Theoretical Economics 18 (2023), 1063–1100 1555-7561/20231063

Collective hold-up

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We consider dynamic processes of coalition formation in which a principal bargains sequentially with a group of agents. This problem is at the core of a variety of applications in economics, including lobbying, exclusive deals, and acquisition of complementary patents. In this context, we study how the allocation of bargaining power between principal and agents affects efficiency and welfare. We show that when the principal's willingness to pay is large relative to agents' payoffs for completion, efficiency requires concentrating bargaining power in the principal. Strengthening the bargaining position of the agents increases inefficient delay and reduces agents' welfare. This occurs in spite of the lack of informational asymmetries or discriminatory offers. When this collective action problem is severe enough, agents are better off when bargaining power is concentrated in the principal.

Keywords. Bargaining, contracting externalities, political economy, vote buying, delay.

JEL classification. C78, D72, D86.

1. Introduction

In this paper, we study dynamic processes of coalition formation in which a principal bargains sequentially with a group of agents. This type of problem, in which one of the players takes a central role in organizing collective action, is pervasive in applications in economics (lobbying, exclusive deals, acquisition of complementary patents, etc.).

A salient feature of these problems is that the principal often bargains with agents sequentially. In legislative politics, for instance, lobbyists or party leaders rarely make proposals simultaneously to all members on the floor of the chamber. Instead, they typically strike individual deals with legislators, gradually accumulating support in favor of

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We thank Nageeb Ali, Laura Doval, Tiberiu Dragu, Federico Echenique, German Gieczewski, Navin Kartik, Stephen Morris, David Myatt, and seminar participants at Berkeley, Caltech, Carlos III, CEMFI, Ecole Polytechnic, Paris School of Economics, University of Essex, University of Nottingham, Princeton, University of Warwick, ETH Zurich, the 2017 ERC Workshop on Voting Theory and Elections, the 2017 LSE/NYU Workshop in Political Economy, and the 2017 Stony Brook Workshop in Political Economy for helpful suggestions. Iaryczower gratefully acknowledges financial support from NSF grant SBE 1757191. Oliveros gratefully acknowledges the support of the Economic and Social Research Council (UK), ES/S01053X/1.

Available at https://econtheory.org. https://doi.org/10.3982/TE4632

their preferred alternative. As Caro (2002) put it in his celebrated account of Lyndon Johnson's reign in the Senate, "From the time he became Majority Leader, Johnson began using talk on the floor as a smoke screen for the maneuvering that was taking place in the cloakrooms, ... as a method of stalling the Senate *to give him time to work out his deals*."[1] In this context, the offers the leader makes to, or receives from, a legislator, will generally depend on how advanced the negotiation process is, as the outside option for both parties changes with the number of legislators whose support the leader still needs to secure. This consideration becomes particularly important if legislators are strategically farsighted, because the leader's ability to successfully negotiate with each member depends on their expectations about the nature of future trades.

In this paper, we seek to understand how the allocation of bargaining power between principal and agents shape equilibrium outcomes in sequential contracting. What is the effect on efficiency and welfare of decentralizing bargaining power from the principal to the agents? Surprisingly, this issue is still unresolved, as the literature on sequential contracting has generally maintained the assumption that the principal has complete proposal power, or that there is a fixed bargaining protocol. In applications, however, it is often reasonable to assume that agents have the ability to propose deals to the principal. In our legislative bargaining example, for instance, individual senators negotiating with the executive or the party leadership in the U.S. context appear to have substantial bargaining power, while in other institutional contexts (e.g., Argentina, Mexico) individual legislators appear weaker vis-à-vis the executive.

At first pass, the answer to this question seems straightforward. Given complete information, a mere reallocation of proposal power should not affect efficiency, and agents' equilibrium welfare should be monotonic in their proposal power. We show, however, that in sequential bargaining, these intuitions can be misguided. First, if the principal's willingness to pay is large relative to agents' payoffs for completion, efficiency requires power to be sufficiently concentrated in the principal; when instead agents have a relatively stronger say in bilateral negotiations, the equilibrium of the decentralized bargaining process entails inefficient delay. Second, due to the destruction of surplus caused by delay, decentralizing bargaining power from the principal to the agents reduces their welfare. These results hold without asymmetric information, discriminatory contracts or deadlines, and irrespective of the presence or the direction of externalities on uncommitted agents (all of which have been identified as sources of inefficiency in sequential bargaining).

In our model, a principal negotiates sequentially with a group of n identical agents over an infinite horizon. At each moment in time, an uncommitted agent is randomly selected to meet the principal. In each meeting, principal and agent bargain over the terms by which the agent would support the principal. If an agreement is reached, the agent commits his support to the principal and exits negotiations, and otherwise remains uncommitted. The principal needs to obtain the agreement of a given number

[1] For more systematic evidence of the prevalence of sequential bargaining in legislative politics, see Canen, Kendall, and Trebbi (2020), who exploit "whip count" votes in the U.S. House of Representatives, containing party leaders' private records of the voting intentions of rank and file members at various times before the bill is considered for a vote.

$q \le n$ of agents to implement a reform, action, or policy change, which affects the payoffs of all players. If and when this happens, the principal obtains a payoff $v > 0$, and agents obtain a payoff $z > 0$, in addition to any transfer payed or received during negotiations.[2] All players have a common discount factor $\delta \in (0, 1)$.

To consider arbitrary allocations of bargaining power between the principal and each agent while maintaining the structure of the game fixed, we assume that in a bilateral meeting the principal makes an offer with probability $\phi \in [0, 1]$, and the agent makes an offer with probability $1 - \phi$. As we show in the paper, this is equivalent to assuming that the outcome of each meeting is determined via generalized Nash bargaining, where the threat points are given by discounted continuation values following disagreement. To rule out discriminatory contracts that exploit coordination failures among agents (see, e.g., Segal and Whinston (2000), Genicot and Ray (2006), Galasso (2008), Chowdhury and Sengupta (2012)), we focus on symmetric Markov perfect equilibria of this game.[3]

The intuition for our main result is easiest to convey by focusing on the extreme cases in which either the agents or the principal have all the bargaining power. Consider first the case in which the principal has full bargaining power. Since agents cannot extract rents from the principal, the unique equilibrium of this game is efficient; i.e., all meetings result in agreement with some transfer that depend on the number of agents still needed for completion, and the coalition is formed without delay.

When agents have full bargaining power, instead, the results change dramatically. To see why this is the case, consider first the *critical* state in which the principal only needs to get the support of a single additional agent. Given discounting, the agent negotiating with the principal in the critical state can extract all the surplus from the principal. As a result, the principal is not willing to pay in previous negotiations to move the process forward, and is reduced to a passive intermediary. In the absence of side payments, the decision of each agent to trade or not depends on the relative value of moving the process along supporting the principal for free, versus holding out support with the goal of extracting the rent in the critical state. In this sense, in early stages of the bargaining process, the game becomes similar to a war of attrition between agents, with the caveat that the relative value of holding out or conceding changes along the path of play. And as in the war of attrition, when the principal's payoff from completion is high relative to that of the agents', the equilibrium involves inefficient delay. As the payoff from completion for the principal relative to that of the agents goes to infinity, the losses to agents become so severe that their payoff goes to zero. Thus, when this ratio is large enough, agents are better off when the principal holds all the bargaining power.

[2] To consider the effect of positive and negative externalities on nontraders, in Iaryczower and Oliveros (2022) we allow the completion payoff for committed and uncommitted agents to differ. In particular, we assume that upon completion, committed agents get $z \in \mathbb{R}_+$, and uncommitted agents get $w \in \mathbb{R}$, where $w > 0$ ($w < 0$) implies that there are positive (negative) externalities on uncommitted agents, and $w = 0$ implies that there are no externalities on uncommitted agents.

[3] In Iaryczower and Oliveros (2022), we consider subgame perfect equilibria with bounded recall; i.e., we allow for history dependent strategies but restrict the dependance to an arbitrary finite number of rounds $k > 0$. We show that the equilibrium outcomes we characterize are the same as those of the unique symmetric subgame perfect equilibrium with trade.

The case in which agents have all the bargaining power illustrates the essence of the collective hold-up problem, but is special in several ways. When both the principal and the agents have bargaining power, the equilibrium of the collective hold-up game can be separated into two phases: a final phase in which the principal is active, and transactions are efficient, and an early phase that resembles a war of attrition among agents, who hold out with positive probability to extract rents in the efficient stage. Crucially, the agent's decision to support the principal for free in the early stages of negotiations depends on the surplus that the agent expects to extract in the second phase of the decentralized bargaining game. As a result, all else constant, any change that increases (decreases) the agent's surplus extraction ability in the later phase will induce more (less) delay, for the same reasons that a higher prize induces players in a war of attrition to fight for longer. In particular, we show that the size of the inefficient phase decreases as more power is concentrated in the principal.

A fundamental assumption in our model—as in the vast majority of the bargaining literature—is the lack of perfect commitment.[4] In the absence of commitment, when the principal's bargaining position is weak, the unbridled competition among agents to extract rents from the principal leads to delay, destroying surplus and reducing the welfare of all players. If the agents had the ability to commit to a plan of action, they would constrain their ability to extract rents from the principal at the last stages of the negotiations. This would increase the principal's willingness to compensate the first agents she meets, and would improve efficiency. A lesson that emerges from our analysis is that the inefficiencies induced by limited commitment can be overcome by shifting bargaining power from the agents to the principal. In other words, concentrating bargaining power in the principal is an imperfect way of approximating the commitment solution.

2. Related literature

At the core, our paper contributes to two strands of literature: (i) on sequential contracting between a principal and a group of agents, and (ii) on legislative bargaining. More broadly, the paper also contributes to—and is informed by—the literature on noncooperative dynamic coalition formation.[5]

Our main contribution to the literature on sequential contracting is to study how changes in the allocation of proposal power between the principal and the agents affect equilibrium outcomes. In fact, the common assumptions in the literature are that the principal has complete proposal power (see, e.g., Rasmusen, Ramseyer, and Wiley (1991), Rasmusen and Ramseyer (1994), Jehiel and Moldovanu (1995a), Jehiel and Moldovanu (1995b), Segal and Whinston (2000), Genicot and Ray (2006), Iaryczower and Oliveros (2017), Chen and Zapal (2021)) or that there is a fixed bargaining protocol (Cai (2000), Chowdhury and Sengupta (2012)).[6] We show that changes in the allocation of

[4]For classic papers focused on commitment see, e.g., Fearon (1995), Acemoglu and Robinson (2000, 2001), and Powell (2004).

[5]We discuss the latter in Section 6.1, where we consider an extension of the model that allows for positive and negative externalities on uncommitted agents (i.e., "nontraders").

[6]The one exception we are aware of is Galasso (2008), who considers the setup of Genicot and Ray (2006), where there are negative externalities across agents and trade is inefficient, but the principal benefits from

bargaining power lead to fundamentally different incentives for agents in the game, and markedly different implications for equilibrium outcomes.

Within the literature on sequential contracting, three papers focus on delay in reaching agreement: Cai (2000), Jehiel and Moldovanu (1995a), and Jehiel and Moldovanu (1995b). In both cases, the mechanisms for delay are fundamentally different than in our paper. In Cai (2000), the principal meets with the agents in a prespecified order, and needs to get the support of all agents (unanimity). The bargaining protocol in each bilateral meeting is a single round of alternating offers. Cai shows that when players are sufficiently patient, there is a multiplicity of SPNE, including equilibria with and without delay. Differently than in our paper, delay here appears as a result of discriminating offers (Segal and Whinston (2000), Genicot and Ray (2006)), which can be constructed using the predetermined order of meetings.[7] We explicitly rule this out by focusing on symmetric MPE, and show that delay can occur in this setup in the absence of discriminating contracts. Moreover, collective hold-up emerges as the unique prediction of symmetric MPE, and does not require unanimity.

Discriminatory contracts are also a key feature of Jehiel and Moldovanu (1995b). In their model, a seller tries to sell a single object to one of several potential buyers, and non-buyers suffer a negative externality that is dependent on the identity of the buyer. The seller meets agents randomly, and has to sell the good to a buyer in $T < \infty$ periods. Jehiel and Moldovanu show that under some conditions there is a unique equilibrium with delay (not necessarily inefficient), in which transactions take place only a few stages before the end of the game. Delay appears here because the threat that the seller sells the object to the agent who induces a larger negative externality on other agents increases as the deadline approaches. This makes it optimal for the seller to wait to extract high prices from other agents. Jehiel and Moldovanu (1995a) extend the model to allow for positive externalities and an infinite horizon. They show that without deadlines, delay can occur with negative externalities—when it is welfare-improving for agents—but not with positive externalities, when it would be inefficient.

Our paper is also related to a fast growing literature focusing on understanding the nature of inefficiencies in legislative bargaining. Our main contribution to this literature is to study a sequential bargaining process in which the offers the principal (lobbyist, party leader) makes or receives from legislators depends on how advanced the negotiation process is. This contrasts with the standard assumption in the workhorse models of legislative bargaining a la Baron and Ferejohn (1989), in which proposals are offered to all potential committee members publicly and simultaneously. In this context, equilibrium is efficient, and agents' equilibrium payoffs are monotonic in proposal power (Eraslan (2002)). Banks and Duggan (2006) show that in a general version of the Baron–Ferejohn model, a stationary equilibrium with (inefficient) delay can only exist if the

trading. In this context, Galasso shows that when agents are sufficiently patient, the principal prefers to enter a finite horizon bargaining game in which she is the last mover, to a one shot game in which she makes a take-it-or-leave-it offer to agents.

[7] An important lesson from the literature on sequential contracting is that the principal's ability to treat agents asymmetrically *can* allow the principal to exploit a subset of agents (see, e.g., Segal and Whinston (2000), Genicot and Ray (2006), Galasso (2008), Chowdhury and Sengupta (2012)). Cai (2000) leverages this result to obtain inefficient delay in this context.

status quo is in the core, which is generally empty in multidimensional policy spaces, or when transfers are possible.[8] Our analysis provides new insights, including a new explanation for delay in legislative settings, and the larger inefficiency of more stringent supermajority rules.[9]

3. The model

There is a principal and a group of n agents who interact in an infinite horizon, $t = 1, 2, \dots$. We say the principal wins if and when she obtains the support of $q \le n$ agents. In each period t before the principal wins, any one of the $k(t)$ agents who remain uncommitted at time t meets the principal with probability $1/k(t) > 0$. In each meeting, principal and agent bargain over the terms of a deal by which i would support the principal. With probability $\phi \in [0, 1]$, the principal makes an offer $p \in \mathbb{R}$ to the agent, and with probability $1 - \phi$ the agent makes an offer $b \in \mathbb{R}$ to the principal. In both cases, the offer is a transfer from the principal to the agent (which can be positive or negative). If the recipient of the offer accepts it, i commits his support to the principal, and the transfer takes place; if the offer is rejected, i remains uncommitted. Upon completion, the principal gets a payoff $v \in \mathbb{R}_+$, and agents get $z \in \mathbb{R}_+$. In any period before completion, all players get a payoff of zero, not including any transfer they have received or paid. Principal and agents have a discount factor $\delta \in (0, 1)$.

The solution concept is symmetric Markov perfect equilibria (MPE). The restriction to symmetric MPE rules out discriminatory contracts, in the spirit of Genicot and Ray (2006). In particular, the strategies of principal and agents only condition on the number of agents $m \le q$ the principal still needs to obtain for completion. We let the state space be $M \equiv \{1, \dots, q\}$, and refer to the final trading state $m = 1$, in which the principal only needs to secure the support of an additional agent, as the *critical* state. We let $\beta(m) \equiv 1/(n + m - q)$ denote the probability that an agent meets the principal in state $m \in M$.

A strategy for the principal is a mapping $\sigma^p(\cdot) = (p(\cdot), \pi(\cdot))$, with $p : M \to \mathbb{R}$ and $\pi : \mathbb{R} \times M \to [0, 1]$, where for any state $m \in M$, $p(m)$ denotes the offer the principal would make to an agent in $m \in M$ when she has the opportunity to propose, and for any $b \in \mathbb{R}$ and state $m \in M$, $\pi(b; m)$ denotes the probability that the principal accepts an offer of b from an agent in $m \in M$. A strategy for an agent $i = 1, \dots, n$ is a mapping $\sigma^a(\cdot) = (b(\cdot), \lambda(\cdot))$, with $b : M \to \mathbb{R}$ and $\lambda : \mathbb{R} \times M \to [0, 1]$, where for any state $m \in M$, $b(m)$ denotes the offer the agent would make to the principal in $m \in M$ when she has the opportunity to propose, and for any $p \in \mathbb{R}$ and state $m \in M$, $\lambda(p; m)$ denotes the probability that the agent accepts an offer of p from the principal in $m \in M$. We let $\sigma \equiv (\sigma^p, \sigma^a)$

[8]For other explanations of delay in bargaining, not directly related to this paper, see Fershtman and Seidmann (1993) and Ma and Manove (1993) (deadlines), Yildiz (2004) and Ali (2006) (heterogeneous priors), Acharya and Ortner (2013) (bargaining over multiple issues with partial agreements), Iaryczower and Oliveros (2016) (intermediaries), and Miettinen and Vanberg (2020) (committing to a bargaining position).

[9]Merlo and Wilson (1995) show that under unanimity, *efficient* delay can emerge when the size of the surplus to be divided evolves stochastically over time. Eraslan and Merlo (2002) show that with rules other than unanimity, the equilibrium need not be efficient, in the sense that agreement may be reached *too soon* (too little delay).

denote the strategy profile. We denote the restriction of σ to the node m as $\sigma(m)$, and the restriction of σ to the subgame beginning in node m as $\sigma^m \equiv (\sigma(1), \dots, \sigma(m))$. We let $W(m|\sigma^m)$ and $W_c(m|\sigma^m)$ denote the continuation values of an uncommitted and a committed agent in state $m \in M$ consistent with σ^m, and $V(m|\sigma^m)$ denote the principal's continuation value in state $m \in M$ consistent with σ^m. We denote an equilibrium strategy profile as σ^*, and equilibrium values as $W^*(m) \equiv W(m|\sigma^*)$, $W_c^*(m) \equiv W_c(m|\sigma^*)$ and $V^*(m) \equiv v(m|\sigma^*)$.

4. Fundamentals of collective hold-up

In this section, we first prove existence and uniqueness of equilibrium outcomes. We then illustrate the main ideas of the collective hold-up problem by studying the two extreme cases, where bargaining power is fully allocated to the principal ($\phi = 1$) or the agents ($\phi = 0$).

We begin by establishing some basic properties of equilibria, assuming this exists. Consider an equilibrium σ^*. Suppose the principal has the opportunity to make an offer to agent i in state $m \in M$. Note that the agent will accept an offer p from the principal only if his continuation value after accepting the offer, $\delta W_c^*(m-1) + p$, is at least as large as his continuation value after rejecting the offer, $\delta W^*(m)$, and will accept the offer with probability one if this inequality holds strictly. When the agent receives an offer $p = \delta[W^*(m) - W_c^*(m-1)]$, he is indifferent, and accepts with probability $\lambda^*(m) \in [0, 1]$.

By the usual arguments, any offer $p > \delta[W^*(m) - W_c^*(m-1)]$ is not optimal for the principal. The principal is willing to make an offer $p = \delta[W^*(m) - W_c^*(m-1)]$ if and only if $p \le \delta[V^*(m-1) - V^*(m)]$, or (substituting) if and only if the bilateral surplus of moving forward, $s^*(m) \equiv s(m|\sigma^*)$, is nonnegative, where

$$s(m|\sigma) \equiv \big[V(m-1|\sigma) - V(m|\sigma)\big] + \big[W_c(m-1|\sigma) - W(m|\sigma)\big]. \tag{1}$$

Thus, in equilibrium, the principal offers

$$p^*(m) = \begin{cases} \delta\big[W^*(m) - W_c^*(m-1)\big] & \text{if } s^*(m) \ge 0, \\ -\infty & \text{if } s^*(m) < 0. \end{cases} \tag{2}$$

By a similar argument, in state $m \in M$ the principal accepts an offer $b = \delta[V^*(m-1) - V^*(m)]$ with probability $\pi^*(m)$, and rejects (accepts) strictly lower (higher) offers with probability one. The agent transacting with the principal in state $m \in M$ thus offers

$$b^*(m) = \begin{cases} \delta\big[V^*(m-1) - V^*(m)\big] & \text{if } s^*(m) \ge 0, \\ \infty & \text{if } s^*(m) < 0. \end{cases} \tag{3}$$

Note that in equilibrium, we must have $\pi^*(m) = \lambda^*(m) = 1$ whenever $s^*(m) > 0$. Suppose for instance $\lambda^*(m) < 1$ and $s^*(m) > 0$. Then the principal could gain by making a slightly larger offer, which would be accepted for sure. The equilibrium probabilities of trade in each state $m \in M$ when the agent and the principal propose are then given by

$$\mu_a^*(m) = \begin{cases} 1 & \text{if } s^*(m) > 0, \\ \pi^*(m) & \text{if } s^*(m) = 0, \\ 0 & \text{if } s^*(m) < 0, \end{cases} \qquad \mu_p^*(m) = \begin{cases} 1 & \text{if } s^*(m) > 0, \\ \lambda^*(m) & \text{if } s^*(m) = 0, \\ 0 & \text{if } s^*(m) < 0, \end{cases}$$

and the (ex ante) equilibrium probability of trade in each state $m \in M$ is

$$\mu^*(m) = \phi\mu_p^*(m) + (1-\phi)\mu_a^*(m) \tag{4}$$

Therefore, we have the following.

Remark 1. Given (2) and (3), an equilibrium is fully characterized by the trading probabilities $\mu_a^*(m)$ and $\mu_p^*(m)$ for each $m \in M$, and the equilibrium conditions[10]

$$\mu^*(m) < 1 \quad \Rightarrow \quad s^*(m) \le 0 \quad \text{and} \quad \mu^*(m) > 0 \quad \Rightarrow \quad s^*(m) \ge 0. \tag{5}$$

Given this, from now on we drop the explicit reference to σ, and simply condition on the profile of trading probabilities μ. We denote the vector of trading probabilities up to state k as $\mu^k \equiv (\mu(1), \dots, \mu(k))$, and the entire profile simply $\mu = \mu^q$.

Remark 2. Note that the principal's *expected gains from trade* in state m is composed of the gains from trade when making the offer, $\delta V(m-1) + p(m) = \delta s(m) + \delta V(m)$, which occurs with probability ϕ, and the gains from trade when receiving the offer, $\delta V(m)$ (w.p. $1-\phi$). Therefore, the expected gains from trade for the principal in state m is given by $x_P^* \equiv \phi\delta s(m) + \delta V(m)$. Similarly, the expected gains from trade for the agent in state m is given by $x_A^* \equiv (1-\phi)\delta s(m) + \delta W(m)$. Note that (x_P^*, x_A^*) is the solution to the generalized Nash bargaining program:

$$\max_{x_P \ge \delta V(m), x_A \ge \delta W(m)} \left(x_P - \delta V(m)\right)^{\phi} \left(x_A - \delta W(m)\right)^{(1-\phi)}$$
$$\text{s.t.} \quad x_P + x_A = \delta V(m-1) + \delta W_c(m-1),$$

with threat points given by continuation values after disagreement.[11]

Since the surplus $s^*(m)$ depends on the continuation values of principal and agent, pinning down the equilibrium probability of trade requires that we learn more about these equilibrium payoffs. Using (2) and (3), and recalling that $\beta(m) \equiv 1/(n+m-q)$ is

[10] Note that since the surplus is determined by the ex ante probability of trade $\mu(m)$, if there exists an equilibrium with $\mu^*(m) \in (0, 1)$ for some $m \in M$, any combination of $\left(\mu_p^*(m), \mu_a^*(m)\right)$ such that $\mu^*(m) = \phi\mu_p^*(m) + (1-\phi)\mu_a^*(m)$ can be supported in equilibrium.

[11] In fact, our bargaining protocol is also equivalent to nesting an infinite horizon bilateral bargaining in our game, where one of the sides decides whether to enter in negotiations or not, and in any period of the negotiation phase after a proposal is rejected, the principal (agent) makes offers with probability ϕ (resp., $1-\phi$).

the probability that an agent meets the principal in state $m \in M$, the values of principal and uncommitted agents can be written as (see Appendix A.1):

$$V^*(m) = \left(\frac{\delta}{1-\delta}\right)\phi \max\{s^*(m), 0\}, \tag{6}$$

and

$$W^*(m) = \left[\frac{\delta\beta(m)}{1-\delta\beta(m)}\right](1-\phi)\max\{s^*(m), 0\} + \left[1 + \left(\frac{1-\delta}{1-\beta(m)}\right)\frac{1}{\delta\mu^*(m)}\right]^{-1} W^*(m-1). \tag{7}$$

As equation (6) shows, the principal's equilibrium payoff in state m is proportional to the surplus $s^*(m)$ by a factor that increases with the principal's nominal bargaining power ϕ. Because delay can only occur in equilibrium if $s^*(m) = 0$, this means that if there is delay in state m in equilibrium, then $V^*(m) = 0$. On the other hand, the agent's equilibrium payoff in state m has two components. The first comes from the events in which the agent is negotiating with the principal, and is proportional to the surplus $s^*(m)$ by a factor that increases with the agents' bargaining power $1-\phi$. But differently to the principal's value, the agent's value $W^*(m)$ is positive even when $s^*(m) = 0$. This second component is increasing in the probability of trade $\mu^*(m)$ and the lagged value $W^*(m-1)$, and is due to the fact that as long as the negotiation process moves forward in state m with positive probability, the agent receives some value even when he does not meet the principal in that state.

The equilibrium payoff of a committed agent, on the other hand, only depends on the probability that the process moves forward or not: if there is a transaction (with probability $\mu^*(m)$), the committed agent gets a continuation payoff $\delta W_c^*(m-1)$, and otherwise gets $\delta W_c^*(m)$. Solving recursively, we obtain

$$W_c^*(m) = \left[\prod_{k=1}^{m}\left(\frac{\delta\mu^*(k)}{1-\delta(1-\mu^*(k))}\right)\right]z \tag{8}$$

We can now present our first main result.

Proposition 1. *(i) A symmetric Markov perfect equilibrium exists.*

(ii) The equilibrium probability of trade in each state m, $\mu^*(m)$, is unique.

(iii) The coalition forms (eventually) with probability one: $\mu^*(m) > 0\ \forall m \in M$.

The proof of Proposition 1 is by induction. Note first that with $v, z > 0$, a critical meeting ($m = 1$) must have trade with positive probability, and thus $V^*(1) + W_c^*(1) > 0$. In fact, as we show in Lemma 9, critical meetings must result in trade with probability one. Now suppose that for all $k < m$ there are transactions with positive probability, and take the implied continuation values $W_c^*(m-1)$, $V^*(m-1)$ and $W^*(m-1)$ as given. Note that since in all states $k < m$ there is trade with positive probability, the values of a committed agent and of the principal in state $m-1$ are positive. Thus, we cannot have that $\mu^*(m)=0$, for then $s^*(m)>0$, giving principal and agent an incentive to trade. We then show that the "one-shot" game in state m, in which payoffs are given by the continuation payoffs, has unique equilibrium outcomes.

Our goal for the rest of the paper is to understand how the allocation of bargaining power between principal and agents affect the efficiency of collective outcomes, and the welfare of principal and agents. To convey the key insights of the paper in the simplest way possible, we begin by analyzing the two extreme cases in which either the principal or the agents have full proposal power in bilateral meetings ($\phi=1$ and $\phi=0$, resp.). In Section 5, we consider intermediate allocations of bargaining power $\phi\in[0,1]$, and discuss several extensions of the model.

4.1 *Principal has full proposal power*

The case in which the principal has all the bargaining power ($\phi=1$) was analyzed in Iaryczower and Oliveros (2017).[12] As that paper shows, when the principal has full bargaining power, the unique equilibrium is efficient.

Proposition 2 (Iaryczower and Oliveros (2017)). *The game with $\phi=1$ has a unique equilibrium, in which there is trade in each state $m\in M$ with probability one; i.e., $\mu^*(m)=1$ for all $m\in M$. In this equilibrium, the payoff of an agent is given by*

$$W^P(m)\equiv\left[\frac{\beta(m)(n-q)}{\displaystyle\prod_{j=1}^{m}\bigl(1-\delta\beta(j)\bigr)}\right]\delta^m z. \tag{9}$$

The intuition for the proof can be seen in two steps. First, fix the proposed equilibrium. Since $v>0$ and $z>0$, when the principal needs to collect the support of only one additional agent ($m=1$), the principal and the agent can create and capture a positive surplus by moving forward. Thus, given full information, there is a price at which this transaction occurs. Now suppose the principal needs to get the support of $m>1$ agents. Since in equilibrium there is trade whenever the principal needs to secure the support of $t<m$ additional agents, then in state m there is also a positive surplus for the principal and the selected agent to obtain if they move forward, and then again a price at which this happens. This shows that the proposed strategy profile is an equilibrium.

The argument for uniqueness is by induction. By the same logic as in the previous paragraph, in any equilibrium there must be a transaction when $m=1$. Suppose then that in equilibrium there is trade whenever the principal needs to secure the support of $t<m$ additional agents. Recall that in state m, in the proposed equilibrium there is a positive surplus for the agent and the principal. Then if the principal with positive probability does not make an offer, or the agent with positive probability does not accept, principal and agent would obtain a lower payoff in this state, and thus the gain

[12] Iaryczower and Oliveros (2017) consider the case of competition among multiple principals. They show that agents are better off facing a single principal than when there is competition between principals.

from moving forward would be higher. It follows that the principal will make an offer, which the agent will accept.

Proposition 2 implies that in equilibrium, provided $q < n$, the principal cannot extract all surplus from the agents. The reason for this is similar to the logic behind underprovision of a public good. Note that since the agents benefit from implementing the alternative to the status quo and they cannot obtain a higher terminal payoff by remaining uncommitted, in equilibrium the agents pay the principal to move on.[13] By rejecting the offer, however, an agent can rely on others to pay the bill. This generates an outside option that gives each agent some bargaining power over the principal. Since the cost of deferring implementation of the proposal decreases with δ, the value of the outside option is increasing in δ, and so is agents' equilibrium payoff. In fact, as δ approaches 1, $W^P(m) \to z$. With unanimity ($q = n$), however, the opportunity to free-ride from other agents disappears, and $W^P(m) = 0$ for all $m \in M$.

4.2 *Agents have full proposal power*

We now consider the case in which agents have full proposal power in bilateral negotiations with the principal; i.e., $\phi = 0$. Recall that the value for the principal is determined solely by the gains from trade (see equation (6)), while the value for the agent also incorporates the possibility of relying on others to trade with the principal (see equation (7)). In state $m \in M$, the agent negotiating with the principal makes an offer so that the principal obtains his outside option. Because the principal never makes an offer and has to participate in all meetings, his outside option in state m is just the discounted value of waiting to receive another offer $\delta V^A(m)$ remaining in the same state. Since the agent extracts all surplus from trade, the value at state m for the principal $V^A(m)$ must be equivalent to his outside option $\delta V^A(m)$, yielding $V^A(m) = 0$ for all $m \geq 1$. Now, recall that by (3), the agent negotiating with the principal in state $m \in M$ offers $b^A(m) = \delta[V^A(m-1) - V^A(m)]$ whenever $s^A(m) \geq 0$, and otherwise makes a nonrelevant offer. Thus, $b^A(1) = \delta v$ and $b^A(m) = 0$ at all $m \geq 2$. When the agents have all the bargaining power, then in equilibrium, the principal's role reduces to that of a passive intermediary.

In the absence of side payments, the probability of trade depends on the relative value for an agent of moving the process along supporting the principal for free, $W_c^A(m-1|\mu)$, versus holding out support with the goal of extracting the rent δv in late trading, $W^A(m)$. In this sense, the game in each state $m > 1$ becomes similar to a war of attrition between agents. The caveat of course is that as agents concede, the system moves to a different state, where the relative value of holding out or conceding are different. To

[13] An indirect consequence of the assumption that agents obtain the same payoff $z > 0$ upon completion independently of whether they committed their support or not is that when $\phi = 1$, transfers from principals to agents are negative, a feature that can be unappealing in some applications. In Iaryczower and Oliveros (2022), we solve a more general version of the model where we allow payoffs to depend on whether each agent supported the principal or remained uncommitted. We show that most relevant applications involve positive transfers from the principal to agents under reasonable assumptions on parameters, even when the principal has full bargaining power.

characterize equilibrium, then we need to understand agents' incentives to hold out in each state $m > 1$.

When agents have all the bargaining power, it is easy to solve for the value function of an uncommitted agent, because the principal is not able to capture rents from the agents, and as a result the value function of an uncommitted agent becomes a standalone difference equation. In particular, note that

$$\begin{aligned} W^A(m) = {} & \mu^A(m)\big[\beta(m)\delta W_c^A(m-1) + \big(1-\beta(m)\big)\delta W^A(m-1)\big] \\ & + \big(1-\mu^A(m)\big)\delta W^A(m). \end{aligned}$$

With probability $\beta(m)\mu^A(m)$, agent i meets with the principal and commits her support to her, getting a discounted payoff $\delta W_c^A(m-1)$. With probability $(1-\beta(m))\mu^A(m)$, an agent $j \neq i$ commits his support to the principal, in which case the state moves to $m-1$ and i remains uncommitted, with a discounted payoff $\delta W^A(m-1)$. With probability, $1-\mu^A(m)$, either i or some other agent j meets the principal, but no agreement is reached, so the system remains in m, and i obtains a discounted payoff $\delta W^A(m)$. Thus, solving recursively,

$$W^A(m) = \left[\prod_{j=1}^{m} \frac{\delta\mu^A(j)}{1-\delta\big(1-\mu^A(j)\big)}\right]\big(z+\beta(m)v\big). \tag{10}$$

Using (10) and the value for committed agents (8), the condition for trade with positive probability at $m > 1$ that $W_c^A(m-1) \geq W^A(m)$ boils down to

$$z \geq \left[\frac{\delta\mu^A(m)}{1-\delta\big(1-\mu^A(m)\big)}\right]\big(z+\beta(m)v\big) \tag{11}$$

Now, consider an equilibrium candidate probability of trade in state m, $\mu(m)$. For delay to occur with positive probability at m, we need (11) to hold with equality. Note that the right-hand side is a continuous increasing function $f(\cdot; m)$ of $\mu(m)$ such that $f(0; m) = 0$ and $f(1; m) = \delta(z+\beta(m)v)$. Since (11) is satisfied with $\mu(m) = 0$, this implies that in equilibrium there is always trade with positive probability in all states $m > 1$. On the other hand, there exists a (unique) solution $\mu(m) \in (0, 1)$ satisfying (11) with equality if and only if

$$z < \delta\big(z+\beta(m)v\big) \quad\Longleftrightarrow\quad m < \frac{\delta}{(1-\delta)}\frac{v}{z} - (n-q) \equiv \widehat{m} \tag{12}$$

It follows immediately from this that there exists a unique cutpoint $\widehat{m} > 2$ such that, in equilibrium, there is delay in each state $m \in M : 2 \leq m < \widehat{m}$, and trade with probability one in any $m \geq \widehat{m}$. Moreover, the set of states in which there is delay is weakly increasing in the relative value of holding out, v/z, and for any $m \in M$ there is a v/z large enough such that $m < \widehat{m}$ (and there is delay in m in equilibrium). The ratio v/z also increases the probability of delay in states below the cutpoint. In fact, note that when there is delay in state m, the equilibrium probability of trade is given by $\mu(m) \in (0, 1)$ solving

$W_c^A(m-1) = W^A(m)$, or

$$\mu^A(m) = \left(\frac{1-\delta}{\delta\beta(m)}\right)\frac{z}{v} \tag{13}$$

Three other features of the solution are noteworthy. First, note that $\beta(m) = 1/(n+m-q)$. Thus, both the set of states in which there is delay (12) and the probability of delay in states below the threshold are increasing in the size of the coalition required to win, q, relative to the size of the group, n. This corresponds well to the intuition that more stringent supermajority requirements are costly because they induce delay. Second, since $\beta(m)$ is decreasing in m, the probability of trade $\mu^A(m)$ is increasing in m. Therefore, transactions occur at a faster pace initially, with the process of negotiations slowing down as it goes along. Third, note that as $\delta \to 1$, the threshold $\widehat{m}$ in (12) goes to $+\infty$, so that in equilibrium there is delay in all states $m > 1$. Moreover, from (13), $\lim_{\delta\to 1}\mu^*(m) = 0$, so that in each state $m > 2$, negotiations slow down almost to a halt. As we show later, these are special features of the model with $\phi = 0$, which do not generalize when the principal has positive bargaining power.

The previous discussion fully characterizes equilibria of the game in which agents have all the bargaining power. We are interested in particular in equilibrium outcomes for large v/z, where the collective hold-up problem is severe. More precisely, we focus on equilibrium outcomes for $v/z \geq K$, for some constant $K < \infty$. The next proposition summarizes our discussion focusing on this case.

Proposition 3. *Consider the game with $\phi = 0$. Suppose $v/z \geq \frac{(1-\delta)}{\delta}n$. Then there is a unique equilibrium outcome, with trading with probability one in the critical state $m = 1$ and delay in all $m : 2 \leq m \leq q$, given by (13). Agents' payoffs are*

$$W^A(q) = \left(\prod_{j=2}^{q-1}\frac{1}{1+\beta(j)v/z}\right)\delta z, \quad \textit{and} \quad \lim_{v\to\infty} W^A(q) = 0.$$

Rearranging (9), on the other hand, the initial equilibrium payoff of an uncommitted agent when the principal has all the bargaining power is

$$W^P(q) = \left(\prod_{j=1}^{q}\frac{1}{1-\delta\beta(j)}\right)\left(\frac{n-q}{n}\right)\delta^q z,$$

which is positive whenever $q < n$, and independent of v. It follows that for v/z sufficiently large, and provided $q < n$, agents are better off when the principal has full bargaining power than when agents have full bargaining power.

Corollary 4. *Let $q < n$. $\exists K > 0$ such that $\forall v/z > K$, $W^P(q) > W^A(q)$.*

The large willingness to pay of the principal poses a tradeoff for agents' welfare: a larger v increases the total surplus from transacting, but also leads to larger delay. As the corollary shows, in equilibrium the larger delay more than compensates for the increase in total surplus and leads to a loss of welfare for the agents. The conclusion of the corollary does *not* hold under unanimity, which is the classic railroad–farmers example considered by Coase (see Cai (2000), Chowdhury and Sengupta (2012)). This is because with $q = n$, $\beta(1) = 1/(n+1-q) = 1$, so in the critical state the agent cannot free-ride on others. Thus, $W^P(1) = \delta W^P(1)$, which implies $W^P(1) = 0$. But then, recursively, $W^P(m) = 0$ for all $m \in M$. Thus, while the agents' equilibrium payoff when $\phi = 0$ approaches 0 as $v \to \infty$, agents are still better off when they have proposal power.

The case in which agents have all the bargaining power illustrates the essence of the collective hold-up problem. As we have seen, though, this case is special, because the principal is reduced to a passive intermediary in all states $m > 1$. In the next section, we solve the model for an arbitrary allocation of bargaining power between principal and agents. We show that the equilibrium of the game for large v/z consists of two phases: a final phase in which the principal is active, and trade occurs with probability one, and an early phase that resembles a war of attrition among agents, who hold out with positive probability to extract rents in the efficient stage. We show, furthermore, that the size of the efficient phase increases as bargaining power shifts from the agents to the principal. In essence, as agents gain bargaining power, the incentives induced by collective hold-up dominate other strategic considerations.

5. Bargaining power and equilibrium outcomes

In this section, we address how the allocation of bargaining power between principal and agents affects the efficiency of collective decisions and agents' welfare. A special feature of the case in which agents have full bargaining power is that in equilibrium the principal is fully expropriated of rents, due to the combination of not having proposal power, and being unable to free-ride on other actors to move the project forward. As a result, the principal is effectively reduced to a passive player in all states other than the critical state $m = 1$.

The situation is qualitatively different when the principal makes proposals with positive probability, because the principal is not fully expropriated in later stages of negotiation. For the principal to be an active player in a state m, we need $s^*(m) > 0$, which in turn implies that in equilibrium, there can be no delay in state m. But when agents anticipate that $\mu^*(m) = 1$, their incentives to hold out are strongest, since a rejection is followed by trade for sure on the equilibrium path. The key consideration for whether the principal can be active in equilibrium then is whether the agents' incentives to hold out can be kept at bay *even when there is trade with probability one* at m.

Since the bilateral surplus $s^*(m)$ is composed of the payoff gain for both the principal $V^*(m-1) - V^*(m)$ and the agent, $W_c^*(m-1) - W^*(m)$, to pin down the conditions under which negotiations in state $m \in M$ are efficient that we need to solve the value functions of the principal and the uncommitted agents. As the previous discussion illustrates, the difficulty here comes from the fact that when both principal and agents make proposals with positive probability, the principal can extract rents from agents, agents can extract rents from the principal, and (through the principal) agents can extract rents from other agents. This implies that—differently to the case in which either the principal or the agents have all the bargaining power—the system of difference equations characterizing equilibrium payoffs cannot be decoupled. To tackle this difficulty, we use a transformation to express the system of value functions as a second-order difference equation, which we then solve. This allows us to obtain a closed-form solution for uncommitted agents' payoffs $W^*(m)$ in each state $m \in M$ as a function of trade probabilities $\mu^{*m} \equiv (\mu^*(1), \dots, \mu^*(m))$ and model primitives (see Lemma 10 in the Appendix).

Using Lemma 10, we obtain a necessary and sufficient condition for equilibrium that links a candidate equilibrium probability of trade in state m, $\mu^*(m)$, with the resulting surplus $s^*(m)$ for *any* given probability of trade in the continuation game (Lemma 11 in the Appendix). In particular, we show that for any state $m \in M$, and an equilibrium candidate probability of trade $\mu^*(m)$, then $s^*(m) \geq (\leq) 0$ if and only if

$$W^*(m) \leq (\geq) W_c^*(m)\left(\frac{z + \beta(m)v}{z}\right); \tag{14}$$

Note that by (8), the value of a committed agent can be written as $W_c^*(m) = A \times z$ for some $A \in [0, 1]$, where A is a coefficient capturing delay and discounting. Thus, the surplus in state m is nonnegative (nonpositive) if and only if the value of remaining uncommitted is not larger than an *adjusted* version of the continuation value for a committed agent, where the terminal payoff z is substituted by $z + \beta(m)v$. The additional term $\beta(m)v$ is the expected value of a lottery that gives a prize v to one of the agents who remain uncommitted in state m.

In our next result, we use these results to establish an important property of equilibria. Note that in principle, delay could be front-loaded (occur at the beginning of the bargaining process), back-loaded, or occur in some interior subset of states. Or the set of states with delay could potentially be unconnected, with regions of delay followed by states in which trade is efficient. Moreover, the probability of trade could be nonmonotonic, having stages in which the negotiation process accelerates after every trade followed by periods in which it slows down with subsequent commitments.[14] In our next result, we show that delay always occurs in a connected set of states. Furthermore, we are able to pin down precisely the equilibrium probability of trade as a function of primitives.

Proposition 5. *Let $m', m'' \in M$, with $m'' > m'$.*

(i) If $\mu^*(m') < 1$ and $\mu^*(m'') < 1$, then $\mu^*(m) < 1$ for all $m : m' \leq m \leq m''$.

(ii) Moreover, for any $m : m' < m \leq m''$,

$$\mu^*(m) = \left(\frac{1 - \delta}{\delta}\right) \frac{1}{\beta(m)} \frac{z}{v} \tag{15}$$

To see why delay occurs in a connected set of states, note that in equilibrium, the bilateral trading surplus is nonincreasing in the state (weakly increasing as the process

[14]For instance, in the context of negotiations between the seller of a good and several potential buyers, Jehiel and Moldovanu (1995a) show that when the seller is sufficiently patient and externalities between buyers are negative, SPNE in pure strategies with bounded recall have the property that long periods of waiting alternate with short periods of activity (when externalities are positive there is no delay).

moves toward completion), due to discounting and the fact that payoffs are realized in terminal states. Thus, if there is delay in states m' and $m'' > m'$ (and hence $s^*(m') = s^*(m'') = 0$), in equilibrium we must have $s^*(m) = 0$ for any $m : m' < m < m''$. For the second part, note that if there is delay in states m and $m-1$, then in equilibrium, $s^*(m) = s^*(m-1) = 0$, and then $V^*(m) = V^*(m-1) = 0$. But then $s^*(m) = W_c^*(m-1) - W^*(m)$, and so we must have $W_c^*(m-1) = W^*(m)$. With no payments from the principal, delay has to be such as to equate the value of committing or staying out, as in the extreme case in which the principal had no bargaining power.

As a result, the expression for the trading probabilities (15) in Proposition 5 is identical to the corresponding expression when agents have all the bargaining power and, therefore, unaffected by the allocation of bargaining power between the principal and the agents. For any allocation of bargaining power, then the properties of delay are inherited from Section 4.2: the probability of trade $\mu^*(m)$ is decreasing with the principal's willingness to pay, v, the discount factor δ, and the number of agents required for approval, q, and increasing in agents' terminal payoff, z, and the size of the pool of agents, n. Moreover, whenever there is delay, the pace of negotiations must *slow down* as the process of negotiations move forward ($\mu^*(m)$ increases with m).

Since delay in $m \in M$ requires $\mu^*(m) < 1$, it follows immediately from equation (15) that if $\delta\beta(m)(v/z) < 1 - \delta$, there cannot be an equilibrium with delay in multiple states for states $m' \geq m$. This directly implies that if v/z is sufficiently low, so that holding out is not attractive for the agents trading early, in equilibrium there can only be delay in at most one state; i.e., if $\delta\beta(2)(v/z) < 1 - \delta$, then $\mu^*(m) = 1$ for all $m \in M$, except possibly in state $m = 2$.

In our next two results, we focus on the case in which the relative value of holding out v/z is large, and provide characterization of the equilibria. We focus on this case because it is then that the collective hold-up problem appears more clearly, as agents compete to extract rents from the principal. Our first result provides a sufficient condition for delay to be *front-loaded* (occurring at the earlier stages of negotiations).

Proposition 6 (Front-loaded delay). *Suppose $\delta\beta(m')(v/z) \geq 1 - \delta$ for $m' > 1$. If there is delay in state $m' - 1$, there is delay in state m'. In particular, for any $m \in M$,*

$$\textit{if } \frac{1}{n}\frac{v}{z} \geq \frac{1-\delta}{\delta} \quad \textit{then } \mu^*(m) < 1 \quad \Rightarrow \quad \mu^*\big(m'\big) < 1 \quad \textit{for all } m' \in \{m, \dots, q\}.$$

The condition for front-loaded delay requires v/z to be sufficiently large. The requirement is more stringent the largest is the size of the group, n, and the lower is the discount factor, δ. Suppose for concreteness that $\delta = 0.95$. Then if $n = 19$, we need $v \geq z$. For small groups, we can have $v < z$: with $n = 5$, it suffices to have $v \geq z/4$. Note also that this is the same condition for delay in all but the critical state when agents have full bargaining power. Thus, when there is full delay in noncritical states under $\phi = 0$, delay is front-loaded for any allocation of bargaining power.

When the principal's valuation is high relative to agents' payoff for completion, then the collective hold-up game can be separated into two phases. The first phase is a modified war of attrition between the agents, who must choose to concede or remain in the game. In this first phase, the principal plays a passive role, and the only benefit of trading for the agents comes from moving the process toward completion. The second phase is a decentralized bargaining game in which trade occurs without delay. In this second phase, trade occurs with transfers to and from the principal, so the role of the principal is key. Crucially, the agent's decision to support the project for free in the "war of attrition" phase depends on the surplus that the agent expects to extract in the second phase of the decentralized bargaining game. As a result, all else constant, any change that increases (decreases) the agent's surplus extraction ability in the later phase will induce more (less) delay, for the same reasons that a higher prize induces players in a war of attrition to fight for longer.

We want to understand how the allocation of bargaining power between principal and agents affects the two phases of the bargaining game, when the principal has a large willingness to pay. From our analysis in Section 4, we know that when bargaining power is fully decentralized to the agents, the efficient bargaining phase is reduced to the critical state $m = 1$. Instead, when bargaining power is fully concentrated in the principal, the efficient bargaining phase encompasses all states, as bargaining is fully efficient. In our next result, we show that for any state $m > 1$, the equilibrium of the m-subgame is efficient if and only if power is sufficiently concentrated in the principal; i.e., there is a unique cutpoint $\hat{\phi}(m)$ such that all states $m' \leq m$ are in the efficient bargaining phase if and only if $\phi \geq \hat{\phi}(m)$. Equivalently, for any allocation of bargaining power $\phi \in [0, 1]$, there exists a unique cutpoint in the state space, $\overline{m}(\phi) \in M$, such that in equilibrium there is delay in each state $m > \overline{m}(\phi)$ (given by equation (15)), and trade with probability one in any state $m \leq \overline{m}(\phi)$. Moreover, $\overline{m}(\phi)$ is weakly increasing in ϕ. Therefore, increasing ϕ expands the size of the efficient bargaining phase.[15]

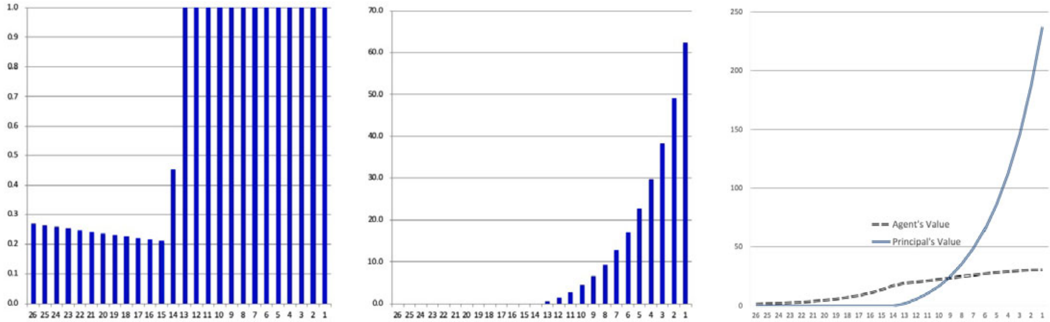
Proposition 7 (Delay and bargaining power). $\exists K > 0$ *such that for all* $v/z \geq K$, *the following is true:*

(i) For any $m \in M \setminus \{1\}$, $\exists \hat{\phi}(m) \in (0, 1)$ *such that the equilibrium of the* m*-subgame is efficient if and only if* $\phi \geq \hat{\phi}(m)$.

(ii) Let $m, m' \in M \setminus \{1\}$. *If* $m' > m$, *then* $\hat{\phi}(m') > \hat{\phi}(m)$.

The proof of part (i) of the proposition has three steps. First, we obtain a necessary and sufficient condition for existence of an efficient equilibrium in the m-subgame in terms of model primitives. To do this, we use Lemma 10 to obtain an expression for payoffs under efficient trading in terms of primitives of the model, which we label $W^{\dagger}(m)$, $W_c^{\dagger}(m)$, and $V^{\dagger}(m)$. In particular, letting $\chi_{jm} \equiv \prod_{j=k}^{m}(1 - \delta + \delta\phi(1 - \beta(j)))$, the payoff of

[15] In the proposition, we take parameters (q, n, ϕ, δ) as given, and consider equilibrium outcomes for v/z large enough, given these parameters. In Section 6.2, we show that for any given (v, z, q, n), if $\phi > 0$, there is a $\overline{\delta} > 0$ such that if $\delta \geq \overline{\delta}$, the unique equilibrium is efficient. In the terminology of the proposition, for any $m \in M \setminus \{1\}$, $\lim_{\delta \to 1} \hat{\phi}(m) = 0$. This contrasts with the case of $\phi = 0$ we analyzed in Section 4.2. We relegate the discussion of this difference in limiting outcomes to this section.



trade probability surplus values

FIGURE 1. Trade probability, equilibrium payoffs, surplus in an example ($v = 300$, $z = 30$, $\delta = 0.95$, $n = 51$, $q = 26$, $\phi = 0.2$).

an uncommitted agent under efficient trading is

$$W^{\dagger}(m) \equiv \delta^m \beta(m) \left\{ \frac{\phi^m}{\chi_{1m}}(n-q)z + (1-\delta)(1-\phi) \sum_{k=1}^{m} \left(\frac{\phi^{m-k}}{\chi_{jm}} \right) \left(v + \frac{z}{\beta(k)} \right) \right\}. \tag{16}$$

Noting that $W_c^{\dagger}(m) = \delta^m z$, condition (14) shows that the equilibrium of the m-subgame is efficient if and only if $W^{\dagger}(m') \leq \delta^{m'}(z + \beta(m')v)$ for all $m' \leq m$. Moreover, by Proposition 6, for large v/z it is enough to require that this condition holds in state m. Thus, for large v/z, the equilibrium of the m-subgame is efficient if and only if the continuation value of an uncommitted agent in state m under efficient trading is not greater than the discounted value of $z + \beta(m)v$. Second, we show that for any $m \in M$, there are $\overline{\phi}(m)$, $\underline{\phi}(m) \in (0, 1)$, which are independent of v, such that for large v, the unique MPE of the m-subgame is efficient if $\phi > \overline{\phi}(m)$, and has delay if $\phi < \underline{\phi}(m)$. Third, we show that if the equilibrium of the m-subgame is efficient, then $W^{\dagger}(m)$ is strictly decreasing in ϕ.[16] We use this result to show that there exists a unique threshold $\hat{\phi}(m) \in (0, 1)$ such that the equilibrium has no delay if and only if $\phi \geq \hat{\phi}(m)$.

Together with our earlier results, Propositions 6 and 7 provide a full characterization of equilibria when the collective hold-up problem is "severe." We should point out, however, that none of our results are limiting results. To fix ideas, consider the following example.

EXAMPLE. Suppose $v = z = 1$. Let $n = 5$, $q = 4$, and suppose $\delta = 0.95$ and $\phi = 0.01$. Then $\mu^*(m) < 1$ for all $m > 1$. If instead $\phi = 0.1$, the equilibrium has delay only in the initial state $m = 4$, and for $\phi = 0.2$, the equilibrium is efficient. $\Diamond$

[16] The result is intuitive, because the direct effect of reducing ϕ is to increase the ability of agents to extract the available surplus from the principal in any state. Reducing the rents of the principal in a state m, however, has the indirect effect of lowering her willingness to pay in states $m' > m$, thus reducing the value of agents transacting early. We show that in a no-delay equilibrium, the direct effect dominates.

5.1 *Collective hold-up and economic outcomes*

We are now in a position to answer the questions we posed in the Introduction.

How does the allocation of bargaining power between principal and agents affect the efficiency of collective decisions? Proposition 7 shows that when the principal's willingness to pay is high, redistributing bargaining power from the principal to the agents creates delay and reduces agents' welfare. In particular, the number of transactions with positive expected delay is decreasing in ϕ, so that giving more power to the agents increases the number of bargaining states in which transactions fail with positive probability. As Proposition 5 shows, however, the probability of trade in all but possibly the last state with delay is independent of the allocation of bargaining power between the principal and the agents. Thus, the allocation of bargaining power affects the number of states with delay, but not the expected delay in each state.

How does this delay appear in the negotiation process? For a given allocation of bargaining power ϕ inducing delay, the expected delay for each transaction increases as we move further along the process in the first $q - \overline{m}(\phi) - 1$ transactions (possibly decreasing in the last transaction with delay). But once the principal obtains the support of $q - \overline{m}(\phi)$ agents, the remaining transactions occur without delay. In the special case in which the agents have full or almost full bargaining power, delay occurs in all but the critical state, and the expected delay is monotonically increasing until the critical state as we move further along the process.

How do agents' and principal's preferences affect this inefficiency? Note that in states with delay, the probability of trade (15) is decreasing in the ratio v/z between the value that the principal and agents put on finishing the project. In fact, for any given allocation of bargaining power ϕ for which there is delay in more than one state, expected delay grows continuously with v/z, and in the limit as $v/z \to \infty$, the expected time for completion goes to infinity. In general, in an equilibrium with delay in L' states, the expected delay to obtain the support of the first $L < L'$ agents is

$$\mathcal{E}(L) = \left(\frac{\delta}{1-\delta}\right)\frac{v}{z}\left[\beta(q) + \cdots + \beta(q-L+1)\right] = \left(\frac{\delta}{1-\delta}\right)\left(\sum_{\ell=1}^{L}\frac{1}{n-\ell+1}\right)\frac{v}{z},$$

increasing in v/z and the discount factor δ, decreasing in the size of the group n, and *independent* of the threshold q.

How does agents' bargaining power affect their welfare? In the efficient equilibrium, agents' welfare increases with their bargaining power. As a result, keeping the strategy profile fixed, agents would be better off retaining as much power as possible. However, as we have seen, decentralizing power to agents also increases the range of states in which negotiations suffer delay. Moreover, in each of these states, delay is increasing in the principal's willingness to pay, v. This poses a tradeoff for agents' welfare: a larger v increases the total surplus from transacting, but also leads to larger delay.

Using our previous results, it is easy to show that the larger delay more than compensates for the increase in total surplus, and leads to a loss of welfare for the agents. This leads to the counterintuitive result that, for large v, agents' welfare is maximized when the principal holds substantial bargaining power. To see this, recall that if in a state

m there is trade with probability $\mu^*(m) < 1$, then $s^*(m) = 0$. Thus, from the recursive expression (7) for the value of uncommitted agents,

$$W^*(m) = \left[1 + \left(\frac{1-\delta}{1-\beta(m)}\right)\frac{1}{\delta\mu^*(m)}\right]^{-1} W^*(m-1) \quad \text{for all } m > \overline{m}(\phi)$$

On the other hand, we know that in all states $m \leq \overline{m}(\phi)$, the equilibrium is efficient. Thus, we can use (23) to obtain an expression for uncommitted agents' payoffs in state $\overline{m}(\phi)$ under efficient trading in terms of primitives of the model, $W^\dagger(\overline{m}(\phi))$. We can then express the equilibrium payoff of an uncommitted agent at the beginning of the game as

$$W^*(q) = \left[\prod_{k=\overline{m}(\phi)+1}^{q} \left(1 + \left(\frac{1-\delta}{1-\beta(k)}\right)\frac{1}{\delta\mu^*(k)}\right)^{-1}\right] W^\dagger\big(\overline{m}(\phi)\big).$$

By Proposition 5, for all $m \leq q$ such that $m > \overline{m}(\phi) + 1$, $\mu^*(m)$ is given by (15), and in particular $\mu^*(m) \to 0$ as $v/z \to \infty$. It follows that if $\overline{m}(\phi) < q-1$, then $W^*(q) \to 0$ as $v/z \to \infty$.[17] Moreover, we have shown that for any $m \in M$ there is a ϕ such that $\overline{m}(\phi) = m$ (Proposition 7). Thus, this is guaranteed to occur if power is not sufficiently concentrated in the principal.

Corollary 8. *$\exists K > 0$ such that for any $v/z > K$, any ϕ such that $\overline{m}(\phi) < q-1$ leads to lower equilibrium payoffs for the agents than giving complete bargaining power to the principal, $\phi = 1$. In particular, agents are better off if the principal has full bargaining power than if agents have full bargaining power.*

In turn, since the no delay payoff $W^\dagger(q)$ is decreasing in ϕ when an efficient equilibrium exists, agents prefer the smallest ϕ such that an efficient equilibrium exists to $\phi = 1$. It follows that for large enough v, agents prefer ϕ such that either $\overline{m}(\phi) = q-1$ or $\overline{m}(\phi) = q$, granting considerable bargaining power to the principal. Overall, the basic intuition of how bargaining power affects outcomes works well in an efficient equilibrium, but when the collective hold- up problem is severe, the intuition breaks down. Instead, concentrating bargaining power in the principal reduces inefficiencies and improves agents' welfare.

6. Extensions

6.1 *Contracting with externalities*

Beginning with Grossman and Hart (1980), one of the central contributions of the literature that focused on understanding contracting problems between a principal and a group of agents is to emphasize the role of externalities. The importance of positive and negative externalities in contracting models was further highlighted in a static setting by Segal (1999) and Segal (2003), and is also a central component in the dynamic setup of

[17] We impose the requirement that there is delay in at least two states because (15) pins down the trading probability in all but the last state with delay.

Jehiel and Moldovanu (1995a) and Jehiel and Moldovanu (1995b). It has also emerged as a key consideration in the literature on noncooperative coalitional bargaining games, which has shown that externalities can lead to breakdown of efficiency (see Bloch (1996), Ray and Vohra (1999, 2001), Gomes (2005), and Gomes and Jehiel (2005)).[18]

To consider the effect of positive and negative externalities on nontraders, in Iaryczower and Oliveros (2022) we allow the completion payoff for committed and uncommitted agents to differ. In particular, we assume that upon completion, committed agents get $z \in \mathbb{R}_+$, and uncommitted agents get $w \in \mathbb{R}$, where $w > 0$ ($w < 0$) implies that there are positive (negative) externalities on uncommitted agents, and $w = 0$ implies that there are no externalities on uncommitted agents. As we show in the working paper, all our main results remain valid in this extended model. In particular, we show that delay can arise in equilibrium with negative externalities, no externalities, or even positive externalities on uncommitted agents. Thus, neither positive nor negative externalities are necessary for our result. Disentangling the terminal payoff of committed and uncommitted agents, however, allows us to better understand the determinants of delay. In particular, a higher value for belonging to the coalition (z large) reduces delay, as it increases the incentive to trade, while a large positive externality on uncommitted agents (large w) has the opposite effect.

The extension also allows us to naturally expand the range of applications of the model, to understand exclusive deals in new technologies with increasing returns to scale ($w > z > 0$; see Katz and Shapiro (1992), Segal and Whinston (2000)), corruption ($z > 0 > w$; see McMillan and Zoido (2004)), or start-ups ($z > w = 0$). We refer the reader to the working paper version for a discussion.[19]

6.2 *Vanishing frictions*

In the paper, we characterized equilibrium outcomes for fixed $\delta < 1$, for v/z sufficiently large. A natural question is how do equilibrium outcomes change for fixed (v, z) as frictions vanish. In fact, the results in the literature on delay in bargaining with complete information have generally been established for large δ. This is the case for delay caused by deadlines in Fershtman and Seidmann (1993), for the monopolist selling a good to heterogeneous buyers in Jehiel and Moldovanu (1995a), for delay through discriminatory contracts in Cai (2000), and in the example provided by Gomes (2005) in a general model of coalitional bargaining.

From the expression for the equilibrium trading probability $\mu^*(m)$ in Proposition 5, one might be tempted to conclude that for fixed v, the probability of trade goes to zero as $\delta \to 1$, so that when bargaining frictions vanish negotiations slow down almost to a halt. This would be incorrect, for the threshold $\hat{\phi}(m)$ in Proposition 7 is a function of

[18]Coalitional bargaining games without externalities, instead, generally have efficient equilibria. See, for example, Okada (1996, 2011). Chatterjee, Dutta, Ray, and Sengupta (1993), Ray and Vohra (1999), and Gomes (2005) provide examples featuring delay in general bargaining models.

[19]In some contexts, the commitment payoff $z > 0$ can be naturally interpreted as a limited form of contingent contracting between principal and agents, further broadening the scope of applications of the model.

δ, and in fact $\lim_{\delta \to 1} \hat{\phi}(m) = 0$ for all $m \in M$. Thus, for any given $\phi > 0$ and parameters (v, z, q, n) there is a $\overline{\delta} > 0$ such that if $\delta \geq \overline{\delta}$ the unique equilibrium is efficient. To see this more directly, note that from (8) and (16), for any $m \in M$,

$$\lim_{\delta \to 1} W^{\dagger}(m) = \lim_{\delta \to 1} W_c^{\dagger}(m) = z.$$

Thus, the condition for existence of an efficient equilibrium (14) boils down to $z \leq z + \beta(m)v$ for all $m \leq q$, which is always satisfied, since $v \geq 0$.

This result can be surprising in light of our results in Section 4.2, where we showed that when the agents have full proposal power ($\phi = 0$), for large v/z, the equilibrium has delay in all but the critical state. To understand the logic, assume $\delta \to 1$, and consider the critical state, $m = 1$. Recall that in equilibrium, the coalition eventually forms with probability one. Since the principal has veto power, and $\phi > 0$, she is willing to wait to get a better deal whenever she is not offered her contribution to surplus. As a result, she can guarantee herself v, making no positive transfers to agents. Because of this, holding out to trade late has no value to agents, and the same logic extends to previous states.[20] Thus, in equilibrium, transactions occur without delay. Since agents can guarantee themselves $z > 0$ upon completion independently of whether they supported the principal or not, patient agents also are willing to wait to get a better deal whenever they are not offered their contribution to surplus. As a result, in equilibrium each agent obtains $z > 0$, making no positive transfers to the principal.

In Iaryczower and Oliveros (2022), we show that if agents get a larger payoff upon completion when they are in the principal's coalition ($z > w$), the equilibrium for $\phi > 0$, fixed v, and $\delta \to 1$ is still efficient, but $b^*(m) = p^*(m) = -(z - w)$ for all $m \in M$. As before, the principal can guarantee herself v, and the agents can guarantee themselves the outside option w. However, since the principal is on the short side of the market, she can capture the differential $z - w > 0$ entirely in each meeting.

The result can be related to the insight from the search literature that small search costs can have drastic effects on appropriation of surplus (Diamond (1982), Pissarides (1985), Mortensen and Pissarides (1994), Albrecht (2011)). To see this, fix $\phi = 0$, and focus on the critical state, $m = 1$. Note that if $\delta = 1$, there is an efficient equilibrium in which the agent gets just the outside option, w, and the principal gets $v + (z - w)$: if the principal only accepts offers of at least this amount, and other agents are expected to trade with the principal at this price, an agent can do no better than to make this offer. Building on this outcome in the critical state, the same outcome can be supported in previous states. On the other hand, as we have shown before, for any $\delta < 1$, the principal gets zero, and the equilibrium is inefficient. Now consider $\phi > 0$. Note again that if $\delta = 1$, multiple efficient equilibria can be supported in the critical state $m = 1$. However, for $\delta \to 1$, there is a unique equilibrium outcome, in which the agents appropriate their outside option w, and the principal, being on the short side of the market, appropriates v and the entire differential $z - w$.

[20] Another way to see this is that as δ increases, the principal recovers bargaining power, since now she is able to wait at low cost until she is able to propose. And we already know from our previous results that efficiency is attained whenever the principal has enough bargaining power.

6.3 *Breakdown of negotiations*

Up to this point, we maintained the assumption that the terminal payoff of an agent who committed his support to the principal is $z > 0$. In some applications, however, it is reasonable to assume that the terminal payoff of an uncommitted agent is $w > 0$, but the payoff of a committed agent is $z = 0$ (e.g., corporate takeovers) or even $z < 0$ (e.g., vote buying with audience costs). Consider, for example, a dynamic version of the corporate takeover model of Grossman and Hart (1980) (GH). GH analyze a problem in which a company (the raider) acquires shares of a target company to control its board of directors. It is assumed that the raider can improve the value of the company. To capture this feature, we can normalize the value of a share under the incumbent management to zero, and assume that the value of a share under the raider's control is $w > 0$.

In Iaryczower and Oliveros (2022), we consider this extension.[21] The main result of this analysis is that when $z \leq 0$, the inefficiency induced by collective hold-up can result in inaction, i.e., $\mu^*(q) = 0$, and not merely delay. In particular, we show that even when it is efficient to complete the project when v is large, if the agents have enough bargaining power, no transactions take place in equilibrium.

The key step in the proof is to show that when $z \leq 0 < w$, there cannot be delay in two contiguous states m' and $m' + 1$. To see why this is the case, note that with no payments from the principal, all incentives to trade have to come from diminishing the value of holding out through delay. But delay can only lower the (positive) value of not trading, and thus by itself is insufficient to induce agents to trade when $z \leq 0$. More formally, suppose toward a contradiction that there is delay in two states m' and $m' + 1$. Then we must have $V^*(m') = V^*(m' + 1) = 0$, and thus $s^*(m' + 1) = 0$ (which is needed for delay in $m' + 1$) if and only if $W^*(m' + 1) = W_c^*(m')$. Since $W^*(m' + 1) \propto w > 0$, while $W_c^*(m') \propto z \leq 0$, this is impossible.

Grossman and Hart show that externalities across shareholders can prevent takeovers that add value to the company. The idea is that since shareholders that do not sell can capture the increase in value brought by the *raider*, no shareholder will tender his shares at a price that would allow the raider to profit from the takeover. In our version of the GH model—where the principal buys shares one at a time and shareholders are fully forward looking and strategic—efficient takeovers are not prevented by externalities when $\delta < 1$ *as long as the raider has enough bargaining power*. But when agents do have enough bargaining power, efficient takeovers can fail to occur due to the collective hold-up problem.

7. Conclusion

In this paper, we consider a dynamic process of coalition formation in which a principal bargains sequentially with a group of agents. In this context, we consider how the allocation of bargaining power between the principal and the agents affect efficiency and

[21] As in Grossman and Hart (1980) and Segal (2003), we assume that shareholders are homogeneous. Unlike Grossman and Hart, we suppose that shareholders are fully aware of the effect of their action on the outcome of the raid attempt.

welfare. We show that when the principal's payoff for completion is large relative to that of the agents, the equilibrium of the bargaining game can be divided into an early war of attrition type phase, which features delay, and an efficient phase, where principal and agents transact at transfers that vary with as the coalition formation process moves forward. Moreover, the size of the efficient region increases as bargaining power shifts from agents to the principal, until full efficiency is achieved. Thus, efficiency requires power to be sufficiently concentrated in the principal.

Our model generates rich empirical implications for a number of diverse applications in economics, including lobbying, bargaining, exclusive deals, endorsements, and some forms of corrupt bureaucracies. While the model abstracts away from some of the details pertinent to each application, the results shed light on a common idea behind these apparently diverse problems: bargaining institutions that decentralize power to agents can be detrimental to agents' welfare by making the coalition formation process inefficient.

In the paper, we considered several extensions of our benchmark model. Two other extensions of the model are left for future work. First, in the model, we assumed that the transfers between principal and agent are a quid pro quo contingent on the behavior of the agent transacting with the principal, but not contingent on the completion of the project. This assumption is by far the most prevalent in the literature, and fits many applications well. In some other cases, however, contingent transfers are paramount. In bankruptcy restructuring proceedings, for example, bilateral deals between the firm and creditors must ultimately be jointly approved by the judge. A natural question is whether our results hold in this modified setting. In Iaryczower and Oliveros (2022), we show that while contingent contracts allow other equilibria, collective hold-up can still occur. In particular, when agents have all the bargaining power, the unique equilibrium outcomes in the benchmark model is still an equilibrium when transfers are contingent on completion of the project. In general, however, understanding how efficiency and welfare change as power is decentralized to agents is an open question.[22]

Second, our model does not allow for more general payoff structures in which payoffs depend on the size of the coalition that supports the principal, and can accrue before the coalition is formed. For example, in industries in which new technologies have a component of learning by doing, earlier sales affect later payoffs. Here, the incentives to hold out compete with the benefits of joining early. This presents an interesting problem, where the principal may optimally front payments and sell at a loss. In that sense, collective hold-up may manifest itself in delayed learning.

[22]A fundamental difference in the contingent transfer model is that by affecting the amount of standing promises, agents can affect the equilibrium play of agents contracting later. This implies, in particular, that the payoff-relevant state has to be extended to include both the number of agents required for completion and the amount of standing promises. Chen and Zapal (2021) consider this problem when the principal has full bargaining power and agents cannot reapproach the principal after rejecting an offer.

Appendix: Proofs

A.1 *Values*

Consider the value of the principal in state m, $V^*(m)$. With probability $\phi\mu_p^*(m)$, the principal has agenda setting power and makes an offer that is accepted by the agent, getting a payoff $\delta V^*(m-1) - p^*(m)$. With probability $1 - \phi\mu_p^*(m)$, either there is no transaction in m or there is a transaction following a proposal by the agent, and the principal obtains a discounted continuation value $\delta V^*(m)$. Thus,

$$V^*(m) = \phi\mu_p^*(m)\big(\delta V^*(m-1) - p^*(m)\big) + \big(1 - \phi\mu_p^*(m)\big)\delta V^*(m).$$

Using (2), and subtracting $\delta V^*(m)$ on both sides, we have

$$V^*(m) = \left(\frac{\delta}{1-\delta}\right)\phi \max\{s^*(m), 0\}. \tag{6}$$

Equation (6.A) says that the value of the principal in state m is proportional to the surplus in state m whenever this is positive, and zero otherwise. The expression eliminates the dependency on the probability of trade $\mu_p^*(m)$ using the fact that if $s^*(m) > 0$ then $\mu_p^*(m) = 1$, if $s^*(m) < 0$ then $\mu_p^*(m) = 0$, and that $s^*(m) = 0$ when $\mu_p^*(m) \in (0, 1)$.

Consider instead the value of an uncommitted agent i in state m, $W^*(m)$, recalling that $\beta(m) \equiv 1/(n+m-q)$ denotes the probability that agent i meets the principal. With probability $\beta(m)(1-\phi)\mu_a^*(m)$, agent i meets the principal, has agenda setting power, and makes an offer $b^*(m)$ (which is accepted), leading to a payoff $\delta W_c^*(m-1) + b^*(m)$, where $b^*(m)$ is given by (3). With probability $(1-\beta(m))\mu^*(m)$, another agent $j \neq i$ meets the principal, and the meeting results in a transaction, leading to a payoff $\delta W^*(m-1)$ for player i. In all other cases (i meets the principal but either the principal has agenda setting power or the transactions falls through, or some other agent $j \neq i$ meets the principal but the transaction falls through), agent i gets a continuation payoff $\delta W^*(m)$:

$$\begin{aligned} W^*(m) = {} & \beta(m)(1-\phi)\mu_a^*(m)\big[\delta W_c^*(m-1) + b^*(m)\big] \\ & + \big(1-\beta(m)\big)\mu^*(m)\delta W^*(m-1) \\ & + \big[\beta(m)\big[\phi + (1-\phi)\big(1-\mu_a^*(m)\big)\big] + \big(1-\beta(m)\big)\big(1-\mu^*(m)\big)\big]\delta W^*(m) \end{aligned}$$

Using (3) for the transfer $b^*(m)$ and simplifying, we have that for all $m \geq 2$,[23]

$$\begin{aligned} W^*(m) = {} & \left[\frac{\delta\beta(m)}{1-\delta\beta(m)}\right](1-\phi)\max\{0, s^*(m)\} \\ & + \left[1 + \left(\frac{1-\delta}{1-\beta(m)}\right)\frac{1}{\delta\mu^*(m)}\right]^{-1} W^*(m-1). \end{aligned} \tag{7}$$

[23]As before, we have used the fact that if $s^*(m) > 0$ then $\mu_a^*(m) = \mu^*(m) = 1$, if $s^*(m) < 0$ then $\mu_a^*(m) = \mu^*(m) = 0$, and that $s^*(m) = 0$ when $\mu^*(m) \in (0, 1)$.

Using (7), we can express the current value for an uncommitted agent as a function of the final payoff z and the sequence of surpluses $[s(k)]$ for $k \leq m$:

$$W^*(m) = (1-\phi)\sum_{k=1}^{m}\left(\frac{\beta(k)}{1-\beta(k)}\right)e_{km}\max\{0, s^*(k)\} + e_{1m}z \quad \forall m \geq 1, \tag{17}$$

where we have defined

$$e_{km} \equiv \left[\prod_{j=k}^{m}\left(1 + \left(\frac{1-\delta}{1-\beta(j)}\right)\frac{1}{\delta\mu^*(j)}\right)\right]^{-1}$$

A.2 *Proofs*

Proof of Proposition 1. We show that a symmetric MPE exists, that the equilibrium probability of trade in each state $m \in M$ is uniquely determined, and satisfies

$$\mu^*(m) = \min\left\{1, \left(\frac{1-\delta}{\delta}\right)\left(\frac{1}{1-\beta(m)}\right)\left(\frac{s^0(m)}{W^*(m-1) - s^0(m)}\right)\right\} > 0 \quad \forall m \in M, \tag{18}$$

where for any $m \in M$, $s^0(m) \equiv V^*(m-1) + W_c^*(m-1)$ denotes the maximum feasible surplus in state m given continuation values $V^*(m-1)$ and $W_c^*(m-1)$. This obtains for $\mu^*(m) = 0$, in which case $W^*(m) = V^*(m) = 0$.

Before proving the proposition, we establish the following lemma, characterizing equilibrium trade probabilities in the critical state, $m = 1$.

Lemma 9. *In equilibrium, there is trade with probability one in state 1:* $\mu^*(1) = 1$.

Proof of Lemma 9. Fix a MPE σ^*. Since principal and agent only make offers if $s^*(m) \geq 0$, (6.A) and (17) imply that $V^*(m) \geq 0$ and $W^*(m) \geq 0$ for all $m \in M$, and in particular $V^*(1) \geq 0$ and $W^*(1) \geq 0$.

Recall that $s^*(1) = v - V^*(1) + z - W^*(1)$. Note that if $\mu^*(1) = 0$, then $V^*(1) = W^*(1) = 0$, and thus $s^*(1) > 0$, which implies $\mu_1^* > 0$, a contradiction. It follows that $\mu^*(1) > 0$. Suppose first that $\mu^*(1) = 1$. Then (6) gives $V^*(1) = \frac{\delta}{1-\delta}\phi s^*(1)$ and (17) gives

$$W^*(1) = \frac{\delta(1-\phi)\beta(1)}{(1-\delta\beta(1))}s^*(1) + \frac{\delta(1-\beta(1))}{(1-\delta\beta(1))}z$$

Substituting,

$$s^*(1)\left[1 + \frac{\delta\phi}{1-\delta} + \frac{\delta(1-\phi)}{(1-\delta\beta(1))}\beta(1)\right] = v + \left(\frac{1-\delta}{1-\delta\beta(1)}\right)z$$

Thus, $s^*(1) \geq 0$, consistent with equilibrium, if and only if

$$v \geq -\left(\frac{1-\delta}{1-\delta\beta(1)}\right)z$$

Now consider the possibility that $\mu^*(1) \in (0, 1)$. Note that with $s^*(1) = 0$, (6) implies $V^*(1) = 0$, and (17) implies that

$$W^*(1) = \left(\frac{\delta\mu^*(1)}{\left(\dfrac{1-\delta}{1-\beta(1)}\right) + \delta\mu^*(1)}\right) z \tag{19}$$

Recalling that $s^*(1) = 0 = v + z - W^*(1)$, and substituting, we obtain

$$1 < \frac{v+z}{z} = \frac{\delta\mu^*(1)}{\left(\dfrac{1-\delta}{1-\beta(1)}\right) + \delta\mu^*(1)} < 1, \tag{20}$$

a contradiction. □

Fix an equilibrium in the subgame starting in state $m-1$, $(\mu^*(1), \dots, \mu^*(m-1))$. This produces continuation values $V^*(m-1)$, $W^*(m-1)$ and $W_c^*(m-1)$. Given these continuation values, let $\hat{v}(m; \mu(m))$ and $\hat{w}(m; \mu(m))$ denote the values of the principal and uncommitted agent in state m when transaction probability $\mu(m)$, and let $\hat{s}(m; \mu(m))$ denote the surplus in state m when transaction probability $\mu(m)$. From (6) and (7), $\hat{v}(m; 0) = \hat{w}(m; 0) = 0$. Thus,

$$\begin{aligned}\hat{s}(m; 0) &\equiv \left[V^*(m-1) - \hat{v}(m; 0)\right] + \left[W_c^*(m-1) - \hat{w}(m; 0)\right] \\ &= V^*(m-1) + W_c^*(m-1) = s^0(m).\end{aligned}$$

It follows that if $V^*(m-1) + W_c^*(m-1) > 0$, then $\mu^*(m) > 0$. Now, note that $V^*(m-1) \geq 0$. Moreover, by (8), we have that if $\mu^*(k) > 0$ for all $k < m$, then $W_c^*(m-1) = [\prod_{k=1}^{m-1}(\frac{\delta\mu^*(k)}{1-\delta(1-\mu^*(k))})]z > 0$. This shows that if $\mu^*(k) > 0$ for all $k < m$, then $\mu^*(m) > 0$.

By Lemma 9, we have (i) $\mu^*(1) > 0$, and we have shown above that (ii) if $\mu^*(k) > 0$ for all $k < m$, then $\mu^*(m) > 0$. An induction argument then establishes that $\mu^*(m) > 0$ for all $m \in M$.

Next, suppose $\mu^*(m) = 1$. Using the expression for the principal's value (6) and the expression for the uncommitted agent's value (7) in the definition of the surplus (1), we have

$$\begin{aligned} s^*(m)&\left[1 + \frac{\delta}{1-\delta}\phi + \frac{\left(\dfrac{\delta}{1-\delta}\right)\beta(m)(1-\phi)}{\left(1 + \left(\dfrac{\delta}{1-\delta}\right)(1-\beta(m))\right)}\right] \\ &= W_c^*(m-1) + V^*(m-1) - \frac{1}{\left[1 + \left(\dfrac{1-\delta}{\delta}\right)\left(\dfrac{1}{1-\beta(m)}\right)\right]} W^*(m-1).\end{aligned}$$

Given $\mu^*(1) > 0$, equilibrium requires $s^*(m) \geq 0$. From the previous expression, $s^*(m) \geq 0$ if and only if

$$1 + \left(\frac{1-\delta}{\delta}\right)\left(\frac{1}{1-\beta(m)}\right) \geq \frac{W^*(m-1)}{W_c^*(m-1) + V^*(m-1)}. \tag{21}$$

Next, suppose $\mu^*(m) \in (0, 1)$. Equilibrium then requires $s^*(m) = 0$, which in turn implies $V^*(m) = 0$ and then (from the surplus condition), $W^*(m) = V^*(m-1) + W_c^*(m-1)$. Also with $s^*(m) = 0$, (7) gives

$$W^*(m) = \left(\frac{\delta\mu^*(m)}{\left(\frac{1-\delta}{1-\beta(m)}\right) + \delta\mu^*(m)} \right) W^*(m-1)$$

Substituting in $W^*(m) = V^*(m-1) + W_c^*(m-1)$, and then solving for $\mu^*(m)$ gives

$$\mu^*(m) = \left(\frac{1-\delta}{\delta}\right)\left(\frac{1}{1-\beta(m)}\right)\left(\frac{V^*(m-1) + W_c^*(m-1)}{W^*(m-1) - \big(V^*(m-1) + W_c^*(m-1)\big)}\right), \tag{22}$$

which is the statement in the proposition. This is less than one if and only if (21) does not hold.

We have shown that the equilibrium trading probabilities are uniquely determined. In each state $m \in M$, we have $\mu^*(m) = 1$ if (21) holds, and $\mu^*(m) \in (0, 1)$ given in (22) if (21) does not hold. □

Lemma 10. *Let* $\theta_{km} \equiv \prod_{j=k}^{m} \left(\frac{\delta\phi\mu^*(j)}{1-\delta+\delta\mu^*(j)\phi(1-\beta(j))}\right)$*. Then for any* $m \in M$*,*

$$\frac{W^*(m)}{\beta(m)} = \theta_{1m}(n-q)z + \left(\frac{1-\phi}{\phi}\frac{1-\delta}{\delta}\right)\sum_{k=1}^{m} \frac{\theta_{km}}{\mu^*(k)}\left(v + \frac{z}{\beta(k)}\right)\frac{W_c^*(k)}{z}, \tag{23}$$

where $W_c^*(k)$ *is given by (8).*

Proof of Lemma 10. The value functions of the principal and agents satisfy

$$V^*(m) = \mu^*(m)\frac{\delta}{1-\delta}\phi s^*(m) \tag{24}$$

and

$$W^*(m) = \frac{\delta\beta(m)(1-\phi)\mu^*(m)}{1-\delta+\delta\big(1-\beta(m)\big)\mu^*(m)} s^*(m) + \frac{\delta\big(1-\beta(m)\big)\mu^*(m)}{1-\delta+\delta\big(1-\beta(m)\big)\mu^*(m)} W^*(m-1) \tag{25}$$

Substituting (24) in the surplus condition (1) and using that $\frac{1-\beta(m)}{\beta(m)} = \frac{1}{\beta(m-1)}$, we have the system of difference equations:

$$\begin{aligned} (1-\phi)s^*(m) &= \left(\frac{1-\delta}{\delta\mu^*(m)} + 1 - \beta(m)\right)\frac{W^*(m)}{\beta(m)} - \frac{W^*(m-1)}{\beta(m-1)} \\ \frac{1-\delta+\delta\phi\mu^*(m)}{1-\delta} s^*(m) &= \mu^*(m-1)\frac{\delta}{1-\delta}\phi s^*(m-1) + W_c^*(m-1) - W^*(m) \end{aligned} \tag{26}$$

Solving the first equation for $s^*(m)$ and substituting in the second equation, we transform the system of first-order difference equations into a second-order difference equation.

Letting $\alpha(m) \equiv \frac{\delta\mu^*(m)}{1-\delta(1-\mu^*(m))}$, and defining

$$H(m) \equiv \frac{\phi}{1-\phi}\frac{\delta}{1-\delta}\left[\left(\frac{1-\delta}{\delta\phi} + \mu^*(m)\big(1-\beta(m)\big)\right)\frac{W^*(m)}{\beta(m)} - \mu^*(m)\frac{W^*(m-1)}{\beta(m-1)}\right], \tag{27}$$

we can write this recursion as

$$H(m) = \alpha(m)H(m-1) + \alpha(m)W_c^*(m-1) \quad \text{for } m: 3 \le m \le m' \tag{28}$$

Solving recursively, and using that $W_c^*(m) = \alpha(m)W_c^*(m-1)$ we have

$$H(m) = \left(\prod_{j=3}^{m}\alpha(j)\right)H(2) + (m-2)W_c^*(m)$$

Therefore, letting $\tau(m) = \frac{1-\delta}{1-\delta+\delta\mu^*(m)\phi(1-\beta(m))}$ for convenience,

$$\begin{aligned}\frac{W^*(m)}{\beta(m)} &= \frac{1-\delta\big(1-\mu^*(m)\big)}{1-\delta}\phi\tau(m)\alpha(m)\frac{W^*(m-1)}{\beta(m-1)} \\ &\quad + \tau(m)(1-\phi)\left[\left(\prod_{j=3}^{m}\alpha(j)\right)H(2) + (m-2)W_c^*(m)\right].\end{aligned}$$

The boundary conditions follow by (26) for $m = 1, 2$, and (27) for $H(2)$, which give

$$\begin{aligned}H(2) &= \alpha(2)\alpha(1)\left(v + 2z + \frac{z}{\beta(0)}\right) \\ \frac{W^*(2)}{\beta(2)} &= \tau(2)\left(\alpha(2)\frac{1}{\tau(1)} + \frac{\delta}{1-\delta}\mu^*(2)\phi\right)\frac{W^*(1)}{\beta(1)} \\ &\quad - \alpha(2)\tau(2)\mu^*(1)\frac{\delta}{1-\delta}\phi\frac{w}{\beta(0)} + \alpha(2)\tau(2)(1-\phi)W_c^*(1) \\ \frac{W^*(1)}{\beta(1)} &= \tau(1)\phi\left[\frac{\delta}{1-\delta}\mu^*(1)\frac{z}{\beta(0)} + \alpha(1)\frac{1-\phi}{\phi}\left(v + z + \frac{z}{\beta(0)}\right)\right]\end{aligned}$$

Using these initial conditions together with $W_c^*(m) = (\prod_{j=1}^{m}\alpha(j))z$, we obtain a simple recursive representation of the value functions

$$\begin{aligned}\frac{W^*(m)}{\beta(m)} &= \frac{1-\delta\big(1-\mu^*(m)\big)}{1-\delta}\phi\tau(m)\alpha(m)\frac{W^*(m-1)}{\beta(m-1)} \\ &\quad + \tau(m)(1-\phi)\left(\prod_{j=1}^{m}\alpha(j)\right)\big(v + (n+m-q)z\big)\end{aligned} \tag{29}$$

Solving recursively, we obtain

$$\begin{aligned}\frac{W^*(m)}{\beta(m)} = {} & \left(\prod_{j=1}^{m} \alpha(j)\right)\left[\prod_{j=1}^{m}\left(\frac{1-\delta\big(1-\mu^*(j)\big)}{1-\delta}\phi\tau(j)\right)\right](n-q)z \\ & + (1-\phi)\left(\prod_{j=1}^{m} \alpha(j)\right) \\ & \times \sum_{k=1}^{m-1}\left\{\left[\prod_{j=k+1}^{m}\left(\frac{1-\delta\big(1-\mu^*(j)\big)}{1-\delta}\phi\tau(j)\right)\right]\tau(k)\big(v+(n+k-q)z\big)\right\} \\ & + \tau(m)(1-\phi)\left(\prod_{j=1}^{m} \alpha(j)\right)\big(v+(n+m-q)z\big), \end{aligned} \tag{30}$$

which is equivalent to (23). □

Lemma 11. *Consider any $m \in M$, and an equilibrium candidate probability of trade $\mu^*(m)$. Then $s^*(m) \geq (\leq) 0$ if and only if $T^*(m) \leq (\geq) 0$, where*

$$T^*(m) \equiv \frac{W^*(m)}{\beta(m)} - W_c^*(m)\left(\frac{1}{\beta(m)} + \frac{v}{z}\right).$$

Proof of Lemma 11. Follows from (29) and the surplus condition (26), noting that

$$\begin{aligned} T^*(m) &= \frac{W^*(m)}{\beta(m)} - \left(\prod_{j=1}^{m-1}\frac{\delta\mu^*(j)}{1-\delta\big(1-\mu^*(j)\big)}\right)\left(\frac{\delta\mu(m)}{1-\delta\big(1-\mu(m)\big)}\right)\left(v+\frac{z}{\beta(m)}\right) \\ &= \frac{W^*(m)}{\beta(m)} - W_c^*(m)\left(\frac{1}{\beta(m)}+\frac{v}{z}\right). \end{aligned}$$

□

Proof of Proposition 5. Part (i). Note that for any $m > 1$:

$$\begin{aligned} T^*(m-1) = {} & \left(\frac{1-\delta+\delta\mu^*(m)\phi\big(1-\beta(m)\big)}{\phi\big[1-\delta\big(1-\mu^*(m)\big)\big]}\right)T^*(m) \\ & + z - \left(\frac{\delta\beta(m)\mu^*(m)}{1-\delta\big(1-\mu^*(m)\big)}\right)\left(v+\frac{z}{\beta(m)}\right) \end{aligned} \tag{31}$$

Now suppose toward a contradiction that $\mu^*(m) < 1$, $\mu^*(m+j) = 1$ for all $j = 1, \ldots, k$, and $\mu^*(m+k+1) < 1$. Then we must have $T^*(m) = T^*(m+k+1) = 0$, and $T^*(m+j) \leq 0$ for all $j = 1, \ldots, k$. Thus, we have

$$\begin{aligned} & -\left(\frac{1-\delta+\delta\phi\big(1-\beta(m+1)\big)}{\phi}\right)T^*(m+1) \\ & = z - \big(\delta\beta(m+1)\big)\left(v+\frac{z}{\beta(m-1)}\right) \end{aligned} \tag{32}$$

$$T^*(m+j) = \left(\frac{1-\delta+\delta\phi\big(1-\beta(m+j+1)\big)}{\phi}\right)T^*(m+j+1)$$
$$+ z - \delta\beta(m+j+1)\left(v + \frac{z}{\beta(m+j+1)}\right) \quad \text{for } j = 1, \dots, k, \tag{33}$$

and

$$T^*(m+k) = z - \left(\frac{\delta\beta(m+k+1)\mu^*(m+k+1)}{1-\delta\big(1-\mu^*(m+k+1)\big)}\right)\left(v + \frac{z}{\beta(m+k+1)}\right) \tag{34}$$

From (32), since we are assuming $T^*(m+1) \le 0$, we need

$$z \ge \big(\delta\beta(m+1)\big)\left(v + \frac{z}{\beta(m+1)}\right) \tag{35}$$

From (34), since we are assuming $T^*(m+k) \le 0$, we also need

$$z \le \left(\frac{\delta\beta(m+k+1)\mu^*(m+k+1)}{1-\delta\big(1-\mu^*(m+k+1)\big)}\right)\left(v + \frac{z}{\beta(m+k+1)}\right) \tag{36}$$

Note that the expression in parenthesis in (36) is increasing in $\mu^*(m+k+1)$. Thus, there is no $\mu^*(m+k+1) \in (0, 1)$ that satisfies this inequality if

$$z > \big(\delta\beta(m+k+1)\big)\left(v + \frac{z}{\beta(m+k+1)}\right) \tag{37}$$

But from (35) we have that

$$\left(v + \frac{z}{\beta(m+1)}\right) \le \frac{z}{\delta\beta(m+1)}$$

Thus, a sufficient condition for (36) not to hold is that

$$\frac{\beta(m+1)}{\beta(m+k+1)} > \big(1 + k\delta\beta(m+1)\big) \quad \Leftrightarrow \quad \delta < 1,$$

where we have used the fact that $\beta(m) = 1/(n+m-q)$ for any $m \in M$.

Part (ii). Note from (31), that if $T^*(m-1) = T^*(m) = 0$, we have

$$z = \left(\frac{\delta\beta(m)\mu^*(m)}{1-\delta\big(1-\mu^*(m)\big)}\right)\big(v + (n+m-q)z\big)$$

Solving for $\mu^*(m)$, we have

$$\mu^*(m) = \frac{(1-\delta)}{\delta\beta(m)}\frac{z}{v} \qquad \square$$

Proof of Proposition 6. We want to show that if $\delta\beta(m)(v/z) \ge 1-\delta$ for $m > 1$, then (i) $s^*(m) > 0 \Rightarrow s^*(m-1) > 0$, and (ii) $s^*(m-1) = 0 \Rightarrow s^*(m) = 0$. Note that

$$T^*(m-1) = \left(\frac{1-\delta+\delta\mu^*(m)\phi\big(1-\beta(m)\big)}{\phi\big[1-\delta\big(1-\mu^*(m)\big)\big]}\right)T^*(m)$$

$$+ z - \left(\frac{\delta\beta(m)\mu^*(m)}{1-\delta\big(1-\mu^*(m)\big)}\right)\left(v + \frac{z}{\beta(m)}\right) \tag{38}$$

so if $T^*(m) < 0$, we have

$$T^*(m-1) < z - \delta\beta(m)\left(v + \frac{z}{\beta(m)}\right),$$

where we have used the fact that $s^*(m) > 0$ implies $\mu^*(m) = 1$. Note that the RHS is less than or equal to zero if and only if $\delta\beta(m)(v/z) \geq 1-\delta$ for $m > 1$. Thus, provided this condition is satisfied, $T^*(m) < 0 \Rightarrow T^*(m-1) < 0$. This proves part (i). Part (ii) follows from part (i), noting that $s^*(m) \geq 0$ for all $m \in M$. □

Proof of Proposition 7, part (i). We want to show that for v/z large enough, for any $m \in M \setminus \{1\}$, there exists $\hat{\phi}(m) \in (0,1)$ such that the equilibrium of the m-subgame is efficient if and only if $\phi \geq \hat{\phi}(m)$. We do this in three steps.

(a) From Lemma 10, with $\mu^*(j) = 1$ for all $j \in M$, it follows that the equilibrium payoffs of an uncommitted agent in an efficient equilibrium, $W^\dagger(m)$, are

$$\frac{W^\dagger(m)}{\beta(m)} \equiv \overline{\theta}_{1m}(n-q)z + \sum_{k=1}^{m} \frac{1-\delta}{\delta}\frac{1-\phi}{\phi}\overline{\theta}_{km}\left(\frac{v}{z} + \frac{1}{\beta(k)}\right)\delta^k z$$

where for convenience we have defined $\overline{\theta}_{km} \equiv \prod_{j=k}^{m}\left(\frac{\delta\phi}{1-\delta+\delta\phi(1-\beta(j))}\right)$. Equivalently, this can be written as (16) in the text. And from (8), with efficient trading,

$$W_c^\dagger(m) = \delta^m z \tag{39}$$

Substituting in (6), and solving the difference equation, we then have that the principal's value in the subgame m under efficient trading is

$$V^\dagger(m) = \kappa_m v + (1-\kappa_m)\frac{\delta\phi}{1-\delta}\big(W_c^\dagger(m-1) - W^\dagger(m)\big), \tag{40}$$

where $\kappa_m \equiv \left(\frac{\delta\phi}{1-\delta(1-\phi)}\right)^m$.

Take any $m \in M$. Suppose the equilibrium of the m-subgame is efficient. Then for any $m' \leq m$, $T(m'|\mu^*(m')) = T^\dagger(m')$, where

$$T^\dagger\big(m'\big) \equiv \frac{W^\dagger\big(m'\big)}{\beta\big(m'\big)} - \left(\frac{v}{z} + \frac{1}{\beta\big(m'\big)}\right)\delta^{m'} z \tag{41}$$

From Proposition 5, equilibrium requires that $T^\dagger(m') \leq 0$ for all $m' \leq m$. Moreover, by Proposition 6, if $\delta\beta(m)(v/z) \geq 1-\delta$, $T^\dagger(m) < 0 \Rightarrow T^\dagger(m') < 0$ for all $m' < m$. It follows that if $T^\dagger(m) < 0$, the equilibrium of the m-subgame is efficient, while if $T^\dagger(m) > 0$, the equilibrium of the m-subgame is inefficient.

(b) We first show that for any $m \in M$, there is a $\overline{\phi}(m) \in (0,1)$, which is independent of v, and a $K > 0$, such that if $\phi > \overline{\phi}(m)$ and $v/z > K$, the unique MPE of the m-subgame is efficient. From expression (16), we have that $T^{\dagger}(m) < 0$ if

$$\sum_{j=1}^{m}\left(\prod_{k=j}^{m}\frac{\delta\phi}{(1-\delta)+\delta\phi\big(1-\beta(k)\big)}\right)\frac{(1-\delta)}{\delta}\frac{(1-\phi)}{\phi}\delta^{j}\left(\frac{v}{z}+\frac{1}{\beta(j)}\right)+C$$
$$< \delta^{m}\left(\frac{v}{z}+\frac{1}{\beta(m)}\right)$$

where

$$C \equiv \max\left\{0, \left(\prod_{j=1}^{m}\frac{\delta\phi}{1-\delta\beta(j)}\right)(n-q)\right\}$$

Note that

$$\frac{\delta\phi}{(1-\delta)+\delta\phi\big(1-\beta(k)\big)} < \frac{\delta}{1-\delta\beta(k)},$$

so $T^{\dagger}(m) < 0$ if

$$\sum_{j=1}^{m}\left(\prod_{k=j}^{m}\frac{\delta}{1-\delta\beta(k)}\right)\left(\frac{1-\delta}{\delta}\right)\left(\frac{1-\phi}{\phi}\right)\delta^{j}\left(\frac{v}{z}+\frac{1}{\beta(j)}\right)+C < \delta^{m}\left(\frac{v}{z}+\frac{1}{\beta(m)}\right)$$

Note that both sides of this inequality are increasing in v/z, and the right-hand side increases at a faster rate than the left-hand side if and only if

$$\frac{1-\phi}{\phi} < \frac{\delta}{1-\delta}\left(\frac{\delta^{m}}{\sum_{j=1}^{m}\left(\prod_{k=j}^{m}\frac{\delta}{1-\delta\beta(k)}\right)\delta^{j}}\right) \equiv \frac{1-\overline{\phi}(m)}{\overline{\phi}(m)},$$

where $\overline{\phi}(m)$ is independent of v. It follows that if $\phi > \overline{\phi}(m)$, for v/z large enough, $T^{\dagger}(m) < 0$.

Next, we show that for any $m \in M \setminus \{1\}$, there exists $\underline{\phi}(m) > 0$, independent of v/z, and $K > 0$ such that if $\phi < \underline{\phi}(m)$ and $v/z > K$, the unique MPE of the m-subgame entails delay (i.e., there is $m' \leq m$ such that $\mu(m') < 1$). From expression (16), we have that

$$\frac{W^{\dagger}(m)}{\beta(m)} = \sum_{j=1}^{m}\left(\prod_{k=j}^{m}\frac{\delta\phi}{(1-\delta)+\delta\phi\big(1-\beta(k)\big)}\right)\frac{1-\delta}{\delta}\frac{(1-\phi)}{\phi}\delta^{j}\left(v+\frac{z}{\beta(j)}\right)$$
$$+\left(\prod_{j=1}^{m}\frac{\delta\phi}{(1-\delta)+\delta\phi\big(1-\beta(j)\big)}\right)(n-q)z \tag{16b}$$

Dropping the first $m-2$ terms of the summation from expression (16b), which are positive, we have

$$\frac{W^{\dagger}(m)}{\beta(m)} > \left(\frac{\delta\phi}{1-\delta+\delta\phi\big(1-\beta(m-1)\big)}\right)\left(\frac{(1-\delta)(1-\phi)}{1-\delta+\delta\phi\big(1-\beta(m)\big)}\right)$$

$$\times\,\delta^{m-1}\left(v+\frac{z}{\beta(m-1)}\right)$$
$$+\left(\frac{(1-\delta)(1-\phi)}{1-\delta+\delta\phi\big(1-\beta(m)\big)}\right)\delta^{m}\left(v+\frac{z}{\beta(m)}\right)+Dz,$$

where

$$D\equiv\min\left\{\left(\prod_{j=1}^{m}\frac{\delta}{1-\delta\beta(j)}\right)(n-q),0\right\}$$

So $T^{\dagger}(m)\equiv\frac{W^{\dagger}(m)}{\beta(m)}-\delta^{m}(v+\frac{z}{\beta(m)})>0$ if

$$\left(\frac{\phi}{1-\delta+\delta\phi\big(1-\beta(m-1)\big)}\right)\left(\frac{(1-\delta)(1-\phi)}{1-\delta\beta(m)}\right)\left(\frac{v}{z}+\frac{1}{\beta(m-1)}\right)+\frac{D}{\delta^{m}}$$
$$>\phi\left(\frac{v}{z}+\frac{1}{\beta(m)}\right) \tag{42}$$

Taking derivatives of both sides with respect to v/z, the LHS increases faster than the RHS if and only if

$$\frac{\phi}{1-\phi}<\frac{(1-\delta)\delta\beta(m)}{\big(1-\delta\beta(m)\big)\big(1-\delta\beta(m-1)\big)}\equiv\frac{\underline{\phi}(m)}{1-\underline{\phi}(m)},$$

where $\underline{\phi}(m)$ is independent of v/z. It follows that if $\phi<\underline{\phi}(m)$, for v/z large enough $T^{\dagger}(m)>0$.

(c) We first show that if the equilibrium of the m-subgame is efficient, then $W^{\dagger}(m)$ and $T^{\dagger}(m)$ are strictly decreasing in ϕ. Note that in an efficient equilibrium, for all $m'\leq m$, $m'>1$, (38) is

$$T^{\dagger}\big(m'-1\big)=\left(\frac{1-\delta}{\phi}+\delta\big(1-\beta\big(m'\big)\big)\right)T^{\dagger}\big(m'\big)+z-\delta\beta\big(m'\big)\left(v+\frac{z}{\beta\big(m'\big)}\right)$$

Taking derivatives, we get

$$\frac{\partial T^{\dagger}\big(m'-1\big)}{\partial\phi}=-\frac{1-\delta}{\phi^{2}}T^{\dagger}\big(m'\big)+\delta\left(\frac{1-\delta}{\delta\phi}+\big(1-\beta\big(m'\big)\big)\right)\frac{\partial T^{\dagger}\big(m'\big)}{\partial\phi},$$

and since $T^{\dagger}(m')\leq 0$, then $\frac{\partial T^{\dagger}(m'-1)}{\partial\phi}\leq 0$ implies $\frac{\partial T^{\dagger}(m')}{\partial\phi}\leq 0$. Thus, it is sufficient to show that $\frac{\partial T^{\dagger}(1)}{\partial\phi}\leq 0$. Using now that from Lemma 9 we have that

$$v+z-\frac{1-\beta(1)}{1-\delta\beta(1)}\delta z=\big(v+z-W^{\dagger}(1)\big)\left(1+\frac{\delta\beta(1)}{1-\delta\beta(1)}\frac{1-\delta}{1-\delta(1-\phi)}\right),$$

it follows that $W^{\dagger}(1)$ decreases with ϕ. And since (41) implies

$$T^{\dagger}(1)\equiv\frac{W^{\dagger}(1)}{\beta(1)}-\delta\left(v+\frac{z}{\beta(1)}\right),$$

it follows that in an efficient equilibrium, both $T^{\dagger}(m)$ and $\frac{W^{\dagger}(m)}{\beta(m)}$ are decreasing in ϕ.

Next, by part (b), for any $m \in M$, there are $\overline{\phi}(m)$, $\underline{\phi}(m) \in (0, 1)$, which are independent of v/z, such that for large v/z, $T^\dagger(m) > 0$ if $\phi < \underline{\phi}(m)$ and $T^\dagger(m) < 0$ if $\phi > \overline{\phi}(m)$. By continuity of $T^\dagger(m)$ in ϕ, there is a $\hat{\phi}(m) \in (0, 1)$ such that $T^\dagger(m) = 0$ at $\phi = \hat{\phi}$. And since $T^\dagger(m)$ is decreasing in ϕ whenever $T^\dagger(m) < 0$, it follows that $T^\dagger(m)$ only crosses zero once. Thus, there is a unique threshold $\hat{\phi}$ such that $T^\dagger(m) = 0$ at $\phi = \hat{\phi}$, $T^\dagger(m) > 0$ for all $\phi < \hat{\phi}$, and $T^\dagger(m) < 0$ for all $\phi > \hat{\phi}$. Finally, note that at $\hat{\phi}$, $s(m|\mu^{*m-1}, 1) = 0$. Since $s(m|\mu^{*m-1}, \mu(m))$ is decreasing in $\mu(m)$, if in equilibrium we had $\mu^*(m) < 1$, this would result in $s(m|\mu^{*m-1}, \mu^*(m)) > 0$, a contradiction. Thus, at $\hat{\phi}$ we must have $\mu^*(m) = 1$. This concludes the proof. □

Proof of Proposition 7, part (ii). We want to show that if $m' > m$, then $\hat{\phi}(m') > \hat{\phi}(m)$. By definition of $\hat{\phi}(m')$, $T^\dagger(m'; \hat{\phi}(m')) = 0$. By Proposition 6, this implies

$$T^\dagger\big(m; \hat{\phi}(m')\big) < 0.$$

Since (a) $T^\dagger(m; \cdot) < 0$ is decreasing in ϕ when $T^\dagger(m; \cdot) < 0$ by the third part of part (i) of the proposition, and (b) $T^\dagger(m; \hat{\phi}(m)) = 0$ by definition of $\hat{\phi}(m)$, then $\hat{\phi}(m') > \hat{\phi}(m)$. □

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Co-editor Simon Board handled this manuscript.

Manuscript received 24 November, 2020; final version accepted 18 August, 2022; available online 26 August, 2022.