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# Digital currency and privacy

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We develop a monetary model in which a private company issues digital currency and uses payment data to estimate consumers' preferences. Sellers purchase preference information to produce goods that better match consumers' preferences. A monopoly arises in the digital currency industry, and digital currency is not issued if the inflation rate is sufficiently high. Due to reinforcing interactions between the value of preference information and trade volume, multiple equilibria (with and without digital currency) can exist, depending on market structures for monetary exchanges. When left to market forces alone, socially efficient uses of payment data may not occur.

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JEL classification. E12, E40, E50, G10.

## 1. Introduction

As our economy has become more digitalized, electronic payments have steadily increased over recent decades (see Stavins (2017)). Although electronic means of payment (henceforth, E-money), such as debit cards, Alipay, and PayPal, differ from traditional cash in many respects, one important difference is privacy: Cash retains user privacy, while digital currency transactions are collected by the company that operates the electronic payment system. Payment histories can indicate individual preferences for certain items, and this preference information, combined with users' personal information, can be used for marketing purposes and to design better goods that are more tailored to consumers' preferences. Thus, payment history data have commercial value and their importance has increased with advances in analytical technologies such as machine learning.

Although economic studies on digital currency have emerged recently since a surge in the Bitcoin price, the practice of using payment data of digital currency has received relatively little attention in academic areas. The following questions still need to be addressed: Under which conditions does the E-money business—issuing digital currency

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and commercially using payment data—exist in equilibrium, and is it good or bad for a society? How does monetary policy affect the E-money business? How do equilibrium outcomes depend on market structures? What are the effects of government's market interventions on real allocations and welfare?

In this paper, we construct a money search model in which a private payment platform company issues E-money that is backed by government-issued cash, similar to PayPal and debit cards, to address the above questions. The company can estimate buyers' preferences using E-money transaction data and sell the preference information to sellers. A seller can use the preference information to produce goods that are customized for buyers' preferences, which increases the total trade surplus. The precision of the preference information increases with the amount of payment data, and the company provides rewards for using E-money to attract more buyers to use it. Buyers incur disutility from providing personal information, including payment histories, to the company and, hence, use E-money only if the rewards are higher than the disutility; otherwise, they use cash.

An increase in the trade volume raises the additional surplus that the seller can obtain by selling customized goods, so it increases the value of preference information and the company's profit. If the inflation rate is sufficiently high, the trade volume is too small for the company to make profits, so the E-money business does not exist. Meanwhile, buyers hold more real balances to buy customized goods when the E-money business exists. Thus, reinforcing interactions exist between the trade volume and the value of preference information. Because of these interactions, multiple equilibria can exist with different transaction patterns (with and without E-money) when buyers and sellers are randomly matched and bargain over the terms of trade. However, the multiplicity disappears in competitive search equilibrium because the posted terms of trade work as a coordinating device.

Once the company has incurred the cost for reward payments to obtain preference information, the marginal cost of selling the information to an additional seller is zero. Thus, a monopoly arises in the E-money industry with Bertrand competition. In the model, the private benefit of the E-money business is lower than its social benefit because of the money holding cost and the externality problem, i.e., the company does not fully internalize the buyers' benefit of consuming customized goods when the E-money business exists. Thus, when left to market forces alone, socially efficient uses of payment data may not occur.[1] Providing a subsidy to sellers directly incentivizes information users, so it is more effective for supporting efficient uses of payment data than subsidizing the E-money issuer.

Although the literature on privacy is extensive, relatively little attention has been given to privacy in monetary economics.[2] Kahn, McAndrews, and Roberds (2005) inves-

[1]The externality problem disappears under competitive search, but the socially efficient E-money business may not exist when the money holding cost exists.

[2]A nonmonetary model that is closely related to our model is Bergemann, Bonatti, and Gan (2022) in which a monopolist intermediary buys preference information directly from individual consumers and resells the information in a product market, and they show the presence of informational externalities. We differ from their model as we focus on the special characteristics of the digital currency industry and the effects of government policies.

tigate the role of privacy in money transactions and Garratt and van Oordt (2021) show that individual customers do not preserve their privacy in payments at the socially optimal level. Guennewig (2023) shows that final goods producers issue digital currency to obtain their consumers' information. More related, Garratt and Lee (2021) show that payment data that can be used to design future goods drive the formation of a market monopoly.

However, in those earlier studies, merchants obtain customers' private data, while the payment platform company obtains the payment data in our model, complementing the previous works. Chiu and Koeppl (2022a) investigate dynamic feedback loops between the data and activity sides of the platform. In their model, the platform company obtains consumers' information, but there are no digital currencies and government issued money. Furthermore, in contrast to the previous studies cited above, our model is based on Lagos and Wright (2005), so it admits the analysis of the effects of monetary policy on the economic uses of payment data. In particular, we derive testable implications regarding the relation between the inflation rate and the profitability of the E-money business.

We also contribute to the growing literature on digital currency. Chiu and Wong (2015) and Carli and Uras (2023) examine how E-money improves the efficiency of the economy. Chiu and Koeppl (2022b) and Kang (2023) investigate double spending incentives in the Bitcoin system, and Choi and Rocheteau (2021) and Pagnotta (2022) study cryptocurrency pricing. Fernández-Villaverde and Sanches (2019), Schilling and Uhlig (2019), and Kang and Lee (forthcoming) explore the macroeconomic implications of cryptocurrencies via currency competition. While these papers focus on analyzing the economic implications of technical features of digital currency, such as blockchain technology, we focus on the privacy issue of digital currency payments.[3]

The rest of the paper is organized as follows. Section 2 presents the model environment. In Section 3, we characterize bargaining equilibrium, and in Section 4, we conduct a welfare analysis. Section 5 investigates competitive search equilibrium. In Section 6, we extend the model with multiple E-money issuers. Section 7 concludes the paper. Appendix A contains proofs.

## 2. Environment

The basic framework of the model is based on Lagos and Wright (2005) and Rocheteau and Wright (2005). Time is indexed by $t = 0, 1, 2, \ldots$, and each time period $t$ is divided into three subperiods: morning ($m$), afternoon ($a$), and evening ($e$). A continuum of buyers and sellers exists, each with unit mass. Each buyer has preferences, given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \big[X_t - H_t - \mathbf{1}_p \delta + v(q_t) + \alpha u(x_t)\big],$$

[3]There also has been extensive research on central bank digital currency (see Fernández-Villaverde, Sanches, Schilling, and Uhlig (2021), Keister and Monnet (2022), and Williamson (2022a,b), and Chiu, Davoodalhosseini, Jiang, and Zhu (2023)).

and each seller has preferences, given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \big[X_t - H_t - c(h_{a,t}) - h_{e,t}\big].$$

Here $\beta \in (0, 1)$ is the discount rate, and $X_t$, $q_t$, and $x_t$ are consumption in the morning, afternoon, and evening, respectively, and $H_t$, $h_{a,t}$, and $h_{e,t}$ are labor supplies in the morning, afternoon, and evening, respectively. We assume that $v$, $u$, and $c$ are twice continuously differentiable with $v(0) = 0$, $v'' < 0 < v'$, $v'(0) = \infty$, $v'(\infty) = 0$, $u(0) = 0$, $u'' < 0 < u'$, $u'(0) = \infty$, $u'(\infty) = 0$, $c(0) = 0$, $c' > 0$, and $c'' > 0$. Furthermore, we assume that $u'(x)u'''(x) > [u''(x)]^2$ for all $x > 0$. Here $\alpha > 0$ is a parameter that affects the buyer's utility in the evening, $\delta > 0$ is a parameter that captures the buyer's disutility by forgoing privacy, the exact definition of which will be provided later, and $\mathbf{1}_p$ is an indicator function that takes the value of 1 if a buyer forgoes privacy and 0 otherwise.

Agents can produce one unit of the perishable consumption good with one unit of labor supply in each subperiod. We call goods produced in the morning, afternoon, and evening morning goods, afternoon goods, and evening goods, respectively, and we set morning goods as the numeraire goods.

In the morning, there is a centralized Walrasian market in which all agents trade numeraire goods and assets. Buyers and sellers meet in large groups trading afternoon goods in a competitive market in the afternoon. Finally, in the evening, there are bilateral meetings between buyers and sellers. In pairwise meetings, a buyer and a seller bargain over the terms of trade, which are determined according to the bargaining solution of Kalai (1977), where the seller's bargaining power is $\theta \in (0, 1)$.

Ideally, buyers would like to borrow output from sellers in the afternoon and evening markets and to repay loans the next morning. Such credit arrangements are ruled out here because agents are anonymous and no device is available to record credit histories. Consequently, any trades between buyers and sellers must occur on a quid pro quo basis through the use of a medium of exchanges.

There exists fiat money that is traded at price $\phi_t$ in terms of numeraire goods in the morning in period $t$. Money is supplied by the government at the beginning of the morning with a lump-sum transfer $\tau_t = (\gamma - 1)\phi_t M_{t-1}$ to each buyer, where $M_{t-1}$ is the money stock in period $t - 1$. Thus, the money stock grows at a constant gross rate $\gamma$. We focus on policies where $\gamma \geq \beta$ because equilibrium does not exist otherwise. When $\gamma = \beta$, we consider equilibrium obtained by taking the limit $\gamma \to \beta$. Furthermore, we assume that $\gamma < \beta/\theta$ because money is not used in the evening otherwise.

At the beginning of the morning, buyers are subject to an idiosyncratic shock, which determines whether they consume early (in the afternoon) or late (in the evening). Let $\rho \in (0, 1)$ denote the probability that a buyer goes to the afternoon market and a buyer goes to the evening market with probability $1 - \rho$. Note that this shock is realized at the beginning of the morning. Thus, buyers know which market they will go to in the subsequent period when they make decisions in the morning. We call buyers who go to the afternoon market early buyers and those who go to the evening market late buyers.

*Individual preference in the evening* In the model economy, $\mathcal{N} \in \mathbb{N}_+$, different tastes exist for evening goods, and a buyer has one of those tastes with probability $1/\mathcal{N}$. The evening taste is realized in the afternoon, and buyers' evening tastes are their private information. If a late buyer consumes customized goods tailored to his/her taste in the evening, then $\alpha = \alpha_H$; otherwise, and $\alpha = \alpha_L$, where $0 < \alpha_L < \alpha_H$. Given the value of $\alpha$, we define the threshold values of trade volume in the evening market and real balances as

$$x_i^* = u'^{-1}\left(\frac{1}{\alpha_i}\right) \quad \text{and} \quad m_i^* = \frac{\theta\alpha_i u\left(x_i^*\right) + (1-\theta)x_i^*}{\beta} \tag{1}$$

for each $i \in \{H, L\}$.

We assume that a seller can produce a customized product tailored to a particular taste only if the seller prepared the production of that product by incurring $\varsigma > 0$ units of labor at the beginning of the evening. Furthermore, there exists a collection of infinite numbers of different evening tastes, and in each period, $\mathcal{N}$ evening tastes are randomly chosen from that collection. Thus, the set of evening tastes changes over time, similar to a passing fad, and the probability that a particular evening taste is realized in a given period is zero. Consequently, sellers cannot prepare the production of a customized product unless they know what evening tastes are realized in the current period.

A seller may attempt to contact individual buyers to learn their realized evening tastes. However, there is no way to verify whether an individual buyer provides the correct information. In particular, we assume that buyers incur some disutility from providing personal information to others due to privacy concerns, which will be discussed further later. Furthermore, a late buyer has a zero probability of meeting the seller to whom he/she provided the preference information because of the random matching process in the evening. Thus, buyers always have an incentive to provide incorrect information to keep their privacy, so sellers cannot obtain information about realized evening tastes by asking individual buyers.

*Digital currency and privacy* In this economy, there is a private company that can issue electronic money (henceforth, E-money) and we assume that E-money must be backed by government issued money.[4] For example, the government can prohibit private sectors from issuing pure fiat money to maintain the efficacy of monetary policy or agents are reluctant to receive money issued by a private company unless it is backed by government issued money.[5] Specifically, if an agent deposits money to the company by

[4]We can construct a model such that the company supports online transactions by issuing credit cards that have two distinctive features: (i) Credit card users purchase goods with credit and make monthly payments on credit card bills, and (ii) the credit card company transfers money to merchants quickly, in contrast to standard loans, on behalf of its users. However, the main implications from the model with the credit card company do not differ from those in the model with the E-money company.

[5]The Chinese government, for example, forbids using any privately issued money in China, while it allows people to use Alipay and WeChat Pay that are basically backed by Renminbi. Furthermore, although it is not illegal to use cryptocurrencies for trades in countries, such as Japan, South Korea, and the United States, cryptocurrencies are not widely used in retail transactions in those countries in contrast to debit cards and PayPal that are backed by government issued money.

opening an account, then the company puts the same amount of E-money into the agent's account. In this sense, E-money is equivalent to debit cards and Alipay in reality.

We assume that the company is a monopoly, but in Section 6 we show that a monopoly arises although multiple companies can issue their own E-money. Agents must open a new account in the morning to use E-money in the current period and the account is closed the next morning. The company cannot restrict the number of users and distributes its profits to buyers in the morning. Finally, we assume that the company can collect each individual's E-money transaction data, which could provide useful information as described below.

In reality, an individual who cannot find products that meet his/her preference perfectly because those products are not yet available in the market can cater to his/her preference to some degree by consuming available goods appropriately. For example, suppose that John wants to enjoy high-quality coffee at home in the morning, but does not have a sophisticated espresso machine and good skills in making coffee. In this case, he can satisfy his needs in part by going to Starbucks in the morning, although what he really wants is to enjoy quality coffee on his balcony seeing the sunrise. As shown in this example, preferences could affect consumption behaviors, so consumption patterns could provide some information about individuals' preferences that can be used to develop more customized products, such as advanced Nespresso capsule coffee machines in our coffee example.

Similar to the example described above, early buyers' evening tastes could affect their consumption behaviors in the afternoon although they do not go to the evening market because they want to consume goods early in the afternoon. This implies that the company can estimate evening tastes realized in the current period by analyzing E-money transactions in the afternoon. Obviously, the accuracy of estimation would increase with the sample size by the fundamental rule in statistics. Based on this rationale, we assume that the company obtains the correct preference information with probability $\kappa(B) \in [0, 1]$ by analyzing E-money transactions in the afternoon, where $B \in [0, \rho]$ is the mass of early buyers who use E-money in the afternoon and $\kappa$ is an increasing function with $\kappa(0) = 0$ and $\kappa(\rho) = 1$.[6,7]

Sellers would buy preference information because they can prepare the production of customized goods tailored to realized evening tastes with preference information, which increases the total trade surplus in the evening. Let $\varphi_t$ denote the price of each taste information in period $t$ in terms of morning goods in the next period $t + 1$. We assume that sellers can commit to making payments for information purchases the next morning.

[6]In the model, preference information of an individual buyer reveals preference information of other buyers who share the same evening taste. In this sense, our model resonates data externality, whose economic implications have been investigated in Bergemann, Bonatti, and Gan (2022) and Ichihashi (2021). However, in contrast to the models of data externality, the company cannot learn individual buyer's evening taste because a buyer always has incentives to provide inaccurate information about himself/herself. Furthermore, analyzing afternoon transaction data does not reveal each buyer's preference and it only provides information about realized evening tastes in the current period.

[7]We assume that $\kappa(\rho) = 1$ to obtain a closed form solution for equilibrium allocations. If $\kappa(\rho) < 1$, we cannot obtain the closed form solution, but main implications do not change based on the numerical analysis.

In reality, many people are inherently reluctant to reveal private information to others. We capture this feature in the model with disutility $\delta$ similar to Choi, Doh-Shin, and Byung-Cheol (2019): When a buyer opens an account at the company, the buyer incurs $\delta > 0$ units of fixed disutility in the morning to agree that the company obtains his/her personal information, including payment histories, and can use the obtained information for commercial purposes.[8] This disutility is associated with, for instance, privacy concerns such as data hacking or privacy costs originating from the agent's own taste for keeping privacy. Consequently, the company must compensate buyers for using E-money, such as the Bounty Payments program of PayPal and assorted benefits provided by debit card issuers.

The reward can have two forms: fixed and proportional rewards. In Appendix B, we show that the profit maximizing company does not provide a proportional reward because it distorts buyers' decisions about money holdings, which generates an additional cost to the company. Thus, in this paper, we assume that the company provides only fixed rewards. This implies that the company does not compensate late buyers for using E-money in the evening because evening transaction data have no value to the company and providing a fixed reward does not affect the quantity of late buyers' money holdings in the evening. On the other hand, the company provides $R_{t+1} \geq 0$ units of numeraire goods to a buyer in the morning in period $t+1$ if the buyer used E-money for afternoon transactions in period $t$.[9]

Figure 1 summarizes the sequence of events in a representative period. Throughout the paper, the E-money business means the business of obtaining and selling evening taste information by issuing E-money. In what follows, we call money supplied by the government P-money to emphasize that it is paper money rather than electronic money.

## 3. Equilibrium

In this section, we characterize equilibrium of the model economy as follows. First, we study agents' value functions in each subperiod. Second, we study the optimal decisions of buyers, sellers, and the company. Third, we study market clearing conditions. Then we characterize equilibrium.

### 3.1 *Value functions*

*Morning market* In the morning, agents consume numeraire goods, supply labor, and readjust their portfolios. We define an indicator variable $\boldsymbol{\iota}_t \in \{0, 1\}$ such that

[8] In the model, we assume that buyers incur privacy cost $\delta$ when they open an account at the company to capture worries that an individual experiences when he/she provides personal information, such as name, phone number, and social security number, to open a new account, for example, at a bank or on social media. Alternatively, we can assume that buyers incur disutility $\delta$ when they use E-money. However, the main results do not change because, in equilibrium, buyers will never accumulate E-money in the morning unless they use it in the afternoon or evening market for transactions.

[9] Alternatively, we can assume that the company transfers $R_{t+1}$ units of money (or E-money) in terms of morning goods in the next period to an early buyer at the end of the afternoon in period $t$ if he/she used E-money for afternoon transactions in period $t$. This alternative assumption raises the company's cost in equilibrium, but the main implications do not change.

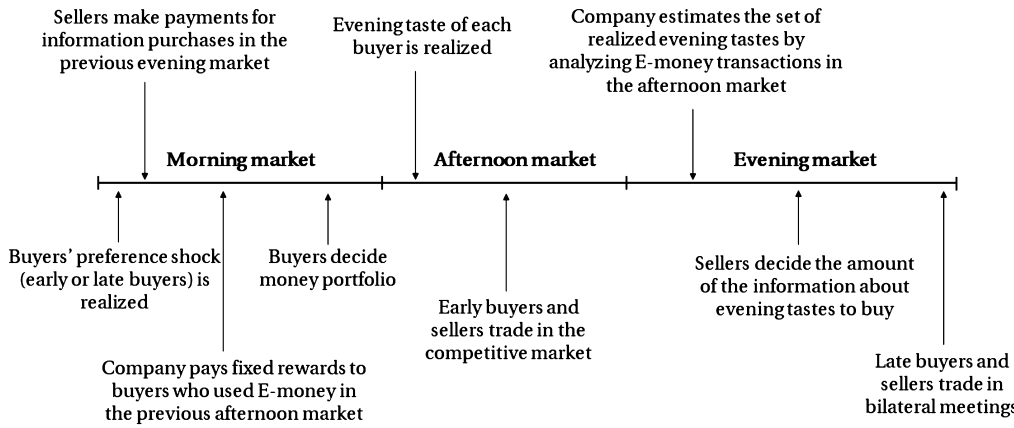


FIGURE 1. Timeline of a representative period.

$\boldsymbol{\iota}_t = 1$ if a buyer used E-money in the afternoon in period $t$ and $\boldsymbol{\iota}_t = 0$ otherwise. Let $V_{m,t}^{b,\text{early}}(m_p, m_e, \boldsymbol{\iota}_{t-1})$ denote the early buyer's value function at the beginning of the morning in period $t$ with $m_p$ units of real P-money, $m_e$ units of real E-money, and previous action $\boldsymbol{\iota}_{t-1}$.

Then, by virtue of the quasi-linearity of preferences, $V_{m,t}^{b,\text{early}}(m_p, m_e, \boldsymbol{\iota}_{t-1})$ is given as

$$V_{m,t}^{b,\text{early}}(m_p, m_e, \boldsymbol{\iota}_{t-1}) = m_p + m_e + \tau_t + \pi_t + R_t \boldsymbol{\iota}_{t-1} + \max_{m'_p, m'_e} \left\{ -\frac{\phi_t}{\phi_{t+1}}(m'_p + m'_e) - \mathbf{1}_{\{m'_e > 0\}}\delta + V_{a,t}^b(m'_p, m'_e) \right\}. \quad (2)$$

Here, $\pi_t$ is dividend payments from the company, $m'_p$ and $m'_e$ are the real balances of P-money and E-money, respectively, both in terms of numeraire goods in the next period, $\mathbf{1}_{\{m'_e > 0\}}$ is an indicator function that takes the value of 1 if $m'_e > 0$ and 0 otherwise, and $V_{a,t}^b(m'_p, m'_e)$ is the value of the early buyer with portfolio $(m'_p, m'_e)$ in the afternoon in period $t$. Similarly, the value function $V_{m,t}^{b,\text{late}}(m_p, m_e, \boldsymbol{\iota}_{t-1})$ of a late buyer with portfolio $(m_p, m_e)$ and previous action $\boldsymbol{\iota}_{t-1}$ at the beginning of the morning in period $t$ is given as

$$V_{m,t}^{b,\text{late}}(m_p, m_e, \boldsymbol{\iota}_{t-1}) = m_p + m_e + \tau_t + \pi_t + R_t \boldsymbol{\iota}_{t-1} + \max_{m'_p, m'_e} \left\{ -\frac{\phi_t}{\phi_{t+1}}(m'_p + m'_e) - \mathbf{1}_{\{m'_e > 0\}}\delta + V_{e,t}^b(m'_p, m'_e) \right\}, \quad (3)$$

where $V_{e,t}^b(m'_p, m'_e)$ is the value of the late buyer in the evening with portfolio $(m'_p, m'_e)$.

Next, the seller's value function $V_{m,t}^s(m_p, m_e, n)$ in the morning in period $t$ with portfolio $(m_p, m_e)$ and the number of unpaid information purchases $n \in \{0, 1, \ldots, \mathcal{N}\}$ in the previous evening is given as

$$V_{m,t}^s(m_p, m_e, n) = m_p + m_e - n\varphi_{t-1} + \max_{m'_p, m'_e} \left\{ -\frac{\phi_t}{\phi_{t+1}}(m'_p + m'_e) + V_{a,t}^s(m'_p, m'_e) \right\}, \quad (4)$$

where $V^s_{a,t}(m'_p, m'_e)$ is the value function of the seller with portfolio $(m'_p, m'_e)$ in the afternoon in period $t$.

*Afternoon market* Sellers are indifferent between P-money and E-money as a medium of exchange given a one-to-one exchange ratio between them. Thus, the price of afternoon goods cannot vary depending on the type of payment method. Let $p_t$ denote the market price of afternoon goods in period $t$ in terms of morning goods in the next period. Then the buyer's value in the afternoon is given as

$$V^b_{a,t}(m'_p, m'_e) = \max_{q_p, q_e} \{ \upsilon(q_p + q_e) + \beta V^b_{m,t+1}(m'_p - p_t q_p, m'_e - p_t q_e, \boldsymbol{\iota}_t) \} \tag{5}$$

subject to

$$m'_p - p_t q_p \geq 0 \tag{6}$$

$$m'_e - p_t q_e \geq 0, \tag{7}$$

where $q_p$ and $q_e$ are the quantities of afternoon goods purchased with P-money and E-money, respectively, $\boldsymbol{\iota}_t = 1$ if $q_e > 0$ and $\boldsymbol{\iota}_t = 0$ otherwise, and $V^b_{m,t+1}(\cdot,\cdot,\cdot)$ is the buyer's value at the beginning of the morning in period $t+1$ before the realization of the shock on the timing of consumption, i.e., $V^b_{m,t+1}(\cdot,\cdot,\cdot) = \rho V^{b,\text{early}}_{m,t+1}(\cdot,\cdot,\cdot) + (1-\rho) V^{b,\text{late}}_{m,t+1}(\cdot,\cdot,\cdot)$.

In the afternoon in period $t$, sellers sell afternoon goods in a competitive market at price $p_t$ in exchange for money. Thus, the seller's value in the afternoon is given as

$$V^s_{a,t}(m'_p, m'_e) = \max_{q^s_p, q^s_e} \{ -c(q^s_p + q^s_e) + V^s_{e,t}(m'_p + p_t q^s_p, m'_e + p_t q^s_e) \}, \tag{8}$$

where $q^s_p$ and $q^s_e$ are the quantities of afternoon goods that the seller sells in exchange for P-money and E-money, respectively, and $V^s_{e,t}(m'_p, m'_e)$ is the seller's value function at the beginning of the evening with asset portfolio $(m'_p, m'_e)$.

*Evening market* Late buyers and sellers are randomly matched in the evening and bargain over the terms of trade. Note that the company does not provide any proportional rewards for using E-money in the evening. This implies that given the one-to-one exchange ratio between P-money and E-money, what matters to a late buyer and a seller in a meeting is the total real balances—sum of P-money and E-money—and the composition does not matter. Thus, the bargaining outcome in a bilateral meeting given the late buyer's portfolio $(m'_p, m'_e)$ is a pair $(x, d)$ that specifies the quantity of evening goods $x$ produced by the seller and the sum of P-money and E-money transfers $d$ from the late buyer to the seller in terms of morning goods in the next period.

Given that the value functions of buyers and sellers in the morning are linear in asset holdings and the seller's bargaining power is $\theta \in (0, 1)$, the terms of trade $(x, d)$ are obtained by solving the problem

$$\max_{x,d} \{ \alpha_i u(x) - x \} \tag{9}$$

subject to

$$d = \frac{\theta\alpha_i u(x) + (1-\theta)x}{\beta} \tag{10}$$

$$d \le m'_p + m'_e, \tag{11}$$

where $\alpha_i = \alpha_H$ if the seller can produce customized goods and otherwise, $\alpha_i = \alpha_L$, (10) is the bargaining rule, i.e., $-x + \beta d = \theta[\alpha_i u(x) - x]$, and (11) is the feasibility constraint. The next lemma solves the maximization problem (9) describing the terms of trade.

Lemma 1. *Given late buyers' real balances* $(m'_p, m'_e)$ *and parameter value* $\alpha_i$ *for* $i \in \{H, L\}$ *in a pairwise meeting, the terms of trade,* $(x, d)$*, are given as*

$$\begin{aligned}(x, d) &= \big(\hat{x}_i(m'_p + m'_e), \hat{d}_i(m'_p + m'_e)\big) \\ &\equiv \begin{cases} (x_i^*, m_i^*) & \text{if } m'_p + m'_e \ge m_i^* \\ \big(\Phi_i^{-1}(m'_p + m'_e), m'_p + m'_e\big) & \text{if } m'_p + m'_e < m_i^*, \end{cases}\end{aligned} \tag{12}$$

*where* $\Phi_i(x) = [\theta\alpha_i u(x) + (1-\theta)x]/\beta$ *for each* $i \in \{H, L\}$.

See Appendix A for proofs.

Given the bargaining solution, the value of late buyers with asset portfolio $(m'_p, m'_e)$ in the evening is

$$\begin{aligned}V^b_{e,t}(m'_p, m'_e) = {}& \omega_t\big[\alpha_H u\big(\hat{x}_H(m'_p + m'_e)\big) - \beta\hat{d}_H(m'_p + m'_e)\big] \\ &+ (1-\omega_t)\big[\alpha_L u\big(\hat{x}_L(m'_p + m'_e)\big) - \beta\hat{d}_L(m'_p + m'_e)\big] \\ &+ \beta V^b_{m,t+1}(m'_p, m'_e, \boldsymbol{\iota}_t),\end{aligned} \tag{13}$$

where $\omega_t$ is the probability that a late buyer meets a seller who can produce customized goods tailored to the buyer's taste in a bilateral meeting and $\boldsymbol{\iota}_t = 0$ because late buyers do not trade in the afternoon market.

When the company sells preference information at the beginning of the evening, a seller optimally chooses the amount of information $n \in \{0, \dots, \mathcal{N}\}$ that he/she will buy at unit price $\varphi_t$. Based on the above arguments, the seller's value function at the beginning of the evening with asset portfolio $(m'_p, m'_e)$ is given as

$$\begin{aligned}&V^s_{e,t}(m'_p, m'_e) \\ &= \max_{n\in\{0,\dots,\mathcal{N}\}} \left\{ \begin{array}{c} \dfrac{(1-\rho)\kappa(B)n}{\mathcal{N}} \displaystyle\int \big[-\hat{x}_H(\tilde{m}_p + \tilde{m}_e) + \beta\hat{d}_H(\tilde{m}_p + \tilde{m}_e)\big]\, dF_t(\tilde{m}_p, \tilde{m}_e) \\ + \dfrac{(1-\rho)\big(\mathcal{N} - \kappa(B)n\big)}{\mathcal{N}} \\ \times \displaystyle\int \big[-\hat{x}_L(\tilde{m}_p + \tilde{m}_e) + \beta\hat{d}_L(\tilde{m}_p + \tilde{m}_e)\big]\, dF_t(\tilde{m}_p, \tilde{m}_e) \\ - n\varsigma + \beta V^s_{m,t+1}(m'_p, m'_e, n) \end{array} \right\},\end{aligned} \tag{14}$$

where $F_t(\tilde{m}_p, \tilde{m}_e)$ is the cumulative distribution of late buyers at the beginning of the evening in period $t$ holding $m'_p \leq \tilde{m}_p$ and $m'_e \leq \tilde{m}_e$ units of real balances in terms of morning goods in the next period. In (14), $1-\rho$ is the probability that a seller has a meeting because there are $1-\rho$ mass of late buyers and unit mass of sellers in the evening.

### 3.2 *Agents' optimal decisions*

In this subsection, we study the optimal behaviors of each economic agent in stationary equilibrium in which all real quantities are constant over time and, thus, $\phi_t/\phi_{t+1} = \gamma$.

*Buyers' choices* In the morning, buyers determine the portfolio of real balances. Note that the company does not provide rewards for using E-money in the evening and using E-money only incurs privacy cost $\delta$ to late buyers. Thus, late buyers will not hold any E-money, i.e., $m_e = 0$, and only use P-money in the evening. However, early buyers receive $R$ units of morning goods in the next period as a reward for using E-money. Thus, if $\beta R \geq \delta$, early buyers will use a positive amount of E-money. Without loss of generality, we assume that early buyers carry only E-money into the afternoon market if $\beta R \geq \delta$. This leads to the following lemma, whose proof is omitted.

Lemma 2. *For afternoon transactions, early buyers use E-money if $\beta R \geq \delta$ and use P-money otherwise. Late buyers always use P-money for evening transactions.*

Buyers will not carry any money into the next morning because the buyer's value functions in the morning are linear in money holdings and $\gamma \geq \beta$. Then, from (2) and (5)–(7), we obtain the early buyer's problem as

$$\max_{q_p, q_e} \{(\beta R - \delta)\mathbf{1}_{\{q_e>0\}} - \gamma p(q_p + q_e) + v(q_p + q_e)\},$$

where $\mathbf{1}_{\{q_e>0\}}$ is an indicator function that takes the value of 1 if $q_e > 0$ and 0 otherwise. Note from Lemma 2 that early buyers will either choose $q_p > 0$ and $q_e = 0$ or $q_e > 0$ and $q_p = 0$. Then the first-order condition is given as

$$\gamma p = \begin{cases} v'(q_p) & \text{if } q_e = 0 \\ v'(q_e) & \text{if } q_p = 0. \end{cases} \tag{15}$$

Letting $m_p$ denote the real P-money in terms of numeraire goods in the next period, we obtain, from Lemmas 1 and 2, (3), and (13), the late buyer's problem in the morning as

$$\max_{m_p} \left\{ \begin{matrix} -(\gamma - \beta)m_p + \omega[\alpha_H u(\hat{x}_H(m_p)) - \beta \hat{d}_H(m_p)] \\ + (1-\omega)[\alpha_L u(\hat{x}_L(m_p)) - \beta \hat{d}_L(m_p)] \end{matrix} \right\},$$

where $\omega$ is the probability that a buyer meets a seller who can produce evening goods tailored to the buyer's evening taste. Then the first-order condition for $m_p$ is given as

$$\begin{aligned} \gamma - \beta = {} & \omega[\alpha_H u'(\hat{x}_H(m_p))\hat{x}'_H(m_p) - \beta \hat{d}'_H(m_p)] \\ & + (1-\omega)[\alpha_L u'(\hat{x}_L(m_p))\hat{x}'_L(m_p) - \beta \hat{d}'_L(m_p)]. \end{aligned} \tag{16}$$

*Sellers' choices* In the morning, sellers spend all real balances to purchase numeraire goods and do not carry any money into the next subperiods due to the money holding cost, given that $\gamma \geq \beta$, as can be verified by (4), (8), and (14).

In the afternoon, sellers optimally supply goods given the market price $p$. Specifically, from (4), (8), and (14), we can write the seller's problem in the afternoon market as

$$\max_{q_p^s, q_e^s} \{-c(q_p^s + q_e^s) + \beta p(q_p^s + q_e^s)\},$$

which gives

$$c'(q_p^s + q_e^s) = \beta p \tag{17}$$

as the optimality condition.

In the evening, sellers decide the amount of information purchases. As shown in (16), all late buyers make the same choice for real balances given the probability $\omega$. Let $m_p$ be the equilibrium real P-money balance of late buyers in the evening. Then, from (4), (12), and (14), the seller's problem of information purchase in the evening can be written as

$$\max_{n \in \{0,\dots,\mathcal{N}\}} \left\{ n \left[ \frac{(1-\rho)\kappa(B)D(m_p)}{\mathcal{N}} - \varsigma - \beta\varphi \right] \right\}, \tag{18}$$

where

$$D(m_p) = \theta\{\alpha_H u(\hat{x}_H(m_p)) - \hat{x}_H(m_p) - [\alpha_L u(\hat{x}_L(m_p)) - \hat{x}_L(m_p)]\} \tag{19}$$

is the difference in the seller's trade surplus in a meeting between when the seller can produce customized goods tailored to the buyer's taste and when he/she cannot. Assuming that sellers purchase preference information if they are indifferent, the seller's optimal choice for the number of information purchases is given as

$$n = \begin{cases} \mathcal{N} & \text{if } D(m_p) \geq \dfrac{\mathcal{N}(\varsigma + \beta\varphi)}{(1-\rho)\kappa(B)} \\ 0 & \text{if } D(m_p) < \dfrac{\mathcal{N}(\varsigma + \beta\varphi)}{(1-\rho)\kappa(B)}. \end{cases} \tag{20}$$

The next lemma describes the properties of the $D(\cdot)$ function, which provides a useful intermediate step for equilibrium characterization.

Lemma 3. *For all $m_p < m_H^*$, $D'(m_p) > 0$, and for all $m_p \geq m_H^*$, $D(m_p) = \overline{D}$, where*

$$\overline{D} \equiv \theta[\alpha_H u(x_H^*) - x_H^*] - \theta[\alpha_L u(x_L^*) - x_L^*]. \tag{21}$$

The main implication of Lemma 3 is that an increase in the seller's trade surplus from being able to produce customized goods in a pairwise meeting increases with the late buyer's real balances that determine the trade volume in the evening market. Thus, sellers' incentives to purchase preference information rise with the trade volume in the evening market.

*Company's choices* The company decides the reward $R$ for using E-money in the afternoon market and the price of each preference information $\varphi$. Given the monopoly power and the results of Lemma 2, the company sets the reward as

$$R = \frac{\delta}{\beta} \tag{22}$$

whenever it runs its business. Then all early buyers use E-money, i.e., $B = \rho$, and, hence, the company obtains the correct preference information with certainty, i.e., $\kappa(B) = 1$.[10] Next, given the seller's choice described in (20) with $\kappa(B) = 1$, the company sets the price for each preference information as

$$\varphi = \frac{(1-\rho)D(m_p) - \mathcal{N}\varsigma}{\beta\mathcal{N}}. \tag{23}$$

### 3.3 *Market clearing conditions*

In equilibrium, asset and goods markets must clear. First, because E-money is backed by P-money, the sum of demands for E-money and P-money should be equal to the supply of money coming from the government. Thus, we obtain

$$\gamma\big[\rho p(q_p + q_e) + (1-\rho)m_p\big] = \phi_t M_t$$

as a market clearing condition in the money market. Second, buyers' demand for afternoon goods should be equal to the supply from sellers. Thus, a market clearing condition in the afternoon is given as

$$\rho q_p = q_p^s \quad \text{and} \quad \rho q_e = q_e^s. \tag{24}$$

### 3.4 *Equilibrium characterization*

Late buyers only use P-money, and early buyers use either P-money or E-money in equilibrium. Thus, there are two relevant cases: (i) equilibrium in which all buyers use P-money, and (ii) equilibrium in which early buyers use E-money. We call the first equilibrium P-equilibrium and the second equilibrium E-equilibrium in what follows. Note from (15), (17), and Lemma 2 that early buyers' demands and sellers' supplies in the afternoon market do not depend on the type of money that is used in the afternoon. Furthermore, late buyers use only P-money. Thus, in what follows, we drop the index $j \in \{e, p\}$ in $q_j$ and $m_j$ that specifies the type of money traded in each subperiod unless it causes any confusion.

From (15), (17), and (24), we obtain

$$q = \widetilde{q}(\gamma), \tag{25}$$

[10]This result hinges on the assumption of the constant disutility $\delta$ across buyers. If we introduce heterogeneity in disutility $\delta$, it is possible that $B \in (0, \rho)$ and $\kappa(B) \in (0, 1)$ in equilibrium. However, the main implications do not change with heterogeneous $\delta$.

where $\widetilde{q}:[\beta,\infty)\to\mathbb{R}_+$ is a decreasing function of $\gamma$ determined by

$$\frac{v'(\widetilde{q}(\gamma))}{c'(\rho\widetilde{q}(\gamma))}=\frac{\gamma}{\beta}. \tag{26}$$

Then we obtain the price of afternoon goods as $p=c'(\rho\widetilde{q}(\gamma))/\beta$ from (17) and (24).

In E-equilibrium, the company obtains correct preference information with certainty and sellers purchase all preference information. Thus, a late buyer meets a seller who can produce customized goods tailored to his/her taste in the evening with certainty, i.e., $\omega=1$. On the other hand, $\omega=0$ in P-equilibrium. Then, from (12) and (16), we obtain the equilibrium real balance $m$ of late buyers and trade volume $x$ in the evening as

$$x=\widetilde{x}_H(\gamma)\quad\text{and}\quad m=\widetilde{d}_H(\gamma)\quad\text{in E-equilibrium} \tag{27}$$

$$x=\widetilde{x}_L(\gamma)\quad\text{and}\quad m=\widetilde{d}_L(\gamma)\quad\text{in P-equilibrium,} \tag{28}$$

where

$$\widetilde{x}_i(\gamma)\equiv u'^{-1}\left(\frac{\gamma(1-\theta)}{\alpha_i(\beta-\theta\gamma)}\right) \tag{29}$$

$$\widetilde{d}_i(\gamma)\equiv\frac{\theta\alpha_i u(\widetilde{x}_i(\gamma))+(1-\theta)\widetilde{x}_i(\gamma)}{\beta} \tag{30}$$

are decreasing functions of $\gamma$ for each $i\in\{H,L\}$.

In summary, we have the following proposition, whose proof is omitted, which describes economic outcomes in each equilibrium.

Proposition 1. *Given monetary policy $\gamma$, real allocations and prices are as follows:*

*(I) In E-equilibrium, $q=\widetilde{q}(\gamma)$, $x=\widetilde{x}_H(\gamma)$, $m=\widetilde{d}_H(\gamma)$, $p=c'(\rho\widetilde{q}(\gamma))/\beta$, and $\varphi=[(1-\rho)D(\widetilde{d}_H(\gamma))-\mathcal{N}\varsigma]/(\beta\mathcal{N})$.*

*(II) In P-equilibrium, $q=\widetilde{q}(\gamma)$, $x=\widetilde{x}_L(\gamma)$, $m=\widetilde{d}_L(\gamma)$, and $p=c'(\rho\widetilde{q}(\gamma))/\beta$.*

In money search models, as the money growth rate $\gamma$ increases, the inflation rate and the money holding cost increase in a stationary equilibrium. Thus, buyers accumulate less money in the morning for transactions in subsequent subperiods, and trade volumes in the afternoon and evening markets fall, as shown in (25)–(30). A decrease in demand for afternoon goods decreases the market price of afternoon goods. Finally, as real balances of late buyers in the evening decrease, the value of preference information decreases by the result of Lemma 3 and, hence, the price of preference information $\varphi$ falls.

The existence of each type of equilibrium depends on whether the company can make positive profits from running its business. The company can make all early buyers use E-money and sell all preference information to all sellers by setting the reward as (22) and the information price as (23). Thus, the company's profit is given as

$$\pi = \frac{(1-\rho)D(m) - \mathcal{N}\varsigma - \rho\delta}{\beta}, \tag{31}$$

where $m$ is the late buyer's real P-money balances in the evening market in equilibrium. Then it must be that $D(m) \geq (\mathcal{N}\varsigma + \rho\delta)/(1-\rho)$ for E-equilibrium to exist because the company has no incentive to run its business otherwise. On the other hand, it must be that $D(m) < (\mathcal{N}\varsigma + \rho\delta)/(1-\rho)$ for P-equilibrium to exist because, otherwise, the company can make nonnegative profits from running its business. These arguments lead to the next proposition.

PROPOSITION 2. *Stationary monetary equilibrium exists as follows:*

(I) *Suppose that $(\mathcal{N}\varsigma + \rho\delta)/(1-\rho) \leq D(m_L^*)$. Then there exist $\gamma_1 > \gamma_2 \geq \beta$ such that (a) for all $\gamma \in [\beta, \gamma_1]$, E-equilibrium exists, and (b) for all $\gamma > \gamma_2$, P-equilibrium exists.*

(II) *Suppose that $D(m_L^*) < (\mathcal{N}\varsigma + \rho\delta)/(1-\rho) \leq \overline{D}$. Then, there exists $\gamma_3 \geq \beta$ such that (a) for all $\gamma \in [\beta, \gamma_3]$, E-equilibrium exists, and (b) for all $\gamma \geq \beta$, P-equilibrium exists.*

(III) *Suppose that $\overline{D} < (\mathcal{N}\varsigma + \rho\delta)/(1-\rho)$. Then, for all $\gamma \geq \beta$, P-equilibrium exists.*

Proposition 2 shows how the value of $(\mathcal{N}\varsigma + \rho\delta)/(1-\rho)$ and the inflation rate $\gamma$ together determine the existence of each type of equilibrium. Figure 2 depicts how the parameter space is subdivided with $(\mathcal{N}\varsigma + \rho\delta)/(1-\rho)$ on the vertical axis and $\gamma$ on the horizontal axis, illustrating Proposition 2 graphically.

The effects of parameters $\delta$, $\varsigma$, $\mathcal{N}$, and $\rho$ on the type of equilibrium are straightforward. Here $\rho\delta$ is the early buyers' disutility from forgoing privacy for using E-money, $\mathcal{N}\varsigma$

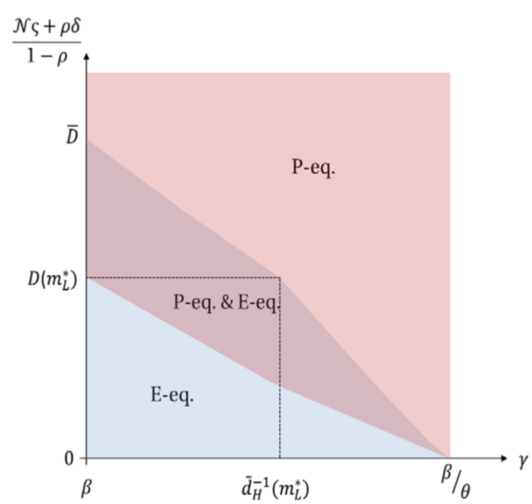


FIGURE 2. Typology of equilibria in $(\gamma, \frac{\mathcal{N}\varsigma+\rho\delta}{1-\rho})$ space.

is the total investment cost that sellers incur to prepare the production of customized goods in the evening, and $1 - \rho$ is the probability that a seller meets a buyer in the evening market. Thus, if $\delta$, $\varsigma$, $\mathcal{N}$, and $\rho$ are sufficiently high, as illustrated in the third case of Proposition 2, the costs of obtaining and harnessing preference information are higher than the expected payoff; hence, the company cannot make nonnegative profits from its business.

Next, Proposition 2 shows that the company is more likely to run the E-money business when $\gamma$ is low, while E-money is not circulated when $\gamma$ is sufficiently high.[11][12] The economic mechanism for this result is in line with our earlier observation. As $\gamma$ increases, late buyers carry less P-money into the evening market, which reduces the value of preference information to sellers by the results of Lemma 3. Consequently, the price of preference information and the company's profit decrease. Thus, it is more likely that the company runs the E-money business making nonnegative profits when $\gamma$ is low and vice versa.[13]

One noticeable result in Proposition 2 is that the model can generate multiple stationary monetary equilibria: For all $\gamma \in (\gamma_2, \gamma_1]$ when $(\mathcal{N}\varsigma + \rho\delta)/(1 - \rho) \leq D(m_L^*)$ or for all $\gamma \in [\beta, \gamma_3]$ when $D(m_L^*) < (\mathcal{N}\varsigma + \rho\delta)/(1 - \rho) \leq \overline{D}$, both E-equilibrium and P-equilibrium can exist. The intuition for this finding is as follows. For intermediate inflation, if late buyers expect that they can buy customized goods in the evening, they accumulate a sufficient amount of money, which motivates sellers to buy preference information from the company. On the other hand, if late buyers expect that they cannot buy customized goods in the evening, they hold too little money in the evening to incentivize sellers to buy preference information, which justifies late buyers' initial expectation. This complementarity leads to multiple equilibria.

A multiplicity of equilibria due to strategic complementarities has been discussed in previous studies. For instance, in the platform model with two-sided markets, such as Rochet and Tirole (2003) and Hagiu and Spulber (2013), one group's benefit of using the platform increases with the size of the other group that uses the platform, which generates strategic complementarities and multiple equilibria. The E-money business in our model is similar to a platform in the sense that it connects suppliers of information (early buyers) and users of information (sellers). However, early buyers' choices do not depend on sellers' decision on information purchases: Early buyers only care about the privacy costs and the rewards for using E-money. Strategic complementarities in our model exist between the seller's investment decision and the real balances of late buyers who do not use E-money.

[11] Berentsen, Camera, and Waller (2007) show that the existence of a monetary equilibrium with credit requires some positive inflation, while E-money and P-money coexist when inflation is sufficiently low in our model.

[12] This result implies that an increase in the inflation rate can decrease the variety of goods that sellers can produce because sellers can produce customized goods only if the E-money business exists. In this sense, our model is related to Shevchenko (2004) and Dong (2010) who investigate how economic environments, such as inflation and search frictions, affect the variety of goods.

[13] Note that our model delivers potentially testable predictions about the relation between the inflation rate $\gamma$ and the profitability of the E-money business: An increase in $\gamma$ reduces the company's profit and thereby drives out the E-money business from the economy.

More related, in the money search literature, Lester, Postlewaite, and Wright (2012) show that if agents can verify the asset's quality at a cost, then strategic complementarities exist between buyers' asset demand and sellers' information investment. This strategic complementarity can generate multiple equilibria similar to our model.[14] However, our model differs from Lester, Postlewaite, and Wright (2012) in the way that sellers obtain information. In their model, sellers pay an exogenous fixed cost to obtain information about the asset's quality, while the price of preference information in our model is determined by the profit maximizing company in equilibrium. Furthermore, the company in our model is a big player, so it may be able to lead to E-equilibrium when multiple equilibria are feasible as follows.

Suppose that when the company sells preference information, it promises to compensate a seller if he/she does not make a sufficient surplus from selling customized goods because the matched buyer in a bilateral meeting holds less than $\widetilde{d}_H(\gamma)$ units of real P-money.[15] Note that the company can correctly infer the sellers' trade surplus in the evening in equilibrium by observing aggregate variables, such as aggregate trade volumes in the evening, although the company cannot directly observe the trade volume of individual sellers in the evening market. Thus, the company can make compensation contingent on those aggregate variables.

Then sellers will buy preference information without the concern of making insufficient surplus in the evening. Anticipating that sellers will prepare to produce customized goods in the evening market, late buyers will carry $\widetilde{d}_H(\gamma)$ units of real P-money into the evening. Thus, the company does not need to compensate sellers in equilibrium, and guaranteeing the seller's surplus can eliminate P-equilibrium when multiple equilibria are feasible, similar to finding that a bank—a big player—can prevent bank run equilibrium by announcing the suspension of convertibility in Diamond and Dybvig (1983).

However, the company cannot use the surplus guarantee if some late buyers do not have an individual taste for evening goods. Suppose that the $\mu \in (0, 1)$ fraction of late buyers does not have evening tastes and that their utility in the evening is $\alpha_L u(x)$. Then the $\mu$ fraction of late buyers will bring $\widetilde{d}_L(\gamma)$ units of real balances into the evening market even though sellers can produce customized goods, and the other $1 - \mu$ faction of late buyers will hold $\widetilde{d}_H(\gamma)$ units of real balances. Thus, there are sellers who met a late buyer with $\widetilde{d}_H(\gamma)$ units of real balances and sellers who met a late buyer with $\widetilde{d}_L(\gamma)$ units of real balances in the evening.[16] In this case, the company needs to provide compensation based on information about trade surpluses reported by individual sellers.

[14]There are other asset exchange models, such as Trejos and Wright (2016), Burdett, Trejos, and Wright (2017), and He and Wright (2019), that show the existence of multiple equilibria. However, in those models, multiple equilibria exist because of a self-fulfilling prophecy of the asset's liquidity—if an agent thinks others value the asset high, then the agent would give high payment to get the asset—not because of a strategic complementarity. Furthermore, in those models, the asset is indivisible and multiple equilibria do not exist if the asset is fiat.

[15]Specifically, what the company does is to guarantee $\theta[\alpha_H u(\hat{x}_H(\widetilde{d}_H(\gamma))) - \hat{x}_H(\widetilde{d}_H(\gamma))]$ units of trade surplus to a seller in the evening if the seller buys preference information.

[16]Otherwise, introducing the assumption that the $\mu$ fraction of late buyers does not have evening taste does not change equilibrium outcomes except that an increase in the seller's trade surplus in the evening by preparing the production of customized goods without the surplus guarantee is now given as $(1-\mu)D(m_p)$.

However, the company cannot verify each seller's trade volume in the evening, so sellers will always misinform the company of their surplus to obtain compensation. Thus, the company cannot use the surplus guarantee as a tool for supporting E-equilibrium when multiple equilibria are feasible.

## 4. Welfare analysis

In this section, we examine the model's normative properties in terms of social welfare, investigate the optimal monetary policy, and explore the effects of government interventions, such as providing subsidies, on real allocations and welfare. We define the sum of expected utilities in a steady state equilibrium across agents with equal weight as our welfare measure,

$$W = \rho v(q) - c(\rho q) + (1-\rho)\big[\alpha_i u(x) - x\big] - \mathbf{1}_{\{e=E\}}(\mathcal{N}\varsigma + \rho\delta),$$

where $i = H$ in E-equilibrium, $i = L$ in P-equilibrium, and $\mathbf{1}_{\{e=E\}}$ is an indicator function that takes the value of 1 if the economy is in E-equilibrium and 0 otherwise. Specifically, given the results of Proposition 1, welfare is given as

$$W = W_P(\gamma) \equiv \rho v\big(\widetilde{q}(\gamma)\big) - c\big(\rho\widetilde{q}(\gamma)\big) + (1-\rho)\big[\alpha_L u\big(\widetilde{x}_L(\gamma)\big) - \widetilde{x}_L(\gamma)\big] \tag{32}$$

in P-equilibrium and

$$W = W_E(\gamma) \equiv \rho v\big(\widetilde{q}(\gamma)\big) - c\big(\rho\widetilde{q}(\gamma)\big) + (1-\rho)\big[\alpha_H u\big(\widetilde{x}_H(\gamma)\big) - \widetilde{x}_H(\gamma)\big] - (\mathcal{N}\varsigma + \rho\delta) \tag{33}$$

in E-equilibrium. Because the company runs the E-money business in E-equilibrium and does not in P-equilibrium, $W_E(\gamma) - W_P(\gamma)$ measures the social net benefit of the E-money business. The next lemma shows how the inflation rate $\gamma$ affects this social net benefit.

Lemma 4. *The social net benefit $W_E(\gamma) - W_P(\gamma)$ decreases with $\gamma$.*

To obtain the intuition for the results of Lemma 4, note that the trade volume $x$ in the evening market falls as $\gamma$ increases. A decrease in $x$, in turn, reduces the contribution of the E-money business to welfare—an increase in the trade surplus $\alpha u(x) - x$ by raising $\alpha$ from $\alpha_L$ to $\alpha_H$—while social cost $\mathcal{N}\varsigma + \rho\delta$ stays at a constant level. Thus, the social net benefit of the E-money business $W_E(\gamma) - W_P(\gamma)$ decreases with $\gamma$.

In the model economy, what the company considers when it decides whether to run the E-money business is the profit from the business, not its social benefit. Thus, the company may run (may not run) the E-money business although it is socially undesirable (desirable). However, the next proposition shows that whenever the economic environment described by $(\mathcal{N}, \varsigma, \rho, \delta, \gamma)$ supports nonnegative profit from the E-money business in E-equilibrium, welfare is higher with the active E-money business than without it.

Proposition 3. *Suppose that the company can make nonnegative profits from its business when late buyers hold $\widetilde{d}_H(\gamma)$ units of real balances, i.e., E-equilibrium can exist. Then $W_E(\gamma) > W_P(\gamma)$, so welfare is higher with the E-money business than without it.*

The intuition for the results of Proposition 3 is as follows. As shown in (31) and (33), the private cost of running the E-money business equals its social costs $\mathcal{N}\varsigma + \rho\delta$, so the company fully internalizes the social cost of the E-money business. On the other hand, the main source of the E-money business' revenue is the increase in the seller's trade surplus by selling customized goods in the evening market (private benefit), while the E-money business' positive effects on welfare are the increase in the total trade surplus from trading customized goods in the evening (social benefit), and there is a wedge between the private and social benefits of the E-money business for the following two reasons.

First, expecting to consume customized goods, late buyers hold more real balances in E-equilibrium than in P-equilibrium. Thus, the trade volume of noncustomized goods, and thereby the trade surplus, is higher in E-equilibrium than in P-equilibrium when the money holding cost exists, i.e., $\hat{x}_L(\widetilde{d}_H(\gamma)) > \widetilde{x}_L(\gamma)$ for all $\gamma > \beta$. This implies a higher social benefit than the private benefit of the E-money business ceteris paribus, because the main source of the private and social benefits of the E-money business is obtained by subtracting the surplus from trading noncustomized goods from the surplus from trading customized goods.[17] Second, in the model, the seller's trade surplus is the $\theta \in (0, 1)$ fraction of the total trade surplus in the evening market given the bargaining rule. Thus, the private benefit is scaled down by $\theta$ in contrast to the social benefit. For these two reasons, the social benefit is higher than the private benefit. Thus, whenever the company makes a nonnegative profit from running the E-money business, it must be that $W_E(\gamma) > W_P(\gamma)$.

Note from (1), (12), (29), and (30) that $\hat{x}_L(\widetilde{d}_H(\beta)) = \widetilde{x}_L(\beta) = x_L^*$. Thus, when $\gamma = \beta$, the surplus from trading noncustomized goods under each type of equilibrium is the same, so the first channel, described above, that generates the wedge between the social and private benefits of the E-money business disappears. Next, the second channel originates from the bargaining assumption in bilateral meetings, and this channel may disappear under a different market structure in the evening. For instance, suppose that late buyers and sellers trade in the evening through a matching platform operated by the company. Then the company can extract the additional surplus that late buyers obtain from trading customized goods by raising the late buyers' fees for using the platform. Furthermore, in Section 5, we show that the company can also extract the late buyer's additional surplus without running the matching platform if the search is directed with the price posting in the evening.

[17] Specifically, note from (19), (31), (32), and (33) that the private and social benefits are obtained by subtracting $\alpha_L u(\hat{x}_L(\widetilde{d}_H(\gamma))) - \hat{x}_L(\widetilde{d}_H(\gamma))$ and $\alpha_L u(\widetilde{x}_L(\gamma)) - \widetilde{x}_L(\gamma)$, respectively, from $\alpha_H u(\widetilde{x}_H(\gamma)) - \widetilde{x}_H(\gamma)$. Because late buyers carry more real balances in E-equilibrium than in P-equilibrium expecting to consume customized goods, it must be that $\hat{x}_L(\widetilde{d}_H(\gamma)) \geq \widetilde{x}_L(\gamma)$ with the strict inequality when $\gamma > \beta$. Thus, we subtract a higher value from $\alpha_H u(\widetilde{x}_H(\gamma)) - \widetilde{x}_H(\gamma)$ when we calculate the private benefit than when we calculate the social benefit.

The alternative interpretation of the results of Proposition 3 is that the company does not fully internalize the positive effects of using payment data on welfare, so the socially efficient E-money business may not exist in equilibrium. Specifically, note from (30), (31), Proposition 1, and Lemmas 3 and 4 that $W_E(\gamma) - W_P(\gamma)$ and the company's profit $\pi$ decrease with $\gamma$. Thus, if $\gamma^*$ and $\gamma^{**}$ exist such that $W_E(\gamma^*) = W_P(\gamma^*)$ and the company earns zero profit when $\gamma = \gamma^{**}$, it must be that $\gamma^* > \gamma^{**}$ by the results of Proposition 3. Then, for all $\gamma \in (\gamma^{**}, \gamma^*)$, the company does not run the E-money business although the E-money business improves social welfare. This result is formally stated in the next proposition.

Proposition 4. *Assume that* $(\mathcal{N}\varsigma + \rho\delta)/(1-\rho) < \overline{D}/\theta$. *Then there exist* $\gamma^* > \beta$ *and* $\gamma^{**} \in [\beta, \gamma^*)$ *such that for all* $\gamma \in (\gamma^{**}, \gamma^*)$, $W_E(\gamma) > W_P(\gamma)$, *but the company does not run the E-money business and the economy is in P-equilibrium.*

As explained earlier, socially efficient uses of payment data may not exist because the private benefit of the E-money business is lower than its social benefit. Thus, the government may fix this problem by subsidizing the E-money industry. Specifically, we consider two schemes for subsidy policy: The government provides $\Delta$ units of numeraire goods to the company the next morning if the company issues E-money (subsidy to the company) and the government provides a seller $\Delta/\mathcal{N}$ units of numeraire goods the next morning for buying each evening taste information (subsidy to sellers).[18,19] We assume that the government raises funds for the subsidy with lump-sum taxes on buyers in the morning.

Note that the company can extract the government's subsidy to sellers by selling preference information at a higher price. However, the identity of the subsidy recipient could affect sellers' incentives to prepare the production of customized goods and, thereby, welfare in a different way, as described in the next proposition.

Proposition 5. *Suppose that* $W_E(\gamma) > W_P(\gamma)$ *and* $D(\widetilde{d}_H(\gamma)) < (\mathcal{N}\varsigma + \rho\delta)/(1-\rho)$, *so the company does not run the socially efficient E-money business. If the government supports the E-money industry with* $\Delta \geq [\mathcal{N}\varsigma + \rho\delta - (1-\rho)D(\widetilde{d}_H(\gamma))]/\beta$ *units of subsidy, then welfare* $W$ *under each subsidy scheme is given as follows:*

*(I) Under the subsidy to the company,* $W = W_E(\gamma)$ *if* $D(\widetilde{d}_H(\gamma)) \geq \mathcal{N}\varsigma/(1-\rho)$ *and* $W = W_P(\gamma) - \rho\delta$ *if* $D(\widetilde{d}_H(\gamma)) < \mathcal{N}\varsigma/(1-\rho)$.

*(II) Under the subsidy to sellers,* $W = W_E(\gamma)$.

When $D(\widetilde{d}_H(\gamma)) \geq \mathcal{N}\varsigma/(1-\rho)$, sellers prepare the production of customized goods unless price $\varphi$ is too high as shown in (20), and the company can make nonnegative

[18]The government can also subsidize early buyers for using E-money in the afternoon. However, its economic effects are the same as those of subsidizing the company, because the company can make early buyers use its E-money with less (even negative) rewards for using it in the afternoon.

[19]Note that the total amount of subsidies under each scheme is $\Delta$, because sellers buy $\mathcal{N}$ number of evening taste information.

profit under each subsidy scheme. However, when $D(\widetilde{d}_H(\gamma)) < \mathcal{N}\varsigma/(1-\rho)$, the increase in the seller's expected surplus from selling customized goods is lower than the fixed investment cost $\mathcal{N}\varsigma$. Thus, sellers would not prepare the production of customized goods without any support. In this case, if the government gives subsidies to the company, the company issues E-money to obtain subsidies without selling preference information. Thus, real allocations are the same as in P-equilibrium except that early buyers incur privacy costs, so $W = W_P(\gamma) - \rho\delta$. On the other hand, if the government subsidizes sellers for buying preference information, they will prepare the production of customized goods. Thus, the economy is in E-equilibrium and $W = W_E(\gamma)$. As shown from the above analysis, the subsidy to sellers is more effective than the subsidy to the company because the former directly incentivizes sellers, who are the final users of preference information, to harness preference information.

We now analyze the effects of monetary policy, i.e., changes in $\gamma$, on welfare. As shown in Proposition 1, $q$ and $x$ decrease with $\gamma$ in each type of equilibrium. Because the trade volumes in the afternoon and evening markets are inefficiently low for all $\gamma > \beta$, welfare $W$ decreases with $\gamma$ in each type of equilibrium. Furthermore, as shown in Proposition 2, a decrease in $\gamma$ tends to change the equilibrium type from P-equilibrium to E-equilibrium, thereby discontinuously increasing welfare by supporting socially efficient uses of payment data. Thus, welfare monotonically decreases with $\gamma$. This implies that the optimal monetary policy is the Friedman rule as stated in the next proposition, whose proof is omitted.

Proposition 6. *Optimal monetary policy is the Friedman rule, i.e.,* $\gamma = \beta$.

The results of Proposition 6, however, do not necessarily imply that the Friedman rule always achieves the highest welfare. Although the Friedman rule eliminates the money holding cost, the socially efficient E-money business may not exist due to the externality problem. Specifically, if $\overline{D} < (\mathcal{N}\varsigma + \rho\delta)/(1-\rho) < \overline{D}/\theta$, the economy is in P-equilibrium under the Friedman rule as shown by Proposition 2, while welfare is higher with the active E-money business than without it as shown by (32) and (33). However, with appropriate use of the subsidy to sellers, the Friedman rule achieves the highest welfare in the model economy.

## 5. Competitive search in the evening

In this section, we adopt the concept of competitive search, where sellers post their terms of trade for evening transactions and late buyers direct their search in the evening, to understand how equilibrium outcomes depend on the market structure in the evening. We assume that the economy is composed of different submarkets in the evening, where a submarket is identified by its terms of trade posted by sellers.[20] We further assume that within any submarket, late buyers and sellers are randomly matched.

[20]In this section, we remove the upper bound $\beta/\theta$ for $\gamma$, because the seller's bargaining power $\theta$ does not matter in the model with competitive search.

Note that the equilibrium outcomes in the afternoon markets are the same as in bargaining equilibrium, so we focus on the equilibrium outcomes in the evening market.

The sequence of events is as follows. At the beginning of the morning, the company announces its reward policy $R$ and price $\varphi$ for each evening taste information. After observing $(R, \varphi)$, each seller decides whether to prepare the production of customized goods in the evening. Given the linear preference of sellers in the morning and evening, a seller will either buy all evening taste information or buy no information.[21] If the seller chooses to sell customized goods in the evening under the assumption that he/she can buy the correct preference information, the seller posts the terms of trade $(x_H, d_H)$ for customized goods in the morning.[22] Otherwise, the seller posts the terms of trade $(x_L, d_L)$ for noncustomized goods in the morning. Finally, based on the observed terms of trade in the morning, late buyers decide which particular submarket identified by $(x_i, d_i)_{i\in\{H,L\}}$ they will visit in the evening and the quantity of real balances that they carry into the evening market.

Let $U^b$ denote the expected surplus of a late buyer in the evening given $(R, \varphi)$, net of the money holding cost. Then, in any active submarket $(x_i, d_i)$ for each $i \in \{H, L\}$, we have

$$-(\gamma-\beta)d_i + \min\left\{1, \frac{1}{n_i}\right\}\left[\alpha_i u(x_i) - \beta d_i\right] = U^b, \tag{34}$$

where $n_i$ denotes the ratio of buyers per seller in a submarket $(x_i, d_i)$.

When a seller posts his/her terms of trade in the morning, the seller takes $U^b$ as given, and $(x_i, d_i)$ determines the length of the queue, $n_i$, in his/her submarket. Specifically, if the seller chooses to sell noncustomized goods, his/her expected surplus is given as

$$V_L^s = \max_{x_L, d_L, n_L} \left\{\min\{1, n_L\}(-x_L + \beta d_L)\right\} \tag{35}$$

subject to (34) with $i = L$. On the other hand, if the seller decides to sell customized goods under the assumption that the seller can buy the correct preference information from the company, then his/her expected surplus is given as

$$V_H^s = \max_{x_H, d_H, n_H} \left\{\min\{1, n_H\}(-x_H + \beta d_H) - \mathcal{N}(\varsigma + \beta\varphi)\right\} \tag{36}$$

subject to (34) with $i = H$.

The type of goods that sellers can sell depends on whether the company actively runs the E-money business. First, if $R \geq \delta/\beta$, then the company will obtain the correct preference information, and sellers will optimally decide the type of evening goods that they will sell. Thus, the seller's expected surplus, denoted by $V^s$, from actively participating in the evening is given as $V^s = \max\{V_H^s, V_L^s\}$. On the other hand, if $R < \delta/\beta$, then

[21] We assume that sellers choose to prepare the production of customized goods if they are indifferent.

[22] If a seller chooses to prepare the production of customized goods, he/she can post the terms of trade for customized goods and noncustomized goods. However, the seller prepares the production of customized goods only if it is better to sell customized goods than noncustomized goods. Thus, we assume that the seller only posts the terms of trade for customized goods in the morning without loss of generality.

the company does not obtain preference information, so sellers have no choice but to sell noncustomized goods in the evening and $V^s = V_L^s$.

The equilibrium value of the late buyer's expected surplus $U^b$ given $(R, \varphi)$ is determined such that the ratio of buyers per seller in the different submarkets is consistent with the mass of late buyers and sellers in the evening. To make the analysis clear, we index each seller by $\mathbf{i} \in [0, 1]$, define $\Omega \subseteq [0, 1]$ as the set of active sellers participating in the evening, and let $n(\mathbf{i})$ denote the measure of late buyers per seller in the submarket of seller $\mathbf{i} \in \Omega$. Then, in competitive search equilibrium, the condition

$$N^d \equiv \int_\Omega n(\mathbf{i})\, d\mathbf{i} = N^s \in [0, 1-\rho]$$

must hold, where $N^d$ is the aggregate demand for active late buyers by sellers and $N^s$ is the aggregate supply of active late buyers in the evening.

To set the stage of equilibrium characterization, we first derive the upper bound of the late buyer's surplus in any equilibrium given $(R, \varphi)$. Let $\overline{U}_L^b$ and $\overline{U}_H^b$ denote the upper bound of the late buyer's surplus when he/she chooses to buy noncustomized goods and customized goods, respectively. Then, from (34)–(36), we obtain

$$\overline{U}_L^b = -\frac{\gamma}{\beta}\bar{x}_L(\gamma) + \alpha_L u\big(\bar{x}_L(\gamma)\big) \tag{37}$$

$$\overline{U}_H^b = -\frac{\gamma}{\beta}\bar{x}_H(\gamma) + \alpha_H u\big(\bar{x}_H(\gamma)\big) - \frac{\gamma}{\beta}\mathcal{N}(\varsigma + \beta\varphi), \tag{38}$$

where

$$\bar{x}_i(\gamma) = u'^{-1}\left(\frac{\gamma}{\beta\alpha_i}\right) \tag{39}$$

for each $i \in \{H, L\}$. Using these definitions, we characterize the competitive search equilibrium given $(R, \varphi)$ in the next proposition.

Proposition 7. *Define the cutoff level of the price of evening taste information as*

$$\varphi^* = \frac{1}{\gamma\mathcal{N}}\left\{-\frac{\gamma}{\beta}\bar{x}_H(\gamma) + \alpha_H u\big(\bar{x}_H(\gamma)\big) - \left[-\frac{\gamma}{\beta}\bar{x}_L(\gamma) + \alpha_L u\big(\bar{x}_L(\gamma)\big)\right]\right\} - \frac{\varsigma}{\beta}. \tag{40}$$

*Then the equilibrium outcomes with competitive search given $(R, \varphi)$ are as follows:*

*(I) If $R \geq \delta/\beta$ and $\varphi \leq \varphi^*$, then all active sellers sell customized goods, $(x_H, d_H) = (\bar{x}_H(\gamma), [\bar{x}_H(\gamma) + \mathcal{N}(\varsigma + \beta\varphi)]/\beta)$, $n_H = 1$, $U^b = \overline{U}_H^b$, $V^s = 0$, and $N^d = N^s = 1 - \rho$.*

*(II) If $R < \delta/\beta$ or $\varphi > \varphi^*$, then all active sellers sell noncustomized goods, $(x_L, d_L) = (\bar{x}_L(\gamma), \bar{x}_L(\gamma)/\beta)$, $n_L \leq 1$, $U^b = \overline{U}_L^b$, $V^s = 0$, and $N^d = N^s = 1 - \rho$.*

In the evening, late buyers are on the short side of the market because a unit mass of sellers go to the evening market while a $1-\rho$ mass of late buyers go there. As a result, late buyers have all the market power and extract the entire match surplus in competitive search equilibrium. Thus, sellers receive zero surplus.[23] Next, the company cannot obtain preference information if $R < \delta/\beta$ because no early buyers use E-money. Furthermore, sellers must buy preference information to prepare for the production of customized goods. Thus, sellers sell customized goods only if $R \geq \delta/\beta$ and the price of preference information is sufficiently low as $\varphi \leq \varphi^*$. Since all active sellers sell either customized goods or noncustomized goods, we call equilibrium where sellers sell noncustomized (customized) goods P-equilibrium (E-equilibrium) similar to the baseline model with the bargaining.[24]

We now investigate the company's optimal decision. Given the results of Proposition 7, the company will set $(R, \varphi) = (\delta/\beta, \varphi^*)$ if it chooses to run the E-money business. Note that $n_H = 1$ and $N^d = 1-\rho$ in E-equilibrium. Thus, the company sells preference information to the $1-\rho$ measure of sellers and the profit is given as $\pi^{\text{cs}} = [(1-\rho)\beta\mathcal{N}\varphi^* - \rho\delta]/\beta$. Using (40), we can express the company's profit as a function of $\gamma$ such that $\pi^{\text{cs}} = \Pi(\gamma)/\beta$, where

$$\Pi(\gamma) = (1-\rho)\left\{-\bar{x}_H(\gamma) + \frac{\beta}{\gamma}\alpha_H u\big(\bar{x}_H(\gamma)\big) + \bar{x}_L(\gamma) - \frac{\beta}{\gamma}\alpha_L u\big(\bar{x}_L(\gamma)\big) - \mathcal{N}\varsigma\right\} - \rho\delta. \quad (41)$$

Then the company will run the E-money business and the economy is in E-equilibrium if and only if $\Pi(\gamma) \geq 0$. This leads to the next proposition.

Proposition 8. *Stationary monetary equilibrium with competitive search exists as follows:*

*(I) If $\alpha_H u(x_H^*) - x_H^* - [\alpha_L u(x_L^*) - x_L^*] \geq [(1-\rho)\mathcal{N}\varsigma + \rho\delta]/(1-\rho)$, there is $\gamma^{\text{cs}} \geq \beta$ such that (a) for all $\gamma \in [\beta, \gamma^{\text{cs}}]$, E-equilibrium exists, and (b) for all $\gamma > \gamma^{\text{cs}}$, P-equilibrium exists.*

*(II) If $\alpha_H u(x_H^*) - x_H^* - [\alpha_L u(x_L^*) - x_L^*] < [(1-\rho)\mathcal{N}\varsigma + \rho\delta]/(1-\rho)$, then for all $\gamma \geq \beta$, P-equilibrium exists.*

The effects of $\delta$, $\mathcal{N}$, $\varsigma$, $\rho$, and $\gamma$ on the equilibrium type are similar to those in the bargaining model by the same rationale. However, in contrast to the bargaining model, there is a unique equilibrium under competitive search. This is because the terms of trade posted by sellers work as a coordinating device and internalize any strategic complementarity between late buyers' money demand and sellers' decisions about the type of goods.

[23] One thing to note is that when active sellers sell customized goods, they can offer late buyers their market expected utility $\overline{U}_H^b$ without generating a negative payoff for themselves only if there is no congestion on the seller's side, i.e., $n_H = 1$, in contrast to the case when they sell noncustomized goods. This is because sellers must incur cost $\mathcal{N}(\varsigma + \beta\varphi)$ before being matched to sell customized goods as shown in (36).

[24] All open submarkets have the same terms of trade because the solutions to (35) and (36) are unique.

Another feature of competitive search equilibrium is that the company harvests the entire gains created by trading customized goods while the company only extracts the increase in the sellers' surplus in bargaining equilibrium. The intuitive explanations are as follows. In competitive search equilibrium, the late buyer's payment to a seller equals the total cost of producing evening goods, including the cost of purchasing preference information when customized goods are traded. Consequently, an increase in the price of preference information $\varphi$ transfers the late buyer's surplus to the company. In particular, $\overline{U}_L^b = \overline{U}_H^b$ when $\varphi = \varphi^*$ as shown by (37), (38), and (40). Thus, the company can fully extract the increase in the match surplus from trading customized goods by setting $\varphi = \varphi^*$.

Does this finding imply that the company runs the E-money business in a socially efficient way? To answer this question, we investigate welfare in competitive search equilibrium. From (25) and Proposition 7, we obtain welfare $W^{\text{cs}}$ under competitive search as

$$W^{\text{cs}} = W_P^{\text{cs}}(\gamma) \equiv \rho v\big(\widetilde{q}(\gamma)\big) - c\big(\rho\widetilde{q}(\gamma)\big) + (1-\rho)\big[\alpha_L u\big(\bar{x}_L(\gamma)\big) - \bar{x}_L(\gamma)\big] \tag{42}$$

in P-equilibrium and

$$\begin{aligned} W^{\text{cs}} = W_E^{\text{cs}}(\gamma) \equiv{}& \rho v\big(\widetilde{q}(\gamma)\big) - c\big(\rho\widetilde{q}(\gamma)\big) + (1-\rho)\big[\alpha_H u\big(\bar{x}_H(\gamma)\big) - \bar{x}_H(\gamma)\big] \\ & - (1-\rho)\mathcal{N}\varsigma - \rho\delta \end{aligned} \tag{43}$$

in E-equilibrium. Then, the E-money business contributes to social welfare whenever $W_E^{\text{cs}}(\gamma) - W_P^{\text{cs}}(\gamma) \geq 0$.

Note from (41)–(43) that for all $\gamma > \beta$, $W_E^{\text{cs}}(\gamma) - W_P^{\text{cs}}(\gamma) > \Pi(\gamma)$. Thus, the socially efficient E-money business does not exist if $W_E^{\text{cs}}(\gamma) - W_P^{\text{cs}}(\gamma) \geq 0 > \Pi(\gamma)$ because the company runs the business only if $\Pi(\gamma) \geq 0$. However, note that $W_E^{\text{cs}}(\beta) - W_P^{\text{cs}}(\beta) = \Pi(\beta)$. Thus, when $\gamma = \beta$, the company runs the E-money business if and only if $W_E^{\text{cs}}(\beta) \geq W_P^{\text{cs}}(\beta)$. Consequently, in competitive search equilibrium, the Friedman rule not only maximizes the trade surplus, but also supports the socially efficient use of payment data. Thus, the Friedman rule always achieves the highest welfare in contrast to the bargaining model. This is because the company completely internalizes the increase in the trade surplus from trading customized goods as the revenue, thereby eliminating the externality problem, and the Friedman rule removes the money holding cost.

Competitive search equilibrium also differs from bargaining equilibrium in terms of the level of welfare. First, for all $\gamma > \beta$, the trade volume in the evening is higher in competitive search equilibrium than in bargaining equilibrium, i.e., $\bar{x}_i(\gamma) > \widetilde{x}_i(\gamma)$ for $i \in \{H, L\}$, as described in Propositions 1 and 7. This is because competitive search equilibrium is basically equivalent to having late buyers and sellers commit to the terms of trade before being matched, which resolves the holdup problem that exists in bargaining equilibrium. Second, as shown in (33) and (43), when the company runs the E-money business, $1-\rho$ mass of sellers incurs $\mathcal{N}\varsigma$ units of cost in competitive search equilibrium, while unit mass of sellers incurs that cost in bargaining equilibrium. For these two reasons, welfare is higher in competitive search equilibrium than in bargaining equilibrium.

## 6. Multiple E-money issuers

We have assumed that the monopoly company runs the E-money business. We now extend the baseline model such that there exists $J > 2$ companies indexed by $j \in \{1, \dots, J\}$ that can issue their own digital currency and estimate evening taste information by analyzing afternoon transactions of their own digital currency.[25] Because buyers must provide personal information to different companies for using different E-moneys, we assume that buyers incur disutility $\delta$ whenever they open an account at each company.[26] Finally, to focus on the effects of introducing competition in the E-money business industry, we assume that all companies have the same technology that the monopoly company has in the baseline model to estimate preference information from afternoon transaction data and we focus on equilibrium in which companies do not cooperate.[27]

We first consider the model with bargaining in the evening. The extended model generates a game among companies and we adopt pure strategy subgame perfect Nash equilibrium as our equilibrium concept for the game. Specifically, at the beginning of the morning, each company $j \in \{1, \dots, J\}$ decides whether to run the E-money business and chooses its reward scheme for using its E-money in the afternoon. Then, in the evening, a company determines the price of preference information that the company obtained from its E-money transaction data in the afternoon.

In the model, providing proportional rewards causes an additional cost for a company by distorting buyers' E-money holdings (see Appendix B), and the probability that a company obtains the correct preference information by analyzing afternoon transaction data only depends on the mass of its E-money users. Consequently, it is optimal for a company to only provide a fixed reward to early buyers if the company decided to run the business. This implies that any active companies will provide $R = \delta/\beta$ units of the fixed reward for using their E-moneys in the afternoon and they will obtain the correct evening taste information with the certainty similar to the monopoly company in the baseline model.

Suppose that only one company issues its E-money and acquires preference information. Then the optimal behavior strategy of that company in the evening is monopoly pricing given by (23), and equilibrium outcomes are exactly the same as in the baseline model. Now suppose that multiple companies obtain preference information in the evening. Then Bertrand competition leads all of them to set their price of preference information equal to the marginal cost, which is zero due to the nonrivalry property of information, and those companies make negative profits. Consequently, if there is company $j \in \{1, \dots, J\}$ whose strategy includes issuing its E-money with the reward $R = \delta/\beta$, then the best responses of other companies are not to run the E-money business. Furthermore, given that other companies do not issue E-money, the best response

[25] For instance, Alibaba Group and Wechat issue Alipay and Wachat pay, respectively, and use transaction data of their own digital currency.

[26] For example, if a buyer opens accounts at two companies to use two types of E-money, then the buyer incurs $2\delta$ units of disutility.

[27] Specifically, a company $j \in \{1, \dots, J\}$ obtains the correct preference information with probability $\kappa(B_j) \in [0, 1]$, where $B_j \in [0, \rho]$ is the mass of company $j$'s E-money users in the afternoon.

of company $j$ is issuing E-money with $R = \delta/\beta$ and selling preference information at the monopoly price in the evening. This argument leads to the next proposition.

Proposition 9. *A monopoly arises in the E-money business industry, i.e., only one company runs the E-money business, and equilibrium allocations and the type of equilibrium that exists are the same as in the baseline model with the monopoly company.*

We now consider the economy with competitive search in the evening. In this case, each company chooses its reward policy $R$ and price of preference information $\varphi$ together in the morning. Any active companies will set the reward as $R = \delta/\beta$ by the same reasoning as in the model with bargaining. If multiple companies run the E-money business making nonnegative profits in equilibrium, then any company can increase its profit by selling preference information at a slightly lower price to attract all active sellers. Thus, equilibrium with multiple active companies cannot exist and a monopoly arises.

However, in contrast to bargaining equilibrium, the monopoly company must earn zero profit in competitive search equilibrium because, otherwise, any company can increase its payoff by posting $\varphi$ slightly lower than that of the existing company to attract all active sellers. Thus, the price of preference information is given as $\varphi = \rho\delta/[(1-\rho)\beta\mathcal{N}]$. Note from Proposition 7 that sellers sell customized goods only if $\varphi = \rho\delta/[(1-\rho)\beta\mathcal{N}] \leq \varphi^*$ in competitive search equilibrium, which requires $\Pi(\gamma) \geq 0$ as can be verified from (40) and (41). Thus, Proposition 8 characterizes the existence of each type of competitive search equilibrium (E-equilibrium or P-equilibrium) in the extended model, and real allocations, except price $\varphi$, are the same as in the competitive search model with the single company.

## 7. Conclusion

In this paper, we have developed a money search model in which an electronic payment platform company issues E-money and estimates consumers' preferences by analyzing E-money transaction data. Sellers purchase preference information to produce goods that better match consumers' preferences. We have shown that a monopoly arises in the E-money industry and that the company does not issue E-money if inflation is sufficiently high. Socially efficient uses of payment data may not occur because of a wedge between the social and private benefits of using payment data.

## Appendix A: Omitted proofs

Proof of Lemma 1. Take any $i \in \{H, L\}$ for $\alpha_i$. The objective function $\alpha_i u(x) - x$ is maximized when $x = x_i^*$ and increases with $x$ for all $x < x_i^*$. Thus, if $m_i^* \leq m_p' + m_e'$, $(x, d) = (x_i^*, m_i^*)$ must be the solution because it satisfies (10) and (11). Next, if $m_p' + m_e' < m_i^*$, then $x = x_i^*$ is not attainable, and $(x, d) = (\Phi_i^{-1}(m_p' + m_e'), m_p' + m_e')$ solves the maximization problem (9). By combining the above two cases, we obtain the results of Lemma 1. □

Proof of Lemma 3. Note from (12) that for all $m_p \geq m_H^*$, $\hat{x}_H(m_p) = x_H^*$ and $\hat{x}_L(m_p) = x_L^*$, and, hence, $D(m_p) = \theta[\alpha_H u(x_H^*) - x_H^*] - \theta[\alpha_L u(x_L^*) - x_L^*] \equiv \overline{D}$ by (19). Next, if $m_L^* \leq m_p < m_H^*$, then $D(m_p) = \theta[\alpha_H u(\hat{x}_H(m_p)) - \hat{x}_H(m_p)] - \theta[\alpha_L u(x_L^*) - x_L^*]$, which strictly increases with $m_p$ for all $m_p < m_H^*$. Finally, suppose that $m_p < m_L^*$. Then (12) gives

$$\theta\alpha_L u\big(\hat{x}_L(m_p)\big) + (1-\theta)\hat{x}_L(m_p) = \theta\alpha_H u\big(\hat{x}_H(m_p)\big) + (1-\theta)\hat{x}_H(m_p) = \beta m_p, \tag{44}$$

and, hence, $\hat{x}_L(m_p) > \hat{x}_H(m_p)$ because $\alpha_H > \alpha_L$. Substituting (44) into (19), we obtain $D(m_p) = \hat{x}_L(m_p) - \hat{x}_H(m_p)$. Taking the derivative and using (44), we obtain

$$D'(m_p) = \frac{\beta}{\theta\alpha_L u'\big(\hat{x}_L(m_p)\big) + 1 - \theta} - \frac{\beta}{\theta\alpha_H u'\big(\hat{x}_H(m_p)\big) + 1 - \theta},$$

which is positive because $\hat{x}_L(m_p) > \hat{x}_H(m_p)$. In summary, $D'(m_p) > 0$ for all $m_p < m_H^*$. $\square$

Proof of Proposition 2. In E-equilibrium, $m = \widetilde{d}_H(\gamma)$ as shown in (27) and by substituting this result into (31), we obtain the company's discounted profit as

$$\beta\pi = (1-\rho)D\big(\widetilde{d}_H(\gamma)\big) - \mathcal{N}\varsigma - \rho\delta. \tag{45}$$

Thus, in E-equilibrium, it must be that $D(\widetilde{d}_H(\gamma)) \geq (\mathcal{N}\varsigma + \rho\delta)/(1-\rho)$ to have $\pi \geq 0$.[28]

For all $\gamma \geq \beta$, we obtain from (12), (19), and (30) that

$$D\big(\widetilde{d}_H(\gamma)\big) = \theta\big[\alpha_H u\big(\widetilde{x}_H(\gamma)\big) - \widetilde{x}_H(\gamma)\big] - \theta\big[\alpha_L u\big(\hat{x}_L\big(\widetilde{d}_H(\gamma)\big)\big) - \hat{x}_L\big(\widetilde{d}_H(\gamma)\big)\big].$$

Note that $D(\widetilde{d}_H(\gamma))$ decreases with $\gamma$ by the results of Lemma 3, $\lim_{\gamma\to\beta/\theta} D(\widetilde{d}_H(\gamma)) = 0$, and $\lim_{\gamma\to\beta} D(\widetilde{d}_H(\gamma)) = \overline{D}$. Thus, if $(\mathcal{N}\varsigma + \rho\delta)/(1-\rho) \leq D(m_L^*)$, there exists $\gamma_1 \in [\widetilde{d}_H^{-1}(m_L^*), \beta/\theta)$ such that $D(\widetilde{d}_H(\gamma_1)) = (\mathcal{N}\varsigma+\rho\delta)/(1-\rho)$, and E-equilibrium exists for all $\gamma \leq \gamma_1$. Next, if $D(m_L^*) < (\mathcal{N}\varsigma + \rho\delta)/(1-\rho) \leq \overline{D}$, there exists $\gamma_3 \in [\beta, \widetilde{d}_H^{-1}(m_L^*))$ such that $D(\widetilde{d}_H(\gamma_3)) = (\mathcal{N}\varsigma + \rho\delta)/(1-\rho)$, and E-equilibrium exists for all $\gamma \leq \gamma_3$. Finally, if $\overline{D} < (\mathcal{N}\varsigma + \rho\delta)/(1-\rho)$, then for all $\gamma \geq \beta$, $D(\widetilde{d}_H(\gamma)) < (\mathcal{N}\varsigma + \rho\delta)/(1-\rho)$, so E-equilibrium cannot exist. In summary, E-equilibrium exists if one of the following conditions holds: (i) $(\mathcal{N}\varsigma+\rho\delta)/(1-\rho) \leq D(m_L^*)$ and $\gamma \in [\beta, \gamma_1]$, or (ii) $D(m_L^*) < (\mathcal{N}\varsigma+\rho\delta)/(1-\rho) \leq \overline{D}$ and $\gamma \in [\beta, \gamma_3]$.

Next, in P-equilibrium, late buyers hold $\widetilde{d}_L(\gamma)$ units of real P-money by (28). Thus, if the company runs the E-money business by setting $(R, \varphi)$ as described in (22) and (23), the company's profit is given as $\pi = [(1-\rho)D(\widetilde{d}_L(\gamma)) - \mathcal{N}\varsigma - \rho\delta]/\beta$ by (31). In P-equilibrium, the company should not be able to make nonnegative profits, so it must be that $D(\widetilde{d}_L(\gamma)) < (\mathcal{N}\varsigma+\rho\delta)/(1-\rho)$ for P-equilibrium to exist. Note that $\partial D(\widetilde{d}_L(\gamma))/\partial\gamma < 0$, $\widetilde{d}_L(\beta) = m_L^*$, and $\lim_{\gamma\to\beta/\theta} D(\widetilde{d}_L(\gamma)) = 0$. Thus, if $D(m_L^*) < (\mathcal{N}\varsigma+\rho\delta)/(1-\rho)$, then for all $\gamma \geq \beta$, $D(\widetilde{d}_L(\gamma)) < (\mathcal{N}\varsigma + \rho\delta)/(1-\rho)$ and P-equilibrium exists. On the other hand,

[28] Note that when we derive (45), we implicitly assume that the company can sell preference information at a positive price, which requires that $D(\widetilde{d}_H(\gamma)) \geq \mathcal{N}\varsigma/(1-\rho)$ by (23). However, whenever the company makes nonnegative profits, it must be that $D(\widetilde{d}_H(\gamma)) > \mathcal{N}\varsigma/(1-\rho)$. Thus, the company sells preference information at a positive price.

if $(\mathcal{N}\varsigma + \rho\delta)/(1-\rho) \le D(m_L^*)$, there exists $\gamma_2 \in [\beta, \beta/\theta)$ such that $D(\widetilde{d}_L(\gamma_2)) = (\mathcal{N}\varsigma + \rho\delta)/(1-\rho)$ and P-equilibrium exists for all $\gamma > \gamma_2$. In summary, P-equilibrium exists if one of the following conditions holds: (i) $(\mathcal{N}\varsigma + \rho\delta)/(1-\rho) \le D(m_L^*)$ and $\gamma > \gamma_2$, or (ii) $D(m_L^*) < (\mathcal{N}\varsigma + \rho\delta)/(1-\rho)$.

Finally, we now show that $\gamma_2 < \gamma_1$ when $(\mathcal{N}\varsigma + \rho\delta)/(1-\rho) \le D(m_L^*)$. In the above analysis, $\gamma_1$ and $\gamma_2$ are defined such that $D(\widetilde{d}_H(\gamma_1)) = D(\widetilde{d}_L(\gamma_2)) = (\mathcal{N}\varsigma + \rho\delta)/(1-\rho)$. Note from (30) that $\widetilde{d}_H(\gamma) > \widetilde{d}_L(\gamma)$ for all $\gamma \ge \beta$. Thus, it must be that $\gamma_1 > \gamma_2$ because $\partial D(\widetilde{d}_i(\gamma))/\partial\gamma < 0$ for each $i \in \{H, L\}$ by the results of Lemma 3 and (30), which finishes the proof. □

Proof of Lemma 4. From (29), (32), and (33), we obtain

$$\frac{\partial\big[W_E(\gamma) - W_P(\gamma)\big]}{\partial\gamma} = \left[\frac{u'\big(\widetilde{x}_H(\gamma)\big)}{u''\big(\widetilde{x}_H(\gamma)\big)} - \frac{u'\big(\widetilde{x}_L(\gamma)\big)}{u''\big(\widetilde{x}_L(\gamma)\big)}\right]\frac{\beta(1-\rho)(\gamma-\beta)}{\gamma(\beta-\theta\gamma)^2}.$$

Because $u'(x)/u''(x)$ decreases with $x$ by the property of the utility function $u(\cdot)$ and $\widetilde{x}_H(\gamma) > \widetilde{x}_L(\gamma)$, it must be that $\partial[W_E(\gamma) - W_P(\gamma)]/\partial\gamma < 0$. Thus, $W_E(\gamma) - W_P(\gamma)$ decreases with $\gamma$. □

Proof of Proposition 3. Assume that $D(\widetilde{d}_H(\gamma)) \ge (\mathcal{N}\varsigma + \rho\delta)/(1-\rho)$, so the company's profit is nonnegative under E-equilibrium allocations. From (12), (19), (29), and (30), we obtain that

$$\begin{aligned} D\big(\widetilde{d}_H(\gamma)\big) &= \theta\big\{\alpha_H u\big(\widetilde{x}_H(\gamma)\big) - \widetilde{x}_H(\gamma) - \big[\alpha_L u\big(\hat{x}_L\big(\widetilde{d}_H(\gamma)\big)\big) - \hat{x}_L\big(\widetilde{d}_H(\gamma)\big)\big]\big\} \\ &\le \theta\big\{\alpha_H u\big(\widetilde{x}_H(\gamma)\big) - \widetilde{x}_H(\gamma) - \big[\alpha_L u\big(\widetilde{x}_L(\gamma)\big) - \widetilde{x}_L(\gamma)\big]\big\} \\ &< \alpha_H u\big(\widetilde{x}_H(\gamma)\big) - \widetilde{x}_H(\gamma) - \big[\alpha_L u\big(\widetilde{x}_L(\gamma)\big) - \widetilde{x}_L(\gamma)\big], \end{aligned}$$

where we use the property that $\widetilde{x}_L(\gamma) \le \hat{x}_L(\widetilde{d}_H(\gamma)) \le x_L^*$ to obtain the first inequality. Then, whenever $D(\widetilde{d}_H(\gamma)) \ge (\mathcal{N}\varsigma + \rho\delta)/(1-\rho)$, it must be that $W_E(\gamma) > W_P(\gamma)$, because $W_E(\gamma) \ge W_P(\gamma)$ if and only if $\alpha_H u(\widetilde{x}_H(\gamma)) - \widetilde{x}_H(\gamma) - [\alpha_L u(\widetilde{x}_L(\gamma)) - \widetilde{x}_L(\gamma)] \ge (\mathcal{N}\varsigma + \rho\delta)/(1-\rho)$. □

Proof of Proposition 4. Assume that $(\mathcal{N}\varsigma + \rho\delta)/(1-\rho) < \overline{D}/\theta$. Then, from (1), (21), (29), (32), and (33), it can be shown that $W_E(\beta) - W_P(\beta) > 0$. Next, note that $\lim_{\gamma\to\beta/\theta}\{W_E(\gamma) - W_P(\gamma)\} = -(\mathcal{N}\varsigma + \rho\delta)$. Then, by the result of Lemma 4, there exists a unique $\gamma^* \in (\beta, \beta/\theta)$ such that $W_E(\gamma^*) = W_P(\gamma^*)$ and $W_E(\gamma) > W_P(\gamma)$ for all $\gamma < \gamma^*$.

Next, define $\gamma^{**}$ such that

$$\gamma^{**} = \begin{cases} \gamma_1 & \text{if } \dfrac{\mathcal{N}\varsigma + \rho\delta}{1-\rho} \le D\big(m_L^*\big) \\ \gamma_3 & \text{if } D\big(m_L^*\big) < \dfrac{\mathcal{N}\varsigma + \rho\delta}{1-\rho} \le \overline{D} \\ \beta & \text{if } \overline{D} < \dfrac{\mathcal{N}\varsigma + \rho\delta}{1-\rho} < \dfrac{\overline{D}}{\theta}, \end{cases}$$

where $\gamma_1$ and $\gamma_3$ are given in Proposition 2. Then, for all $\gamma > \gamma^{**}$, E-equilibrium does not exist as shown in Proposition 2. Furthermore, $\gamma_1$ and $\gamma_3$ are defined such that the company's profit is zero (see the proof of Proposition 2). Thus, by results of Lemma 4 and Proposition 3, it must be that $\gamma^* > \gamma^{**}$, which finishes the proof. □

Proof of Proposition 5. Take any $\Delta \geq [\mathcal{N}\varsigma + \rho\delta - (1-\rho)D(\widetilde{d}_H(\gamma))]/\beta$. Consider the subsidy to the company. If $(1-\rho)D(\widetilde{d}_H(\gamma)) \geq \mathcal{N}\varsigma$, then $\varphi \geq 0$ by (23) when $m_p = \widetilde{d}_H(\gamma)$, and $\pi = [(1-\rho)D(\widetilde{d}_H(\gamma)) - \mathcal{N}\varsigma - \rho\delta]/\beta + \Delta \geq 0$. Thus, the economy is in E-equilibrium and $W = W_E(\gamma)$. Next, if $(1-\rho)D(\widetilde{d}_H(\gamma)) < \mathcal{N}\varsigma$, then $\varphi < 0$ for all $m_p \leq \widetilde{d}_H(\gamma)$ by (23). However, the company's profit without selling preference information is positive as $\pi = -\rho\delta/\beta + \Delta > 0$ due to the subsidy. Thus, the company issues E-money without selling preference information. As a result, real allocations are the same as in P-equilibrium except that early buyers incur privacy costs, so $W = W_p(\gamma) - \rho\delta$.

Now consider the subsidy to sellers. Assuming that $m_p = \widetilde{d}_H(\gamma)$, (18) shows that sellers buy all evening taste information if $\beta(\mathcal{N}\varphi - \Delta) \leq (1-\rho)D(\widetilde{d}_H(\gamma)) - \mathcal{N}\varsigma$ given the subsidy. Then the company will set the price as $\varphi = [(1-\rho)D(\widetilde{d}_H(\gamma)) + \beta\Delta - \mathcal{N}\varsigma]/\beta\mathcal{N}$, and $\pi = [(1-\rho)D(\widetilde{d}_H(\gamma)) - \mathcal{N}\varsigma - \rho\delta]/\beta + \Delta \geq 0$. Thus, the economy is in E-equilibrium and $W = W_E(\gamma)$. □

Proof of Proposition 7. Note from (34) that $\alpha_i u(x_i) - \beta d_i > 0$ must hold because $U^b \geq 0$. Suppose that an active submarket $(x_i, d_i)$ exists with $n_i > 1$. Then there exist $n_i' \in (1, n_i)$ and $d_i' > d_i$ such that $-(\gamma - \beta)d_i' + [\alpha_i u(x_i) - \beta d_i]/n_i' = U^b$, so the seller can find a profitable deviation, which is a contradiction. Thus, in any active submarket, it must be that $n_i \leq 1$. We focus on the case with $R \geq \delta/\beta$ and show that equilibrium outcomes when $R < \delta/\beta$ are equivalent to the case with $R \geq \delta/\beta$ and $\varphi > \varphi^*$. Note from (37), (38), and (40) that $\overline{U}_H^b \geq \overline{U}_L^b$ if and only if $\varphi \leq \varphi^*$, and we divide the analysis depending on the relative value of $\overline{U}_H^b$ and $\overline{U}_L^b$.

Case 1. Assume that $\varphi \leq \varphi^*$, so $\overline{U}_H^b \geq \overline{U}_L^b$. If $U^b > \overline{U}_H^b$, sellers cannot offer late buyers their market expected surplus without suffering a negative payoff, so no sellers will participate in the evening. In what follows, we focus on the case with $U^b \leq \overline{U}_H^b$.

(i) Suppose that $U^b = \overline{U}_H^b$. From (34) with $i = H$ and (36), we obtain

$$V_H^s = \max_{n_H}\left\{n_H \max_{x_H}\left[-x_H + \frac{\beta}{\gamma}\alpha_H u(x_H) - \frac{\beta}{\gamma}U^b\right] - \mathcal{N}(\varsigma + \beta\varphi)\right\}. \tag{46}$$

Since $U^b = \overline{U}_H^b$, the solution to (46) is $(x_H, n_H) = (\bar{x}_H(\gamma), 1)$ and $V_H^s = 0$. Then, from (34) with $i = H$, we obtain $d_H = [\bar{x}_H(\gamma) + \mathcal{N}(\varsigma + \beta\varphi)]/\beta$. Next, from (34) with $i = L$ and (35), we obtain

$$V_L^s = \max_{n_L}\left\{n_L \max_{x_L}\left[-x_L + \frac{\beta}{\gamma}\alpha_L u(x_L) - \frac{\beta}{\gamma}U^b\right]\right\}. \tag{47}$$

Given that $U^b = \overline{U}_H^b \geq \overline{U}_L^b$, it must be that $n_L = 0$, so an active submarket for noncustomized goods does not exist.[29] Because $V^s = V_H^s = 0$, some sellers may be inactive, so $N^d \in [0, 1]$. On the other hand, $N^s = 1 - \rho$ because $U^b > 0$.

(ii) Suppose that $U^b \in (0, \overline{U}_H^b)$. Then, from (34) with $i = H$ and (36), we obtain (46), and it must be that $n_H = 1$ because $U^b < \overline{U}_H^b$. Next, from (34) with $i = L$ and (35), we obtain (47). Given that $\overline{U}_H^b \geq \overline{U}_L^b$, it must be that $V_H^s \geq V_L^s$ with strict inequality if $n_L < 1$ or $\overline{U}_H^b > \overline{U}_L^b$. Thus, all active sellers sell customized goods, $(x_H, d_H, n_H) = (\bar{x}_H(\gamma), [\alpha_H u(\bar{x}_H(\gamma)) - U^b]/\gamma, 1)$ by (34) and (46), and $V^s = V_H^s > 0$. Since $V^s > 0$ and $U^b > 0$, all sellers and late buyers participate in the evening, i.e., $N^d = 1$ and $N^s = 1 - \rho$.

(iii) Suppose that $U^b = 0$. Then late buyers are indifferent between actively participating or not in the evening. If some late buyers participate, all active sellers sell customized goods, $(x_H, d_H, n_H) = (\bar{x}_H(\gamma), [\alpha_H u(\bar{x}_H(\gamma)) - U^b]/\gamma, 1)$, $V^s = V_H^s > 0$, and $N^d = 1$ by the same reasoning as the case where $U^b \in (0, \overline{U}_H^b)$. Since $U^b = 0$, we have $N^s \leq 1 - \rho$.

From the analysis of three cases above, one can show that $N^d = N^s$ only if $U^b = \overline{U}_H^b$. Thus, if $\varphi \leq \varphi^*$, then all active sellers sell customized goods, $(x_H, d_H, n_H) = (\bar{x}_H(\gamma), [\bar{x}_H(\gamma) + \mathcal{N}(\varsigma + \beta\varphi)]/\beta, 1)$, $U^b = \overline{U}_H^b$, $V^s = 0$, and $N^d = N^s = 1 - \rho$ in competitive search equilibrium.

Case 2. Assume that $\varphi > \varphi^*$, so $\overline{U}_H^b < \overline{U}_L^b$. In what follows, we focus on the case where $U^b \leq \overline{U}_L^b$ because otherwise, no sellers will participate in the evening.

(i) Suppose that $U^b = \overline{U}_L^b$. From (34) with $i = H$ and (36), we obtain (46). Since $\overline{U}_H^b < \overline{U}_L^b$, it must be that $n_H = 0$ and there exists no active submarket in which customized goods are traded. Next, the solution to (35) is $x_L = \bar{x}_L(\gamma)$, $d_L = \bar{x}_L(\gamma)/\beta$, and $n_L \leq 1$. Thus, any active sellers post $(\bar{x}_L(\gamma), \bar{x}_L(\gamma)/\beta)$ to sell noncustomized goods in the evening and $V^s = V_L^s = 0$. Since $V^s = 0$ and $U^b > 0$, $N^d \in [0, 1]$ and $N^s = 1 - \rho$.

(ii) Suppose that $U^b \in (0, \overline{U}_L^b)$. Then the solution to (35) is $(x_L, d_L, n_L) = (\bar{x}_L(\gamma), \bar{x}_L(\gamma)/\beta, 1)$ and $V_L^s > 0$. Furthermore, given that $\overline{U}_H^b < \overline{U}_L^b$, it can be verified from (34)–(36) that $V_L^s > V_H^s$. Thus, any active sellers sell noncustomized goods and $V^s = V_L^s$. Because $V_L^s > 0$ and $U^b > 0$, we have $N^d = 1$ and $N^s = 1 - \rho$.

(iii) Suppose that $U^b = 0$. Then late buyers are indifferent between actively participating or not in the evening. If some late buyers participate, all active sellers sell noncustomized goods, $(x_L, d_L, n_L) = (\bar{x}_L(\gamma), \bar{x}_L(\gamma)/\beta, 1)$, $V^s = V_L^s > 0$, and $N^d = 1$ for the same reasons as the case where $U^b \in (0, \overline{U}_L^b)$. Because $U^b = 0$, we have $N^s \leq 1 - \rho$.

The analysis of the three cases above shows that $N^d = N^s$ only if $U^b = \overline{U}_L^b$. Thus, if $\varphi > \varphi^*$, then $U^b = \overline{U}_L^b$, $V^s = 0$, all active sellers post $(x_L, d_L) = (\bar{x}_L(\gamma), \bar{x}_L(\gamma)/\beta)$ to sell noncustomized goods, $n_L \leq 1$, and $N^d = N^s = 1 - \rho$ in competitive search equilibrium.

[29] In the knife edge case with $\overline{U}_H^b = \overline{U}_L^b$, $n_L \geq 0$ is feasible and $V_L^s = 0$. However, we assume that active sellers sell customized goods when they are indifferent between selling customized goods and noncustomized goods. Thus, no sellers sell noncustomized goods when $U^b = \overline{U}_H^b \geq \overline{U}_L^b$.

Now suppose that $R < \delta/\beta$. Then the company cannot obtain evening taste information in the evening, so any active sellers sell noncustomized goods. Thus, equilibrium outcomes are equivalent to the case with $R \geq \delta/\beta$ and $\varphi > \varphi^*$. By combining the equilibrium outcomes in Cases 1 and 2, and the case with $R < \delta/\beta$, we obtain the results of Proposition 7. $\square$

Proof of Proposition 8. Note from (1), (39), and (41) that $\lim_{\gamma\to\infty}\Pi(\gamma) = -(1-\rho)\mathcal{N}\varsigma - \rho\delta$, $\Pi(\beta) = (1-\rho)\{\alpha_H u(x_H^*) - x_H^* - [\alpha_L u(x_L^*) - x_L^*]\} - (1-\rho)\mathcal{N}\varsigma - \rho\delta$ and $\Pi'(\gamma) < 0$. Thus, if $\alpha_H u(x_H^*) - x_H^* - [\alpha_L u(x_L^*) - x_L^*] \geq [(1-\rho)\mathcal{N}\varsigma + \rho\delta]/(1-\rho)$, there exists $\gamma^{\text{cs}} \geq \beta$ such that $\Pi(\gamma^{\text{cs}}) = 0$. Then, for all $\gamma \in [\beta, \gamma^{\text{cs}}]$, $\Pi(\gamma) \geq 0$ and E-equilibrium exists, and for all $\gamma > \gamma^{\text{cs}}$, $\Pi(\gamma) < 0$ and P-equilibrium exists. On the other hand, if $\alpha_H u(x_H^*) - x_H^* - [\alpha_L u(x_L^*) - x_L^*] < [(1-\rho)\mathcal{N}\varsigma + \rho\delta]/(1-\rho)$, then for all $\gamma \geq \beta$, $\Pi(\gamma) < 0$, so P-equilibrium exists. $\square$

Proof of Proposition 9. A behavior strategy of company $j \in \{1, \dots, J\}$ in the morning is the fixed reward $R_j$ for using its E-money in the afternoon and a behavior strategy in the evening is the price of evening taste information $\varphi_j$. Note that setting $R_j < \delta/\beta$ is equivalent to not running the E-money business because no early buyers use $j$'s E-money, so company $j$ does not obtain evening taste information. Thus, $\varphi_j$ is irrelevant. Without loss of generality, we set $R_j = 0$ and $\varphi_j = \varepsilon > 0$ if company $j$ chooses not to run the E-money business. Let $\mathcal{L} \subseteq \{1, \dots, J\}$ denote the set of companies that issue their E-money with the fixed reward $R \geq \delta/\beta$. In what follows, we solve subgame perfect Nash equilibrium by using backward induction starting from a game in the evening.

In the subgame in the evening, suppose that $|\mathcal{L}| = 1$, where $|\mathcal{L}|$ is the cardinality of $\mathcal{L}$. Then the optimal behavior strategy of company $j \in \mathcal{L}$ in the evening is monopoly pricing given by (23) and it is Nash equilibrium of this subgame. Next, suppose that $|\mathcal{L}| > 1$. Sellers will buy preference information from one of the companies that sell the information at the lowest price, because all companies in the set $\mathcal{L}$ have the correct information. Suppose that $\varphi_{\min} \equiv \min_{j\in\mathcal{L}} \varphi_j > 0$. Then any company $j \in \mathcal{L}$ can raise its revenue by setting the price of preference information as $\varphi_j' = \varphi_{\min} - \varepsilon$ for a sufficiently small $\varepsilon > 0$ to attract all sellers. Thus, the only Nash equilibrium in the subgame in the evening with $|\mathcal{L}| > 1$ consists of $\varphi_j = 0$ for all $j \in \mathcal{L}$.

We now analyze companies' optimal decisions in the morning. The above analysis shows that whenever $|\mathcal{L}| > 1$, companies in the set $\mathcal{L}$ end up making negative profits as $\pi = -\rho\delta/\beta$. Suppose that there is one company $j \in \{1, \dots, J\}$ with the strategy $\{R_j, \varphi_j\} = \{\delta/\beta, \widehat{\varphi}\}$, where $\widehat{\varphi} = \begin{cases} [(1-\rho)D(m_p) - \mathcal{N}\varsigma]/\beta\mathcal{N} & \text{if } |\mathcal{L}| = 1 \\ 0 & \text{if } |\mathcal{L}| > 1 \end{cases}$. Then the best responses of other companies $j' \neq j$ are not to run the E-money business. Given that all other companies $j' \neq j$ do not run the E-money business, company $j$'s strategy $\{R_j, \varphi_j\} = \{\delta/\beta, \widehat{\varphi}\}$ is the best response. Thus, for any $j \in \{1, \dots, J\}$, a profile of strategies $\{\{R_j, \varphi_j\}, \{R_i, \varphi_i\}_{i\neq j}\} = \{\{\delta/\beta, \widehat{\varphi}\}, \{0, \varepsilon\}\}$ constitutes subgame perfect Nash equilibrium. $\square$

## Appendix B: Reward policy for using E-money

In the main body, we assumed that the company provides only the fixed reward to early buyers for using E-money. However, the company can also provide proportional rewards: The company can subsidize the $\varkappa_a \in [0, 1]$ and $\varkappa_e \in [0, 1]$ fractions of the E-money payments in the afternoon and evening, respectively.[30] In this appendix, we show that the assumption that the company only provides the fixed reward is without loss of generality by illustrating that the profit maximizing company does not provide proportional rewards.

Given the fixed reward $R$, privacy cost $\delta$, and proportional reward $\varkappa_a \geq 0$, early buyers will either use E-money or P-money. If the early buyer chooses to use E-money, then his/her surplus is

$$S_e^{\text{early}} = \max_{q_e} \bigl\{ \beta R - \delta - \gamma p (1 - \varkappa_a) q_e + v(q_e) \bigr\}, \tag{48}$$

and if the early buyer chooses to use P-money, then his/her surplus is

$$S_p^{\text{early}} = \max_{q_p} \bigl\{ -\gamma p q_p + v(q_p) \bigr\}. \tag{49}$$

Given the monopoly power, the company will set $R$ and $\varkappa_a$ such that $S_e^{\text{early}} = S_p^{\text{early}}$. By substituting the optimality conditions, $\gamma p(1 - \varkappa_a) = v'(q_e)$ and $\gamma p = v'(q_p)$ into (48) and (49), respectively, the condition that $S_e^{\text{early}} = S_p^{\text{early}}$ gives

$$\begin{aligned} \beta R = \delta &- \bigl\{ -\gamma p (1 - \varkappa_a) v'^{-1}\bigl(\gamma p (1 - \varkappa_a)\bigr) + v\bigl(v'^{-1}\bigl(\gamma p (1 - \varkappa_a)\bigr)\bigr) \bigr\} \\ &+ \bigl\{ -\gamma p v'^{-1}(\gamma p) + v\bigl(v'^{-1}(\gamma p)\bigr) \bigr\}. \end{aligned}$$

Next, the company pays $p\varkappa_a q_e$ units of E-money, which is backed by P-money, to sellers in the afternoon as subsidies for buying goods with E-money in the afternoon market. Combined with the fixed reward, the total cost of attracting each early buyer is given as

$$\delta + \bigl[ \gamma p v'^{-1}\bigl(\gamma p (1 - \varkappa_a)\bigr) - v\bigl(v'^{-1}\bigl(\gamma p (1 - \varkappa_a)\bigr)\bigr) - \gamma p v'^{-1}(\gamma p) + v\bigl(v'^{-1}(\gamma p)\bigr) \bigr].$$

Note that the term in the square bracket is strictly positive for all $\varkappa_a > 0$ and zero with $\varkappa_a = 0$. Thus, it is optimal for the company to set $\varkappa_a = 0$.

We now show that a company's profit decreases with $\varkappa_e$, so it is optimal for the company to set $\varkappa_e = 0$. Note that there is no reason for the company to provide any rewards to late buyers for using E-money in the evening if early buyers do not use E-money in the afternoon. Thus, we assume that early buyers use E-money. Similar to early buyers, late buyers will either use P-money or E-money for evening transactions given the proportional reward $\varkappa_e \geq 0$ and the fixed privacy cost $\delta$. To analyze how $\varkappa_e$ affects a company's profit, we assume that late buyers choose to use E-money for evening transactions.

[30] Because evening transaction data have no value to the company, if the company provides any reward to late buyers, it must be a proportional reward that could affect late buyers' trade volume in the evening market.

Given that late buyers hold E-money, we can rewrite the bargaining problem in the evening as

$$\max_{x,d}\big\{\alpha_i u(x) - x + \beta\varkappa_e d\big\} \tag{50}$$

subject to

$$-x + \beta d = \theta\big\{\alpha_i u(x) - x + \beta\varkappa_e d\big\} \tag{51}$$

$$(1-\varkappa_e)d \le m_e \tag{52}$$

for each $i \in \{H, L\}$. Let $\hat{x}_i^e(m_e)$ and $\hat{d}_i^e(m_e)$ denote the solution to the above maximization problem given $i \in \{H, L\}$ and the late buyer's E-money holdings $m_e$. Note that $\hat{x}_i^e(m_e)$ increases with $\varkappa_e$ whenever constraint (52) binds.

Given the proportional reward $\varkappa_e$ and late buyer's E-money holdings $m_e$, sellers buy all preference information if $D^e(m_e) \ge \mathcal{N}(\varsigma + \beta\varphi)/[(1-\rho)\kappa(B)]$, where

$$D^e(m_e) = \theta\left\{\begin{array}{c}\alpha_H u\big(\hat{x}_H^e(m_e)\big) - \hat{x}_H^e(m_e) + \beta\varkappa_e\hat{d}_H^e(m_e)\\ -\big[\alpha_L u\big(\hat{x}_L^e(m_e)\big) - \hat{x}_L^e(m_e) + \beta\varkappa_e\hat{d}_L^e(m_e)\big]\end{array}\right\} \tag{53}$$

is an increase in the seller's trade surplus in the evening by preparing the production of customized goods when the company provides proportional rewards for using E-money in the evening.

Using the monopoly power, the company will set the price of preference information as $\varphi = [(1-\rho)\kappa(B)D^e(m_e) - \mathcal{N}\varsigma]/\beta\mathcal{N}$ and sell all preference information to all sellers. The cost of running the E-money business consists of the fixed rewards to early buyers for using E-money in the afternoon and $\varkappa_e d^e$ units of E-money transfers in each bilateral meeting in the evening as proportional rewards to late buyers. Then we obtain discounted net profit as

$$\beta\pi = (1-\rho)D^e(m_e) - \mathcal{N}\varsigma - \big[\rho\delta + (1-\rho)\gamma\varkappa_e d^e\big], \tag{54}$$

where we impose the condition that $\kappa(B) = 1$, because all early buyers use E-money in the afternoon market, i.e., $B = \rho$, given that $R = \delta/\beta$.

Note that late buyers will minimize idle E-money that is not used in the evening market. This implies that $(1-\varkappa_e)\hat{d}_H^e(m_e) = m_e$ by (52). Then, from (51)–(53), we obtain

$$D^e(m_e) = -\hat{x}_H^e(m_e) + \frac{\beta m_e}{1-\varkappa_e} - \theta\big[\alpha_L u\big(\hat{x}_L^e(m_e)\big) - \hat{x}_L^e(m_e) + \beta\varkappa_e\hat{d}_L^e(m_e)\big]. \tag{55}$$

Note that the term $\alpha_L u(\hat{x}_L^e(m_e)) - \hat{x}_L^e(m_e) + \beta\varkappa_e\hat{d}_L^e(m_e)$ increases with $\varkappa_e$ as shown in (50)–(52). Then, from (51), the binding (52), (54), and (55), we obtain

$$\frac{\partial(\beta\pi)}{\partial\varkappa_e} \approx -\frac{\partial\hat{x}_H^e(m_e)}{\partial\varkappa_e} - \frac{m_e(\gamma-\beta)}{(1-\varkappa_e)^2} - \theta\frac{\partial\big[\alpha_L u\big(\hat{x}_L^e(m_e)\big) - \hat{x}_L^e(m_e) + \beta\varkappa_e\hat{d}_L^e(m_e)\big]}{\partial\varkappa_e} < 0.$$

This implies that it is optimal for the company to set $\varkappa_e = 0$. Thus, the profit maximizing company does not provide any proportional rewards: $\varkappa_a = \varkappa_e = 0$.

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