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Surprise and default in general equilibrium

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I model an incomplete markets economy where unaware agents do not perceive all states of nature, so unintended default can occur when asset returns differ from what was perceived. The presence of default plays a crucial role in the proof of existence—particularly in economies where beliefs are biased—by removing perceived arbitrage opportunities with respect to delivery-adjusted asset returns. The First Fundamental Welfare Theorem fails because of default and pecuniary inefficiencies, but the Second Fundamental Welfare Theorem holds for economies with no aggregate risk. Welfare is shown to not necessarily be monotonic in discovery or the increasing of awareness.

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1. Introduction

Unawareness refers to the inability of agents to conceive of all possible future states of nature. Schipper (2014) provides a useful comparison: under risk, the decision maker conceives of the space of all relevant contingencies and is able to assign probabilities to them. Under ambiguity, the agent still conceives of the space of all relevant contingencies, but has difficulty evaluating them probabilistically. Under unawareness, the agent cannot even conceive of all relevant contingencies. In some sense, unawareness captures an even deeper notion of uncertainty than risk or ambiguity. In this paper, I model unintended default using the notion of unawareness: households find themselves in a state of nature that they failed to plan for, where liabilities surpass assets. Not only does this paper present a particularly simple and tractable way to model unawareness, but it also sheds light on the welfare implications of unintended default.

I consider states of nature that are described by a product space: for example, one dimension might be temperature (hot versus cold) and another might be climate (rainy versus sunny). This creates four possible states. Unaware agents only understand a subset of the dimensions of this product space, while any random variable Z is defined over the entire product space. An important assumption I make is that an agent's perception of the random variable agrees with its expected value. So an agent only aware of temperature perceives a random variable that can take on only two possible values: $E[Z|\text{hot}]$ and $E[Z|\text{cold}]$. Notably, I do not impose that expected values incorporate true probabilities; that is, I allow for *biased* perceptions. If perceptions happen to incorporate the

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true objective probabilities, individuals are correct in expectation about states they do not perceive.

Using this formulation, agents face a perceived solvency problem. They choose asset positions based on perceived endowments and asset returns, which are specified exogenously, and perceived prices and delivery rates, which are determined in equilibrium. The perceived problem generally differs from the actual solvency problem. In such scenarios, agents may involuntarily find themselves in a state of nature where their net wealth is strictly negative. I assume there is no penalty for default other than the complete seizure of wealth, which is pooled together across agents to calculate the delivery rate on a particular asset.

Why might an agent be unaware of future endowments? Consider medical expenses resulting from rare diseases, which doctors only discuss with the agent at a high level. What about unawareness regarding asset returns? Consider complex financial contracts that are summarized for the trader by her financial advisor. When it comes to the exogenous random variables in this economy, the narrative I endorse is one of learning from experts. That *someone* knows the entire distribution of endowments or asset returns is reasonable. That *households* know this information is rather far-fetched. And while learning from experts may seem like a narrow explanation at first glance, I argue that a large proportion of information households read in articles, research online, or are told in person comes in (potentially biased) summarized form.

Why might an agent be unaware of endogenous random variables, such as delivery rates? Forming perceptions over delivery rates is vital to my existence result, which holds for any full-support expectations. Consider a risky asset and a risk-less bond, and consider an optimistic agent and a pessimistic one (in terms of perceived payout of the risky asset). As the optimistic agent goes long and the pessimistic agent goes short on the risky asset, the pessimistic agent eventually defaults until the effective (delivery-adjusted) return of the risky asset is reduced to a point where the optimistic agent sees no arbitrage opportunity.[1] Although unaware, optimistic agents must realize that their arbitrarily large long position cannot be delivered in full, and that, therefore, their returns must be reduced. I argue that this line of reasoning—forming perceptions over others' default—is not only an essential, but realistic, feature of financial markets. As an example, a bank may choose to extend fewer loans during recessions due to concerns about the solvency of borrowers (as opposed to simply charging higher rates). I emphasize the existence result because it stands in contrast to the past literature on unawareness in general equilibrium.

The key reference for general equilibrium models augmented with unawareness is Modica, Rustichini, and Tallon (1998),[2] who use the following definition of unawareness: with finite state space S, each agent i only sees a subset of states $S^i \subseteq S$. To guarantee existence, the authors require strong assumptions. They require all agents to agree on a subset of states $C \subseteq S^i$, with the cardinality of C equal to the number of assets A

[1]This parallels work by Daher, Martins-da Rocha, Pascoa, and Vailakis (2007), who solve the problem of expectation error through default and collateral.

[2]Extended to production economies by Kawamura (2005).

($|C| = A$). These assumptions are difficult to justify, because the entry of a novice financial trader shrinks C and financial innovation increases A. In fact, the authors do not attempt to defend the assumption on the grounds of realism, and this lack of existence is one of the motivations for the present work.

To contrast the model of Modica, Rustichini, and Tallon (1998) with the current one, imagine an asset that pays 100 in the first state of nature (the state occurs with probability $q > 0$) and 0 otherwise. There is also a risk-free asset. In Modica, Rustichini, and Tallon (1998), the presence of fully aware agents forces the risky asset price to be positive, but if any agent is unaware of the first state ($s \notin S^i$), she has an incentive to take out a maximal short position on the asset and an equilibrium fails to exist. Now consider the current setting. Unaware agents trade the risky asset as if it were risk-free with payout $100q$. Default may occur if unaware agents short the risky asset, but now the incentive to maximally short is gone.

While the correct-in-expectations benchmark is just a special case of my model, it serves as an important benchmark. Unawareness with beliefs that are correct in expectations is employed by Carvajal, Rostek, Schipper, and Sublet (2019), but not in a general equilibrium setting. Auster, Kettering, and Kochov (2021) use the same assumption in a general equilibrium setting, but they assume that all unawareness is dispelled one period before the random variable is realized. With only one-period-ahead assets, traders are effectively restricted to trading only assets they understand. To a similar end, Guerdjikova and Quiggin (2021) assume that agents are infinitely averse to "surprise" and, hence, do not trade on partitions of the state space finer than those consistent with their level of awareness.

Generally speaking, unawareness introduces two effects. The first is that the cardinality of the agent's state space is reduced. The second is that in the remaining states, the agent's expectations are now biased because part of the support is missing. The existence problems encountered in Modica, Rustichini, and Tallon (1998) are of the second type. That is, they are the same existence issues one would encounter with zero probabilities.[3] Using the correct-in-expectations benchmark, I am able to introduce the first effect without the second.

With existence guaranteed, I move on to novel questions of welfare under unawareness. I find that default is inefficient for the following reason. Although agents realize there may be equilibrium delivery rates less than 1, they never believe they will be the ones defaulting. This results in long positions being down-weighted by delivery rates, but not short positions, and, hence, default makes assets look worse in perceptions. Removing default and bringing asset delivery rates back to 1 is shown to be Pareto improving. In economies with no aggregate risk, I can guarantee at least one equilibrium with no default. With no aggregate risk, households simply attempt to equate wealth levels across states of nature. But with constant wealth levels and resulting constant prices across states, the households' unawareness becomes irrelevant (because they should make the same plan across different states anyway). Hence, they do not default.

[3] See Heifetz, Meier, and Schipper (2013) for a discussion of the difference between zero probability and unawareness.

Even without default, the economy fails to be efficient. This is because agents do not equate marginal rates of substitution across states they do not perceive, resulting in a pecuniary inefficiency standard in incomplete markets. In standard settings, agents cannot equate marginal rates of substitution because assets are missing. Here, they do not equate marginal rates of substitution because they do not perceive the full state space. Hence, with unawareness, the inefficiency persists even when markets are complete.

The Second Fundamental Welfare Theorem holds only in economies with no aggregate risk. Imagine a planner using lump sum wealth transfers to implement a Pareto efficient allocation. The presence of aggregate risk can be problematic because unaware agents willingly overexpose themselves to such risks. They inadvertently trade away Pareto efficient endowments, making lump sum transfers an ineffective policy tool in such settings.

Finally, I also show that financial education, or the increasing of awareness, can be counterproductive when unawareness is not fully dispelled. Although the complete markets, full awareness benchmark is socially desirable (see Geanakoplos and Polemarchakis (1986)), the lesson is precautionary: a little financial education may be worse than none at all.

The model helps us understand a new phenomenon: unintended default. The traditional perspective has been that households weigh the costs and benefits of default and maximize their gain from bankruptcy.[4] Fay, Hurst, and White (2002) find a positive empirical relationship between filing for bankruptcy and financial benefit for filing. They rule in favor of strategic default, ending the debate, at least for the time. In most theoretical rational expectations models, unintended default is an impossibility because all future contingencies are probabilistically and correctly assessed. Unsurprisingly, the general equilibrium literature has modelled default as a conscious decision, where agents weigh the benefits of defaulting on promised payments against various costs. For example, in Dubey, Geanakoplos, and Shubik (2005), the cost is a direct utility loss; in Araujo, Kubler, and Schommer (2012), it is the seizure of collateral; in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), it is a restriction on future access to financial markets. Other authors have cited the stigma, or social cost, of default. In Ben-Ami and Geanakoplos (2021), agents have no choice but to default because they begin with debt; however, the authors are silent on the initial source of debt.

More recent empirical work has called into question this conventional view, and proposes that households find themselves in situations where liabilities may surpass assets and they have no choice but to default. Zhang, Sabarwal, and Gan (2015) point out that adverse events such as income shocks can increase the financial benefits of filing, leading to bankruptcy. While compatible in the econometric sense with the results of Fay, Hurst, and White (2002), this story is consistent with nonstrategic behavior. Further tests for households manipulating debt to maximize financial benefits from bankruptcy give negative results, reinforcing the nonstrategic view. Results from Keys (2018) also

[4]Chapter 7 is the most common personal bankruptcy procedure filed in the United States, and is the main focus of this paper. After filing, debtors are not obliged to use any of their future earnings to repay their debt, but they are obliged to turn over all of their assets above a fixed exemption level.

confirm the nonstrategic motive, showing that bankruptcy is three times more likely immediately after job loss.

Thus, the empirical debate over the *average* American household's bankruptcy motive, strategic or nonstrategic, is not over. However, it is uncontroversial to claim that *some* American households default involuntarily. Unaccounted-for contingencies arise in which liabilities may surpass assets. These households file for bankruptcy because they have no other choice. The distinction is an important one from a welfare perspective. Strategic default may be welfare improving. As previously discussed, unintended default is unequivocally welfare reducing.

The remainder of the paper is organized as follows. In Section 2, I provide two motivating examples that highlight the inefficiencies in this economy as well as the role of default when perceptions are biased. In Section 3, I define the formal model. In Section 4, I briefly discuss no-arbitrage prices and provide a proof of existence. In Section 5, I discuss the First and Second Welfare Theorems, as well as two additional examples that display the pecuniary inefficiency and the non-monotonic effect of increasing awareness on welfare. Section 6 concludes.

2. Motivating examples

2.1 *Example (improving allocations)*

Suppose the state space, $\mathcal{S}$, consists of two equally likely rows, T and B, and two equally likely columns, L and R. Rows and columns are independent. For each state, there is an asset that pays 2 if the state occurs and nothing otherwise. There is only one commodity. There are two agents who are only aware of rows, denoted $R1$ and $R2$, and two who are only aware of columns, $C1$ and $C2$. Their endowments are

$$e^{R1} = \begin{bmatrix} 1 & 3 \\ 3 & 5 \end{bmatrix}, \qquad e^{R2} = \begin{bmatrix} 3 & 1 \\ 5 & 3 \end{bmatrix}, \qquad e^{C1} = e^{C2} = \begin{bmatrix} 4 & 4 \\ 2 & 2 \end{bmatrix}.$$

The two row agents wrongly perceive that they are identical and have endowments

$$e^{R} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}.$$

The two column agents are identical and perceive endowments

$$e^{C} = \begin{bmatrix} 3 & 3 \end{bmatrix}.$$

Each agent can trade in the four assets, but each sees them as redundant in pairs. For example, row agents perceive the return of the $[T, L]$ and $[T, R]$ assets as both being equal to

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and the return of the $[B, L]$ and $[B, R]$ assets as both being equal to

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

The columns agents make similar mistakes, perceiving asset returns as either $[1 \ \ 0]$ or $[0 \ \ 1]$. In equilibrium, all assets are traded at the same price; row agents demand the two T assets and short the two B assets; column agents are indifferent between no trade and the trades required to accommodate these demands. Because of their wrong perceptions, given any equilibrium, I can construct another one where agent $R1$ only buys asset $[T, R]$ while agent $R2$ only demands $[T, L]$. A policy swapping these demands makes them better off in reality.

The perceived indeterminacy of assets from the example above will be a general characteristic of this economy. As unawareness deepens, intuition suggests that the indeterminacy necessarily worsens, as agents perceive fewer and fewer states. However, this need not always be the case, because unawareness can lead to unintended default. Such default can differentiate otherwise indistinguishable assets, alleviating indeterminacy issues even in perceptions.[5]

2.2 *Example (biased perceptions)*

Suppose the state space, $\mathcal{S}$, consists of two states. The first asset is risk-free and the second asset is risky, with respective returns

$$r_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}, \qquad r_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

There is only one commodity. There are two agents, both of whom are completely unaware. The optimistic agent, denoted O, forms perceptions using a weight of $\frac{9}{16}$ on the second state. The pessimistic agent, denoted P, forms perceptions using a weight of $\frac{1}{2}$ on the second state. Both agents agree on the perceived return of the risk-free asset by construction, $E_O[r_1] = E_P[r_1] = 1$, but perceptions of the risky asset differ:

$$E_O[r_2] = \frac{9}{16}, \qquad E_P[r_2] = \frac{1}{2}.$$

There is an incentive for the optimistic agent to buy the second asset (while shorting the risk-free), and for the pessimistic agent to short it (while buying the risk-free). Endowments are set to 2 for the pessimist across both states and to 4 for the optimist across both states. In equilibrium, the pessimistic agent defaults on the risky asset in the second state at a rate of $\delta = \frac{8}{9}$. Note that this is precisely the rate at which *delivery-adjusted*

[5]As a concrete example, consider an economy with two assets with returns of

$$\begin{bmatrix} 2 & 0 \\ 2 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix}.$$

A row agent views these two assets as redundant. Default of either asset in any state leads to lower delivery-adjusted returns in that state, breaking this perceived redundancy.

asset returns equalize across agents, effectively removing any perceived arbitrage opportunities:

$$E_O[\delta r_2] = \frac{1}{2}.$$

Asset prices reflect these effective returns: $q_1 = 1$ and $q_2 = \frac{1}{2}$. While these prices and returns make agents indifferent between any asset position, a particular selection is consistent with the previously mentioned default rate.[6] These are

$$y_1^O = -\frac{18}{7}, \qquad y_2^O = \frac{36}{7}, \qquad y_1^P = \frac{18}{7}, \qquad y_2^P = -\frac{36}{7}.$$

The optimist does not default because she has ample endowment; neither does the pessimist in state one, because he has long positions only. Both assets pay out in the second state and, hence, the pessimistic agent's return in state two is $y_1^P + y_2^P$. He owes more than his endowment of 2 in this second state, so he delivers only what he can, $\frac{8}{9}$, of his promise; his wealth in state two is $2 + y_1^P + \frac{8}{9} y_2^P = 0$.

3. General model

3.1 *Small worlds*

There are two periods and a finite set $\mathcal{S}$ of states of nature in the second period, with elements $s = 1, \dots, S$. There is a probability distribution μ over space $\mathcal{S}$, and any random variable is defined over $\mathcal{S}$. Set $\mathcal{S}$ is a product space, $\mathcal{S} = \times_d \mathcal{S}_d$, with dimensions $d = 1, \dots, D$. Let $\mathcal{D}$ denote the set of dimensions. Each individual i only understands her "small world" $\mathcal{S}^i$, which is itself a product space $\mathcal{S}^i = \times_m \mathcal{S}_m$ for $m \in \mathcal{M}^i \subseteq \mathcal{D}$. As an extreme case, consider when $\mathcal{M}^i$ are disjoint singletons $\forall i$. Then $\mathcal{S} = \times_i \mathcal{S}^i$; an example of this would be an economy with one row agent and one column agent. At the other extreme, consider $\mathcal{M}^i \subset \mathcal{M}^j = \mathcal{D}$, which would correspond to an agent j who understands both rows and columns, as well as a row (or column) agent i.

With the assumption of price taking behavior, individuals need not form beliefs about others, only about equilibrium variables. Individual i's beliefs over $\mathcal{S}^i$ are given by μ^i, and given a random variable Z, individual i forms a perception at each perceived state s^i, which I denote $E_i[Z(s^i)]$. I assume that these perceptions are given by a convex combination of the random variable:[7]

$$E_i\big[Z\big(s^i\big)\big] = \sum_{s \in (s^i \times \mathcal{S}^{-i})} \beta_i(s) Z(s), \qquad \sum_{s \in (s^i \times \mathcal{S}^{-i})} \beta_i(s) = 1.$$

One special case, the correct-in-expectation benchmark, is when μ^i is the marginal distribution of μ on $\mathcal{S}^i$ and perceptions are given by the conditional expectation $E_i[Z(s^i)] = E[Z(s)|s^i]$.

[6]These positions must also respect budgets, $q_1 y_1 + q_2 y_2 = 0$, and clear markets.

[7]The notation $\times \mathcal{S}^{-i}$ denotes states in all dimensions a particular agent does not perceive, $\times_m \mathcal{S}_m$ for $m \in (\mathcal{D} \setminus \mathcal{M}^i)$.

3.2 *The economy*

Let $i = 1, \dots, I$ denote consumers and let $\ell = 1, \dots, L$ denote commodities. Individual i's preferences over commodities are represented by $u^i : \mathbb{R}^L_+ \to \mathbb{R}$; her future endowment is the random variable e^i, which maps $\mathcal{S}$ into $\mathbb{R}^L_+$. Let $a = 1, \dots, A$ denote assets. The promised payoff of asset a is a random variable r_a, defined on $\mathbb{R}$. The term $r(s)$ denotes the vector $[r_1(s), \dots, r_A(s)]$. All payoffs are denominated in units of commodity 1.

In the first period, the agents only trade the A assets at prices $q = [q_1, \dots, q_A]$. Individual i's portfolio is $y^i \in \mathbb{R}^A$, with $y^i_{+a} = \max\{y^i_a, 0\}$ being her individual holdings of asset a and $y^i_{-a} = -\min\{y^i_a, 0\}$ being her short sales. Her vector of holdings is $y^i_+ = [y^i_{+1}, \dots, y^i_{+A}]$ and, similarly, her vector of short sales is $y^i_- = [y^i_{-1}, \dots, y^i_{-A}]$. These assets, which can be thought of as dividends of an unmodelled firm, are not traded as contingent contracts, but simply as random variables. The distinction is important when assets pay out a different value than expected; these scenarios are meant to capture financial contracts that are difficult to spell out explicitly. In state s, individual i's consumption bundle is $x^i(s)$. Because of her limited understanding, she does not make consumption plans over $\mathcal{S}$. Instead she perceives that her consumption for her state s^i is $\hat{X}(s^i)$. Nothing guarantees that $\hat{X}(s^i) = E_i[x(s^i)]$.

3.3 *Ex post trade*

Let $y = [y^1, \dots, y^I]$ be given. At state $s \in \mathcal{S}$, commodity prices are derived from equilibrium in the spot commodity markets. Default occurs, but only involuntarily. For all $s \in \mathcal{S}$, let $(p(s), \delta(s))$ be a solution on $\mathbb{R}^L_+ \times [0, 1]^A$ to the system of equalities

$$\delta_a(s) = \begin{cases} 1 \quad \text{if } \sum_i y^i_{+a} = 0 \\ \dfrac{1}{\sum_i y^i_{+a}} \sum_i y^i_{-a} \min\left\{1, \dfrac{p(s) \cdot e^i(s) + p_1(s) \sum_{\alpha=1}^{A} \delta_\alpha(s) r_\alpha(s) y^i_{+\alpha}}{p_1(s) \sum_{\alpha=1}^{A} r_\alpha(s) y^i_{-\alpha}}\right\} \\ \text{otherwise} \end{cases} \tag{1}$$

$$\sum_i x^i(s) = \sum_i e^i(s), \tag{2}$$

where $x^i(s)$ is the solution to the problem of maximizing $u^i(x)$ subject to

$$p(s) \cdot x = \max\left\{0,\ p(s) \cdot e^i(s) + p_1(s) \sum_{\alpha=1}^{A} \delta_\alpha(s) r_\alpha(s) y^i_{+\alpha} - p_1(s) \sum_{\alpha=1}^{A} r_\alpha(s) y^i_{-\alpha}\right\}. \tag{3}$$

Equation (1) establishes delivery rates for the assets. Inside of the $\min\{\cdot\}$ function, the term in the numerator denotes the value of her endowment plus what is owed to her, and the term in the denominator denotes the value of what she owes. If she has more than she owes, the $\min\{\cdot\}$ equals 1; if this is true for all agents, delivery rates equal 1.

Otherwise, the proportion of what each agent is able to repay is pooled together to calculate a delivery rate strictly less than 1 for that particular asset. Note the effective real payoff for asset a in state s is $\rho_a(s) = \delta_a(s) r_a(s)$. For simplicity of notation, I write $\rho(s) = [\rho_1(s), \dots, \rho_A(s)]$. Equation (2) is more straightforward: it imposes market clearing in the spot markets. In determining the optimal demands of the individuals, their budget constraints, given by (3), establish that there is no penalty for agents who default other than the complete seizure of their wealth. Across states of nature, this mechanism defines random variables x, p, δ, and ρ.

3.4 *Ex ante trade*

Let random variables p, δ, and ρ be given. At asset prices q, individual i chooses a portfolio y^i and formulates a perceived consumption plan $\hat{X} : \mathcal{S}^i \to \mathbb{R}^L_+$ so as to maximize

$$\sum_{s^i \in \mathcal{S}^i} \mu^i(s^i) \cdot u^i(\hat{X}(s^i)) \tag{4}$$

subject to the constraints

$$q \cdot y^i = 0 \tag{5}$$

and for each $s^i \in \mathcal{S}^i$,

$$E_i[p(s^i)] \cdot \hat{X}(s^i) = E_i[p(s^i) \cdot e^i(s^i)] + E_i[p_1(s^i)\rho(s^i)] \cdot y^i_+ - E_i[p_1(s^i) r(s^i)] \cdot y^i_-. \tag{6}$$

The ex ante budget constraint (5) is straightforward because there is no consumption in the first period. The budget constraints given by (6) show the individual's perception of her solvency problem. Importantly, she plans to remain solvent, as far as she can see, but recognizes that the payoffs she receives in her purchases may differ from those promised; as with other random variables, her perception of effective payoffs is given by her (potentially biased) perceptions.[8] Note that delivery rates multiply long positions, but not short. While she realizes assets may not be delivered to her in full, she herself never believes she will be the one defaulting; this reflects the unintended nature of default. Asset markets clear when

$$\sum_i y^i = 0. \tag{7}$$

3.5 *Equilibrium*

Equilibrium consists of a tuple $(p, \delta, \rho, x, \hat{X}, q, y) \in \mathbb{R}^{SL}_+ \times [0, 1]^{SA} \times \mathbb{R}^{SA} \times \mathbb{R}^{SLI}_+ \times \mathbb{R}^{\overline{S}LI}_+ \times \mathbb{R}^A_+ \times \mathbb{R}^{AI}$, where $\overline{S} = \frac{1}{I} \sum_i |\mathcal{S}^i|$, such that

(i) at each s, $(p(s), \delta(s), \rho(s), x^i(s))$ solves (1)–(3) given $y = [y^1, \dots, y^I]$

(ii) $(y, \hat{X}, q)$ solves (4)–(7) given (p, δ, ρ).

[8]One alternative, which is not equivalent, would be to compute the perceptions of both r and δ, and then compute their product. This would make agents unaware of the correlation between the two random variables.

This is a rational expectations equilibrium that allows for both unawareness and bias in beliefs. While perceptions over endogenous delivery rates play the important role of removing perceived arbitrage opportunities as shown in Section 2.2, perceptions over endogenous commodity prices are nonessential in the following sense. As already demonstrated in the motivating examples in Section 2, the economy functions properly with only one commodity. Furthermore, as a corollary immediately following the proof of existence, I show that *any* price perceptions—even those entirely disconnected from true ex post prices—guarantee existence.

In equilibrium agents realize that others may default and that institutions are set up to handle such default. But precisely due to the competitive nature of the economy, agents need not form beliefs over others' preferences, awareness levels, or endowments. Therefore, agents never realize that another's default is due to their unawareness. Default could, for example, be due to preexisting debt à la Ben-Ami and Geanakoplos (2021). Agents believe that they are the "lucky ones" safe from the perils of bankruptcy, so the existence of default institutions does *not* raise individuals' levels of awareness of their own possibility of default. Furthermore, the two period nature of the model rules out (interesting) questions about raising awareness through information contained in equilibrium variables. That is, the setup effectively assumes away any subjective state space revision.

I previously mentioned that default occurs only involuntarily, which is akin to assuming infinite utility costs of default. Because of this, agents never attempt to default ex ante; default only occurs involuntarily ex post. This assumption not only focuses the paper on the novel phenomenon of unintended default, but it also sidesteps problems with equilibrium existence when agents take advantage of limited liability laws (they would attempt to default maximally).[9]

4. Existence

Beyond the environment laid out in Section 3, several assumptions below will be sufficient for the proof of existence. I define the S-by-A return matrix R as $R_{sa} = r_a(s)$.

Assumption 1. *Utility functions are continuous, concave, and strictly monotonic.*

Assumption 2. *Beliefs $\mu^i(s^i)$ and perceptions $\beta_i(s)$ have full support $\forall i, s, s^i$.*

Assumption 3. *Endowments are strictly positive $e^i(s) > 0$ $\forall i, s$.*

Assumption 4. *Asset returns are nonnegative $R \geq 0$ and there are no redundant assets* $\text{rank}(R) = A$.

[9]This rules out alternative interpretations of the model that involve bundling: agents bundle states together to reduce the cognitive complexity of the problem at the expense of suboptimal allocations and default. This interpretation is reminiscent of coarse competitive equilibria from Gul, Pesendorfer, and Strzalecki (2017).

I will first comment informally on the necessity of these assumptions. Continuity and concavity from Assumption 1 allow use of the theorem of the maximum. Strict monotonicity from Assumption 1 and positive endowments from Assumption 3 together ensure positive commodity prices. Assumption 4 ensures that asset prices are positive. Assumption 2 has considerable "bite": when perceptions have full support, agents choose not to take out maximal short positions on assets, for if they did, this would enter into their perceived budget constraint (6) through the perceptions operator, destroying perceived solvency. When agents use the correct probabilities (the correct-in-expectation benchmark), Assumption 2 simply collapses to the requirement that μ have full support.

With full awareness and correct beliefs, $\mu^i = \mu$ for all i, the ex ante budget equation (6) collapses to the ex post one (3), deliveries are all fulfilled $\delta_a(s) = 1$ for all a, s, and the problem is equivalent to the standard incomplete markets problem. Compared to the assumptions of Geanakoplos and Polemarchakis (1986), Assumption 1 is strengthened from strict monotonicity in the numeraire only to strict monotonicity in all goods. Quasi-concavity is strengthened to concavity to ensure the object (4) is quasi-concave. Assumption 4 is strengthened to include $R \geq 0$, which rules out negative asset prices.

4.1 *No-arbitrage prices*

I discuss no-arbitrage prices in the correct-in-expectations benchmark for intuition. With different levels of awareness across individuals, no arbitrage is a personalized condition to each agent in the sense that no agent can perceive arbitrage in equilibrium. The no-arbitrage set NA should read

$$\mathrm{NA} = \cap_i \{ q \in \mathbb{R}_+^A \mid E_i[\rho(s^i)] \cdot y_+ - E_i[r(s^i)] \cdot y_- > 0 \; \forall s^i \implies q \cdot y > 0 \}.$$

To see that NA is nonempty, consider the point $q = E[r(s)]$. For each individual i, say the premise in the no-arbitrage definition is satisfied:

$$E_i[\rho(s^i)] \cdot y_+ - E_i[r(s^i)] \cdot y_- > 0 \quad \forall s^i \implies E_i[r(s^i)] \cdot y > 0 \quad \forall s^i \implies E[r(s)] \cdot y > 0.$$

The first implication is true because deliveries are only applied to long positions. The second implication follows from the law of iterated expectations. By similar logic, $q = E[\rho(s)]$ is in the NA set, and by convexity of NA, so is any convex combination between the two price vectors. The purpose of this discussion is to associate the correct-in-expectation assumption with asset pricing; prices should lie somewhere between expected returns for long and short positions.

4.2 *Existence*

First I define the bounded problem, which solves (1)–(7) with two exceptions. First and most notably, commodity and asset positions $(x, \hat{X}, y)$ are restricted to a compact cube of size n. Second, market clearing conditions (2) and (7) are replaced with explicit market makers who maximize the value of excess demand.[10] Lemmas 2 and 3 ensure that these market makers do indeed clear markets. But first, I show that the bounded problem has a solution.

Lemma 1 (Bounded Problem). *The bounded problem has at least one solution.*

See the Appendix for all proofs.

The result follows from standard fixed point arguments because the problem is continuous and convex. With compactness guaranteed by bounds of size n, the result is obtained. Interestingly, the pessimistic equilibrium as in Dubey, Geanakoplos, and Shubik (2005), where zero expected deliveries can lead to zero asset purchases, which can then lead to zero deliveries, cannot occur because deliveries here are always nonzero (agents deliver if they are able). Moving forward, I denote the solution to the bounded problem with a subscript n.

In the next lemma, I prove Walras' law for ex post commodity markets, i.e., $\sum_i p_n(s) \cdot x_n^i(s) = \sum_i p_n(s) \cdot e^i(s)$, despite the presence of bankruptcy. Intuitively, no wealth is lost in the process of default and imperfect asset delivery.

Lemma 2 (Walras' Law). *If $\sum_i y_n^i \leq 0$, then $p_n(s) \cdot \sum_i (x_n^i(s) - e^i(s)) \leq 0$ for all s, and if $\sum_i y_n^i = 0$, then $p_n(s) \cdot \sum_i (x_n^i(s) - e^i(s)) = 0$ for all s.*

Next I show that asset markets and ex post commodity markets must clear in the n-bounded problem. Intuitively, the result follows from Walras' law combined with monotonic utility and positive asset returns, which rule out zero commodity and asset prices.

Lemma 3 (Market Clearing). *Asset markets clear, $\sum_i y_n^i = 0$, and ex post commodity markets clear, $\sum_i x_n^i(s) = \sum_i e^i(s)$ for all s.*

Up to this point I have evaluated each n-bounded problem on its own. Now I consider what happens to the sequence $(y_n, x_n, \delta_n, q_n, p_n)_{n\in\mathbb{N}}$ as I take n to be large. In my final lemma, I derive a lower bound for the price of commodity one. Intuitively, prices of two commodities cannot differ by arbitrary amounts when utilities are monotonic and endowments are positive; this creates a lower bound when prices are restricted to the simplex.

Lemma 4 (Uniform Bound). *There exists $\epsilon > 0$ such that $p_{n1}(s) \geq \epsilon$ $\forall s, n$.*

In the following theorem, I expand the cubes of Lemma 1 by sending $n \to \infty$, and verify that the limit of these fixed points is an equilibrium.

Theorem 1 (Existence). *There exists at least one equilibrium.*

[10]The delivery condition is also altered to something that collapses to (1) when asset markets clear. This ensures continuity; for details, see Appendix A.

Although agents can default in some state s, the full support assumption in the ex ante problem bounds the degree of bankruptcy. In other words, agents do not take out maximal short positions on assets because doing so would destroy their perceived solvency. The problem of an endogenous return matrix dropping rank à la Hart (1975) is not an issue because deliveries are applied only to long positions. The proof would fail if this were not the case.

As explained in the example in Section 2.2, perceptions of delivery rates are essential to the proper functioning of this economy. However, perceptions of commodity prices are not. To emphasize this last point, I have the following corollary. It says that *any* perception of commodity prices suffices to guarantee existence. These perceptions need to be positive-valued, but need not be connected to true ex post values in any way.

Corollary 1 (Price Perceptions). *Say that the perceived solvency problem (6) were, instead, given by*

$$\hat{p}(s^i)\cdot\hat{X}(s^i)=\hat{p}(s^i)\cdot E_i\big[e^i(s^i)\big]+E_i\big[p_1(s^i)\rho(s^i)\big]\cdot y^i_+-E_i\big[p_1(s^i)r(s^i)\big]\cdot y^i_-, \tag{8}$$

where $\hat{p}_\ell(s^i)>0$ *are fixed for all* ℓ, s^i. *Then an equilibrium exists.*

While the goal of Corollary 1 is to deemphasize price perceptions, it also highlights how ex ante markets function. Even in the standard problem (6), perceived consumption $\hat{X}(s^i)$ need not clear markets; perceived prices $E_i[p(s^i)]$ are not being chosen in a way to guarantee ex ante market clearing. This is why arbitrary prices $\hat{p}(s^i)$ can supplant perceived prices, while still keeping the problem intact.[11]

I was able to guarantee existence even when perceptions are incorrect (as long as they have full support). In the upcoming welfare section, I impose the correct-in-expectations assumption as a best-case scenario. I will show that inefficiencies exist *even if* agents incorporate true probabilities. Any further bias can be viewed as an additional deviation from efficiency.

5. Welfare

In the second half of the paper, I cover the First and Second Welfare Theorems, and then finish with a section on the welfare impact of discovery or the gaining of awareness. As previously discussed, I analyze the correct-in-expectation best-case scenario. Also, because much is already known about the inefficiencies caused by incomplete markets, I choose to shut that channel down.

Assumption 5. *The term* μ^i *is the marginal distribution of* μ *on* S^i *and* $E_i[Z(s^i)]=E[Z(s)|s^i]$.

Assumption 6. *Financial markets are complete* $\operatorname{rank}(R)=S$.

[11]Note that expression (8) does *not* allow agents to have arbitrary perceptions of numeraire prices $p_1(s^i)$. This ensures that the ex ante problem is "not too different" from the well behaved ex post problem.

Typically in these settings, a constrained version of Pareto efficiency is used, where the planner is constrained to the same incomplete financial market as agents. Without such a worry, I use the standard Pareto efficiency definition provided in undergraduate texts.

Definition 1. An allocation is Pareto efficient *in reality* if there does not exist a reallocation of commodities that makes all agents weakly better off in $\sum_s \mu(s) u^i(x^i(s))$, with at least one agent strictly better off.

The object referenced in the definition above is ex ante utility if agents were perfectly aware. It is what agents wished they had maximized after all unawareness has been dispelled. Another interpretation is that the social planner is perfectly aware and, furthermore, takes a paternalistic view: she knows what is best for her constituents. I will assume the social planner does not have access to technology that can force agents to consume particular commodity bundles. After all, unaware agents do not even perceive the correct state space. I will consider asset reallocations implemented by the planner and the resulting "equilibria" induced by these reallocations. To make the idea concrete, I introduce the idea of a quasi-equilibrium.

Definition 2. Given asset allocations y, a quasi-equilibrium solves (1)–(4) and (6).

In words, it is an ex post equilibrium only. Now I ask a slightly different question: "Is the object in Definition 1 truly what we, as a society, want to be maximizing?" We should care about an agent's happiness, even in her own small world. Hence, I introduce a third and final definition.

Definition 3. An allocation is Pareto efficient *in perceptions* if there does not exist a reallocation of assets that makes all agents weakly better off in $\sum_{s^i} \mu^i(s^i) u^i(\hat{X}(s^i))$ in the induced quasi-equilibrium, with at least one agent strictly better off.

If agents are perfectly aware, then with complete markets and unbiased beliefs, Definitions 1 and 3 become equivalent. As previously discussed, unawareness differentiates the ex ante and ex post problems.

5.1 *First fundamental welfare theorem*

There is no reason to even suspect that the economy should be efficient in reality, and the example in Section 2.1 confirms this intuition. Let me ask a more difficult question: "Are equilibria efficient in perceptions?" In fact, I will use Definition 3 as my welfare criterion in all of the upcoming results unless stated otherwise. The next theorem guarantees inefficiency in perceptions when there is default, even under the assumption of complete financial markets.

Theorem 2 (Inefficiency in Perceptions). *Given Assumptions 1–6 and strict default $\delta_a(s) < 1$ for some a, s, competitive equilibrium is inefficient in perceptions.*

The intuition is as follows. Imagine taking each agent's perceived wealth and taking a weighted sum over all perceived states, weighted by $\mu^i(s^i)$. Then taking a sum over all agents, I get an expression for total expected wealth:

$$\sum_i E\big[w^i(s)\big] = \sum_i E\big[p(s) \cdot e^i(s) + p_1(s)\rho(s) \cdot y^i_+ - p_1(s)r(s) \cdot y^i_-\big].$$

Although asset markets do clear in equilibrium, the last two terms on the right-hand side of the expression above do not cancel. The reason is that long positions pay out delivery-adjusted returns, while short positions pay out in full. This is not happening in reality; it is entirely inside the agent's mind. The agent believes that she will never be the one defaulting, although she does realize there may be some equilibrium rate of delivery less than 1. By removing strict default, $\rho(s) = r(s)$ in the expression above, and the two problematic terms cancel out. Total perceived wealth in the economy increases, but only in the agent's mind.

Why are agents unable to correct such simple mistakes? The answer is that agents do not realize they can affect delivery rates advantageously in this way. Moving forward, I will refer to this inefficiency as the default inefficiency. It turns out that there are special classes of economies where there is no default and, hence, no default inefficiency; however, this discussion will need to be postponed until additional results have been established.

5.2 *Example (pecuniary inefficiency)*

Theorem 2 outlined the intuition behind the default inefficiency present in this economy. The next natural question might be "is the economy efficient in perceptions when there is no default?" The answer is again no, even with complete markets. I set up an example to investigate this second source of inefficiency. There are two agents who live in the same small world, where they are unable to differentiate between two equally likely states of nature. Their endowments of the three commodities are

$$e^1 = \big[(5, 3, 1), (4, 2, 2)\big], \qquad e^2 = \big[(1, 1, 7), (2, 2, 6)\big],$$

where, for example, the vector $(5, 3, 1)$ denotes five units of good one in state one, three units of good two in state one, and one unit of good three in state one. Their preferences are

$$u^i(x^i) = \sqrt{a^i x^i_1} + \sqrt{b^i x^i_2} + \sqrt{x^i_3}, \quad i \in \{1, 2\},$$

where $a^1 = 10$, $b^1 = 5$, $a^2 = 0.1$, and $b^2 = 10$. The asset market is complete with two Arrow assets that pay 2 in a particular state and 0 in the other. The setup is particularly appealing because agents perceive these assets as redundant ex ante, forcing their prices to equate. Their ex ante budget constraint then collapses to

$$E_i\big[p(s^i) \cdot e^i(s^i)\big] + y^i_1 + y^i_2 = E_i\big[p(s^i) \cdot e^i(s^i)\big].$$

On the left-hand side of the equality, asset positions are multiplied by their perceived returns, which both equal 1. On the right-hand side of the equality, the period one budget constraint $q_1 y_1^i + q_2 y_2^i = 0$ forces $y_1^i + y_2^i = 0$. An immediate corollary is that any asset position will be ex ante optimal for these agents, and I will exploit this degree of freedom. However, the choice of y is not completely irrelevant to the ex ante problem. A change in y affects ex post prices, which then enter into the ex ante problem via the expectations operator. Agents do not realize they can affect prices advantageously in this way.

Say there were no unawareness. Because here there is no aggregate risk, agents simply trade assets to equate wealth levels across states. Marginal rates of substitution across states are equated so that any price change hurts one individual as much as it helps the other; there is no pecuniary inefficiency. With unawareness, neither wealth levels nor marginal rates of substitution are equated across states $s \in \mathcal{S}$. Price changes may have a differential effect across individuals, and so it is possible that expected price changes help both individuals in perceptions. As opposed to standard incomplete markets where marginal rates of substitution are not equated because assets are missing, here, marginal rates of substitution are not equated because agents do not perceive the complete set of states. Importantly, the pecuniary inefficiency in an unawareness setting can persist even when markets are complete.

To be clear, there is only 1 degree of freedom in the example, and no more. Once the first individual's holdings of asset one, y_1^1, is fixed, y_2^1 is pinned down by the budget constraint and the second individual's asset holdings are pinned down by asset market clearing. Figure 1 graphs different choices of y_1^1, all of which lead to equilibria, and the corresponding ex ante utility levels for each individual. Again, these utility levels are affected not directly by the choice of y, but indirectly through ex post prices. In certain ranges, approximately $1.2 \leq y_1^1 \leq 1.8$, the equilibria can be Pareto ranked. A social planner can be of service when equilibria fall into this range.

In Auster, Kettering, and Kochov (2021), the authors conclude that their economy is Pareto efficient when there is no aggregate risk, $\sum_i e^i(s) = \sum_i e^i(s')$ for all s, s'. Notice that Example 5.2 was constructed with no aggregate risk, yet inefficiencies still arise. Two key differences in modelling assumptions explain this discrepancy. First, in their setting, there are more than two time periods and agents become aware of all contingencies one period ahead of their realization. The inefficiency that arises in their setting is a "savings mistake"; agents bear the cost of insurance at once rather than spread over time. Such a mistake cannot occur here in a two period model. Second, in my setting, there are multiple commodities so a pecuniary inefficiency is present. In their setting, with only one commodity, the pecuniary inefficiency is shut down by construction. In this sense, the inefficiencies in the two models are orthogonal.

One well known way to shut down this pecuniary inefficiency is by imposing assumptions on trader preferences. Because the argument is standard, it is stated without proof (see Araujo, Kubler, and Schommer (2012)). When agents all have identical homothetic utility functions, commodity prices do not depend on the distribution of wealth. Then the planner cannot affect prices using asset reallocations, which only affect the wealth distribution; hence, there can be no pecuniary inefficiency.

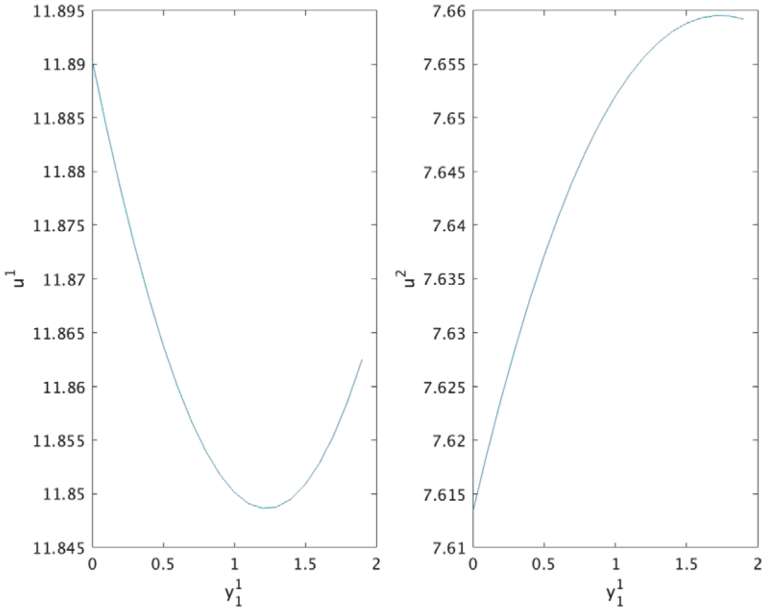

FIGURE 1. Pareto ranked ex ante utilities.

5.3 *Second fundamental welfare theorem*

Next I prove the Second Fundamental Welfare Theorem: any Pareto optimal allocation can be implemented as a competitive equilibrium of an economy with perturbed endowments or, equivalently, lump sum wealth transfers. I am forced to prove the result for Pareto efficient allocations in reality, as opposed to in perceptions, for the reason previously described: perceived allocations $\hat{X}(s^i)$ are not even allocations in general (there are no prices clearing the perceived market). When $\hat{X}(s^i)$ are not constrained to be allocations, they cause the planner's problem to be ill-posed or at the very least, extremely difficult. For the upcoming result to hold, I must restrict my space of economies.

ASSUMPTION 7. *No aggregate risk:* $\sum_i e^i_\ell(s) = \sum_i e^i_\ell(s')$ *for all* s, s', ℓ.

My final theorem is next.

THEOREM 3 (Second Fundamental). *Given Assumptions 1–5 and 7, any Pareto efficient allocation in reality* $\overline{x} > 0$ *is an equilibrium of an economy with perturbed endowments* ω.

In the proof, I rely heavily on perceptions being correct in expectation. Without enough agreement on asset returns $E[r(s)]$, some agents may be better off in perceptions by buying assets they like and selling assets they dislike. They may choose to trade away Pareto efficient endowments, inadvertently destroying efficiency.

The absence of aggregate risk ensures that Pareto allocations are constant across states. When implemented as endowments, these allocations guarantee that prices are constant across states and, hence, perceived prices equal actual prices. The ex ante problem collapses to the ex post one, which ensures that unaware agents do not trade. The example in Appendix I shows how the result breaks down when aggregate risk is present. Intuitively, problems arise when aggregate risk is borne by unaware agents. Because risky allocations are perceived as their expected value, unaware agents willingly take on too much aggregate risk, which often contradicts Pareto requirements.

Theorem 3 has a secondary use as an existence proof for economies without default, and I can use this to answer a previously open question: For which economies can I guarantee no default?

Corollary 2 (No Default). *Given Assumptions 1–7, there exists an equilibrium with no default: $\delta_a(s) = 1$ for all a, s.*

The idea behind Corollary 2 is similar to that of Theorem 3. Instead of using perturbed endowments to equate wealth levels across states, here I use assets to equate wealth levels across states. Equating wealth across states is feasible by Assumption 6 and desirable by Assumption 7. In the proof, I use assets to construct wealth levels that would prevail if agents had endowments $E[e^i(s)]$ instead of $e^i(s)$. As in Theorem 3, this creates constant prices and wealth across states s, and, hence, the ex ante problem collapses to the ex post one and agents do not trade *beyond* the asset positions described in the construction above. Reminiscent of the no-arbitrage discussion from Section 4, I let asset prices equal expected returns, $q = p_1 E[r(s)]$, in both Theorem 3 and Corollary 2. Notice that Corollary 2 does not rule out default altogether; it merely states that a no-default equilibrium is guaranteed to exist.

In my final example, I consider the welfare impact of discovery or the increasing of an agent's awareness. From the perspective of policy, this could be considered a form of financial education. I find that education is effective if all unawareness is dispelled; however, it can be counterproductive if unawareness is dispelled only partially.

5.4 *Example (discovery)*

As previously mentioned, if agents are aware and unbiased, the economy collapses to the complete markets financial economy of Geanakoplos and Polemarchakis (1986) with Assumption 6, and both ex ante and ex post welfare are maximized. What I am interested in are intermediate cases where awareness is increased, but not fully. Because comparing perceived welfare across different awareness levels is nonsensical, I use real welfare as the fixed benchmark.

To set up the investigation, notice that there is a partial ordering over perceived states $\mathcal{S}^i$. State $\mathcal{S}^i$ is said to be "more expressive" than $\mathcal{S}^j$ if $\mathcal{M}^j \subseteq \mathcal{M}^i$. It is a partial ordering because $\mathcal{M}^i$ and $\mathcal{M}^j$ may be disjoint. In the following example, I explore an economy where increasing awareness drastically reduces welfare for all agents. There are two agents with the endowments[12]

$$e^R = \begin{bmatrix} 2 & 2 \\ 7 & 1 \end{bmatrix}, \qquad e^C = \begin{bmatrix} 1 & 2 \\ 7 & 2 \end{bmatrix}.$$

[12]Notice that this example is characterized by aggregate risk. It turns out that aggregate risk is *not* a necessary feature of economies for which increasing awareness reduces welfare for all agents.

As before, rows and columns are independent and equally likely. There is a complete asset market with an asset that pays 2 if the state occurs and nothing otherwise. There is only one commodity. Although the agents are labeled as row and column agents, first consider a case of complete unawareness. Both agents perceive their endowment as a singleton, cannot distinguish between any of the four assets, and, hence, do not trade.

Next consider a scenario where awareness is increased, but not fully. The row agent is made aware of the row dimension, and the column agent is made aware of the column dimension. They perceive endowments

$$e^R = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \qquad e^C = \begin{bmatrix} 4 & 2 \end{bmatrix}.$$

The resulting equilibrium will be characterized by both agents defaulting. To see why, neither agent wishes to trade the asset $[T, R]$, which pays out in a state where they are both perceived as poor, or $[B, L]$, where they are both perceived as rich. The row agent will demand $[T, L]$ due to her low perceived endowment, and the column agent is more than willing to accommodate her due to his large perceived endowment. But notice that, in reality, the column agent is not well suited to deliver on this promise. The opposite scenario unfolds with $[B, R]$: the column agent demands this asset, yet the row agent would be unwise to deliver in reality. Equilibrium asset demands are

$$y^R_{[T,L]} = \frac{2}{3}, \qquad y^R_{[B,R]} = -\frac{2}{3}, \qquad y^C_{[T,L]} = -\frac{2}{3}, \qquad y^C_{[B,R]} = \frac{2}{3},$$

and 0 for all other assets. The delivery rate on both assets is $\frac{3}{4}$. In perceptions, the agents have done a reasonable job at smoothing consumption across states:

$$\hat{X}^R = \begin{bmatrix} 2.5 \\ 3.33 \end{bmatrix}, \qquad \hat{X}^C = \begin{bmatrix} 3.33 & 2.5 \end{bmatrix}.$$

In reality, however, the row agent defaults in state BR. She delivers $\frac{3}{4}$ of the promise of $\frac{2}{3}$ units of an asset that pays out 2. This equals her entire endowment of 1 unit in state BR. The column agent defaults in state TL in the exact same way. With natural log utility functions, the effect on welfare in reality is disastrous. Intuitively, full unawareness creates a hesitance to trade, which can be beneficial compared to partial awareness, where agents trade assets they do not fully understand. There is an analogous result in Guerdjikova and Quiggin (2021), where unawareness can be beneficial by restricting bad trades. In their setting, however, traders are restricted from trading assets they do not fully understand due to an infinite degree of pessimism. Here, the perceived multiplicity of assets leads to no trade.[13]

[13] In my example with complete unawareness, any no-default allocation that satisfies budget constraints and clears markets is an equilibrium. The result—that discovery can be Pareto worsening—holds for any of these equilibria other than a measure zero set where both agents have exactly zero nominal wealth in some state.

Table 1. Summary of welfare results.

Implication	Assumption		
	Fully Aware	No Agg Risk	Homothetic
Discovery cannot be Pareto worsening	✓	×	×
No default	✓	✓	×
No pecuniary inefficiency	✓	×	✓
Second Fundamental Welfare Theorem	✓	✓	×

5.5 *Discussion*

To summarize my welfare findings, there are two unrelated sources of inefficiency: through a default and a pecuniary channel. Default is inefficient because it effectively makes assets look worse to agents, and a pecuniary inefficiency arises because agents do not equate marginal rates of substitution across states they are unaware of. An equilibrium with no default is guaranteed when there is no aggregate endowment risk; when prices are constant across states, the ex ante problem collapses to the ex post one. In addition, the pecuniary inefficiency disappears when traders have identical homothetic utility functions, because asset trades, which affect only the distribution of wealth, do not affect commodity prices.

I then consider two courses of action: lump sum transfers and financial education. When agents are endowed with Pareto efficient allocations via lump sum transfers, no aggregate risk is again needed to collapse the ex ante problem to the ex post one. This guarantees that unaware traders do not inadvertently trade away Pareto efficient endowments. Financial education, or the gaining of awareness, is effective if all unawareness is dispelled, but can have disastrous effects otherwise. These results are summarized in Table 1.

Notice that full awareness implies all other welfare results: increasing awareness fully cannot be Pareto worsening (else there would exist Pareto improvements from fully aware equilibria), there can be no default, there can be no pecuniary inefficiency with complete markets, and the Second Fundamental Welfare Theorem holds. Furthermore, Table 1 clarifies what each assumption does *not* imply. Under the assumption of no aggregate risk, discovery can be Pareto worsening (see footnote 12) and pecuniary inefficiencies can still exist (see Example 5.2). Under the assumption of identical homothetic utility, discovery can be Pareto worsening (see Example 5.4), default can still occur (again see Example 5.4), and the Second Fundamental Welfare Theorem can fail (see Appendix I).

6. Conclusion

I have established the existence of equilibrium in a setting characterized by unawareness and potentially biased beliefs. Despite being a generalization of Geanakoplos and Polemarchakis (1986), the assumptions required for existence are not significantly expanded. When perceptions are incorrect, unanticipated default plays a key role in the existence proof because agents realize that others will default trying to deliver on arbitrarily large positions. On the other hand, the special correct-in-expectations case is a natural way to study general equilibrium with unawareness without introducing any bias.

Under this additional assumption, the economy is inefficient for two unrelated reasons. The default inefficiency stems from the unexpected nature of default, and the pecuniary inefficiency stems from the agent's failure to equate marginal rates of substitution across states they do not perceive. The Second Fundamental Welfare Theorem holds in economies with no aggregate risk. Finally, educating agents about the larger state space maximizes welfare if all unawareness is dissolved, but it can have disastrous results otherwise. While other papers suggest that strategic default can be welfare improving, here—more in line with conventional thinking—unintended default is at the heart of this economy's inefficiencies.

While doing so is not the main goal of the paper, I briefly draw parallels between this model and the 2008 financial crisis. Gennaioli and Schleifer (2020) argue that many of the bankruptcies that took place during the crisis can only be understood in the context of the overly sanguine beliefs held by market participants. The problem with modelling such optimistic beliefs using zero probabilities (Gennaioli, Shleifer, and Vishny (2015)) has been discussed in Section 1: agents can bet against zero probability states. A description of Michael Burry, whose hedge fund famously made close to \$1 billion in profits during the crisis, suggests another interpretation:

> Burry had devoted himself to finding exactly the right ones to bet against. He'd read dozens of prospectuses and scoured hundreds more, looking for the dodgiest pools of mortgages, and was still pretty certain even then (and dead certain later) that he was the only human being on earth who read them, apart from the lawyers who drafted them (Lewis (2010)).

That is, a large group of traders was unaware of all of the risks associated with complex financial instruments. Other studies reinforce this view, finding that investors did not even contemplate the magnitude of the home price declines that materialized (Foote, Gerardi, and Willen (2012)). This paper takes a step toward modelling agents who "did not read the fine print"; they trade assets that they do not fully understand, leading to unintended default and other adverse consequences.

Appendix A: Proof of Lemma 1

To prove Lemma 1, first I define the ex ante budget set:

$$B_1^i(\delta, q, p) = \{(\hat{X}, y) \in \mathbb{R}_+^{L|\mathcal{S}^i|} \times \mathbb{R}^A \mid (5) \text{ holds and } (6) \text{ holds for each } s^i\}.$$

To deal with the non-compactness of the set, I define the compact, convex cube similar to Geanakoplos and Polemarchakis (1986):

$$Q^{in} = \Big\{(x, y) \in \mathbb{R}_+^{L|\mathcal{S}^i|} \times \mathbb{R}^A \,\Big\|\, \max_k\{|x_k|, |y_k|\} \le n\Big\}.$$

Their intersection $B_1^i(\delta, q, p) \cap Q^{in}$ is both convex and compact. For sufficiently large n, the intersection is nonempty. For each individual, I define the bounded ex ante optimality correspondence:

$$\big(\hat{X}(s^i), \Phi_1^{in}(\delta, q, p)\big) = \operatorname{argmax}_{\hat{X}(s^i), y} \sum_{s^i \in \mathcal{S}^i} \mu^i(s^i) \cdot u^i\big(\hat{X}(s^i)\big)$$

$$\text{such that } \big(\hat{X}(s^i), y\big) \in B_1^i(\delta, q, p) \cap Q^{in}.$$

Then define $\Phi_1^n(\delta, q, p) = (\Phi_1^{1n}(\delta, q, p), \dots, \Phi_1^{In}(\delta, q, p))$ for all individuals. Next I define the ex post budget set for each state s:

$$B_2^{is}\big(y^i, \delta, p\big) = \big\{x \in \mathbb{R}_+^L \mid (3) \text{ holds}\big\}.$$

Its intersection with a large enough cube Q^n of dimension $\mathbb{R}_+^L$, $B_2^{is}(y^i, \delta, p) \cap Q^n$, is convex, compact, and nonempty. For each individual and state, I define the bounded ex post optimality correspondence:

$$\Phi_2^{ins}\big(y^i, \delta, p\big) = \operatorname{argmax}_x u^i(x)$$

$$\text{such that } x \in B_2^{is}\big(y^i, \delta, p\big) \cap Q^n.$$

Define $\Phi_2^n(y, \delta, p) = (\Phi_2^{1n1}(y^1, \delta, p), \dots, \Phi_2^{1nS}(y^1, \delta, p), \dots, \Phi_2^{In1}(y^I, \delta, p), \dots, \Phi_2^{InS}(y^I, \delta, p))$ for all individuals and states, where $y = (y^1, \dots, y^I)$. The delivery function $\Phi_3(y, \delta, p)$ defined below will collapse to (1) in equilibrium. For each a, s,

$$\Phi_3^{as}(y, \delta, p) = \begin{cases} 1 \quad \text{if} \sum_i y_{-a}^i = 0 \\ \dfrac{1}{\sum_i y_{-a}^i} \sum_i y_{-a}^i \min\left\{1, \dfrac{p(s) \cdot e^i(s) + p_1(s) \sum_{\alpha=1}^{A} \delta_\alpha(s) r_\alpha(s) y_{+\alpha}^i}{p_1(s) \sum_{\alpha=1}^{A} r_\alpha(s) y_{-\alpha}^i}\right\} \\ \quad \text{if } 0 < \sum_i y_{-a}^i < \infty, \end{cases}$$

$p(s)$ will be restricted to the simplex, $e^i(s) > 0$ by Assumption 3, and asset returns are nonnegative by Assumption 4. Therefore, the numerator inside the minimum function is strictly positive, which makes $\Phi_3^{as}(\cdot)$ continuous at $\sum_i y_{-a}^i = 0$. Also notice that $0 < \Phi_3^{as}(\cdot) \leq 1$. While the open lower bound may seem problematic, the cubes Q^{in} implicitly define a closed lower bound for $\Phi_3^{as}(\cdot)$. Across all assets and states, define $\Phi_3(y, \delta, p) = (\Phi_3^{11}(y, \delta, p), \dots, \Phi_3^{A1}(y, \delta, p), \dots, \Phi_3^{1S}(y, \delta, p), \dots, \Phi_3^{AS}(y, \delta, p))$. Next I move to the market maker. The asset market maker solves

$$\Phi_4(y) = \operatorname{argmax}_{q \in \Delta^A} q \cdot \sum_i y^i,$$

where the simplex $\Delta^A = \{q \in \mathbb{R}^A_+ \mid \sum_a q_a = 1\}$. The last correspondence solves, for each state s, the problem of the commodity market maker (Mas Colell, Whinston, and Green (1995)),

$$\Phi_5^s(x) = \operatorname{argmax}_{p \in \Delta^L} p \cdot \left(\sum_i x^i(s) - \sum_i e^i(s) \right),$$

where $\Delta^L = \{p \in \mathbb{R}^L_+ \mid \sum_\ell p_\ell = 1\}$, and across states I have $\Phi_5(x) = (\Phi_5^1(x), \dots, \Phi_5^S(x))$. I define the vector-valued correspondence

$$\Phi^n(y, x, \delta, q, p) = \big(\Phi_1^n(\delta, q, p), \Phi_2^n(y, \delta, p), \Phi_3(y, \delta, p), \Phi_4(y), \Phi_5(x)\big),$$

which consists of the two utility maximization problems, the delivery problem, and the two market maker problems. By Assumption 1 and the maximum theorem, $\Phi_1^n(\cdot)$, $\Phi_2^n(\cdot)$, $\Phi_4(\cdot)$, and $\Phi_5(\cdot)$ are nonempty, upper hemi-continuous, and convex-valued. I am looking for fixed points of the form $(y_n, x_n, \delta_n, q_n, p_n) \in \Phi^n(y_n, x_n, \delta_n, q_n, p_n)$. I have already shown that (y, x, δ, q, p) lives in a nonempty, compact and convex subset of a Euclidean space. Then by Kakutani's theorem $\Phi^n(\cdot)$ has a fixed point.

Appendix B: Proof of Lemma 2

To prove Lemma 2, I begin with each individual's ex post budget constraint, which is nonempty for large enough n. I suppress notation on the state s for brevity. Summing over individuals,

$$\begin{aligned}
\sum_i p_n \cdot x_n^i &= \sum_i \max\{0, p_n \cdot e^i + p_{n1} c_n^i\} \\
&= \sum_i p_n \cdot e^i + \sum_i p_{n1} c_n^i - \sum_i 1_{\{w_n^i < 0\}} [p_n \cdot e^i + p_{n1} c_n^i] \\
&= \sum_i p_n \cdot e^i + \sum_i 1_{\{w_n^i \geq 0\}} p_{n1} c_n^i - \sum_i 1_{\{w_n^i < 0\}} p_n \cdot e^i,
\end{aligned}$$

where nominal wealth is given by $w_n^i = p_n \cdot e^i + p_{n1} c_n^i$, and real net asset position is given by $c_n^i = \sum_\alpha \delta_{n\alpha} r_\alpha y^i_{+n\alpha} - \sum_\alpha r_\alpha y^i_{-n\alpha}$. The goal is to show that the second and third terms on the right-hand side of the above expression sum to zero. The intuition is that money seized from the insolvent individuals must be paid out to the solvent ones; no wealth is lost. The net asset position c_n^i is a function of the delivery rate δ_{na} defined by $\Phi_3^{as}(\cdot)$ from Appendix A:

$$\delta_{na} = \frac{1}{\sum_i y^i_{-na}} \sum_i y^i_{-na} \min \left\{ 1, \frac{p_n \cdot e^i + p_{n1} \sum_{\alpha=1}^A \delta_{n\alpha} r_\alpha y^i_{+n\alpha}}{p_{n1} \sum_{\alpha=1}^A r_\alpha y^i_{-n\alpha}} \right\}$$

$$= \frac{1}{\sum_i y^i_{-na}} \left[\sum_i y^i_{-na} 1_{\{w^i_n \geq 0\}} + \sum_i y^i_{-na} 1_{\{w^i_n < 0\}} \frac{p_n \cdot e^i + p_{n1} \sum_{\alpha=1}^{A} \delta_{n\alpha} r_\alpha y^i_{+n\alpha}}{p_{n1} \sum_{\alpha=1}^{A} r_\alpha y^i_{-n\alpha}} \right].$$

Some algebra yields

$$\delta_{na} \sum_i y^i_{-na} - \sum_i y^i_{-na} 1_{\{w^i_n \geq 0\}} = \sum_i y^i_{-na} 1_{\{w^i_n < 0\}} \frac{p_n \cdot e^i + p_{n1} \sum_{\alpha=1}^{A} \delta_{n\alpha} r_\alpha y^i_{+n\alpha}}{p_{n1} \sum_{\alpha=1}^{A} r_\alpha y^i_{-n\alpha}}$$

$$\delta_{na} \sum_i y^i_{-na} - \sum_i y^i_{-na} = \sum_i y^i_{-na} 1_{\{w^i_n < 0\}} \frac{p_n \cdot e^i + p_{n1} c^i_n}{p_{n1} \sum_{\alpha=1}^{A} r_\alpha y^i_{-n\alpha}}$$

$$r_a \delta_{na} \sum_i y^i_{-na} - r_a \sum_i y^i_{-na} = r_a \sum_i y^i_{-na} 1_{\{w^i_n < 0\}} \frac{p_n \cdot e^i + p_{n1} c^i_n}{p_{n1} \sum_{\alpha=1}^{A} r_\alpha y^i_{-n\alpha}}.$$

If asset markets clear, I have, summing over all A assets, the equality

$$\sum_i \sum_a \delta_{na} r_a(s) y^i_{+na} - \sum_i \sum_a r_a y^i_{-na} = \sum_i \sum_a r_a y^i_{-na} 1_{\{w^i_n < 0\}} \frac{p_n \cdot e^i + p_{n1} c^i_n}{p_{n1} \sum_{\alpha=1}^{A} r_\alpha y^i_{-n\alpha}}$$

$$p_{n1} \sum_i c^i_n = \sum_i 1_{\{w^i_n < 0\}} [p_n \cdot e^i + p_{n1} c^i_n]$$

$$p_{n1} \sum_i 1_{\{w^i_n \geq 0\}} c^i_n = \sum_i 1_{\{w^i_n < 0\}} p_n \cdot e^i,$$

which is the desired equality. If asset markets only clear with inequality $\sum_i y^i_n \leq 0$, then the above equality becomes

$$p_{n1} \sum_i 1_{\{w^i_n \geq 0\}} c^i_n \leq \sum_i 1_{\{w^i_n < 0\}} p_n \cdot e^i,$$

which implies the desired inequality

$$\sum_i p_n \cdot x^i_n \leq \sum_i p_n \cdot e^i.$$

Appendix C: Proof of Lemma 3

To prove Lemma 3, first I show that Walras' law holds for assets. For large enough n, the ex ante budget set is nonempty. Then the period 1 budget constraint (5) implies, for each individual i,

$$\sum_{a=1}^{A} q_{na} y_{na}^i = 0.$$

Summing over individuals,

$$\sum_{i=1}^{I} \sum_{a=1}^{A} q_{na} y_{na}^i = 0.$$

I claim $\sum_i y_n^i \leq 0$, for if it were not, $\exists a$ such that $\sum_i y_{na}^i > 0$. Then I could define $q' = [0, \dots, 1, \dots, 0]$ with a 1 in the ath coordinate, which contradicts the optimality of $\Phi_4(\cdot)$ from Appendix A and Walras' law in assets. Next I argue $\sum_i x_n^i(s) - \sum_i e^i(s) \leq 0$ for all s. If this were not the case, there exists some s, ℓ for which $\sum_i x_{n\ell}^i(s) - \sum_i e_\ell^i(s) > 0$. But then I define $p' = [0, \dots, 0, 1, 0, \dots, 0]$ with a 1 in the ℓth coordinate, which can be shown to contradict the optimality of $\Phi_5(\cdot)$ from Appendix A and Walras' inequality in ex post commodities, which I can now invoke because $\sum_i y_n^i \leq 0$.

Because commodity demands are nonnegative, each individual demand can now be bounded $0 \leq x_n^i(s) \leq \sum_i e^i(s)$. Next I argue that $p_{n\ell}(s) > 0$ $\forall s, \ell$ and for large $n > \sup_{\ell,s} \sum_i e_\ell^i(s)$. Suppose otherwise. Then any individual i could take out the commodity position $x_n^i(s) + [0, \dots, 1, \dots, 0]$, which is inside the bounded budget set and yields higher ex post utility by Assumption 1, a contradiction to the optimality of $\Phi_2^{ins}(\cdot)$ from Appendix A. Next I argue that if ex post consumption $x_n^i(s)$ is bounded, so is ex ante perceived consumption $\hat{X}_n(s^i)$. To see why, I fix s^i and consider the perception over ex post budget constraints:

$$\begin{aligned}
&E_i\big[p_n(s^i) \cdot x_n^i(s^i)\big] \\
&\quad = E_i\left[1_{\{w_n^i(s) \geq 0\}} \left\{ p_n(s^i) \cdot e^i(s^i) + p_{n1}(s^i) \sum_{\alpha=1}^{A} \delta_{n\alpha}(s^i) r_\alpha(s^i) y_{+n\alpha}^i - p_{n1}(s^i) \sum_{\alpha=1}^{A} r_\alpha(s^i) y_{-n\alpha}^i \right\}\right] \\
&\quad \geq E_i\left[p_n(s^i) \cdot e^i(s^i) + p_{n1}(s^i) \sum_{\alpha=1}^{A} \delta_{n\alpha}(s^i) r_\alpha(s^i) y_{+n\alpha}^i - p_{n1}(s^i) \sum_{\alpha=1}^{A} r_\alpha(s^i) y_{-n\alpha}^i \right] \\
&\quad = E_i\big[p_n(s^i)\big] \cdot \hat{X}_n(s^i).
\end{aligned}$$

I then use the bound on ex post demand:

$$E_i\big[p_n(s^i)\big] \cdot \hat{X}_n(s^i) \leq \sup_{\{s,\, p \in \Delta^L\}} \sum_k p \cdot e^k(s).$$

Now there must exist some good ℓ such that $E_i[p_{n\ell}(s^i)] \geq \frac{1}{L}$ because prices live in the simplex. Then for good ℓ, I have my upper bound

$$\hat{X}_{n\ell}(s^i) \leq \sup_{\{s,\, p \in \Delta^L\}} \sum_k Lp \cdot e^k(s).$$

Going back to asset market clearing, I need to rule out $\sum_i y_n^i < 0$. If an asset price is positive $q_{na} > 0$, then I must have market clearing $\sum_i y_{na}^i = 0$ by Walras' law for assets. Say an asset price is zero, $q_{na} = 0$, yet that asset market does not clear $\sum_i y_{na}^i < 0$. There must exist some individual i who shorted the asset $y_{na}^i < 0$. By Assumption 4, assets pay out a strictly positive return in some state, so the asset position $[y_{n1}^i, \dots, 0, \dots, y_{nA}^i]$ with a 0 in the ath coordinate yields strictly more perceived income than y_n^i for some state s^i. This last statement requires positive numeraire commodity prices. Then by the argument above, for $n > \sup_{\{s,\, p \in \Delta^L\}} \sum_k Lp \cdot e^k(s)$, the additional perceived income can be used to buy more perceived good ℓ, contradicting the optimality of $\Phi_1^{in}(\cdot)$ from Appendix A, Assumption 1, and Assumption 2. Hence, asset markets must clear. If asset markets clear, I have Walras' law for ex post consumption by Lemma 2; therefore, ex post commodity markets clear $\sum_i x_n^i(s) = \sum_i e^i(s)$.

Appendix D: Proof of Lemma 4

To prove Lemma 4, say $\lim_{n \to \infty} p_{n1}(s) = 0$. By Walras' law and Assumption 3, I have

$$\sum_i p_n(s) \cdot x_n^i(s) = \sum_i p_n(s) \cdot e^i(s) \geq \inf_{s,\, p \in \Delta^L} \sum_i p \cdot e^i(s) > 0.$$

I call this lower bound K. I must have at least one individual i for whom $p_n(s) \cdot x_n^i(s) \geq \frac{K}{I}$. By Assumption 1, for that individual,

$$u^i\big(\big[1 - p_{n1}(s)\big]x_n^i(s) + p_n(s) \cdot x_n^i(s)[1, 0, \dots, 0]\big) > u^i\big(x_n^i(s)\big),$$

which holds for large enough n. However, the new bundle $([1 - p_{n1}(s)]x_n^i(s) + p_n(s) \cdot x_n^i(s)[1, 0, \dots, 0])$ is affordable and inside the required bounds for large $n > \sup_{\ell, s} \sum_i e_\ell^i(s) + \sup_{s,\, p \in \Delta^L} \sum_i p \cdot e^i(s)$, contradicting the optimality of $\Phi_2^{ins}(\cdot)$ from Appendix A. Let $\epsilon(i, x)$ implicitly solve for each individual:

$$u^i\left(\big[1 - \epsilon(i, x)\big]x + \frac{K}{I}[1, 0, \dots, 0]\right) = u^i(x).$$

Then I take $\epsilon = \inf_{\{i, 0 \leq x \leq \sup_s \sum_i e^i(s)\}} \epsilon(i, x)$.

Appendix E: Proof of Theorem 1

In this appendix, I prove Theorem 1. For each $n \in \mathbb{N}$, I can apply Lemma 1, resulting in a sequence of fixed points $(y_n, x_n, \delta_n, q_n, p_n)_{n \in \mathbb{N}}$. I have shown that the last four variables are always bounded independent of n or any other endogenous variables; therefore, I can extract a convergent subsequence with limit $(\overline{x}, \overline{\delta}, \overline{q}, \overline{p})$. These limits of subsequences will be my candidates for equilibrium.

Next I argue $R\sum_i y_n^i \leq 0$. If this were not the case, there would exist some row of the return matrix $r(s)$ for which $r(s)\sum_i y_n^i > 0$. However, then I define $q' = (q_n + r(s))/(1 + \sum_a r_a(s))$, which can be shown to contradict the optimality of $\Phi_4(\cdot)$ from Appendix A and Walras' law in assets. The difficult task is to find a lower bound for each Ry_n^i. The idea is as follows: individuals can go bankrupt in some state s, but the ex ante budget constraint helps put a bound on the degree of bankruptcy. Because of Assumption 2, for an individual to go strictly insolvent in some bad state $s \in \mathcal{B}^{in} \subseteq \mathcal{S}$, where $w_n^i(s) < 0$, there must be another good state $s' \in \mathcal{G}^{in} \subseteq \mathcal{S}$, where the individual is strictly solvent and $w_n^i(s') > 0$; otherwise, the perceived solvency problem (6) cannot be satisfied. For each s^i, perceptions in (6) require

$$\sum_{s \in (s^i \times \mathcal{S}^{-i})} \beta_i(s) w_n^i(s) \geq 0.$$

Recall that $\beta_i(s)$ represent the potentially incorrect, yet full support, probabilities for agent i. Here I derive my lower bound for wealth in bad states, $w_n^i(s)$, where $s \in \mathcal{B}^{in}$:

$$0 \leq \sum_{s' \in (s^i \times \mathcal{S}^{-i})} \beta_i(s') w_n^i(s') \leq \beta_i(s) w_n^i(s) + \sum_{s' \in (s^i \times \mathcal{S}^{-i}) \cap \mathcal{G}^{in}} \beta_i(s') w_n^i(s').$$

During solvent states, individuals use their wealth on consumption:

$$-\sum_{s' \in (s^i \times \mathcal{S}^{-i}) \cap \mathcal{G}^{in}} \frac{\beta_i(s')}{\beta_i(s)} p_n(s') \cdot x_n^i(s') \leq w_n^i(s).$$

By Assumption 2, this expression is well defined. Using my bound on $x_n^i(s)$,

$$-\sum_{s' \in (s^i \times \mathcal{S}^{-i}) \cap \mathcal{G}^{in}} \frac{\beta_i(s')}{\beta_i(s)} p_n(s') \cdot \sum_i e^i(s') \leq w_n^i(s).$$

To lose the dependence on good and bad states, I define $\beta_{\max} = \max_{i,s} \beta_i(s)$ and $\beta_{\min} = \min_{i,s} \beta_i(s)$:

$$-\sum_{s' \in \mathcal{S}} \frac{\beta_{\max}}{\beta_{\min}} p_n(s') \cdot \sum_i e^i(s') \leq w_n^i(s).$$

The same argument holds for any s, s^i, and i. Taking the argument one step further, I apply the definition of nominal wealth $w_n^i(s)$ and take the infimum:

$$\begin{aligned} &\inf_{p \in \Delta^L} \left[-\sum_{s' \in \mathcal{S}} \frac{\beta_{\max}}{\beta_{\min}} p \cdot \sum_i e^i(s') - p \cdot e^i(s) \right] \\ &\quad \leq p_{n1}(s) \sum_{\alpha=1}^{A} \delta_{n\alpha}(s) r_\alpha(s) y_{+n\alpha}^i - p_{n1}(s) \sum_{\alpha=1}^{A} r_\alpha(s) y_{-n\alpha}^i. \end{aligned}$$

I call the infimum of the left-hand side over all s, i the lower bound $-L$. While further losing some tightness of the bound, I have

$$-L \leq p_{n1}(s) \sum_{\alpha=1}^{A} r_\alpha(s) y^i_{na},$$

which holds for all individuals i and states s. By Lemma 4, $p_{n1}(s) \geq \epsilon$ for all s, so

$$-\frac{L}{\epsilon} \leq R y^i_n.$$

With an upper and a lower bound, I can finally bound each individual's asset returns:

$$-\frac{L}{\epsilon} \leq R y^i_n \leq (I-1)\frac{L}{\epsilon}.$$

I consider $(Ry^i_n)_{n\in\mathbb{N}}$ as a bounded sequence for each i and extract a convergent subsequence. Call the limit point $k^i \in \mathbb{R}^S$. By Assumption 4, R has a left inverse, so I let $\overline{y}^i = R^{-1}k^i$.[14] Since $Ry^i_n \to k^i$, I have $R^{-1}Ry^i_n \to R^{-1}k^i$ or $y^i_n \to \overline{y}^i$. Notice that conditions (3)–(6) are satisfied by construction. Say an asset market did not clear in the limit, $\sum_i \overline{y}^i = [0, \dots, \pm\epsilon, \dots, 0]$. By the definition of convergence, for every ϵ there exists N such that for $n \geq N$, $\| \sum_i \overline{y}^i - \sum_i y^i_n \|_2 < \epsilon$. However, it was shown in Lemma 3 that asset markets clear for the n-bounded problem, so I have

$$\left\| \sum_i \overline{y}^i \right\|_2 < \epsilon,$$

which contradicts the original statement. The same argument applies to commodity markets, and finally $\Phi^{as}_3(\cdot)$ from Appendix A collapses to (1) if asset markets clear.

Appendix F: Proof of Corollary 1

To prove Corollary 1, reapply Lemmas 1–4 and Theorem 1 with the following three adjustments. In Lemma 1, the ex ante budget set changes to

$$B^i_1(\delta, q, p) = \left\{(\hat{X}, y) \in \mathbb{R}^{L|\mathcal{S}^i|}_+ \times \mathbb{R}^A \mid (5) \text{ holds and } (8) \text{ holds for each } s^i\right\}.$$

In Lemma 3, I must adjust the argument for bounding $\hat{X}_n(s^i)$. Fix s^i and consider the perceived ex post budget constraint:

$$\begin{aligned} &E_i\big[p_n(s^i) \cdot x^i_n(s^i)\big] \\ &\quad = E_i\left[1_{\{w^i_n(s) \geq 0\}} \left\{ p_n(s^i) \cdot e^i(s^i) + p_{n1}(s^i) \sum_{\alpha=1}^{A} \delta_{n\alpha}(s^i) r_\alpha(s^i) y^i_{+n\alpha} - p_{n1}(s^i) \sum_{\alpha=1}^{A} r_\alpha(s^i) y^i_{-n\alpha} \right\}\right] \end{aligned}$$

[14] To define a unique left inverse, use the Moore–Penrose inverse R^+.

$$\geq E_i\left[p_n(s^i)\cdot e^i(s^i)+p_{n1}(s^i)\sum_{\alpha=1}^{A}\delta_{n\alpha}(s^i)r_\alpha(s^i)y^i_{+n\alpha}-p_{n1}(s^i)\sum_{\alpha=1}^{A}r_\alpha(s^i)y^i_{-n\alpha}\right]$$
$$=\hat{p}(s^i)\cdot\hat{X}_n(s^i)+E_i\big[p_n(s^i)\cdot e^i(s^i)\big]-\hat{p}(s^i)\cdot E_i\big[e^i(s^i)\big].$$

I then use the bound on ex post demand:

$$\hat{p}(s^i)\cdot\hat{X}_n(s^i)\leq\sup_{\{s,j,p\in\Delta^L\}}\sum_k p\cdot e^k(s)+\hat{p}(s^j)\cdot e^j(s).$$

Then for any good ℓ, I have an upper bound

$$\hat{X}_{n\ell}(s^i)\leq\frac{1}{\hat{p}_\ell(s^i)}\left[\sup_{\{s,j,p\in\Delta^L\}}\sum_k p\cdot e^k(s)+\hat{p}(s^j)\cdot e^j(s)\right],$$

which is a well defined bound because $\hat{p}_\ell(s^i)>0$. In Theorem 1, I must adjust the argument for bounding perceived wealth. For each s^i, (8) requires

$$\sum_{s\in(s^i\times\mathcal{S}^{-i})}\beta_i(s)w^i_n(s)\underbrace{-E_i\big[p_n(s^i)\cdot e^i(s^i)\big]+\hat{p}(s^i)\cdot E_i\big[e^i(s^i)\big]}_{\text{error}}\geq 0.$$

Recall that $\beta_i(s)$ represents the potentially incorrect, yet full support, probabilities for agent i. Note that I can bound my new error term,

$$-E_i\big[p_n(s^i)\cdot e^i(s^i)\big]+\hat{p}(s^i)\cdot E_i\big[e^i(s^i)\big]\leq\sup_{j,s}\hat{p}(s^j)\cdot e^j(s),$$

and I call the right-hand side of the above inequality E. Next I derive my lower bound for wealth in bad states, $w^i_n(s)$ where $s\in\mathcal{B}^{in}$:

$$-E\leq\sum_{s'\in(s^i\times\mathcal{S}^{-i})}\beta_i(s')w^i_n(s')\leq\beta_i(s)w^i_n(s)+\sum_{s'\in(s^i\times\mathcal{S}^{-i})\cap\mathcal{G}^{in}}\beta_i(s')w^i_n(s').$$

During solvent states, individuals use their wealth on consumption:

$$-\frac{E}{\beta_i(s)}-\sum_{s'\in(s^i\times\mathcal{S}^{-i})\cap\mathcal{G}^{in}}\frac{\beta_i(s')}{\beta_i(s)}p_n(s')\cdot x^i_n(s')\leq w^i_n(s).$$

By Assumption 2, this expression is well defined. Using my bound on $x^i_n(s)$,

$$-\frac{E}{\beta_i(s)}-\sum_{s'\in(s^i\times\mathcal{S}^{-i})\cap\mathcal{G}^{in}}\frac{\beta_i(s')}{\beta_i(s)}p_n(s')\cdot\sum_i e^i(s')\leq w^i_n(s).$$

To lose the dependence on good and bad states, I define $\beta_{\max}=\max_{i,s}\beta_i(s)$ and $\beta_{\min}=\min_{i,s}\beta_i(s)$:

$$-\frac{E}{\beta_{\min}}-\sum_{s'\in\mathcal{S}}\frac{\beta_{\max}}{\beta_{\min}}p_n(s')\cdot\sum_i e^i(s')\leq w^i_n(s).$$

The same argument holds for any s, s^i, and i. Taking the argument one step further, I apply the definition of nominal wealth $w_n^i(s)$ and take the infimum:

$$\inf_{p\in\Delta^L}\left[-\frac{E}{\beta_{\min}}-\sum_{s'\in\mathcal{S}}\frac{\beta_{\max}}{\beta_{\min}}p\cdot\sum_i e^i(s')-p\cdot e^i(s)\right]$$
$$\leq p_{n1}(s)\sum_{\alpha=1}^{A}\delta_{n\alpha}(s)r_\alpha(s)y_{+n\alpha}^i-p_{n1}(s)\sum_{\alpha=1}^{A}r_\alpha(s)y_{-n\alpha}^i.$$

I call the infimum of the left-hand side over all s, i the lower bound $-L$.

Appendix G: Proof of Theorem 2

To prove Theorem 2, let $w^i(s,\delta(s))$ denote the nominal wealth of individual i in state s and with delivery rates $\delta(s)$. I implicitly define the reallocation $\tilde{y}$ such that

$$\tilde{w}^i(s,1)=\begin{cases}w^i(s,\delta(s)) & \text{if } w^i(s,\delta(s))\geq 0\\ 0 & \text{otherwise.}\end{cases}$$

In words, I have removed all strict default, so delivery rates will be perfect. By construction, ex post wealth levels do not change; hence, neither ex post allocations $\tilde{x}$ nor prices $\tilde{p}$ change. Under $\tilde{y}$ there is higher ex ante perceived income

$$E_i[\tilde{w}^i(s,1)]=E_i[1_{\{w^i(s,\delta(s))\geq 0\}}w^i(s,\delta(s))]\geq E_i[w^i(s,\delta(s))],$$

where the inequality is strict when there is strict default. The existence of such a $\tilde{y}$ follows from Assumption 6. It remains to be shown that $\tilde{y}$ is an allocation. I start with the definition of $\tilde{w}^i(s,1)$, summed over individuals,

$$\begin{aligned}&\sum_i\left[p(s)\cdot e^i(s)+p_1(s)\sum_a r_a(s)\tilde{y}_a^i\right]\\&=\sum_i\left[p(s)\cdot e^i(s)+p_1(s)\sum_a\delta_a(s)r_a(s)y_{+a}^i-p_1(s)\sum_a r_a(s)y_{-a}^i\right]1_{\{w^i(s)\geq 0\}}\\&=\sum_i p(s)\cdot e^i(s)1_{\{w^i(s)\geq 0\}}+\sum_i p(s)\cdot e^i(s)1_{\{w^i(s)<0\}}\\&=\sum_i p(s)\cdot e^i(s),\end{aligned}$$

where the second equality follows from arguments laid out in Appendix B. I then have, for each s,

$$p_1(s)r(s)\cdot\sum_i\tilde{y}^i=0.$$

First, prices are nonzero. Second, I can write in matrix form

$$R\sum_i \tilde{y}^i = 0.$$

By Assumption 6, the matrix R has an inverse and the proof is complete.

Appendix H: Proof of Theorem 3

To prove Theorem 3, first imagine an economy with perturbed endowments equal to the convex combination of the desired allocation:

$$\Omega^i(s) = \sum_{s'} \mu(s')\bar{x}^i(s') \quad \forall i, s.$$

By Assumption 7, the allocation is feasible. It is Pareto efficient in reality by concavity (Assumption 1) and must provide the same utility as $\bar{x}^i(s)$ by the original assumption of Pareto optimality of $\bar{x}^i(s)$:

$$\sum_s \mu(s)u^i\big(\bar{x}^i(s)\big) = u^i\bigg(\sum_s \mu(s)\bar{x}^i(s)\bigg) \quad \forall i. \tag{9}$$

Next I claim that $\Omega^i(s)$ is also Pareto efficient for each state individually. If it were not, there would exist a Pareto improvement $z^i(s')$ in some state s'. However, by Assumption 7, $z^i(s')$ is a feasible allocation for every state, contradicting the original optimality of $\Omega^i(s)$. Therefore, it is Pareto efficient per state and I could hypothetically apply the standard Second Fundamental Welfare Theorem, which requires Assumption 1 and $\bar{x} > 0$, to conclude that there exist price vectors $p(s)$ that support no trade as an ex post equilibrium. Explicitly, for each i, s,

$$\Omega^i(s) \in \operatorname{argmax} u^i(x) \quad \text{s.t. } p \cdot x = p \cdot \Omega^i(s). \tag{10}$$

Prices are equated across states because the problem is identical across states. This hypothetical experiment is not my true construction, but (9) and (10) will become useful later on. To begin my construction, I set the perturbed endowments $\omega^i(s) = \bar{x}^i(s)$ for all i, s. The goal is to explicitly show that conditions (1)–(7) are satisfied for my constructed equilibrium. I begin with ex post commodity allocations, asset positions, and delivery rates: $x^i(s) = \bar{x}^i(s)\ \forall i, s$, $y^i = 0\ \forall i$, and $\delta_a(s) = 1\ \forall a, s$. Conditions (1), (2), and (7) are immediately satisfied. Next, given $y^i = 0\ \forall i$, I must show ex post optimality (3) for some price vector $p(s)$. The price vector I choose is the one from (10): $p(s) = p\ \forall s$. I claim that no trade is optimal for every i, s:

$$\bar{x}^i(s) \in \operatorname{argmax} u^i(x) \quad \text{such that } p \cdot x = p \cdot \omega^i(s).$$

If this were not the case, there would exist i, s' where $u^i(z^i(s')) > u^i(\bar{x}^i(s'))$. However, I could then take the convex combination of $\bar{x}^i(s)$, replacing $z^i(s')$ in state s' to derive a contradiction to (10),

$$u^i\left(\mu(s')z^i(s') + \sum_{s\neq s'} \mu(s)\bar{x}^i(s)\right) \geq \mu(s')u^i\big(z^i(s')\big) + \sum_{s\neq s'} \mu(s)u^i\big(\bar{x}^i(s)\big)$$

$$> \sum_s \mu(s)u^i\big(\bar{x}^i(s)\big)$$

$$= u^i\left(\sum_s \mu(s)\bar{x}^i(s)\right),$$

where the strict inequality follows from Assumption 2, and the last equality follows from (9). The allocation is affordable under the budget constraint of (10):

$$p\cdot\left(\mu(s')z^i(s') + \sum_{s\neq s'} \mu(s)\bar{x}^i(s)\right) = p\cdot\left(\mu(s')\omega^i(s') + \sum_{s\neq s'} \mu(s)\bar{x}^i(s)\right)$$

$$= p\cdot\sum_s \mu(s)\bar{x}^i(s).$$

Hence, (3) holds. The last step is showing ex ante optimality of $y^i = 0\ \forall i$ in (4)–(6) for some q, given $p(s) = p(s')$ and $\delta_a(s) = 1$. I define

$$\hat{X}^i(y) \in \operatorname{argmax}_{\hat{X}(s^i)} \sum_{s^i} \mu^i(s^i)u^i\big(\hat{X}(s^i)\big)$$

$$\text{such that } p\cdot\hat{X}(s^i) = p\cdot E_i\big[\omega^i(s^i)\big] + p_1E_i\big[r(s^i)\big]\cdot y^i \quad \forall s^i \text{ and } q\cdot y^i = 0,$$

which is the ex ante problem, incorporating perfect deliveries and constant prices across states. Take any other $\tilde{y}\neq 0$. Then if the following inequalities hold $\forall i$, my proof is complete:

$$\sum_{s^i} \mu^i(s^i)u^i\big(\hat{X}^i(\tilde{y})\big) \leq u^i\left(\sum_{s^i} \mu^i(s^i)\hat{X}^i(\tilde{y})\right)$$

$$\leq \sum_{s^i} \mu^i(s^i)u^i\big(\hat{X}^i(0)\big).$$

The first inequality is by concavity (Assumption 1). For the second inequality, say otherwise; i.e.,

$$u^i\left(\sum_{s^i} \mu^i(s^i)\hat{X}^i(\tilde{y})\right) > \sum_{s^i} \mu^i(s^i)u^i\big(\hat{X}^i(0)\big)$$

$$\geq \sum_{s^i} \mu^i(s^i)u^i\big(E_i[\bar{x}^i(s^i)]\big)$$

$$\geq \sum_s \mu(s)u^i\big(\bar{x}^i(s)\big)$$

$$= u^i\left(\sum_s \mu(s)\bar{x}^i(s)\right),$$

where the third inequality is by concavity (Assumption 1) and the last equality is again by (9). Potentially looking for a contradiction with (10), I examine the budget constraints. For each s^i,

$$
\begin{aligned}
p \cdot \sum_{s^i} \mu^i(s^i)\hat{X}^i(\tilde{y}) &= \sum_{s^i} \mu^i(s^i) p \cdot \hat{X}^i(\tilde{y}) \\
&= \sum_{s^i} \mu^i(s^i)\big(p \cdot E_i\big[\omega^i(s^i)\big] + p_1 E_i\big[r(s^i)\big] \cdot \tilde{y}^i\big) \\
&= p \cdot E\big[\omega^i(s)\big] + p_1 E\big[r(s)\big] \cdot \tilde{y}^i.
\end{aligned}
$$

Finally I let $q = p_1 E[r(s)]$, much like the no-arbitrage discussion from Section 4. Then $p_1 E[r(s)] \cdot \tilde{y}^i = 0\ \forall i$, so I conclude that the new allocation was affordable under (10) and the proof is complete.

Appendix I: Necessity of Assumption 7

I use the next example to informally argue the necessity of condition Assumption 7 in Theorem 3. Two agents have the aggregate endowment of a single commodity:

$$
e^S + e^L = \begin{bmatrix} 2 & 4 \end{bmatrix}.
$$

The small world agent cannot distinguish between the two equally likely states, while the large world agent can perfectly distinguish between the two. There are two Arrow assets. A Pareto efficient allocation that is not implementable is

$$
\overline{x}^S = \overline{x}^L = \begin{bmatrix} 1 & 2 \end{bmatrix},
$$

which is an equal split of the aggregate endowment. The small world agent sees her endowment as the singleton average of the two state endowments. If there is default, the desired Pareto optimum cannot be implemented. So assume there is no default. Then the small world agent is weakly willing to accept any asset trade at no-arbitrage asset prices, which must be equated when there is no default. At these fair prices, the large world agent will always trade to equate allocations across states if they are strictly risk averse, making the given Pareto allocation impossible to implement.

Appendix J: Proof of Corollary 2

To prove Corollary 2, let $p(s)$ denote ex post equilibrium prices for a hypothetical riskless exchange economy with endowments $E[e^i(s)] > 0$. Note that $p(s) = p(s')$ because these new endowments are constant across states; for brevity, I suppress dependence on s. Let $x^i(s) = x^i(s')$ denote the ex post allocation; that is,

$$
x^i(s) \in \operatorname{argmax} u^i(x) \quad \text{such that } p \cdot x = p \cdot E\big[e^i(s)\big]. \tag{11}
$$

Let $\delta_a(s) = 1$ for all a, s. Now fix i and implicitly define asset positions y^i to be the ones that satisfy, for all s,

$$p \cdot E\big[e^i(s)\big] = p \cdot e^i(s) + p_1 \sum_\alpha r_\alpha(s) y^i_a, \tag{12}$$

so that asset positions are being used to "construct" the hypothetical exchange economy. Such a y^i exists by Assumption 6. To see that y^i is indeed an allocation, I take the sum of (12) over all i,

$$p \cdot \sum_i E\big[e^i(s)\big] = p \cdot \sum_i e^i(s) + p_1 \sum_\alpha r_\alpha(s) \sum_i y^i_a,$$

and the first two terms cancel by Assumption 7. I rewrite the remaining condition in matrix form,

$$R \sum_i y^i = 0,$$

which requires that numeraire commodity prices are nonzero. By Assumption 6, the matrix R has an inverse and so asset markets clear. Let $q = p_1 E[r(s)]$ as in Theorem 3. To see that ex ante budgets are satisfied, I take an expectation of (12),

$$p \cdot E\big[e^i(s)\big] = p \cdot E\big[e^i(s)\big] + p_1 \sum_\alpha E\big[r_\alpha(s)\big] y^i_a,$$

which simplifies to

$$q \cdot y^i = 0.$$

The final condition that must be verified is the optimality of assets y^i. Arguments are similar to those provided in the proof of Theorem 3. I define, as a function of z,

$$\hat{X}^i(z) \in \operatorname{argmax}_{\hat{X}(s^i)} \sum_{s^i} \mu^i(s^i) u^i\big(\hat{X}(s^i)\big)$$
$$\text{such that } p \cdot \hat{X}(s^i) = p \cdot E_i\big[e^i(s^i)\big] + p_1 E_i\big[r(s^i)\big] \cdot z^i \quad \forall s^i \text{ and } q \cdot z^i = 0,$$

and I claim that, for any $\tilde{y}$,

$$\sum_{s^i} \mu^i(s^i) u^i\big(\hat{X}^i(y)\big) \geq \sum_{s^i} \mu^i(s^i) u^i\big(\hat{X}^i(\tilde{y})\big).$$

Say otherwise; that is,

$$\begin{aligned}\sum_{s^i} \mu^i(s^i) u^i\big(\hat{X}^i(y)\big) &< \sum_{s^i} \mu^i(s^i) u^i\big(\hat{X}^i(\tilde{y})\big) \\ &\leq u^i\Big(\sum_{s^i} \mu^i(s^i) \hat{X}^i(\tilde{y})\Big),\end{aligned}$$

where the weak inequality follows from concavity. Notice that because ex post wealth levels and prices are equated across states s, perceived wealth levels and perceived prices must be equated across perceived states s^i. Hence, $\hat{X}^i(y)$ can be selected so it is constant across states:

$$\sum_{s^i} \mu^i(s^i) u^i(\hat{X}^i(y)) = u^i(\hat{X}^i(y)) = u^i(x^i(s)).$$

The second equality follows from the fact that the ex ante and ex post problems are now identical. Combining the inequalities with equalities,

$$u^i(x^i(s)) < u^i\left(\sum_{s^i} \mu^i(s^i) \hat{X}^i(\tilde{y})\right),$$

which can be shown to contradict the optimality of (11). To see this, I show that this preferred allocation is affordable,

$$\begin{aligned} p \cdot \sum_{s^i} \mu^i(s^i) \hat{X}^i(\tilde{y}) &= \sum_{s^i} \mu^i(s^i) p \cdot \hat{X}^i(\tilde{y}) \\ &= \sum_{s^i} \mu^i(s^i) \left[p \cdot E_i[e^i(s^i)] + p_1 E_i[r(s^i)] \cdot \tilde{y}^i \right] \\ &= p \cdot E[e^i(s)], \end{aligned}$$

where the last equality follows from the ex ante budget constraint, $q \cdot \tilde{y}^i = 0$. This completes the proof.

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