On the Strategic Use of Attention Grabbers: Corrigendum

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1 Introduction

In this note we correct a few minor errors in Eliaz and Spiegler (2011). One error concerns the "no perfect substitutes" assumption stated in Section 2 of the paper. The others concerns the proofs of Proposition 1 and Lemma 2.

2 The "No Perfect Substitutes" Assumption

Eliaz and Spiegler (2011) assume that the preference relation $\succsim$ over the set of menus $P(X)$ satisfies the following property:

No Perfect Substitutes: For every $M, M' \in P(X)$, if $M \sim M'$, then $M \subseteq M'$ or $M' \subseteq M$.

This property is far too strong, in two respects. First, in conjunction with the monotonicity axiom, it excludes many interesting cases that should belong to the model's domain. For example, it rules out max-max preferences: $X = \{1, ..., n\}$; for every $M, M' \subseteq X$, $M \succ M'$ if and only if $\max(M) > \max(M')$. Second, the property is stronger than needed. In addition to monotonicity, the only property that Eliaz and Spiegler (2011) actually use in the analysis is the following, weaker version.

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Unique Minimal Equivalent Subset: For every menu $M \in P(X)$ and every $M', M'' \subseteq M$, if $M' \sim M''$ then $M' \subseteq M''$ or $M'' \subseteq M'$.

This axiom means that every menu M has a unique minimal preference-equivalent (weak) subset, and that is denoted $L(M)$. In other words, every menu can be unambiguously partitioned into a subset of preference-relevant alternatives $L(M)$ and a subset of pure attention grabbers $M - L(M)$. This is the property that Eliaz and Spiegler (2011) rely on in the analysis.

Thus, the no-perfect-substitutes property was overly strong, both in terms of prior motivation and in terms of what is actually needed for the analysis. It was a misguided attempt to provide a "foundation" for the property that every menu can be unambiguously partitioned into relevant alternatives and pure attention grabbers.

We do not think that a sensible alternative foundation is feasible. To see why, consider a behavioral model that plausibly governs $\succsim$ - namely, the "preference for flexibility" representation due to Kreps (1979). Specifically, assume a finite collection of states $\{1, ..., K\}$, and associate with each state k a strict primitive preference relation $>_k$ over X. The set of top elements in M according to the K primitive preference relations is

$$T(M) = \{x \in M \mid x \succ_k y \text{ for all } y \in M - \{x\} \text{ and some } k = 1, ..., K\}$$

According to Kreps's representation, $M \sim T(M)$. The reason is that the consumer does not expect to choose any element in $M - T(M)$ in any state, and therefore these elements are irrelevant for his evaluation of the menu.

Given the Krepsian representation, an indifference between two menus $M' \sim M''$ can arise from two scenarios. First, it is possible that $T(M') = T(M'')$, yet M' or M'' contain additional, dominated elements. This type of indifference falls within the scope of the model of Eliaz and Spiegler (2011) - the dominated alternatives are irrelevant in terms of the consumer's preferences, but they may function as pure attention grabbers. The second scenario is that $T(M') \neq T(M'')$ (so that the indifference follows from the fact that the subjective probabilities of the states and the utility function over X are such that the expected utility from each menu is the same). This type of indifference is "non-generic" in the sense that slight perturbation of the utility functions that represent the primitive preference relation would break the indifference. It is this type of indifference that Eliaz and Spiegler (2011) attempted to rule out. However, because it is ultimately a genericity assumption, it is unlikely to be captured by a convincing behavioral axiom.

3 The Proofs of Proposition 1 and Lemma 2

The proof of part (*ii*) of Proposition 1 in Eliaz and Spiegler (2011) contains an error because the inequality that tests the profitability of the deviation from M_* to M^* misses a term. However, it is easy to fix the error by considering a deviation to $M_* \cup M^*$ instead.

Another error is in the proof of Lemma 2, which takes it for granted that if M^* is in the support of the equilibrium strategy, it cannot beat any other menu in the support. Of course, this claim requires proof. As it happens, the claim can be proven in Section 2, as part of the general analysis, using the same (corrected) argument as in the proof of part (*ii*) of Proposition 1.

The following is a restatement of Proposition 1 and its proof, which fixes the proof of part (*ii*) and adds a third part that closes the gap in Lemma 2.

Proposition 1 *Let σ be a symmetric Nash equilibrium strategy. Then: (i) $\beta_\sigma(M^*) \in (0,1)$; (ii) There exists $M \in S(\sigma)$ such that $M^* \subset M$; (iii) If $M^* \in S(\sigma)$, then M^* beats no other menu in $S(\sigma)$.*

Proof. (*i*) Suppose that $\beta_\sigma(M^*) = 0$. Consider a menu $M \in S(\sigma)$ such that $M' \succsim M$ for all $M' \in S(\sigma)$. Then, M beats no menu in $S(\sigma)$. Therefore, M generates a market share of at most $\frac{1}{2}$. If a firm deviates from M to X, the deviation is profitable. By (A2), it raises the firm's market share from $\frac{1}{2}$ to 1, whereas by (A3), it changes its cost by $c(X) - c(M) < \frac{1}{2}$. Now suppose that $\beta_\sigma(M^*) = 1$. Since M^* is the (unique) least costly menu M such that $M \sim M^*$, each firm must offer M^* with probability one. By (A1) and (A4), there exists a menu M' such that M' is less costly than M^* and M^* does not beat M', it is profitable for a firm to deviate to M'. It follows that $\beta_\sigma(M^*) \in (0,1)$.

(*ii*) Assume the contrary. By (*i*), $\beta_\sigma(M^*) > 0$, hence $\beta_\sigma(M^*) = \sigma(M^*)$. Thus, $M^* \in S(\sigma)$. Let $\mathcal{M}_1$ denote the set of menus in $S(\sigma)$ that M^* beats, and let $\mathcal{M}_0$ denote the set of menus $M \in S(\sigma)$ for which $M^* \succ M$ yet M^* does not beat M. Recall that all menus are weakly worse than M^*, hence, the set $\mathcal{M}_0 \cup \mathcal{M}_1$ includes all the menus other than M^*.

Suppose $\mathcal{M}_1$ is empty. Then M^* generates a payoff of $\frac{1}{2} - c(M^*)$. Let $\tilde{M} \in S(\sigma)$ be a $\succsim$-maximal menu in $\mathcal{M}_0$. By (A1), $c(L(\tilde{M})) < c(M^*)$. Moreover, by the definition of the beating relation, no menu in $S(\sigma)$ beats $L(\tilde{M})$. Therefore, if a firm deviated to $L(\tilde{M})$, it would generate a market share of at least $\frac{1}{2}$ while costing less than $c(M^*)$, hence the deviation would be profitable, a contradiction.

Now suppose $\mathcal{M}_1$ is non-empty. Let M_* denote some $\succsim$-minimal menu in $\mathcal{M}_1$. Thus, M_* does not beat any menu in $\mathcal{M}_1$. Suppose that a firm deviates from M_* to $M_* \cup M^*$. This deviation is unprofitable only if the following inequality holds:

$$\frac{1}{2}\sigma(M^*) + \frac{1}{2}\sum_{M \in \mathcal{M}_1} \sigma(M) - c(M_* \cup M^*) + c(M_*) \leq 0$$

By the assumption of costs are additive,

$$c(M_* \cup M^*) = c(M_*) + c(M^*) - c(M_* \cap M^*) \leq c(M_*) + c(M^*)$$

This leads to the following necessary condition for the unprofitability of deviating from M_* to $M_* \cup M^*$:

$$\frac{1}{2}\sigma(M^*) + \frac{1}{2}\sum_{M \in \mathcal{M}_1} \sigma(M) - c(M^*) \leq 0$$

Now suppose that a firm deviates from M^* to X. By (A2), this deviation is unprofitable only if the following inequality holds:

$$\frac{1}{2}\sum_{M \in \mathcal{M}_0} \sigma(M) - c(X) + c(M^*) \leq 0$$

Note that $S(\sigma) = \{M^*\} \cup \mathcal{M}_0 \cup \mathcal{M}_1$. Therefore, combining the final pair of inequalities, we obtain

$$\frac{1}{2} \leq c(X)$$

in contradiction to the assumption that $c(X) < \frac{1}{2}$.

(*iii*) Assume the contrary - i.e., $M^* \in S(\sigma)$ and $\mathcal{M}_1$ is non-empty. The proof proceeds exactly like the proof of part (*ii*) - i.e. showing that either a deviation from M^* to X or from M_* to $M_* \cup M^*$ must be profitable. ∎

4 Typo in Proof of Proposition 4

In the paper, the elements $y^*(x)$ and $y_*(x)$ should be redefined as the largest and smallest elements in X that belong to $I(x)$. However, the proof later implicitly takes it for granted that these elements belong to $\cup_{M \in \mathcal{M}} B_\sigma(M)$. Therefore, $y^*(x)$ and $y_*(x)$ should be redefined as the largest and smallest elements in $\cup_{M \in \mathcal{M}} B_\sigma(M)$ that belong to $I(x)$.

References

[1] Eliaz, K. and R. Spiegler (2011), "On the Strategic Use of Attention Grabbers," Theoretical Economics 6, 127-155.

[2] Kreps, D. (1979), "A Representation Theorem for 'Preference for Flexibility'", Econometrica 47, 565-577.