Theoretical Economics 18 (2023), 561–596 1555-7561/20230561

# Loss aversion in sequential auctions

Antonio Rosato
School of Economics, University of Queensland and CSEF

I analyze sequential auctions with expectations-based loss-averse bidders who have independent private values and unit demand. Equilibrium bids are history dependent and subject to a "discouragement effect": the higher is the winning bid in the current round, the less aggressive are the bids of the remaining bidders in the next round. Moreover, because they experience a loss in each round in which they fail to obtain an object, bidders are willing to pay a premium to win sooner rather than later. This desire to win earlier leads prices to decline in equilibrium. I also show how various disclosure policies regarding the outcome of earlier auctions affect equilibrium bids, and that sequential and simultaneous auctions are neither bidder-payoff equivalent nor revenue equivalent.

Keywords. Loss aversion, sequential auctions, afternoon effect.

JEL classification. D03, D44, D81, D82.

> Now that I've won a slam, I know something very few people on earth are permitted to know. A win doesn't feel as good as a loss feels bad, and the good feeling doesn't last as long as the bad. Not even close.
>
> Andre Kirk Agassi

## 1. Introduction

There is abundant evidence of overbidding in auctions. While this has often been attributed to a desire to win, and to related concepts like "joy of winning" (Cox, Smith, and Walker (1992), Goeree, Holt, and Palfrey (2002), and Cooper and Fang (2008)) and "bidding fever" (Heyman, Orhun, and Ariely (2004) and Ehrhart, Ott, and Abele (2015)), there

Antonio Rosato: a.rosato@uq.edu.au
This paper is a revised version of the second chapter of my doctoral dissertation, submitted to UC Berkeley. I am indebted to Benjamin Hermalin, Botond Kőszegi and, especially, Matthew Rabin for their invaluable advice and encouragement. I am grateful to three anonymous referees for several suggestions that vastly improved the paper. For helpful comments, I also thank Benjamin Balzer, Simon Board, Juan Carlos Carbajal, Haluk Ergin, Erik Eyster, Juan-José Ganuza, Jacob Goeree, Fabian Herweg, Maciej Kotowski, George Mailath, Flavio Menezes, Claudio Mezzetti, Paul Milgrom, Takeshi Murooka, Aniko Öry, Marco Pagnozzi, Toru Suzuki, John Wooders, Jun Xiao, and audiences at UC Berkeley, UNSW, UTS, University of Melbourne, University of Adelaide, CSEF, Yale, UNC Chapel Hill, University of Bern, NYU Abu Dhabi, HEC Montreal, Imperial College London, Royal Holloway University of London, the 2013 Asian Meeting of the Econometric Society in Singapore, the 2014 RES conference in Manchester, the 2015 MaCCI Workshop on Behavioral Industrial Organization in Bad Homburg, and the 2015 CSEF-IGIER Symposium on Economics and Institutions in Anacapri. Yanlin Chen provided good research assistance. Financial support from ARC Grant DE180100190 is gratefully acknowledged.


Available at https://econtheory.org. https://doi.org/10.3982/TE4096

is also evidence that bidders may actually be driven by a fear or frustration of losing, as shown, for instance, by Delgado, Schotter, Ozbay, and Phelps (2008) and Cramton, Filiz-Ozbay, Ozbay, and Sujarittanonta (2012). Such a frustration of losing is consistent with the notion of loss aversion introduced by Kahneman and Tversky (1979), according to which people tend to evaluate outcomes relative to a reference point, with losses (relative to this reference point) looming larger than equal-size gains. Therefore, in this paper, I explore the implications of loss aversion for multi-unit auctions when bidders have independent private values and unit demand.

Following the work of Kőszegi and Rabin (2006, 2007, 2009), I consider expectations-based reference points, whereby an individual compares his realized material outcomes to a reference point equal to his expectations about those same outcomes. With this formulation of the reference point, the higher are a bidder's value for an item and the probability with which he expects to obtain it, the greater the psychological loss if he fails to do so. In particular, in a sequential auction, the bidder updates his subjective probability of obtaining an item based on the outcome of the previous rounds. This updating of the reference point introduces an endogenous form of interdependence in the bidders' payoffs, even when values are private and independent. The reason is that even though a bidder's value does not depend directly on his competitors' types, they do affect his probability of winning the auction and, hence, his reference point.

Section 2 introduces the environment and the bidders' preferences. There are $K \geq 2$ identical items to be sold one by one using a sequence of sealed-bid auctions. I consider a standard symmetric environment where bidders have independent private values and are interested in buying at most one item. At the beginning of the auction sequence, a bidder forms a subjective probability of obtaining an item in any given round based on his private value, his bidding strategy, and his beliefs about his opponents' values and strategies. Then, at the start of each round in which he is still active, the bidder updates this probability based on the observed history of the game. When he obtains an item, a bidder's utility simply equals his material payoff; that is, his intrinsic valuation for the item minus the price paid to acquire it. In each round in which he fails to obtain an item, however, the bidder suffers a psychological loss that is proportional to his intrinsic valuation and to the probability with which, at the beginning of the round, he expected to obtain the item in that same round.

Section 3 gathers the paper's main analysis and results. I begin in Section 3.1 by analyzing sequential second-price auctions. In the last round, bidders bid their "loss-adjusted" willingness to pay; that is, their intrinsic value for the item plus the value of avoiding the psychological loss they would experience if they were to lose the auction. Since the value of avoiding losses is always positive, loss-averse bidders bid more aggressively than risk-neutral ones. Such aggressive bidding also arises in earlier rounds, where a bidder bids his expectation of the next round's price plus the value of avoiding the psychological loss from losing in the current round. The reason is that in equilibrium, bidders must be indifferent between winning in the current round or the next; yet, to win in the next round, a bidder must endure a loss in the current one, whereas by winning in the current round, he avoids such loss. Hence, loss-averse bidders are willing to pay a premium above the next round's expected price to win in the current round, thereby reducing the uncertainty over whether they will obtain the item. Moreover, because in each round this premium depends on the bidder's updated probability of getting the item, expectations-based loss aversion creates an informational externality that renders equilibrium bids history dependent, even if bidders have independent private values. In particular, I identify a "discouragement effect": the higher is the winning bid in the current round, the less aggressive is the bidding strategy of the remaining bidders in the next round. Indeed, from the point of view of a bidder who lost the current round, the higher is the type of the winner, the less likely he is to win in the next round; this, in turn, lowers the reference point of the bidder, who thus bids less aggressively. This is the frustrating effect of losing, which lowers a loss-averse bidder's willingness to pay.

The preference of loss-averse bidders for winning in the current round rather than the next leads them to bid more aggressively in earlier rounds; this, in turn, implies that equilibrium prices must follow a declining path. Hence, expectations-based loss aversion provides a novel explanation for the "declining price anomaly" or "afternoon effect" (as later auctions often take place in the afternoon, whereas earlier ones are in the morning) in sequential auctions. Weber (1983) and Milgrom and Weber (2000) showed that with symmetric, risk-neutral bidders having unit demand and independent private values, the law of one price should hold and on average prices should be the same across rounds.[1] Intuitively, if they were not, then demand from rounds with a higher expected price would shift toward those with a lower expected price, due to arbitrage opportunities. Yet, evidence from both the lab and the field does not seem to support this prediction, as declining prices have been reported across many different goods and auction formats; see Ashenfelter (1989), Ashenfelter and Genesove (1992), McAfee and Vincent (1993), Beggs and Graddy (1997), Ginsburgh (1998), Van den Berg, Van Ours, and Pradhan (2001), Lambson and Thurston (2006), Février, Linnemer, and Visser (2007), and Neugebauer and Pezanis-Christou (2007). Moreover, while declining prices are more common, increasing prices have also been documented; see Gandal (1997) and Deltas and Kosmopoulou (2004).[2] Overall, while they do not occur in every auction, declining prices seem to be an empirically robust feature of sequential auctions.

In Section 3.2, I compare sequential auctions to simultaneous auctions. With risk-neutral bidders and independent private values, these auction formats are revenue equivalent for the seller and payoff equivalent for the bidders; however, both these equivalences break down if bidders are expectations-based loss averse. The reason is that in simultaneous auctions, the resolution of uncertainty happens all at once, whereas it is more gradual in sequential auctions. This has two implications. First, as in sequential auctions, bidders suffer a psychological loss in every round in which they expected to win with positive probability but instead lose; they can suffer multiple losses

[1] Technically, with independent private values, the price sequence of any standard auction is a martingale; i.e., the conditional expectation of the next-round price is equal to the current price.

[2] Milgrom and Weber (2000) showed that with interdependent values and affiliated signals, the equilibrium price sequence is a submartingale and the expected value of $p_{k+1}$, conditional on $p_k$, is higher than $p_k$. Mezzetti (2011) showed that affiliated signals are not necessary for increasing prices: interdependent values with informational externalities—that is, when a bidder's value is increasing in all bidders' private signals—push prices to increase between rounds, even with independent signals.

on their way to eventually obtaining an item. By contrast, in simultaneous auctions a bidder suffers a loss only if he fails to obtain an item. Second, the loss-aversion premium component of a bidder's willingness to pay differs across the two formats: in a simultaneous auction, it is proportional to the bidder's ex ante probability of being one of the top $K$ bidders, whereas in each round of a sequential auction, it is proportional to the bidder's probability of winning in that round conditional on having lost the previous round. Indeed, I show that bidders with high (resp. low) values prefer simultaneous (sequential) auctions and that sequential auctions raise more revenue than simultaneous ones. This is consistent with experimental evidence from Betz, Greiner, Schweitzer, and Seifert (2017), who find that sequential multi-item auctions raise more revenue than simultaneous ones. Moreover, it is well known that sequential auctions are more vulnerable to bidder collusion than simultaneous ones; see, for instance, Klemperer (2002) and Sherstyuk and Dulatre (2008). As collusion tends to reduce the seller's revenue, we would expect sellers to prefer simultaneous auctions over sequential ones. Yet, loss aversion provides an alternative reason why sellers might prefer sequential auctions.

When analyzing sequential auctions in Section 3, I assume that the winning bid in each round is publicly announced by the seller prior to the next round. Section 4 considers two alternative disclosure policies. First, I analyze sequential auctions with no announcement and I show that the equilibrium strategies are radically different. If the winning bid from the previous round is not publicly revealed, a losing bidder must use his own past bid to update his beliefs about how likely he is to win in the current round. In this case, the discouragement effect takes on a different form that depends on an individual bidder's (private) bidding history. Nevertheless, since loss-averse bidders still prefer to win in the current round rather than the next, equilibrium prices continue to follow a declining path. Next, I consider sequential second-price auctions with announcement of the previous round's price and I argue that in this case, existence of a symmetric equilibrium in increasing strategies is not guaranteed. The reason is that, just as in the classical model with interdependent values, revealing the previous round's price makes the game highly asymmetric, as in the next round one of the remaining bidders would have his exact bid known to the others.

Section 5 discusses the related literature, while Section 6 gathers concluding remarks. All proofs are relegated to the Appendix.

## 2. Model

Suppose $K \geq 2$ identical items are sold one by one to $N \geq K+1$ bidders via a series of sealed-bid auctions with no reserve price. Bidders demand one unit and have independent private values. Each bidder's value (or type) $\theta_i$, $i = 1, \dots, N$, is drawn from the same continuous and strictly increasing distribution $F$ that admits a continuous and positive density $f$ everywhere on the support $[0, 1]$. I will consider two canonical selling mechanisms: first-price auctions (FPA) and second-price auctions (SPA). In each round $k = 1, \dots, K$, the highest bidder obtains an item and pays price $p_k$ that equals either his bid or the highest losing bid in that round, depending on the auction format. The winner leaves the auction and his bid is publicly announced at the beginning of the next round, where the remaining bidders compete using the same procedure. Let $h^0$ be the empty history at the beginning of the first round and denote by $w_k$ the winning bid in round $k$. Then, for $k \in \{2, \dots, K\}$, a public history at the beginning of round $k$ is a sequence of winning bids $h^{k-1} = (w_1, \dots, w_{k-1})$.

A strategy for bidder $i$ is a sequence of bidding functions $\beta_i = (\beta_{i,1}, \dots, \beta_{i,k}, \dots, \beta_{i,K})$, one for each auction, where $\beta_{i,k}(\theta_i; h^{k-1})$ denotes bidder $i$'s bid in round $k$ as a function of his type $\theta_i$ and of the public history of winning bids. A strategy $\beta_i$ is monotone if for each $k = 1, \dots, K$, $\beta_{i,k}$ is increasing in $\theta_i$ for any $h^{k-1}$. Restricting attention to symmetric equilibria in pure and monotone strategies, hereafter I will drop the subscript indexing bidders and, slightly abusing notation, I will use $\beta_k$ to denote a symmetric bidding function and $\beta$ to denote a symmetric strategy profile.

Let the random variable $Y_k^{(n)}$, an order statistic, denote the $k$th highest value out of $n$. Since strategies are monotone, the winner in round $k$ is the bidder with the $k$th highest value. Hence, the winning bids in previous rounds, e.g., $w_1, \dots, w_{k-1}$, can be mapped back to the realized values of the order statistics, e.g., $Y_1^{(N)} = y_1 \geq \dots \geq y_{k-1} = Y_{k-1}^{(N)}$. Therefore, the bidding function in round $k$ can be written as $\beta_k(\theta; y_{k-1})$ since at the beginning of round $k$, it is common knowledge that all remaining bidders' types are lower than $y_{k-1}$, with $y_0 \equiv 1$.

Bidders are expectations-based loss averse à la Kőszegi and Rabin (2006, 2007, 2009). For $K \geq t \geq k$, let $Q_{i,k}^t := Q^t(\theta_i; y_{k-1}|\beta)$ denote $i$'s subjective probability at the start of round $k$ of obtaining the item in round $t$, conditional on the history of the winning bids up to round $k-1$ and taking as given the strategy profile $\beta$. Moreover, for $k = 1, \dots, K$ and $i = 1, .., N$, let $b_{i,k}$ denote $i$'s bid in round $k$. Then, ignoring ties, as they are measure-zero events, $i$'s *realized* utility in round $k$ is equal to

$$u_k(\theta_i, b_{i,k}, p_k; y_{k-1}|\beta) = \begin{cases} \theta_i - p_k & \text{if } b_{i,k} > \max\limits_{j \neq i} b_{j,k} \\ -\Lambda \theta_i Q_{i,k}^k & \text{if } b_{i,k} < \max\limits_{j \neq i} b_{j,k}, \end{cases} \tag{1}$$

where $Q_{i,k}^{K+1} = 0 \; \forall i, k$ and $\Lambda \geq 0$. The parameter $\Lambda$ represents the coefficient of loss aversion, with $\Lambda = 0$ corresponding to the risk-neutral benchmark.[3] In words, if a bidder wins the auction in round $k$, his utility equals his standard material payoff; if, instead, he fails to win the auction, the bidder experiences a psychological loss that is proportional to his type and to the probability with which, at the beginning of round $k$, he expected to obtain the item in that same round. Hence, throughout the auction sequence, a bidder updates his reference point to the probability of winning in the current round conditional on the outcome of the previous rounds. The timing of events and payoffs within

[3]There are two minor differences between expression (1) and the original formulation of Kőszegi and Rabin (2006, 2007, 2009). First, the bidder experiences psychological (dis-)utility from losses but not from gains. This is a simple normalization that can be interpreted as capturing a limit case where bidders weigh losses much more strongly than same-size gains. Second, bidders are loss averse only with respect to their value for the item, but not with respect to the price they might pay; in other words, bidders are risk neutral over money. As argued by Kőszegi and Rabin (2009), this assumption is reasonable if bidders' income is already subject to large background risk. In related work, Novemsky and Kahneman (2005) propose that money given up in purchases is not generally subject to loss aversion.

each round are the following. At the start of round $k$, the seller reveals the previous round's winning bid; thereafter, bidder $i$ updates his reference point to $Q_{i,k}^k$ and submits his bid for the current round. Then the winner is selected and payoffs are realized.

Bidders cannot commit to a sequence of bids at the outset. Instead, they make a state-contingent plan whereby in each round their current bid is consistent with their future bidding behavior and maximizes their total expected reference-dependent utility going forward. Let $U_k(\theta_i, b_{i,k}; y_{k-1}|\beta) := \mathbb{E}_k[\sum_{m=k}^K u_m(\theta_i, b_{i,m}, p_m; y_{m-1}|\beta)]$ denote bidder $i$'s total expected payoff at the start of round $k$, given a strategy profile $\beta$. If bidder $i$ wins in round $k$, then $b_{i,k+l} = 0$ and $u_{k+l}(\theta_i, b_{i,k+l}, p_{k+l}; y_{k+l-1}|\beta) = 0$ for $l = 1, \dots, K-k$. Hence, at the beginning of each round in which he is active, a bidder updates his subjective probability of obtaining the item based on the history of the winning bids and then chooses a bid to maximize the expectation of the sum of his instantaneous reference-dependent payoffs, keeping his reference point fixed and fully anticipating the losses he might experience in future rounds, with each loss weighted by the corresponding probability. Fixing his competitors' strategies and the expectations induced by the strategy profile $\beta$, if he follows his plan, bidder $i$ bids $b_{i,k} := \beta_k(\theta_i; y_{k-1})$ for $k \in \{1, 2, \dots, K\}$. Then the solution concept is as follows.

Definition 1. A strategy profile $\beta^*$ constitutes a sequential personal equilibrium (SPE) if for all $i$, for all $\theta_i$, for all $y_{k-1}$, and for $k = 1, \dots, K$,

$$U_k(\theta_i, b_{i,k}^*; y_{k-1}|\beta^*) \geq U_k(\theta_i, \widetilde{b}_{i,k}; y_{k-1}|\beta^*)$$

for any $\widetilde{b}_{i,k} \neq b_{i,k}^*$.

In an SPE, a bidder has to think backward. First, in round $K$, for each possible value of $y_{K-1}$, he chooses a bid that maximizes his utility in the last round; then, in each previous round, he chooses a bid that maximizes his expected reference-dependent utility given the expectations generated by his strategy and correctly anticipating how he will bid in later rounds.[4] Notice that by restricting attention to one-round deviations, Definition 1 also embeds the single-deviation property as part of the solution concept. This differs from standard game-theoretic solution concepts based on subgame perfection or sequential rationality, which obtain the single-deviation property as a result. I include the single-deviation property as part of Definition 1 because nonlocal deviations, such as planning in round $k$ to deviate at a later round $k+l$, for $l \in \{1, \dots, K-k\}$, can affect the bidder's expectations and, hence, his reference point, making the problem much more intricate. This additional restriction differentiates SPE from the dynamic version of PE in Kőszegi and Rabin (2009), according to which a person can affect his current utility by planning to change his future actions. Moreover, it is worthwhile to point out that because a decision maker with expectations-based reference-dependent preferences is prone to self-fulfilling expectations, the restriction to pure, strictly increasing bidding strategies also implicitly entails an equilibrium selection; in this sense, ruling out mixed strategies might not be without loss of generality. For the remainder of the paper, I will refer to an SPE simply as an equilibrium.

[4]For static problems, SPE reduces to the personal equilibrium (PE) of Kőszegi and Rabin (2006).

## 3. Analysis

This section gathers the paper's main results. I begin in Section 3.1 by deriving the bidding strategies in the sequential SPA and showing that equilibrium prices follow a declining path. In Section 3.2, I analyze the uniform-price simultaneous auction and show that it is not revenue equivalent to the sequential second-price auction.

### 3.1 *Sequential auctions*

Suppose $K \geq 2$ identical items are sold sequentially via second-price auctions. The first-round bidding strategy is a function that depends only on the bidder's type. The bids in later rounds, however, might depend also on the public history of the winning bids. Since we are focusing on a symmetric equilibrium, it is useful to take the point of view of one of the bidders, say bidder $i$ with type $\theta$, and to consider the order statistics associated with the types of the other $(N-1)$ bidders. Hence, let $Y_k^{(N-1)} \equiv Y_k$ be the $k$th highest of $N-1$ values, and denote by $F_k(\cdot)$ and $f_k(\cdot)$ its cumulative distribution function (CDF) and corresponding probability distribution function (PDF), respectively. Moreover, let $F_k(\cdot|x)$ and $f_k(\cdot|x)$, respectively, denote its CDF and PDF conditional on $Y_{k-1} = x$. Notice that because the different values are drawn independently, it follows that $F_k(\theta|x) = F(\theta)^{N-k}/F(x)^{N-k}$; hence, $F_k(\theta|x)$ is decreasing in $x$. The following proposition characterizes the symmetric equilibrium strategies.

Proposition 1. *In the sequential SPA, the symmetric equilibrium bidding strategies are given by*

$$\beta_k^{\text{SPA}}(\theta; y_{k-1}) = \int_0^\theta \beta_{k+1}^{\text{SPA}}(y_{k+1}; \theta) f_{k+1}(y_{k+1}|\theta)\, dy_{k+1} + \Lambda\theta F_k(\theta|y_{k-1})$$

*for* $k = 1, \dots, K-1$ *and*

$$\beta_K^{\text{SPA}}(\theta; y_{K-1}) = \theta + \Lambda\theta F_K(\theta|y_{K-1}).$$

*For* $k = 2, \dots, K$, *the complete bidding strategy is to bid* $\beta_k^{\text{SPA}}(\theta; y_{k-1})$ *if* $\theta < y_{k-1}$ *and* $\beta_k^{\text{SPA}}(y_{k-1}; y_{k-1})$ *if* $\theta \geq y_{k-1}$.[5]

To understand the bidding functions in the sequential second-price auction, it is easier to start from round $K$, the last one. In this round, a loss-averse bidder with type $\theta$ bids his loss-adjusted willingness to pay. This modified willingness to pay takes into account the bidder's intrinsic value for the good (i.e., $\theta$), as well as the value of avoiding the psychological feeling of loss he would experience by failing to win the auction (i.e., $\Lambda\theta F_K(\theta|y_{K-1})$). Indeed, for $\Lambda = 0$, $\beta_K^{\text{SPA}}(\theta; y_{K-1})$ reduces to the standard (weakly) dominant strategy of bidding one's intrinsic value. Hence, loss aversion induces all types to overbid compared to the risk-neutral benchmark.[6]

[5]The latter event may occur "off path" if a type-$\theta$ bidder underbid in round $k-1$, thereby causing a bidder with a lower type to win.

[6]Such straight overbidding compared to the risk-neutral benchmark is due to the assumption of no loss aversion over money, which reduces the weight over the money dimension relative to the item dimension

The bidding functions in the earlier rounds are pinned down by the condition that, in equilibrium, a bidder must be indifferent between winning in the current round or the next one. Say that bidder $i$ with type $\theta$ is pivotal in round $k$ if he has the same type as the $k$th highest of his opponents, and, hence, he is in a tie with such opponent as the remaining bidder with the highest type. Then, in round $k$, bidder $i$ bids his expectation of the next round's price conditional on being pivotal (i.e., $\int_0^\theta \beta_{k+1}^{\mathrm{SPA}}(y_{k+1};\theta) f_{k+1}(y_{k+1}|\theta)\,dy_{k+1}$) plus the value of avoiding the psychological loss from losing in round $k$ (i.e., $\Lambda\theta F_k(\theta|y_{k-1})$). Hence, compared to the risk-neutral benchmark, loss-averse bidders are willing to pay a premium to win earlier rather than later. Intuitively, by winning in an earlier round, a bidder reduces the uncertainty of the auction and, thus, experiences fewer psychological losses. Moreover, this premium is decreasing in the type of the previous round's winner, implying that the equilibrium bidding function is decreasing in the previous round's winning bid, as the following lemma shows.

Lemma 1 (Discouragement Effect). *For $k > 1$, it holds that $\partial\beta_k^{\mathrm{SPA}}(\theta; y_{k-1})/\partial y_{k-1} < 0\ \forall\theta$.*

Lemma 1 says that the higher is the type of the previous-round winner, and, hence, his bid, the less aggressively will the remaining bidders bid in the current round. The rationale for this discouragement effect is as follows. From the perspective of a bidder who lost the previous round, the higher is the type of the winner, the less likely he is to win in the current round. With expectations-based reference-dependent preferences, a bidder who thinks that most likely he is not going to win does not feel a strong attachment to the item and, thus, bids more conservatively. Hence, revealing the previous-round winner's bid creates an informational externality. Yet, notice that the direction of this informational externality is exactly opposite to that arising with interdependent (or common) values, where the higher is the type of the previous-round winner, the higher is the value of the object to all remaining bidders, who in turn bid more aggressively in the current round. The discouragement effect represents a testable implication that differentiates my model from those with risk-neutral (Milgrom and Weber (2000), Weber (1983)) and risk-averse bidders (McAfee and Vincent (1993), Mezzetti (2011), Hu and Zou (2015)), where previous winning bids have no influence on the remaining bidders' strategies.

The willingness of loss-averse bidders to pay a premium to win earlier implies that equilibrium prices follow a declining path, as summarized by the following proposition.

Proposition 2 (Afternoon Effect). *The price sequence in the sequential SPA is a supermartingale; that is, for $k \in \{1, \dots, K-1\}$, the expected price in round $k+1$ conditional on the price in round $k$ is lower than the price in round $k$. Hence, the afternoon effect arises in equilibrium.*

---

in a bidder's overall utility. In the more general case where bidders are loss averse in both dimensions, only those with relatively high types will overbid; see Lange and Ratan (2010), Balzer and Rosato (2021), and von Wangenheim (2021). Yet, a similar notion of loss-adjusted willingness to pay also applies if bidders are loss averse in both dimensions.

The intuition behind Proposition 2 is that in equilibrium, bidders must be indifferent between winning in the current round or the next. In the risk-neutral benchmark, this implies that equilibrium prices are constant in expectation; i.e., the price sequence is a martingale. Yet, if the expected prices were constant across two rounds, loss-averse bidders would strictly prefer to win in the earlier round; hence, in the current round, they are willing to pay a premium above the next round's price. But why would not a bidder deviate and lose in the current round so as to wait for the price to drop in the next round? The reason is that when winning in round $k$, the premium that a bidder expects to pay is equal to the psychological loss of his strongest remaining opponent, the price setter; in equilibrium, however, this premium is smaller than the psychological loss the bidder himself would suffer from losing in round $k$. Thus, while in equilibrium, the next round's expected price is indeed lower than the current one's, a bidder cannot profitably deviate by lowering his bid in the current round. Moreover, notice that while the decline in equilibrium prices is due to the preference of loss-averse bidders to win earlier so as to reduce the uncertainty and avoid additional losses, the magnitude of the decline varies with the informational externality; i.e., the stronger the discouragement effect, the steeper the decline in price.

I conclude this section with a brief discussion of the sequential FPA. Continuing to assume the winning bids are publicly disclosed, the following proposition characterizes its symmetric equilibrium strategies.[7]

Proposition 3. *In the sequential FPA, the symmetric equilibrium bidding strategies are given by*

$$\beta_k^{\mathrm{FPA}}(\theta; y_{k-1}) = \int_0^\theta \big[\beta_{k+1}^{\mathrm{FPA}}(x; x) + \Lambda x F_k(x|y_{k-1})\big] f_k(x|\theta)\, dx$$

*for* $k = 1, \dots, K-1$ *and*

$$\beta_K^{\mathrm{FPA}}(\theta; y_{K-1}) = \int_0^\theta \big[x + \Lambda x F_K(x|y_{K-1})\big] f_K(x|\theta)\, dx.$$

*For* $k = 2, \dots, K$*, the complete bidding strategy is to bid* $\beta_k^{\mathrm{FPA}}(\theta; y_{k-1})$ *if* $\theta < y_{k-1}$ *and* $\beta_k^{\mathrm{FPA}}(y_{k-1}; y_{k-1})$ *if* $\theta \geq y_{k-1}$.

In round $K$, a bidder with type $\theta$ bids his expectation of the loss-adjusted willingness to pay of his closest opponent, conditional on himself having the highest type among all remaining bidders. In earlier rounds, the bidding functions are again pinned down by the condition that, in equilibrium, a bidder must be indifferent between winning in the current round or the next round. To see the intuition, suppose bidder $i$ wins in round $k$ if he bids as his type; that is, suppose $Y_k \leq \theta_i$. Bidder $i$ also has the option to bid as low as to lose in round $k$ and discover the value of $Y_k$; then, he can win for sure in the next round by bidding as if his type were $Y_k$. Hence, in round $k$, bidder $i$ bids the expectation of the next round's price as though he was tied with his closest competitor (i.e., $\beta_{k+1}^{\mathrm{FPA}}(Y_k; Y_k)$)

[7]The proof of Proposition 3 follows similar steps to those of Proposition 1 and is, hence, omitted. Details are available from the author upon request.

plus the value for his closest competitor of avoiding the psychological loss from losing in round $k$ (i.e., $\Lambda Y_k F_k(Y_k|y_{k-1})$), conditional on himself having the highest type among all remaining bidders. Therefore, as in the SPA, loss-averse bidders are willing to pay a premium to win sooner rather than later. Furthermore, it is easy to verify that also in the FPA, equilibrium bids are subject to the discouragement effect: the higher is the previous round's winning bid, the less aggressively the remaining bidders will bid in the current round.

Finally, as shown by Lange and Ratan (2010) and Balzer and Rosato (2021), the FPA and SPA are revenue equivalent if bidders are not loss averse over money. The reason is that, in this case, the psychological losses depend only on the probability with which a bidder expects to win the auction and, with symmetric strategies, this is the same in both formats. Thus, in each round, the seller's expected revenue from the FPA is the same as that from the SPA, and the afternoon effect arises in equilibrium.

### 3.2 *Sequential versus simultaneous auctions*

In this section, I analyze simultaneous auctions; that is, auctions in which all items are allocated after only one round of bidding. In particular, I derive the equilibrium bidding strategy in a uniform-price auction, where bidders submit sealed bids, the $K$ highest bidders each receive one item and pay a price equal to the $(K+1)$th highest bid. This procedure generalizes the single-item SPA.[8] The following proposition characterizes the symmetric equilibrium strategies.

Proposition 4. *In the uniform-price auction, the symmetric equilibrium bidding strategies are given by*

$$\beta^{\mathrm{UPA}}(\theta) = \theta + \Lambda\theta F_K(\theta).$$

In the classical model without reference dependence, analogous to the single-item SPA, it is weakly dominant for bidders to bid their intrinsic values. By contrast, a loss-averse bidder bids his loss-adjusted willingness to pay, which consists of his intrinsic value for an item (i.e., $\theta$) as well as the value of avoiding the psychological loss he would experience by failing to win the auction (i.e., $\Lambda\theta F_K(\theta)$).[9] Notice that this value depends on the bidder's probability of being among the top $K$ bidders, as it is not necessary for the bidder to submit the highest bid to obtain an item; it is enough to outbid his $K$th highest competitor.

Next I compare the bidders' equilibrium utility and the seller's expected revenue in simultaneous and sequential auctions.[10] Under risk neutrality, simultaneous and sequential formats are payoff-equivalent for the bidders and revenue-equivalent for the

[8]An alternative procedure is the discriminatory (or "pay-your-bid") auction, where bidders submit sealed bids, and the $K$ highest bidders each receive one item and pay their own bid. This procedure generalizes the single-item FPA.

[9]Similarly, it is easy to verify that the symmetric equilibrium strategy in a discriminatory auction is $\beta^{\mathrm{DPA}}(\theta) = \int_0^\theta [x + \Lambda x F_K(x)] f_K(x|\theta)\, dx$.

[10]I do the comparison for uniform-price and sequential second-price auctions, but the same results apply to discriminatory and sequential first-price auctions by revenue equivalence.

seller. Indeed, in both formats, a bidder wins an item with the same probability (i.e., if his type is higher than that of his $K$th highest opponent) and, in expectation, pays the same price (i.e., the value of his $K$th highest opponent). With expectations-based loss aversion, however, both equivalences break down since bidders' expected payments as well as their expected psychological losses differ across the two formats. The following proposition characterizes the difference in the expected payments.

Proposition 5 (Revenue Non-Equivalence). *For $K \geq 2$, let $P_K^{\mathrm{UPA}}(\theta)$ and $P_K^{\mathrm{SPA}}(\theta)$ denote a type-$\theta$ equilibrium expected payment in (simultaneous) uniform-price auctions (UPA) and (sequential) second-price auctions, respectively. Then $P_K^{\mathrm{UPA}}(\theta) \leq P_K^{\mathrm{SPA}}(\theta)$ $\forall\theta$.*

In either a simultaneous or sequential auction, a bidder of type $\theta$ expects to pay a price equal to the loss-adjusted willingness to pay of his marginal opponent; i.e., the one with the $K$th highest value (among $N-1$). Yet, despite the marginal opponent having the same intrinsic value, his loss-adjusted willingness to pay is not the same in the two auction formats. In a simultaneous auction, the willingness to pay depends on the ex ante probability with which the bidder expects to obtain an item; in a sequential auction, instead, the willingness to pay of the marginal bidder depends on his updated probability of winning in the last round based on the outcome of the previous rounds. Furthermore, recall that in each round of a sequential auction loss-averse bidders are actually paying a premium over the next round's expected price; therefore, their expected payments are higher in sequential auctions than in simultaneous ones. Moreover, an immediate implication of Proposition 5 is that the seller's revenue is higher in sequential auctions compared to simultaneous ones. This is in line with the experimental results of Betz et al. (2017), who find that sequential multi-item auctions raise more revenue than simultaneous ones, with the source of this difference being fiercer competition on the item(s) auctioned first.

Based on Proposition 5, revenue equivalence between sequential and simultaneous auctions no longer holds if bidders are expectations-based loss averse, and a revenue-maximizing auctioneer should always favor the former. However, this does not necessarily imply that loss-averse bidders prefer simultaneous auctions over sequential ones, as the two formats provide the bidders with different information about their likelihood of winning. In a simultaneous auction, a bidder only learns whether he is among the top $K$ bidders, so that there is just one opportunity for disappointment; by contrast, in a sequential auction, a bidder will experience a psychological loss in each round in which he learns that he is not the highest remaining bidder. Indeed, as the following proposition shows, bidders' expected psychological losses may differ across formats.

Proposition 6 (Bidder-Losses Non-Equivalence). *For $K \geq 2$, let $L_K^{\mathrm{UPA}}(\theta)$ and $L_K^{\mathrm{SPA}}(\theta)$ denote type-$\theta$ equilibrium expected psychological losses in (simultaneous) uniform-price and (sequential) second-price auctions, respectively. Then $L_K^{\mathrm{SPA}}(0) - L_K^{\mathrm{UPA}}(0) = 0 = L_K^{\mathrm{SPA}}(1) - L_K^{\mathrm{UPA}}(1)$. Moreover, for $\theta \in (0, 1)$ there exists a cutoff type $\theta_K^*$ such that $L_K^{\mathrm{SPA}}(\theta) \geq L_K^{\mathrm{UPA}}(\theta)$ if and only if $\theta \geq \theta_K^*$.*

The proof of Proposition 6 in Appendix provides formal definitions for $L_K^{\text{UPA}}(\theta)$ and $L_K^{\text{SPA}}(\theta)$; here, I will describe these terms intuitively. Since the uniform-price auction is a static format, a type-$\theta$ expected psychological loss in equilibrium depends only on his ex ante probability of obtaining one of the items; i.e., $F_K(\theta)$. By contrast, at the start of a $K$-round sequential second-price auction, a type-$\theta$ bidder expects to experience a psychological loss in each round in which he might not win, with each loss weighted by the corresponding probability; i.e., $F_1(\theta)$, $F_2(\theta|y_1)$, etc. Notice that a bidder with the lowest (resp. highest) type expects to lose (resp. win) for sure in either a sequential auction or a simultaneous one, and as the realized outcome exactly matches his expectations, his psychological losses are equal to zero in both formats. Yet, bidders with interior types are ex ante uncertain about their outcomes; hence, for these bidders, it makes a difference whether uncertainty is resolved sequentially or all at once. In particular, high-type bidders are unlikely to experience a psychological loss, but if they do, they suffer a rather large one; in contrast, low-type bidders are likely to suffer losses, but these are relatively small. This trade-off between the likelihood and the magnitude of psychological losses is what drives a loss-averse bidder's preference between the two auction formats. Consider first a bidder with a relatively high type; e.g., higher than $\theta_K^*$. Such a bidder has a high chance to obtain an item and is, therefore, unlikely to suffer a loss in a simultaneous auction. In a sequential auction, however, the bidder could still suffer a (partial) loss even conditional on obtaining an item; e.g., if he loses in the first round but wins in the second. Hence, for high-type bidders, the expected disutility from losses is smaller in simultaneous auctions than in sequential ones. Conversely, consider a bidder with a relatively low type; e.g., lower than $\theta_K^*$. Such a bidder is unlikely to obtain an item in either format; hence, he expects to suffer psychological losses in both auction formats. Yet, while a sequential auction exposes the bidder to the possibility of experiencing multiple psychological losses, it also allows him to adjust his reference point downward in between rounds so that successive losses become smaller and hurt less. Thus, high-value bidders are more concerned with reducing the probability of experiencing psychological losses altogether; low-value bidders, on the other hand, are more concerned with reducing the magnitude of their losses.

By Proposition 5 and Proposition 6, for bidder types with $\theta \geq \theta_K^*$, the uniform-price auction entails both a lower expected payment and lower expected psychological losses than the sequential second-price auction; hence, they prefer the uniform-price auction.

Corollary 1 (Bidder-Payoff Non-Equivalence). *For $K \geq 2$, let $V_K^{\text{UPA}}(\theta)$ and $V_K^{\text{SPA}}(\theta)$ denote a type-$\theta$ equilibrium total expected payoff in (simultaneous) uniform-price and (sequential) second-price auctions, respectively. Furthermore, let $\theta_K^*$ be defined as in Proposition 6. Then $V_K^{\text{UPA}}(\theta) > V_K^{\text{SPA}}(\theta)$ for $\theta \geq \theta_K^*$.*

Low-type bidders, however, might prefer sequential auctions to simultaneous ones if the higher expected psychological losses in the latter more than outweigh the higher expect payment in the former.

## 4. Alternative disclosure policies

A delicate issue in sequential auctions is what information should the auctioneer reveal between rounds. Following the literature (McAfee and Vincent (1993), Mezzetti (2011), Hu and Zou (2015)), I assumed the seller publicly discloses each round's winning bid; yet, other disclosure policies are possible. Milgrom and Weber (2000) showed that with risk-neutral bidders having independent private values, the seller's disclosure policy is inconsequential and equilibrium bids are the same no matter what information (if any) the seller discloses in between rounds. In this section, I show that different disclosure policies result in different equilibrium bids when bidders are loss averse.

### 4.1 *Sequential auctions with no announcement*

With expectations-based reference-dependent preferences, the bidding strategy depends on the (public) history of the winning bids. Hence, some questions naturally arise: Is the bidding strategy different if the seller commits not to reveal the history of winning bids? Does the rationale for declining prices rely on the history of winning bids being publicly available? I answer these questions in the context of sequential second-price auctions and, for simplicity, I restrict attention to $K = 2$.

Because the seller does not reveal the first-round winning bid, in the second round, a bidder will have to use his own past bid to infer where he stands in the ranking of the remaining bidders' values and update his reference point accordingly. Consider, for instance, a bidder with type $\theta$ who in the first round bid as if his type were $\widetilde{\theta}_1 \neq \theta$ and lost. Then, in the second round, if he bids according to his true type, he expects to win with probability $F_2(\theta | Y_1 > \widetilde{\theta}_1) := (N-1)[1 - F(\widetilde{\theta}_1)]F(\theta)^{N-2}/[1 - F(\widetilde{\theta}_1)^{N-1}]$. The following proposition characterizes the symmetric equilibrium strategies.

Proposition 7. *In the two-round sequential SPA without announcement of the winning bid, the symmetric equilibrium bidding strategies are given by*

$$\beta_1^{\text{SPA-w/o}}(\theta) = \int_0^\theta \beta_2^{\text{SPA-w/o}}(x; x) f_2(x|\theta)\, dx + \Lambda\theta F_1(\theta)$$

*and*

$$\beta_2^{\text{SPA-w/o}}(\theta; \widetilde{\theta}_1) = \theta + \Lambda\theta F_2(\theta | Y_1 > \widetilde{\theta}_1).$$

Comparing the bidding functions in Proposition 7 with those in Proposition 1, it is easy to see some similarities. Namely, in the first round, a bidder with type $\theta$ bids his expectation of the second-round price conditional on being pivotal (i.e., $\int_0^\theta \beta_2^{\text{SPA-w/o}}(x; x) f_2(x|\theta)\, dx$) plus the value of avoiding the psychological loss from failing to win in the first round (i.e., $\Lambda\theta F_1(\theta)$). Hence, as in the case with announcement of the winning bid, loss-averse bidders are willing to pay a premium to win sooner rather than later. Moreover, in the second round, a loss-averse bidder with type $\theta$ bids his loss-adjusted willingness to pay. This modified willingness to pay takes into account the bidder's intrinsic value for the good (i.e., $\theta$) as well as the value of avoiding the psychological feeling of loss he would experience by losing the auction (i.e., $\Lambda\theta F_2(\theta|Y_1 > \widetilde{\theta}_1)$). Yet, now the feeling of loss does not depend on the public history of the game, but rather on the individual bidder's private history; that is, the bidding function in the second round depends on $\widetilde{\theta}_1$—the type the bidder mimicked in the previous auction. Hence, a different form of discouragement effect arises.

Lemma 2 (Discouragement Effect II). *The second-round bidding function is decreasing in the type that a bidder mimicked in the first round; that is,* $\partial\beta_2^{\text{SPA-w/o}}(\theta; \widetilde{\theta}_1)/\partial\widetilde{\theta}_1 < 0\ \forall\theta$.

When the winning bid from the first round is not publicly revealed, a bidder can only use his own first-round bid to assess how likely he is to win in the second one. The higher is the type he pretended to be in the first auction, the less likely he is to win in the second auction, since not winning the first auction, given that he pretended to have a high type, is bad news about how fierce competition is. This, in turn, implies that the higher is the type a bidder mimicked in the first auction, the less he will bid in the second one. Comparing the second-round equilibrium strategies with and without bid announcement yields the following result.

Lemma 3 (Effect of Information I). *Equilibrium bidding in the second round is more aggressive when the seller does not reveal the first-round winning bid if and only if*

$$F(\widetilde{\theta}_1)^{N-1} + (N-1)\big[1 - F(\widetilde{\theta}_1)\big]F(y_1)^{N-2} > 1. \tag{2}$$

Notice that condition (2) can hold only if $y_1 > \widetilde{\theta}_1$. In a sequential auction without revelation of the winning bid, when losing the first round, a bidder only learns that that the first-round winner's type is above $\widetilde{\theta}_1$. By contrast, if the winning bid is announced, the bidder learns that all remaining bidders' types are below $y_1$. Hence, with no bid announcement, bidders are asymmetrically informed about the intensity of competition in the second round, whereas with bid announcement, they all have the same information. Therefore, whether second-round bidding is more aggressive with or without announcement of the first-round winning bid depends on how discouraged a bidder is in each format. Of course, on the equilibrium path, a bidder will behave according to his true type in the first round, so that condition (2) must be evaluated at $\widetilde{\theta}_1 = \theta$. The following lemma compares the first-round equilibrium strategies with and without bid announcement.

Lemma 4 (Effect of Information II). *There exists a threshold type* $\widehat{\theta} \in (0, 1)$ *such that equilibrium bidding in the first round is more aggressive when the seller commits not to reveal the winning bid prior to the second round if and only if* $\theta \geq \widehat{\theta}$.

Hence, high-type bidders bid more in the first round of a sequential SPA without winning bid announcement, whereas low-type bidders do the opposite. To see the intuition, recall that in the first round, a bidder bids his expectation of the second-round price conditional on being pivotal; hence, when anticipating that the winning bid will be announced, a bidder effectively bids as if his own first-round bid will determine the intensity of the discouragement effect for all remaining bidders in the second round. Consider then a bidder with a relatively high value. Such a bidder anticipates that his strongest remaining opponent in the second round (i.e., the second-round price setter) will be more discouraged when the first-round winning bid is announced than when it is not. This, in turn, implies that his expectation of the second-round price is higher without bid announcement. Conversely, a bidder with a relatively low value knows that in the event he is pivotal, in the second round his first-round bid will not generate a strong discouragement effect for his remaining opponents, who will then bid rather aggressively; therefore, such a bidder expects a higher second-round price when the first-round winning bid is revealed. In essence, if the first-round winning bid is publicly disclosed, a bidder accounts for how his first-round bid, in the event of being pivotal, will affect the reference point of his opponents in the second round; this effect of information revelation tends to benefit high-value bidders more than low-value ones.

Lemmas 3 and 4 show that the bidding strategies of loss-averse bidders depend on the seller's disclosure policy. Yet, as the following proposition shows, even without announcement of the winning bid, equilibrium prices are still declining.

Proposition 8 (Afternoon Effect II). *The price sequence in a two-round sequential second-price auction without announcement of the winning bid is a supermartingale; that is, the expected price in the second round conditional on the first-round price is lower than the price in the first round. Hence, the afternoon effect arises in equilibrium.*

The intuition for the above result is the same as that behind Proposition 2: even though there is no informational externality between rounds—as the prior winning bid is not publicly disclosed—loss-averse bidders are still willing to pay a premium above the next round's expected price to avoid the loss they would feel by losing in the current round. Therefore, it is the direct effect of loss aversion on the bidding function, even without revelation of the prior winning bids, that causes prices to decline in equilibrium.

### 4.2 *Revealing the winning price in the sequential SPA*

Unlike the sequential FPA, announcing the winning price in a sequential SPA entails revealing the bid of a bidder who will be present in the next round. In the classical model with private values, this is inconsequential since strategies are history independent. Hence, in the last round, it is still a (weakly) dominant strategy for all remaining bidders to bid their value, and bids in earlier rounds are determined recursively via the usual indifference condition. For loss-averse bidders, however, the history of the game matters and bidding one's value in the last round is not a dominant strategy. Therefore, the existence of a symmetric equilibrium in monotone strategies is not warranted.

Suppose such an equilibrium existed, and consider a bidder who knows that he will likely be the price setter in round $k$ and then win in round $k+1$. Such a bidder has an incentive to raise his bid in round $k$ to discourage his opponents in round $k+1$ and win at a lower price. Indeed, as shown by De Frutos and Rosenthal (1998) and Mezzetti (2011), a similar issue applies to risk-neutral bidders with interdependent values. However, the incentives to deviate for loss-averse bidders with private values are exactly opposite to those of risk-neutral bidders with interdependent values. In the latter case, bidders with relatively low types have an incentive to deviate by decreasing their bid in the current round so as to pay a lower price in the next round in the unlikely (but germane) event that they were to be the price setter. Conversely, with private values and loss aversion, the bidders with relatively high types have an incentive to bid more in the current round to discourage their opponents in the next round in the unlikely (but germane) event that they were to be the price setter.

## 5. Related literature

This paper contributes to two strands of literature. The first is a recent literature on how expectations-based loss aversion affects bidding and revenue in auctions. Lange and Ratan (2010) analyze the FPA and SPA with independent private values using the solution concept of "choice-acclimating personal equilibrium" (CPE) introduced in Kőszegi and Rabin (2007), and show that the FPA raises (weakly) more revenue than the SPA. Using the same equilibrium concept, Eisenhuth (2019) shows that with independent private values, the all-pay auction raises the most revenue among all sealed-bid formats. Balzer and Rosato (2021) analyze the FPA and SPA with interdependent values under both PE and CPE, showing that these two formats are revenue equivalent under the former equilibrium concept, but not under the latter; moreover, they show that the ex ante uncertainty in their valuations leads loss-averse bidders to overbid, thereby exposing them to the "winner's curse" in equilibrium. All the papers mentioned thus far restrict attention to static auctions. Two recent papers compare static and dynamic auctions with loss-averse bidders: Balzer, Rosato, and von Wangenheim (2022) show that the Dutch auction raises more revenue than the FPA, while von Wangenheim (2021) shows that with independent private values, the SPA raises more revenue than the English auction. Different from these previous contributions, my paper is the first to study the role of loss aversion in multi-unit auctions.

The second strand of literature to which this paper contributes is the one on the afternoon effect. Ashenfelter (1989) hypothesized risk aversion as a plausible explanation for declining prices. Yet, McAfee and Vincent (1993) show that equilibrium prices decline only if bidders display increasing absolute risk aversion: under the more plausible assumption of decreasing absolute risk aversion, a monotone pure-strategy equilibrium fails to exist and prices need not decline. Eyster (2002) models the behavior of an agent who has a taste for rationalizing past actions by taking current actions for which those past actions were optimal, and shows that this taste for consistency rationalizes declining prices in sequential auctions. Mezzetti (2011) introduces a special case of risk aversion, called aversion to price risk, according to which a bidder prefers to win an object at a certain price rather than at a random price with the same expected value. Under this different notion of risk aversion, a monotone pure-strategy equilibrium always exists in sequential auctions and prices decline.[11] Although both his model and mine can explain

[11] Hu and Zou (2015) generalize the analysis in Mezzetti (2011) by considering bidders who are heterogeneous in exposure to background risk.

the afternoon effect, the intuition behind the result is quite different. In Mezzetti (2011), the afternoon effect arises because bidders dislike risk in their payment; in my model, instead, the afternoon effect arises because bidders dislike risk over whether they get the good. Finally, another recent paper that explains declining prices by appealing to preferences outside the standard expected utility theory (EUT) framework is Ghosh and Liu (2021), who analyze sequential auctions with ambiguity-averse bidders.[12]

## 6. Conclusions

Sequential auctions are often used by auction houses and internet retailers to sell identical or similar goods. In this paper, I have explored the implications of expectations-based loss aversion for these auctions. Loss-averse bidders update their probability of obtaining a good as the auction progresses and suffer a psychological loss in each round in which they expect to obtain the good but fail to do so; the desire to avoid such losses then leads them to bid more aggressively in earlier rounds. Hence, loss aversion provides an explanation for the declining prices often observed in sequential auctions. Moreover, expectations-based loss aversion creates an informational externality—the discouragement effect—that renders equilibrium strategies history dependent: the higher is the winning bid in the current round, the less aggressively will the remaining bidders bid in the next round. Such a discouragement effect can be used to empirically test the implications of loss aversion against those of the standard model with either private or common values.

In addition to rationalizing declining prices, loss aversion delivers new implications for the design of multi-unit auctions that are of independent interest for theorists and practitioners alike. For example, if bidders are loss averse, sequential auctions raise more revenue than simultaneous ones. Furthermore, depending on the distribution of bidders' values, in sequential auctions, a seller may achieve a higher revenue by concealing winning bids from earlier rounds.

## Appendix: Proofs

*Preliminary Observations.* For most of the proofs in this appendix, it will prove helpful to rewrite $i$'s total expected payoff in round $k$ as the sum of his current-round expected payoff plus his expected payoff in later rounds. For $k \in \{1, \dots, K\}$, using the definition of $U_k(\theta_i, b_{i,k}; y_{k-1}|\beta)$ in Section 2, we have

$$U_k(\theta_i, b_{i,k}; y_{k-1}|\beta)$$
$$= \mathbb{E}_k\left[\sum_{m=k}^{K} u_m(\theta_i, b_{i,m}, p_m; y_{m-1}|\beta)\right]$$

[12]Other authors have proposed non-preference-based explanations for declining prices, such as demand complementarities (Menezes and Monteiro (2003)), supply uncertainty (Jeitschko (1999)), heterogeneous objects (Bernhardt and Scoones (1994), Gale and Stegeman (2001)), order-of-sale effects (Gale and Hausch (1994), Chakraborty, Gupta, and Harbaugh (2006)), and budget constraints (Ghosh and Liu (2019)).

$$
\begin{aligned}
&= \mathbb{E}_k\big[u_k(\theta_i, b_{i,k}, p_k; y_{k-1}|\beta)\big] + \mathbb{E}_k\left[\sum_{m=k+1}^{K} u_m(\theta_i, b_{i,m}, p_m; y_{m-1}|\beta)\right] \\
&= \Pr\Big[b_{i,k} > \max_{j\neq i} b_{j,k} \big| \max_{j\neq i} b_{j,k-1} = \beta_{k-1}(y_{k-1}; y_{k-2})\Big]\Big(\theta_i - \mathbb{E}\Big[p_k \big| b_{i,k} > \max_{j\neq i} b_{j,k}\Big]\Big) \\
&\quad - \Pr\Big[b_{i,k} < \max_{j\neq i} b_{j,k} \big| \max_{j\neq i} b_{j,k-1} = \beta_{k-1}(y_{k-1}; y_{k-2})\Big]\Lambda\theta_i Q_{i,k}^k \\
&\quad + \Pr\Big[b_{i,k} < \max_{j\neq i} b_{j,k} \big| \max_{j\neq i} b_{j,k-1} = \beta_{k-1}(y_{k-1}; y_{k-2})\Big] \\
&\quad \times \mathbb{E}_k\left[\sum_{m=k+1}^{K} u_m(\theta_i, b_{i,m}, p_m; y_{m-1}|\beta)\right] \\
&= \Pr\Big[b_{i,k} > \max_{j\neq i} b_{j,k} \big| \max_{j\neq i} b_{j,k-1} = \beta_{k-1}(y_{k-1}; y_{k-2})\Big]\Big(\theta_i - \mathbb{E}\Big[p_k \big| b_{i,k} > \max_{j\neq i} b_{j,k}\Big]\Big) \\
&\quad - \Pr\Big[b_{i,k} < \max_{j\neq i} b_{j,k} \big| \max_{j\neq i} b_{j,k-1} = \beta_{k-1}(y_{k-1}; y_{k-2})\Big]\Lambda\theta_i Q_{i,k}^k \\
&\quad + \Pr\Big[b_{i,k} < \max_{j\neq i} b_{j,k} \big| \max_{j\neq i} b_{j,k-1} = \beta_{k-1}(y_{k-1}; y_{k-2})\Big]\mathbb{E}_k\big[U_{k+1}(\theta_i, b_{i,k+1}; y_k|\beta)\big].
\end{aligned}
$$

Moreover, with symmetric strategies and winning-bid announcements, it holds that

$$
\begin{aligned}
Q_{i,k}^k &= \Pr\Big[b_{i,k} > \max_{j\neq i} b_{j,k} \big| \max_{j\neq i} b_{j,k-1} = \beta_{k-1}(y_{k-1}; y_{k-2})\Big] \\
&= \Pr\big[Y_k^{(N-1)} \leq \theta_i | Y_{k-1}^{(N-1)} = y_{k-1}\big] \\
&= F_k(\theta_i | y_{k-1}).
\end{aligned}
$$

In the proofs, to simplify the notation, I am going to suppress the dependence of the bidder's payoff on the bid and strategy profile; hence, slightly abusing notation, I will denote a type-$\theta$ equilibrium expected utility in round $k$ by $U_k^*(\theta; y_{k-1})$, and denote the utility associated with a deviation in round $k$ by $U_k(\theta, \widetilde{\theta}; y_{k-1})$.

Proof of Proposition 1. Let $\beta_k^{\mathrm{SPA}}(\theta; y_{k-1})$ be the round-$k$ posited equilibrium bidding function and denote by $U_k^*(\theta; y_{k-1})$ the expected utility of a type-$\theta$ bidder in the continuation equilibrium at the beginning of round $k$, conditional on having lost all previous $k-1$ auctions and on the history of the winning bids. Suppose all the other bidders follow their equilibrium strategies, while bidder $i$ is considering deviating in round $k$ (only). Then the payoff of bidder $i$ with type $\theta$ if he bids as if he had type $\widetilde{\theta}$ is

$$
\begin{aligned}
U_k(\theta, \widetilde{\theta}; y_{k-1}) = &\int_0^{\widetilde{\theta}} \big[\theta - \beta_k^{\mathrm{SPA}}(y_k; y_{k-1})\big] f_k(y_k | y_{k-1})\, dy_k \\
&- \Lambda \int_{\widetilde{\theta}}^{y_{k-1}} \theta F_k(\theta | y_{k-1}) f_k(y_k | y_{k-1})\, dy_k \\
&+ \int_{\widetilde{\theta}}^{y_{k-1}} U_{k+1}^*(\theta; y_k) f_k(y_k | y_{k-1})\, dy_k,
\end{aligned} \tag{3}
$$

where

$$U^*_{k+1}(\theta; y_k) = \begin{cases} \displaystyle\int_0^{\theta} \big[\theta - \beta^{\mathrm{SPA}}_{k+1}(y_{k+1}; y_k)\big] f_{k+1}(y_{k+1}|y_k)\, dy_{k+1} & \\ \displaystyle\quad - \Lambda \int_{\theta}^{y_k} \theta F_{k+1}(\theta|y_k) f_{k+1}(y_{k+1}|y_k)\, dy_{k+1} & \\ \displaystyle\quad + \int_{\theta}^{y_k} U^*_{k+2}(\theta; y_{k+1}) f_{k+1}(y_{k+1}|y_k)\, dy_{k+1} & \text{if } \theta < y_k \\ \displaystyle\theta - \int_0^{y_k} \beta^{\mathrm{SPA}}_{k+1}(y_{k+1}; y_k) f_{k+1}(y_{k+1}|y_k)\, dy_{k+1} & \text{if } \theta \geq y_k \end{cases}$$

for $k \in \{1, \dots, K-1\}$, as he wins for sure in round $k+1$ if $\theta \geq y_k$, and $U^*_{k+1}(\theta; y_k) = 0$ for $k = K$. Differentiating (3) with respect to $\widetilde{\theta}$ yields

$$\begin{aligned} \frac{\partial U_k(\theta, \widetilde{\theta}; y_{k-1})}{\partial \widetilde{\theta}} &= \big[\theta - \beta^{\mathrm{SPA}}_k(\widetilde{\theta}; y_{k-1})\big] f_k(\widetilde{\theta}|y_{k-1}) + \Lambda \theta F_k(\theta|y_{k-1}) f_k(\widetilde{\theta}|y_{k-1}) \\ &\quad - U^*_{k+1}(\theta; \widetilde{\theta}) f_k(\widetilde{\theta}|y_{k-1}). \end{aligned}$$

The first-order condition requires that $\frac{\partial U_k(\theta, \widetilde{\theta}; y_{k-1})}{\partial \widetilde{\theta}}|_{\widetilde{\theta}=\theta} = 0$; hence, we obtain the necessary condition

$$\big\{\theta - \beta^{\mathrm{SPA}}_k(\theta; y_{k-1}) + \Lambda \theta F_k(\theta|y_{k-1}) - U^*_{k+1}(\theta; \theta)\big\} f_k(\theta|y_{k-1}) = 0. \tag{4}$$

Substituting for $U^*_{k+1}(\theta; \theta)$ into condition (4) and rearranging it yields

$$\beta^{\mathrm{SPA}}_k(\theta; y_{k-1}) = \int_0^{\theta} \beta^{\mathrm{SPA}}_{k+1}(y_{k+1}; \theta) f_{k+1}(y_{k+1}|\theta)\, dy_{k+1} + \Lambda \theta F_k(\theta|y_{k-1})$$

for $k \in \{1, \dots, K-1\}$ and

$$\beta^{\mathrm{SPA}}_K(\theta; y_{K-1}) = \theta + \Lambda \theta F_K(\theta|y_{K-1})$$

for $k = K$. Moreover, it is easy to verify that the bidding functions are increasing in $\theta$. Hence, for $k = 2, \dots, K$, the bidder is guaranteed to win in round $k$ by bidding $\beta^{\mathrm{SPA}}_k(y_{k-1}; y_{k-1}) \leq \beta^{\mathrm{SPA}}_k(\theta; y_{k-1})$ if $y_{k-1} \leq \theta$.

Hence, it only remains to show that the first-order conditions are sufficient for equilibrium. First, let $k = K$. Using condition (4) to substitute for $\beta^{\mathrm{SPA}}_K(\widetilde{\theta}; y_{K-1})$ into the expression for $\partial U_K(\theta, \widetilde{\theta}; y_{K-1})/\partial \widetilde{\theta}$ yields

$$\frac{\partial U_K(\theta, \widetilde{\theta}; y_{K-1})}{\partial \widetilde{\theta}} = \big\{\theta - \widetilde{\theta} + \Lambda\big[\theta F_K(\theta|y_{K-1}) - \widetilde{\theta} F_K(\widetilde{\theta}|y_{K-1})\big]\big\} f_K(\widetilde{\theta}|y_{K-1}).$$

Hence, $\partial U_K(\theta, \widetilde{\theta}; y_{K-1})/\partial \widetilde{\theta}$ has the same sign as $(\theta - \widetilde{\theta})$; thus, $\widetilde{\theta} = \theta$ is optimal.

Next, let $k \in \{1, \dots, K-1\}$. Using condition (4) to substitute for $\beta^{\mathrm{SPA}}_k(\widetilde{\theta}; y_{k-1})$ into the expression for $\partial U_k(\theta, \widetilde{\theta}; y_{k-1})/\partial \widetilde{\theta}$ yields

$$\begin{aligned} &\frac{\partial U_k(\theta, \widetilde{\theta}; y_{k-1})}{\partial \widetilde{\theta}} \\ &\quad = \big\{\theta - \widetilde{\theta} + \Lambda\big[\theta F_k(\theta|y_{k-1}) - \widetilde{\theta} F_k(\widetilde{\theta}|y_{k-1})\big] + U^*_{k+1}(\widetilde{\theta}; \widetilde{\theta}) - U^*_{k+1}(\theta; \widetilde{\theta})\big\} f_k(\widetilde{\theta}|y_{k-1}). \end{aligned}$$

Take first the case $\widetilde{\theta} \leq \theta$. In this case,

$$U^*_{k+1}(\theta; \widetilde{\theta}) = \theta - \int_0^{\widetilde{\theta}} \beta^{\text{SPA}}_{k+1}(y_{k+1}; \widetilde{\theta}) f_{k+1}(y_{k+1}|\widetilde{\theta})\, dy_{k+1}$$

and, therefore,

$$U^*_{k+1}(\widetilde{\theta}; \widetilde{\theta}) - U^*_{k+1}(\theta; \widetilde{\theta}) = \widetilde{\theta} - \theta.$$

Hence, $\partial U_k(\theta, \widetilde{\theta}; y_{k-1})/\partial\widetilde{\theta} \geq 0$ for $\widetilde{\theta} \leq \theta$. Next consider the case $\widetilde{\theta} > \theta$. In this case,

$$\begin{aligned}
&U^*_{k+1}(\widetilde{\theta}; \widetilde{\theta}) - U^*_{k+1}(\theta; \widetilde{\theta}) \\
&\quad = \widetilde{\theta} - \int_0^{\theta} \theta f_{k+1}(y_{k+1}|\widetilde{\theta})\, dy_{k+1} - \int_{\theta}^{\widetilde{\theta}} \beta^{\text{SPA}}_{k+1}(y_{k+1}; \widetilde{\theta}) f_{k+1}(y_{k+1}|\widetilde{\theta})\, dy_{k+1} \\
&\qquad + \Lambda \int_{\theta}^{\widetilde{\theta}} \theta F_{k+1}(\theta|\widetilde{\theta}) f_{k+1}(y_{k+1}|\widetilde{\theta})\, dy_{k+1} - \int_{\theta}^{\widetilde{\theta}} U^*_{k+2}(\theta; y_{k+1}) f_{k+1}(y_{k+1}|\widetilde{\theta})\, dy_{k+1}.
\end{aligned}$$

Hence, substituting and rearranging, we have

$$\begin{aligned}
&\frac{\partial U_k(\theta, \widetilde{\theta}; y_{k-1})}{\partial\widetilde{\theta}} \\
&\quad = \left\{\int_{\theta}^{\widetilde{\theta}} \theta f_{k+1}(y_{k+1}|\widetilde{\theta})\, dy_{k+1} - \int_{\theta}^{\widetilde{\theta}} \beta^{\text{SPA}}_{k+1}(y_{k+1}; \widetilde{\theta}) f_{k+1}(y_{k+1}|\widetilde{\theta})\, dy_{k+1}\right\} f_k(\widetilde{\theta}|y_{k-1}) \\
&\qquad + \left\{\Lambda \int_{\theta}^{\widetilde{\theta}} \theta F_{k+1}(\theta|\widetilde{\theta}) f_{k+1}(y_{k+1}|\widetilde{\theta})\, dy_{k+1} - \int_{\theta}^{\widetilde{\theta}} U^*_{k+2}(\theta; y_{k+1}) f_{k+1}(y_{k+1}|\widetilde{\theta})\, dy_{k+1}\right\} \\
&\qquad \times f_k(\widetilde{\theta}|y_{k-1}) \\
&\qquad + \Lambda\big[\theta F_k(\theta|y_{k-1}) - \widetilde{\theta} F_k(\widetilde{\theta}|y_{k-1})\big] f_k(\widetilde{\theta}|y_{k-1}).
\end{aligned}$$

Moreover, notice that

$$\begin{aligned}
\beta^{\text{SPA}}_{k+1}(y_{k+1}; \widetilde{\theta}) &= y_{k+1} + \Lambda y_{k+1} F_{k+1}(y_{k+1}|\widetilde{\theta}) - U^*_{k+2}(y_{k+1}; y_{k+1}) \\
&\geq \theta + \Lambda\theta F_{k+1}(\theta|\widetilde{\theta}) - U^*_{k+2}(\theta; y_{k+1}),
\end{aligned}$$

where the equality follows from (4), while the inequality follows from the fact that $y_{k+1} \in [\theta, \widetilde{\theta}]$. Hence, $\partial U_k(\theta, \widetilde{\theta}; y_{k-1})/\partial\widetilde{\theta} < 0$ for $\widetilde{\theta} > \theta$. Therefore, $\partial U_k(\theta, \widetilde{\theta}; y_{k-1})/\partial\widetilde{\theta}$ has the same sign as $(\theta - \widetilde{\theta})$; thus, $\widetilde{\theta} = \theta$ is optimal. □

The proof of Lemma 1 follows since $F_k(\theta|y_{k-1})$ is decreasing in $y_{k-1}$.

Proof of Proposition 2. So as to compute prices and expected prices in each round, we need to take the point of view of the seller. Hence, we will use $Y_k^{(N)}$ to denote the $k$th highest order statistic among $N$ (i.e., when taking the point of view of the seller) and use $Y_k^{(N-1)}$ to denote the $k$th highest order statistic among $N-1$ (i.e., when taking the point of view of a bidder). We will use a similar notation for the CDFs and PDFs of $Y_k^{(N)}$ and

$Y_k^{(N-1)}$, respectively. In a symmetric equilibrium, the winner in round $k$ is the bidder with the $k$th highest type and the price setter is the bidder with the next highest type. Hence, we have

$$\begin{aligned} p_k &= \beta_k^{\mathrm{SPA}}\big(Y_{k+1}^{(N)} = y_{k+1}; Y_{k-1}^{(N)} = y_{k-1}\big) \\ &= \int_0^{y_{k+1}} \beta_{k+1}^{\mathrm{SPA}}(\theta; y_{k+1}) f_{k+1}^{(N-1)}(\theta|y_{k+1})\, d\theta + \Lambda y_{k+1} F_k^{(N-1)}(y_{k+1}|y_{k-1}) \\ &> \int_0^{y_{k+1}} \beta_{k+1}^{\mathrm{SPA}}(\theta; y_k) f_{k+2}^{(N)}(\theta|y_{k+1})\, d\theta \\ &= \mathbb{E}\big[p_{k+1}|p_k = \beta_k^{\mathrm{SPA}}(y_{k+1}; y_{k-1})\big], \end{aligned}$$

where the inequality follows since $f_{k+1}^{(N-1)}(\cdot|y_{k+1}) = f_{k+2}^{(N)}(\cdot|y_{k+1})$, $\beta_{k+1}^{\mathrm{SPA}}(\theta; y_{k+1}) > \beta_{k+1}^{\mathrm{SPA}}(\theta; y_k)$ by Lemma 1 and $\Lambda y_{k+1} F_k^{(N-1)}(y_{k+1}|y_{k-1}) > 0$. □

Proof of Proposition 4. Let $\beta^{\mathrm{UPA}}(\theta)$ be the posited equilibrium bidding function and suppose that all other bidders follow their equilibrium strategies, while bidder $i$ considers deviating. The payoff of bidder $i$ with type $\theta$ if he bids as if he had type $\widetilde{\theta}$ is

$$U(\theta, \widetilde{\theta}) = \int_0^{\widetilde{\theta}} \big[\theta - \beta^{\mathrm{UPA}}(y_K)\big] f_K(y_K)\, dy_K - \Lambda\theta \int_{\widetilde{\theta}}^1 F_K(\theta) f_K(y_K)\, dy_K. \tag{5}$$

Differentiating (5) with respect to $\widetilde{\theta}$ yields the first-order condition

$$\big[\theta - \beta^{\mathrm{UPA}}(\widetilde{\theta})\big] f_K(\widetilde{\theta}) + \Lambda\theta F_K(\theta) f_K(\widetilde{\theta}) = 0.$$

In equilibrium, $\widetilde{\theta} = \theta$ must hold; hence, we obtain the necessary condition

$$\big[\theta - \beta^{\mathrm{UPA}}(\theta) + \Lambda\theta F_K(\theta)\big] f_K(\theta) = 0. \tag{6}$$

Simplifying and rearranging the above condition yields

$$\beta^{\mathrm{UPA}}(\theta) = \theta + \Lambda\theta F_K(\theta).$$

It is easy to verify that the bidding functions are increasing in $\theta$. Hence, it only remains to show that the first-order conditions are sufficient for equilibrium. Using condition (6) to substitute for $\beta^{\mathrm{UPA}}(\widetilde{\theta})$ into the expression for $\partial U(\theta, \widetilde{\theta})/\partial\widetilde{\theta}$ yields

$$\begin{aligned} \frac{\partial U(\theta, \widetilde{\theta})}{\partial \widetilde{\theta}} &= \big[\theta - \widetilde{\theta} - \Lambda\widetilde{\theta} F_K(\widetilde{\theta})\big] f_K(\widetilde{\theta}) + \Lambda\theta F_K(\theta) f_K(\widetilde{\theta}) \\ &= \big\{\theta - \widetilde{\theta} + \Lambda\big[\theta F_K(\theta) - \widetilde{\theta} F_K(\widetilde{\theta})\big]\big\} f_K(\widetilde{\theta}). \end{aligned}$$

Hence, $\partial U(\theta, \widetilde{\theta})/\partial\widetilde{\theta}$ is the same sign as $(\theta - \widetilde{\theta})$; thus, $\widetilde{\theta} = \theta$ is optimal. □

Proof of Proposition 5. We prove the result by induction on $K$.

*Base Case*: $K = 2$. We show that $P_2^{\mathrm{SPA}}(\theta) \geq P_2^{\mathrm{UPA}}(\theta)$ for all $\theta \in [0, 1]$. In equilibrium, the expected payment of a type-$\theta$ bidder in a two-round sequential SPA is

$$\begin{aligned}
P_2^{\mathrm{SPA}}(\theta) &= \int_0^\theta \beta_1^{\mathrm{SPA}}(y_1) f_1(y_1)\, dy_1 + \int_\theta^1 \int_0^\theta \beta_2^{\mathrm{SPA}}(y_2; y_1) f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1 \\
&= \int_0^\theta \left[\int_0^{y_1} \beta_2^{\mathrm{SPA}}(y_2; y_1) f_2(y_2|y_1)\, dy_2 + \Lambda y_1 F_1(y_1)\right] f_1(y_1)\, dy_1 \\
&\quad + \int_\theta^1 \left[\int_0^\theta \left[y_2 + \Lambda y_2 F_2(y_2|y_1)\right] f_2(y_2|y_1)\, dy_2\right] f_1(y_1)\, dy_1 \\
&= \int_0^\theta \Lambda y_1 F_1(y_1) f_1(y_1)\, dy_1 + \int_0^\theta \left[\int_0^{y_1} \left[y_2 + \Lambda y_2 F_2(y_2|y_1)\right] f_2(y_2|y_1)\, dy_2\right] f_1(y_1)\, dy_1 \\
&\quad + \int_\theta^1 \left[\int_0^\theta \left[y_2 + \Lambda y_2 F_2(y_2|y_1)\right] f_2(y_2|y_1)\, dy_2\right] f_1(y_1)\, dy_1.
\end{aligned}$$

Similarly, the expected payment of a type-$\theta$ bidder in a uniform-price auction is

$$\begin{aligned}
P_2^{\mathrm{UPA}}(\theta) &= \int_0^\theta \beta^{\mathrm{UPA}}(y_2) f_2(y_2)\, dy_2 \\
&= \int_0^\theta \left[y_2 + \Lambda y_2 F_2(y_2)\right] f_2(y_2)\, dy_2.
\end{aligned}$$

Let $\phi_2(\theta, N) := P_2^{\mathrm{SPA}}(\theta) - P_2^{\mathrm{UPA}}(\theta)$. Since

$$\int_0^\theta \int_0^{y_1} y_2 f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1 + \int_\theta^1 \int_0^\theta y_2 f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1 = \int_0^\theta y_2 f_2(y_2)\, dy_2,$$

it follows that

$$\begin{aligned}
\phi_2(\theta, N) = &\int_0^\theta \int_0^{y_1} \Lambda y_2 F_2(y_2|y_1) f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1 \\
&+ \int_\theta^1 \int_0^\theta \Lambda y_2 F_2(y_2|y_1) f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1 \\
&+ \int_0^\theta \Lambda y_1 F_1(y_1) f_1(y_1)\, dy_1 - \int_0^\theta \Lambda y_2 F_2(y_2) f_2(y_2)\, dy_2.
\end{aligned}$$

Furthermore, notice that $\phi_2(\theta, N)|_{\theta=0} = 0$, while

$$\begin{aligned}
\phi_2(\theta, N)|_{\theta=1} = &\int_0^1 \int_0^{y_1} \Lambda y_2 F_2(y_2|y_1) f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1 \\
&+ \int_0^1 \Lambda y_1 F_1(y_1) f_1(y_1)\, dy_1 - \int_0^1 \Lambda y_2 F_2(y_2) f_2(y_2)\, dy_2
\end{aligned}$$

$$= \int_0^1 \int_0^{y_1} \Lambda y_2 F_2(y_2|y_1) f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1 + \frac{1}{2}\int_0^1 \Lambda\big[F_2(x)^2 - F_1(x)^2\big]\, dx$$
$$> 0,$$

where the second equality follows since, using integration by parts, $\int_0^1 y_1 F_1(y_1) \times f_1(y_1)\, dy_1 = \frac{1-\int_0^1 F_1(y_1)^2\, dy_1}{2}$ and $\int_0^1 y_2 F_2(y_2) f_2(y_2)\, dy_2 = \frac{1-\int_0^1 F_2(y_2)^2\, dy_2}{2}$.

Hence, a sufficient condition for $\phi_2(\theta, N) \geq 0$ to hold for all $\theta \in [0, 1]$ is that $\frac{\partial \phi_2(\theta,N)}{\partial \theta} \geq 0$. We have

$$\begin{aligned}\frac{\partial \phi_2(\theta, N)}{\partial \theta} &= \int_\theta^1 \Lambda\theta F_2(\theta|y_1) f_2(\theta|y_1) f_1(y_1)\, dy_1 + \Lambda\theta F_1(\theta) f_1(\theta) - \Lambda\theta F_2(\theta) f_2(\theta) \\ &= \Lambda\theta(N-1)(N-2)F(\theta)^{2N-5} f(\theta) \int_\theta^1 \frac{f(y_1)}{F(y_1)^{N-2}}\, dy_1 \\ &\quad + \Lambda\theta F(\theta)^{N-1}(N-1)F(\theta)^{N-2} f(\theta) \\ &\quad - \Lambda\theta\big[F(\theta)^{N-1} + (N-1)\big[1-F(\theta)\big]F(\theta)^{N-2}\big] \\ &\quad \times (N-1)\big[1-F(\theta)\big](N-2)F(\theta)^{N-3} f(\theta).\end{aligned}$$

Next, notice that

$$\int \frac{f(x)}{F(x)^{N-2}}\, dx = \begin{cases} \ln F(x) & \text{if } N = 3 \\ \dfrac{F(x)^{3-N}}{3-N} & \text{if } N \geq 4. \end{cases}$$

Hence,

$$\frac{\partial \phi_2(\theta, N)}{\partial \theta} = \Lambda\theta 2F(\theta) f(\theta)\big\{F(\theta)^2 - \ln F(\theta) - \big[1-F(\theta)\big]\big[2-F(\theta)\big]\big\} \quad \text{for } N = 3$$

and

$$\begin{aligned}&\frac{\partial \phi_2(\theta, N)}{\partial \theta} \\ &\quad = \Bigg(F(\theta)^{N-1} \\ &\qquad + \frac{N-2}{N-3}\big\{1 - F(\theta)^{N-3} - (N-3)\big[1-F(\theta)\big]F(\theta)^{N-3}\big[N-1-(N-2)F(\theta)\big]\big\}\Bigg) \\ &\qquad /\big[\Lambda\theta(N-1)F(\theta)^{N-2} f(\theta)\big]^{-1} \quad \text{for } N \geq 4.\end{aligned}$$

Consider first the case $N = 3$. Notice that $\frac{\partial \phi_2(\theta,3)}{\partial \theta}|_{\theta=0} = 0$ and $\frac{\partial \phi_2(\theta,3)}{\partial \theta}|_{\theta=1} = 2\Lambda f(1) > 0$. Moreover, for $\theta \in (0, 1)$, the sign of $\frac{\partial \phi_2(\theta,3)}{\partial \theta}$ is equal to the sign of $F(\theta)^2 - \ln F(\theta) - [1 - F(\theta)][2 - F(\theta)]$; this expression is minimized for $F(\theta) = \frac{1}{3}$ and $F(\theta)^2 - \ln F(\theta) - [1 - F(\theta)][2 - F(\theta)]|_{\theta = F^{-1}(\frac{1}{3})} = \ln 3 - 1 > 0$. Hence, $\frac{\partial \phi_2(\theta,3)}{\partial \theta} \geq 0$.

Next, consider the case $N \geq 4$. Notice that $\frac{\partial \phi_2(\theta,N)}{\partial\theta}|_{\theta=0} = 0$ and $\frac{\partial \phi_2(\theta,N)}{\partial\theta}|_{\theta=1} = \Lambda(N-1)f(1) > 0$. Moreover, for $\theta \in (0,1)$, the sign of $\frac{\partial \phi_2(\theta,N)}{\partial\theta}$ is equal to the sign of

$$F(\theta)^{N-1} + \frac{N-2}{N-3}\big\{1 - F(\theta)^{N-3} - (N-3)\big[1-F(\theta)\big]F(\theta)^{N-3}\big[N-1-(N-2)F(\theta)\big]\big\}. \quad (7)$$

Expression (7) is minimized for $F(\theta) = \frac{20N-11N^2+2N^3-12-\sqrt{(5N-6)(N-2)^3}}{2(N-1)^2(N-3)}$ and it can be verified numerically or with a plot that for $\theta = F^{-1}(\frac{20N-11N^2+2N^3-12-\sqrt{(5N-6)(N-2)^3}}{2(N-1)^2(N-3)})$, expression (7) is strictly positive for any $N \geq 4$.

Therefore, for $N \geq 3$, it holds that $\frac{\partial \phi_2(\theta,N)}{\partial\theta} \geq 0$, which in turn implies that $\phi_2(\theta,N) := P_2^{\mathrm{SPA}}(\theta) - P_2^{\mathrm{UPA}}(\theta) \geq 0$ for all $\theta \in [0,1]$.

*Induction Step.* For $K > 2$, we show that if $P_K^{\mathrm{SPA}}(\theta) \geq P_K^{\mathrm{UPA}}(\theta)$ for all $\theta \in [0,1]$, then $P_{K+1}^{\mathrm{SPA}}(\theta) \geq P_{K+1}^{\mathrm{UPA}}(\theta)$ for all $\theta \in [0,1]$. Begin by noticing that the expected payment of a type-$\theta$ bidder in a uniform-price auction with $K$ units is

$$\begin{aligned} P_K^{\mathrm{UPA}}(\theta) &= \int_0^\theta \beta^{\mathrm{UPA}}(y_K) f_K(y_K)\, dy_K \\ &= \int_0^\theta \big[y_K + \Lambda y_K F_K(y_K)\big] f_K(y_K)\, dy_K. \end{aligned}$$

The expression for the expected payment of a type-$\theta$ bidder in a $K$-round sequential SPA is rather cumbersome, so I will briefly describe it in words first. From the point of view of a bidder with type $\theta$, his marginal opponent—the bidder that he must outbid in order to win an item—is the one with the $K$th highest value (among $N-1$). Hence, in each round $k \in \{1,\dots,K\}$ in which he is active, a type $\theta$ expects to pay a price equal to the expected value, conditional on the information available in round $k$, of the loss-adjusted willingness to pay of the marginal opponent. There are $K$ such terms. Moreover, as he prefers to win sooner rather than later, in each round, the bidder also expects to pay a premium equal to the future avoided losses. For example, in the first round, the bidder expects to pay a premium, if he wins, equal to the avoided losses in rounds 1 through $K$. Similarly, in the second round, the bidder expects to pay a premium equal to the avoided losses in rounds 2 through $K$ and so on. Hence, the number of premia per round decreases throughout the auction and there are $\frac{K(K-1)}{2}$ such terms in total. Therefore, the expected payment of a type-$\theta$ bidder in a $K$-round sequential SPA consists of $\frac{K(K+1)}{2}$ terms and can be written as

$$\begin{aligned} &P_K^{\mathrm{SPA}}(\theta) \\ &\quad= \int_0^\theta \beta_1^{\mathrm{SPA}}(y_1) f_1(y_1)\, dy_1 + \int_\theta^1 \int_0^\theta \beta_2^{\mathrm{SPA}}(y_2; y_1) f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1 \\ &\qquad+ \underbrace{\cdots}_{\text{expected payments in rounds 3 through } K-1} \\ &\qquad+ \int_\theta^1 \cdots \int_0^\theta \beta_K^{\mathrm{SPA}}(y_K; y_{K-1}) f_K(y_K|y_{K-1})\, dy_K \cdots f_1(y_1)\, dy_1 \end{aligned}$$

$$= \int_0^\theta \Lambda y_1 F_1(y_1) f_1(y_1)\, dy_1 + \int_0^\theta \int_0^{y_1} \Lambda y_2 F_2(y_2|y_1) f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1$$
$$+ \int_\theta^1 \int_0^\theta \Lambda y_2 F_2(y_2|y_1) f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1$$
$$+ \underbrace{\cdots}_{\text{expected losses avoided in rounds 3 through } K-1}$$
$$+ \int_0^\theta \int_0^{y_1} \cdots \int_0^{y_{K-1}} \big[y_K + \Lambda y_K F_K(y_K|y_{K-1})\big]$$
$$\times f_K(y_K|y_{K-1})\, dy_K \cdots f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1$$
$$+ \underbrace{\cdots}_{\text{expectations of the loss-adjusted type of the marginal opponent in rounds 2 through } K-1}$$
$$+ \int_\theta^1 \int_\theta^{y_1} \cdots \int_0^\theta \big[y_K + \Lambda y_K F_K(y_K|y_{K-1})\big] f_K(y_K|y_{K-1})\, dy_K \cdots f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1.$$

Since

$$\int_0^\theta y_K f_K(y_K)\, dy_K = \int_0^\theta \int_0^{y_1} \cdots \int_0^{y_{K-1}} y_K f_K(y_K|y_{K-1})\, dy_K \cdots f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1$$
$$+ \cdots + \int_\theta^1 \int_\theta^{y_1} \cdots \int_0^\theta y_K f_K(y_K|y_{K-1})\, dy_K \cdots f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1,$$

it follows that the sign of $P_K^{\mathrm{SPA}}(\theta) - P_K^{\mathrm{UPA}}(\theta)$ depends only on the terms multiplied by $\Lambda$. Then, by the induction hypothesis, we have that

$$P_K^{\mathrm{SPA}}(\theta) - P_K^{\mathrm{UPA}}(\theta) \geq 0$$
$$\Leftrightarrow \quad \int_0^\theta \Lambda y_1 F_1(y_1) f_1(y_1)\, dy_1 + \int_0^\theta \int_0^{y_1} \Lambda y_2 F_2(y_2|y_1) f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1$$
$$+ \int_\theta^1 \int_0^\theta \Lambda y_2 F_2(y_2|y_1) f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1$$
$$+ \underbrace{\cdots}_{\text{expected losses avoided in rounds 3 through } K-1}$$
$$+ \int_0^\theta \int_0^{y_1} \cdots \int_0^{y_{K-1}} \Lambda y_K F_K(y_K|y_{K-1})$$
$$\times f_K(y_K|y_{K-1})\, dy_K \cdots f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1 + \cdots$$
$$+ \int_\theta^1 \int_\theta^{y_1} \cdots \int_0^\theta \Lambda y_K F_K(y_K|y_{K-1})$$
$$\times f_K(y_K|y_{K-1})\, dy_K \cdots f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1$$
$$\geq \int_0^\theta \Lambda y_K F_K(y_K) f_K(y_K)\, dy_K. \tag{8}$$

Next, we have that

$$P_{K+1}^{\mathrm{UPA}}(\theta) = \int_0^\theta \big[y_{K+1} + \Lambda y_{K+1} F_{K+1}(y_{K+1})\big] f_{K+1}(y_{K+1})\,dy_{K+1}$$

and

$$\begin{aligned}
P_{K+1}^{\mathrm{SPA}}(\theta) = {} & \int_0^\theta \Lambda y_1 F_1(y_1) f_1(y_1)\,dy_1 + \int_0^\theta \int_0^{y_1} \Lambda y_2 F_2(y_2|y_1) f_2(y_2|y_1)\,dy_2 f_1(y_1)\,dy_1 \\
& + \int_\theta^1 \int_0^\theta \Lambda y_2 F_2(y_2|y_1) f_2(y_2|y_1)\,dy_2 f_1(y_1)\,dy_1 \\
& + \underbrace{\cdots}_{\text{expected losses avoided in rounds 3 through } K} \\
& + \int_0^\theta \int_0^{y_1} \cdots \int_0^{y_K} \big[y_{K+1} + \Lambda y_{K+1} F_{K+1}(y_{K+1}|y_K)\big] \\
& \times f_{K+1}(y_{K+1}|y_K)\,dy_{K+1} \cdots f_2(y_2|y_1)\,dy_2 f_1(y_1)\,dy_1 \\
& + \underbrace{\cdots}_{\text{expectations of the loss-adjusted type of the marginal opponent in rounds 2 through } K} \\
& + \int_\theta^1 \int_\theta^{y_1} \cdots \int_0^\theta \big[y_{K+1} + \Lambda y_{K+1} F_{K+1}(y_{K+1}|y_K)\big] \\
& \times f_{K+1}(y_{K+1}|y_K)\,dy_{K+1} \cdots f_2(y_2|y_1)\,dy_2 f_1(y_1)\,dy_1.
\end{aligned}$$

Hence,

$$\begin{aligned}
& P_{K+1}^{\mathrm{SPA}}(\theta) - P_{K+1}^{\mathrm{UPA}}(\theta) \geq 0 \\
\Leftrightarrow \quad & \int_0^\theta \Lambda y_1 F_1(y_1) f_1(y_1)\,dy_1 + \int_0^\theta \int_0^{y_1} \Lambda y_2 F_2(y_2|y_1) f_2(y_2|y_1)\,dy_2 f_1(y_1)\,dy_1 \\
& \quad + \int_\theta^1 \int_0^\theta \Lambda y_2 F_2(y_2|y_1) f_2(y_2|y_1)\,dy_2 f_1(y_1)\,dy_1 \\
& \quad + \underbrace{\cdots}_{\text{expected losses avoided in rounds 3 through } K} \\
& \quad + \int_0^\theta \int_0^{y_1} \cdots \int_0^{y_K} \Lambda y_{K+1} F_{K+1}(y_{K+1}|y_K) \\
& \quad \times f_{K+1}(y_{K+1}|y_K)\,dy_{K+1} \cdots f_2(y_2|y_1)\,dy_2 f_1(y_1)\,dy_1 + \cdots \\
& \quad + \int_\theta^1 \int_\theta^{y_1} \cdots \int_0^\theta \Lambda y_{K+1} F_{K+1}(y_{K+1}|y_K) \\
& \quad \times f_{K+1}(y_{K+1}|y_K)\,dy_{K+1} \cdots f_2(y_2|y_1)\,dy_2 f_1(y_1)\,dy_1 \\
& \geq \int_0^\theta \Lambda y_{K+1} F_{K+1}(y_{K+1}) f_{K+1}(y_{K+1})\,dy_{K+1}.
\end{aligned} \tag{9}$$

Notice that the first $\frac{(K+1)K}{2}$ terms on the left-hand side of (9) are exactly the same as the terms on the left-hand side of (8). Hence, by the induction hypothesis, the sufficient condition for (9) to hold is

$$\begin{aligned}
&\int_0^\theta \int_0^{y_1} \cdots \int_0^{y_K} \Lambda y_{K+1} F_{K+1}(y_{K+1}|y_K) \\
&\quad \times f_{K+1}(y_{K+1}|y_K)\, dy_{K+1} \cdots f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1 + \cdots \\
&\quad + \int_\theta^1 \int_\theta^{y_1} \cdots \int_0^\theta \Lambda y_{K+1} F_{K+1}(y_{K+1}|y_K) \\
&\quad \times f_{K+1}(y_{K+1}|y_K)\, dy_{K+1} \cdots f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1 \\
&\geq \int_0^\theta \Lambda y_{K+1} F_{K+1}(y_{K+1}) f_{K+1}(y_{K+1})\, dy_{K+1} - \int_0^\theta \Lambda y_K F_K(y_K) f_K(y_K)\, dy_K.
\end{aligned}$$

Let

$$\begin{aligned}
&\phi_{K+1}(\theta, N) \\
&\quad := \int_0^\theta \int_0^{y_1} \cdots \int_0^{y_K} \Lambda y_{K+1} F_{K+1}(y_{K+1}|y_K) \\
&\quad \times f_{K+1}(y_{K+1}|y_K)\, dy_{K+1} \cdots f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1 + \cdots \\
&\quad + \int_\theta^1 \int_\theta^{y_1} \cdots \int_0^\theta \Lambda y_{K+1} F_{K+1}(y_{K+1}|y_K) \\
&\quad \times f_{K+1}(y_{K+1}|y_K)\, dy_{K+1} \cdots f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1 \\
&\quad - \int_0^\theta \Lambda y_{K+1} F_{K+1}(y_{K+1}) f_{K+1}(y_{K+1})\, dy_{K+1} + \int_0^\theta \Lambda y_K F_K(y_K) f_K(y_K)\, dy_K.
\end{aligned}$$

It is easy to verify that $\phi_{K+1}(\theta, N)|_{\theta=0} = 0$. Moreover, we have that

$$\begin{aligned}
&\phi_{K+1}(\theta, N)|_{\theta=1} \\
&= \int_0^1 \int_0^{y_1} \cdots \int_0^{y_K} \Lambda y_{K+1} F_{K+1}(y_{K+1}|y_K) f_{K+1}(y_{K+1}|y_K)\, dy_{K+1} \cdots f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1 \\
&\quad - \int_0^1 \Lambda y_{K+1} F_{K+1}(y_{K+1}) f_{K+1}(y_{K+1})\, dy_{K+1} + \int_0^1 \Lambda y_K F_K(y_K) f_K(y_K)\, dy_K \\
&= \int_0^1 \int_0^{y_1} \cdots \int_0^{y_K} \Lambda y_{K+1} F_{K+1}(y_{K+1}|y_K) f_{K+1}(y_{K+1}|y_K)\, dy_{K+1} \cdots f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1 \\
&\quad + \frac{1}{2} \int_0^1 \Lambda \big[F_{K+1}(x)^2 - F_K(x)^2\big]\, dx \\
&> 0,
\end{aligned}$$

where the second equality follows by using integration by parts. Next, differentiating $\phi_K(\theta, N)$ with respect to $\theta$ yields

$$\begin{aligned}\frac{\partial \phi_{K+1}(\theta, N)}{\partial \theta} = &\int_\theta^1 \int_\theta^{y_1} \cdots \int_0^{y_{K-1}} \Lambda\theta F_{K+1}(\theta|y_K) \\ &\times f_{K+1}(\theta|y_K) f_K(y_K|y_{K-1})\, dy_K \cdots f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1 \\ &- \Lambda\theta F_{K+1}(\theta) f_{K+1}(\theta) + \Lambda\theta F_K(\theta) f_K(\theta).\end{aligned}$$

Notice that $\frac{\partial \phi_{K+1}(\theta,N)}{\partial \theta}|_{\theta=0} = 0 = \frac{\partial \phi_{K+1}(\theta,N)}{\partial \theta}|_{\theta=1}$. Moreover, since $\Lambda\theta F_K(\theta) f_K(\theta) - \Lambda\theta F_{K+1}(\theta) f_{K+1}(\theta)$ crosses zero only once for $\theta \in (0, 1)$ and from above, $\frac{\partial \phi_{K+1}(\theta,N)}{\partial \theta}$ can change sign at most once, from positive to negative. Therefore, given that $\phi_{K+1}(\theta, N)|_{\theta=1} > 0$ and $\frac{\partial \phi_{K+1}(\theta,N)}{\partial \theta}|_{\theta=1} = 0$, it follows that $P^{\text{SPA}}_{K+1}(\theta) - P^{\text{UPA}}_{K+1}(\theta) \geq 0$ for all $\theta \in [0, 1]$. □

Proof of Proposition 6. We show that for $K \geq 2$, there exists a $\theta^*_K \in (0, 1)$ such that $L^{\text{SPA}}_K(\theta) \geq L^{\text{UPA}}_K(\theta)$ if and only if $\theta \geq \theta^*_K$. In equilibrium, the expected psychological loss of a type-$\theta$ bidder in a $K$-round sequential SPA is

$$\begin{aligned}L^{\text{SPA}}_K(\theta) = &\Lambda\theta \int_\theta^1 F_1(\theta) f_1(y_1)\, dy_1 + \Lambda\theta \int_\theta^1 \int_\theta^{y_1} F_2(\theta|y_1) f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1 \\ &+ \underbrace{\cdots}_{\text{expected losses in rounds 3 through } K-1} \\ &+ \Lambda\theta \int_\theta^1 \cdots \int_\theta^{y_{K-1}} F_K(\theta|y_{K-1}) f_K(y_K|y_{K-1})\, dy_K \cdots f_1(y_1)\, dy_1.\end{aligned}$$

Similarly, the expected psychological loss of a type-$\theta$ bidder in a uniform-price auction is

$$L^{\text{UPA}}_K(\theta) = \Lambda\theta F_K(\theta)\big[1 - F_K(\theta)\big].$$

It is easy to verify that $L^{\text{SPA}}_K(0) - L^{\text{UPA}}_K(0) = 0 = L^{\text{SPA}}_K(1) - L^{\text{UPA}}_K(1)$. Moreover, simplifying and rearranging the terms in $L^{\text{SPA}}_K(\theta)$, the difference $L^{\text{SPA}}_K(\theta) - L^{\text{UPA}}_K(\theta)$ can be rewritten as

$$\begin{aligned}&\Lambda\theta\left\{F_1(\theta)\big[1 - F_1(\theta)\big] + \sum_{k=2}^{K}\big[F_k(\theta) - F_{k-1}(\theta)\big] - F_K(\theta)\big[1 - F_K(\theta)\big]\right\} \\ &\quad - \Lambda\theta\left\{\int_\theta^1 F_2(\theta|y_1)^2 f_1(y_1)\, dy_1 + \cdots \right. \\ &\quad \left. + \int_\theta^1 \cdots \int_\theta^{y_{K-2}} F_K(\theta|y_{K-1})^2 f_{K-1}(y_{K-1}|y_{K-2})\, dy_{K-1} \cdots f_1(y_1)\, dy_1\right\} \\ &= \Lambda\theta\big\{F_1(\theta)\big[1 - F_1(\theta)\big] + F_K(\theta) - F_1(\theta) - F_K(\theta)\big[1 - F_K(\theta)\big]\big\} \\ &\quad - \Lambda\theta\left\{\int_\theta^1 F_2(\theta|y_1)^2 f_1(y_1)\, dy_1 + \cdots\right.\end{aligned}$$

$$
\begin{aligned}
&\quad + \int_\theta^1 \cdots \int_\theta^{y_{K-2}} F_K(\theta|y_{K-1})^2 f_{K-1}(y_{K-1}|y_{K-2})\,dy_{K-1}\cdots f_1(y_1)\,dy_1\Bigg\} \\
&= \Lambda\theta\big\{F_K(\theta)^2 - F_1(\theta)^2\big\} \\
&\quad - \Lambda\theta\Bigg\{\int_\theta^1 F_2(\theta|y_1)^2 f_1(y_1)\,dy_1 + \cdots \\
&\quad + \int_\theta^1 \cdots \int_\theta^{y_{K-2}} F_K(\theta|y_{K-1})^2 f_{K-1}(y_{K-1}|y_{K-2})\,dy_{K-1}\cdots f_1(y_1)\,dy_1\Bigg\}.
\end{aligned}
$$

Hence, for $\theta \in (0, 1)$, the sign of $L_K^{\mathrm{SPA}}(\theta) - L_K^{\mathrm{UPA}}(\theta)$ is equal to the sign of the expression

$$
\underbrace{
\begin{aligned}
&F_K(\theta)^2 - F_1(\theta)^2 - \int_\theta^1 F_2(\theta|y_1)^2 f_1(y_1)\,dy_1 - \cdots \\
&\quad - \int_\theta^1 \cdots \int_\theta^{y_{K-2}} F_K(\theta|y_{K-1})^2 f_{K-1}(y_{K-1}|y_{K-2})\,dy_{K-1}\cdots f_1(y_1)\,dy_1.
\end{aligned}
}_{\varphi_K(\theta,N)}
$$

Notice that $\lim_{\theta\searrow 0}\varphi_K(\theta, N) < 0$; hence, at $\theta = 0$, the function $L_K^{\mathrm{SPA}}(\theta) - L_K^{\mathrm{UPA}}(\theta)$ approaches 0 from below. Moreover, we have that

$$
\begin{aligned}
&\frac{\partial\varphi_K(\theta, N)}{\partial\theta} \\
&\quad = 2F_K(\theta) f_K(\theta) - 2F_1(\theta) f_1(\theta) + \sum_{k=1}^{K-1} f_k(\theta) - \int_\theta^1 2F_2(\theta|y_1) f_2(\theta|y_1) f_1(y_1)\,dy_1 - \cdots \\
&\qquad - \int_\theta^1 \cdots \int_\theta^{y_{K-2}} 2F_K(\theta|y_{K-1}) f_K(\theta|y_{K-1}) f_{K-1}(y_{K-1}|y_{K-2})\cdots f_1(y_1)\,dy_1.
\end{aligned}
$$

Evaluating the above at $\theta = 1$ yields

$$
\begin{aligned}
\frac{\partial\varphi_K(\theta, N)}{\partial\theta}\bigg|_{\theta=1} &= 2f_K(1) - 2f_1(1) + \sum_{k=1}^{K-1} f_k(1) \\
&= -f_1(1) \\
&< 0.
\end{aligned}
$$

Hence, at $\theta = 1$, the function $L_K^{\mathrm{SPA}}(\theta) - L_K^{\mathrm{UPA}}(\theta)$ approaches 0 from above. Therefore, $L_K^{\mathrm{SPA}}(\theta) - L_K^{\mathrm{UPA}}(\theta)$ must cross zero from below at least once for $\theta \in (0, 1)$. To show that this is the only crossing point, we argue that once $\varphi_K(\theta, N)$ becomes positive, it never changes sign again. Indeed, we have

$$
\begin{aligned}
\varphi_K(\theta, N) &= F_K(\theta)^2 - F_1(\theta)^2 - \int_\theta^1 F_2(\theta|y_1)^2 f_1(y_1)\,dy_1 - \cdots \\
&\quad - \int_\theta^1 \cdots \int_\theta^{y_{K-2}} F_K(\theta|y_{K-1})^2 f_{K-1}(y_{K-1}|y_{K-2})\,dy_{K-1}\cdots f_1(y_1)\,dy_1
\end{aligned}
$$

$$
\begin{aligned}
&> F_K(\theta)^2 - F_1(\theta)^2 - \int_\theta^1 F_2(\theta|y_1) f_1(y_1)\, dy_1 - \cdots \\
&\quad - \int_\theta^1 \cdots \int_\theta^{y_{K-2}} F_K(\theta|y_{K-1}) f_{K-1}(y_{K-1}|y_{K-2})\, dy_{K-1} \cdots f_1(y_1)\, dy_1 \\
&= F_K(\theta)^2 - F_1(\theta)^2 - \sum_{k=2}^{K} \big[F_k(\theta) - F_{k-1}(\theta)\big] \\
&= F_K(\theta)^2 - F_1(\theta)^2 - \big[F_K(\theta) - F_1(\theta)\big] \\
&= \big[F_K(\theta) - F_1(\theta)\big]\big[F_K(\theta) + F_1(\theta) - 1\big].
\end{aligned}
$$

Since $[F_K(\theta) - F_1(\theta)][F_K(\theta) + F_1(\theta) - 1]$ switches sign only once for $\theta \in (0, 1)$, so does $\varphi_K(\theta, N)$. Therefore, there exists a unique $\theta_K^* \in (0, 1)$ such that $L_K^{\mathrm{SPA}}(\theta) \geq L_K^{\mathrm{UPA}}(\theta)$ if and only if $\theta \geq \theta_K^*$. □

Proof of Proposition 7. The proof proceeds as follows. First, we will derive the second-round symmetric equilibrium bidding function for a type-$\theta$ bidder taking as given what the bidder bid in the first round; then, we are going to derive the first-round symmetric equilibrium bidding function for a type-$\theta$ bidder who correctly anticipates how he will bid in the second round. Recall that the joint distribution of the highest and second-highest order statistics (among $N-1$) is given by

$$
f_{1,2}(y_1, y_2) = (N-1)(N-2) f(y_1) f(y_2) F(y_2)^{N-3}. \tag{10}
$$

Using (10), we can easily derive an expression for the CDF of $Y_2$ conditional on $Y_1$ being lager than $y$:

$$
F_2(z|Y_1 > y) = \frac{\int_y^1 \int_0^z f_{1,2}(y_1, y_2)\, dy_2\, dy_1}{\int_y^1 \int_0^{y_1} f_{1,2}(y_1, y_2)\, dy_2\, dy_1} = \frac{(N-1)\big[1 - F(y)\big] F(z)^{N-2}}{1 - F(y)^{N-1}}.
$$

Similarly, the PDF of $Y_2$ conditional on $Y_1 = y$ is equal to

$$
f_2(z|Y_1 = y) = \frac{f_{1,2}(y, z)}{f_1(y)} = \frac{(N-2) f(z) F(z)^{N-3}}{F(y)^{N-2}}.
$$

Let $\beta_2^{\text{SPA-w/o}}(\theta; \widetilde{\theta}_1)$ be the second-round equilibrium bidding function, where $\widetilde{\theta}_1$ denotes the type that the bidder mimicked in the first round. Suppose that all the other bidders follow their equilibrium strategies, while bidder $i$ is considering deviating in round 2. The payoff of bidder $i$ with type $\theta$ when he bids as if he had type $\widetilde{\theta}_2$ is

$$
\begin{aligned}
U_2(\theta, \widetilde{\theta}_2; \widetilde{\theta}_1) = &\int_{\widetilde{\theta}_1}^1 \int_0^{\widetilde{\theta}_2} \big[\theta - \beta_2^{\text{SPA-w/o}}(y_2; \widetilde{\theta}_1)\big] f_2(y_2|y_1)\, dy_2 f_1(y_1|y_1 > \widetilde{\theta}_1)\, dy_1 \\
&- \Lambda \int_{\widetilde{\theta}_1}^1 \int_{\widetilde{\theta}_2}^{y_1} \theta F_2(\theta|Y_1 > \widetilde{\theta}_1) f_2(y_2|y_1)\, dy_2 f_1(y_1|y_1 > \widetilde{\theta}_1)\, dy_1,
\end{aligned} \tag{11}
$$

where $f_1(y_1|y_1 > \widetilde{\theta}_1) := \frac{f_1(y_1)}{1-F(\widetilde{\theta}_1)^{N-1}}$. Differentiating (11) with respect to $\widetilde{\theta}_2$ yields the first-order condition

$$0 = \int_{\widetilde{\theta}_1}^{1} \big[\theta - \beta_2^{\text{SPA-w/o}}(\widetilde{\theta}_2; \widetilde{\theta}_1)\big] f_2(\widetilde{\theta}_2|y_1) f_1(y_1|y_1 > \widetilde{\theta}_1)\, dy_1 + \Lambda \int_{\widetilde{\theta}_1}^{1} \theta F_2(\theta|Y_1 > \widetilde{\theta}_1) f_2(\widetilde{\theta}_2|y_1) f_1(y_1|y_1 > \widetilde{\theta}_1)\, dy_1.$$

In equilibrium, $\widetilde{\theta}_2 = \theta$ must hold; hence, we obtain the necessary condition

$$\int_{\widetilde{\theta}_1}^{1} \big[\theta - \beta_2^{\text{SPA-w/o}}(\theta; \widetilde{\theta}_1) + \Lambda\theta F_2(\theta|Y_1 > \widetilde{\theta}_1)\big] f_2(\theta|y_1) f_1(y_1|y_1 > \widetilde{\theta}_1)\, dy_1 = 0. \tag{12}$$

Simplifying and rearranging the above condition yields

$$\beta_2^{\text{SPA-w/o}}(\theta; \widetilde{\theta}_1) = \theta + \Lambda\theta F_2(\theta|Y_1 > \widetilde{\theta}_1).$$

Using condition (12) to substitute for $\beta_2^{\text{SPA-w/o}}(\widetilde{\theta}_2; \widetilde{\theta}_1)$ into the expression for $\partial U_2(\theta, \widetilde{\theta}_2; \widetilde{\theta}_1)/\partial\widetilde{\theta}_2$ yields

$$\frac{\partial U_2(\theta, \widetilde{\theta}_2; \widetilde{\theta}_1)}{\partial \widetilde{\theta}_2} = \int_{\widetilde{\theta}_1}^{1} \big\{\theta - \widetilde{\theta}_2 + \Lambda\big[\theta F_2(\theta|Y_1 > \widetilde{\theta}_1) - \widetilde{\theta}_2 F_2(\widetilde{\theta}_2|Y_1 > \widetilde{\theta}_1)\big]\big\} f_2(\widetilde{\theta}_2|y_1) f_1(y_1|y_1 > \widetilde{\theta}_1)\, dy_1.$$

Thus, $\partial U_2(\theta, \widetilde{\theta}_2; \widetilde{\theta}_1)/\partial\widetilde{\theta}_2$ has the same sign as $(\theta - \widetilde{\theta}_2)$; hence, $\widetilde{\theta}_2 = \theta$ is optimal.

Next, let $\beta_1^{\text{SPA-w/o}}(\theta)$ be the first-round equilibrium bidding function. Suppose all other bidders follow their equilibrium strategies, while bidder $i$ considers deviating in the first round only. The payoff of bidder $i$ with type $\theta$ if he bids as if he had type $\widetilde{\theta}_1$ is

$$\begin{aligned} U_1(\theta, \widetilde{\theta}_1) = {} & \int_0^{\widetilde{\theta}_1} \big[\theta - \beta_1^{\text{SPA-w/o}}(y_1)\big] f_1(y_1)\, dy_1 - \Lambda \int_{\widetilde{\theta}_1}^{1} \theta F_1(\theta) f_1(y_1)\, dy_1 \\ & + \int_{\widetilde{\theta}_1}^{1} \int_0^{\theta} \big[\theta - \beta_2^{\text{SPA-w/o}}(y_2; y_2)\big] f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1 \\ & - \Lambda \int_{\widetilde{\theta}_1}^{1} \int_{\theta}^{y_1} \theta F_2(\theta|Y_1 > \theta) f_2(y_2|y_1)\, dy_2 f_1(y_1)\, dy_1. \end{aligned} \tag{13}$$

Differentiating (13) with respect to $\widetilde{\theta}_1$ yields the first-order condition

$$\begin{aligned} 0 = {} & \big[\theta - \beta_1^{\text{SPA-w/o}}(\widetilde{\theta}_1)\big] f_1(\widetilde{\theta}_1) + \Lambda\theta F_1(\theta) f_1(\widetilde{\theta}_1) \\ & - \int_0^{\theta} \big[\theta - \beta_2^{\text{SPA-w/o}}(y_2; y_2)\big] f_2(y_2|\widetilde{\theta}_1)\, dy_2 f_1(\widetilde{\theta}_1) \\ & + \Lambda \int_{\theta}^{\widetilde{\theta}_1} \theta F_2(\theta|Y_1 > \theta) f_2(y_2|\widetilde{\theta}_1)\, dy_2 f_1(\widetilde{\theta}_1). \end{aligned}$$

In equilibrium, $\widetilde{\theta}_1 = \theta$ must hold; hence, we obtain the necessary condition

$$\left\{\theta - \beta_1^{\text{SPA-w/o}}(\theta) + \Lambda\theta F_1(\theta) - \int_0^\theta \big[\theta - \beta_2^{\text{SPA-w/o}}(y_2; y_2)\big] f_2(y_2|\theta)\, dy_2\right\} f_1(\theta) = 0. \tag{14}$$

Simplifying and rearranging the above condition yields

$$\beta_1^{\text{SPA-w/o}}(\theta) = \int_0^\theta \beta_2^{\text{SPA-w/o}}(y_2; y_2) f_2(y_2|\theta)\, dy_2 + \Lambda\theta F_1(\theta).$$

Finally, to show that the first-order condition is also sufficient, we substitute for $\beta_1^{\text{SPA-w/o}}(\widetilde{\theta}_1)$, using condition (14), into the expression for $\partial U_1(\theta, \widetilde{\theta}_1)/\partial\widetilde{\theta}_1$ and obtain

$$\begin{aligned}
&\frac{\partial U_1(\theta, \widetilde{\theta}_1)}{\partial\widetilde{\theta}_1} \\
&\quad= \left[\theta - \int_0^{\widetilde{\theta}_1} \beta_2^{\text{SPA-w/o}}(y_2; y_2) f_2(y_2|\widetilde{\theta}_1)\, dy_2 - \Lambda\widetilde{\theta}_1 F_1(\widetilde{\theta}_1)\right] f_1(\widetilde{\theta}_1) + \Lambda\theta F_1(\theta) f_1(\widetilde{\theta}_1) \\
&\qquad - \int_0^\theta \big[\theta - \beta_2^{\text{SPA-w/o}}(y_2; y_2)\big] f_2(y_2|\widetilde{\theta}_1)\, dy_2 f_1(\widetilde{\theta}_1) \\
&\qquad + \Lambda \int_\theta^{\widetilde{\theta}_1} \theta F_2(\theta|Y_1 > \theta) f_2(y_2|\widetilde{\theta}_1)\, dy_2 f_1(\widetilde{\theta}_1) \\
&\quad= \int_\theta^{\widetilde{\theta}_1} \theta f_2(y_2|\widetilde{\theta}_1)\, dy_2 f_1(\widetilde{\theta}_1) + \Lambda\big[\theta F_1(\theta) - \widetilde{\theta}_1 F_1(\widetilde{\theta}_1)\big] f_1(\widetilde{\theta}_1) \\
&\qquad + \int_{\widetilde{\theta}_1}^\theta \beta_2^{\text{SPA-w/o}}(y_2; y_2) f_2(y_2|\widetilde{\theta}_1)\, dy_2 f_1(\widetilde{\theta}_1) \\
&\qquad + \Lambda \int_\theta^{\widetilde{\theta}_1} \theta F_2(\theta|Y_1 > \theta) f_2(y_2|\widetilde{\theta}_1)\, dy_2 f_1(\widetilde{\theta}_1).
\end{aligned}$$

Since in equilibrium $\beta_2^{\text{SPA-w/o}}(y_2; y_2) = y_2 + \Lambda y_2 F_2(y_2|Y_1 > y_2)$, it follows that

$$\begin{aligned}
\frac{\partial U_1(\theta, \widetilde{\theta}_1)}{\partial\widetilde{\theta}_1} &= \int_\theta^{\widetilde{\theta}_1} (\theta - y_2) f_2(y_2|\widetilde{\theta}_1)\, dy_2 f_1(\widetilde{\theta}_1) + \Lambda\big[\theta F_1(\theta) - \widetilde{\theta}_1 F_1(\widetilde{\theta}_1)\big] f_1(\widetilde{\theta}_1) \\
&\quad + \Lambda \int_\theta^{\widetilde{\theta}_1} \big[\theta F_2(\theta|Y_1 > \theta) - y_2 F_2(y_2|Y_1 > y_2)\big] f_2(y_2|\widetilde{\theta}_1)\, dy_2 f_1(\widetilde{\theta}_1).
\end{aligned}$$

Hence, $\partial U_1(\theta, \widetilde{\theta}_1)/\partial\widetilde{\theta}_1$ has the sign same as $(\theta - \widetilde{\theta}_1)$; thus, $\widetilde{\theta}_1 = \theta$ is optimal. □

The proof of Lemma 2 follows since $F_2(\theta|Y_1 > \widetilde{\theta}_1)$ is decreasing in $\widetilde{\theta}_1$.

Proof of Lemma 3. We have

$$\begin{aligned}
\beta_2^{\text{SPA-w/o}}(\theta; \widetilde{\theta}_1) - \beta_2^{\text{SPA}}(\theta; y_1) &= \Lambda\theta F_2(\theta|Y_1 > \widetilde{\theta}_1) - \Lambda\theta F_2(\theta|y_1) \\
&= \Lambda\left\{\frac{(N-1)\big[1 - F(\widetilde{\theta}_1)\big] F(\theta)^{N-2}}{1 - F(\widetilde{\theta}_1)^{N-1}} - \frac{F(\theta)^{N-2}}{F(y_1)^{N-2}}\right\}.
\end{aligned}$$

Hence, $\beta_2^{\text{SPA-w/o}}(\theta; \widetilde{\theta}_1) - \beta_2^{\text{SPA}}(\theta; y_1) > 0$ if and only if

$$\frac{(N-1)\big[1-F(\widetilde{\theta}_1)\big]F(\theta)^{N-2}}{1-F(\widetilde{\theta}_1)^{N-1}} > \frac{F(\theta)^{N-2}}{F(y_1)^{N-2}}.$$

Rearranging the above inequality yields the condition provided in the text. □

Proof of Lemma 4. We have

$$\begin{aligned}
&\beta_1^{\text{SPA-w/o}}(\theta) - \beta_1^{\text{SPA}}(\theta)\\
&\quad = \int_0^\theta \big[\beta_2^{\text{SPA-w/o}}(y_2; y_2) - \beta_2^{\text{SPA}}(y_2; \theta)\big] f_2(y_2|\theta)\, dy_2\\
&\quad = \Lambda \int_0^\theta y_2 \Bigg\{ \underbrace{\frac{(N-1)\big[1-F(y_2)\big]F(y_2)^{N-2}}{1-F(y_2)^{N-1}}}_{F_2(y_2|Y_1>y_2)} - \underbrace{\frac{F(y_2)^{N-2}}{F(\theta)^{N-2}}}_{F_2(y_2|\theta)} \Bigg\} f_2(y_2|\theta)\, dy_2.
\end{aligned}$$

Next notice that

$$\lim_{\theta\searrow 0}\big(\beta_1^{\text{SPA-w/o}}(\theta) - \beta_1^{\text{SPA}}(\theta)\big) < 0 < \big(\beta_1^{\text{SPA-w/o}}(\theta) - \beta_1^{\text{SPA}}(\theta)\big)|_{\theta=1}.$$

By Lemma 3, $F_2(y_2|Y_1 > y_2) - F_2(y_2|\theta)$ crosses zero at most once for any $\theta \in (0, 1)$; hence, $\beta_1^{\text{SPA-w/o}}(\theta) - \beta_1^{\text{SPA}}(\theta)$ changes sign from negative to positive only once. Thus, there exists a $\widehat{\theta} \in (0, 1)$ such that $\beta_1^{\text{SPA-w/o}}(\theta) - \beta_1^{\text{SPA}}(\theta) \geq 0$ if and only if $\theta \geq \widehat{\theta}$. □

Proof of Proposition 8. Let $Y_k^{(N)}$ denote the $k$th highest order statistic among $N$ (i.e., when taking the point of view of the seller) and let $Y_k^{(N-1)}$ denote the $k$th highest order statistic among $N-1$ (i.e., when taking the point of view of a bidder). We will use a similar notation for the CDFs and PDFs of $Y_k^{(N)}$ and $Y_k^{(N-1)}$, respectively. In a symmetric equilibrium, the winner in the first round is the bidder with the highest type and the price setter is the bidder with the next highest type. Hence, we have

$$\begin{aligned}
p_1 &= \beta_1^{\text{SPA-w/o}}\big(Y_2^{(N)} = y_2\big)\\
&= \int_0^{y_2} \beta_2^{\text{SPA-w/o}}(\theta; \theta) f_2^{(N-1)}(\theta|y_2)\, d\theta + \Lambda y_2 F_1^{(N-1)}(y_2)\\
&> \int_0^{y_2} \beta_2^{\text{SPA-w/o}}(\theta; \theta) f_3^{(N)}(\theta|y_2)\, d\theta\\
&= \mathbb{E}\big[p_2 | p_1 = \beta_1^{\text{SPA-w/o}}(y_2)\big],
\end{aligned}$$

where the inequality follows since $f_2^{(N-1)}(\cdot|y_2) = f_3^{(N)}(\cdot|y_2)$ and $\Lambda y_2 F_1^{(N-1)}(y_2) > 0$. □

## References

Ashenfelter, Orley (1989), "How auctions work for wine and art." *Journal of Economic Perspectives*, 3, 23–36. [563, 576]

Ashenfelter, Orley and David Genesove (1992), "Testing for price anomalies in real-estate auctions." *American Economic Review P&P*, 82, 501–505. [563]

Balzer, Benajmin, Antonio Rosato and Jonas von Wangenheim (2022), "Dutch versus first-price auctions with dynamic reference-dependent preferences." *Journal of Economic Theory*, 205, 105545. [576]

Balzer, Benjamin and Antonio Rosato (2021), "Expectations-based loss aversion in auctions with interdependent values: Extensive vs. intensive risk." *Management Science*, 67, 1056–1074. [568, 570, 576]

Beggs, Alan and Kathryn Graddy (1997), "Declining values and the afternoon effect: Evidence from art auctions." *RAND Journal of Economics*, 28, 544–565. [563]

Bernhardt, Dan and David Scoones (1994), "A note on sequential auctions." *American Economic Review*, 84, 501–505. [577]

Betz, Regina, Ben Greiner, Sascha Schweitzer, and Stefan Seifert (2017), "Auction format and auction sequence in multi-item multi-unit auctions: An experimental study." *Economic Journal*, 127, 351–371. [564, 571]

Chakraborty, Archishman, Nandini Gupta, and Rick Harbaugh (2006), "Best foot forward or best for last in a sequential auction?" *RAND Journal of Economics*, 37, 176–194. [577]

Cooper, David and Hanming Fang (2008), "Understanding overbidding in second price auctions: An experimental study." *Economic Journal*, 118, 1572–1595. [561]

Cox, James, Vernon Smith, and James Walker (1992), "Theory and misbehavior of first-price auctions: Comment." *American Economic Review*, 82, 1392–1412. [561]

Cramton, Peter, Emel Filiz-Ozbay, Erkut Ozbay, and Pacharasut Sujarittanonta (2012), "Fear of losing in a clock auction." *Review of Economic Design*, 16, 119–134. [562]

De Frutos, Maria and Robert Rosenthal (1998), "On some myths about sequenced common-value auctions." *Games and Economic Behavior*, 23, 201–221. [575]

Delgado, Mauricio, Andrew Schotter, Erkut Ozbay, and Elizabeth Phelps (2008), "Understanding overbidding: Using the neural circuitry of reward to design economic auctions." *Science*, 321, 1849–1852. [562]

Deltas, George and Georgia Kosmopoulou (2004), "Bidding in sequential auctions: 'Catalogue' vs. 'order-of-sale' effects." *Economic Journal*, 114, 28–54. [563]

Ehrhart, Karl-Martin, Marion Ott, and Susanne Abele (2015), "Auction fever: Rising revenue in second-price auction formats." *Games and Economic Behavior*, 92, 206–227. [561]

Eisenhuth, Roland (2019), "Reference dependent mechanism design." *Economic Theory Bulletin*, 7, 77–103. [576]

Eyster, Erik (2002), "Rationalizing the past: A taste for consistency." Working Paper. [576]

Février, Philippe, Laurent Linnemer, and Michael Visser (2007), "Buy or wait, that is the option: The buyer's option in sequential laboratory auctions." *RAND Journal of Economics*, 38, 98–118. [563]

Gale, Ian and Donal Hausch (1994), "Bottom-fishing and declining prices in sequential auctions." *Games and Economic Behavior*, 7, 318–331. [577]

Gale, Ian and Mark Stegeman (2001), "Sequential auctions of endogenously valued objects." *Games and Economic Behavior*, 36, 74–103. [577]

Gandal, Neil (1997), "Sequential auctions of interdependent objects: Israeli cable television licenses." *Journal of Industrial Economics*, 45, 227–244. [563]

Ghosh, Gagan and Heng Liu (2019), "Sequential second-price auctions with private budgets." *Games and Economic Behavior*, 113, 611–632. [577]

Ghosh, Gagan and Heng Liu (2021), "Sequential auctions with ambiguity." *Journal of Economic Theory*, 197, 105324. [577]

Ginsburgh, Victor (1998), "Absentee bidders and the declining price anomaly in wine auctions." *Journal of Political Economy*, 106, 1302–1319. [563]

Goeree, Jacob, Charles Holt, and Thomas Palfrey (2002), "Quantal response equilibrium and overbidding in private-value auctions." *Journal of Economic Theory*, 104, 247–272. [561]

Heyman, James, Yesim Orhun, and Dan Ariely (2004), "Auction fever: The effect of opponents and quasi-endowment on product valuations." *Journal of Interactive Marketing*, 18, 7–21. [561]

Hu, Audrey and Liang Zou (2015), "Sequential auctions, price trends, and risk preferences." *Journal of Economic Theory*, 158, 319–335. [568, 573, 576]

Jeitschko, Thomas (1999), "Equilibrium price paths in sequential auctions with stochastic supply." *Economics Letters*, 64, 67–72. [577]

Kahneman, Daniel and Amos Tversky (1979), "Prospect theory: An analysis of decision under risk." *Econometrica*, 47, 263–291. [562]

Klemperer, Paul (2002), "What really matters in auction design." *Journal of Economic Perspectives*, 16, 169–189. [564]

Kőszegi, Botond and Matthew Rabin (2006), "A model of reference-dependent preferences." *Quarterly Journal of Economics*, 121, 1133–1165. [562, 565, 566]

Kőszegi, Botond and Matthew Rabin (2007), "Reference-dependent risk attitudes." *American Economic Review*, 97, 1047–1073. [562, 565, 576]

Kőszegi, Botond and Matthew Rabin (2009), "Reference-dependent consumption plans." *American Economic Review*, 99, 909–936. [562, 565, 566]

Lambson, Val and Norman Thurston (2006), "Sequential auctions: Theory and evidence from the Seattle fur exchange." *RAND Journal of Economics*, 37, 70–80. [563]

Lange, Andreas and Anmol Ratan (2010), "Multi-dimensional reference-dependent preferences in sealed-bid auctions: How (most) laboratory experiments differ from the field." *Games and Economic Behavior*, 68, 634–645. [568, 570, 576]

McAfee, Randolph P. and Daniel Vincent (1993), "The declining price anomaly." *Journal of Economic Theory*, 60, 191–212. [563, 568, 573, 576]

Menezes, Flavio and Paulo Monteiro (2003), "Synergies and price trends in sequential auctions." *Review of Economic Design*, 8, 85–98. [577]

Mezzetti, Claudio (2011), "Sequential auctions with informational externalities and aversion to price risk: Decreasing and increasing price sequences." *Economic Journal*, 121, 990–1016. [563, 568, 573, 575, 576, 577]

Milgrom, Paul and Robert Weber (2000), "A theory of auctions and competitive bidding, II." In *The Economic Theory of Auctions*, Vol. 1 (Paul Klemperer, ed.), 179–194. Edward Edgar, Cambridge, UK. [563, 568, 573]

Neugebauer, Tibor and Paul Pezanis-Christou (2007), "Bidding behavior at sequential first-price auctions with(out) supply uncertainty: A laboratory analysis." *Journal of Economic Behavior & Organization*, 63, 55–72. [563]

Novemsky, Nathan and Daniel Kahneman (2005), "The boundaries of loss aversion." *Journal of Marketing Research*, 42, 119–128. [565]

Sherstyuk, Katerina and Jeremy Dulatre (2008), "Market performance and collusion in sequential and simultaneous multi-object auctions: Evidence from an ascending auctions experiment." *International Journal of Industrial Organization*, 26, 557–572. [564]

Van den Berg, Gerard, Jan Van Ours, and Menno Pradhan (2001), "The declining price anomaly in Dutch Dutch Rose auctions." *American Economic Review*, 91, 1055–1062. [563]

von Wangenheim, Jonas (2021), "English auctions versus Vickrey auctions with loss-averse bidders." *Journal of Economic Theory*, 197, 105328. [568, 576]

Weber, Robert (1983), "Multi-object auctions." In *Auctions, Bidding and Contracting: Uses and Theory* (Richard Engelbrecht-Wiggans, Martin Shubik, and Robert Stark, eds.), 165–194. New York University Press, New York. [563, 568]

---

Co-editor Ran Spiegler handled this manuscript.

Manuscript received 23 December, 2019; final version accepted 15 April, 2022; available online 10 May, 2022.