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Generalized compensation principle

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Economic disruptions generally create winners and losers. The compensation problem consists of designing a reform of the existing income tax system that offsets the welfare losses of the latter by redistributing the gains of the former. We derive a formula for the compensating tax reform and its impact on the government budget when only distortionary tax instruments are available and wages are determined endogenously in general equilibrium. We apply this result to the compensation of robotization in the United States.

Keywords. Compensation principle, distortionary taxation, general equilibrium, wage disruption.

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Introduction

Economic disruptions, for instance, technological change, opening to international trade, inflows of immigration, or exogenous price shocks, generally create winners and losers, i.e., real wage and welfare gains for some individuals and welfare losses for others. The welfare compensation problem consists of designing a reform of the tax-and-transfer system that offsets the losses by redistributing the winners' gains. We solve this

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problem in an environment where only distortionary taxes are available, and wages are determined endogenously in general equilibrium.

The traditional public finance literature (Kaldor (1939), Hicks (1939, 1940)) shows that in an economy where individualized lump-sum taxes are available, the tax reform that redistributes the welfare gains and losses caused by a disruption is straightforward: It simply consists of raising (resp., lowering) the lump-sum tax liability of agents whose welfare increases (resp., decreases) from the shock by an amount equal to their compensating variation. This standard Kaldor–Hicks approach is flawed, however. First, because of asymmetric information, as in Mirrlees (1971), the only tax instrument at the government's disposal, the labor income tax, is distortionary. Second, many economic shocks require explicitly modeling the endogeneity of wages.

Consider, for example, an inflow of low-skilled immigration, i.e., an exogenous (relative) increase in the total supply of low-skilled labor. In partial equilibrium, i.e., if wages were exogenous, this would not affect the individual utility of resident workers. However, in general equilibrium, this disruption lowers the wage of low-skilled workers whose marginal product of labor is decreasing and raises the wage of high-skilled workers whose labor is complementary to the tasks performed by the incoming workers; see, e.g., Card (2009). Therefore, immigration flows have nontrivial welfare consequences only because the endogeneity of wages is explicitly taken into account. Similarly, the impact of automation on inequality can be understood as a race between education—the supply of high-skilled workers—and technology; see, e.g., Katz and Murphy (1992). In both of these examples, movements in the relative *labor supplies* of different skills fundamentally drive trends in relative wages. As a result, standard public finance models in which labor supply is endogenous but wages are exogenous cannot properly account for the welfare implications of these disruptions.

Now suppose that in response to the disruption, the government implements a tax reform that aims to compensate the welfare losses of agents whose wages are adversely impacted. Since the only available policy tools are distortionary taxes, such a reform affects workers' labor supply choices. These labor supply adjustments impact individuals' wages and utility by the same general equilibrium forces we just described. The resulting welfare effects themselves need to be accounted for and compensated. But this can only be done through the distortionary tax code, which creates further welfare gains and losses, and so on. Hence the combination of distortionary taxes and endogenous wages leads to an a priori complex fixed point problem for the compensating tax reform.

We start by analyzing the welfare compensation problem in a partial-equilibrium environment where wages are exogenous. We show that the design of the compensating tax reform that brings every agent's utility back to its pre-disruption level is simple, even when distortionary income taxes are the only available instrument. The key insight here is that individual utility is only affected by the *average* tax rates of the reform; that is, the changes in *marginal* tax rates do not impact welfare. This follows from an envelope theorem argument: The marginal tax rate that individuals face affects their indirect utility only through their optimal labor supply decision so that the corresponding welfare effect is second order. As a consequence, it is straightforward to show that a suitably designed adjustment in the average tax rate is sufficient to achieve exact welfare compensation. Namely, one that exactly cancels out the after-tax income gain or loss caused by the exogenous disruption, regardless of the marginal tax rate changes it induces.

The analysis becomes significantly more complex when distortionary taxes are coupled with general-equilibrium forces. In this case, despite the envelope theorem, endogenous changes in labor supply do matter for welfare through their impact on wages, resulting from the decreasing marginal productivities and the production complementarities. Therefore, in general equilibrium, because of the labor supply responses that they generate, the tax reform's marginal rates directly affect the agent's utility, even conditional on the average tax rate change. As a result, to determine the compensating tax reform, we must solve for its average and marginal rates simultaneously. This is the key difference from the partial-equilibrium environment and the main technical challenge of our paper. We show that the solution to the welfare compensation problem can be formalized as the solution to an integro-differential equation.

Our first main result is to derive a formula for the compensating tax reform in general equilibrium in terms of elasticity variables that can be measured empirically. This formula is valid for arbitrary preferences, initial tax code, production function, and wage disruptions as long as they are marginal; that is, our tax reform compensates for the first-order welfare effects caused by general disruptions. Our second main result is to derive a formula for the fiscal surplus (or deficit), i.e., the impact of the disruption and its compensation on the government budget. Thus, our analysis generalizes the traditional Kaldor–Hicks criterion and provides a simple test to determine whether economic shocks or policies are *compensable*, that is, whether offsetting the individual welfare changes using only distortionary tax instruments is budget-feasible. More generally, the value of the fiscal surplus (not only its sign) provides a relevant monetary measure of the aggregate welfare gains or losses from the disruption.

The main economic insight of our general-equilibrium compensation formula is that whenever the exogenous disruption features a sharp nonlinearity around some income level (say, a large wage drop), the compensating policy smoothes out the distortions by spreading the tax rebates over the entire range of incomes *below* that level. More specifically, exact compensation is achieved via a *progressive* tax reform over that range of incomes, with monotonic reductions in marginal and average tax rates. The rate of progressivity of the compensating tax reform—i.e., how fast the average tax rate grows with income—is given by the ratio between the labor demand and labor supply elasticities, net of the rate of progressivity of the initial tax code. These results stand in contrast to the partial-equilibrium compensation, which tracks the nonlinearities of the wage disruption one-for-one.

To understand this result and derive further analytical properties of the compensation, we apply our formula to several simple disruptions. We assume that the production function is constant elasticity of substitution (CES) and consider first a disruption that affects all wages uniformly. In this case, the compensating tax reform in general equilibrium coincides with the partial-equilibrium compensation. This follows from the fact that the endogenous wage responses caused by the decreasing marginal product of labor and the skill complementarities in production exactly offset each other—a consequence of Euler's homogeneous function theorem—thus removing the need to adjust the partial-equilibrium policy.

Now consider the polar opposite case, where a single skill is adversely affected by the disruption, thus creating a sharp nonlinearity in wage losses. In partial equilibrium, the compensation would grant a large tax rebate to the corresponding income level. However, doing so would involve large movements in the *marginal* tax rates around that income level, which would cause sizeable unintended welfare consequences in general equilibrium. Instead, to offset these welfare effects from wage responses, the appropriate policy smoothes the tax changes by progressively reducing the tax liabilities of all incomes *below* that of the disrupted agent. When the marginal product of labor is decreasing, the tax reform must ensure that the (negative) welfare effects caused by a reduction in any worker's marginal tax rate are offset by the (positive) welfare effects of reducing her average tax rate. If, as empirically relevant, the ratio of the elasticities of labor demand and labor supply is larger than the rate of progressivity of the preexisting tax code, the reduction in the marginal tax rate at each income level must be compensated by an even larger reduction in the average tax rate. Thus, the tax rebates on earnings below that of the disrupted worker are exponentially growing, i.e., progressive.

Next, skill complementarities in production generate additional indirect wage adjustments that also need to be compensated. The marginal tax rates of this second round of compensation cause, in turn, further wage and welfare changes, which themselves require compensation, and so on. We generally solve this fixed point problem by defining inductively a sequence of functions that each capture a round of general-equilibrium wage changes and their compensation. In other words, when the shock hits, we adjust the tax schedule to compensate for it, ignoring production complementarities. We then compute the first round of general equilibrium effects on wages, compensate for them again, and so forth until convergence. If the production function is CES, this series boils down to a uniform shift of the marginal tax rates, adding to the progressive component described in the previous paragraph.

We then apply our compensating formula, under a CES technology, to disruptions that affect all incomes uniformly above (or below) a threshold or over an interior range of incomes. We show that one can analytically decompose the compensation of such disruptions into the sum of three elements: first, the partial-equilibrium reform that tracks income gains and losses one for one; second, a correction for the decreasing marginal product of labor that features progressively growing tax changes—at a rate given by the simple combination of elasticities described above—on all incomes below each sharp nonlinearity in wage gains and losses; third, a correction for the cross-wage complementarities that amounts to a uniform shift in tax rates. We then quantitatively explore the robustness of the compensating tax reform to the size of the labor supply and demand elasticities, the initial tax schedule, and the (nonmarginal) size of the disruption. We show, in particular, that our tax reform compensates for at least 95% (resp., 78%, 53%) of the welfare losses of a disruption that leads to 1% (resp., 5%, 10%) wage losses.

We finally apply our theory in the context of the robotization of the U.S. economy between 1990 and 2007. Acemoglu and Restrepo (2020) estimate the impact of an additional robot per one thousand workers on the wages of different skills—roughly the amount observed in the United States between these dates. The closed-form solution we derive allows us to easily evaluate the compensating reform quantitatively. For instance, we find that an additional robot per thousand workers requires compensating agents at the 10th (resp., 85th) percentile of the wage distribution by 97% of their income loss (resp., 132% of the income gain) from the disruption. This represents a 0.7 percentage point (resp., 0.08 percentage point (pp)) decrease in their average tax rate and generates a $145 budget deficit for the government.

Related literature Our theoretical analysis builds on Kaplow (2004, 2012) and Hendren (2020), who extend the Kaldor–Hicks principle to the case of distortionary taxes in partial equilibrium using inverse-optimum weights (see, e.g., Jacobs, Jongen, and Zoutman (2017)). Our main contribution is the analysis of the general equilibrium environment in which wages are endogenous. Guesnerie (1998), Itskhoki (2008), and Antras, de Gortari, and Itskhoki (2016) study compensating tax reforms and the welfare implications of trade liberalization in a general-equilibrium framework similar to ours. They restrict the analysis to specific classes of distortionary taxes and tax reforms, however: linear for Guesnerie (1998) and with a constant rate of progressivity (as in Bénabou (2002), Heathcote, Storesletten, and Violante (2017)) for Antras, de Gortari, and Itskhoki (2016). While we do not consider a sophisticated trade model, we solve the compensation problem by allowing for arbitrarily nonlinear tax schedules and nonlinear tax reforms. The generality of the tax reforms, in particular, is necessary to ensure that every agent's welfare is compensated for. Andersen and Bhattacharya (2017, 2020) and Andersen, Bhattacharya, and Liu (2020) extend the Kaldor–Hicks approach to dynamic overlapping generations (OLG) settings; they focus on achieving generation-by-generation Pareto neutrality via taxation and debt, and do not consider intra-generational heterogeneity. More broadly, our model is within the class of Mirrleesian economies in general equilibrium. Stiglitz (1982), Rothschild and Scheuer (2013), and Sachs, Tsyvinski, and Werquin (2020) study optimal taxes in this environment for given production and social welfare functions. Ales, Kurnaz, and Sleet (2015), Guerreiro, Rebelo, and Teles (2017), Uwe (2018), Costinot and Werning (2018), Hosseini and Shourideh (2018), Beraja and Zorzi (2021) characterize optimal income taxes, robot taxation, or trade policies following disruptions. Costinot and Werning (2018), in particular, derive optimal robot taxes by studying, like us, tax changes that keep utility unchanged. In contrast to these papers, our goal is to study the specific tax reform that achieves such compensation in general equilibrium. Finally, our paper is related to the literature that analyzes the set of Pareto efficient taxes—an important alternative to the standard optimal tax problem that does not require positing a social welfare function; see, e.g., Werning (2007), Scheuer and Werning (2017), Bierbrauer and Boyer (2014), Lorenz and Sachs (2016), Bierbrauer, Boyer, and Hansen (2020). We discuss in more detail the relationship to the optimal and Pareto efficient taxation literature in Section 4.1. Finally, from a technical viewpoint, our derivations are based on the general-equilibrium tax incidence analysis of Sachs, Tsyvinski, and Werquin (2020). However, this paper does not address the compensation problem, which requires solving not only for labor supply changes in response to a given tax reform, but also for the tax reform itself.

Outline In Section 1, we set up the model and define the welfare compensation problem. In Section 2, we solve for the compensating tax reform and the fiscal surplus in partial and general equilibrium. In Section 3, we analyze the compensating tax reform considering various examples of disruptions and an empirical application to the robot disruption. Section 4 concludes with a discussion of the differences between the compensation approach and the standard optimal taxation approach. The proofs are gathered in the Appendix.

1. Welfare compensation problem

1.1 *Initial equilibrium*

There is a continuum of measure 1 of individuals indexed by their skill $i \in [0, 1]$. In the initial (undisrupted) economy, agents i earn a pre-tax wage rate $w_i \in \mathbb{R}_+$ that they take as given. Without loss of generality, we order skills so that wages w_i are increasing in i. Thus, the skill index $i \in [0, 1]$ can be interpreted as the agent's percentile in the wage distribution of the initial economy.

Agents with skill i have preferences over consumption c and labor supply l that are represented by the utility function $u_i(c, l)$. They choose effort l_i and earn pre-tax income $y_i = w_i l_i$. Under standard assumptions on preferences, income $y_i = w_i l_i$ is strictly increasing in i, so that there are one-to-one maps between skills i, wages w_i, and incomes y_i in the initial equilibrium.[1] We assume that incomes y_i belong to an interval $[\underline{y}, \bar{y}] \subset \mathbb{R}_+$ and have a continuous density $f(\cdot)$.

The government levies a nonlinear income tax. The tax schedule $T : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable. Agents i consume their after-tax income $c_i = y_i - T(y_i)$. Their indirect utility U_i is thus given by

$$U_i = u_i\big(w_i l_i - T(w_i l_i), l_i\big), \tag{1}$$

where the labor supply l_i satisfies the first-order condition[2]

$$-\frac{\frac{\partial u_i}{\partial l}\big(w_i l_i - T(w_i l_i), l_i\big)}{\frac{\partial u_i}{\partial c}\big(w_i l_i - T(w_i l_i), l_i\big)} = \big(1 - T'(w_i l_i)\big) w_i. \tag{2}$$

There is a continuum of mass 1 of identical firms whose inputs in production are the aggregate labor supplies L_j of all types $j \in [0, 1]$. The production function has constant

[1] This is the case, for instance, if agents have a common utility function u that satisfies the Spence–Mirrlees condition. Importantly, because we focus on marginal perturbations, this ordering of wages need not be preserved by the disruption and the tax reform.

[2] We assume that this equation has a unique solution.

returns to scale and is denoted by $\mathcal{F}(\boldsymbol{L})$, where $\boldsymbol{L} \equiv \{L_j\}_{j\in[0,1]}$. In equilibrium, firms earn no profits and the wage w_i is equal to the marginal product of labor of skill i,

$$w_i = \frac{\partial \mathcal{F}}{\partial L_i}(\boldsymbol{L}). \tag{3}$$

We finally denote government revenue by

$$R = \int_0^1 T(w_i l_i)\, di. \tag{4}$$

For future reference, we define the local rate of progressivity of the tax schedule at income y_i as (minus) the elasticity of the retention rate $r_i = 1 - T'(y_i)$ with respect to gross income y_i, that is, $p(y_i) \equiv -\partial \ln(1 - T'(y_i))/\partial \ln y_i$.

1.2 *Wage disruption and tax reform*

Consider an exogenous perturbation $\hat{\boldsymbol{w}}^E = \{\hat{w}_i^E\}_{i\in[0,1]}$ of the wage distribution $\boldsymbol{w} = \{w_i\}_{i\in[0,1]}$, where $\hat{w}_i^E \in \mathbb{R}$ for all i. That is, the wage of agent i changes, on impact, from w_i to $w_i(1 + \mu\hat{w}_i^E)$, where $\mu > 0$ is a constant. Such a disruption can be caused by various exogenous shocks, e.g., technological change, which affects the production function $\mathcal{F}$, or immigration flows, which modify the relative shares of different skills in the economy.[3] Without loss of generality, we normalize $\sup_{i\in[0,1]} |\hat{w}_i^E| = 1$.[4] Thus, the map $\{\hat{w}_i^E\}_{i\in[0,1]}$ defines the (infinite-dimensional) direction of the disruption, while the scalar μ parametrizes its size.

Following the disruption, the government can implement an arbitrarily nonlinear tax reform $\hat{T}(\cdot)$, whereby the statutory tax payment at income y_i changes from $T(y_i)$ to $T(y_i) + \mu\hat{T}(y_i)$.[5]

In response to the wage disruption $\hat{\boldsymbol{w}}^E$ and the tax reform $\hat{T}$, individuals optimally adjust their labor supply. In general equilibrium, these decisions impact their wages, which in turn further modify their labor supply choices, and so on. We denote by $\mu\hat{w}_i$ and $\mu\hat{l}_i$ the total endogenous percentage changes in the wage and labor supply of individual i between the initial and the perturbed equilibria. Thus, the wages and labor supplies in the disrupted economy are, respectively, given by $w_i(1 + \mu\hat{w}_i^E + \mu\hat{w}_i)$ and $l_i(1 + \mu\hat{l}_i)$.

We define agent i's compensating variation $\mu\hat{U}_i$ as the change in utility between the initial and the perturbed equilibria, normalized by the (initial) marginal utility of consumption $\partial u_i/\partial c$ so as to obtain a monetary measure of the welfare gains and losses. Finally, we denote by $\mu\hat{R}$ the change in government revenue caused by the disruption and the tax reform, or *fiscal surplus*.

[3]For instance, the wage disruption implied by a change in the production function from $\mathcal{F}$ to $\tilde{\mathcal{F}}$ is given by $\mu\hat{w}_i^E \equiv \frac{1}{w_i}[\partial\tilde{\mathcal{F}}/\partial L_i - \partial\mathcal{F}/\partial L_i]$ for all i.

[4]Throughout the paper, we focus on continuously differentiable functions $i \mapsto \hat{w}_i^E$ on $[0, 1]$.

[5]In Section 2.4, we assume that the tax reforms $\hat{T}$ that the government can implement belong to the Banach space of functions that are continuously differentiable and bounded, with bounded first derivative.

1.3 *Compensation problem*

Compensating tax reform The welfare compensation problem consists of designing a reform $\hat{T}$ of the existing tax code that offsets the welfare gains and losses of the wage disruption $\mu\hat{\boldsymbol{w}}^E$. Hence, the tax reform $\hat{T}$ must be designed such that each agent's compensating variation is equal to zero:

$$\hat{U}_i = 0 \quad \forall i \in [0, 1]. \tag{5}$$

We say that the disruption $\{\hat{w}_i^E\}_{i\in[0,1]}$ is compensable if the fiscal surplus is nonnegative, i.e., $\hat{R} \geq 0$.

Marginal wage disruptions In this paper, we characterize analytically the solution to the welfare compensation problem for marginal wage disruptions, i.e., as $\mu \to 0$. Thus, our exercise consists of designing and evaluating the fiscal impact of a tax reform $\hat{T}$ that compensates the first-order welfare effects of a small wage disruption in the direction $\hat{\boldsymbol{w}}^E$. In Section 3.4, we explore quantitatively how our compensating tax reform fares against large shocks.

Aggregate gains of disruptions If a disruption is compensable, then it is possible to find a reform of the initial tax code T that achieves a strict Pareto improvement.[6] Conversely, it is possible that a disruption generates strictly positive aggregate gains, both in terms of gross incomes and government revenue, but that these gains are not compensable (i.e., the fiscal surplus $\hat{R}$ is negative) if the labor supply distortions that the compensation would generate outweigh these gains. More generally, the value of the fiscal surplus, not only its sign, carries important information: It provides a metric that allows us to compare, in monetary units, the aggregate welfare gains (or losses) of different economic shocks. For example, suppose that a given disruption (say, automation) generates more revenue, after implementing the compensating tax reform, than another (say, an inflow of immigration). It follows that the government can achieve a strictly better Pareto improvement from the former shock.

Remark: A more general problem It is natural to wonder what a compensating tax reform would be if the government's objective were to compensate all agents to make their welfare at least as large (rather than exactly as large) as in the initial economy, i.e., $\hat{U}_i \geq 0$ for all i. To address this problem, we can directly specify the nonzero welfare improvements (or losses) $\hat{U}_i = h_i \in \mathbb{R}$ that one wants to achieve for each skill level. We then solve the compensation problem by replacing 0 with h_i in the right-hand side of (5). The differential equation derived in Lemma 2 below now features the exogenous function h. The corresponding tax reform and fiscal surplus can then be straightforwardly derived following identical steps as in the proofs of Propositions 1 and 2.

[6] For instance, the government can redistribute lump sum the budget surplus uniformly to all workers.

2. Compensating tax reform and fiscal surplus

2.1 *Elasticity concepts*

As a preliminary step, we start by defining the elasticities of labor supply, labor demand, and substitution on which the solution to the compensation problem depends. All of them are standard and can be naturally mapped to empirical estimates.

Elasticities of labor supply We decompose the uncompensated (Marshallian) elasticity $\partial \ln l_i / \partial \ln r_i$ of labor supply of skill i with respect to the retention rate r_i as $e_i^r - e_i^n$, where $e_i^r \equiv \partial \ln l_i^c / \partial \ln r_i > 0$ is the compensated (Hicksian) elasticity, or substitution effect, and $e_i^n \equiv r_i \, \partial \ln l_i / \partial(-n_i) > 0$ is the income effect parameter—i.e., (minus) the semi-elasticity of labor supply with respect to non-labor income n_i. The elasticity of labor supply with respect to the wage w_i is then equal to $e_i^w = (1 - p(y_i))e_i^r - e_i^n$. We define the corresponding elasticities along the nonlinear budget constraint[7] by $\varepsilon_i^x \equiv e_i^x / (1 + p(y_i)e_i^r)$ for $x \in \{r, n, w\}$. The scaling factor $1 + p(y_i)e_i^r$ accounts for the fact that the direct labor supply response e_i^x endogenously affects the agent's marginal tax rate $T'(y_i)$ by the rate of progressivity $p(y_i)$, which in turn causes a further labor supply adjustment given by $p(y_i)e_i^r$.

Elasticities of labor demand and substitution We define the cross-wage elasticity γ_{ij} of the wage of skill i with respect to the aggregate labor of skill j and the own-wage elasticity (or inverse elasticity of labor demand) $1/\varepsilon_j^d$ of the wage of skill j with respect to L_j by $\partial \ln w_i / \partial \ln L_j = \gamma_{ij} - (1/\varepsilon_j^d)\,\delta(i - j)$, where $\delta(\cdot)$ is the Dirac delta function.[8] For instance, if the production function has a constant elasticity of substitution (CES) between skills,[9] the own-wage elasticity $1/\varepsilon^d$ is constant and the cross-wage elasticity is equal to $\gamma_{ij} = (1/\varepsilon^d) y_j / \mathbb{E} y$. In this case, we have $\gamma_{ij} > 0$ for all i, j, so that different skills are Edgeworth complements in production. Moreover, γ_{ij} does not depend on i, implying that an increase in the labor supply of type j raises the wages of all types $i \neq j$ by the same percentage amount.

2.2 *Incidence of disruptions and tax reforms*

To characterize the compensating tax reform and the fiscal surplus, we derive first-order Taylor expansions as $\mu \to 0$ of the perturbed equilibrium conditions ((23)–(26) in the Appendix) around the initial equilibrium (1)–(4). This variational approach was pioneered by Saez (2001) and extended to general-equilibrium environments by Sachs, Tsyvinski, and Werquin (2020).

Welfare changes The (normalized) change $\hat{U}_i$ in the utility of agent i induced by the wage disruption $\hat{\boldsymbol{w}}^E$ and the tax reform $\hat{T}$ is given by

$$\hat{U}_i = \big(1 - T'(y_i)\big) y_i \big[\hat{w}_i^E + \hat{w}_i\big] - \hat{T}(y_i) = 0, \tag{6}$$

[7] See, e.g., Scheuer and Werning (2017), Jacquet and Lehmann (2021).

[8] The Dirac notation ensures that the Euler theorem holds: $\int_0^1 w_i L_i \times (L_j / w_i)(\partial w_i / \partial L_j)\, di = 0$.

[9] The CES production function is defined by $\mathcal{F}(\boldsymbol{L}) = [\int_0^1 \theta_j L_j^{1-1/\varepsilon^d}\, dj]^{\varepsilon^d/(\varepsilon^d - 1)}$.

where the second equality imposes that once the new tax schedule is implemented, agent i keeps the same level of welfare in the disrupted economy as in the initial equilibrium. The first term on the right-hand side of (6) shows that the change in the utility of agents i is equal to their total income gain or loss $y_i[\hat{w}_i^E + \hat{w}_i]$ caused by both the exogenous shock $\hat{w}_i^E$ and the general-equilibrium adjustments $\hat{w}_i$,[10] weighted by the share $(1 - T'(y_i))$ of this income change that they keep after paying taxes. The second term shows that their utility also responds to the change in their tax liability $\hat{T}(y_i)$, which makes them poorer (resp., richer) if $\hat{T}(y_i) > 0$ (resp., < 0).

Labor supply changes The disruption $\hat{\boldsymbol{w}}^E$ and the tax reform $\hat{T}$ induce changes in labor supply equal to

$$\hat{l}_i = \varepsilon_i^w\big[\hat{w}_i^E + \hat{w}_i\big] - \varepsilon_i^r \frac{\hat{T}'(y_i)}{1 - T'(y_i)} + \varepsilon_i^n \frac{\hat{T}(y_i)}{\big(1 - T'(y_i)\big)y_i}. \tag{7}$$

This equation shows that agents i adjust their effort upward, $\hat{l}_i > 0$, if their wage increases (first term on the right-hand side of (7)), their marginal tax rate decreases (second term), and their total tax liability—or average tax rate—increases (third term). For future reference, we denote the labor supply response to a disruption and a tax reform absent any endogenous wage adjustment $\hat{w}_i$ (i.e., in partial equilibrium) by

$$\hat{l}_i^E = \varepsilon_i^w \hat{w}_i^E - \varepsilon_i^r \frac{\hat{T}'(y_i)}{1 - T'(y_i)} + \varepsilon_i^n \frac{\hat{T}(y_i)}{\big(1 - T'(y_i)\big)y_i}. \tag{8}$$

Endogenous wage changes The disruption $\hat{\boldsymbol{w}}^E$ and the tax reform $\hat{T}$ lead to endogenous wage changes equal to

$$\hat{w}_i = -\frac{1}{\varepsilon_i^d}\hat{l}_i + \int_0^1 \gamma_{ij}\hat{l}_j \, dj. \tag{9}$$

Intuitively, a 1% increase in the labor supply of individuals with skill i leads to a $-1/\varepsilon_i^d$ percent change in their own wage, because the marginal product of labor is decreasing. A 1% increase in the labor supply of agents with skill $j \in [0, 1]$ leads to a γ_{ij} percent change in the wage of type i through complementarities between skills in production.

Fiscal surplus Finally, the fiscal surplus generated by the disruption $\hat{\boldsymbol{w}}^E$ and the tax reform $\hat{T}$ is given by

$$\hat{R} = \int_0^1 \big[\hat{T}(y_i) + T'(y_i)y_i\big(\hat{w}_i^E + \hat{w}_i + \hat{l}_i\big)\big] \, di. \tag{10}$$

The first term in square brackets is the mechanical effect of the compensation on government revenue due to the statutory changes in tax rates. The second term accounts for the fiscal externalities from changes in workers' earnings y_i via wage adjustments and labor supply choices. The marginal tax rate $T'(y_i)$ captures the share of these earnings gains or losses that accrues to the government.

[10]Recall that $\hat{w}_i$ is a percentage wage change, so that $w_i\hat{w}_i$ is the absolute wage change, and $l_i \times (w_i\hat{w}_i)$ is the gross income change.

2.3 *Compensation in partial equilibrium*

In this section, we show that the solution to the compensation problem takes a simple form in partial equilibrium, even when taxes are distortionary. Suppose that the production function is given by $\mathcal{F}(\boldsymbol{L}) = \int_0^1 \theta_i L_i \, di$, so that for any i, the wage w_i is equal to the exogenous technological parameter θ_i. The marginal product of labor is then constant ($\varepsilon_i^d \to \infty$) and skills are infinitely substitutable in production ($\gamma_{ij} = 0$). In this case, a disruption $\hat{\boldsymbol{w}}^E$ generates no further endogenous adjustment in the wage: $\hat{w}_i = 0$ for all i. Equation (6) thus gives immediately the compensating tax reform $\hat{T}$. Since there is a one-to-one map between skills i and incomes $y \equiv y_i$ in the initial equilibrium, we denote $\hat{w}^E(y) \equiv \hat{w}_i^E$ and $\varepsilon_y^x \equiv \varepsilon_i^x$ for $x = r, n, w$.

Proposition 1. *In partial equilibrium, the tax reform that compensates a marginal wage disruption in the direction $\hat{\boldsymbol{w}}^E$ is given by*

$$\frac{\hat{T}(y)}{y} = \big(1 - T'(y)\big)\, \hat{w}^E(y). \tag{11}$$

The fiscal surplus generated by the disruption and the compensating tax reform is given by

$$\hat{R} = \mathbb{E}\big[y\, \hat{w}^E(y)\big] - \mathbb{E}\big[T'(y) y\, \varepsilon_y^r \hat{\psi}(y)\big], \tag{12}$$

where $\hat{\psi}(y) \equiv \frac{d\hat{w}^E(y)}{d \ln y}$ measures the local variation of the exogenous wage disruption along the income distribution.

Equation (11) shows that if wages are exogenous, the compensating tax reform simply consists of increasing or decreasing the average tax rate (ATR) $\hat{T}(y_i)/y_i$ of each agent i by an amount equal to her net-of-tax wage gain or loss resulting from the disruption, $(1 - T'(y_i))\hat{w}_i^E$. This makes them just as well off as if the disruption had not occurred. Equation (12) allows us to determine whether a given economic shock $\{\hat{w}_i^E\}_{i \in [0,1]}$ is compensable. Note in particular that calculating the fiscal surplus does not require actually implementing or even computing the compensating tax reform: The expression for $\hat{R}$ depends only on the exogenous disruption and the characteristics—tax rates, income distribution, and labor supply elasticities—of the initial (undisrupted) economy.

Taking stock The feature that allowed us to solve trivially for the compensating tax reform $\hat{T}$ in partial equilibrium is that, in the absence of endogenous wage adjustments ($\hat{w}_i = 0$), the changes in *marginal tax rates* (MTR), $\hat{T}'(y_i)$, do not enter (6). That is, conditional on the total tax change $\hat{T}(y_i)$, the MTR does not matter for welfare. This follows from the envelope theorem: The MTR that individuals face affects their utility only through their labor supply decision (2), but since labor supply is initially chosen optimally, these behavioral responses induce no first-order effect on welfare. As a result, it is sufficient to adjust all agents' total tax payment (or ATR) to neutralize their income gain or loss due to the exogenous disruption, regardless of the changes in MTR that such a reform implies. Of course, while the endogenous labor supply responses (8) are irrelevant for the welfare compensating tax reform, they determine the deadweight loss of the reform and, therefore, the fiscal surplus $\hat{R}$.

2.4 *Compensation in general equilibrium*

We now characterize the compensating tax reform and the fiscal surplus when wages are endogenous. In general equilibrium, (6) no longer gives directly the tax reform $\hat{T}$ that compensates the exogenous disruption $\hat{\boldsymbol{w}}^E$, because the wage changes $\hat{w}_i$ are endogenous to the tax reform. Specifically, these wage responses are determined by the labor supply responses via (9). In turn, these labor supply changes are driven by the changes in marginal and average tax rates via (7).

Labor supply changes The first step of the analysis is to solve for the total labor supply changes following the disruption and tax reform. Substituting for $\hat{w}_i$ into (7) using (9) implies that the labor supply adjustments $\{\hat{l}_i^E\}_{i\in[0,1]}$ are the solution to an integral equation. The following lemma follows from Proposition 1 in Sachs, Tsyvinski, and Werquin (2020) and is proved in the Appendix.

Lemma 1. *Assume that $\int_{[0,1]^2} |\phi_i \varepsilon_i^w \gamma_{ij}|^2 \, di \, dj < 1$, where $\phi_i \equiv 1/(1+\varepsilon_i^w/\varepsilon_i^d)$.*[11] *The change in labor supply of agent i in response to a wage disruption and a tax reform is given by*

$$\hat{l}_i = \phi_i \hat{l}_i^E + \phi_i \varepsilon_i^w \int_0^1 \Gamma_{ij} \phi_j \hat{l}_j^E \, dj, \tag{13}$$

where $\hat{l}_i^E$ is defined by (8). If the production function is CES, we have $\Gamma_{ij} = \gamma_{ij}/(\int_0^1 \phi_k \frac{y_k}{\mathbb{E}y} \, dk)$ with $\gamma_{ij} = (1/\varepsilon^d) y_j/\mathbb{E}y$. More generally, we have $\Gamma_{ij} \equiv \sum_{n=0}^{\infty} \Gamma_{ij}^{(n)}$ with $\Gamma_{ij}^{(0)} = \gamma_{ij}$ and for all $n \geq 1$, $\Gamma_{ij}^{(n)} = \int_0^1 \Gamma_{ik}^{(n-1)} \phi_k \varepsilon_k^w \gamma_{kj} \, dk$.

The first term on the right-hand side of (13) is the partial-equilibrium change in labor supply $\hat{l}_i^E$, scaled by a factor ϕ_i. This scaling factor accounts for the fact that the marginal product of labor is decreasing, so that the agent's initial (say, positive) labor supply adjustment lowers her wage by a factor $1/\varepsilon_i^d$, which in turn leads her to reduce her labor supply by a factor $\varepsilon_i^w/\varepsilon_i^d$, thus dampening her initial response by $\phi_i \equiv 1/[1 + \varepsilon_i^w/\varepsilon_i^d]$.

The second term on the right-hand side of (13) captures the change in the labor supply of agent i caused by the behavioral responses of all other agents $j \in [0, 1]$ through complementarities in production. An increase in the labor supply of skill j raises the wage of skill i by the cross-wage elasticity Γ_{ij}, which in turn raises the labor supply of skill i proportionately to ε_i^w. If the production function is CES, Γ_{ij} is simply proportional to the structural elasticity γ_{ij}. For a general production function, it is defined by a series $\sum_{n=0}^{\infty} \Gamma_{ij}^{(n)}$ that comprises the direct effect $\Gamma_{ij}^{(0)} = \gamma_{ij}$ of the labor supply l_j on the wage w_i, as well as the infinite sequence of indirect effects that occur in general equilibrium: For each $n \geq 1$, $\Gamma_{ij}^{(n)}$ accounts for the impact of l_j on w_i via the wage and, hence, labor supply adjustments of n intermediate types; e.g., for $n = 1$, $l_j \xrightarrow{\gamma_{kj}} w_k \xrightarrow{\varepsilon_k^w} l_k \xrightarrow{\gamma_{ik}} w_i$.

[11]This condition ensures that the series defining Γ_{ij} converges. It is straightforward to derive sufficient conditions on primitives for this condition to hold: e.g., a CES production function, a constant rate of progressivity (CRP) tax schedule, and a quasilinear utility function with isoelastic disutility of labor.

Integro-differential equation characterization Combining (6), (9), and (13) yields an implicit characterization of the compensating tax reform.

LEMMA 2. *The compensating tax reform $\hat{T}$ is the solution to*

$$\frac{\hat{T}(y_i)}{y_i} = \big(1 - T'(y_i)\big)\hat{\Omega}_i^E + \frac{\phi_i}{\varepsilon_i^d}\left[\varepsilon_i^r \hat{T}'(y_i) - \varepsilon_i^n \frac{\hat{T}(y_i)}{y_i}\right] + \big(1 - T'(y_i)\big)\phi_i \Lambda_i, \tag{14}$$

where $\hat{\Omega}_i^E$ is the total wage disruption faced by agent i and is defined by

$$\hat{\Omega}_i^E = \phi_i \hat{w}_i^E + \phi_i \int_0^1 \Gamma_{ij} \phi_j \varepsilon_j^w \hat{w}_j^E \, dj, \tag{15}$$

and where Λ_i is equal to

$$\Lambda_i = \int_0^1 \Gamma_{ij} \phi_j \left[-\varepsilon_j^r \frac{\hat{T}'(y_j)}{1 - T'(y_j)} + \varepsilon_j^n \frac{\hat{T}(y_j)}{\big(1 - T'(y_j)\big) y_j}\right] dj. \tag{16}$$

Moreover, the (income-weighted) mean change in average tax rates is equal to the mean exogenous disruption: $\mathbb{E}[(y/\mathbb{E}y)\,\hat{T}(y)/((1 - T'(y))y)] = \mathbb{E}[(y/\mathbb{E}y)\,\hat{\Omega}_y^E]$.

The interpretation of (14) is again that the ATR change $\hat{T}(y_i)/y_i$ must compensate the wage (and, hence, welfare) gains or losses incurred by agent i. The first term on the right-hand side, $(1 - T'(y_i))\hat{\Omega}_i^E$, is the net-of-tax wage change caused by the exogenous disruption. In partial equilibrium, we have $\hat{\Omega}_i^E = \hat{w}_i^E$, so that (14) reduces to (11). In general equilibrium, $\hat{\Omega}_i^E$ accounts for the full incidence of the initial shock—absent any tax reform—on the wage of agent i. Equation (15) shows that this total disruption comprises the direct impact $\hat{w}_i^E$ scaled by the own-wage dampening factor ϕ_i, plus all of the indirect effects caused by the wage adjustments $\{\hat{w}_j^E\}_{j\in[0,1]}$, which affect the wage of skill i via cross-skill complementarities Γ_{ij}. Empirical studies that evaluate the impact of a disruption on the wage distribution may already account for these labor demand spillovers. In this case, the compensation formula we derive below can be applied using $\{\hat{\Omega}_i^E\}_{i\in[0,1]}$, rather than $\{\hat{w}_i^E\}_{i\in[0,1]}$, as a primitive.

The remaining terms on the right-hand side of (14) account for the welfare gains and losses triggered by changes in the tax rates themselves. The key observation is that, in general equilibrium, both the average and—this is new—the marginal tax rates impact welfare: This is because they generate labor supply distortions that, despite the envelope theorem, have a first-order effect on utility through their impact on wages. The welfare consequences of a given tax reform are thus much richer, and the design of the compensation is correspondingly more complex, than in partial equilibrium. The second term (in square brackets) on the right-hand side of (14) captures the welfare effects of agent i's own tax rate changes, while the third term captures the welfare effects caused by the tax changes of all other agents $j \neq i$.

Formally, an increase in the MTR of agents i by $\hat{T}'(y_i)$ reduces their labor supply (substitution effect) by $\phi_i \varepsilon_i^r \hat{T}'(y_i)$ and, hence, because the marginal product of labor is decreasing, raises their own wage by $(\phi_i\varepsilon_i^r/\varepsilon_i^d)\hat{T}'(y_i)$. Analogously, an increase in the ATR of agents i by $\hat{T}(y_i)/y_i$ raises their labor supply (income effect) by $\phi_i\varepsilon_i^n\,\hat{T}(y_i)/y_i$ and, hence, reduces their own wage by $(\phi_i\varepsilon_i^n/\varepsilon_i^d)\,\hat{T}(y_i)/y_i$. Therefore, while a higher *average* tax rate at income y_i hurts the welfare of agents i by directly making them poorer (as in partial equilibrium) and by triggering increases in labor supply, a higher *marginal* tax rate, by discouraging effort and consequently raising wages, *increases* the welfare of agents i.[12] Analogously, an increase in the marginal (respectively, average) tax rate of any agents $j \neq i$ by $\hat{T}'(y_j) > 0$ (resp., $\hat{T}(y_j) > 0$) leads to a reduction (resp., increase) in their labor supply and, hence in the wage of agents i whenever these skills are complementary in production ($\Gamma_{ij} > 0$). Summing over all j leads to the term Λ_i in (16).

Taking stock To sum up, (14) formalizes the insight that, in general equilibrium, the ATR and the MTR of the tax reform have to be determined simultaneously as they both affect welfare: The compensating policy must be designed such that the total effect of these two instruments exactly offsets that of the exogenous disruption. Suppose, for instance, that the planner implements the tax reform (11) that would compensate every agent's welfare in partial equilibrium. This tax reform is constructed so that its average tax rates exactly compensate the wage gains or losses of the disruption. While the implied adjustments in marginal tax rates can be ignored in partial equilibrium because of the envelope theorem, in general equilibrium they lead to additional, unintended welfare consequences. These first-order welfare effects themselves need to be compensated (second term on the right-hand side of (14)), which requires further changes in marginal tax rates, and so on. As a result, the combination of distortionary tax instruments and elastic labor supply (whereby marginal tax rates affect labor supply behavior) and general equilibrium (whereby labor supply decisions determine wages) leads to a fixed point problem for the compensating tax reform, formalized by expressing the compensating reform $\hat{T}$ as the solution to the integro-differential equation (14).

Solution to the compensation problem The next proposition gives the solution to (14): It is the main result of this paper. As before, since there is a one-to-one map between skills i and incomes $y \equiv y_i$, we can change variables and denote the wage disruption (15) by $\hat{\Omega}^E(y)$, the welfare effects due to production complementarities (15) by $\Lambda(y)$, the labor supply and demand elasticities by ε_y^x for $x = r, n, w, d$, and the cross-wage elasticities in (13) by $\Gamma_{y,z}$.[13]

Proposition 2. *The tax reform that compensates a marginal wage disruption in the direction $\hat{\boldsymbol{w}}^E$ is given by*

$$\frac{\hat{T}(y)}{y} = \big(1 - T'(y)\big) \int_y^{\overline{y}} \Pi(y, z)\big[\phi_z^{-1}\hat{\Omega}^E(z) + \Lambda(z)\big]\,dz \tag{17}$$

[12]The fact that an agent is made better off from a higher marginal tax rate (conditional on a total tax payment) follows from the same logic as the "trickle-down" result of Stiglitz (1982) that implies lower optimal high-income tax rates than in partial equilibrium.

[13]The relevant change of variables for the cross-wage elasticities reads $\gamma_{y_i,y_j} \equiv \gamma_{ij}/y_j'$ and $\Gamma_{y_i,y_j} \equiv \Gamma_{ij}/y_j'$, where $y_j' \equiv \partial y_j/\partial j$. In particular, if the production function is CES, we have $\gamma_{ij} = (1/\varepsilon^d)y_j/\mathbb{E}y$ and $\gamma_{y_i,y_j} = (1/\varepsilon^d)y_j f(y_j)/\mathbb{E}y$, where f is the density of incomes.

and the fiscal surplus reads as[14]

$$\hat{R} = \mathbb{E}\big[y\,\hat{\Omega}^E(y)\big] - \mathbb{E}\left[T'(y)y\int_y^{\bar{y}} \frac{\varepsilon_y^d}{\varepsilon_z^d}\Pi(y,z)\varepsilon_z^r \hat{\Psi}(z)\,dz\right], \tag{18}$$

with $\hat{\Psi}(z) \equiv d[\phi_z^{-1}\hat{\Omega}^E(z) + \Lambda(z)]/d\ln z$. *In these expressions, we let*

$$\Pi(y,z) = \frac{\varepsilon_z^d}{\varepsilon_z^r z}\exp\left(-\int_y^z \frac{\varepsilon_x^d}{\varepsilon_x^r x}\,dx\right)$$

and $\Lambda(z) = \sum_{n=1}^{\infty}\Lambda^{(n)}(z)$ *is defined inductively, for all* $n \geq 1$, *as*

$$\Lambda^{(n)}(z) = \int_{\underline{y}}^{\bar{y}} \Gamma_{z,y}\varepsilon_y^d\left[\phi_y\Lambda^{(n-1)}(y) - \int_z^{\bar{y}}\Pi(y,x)\Lambda^{(n-1)}(x)\,dx\right]dy,$$

with $\Lambda^{(0)}(z) \equiv \phi_z^{-1}\hat{\Omega}^E(z)$. *If the production function is CES,* $\Lambda(z)$ *is a constant* $\mathbb{E}[y(\phi_y\Lambda^{(0)}(y) - \int_y^{\bar{y}}\Pi(y,x)\Lambda^{(0)}(x)\,dx)]/\mathbb{E}[y\int_y^{\bar{y}}\Pi(y,x)\,dx]$.

Formulas (17) and (18) depend only on the exogenous wage disruption $\hat{\Omega}^E$ (or $\hat{\boldsymbol{w}}^E$) and on variables that are observed in the pre-disruption economy: statutory marginal tax rates, elasticities of labor supply ε_y^r, ε_y^n, ε_y^w, elasticities of labor demand ε_y^d, and elasticities of substitution between skills $\Gamma_{y,z}$ (or $\gamma_{y,z}$). It is thus straightforward to implement such a tax reform in practice. In Section 3, we analyze the shape of the compensation (17) in detail, and study various examples and an empirical application.

Before proceeding, we describe the structure of the compensation formula (17). It expresses the tax reform as a series of partial compensations. Suppose first that the marginal product of labor is decreasing but that skills are perfect substitutes in production, so the cross-wage elasticities $\Gamma_{z,y}$ are equal to 0. In this case, the compensation of the exogenous wage disruption $\Lambda^{(0)}(z) \equiv \phi_z^{-1}\hat{\Omega}^E(z)$ reduces to $(1 - T'(y))y\int_y^{\bar{y}}\Pi(y,z)\Lambda^{(0)}(z)\,dz$ (we analyze this expression in the next section). For a general production function, this compensation and the cross-wage effects $\Gamma_{z,y}$ generate further wage changes for agent z given by $\Lambda^{(1)}(z)$. These must be compensated by $(1 - T'(y))y\int_y^{\bar{y}}\Pi(y,z)\Lambda^{(1)}(z)\,dz$ (second round of "compensating the compensation"), thus leading to further changes in wages and so on. Repeating this procedure for all $n \geq 2$ yields the full compensation (17), where each term $\Lambda^{(n)}(z)$ in the series captures the (cross-)wage changes caused by the $(n-1)$th round of partial compensation. In other words, when the exogenous shock hits, we adjust the tax schedule to compensate for it, ignoring the endogeneity of wages due to production complementarities, i.e., treating each labor market with its own labor demand curve and decreasing marginal product of labor, independently of the others. We then compute the first round of general-equilibrium effects on wages, naively compensate for them again, and so forth, until we have settled. When the production function is CES, this iterative procedure

[14]This expressions assumes for simplicity that $\bar{y} \to \infty$; the general expression is derived in the proof in the Appendix.

becomes particularly simple: In this case, each intermediate round of compensation leads to uniform wage changes across workers (i.e., constant $\Lambda^{(n)}(\cdot)$), so the series $\Lambda(\cdot)$ collapses to a constant.

Remark: Extensive margin of labor supply Our results extend to a setting where, in addition to adjusting their labor effort on the intensive margin, workers can respond to wage disruptions and tax changes by deciding to enter or exit the labor force. Suppose that agents differ along two dimensions: their skill $i \in [0, 1]$, as in the previous sections, and their fixed cost of participating in the labor force $\kappa \in \mathbb{R}_+$. These two characteristics can be arbitrarily correlated in the population. An agent with types (i, κ) has idiosyncratic preferences over consumption c and labor supply l described by $u_i(c, l) - \kappa \mathbb{I}_{\{l>0\}}$, where $\mathbb{I}_{\{l>0\}}$ is an indicator function equal to 1 if the agent is employed. Agents i participate if their fixed cost of work κ is smaller than a threshold κ_i equal to the difference between the utility conditional on employment, $u_i[w_i l_i - T(w_i l_i), l_i]$, and the utility conditional on unemployment, $u_i[-T(0), 0]$. We can easily show that the tax reform derived in Proposition 2, along with a fixed unemployment transfer $-T(0)$, continues to solve the compensation problem in this setting. Indeed, this reform leaves unchanged the worker's utility both conditional on employment and on unemployment, so that no agent switches participation status after the disruption and its compensation.[15]

3. Analysis of the compensating tax reform

In this section, we analyze the economic implications of Proposition 2 by applying the compensation (17) to various disruptions. In Sections 3.1, 3.2, and 3.3, we study three benchmark classes of disruptions: first, those that affect all skills uniformly; second, those that change the wage of a single skill; third, those that involve an interval of skills (e.g., the middle class or the top decile). These special cases help establish the main principles of welfare compensation in general equilibrium. In Section 3.4, we evaluate the robustness of our results to the size of the behavioral elasticities, the shape of the initial tax schedule, and the size of (nonmarginal) disruptions. Finally, Section 3.5 turns to a concrete empirical application: the compensation of robots in the United States. Unless stated otherwise, we impose the following assumption throughout this section.

Assumption 1. *The initial (pre-disruption) production function is CES. Preferences take the form* $u(c, l) = c^{1-\eta}/(1-\eta) - l^{1+1/e}/(1+1/e)$ *with* $e > 0$ *and* $\eta \geq 0$. *The initial tax schedule has a constant rate of progressivity (CRP), i.e.,* $T(y) = y - ((1-\tau)/(1-p))y^{1-p}$ *with* $\tau \in \mathbb{R}$ *and* $p < 1$.

Assumption 1 ensures that the rate of progressivity $p(y) = p$, the Hicksian elasticity $e_i^r = e/(1 + (1-p)e\eta)$, the income effect parameter $e_i^n = (1-p)\eta e_i^r$, the labor supply elasticities ε_i^r, ε_i^n, ε_i^w, the labor demand elasticity ε_i^d, and the elasticity of substitution between skills, are all constant.

[15]This argument implies in particular that the values of the elasticities of participation with respect to the tax rates (which otherwise would matter to determine the endogenous wage adjustments $\hat{w}_i$) are irrelevant for the construction of the compensating tax reform.

3.1 *Uniform disruptions*

We first study a perturbation that reduces the wages of all workers by the same amount in percentage terms.

Corollary 1. *Suppose that Assumption 1 holds. Consider a uniform wage disruption, so that* $\hat{w}_i^E \equiv \hat{w}^E$ *for all* $i \in [0, 1]$. *Then the general-equilibrium compensation (17) coincides with the partial-equilibrium compensation (11).*

To show this result, notice first that the partial- and general-equilibrium compensations coincide if and only if the endogenous wage adjustments $\hat{w}_i$ in (9) vanish; that is, if $-(1/\varepsilon^d)\hat{l}_i + \int_0^1 \gamma_{ij}\hat{l}_j\,dj = 0$. Now Euler's homogeneous function theorem imposes that $-1/\varepsilon^d + \int_0^1 \gamma_{ij}\,dj = 0$. Thus, it suffices to prove that, under Assumption 1, the labor supply response to the disruption and the compensation is uniform, i.e., $\hat{l}_i^E = \hat{l}_j^E$ for all i, j. But this is straightforward to show by plugging (11) into (8). Therefore, for a uniform wage disruption and the assumed preferences and tax schedule, the own- and cross-wage effects just offset each other, thus yielding zero general-equilibrium wage adjustments. As a result, the partial-equilibrium (PE) tax reform achieves exact welfare compensation even in the general-equilibrium (GE) environment.

The uniform disruption and its compensation are represented in Figure 1. We calibrate the elasticity of labor supply to $e = 0.33$ (Chetty (2012)) and the elasticity of substitution between skills to $\varepsilon^d = \infty$ (partial equilibrium) or $\varepsilon^d = 1.5$ (Katz and Murphy (1992), Card and Lemieux (2001), Card (2009)). We suppose moreover that there are no income effects on labor supply: $\eta = 0$. We take a rate of progressivity of the initial tax schedule equal to $p = 0.15$ and a level parameter of $\tau = -3$ (Heathcote, Storesletten, and Violante (2017)). The marginal tax rate is thus increasing with income: It is equal to 9% at \$20,000, to 23% at \$60,000, and to 29% at \$100,000. We match the U.S. annual earnings distribution by positing a (truncated) log-normal distribution below \$150,000 with mean 10 and variance 0.95 and appending a Pareto distribution with a tail parameter that decreases from $a = 2.5$ at \$150,000 to $a = 1.5$ for all incomes above \$350,000 (Diamond and Saez (2011)). As in Saez (2001), we infer the wage distribution from the observed earnings distribution and the individuals' first-order conditions (see Sachs, Tsyvinski, and Werquin (2020) for details on the extension of this method to the general-equilibrium setting).

The left panel shows that the disruption reduces the wage of all agents by 1%. These wage losses translate into pre-tax income losses represented by the black curve in the right panel: e.g., workers with income equal to \$60,000 (respectively, \$100,000, \$500,000) before the disruption suffer pre-tax earnings losses of \$600 (resp., \$1000, \$5000). The blue and red curves in the right panel show the compensation in partial and general equilibrium, respectively. Recall that the decrease in the agent's average tax rate implied by the tax reform mirrors the *after-tax* income losses due to the wage disruption. Since the initial tax schedule is progressive, this implies that the compensation is flatter than the gross income losses: Losing a dollar of pre-tax income does not hurt higher-paid workers as much, since they retain a smaller share $1 - T'(y)$ of that dollar. Quantitatively,

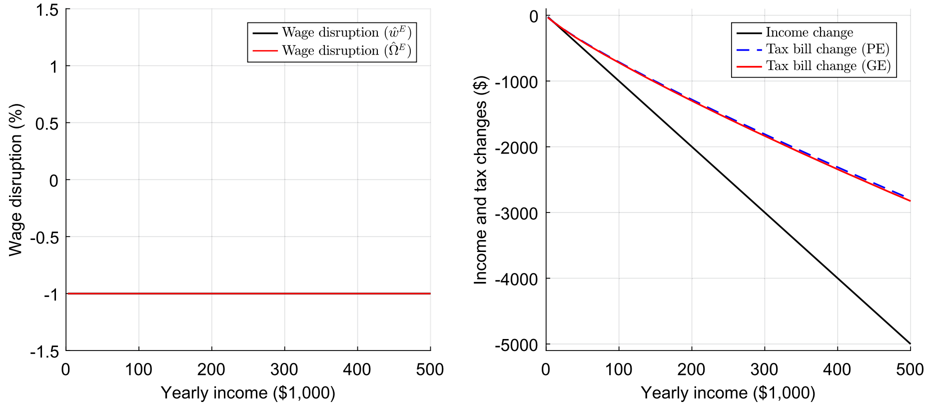

FIGURE 1. Uniform disruption and compensation.

a pre-tax income loss of $1000 at $100,000 (respectively, $5000 at $500,000) translates into an after-tax income loss—and thus requires a reduction in tax payment—of $712 (resp., $2796).

3.2 *Dirac disruptions*

Our second polar case consists of a disruption that affects only the wage of agents with a given skill i_* and corresponding income y_*. Formally, we let $\hat{w}^E(y) = -\delta(y - y_*)$, where $\delta(\cdot)$ is the Dirac delta function.[16]

COROLLARY 2. *Suppose that Assumption 1 holds and let $\bar{y} \to \infty$. Consider a Dirac wage disruption at income y_*, so that $\hat{w}_i^E \equiv -\delta(y_i - y_*)$ for all $i \in [0, 1]$. Then the general-equilibrium compensation is given by*

$$\frac{\hat{T}(y)}{y} = -\frac{\varepsilon^d}{\varepsilon^r}(1-\tau)\frac{y^{\varepsilon^d/\varepsilon^r - p}}{y_*^{\varepsilon^d/\varepsilon^r + 1}}\mathbb{I}_{\{y \le y_*\}} + C(1-\tau)y^{-p}, \tag{19}$$

where $C = y_* f(y_*)/\mathbb{E}z - (\varepsilon^d/\varepsilon^r)\mathbb{E}[(z/\mathbb{E}z)(z/y_*)^{\varepsilon^d/\varepsilon^r+1}\mathbb{I}_{\{z \le y_*\}}]$ is a constant.

To understand this result, first ignore the cross-wage complementarities. In this case, (19) with $C = 0$ follows from the first-order linear ordinary differential equation (ODE) (14), which reduces to

$$\frac{\hat{T}(y)}{y} = \frac{\varepsilon^d}{\phi \varepsilon^r}\hat{T}'(y) \quad \forall y < y_*. \tag{20}$$

This equation requires that the change in average tax rates is proportional to the change in marginal tax rates at every income level below y_*. Intuitively, suppose that the government naively compensates for the partial-equilibrium disruption by reducing the tax

[16]Note that this perturbation is not differentiable. We approximate it with a sequence of smooth wage disruptions centered around income y_*.

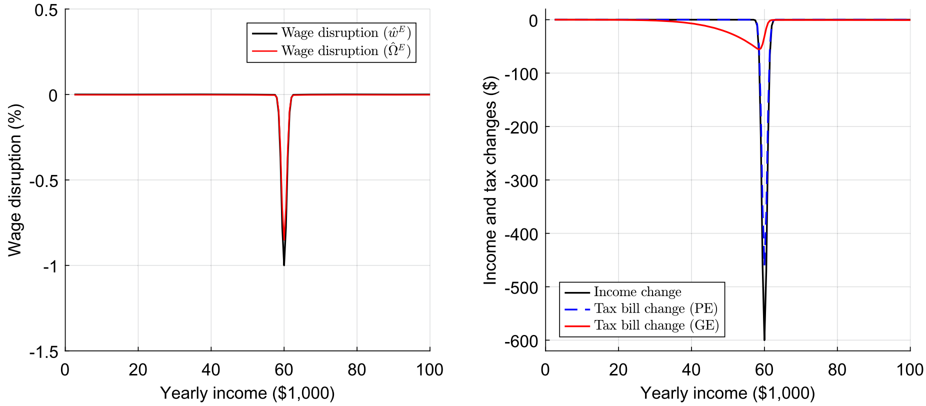

FIGURE 2. Dirac disruption and compensation.

liability of agent y_*. It must then reduce the marginal tax rates of those with lower incomes $y < y_*$, i.e., $\hat{T}'(y) < 0$. However, in general equilibrium, this reduction in MTR raises their labor supply and, hence, lowers their wage, thereby causing welfare losses that are proportional to the ratio of elasticities of labor supply and demand. These welfare losses must be offset by welfare gains of equal magnitude through reductions in their average tax rates $\hat{T}(y)/y < 0$.

If $\varepsilon^d/\varepsilon^r > p$ or, equivalently $\varepsilon^d/\phi\varepsilon^r > 1$,[17] the average tax rate must fall more than one-for-one in response to a marginal tax rate cut. However, this mechanically lowers the tax bill of agents with slightly higher income, thus requiring an even larger cut in their marginal tax rate, and so on. This "race" between the MTR and the ATR leads to exponentially decreasing tax rates on $[\underline{y}, y_*)$, captured by $\hat{T}(y)/y \propto -y^{\varepsilon^d/\varepsilon^r - p}$ in the solution to the ODE (19). That is, the compensating tax reform is *progressive* at a rate given by the ratio of elasticities of labor demand and labor supply $\varepsilon^d/\varepsilon^r$, net of the rate of progressivity p of the preexisting tax code.

Finally, accounting for the cross-wage effects adds the correction $C(1 - T'(y))y$ to the compensating tax reform in (19). It is easy to show that this amounts to raising the parameter τ of the baseline CRP tax schedule by an amount $\hat{\tau}/(1-\tau) = (1-p)C$. That is, skill complementarities require a uniform percentage shift in tax rates over the entire income distribution.

Figure 2 illustrates these results. We construct a 1% wage disruption $\mu\hat{w}^E(y_*)$ at income level $y_* = \$60{,}000$.[18] This leads to a pre-tax income loss of $y_*\mu\hat{w}^E(y_*) = \$600$ (black curve in the right panel) and an after-tax income loss of $(1 - T'(y_*))y_*\mu\hat{w}^E(y_*) =$ \$461 (blue curve in the right panel). The compensating tax reform in partial equilibrium tracks the after-tax income losses: It leaves the tax liabilities of all agents $y \neq y_*$ unchanged while reducing the tax bill of income y_* by a large amount (blue curve). In general equilibrium, the compensation (red curve) accounts for the additional wage adjustments induced by the disruption and tax changes. In particular, the wage loss at

[17]Empirically, the inequality $\varepsilon^d/\varepsilon^r > p$ is clearly satisfied, since we have $p \approx 0.15$, $\varepsilon^r \approx 0.3$, and $\varepsilon^d \geq 0.5$.
[18]The calibration is the same as in Figure 1.

income y_* lowers these agents' labor supply, marginally increasing their wages and reducing the compensation necessary to keep utility unchanged. At the same time, all other income levels need to be compensated because the labor supply reductions at y_* adversely affect their wages via production complementarities.

The key insight from Figure 2 is that the partial-equilibrium compensation creates large movements in marginal tax rates around income y_*, which yield sizeable unintended welfare consequences in general equilibrium. For instance, such a tax reform would make agents with income just below y_* strictly worse off because of the very sharp decrease in their marginal tax rate, which raises labor supply and lowers their wages and welfare. Instead, the accurate compensation reduces the tax payment of the disrupted agent at y_* by a much smaller amount while at the same time granting substantial tax rebates to incomes below y_* even though the disruption did not initially hurt them.[19] Finally, agents with an income higher than y_* also face tax cuts; these are barely noticeable in Figure 2, however, since the disruption affects a small number of workers and, thus, generates minor cross-wage effects.

3.3 *Interval disruptions*

We finally consider disruptions intermediate between the two polar (uniform and Dirac) cases studied above, and affect a nontrivial range of workers; e.g., all incomes above a threshold or all incomes within a given interval.

Corollary 3. *Suppose that Assumption 1 holds and let $\bar{y} \to \infty$. Consider a wage disruption that affects uniformly all skills above i_*, with corresponding income y_*; thus, $\hat{w}^E(y) \equiv -\mathbb{I}_{\{y \geq y_*\}}$. Then the general-equilibrium compensation is given by*

$$\frac{\hat{T}(y)}{y} = \big(1 - T'(y)\big)\left[\hat{w}^E(y) - \left(\frac{y}{y_*}\right)^{\varepsilon^d/\varepsilon^r} \mathbb{I}_{\{y \leq y_*\}} + C\right], \tag{21}$$

where $C = \mathbb{E}[(z/\mathbb{E}z)(z/y_*)^{\varepsilon^d/\varepsilon^r} \mathbb{I}_{\{z \leq y_*\}}]$ is a positive constant. More generally, consider a disruption that affects all skills in an interval $[i_L, i_H]$ uniformly, with corresponding incomes $[y_L, y_H]$; thus, $\hat{w}^E(y) \equiv -\mathbb{I}_{\{y_L \leq y \leq y_H\}}$. Then the general-equilibrium compensation is given by

$$\frac{\hat{T}(y)}{y} = \big(1 - T'(y)\big)\left[\hat{w}^E(y) - \left(\frac{y}{y_L}\right)^{\varepsilon^d/\varepsilon^r} \mathbb{I}_{\{y \leq y_L\}} + \left(\frac{y}{y_H}\right)^{\varepsilon^d/\varepsilon^r} \mathbb{I}_{\{y \leq y_H\}} + C\right], \tag{22}$$

where $C = \mathbb{E}[(z/\mathbb{E}z)(z/y_L)^{\varepsilon^d/\varepsilon^r} \mathbb{I}_{\{z \leq y_L\}}] - \mathbb{E}[(z/\mathbb{E}z)(z/y_H)^{\varepsilon^d/\varepsilon^r} \mathbb{I}_{\{z \leq y_H\}}]$ is a constant.

---

[19] Note also that the compensation peaks at an income y_{**} that is strictly below the income y_* that incurs the largest wage loss. Indeed, by definition, the agent y_{**} with the highest tax reduction has a zero marginal tax rate change. Thus, an agent with a slightly higher income gets almost the same total tax rebate (the difference between the two is second order since $\hat{T}'(y_{**}) = 0$) and a strictly higher marginal tax rate change (the difference is first order if $\hat{T}''(y_{**}) > 0$), and, hence, a strictly higher compensation. This explains why we must have $y_{**} < y_*$.

Formula (21) characterizes the compensation for a disruption that hurts all workers above an income threshold y_*. The compensation can be decomposed as the sum of three terms. The first is the partial-equilibrium compensation derived in Proposition 1. Appropriately normalized by the net-of-tax rate $(1 - T'(y))y$ to account for the redistribution already achieved by the existing tax code, this term tracks the exogenous wage losses $\hat{w}^E(y)$ one-for-one.

The second term in (21) corrects for the own-wage effects caused by the decreasing marginal product of labor. It reduces the tax liabilities below the disrupted incomes, i.e., on $[0, y_*]$, and has the same shape as in the case of a Dirac disruption in Corollary 2. In particular, its rate of progressivity—that is, the rate at which the compensation's average and marginal rates fall with income—is equal to the ratio of elasticities of labor demand and labor supply, $\varepsilon^d/\varepsilon^r$, net of the rate of progressivity p of the initial tax code.

Finally, the third term in (21) compensates for the cross-wage effects caused by skill complementarities in production. It amounts to a uniform increase in average tax rates at all income levels, above and beyond the partial-equilibrium compensation and the progressive correction we just described. Again, this last element of the compensation is similar to the corresponding term in Corollary 2.

The compensation for a disruption that affects an interior interval of skills, given by (22), follows again from a similar logic, except that there are now two progressive corrections due to own-wage effects, captured by the second and third terms on the right-hand side: The former spreads tax cuts across workers $y \leq y_L$ to compensate for the sharp wage loss at income y_L, while the latter has the opposite sign and compensates for the sharp reversal at income y_H.

Figure 3 shows this decomposition graphically for a uniform interval disruption that affects workers in the income range $y_L = \$20{,}000$ and $y_H = \$100{,}000$, with corresponding income losses shown in black. The left panel shows the partial-equilibrium (dashed blue) and general-equilibrium (solid red) compensations. The right panel decomposes the latter into its four components: the partial-equilibrium compensation (dashed blue), the progressive corrections for the decreasing marginal product of labor (solid and dotted curves), and the uniform correction for the cross-wage complementarities (dashed red).

Figure 4 constructs the compensation of smoothed-out versions of the same perturbation. In the top panels, a smooth disruption hurts middle-class workers and peaks at $y_* = \$60{,}000$. In the bottom panels, the disruption uniformly affects all workers with earnings higher than \$120,000. The top and bottom left panels depict the direct and total wage losses $\hat{w}^E(y)$ and $\hat{\Omega}^E(y)$, respectively. The black curves in the top and bottom right panels show the corresponding gross earnings losses.

The key insights described in the previous sections carry over to these cases. First, the compensation in partial equilibrium tracks the shape of the after-tax earnings losses due to the exogenous disruption. These are represented by the dashed blue curves in the top and bottom right panels. As before, a given percentage change in the wage leads to more considerable earnings losses at higher income levels, although these losses are dampened by the progressivity of the initial tax code. In general equilibrium, the compensation—represented by the solid red curves in the top and bottom

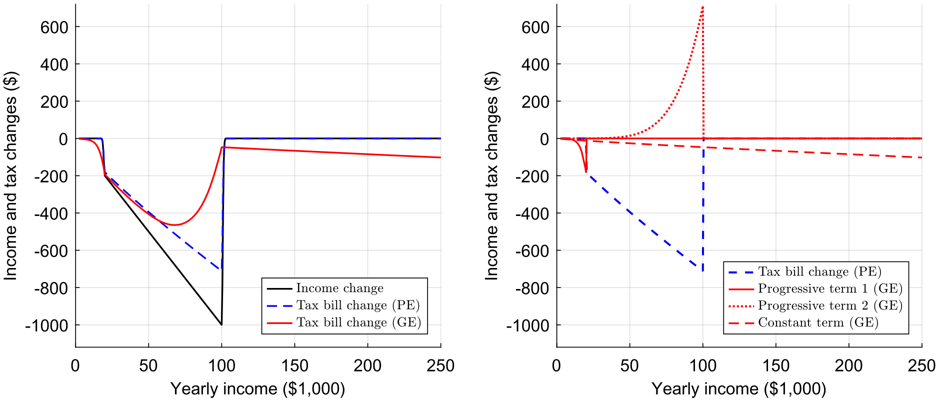

FIGURE 3. Compensation of an interval disruption: decomposition.

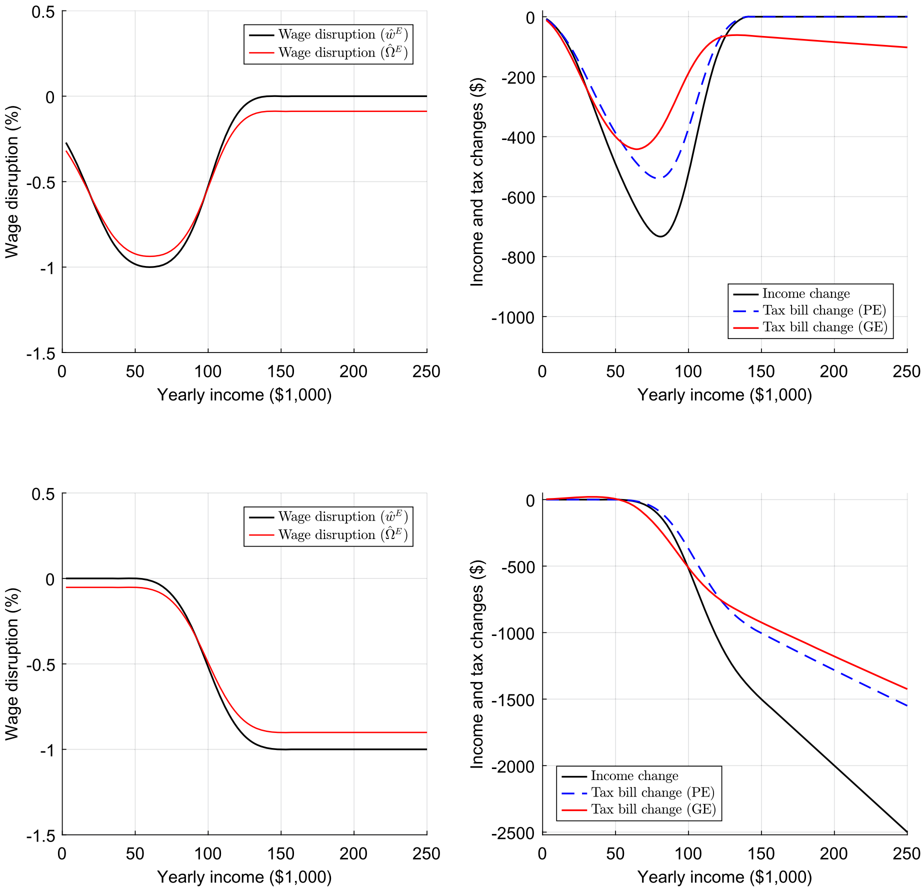

FIGURE 4. Interval disruptions and compensation.

right panels—accounts for the additional wage adjustments induced by the disruption and tax changes. The robust finding is that whenever the partial-equilibrium compensation implies sharp changes in marginal tax rates and substantial unintended welfare effects, the general-equilibrium forces smooth out such nonlinearities. They spread the tax changes over all the lower income levels, as captured by the progressive terms in (21) and (22). The top right panel of Figure 4 also shows that the general-equilibrium compensation reduces the tax rates even for the indirectly affected, high-income workers. This reflects the compensation for the general-equilibrium wage changes due to cross-skill complementarities.

3.4 *Robustness of the results*

In this section, we evaluate the robustness of our results to the values of the labor supply and demand elasticities, the shape of the baseline tax schedule, and the size of the exogenous disruption. Throughout this section, we focus on the middle-class disruption studied in the top panel of Figure 4.

Behavioral elasticities The top left panel of Figure 5 displays the compensation for different values of the Frisch elasticity $e \in \{0.25, 0.33, 0.5\}$, otherwise keeping the same calibration as in the previous sections. The top right panel of Figure 5 plots the compensation for various values of the income effect on labor supply, $\eta \in \{0, 0.25, 0.5\}$. While the partial-equilibrium compensation is unaffected by these different behavioral elasticities, the general-equilibrium compensation is sensitive to the values of e and η. Recall from Lemma 2 that income effects increase the welfare cost of raising an individual's total tax liability: They make the agents work more, which reduces their wage. As a result, higher income effects move the general-equilibrium compensation closer to the partial-equilibrium one. Moreover, the endogenous wage adjustments are driven by the magnitude of the labor supply responses to tax and wage changes, which are in turn determined by the Frisch elasticity. Accordingly, the compensation in general equilibrium is closer to the partial-equilibrium compensation for smaller values of e. The bottom panel of Figure 5 displays the compensations for different labor demand elasticities $\varepsilon^d \in \{0.5, 1.5, 2.5, \infty\}$. This exercise shows that the magnitude of general-equilibrium effects plays a critical role for the compensation of the middle-class disruption. A smaller value of ε^d implies stronger own- and cross-wage effects, lowering the middle class's compensation and raising the tax cuts for higher incomes. Conversely, as ε^d grows larger, the compensation converges to the partial-equilibrium case ($\varepsilon^d = \infty$).

Baseline tax schedule In our previous simulations, we assumed that the initial tax schedule had a constant rate of tax progressivity (CRP). This may be unrealistic for at least two reasons: The phasing out of low-income transfers may lead to high marginal tax rates at the bottom (rather than negative tax rates in the case of a CRP tax code), and the tax rates converge to a value lower than 100% at the top. We now evaluate (17) for alternative tax codes. To illustrate the impact of these two features, we use the optimal Mirrlees tax schedule (in partial or general equilibrium) as the baseline tax code. As is well known, the optimal tax schedule has high marginal tax rates at the bottom, and the

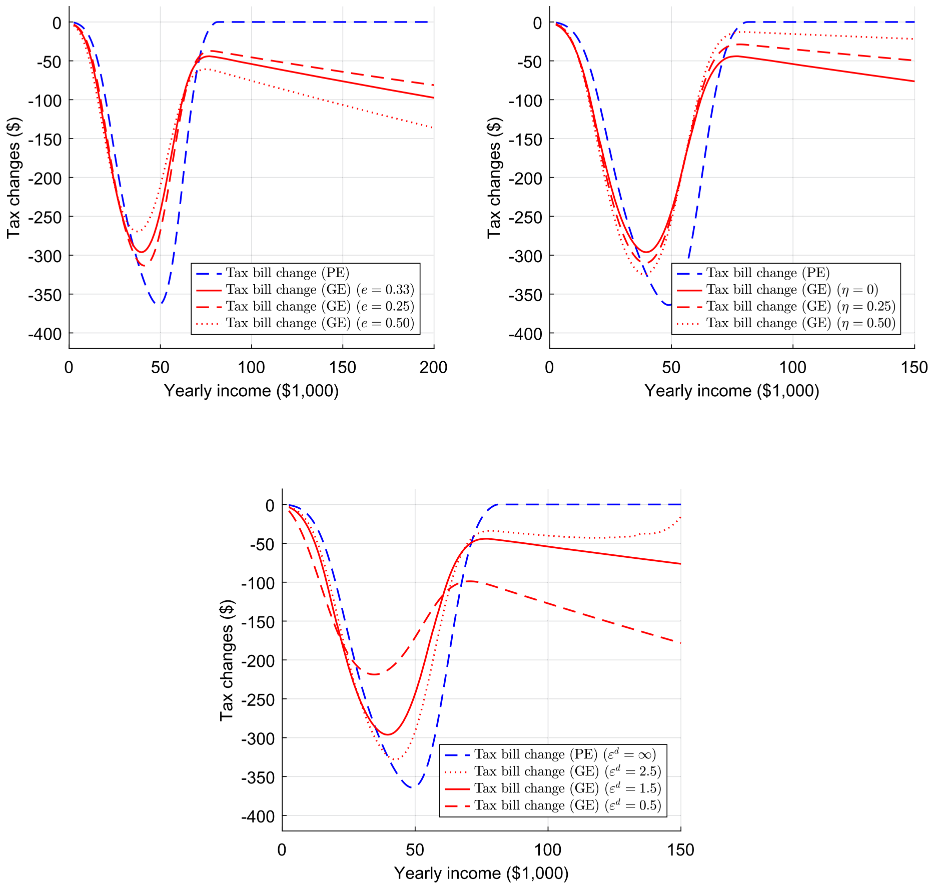

FIGURE 5. Robustness to the behavioral elasticities.

tax rate at the top is bounded away from 1, with an overall U-shape for the marginal tax rates. Figure 6 plots the compensation of the middle-class disruption studied above. The left panel (resp., right panel) shows the compensation in partial equilibrium (resp., general equilibrium) for these alternative baseline tax schemes. The shape of the compensating tax reform is qualitatively robust to the initial tax scheme: it always follows the shape of the disruption in dollar values. However, the size of the compensation is sensitive to the preexisting tax rates. The CRP tax scheme has the lowest marginal tax rates in the depicted income range and, hence, the highest retention rates. Accordingly, the tax cuts are largest in this case. In contrast, the tax changes under the optimal Mirrlees tax schedules are substantially smaller. The optimal tax schedule in general equilibrium features lower marginal tax rates and, hence, higher retention rates than the Mirrlees optimum in partial equilibrium, so that the compensation under the former tax code is slightly closer to that obtained under a CRP tax scheme. Properly accounting for the schedule of marginal tax rates in the preexisting economy is therefore important for the design of the welfare compensation.

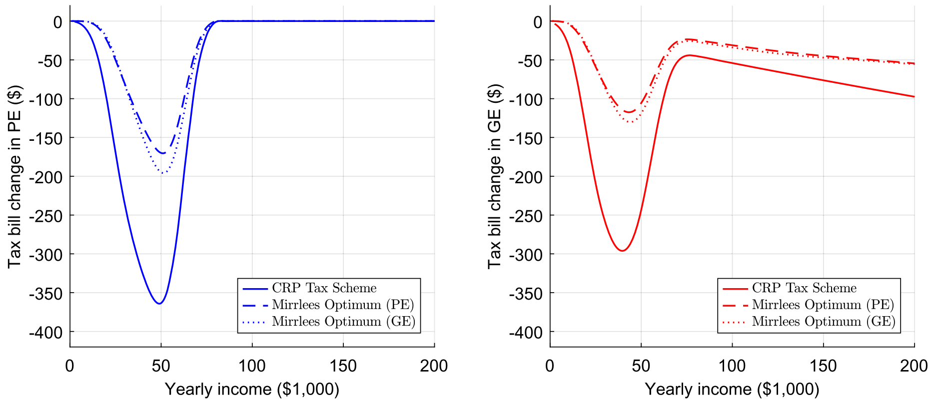

FIGURE 6. Robustness to the baseline tax schedule.

Large disruptions A key assumption we have made throughout this paper is that the disruption is marginal. In other words, our compensation scheme only compensates for the first-order effects of a disruption of the wage distribution. A natural question is whether appropriately scaling up our compensating tax reform accurately compensates workers against a large disruption in a given direction. We study this question in Figure 7. Again, we consider the middle-class disruption studied above, but contrast a 1% with a 5% wage shock in that direction (solid versus dashed curves). The figure displays the utility changes, expressed as a percentage of pre-tax income, that result from the correspondingly scaled-up compensations in partial equilibrium (blue curves in the left panel) and in general equilibrium (red curves in the right panel).[20] That is, a value

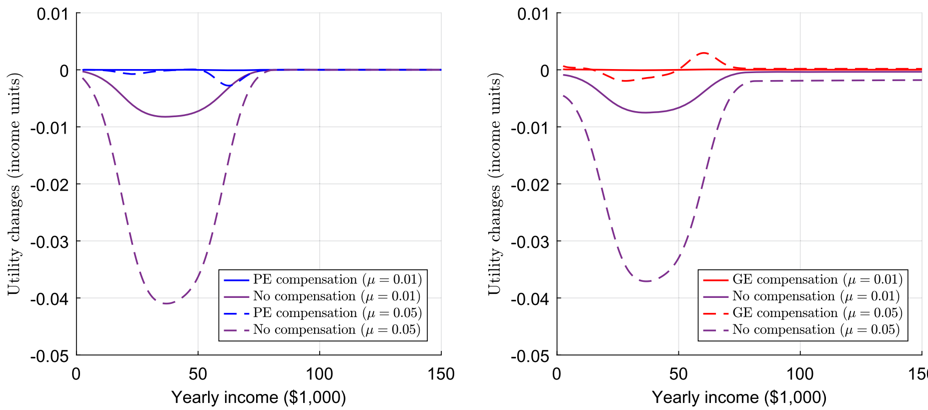

FIGURE 7. Robustness to the size of the disruption.

[20]The figures depict the actual utility gains and losses of the disruption and the compensation, that is, $\mu \hat{U}_i$ (scaled by income); these are proportional to the size of the disruption μ and are, consequently, larger for the 5% disruption than for the 1% disruption. However, when we compare the utility losses between disruptions, we report the values of the *unweighted* utility changes $\hat{U}_i$, so that the compensation does not get mechanically more accurate as $\mu \to 0$.

of −0.01 means that the utility loss is equal to 1% of income. We compare these utility changes to those we would obtain in the absence of compensation (purple curves in both panels). For the 1% disruption, the compensation approach performs very well: It offsets almost exactly (at least 95.8% of) the utility losses from the disruption.[21] For the 5% disruption, the compensation is no longer exact, but still offsets at least 78% of the utility losses in general equilibrium and at least 97.3% of the losses of those workers whose utility after compensation remains strictly lower than in the initial economy. For a very large disruption of 10% of initial wages (not represented in the figure), our tax reform still offsets at least 53.6% of the utility losses from the disruption, and at least 87.4% of the losses of the workers who remain worse off than in the initial equilibrium.

3.5 *Empirical application: Robots*

In this section, we show how our theoretical results can be implemented in an empirical application: compensating for the welfare consequences of robotization in the United States. Using the 1990 and 2007 U.S. Census data, Acemoglu and Restrepo (2020) have estimated the impact of one additional robot per thousand workers[22] on wages, employment, and hours worked. These estimates are obtained by comparing people in the same skill cell, but who reside in commuting zones with different exposure to robots. They include both the direct effects of robots on employment and wages, and any indirect spillover effects that might arise because of a resulting decline in local demand; in other words, they estimate the total disruption $\hat{\Omega}^E$ rather than the direct impact $\hat{\boldsymbol{w}}^E$.

The left panel of Figure 8 plots the wage disruption, i.e., the percentage change in the wage, along the baseline earnings distribution, as well as the standard errors. We use panel A in Figure 10 of Acemoglu and Restrepo (2020) to calculate the impact of robots throughout the income distribution. Since the effects are only calculated for the 5th, 10th, …, and 95th wage percentiles, we use a linear extrapolation to estimate the disruption for the entire wage distribution and keep constant the disruption for all workers above the 95th percentile. The figure shows that, for the bottom and middle percentiles, the change in wages is increasing with the agent's position in the income distribution. At the top, wages start to decline again. The wage of the 10th wage percentile in 1990 was reduced by 0.76%, while the 85th percentile experienced an estimated increase in their wage of 0.06%. The wage at the 90th percentile declines by 0.05%. The solid black curve in the right panel of Figure 8 gives the corresponding changes in annual income.

In the right panel of Figure 8, we plot the compensating tax reform (dashed blue curve) obtained in the partial-equilibrium environment (11). (We use the same baseline calibration as in Section 3.1.) The partial-equilibrium compensation tracks one-for-one the shape of the income gains and losses (solid black curve), correcting only for the fact

[21]The value 95.8% is computed as (1 minus) the maximum value of the ratio of utility changes (in absolute value) without versus with compensation. Note that some adversely impacted workers realize a net utility gain after the compensation. If we exclude these workers, the remaining utility losses represent less than 0.6% of the losses due to the disruption.

[22]This corresponds to the increase in robots observed in the United States between 1990 and 2007.

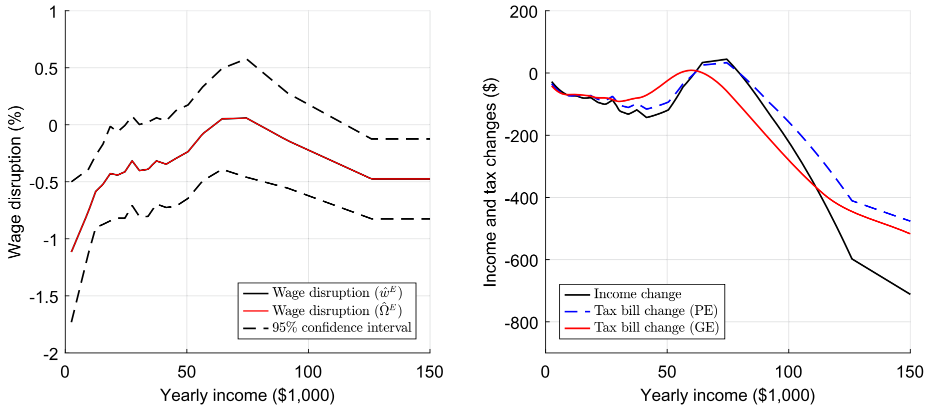

FIGURE 8. Robots disruption and compensation.

that the initial tax schedule is progressive so that gross income changes differ from net income changes. Workers in the 10th income percentile ($9500 per year) have their tax bill reduced by $73 (i.e., 101% of their income loss), while in the 85th income percentile ($74,500 per year) they face a tax increase of $33 per year (i.e., 74% of their income gain).

The solid red curve in the right panel of Figure 8 plots the compensation in general equilibrium. Up to an income of $80,000, the wage losses of the disruption are declining with income. The compensation is achieved by smoothing-out the tax changes: Middle-class workers face tax cuts that are smaller than their income losses. They are made indifferent by the joint effects of reduced total tax liabilities and higher marginal tax rates. This front-loading avoids the steep decline in marginal tax rates at low and middle incomes that the partial-equilibrium compensation would create. Symmetrically, the compensation in the upper regions of the income distribution is flatter than the disruption. Moreover, in general equilibrium, the decline in marginal tax rates resulting from the shape of the disruption reduces the welfare of these workers. The compensation therefore lowers their total tax payments (red curve) strictly more than in partial equilibrium (dashed blue curve).

Quantitatively, a low-income worker in the 10th percentile (whose annual income is $9500) now receives a compensation of $70 (97% of the income loss). At the same time, a high-income worker in the 85th percentile (whose annual income is $75,500) also experiences a tax cut of $58 (−132% of the income gain). The average tax payment of the low-income (resp., high-income) worker declines by 0.7 percentage points (resp., 0.08 percentage points), versus 0.8 percentage points (resp., 0.04 percentage points) in partial equilibrium.[23] The disruption and the compensation generate a fiscal deficit (−$145 in partial and general equilibrium), which is only partly due to the disruption itself, whose fiscal cost without compensation is −$47.

[23] Recall that these numbers are for one additional robot per thousand workers. The compensation should be scaled accordingly when more robots are introduced.

4. Discussion

4.1 *Comparison with optimal taxation*

The compensation and optimal tax approaches address conceptually different questions. The optimal taxation problem starts by positing a social welfare—typically weighted utilitarian—objective and proceeds by characterizing the tax schedule that maximizes this objective subject to a government budget constraint. In response to a given disruption, this approach would compare the optimal tax-and-transfer system before and after the disruption, keeping the social welfare function fixed, and infer how the optimal tax rates should be adjusted; for an illustration of this exercise in a similar setting as this paper, see, e.g., Ales, Kurnaz, and Sleet (2015). An important alternative approach, explored by Werning (2007), Bierbrauer and Boyer (2014), Lorenz and Sachs (2016), Scheuer and Werning (2017), Bierbrauer, Boyer, and Hansen (2020), avoids taking an explicit stand on the social welfare objective and characterizes the set of Pareto efficient tax systems instead. That is, the goal is to provide bounds on tax rates below which any redistribution necessarily entails winners and losers; in response to a disruption, the boundary of this Pareto set adjusts.

The compensation problem studied in this paper contrasts with and complements both of these alternatives. It places constraints on the realized, individual-level utility gains and losses rather than on the fiscal surplus functional. On the one hand, the main benefit of our approach is that, as in the latter set of papers, it relies only on the Pareto principle: We do not need to choose a social welfare function or make interpersonal comparisons of welfare. Yet, as the optimum approach, it pins down a unique tax reform in response to a disruption rather than a set of possible reforms. A practical advantage of our solution is that the policy response to an economic disruption is given by a reform of the actual (e.g., U.S.) tax schedule rather than a fictitious, optimal one that was not implemented in the first place.[24] On the other hand, the main drawback of our approach is that it is silent about how to redistribute the resulting fiscal surplus or deficit if any.

Figure 9 illustrates how the policy prescriptions of the optimum and the compensation approaches differ, employing a concrete example. Consider the interval disruption studied in Section 3.3 that reduces middle-class workers' wages (top panel of Figure 4). To make the comparison between both approaches transparent, suppose that the tax schedule in the initial (undisrupted) economy is the Rawlsian optimum.[25] The solid red curve in the left panel depicts the compensating tax reform. The dashed blue curve plots the change in tax payments required to implement the new Rawlsian optimum, i.e., the optimum associated with the perturbed wage distribution.[26] While both tax reforms cut the tax liabilities of the most adversely affected (middle-class) workers, the optimum approach responds much more sensitively to the disruption and involves large tax hikes (resp., cuts) at the bottom (resp., top) of the income distribution. The re-

[24]The compensation formula also depends on sufficient statistics (endogenous elasticities and income distribution) estimated with current data rather than evaluated at the optimum—unobserved—tax system.

[25]The rest of the calibration is the same as in Sections 3.1–3.3.

[26]We construct the figure by backing out the changes in the exogenous labor productivity parameters of the CES production function that rationalize the wage disruption.

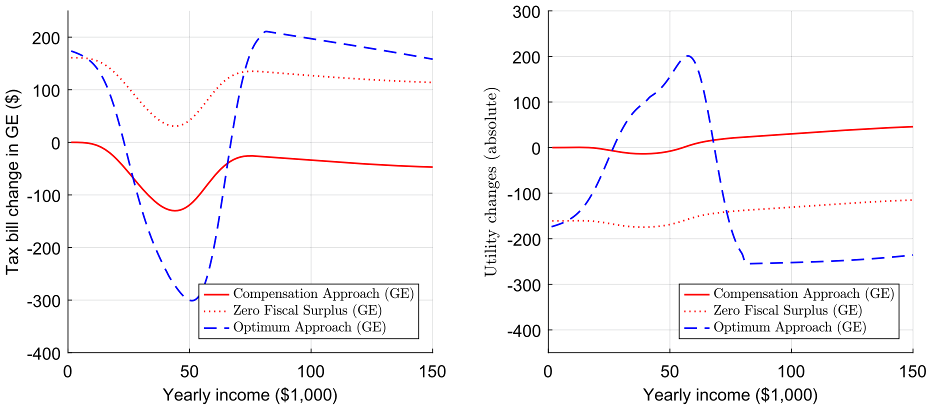

FIGURE 9. Compensation approach versus optimum approach.

sulting utility gains and losses, depicted in the right panel of Figure 9, reflect the shape of these reforms: While the compensation does not induce any welfare changes—up to small approximation errors—by construction (solid red curve), the Rawlsian optimal reform causes large welfare losses among low- and high-income workers, and large welfare gains for the middle class. Notice that the compensating tax reform generates a fiscal deficit of −$161 since everyone faces wage losses. To facilitate the comparison between the two approaches, the dotted line in the left panel plots a uniform upward shift of the tax payments that restores the budget balance. The corresponding utility losses in the right panel are also uniform since we assumed a quasilinear utility function.[27]

The preceding discussion naturally raises the question of how to redistribute the fiscal surplus or allocate the burden of higher taxes if the compensation leads to a fiscal deficit. In the latter case, this question becomes particularly important because, otherwise, the compensation is not feasible. One possibility, of course, would consist of choosing a social welfare function and optimally redistributing the net surplus (whether positive or negative) according to this objective. However, there would then be little benefit to using the compensation rather than the optimum approach to begin with. However, we can also let the policy-maker redistribute the surplus or allocate the losses according to other, not necessarily explicit, objectives. For instance, the redistribution depicted by the dotted curves of Figure 9, which equalizes the welfare losses from the disruption across the population, can be rationalized as the maximization of some underlying social welfare function. However, this social welfare function does not need to be known ex ante, let alone specified analytically, to implement this redistributive objective. In the same vein, the compensation approach allows a policy-maker to design redistributive schemes that respond to various political economy considerations in response to the disruption, for instance, ensuring that the welfare of a given coalition that

[27] Note that the y-axis of Figure 9 gives the absolute utility gains and losses (in dollars, since preferences are quasilinear in consumption). In contrast, Figure 7 represents these utility changes as a percentage of the worker's initial income.

amounts to half of the electorate improves.[28] Rather than specifying a social welfare objective ex ante without knowing a priori how to distribute the resulting utility gains and losses, our approach directly targets the ex post levels of welfare gains and losses across the income distribution. This allows us to achieve specific objectives that the standard approach would find more challenging to handle.

Finally, even though we think that the shape of the compensating tax reform is interesting in its own right—it has recently been a particularly salient policy question—one can also view our paper in a more positive (as opposed to normative) light. The fiscal surplus defines a relevant notion of aggregate welfare gains or losses of a disruption, e.g., a measure of the "gains from trade" (or gains from automation, etc.) that accounts for the distortionary nature of redistributive tax instruments and does not rely on the choice of a social welfare function. In this light, our paper generalizes the analysis of Hendren (2020) to the case where taxes have general-equilibrium effects.

4.2 *Directions for future research*

Our analysis abstracts from several important dimensions that we view as fruitful directions for future research. First, throughout most of our analysis, we have focused on marginal disruptions. Our formulas are thus well suited for compensating the impact of, say, the progressive introduction of robots into the economy (see Section 3.5), less so for the impact of a large one-time event such as the "China shock." A systematic analysis of the compensation for large disruptions would require accounting for the second- and higher-order effects of tax changes on labor supply, wages, and welfare.

Second, our analysis does not allow for multidimensional worker heterogeneity. In particular, the disruptions we consider do not have heterogeneous effects conditional on income. A one-dimensional income tax instrument would no longer be able to compensate for such multidimensional shocks. We can easily add "tags" and implement tax reforms that target specific sectors (say), as long as there is no endogenous switching between sectors. However, in its full generality, the multidimensional compensation problem would require richer policy instruments.

Third, we also ignored the dynamic effects that an economic disruption and its compensation may cause. For instance, the literature on optimal taxation highlights the importance of incorporating the endogenous accumulation of human capital. An economic disruption, e.g., caused by automation, may alter the incentives for workers to obtain higher education. At the same time, the compensation of low-income households may disincentivize them from acquiring human capital if they anticipate tax relief. The level of compensation we derived in our static setting is a first step toward understanding the design of tax changes over the life cycle or over time. Andersen and Bhattacharya (2017, 2020), Andersen, Bhattacharya, and Liu (2020), and, more recently, Dávila and Schaab (2021), who extend the generalized marginal social welfare weights

[28] In a similar vein, Scheuer and Wolitzky (2016) characterize optimal capital taxes under the constraint that a policy must attract the support of a large enough coalition of citizens to be sustainable. Bierbrauer, Boyer, and Peichl (2021) study a set of reforms of a status quo tax schedule that are able to gather majority support.

approach of Saez and Stantcheva (2016) to dynamic environments, provide useful steps in these directions.

Conclusion

The classic policy question of compensating winners and losers from an economic disruption becomes quite involved when the environment features distortionary taxes and general-equilibrium responses. At the same time, both of these considerations are important in many applied and policy settings (e.g., to compensate for the adverse effects of technical change). We derive and analyze a general closed-form formula for the design of the welfare-compensating tax reform and its impact on the government budget. This equation is straightforward to implement in practical applications.

Appendix

Definition of the perturbed equilibrium. After a disruption and a tax reform, the perturbed indirect utility of agent i is given by

$$\tilde{U}_i = u_i\big[\tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i), \tilde{l}_i\big], \tag{23}$$

where the equilibrium labor supplies $\tilde{l}_i = l_i(1+\mu\hat{l}_i)$ and wages $\tilde{w}_i = w_i(1+\mu\hat{w}_i^E + \mu\hat{w}_i)$ are defined by the perturbed first-order condition

$$-\frac{u'_{i,l}\big[\tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i), \tilde{l}_i\big]}{u'_{i,c}\big[\tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i), \tilde{l}_i\big]} = \big[1 - T'(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}'(\tilde{w}_i\tilde{l}_i)\big]\tilde{w}_i \tag{24}$$

and the perturbed wage equation

$$\tilde{w}_i = \tilde{\mathcal{F}}'_i\big(\big\{L_j(1+\mu\hat{l}_j)\big\}_{j\in[0,1]}\big). \tag{25}$$

The perturbed government revenue is given by

$$\tilde{\mathcal{R}} = \int_0^1 \big[T(\tilde{w}_i\tilde{l}_i) + \mu\hat{T}(\tilde{w}_i\tilde{l}_i)\big]\mathrm{d}i. \tag{26}$$

□

Proof of (6). The change in utility of agent i in response to the disruption and tax reform is given by

$$\mu\hat{U}_i \equiv \tilde{U}_i - U_i = u_i\big[\tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i), \tilde{l}_i\big] - u_i\big[w_il_i - T(w_il_i), l_i\big],$$

where $\tilde{w}_i = w_i(1+\mu\hat{w}_i^E + \mu\hat{w}_i)$ and $\tilde{l}_i = l_i(1+\mu\hat{l}_i)$. A first-order Taylor expansion of this equation around the initial equilibrium (as $\mu \to 0$) yields

$$\tilde{U}_i - U_i = \mu\big[\big(1-T'(y_i)\big)\big(y_i\hat{l}_i + y_i\hat{w}_i^E + y_i\hat{w}_i\big) - \hat{T}(y_i)\big]u'_{i,c} + \mu l_i\hat{l}_iu'_{i,l} + o(\mu). \tag{27}$$

However, the first-order condition (2), or the envelope theorem, implies $(1 - T'(y_i))y_i\hat{l}_iu'_{i,c} + l_i\hat{l}_iu'_{i,l} = 0$. We thus obtain (6). □

Proof of (7). The perturbed first-order condition of agent i in response to the disruption and tax reform is given by

$$0 = \left[1 - T'(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}'(\tilde{w}_i\tilde{l}_i)\right]\tilde{w}_i u'_{i,c}\left[\tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i), \tilde{l}_i\right] + u'_{i,l}\left[\tilde{w}_i\tilde{l}_i - T(\tilde{w}_i\tilde{l}_i) - \mu\hat{T}(\tilde{w}_i\tilde{l}_i), \tilde{l}_i\right].$$

A first-order Taylor expansion of this equation around the initial equilibrium (as $\mu \to 0$) gives

$$\begin{aligned}0 = {} & \left[\left(1 - T'(y_i)\right)^2 w_i^2 u''_{i,cc} + 2\left(1 - T'(y_i)\right) w_i u''_{i,cl} + u''_{i,ll} - w_i^2 T''(y_i) u'_{i,c}\right] l_i \hat{l}_i \\ & + \left[\left(1 - T'(y_i)\right)^2 w_i l_i u''_{i,cc} + \left(1 - T'(y_i)\right) l_i u''_{i,cl} + \left(1 - T'(y_i) - w_i l_i T''(y_i)\right) u'_{i,c}\right] \\ & \times w_i\left(\hat{w}_i^E + \hat{w}_i\right) \\ & - w_i u'_{i,c}\hat{T}'(y_i) - \left[\left(1 - T'(y_i)\right) w_i u''_{i,cc} + u''_{i,cl}\right]\hat{T}(y_i).\end{aligned}$$

The Hicksian (compensated) labor supply elasticity e_i^r and the income effect parameter e_i^n are, respectively, equal to (see, e.g., Saez (2001, p. 227))

$$\begin{aligned} e_i^r &= \frac{\dfrac{u'_{i,l}}{l_i}}{\left(\dfrac{u'_{i,l}}{u'_{i,c}}\right)^2 u''_{i,cc} - 2\left(\dfrac{u'_{i,l}}{u'_{i,c}}\right) u''_{i,cl} + u''_{i,ll}}, \\ e_i^n &= \frac{\left(\dfrac{u'_{i,l}}{u'_{i,c}}\right)^2 u''_{i,cc} - \left(\dfrac{u'_{i,l}}{u'_{i,c}}\right) u''_{i,cl}}{\left(\dfrac{u'_{i,l}}{u'_{i,c}}\right)^2 u''_{i,cc} - 2\left(\dfrac{u'_{i,l}}{u'_{i,c}}\right) u''_{i,cl} + u''_{i,ll}}. \end{aligned} \tag{28}$$

Solving the previous equation for $\hat{l}_i$ then implies

$$\hat{l}_i = \frac{\left(1 - p(y_i)\right)e_i^r - e_i^n}{1 + p(y_i)e_i^r}\left(\hat{w}_i^E + \hat{w}_i\right) - \frac{e_i^r}{1 + p(y_i)e_i^r}\frac{\hat{T}'(y_i)}{1 - T'(y_i)} + \frac{e_i^n}{1 + p(y_i)e_i^r}\frac{\hat{T}(y_i)}{\left(1 - T'(y_i)\right)y_i}.$$

Using the definitions of the elasticities along the nonlinear budget constraint ε_i^r, ε_i^n, ε_i^w leads to (7). □

Proof of (9). Consider an exogenous disruption $\mu\hat{\mathcal{F}}^E$ of the production function and a tax reform $\mu\hat{T}$, with $\mu > 0$. The corresponding wage disruption is defined by

$$\hat{w}_i^E = \frac{\partial \hat{\mathcal{F}}^E}{\partial L_i}\left(\{L_j\}_{j\in[0,1]}\right).$$

Denote by $\mu\hat{w}_i$ and $\mu\hat{l}_i$ the first-order endogenous percentage changes as $\mu \to 0$ in the wage and labor supply of type i, and let $\tilde{w}_i = w_i(1+\mu\hat{w}_i^E + \mu\hat{w}_i)$ and $\tilde{l}_i = l_i(1+\mu\hat{l}_i)$. In the perturbed equilibrium, the wage is equal to the marginal product of the labor of the corresponding type:

$$\tilde{w}_i = \frac{\partial\big[\mathcal{F}+\mu\hat{\mathcal{F}}^E\big]}{\partial L_i}\big(\{L_j(1+\mu\hat{l}_j)\}_{j\in[0,1]}\big).$$

The Gateaux derivative of the wage functional is given by

$$\begin{aligned}\hat{w}_i &\equiv \lim_{\mu\to 0}\frac{1}{\mu w_i}\big[\tilde{w}_i - w_i - \mu\hat{w}_i^E\big] \\ &= \lim_{\mu\to 0}\frac{1}{\mu w_i}\bigg\{\frac{\partial\big[\mathcal{F}+\mu\hat{\mathcal{F}}^E\big]}{\partial L_i}\big(\{L_j(1+\mu\hat{l}_j)\}_{j\in[0,1]}\big) \\ &\quad - \frac{\partial\mathcal{F}}{\partial L_i}\big(\{L_j\}_{j\in[0,1]}\big) - \mu\frac{\partial\hat{\mathcal{F}}^E}{\partial L_i}\big(\{L_j\}_{j\in[0,1]}\big)\bigg\}.\end{aligned}$$

This expression is equal to

$$\hat{w}_i = \frac{1}{w_i}\int_0^1 \hat{l}_j L_j\frac{\partial^2\mathcal{F}(\boldsymbol{L})}{\partial L_i\partial L_j}\,dj.$$

The own-wage (or inverse labor demand) and cross-wage elasticities are defined by

$$\frac{L_j}{w_i}\frac{\partial w_i}{\partial L_j} \equiv \gamma_{ij} - \frac{1}{\varepsilon_j^d}\delta(j-i)$$

for all i, j. In particular, when the production function is CES, the cross-wage elasticities are given by, for $i \neq j$,

$$\begin{aligned}\frac{L_j}{w_i}\frac{\partial^2\mathcal{F}(\boldsymbol{L})}{\partial L_i\partial L_j} &= \frac{L_j}{w_i}\frac{\partial}{\partial L_j}\left\{\theta_i L_i^{-1/\varepsilon^d}\left[\int_0^1 \theta_j L_j^{1-1/\varepsilon^d}\,dj\right]^{\frac{1}{\varepsilon^d-1}}\right\} \\ &= \frac{1}{\varepsilon^d}\frac{\theta_j L_j^{1-1/\varepsilon^d}}{\displaystyle\int_0^1 \theta_k L_k^{1-1/\varepsilon^d}\,dk} = \frac{1}{\varepsilon^d}\frac{w_j L_j}{\mathcal{F}(\boldsymbol{L})} \equiv \gamma_j,\end{aligned}$$

and the own-wage elasticities by

$$\begin{aligned}\frac{L_i}{w_i}\frac{\partial^2\mathcal{F}(\boldsymbol{L})}{\partial L_i^2} &= \frac{L_i}{w_i}\frac{\partial}{\partial L_i}\left\{\theta_i L_i^{-1/\varepsilon^d}\left[\int_0^1 \theta_j L_j^{1-1/\varepsilon^d}\,dj\right]^{\frac{1}{\varepsilon^d-1}}\right\} \\ &= \gamma_i - \frac{1}{\varepsilon^d}\frac{1}{w_i}\theta_i L_i^{-1/\varepsilon^d}\left[\int_0^1 \theta_j L_j^{1-1/\varepsilon^d}\,dj\right]^{\frac{1}{\varepsilon^d-1}}\delta(0) = \gamma_i - \frac{1}{\varepsilon^d}\delta(0).\end{aligned}$$

Substituting into the formula for $\hat{w}_i$ leads to

$$\hat{w}_i = \int_0^1 \hat{l}_j \left\{ \gamma_{ij} - \frac{1}{\varepsilon_j^d} \delta(j-i) \right\} dj,$$

which leads to (9). □

Proof of (10). The effect of the wage disruption and the corresponding compensating tax reform on government budget is given by

$$\hat{R} = \lim_{\mu \to 0} \frac{1}{\mu} \left\{ \int_0^1 \left[T(\tilde{w}_i \tilde{l}_i) + \mu \hat{T}(\tilde{w}_i \tilde{l}_i) \right] \mathrm{d}i - \int_0^1 T(w_i l_i) \mathrm{d}i \right\}.$$

A first-order Taylor expansion around the initial equilibrium easily leads to (10). □

Proof of Lemma 1. This lemma follows from Sachs, Tsyvinski, and Werquin (2020); for completeness, we give its proof here. Substituting for $\hat{w}_i$ into (7) using (9) leads to

$$\hat{l}_i = \phi_i \hat{l}_i^E + \phi_i \varepsilon_i^w \int_0^1 \gamma_{ij} \hat{l}_j \, dj, \tag{29}$$

where we let $\phi_i = \frac{1}{1+\varepsilon_i^w/\varepsilon_i^d}$ and

$$\hat{l}_i^E = \varepsilon_i^w \hat{w}_i^E - \varepsilon_i^r \frac{\hat{T}'(y_i)}{1-T'(y_i)} + \varepsilon_i^n \frac{\hat{T}(y_i)}{\left(1-T'(y_i)\right) y_i}. \tag{30}$$

This is a Fredholm integral equation in $\{\hat{l}_i\}_{i \in [0,1]}$. To solve for the labor supply changes for a general production function, substitute for $\hat{l}_j$ in the integral to obtain

$$\begin{aligned}
\hat{l}_i &= \phi_i \hat{l}_i^E + \phi_i \varepsilon_i^w \int_0^1 \gamma_{ij} \left[\phi_j \hat{l}_j^E + \phi_j \varepsilon_j^w \int_0^1 \gamma_{jk} \hat{l}_k \, dk \right] dj \\
&= \left[\phi_i \hat{l}_i^E + \phi_i \varepsilon_i^w \int_0^1 \gamma_{ij} \phi_j \hat{l}_j^E \, dj \right] + \phi_i \varepsilon_i^w \int_0^1 \left[\int_0^1 \gamma_{ik} \phi_k \varepsilon_k^w \gamma_{kj} \, dk \right] \hat{l}_j \, dj \\
&\equiv \left[\phi_i \hat{l}_i^E + \phi_i \varepsilon_i^w \int_0^1 \gamma_{ij} \phi_j \hat{l}_j^E \, dj \right] + \phi_i \varepsilon_i^w \int_0^1 \Gamma_{ij}^{(1)} \hat{l}_j \, dj,
\end{aligned}$$

where $\Gamma_{ij}^{(0)} = \gamma_{ij}$ and $\Gamma_{ij}^{(1)} = \int_0^1 \Gamma_{ik}^{(0)} \phi_k \varepsilon_k^w \gamma_{kj} \, dk$. By induction, it is easy to show that, for all $N \geq 0$,

$$\hat{l}_i = \left[\phi_i \hat{l}_i^E + \phi_i \varepsilon_i^w \int_0^1 \left\{ \sum_{n=0}^N \Gamma_{ij}^{(n)} \right\} \phi_j \hat{l}_j^E \, dj \right] + \phi_i \varepsilon_i^w \int_0^1 \Gamma_{ij}^{(N+1)} \hat{l}_j \, dj,$$

where, for all $n \geq 0$, $\Gamma_{ij}^{(n+1)} = \int_0^1 \Gamma_{ik}^{(n)} \phi_k \varepsilon_k^w \gamma_{kj} \, dk$. The condition $\int_0^1 \int_0^1 | \phi_i \varepsilon_i^w \gamma_{ij} |^2 \, di \, dj < 1$ ensures that the series $\sum_{n=0}^N \Gamma_{ij}^{(n)}$ converges as $N \to \infty$. This implies (13). Note that we can write the endogenous wage changes as

$$\hat{w}_i = \frac{\phi_i}{\varepsilon_i^d}\left[-\varepsilon_i^w \hat{w}_i^E + \varepsilon_i^r \frac{\hat{T}'(y_i)}{1-T'(y_i)} - \varepsilon_i^n \frac{\hat{T}(y_i)}{(1-T'(y_i))y_i}\right]$$
$$+ \phi_i \int_0^1 \Gamma_{ij}\phi_j\left[\varepsilon_j^w \hat{w}_j^E - \varepsilon_j^r \frac{\hat{T}'(y_j)}{1-T'(y_j)} + \varepsilon_j^n \frac{\hat{T}(y_j)}{(1-T'(y_j))y_j}\right] dj, \tag{31}$$

which follows from (7) and (13). Finally, if the initial production function is CES, the cross-wage elasticities γ_{ij} depend only on j. Multiplying both sides of (29) by γ_i and integrating from 0 to 1 then leads to

$$\int_0^1 \gamma_i \hat{l}_i \, di = \int_0^1 \gamma_i \phi_i \hat{l}_i^E \, di + \left(\int_0^1 \gamma_i \phi_i \varepsilon_i^w \, di\right)\left(\int_0^1 \gamma_j \hat{l}_j \, dj\right)$$
$$= \frac{\int_0^1 \gamma_i \phi_i \hat{l}_i^E \, di}{1 - \int_0^1 \gamma_i \phi_i \varepsilon_i^w \, di}.$$

Substituting this expression into (29) yields

$$\hat{l}_i = \phi_i \hat{l}_i^E + \phi_i \varepsilon_i^w \int_0^1 \Gamma_j \phi_j \hat{l}_j^E \, dj,$$

where $\Gamma_j \equiv \gamma_j/(1 - \int_0^1 \gamma_k \phi_k \varepsilon_k^w \, dk)$. Using the expression of the cross-wage elasticities $\gamma_k = y_k/(\varepsilon^d \mathbb{E}y)$, we can write $1 - \int_0^1 \gamma_k \phi_k \varepsilon_k^w \, dk = \int_0^1 (y_k/\mathbb{E}y)(1 - \phi_k \varepsilon_k^w/\varepsilon^d) \, dk$. Using $\phi_k = 1/(1 + \varepsilon_k^w/\varepsilon^d)$ finally gives $\Gamma_j = \gamma_j/(\int_0^1 \phi_k y_k/\mathbb{E}y \, dk)$. □

Proof of Lemma 2. Substitute for $\hat{w}_i^E + \hat{w}_i$ in (6) using (7) to get

$$\hat{T}(y_i) = \frac{1}{\varepsilon_i^w}(1-T'(y_i))y_i \hat{l}_i + \frac{\varepsilon_i^r}{\varepsilon_i^w} y_i \hat{T}'(y_i) - \frac{\varepsilon_i^n}{\varepsilon_i^w}\hat{T}(y_i).$$

Using the expression we derived above for $\hat{l}_i$ leads to

$$\hat{T}(y_i) = \left[\frac{1}{\varepsilon_i^w}(1-T'(y_i))y_i \phi_i \hat{l}_i^E + \frac{\varepsilon_i^r}{\varepsilon_i^w} y_i \hat{T}'(y_i) - \frac{\varepsilon_i^n}{\varepsilon_i^w}\hat{T}(y_i)\right]$$
$$+ (1-T'(y_i))y_i \phi_i \int_0^1 \Gamma_{ij} \phi_j \hat{l}_j^E \, dj.$$

Replacing the partial-equilibrium labor supply changes $\hat{l}_i^E$ with their expression (30) allows us to rewrite this equation as

$$\hat{T}(y_i) = (1-T'(y_i))y_i \phi_i \left[\hat{w}_i^E + \int_0^1 \Gamma_{ij} \phi_j \varepsilon_j^w \hat{w}_j^E \, dj\right]$$
$$+ \frac{\varepsilon_i^r/\varepsilon_i^d}{1+\varepsilon_i^w/\varepsilon_i^d} y_i \hat{T}'(y_i) - \frac{\varepsilon_i^n/\varepsilon_i^d}{1+\varepsilon_i^w/\varepsilon_i^d}\hat{T}(y_i)$$

$$- \left(1 - T'(y_i)\right) y_i \phi_i \int_0^1 \Gamma_{ij} \phi_j \left[\varepsilon_j^r \frac{\hat{T}'(y_j)}{1 - T'(y_j)} - \varepsilon_j^n \frac{\hat{T}(y_j)}{\left(1 - T'(y_j)\right) y_j} \right] dj.$$

This leads to (14). Rearranging and summing over all agents leads to

$$\int_0^1 \frac{\hat{T}(y_i)}{1 - T'(y_i)} = \int_0^1 y_i \hat{\Omega}_i^E \, di + \int_0^1 \frac{\phi_i}{\varepsilon_i^d} y_i \left[\varepsilon_i^r \frac{\hat{T}'(y_i)}{1 - T'(y_i)} - \varepsilon_i^n \frac{\hat{T}(y_i)}{\left(1 - T'(y_i)\right) y_i} \right] di$$
$$- \int_0^1 \phi_i y_i \Lambda_i \, di.$$

The last integral in this expression can be rewritten as

$$\int_0^1 \phi_i y_i \Lambda_i \, di = \int_0^1 \left\{ \int_0^1 \phi_i y_i \Gamma_{ij} \, di \right\} \phi_j \left[\varepsilon_j^r \frac{\hat{T}'(y_j)}{1 - T'(y_j)} - \varepsilon_j^n \frac{\hat{T}(y_j)}{\left(1 - T'(y_j)\right) y_j} \right] dj.$$

An application of Euler's homogeneous function theorem (see Lemma 2, (24) in Sachs, Tsyvinski, and Werquin (2020)) implies that $\int_0^1 \phi_i y_i \Gamma_{ij} \, di = (1/\varepsilon_j^d) y_j$. We thus obtain $\mathbb{E}[\hat{T}(y_i)/(1 - T'(y_i))] = \mathbb{E}[y_i \hat{\Omega}_i^E]$. □

Proof of Proposition 1. Equation (11) is a special case of (14) obtained by setting $\Gamma_{ij} = 0$ and letting $\varepsilon^d \to \infty$. Using this formula for the compensating tax reform in partial equilibrium along with $\hat{w}_i = 0$, the fiscal surplus (10) can be expressed as

$$\hat{R} = \int_0^1 \left[\hat{w}_i^E + T'(y_i) \hat{l}_i \right] y_i \, di.$$

Differentiate $\hat{T}(y)$ with respect to y in (11) to obtain the marginal tax rates of the compensating tax reform. Letting $y_i' \equiv dy_i/di$, we obtain

$$\hat{T}'(y_i) = \frac{1}{y_i'} \left[-y_i' T''(y_i) y_i \hat{w}_i^E + \left(1 - T'(y_i)\right) y_i' \hat{w}_i^E + \left(1 - T'(y_i)\right) y_i \frac{d\hat{w}_i^E}{di} \right].$$

Using $p(y_i) = y_i T''(y_i)/(1 - T'(y_i))$, we can thus write

$$\hat{l}_i = \varepsilon_i^w \hat{w}_i^E - \varepsilon_i^r \left[\left(1 - p(y_i)\right) \hat{w}_i^E + \frac{y_i}{y_i'} \frac{d\hat{w}_i^E}{di} \right] + \varepsilon_i^n \hat{w}_i^E = -\varepsilon_i^r \frac{y_i}{y_i'} \frac{d\hat{w}_i^E}{di},$$

where we used the fact that $\varepsilon_i^w = (1 - p(y_i)) \varepsilon_i^r - \varepsilon_i^n$. Substituting into the above expression for $\hat{R}$ and changing variables from skills to incomes leads to (12). □

Proof of Proposition 2. Since there is a one-to-one map between skills i and incomes y_i, we can change variables to express the ODE (14) in terms of incomes. We obtain

$$\hat{T}'(y) - \left(1 - p(y) + \frac{\varepsilon_y^d}{\varepsilon_y^r}\right) \frac{\hat{T}(y)}{y} = -\left(1 - T'(y)\right) \frac{\varepsilon_y^d}{\varepsilon_y^r} \mathcal{A}(y),$$

where we used $1/(\phi_i \varepsilon_i^r/\varepsilon_i^d) = 1 - p(y_i) + (\varepsilon_i^d - \varepsilon_i^n)/\varepsilon_i^r$ and where

$$\begin{aligned}\mathcal{A}(y) &\equiv \phi_y^{-1}\hat{\Omega}^E(y) + \int_{\underline{y}}^{\bar{y}} \Gamma_{y,z}\phi_z \left[-\varepsilon_z^r \frac{\hat{T}'(z)}{1-T'(z)} + \varepsilon_z^n \frac{\hat{T}(z)}{\big(1-T'(z)\big)z}\right] dz \\ &= \phi_y^{-1}\hat{\Omega}^E(y) + \Lambda(y),\end{aligned}$$

with

$$\hat{\Omega}^E(y) = \phi_y \hat{w}^E(y) + \phi_y \int_{\underline{y}}^{\bar{y}} \Gamma_{y,z}\phi_z \varepsilon_z^w \hat{w}^E(y)\, dz.$$

We can solve this equation as a first-order ODE. The general solution to the homogeneous equation is given by

$$\hat{T}_H(y) = Ce^{-\int_y^{\bar{y}}(1-p(z)+\frac{\varepsilon_z^d}{\varepsilon_z^r})\frac{dz}{z}} = C\frac{\big(1-T'(y)\big)y}{\big(1-T'(\bar{y})\big)\bar{y}} e^{-\int_y^{\bar{y}}\frac{\varepsilon_z^d}{\varepsilon_z^r}\frac{dz}{z}},$$

where C is a constant and where the second equality uses the fact that $p(z)/z = T''(z)/(1-T'(z))$, so that $\int_x^y (1-p(z))\,dz/z = \log[((1-T'(y))y)/((1-T'(x))x)]$. Using the method of variation of the parameter, we find a particular solution of the form

$$\hat{T}_P(y) = C(y)\frac{\big(1-T'(y)\big)y}{\big(1-T'(\bar{y})\big)\bar{y}} e^{-\int_y^{\bar{y}}\frac{\varepsilon_z^d}{\varepsilon_z^r}\frac{dz}{z}},$$

where the function $C(y)$ satisfies

$$\frac{C(y)}{\big(1-T'(\bar{y})\big)\bar{y}} = \int_y^{\bar{y}} \frac{\varepsilon_x^d}{\varepsilon_x^r} e^{\int_x^{\bar{y}}\frac{\varepsilon_z^d}{\varepsilon_z^r}\frac{dz}{z}} \mathcal{A}(x)\frac{dx}{x}.$$

The general solution to (14) is thus equal to

$$\hat{T}(y) = \big(1-T'(y)\big)y\int_y^{\bar{y}} \Pi(y,x)\mathcal{A}(x)\,dx + C\frac{\big(1-T'(y)\big)y}{\big(1-T'(\bar{y})\big)\bar{y}} e^{-\int_y^{\bar{y}}\frac{\varepsilon_z^d}{\varepsilon_z^r}\frac{dz}{z}},$$

where $\Pi(y,x) = (\varepsilon_x^d/\varepsilon_x^r x)e^{-\int_y^x(\varepsilon_z^d/\varepsilon_z^r)\,dz/z}$. If the initial tax schedule is Pareto efficient, the tax reform should be $\hat{T}(\cdot) = 0$ in the absence of a disruption ($\hat{\Omega}^E(\cdot) = 0$). (Note that as $\bar{y}\to\infty$, the last term in the previous expression converges to zero for any value of C.)

If the production function is CES, $\Gamma_{y,z}$ does not depend on y and, hence, $\Lambda(y)$ is equal to a constant $\Lambda \in \mathbb{R}$. To find Λ, recall that $\mathbb{E}[\hat{T}(y)/(1-T'(y))] = \mathbb{E}[y\hat{\Omega}^E(y)]$. Substituting the solution to the ODE into this condition (setting $C = 0$) yields

$$\Lambda = \frac{\mathbb{E}\big[y\hat{\Omega}^E(y)\big] - \mathbb{E}\Big[y\int_y^{\bar{y}}\Pi(y,x)\phi_x^{-1}\hat{\Omega}^E(x)\,dx\Big]}{\mathbb{E}\Big[y\int_y^{\bar{y}}\Pi(y,x)\,dx\Big]}.$$

For a general production function, we can use the ODE and insert its solution into the definition of the auxiliary function $\mathcal{A}(\cdot)$ to rewrite it as

$$\begin{aligned}\mathcal{A}(y) &= \phi_y^{-1}\hat{\Omega}^E(y) - \int_{\underline{y}}^{\bar{y}} \Gamma_{y,z}\phi_z\left[\left(\varepsilon_z^w + \varepsilon_z^d\right)\frac{\hat{T}(z)}{\left(1-T'(z)\right)z} - \varepsilon_z^d\mathcal{A}(z)\right]dz \\ &= \phi_y^{-1}\hat{\Omega}^E(y) + \int_{\underline{y}}^{\bar{y}} \Gamma_{y,z}\phi_z\varepsilon_z^d\mathcal{A}(z)\,dz - \int_{\underline{y}}^{\bar{y}} \Gamma_{y,z}\varepsilon_z^d\left[\int_z^{\bar{y}} \Pi(z,x)\mathcal{A}(x)\,dx\right]dz,\end{aligned}$$

where the second equality uses $\phi_z(\varepsilon_z^w + \varepsilon_z^d) = \varepsilon_z^d$. Inverting the order of the two integrals in the last line implies that this expression can be rewritten as

$$\mathcal{A}(y) = \phi_y^{-1}\hat{\Omega}^E(y) + \int_{\underline{y}}^{\bar{y}}\left[\Gamma_{y,z}\phi_z\varepsilon_z^d - \int_{\underline{y}}^{z} \Pi(x,z)\Gamma_{y,x}\varepsilon_x^d\,dx\right]\mathcal{A}(z)\,dz.$$

However, this is a standard linear Fredholm integral equation, with kernel equal to $K_{y,z}^{(0)}$, where

$$K_{y,z}^{(0)} \equiv \Gamma_{y,z}\varepsilon_z^d\phi_z - \int_{\underline{y}}^{z} \Pi(x,z)\Gamma_{y,x}\varepsilon_x^d\,dx.$$

Assume that

$$\int_{[\underline{y},\bar{y}]^2} \left|K_{y,z}^{(0)}\right|^2 dy\,dz < 1,$$

which ensures the convergence of the series $\sum_{n=0}^{\infty} K_{y,z}^{(n)}$ defined below. Following steps analogous to the proof of Lemma 1, we get

$$\mathcal{A}(y) = \phi_y^{-1}\hat{\Omega}^E(y) + \int_{\underline{y}}^{\bar{y}}\left\{\sum_{n=0}^{\infty} K_{y,z}^{(n)}\right\}\phi_z^{-1}\hat{\Omega}^E(z)\,dz,$$

with $K_{y,z}^{(n)} = \int_{\underline{y}}^{\bar{y}} K_{y,x}^{(n-1)}K_{x,z}^{(0)}\,dx$ for all n. Inverting the integrals one more time leads to

$$\begin{aligned}&\int_{\underline{y}}^{\bar{y}} K_{y,z}^{(0)}\phi_z^{-1}\hat{\Omega}^E(z)\,dz \\ &\quad= \int_{\underline{y}}^{\bar{y}} \Gamma_{y,z}\varepsilon_z^d\hat{\Omega}^E(z)\,dz - \int_{\underline{y}}^{\bar{y}} \Gamma_{y,z}\varepsilon_z^d\left[\int_z^{\bar{y}} \Pi(z,x)\phi_x^{-1}\hat{\Omega}^E(x)\,dx\right]dz \\ &\quad\equiv \int_{\underline{y}}^{\bar{y}} \lambda^{(0)}(y,z)\,dz,\end{aligned}$$

where we denote

$$\lambda^{(0)}(y,z) = \Gamma_{y,z}\varepsilon_z^d\left[\phi_z\left(\phi_z^{-1}\hat{\Omega}^E(z)\right) - \int_z^{\bar{y}} \Pi(z,x)\left(\phi_x^{-1}\hat{\Omega}^E(x)\right)dx\right].$$

Now, for any $n \geq 1$, we can write

$$\begin{aligned}\int_{\underline{y}}^{\bar{y}} K_{y,z}^{(n)}\big[\phi_z^{-1}\hat{\Omega}^E(z)\big]\,dz &= \int_{\underline{y}}^{\bar{y}} K_{y,x}^{(n-1)}\left[\int_{\underline{y}}^{\bar{y}} K_{x,z}^{(0)}\phi_z^{-1}\hat{\Omega}^E(z)\,dz\right]dx \\ &= \int_{\underline{y}}^{\bar{y}} K_{y,z}^{(n-1)}\left[\int_{\underline{y}}^{\bar{y}} \lambda^{(0)}(z,x)\,dx\right]dz,\end{aligned}$$

so that

$$\begin{aligned}&\sum_{n=0}^{\infty}\left\{\int_{\underline{y}}^{\bar{y}} K_{y,z}^{(n)}\phi_z^{-1}\hat{\Omega}^E(z)\,dz\right\} \\ &\quad= \int_{\underline{y}}^{\bar{y}} \lambda^{(0)}(y,z)\,dz + \sum_{n=1}^{\infty}\left\{\int_{\underline{y}}^{\bar{y}} K_{y,z}^{(n-1)}\left[\int_{\underline{y}}^{\bar{y}} \lambda^{(0)}(z,x)\,dx\right]dz\right\} \\ &\quad= \Lambda^{(1)}(y) + \sum_{n=0}^{\infty}\left\{\int_{\underline{y}}^{\bar{y}} K_{y,z}^{(n)}\Lambda^{(1)}(z)\,dz\right\},\end{aligned}$$

where we denote

$$\begin{aligned}\Lambda^{(1)}(y) &\equiv \int_{\underline{y}}^{\bar{y}} \lambda^{(0)}(y,z)\,dz \\ &= \int_{\underline{y}}^{\bar{y}} \Gamma_{y,z}\varepsilon_z^d\left[\phi_z\big(\phi_z^{-1}\hat{\Omega}^E(z)\big) - \int_z^{\bar{y}} \Pi(z,x)\big(\phi_x^{-1}\hat{\Omega}^E(x)\big)\,dx\right]dz.\end{aligned}$$

By induction, repeating the above steps for $n \geq 2$ leads to

$$\sum_{n=0}^{\infty}\left\{\int_{\underline{y}}^{\bar{y}} K_{y,z}^{(n)}\phi_z^{-1}\hat{\Omega}^E(z)\,dz\right\} = \sum_{n=1}^{N}\Lambda^{(n)}(y) + \sum_{n=0}^{\infty}\left\{\int_{\underline{y}}^{\bar{y}} K_{y,z}^{(n)}\Lambda^{(N)}(z)\,dz\right\}$$

for all N, where, for all $n \geq 2$,

$$\begin{aligned}\Lambda^{(n)}(y) &= \int_{\underline{y}}^{\bar{y}} K_{y,z}^{(0)}\Lambda^{(n-1)}(z)\,dz \\ &= \int_{\underline{y}}^{\bar{y}} \Gamma_{y,z}\varepsilon_z^d\left[\phi_z\Lambda^{(n-1)}(z) - \int_z^{\bar{y}} \Pi(z,x)\Lambda^{(n-1)}(x)\,dx\right]dz.\end{aligned}$$

Assuming that the series converges as $N \to \infty$, we finally obtain

$$\mathcal{A}(y) = \phi_y^{-1}\hat{\Omega}^E(y) + \sum_{n=1}^{\infty}\Lambda^{(n)}(y).$$

For completeness, let us compute $\Lambda(z)$ from the series representation when the production is CES. In this case, recall that $\Gamma_{y,z} = 1/(\varepsilon^d \mathbb{E}[y\phi_y])zf(z)$, so that

$$\Lambda^{(1)}(y) \equiv \frac{1}{\mathbb{E}[y\phi_y]} \int_{\underline{y}}^{\bar{y}} z\left[\phi_z\left(\phi_z^{-1}\hat{\Omega}^E(z)\right) - \int_z^{\bar{y}} \Pi(z,x)\left(\phi_x^{-1}\hat{\Omega}^E(x)\right) dx\right] f(z)\, dz$$

$$= \frac{1}{\mathbb{E}[y\phi_y]}\mathbb{E}\left[z\hat{\Omega}^E(z)\right] - \frac{\mathbb{E}\left[z\int_z^{\bar{y}} \Pi(z,x)\left(\phi_x^{-1}\hat{\Omega}^E(x)\right) dx\right]}{\mathbb{E}[y\phi_y]}.$$

Note that $\Lambda^{(1)}(y) \equiv \Lambda^{(1)}$ is a constant that does not depend on y. By induction, assuming that $\Lambda^{(n-1)}(z)$ is a constant, we get, for any $n \geq 2$,

$$\Lambda^{(n)}(y) = \frac{1}{\mathbb{E}[y\phi_y]} \int_{\underline{y}}^{\bar{y}} z\left[\phi_z \Lambda^{(n-1)} - \int_z^{\bar{y}} \Pi(z,x)\Lambda^{(n-1)}\, dx\right] f(z)\, dz$$

$$= \Lambda^{(n-1)} \frac{\mathbb{E}[y\phi_y] - \mathbb{E}\left[z\int_z^{\bar{y}} \Pi(z,x)\, dx\right]}{\mathbb{E}[y\phi_y]},$$

which is a constant. We thus obtain

$$\sum_{n=1}^{\infty} \Lambda^{(n)} = \frac{\mathbb{E}\left[z\hat{\Omega}^E(z)\right] - \mathbb{E}\left[z\int_z^{\bar{y}} \Pi(z,x)\left(\phi_x^{-1}\hat{\Omega}^E(x)\right) dx\right]}{\mathbb{E}[y\phi_y]}$$

$$+ \sum_{n=2}^{\infty}\left(1 - \frac{\mathbb{E}\left[z\int_z^{\bar{y}} \Pi(z,x)\, dx\right]}{\mathbb{E}[y\phi_y]}\right)\Lambda^{(n-1)}.$$

Solving for $\Lambda \equiv \sum_{n=1}^{\infty} \Lambda^{(n)}$ leads to

$$\sum_{n=1}^{\infty} \Lambda^{(n)} = \frac{\mathbb{E}\left[z\hat{\Omega}^E(z)\right] - \mathbb{E}\left[z\int_z^{\bar{y}} \Pi(z,x)\left(\phi_x^{-1}\hat{\Omega}^E(x)\right) dx\right]}{\mathbb{E}\left[z\int_z^{\bar{y}} \Pi(z,x)\, dx\right]},$$

which is indeed the expression we found above.

We finally compute the fiscal surplus (10). Substituting for $\hat{l}_i$ using (7) and for $\hat{w}_i^E + \hat{w}_i$ using (6) in this expression, we can write

$$\hat{R} = \int_0^1 \hat{T}(y_i)\, di - \int_0^1 T'(y_i)y_i\left[\varepsilon_i^r \frac{\hat{T}'(y_i)}{1 - T'(y_i)} - \left(1 + \varepsilon_i^w + \varepsilon_i^n\right)\frac{\hat{T}(y_i)}{\left(1 - T'(y_i)\right)y_i}\right] di.$$

The ODE (14) can be rewritten as

$$\frac{\hat{T}'(y_i)}{1 - T'(y_i)} = \left(1 - p(y_i) + \frac{\varepsilon_i^d}{\varepsilon_i^r}\right)\frac{\hat{T}(y_i)}{\left(1 - T'(y_i)\right)y_i} - \frac{\varepsilon_i^d}{\varepsilon_i^r}\phi_i^{-1}\hat{\Omega}_i^E - \frac{\varepsilon_i^d}{\varepsilon_i^r}\Lambda_i.$$

Using this equation to substitute for $\hat{T}'(y_i)$ in the fiscal surplus expression yields

$$\hat{R} = \int_0^1 \left[1 + (1 - \varepsilon_i^d)\frac{T'(y_i)}{1 - T'(y_i)}\right]\hat{T}(y_i)\,di + \int_0^1 T'(y_i)y_i\varepsilon_i^d\big[\phi_i^{-1}\hat{\Omega}_i^E + \Lambda_i\big]\,di.$$

Using the relationship $\mathbb{E}[\hat{T}(y)/(1-T'(y))] = \mathbb{E}[y\hat{\Omega}_y^E]$ allows us to rewrite the first integral on the right-hand side as $\int_0^1 y_i\hat{\Omega}_i^E\,di - \int_0^1 T'(y_i)/(1-T'(y_i))\varepsilon_i^d\hat{T}(y_i)\,di$. Using the solution for $\hat{T}$ leads to

$$\begin{aligned}\hat{R} = &\int_0^1 y_i\hat{\Omega}_i^E\,di \\ &+ \int_0^1 T'(y_i)y_i\varepsilon_i^d\left\{\big[\phi_i^{-1}\hat{\Omega}_i^E + \Lambda_i\big] - \int_{y_i}^{\bar{y}} \Pi(y_i, y_j)\big[\phi_j^{-1}\hat{\Omega}_j^E + \Lambda(y_j)\big]\,dy_j\right\}di.\end{aligned}$$

Changing variables from skills to incomes and integrating the last term by parts, letting $X(z) \equiv \phi_z^{-1}\hat{\Omega}^E(z) + \Lambda(z)$, leads to

$$\begin{aligned}\hat{R} = &\,\mathbb{E}\big[y\hat{\Omega}^E(y)\big] - \mathbb{E}\left[T'(y)y\left(\int_y^{\bar{y}} \frac{\varepsilon_y^d}{\varepsilon_z^d}\Pi(y,z)\varepsilon_z^r zX'(z)\,dz\right)\right] \\ &+ \mathbb{E}\big[T'(y)y\varepsilon_y^d e^{-\int_y^{\bar{y}} \frac{\varepsilon_x^d}{\varepsilon_x^r x}\,dx}\big]X(\bar{y}).\end{aligned}$$

If the ratio $\varepsilon_x^d/\varepsilon_x^r$ is constant, then the last term in this expression is proportional to $\bar{y}^{-\varepsilon^d/\varepsilon^r}$ and converges to zero as $\bar{y} \to \infty$. This holds more generally as long as $\varepsilon_y^d/\varepsilon_y^r$ is bounded away from zero, since in this case $0 < e^{-\int_y^{\bar{y}}(\varepsilon_x^d/\varepsilon_x^r)\,dx/x} \to 0$ as $\bar{y} \to \infty$. □

Proof of Corollary 1. Suppose that the production function is CES, the tax schedule is CRP, and the labor supply elasticities are constant. Consider a uniform wage disruption, i.e., $\hat{w}^E(y) = \hat{w}^E\ \forall y \in [\underline{y}, \bar{y}]$. The partial-equilibrium compensation reads

$$\hat{T}_{PE}(y) = \big(1 - T'(y)\big)y\hat{w}^E$$
$$\hat{T}'_{PE}(y) = \big(1 - T'(y)\big)(1-p)\hat{w}^E.$$

The general-equilibrium wage disruption—absent any compensation—is given by

$$\hat{\Omega}_i^E = \phi_i\left\{\hat{w}_i^E + \frac{\int_0^1 \phi_j\varepsilon_j^w\gamma_j\hat{w}_j^E\,dj}{\int_0^1 \phi_j\frac{y_j}{\mathbb{E}y}\,dj}\right\} = \phi\hat{w}^E + \phi\frac{\varepsilon^w}{\varepsilon^d}\hat{w}^E = \hat{w}^E,$$

where the last equality follows from $\phi\varepsilon^w/\varepsilon^d = 1 - \phi$. The ODE (14) thus simplifies to

$$\begin{aligned}\hat{w}^E = &\left(1 + \phi\frac{\varepsilon^n}{\varepsilon^d}\right)\frac{\hat{T}(y_i)}{\big(1 - T'(y_i)\big)y_i} - \phi\frac{\varepsilon^r}{\varepsilon^d}\frac{\hat{T}'(y_i)}{1 - T'(y_i)} \\ &+ \phi\int_0^1 \frac{y_j}{\mathbb{E}y}\left[\frac{\varepsilon^r}{\varepsilon^d}\frac{\hat{T}'(y_j)}{1 - T'(y_j)} - \frac{\varepsilon^n}{\varepsilon^d}\frac{\hat{T}(y_j)}{\big(1 - T'(y_j)\big)y_j}\right]dj.\end{aligned}$$

Plugging in the partial-equilibrium compensation in the right-hand side leads to

$$\left(1+\phi\frac{\varepsilon^n}{\varepsilon^d}\right)\hat{w}^E-\phi\frac{\varepsilon^r}{\varepsilon^d}(1-p)\hat{w}^E+\phi\int_0^1\frac{y_j}{\mathbb{E}y}\left[\frac{\varepsilon^r}{\varepsilon^d}(1-p)\hat{w}^E-\frac{\varepsilon^n}{\varepsilon^d}\hat{w}^E\right]dj=\hat{w}^E.$$

Therefore, $\hat{T}_{PE}$ satisfies (14). □

Proof of Corollary 2. Under Assumption 1, we have

$$\Pi(y,z)=\frac{\varepsilon^d}{\varepsilon^r}\frac{y^{\varepsilon^d/\varepsilon^r}}{z^{\varepsilon^d/\varepsilon^r+1}}$$

and $\Gamma_{ij}=\phi^{-1}\gamma_{ij}$, with $\phi^{-1}=1+\varepsilon^w/\varepsilon^d$ and $\gamma_{ij}=(1/\varepsilon^d)y_j/\mathbb{E}[y]$, or

$$\gamma(y_i,y_j)=\frac{1}{\varepsilon^d}\frac{y_jf(y_j)}{\mathbb{E}[y]}.$$

Thus, we get

$$\hat{\Omega}^E(y)=\phi\hat{w}^E(y)+(1-\phi)\frac{\mathbb{E}[y\hat{w}^E(y)]}{\mathbb{E}[y]}$$

and, as $\bar{y}\to\infty$,

$$\Lambda=-\frac{\varepsilon^d/\varepsilon^r}{\mathbb{E}[y]}\mathbb{E}\left[\int_y^\infty\left(\frac{y}{x}\right)^{\varepsilon^d/\varepsilon^r+1}\hat{w}^E(x)\,dx\right]+\left(1-\frac{\varepsilon^w}{\varepsilon^d}\right)\frac{\mathbb{E}[y\hat{w}^E(y)]}{\mathbb{E}[y]}.$$

Substituting these expressions into formula (17), we obtain that the compensation is given by

$$\frac{\hat{T}(y)}{(1-T'(y))y}=\int_y^\infty\Pi(y,z)\hat{w}^E(z)\,dz+\frac{\mathbb{E}[z\hat{w}^E(z)]-\mathbb{E}\left[z\int_z^\infty\Pi(z,x)\hat{w}^E(x)\,dx\right]}{\mathbb{E}[z]}. \quad (32)$$

It is easy to check that $\int_y^\infty\Pi(y,z)\,dz=1$, so that $\hat{T}(y)/((1-T'(y))y)=1$ for a uniform disruption $\hat{w}^E(\cdot)=1$. For a Dirac disruption $\hat{w}^E(y)=-\delta(y-y_*)$, we get

$$\begin{aligned}\frac{\hat{T}(y)}{(1-T'(y))y}&=-\int_y^\infty\Pi(y,z)\delta(z-y_*)\,dz-\frac{\mathbb{E}[z\delta(z-y_*)]-\mathbb{E}\left[z\int_z^\infty\Pi(z,x)\delta(x-y_*)\,dx\right]}{\mathbb{E}[z]}\\&=-\Pi(y,y_*)\mathbb{I}_{\{y\le y_*\}}-\frac{1}{\mathbb{E}[z]}\left[y_*f(y_*)-\frac{\varepsilon^d}{\varepsilon^r}\int_{z\le y_*}\left(\frac{z}{y_*}\right)^{1+\varepsilon^d/\varepsilon^r}f(z)\,dz\right].\end{aligned}$$

In particular, as $\varepsilon^d\to\infty$, the second term in this expression (in square brackets) converges to 0, and the first term converges to 0 for all $y<y_*$ and to $-\infty$ for $y=y_*$; we thus recover the partial-equilibrium compensation. □

Proof of Corollary 3. Applying the compensating tax reform (32) to the disruption $\hat{w}^E(y) \equiv -\mathbb{I}_{\{y \geq y_*\}}$ leads to

$$\begin{aligned}\frac{\hat{T}(y)}{\big(1-T'(y)\big)y} &= -\left[\int_y^\infty \Pi(y,z)\,dz\right]\mathbb{I}_{\{y\geq y_*\}} - \left[\int_{y_*}^\infty \Pi(y,z)\right]\mathbb{I}_{\{y\leq y_*\}} \\ &\quad - \frac{\mathbb{E}[z\mathbb{I}_{\{z\geq y_*\}}] - \mathbb{E}\left[z\left(\int_{y_*}^\infty \Pi(z,x)\,dx\right)\mathbb{I}_{\{z\leq y_*\}} + z\left(\int_z^\infty \Pi(z,x)\,dx\right)\mathbb{I}_{\{z\geq y_*\}}\right]}{\mathbb{E}[z]} \\ &= -\mathbb{I}_{\{y\geq y_*\}} - \left(\frac{y}{y_*}\right)^{\varepsilon^d/\varepsilon^r}\mathbb{I}_{\{y\leq y_*\}} \\ &\quad - \frac{\mathbb{E}[z\mathbb{I}_{\{z\geq y_*\}}] - \mathbb{E}\left[z\left(\frac{z}{y_*}\right)^{\varepsilon^d/\varepsilon^r}\mathbb{I}_{\{z\leq y_*\}} + z\mathbb{I}_{\{z\geq y_*\}}\right]}{\mathbb{E}[z]},\end{aligned}$$

which easily leads to formula (21). In particular, as $\varepsilon^d \to \infty$, the second and third terms in this expression converge to 0, leaving only the first term $-\mathbb{I}_{\{y\geq y_*\}}$; we thus recover the partial-equilibrium compensation.

Similarly, for the disruption $\hat{w}^E(y) \equiv -\mathbb{I}_{\{y_L\leq y\leq y_H\}}$, we get

$$\begin{aligned}&\frac{\hat{T}(y)}{\big(1-T'(y)\big)y} \\ &\quad = -\left[\int_{y_L}^{y_H} \Pi(y,z)\,dz\right]\mathbb{I}_{\{y\leq y_L\}} - \left[\int_y^{y_H} \Pi(y,z)\,dz\right]\mathbb{I}_{\{y_L\leq y\leq y_H\}} \\ &\quad\quad - \frac{\mathbb{E}[z\mathbb{I}_{\{y_L\leq z\leq y_H\}}] - \mathbb{E}\left[z\left\{\left(\int_{y_L}^{y_H} \Pi(z,x)\,dx\right)\mathbb{I}_{\{z\leq y_L\}} + \left(\int_z^{y_H} \Pi(z,x)\,dx\right)\mathbb{I}_{\{y_L\leq z\leq y_H\}}\right\}\right]}{\mathbb{E}[z]}.\end{aligned}$$

Straightforward algebra leads to

$$\begin{aligned}\frac{\hat{T}(y)}{\big(1-T'(y)\big)y} &= \left(\left(\frac{y}{y_H}\right)^{\varepsilon^d/\varepsilon^r} - \left(\frac{y}{y_L}\right)^{\varepsilon^d/\varepsilon^r}\right)\mathbb{I}_{\{y\leq y_L\}} + \left(\left(\frac{y}{y_H}\right)^{\varepsilon^d/\varepsilon^r} - 1\right)\mathbb{I}_{\{y_L\leq y\leq y_H\}} \\ &\quad + \frac{\mathbb{E}\left[z\left(\left(\frac{z}{y_L}\right)^{\varepsilon^d/\varepsilon^r} - \left(\frac{z}{y_H}\right)^{\varepsilon^d/\varepsilon^r}\right)\mathbb{I}_{\{z\leq y_L\}} - z\left(\frac{z}{y_H}\right)^{\varepsilon^d/\varepsilon^r}\mathbb{I}_{\{y_L\leq z\leq y_H\}}\right]}{\mathbb{E}[z]},\end{aligned}$$

which in turn yields formula (22). In particular, as $\varepsilon^d \to \infty$, the first and third terms in this expression converge to 0, and the second term converges to $-\mathbb{I}_{\{y_L\leq y\leq y_H\}}$; we thus recover the partial-equilibrium compensation. □

References

Acemoglu, Daron and Pascual Restrepo (2020), "Robots and jobs: Evidence from us labor markets." *Journal of Political Economy*, 128, 2188–2244. [1669, 1690]

Ales, Laurence, Musab Kurnaz, and Christopher Sleet (2015), "Technical change, wage inequality, and taxes." *The American Economic Review*, 105, 3061–3101. [1669, 1692]

Andersen, Torben M. and Joydeep Bhattacharya (2017), "The intergenerational welfare state and the rise and fall of pay-as-you-go pensions." *The Economic Journal*, 127, 896–923. [1669, 1694]

Andersen, Torben M. and Joydeep Bhattacharya (2020), "Intergenerational debt dynamics without tears." *Review of Economic Dynamics*, 35, 192–219. [1669, 1694]

Andersen, Torben M., Joydeep Bhattacharya, and Pan Liu (2020), "Resolving intergenerational conflict over the environment under the Pareto criterion." *Journal of Environmental Economics and Management*, 100, 102290. [1669, 1694]

Antras, Pol, Alonso de Gortari, and Oleg Itskhoki (2016), "Globalization, inequality and welfare." *Journal of International Economics*, 108, 387–412. [1669]

Bénabou, Roland (2002), "Tax and education policy in a heterogenous-agent economy: What of levels of redistribution maximize growth and efficiency?" *Econometrica*, 70, 481–517. [1669]

Beraja, Martin and Nathan Zorzi (2021), "Inefficient automation." Technical report, National Bureau of Economic Research. [1669]

Bierbrauer, Felix, Pierre Boyer, and Emanuel Hansen (2020), "Pareto-improving tax reforms and the earned income tax credit." *Econometrica*, 91, 1077–1103. [1669, 1692]

Bierbrauer, Felix J. and Pierre C. Boyer (2014), "The Pareto-frontier in a simple mirrleesian model of income taxation." *Annals of Economics and Statistics/Annales d'Économie et de Statistique*, 113/114, 185–206. [1669, 1692]

Bierbrauer, Felix J., Pierre C. Boyer, and Andreas Peichl (2021), "Politically feasible reforms of nonlinear tax systems." *American Economic Review*, 111, 153–191. [1694]

Card, David (2009), "Immigration and inequality." *American Economic Review*, 99, 1–21. [1666, 1681]

Card, David and Thomas Lemieux (2001), "Can falling supply explain the rising return to college for younger men? A cohort-based analysis." *Quarterly Journal of Economics*, 116, 705–746. [1681]

Chetty, Raj (2012), "Bounds on elasticities with optimization frictions: A synthesis of micro and macro evidence on labor supply." *Econometrica*, 80, 969–1018. [1681]

Costinot, Arnaud and Iván Werning (2018), "Robots, trade, and luddism: A sufficient statistic approach to optimal technology regulation." *The Review of Economic Studies*, 90, 2261–2291. [1669]

Dávila, Eduardo and Andreas Schaab (2021), "Welfare assessments with heterogeneous individuals." Technical report, National Bureau of Economic Research. [1694]

Diamond, Peter and Emmanuel Saez (2011), "The case for a progressive tax: From basic research to policy recommendations." *Journal of Economic Perspectives*, 25, 165–190. [1681]

Guerreiro, Joao, Sergio Rebelo, and Pedro Teles (2017), "Should robots be taxed?" *The Review of Economic Studies*, 89, 279–311. [1669]

Guesnerie, Roger (1998), "Peut-on toujours redistribuer les gains à la spécialisation et à l'échange? un retour en pointillé sur ricardo et heckscher-ohlin." *Revue Économique*, 49, 555–579. [1669]

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante (2017), "Optimal tax progressivity: An analytical framework." *The Quarterly Journal of Economics*, 132, 1693–1754. [1669, 1681]

Hendren, Nathaniel (2020), "Measuring economic efficiency using inverse-optimum weights." *Journal of Public Economics*, 187, 104198. [1669, 1694]

Hicks, John R. (1939), "The foundations of welfare economics." *The Economic Journal*, 49, 696–712. [1666]

Hicks, John R. (1940), "The valuation of the social income." *Economica*, 7, 105–124. [1666]

Hosseini, Roozbeh and Ali Shourideh (2018), "Inequality, redistribution and optimal trade policy: A public finance approach." Available at SSRN 3159475. [1669]

Itskhoki, Oleg (2008), "Optimal redistribution in an open economy." Working Paper. [1669]

Jacobs, Bas, Egbert L. W. Jongen, and Floris T. Zoutman (2017), "Revealed social preferences of Dutch political parties." *Journal of Public Economics*, 156, 81–100. [1669]

Jacquet, Laurence and Etienne Lehmann (2021), "Optimal income taxation with composition effects." *Journal of the European Economic Association*, 19, 1299–1341. [1673]

Kaldor, Nicholas (1939), "Welfare propositions of economics and interpersonal comparisons of utility." *The Economic Journal*, 49, 549–552. [1666]

Kaplow, Louis (2004), "On the (ir)relevance of distribution and labor supply distortion to government policy." *Journal of Economic Perspectives*, 18, 159–175. [1669]

Kaplow, Louis (2012), "Optimal control of externalities in the presence of income taxation." *International Economic Review*, 53, 487–509. [1669]

Katz, Lawrence F. and Kevin M. Murphy (1992), "Changes in relative wages, 1963–1987: Supply and demand factors." *The Quarterly Journal of Economics*, 107, 35–78. [1666, 1681]

Lorenz, Normann and Dominik Sachs (2016), "Identifying Laffer bounds: A sufficient-statistics approach with an application to Germany." *The Scandinavian Journal of Economics*, 118, 646–665. [1669, 1692]

Mirrlees, James (1971), "An exploration in the theory of optimum income taxation." *The Review of Economic Studies*, 38, 175–208. [1666]

Rothschild, Casey and Florian Scheuer (2013), "Redistributive taxation in the Roy model." *The Quarterly Journal of Economics*, 128, 623–668. [1669]

Sachs, Dominik, Aleh Tsyvinski, and Nicolas Werquin (2020), "Nonlinear tax incidence and optimal taxation in general equilibrium." *Econometrica*, 88, 469–493. [1669, 1673, 1676, 1681, 1698, 1700]

Saez, Emmanuel (2001), "Using elasticities to derive optimal income tax rates." *Review of Economic Studies*, 68, 205–229. ISSN 1467-937X. [1673, 1681, 1696]

Saez, Emmanuel and Stefanie Stantcheva (2016), "Generalized social marginal welfare weights for optimal tax theory." *American Economic Review*, 106, 24–45. [1695]

Scheuer, Florian and Iván Werning (2017), "The taxation of superstars." *The Quarterly Journal of Economics*, 132, 211–270. [1669, 1673, 1692]

Scheuer, Florian and Alexander Wolitzky (2016), "Capital taxation under political constraints." *American Economic Review*, 106, 2304–2328. [1694]

Stiglitz, Joseph E. (1982), "Self-selection and Pareto efficient taxation." *Journal of Public Economics*, 17, 213–240. [1669, 1678]

Thuemmel, Uwe (2018), "Optimal taxation of robots." *Journal of the European Economic Association*, 21, 1154–1190. [1669]

Werning, Iván (2007), "Pareto efficient income taxation." Technical report, MIT. [1669, 1692]

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