption is not necessary for the gist of our result but simplifies statement and proof: Without (RESP), the result's conclusion must be slightly weakened, making the result harder to communicate. See Feng and Wu (2020) for an earlier version of the result without (RESP).

denote the set of agents who are indifferent between this pair in state θ. If $I^{ab}(\theta)$ is nonempty, we say that (a, b) is an *indifference pair* of θ. Moreover, we say that θ is an *indifference state* if it has at least one indifference pair.

Theorem 1. *Suppose for any indifference state θ and any of its indifference pairs (a, b), there exists an agent $i \in I^{ab}(\theta)$ and another agent $j \in N$ such that:*

(i) (local interdependence) $\nabla_{\theta_j} v_i^{ab}(\theta) \neq \mathbf{0}$;

(ii) (local heterogeneity) $j \notin I^{ab}(\theta)$ or $\nabla v_i^{ab}(\theta) \neq \lambda \nabla v_j^{ab}(\theta)$ for any $\lambda \geq 0$.

Then all EPIC social choice functions are trivial.

To better understand the result, let us parse the statement. Note first that the sufficient condition only constrains indifference states regarding their indifference pairs. That is, only for the indifference states θ and their indifference pairs (a, b) do we need to find two agents i and j whose preferences regarding (a, b) are interdependent but nonetheless heterogeneous locally around θ? More precisely, local interdependence means that i, who is indifferent between (a, b) in θ, is no longer indifferent following some small change in j's type, i.e., the *ordinal preference* of i depends on j's information around θ. Local interdependence is satisfied in Figure 1a but not in Figure 1b because it requires agent 1's indifference curve IC_1 to not be entirely vertical and agent 2's indifference curve IC_2 to not be entirely horizontal. Local heterogeneity means that i and j disagree on whether a or b is better in or near state θ. Specifically, if $j \notin I^{ab}(\theta)$, then heterogeneity in θ is immediate: i is indifferent, but j is not. On the other hand, if j is also indifferent in θ, then the condition that $\nabla v_i^{ab}(\theta) \neq \lambda \nabla v_j^{ab}(\theta)$ for any $\lambda \geq 0$, i.e., that the two gradients are not codirectional at θ, implies that there is an arbitrarily close state in which i and j rank (a, b) differently.[8] Local heterogeneity is satisfied in Figure 1a but not in Figure 1c because it requires that IC_1 and IC_2 cross each other when they intersect,[9] as only then would the gradients, which are respectively normal to the indifference curves, be misaligned at the intersection.

We view Theorem 1 as a strong negative result—an "impossibility" theorem— for the following reasons. First, its sufficient condition only puts restrictions on indifference states, which typically compose a very small subset of all states.[10] Second, local interdependence only rules out the knife-edge case that $\nabla_{\theta_j} v_i^{ab}(\theta)$ is exactly equal to $\mathbf{0}$, and likewise, in case $j \in I^{ab}(\theta)$, local heterogeneity only rules out the knife-edge case that $\nabla v_i^{ab}(\theta)$ and $\nabla v_j^{ab}(\theta)$ are exactly codirectional. In other words, the sufficient condition is satisfied even if, locally around θ, there is only a minimal amount of preference interdependence and heterogeneity. Third, for there to be local interdependence and heterogeneity, we only need two agents whose preferences jointly satisfy the respective

[8] Another way to think of this condition is that there are two pieces of information in state θ between which i and j have different marginal rates of substitution.

[9] For local heterogeneity to hold at the intersection, the indifference curves of i and j may be tangent only when their preferences regarding (a, b) are diametrically opposed in a neighborhood of the intersection, which is not possible in the example for any $\beta \in [0, 1]$.

[10] Clearly, for any continuous distribution on Θ, the set of indifference states has measure zero.

requirements, and these agents need not be the same across indifference states or even pairs. In particular, our result still holds if subsets of agents, say, parties in a parliament, have identical preferences as long as there is preference interdependence and heterogeneity between parties.

In fact, the result can be further strengthened. First, what we prove in the Appendix is actually stronger (Theorem 2): Nontrivial social choice functions do not exist even under the weaker notion of *local ex post incentive compatibility*, which requires that no agent i has an incentive to slightly misrepresent her true type θ_i as some $\tilde{\theta}_i$ that is close to θ_i. Moreover, the presence of local interdependence and heterogeneity in *every* indifference state is an overkill for deriving the impossibility result. All that is needed is a specific discrete set of indifference states satisfying the conditions; see Remark 1 in the Appendix.

Why is the existence of a minimal amount of preference interdependence and heterogeneity in some indifference states already enough to disrupt even local ex post implementation? With transfers absent and mechanisms deterministic, incentives are determined by preference rankings only. Thus, it is local incentives around indifference states that matter most to implementation because indifference states are precisely those where preference rankings change. Moreover, since minimal movements around an indifference state are enough to change an agent's preference ranking, EPIC admits no "margin of error" there when it comes to the magnitude of preference interdependence or heterogeneity. The implied discontinuity in implementability between pure private/common value environments and interdependent value environments reflects how chokingly stringent EPIC is as a constraint on mechanism design.

5. Discussion

5.1 *An ordinal framework*

As we have mentioned earlier, when transfers are absent and mechanisms are deterministic, ex post incentives are determined by preference rankings only, whereas cardinal valuations per se are irrelevant. Although the key conditions for our analysis—those about local interdependence and local heterogeneity—are formulated in terms of cardinal valuations, they are essentially about how ordinal preferences change from an indifference state to nearby states. In principle, these conditions can be alternatively defined in terms of ordinal preferences, but one can imagine that such definitions would be more tedious to formulate and use for our analysis. Since our result hinges on preferences in and around indifference states, we have imposed mild regularity conditions on valuation functions to ensure that the set of indifference states is well behaved. In the ordinal model, if we were to impose analogous conditions on the boundaries that separate the regions where a given agent's preferences are constant, then our analysis would go through analogously with the appropriately modified notions of local interdependence and heterogeneity.[11]

[11]With continuous preferences, these boundaries are nothing but the agent's indifference curves. Whether or not the agent is actually indifferent in states where her preferences change is not relevant for the result.

5.2 *Transfers*

In the Introduction, we mentioned that transfers facilitate ex post implementation. If transfers are allowed and an agent only cares about her own transfer, as is typically assumed, then she is indifferent between any two outcomes where the chosen nonmonetary alternative and her own transfer are the same, despite differences in the other agents' transfers. These indifferences persist across states, and thus violate both local interdependence and (RESP), rendering our result silent. Transfers can be used to overcome preference interdependence or heterogeneity—the two roadblocks to ex post implementation suggested by our result—by either making values effectively private or by aligning the agents' interests.[12] In the following, we illustrate these two possibilities in the context of our leading example.

Example (continued from Section 2). Suppose monetary transfers are now allowed and agents have quasilinear utilities: $u_i(\theta) = v_i^X(\theta) + t_i(\theta)$, where X is the chosen alternative and t_i is the transfer agent i receives.

First, consider the transfer scheme $(t_i)_{i=1,2}$ where $t_i(\theta) = -\beta\theta_{-i}$ if R is chosen and $t_i = 0$ if S is chosen. Agent i's "post-transfer" utility is then θ_i if R is chosen and 0 if S is chosen. Thus, transfers eliminate preference interdependence and transform the environment into one of private values as in Figure 1b. Consequently, mechanisms such as dictatorship or unanimity voting are EPIC.

Second, consider the transfer scheme $(t'_i)_{i=1,2}$ where $t'_i(\theta) = (1-\beta)\theta_{-i}$ if R is chosen and $t_i = 0$ if S is chosen. Both agents have the same "post-transfer" utility, namely $\theta_1 + \theta_2$ if R is chosen and 0 if S is chosen. Thus, transfers eliminate preference heterogeneity and transform the environment into one of common values as in Figure 1c. Consequently, the mechanism that chooses R if and only if $\theta_1 + \theta_2 > 0$ is EPIC.

5.3 *Assignment and matching problems*

A common assumption in matching is that each agent only cares about her own assigned object or match. Thus, similar to the case of transfers, local interdependence and (RESP) generally fail to hold in such problems.[13] It is therefore not surprising that nontrivial EPIC social choice functions exist even when preferences are interdependent.[14] However, as Che, Kim, and Kojima (2015) show, such EPIC social choice functions cannot be efficient, at least in the housing allocation problem where each agent is assigned exactly one object. Moreover, our negative result can still apply to assignment or matching problems with allocative externalities, e.g., when students not only care about which dorm room they get but also which rooms their friends get.

[12] See Section 5 in Chung and Ely (2003) for further discussion on how transfers can be used to align individual interests in collective choice problems.

[13] The housing allocation problem with two objects and two agents is an exception since the assignment of one object to one agent implies that the remaining object must be assigned to the other agent. See also the illustrative example in Che, Kim, and Kojima (2015).

[14] This observation echoes how Jehiel et al. (2006) relies on allocative externalities; also see Bikhchandani (2006).

5.4 *Discrete state spaces*

We have assumed that the state space is a connected subset of a Euclidean space. If instead the state space is discrete, then counterexamples to our result are easy to find; for instance, see Feng and Wu (2020). One way to understand the discrepancy between discrete and continuous state spaces is to think of a discrete state space as a low-resolution discretization of a continuous space. For example, suppose each agent's underlying type can be any number between -1 and 1, yet each agent is only aware of whether her type is above or below 0, making her effective type space binary. Since the agents' indifference curves are then being squeezed into a discrete grid, they tend to become more aligned, and this alignment gives leeway to nontrivial ex post implementation.

5.5 *Stochastic social choice functions*

What if we allow for randomization so that the collective choice can be a *lottery* over alternatives? It turns out that Theorem 1 still holds as long as there are only two alternatives. The reason is simple: An agent is indifferent between lotteries if and only if she is indifferent between the two underlying alternatives, and she otherwise prefers lotteries in which her preferred alternative is chosen with a higher rather than lower probability. Thus, our arguments immediately extend to stochastic implementation with two alternatives. However, if there are three or more alternatives, then an agent can get the same expected utility from different lotteries despite having strict preferences over the underlying alternatives. In the following example, these indifferences can indeed be used to construct a nontrivial stochastic EPIC social choice function.

Example (continued from Section 2). Agents 1 and 2 now decide between three alternatives, R, S, and P. For $i = 1, 2$, still assume $\Theta_i = [-1, 1]$, $v_i^R = \theta_i + \theta_{-i}/2$, and $v_i^S = 0$. Additionally, assume $v_i^P = -1$. Theorem 1 applies here, so any deterministic EPIC social choice function must be trivial. However, consider the stochastic social choice function $\phi = (\phi^R, \phi^P, \phi^S)$ given by

$$\phi^R(\theta) = \frac{4 + 2\theta_1 + 2\theta_2}{11}, \qquad \phi^P(\theta) = \frac{\theta_1^2 + \theta_1\theta_2 + \theta_2^2}{11}, \qquad \phi^S(\theta) = 1 - \phi^R(\theta) - \phi^P(\theta),$$

where $\phi^X(\theta)$ denotes the probability that alternative X will be chosen in state θ. It is readily verified that ϕ is EPIC.

Appendix: Proof of Theorem 1

Endow Θ with the norm topology. Let $B_\varepsilon(\theta)$ denote the open ball with radius $\varepsilon > 0$ centered at θ. A social choice function ϕ is said to be *locally EPIC* if there exists some $\varepsilon > 0$ such that for any $\theta \in \Theta$, ϕ restricted to $B_\varepsilon(\theta)$ is EPIC, i.e.,

$$\forall i \in N, \forall \theta \in \Theta, \forall \tilde{\theta}_i \in \Theta_i : \quad (\tilde{\theta}_i, \theta_{-i}) \in B_\varepsilon(\theta) \implies v_i^{\phi(\theta_i, \theta_{-i})}(\theta) \geq v_i^{\phi(\tilde{\theta}_i, \theta_{-i})}(\theta). \qquad \text{(LEPIC)}$$

Let $\overline{\Theta} := \{\theta \in \operatorname{int}\Theta | \forall i \in N, \forall a, b \in A : a \neq b, v_i^{ab}(\theta) \neq 0\}$ denote the set of interior states where all agents have strict preferences, and let $\mathcal{C}$ denote the set of all connected components of $\overline{\Theta}$. $\overline{\Theta}$ is open because valuation functions are continuous. Similarly, each connected component $C \in \mathcal{C}$ is open. Note that the ordinal preferences of all agents are strict and constant on each $C \in \mathcal{C}$.

Lemma 1. *If ϕ is locally EPIC, then ϕ is constant on each $C \in \mathcal{C}$.*

Proof. Suppose ϕ satisfies (LEPIC) for some $\varepsilon > 0$. Pick any $C \in \mathcal{C}$. Suppose for the sake of contradiction that ϕ is not constant on C, then there exists some $\theta \in C$ and $\tilde{\varepsilon} \in (0, \varepsilon)$ such that $B_{\tilde{\varepsilon}}(\theta) \subset C$ and $\phi(\theta) \neq \phi(\theta')$ for some $\theta' \in B_{\tilde{\varepsilon}}(\theta)$. Clearly, we can find a sequence of states $(\theta^0, \ldots, \theta^n)$ in $B_{\tilde{\varepsilon}}(\theta)$ where $\theta^0 = \theta$, $\theta^n = \theta'$, and for every $k = 0, \ldots, n-1$, θ^k and θ^{k+1} differ at most in the $k+1$th entry. Thus, $\phi(\theta) \neq \phi(\theta')$ implies that $\phi(\theta^k) \neq \phi(\theta^{k+1})$ for some k. By construction, θ^k and θ^{k+1} differ only in the type of agent $k+1$ who has the same strict ordinal preferences in both states. Therefore, she either could profit from misreporting her type as θ^k_{k+1} in state θ^{k+1} or from misreporting her type as θ^{k+1}_{k+1} in state θ^k, contradicting (LEPIC). □

Given Lemma 1, it causes no confusion to write $\phi(C)$ for the choice by ϕ on $C \in \mathcal{C}$.

Distinct $C, C' \in \mathcal{C}$ are said to be *adjacent at* $\theta \in \operatorname{int}\Theta$ if (1) $\theta \in [\operatorname{cl} C \cap \operatorname{cl} C']$ and, moreover, (2) $B_\varepsilon(\theta) \subset [\operatorname{cl} C \cup \operatorname{cl} C']$ for some $\varepsilon > 0$. In addition, we consider every $C \in \mathcal{C}$ as being adjacent to itself (at every $\theta \in \operatorname{cl} C$).

A collection $\mathbf{X}$ of vectors are said to be *collinear* if for any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $\mathbf{x} = \lambda\mathbf{y}$ for some $\lambda \in \mathbb{R}$, i.e., these vectors lie on a common line passing through the origin. If, in addition, for any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $\mathbf{x} = \lambda\mathbf{y}$ for some $\lambda \geq 0$, i.e., these vectors lie on a common ray emanating from the origin, then they are said to be *codirectional.*

Lemma 2. *Suppose ϕ is locally EPIC. If $C, C' \in \mathcal{C}$ are adjacent at $\theta \in \operatorname{int}\Theta$, and $\phi(C) := a \neq b =: \phi(C')$, then:*

(i) $\nabla_{\theta_j} v_i^{ab}(\theta) = \mathbf{0}$ *for any* $i \in I^{ab}(\theta)$ *and* $j \in N \setminus I^{ab}(\theta)$, *and*

(ii) $(\nabla v_i^{ab}(\theta))_{i \in I^{ab}(\theta)}$ *are codirectional.*

Proof. The lemma's premises imply that we can find $\varepsilon > 0$ such that (1) (LEPIC) holds for $B_\varepsilon(\theta)$, (2) $B_\varepsilon(\theta) \subset [\operatorname{cl} C \cup \operatorname{cl} C']$, and (3) for any agent i and any distinct pair of alternatives (x, y), if i strictly prefers x to y in θ, then she strictly prefers x to y in every state in $B_\varepsilon(\theta)$.

Arbitrarily pick alternatives $x, y, w, z \in A$ where $x \neq y$ and $w \neq z$ and agents $i \in I^{xy}(\theta)$ and $j \in I^{wz}(\theta)$. Claim that $\nabla v_i^{xy}(\theta)$ and $\nabla v_j^{wz}(\theta)$ are collinear. Indeed, if not, then we can find $\theta', \theta'' \in B_\varepsilon(\theta)$ such that $v_i^{xy}(\theta') = 0$ but $v_j^{wz}(\theta') \neq 0$, and $v_i^{xy}(\theta'') \neq 0$ but $v_j^{wz}(\theta'') = 0$.

By (RESP), we can find two states arbitrarily close to θ' (hence within $B_\varepsilon(\theta)$) in which j has the same strict preference regarding (w, z) but i has different strict preferences regarding (x, y). Similarly, we can find two states arbitrarily close to θ'' (hence also within $B_\varepsilon(\theta)$) in which i has the same strict preference regarding (x, y) but j has different strict preferences regarding (w, z). Thus, $B_\varepsilon(\theta)$ must intersect at least three distinct connected components of $\overline{\Theta}$ as it contains at least three profiles of strict preferences of the agents. This contradicts that $B_\varepsilon(\theta)$ only intersects two such components, namely C and C'.

Toward proving part (1), suppose for the sake of contradiction that there exists $i \in I^{ab}(\theta)$ and $j \in N \setminus I^{ab}(\theta)$ such that $\nabla_{\theta_j} v_i^{ab} \neq \mathbf{0}$. Thus, we can find $\rho > 0$ sufficiently small such that $\theta' := (\theta_j + \rho \nabla_{\theta_j} v_i^{ab}(\theta), \theta_{-j}) \in B_\varepsilon(\theta)$, $\theta'' := (\theta_j - \rho \nabla_{\theta_j} v_i^{ab}(\theta), \theta_{-j}) \in B_\varepsilon(\theta)$, and

$$v_i^{ab}(\theta') v_i^{ab}(\theta'') < 0, \qquad v_j^{ab}(\theta') v_j^{ab}(\theta'') > 0.$$

In other words, agent i has different strict preferences regarding (a, b) in θ' and θ'', whereas agent j has the same strict preference. By the collinearity observation above, we can further conclude that, for ρ small enough, any agent k who is indifferent between any pair (x, y) in θ has different strict preferences regarding this pair in θ' and θ''. Together with $\theta', \theta'' \in B_\varepsilon(\theta)$, we thus establish $\theta', \theta'' \in \overline{\Theta}$, i.e., all agents have strict preferences in both states. Moreover, θ' and θ'' must be in distinct connected components of $\overline{\Theta}$—one in C, the other in C'—because i's preferences differ across the two states. Since agent j has the same strict preference regarding (a, b) in θ' and θ'' and since the two states differ only in j's type, (LEPIC) implies $\phi(\theta') = \phi(\theta'')$, a contradiction.

Now we show part (2). From the collinearity observation, we conclude that $\nabla v_i^{ab}(\theta)$ and $\nabla v_j^{ab}(\theta)$ are collinear for any $i, j \in I^{ab}(\theta)$. If, for the sake of contradiction, for some $i, j \in I^{ab}(\theta)$ the two gradients are not also codirectional, then they must be diametrically opposed. By (RESP), we can find $\rho > 0$ sufficiently close to 0 such that the following three statements are true. First, i strictly prefers a to b and j strictly prefers b to a in both of the following two states:

$$\hat{\theta} := (\theta_i + \rho \nabla_{\theta_i} v_i^{ab}(\theta), \theta_{-i}) \quad \text{and} \quad \tilde{\theta} := (\theta_j - \rho \nabla_{\theta_j} v_j^{ab}(\theta), \theta_{-j}).$$

Second, i strictly prefers b to a and j strictly prefers a to b in both of the following two states:

$$\hat{\theta}' := (\theta_i - \rho \nabla_{\theta_i} v_i^{ab}(\theta), \theta_{-i}) \quad \text{and} \quad \tilde{\theta}' := (\theta_j + \rho \nabla_{\theta_j} v_j^{ab}(\theta), \theta_{-j}).$$

Third, the above two pairs of states are in $B_\varepsilon(\theta)$. Following the argument in the previous paragraph, the four states are also in $\overline{\Theta}$, and thus either in C or in C'. In addition, one pair must fall in C and the other pair must fall in C' because the preferences of agent i (equivalently, j) regarding (a, b) are the same within each pair but differ across pairs. Therefore, $\phi(\hat{\theta}) = \phi(\tilde{\theta})$ but $\phi(\hat{\theta}) \neq \phi(\hat{\theta}')$. (LEPIC) implies that $\phi(\hat{\theta}) = a$ for otherwise i would misreport her type as $\hat{\theta}'_i$ in state $\hat{\theta}$. Similarly, $\phi(\tilde{\theta}) = b$ for j not to misreport, but then $\phi(\hat{\theta}) \neq \phi(\tilde{\theta})$, a contradiction. □

Lemma 3. *For any $C, C' \in \mathcal{C}$, there exists a finite sequence of connected components $C^0, \dots, C^K \in \mathcal{C}$ and a finite sequence of indifference states $\theta^1, \dots, \theta^K \in \operatorname{int}\Theta$ such that $C^0 = C$, $C^K = C'$, and C^k and C^{k+1} are adjacent at θ^{k+1} for every $k = 0, \dots, K-1$.*

Proof. Pick any $\overline{C} \in \mathcal{C}$. Let $\mathcal{C}'$ denote the set of all $C' \in \mathcal{C}$ that can be linked to $\overline{C}$ through a finite sequence of connected components with the same properties as in the statement of the lemma. Clearly, the lemma is established if $\mathcal{C} \setminus \mathcal{C}'$ is empty; thus, for the sake of contradiction, suppose $\mathcal{C} \setminus \mathcal{C}'$ is nonempty.

Define

$$S := \operatorname{int}\Theta \cap \Big[\operatorname{cl} \bigcup_{\tilde{C}\in\mathcal{C}\setminus\mathcal{C}'} \tilde{C}\Big] \cap \Big[\operatorname{cl} \bigcup_{\tilde{C}\in\mathcal{C}'} \tilde{C}\Big].$$

Geometrically speaking, S is the frontier separating the components in $\mathcal{C}'$ from those in $\mathcal{C} \setminus \mathcal{C}'$. Note that $S \subset [\operatorname{int}\Theta \setminus \overline{\Theta}]$. Moreover, S is nonempty, for otherwise, the two (relatively) closed sets $\operatorname{int}\Theta \cap [\operatorname{cl}\bigcup_{\tilde{C}\in\mathcal{C}\setminus\mathcal{C}'} \tilde{C}]$ and $\operatorname{int}\Theta \cap [\operatorname{cl}\bigcup_{\tilde{C}\in\mathcal{C}'} \tilde{C}]$ would partition $\operatorname{int}\Theta$, which contradicts that $\operatorname{int}\Theta$ is connected.

For any agent $i \in N$ and distinct alternatives (a, b), let $\mathrm{IC}_i^{ab} := \{\tilde{\theta} \in \Theta | v_i^{ab}(\tilde{\theta}) = 0\}$ denote the set of states where $i \in N$ is indifferent between (a, b). As an intermediate step, we will show that there exists a state $\theta \in S$ such that if an open ball B centered at θ is sufficiently small, then for any agent $i \in N$ and pair of distinct alternatives (a, b), $[B \cap S] \subset [B \cap \mathrm{IC}_i^{ab}]$ if $B \cap \mathrm{IC}_i^{ab}$ is nonempty.

The desired state θ can be obtained constructively as follows. Fix an arbitrary state $\theta' \in S$. Since valuation functions are continuous, if an open ball B' centered at θ' is sufficiently small, then $\theta' \in \mathrm{IC}_i^{ab}$ for any $i \in N$ and alternatives (a, b) such that $B' \cap \mathrm{IC}_i^{ab}$ is nonempty. Now we look for a state $\theta'' \in B' \cap S$ such that for some $i \in N$ and alternatives (a, b), $\theta' \in \mathrm{IC}_i^{ab}$ whereas $\theta'' \notin \mathrm{IC}_i^{ab}$. If such θ'' does not exist, then θ' is the desired state θ. If such θ'' exists, then we proceed analogously with θ'' in place of θ'. The procedure terminates after finitely many iterations because there are only finitely many agents and pairs of distinct alternatives, thus eventually yielding the desired state θ.

Observe that there is a sufficiently small open ball B centered at θ such that each $B \cap \mathrm{IC}_i^{ab}$, if nonempty, not only satisfies $[B \cap S] \subset [B \cap \mathrm{IC}_i^{ab}]$ (established above) but also is diffeomorphic to a hyperplane (by (RESP) and the inverse function theorem). Hence, $B \setminus \mathrm{IC}_i^{ab}$ consists of two open connected components, $U = \{\tilde{\theta} \in B | v_i^{ab}(\tilde{\theta}) < 0\}$ and $U' = \{\tilde{\theta} \in B | v_i^{ab}(\tilde{\theta}) > 0\}$, with common boundary $B \cap \mathrm{IC}_i^{ab}$.[15]

Now we show that $[B \cap S] = [B \cap \mathrm{IC}_i^{ab}]$ if $B \cap \mathrm{IC}_i^{ab}$ is nonempty. Since $\theta \in S$, both $\operatorname{int}\Theta \cap [\operatorname{cl}\bigcup_{\tilde{C}\in\mathcal{C}\setminus\mathcal{C}'} \tilde{C}]$ and $\operatorname{int}\Theta \cap [\operatorname{cl}\bigcup_{\tilde{C}\in\mathcal{C}'} \tilde{C}]$ must intersect $B \setminus S$, which implies that $B \setminus S$ is disconnected because the two sets are relatively closed and disjoint in $B \setminus S$. If there exists $\theta' \in B \cap \mathrm{IC}_i^{ab}$ such that $\theta' \notin B \cap S$ for some $i \in N$ and alternatives (a, b), then S would be diffeomorphic to a hyperplane missing some points, hence $B \setminus S$ would have to be connected, a contradiction.

It follows that $B \cap \overline{\Theta}$ intersects exactly two connected components in $\mathcal{C}$ because all nonempty $B \cap \mathrm{IC}_i^{ab}$ coincide by the previous paragraph. One of these connected components is some $C \in \mathcal{C} \setminus \mathcal{C}'$ and the other is some $C' \in \mathcal{C}'$ because $\theta \in S$ by construction. Moreover, $\theta \in [\operatorname{cl} C] \cap [\operatorname{cl} C']$. Thus, $C \in \mathcal{C} \setminus \mathcal{C}'$ and $C' \in \mathcal{C}'$ are adjacent at θ, contradicting the initial assumption that no component in $\mathcal{C}'$ is adjacent to a component in $\mathcal{C} \setminus \mathcal{C}'$. $\square$

[15]Specifically, using (RESP), suppose without loss of generality that $\partial v_i^{ab}(\theta)/\partial\theta_{is} \neq 0$, where θ_{is} is the sth entry of θ_i. Then the Jacobian of $h(\theta) = (\theta_{-is}, v_i^{ab}(\theta))$ is invertible, hence h is the desired local diffeomorphism: h^{-1} maps the hyperplane defined by the equation $\theta_{is} = 0$ to IC_i^{ab} and maps the half-spaces separated by that hyperplane to U and U', respectively. Finally, recall that connectivity is preserved under the continuous map h^{-1}.

We will now state and prove a stronger impossibility theorem that immediately implies Theorem 1 as a corollary.

THEOREM 2. *Suppose the premises of Theorem 1 hold. Then all* locally *EPIC social choice functions are trivial.*

PROOF. Fix any ϕ that is locally EPIC for radius $\varepsilon > 0$. Let $\Theta^k \subset \operatorname{int}\Theta$ denote the set of interior states where exactly k agents have indifferences in their preferences. Thus, $\operatorname{int}\Theta = \bigcup_{k=0}^{n} \Theta^k$. It suffices to show that ϕ is constant on Θ^k for every $k = 0, \dots, n$ and, moreover, that $\phi(\Theta^0) = \cdots = \phi(\Theta^n)$. We proceed by induction on k.

For $k = 0$, note that $\Theta^k = \overline{\Theta}$. Suppose, for the sake of contradiction, that ϕ is not constant on $\overline{\Theta}$. By Lemma 3, there exist two connected components C and C' of $\overline{\Theta}$ adjacent at some indifference state θ such that $\phi(C) \neq \phi(C')$. For any indifference pair (a, b) of θ, one of the following two cases must hold by assumption: (1) There is $i \in I^{ab}(\theta)$ and $j \notin I^{ab}(\theta)$ such that $\nabla_{\theta_j} v_i^{ab}(\theta) \neq \mathbf{0}$. (2) There are $i, j \in I^{ab}(\theta)$ where $\nabla v_i^{ab}(\theta)$ and $\nabla v_j^{ab}(\theta)$ are not codirectional. Hence, we have $\phi(C) = \phi(C')$ by the contrapositive of Lemma 2, a contradiction. Thus, ϕ must be constant on $\overline{\Theta} = \Theta^0$.

Now suppose ϕ is constant on Θ^ℓ for every $\ell < k$ and, moreover, $\phi(\Theta^0) = \cdots = \phi(\Theta^{k-1})$. Pick any $\theta \in \Theta^k$. By iteratively using (RESP), we can find states $\theta', \theta'' \in B_\varepsilon(\theta)$ arbitrarily close to θ such that (1) θ, θ' and θ'' differ from each other only in some agent i's type, (2) agent i is indifferent between one or more pairs of distinct alternatives in θ, and in addition, for any such pair she has strict and opposite preferences in θ' and θ'', (3) for any agent whose preference regarding any given pair of distinct alternatives is strict in θ, her preference regarding this pair remains the same in θ' and θ''. Thus, $\theta' \in \Theta^\ell$ and $\theta'' \in \Theta^{\ell'}$ for $\ell, \ell' < k$. Consequently, the inductive hypothesis implies that $\phi(\theta') = \phi(\theta'') = \phi(\Theta^0)$. Suppose for the sake of contradiction that $\phi(\theta) = a$ but $\phi(\Theta^0) = b \neq a$. On the one hand, if i has a strict preference regarding (a, b) in θ, then she has the same strict preference in θ and θ', and hence by (LEPIC), we must have $a = \phi(\theta) = \phi(\theta') = b$ for there to be no incentive for i to misreport, a contradiction. On the other hand, if i is indifferent between (a, b) in θ, then, by construction, i strictly prefers a over b in one of θ' and θ'', and in that state, she has an incentive to misreport her type as θ_i, also a contradiction. Thus, $\phi(\theta) = \phi(\Theta^0)$. Since θ was arbitrarily chosen from Θ^k, we conclude that ϕ must be constant on Θ^k and, moreover, $\phi(\Theta^k) = \phi(\Theta^0)$. □

REMARK 1. The sufficient condition for Theorems 1 and 2 can be weakened. Indeed, Lemma 3 guarantees the existence of a *discrete* set of indifference states Θ^* such that for any $C, C' \in \mathcal{C}$ there is a finite sequence of connected components $C^0, \dots, C^K \in \mathcal{C}$ where $C^0 = C$, $C^K = C'$, and C^k and C^{k+1} are adjacent at some $\theta \in \Theta^*$ for every $k = 0, \dots, K-1$. The proof of Theorem 2 goes through as long as local interdependence and heterogeneity are present in such a set of indifference states. Importantly, if $\mathcal{C}$ is finite, then Θ^* can be chosen as a finite set.[16] Thus, the set of indifference states where local interdependence and heterogeneity need actually be present is much smaller than the set of all indifference states.

[16]One can imagine that preferences must be rather special for $\mathcal{C}$ to be infinite, but such preferences do exist. We are grateful to an anonymous referee for suggesting the following illustrative example: Suppose there are two agents, two alternatives, and one-dimensional types $\theta = (\theta_1, \theta_2)$ as in our example from

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Section 2. If relative valuations are given by

$$v_1(\theta) = \theta_1^3 \sin(1/\theta_1) - \theta_2 \quad \text{and} \quad v_2(\theta) = \theta_2,$$

where $v_1(0, \theta_2) = -\theta_2$, then any neighborhood of $\theta = (0, 0)$ contains infinitely many components in $\mathcal{C}$. The example can be modified to satisfy all of our assumptions, including (RESP), as well as the premises of Theorem 1 by rotating the state space.

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