Supplement to "Informed intermediaries"

(*Theoretical Economics*, Vol. 17, No. 1, January 2022, 57–87)

Paula Onuchic
Department of Economics, New York University

Appendix: Proof of Proposition 2 (continued)

S.1 *Guesses*

Surplus sharing:

$$\beta(I, v_s, U, v_b) = \beta^I \geq \frac{1}{2}, \qquad \beta(U, v_s, I, v_b) = 1 - \beta^I \leq \frac{1}{2},$$
$$\beta(I, v_s, I, v_b) = \frac{1}{2}.$$

Efficient trading if and only if an informed agent is involved:

$$\mathcal{I}(i_s, v_s, i_b, v_b) = 1 \quad \Leftrightarrow \quad (i_s, i_b) \neq (U, U) \quad \text{and} \quad V^{i_s}_{v_s 0} - V^{i_s}_{v_s 1} + V^{i_b}_{v_b 1} - V^{i_b}_{v_b 0} > 0.$$

S.2 *Stationary distribution*

Given the guesses for $\mathcal{I}$, the inflow equal to outflow equations for the stationary distribution become

$$\mu^U_{L1}\big(\eta + \lambda(\mu^I_{H0} + \mu^I_{L0})\big) = \eta\mu^U_{L0}, \tag{S.1}$$
$$\mu^U_{H0}\big(\eta + \lambda(\mu^I_{L1} + \mu^I_{H1})\big) = \eta\mu^U_{H1}, \tag{S.2}$$
$$\mu^I_{L1}\big(\eta + \lambda(\mu^U_{H0} + \mu^I_{H0})\big) = \big(\eta + \lambda\mu^U_{L1}\big)\mu^I_{L0}, \tag{S.3}$$
$$\mu^I_{H0}\big(\eta + \lambda(\mu^U_{L1} + \mu^I_{L1})\big) = \big(\eta + \lambda\mu^U_{H0}\big)\mu^I_{H1}. \tag{S.4}$$

Combine (S.1) and (S.2), and using $\mu^U_{L1} + \mu^U_{L0} = \mu^U_{H0} + \mu^U_{H1} = \frac{1-\phi}{2}$ (since half of the uninformed agents have high valuation and half have low valuation), I get

$$\mu^U_{L1}\big(2\eta + \lambda(\mu^I_{H0} + \mu^I_{L0})\big) = \mu^U_{H0}\big(2\eta + \lambda(\mu^I_{L1} + \mu^I_{H1})\big) = \frac{\eta(1-\phi)}{2}. \tag{S.5}$$

Similarly use (S.3) and (S.4) and $\mu^I_{L1} + \mu^I_{L0} = \mu^I_{H0} + \mu^I_{H1}$ to get

$$\big(\eta + \lambda\mu^U_{L1}\big)\big(\mu^I_{H0} + \mu^I_{L0}\big) = \big(\eta + \lambda\mu^U_{H0}\big)\big(\mu^I_{H1} + \mu^I_{L1}\big). \tag{S.6}$$

Paula Onuchic: p.onuchic@nyu.edu

Available at https://econtheory.org. https://doi.org/10.3982/TE4072

Use (S.5) and (S.6) to get $\mu^U_{L1}+\mu^I_{L1}+\mu^I_{H1}=\mu^U_{H0}+\mu^I_{H0}+\mu^I_{L0}$. Combining this with (S.5) yet again, conclude that $\mu^U_{L1}=\mu^U_{H0}\equiv\hat{\mu}^U$. This, along with (S.6), implies $(\mu^I_{H0}+\mu^I_{L0})=(\mu^I_{L1}+\mu^I_{H1})$; hence, $\mu^I_{H0}=\mu^I_{L1}\equiv\hat{\mu}^I$. Rewrite the inflow equals outflow conditions now as

$$\hat{\mu}^U\big(\eta+\lambda(\phi/2)\big)=\eta\frac{\big(1-\phi-2\hat{\mu}^U\big)}{2}\hat{\mu}^I,\qquad \big(\eta+\lambda(\hat{\mu}^U+\hat{\mu}^I)\big)=\frac{\phi-2\hat{\mu}^I}{2}\big(\eta+\lambda\hat{\mu}^U\big).$$

Solving these, I get

$$\hat{\mu}^U=\frac{1-\phi}{4+\dfrac{\lambda}{\eta}},\qquad \hat{\mu}^I=-\frac{\eta+\lambda,\hat{\mu}^U}{\lambda}+\sqrt{\left(\frac{\eta+\lambda\hat{\mu}^U}{\lambda}\right)^2+\frac{\phi}{2}\frac{\eta+\lambda\hat{\mu}^U}{\lambda}}.$$

S.3 *Unflagged values*

Again taking into account the guesses for $\mathcal{I}$ and β, unflagged values are given by the system

$$\begin{aligned}
rV^I_{H0}&=\eta\big(V^I_{H1}-V^I_{H0}\big)+\lambda\hat{\mu}^I\frac{V^I_{H1}-V^I_{H0}+V^I_{L0}-V^I_{L1}}{2}+\lambda\hat{\mu}^U\beta^I\big(V^I_{H1}-V^I_{H0}+V^U_{L0}-V^U_{L1}\big),\\
rV^I_{H1}&=\delta_H+\eta\big(V^I_{H0}-V^I_{H1}\big)+\lambda\hat{\mu}^U\beta^I\big(V^I_{H0}-V^I_{H1}+V^U_{H1}-V^U_{H0}\big),\\
rV^I_{L1}&=\delta_L+\eta\big(V^I_{L0}-V^I_{L1}\big)+\lambda\hat{\mu}^I\frac{V^I_{L0}-V^I_{L1}+V^I_{H1}-V^I_{H0}}{2}\\
&\quad+\lambda\hat{\mu}^U\beta^I\big(V^I_{L0}-V^I_{L1}+V^U_{H1}-V^U_{H0}\big),\\
rV^I_{L0}&=\eta\big(V^I_{L1}-V^I_{L0}\big)+\lambda\hat{\mu}^U\beta^I\big(V^I_{L1}-V^I_{00}+V^U_{L0}-V^U_{L1}\big),\\
rV^U_{H0}&=\eta\big(V^U_{H1}-V^U_{H0}\big)+\lambda\hat{\mu}^I\big(1-\beta^I\big)\big(V^U_{H1}-V^U_{H0}+V^I_{L0}-V^I_{L1}\big)\\
&\quad+\lambda\hat{\mu}^I\big(1-\beta^I\big)\big(V^U_{H1}-V^U_{H0}+V^I_{H0}-V^I_{H1}\big),\\
rV^U_{H1}&=\delta_H+\eta\big(V^U_{H0}-V^U_{H1}\big),\\
rV^U_{L1}&=\delta_L+\eta\big(V^U_{L0}-V^U_{L1}\big)+\lambda\hat{\mu}^I\big(1-\beta^I\big)\big(V^U_{L0}-V^U_{L1}+V^I_{H1}-V^I_{H0}\big)\\
&\quad+\lambda\hat{\mu}^I\big(1-\beta^I\big)\big(V^U_{L0}-V^U_{L1}+V^I_{L1}-V^I_{L0}\big),\\
rV^U_{L0}&=\eta\big(V^U_{L1}-V^U_{L0}\big).
\end{aligned}$$

In terms of the values of holding an asset, this system becomes

$$\begin{aligned}
rS^I_H&=\delta_H-2\eta S^I_H+\lambda\hat{\mu}^U\beta^I\big(S^U_H-S^I_H\big)+\lambda\hat{\mu}^I\frac{\big(S^I_L-S^I_H\big)}{2}+\lambda\hat{\mu}^U\beta^I\big(S^U_L-S^I_H\big),\\
rS^I_L&=\delta_L-2\eta S^I_L+\lambda\hat{\mu}^U\beta^I\big(S^U_L-S^I_L\big)+\lambda\hat{\mu}^I\frac{\big(S^I_H-S^I_L\big)}{2}+\lambda\hat{\mu}^U\beta^I\big(S^U_H-S^I_L\big),\\
rS^U_H&=\delta_H-2\eta S^U_H+\lambda\hat{\mu}^I\big(1-\beta^I\big)\big(S^I_L-S^U_H\big)+\lambda\frac{\phi-2\hat{\mu}^I}{2}\big(1-\beta^I\big)\big(S^I_H-S^U_H\big),
\end{aligned}$$

$$rS_L^U = \delta_L - 2\eta S_L^U + \lambda\hat{\mu}^I\left(1-\beta^I\right)\left(S_H^I - S_L^U\right) + \lambda\frac{\phi - 2\hat{\mu}^I}{2}\left(1-\beta^I\right)\left(S_L^I - S_L^U\right).$$

Add up the first two lines and the last two lines to get

$$(r+2\eta)\left(S_H^I + S_L^I\right) = \delta_H + \delta_L + 2\lambda\hat{\mu}^U\beta^I\left(S_H^U + S_L^U\right) - 2\lambda\hat{\mu}^U\beta^I\left(S_H^I + S_L^I\right),$$
$$(r+2\eta)\left(S_H^U + S_L^U\right) = \delta_H + \delta_L + \frac{\lambda\phi}{2}\left(1-\beta^I\right)\left(S_H^I + S_L^I\right) - \frac{\lambda\phi}{2}\left(1-\beta^I\right)\left(S_H^U + S_L^U\right).$$

These imply $(S_H^I + S_L^I) = (S_H^U + S_L^U) = \frac{\delta_H+\delta_L}{r+2\eta}$. Now from the original system, subtract the second equation from the first and the fourth from the third to find

$$(r+2\eta)\left(S_H^I - S_L^I\right) = \delta_H - \delta_L - \lambda\left(2\hat{\mu}^U\beta^I + \hat{\mu}^I\right)\left(S_H^I - S_L^I\right),$$
$$(r+2\eta)\left(S_H^U - S_L^U\right) = \delta_H - \delta_L - \frac{\lambda\phi}{2}\left(1-\beta^I\right)\left(S_H^U - S_L^U\right)$$
$$+\left(\frac{\lambda\phi}{2} - \hat{\mu}^I\right)\left(1-\beta^I\right)\left(S_H^I - S_L^I\right).$$

Rearrange these to get the expressions

$$\begin{aligned}\left(S_H^I - S_L^I\right) &= \hat{\alpha}^I(\delta_H - \delta_L),\\ \left(S_H^I - S_L^I\right) &= \hat{\alpha}^U(\delta_H - \delta_L),\end{aligned} \tag{S.7}$$

where

$$\hat{\alpha}^I = \frac{1}{2\left(r+2\eta+\lambda\left(2\hat{\mu}^U\beta^I + \hat{\mu}^I\right)\right)},$$
$$\alpha^U = \hat{\alpha}^U = \left[\frac{r+2\eta+\lambda\hat{\mu}^U + \dfrac{\lambda\phi}{4}}{r+2\eta+\dfrac{\lambda\phi}{4}}\right]\hat{\alpha}^I,$$

which finally implies

$$S_H^i = \left[\frac{1}{2(r+2\eta)}\right](\delta_H + \delta_L) + \frac{\hat{\alpha}^i}{2}(\delta_H - \delta_L),$$
$$S_L^i = \left[\frac{1}{2(r+2\eta)}\right](\delta_H + \delta_L) - \frac{\hat{\alpha}^i}{2}(\delta_H - \delta_L).$$

S.4 *Flagged values*

I solve for D_{H1}^I, D_{L1}^I, and D_{H1}^U. The system defining $\{D_{\text{va}}^I\}$ is

$$rD_{H1}^I = \left(\eta + \lambda\hat{\mu}^U\right)\left(D_{H0}^I - D_{H1}^I\right),$$
$$rD_{L1}^I = \left(\eta + \lambda\hat{\mu}^U\right)\left(D_{L0}^I - D_{L1}^I\right) + \lambda\hat{\mu}^I\frac{S_H^I - S_L^I}{2},$$

$$rD^I_{L0} = (\eta + \lambda\hat{\mu}^U)(D^I_{L1} - D^I_{L0}) - \lambda\hat{\mu}^U(\beta^I S^U_L + (1-\beta^I)S^I_L),$$
$$rD^I_{H0} = (\eta + \lambda\hat{\mu}^U)(D^I_{H1} - D^I_{H0}) + \lambda\hat{\mu}^I\frac{S^I_H - S^I_L}{2} - \lambda\hat{\mu}^U(\beta^I S^I_H + (1-\beta^I)S^U_L).$$

Combining the first equality with the fourth and the second equality with the third yields

$$r(D^I_{H1} - D^I_{H0}) = -2(\eta + \lambda\hat{\mu}^U)(D^I_{H1} - D^I_{H0}) - \lambda\hat{\mu}^I\frac{S^I_H - S^I_L}{2} + \lambda\hat{\mu}^U(\beta^I S^I_H + (1-\beta^I)S^U_L),$$
$$r(D^I_{L1} - D^I_{L0}) = -2(\eta + \lambda\hat{\mu}^U)(D^I_{L1} - D^I_{L0}) + \lambda\hat{\mu}^I\frac{S^I_H - S^I_L}{2} + \lambda\hat{\mu}^U(\beta^I S^U_L + (1-\beta^I)S^I_L).$$

Solve to find

$$(D^I_{H1} - D^I_{H0}) = -\frac{\lambda\hat{\mu}^I}{(r + 2\eta + 2\lambda\hat{\mu}^U)}\frac{S^I_H - S^I_L}{2} + \frac{\lambda\hat{\mu}^U}{(r + 2\eta + 2\lambda\hat{\mu}^U)}(\beta^I S^I_H + (1-\beta^I)S^U_L),$$
$$(D^I_{L1} - D^I_{L0}) = \frac{\lambda\hat{\mu}^I}{(r + 2\eta + 2\lambda\hat{\mu}^U)}\frac{S^I_H - S^I_L}{2} + \frac{\lambda\hat{\mu}^U}{(r + 2\eta + 2\lambda\hat{\mu}^U)}(\beta^I S^U_L + (1-\beta^I)S^I_L).$$

Plug this back into the original system to get

$$D^I_{H1} = \frac{\eta + \lambda\hat{\mu}^U}{r(r + 2\eta + 2\lambda\hat{\mu}^U)}\left[\lambda\hat{\mu}^I\frac{S^I_H - S^I_L}{2} - \lambda\hat{\mu}^U(\beta^I S^I_H + (1-\beta^I)S^U_L)\right], \quad \text{(S.8)}$$
$$D^I_{L1} = \frac{r + \eta + \lambda\hat{\mu}^U}{r(r + 2\eta + 2\lambda\hat{\mu}^U)}\lambda\hat{\mu}^I\frac{S^I_H - S^I_L}{2} - \frac{\lambda\hat{\mu}^U(\eta + \lambda\hat{\mu}^U)}{r(r + 2\eta + 2\lambda\hat{\mu}^U)}(\beta^I S^U_L + (1-\beta^I)S^I_L). \quad \text{(S.9)}$$

The system defining $\{D^U_{va}\}$ is

$$rD^U_{H1} = \eta(D^U_{H0} - D^U_{H1}),$$
$$rD^U_{H0} = \eta(D^U_{H1} - D^U_{H0}) + \lambda\hat{\mu}^I(1-\beta^I)(S^U_H - S^I_L) + \lambda(\phi/2 - \hat{\mu}^I)(1-\beta^I)(S^U_H - S^I_H),$$
$$rD^U_{L1} = \eta(D^U_{L0} - D^U_{L1}) + \lambda\hat{\mu}^I(1-\beta^I)(S^I_H - S^U_L) + \lambda(\phi/2 - \hat{\mu}^I)(1-\beta^I)(S^I_L - S^U_L),$$
$$rD^U_{L0} = \eta(D^U_{L1} - D^U_{L0}).$$

Subtract the second equality from the first to get

$$r(D^U_{H1} - D^U_{H0}) = -2\eta(D^U_{H1} - D^U_{H0}) - \lambda\hat{\mu}^I(1-\beta^I)(S^U_H - S^I_L) - \lambda(\phi/2 - \hat{\mu}^I)(1-\beta^I)(S^U_H - S^I_H)$$
$$\Rightarrow \quad (D^U_{H1} - D^U_{H0}) = -\frac{\lambda\hat{\mu}^I(1-\beta^I)}{r + 2\eta}(S^U_H - S^I_L) - \frac{\lambda(\phi/2 - \hat{\mu}^I)(1-\beta^I)}{r + 2\eta}(S^U_H - S^I_H).$$

Finally plug back into the original system to get

$$D_{H1}^{U}=\frac{\eta\lambda\hat{\mu}^{I}(1-\beta^{I})}{r(r+2\eta)}(S_{H}^{U}-S_{L}^{I})+\frac{\eta\lambda(\phi/2-\hat{\mu}^{I})(1-\beta^{I})}{r(r+2\eta)}(S_{H}^{U}-S_{H}^{I}). \qquad \text{(S.10)}$$

Co-editor Florian Scheuer handled this manuscript.

Manuscript received 11 December, 2019; final version accepted 8 March, 2021; available online 17 March, 2021.