The Loser's Curse in the Search for Advice

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Abstract

An agent searches sequentially for advice from multiple experts concerning the payoff of taking an operation. After incurring a positive search cost, the agent can consult an expert, whose interest is partially aligned with him. There are infinitely many experts, each has access to an identically and conditionally independent signal structure about the payoff, and each makes a recommendation after observing the signal realization. We find that the experts face a loser's curse, which could hamper the quality of information transmission. This effect is illustrated by studying the limit of equilibria with vanishing search cost. The main findings are as follows. First, there are signal structures with which both the agent's payoff and social welfare are strictly lower than the alternative scenario in which the agent commits to consulting a single expert only. Second, under some signal structures, no information can be transmitted in equilibrium, even though informative recommendation is possible if the agent could commit to a single expert. Finally, we identify the necessary and sufficient condition that ensures perfect information aggregation in the limit.

Keywords Search, Expert Advice, Information Transmission, Information Aggregation

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1 Introduction

A patient with some medical symptoms is unsure whether surgery is an appropriate treatment. For advice, he consults a doctor, who diagnoses the case and makes a recommendation. Suppose the interests

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of the doctor and the patient are aligned, so the patient's only concern is that the doctor's diagnosis is not accurate (rather than worrying that the doctor may have incentives for lying). Without sufficient confidence, he may consult another doctor for a second opinion. If the recommendations of the two doctors match, then he is more certain about the appropriate course of treatment. Nonetheless, he may continue the process of consulting other doctors for more accurate information. If he is eventually confident enough that the surgery is necessary, then he undergoes the surgery with one of the doctors that recommended the surgery to him. It is natural to expect that by consulting more doctors, the patient could gather better information about the appropriate treatment. A similar scenario arises in the process of a customer looking for a repair service, an entrepreneur looking for financial investment of a venture capitalist, and a claimant looking for legal advice and service of a lawyer.

In this paper, we analyze a model in which an (male) agent sequentially consults (female) experts for advice on whether to undergo an operation or not. His payoff of having the operation is uncertain, taking a positive value if the operation is suitable for him (an event denoted by the state $\omega = 1$), and a negative value if the operation is unsuitable for him (an event denoted by the state $\omega = 0$). There are infinitely many potential experts for the agent to consult, and each expert has access to an identical and conditionally independent signal structure for learning about the agent's state ω. For each consultation, the agent has to incur a positive search cost. A consulted expert makes a recommendation after privately learning her own signal realization, but not those of previously consulted experts. If she recommends the operation, the agent can decide whether to undergo the operation with her, or seek more advice from other experts. If she recommends against the operation, the agent cannot have the operation with her (but he may still consult other experts). Eventually, the agent may undergo the operation with an expert that recommends him to do so, or stop seeking advice without taking the operation.

The payoffs of the agent and the experts depend on the state and whether the operation is performed. We assume the interests of the experts and the agent are partially aligned. More specifically, if the agent undergoes the operation with an expert, then the expert would get a payoff with the same sign as the agent's. On the other hand, she gets a zero payoff if the agent does not undergo the operation with her. For simplicity, the payoff structure is exogenously fixed, so we have abstracted away from considerations such as the pricing of the operation and bargaining over the division of liability of failed operation (i.e., the operation is carried out with the agent's state being $\omega = 0$).

Our objective is to investigate the equilibrium outcome and welfare consequence of allowing the agent to sequentially search for experts' advice. In particular, we would like to compare the agent's payoff and the social welfare under two scenarios: (i) a benchmark setting in which there is only one expert available for consultation; and (ii) a setting in which there are infinitely many experts available for consultation at an infinitesimal search cost. At first sight, it seems that the agent would prefer scenario (ii), as he would be able to learn the state with almost no cost. Our analysis shows that whether this conjecture holds or not crucially depends on the experts' signal structure. The main findings are as follows. First, there are signal structures with which both the agent's payoff and the social welfare are strictly lower in scenario (ii) than in scenario (i). Somewhat strikingly, under some signal structures, no information can be transmitted in equilibrium in scenario (ii), even though information transmission is possible in scenario (i). Finally, we identify the necessary and sufficient condition on the signal structures that allow the agent to learn the true state perfectly in scenario (ii).

The key driving force behind our results is a *loser's curse* effect: in an equilibrium in which the experts' advice is informative, each expert understands that the agent decides to undergo the operation if and only if he has received sufficiently favorable information from other experts, and that her recommendation affects her own payoff if and only if she is pivotal. In other words, in deciding her recommendation, an expert conditions her payoff calculations on being pivotal, which is a piece of news that is supportive of the operation. As a result, the loser's curse consideration leads her to recommend the operation more often than she would, were she base her recommendation only on her privately observed signal. This in turn hurts the agent because it worsens the quality of information transmitted in each expert consultation.

The emergence of the loser's curse in our setting hinges crucially on the sequential optimality of the agent's search behaviors. If the agent were able to commit to a predetermined rule of sampling experts' advice and making the operation decision, then from an individual expert's perspective, being pivotal no longer means the operation is likely suitable — it just means the decision is a close call — so her recommendation would not change systematically by conditioning on the pivotal event.

A lower search cost could potentially exacerbate the loser's curse. Intuitively, with a lower search cost, the experts anticipate that the agent can afford to consult more experts and will decide to undergo the operation only after he becomes more confident. Each expert is therefore willing to further lower her own standard for recommending the operation, making such a recommendation even less informative. This effect can be illustrated most transparently by studying the limiting equilibrium with vanishing search cost. We show that at a negligible search cost, the loser's curse becomes extremely severe: every expert almost always discards her private signal and recommends the operation.

Using the observation above, we show that if the expert's signal structure does not contain a signal that fully reveals $\omega = 0$, and if the experts' loss in a failed operation (relative to the gain in a successful operation) is less than that of the agent, it is possible that the agent's payoff is strictly lower than those in the single-expert benchmark. The intuition is as follows. Suppose the experts adopt a partially informative recommendation rule. In the absence of a signal that fully reveals $\omega = 0$, the agent can never be very confident that $\omega = 1$. Otherwise, being pivotal would be extremely good news and the experts would always recommend the operation, thus making her recommendation completely uninformative. The agent's posterior belief when he decides to undergo the operation is therefore bounded away from one, and this observation provides an upper bound on the agent's limit equilibrium payoff. Furthermore, we find that this upper bound decreases with the experts' liability. Intuitively, if the experts suffer little from a failed operation, they would inherently adopt a low standard for recommending the operation. This worsens the informativeness of their recommendations, making the agent's search for advice less effective.

Somewhat surprisingly, we find that if the experts' liability in a failed operation is sufficiently small, the only equilibrium outcome involves experts adopting a completely uninformative recommendation rule. We identify the conditions on information and payoff structure, and provide a specific example, that allow partial information transmission in the single-expert benchmark, but no information transmission is possible with infinitely many experts available at a sufficiently low search cost. That is, the loser's curse could lead the market for advice to completely break down.

We also investigate the effect of the loser's curse on the effectiveness of aggregating information dispersedly held by experts. As each expert observes a conditionally independent signal about the state ω, if the agent can consult a large enough number of experts whose recommendations are sufficiently informative about their signals, he is able to learn the true state ω and hence take the ex-post correct decision concerning the operation almost surely —— information is perfectly aggregated. As explained above, a necessary condition for perfect information aggregation in the limit is the existence of a signal that fully reveals $\omega = 0$. It turns out the condition is also sufficient, and the reason is as follows. As noted above, the loser's curse makes each expert almost always recommend the operation in the limit. A recommendation against the operation thus implies that the expert must have seen a signal that fully (or almost fully) reveals that $\omega = 0$. As the search cost is negligibly small, the agent can afford to sample a large number of recommendations, and is bounded to receive quite a number of recommendations against the operation if indeed $\omega = 0$. Therefore, the agent can learn the true state almost surely.

Our analysis suggests that the loser's curse can potentially be a significant source of inefficiency in settings involving search for advice. By mitigating the loser's curse, the agent's welfare as well as social welfare may be improved using policies that mandate a higher consultation fee, or enforce contracts that prohibit the agent from seeking second opinions.

1.1 Related Literature

This paper is related to the following four lines of literature.

Search with adverse selection Our model features an agent sequentially consulting multiple experts, who hold private information concerning a common state of nature. Therefore, it is related to a body of work that studies the interaction of search and information asymmetry, such as Lauermann and Wolinsky (2016), Inderst (2005) and Guerrieri, Shimer, and Wright (2010). In Lauermann and Wolinsky (2016), a buyer has private information about the cost of the transaction (incurred by the seller), and he sequentially samples sellers who observe conditionally independent signals about his cost. The buyer has incentives to search for a seller who observes a favorable signal and hence is willing to accept a low price. Lauermann and Wolinsky (2016) identify a strong winner's curse, which they call the sampling curse, in their setting. The sampling curse implies that perfect information aggregation (in the limit as search cost vanishes) requires strong conditions on the signal structure. Our setting is similar to theirs, as the agent's (buyer's) type distribution, from the expert's (seller's) perspective, is endogenously determined by his search behavior. However, the reasons for search are quite different: the agent in our model consults experts to learn about a payoff-relevant state, whereas the buyer in their setting searches for a favorable deal. Moreover, being sampled is bad news for a seller in their setting, as a high-cost buyer tends to search longer. Experts in our setting may find it either good news or bad news, depending on the expected duration of search in each state.

Large election Feddersen and Pesendorfer (1997) analyze two-candidate elections in which voters receive conditionally independent signals about a state variable that affects the utility of all voters. They show that as the size of the electorate goes to infinity, information is perfectly aggregated, in the sense that the election outcome would not change were all private signals become public. Similarly, we consider an agent deciding between two options (whether to have an operation or not) and soliciting recommendations (analogous to votes) from partially informed experts (analogous to voters), and we are also interested in the scenario in which the agent's search cost vanishes (analogous to having infinitely many voters). Our model can therefore be viewed as a sequential search version of aggregating information held dispersedly in some population. An expert in our search setting faces strategic consideration similar to a voter in an election as they both evaluate their payoffs conditional on being pivotal. However, we find that information is perfectly aggregated if and only if the experts' signal space contains a perfectly-revealing signal, whereas the existence of such signal is not needed for large elections. The difference arises because the agent's stopping rule in our search setting is endogenous, whereas the decision rule (the size of electorate and the fraction of votes a candidate needs to win) is exogenously fixed in an election.[1] More discussion on the relation between our results and Feddersen and Pesendorfer (1997) can be found in Section 6.2. Other notable studies on information aggregation in elections include Dekel and Piccione (2000) and Feddersen and Pesendorfer (1996).[2]

Common-value auction Our model has some flavor of a common-value auction, as the state of the agent is common to all experts, and he eventually picks one expert (if any) to carry out the operation. However, our model does not involve bidding, and each expert only makes a binary recommendation.

The literature in common-value auctions has studied whether the price paid by the winning bidder converges, in the limit as the number of bidders goes to infinity, to the value of the object being auctioned. We consider a related but different question on information aggregation: whether the advice of all consulted experts would collectively reflect the underlying state.[3] Milgrom (1979) identifies a necessary and sufficient condition for perfect information aggregation for single-object auctions. If the object has only two possible values, Milgrom's condition is the existence of a signal that (almost) perfectly reveals that the object has a high value. His result is driven primarily by the intense competition among a large number of bidders. Although our condition for perfect information aggregation is similar, our result is driven by the experts' loser's curse and the agent's endogenous stopping rule, rather than competition among experts. Moreover, when Milgrom's condition fails, the information aggregation properties in the two settings are drastically different. A more detailed discussion on the relation between our results and Milgrom (1979) can be found in Section 6.1.

Other notable studies on information aggregation in large auctions are as follows. Pesendorfer and Swinkels (1997) consider a common-value auction with a large supply, and show that the winner's curse

[1] Another difference is that in the large elections considered in Feddersen and Pesendorfer (1997), the election outcome is almost always very close. In contrast, the endogeneity of the agent's stopping rule in our search setting implies that only when he has received sufficient information would he stop searching, at which point the advice collected would clearly favor the chosen option.

[2] Dekel and Piccione (2000) show that every symmetric equilibrium of a simultaneous voting game remains to be an equilibrium if voters cast their votes sequentially. Feddersen and Pesendorfer (1996) identify the swing voter's curse: less-informed, indifferent voters strictly prefer to abstain from voting. They show that while a substantial fraction of the electorate might abstain, information is still perfectly aggregated in the limit.

[3] If an observer of an (first-price or second-price) auction has access to not only the winning price, but also all bids submitted, then she can recover the object's value as the number of bidders goes to infinity. This is because the signal of each bidder can be backed out from the submitted bid, as the unique symmetric equilibrium bidding function is strictly monotone (see Krishna (2009)).

and the loser's curse balance each other out, leading to perfect information aggregation. Kremer (2002) shows that the competitiveness of a large auction forces the limiting price to approach the object's expected value conditional on the pivotal bidder's information.

Our finding that the agent may suffer from a decrease in his search cost because of an exacerbation of the experts' loser's curse is related to the finding in the auction literature that the auctioneer's expected revenue may decrease in the number of bidders because of the bidders' winner's curse. The latter possibility is illustrated in Bulow and Klemperer (2002) and Hong and Shum (2002).[4] The reason for our finding is that the exacerbation of the loser's curse induces the experts to make less informative recommendations, thus lowering the agent's expected payoff. In contrast, the auctioneer may suffer from having more bidders because the exacerbation of the winner's curse makes bidders shade their bids more, without any impact on the information content of their bids.[5] Moreover, the implications of the loser's curse on the experts' payoffs, as well as the social welfare, are different from the counterparts in auction settings. We will discuss this in more detail in Section 5.3.

Credence goods provision In the credence goods market, the expert becomes more informed about the type and/or quality of the service the customer needs after performing a diagnosis. The expert then recommends and provides, subject to the customer's approval, the recommended service (see Dulleck and Kerschbamer (2006) for a survey). The literature on credence goods provision has studied whether competition among experts can overcome market inefficiency due to the information asymmetry. Wolinsky (1993) shows that competition can mitigate experts' incentives to prescribe overtreatment (i.e., providing unnecessarily expensive treatment) if there are firms that specialize in providing low-cost repair. Alger and Salanié (2006) show that price competition in the low-cost repair induces overtreatment. In contrast, our model does not feature any price competition, and overtreatment is not a problem. Pesendorfer and Wolinsky (2003) analyze a setting in which experts need to exert diagnostic effort to identify the treatment needed, and show that price competition leads to inefficient effort exertion. Wolinsky (2005) analyzes a setting in which experts exert effort to devise an appropriate plan for the customer, and show that inefficiency arises because the customer does not internalize the experts' effort cost. Our model, on the other hand, does not have any moral hazard.

[4] Bulow and Klemperer (2002) illustrate this possibility in two settings: (i) symmetric bidders with decreasing hazard rate; and (ii) asymmetric bidders with increasing hazard rate. In contrast, we consider symmetric experts without any assumption on the hazard rate. Hong and Shum (2002) find an inverse relation between the auctioneer's revenue and the number of bidders in procurement auctions.

[5] In either a first or second-price common-value auction, the unique symmetric equilibrium bidding function is strictly monotone (see Krishna (2009)).

The outline of the paper is as follows. The model is set up in Section 2. In Section 3, we consider the benchmark case in which there is only one available expert. In analyzing the main model in Section 4, we establish equilibrium existence and explain the effect of the losers' curse. We show that the loser's curse can lead to inefficiency by studying the limit equilibrium with a negligible search cost in Section 5. In Section 6, we identify conditions under which dispersed information is perfectly aggregated in the limit. Section 7 discusses a few alternative settings. Lengthy proofs are relegated to the appendix.

2 Model

An (male) agent can either undergo an operation (denoted by $a = 1$) or not (denoted by $a = 0$). His payoff of undergoing the operation depends on a binary state of the world $\omega \in \{0, 1\}$. If the state is $\omega = 1$ ($\omega = 0$), then the operation is suitable (unsuitable) for him. His prior belief about the state is denoted by $\pi \equiv \Pr(\omega = 1) \in (0, 1)$. The operation must be carried out by an (female) expert. There are infinitely many ex-ante identical experts. In each of the infinitely many periods, the agent can visit one expert. Upon a visit, each expert conducts a test which generates an informative signal about ω. After privately observing the signal, she makes a recommendation to the agent. The experts have a common payoff function, as well as common information acquisition technology, which will be discussed below. The agent has free access to one expert, and he always consults this first expert. For each additional consultation of other experts, the agent has to incur a search cost of $c \in (0, 1)$. Each of the infinitely many experts is drawn (without recall) with equal probability in every period.

Each expert can run a costless test to obtain a signal s about the state ω. The signal of each expert is distributed identically and independently (conditional on the state ω). More specifically, the signal space is a closed interval, denoted by $[\underline{s}, \bar{s}] \subset [0, \infty]$, and the signal s is generated according to conditional density function $f(s|\omega)$, with the corresponding conditional distribution function $F(s|\omega)$. We further assume that $f(s|\omega)$ has full support on $[\underline{s}, \bar{s}]$ for each ω. We say a signal structure is permissible if it satisfies the assumptions above. In the subsequent analysis, we will label the signals as their likelihood ratios, i.e., $s \equiv \frac{f(s|1)}{f(s|0)}$. With this labelling, a high signal is more indicative of $\omega = 1$, and the signal structure is informative if and only if $\underline{s} \in [0, 1)$ and $\bar{s} \in (1, \infty]$.

The signal realization of the test is unverifiable and observed privately by the consulted expert. Moreover, we assume that the signal realization is so complicated that it is infeasible to communicate its full content to the agent. Instead, each expert makes a binary recommendation of having the operation or not. Denote the set of recommendations by $\{Y, N\}$, where Y stands for a non-binding recommendation for the operation, and N stands for a recommendation against the operation. If the expert recommends N, it means that she refuses to perform the operation for the agent, who must then part with the expert, and that period is over. On the other hand, if the expert recommends Y, it means that she is willing to perform the operation for the agent, who can choose whether or not to have the operation with her. In other words, a recommendation N is a rejection by the expert, whereas a recommendation Y means the expert provides an option for the agent to undergo the operation with her. If the expert makes recommendation Y and the agent agrees to have the operation with her, then they collect their respective payoffs described below, and the game is over. If the agent chooses not to have the operation with the expert, he parts with her and that period is over. He can then either consult another expert in the next period, or stop the search for advice altogether.

The respective payoffs received by the agent and the expert for different scenarios are as follows:

		Action	
		$a = 1$	$a = 0$
State	$\omega = 1$	$1, 1$	$0, 0$
	$\omega = 0$	$-L, -l$	$0, 0$

The agent receives a positive payoff normalized to one if he undergoes the operation and the state is $\omega = 1$. He suffers a loss L if he undergoes the operation but the state is $\omega = 0$. His payoff of not having the operation is state-independent and normalized to zero. The expert who performs the operation for the agent has a partially-aligned payoff function: a positive payoff normalized to one if the operation is indeed suitable for the agent, and a negative payoff equals $-l$ if the operation is not suitable. Her payoff is normalized to zero if she does not perform the operation for the agent.[6] It is clear that if the state ω is publicly known, the agent and all experts would agree on the operation decision. Throughout our analysis, we assume $L > l > 0$, so that the agent suffers more than the expert in the case of a failed operation. We believe that this is the relevant parameter configurations in most applications, including the case of a patient looking for advice on surgery and a claimant looking for legal advice.

The agent's search process is assumed to be without recall. Therefore, once the agent parts with an expert (either because the expert recommends N, or the agent refuses to undergo the operation with her), the latter's role in the game is over. Moreover, the agent cannot communicate his history of recommendations received from previously consulted experts. Finally, we assume that the experts do not know the number of previous experts the agent has consulted. This assumption is natural if

[6] This payoff specification is a normalization because the preference represented in the form of expected utility is preserved under a positive affine transformation of the von Neumann-Morgenstern utility function.

the agent becomes aware of his problem at a random time, which is not observable by the experts. Discussion on these assumptions can be found in Section 7.

A pure strategy of an expert is the set of signals under which she recommends the operation. A history of the agent after consulting n experts is a sequence of n recommendations. Denote the set of all possible recommendation histories by $H \equiv \{\varnothing\} \cup (\cup_{n \in \mathbb{N}} \{Y, N\}^n)$. At the beginning of a period, the agent decides whether to visit an expert that he has not consulted before. At the end of a period, if he has consulted an expert and the expert recommends Y, he decides whether or not to undergo the operation with her. Therefore, a generic behavioral strategy of the agent, denoted by $b = (b_0, b_1)$, consists of two components, both of which are mappings from H to $[0, 1]$. First, $b_0(h)$ is the probability that the agent chooses not to consult any expert for that period, provided that the current history is h. Second, $b_1(h)$ is the probability that the agent decides to undergo the operation at the end of a period after being recommended by the current expert to do so.[7]

The solution concept is weak perfect Bayesian equilibrium. We say an equilibrium is **informative** if the operation outcome varies stochastically with the state ω. In an informative equilibrium, it is necessary that the agent's strategy satisfies $b_1(h, Y) > 0$ for some history $h \in H$ on the equilibrium path. On the other hand, an equilibrium is said to be uninformative if the operation outcome is independent of the state: either the operation is always carried out, or it is never carried out. While informative and uninformative equilibria may coexist for some parameters, our subsequent analysis will put more emphasis on informative equilibria (whenever they exist) because they have more interesting welfare and information properties.

3 Benchmark Model: Single Expert

For comparison of results later, we consider a benchmark model in which the agent can consult only one expert. The specific objective is to compute the maximum payoff of the agent and the expert among all permissible signal structures.

After learning a signal s, the expert updates her belief on ω according to Bayes' rule: $\Pr(\omega = 1|s) = \left(1 + \frac{1-\pi}{\pi}\frac{1}{s}\right)^{-1}$. If the agent follows her recommendation with a positive probability, she finds it optimal to recommend the operation if and only if $\Pr(\omega = 1|s) - l \Pr(\omega = 0|s) \geq 0$, or equivalently,

$$s \geq \frac{1-\pi}{\pi} l \equiv \tilde{s}. \tag{1}$$

[7]In our notation, for $i = 0, 1$, $b_i(h) = 1$ stands for stopping the search, and $b_i(h) = 0$ stands for continuing the search.

Consequently, in an informative equilibrium, the expert adopts the cutoff strategy of recommending Y if and only if $s \geq \tilde{s}$ and it is necessary that $\tilde{s} \in (\underline{s}, \bar{s})$.

Upon receiving a positive recommendation, the agent's payoff of taking the operation is $\Pr(\omega = 1 | s \geq \tilde{s}) - L \Pr(\omega = 0 | s \geq \tilde{s})$. He follows the recommendation if this payoff is nonnegative, or equivalently,

$$\frac{\pi}{1-\pi} \frac{1 - F(\tilde{s}|1)}{1 - F(\tilde{s}|0)} \geq L. \tag{2}$$

An informative equilibrium requires that the expert makes her recommendation based on the observed signal, and that the agent follows her recommendation. Thus, an informative equilibrium exists if and only if both $\pi \in \left(\frac{l}{l+\bar{s}}, \frac{l}{l+\underline{s}} \right)$ and inequality (2) holds.[8] If either one of these conditions fails, then the equilibrium is necessarily uninformative.[9]

Denote by $U_1(F)$ and $T_1(F)$ the highest equilibrium (ex-ante) payoffs of the agent and the expert respectively, given a signal structure F. While an informative equilibrium and an uninformative equilibrium in which the agent never takes the operation may coexist, the former strictly Pareto dominates the latter. This follows immediately from the observation that either the expert or the agent can veto the operation, so their respective expected payoffs must be nonnegative whenever the operation is carried out. Consequently, $U_1(F)$ and $T_1(F)$ are determined by the informative equilibrium whenever it exists.

3.1 Bounds on Payoffs

In this subsection, we derive tight upper bounds on $U_1(F)$ and $T_1(F)$ **among all permissible signal structures, fixing the lower bound of the signal space $\underline{s} \in [0, 1)$.** The main purpose of this derivation is to compare the welfare between the single-expert benchmark, and that of allowing the agent to sequentially seek advice from multiple experts (to be considered in Sections 4 to 6).

Given a fixed lower bound of the signal space $\underline{s}$, the most informative signal structure is one that has a binary support, say $\{\underline{s}, \bar{s}\}$, where $\bar{s}$ fully reveals $\omega = 1$. This signal structure, denoted by $F_{\underline{s}}$, generates signals with conditional probabilities $\Pr(\underline{s}|\omega = 0) = 1$, and $\Pr(\underline{s}|\omega = 1) = \underline{s} = 1 - \Pr(\bar{s}|\omega = 1)$.[10] It is easy to see that if $\underline{s}$ is sufficiently low, in particular $\pi < \frac{l}{l+\underline{s}}$, then the expert is willing to recommend N after observing signal $\underline{s}$. In this case, an informative equilibrium exists, and the ex-ante expected payoffs of the agent and the expert are both $\pi(1 - \underline{s})$. While $F_{\underline{s}}$ does not have a conditional density

[8] The condition $\pi \in \left(\frac{l}{l+\bar{s}}, \frac{l}{l+\underline{s}} \right)$ is equivalent to $\tilde{s} \in (\underline{s}, \bar{s})$.

[9] In particular, there is an uninformative equilibrium in which the expert always recommends Y and the agent always takes the operation regardless of signals if and only if $\pi \geq \max\left\{ \frac{l}{l+\underline{s}}, \frac{L}{1+L} \right\}$. It is clear that there is no informative equilibrium in this case.

[10] That is, $F_{\underline{s}}(\underline{s}|0) = 1$, and $F_{\underline{s}}(s|1) = 0$ for $s < \underline{s}$. Also, $F_{\underline{s}}(s|1) = \underline{s}$ for $s \in [\underline{s}, \bar{s})$; and $F_{\underline{s}}(\bar{s}|1) = 1$.

and is thus not permissible, it provides a tight upper bound for equilibrium payoffs of the single-expert benchmark.

Proposition 1 *Fix the lower bound of the expert's signal space at* $\underline{s} \in [0,1)$ *and denote by* $\Sigma_{\underline{s}}$ *the set of permissible signal structures with a lower bound of the signal space* $\underline{s}$*. If* $\pi < \frac{l}{l+\underline{s}}$*, then*

$$\sup_{F \in \Sigma_{\underline{s}}} U_1(F) = \sup_{F \in \Sigma_{\underline{s}}} T_1(F) = \pi(1-\underline{s}).$$

Proposition 1 is quite intuitive.[11] As the interests of the agent and the expert are partially aligned, it is not surprising that the agent would prefer an expert endowed with a more informative signal structure. In the proof of Proposition 1, we first consider the case in which discrete signal structures are permitted, and show that the optimal signal structure is $F_{\underline{s}}$. The proof is completed by noting that $F_{\underline{s}}$ can be approximated arbitrarily well using signal structures with conditional density functions that have a support $[\underline{s}, \infty]$.

4 Equilibrium Existence and Characterization

In this section, we analyze the main model, in which the agents can sequentially consult multiple experts. We focus on the informative equilibrium, identify parameter configurations that ensure its existence, and provide some characterization.

Consider first the experts' problem. A key observation is that an expert's recommendation matters to her payoff if and only if she is pivotal. Specifically, a recommendation N gives her a sure payoff of zero. A recommendation Y gives her a non-zero payoff if and only if the agent follows her recommendation and takes the operation with her. Therefore, when deciding her recommendation, the expert should compare her payoffs conditional on the pivotal event that the agent would follow her recommendation Y and take the operation with her. Denote the pivotal event by $piv(b) \equiv \{h \in H : b_1(h, Y) > 0 \text{ and } h \text{ is on the equilibrium path}\}$. By definition, $piv(b)$ has a positive probability in an informative equilibrium.

We explain below that the experts necessarily adopt an identical cutoff strategy in an informative equilibrium. By assumption, each expert's signal s is, conditional on ω, independent of other events in the game. This implies that her belief that $\omega = 1$, conditional on the signal s, being pivotal, and the strategy profile of other players, is strictly increasing in s. Consequently, each expert would find

[11] The proof can be found in the working-paper version of this article..

it optimal to adopt a cutoff strategy of recommending Y if and only if the observed signal s exceeds a certain cutoff. Moreover, as the experts cannot observe the consultation history of the agent, they all face an identical problem, so they necessarily adopt a common cutoff strategy in equilibrium. Therefore, without loss of generality, we will simply use the common cutoff s^* to stand for an expert's strategy. In an informative equilibrium, the experts adopt cutoff $s^* \in (\underline{s}, \bar{s})$ so that their recommendations vary with their observed signals.

Using the observations above, in an informative equilibrium, cutoff strategy s^* is an individual expert's best response to the strategy profile (s^*, b) of other experts and the agent if and only if

$$\Pr(\omega = 1 | s^*, piv(b); s^*, b) = \frac{l}{1+l}. \tag{3}$$

The conditional probability $\Pr(\omega = 1 | s, piv(b); s^*, b)$ can be expressed more explicitly in terms of the strategy profile (s^*, b). Specifically, denote by $q_\omega(h; s^*, b)$ the ex-ante probability that the agent arrives at history $h \in H$ prior to a consultation, given a strategy profile (s^*, b) and a state ω. Then

$$\begin{aligned} \Pr(\omega = 1 | s, piv(b); s^*, b) &= \left(1 + \frac{1}{s}\frac{1-\pi}{\pi}\frac{\Pr(piv(b) | \omega = 0; s^*, b)}{\Pr(piv(b) | \omega = 1; s^*, b)}\right)^{-1} \\ &= \left(1 + \frac{1}{s}\frac{1-\pi}{\pi}\frac{\sum_{h \in H} q_0(h; s^*, b)\, b_1(h, Y)}{\sum_{h \in H} q_1(h; s^*, b)\, b_1(h, Y)}\right)^{-1}. \end{aligned} \tag{4}$$

The first equality is a straightforward application of Bayes' rule, using the fact that each expert's signal s is independently drawn conditional on ω. The second equality computes the likelihood ratio $\frac{\Pr(piv(b)|\omega=0;s^*,b)}{\Pr(piv(b)|\omega=1;s^*,b)}$ of the pivotal event $piv(b)$. This likelihood ratio is determined by the ratio of the ex-ante expected number of pivotal experts in each state.[12] In state ω, $\sum_{h \in H} q_\omega(h; s^*, b)\, b_1(h, Y)$ is the expected number of pivotal experts in the agent's equilibrium search.

Next, we consider the agent's search strategy. The agent's posterior belief associated with history $h \in H$ depends only on the experts' strategy s^*, and can be computed with a simple application of Bayes' rule. We write $p(h; s^*)$ to stand for the induced posterior belief that $\omega = 1$. The lemma below states that the agent's best response to any common strategy of the experts has a Markov structure, with the state variable being the agent's current belief.[13]

Lemma 1 *For any strategy s^* of the experts, the agent's optimal search strategy, characterized by a unique pair of beliefs $p_0(s^*), p_1(s^*) \in [0, 1]$, is as follows. Let h be a beginning-of-period history.*

[12] Lemma 6 in Appendix A.2 ensures that, in an informative equilibrium, the likelihood ratio of $piv(b)$, and thus the conditional probability $\Pr(\omega = 1 | s, piv(b); s^*, b)$, are well-defined.

[13] For a proof of Lemma 1, see Ross (1983).

(i) At the beginning of a period (other than the first period), the agent quits searching (i.e., $b_0(h) = 1$) if $p(h; s^*) < p_0(s^*)$, and consults an expert (i.e., $b_0(h) = 0$) if $p(h; s^*) > p_0(s^*)$. At $p(h; s^*) = p_0(s^*)$, he is indifferent and may randomize (i.e., $b_0(h) \in [0, 1]$).

(ii) Suppose the current expert recommends Y. The agent takes the operation (i.e., $b_1(h, Y) = 1$) if $p(h, Y; s^*) > p_1(s^*)$, and does not take (i.e., $b_1(h, Y) = 0$) if $p(h, Y; s^*) < p_1(s^*)$. At $p(h, Y; s^*) = p_1(s^*)$, he is indifferent and may randomize (i.e., $b_1(h, Y) \in [0, 1]$).

Appendix A.1 contains the details of characterizing the cutoffs $p_0(s^*)$ and $p_1(s^*)$ using the agent's Bellman equation. If $\pi \in (p_0(s^*), p_1(s^*))$, the agent would typically solicit a number of advice, stopping the search whenever his posterior belief falls below $p_0(s^*)$ or rises above $p_1(s^*)$. Note that it is possible for his belief prior to meeting an expert to exceed $p_1(s^*)$; this can happen if $\pi > p_1(s^*)$ and he has not received any recommendation Y so far.

In sum, a strategy profile (s^*, b) constitutes an informative equilibrium if and only if

1. (experts' best response) equation (3) holds;

2. (agent's best response) b is as described in Lemma 1; and

3. (informative outcome) $s^* \in (\underline{s}, \bar{s})$ and $piv(b)$ is a positive-probability event.

In addition to the informative equilibria discussed above, an uninformative equilibrium may also exist. In such an equilibrium, the outcome is not informative of the state ω in that (i) $s^* \in \{\underline{s}, \bar{s}\}$, and/or (ii) $b_1(h, Y) = 0$ for all on-the-equilibrium-path history h.

The following proposition establishes the existence of an equilibrium.

Proposition 2 *An equilibrium exists. Moreover, if $\pi \in \left(\frac{L}{1+L}, \frac{l}{l+\underline{s}}\right)$, an informative equilibrium exists.*

The proof of Proposition 2 can be found in Appendix A.2. An informative equilibrium may still exist even if $\pi \notin \left(\frac{L}{1+L}, \frac{l}{l+\underline{s}}\right)$, but its existence is not guaranteed by Proposition 2. Although we cannot establish equilibrium uniqueness for the general case,[14] we show in the next section that the equilibrium is essentially unique in the limit when the search cost is vanishingly small.

In the rest of this section, we explain the implication of the pivotal nature of the experts' problem by considering equations (3) and (4). If the agent is expected to consult many experts, being pivotal

[14] Thus, if $\pi \in \left(\frac{L}{1+L}, \frac{l}{l+\underline{s}}\right)$, there may be multiple informative equilibria. If $\pi \notin \left(\frac{L}{1+L}, \frac{l}{l+\underline{s}}\right)$, informative and uninformative equilibria may coexist.

is good news to the expert because the agent must have collected sufficiently favorable information from other experts that warrant implementing the operation with a single extra recommendation Y. In other words, the agent's belief prior to consulting her must be sufficiently close to (or above) $p_1(s^*)$. Specifically, denote by $\tilde{p}_1(s^*)$ the minimum belief for the consulted expert to be pivotal. That is, with a beginning-of-period belief $\tilde{p}_1(s^*)$, the agent's posterior would jump up to $p_1(s^*)$ after receiving a recommendation Y from the current expert. Thus, $p_1(s^*)$ and $\tilde{p}_1(s^*)$ are related by

$$p_1(s^*) = \left(1 + \frac{1 - \tilde{p}_1(s^*)}{\tilde{p}_1(s^*)} \frac{1 - F(s^*|0)}{1 - F(s^*|1)}\right)^{-1}. \tag{5}$$

We can derive an upper bound on the likelihood ratio of the pivotal event $piv(b)$ using $\tilde{p}_1(s^*)$. First, as the event $piv(b)$ is made up of the agent's on-path histories such that $b_1(h, Y) > 0$, its likelihood ratio is bounded from above by that associated with the most "unfavorable" history within the event:

$$\frac{\Pr(piv(b)|\omega = 0; s^*, b)}{\Pr(piv(b)|\omega = 1; s^*, b)} \leq \sup_{h \in piv(b)} \frac{q_0(h; s^*, b)}{q_1(h; s^*, b)}. \tag{6}$$

Now suppose the agent has a history h and the current consultation is pivotal. By the definition of $\tilde{p}_1(s^*)$, his beginning-of-period belief $\left(1 + \frac{1-\pi}{\pi} \frac{q_0(h;s^*,b)}{q_1(h;s^*,b)}\right)^{-1}$ must be no lower than $\tilde{p}_1(s^*)$. This implies that $\frac{q_0(h;s^*,b)}{q_1(h;s^*,b)} \leq \frac{\pi}{1-\pi} \frac{1-\tilde{p}_1(s^*)}{\tilde{p}_1(s^*)}$. Together with inequality (6), the likelihood ratio of $piv(b)$ is thus bounded from above by $\frac{\pi}{1-\pi} \frac{1-\tilde{p}_1(s^*)}{\tilde{p}_1(s^*)}$.

Using this upper bound, the equilibrium condition on the experts' strategy (3) implies

$$\left(1 + \frac{1}{s^*} \frac{1 - \tilde{p}_1(s^*)}{\tilde{p}_1(s^*)}\right)^{-1} \leq \frac{l}{1 + l}. \tag{7}$$

Therefore, when deciding her recommendation, it is as if the expert replaces the prior belief π with some belief higher than $\tilde{p}_1(s^*)$. If $\tilde{p}_1(s^*) > \pi$, then being pivotal is necessarily good news. A failure to take this into account would make an individual expert suffer the **loser's curse**: by adopting an excessively high cutoff, she may end up recommending N, even though the expected payoff of carrying out the operation (which would happen were she recommend Y) is positive. In the next section, we will show that $\tilde{p}_1(s^*) > \pi$ arises in equilibrium if the search cost is sufficiently small and π is not too high.

5 Welfare Loss with Vanishing Search Cost

In this section, we illustrate the welfare loss resulting from the experts' loser's curse. To this end, we consider a scenario in which the agent's search cost c is vanishingly small. Specifically, take an arbitrary sequence of (positive) search cost $\{c_n\}$, such that $\lim_{n\to\infty} c_n = 0$, and a corresponding sequence of equilibria. In the absence of the loser's curse (for instance, if all experts naively believe that they are the only expert being consulted) so that the experts adopt the cutoff defined in (1), it is straightforward that the agent is able to learn the true state ω with probability arbitrarily close to one as $c_n \to 0$.[15] In this case, there is essentially full efficiency, as the payoff of the agent and expert are both arbitrarily close to the highest possible level, which is equal to π. Therefore, in this hypothetical setting without the loser's curse, the agent and the social planner would strictly prefer having a negligible search cost, rather than a search cost so high that the agent can afford to consult only one expert.

One of our main findings is that the conclusions above can be very different once the loser's curse is taken into account. By computing upper bounds for the equilibrium payoffs of the agent and experts with vanishing search cost, and comparing them with those of the single-expert benchmark (in Proposition 1), we find that there are circumstances under which the agent and the experts would all strictly benefit if the agent could commit to consulting only a single expert. In deriving this result, we show that under some signal structures, the loser's curse prevents the agent from perfectly learning the state even if the search cost is negligible. In some extreme cases, this effect could be so severe that informative equilibria do not exist, even though information transmission is possible in the single-expert benchmark.

Denote by $U(c)$ the highest equilibrium payoff of the agent in the game with a search cost of c. To be specific, $U(c)$ is given by the agent's ex-ante expected payoff from the operation decision, less the total search cost incurred. Also, denote by $T(c)$ the highest equilibrium joint payoff of the experts in the corresponding game.[16] The main result of this section, formally stated in the proposition below, identifies upper bounds on $U(c)$ and $T(c)$ in the limit equilibrium with vanishing search cost. To facilitate the comparison with the single-expert benchmark, we focus on the case $\pi < \frac{l}{l+\underline{s}}$ (as in Proposition 1).

Proposition 3 *Suppose* $\pi < \frac{l}{l+\underline{s}}$.

(i) If $\underline{s} \in \left(0, \frac{l}{L}\right]$, *then* $\limsup_{n\to\infty} U(c_n) \leq \pi\left(1 - \frac{L}{l}\underline{s}\right)$, *and* $\limsup_{n\to\infty} T(c_n) < \pi(1-\underline{s})$.[17]

(ii) If $\underline{s} > \frac{l}{L}$, *then for all sufficiently small values of* c_n, *there is no informative equilibrium, and* $U(c_n) = T(c_n) = 0$.

The proof of Proposition 3 can be found in Appendix A.4. Except for the lower bound $\underline{s}$ of the

[15] This is a consequence of the law of large numbers.

[16] In the proof of Proposition 2, we establish the upper semi-continuity of the experts' best response correspondence (to other experts' common strategy), so the set of equilibrium experts' cutoffs is closed. Together with the fact that the agents' optimal cutoff beliefs (p_0, p_1) are continuous in experts' cutoff, $U(c)$ and $T(c)$ are well-defined.

[17] As $\underline{s} \in \left(0, \frac{l}{L}\right]$, both upper bounds are non-negative.

signal space, Proposition 3 makes no assumption on the experts' signal structure. The most interesting implication of part (i) of the proposition arises when the experts' signal structure F is close to $F_{\underline{s}}$. Recall that in the single-expert benchmark in Section 3, Proposition 1 states that the expected payoffs of the agent and the expert are both approximately $\pi(1-\underline{s})$. On the other hand, Proposition 3 states that with access to infinitely many experts at an infinitesimal search cost, the agent's payoff is no more than $\max\left\{\pi\left(1-\frac{L}{l}\underline{s}\right), 0\right\}$, which is strictly below $\pi(1-\underline{s})$. Moreover, the limit of the experts' joint payoff is strictly below $\pi(1-\underline{s})$. Therefore, Proposition 3 implies that if the experts' signal structure is sufficiently close to $F_{\underline{s}}$, the single-expert benchmark Pareto dominates the case of allowing the agent to access infinitely many experts at a negligible search cost. Part (ii) of Proposition 3 states that no information is transmitted if $\underline{s} > \frac{l}{L}$ and the search cost is sufficiently small. The condition $\pi < \frac{l}{l+\underline{s}}$ implies that the only uninformative equilibria are those in which the agent does not take the operation, and all players get a zero payoff.

In Section 5.1, we identify the key properties of the informative equilibria in the limit, which allow us to derive the upper bounds on the equilibrium payoffs in part (i) of Proposition 3.[18] In Section 5.2, we explain why informative equilibria do not exist in the limit when $\underline{s} > \frac{l}{L}$. In Section 5.3, we discuss some further welfare implications of our findings.

5.1 Limiting Properties of Informative Equilibria

In an informative equilibrium with negligible search cost, the quality of information transmission can be severely impeded by the loser's curse. The intuition is as follows. If an agent with negligible search cost shows up for a consultation, it is very likely that he has already consulted a large number of other experts. Moreover, in the event that the consultation is pivotal, he is just looking for a final proof that the operation is appropriate, as previous evidence collected from other experts supports the need for the operation. This in turn implies that the pivotal expert would be inclined to easily recommend the operation. As a result, the experts' recommendation rule is highly uninformative, making it hard for the agent to collect useful information from the consultations.

Lemma 2 below states that as the search cost vanishes, the experts' recommendation becomes almost completely uninformative. The lack of information content in their advice in turn limits the agent's ability to learn the state effectively, and he stops his search before he is certain of the state.

[18] The reason why it suffices to focus on informative equilibria is that the condition $\pi < \frac{l}{l+\underline{s}}$ ensures that in any uninformative equilibrium, the agent does not take the operation, so all players receive a zero payoff.

Lemma 2 *Denote by $s^*(c_n)$ the experts' cutoff in an informative equilibrium of the game with search cost c_n.*[19] *Suppose $\pi < \frac{l}{l+\underline{s}}$. For every sequence of informative equilibria,*

(i) $\lim_{n\to\infty} s^*(c_n) = \underline{s}$*;*

(ii) $\lim_{n\to\infty} p_1(s^*(c_n)) = \frac{l}{l+\underline{s}}$*; and*

(iii) $\liminf_{n\to\infty} p_0(s^*(c_n)) > 0$ *if* $\underline{s} > 0$.

The proof of Lemma 2 can be found in Appendix A.3. The intuition for part (i) is as follows. Suppose $\lim_{n\to\infty} s^*(c_n) > \underline{s}$, then there is a positive measure of unfavorable signals over which the experts recommend N even as the search cost vanishes. As the search cost is very low and the experts' advice is strictly informative, the agent would sample advice until he is almost sure that $\omega = 1$ before taking the operation. This in turn implies that regardless of the signal s, as long as the agent takes the operation following her recommendation Y, the expert's belief that the agent has $\omega = 1$ is very close to one. This contradicts that $\lim_{n\to\infty} s^*(c_n) > \underline{s}$.

Part (ii) and (iii) of Lemma 2 concerns the agent's equilibrium search strategy in the limit. The intuition for part (ii) is as follows. Suppose for simplicity that $\pi < p_1(s^*(c_n))$ for all n. Denote by q_n the pivotal expert's belief prior to observing her private signal if the search cost is c_n. By part (i), at a small search cost, she is willing to recommend the operation even if the private signal is very close to $\underline{s}$, i.e., the posterior $\left(1 + \frac{1}{\underline{s}}\frac{1-q_n}{q_n}\right)^{-1}$ must be (smaller but) very close to $\frac{l}{1+l}$. This implies that q_n must be very close to $\frac{l}{l+\underline{s}}$. Moreover, as the expert's recommendation Y is almost completely uninformative, the pivotal consultation only increases the agent's belief marginally; $\tilde{p}_1(s^*(c_n))$ and $p_1(s^*(c_n))$ are very close (recall equation (5)). As $q_n \in [\tilde{p}_1(s^*(c_n)), p_1(s^*(c_n))]$, $p_1(s^*(c_n))$ must converge to $\frac{l}{l+\underline{s}}$.

Part (iii) of the lemma states that there is some sufficiently low belief that when the search cost gets small enough, the agent would have no incentives to search for advice. The reason is that the quality of information that can be obtained by consulting experts is so low that it does not justify the overall search cost involved. The key condition for this finding is $\underline{s} > 0$. Specifically, as experts adopt a cutoff very close to $\underline{s}$, a recommendation Y is almost completely uninformative, whereas a recommendation N implies that the expert has observed a signal close to $\underline{s}$. The higher the value of $\underline{s}$, the less a recommendation N is indicative of $\omega = 0$, so the less the information the agent can infer from the experts' advice.[20]

An implication of Lemma 2 is that if $\underline{s} > 0$, the agent is unable to learn the state perfectly even if the search cost is negligibly small. Specifically, part (ii) and (iii) imply that $\lim_{n\to\infty} p_1(s^*(c_n)) < 1$ and

[19] If the game has more than one informative equilibrium, the function $s^*(c_n)$ selects an arbitrary equilibrium cutoff.

[20] In Proposition 4, we show that if $\underline{s} = 0$, then $\lim_{n\to\infty} p_0(s^*(c_n)) = 0$.

$\liminf_{n\to\infty} p_0\left(s^*\left(c_n\right)\right) > 0$, so he makes the wrong operation decision with a positive probability. The issue of information aggregation with sequential search is revisited in Section 6, where we show that the state can be perfectly learned if $\underline{s} = 0$ and the search cost is negligible.

We can use Lemma 2 to derive the upper bounds for the limit equilibrium payoffs reported in part (i) of Proposition 3. For simplicity, suppose $p_0\left(s^*\left(c_n\right)\right) < \pi < p_1\left(s^*\left(c_n\right)\right)$ for all c_n. Recall that when the agent's belief falls below $p_0\left(s^*\left(c_n\right)\right)$, it is optimal to quit the search without the operation. Conversely, when his belief reaches $p_1\left(s^*\left(c_n\right)\right)$, it is optimal for him to take the operation, which gives him an expected payoff $-L + (1+L)\, p_1\left(s^*\left(c_n\right)\right)$. As his updated belief in the search process is a martingale, the probability that his search ends with belief $p_1\left(s^*\left(c_n\right)\right)$ is approximately $\frac{\pi - p_0(s^*(c_n))}{p_1(s^*(c_n)) - p_0(s^*(c_n))}$. Therefore, his ex-ante payoff $U\left(c_n\right)$ is approximately the product of the two terms above. As the search cost vanishes, the approximation becomes exact. Using Lemma 2, the limiting value of $U\left(c_n\right)$ is $\frac{\pi - \underline{p}_0}{1 - \left(\frac{l+\underline{s}}{l}\right)\underline{p}_0}\left(1 - \frac{L}{l}\underline{s}\right)$, where $\underline{p}_0 \equiv \liminf_{n\to\infty} p_0\left(s^*\left(c_n\right)\right)$. As $\underline{p}_0 > 0$ by part (iii) of Lemma 2, $U\left(c_n\right)$ is strictly smaller than $\pi\left(1 - \frac{L}{l}\underline{s}\right)$ in the limit. The upper bound on the experts' joint payoff can be calculated analogously. Conditional on performing the operation, an expert's expected payoff is approximately $-l + (1+l)\Pr\left(\omega = 1 | q = p_1\left(s^*\left(c_n\right)\right), s \geq s^*\left(c_n\right)\right)$. Following a similar computation as above, the ex-ante joint payoff of the experts has a limiting value that is strictly smaller than $\pi\left(1 - \underline{s}\right)$.

5.2 Nonexistence of Informative Equilibria when $\underline{s} > \frac{l}{L}$

The case of $\underline{s} > \frac{l}{L}$ has an extreme equilibrium outcome when the search cost is small: no information is transmitted and the agent never takes the operation. We explain below the intuition for why an informative equilibrium does not exist in this case. Recall that the agent is willing to take the operation only if his posterior belief is no less than $\frac{L}{1+L}$, so $p_1\left(s^*\right) \geq \frac{L}{1+L}$. Consequently, in the pivotal consultation, the agent's belief prior to getting the recommendation is close to, or larger than, $\frac{L}{1+L}$. If L is large relative to l, the fact that the consultation is pivotal would make the expert so confident that she is willing to recommend the operation even if she observes the lowest signal $\underline{s}$. This, however, contradicts the requirement that information is transmitted through recommendation in an informative equilibrium.

More specifically, part (i) of Lemma 2 implies that in an informative equilibrium with a negligible search cost, a recommendation Y in the pivotal consultation only increases the agent's belief marginally. The pivotal expert's belief prior to observing her signal is thus close to $p_1\left(s^*\right)$, which is no less than $\frac{L}{1+L}$. Consequently, her belief $\Pr\left(\omega = 1 | \underline{s}, piv\left(b\right)\right)$ conditional on being pivotal and observing signal the worst possible signal $\underline{s}$ is no less than $\left(1 + \frac{1}{\underline{s}}\frac{1-\frac{L}{1+L}}{\frac{L}{1+L}}\right)^{-1}$, so her expected payoff of recommending the operation exceeds $\frac{L}{L\underline{s}+1}\left(\underline{s}-\frac{l}{L}\right)>0$. Therefore, she is willing to completely ignore her signal and always recommend the operation.

It is useful to note that the condition $\underline{s}>\frac{l}{L}$ does not preclude information transmission in the single-expert benchmark, as illustrated by the following example.

Example 1 *Let the expert's signal space be* $[\underline{s},\infty]$ *and her signal structure be* $f(s|0)=\frac{\pi(1-\pi+\pi\underline{s})}{(1-\pi+\pi s)^3}$ *and* $f(s|1)=sf(s|0)$. *Suppose further that* $\underline{s}>\frac{l}{L}$, $\pi\in\left(0,\frac{l}{l+\underline{s}}\right)$, *and* $\frac{1}{2}\left(1+\frac{l}{l+1}\right)>\frac{L}{1+L}$ *hold.*[21] *The signal structure specified induces a distribution of the expert's posterior belief (after observing the signal) that is uniform between* $\left(1+\frac{1}{\underline{s}}\frac{1-\pi}{\pi}\right)^{-1}$ *and 1.*

Given the expert's cutoff rule in (1), the agent's belief conditional on the expert's recommendation Y *is* $\frac{1}{2}\left(1+\frac{l}{l+1}\right)$. *The assumption* $\frac{1}{2}\left(1+\frac{l}{l+1}\right)>\frac{L}{1+L}$ *therefore ensures that inequality (2) holds, and an informative equilibrium exists in the single-expert benchmark. It is immediate that the ex-ante expected payoffs of both the agent and the expert are positive in the informative equilibrium.*

The key difference between the single-expert benchmark and the case of multiple experts lies in the private information the agent holds prior to a consultation. In the single-expert benchmark, the agent has no private information, so the expert relies only on her signal in making a recommendation. In contrast, with access to multiple experts, the agent is expected to have private information about recommendations he previously received. If these recommendations are informative about the state, a consulted expert can blindly recommend the operation, knowing that the agent would agree only if his private information is very favorable, especially when he is sufficiently conservative (i.e., L is sufficiently large). This consideration, however, destroys any information content in an expert's recommendation.

5.3 Welfare Implications

Proposition 3 implies that a market for advice may be inherently inefficient. Suppose we augment our model with a preceding stage in which experts compete by choosing their publicly-posted consultation fees. The consultation fee would enter into our model as a markup on the agent's search cost. Moreover, as the fee is independent of the recommendation and operation outcome, it would not affect the experts' incentives in choosing their cutoffs. Then by a standard argument for Bertrand competition, the consultation fees of all experts would be driven to zero, as the signals are assumed to be costless to the experts. If the agent's intrinsic search cost is very small, Proposition 3 implies that the equilibrium outcome could be inefficient. In the extreme case, the market completely breaks down if $\underline{s}\in\left(\frac{l}{L},1\right)$.

[21] For example, if $l=1$, $L=2$, and $\pi=0.5$, then any $\underline{s}\in(0.5,1)$ would ensure all conditions above are satisfied.

Suppose a social planner, whose objective is maximizing social welfare, can decide a consultation fee that all experts must charge. Proposition 3 shows that the efficient level of consultation fee can be strictly positive.[22] Similarly, suppose a trade organization (say, that of lawyers and physicians) can decide the consultation fee of its members. Our analysis implies that if the trade organization sets a higher consultation fee, it is possible that not only would its members benefit, but also the agent seeking their advice and service. Computing the optimal or socially efficient consultation fee is a challenging problem that is left for future research; our analysis highlights that such computation must take into account the effect of loser's curse in information transmission.

We conclude this section by discussing the relation between Proposition 3 and the finding of Bulow and Klemperer (2003) that the auctioneer's expected revenue may decrease with the number of bidders in a common-value auction. The driving forces are respectively the experts' loser's curse and the bidders' winner's curse. A notable difference in welfare implications is that whereas the winner's (loser's) curse unambiguously hurts the auctioneer (agent), their respective effects on the bidders and experts are in the opposite direction. Specifically, by taking the winner's curse into account, a bidder would lower her bid, thus imposing a positive externality on other bidders. The bidders' joint payoff increases due to the winner's curse. On the other hand, an expert taking the loser's curse into account imposes a negative externality on other experts. The reason is that when facing the loser's curse, each expert lowers her recommendation cutoff, making her advice less informative (in expectation). This makes it difficult for the agent to gather precise information about his case, which in turn harms the experts, as the chance of reaching an ex-post incorrect operation decision increases. Consequently, the loser's curse may cause the experts' joint payoff, as well as the social welfare, to go down.

6 Information Aggregation

This section investigates the effectiveness of sequential search in collecting information dispersedly held by experts. Given any common cutoff $s^* \in (\underline{s}, \bar{s})$ adopted by experts, if the agent has free access to all recommendations, he can learn the true state, as these recommendations are conditionally independent and identically distributed. Strategic behavior by experts makes the agent's problem less trivial, because as shown in Lemma 2, the experts' advice becomes completely uninformative as the search cost vanishes.

If information is almost perfectly aggregated, the agent takes the ex-post correct operation decision with probability close to one. That is, $p_0(s^*)$ and $p_1(s^*)$ are close to 0 and 1 respectively. Lemma 2

[22] Wolinsky (2005) has a related finding in a moral hazard setting.

implies that if $\underline{s} > 0$, the limiting values of $p_0(s^*)$ and $p_1(s^*)$ are strictly bounded away from 0 and 1. These results imply that $\underline{s} = 0$, i.e., the existence of a signal that perfectly reveals the state is $\omega = 0$, is a necessary condition for perfect information aggregation in the limit. The proposition below shows that this condition is also sufficient.[23]

Proposition 4 *If $\pi > \frac{L}{1+L}$, then perfect information aggregation arises as a limit equilibrium outcome if and only if $\underline{s} = 0$.*

The proof of Proposition 4 can be found in Appendix A.5. The condition $\pi > \frac{L}{1+L}$ ensures the existence of an informative equilibrium (recall Proposition 2). In the proof, we compute the limiting payoff of the following simple and necessarily suboptimal search strategy: sample a fixed number of experts and have the operation in the end if and only if all of them recommend Y. It is shown that if $\underline{s} = 0$, by choosing the fixed number of experts appropriately, the agent can attain an ex-ante payoff arbitrarily close to the highest possible level, which equals π. This means that in the limit, the agent necessarily learns the true state with a negligible total search cost. Intuitively, as each expert adopts a cutoff arbitrarily close to $\underline{s} = 0$, it is highly likely that the agent will receive a recommendation N if and only if the state ω is 0. Consequently, the strategy under consideration is highly effective in learning the state.

6.1 Relation to Milgrom (1979)

The condition in Proposition 4 for perfect information aggregation requires the existence of a signal that perfectly reveals $\omega = 0$. This echoes the condition for perfect information aggregation identified by Milgrom (1979) in the context of a sealed-bid first-price auction for an object of common value. To illustrate Milgrom's finding, suppose the common value V of the object is either 1 or 0. Moreover, each bidder receives an identically and conditionally independently distributed private signal concerning V, and for simplicity, the signal space is closed. Milgrom finds that the winning bid perfectly reflects the true value of V in the limit (as the number of bidders goes to infinity) if and only if there is a signal which perfectly reveals $V = 1$.

Though the condition for perfect information aggregation identified in Proposition 4 is similar to that in Milgrom (1979), the driving force is quite different. In a common-value auction, competition among bidders plays a key role in shaping the information content of the winning bid. Specifically,

[23] As part (ii) of Lemma 2 implies that the limiting value of $p_1(s^*)$ is 1 when $\underline{s} = 0$, it suffices to show that the limiting value of $p_0(s^*)$ is 0.

suppose Milgrom's condition holds, then in the event that $V = 1$, many bidders in a large auction would observe signals highly indicative of a high object value. Expecting intense competition from others who observe similar signals, each of them will bid very close to 1. This explains the sufficiency of Milgrom's condition. Its necessity is straightforward: no bidder submits a bid exceeding the object's conditional expected value given his own signal, so the winning bid would always be strictly below 1 when Milgrom's condition fails.[24] It is clear that Milgrom's result is unrelated to the winner's curse in bid formation.

On the other hand, Proposition 4 is driven by the loser's curse and the agent's optimal search behavior. If $\underline{s} > 0$, the agent cannot have $p_1 = 1$ in equilibrium. Otherwise, the experts always recommend the operation, as being pivotal implies $\omega = 1$ with almost certainty. This explains the necessity in Proposition 4. In contrast, if $\underline{s} = 0$, then even if the agent sets p_1 close to 1, an expert who observes a signal extremely close to 0 would still be willing to recommend N. Moreover, when $\omega = 0$, many experts would observe such low signals. Thus, by sampling a large number of experts, the agent must get some N recommendations and learn that $\omega = 0$. This explains the sufficiency in Proposition 4. It is clear that Proposition 4 is unrelated to competition among experts. In fact, when making their recommendations, the experts do not take into account the probability of serving the agent.

It is also interesting to contrast the equilibrium outcome of our setting to that of common-value auctions when $\underline{s} > 0$ (and correspondingly, when Milgrom's condition fails). Kremer (2002) shows that if Milgrom's condition fails, the limit winning bid is $E[V]$ and hence uncorrelated with the true value of V.[25] On the other hand, in our search setting, even if $\underline{s} > 0$, the agent's eventual operation decision could still partially reflect the dispersed information held by the experts. In particular, with $\pi \in \left(\frac{L}{1+L}, \frac{l}{l+\underline{s}}\right)$, Proposition 2 guarantees that an informative equilibrium exists.[26]

6.2 Relation to Feddersen and Pesendorfer (1997)

In this subsection, we explain that the welfare loss and the failure of information aggregation when $\underline{s} > 0$ can be interpreted as a commitment problem of the agent. Suppose the agent commits to the following

[24] Section 2 of Milgrom (1979) discusses the intuition of his finding.

[25] This result is again due to the intense competition among bidders, which drives the winning bid towards the object's expected value conditional on the winning bidder's information. If the most favorable signal does not preclude the possibility that $V = 0$, then the event that someone, among a large number of bidders, observing this most favorable signal is almost completely uninformative.

[26] As $\lim_{n\to\infty} p_1(s^*(c_n)) = \frac{l}{l+\underline{s}}$ and $\lim_{n\to\infty} p_0(s^*(c_n)) < \frac{L}{1+L}$, the agent's eventual operation decision is positively correlated with the state in the limit.

decision rule. Fixing the threshold ratio at $q \in (0,1)$ and the number of experts consulted at n, the agent undergoes the operation (with a randomly chosen expert who recommends him to do so) whenever more than qn experts recommend Y. Thus, he is effectively adopting a voting mechanism. Feddersen and Pesendorfer (1997) study a similar setup of two-candidate election and show that information dispersedly held by voters is perfectly aggregated as the number of voters go to infinity. A similar result holds in our setting if the agent could commit to a voting mechanism.

Claim 1 *Suppose the agent commits to the voting mechanism above (with any threshold ratio). An informative equilibrium exists if the number of experts consulted is sufficiently large. Moreover, as the number of experts approaches infinity, the probability that the agent arrives at the ex-post correct operation decision converges to one.*

The proof of Claim 1 can be found in Appendix A.6. Note that Claim 1 does not make any assumption on the signal space. Therefore, comparing Claim 1 with Proposition 4, we find that committing to a voting mechanism facilitates information aggregation. The intuition is as follows. In the voting mechanism, although experts are strategic and decide their recommendations conditional on being pivotal, the loser's curse is absent because being pivotal is no longer good news. In fact, in the limit with infinitely many experts, being pivotal is completely neutral. Consequently, the experts are willing to adopt a strictly informative recommendation rule in the limit.

The discussion above illustrates that the extent to which information is aggregated, and thus the agent and social welfare, could be improved if the agent engages in non-sequential search by committing in advance to a fixed sample of experts and a specific decision rule. It also highlights that the source of inefficiency in our model is the agent's sequentially optimal search behavior, which generates the experts' loser's curse.

7 Discussion

Uninformative equilibria Recall that in an uninformative equilibrium, the operation decision is independent of the state ω, and the agent does not search beyond the first (free) expert. More specifically, an uninformative equilibrium in which the operation is carried out with certainty exists if and only if $\pi \geq \max\left\{\frac{L}{1+L}, \frac{l}{l+\underline{s}}\right\}$: this condition ensures a nonnegative payoff to the first consulted expert for performing the operation for all signals, and to the agent for accepting the recommendation. On the other hand, an uninformative equilibrium in which the operation is never carried out can arise because of a "coordination failure". For example, the experts recommend Y if and only if s is very close to $\underline{s}$, and the agent always rejects their recommendation.[27] However, the latter class of uninformative equilibria brings a zero payoff to all players, and is thus Pareto dominated by an informative equilibrium whenever the latter exists.

Conservative experts Throughout our analysis, we have focused on the case $l < L$, i.e., experts suffering less from a failed operation than the agent. While we believe that this is the relevant case in most applications, it is natural to ask what would happen if $l \geq L$. First, there is always an equilibrium, but an informative equilibrium is guaranteed to exist if and only if $\pi \in \left(\max\left\{\frac{l}{l+\bar{s}}, \frac{L}{1+L}\right\}, \frac{l}{l+\underline{s}}\right)$. Second, part (i) of Proposition 3 still holds.[28] While we can still guarantee the existence of a sequence of informative equilibria with $\lim_{n\to\infty} s^*(c_n) = \underline{s}$ provided that $\pi \in \left(\max\left\{\frac{l}{l+\bar{s}}, \frac{L}{1+L}\right\}, \frac{l}{l+\underline{s}}\right)$, we cannot rule out the possibility that there could be a sequence of informative equilibria with $\lim_{n\to\infty} s^*(c_n) = \bar{s}$. However, even if the latter sequence exists, it can be shown that $\limsup_{n\to\infty} p_1(s^*(c_n)) \leq \frac{l}{1+l} < \frac{l}{l+\underline{s}}$. Following the argument in Section 5, the agent's limit payoff (in such a sequence) is bounded from above by $\pi\left(1 - \frac{L}{l}\right)$, which is less than $\pi\left(1 - \frac{L}{l}\underline{s}\right)$, the bound in Proposition 3. A similar conclusion holds for the experts' joint payoff. Finally, Proposition 4 holds with $\pi > \max\left\{\frac{l}{l+\bar{s}}, \frac{L}{1+L}\right\}$, as its proof requires only the existence of a sequence of informative equilibria with $s^*(c_n) \to \underline{s}$. The formal derivations for the results discussed above can be found in Appendix A.7.

Observability of agent's history The key ingredient of the loser's curse is that each expert believes there is a positive probability that she is pivotal in the agent's final decision. In our model, this is achieved by assuming that the experts do not know, and cannot learn, the history of the agent. In applications such as a patient seeking doctors' advice and a client looking for lawyers' advice, it is quite natural that the experts do not have much knowledge about the advice that the agent received in his previous consultations. Any modification of our model that removes the positive probability of being pivotal would also eliminate the loser's curse effect. For instance, if the experts can observe the agent's history of received recommendations fully or partially (e.g., his time on the market), then experts consulted in the early stage of the search process are certain that they are not pivotal, so adopting any recommendation rule is optimal (including one that is most informative to the agent).

A concern about non-pivotal experts is that they do not have strict incentives to perform informative diagnoses. For instance, in a more realistic setting, experts need to exert unobservable and costly effort in running tests and diagnosis in order to obtain the signal. In this case, an expert is willing to exert

[27] This strategy profile constitutes an equilibrium if π is smaller than $\frac{L}{L+\underline{s}}$.

[28] Part (ii) of Proposition 3 becomes irrelevant as $\underline{s} < 1 \leq \frac{l}{L}$.

effort only if he believes that there is a sufficiently high probability that he is pivotal. Therefore, even if the agent's history is partially observable, the loser's curse consideration must still be present in an informative equilibrium.[29] Moreover, if the non-performance of non-pivotal experts is a significant concern, the agent has incentives to hide his history.

Search with recall In the main model, we assume the agent undergoes the operation, if he ever chooses to, with the last expert he visits. This assumption can be easily justified if there is a cost of returning to a previously visited expert, as all experts are identical. An alternative assumption is to allow costless recall. For instance, when the agent decides to have the operation, he would randomly choose, with equal probability, any expert that recommended him to do so. Suppose, for simplicity, the experts could not revise their recommendation upon recall. All of our results remains valid in this alternative setting with recall. The intuition is that the loser's curse is still present and it affects the experts' incentives in the same way as our main model without recall. More specifically, note first that the availability of the recall option would not affect the agent's optimal stopping decisions (which depend only on the experts' cutoff). Moreover, with recall, the agent would come back to an expert only if he receives recommendations from other experts that are so supportive that, together with her own positive recommendation, are just sufficient to persuade the agent to undergo the operation. This is exactly the scenario captured by the pivotal event $piv(b)$ in the analysis of our no-recall model. Therefore, equation (3) and (4) continue to characterize the experts' best responses in an informative equilibrium. Appendix A.8 contains a more formal derivation for this observation.[30]

A consequence of allowing recall is that every consulted expert is pivotal with a positive probability. Therefore, with recall, even if the agent's history is observable to the experts, the loser's curse is still present. In such a setting, an expert's cutoff would, in general, depend on the agent's history. This is because the pivotal event that the agent comes back to her for an operation, denoted by $piv(b, h)$, is dependent on the agent's current history h. The expert would still be inclined towards recommending the operation, provided that the likelihood ratio of $piv(b, h)$ is less than 1, a condition that is likely to hold if the agent takes the operation only if he has received very supportive evidence from the experts.[31]

[29] See Pesendorfer and Wolinsky (2003) for a related model that highlights the inefficiency resulting from the experts' moral hazard problem in diagnosis.

[30] Another possibility is to assume that the experts can revise their recommendation upon being recalled. In this case, there is an equilibrium in which the experts always stick with their initial recommendations, and hence no recall takes place. The analysis of equilbria with recall and recommendation revision is complicated as the agent's search strategy would no longer be Markov, and an expert's strategy would depend on the number of previous consultations the agent had with her. We leave this analysis for future research.

[31] As there is a lot more freedom in specifying experts' off-path belief in this setting, equilibria with flavor distinct from

Communication of agent's history In the main model, we assume the agent cannot communicate his history to the experts. One justification for this assumption is that if the message about this history is cheap talk, there is always an equilibrium in which such communication is babbling. Moreover, we argue below that the endogenous conflict of interest induced by the loser's curse makes information transmission via cheap talk very difficult, if not impossible. As shown in Section 5, if the search cost is small, the loser's curse leads each expert to condition her payoff calculation on the goods news of being pivotal, and it makes their recommendation extremely uninformative. To combat the loser's curse, the agent would like to induce a pessimistic belief by the expert. More specifically, if the equilibrium is such that non-pivotal experts are willing to adopt a more informative recommendation strategy than that of pivotal experts, then the agent would claim that the current consultation is non-pivotal in his decision. If the equilibrium is such that only pivotal experts are willing to adopt an informative recommendation strategy,[32] then the agent would always send a message that induces a belief closest to $\tilde{p}_1(\cdot)$.

Transfers Our main model has abstracted away from pricing and liability. We have explored one extension of introducing a consultation fee in Section 5.3. Below we discuss other possible ways to introduce transfers and their impacts on equilibrium outcomes.

Suppose only an outcome-independent operation fee can be charged. In a sequential search setting in which the agent can observe the operation fee only after visiting an expert, each of them would charge a monopoly fee that fully extracts the agent's surplus. Consequently, there is no search in equilibrium, as in the Diamond paradox. Alternatively, one can consider a directed search setting in which experts' operation fees are publicly posted before the agent begins his search. Then Bertrand competition would force the fees to zero, provided that the agent's search cost is sufficiently small. Loosely speaking, if the search cost is low enough, part (i) of Lemma 2 shows that the experts' cutoff is too low for effective information collection. Charging a positive operation fee would only push this cutoff even lower, thus unambiguously making visiting such an expert less desirable than those charging a zero operation fee.

A natural question to consider, once we allow for transfers between the agent and expert, is whether the expert can signal her observed signal s through offering different contracts. For signaling through transfers to work, it is necessary that the contract can be made contingent on the operation outcome. Consider a fee-and-compensation contract (ϕ, ψ) under which the expert collects a fee ϕ for carrying

those we analyzed would emerge. Nonetheless, it is conjectured that the informative equilibrium (or its perturbation) we characterize would continue to exist for a sufficiently small search cost, as $\Pr(\omega = 1 | s, piv(b,h), s^*, b)$ is almost history independent in the limit.

[32] This would be the case if experts need to exert costly diagnostic effort for the signal. Alternatively, as pointed out above, if the agent can recall previously consulted expert, then every expert is pivotal with a positive probability.

out the operation and pays a compensation ψ if the operation fails. The expert can potentially signal a high s by offering a contract with high values of ϕ and ψ. However, in a sequential search setting in which the agent observes the offered contract only after visiting an expert, the expert's optimal contract does not involve signaling at all. Specifically, by offering a contract $(\phi, \psi) = (1, 1 + L)$ regardless of the observed signal, the consulted expert effectively "buys" the problem from the agent. Consequently, the agent is left with zero surplus and does not search beyond the first (free) expert.

The analysis becomes more challenging in a directed search setting in which the experts publicly post and commit to a menu of fee-and-compensation contracts before the agent begins his search. In this case, experts are competing not only in prices but also in the implied information service (i.e., recommendation rule). An equilibrium may involve each expert posting a menu of contracts, and signaling his observed s through the chosen contract. We leave the analysis for future research.

Communication of experts' signal In the main model, we impose a restrictive binary message space for the experts. One may consider a more flexible message space, for example, a cheap-talk message could be sent along with the recommendation. Whereas the equilibrium we consider and characterize still exists, there are also equilibria in which more information is transmitted. A simple example is for experts who do not recommend the operation to fully reveal her signal through her message: truthful revelation of signal is incentive compatible as her payoff is constant at zero. However, as Lemma 2 shows, the experts almost always recommend the operation as the search cost vanishes. This use of the cheap-talk message therefore has little impact on our limit results.

A possible way for the agent to solicit more informative advice from an expert is to commit not to undergoing the operation with her. If such commitment is possible, then the expert is willing to truthfully report her signal. Such commitment, however, may be difficult in some real-world scenarios, such as a patient consulting a doctor and a client consulting a lawyer. A related avenue for improving information aggregation and hence market efficiency is for some experts to commit to providing advising service only.[33] As these experts never carry out the operation, truthful reporting is incentive compatible, though they may suffer from the moral hazard problem of providing diagnostic effort discussed above. This extension is a potential avenue for future research.

Heterogeneity of experts In the main model, all experts are assumed to be ex-ante identical, so it is reasonable for the agent to sample experts in a random order. Suppose, instead, there are k prominent experts whose signal structure $F'(s|\omega)$ are more informative than others, and that these prominent experts are consulted first (but the search among the k prominent experts remains random).

[33] A related idea has been studied in Wolinsky (1993) in the context of credence goods provision.

Suppose also that k is large and/or F' is informative so each prominent expert is pivotal with positive probability. It is clear that the prominent and non-prominent experts would adopt different cutoffs. Moreover, the agent's cutoff pair $(p_{0,t}, p_{1,t})$ would now depend on the number of unconsulted prominent experts $t \leq k$ (with $(p_{0,0}, p_{1,0})$ meaning that all prominent experts have been consulted). Suppose that in equilibrium, the prominent experts' advice is indeed more informative (so that it is reasonable to seek their advice first). Then $p_{0,t}$ is increasing in t, and $p_{1,t}$ is decreasing in t. It is thus possible that the prominent experts would suffer from a more severe loser's curse than nonprominent experts, as $p_{1,t} > p_{1,0}$ for all $t > 1$. If the search cost is sufficiently small, they are thus likely to adopt a cutoff lower than that of non-prominent experts, partially cancelling out the improvement in the quality of information the agent gathers.[34] Finally, the limit result for vanishing search cost would remain unchanged, as the agent almost surely searches beyond the prominent experts.

Alternative expert preference In the model, we assume that an expert gets a non-zero payoff if and only if the agent undergoes the operation with her. Alternatively, each expert may, due to altruism or reputational concern, care about the agent's eventual payoff, provided that she has recommended the operation to him. Suppose each expert's payoff is as follows:

	$\omega = 1$	$\omega = 0$
Recommend Y and the agent takes the operation eventually (may not be with her)	1	$-l$
Recommend N, or the agent does not take the operation in the end	0	0

With such payoffs, the experts face essentially the same incentive structure as in the case of search with recall discussed above. Therefore, all of our results remain valid, except for those concerning the social welfare, as we have changed the experts' payoff.

Appendix

Appendix A.1: Characterizing $p_0(\cdot)$ and $p_1(\cdot)$

This appendix provides the details of characterizing $p_0(\cdot)$ and $p_1(\cdot)$ using the agent's Bellman equation. Fix the cutoff strategy of all experts at $\hat{s} \in [\underline{s}, \bar{s}]$. Suppose the agent approaches an expert with a prior belief p (that $\omega = 1$). If the expert recommends Y, then his posterior belief becomes $\left(1 + \frac{1-p}{p}\frac{1-F(\hat{s}|0)}{1-F(\hat{s}|1)}\right)^{-1}$. If the expert recommends N, then his posterior belief becomes $\left(1 + \frac{1-p}{p}\frac{F(\hat{s}|0)}{F(\hat{s}|1)}\right)^{-1}$. Denote by $V : [0, 1] \rightarrow$

[34]Note that the agent may still find it wothwhile to seek the advice of the prominent experts first because their signal structure is more informative.

$\mathbb{R}$ the agent's beginning-of-period continuation value, as a function of his current belief p, assuming that he decides to search this period. Using the observations above, $V(p)$ can be recursively defined by

$$\begin{aligned} V(p;\hat{s}) &= -c+\left[p\left(1-F(\hat{s}|1)\right)+(1-p)\left(1-F(\hat{s}|0)\right)\right] \\ &\quad \times \max\left\{0, V\left(\left(1+\frac{1-p}{p}\frac{1-F(\hat{s}|0)}{1-F(\hat{s}|1)}\right)^{-1};\hat{s}\right), -L+\left(1+\frac{1-p}{p}\frac{1-F(\hat{s}|0)}{1-F(\hat{s}|1)}\right)^{-1}(1+L)\right\} \\ &\quad +\left[pF(\hat{s}|1)+(1-p)F(\hat{s}|0)\right]\max\left\{0, V\left(\left(1+\frac{1-p}{p}\frac{F(\hat{s}|0)}{F(\hat{s}|1)}\right)^{-1};\hat{s}\right)\right\}. \end{aligned} \tag{8}$$

To understand equation (8), note that after paying a search cost c, the agent may receive a recommendation Y, at which point, he can either (i) leave the current expert without undergoing the operation, which gives him a payoff $\max\left\{0, V\left(\left(1+\frac{1-p}{p}\frac{1-F(\hat{s}|0)}{1-F(\hat{s}|1)}\right)^{-1};\hat{s}\right)\right\}$; or (ii) agree to have the operation with the current expert, which gives him a payoff $-L+\left(1+\frac{1-p}{p}\frac{1-F(\hat{s}|0)}{1-F(\hat{s}|1)}\right)^{-1}(1+L)$. If he receives a recommendation N, then he must leave the current expert without undergoing the operation.

Lemma 3 below establishes the existence of the value function V and identifies some of its properties.

Lemma 3 *There exists a unique function $V:[0,1]\to\mathbb{R}$ that satisfies equation (8). Moreover, V is nondecreasing and weakly convex.*

Proof. See Lemma 3.1 and Theorem 3.2 of Ross (1983). ■

Given the value function $V(p;\hat{s})$, the agent's optimal search strategy can be computed by solving for cutoffs $p_0(\hat{s})$ and $p_1(\hat{s})$. Specifically, if $V(1;\hat{s})>0$, then $p_0(\hat{s})$ is the unique solution to $V(p;\hat{s})=0$. If $V(1;\hat{s})\leq 0$, then $p_0(\hat{s})=1$. Also,

$$p_1(\hat{s})=\min\left\{p\in\left[\frac{L}{1+L},1\right]: V(p;\hat{s})\leq -L+p(1+L)\right\}. \tag{9}$$

The value $p_0(\hat{s})$ is well-defined. To see this, note that $V(\cdot;\hat{s})$ is nondecreasing and convex (thus continuous). As $V(0)=-c$, the equation $V(p;\hat{s})=0$ has a unique solution if $V(1;\hat{s})>0$. Moreover, as $\left\{p\in\left[\frac{L}{1+L},1\right]: V(p;\hat{s})\leq -L+p(1+L)\right\}$ is a non-empty compact interval, $p_1(\hat{s})$ is well-defined.

The following observations on $p_0(\hat{s})$ and $p_1(\hat{s})$ are useful for the subsequent analysis.

Lemma 4 $p_0(\hat{s})<\frac{L}{1+L}$ *if and only if* $p_1(\hat{s})>\frac{L}{1+L}$. *In this case,* $V(p_1(\hat{s});\hat{s})=-L+p_1(\hat{s})(1+L)$.

Proof. If $V(1;\hat{s})\leq 0$, then it is immediate from the definitions above that of $p_0(\hat{s})=1$ and $p_1(\hat{s})=\frac{L}{1+L}$. Suppose instead, $V(1;\hat{s})>0$. As $V(p_0(\hat{s});\hat{s})=0$ and $V(0;\hat{s})=-c$, the convexity of $V(\cdot;\hat{s})$ implies that it is strictly increasing at $p_0(\hat{s})$. Therefore, $p_0(\hat{s})<\frac{L}{1+L}$ if and only if $V\left(\frac{L}{1+L};\hat{s}\right)>0$. By equation

(9), $V\left(\frac{L}{1+L};\hat{s}\right)>0$ if and only if $p_1(\hat{s})>\frac{L}{1+L}$. In this case, $V(p_1(\hat{s});\hat{s})=-L+p_1(\hat{s})(1+L)$ holds, as $V(p;\hat{s})$ is weakly convex in p and $-L+p(1+L)$ is linear in p. ■

The case $p_0(\hat{s})>p_1(\hat{s})$ can potentially arise if the search cost is sufficiently high and/or the experts' recommendations are sufficiently uninformative. Suppose, in addition, $\pi<p_0(\hat{s})$, then the agent will not search beyond the first expert. On the other hand, if π is a lot larger than $p_0(\hat{s})$, then the agent may search beyond the first expert, and stop either when his posterior falls below $p_0(\hat{s})$, or when some expert recommends Y before his posterior falls below $p_0(\hat{s})$.

Appendix A.2: Proof of Proposition 2

The following lemma considers the existence of uninformative equilibria.

Lemma 5 *(i) There exists an uninformative equilibrium in which $s^*=\underline{s}$ and the agent follows the first expert's recommendation Y if and only if $\pi\geq\max\left\{\frac{L}{1+L},\frac{l}{l+\underline{s}}\right\}$.*

(ii) There exists an uninformative equilibrium in which the operation is never carried out if $\pi\leq\frac{L}{1+L}$.

Proof. (i) An uninformative equilibrium in which $s^*=\underline{s}$ and the agent always accepts the first recommendation Y requires $\pi\geq\max\left\{\frac{L}{1+L},\frac{l}{l+\underline{s}}\right\}$. The condition ensures a nonnegative payoff to the first expert for performing the operation for all signals, and to the agent for accepting the recommendation.

(ii) Suppose $\pi\leq\frac{L}{1+L}$, and consider the following strategy profile: the experts always recommend Y and the agent always rejects the recommendation. As the agent never takes the operation, the experts' payoff is constant at zero regardless of strategies. On the other hand, the experts' recommendation is uninformative, so it is optimal for the agent not to take the operation, as doing so would yield a non-positive expected payoff. ■

The rest of the proof focuses on the case $\pi\in\left(\frac{L}{1+L},\frac{l}{l+\underline{s}}\right)$, and we show the existence of an informative equilibrium for this range of π.

Denote by Λ the set of all behavioral strategies of the agent. Define $J:\mathbb{R}_+\times[\underline{s},\bar{s})\times\Lambda\to\mathbb{R}_+$ by

$$J(s,\hat{s},b)\equiv\begin{cases}\left(1+\frac{1}{s}\frac{1-\pi}{\pi}\frac{\sum_{h\in H}b_1(h,Y)q_0(h;\hat{s},b)}{\sum_{h\in H}b_1(h,Y)q_1(h;\hat{s},b)}\right)^{-1} & \text{if } \hat{s}\in[\underline{s},\bar{s})\\ \left(1+\frac{1}{s}\frac{1-\pi}{\pi}\right)^{-1} & \text{if } \hat{s}=\bar{s}\end{cases}. \tag{10}$$

By equation (4), for $\hat{s}>\underline{s}$, $J(s,\hat{s},b)$ is an individual expert's belief that $\omega=1$ conditional on signal s and being pivotal, assuming that all other experts adopt cutoff $\hat{s}$ and the agent uses strategy b. Denote by $\Psi(\hat{s})$ the set of the agent's best response to all experts adopting cutoff $\hat{s}$. We first show that as long as $b \in \Psi(\hat{s})$, the probability $J(s, \hat{s}, b)$ in (10) is well-defined.

Lemma 6 *$J(s, \hat{s}, b)$ is well-defined for each $b \in \Psi(\hat{s})$ and $\hat{s} \in [\underline{s}, \bar{s})$.*

Proof. Fix a $\hat{s} \in (\underline{s}, \bar{s}]$ and a $b \in \Psi(\hat{s})$. It suffices to show that the likelihood ratio of $piv(b)$ is well-defined. Below we show that (i) at least one of $\sum_{h \in H} b_1(h, Y) q_0(h; \hat{s}, b)$ or $\sum_{h \in H} b_1(h, Y) q_1(h; \hat{s}, b)$ is positive, and (ii) $\sum_{h \in H} b_1(h, Y) q_\omega(h; \hat{s}, b) < \infty$ for each $\omega \in \{0, 1\}$.

(i) Suppose both $\sum_{h \in H} b_1(h, Y) q_0(h; \hat{s}, b)$ and $\sum_{h \in H} b_1(h, Y) q_1(h; \hat{s}, b)$ are zero. Then the agent never takes the operation, despite the experts recommending it with a positive probability (as $\hat{s} < \bar{s}$). This implies that the agent does not search beyond the first expert. However, his rejection of the first expert's recommendation Y is then suboptimal as $\pi > \frac{L}{1+L}$.

(ii) Observe that $\sum_{h \in H} q_\omega(h; \hat{s}, b)$ is the expected number of experts that the agent consults, provided that the state is ω, the experts adopt cutoff $\hat{s}$, and the agent plays strategy b. As $b \in \Psi(\hat{s})$, it is necessary that the expected total search cost is less than 1, so $c\left[\pi \sum_{h \in H} q_1(h; \hat{s}, b) + (1-\pi) \sum_{h \in H} q_0(h; \hat{s}, b)\right] \leq 1$. This implies that $\sum_{h \in H} q_\omega(h; \hat{s}, b) \leq \frac{1}{c \min\{\pi, 1-\pi\}}$ for both $\omega \in \{0, 1\}$. Therefore, $\sum_{h \in H} b_1(h, Y) q_\omega(h; \hat{s}, b) \leq \frac{1}{c \min\{\pi, 1-\pi\}}$, as $b_1(h, Y) \leq 1$ for all $h \in H$. ■

Define $x : [\underline{s}, \bar{s}) \times \Lambda \to \mathbb{R}_+$ as the unique solution to the equation $J(\cdot, \hat{s}, b) = \frac{l}{1+l}$. If $x(\hat{s}, b) \in [\underline{s}, \bar{s}]$, then an individual expert finds it optimal to adopt cutoff $x(\hat{s}, b)$, given all other experts' adopting cutoff $\hat{s}$ and the agent playing strategy b. If $x(\hat{s}, b) > \bar{s}$, then an individual expert's best response is to always recommend N, i.e., adopting cutoff $\bar{s}$. Likewise, if $x(\hat{s}, b) < \underline{s}$, then an individual expert's best response is to always recommend Y, i.e., adopting cutoff $\underline{s}$. Using these observations, we define an individual expert's best-response correspondence $Z : [\underline{s}, \bar{s}] \rightrightarrows [\underline{s}, \bar{s}]$ by

$$Z(\hat{s}) \equiv \begin{cases} \{\max\{\underline{s}, \min\{\bar{s}, x(\hat{s}, b)\}\} : b \in \Psi(\hat{s})\} & \text{if } \hat{s} \in [\underline{s}, \bar{s}) \\ \frac{1-\pi}{\pi} l & \text{if } \hat{s} = \bar{s} \end{cases}.$$

The correspondence Z is the set of best responses of an individual expert, given all other experts' adopting cutoff $\hat{s}$, and the agent playing a best response to $\hat{s}$. Note that the definition $Z(\bar{s}) \equiv \frac{1-\pi}{\pi} l$ ensures that Z has a closed graph at $\bar{s}$. To see this, note that if $\hat{s}$ is sufficiently close to $\bar{s}$, then the agent would find it suboptimal to search beyond the first expert, so $b_0(h) = 1$ for all $h \neq \varnothing$. As $\pi > \frac{L}{1+L}$, we have $b_1(Y) = 1$ for all $b \in \Psi(\hat{s})$. Therefore, $J(s, \hat{s}, b) = \left(1 + \frac{1}{s} \frac{1-\pi}{\pi}\right)^{-1}$, giving $x(\hat{s}, b) = \frac{1-\pi}{\pi} l$.

By the definition of the informative equilibrium in Section 4, we are done if we can show that Z has a fixed point $s^* \in (\underline{s}, \bar{s})$. Note that any fixed point of Z (if any) would not occur at the boundary points. First, as $\pi > \frac{L}{1+L} > \frac{l}{l+\bar{s}}$, $Z(\bar{s}) = \frac{1-\pi}{\pi} l < \bar{s}$. Moreover, any $b \in \Psi(\underline{s})$ must satisfy $b_1(Y) = 1$ (i.e., taking the operation at the first expert). Consequently, $J(s, \underline{s}, b) = \left(1 + \frac{1}{s}\frac{1-\pi}{\pi}\right)^{-1}$ and $Z(\underline{s}) = \frac{1-\pi}{\pi} l > \underline{s}$.

Below we invoke the Kakutani's fixed point theorem to show that Z has a fixed point. To this end, it suffices to show that Z is (i) a non-empty-valued self-map, (ii) convex-valued, and (iii) upper semi-continuous.

Z is a non-empty-valued self-map

Lemma 1 established the existence of the agent's best response. Moreover, it follows from Lemma 6 that $x(\hat{s}, b)$ is well-defined for all $b \in \Psi(\hat{s})$. The fact that Z is a self-map is clear from its definition.

Z is convex-valued

It is immediate that $Z(\bar{s})$ is convex-valued. Consider a $\hat{s} < \bar{s}$. Suppose $z', z'' \in Z(\hat{s})$, and $z^{\#} \in (z', z'')$. Denote by b' and b'' the corresponding optimal strategies respectively. As this is a game of perfect recall, by the Kuhn's Theorem (Kuhn (1953)), every behavioral strategy is equivalent to some mixed strategy. Denote by α' and α'' the mixed-strategy equivalents of b' and b'' respectively. With a slight abuse of notation, $J(s, \hat{s}, \alpha)$ is defined analogous to (10) for each mixed strategy α of the agent as follows:

$$J(s, \hat{s}, \alpha) \equiv \int \left(1 + \frac{1}{s}\frac{1-\pi}{\pi} \frac{\sum_{h \in H} \beta_1(h, Y)\, q_0(h; \hat{s}, \beta)}{\sum_{h \in H} \beta_1(h, Y)\, q_1(h; \hat{s}, \beta)} \right)^{-1} d\alpha(\beta),$$

where β are the agent's pure strategies on the support of α. Define $T : [0,1] \to \mathbb{R}$ by $T(\gamma) \equiv \gamma J\left(z^{\#}, \hat{s}, \alpha'\right) + (1-\gamma) J\left(z^{\#}, \hat{s}, \alpha''\right) - \frac{l}{1+l}$. It is clear that $T(1) > 0$ and $T(0) < 0$. As T is continuous and increasing, by the intermediate value theorem, there exists a unique $\gamma^* \in (0,1)$ such that $T(\gamma^*) = 0$. As every pure strategy on the support of α' and α'' is a best response to $\hat{s}$, it is clear that $\gamma^* \alpha' + (1-\gamma^*)\alpha''$ is also a best response to $\hat{s}$. Using Kuhn's Theorem again, $z^{\#} \in Z(\hat{s})$.

Z is upper semi-continuous

We begin by showing that the agent's best response $\Psi(\hat{s})$ is upper semi-continuous.

Lemma 7 *$\Psi(\hat{s})$ is upper semi-continuous.*

Proof. Recall the agent's best response to experts' strategy $\hat{s}$ is characterized by a pair of cutoffs $p_0(\hat{s})$ and $p_1(\hat{s})$ (defined in Appendix A.1). We first show these two functions are continuous.

First consider their continuity at $\bar{s}$. Fix a $p \in (0,1)$. For $\hat{s}$ sufficiently close to $\bar{s}$, $p(1-F(\hat{s}|1)) + (1-p)(1-F(\hat{s}|0)) < \frac{c}{2}$. The definition of $V(\cdot;\hat{s})$ in equation (8) then implies $V(p;\hat{s}) \leq -c + \frac{c}{2} + \left(1-\frac{c}{2}\right)\max\{0, V(p;\hat{s})\}$. It is immediate that $V(p;\hat{s}) < 0$. Consequently, $p_0(\hat{s}) = 1$ and $p_1(\hat{s}) = \frac{L}{1+L}$.for all $\hat{s}$ sufficiently close to $\bar{s}$.

Next we show the continuity of $p_0(\cdot)$ and $p_1(\cdot)$ in the interval $[\underline{s}, \bar{s})$. Fix a $\hat{s} \in [\underline{s}, \bar{s})$ and an $\varepsilon \in (0, \bar{s} - \hat{s})$. Take an arbitrary sequence $\{\hat{s}_n\}$ such that $\lim_{n\to\infty} \hat{s}_n = \hat{s}$ and that $\hat{s}_n < \hat{s} + \varepsilon$ for all $n \in \mathbb{N}$. This gives two sequences of cutoffs $\{p_0(\hat{s}_n)\}_n$ and $\{p_1(\hat{s}_n)\}$. Suppose they are both convergent (otherwise take some convergent subsequences). It suffices to show that $\lim_{n\to\infty} p_0(\hat{s}_n) = p_0(\hat{s})$ and $\lim_{n\to\infty} p_1(\hat{s}_n) = p_1(\hat{s})$.

We show below that the family of functions $\{V(\cdot;\hat{s}_n)\}_{n\in\mathbb{N}}$ is Lipschitz continuous. As $V(\cdot;\hat{s}_n)$ is weakly convex (by Lemma 3), $V(\cdot;\hat{s}_n)$ is differentiable for almost all $p \in [0,1]$, and its derivative is nondecreasing in p. Suppose $V(1;\hat{s}_n) > 0$. For a sufficiently large p, equation (8) can be written as

$$\begin{aligned} V(p;\hat{s}_n) &= -c + \{-L(1-p)[1-F(\hat{s}_n|0)] + p[1-F(\hat{s}_n|1)]\} \\ &\quad + [pF(\hat{s}_n|1) + (1-p)F(\hat{s}_n|0)]\, V\left(\left(1 + \frac{1-p}{p}\frac{F(\hat{s}_n|0)}{F(\hat{s}_n|1)}\right)^{-1}; \hat{s}_n\right). \end{aligned}$$

To obtain an upper bound on the derivative $\frac{\partial V(p;\hat{s}_n)}{\partial p}$, suppose $V(p;\hat{s}_n)$ is differentiable at $p=1$. Differentiating both sides of the equation above with respect to p gives

$$\begin{aligned} \frac{\partial V(p;\hat{s}_n)}{\partial p} &= L[1-F(\hat{s}_n|0)] + [1-F(\hat{s}_n|1)] - [F(\hat{s}_n|0) - F(\hat{s}_n|1)]\, V\left(\left(1 + \frac{1-p}{p}\frac{F(\hat{s}_n|0)}{F(\hat{s}_n|1)}\right)^{-1}; \hat{s}_n\right) \\ &\quad + \frac{F(\hat{s}_n|1)F(\hat{s}_n|0)}{pF(\hat{s}_n|1) + (1-p)F(\hat{s}_n|0)} \frac{\partial}{\partial p} V\left(\left(1 + \frac{1-p}{p}\frac{F(\hat{s}_n|0)}{F(\hat{s}_n|1)}\right)^{-1}; \hat{s}_n\right). \end{aligned}$$

Evaluating the equation above at $p=1$ gives $\left.\frac{\partial V(p;\hat{s}_n)}{\partial p}\right|_{p=1} = 1 + L + \left(\frac{F(\hat{s}_n|0)-F(\hat{s}_n|1)}{1-F(\hat{s}_n|0)}\right)c$. As $\frac{F(\hat{s}_n|0)-F(\hat{s}_n|1)}{1-F(\hat{s}_n|0)}$ is increasing in $\hat{s}_n$, $\left.\frac{\partial V(p;\hat{s}_n)}{\partial p}\right|_{p=1}$ is bounded from above by $1+L+\left(\frac{F(\hat{s}+\varepsilon|0)-F(\hat{s}+\varepsilon|1)}{1-F(\hat{s}+\varepsilon|0)}\right)c$. On the other hand, if $V(1;\hat{s}_n) \leq 0$, it is immediate that $\left.\frac{\partial V(p;\hat{s}_n)}{\partial p}\right|_{p=1} = 1 + L$. Thus, the families of functions $\{V(p;\hat{s}_n)\}_{n\in\mathbb{N}}$ is Lipschitz continuous.

As $\{V(p;\hat{s}_n)\}_{n\in\mathbb{N}}$ are equicontinuous and uniformly bounded (by $[-c,1]$), by the Arzela-Ascoli Theorem, there exists a subsequence $\{\hat{s}_{n_k}\}$ such that $V(p;\hat{s}_{n_k})$ converge uniformly. It is clear that the limiting functions is $V(p;\hat{s})$, as by Lemma 3, there exists a unique function that satisfies equation (8). If, for all sufficiently large k, $V(p;\hat{s}_{n_k}) \leq 0$, so that $p_0(\hat{s}_{n_k}) = 1$ and $p_1(\hat{s}_{n_k}) = \frac{L}{1+L}$, then it is immediate that $\lim_{k\to\infty} p_0(\hat{s}_{n_k}) = p_0(\hat{s}) = 1$ and $\lim_{k\to\infty} p_1(\hat{s}_{n_k}) = p_1(\hat{s}) = \frac{L}{1+L}$. Thus suppose there is a further subsequence $\{m_k\}$ of $\{n_k\}$ such that $V(1; \hat{s}_{m_k}) > 0$. Then $p_0(\hat{s}_{m_k})$ is given by $V(p_0(\hat{s}_{m_k}); \hat{s}_{m_k}) = 0$. Passing the equation to limit gives $V(\lim_{k\to\infty} p_0(\hat{s}_{m_k}); \hat{s}) = 0$. By Lemma 1, the only subsequential limit of $\{p_0(\hat{s}_n)\}$ is thus $p_0(\hat{s})$, so $\lim_{n\to\infty} p_0(\hat{s}_n) = p_0(\hat{s})$.

We establish $\lim_{n\to\infty} p_1(\hat{s}_n) = p_1(\hat{s})$ below. If, for all sufficiently large k, $p_1(\hat{s}_{n_k}) = \frac{L}{1+L}$, then it is immediate that $\lim_{k\to\infty} p_1(\hat{s}_{n_k}) = p_1(\hat{s}) = \frac{L}{1+L}$. Next suppose there is a further subsequence $\{m'_k\}$ of $\{n_k\}$ such that $p_1\left(\hat{s}_{m'_k}\right) > \frac{L}{1+L}$. By Lemma 4, $V\left(p_1\left(\hat{s}_{m'_k}\right); \hat{s}_{m'_k}\right) = -L + (1+L)\, p_1\left(\hat{s}_{m'_k}\right)$. Passing the equation to the limit gives $V\left(\lim_{k\to\infty} p_1\left(\hat{s}_{m'_k}\right); \hat{s}\right) = -L + (1+L) \lim_{k\to\infty} p_1\left(\hat{s}_{m'_k}\right)$. By Lemma 1, the only subsequential limit of $\{p_1(\hat{s}_n)\}$ is thus $p_1(\hat{s})$, so $\lim_{n\to\infty} p_1(\hat{s}_n) = p_1(\hat{s})$. This finishes the proof for the continuity of $p_0(\cdot)$ and $p_1(\cdot)$.

We are ready to establish the upper semi-continuity of $\Psi(\cdot)$. Take a sequence of experts' cutoffs $\{\hat{s}_n\}$ and the agent's best-responses $\{b^n\}$ such that $\hat{s}_n \to \hat{s}$, $b^n \in \Psi(\hat{s}_n)$, and $b^n \to b^{\#}$ for some $b^{\#} \in \Lambda$. We show that $b^{\#}$ is a best response to $\hat{s}$. Suppose not. Then there exists a $h \in H$ such that either one of the following holds. (i) $p(h; \hat{s}) \in (p_0(\hat{s}), p_1(\hat{s}))$ but either $b_0^{\#}(h) > 0$ or $b_1^{\#}(h) > 0$; or (ii) $p(h; \hat{s}) < p_0(\hat{s})$ but $b_0^{\#}(h) < 1$; or (iii) $p(h; \hat{s}) > p_1(\hat{s})$ but $b_1^{\#}(h) < 1$. Suppose case (i) arises. For either $i = 0, 1$, $b_i^{\#}(h) > 0$ implies that for all n sufficiently large, we have $b_i^n(h) > 0$ and $p(h; \hat{s}_n) \notin (p_0(\hat{s}_n), p_1(\hat{s}_n))$. As $p_0(\cdot)$ and $p_1(\cdot)$ are continuous, taking limit gives $p(h; \hat{s}) \notin (p_0(\hat{s}), p_1(\hat{s}))$, a contradiction. Suppose case (ii) arises. As $p(h; \hat{s}_n) \to p(h; \hat{s})$ and $p_0(\hat{s}_n) \to p_0(\hat{s})$, we have that for all n sufficiently large, $p(h; \hat{s}_n) < p_0(\hat{s}_n)$, so $b_0^n(h) = 1$. Thus, $b_0^{\#}(h) = 1$, a contradiction. Case (iii) is symmetric to case (ii).

■

Take a pair of sequences $\{\hat{s}_m\}, \{z_m\}$ such that $\hat{s}_m \to \hat{s}$, $z_m \in Z(\hat{s}_m)$, and $z_m \to z$. To prove the upper semi-continuity of Z, we need to show that $z \in Z(\hat{s})$. As we have explained in the definition of correspondence Z, it is continuous at $\hat{s} = \bar{s}$. Below we consider the case $\hat{s} < \bar{s}$.

Suppose first that $z \in (\underline{s}, \bar{s})$. Then it is without loss to assume $z_m \in (\underline{s}, \bar{s})$ for all $m \in \mathbb{N}$ (otherwise, take a subsequence). Consequently, for all $m \in \mathbb{N}$, $J(z_m, \hat{s}_m, b^m) = \frac{l}{1+l}$ for some $b^m \in \Psi(\hat{s}_m)$. As the set of histories H is countable, following a standard diagonalization argument, one can construct a subsequence $\{b^{m_k}\}$ that converges pointwise to some $b^{\#} : H \to [0,1]^2$. By Lemma 7 above, $b^{\#} \in \Psi(\hat{s})$.

Next observe that for $h \neq \varnothing$, the probability $q_\omega(h; \hat{s}_{m_k}, b^{m_k})$ can be decomposed as follows.

$$\begin{aligned} q_\omega(h; \hat{s}_{m_k}, b^{m_k}) \quad &= \quad \Pr(r_1 | \hat{s}_{m_k}, \omega)\left(1 - b_1^{m_k}(r_1)\right)\left(1 - b_0^{m_k}(r_1)\right) \\ &\quad \times \Pr(r_2 | \hat{s}_{m_k}, \omega)\left(1 - b_1^{m_k}(r_1, r_2)\right)\left(1 - b_0^{m_k}(r_1, r_2)\right) \\ &\quad \times .... \times \Pr\left(r_{|h|} | \hat{s}_{m_k}, \omega\right)\left(1 - b_1^{m_k}(h)\right)\left(1 - b_0^{m_k}(h)\right), \end{aligned}$$

where $h = \left(r_1, r_2, ..., r_{|h|}\right)$ and $|h|$ stands for the length of the history h. Note that as $\Pr\left(r|\hat{s}_{m_k}, \omega\right)$ is either $F\left(\hat{s}_{m_k}|\omega\right)$ or $1 - F\left(\hat{s}_{m_k}|\omega\right)$, and $F\left(\cdot|\omega\right)$ is continuous, $\Pr\left(r|\hat{s}_{m_k}, \omega\right)$ converges to $\Pr\left(r|\hat{s}, \omega\right)$. Together with the fact that $b_0^{m_k}\left(\cdot\right)$ and $b_1^{m_k}\left(\cdot\right)$ converge pointwise to $b_0^{\#}\left(\cdot\right)$ and $b_1^{\#}\left(\cdot\right)$ respectively, $q_\omega\left(h; \hat{s}_{m_k}, b^{m_k}\right)$ converges to $q_\omega\left(h; \hat{s}, b^{\#}\right)$ for each $h \in H$.

We are done if we can show that along some subsequence $\{l_k\}$ of $\{m_k\}$,

$$\lim_{k\to\infty} \sum_{h\in H} b_1^{l_k}\left(h, Y\right) q_\omega\left(h; \hat{s}_{l_k}, b^{l_k}\right) = \sum_{h\in H} b_1^{\#}\left(h, Y\right) q_\omega\left(h; \hat{s}, b^{\#}\right), \tag{11}$$

as this implies that $\lim_{m\to\infty} J\left(z_m, \hat{s}_m, b^m\right) = J\left(z, \hat{s}, b^{\#}\right) = \frac{l}{1+l}$.

Denote by $Q_\omega\left(n; \hat{s}, b\right) \equiv \sum_{h\in H:|h|=n} b_1\left(h, Y\right) q_\omega\left(h; \hat{s}, b\right)$ the probability that the agent takes the operation in period n, assuming that the state is ω and the strategy profile $(\hat{s}, b)$ is played. Using this definition, we can write $\sum_{h\in H} b_1\left(h, Y\right) q_\omega\left(h; \hat{s}, b\right) = \sum_{n=0}^{\infty} Q_\omega\left(n; \hat{s}, b\right)$, and equation (11) is equivalent to $\sum_{n=0}^{\infty} Q_\omega\left(n; \hat{s}_{l_k}, b^{l_k}\right) \to \sum_{n=0}^{\infty} Q_\omega\left(n; \hat{s}, b\right)$. To invoke Lebesgue's dominated convergence theorem, we need to show that there exists $N', K \in \mathbb{N}$ such that for all $n > N'$ and $k > K$, we have $Q_\omega\left(n; \hat{s}_{l_k}, b^{l_k}\right) \leq \Phi\left(n\right)$ for some function $\Phi\left(n\right)$ such that $\sum_{n=0}^{\infty} \Phi\left(n\right) < \infty$.

Concerning how sequences $\{p_0\left(\hat{s}_{m_k}\right)\}$ and $\{p_1\left(\hat{s}_{m_k}\right)\}$ approach their respective limits, either one of the following two subsequences of $\{\hat{s}_{m_k}\}$ must exist: (i) a subsequence $\{\hat{s}_{l_k}\}$ such that $p_0\left(\hat{s}_{l_k}\right) \geq p_1\left(\hat{s}_{l_k}\right)$ for all $k \in \mathbb{N}$; (ii) a subsequence $\{\hat{s}_{l_k}\}$ such that $p_0\left(\hat{s}_{l_k}\right) < p_1\left(\hat{s}_{l_k}\right)$ for all $k \in \mathbb{N}$.

Consider the first possibility. For all $k \in \mathbb{N}$, period $n > 1$ is reached only if $\pi > p_1\left(\hat{s}_{l_k}\right)$, but the agent has always received recommendation N in all periods up to $n-1$. Thus, $Q_\omega\left(n; \hat{s}_{l_k}, b^{l_k}\right) \leq F\left(\hat{s}_{l_k}|\omega\right)^{n-1}$. Let $\varepsilon \in \left(0, 1 - F\left(\hat{s}|0\right)\right)$. As $\hat{s} < \bar{s}$, there exists a K' such that for all $k > K'$, $F\left(\hat{s}_{l_k}|\omega\right) < F\left(\hat{s}|\omega\right) + \varepsilon$, so $Q_\omega\left(n; \hat{s}_{l_k}, b^{l_k}\right) \leq \left(F\left(\hat{s}|\omega\right) + \varepsilon\right)^{n-1}$. It is clear that $\sum_{n=0}^{\infty} \left(F\left(\hat{s}|\omega\right) + \varepsilon\right)^{n-1} = \frac{1}{1-F(\hat{s}|\omega)-\varepsilon} < \infty$.

Consider the second possibility. For each $k \in \mathbb{N}$, period $n > 1$ is reached only if one of the following events occur: (a) the agent's posterior belief at the end of period $n-1$ is in the interval $\left[p_0\left(\hat{s}_{l_k}\right), p_1\left(\hat{s}_{l_k}\right)\right]$, or (b) $\pi > p_1\left(\hat{s}_{l_k}\right)$ but the agent has always received recommendation N in all periods up to period $n-1$. Thus, $Q_\omega\left(n; \hat{s}_{l_k}, b^{l_k}\right)$ is bounded by

$$\begin{aligned} Q_\omega\left(n; \hat{s}_{l_k}, b^{l_k}\right) &\leq F\left(\hat{s}_{l_k}|\omega\right)^{n-1} + \sum_{h\in H:|h|=n-1} \Pr\left(p\left(h, \hat{s}_{l_k}\right) \in \left[p_0\left(\hat{s}_{l_k}\right), p_1\left(\hat{s}_{l_k}\right)\right] |\omega\right) \\ &\leq \begin{cases} F\left(\hat{s}_{l_k}|1\right)^{n-1} + \sum_{h\in H:|h|=n-1} \Pr\left(p\left(h, \hat{s}_{l_k}\right) \leq p_1\left(\hat{s}_{l_k}\right) |\omega = 1\right) & \text{if } \omega = 1 \\ F\left(\hat{s}_{l_k}|0\right)^{n-1} + \sum_{h\in H:|h|=n-1} \Pr\left(p\left(h, \hat{s}_{l_k}\right) \geq p_0\left(\hat{s}_{l_k}\right) |\omega = 0\right) & \text{if } \omega = 0 \end{cases}. \end{aligned}$$

As $\hat{s} < \bar{s}$, there exists an integer K' such that $F\left(\hat{s}_{l_k}|\omega\right) < F\left(\hat{s}|\omega\right) + \varepsilon$ for all $k > K'$ and ω. An upper bound on $\sum_{h\in H:|h|=n-1} \Pr\left(p\left(h, \hat{s}_{l_k}\right) \leq p_1\left(\hat{s}_{l_k}\right) |\omega = 1\right)$ can be obtained by noting that for each $\hat{s}_{l_k}$, the expert's recommendation $r \in \{Y, N\}$ is a Bernoulli random variable, with $\Pr(r = Y) = 1 - F(\hat{s}_{l_k}|\omega)$. The agent's posterior after receiving $n-1$ recommendations is weakly less than $p_1(\hat{s}_{l_k})$ if and only if the number y of recommendation Y is smaller than $\frac{(n-1)\ln\left(\frac{F\left(\hat{s}_{l_k}|0\right)}{F\left(\hat{s}_{l_k}|1\right)}\right)-\ln\left(\frac{1}{p_1\left(\hat{s}_{l_k}\right)}-1\right)-\ln\frac{\pi}{1-\pi}}{\ln\left(\frac{1-F\left(\hat{s}_{l_k}|1\right)}{F\left(\hat{s}_{l_k}|1\right)}\right)-\ln\left(\frac{1-F\left(\hat{s}_{l_k}|0\right)}{F\left(\hat{s}_{l_k}|0\right)}\right)}$.[35] We can apply Hoeffding's inequality to bound the probability of the event above. Hoeffding's inequality states that if $\{r_i\}_{i=1,2,\ldots,n-1}$ is a sequence of $n-1$ independently and identically distributed Bernoulli random variables with $\Pr(r_i = 1) = 1 - \Pr(r_i = 0) = p \in (0,1)$, then for all $\varepsilon > 0$, the probability that the sum $\sum_{i=1}^{n} r_i$ is less than $p(n-1) - \varepsilon$ is no more than $\exp\left(-2\varepsilon^2(n-1)\right)$. Applying Hoeffding's inequality,

$$\begin{aligned}
&\Pr\left(y \le \left. \frac{\ln\left(\frac{F\left(\hat{s}_{l_k}|0\right)}{F\left(\hat{s}_{l_k}|1\right)}\right) - \frac{1}{n-1}\left(\ln\left(\frac{1}{p_1\left(\hat{s}_{l_k}\right)}-1\right)+\ln\frac{\pi}{1-\pi}\right)}{\ln\left(\frac{1-F\left(\hat{s}_{l_k}|1\right)}{F\left(\hat{s}_{l_k}|1\right)}\right) - \ln\left(\frac{1-F\left(\hat{s}_{l_k}|0\right)}{F\left(\hat{s}_{l_k}|0\right)}\right)} (n-1) \right| \omega = 1 \right) \\
\le\ & \exp\left(-2\left[\left(1 - F\left(\hat{s}_{l_k}|1\right)\right) - \frac{\ln\left(\frac{F\left(\hat{s}_{l_k}|0\right)}{F\left(\hat{s}_{l_k}|1\right)}\right) - \frac{1}{n-1}\left(\ln\left(\frac{1}{p_1\left(\hat{s}_{l_k}\right)}-1\right)+\ln\frac{\pi}{1-\pi}\right)}{\ln\left(\frac{1-F\left(\hat{s}_{l_k}|1\right)}{F\left(\hat{s}_{l_k}|1\right)}\right) - \ln\left(\frac{1-F\left(\hat{s}_{l_k}|0\right)}{F\left(\hat{s}_{l_k}|0\right)}\right)} \right]^2 (n-1) \right).
\end{aligned}$$

As $n, k \to \infty$, the bracketed term approaches $\left(1 - F(\hat{s}|1)\right) - \frac{\ln F(\hat{s}|0) - \ln F(\hat{s}|1)}{\ln\left(\frac{1-F(\hat{s}|1)}{F(\hat{s}|1)}\right)-\ln\left(\frac{1-F(\hat{s}|0)}{F(\hat{s}|0)}\right)} \equiv L_{\hat{s}} > 0$.[36] Therefore, there exists a pair of sufficiently large integers $N_1, K_1 > K'$ such that $Q_1\left(n; \hat{s}_{l_k}, b^{l_k}\right) \le \left(F(\hat{s}|1) + \varepsilon\right)^{n-1} + \exp\left(-L_{\hat{s}}^2(n-1)\right)$ for all $n > N_1$ and $k > K_1$. Define a dominating function $\Phi_1 : \mathbb{N} \to [0,1]$ by

$$\Phi_1(n) \equiv \begin{cases} 1 & \text{if } n < N_1 \\ \left(F(\hat{s}|1)+\varepsilon\right)^{n-1} + \exp\left(-L_{\hat{s}}^2(n-1)\right) & \text{if } n \ge N_1 \end{cases}.$$

It is clear that $\sum_{n=1}^{\infty} \Phi_1(n) = N_1 + \frac{1}{1-(F(\hat{s}|1)+\varepsilon)} + \frac{\exp\left(-L_{\hat{s}}^2(N_1-1)\right)}{1-\exp\left(-L_{\hat{s}}^2\right)} < \infty$. Therefore by Lebesgue's dominated convergence theorem, $\sum_{n=0}^{\infty} Q_1\left(n; \hat{s}_{l_k}, b^{l_k}\right) \to \sum_{n=0}^{\infty} Q_1\left(n; \hat{s}, b^{\#}\right)$ as $k \to \infty$.

The upper bound on $\sum_{h \in H:|h|=n-1} \Pr\left(p\left(h, \hat{s}_{l_k}\right) \ge p_0\left(\hat{s}_{l_k}\right)|\omega = 0\right)$ can be derived similarly. The agent's posterior after receiving $n-1$ recommendations is weakly larger than $p_0(\hat{s}_{l_k})$ if and only if the number y of recommendation Y is larger than $\frac{(n-1)\ln\left(\frac{F\left(\hat{s}_{l_k}|0\right)}{F\left(\hat{s}_{l_k}|1\right)}\right)-\ln\left(\frac{1}{p_0\left(\hat{s}_{l_k}\right)}-1\right)-\ln\frac{\pi}{1-\pi}}{\ln\left(\frac{1-F\left(\hat{s}_{l_k}|1\right)}{F\left(\hat{s}_{l_k}|1\right)}\right)-\ln\left(\frac{1-F\left(\hat{s}_{l_k}|0\right)}{F\left(\hat{s}_{l_k}|0\right)}\right)}$. Using Hoeffd-

[35] The agent's posterior after receiving y positive recommendations and $n-1-y$ negative recommendations is $\left(1 + \frac{1-\pi}{\pi}\frac{\left(1-F\left(\hat{s}_{l_k}|0\right)\right)^y\left(F\left(\hat{s}_{l_k}|0\right)\right)^{n-1-y}}{\left(1-F\left(\hat{s}_{l_k}|1\right)\right)^y\left(F\left(\hat{s}_{l_k}|1\right)\right)^{n-1-y}}\right)^{-1}$. This posterior is less than $p_1(\hat{s}_{l_k})$ if and only if y is smaller than the number stated.

[36] To see $L_{\hat{s}} > 0$, note that the ratio $\frac{\ln F(\hat{s}|0)-\ln F(\hat{s}|1)}{\ln\left(\frac{1-F(\hat{s}|1)}{F(\hat{s}|1)}\right)-\ln\left(\frac{1-F(\hat{s}|0)}{F(\hat{s}|0)}\right)}$ decreases in $F(\hat{s}|0)$ and converges to $1 - F(\hat{s}|1)$ as $F(\hat{s}|0) \to F(\hat{s}|1)$.

ing's inequality again,

$$\Pr\left(y\geq\frac{\ln\left(\frac{F\left(\hat{s}_{l_k}|0\right)}{F\left(\hat{s}_{l_k}|1\right)}\right)-\frac{1}{n-1}\left(\ln\left(\frac{1}{p_0\left(\hat{s}_{l_k}\right)}-1\right)+\ln\frac{\pi}{1-\pi}\right)}{\ln\left(\frac{1-F\left(\hat{s}_{l_k}|1\right)}{F\left(\hat{s}_{l_k}|1\right)}\right)-\ln\left(\frac{1-F\left(\hat{s}_{l_k}|0\right)}{F\left(\hat{s}_{l_k}|0\right)}\right)}(n-1)\middle|\,\omega=0\right)$$

$$\leq\exp\left(-2\left[\frac{\ln\left(\frac{F\left(\hat{s}_{l_k}|0\right)}{F\left(\hat{s}_{l_k}|1\right)}\right)-\frac{1}{n-1}\left(\ln\left(\frac{1}{p_0\left(\hat{s}_{l_k}\right)}-1\right)+\ln\frac{\pi}{1-\pi}\right)}{\ln\left(\frac{1-F\left(\hat{s}_{l_k}|1\right)}{F\left(\hat{s}_{l_k}|1\right)}\right)-\ln\left(\frac{1-F\left(\hat{s}_{l_k}|0\right)}{F\left(\hat{s}_{l_k}|0\right)}\right)}-\left(1-F\left(\hat{s}_{l_k}|0\right)\right)\right]^2(n-1)\right).$$

As $n,k\to\infty$, the bracketed term approaches $\frac{\ln F(\hat{s}|0)-\ln F(\hat{s}|1)}{\ln\left(\frac{1-F(\hat{s}|1)}{F(\hat{s}|1)}\right)-\ln\left(\frac{1-F(\hat{s}|0)}{F(\hat{s}|0)}\right)}-\left(1-F\left(\hat{s}|0\right)\right)\equiv L'_{\hat{s}}>0$.[37] Therefore, there exists a sufficiently large integer $N_0,K_0>K'$ such that $Q_0\left(n;\hat{s}_{l_k},b^{l_k}\right)\leq\left(F\left(\hat{s}|0\right)+\varepsilon\right)^{n-1}+\exp\left(-L'^2_{\hat{s}}\left(n-1\right)\right)$ for all $n>N_0$ and $k>K_0$. Define a dominating function $\Phi_0:\mathbb{N}\to[0,1]$ by

$$\Phi_0\left(n\right)\equiv\begin{cases}1 & \text{if } n<N_0\\ \left(F\left(\hat{s}|0\right)+\varepsilon\right)^{n-1}+\exp\left(-L'^2_{\hat{s}}\left(n-1\right)\right) & \text{if } n\geq N_0\end{cases}.$$

It is clear that $\sum_{n=1}^{\infty}\Phi_0\left(n\right)=N_0+\frac{1}{1-F(\hat{s}|0)-\varepsilon}+\frac{\exp\left(-L'^2_{\hat{s}}(N_0-1)\right)}{1-\exp\left(-L'^2_{\hat{s}}\right)}<\infty$. Therefore by Lebesgue's dominated convergence theorem, $\sum_{n=0}^{\infty}Q_0\left(n;\hat{s}_{l_k},b^{l_k}\right)\to\sum_{n=0}^{\infty}Q_0\left(n;\hat{s},b^{\#}\right)$ as $k\to\infty$.

Finally, we consider the cases of $z\in\{\bar{s},\underline{s}\}$. If $z=\bar{s}$, then $J\left(z_m,\hat{s}_m,b^m\right)\leq\frac{l}{1+l}$ for some $b^m\in\Psi\left(\hat{s}_m\right)$. By the upper semi-continuity of Ψ, there exists a subsequence $\{b^{m_k}\}$ such that $\lim_{k\to\infty}b^{m_k}\in\Psi\left(\hat{s}\right)$. By the analysis above, $J\left(\bar{s},\hat{s},b\right)\leq\frac{l}{1+l}$ for some $b\in\Psi\left(\hat{s}\right)$. Therefore, $\bar{s}\in Z\left(\hat{s}\right)$. On the other hand, if $z=\underline{s}$, then $J\left(z_m,\hat{s}_m,b^m\right)\geq\frac{l}{1+l}$ for some $b^m\in\Psi\left(\hat{s}_m\right)$. A symmetric argument as above shows $J\left(\bar{s},\hat{s},b\right)\geq\frac{l}{1+l}$ for some $b\in\Psi\left(\hat{s}\right)$, so $\underline{s}\in Z\left(\hat{s}\right)$. Q.E.D.

Appendix A.3: Proof of Lemma 2

To ease notation, denote $s_n^*\equiv s^*\left(c_n\right)$.

(i) We first show that it is impossible to have $\{s_n^*\}$ converging to $\bar{s}$. Suppose there exists a sequence of informative equilibria such that $\lim_{n\to\infty}s_n^*=\bar{s}$. As the agent takes the operation only if his belief is no less than $\frac{L}{1+L}$, it is necessary that $p_1\left(s_n^*\right)\geq\frac{L}{1+L}$. Together with equation (5), we can derive a lower bound on $\tilde{p}_1\left(s_n^*\right)$ as follows:

$$\tilde{p}_1\left(s_n^*\right)=\left(1+\left(\frac{1}{p_1\left(s_n^*\right)}-1\right)\frac{1-F\left(s_n^*|1\right)}{1-F\left(s_n^*|0\right)}\right)^{-1}\geq\left(1+\frac{1}{L}\frac{1-F\left(s_n^*|1\right)}{1-F\left(s_n^*|0\right)}\right)^{-1}.$$

[37] To see $L'_{\hat{s}}>0$, note that the ratio $\frac{\ln F(\hat{s}|0)-\ln F(\hat{s}|1)}{\ln\left(\frac{1-F(\hat{s}|1)}{F(\hat{s}|1)}\right)-\ln\left(\frac{1-F(\hat{s}|0)}{F(\hat{s}|0)}\right)}$ is decreasing in $F\left(\hat{s}|1\right)$ and converges to $1-F\left(\hat{s}|1\right)$ as $F\left(\hat{s}|1\right)\to F\left(\hat{s}|0\right)$.

Using the upper bound on $\tilde{p}_1(s_n^*)$ from inequality (7), we have

$$\frac{L}{l} \leq \frac{1}{s_n^*}\frac{1-F(s_n^*|1)}{1-F(s_n^*|0)}.$$

If $s_n^* \to \bar{s}$, the right-hand side of the inequality above converges to 1, implying that $L \leq l$, a contradiction to the assumption $l < L$.

Suppose $\{s_n^*\}$ does not converge to $\underline{s}$. As shown in the proof of Lemma 7, $\{V(\cdot; s_n^*)\}$ is a family of Lipschitz continuous functions. Thus, there exists a $\varepsilon \in (0, \bar{s} - \underline{s})$ and a subsequence $\{s_{n_k}^*\}$ of $\{s_n^*\}$ such that $s_{n_k}^* \to \underline{s} + \varepsilon$ and $V(\cdot; s_{n_k}^*)$ converges uniformly to $V(\cdot; \underline{s}+\varepsilon)$. Passing equation (8) with $\hat{s} = s_{n_k}^*$, to the limit gives

$$\begin{aligned} & V(p; \underline{s}+\varepsilon) \\ = \; & [p(1-F(\underline{s}+\varepsilon|1)) + (1-p)(1-F(\underline{s}+\varepsilon|0))] \max \left\{ \begin{array}{c} 0, V\left(\left(1+\frac{1-p}{p}\frac{1-F(\underline{s}+\varepsilon|0)}{1-F(\underline{s}+\varepsilon|1)}\right)^{-1}; \underline{s}+\varepsilon\right), \\ -L + \left(1+\frac{1-p}{p}\frac{1-F(\underline{s}+\varepsilon|0)}{1-F(\underline{s}+\varepsilon|1)}\right)^{-1}(1+L) \end{array} \right\} \\ & + [pF(\underline{s}+\varepsilon|1) + (1-p)F(\underline{s}+\varepsilon|0)] \max\left\{0, V\left(\left(1+\frac{1-p}{p}\frac{F(\underline{s}+\varepsilon|0)}{F(\underline{s}+\varepsilon|1)}\right)^{-1}; \underline{s}+\varepsilon\right)\right\}. \end{aligned} \tag{12}$$

It is straightforward to verify that $V(p; \underline{s}+\varepsilon) = p$ is a solution. Moreover, it is unique by Proposition 3. Therefore, $p_1(s_{n_k}^*) \to 1$.

On the other hand, in equilibrium, inequality (7) holds. It can be rearranged into $s_{n_k}^* \leq \frac{1-\tilde{p}_1(s_{n_k}^*)}{\tilde{p}_1(s_{n_k}^*)} l$, where $\tilde{p}_1(s_{n_k}^*)$ is defined in (5). Using the definition, the right-hand side of the inequality is equal to $\left(\left(\frac{1}{p_1(s_{n_k}^*)} - 1\right)\frac{1-F(s_{n_k}^*|1)}{1-F(s_{n_k}^*|0)}\right) l$, which converges to 0 as $p_1(s_{n_k}^*) \to 1$. This is a contradiction to $s_{n_k}^* \to \underline{s} + \varepsilon$.

(ii) Suppose $\{p_1(s_n^*)\}$ converges (otherwise, take a convergent subsequence). By part (i) above, $s_n^* \to \underline{s}$. Thus, the difference between $p_1(s_n^*)$ and $\tilde{p}_1(s_n^*)$ vanishes in the limit:

$$p_1(s_n^*) - \tilde{p}_1(s_n^*) = \frac{p_1(s_n^*)(1-p_1(s_n^*))[F(s_n^*|0) - F(s_n^*|1)]}{1-F(s_n^*|1) - (F(s_n^*|0) - F(s_n^*|1))p_1(s_n^*)} \to 0.$$

Rearranging inequality (7) gives $\tilde{p}_1(s_n^*) \leq \frac{l}{l+s_n^*}$. Therefore, $\lim_{n\to\infty} p_1(s_n^*) \leq \frac{l}{l+\underline{s}}$. On the other hand, conditional on being pivotal, the agent's belief prior to learning the current expert's recommendation is bounded from above by $\max\{\pi, p_1(s_n^*)\}$. Condition (3) then implies that $\left(1+\frac{1}{s_n^*}\frac{1-\max\{\pi, p_1(s_n^*)\}}{\max\{\pi, p_1(s_n^*)\}}\right)^{-1} \geq \frac{l}{1+l}$, or equivalently,

$$\max\{\pi, p_1(s_n^*)\} \geq \frac{l}{l+s_n^*}. \tag{13}$$

Taking limit on both sides of the inequality then gives $\max\{\pi, \lim_{n\to\infty} p_1(s_n^*)\} \geq \frac{l}{l+\underline{s}}$. As $\pi < \frac{l}{l+\underline{s}}$ by assumption, it is necessary that $\lim_{n\to\infty} p_1(s_n^*) \geq \frac{l}{l+\underline{s}}$. Therefore, $\lim_{n\to\infty} p_1(s_n^*) = \frac{l}{l+\underline{s}}$.

(iii) Fix a $\underline{s} > 0$. We show that if the sequence $\{p_0(s_n^*)\}$ converges, its limit strictly exceeds 0. Suppose instead $\lim_{n\to\infty} p_0(s_n^*) = 0$. Fix a $q \in \left(0, \frac{L}{1+L}\right)$. There exists an integer N' such that for all $n > N'$, $p_0(s_n^*) < q < \frac{L}{1+L} < p_1(s_n^*)$.[38] As $q > p_0(s_n^*)$, the agent's continuation value function evaluated at q, $V(q; s_n^*)$, is strictly positive for all $n > N'$.

An upper bound for $V(q; s_n^*)$ can be derived as follows. In the best conceivable scenario, the agent with $\omega = 0$ learns the state immediately and gets a payoff of 0; whereas the agent with $\omega = 1$ gets a consecutive sequence of Y recommendations, leading to a posterior $p_1(s_n^*)$ and taking the operation. Therefore,

$$V(q; s_n^*) \leq q\left[(1 - F(s_n^*|1))^{M_n} - c_n M_n\right], \tag{14}$$

where M_n denotes the number of consecutive Y recommendations needed to convince the agent to take the operation. It necessarily satisfies $\left(1 + \frac{1-q}{q}\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)^{M_n-1}\right)^{-1} \leq p_1(s_n^*) \leq \left(1 + \frac{1-q}{q}\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)^{M_n}\right)^{-1}$, or equivalently,

$$M_n \in \left[\frac{\ln\left(\frac{q}{1-q}\left(\frac{1}{p_1(s_n^*)} - 1\right)\right)}{\ln\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)}, \frac{\ln\left(\frac{q}{1-q}\left(\frac{1}{p_1(s_n^*)} - 1\right)\right)}{\ln\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)} + 1\right]. \tag{15}$$

Below we derive a contradiction by showing that the upper bound in inequality (14) cannot be positive for all $q \in (0, 1)$.

To this end, note first that, as $p_1(s_n^*) > \frac{L}{1+L}$, the lower bound in (15) implies that

$$M_n \geq \frac{\ln\left(\frac{1-q}{q}\frac{1}{L}\right)}{\ln\left(\frac{1-F(s_n^*|1)}{1-F(s_n^*|0)}\right)}. \tag{16}$$

Next, by Lemma 4, $V(p_1(s_n^*); s_n^*) = -L + (1+L) p_1(s_n^*)$. Using equation (8),

$$\begin{aligned} -L + p_1(s_n^*)(1+L) = \; & -c_n + \left[p_1(s_n^*)(1 - F(s_n^*|1)) + (1 - p_1(s_n^*))(1 - F(s_n^*|0))\right] \\ & \times \left(-L + \left(1 + \frac{1 - p_1(s_n^*)}{p_1(s_n^*)}\frac{1 - F(s_n^*|0)}{1 - F(s_n^*|1)}\right)^{-1}(1+L)\right) \\ & + \left[p_1(s_n^*) F(s_n^*|1) + (1 - p_1(s_n^*)) F(s_n^*|0)\right] \\ & \times \max\left\{0, V\left(\left(1 + \frac{1 - p_1(s_n^*)}{p_1(s_n^*)}\frac{F(s_n^*|0)}{F(s_n^*|1)}\right)^{-1}; s_n^*\right)\right\}. \end{aligned} \tag{17}$$

In the equation above, we have used the fact that the definition of $p_1(s_n^*)$ implies $V(p; s_n^*) \leq -L +$

[38] Such an N exists because, by Lemma 4, $p_0(s_n^*) < \frac{L}{1+L}$ implies that $p_1(s_n^*) > \frac{L}{1+L}$.

$p(1+L)$ for all $p > p_1(s_n^*)$. Using the equation above,

$$\begin{aligned} c_n &\geq LF(s_n^*|0) - p_1(s_n^*)\left[LF(s_n^*|0) + F(s_n^*|1)\right] \\ &\geq \frac{LF(s_n^*|0)\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\frac{s_n^*}{l} - F(s_n^*|1)}{1+\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\frac{s_n^*}{l}}, \end{aligned} \tag{18}$$

where the first inequality uses the fact that the max operator returns a nonnegative value, and the second inequality makes use of the fact that

$$p_1(s_n^*) \leq \left(1+\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\frac{s_n^*}{l}\right)^{-1}. \tag{19}$$

Inequality (19) is obtained by substituting equation (5) into inequality (7). It is straightforward to check that the lower bound of c_n in inequality (18) is positive for s_n^* sufficiently close to $\underline{s}$.

Combining (16) and (18), we get

$$c_n M_n \geq \frac{\ln\left(\frac{1-q}{q}\frac{1}{L}\right)}{1+\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\frac{s_n^*}{l}} \times \left(-\frac{F(s_n^*|1)}{F(s_n^*|0)} + \frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}s_n^*\frac{L}{l}\right) \times \frac{F(s_n^*|0)}{\ln\left(\frac{1-F(s_n^*|1)}{1-F(s_n^*|0)}\right)}.$$

We now take $\limsup_{n\to\infty}$ on both sides of the inequality above. Using L'Hospital rule and the fact that $s_n^* \to \underline{s}$ by part (i), we get

$$\begin{aligned} \limsup_{n\to\infty} c_n M_n &\geq \frac{\ln\left(\frac{1-q}{q}\frac{1}{L}\right)}{1+\frac{\underline{s}}{l}} \times \lim_{n\to\infty}\left(-\frac{F(s_n^*|1)}{F(s_n^*|0)} + \frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}s_n^*\frac{L}{l}\right) \times \lim_{n\to\infty}\frac{F(s_n^*|0)}{\ln\left(\frac{1-F(s_n^*|1)}{1-F(s_n^*|0)}\right)} \\ &= \frac{\ln\left(\frac{1-q}{q}\frac{1}{L}\right)}{1+\frac{\underline{s}}{l}} \times \underline{s}\left(\frac{L}{l}-1\right) \times \lim_{n\to\infty}\frac{f(s_n^*|0)}{\frac{f(s_n^*|0)(1-F(s_n^*|1))-f(s_n^*|1)(1-F(s_n^*|0))}{(1-F(s_n^*|0))(1-F(s_n^*|1))}} \\ &= \ln\left(\frac{1-q}{q}\frac{1}{L}\right)\frac{L-l}{l+\underline{s}}\frac{\underline{s}}{1-\underline{s}}. \end{aligned}$$

Finally, taking $\limsup_{n\to\infty}$ on both sides of inequality (14), and using the fact that $(1-F(s_n^*|1))^{M_n} \leq 1$ and $V(q;s_n^*) > 0$, we get

$$0 \leq q\left(1-\frac{L-l}{l+\underline{s}}\frac{\underline{s}}{1-\underline{s}}\ln\left(\frac{1-q}{q}\frac{1}{L}\right)\right).$$

However, this is a contradiction as q can be an arbitrarily small positive number. Q.E.D.

Appendix A.4: Proof of Proposition 3

Following Appendix A.3, denote $s_n^* \equiv s^*(c_n)$.

(i) As explained in Section 5.1, it is without loss to focus on informative equilibria. It is also without loss to assume $p_0(s_n^*) < p_1(s_n^*)$ and $\pi < p_1(s_n^*)$. The reason is as follows. First, the assumption

$\pi < \frac{l}{l+\underline{s}}$ implies that $\pi < p_1(s_n^*)$ for all sufficiently large n. Next, suppose there is a subsequence $\{s_{n_k}^*\}$ of $\{s_n^*\}$ such that $p_0(s_{n_k}^*) \geq p_1(s_{n_k}^*)$. Then by Lemma 4, $p_1(s_{n_k}^*) = \frac{L}{1+L}$. Moreover, by part (i) of Lemma 2, $\lim_{k\to\infty} s_{n_k}^* = \underline{s}$, so the first (free) expert almost always recommends Y for k sufficiently large. Consequently, $U(c_{n_k})$ converges to $\max\{0, -L + \pi(1+L)\}$, and $T(c_{n_k})$ converges to $-l + \pi(1+l)$ if $\pi \geq \frac{L}{1+L}$ and 0 otherwise. It can be readily checked that these subsequential limits are strictly below $\left(1 - \frac{L}{l}\underline{s}\right)\pi$ and $\pi(1-\underline{s})$ respectively.

By the argument above, we will focus on a sequence of informative equilibria such that $\max\{p_0(s_n^*), \pi\} < p_1(s_n^*)$ for all n. First consider the agent's equilibrium payoff. His payoff $U(c_n)$ at search cost c_n is equal to $V(\pi; s_n^*) + c_n$. By Lemma 4, $V(p_1^*(s_n); s_n^*) = -L + (1+L)p_1(s_n^*)$. Moreover, as each $V(\cdot; s_n^*)$ is weakly convex and $V(p_0(s_n^*); s_n^*) = 0$, an upper bound on $V(\pi; s_n^*)$ is given by

$$V_n(\pi; s_n^*) \leq \max\left\{0, [-L + (1+L)p_1(s_n^*)]\frac{\pi - p_0(s_n^*)}{p_1(s_n^*) - p_0(s_n^*)}\right\}.$$

Define $\underline{p}_0 \equiv \liminf_{n\to\infty} p_0(s_n^*)$. By Lemma 2, $\lim_{n\to\infty} p_1(s_n^*) = \frac{l}{l+\underline{s}}$, and $\underline{p}_0 > 0$. Taking limsup on both sides of the inequality above gives

$$\limsup_{n\to\infty} V_n(\pi; s_n^*) \leq \max\left\{0, \frac{\pi - \underline{p}_0}{1 - \left(\frac{l+\underline{s}}{l}\right)\underline{p}_0}\left(1 - \frac{L}{l}\underline{s}\right)\right\}.$$

The right-hand side of the last inequality is no larger than $\left(1 - \frac{L}{l}\underline{s}\right)\pi$. Therefore, $\limsup_{n\to\infty} U(c_n) = \limsup_{n\to\infty} V_n(\pi; s_n^*) \leq \left(1 - \frac{L}{l}\underline{s}\right)\pi$.

Next we consider the experts' joint payoff. Denote by E_n the expected payoff of the expert who carries out the operation, in an equilibrium of the game in which the search cost is c_n and the experts' cutoff is s_n^*. Recall by assumption $\pi < p_1(s_n^*)$, payoff E_n is thus bounded from above as follows:

$$\begin{aligned} E_n &\leq -l + (1+l)\Pr(\omega = 1 | q = p_1(s_n^*), s \geq s_n^*) \\ &= -l + (1+l)\left(1 + \frac{1 - p_1(s_n^*)}{p_1(s_n^*)}\frac{1 - F(s_n^*|0)}{1 - F(s_n^*|1)}\right)^{-1}, \end{aligned}$$

where $q = p_1(s_n^*)$ denotes the event that the agent holds a belief $p_1(s_n^*)$ prior to the pivotal consultation.

Define $\rho(s_n^*) \equiv \left(1 + \frac{1-\pi}{\pi}\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)^{-1}$, so that $\rho(s_n^*)$ denotes the agent's posterior belief after the getting a recommendation Y from the first (free) expert, assuming that the experts adopt cutoff s_n^*. The agent searches beyond the first expert after a recommendation Y if and only if $\rho(s_n^*) > p_0(s_n^*)$. In deriving the upper bound on the experts' equilibrium payoff, it is without loss to assume that $\rho(s_n^*) > p_0(s_n^*)$, for otherwise, $T(c_n) = 0$.

Further define $\tilde{p}_0(s_n^*) \equiv \left(1 + \frac{1-p_0(s_n^*)}{p_0(s_n^*)}\frac{F(s_n^*|0)}{F(s_n^*|1)}\right)^{-1}$, so that $\tilde{p}_0(s_n^*)$ is a lower bound on the agent's posterior belief conditional on quitting the search without taking the operation, assuming that experts adopt cutoff s_n^*. As the agent's updated belief in the search process is a martingale, starting with a prior belief π, the probability that the agent's posterior reaches $p_1(s_n^*)$ is no more than $\frac{\rho(s_n^*)-\tilde{p}_0(s_n^*)}{p_1(s_n^*)-\tilde{p}_0(s_n^*)}$. The experts' joint payoff $T(c_n)$ is therefore bounded from above by:

$$T(c_n) \le \frac{\rho(s_n^*) - \tilde{p}_0(s_n^*)}{p_1(s_n^*) - \tilde{p}_0(s_n^*)} \left(-l + (1+l) \left(1 + \frac{1 - p_1(s_n^*)}{p_1(s_n^*)} \frac{1 - F(s_n^*|0)}{1 - F(s_n^*|1)} \right)^{-1} \right).$$

Note that $\liminf_{n\to\infty} \tilde{p}_0(s_n^*) = \left(1 + \frac{1-\underline{p}_0}{\underline{p}_0}\frac{1}{\underline{s}}\right)^{-1} > 0$ and $\lim_{n\to\infty} \rho(s_n^*) = \pi$. Taking limsup on both sides of the inequality above gives

$$\limsup_{n\to\infty} T(c_n) \le \frac{\pi - \liminf_{n\to\infty} \tilde{p}_0(s_n^*)}{1 - \frac{l+\underline{s}}{l} \liminf_{n\to\infty} \tilde{p}_0(s_n^*)} (1 - \underline{s}).$$

As $\frac{l}{l+\underline{s}} > \pi \ge \underline{p}_0 > \liminf_{n\to\infty} \tilde{p}_0(s_n^*) > 0$, the right-hand side of the inequality above is strictly less than $\pi(1-\underline{s})$.

(ii) Suppose an informative equilibrium exists for each c_n. Then from the proof of part (ii) of Lemma 2, $\lim_{n\to\infty} p_1(s_n^*) \le \frac{l}{l+\underline{s}}$. As $\underline{s} > \frac{l}{L}$ implies $\frac{l}{l+\underline{s}} < \frac{L}{1+L}$, we have $-L + p_1(s_n^*)(1+L) < 0$ for sufficiently large n. This contradicts that the agent is willing to undergo the operation at $p_1(s_n^*)$. Therefore, there are only uninformative equilibria when c_n is sufficiently small. As $\pi < \frac{l}{l+\underline{s}} < \frac{L}{1+L}$, the only uninformative-equilibrium outcome involves the agent not taking the operation, so $U(c_n) = T(c_n) = 0$. Q.E.D.

Appendix A.5: Proof of Proposition 4

Following the appendices above, denote $s_n^* \equiv s^*(c_n)$. Note first that the proofs of part (i) and part (iii) of Lemma 2 do not rely on the condition $\pi < \frac{l}{l+\underline{s}}$. Thus, whenever $\underline{s} > 0$, $p_0(s_n^*)$ does not converge to 0 in any sequence of informative equilibria, so information is not perfectly aggregated in the limit.

The rest of the proof shows the sufficiency of the condition $\underline{s} = 0$ for perfect information aggregation. Suppose $\underline{s} = 0$ and $\pi > \frac{L}{1+L}$. By Proposition 2, an informative equilibrium exists. Moreover, by Lemma 2, every sequence of informative equilibria has $\lim_{n\to\infty} s_n^* = 0$ and $\lim_{n\to\infty} p_1(s_n^*) = 1$.

Consider, for each $n \in \mathbb{N}$, the following (necessarily suboptimal) search strategy of the agent: sample a fixed number M_n of experts and take the operation in the end if and only if all of them recommend Y. Here, M_n is chosen such that the posterior reaches $p_1(s_n^*)$ if all M_n experts recommends Y. The agent's equilibrium payoff $V(\pi; s_n^*)$ is bounded from below by the expected payoff of this strategy, i.e.,

$$V(\pi; s_n^*) \ge \pi (1 - F(s_n^*|1))^{M_n} + (1-\pi)(1 - F(s_n^*|0))^{M_n}(-L) - c_n M_n.$$

As $V(\cdot; s_n^*)$ is weakly convex, $V(\pi; s_n^*) \leq \pi$. To prove the required result, it suffices to show that the lower bound of $V(\pi; s_n^*)$ above converges to π, as $\pi = \lim_{n\to\infty} V(\pi; s_n^*)$ implies $\lim_{n\to\infty} p_0(s_n^*) = 0$. The subsections below show respectively that $\lim_{n\to\infty} c_n M_n = 0$ and $\lim_{n\to\infty} \pi(1 - F(s_n^*|1))^{M_n} + (1-\pi)(1 - F(s_n^*|0))^{M_n}(-L) = \pi$.

Computation for $\lim_{n\to\infty} c_n M_n = 0$

Note first that M_n must lie between the bounds identified in inequality (15) with $q = \pi$. Together with the upper bound on $p_1(s_n^*)$ identified in inequality (19), we have the following upper bound on M_n:

$$M_n \leq \frac{\ln\left(\frac{\pi}{1-\pi}\frac{s_n^*}{l}\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)}{\ln\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)} + 1. \tag{20}$$

As $\lim_{n\to\infty} p_1(s_n^*) = 1 > \frac{L}{1+L}$, Lemma 4 implies that $V(p_1(s_n^*); s_n^*) = -L + (1+L)p_1(s_n^*) > 0$ and $p_0(s_n^*) < \frac{L}{1+L}$ for n sufficiently large. Therefore, equation (17) holds. Rearranging equation (17) gives

$$\begin{aligned} c_n &= -p_1(s_n^*)F(s_n^*|1)(1+L) + [p_1(s_n^*)F(s_n^*|1) + (1-p_1(s_n^*))F(s_n^*|0)] \\ &\quad \times \left[\max\left\{0, V\left(\left(1 + \frac{1-p_1(s_n^*)}{p_1(s_n^*)}\frac{F(s_n^*|0)}{F(s_n^*|1)}\right)^{-1}; s_n^*\right)\right\} + L\right]. \end{aligned} \tag{21}$$

Moreover, $p_1(s_n^*) > \pi$ for n sufficiently large, as $\lim_{n\to\infty} p_1(s_n^*) = 1$. Therefore, inequality (13) implies

$$p_1(s_n^*) \geq \frac{l}{l + s_n^*}. \tag{22}$$

Together with the fact that the max operator gives a value less than 1, equation (21) implies that

$$c_n \leq (1+L)\frac{s_n^*}{s_n^* + l}F(s_n^*|0). \tag{23}$$

Using both inequality (20) and (23), we get

$$c_n M_n \leq \left[\frac{1+L}{l+s_n^*}\frac{F(s_n^*|0)}{\ln\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)}\right]\left[s_n^*\ln\left(\frac{\pi}{(1-\pi)l}\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}s_n^*\right)\right] + (1+L)\frac{s_n^*}{l+s_n^*}F(s_n^*|0).$$

Now we take limit on both sides of the inequality above. Consider the first bracketed term on the right hand side:

$$\lim_{n\to\infty}\left[\frac{1+L}{l+s_n^*}\frac{F(s_n^*|0)}{\ln\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)}\right] = \frac{1+L}{l} \times \lim_{s^*\to 0}\frac{(1-F(s^*|0))(1-F(s^*|1))}{s^*(1-F(s^*|0)) - (1-F(s^*|1))} = -\frac{1+L}{l},$$

where we have used L'Hospital rule. Consider the second bracketed term:

$$\lim_{n\to\infty} s_n^*\ln\left(\frac{\pi}{(1-\pi)l}\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}s_n^*\right) = \lim_{s^*\to 0}\left[s^*\ln\frac{\pi}{(1-\pi)l} + s^*\ln\frac{1-F(s^*|0)}{1-F(s^*|1)} + s^*\ln s^*\right] = 0,$$

where we have used the fact that $\lim_{s^* \to 0} s^* \ln s^* = 0$. The last term $(1+L)\frac{s_n^*}{l+s_n^*}F(s_n^*|0)$ has a limit of 0. Therefore, $\lim_{n\to\infty} c_n M_n = 0$.

Computation for $\lim_{n\to\infty} \pi\left(1-F\left(s_n^*|1\right)\right)^{M_n} + (1-\pi)\left(1-F\left(s_n^*|0\right)\right)^{M_n}(-L) = \pi$

Note first that for n sufficiently large,

$$\begin{aligned}
\pi &\geq \pi\left(1-F\left(s_n^*|1\right)\right)^{M_n} + (1-\pi)\left(1-F\left(s_n^*|0\right)\right)^{M_n}(-L) \\
&\geq (1-\pi)\left(\frac{p_1\left(s_n^*\right)}{1-p_1\left(s_n^*\right)} - L\right)\left(1-F\left(s_n^*|0\right)\right)^{M_n} \\
&\geq (1-\pi)\left(\frac{l}{s_n^*} - L\right)\left(1-F\left(s_n^*|0\right)\right)^{M_n} \\
&\geq (1-\pi)\left(\frac{l}{s_n^*} - L\right)\left(1-F\left(s_n^*|0\right)\right)^{\frac{\ln\left(\frac{\pi}{1-\pi}\frac{s_n^*}{l}\frac{1-F\left(s_n^*|0\right)}{1-F\left(s_n^*|1\right)}\right)}{\ln\left(\frac{1-F\left(s_n^*|0\right)}{1-F\left(s_n^*|1\right)}\right)}+1} \equiv \Gamma_n.
\end{aligned}$$

In the computation above, the second inequality uses the lower bound in (15) with $q = \pi$, the third inequality uses (22), and the last inequality uses (20).

Below we show the required result by establishing that $\lim_{n\to\infty}\Gamma_n = \pi$. Upon rearranging, we express Γ_n as follows:

$$\Gamma_n = (1-\pi)\left(1-F\left(s_n^*|0\right)\right)\left(\frac{l}{s_n^*} - L\right)\left[\left(\frac{\pi}{1-\pi}\frac{s_n^*}{l}\frac{1-F\left(s_n^*|0\right)}{1-F\left(s_n^*|1\right)}\right)^{\left(1-\frac{\ln\left(1-F\left(s_n^*|1\right)\right)}{\ln\left(1-F\left(s_n^*|0\right)\right)}\right)^{-1}}\right].$$

As $\lim_{s_n^*\to 0}\frac{\ln(1-F(s_n^*|1))}{\ln(1-F(s_n^*|0))} = 0$ by L'Hospital rule, the term in the bracket has a limit of 0. It remains to evaluate the following limit:

$$\begin{aligned}
&\lim_{n\to\infty}\frac{1}{s_n^*}\left(\frac{\pi}{1-\pi}\frac{s_n^*}{l}\frac{1-F\left(s_n^*|0\right)}{1-F\left(s_n^*|1\right)}\right)^{\left(1-\frac{\ln\left(1-F\left(s_n^*|1\right)\right)}{\ln\left(1-F\left(s_n^*|0\right)\right)}\right)^{-1}} \\
=\ &\lim_{n\to\infty}\left(\frac{\pi}{1-\pi}\frac{1}{l}\frac{1-F\left(s_n^*|0\right)}{1-F\left(s_n^*|1\right)}\right)^{\left(1-\frac{\ln\left(1-F\left(s_n^*|1\right)\right)}{\ln\left(1-F\left(s_n^*|0\right)\right)}\right)^{-1}} \times \exp\left(\frac{\ln\left(s_n^*\right)}{\frac{\ln(1-F(s_n^*|0))}{\ln(1-F(s_n^*|1))}-1}\right).
\end{aligned}$$

For the last expression, the limit of the first term is $\frac{\pi}{1-\pi}\frac{1}{l}$. The limit of the exponent can be evaluated by L'Hospital rule:

$$\lim_{s^*\to 0}\frac{\ln s^*}{\frac{\ln(1-F(s^*|0))}{\ln(1-F(s^*|1))}-1} = \lim_{s^*\to 0}\left(\begin{array}{c}\frac{\ln(1-F(s^*|1))}{-(1-F(s^*|1))\ln(1-F(s^*|1))+s^*(1-F(s^*|0))\ln(1-F(s^*|0))} \\ \times\left[\frac{(1-F(s^*|0))(1-F(s^*|1))}{f(s^*|0)}\frac{\ln(1-F(s^*|1))}{s^*}\right]\end{array}\right).$$

As $f(0|0) > 0$ and $\lim_{s^* \to 0} \frac{\ln(1-F(s^*|1))}{s^*} = -f(0|1) = 0$, the limit of the bracketed term above is 0. The limit of the unbracketed term above can be computed by L'Hospital rule again:[39]

$$\begin{aligned} & \lim_{s^* \to 0} \frac{\ln(1-F(s^*|1))}{-(1-F(s^*|1))\ln(1-F(s^*|1)) + s^*(1-F(s^*|0))\ln(1-F(s^*|0))} \\ = & \lim_{s^* \to 0} \frac{1}{\ln(1-F(s^*|1)) - \ln(1-F(s^*|0)) + \frac{1-F(s^*|0)}{f(s^*|0)} \frac{\ln(1-F(s^*|0))}{s^*}} \frac{-1}{1-F(s^*|1)} \\ = & \; 1. \end{aligned}$$

Therefore, $\lim_{n \to \infty} \Gamma_n = (1-\pi) l \times \left(\frac{\pi}{1-\pi} \frac{1}{l}\right) = \pi$. Q.E.D.

Appendix A.6: Proof of Claim 1

Denote by piv the pivotal event for an individual expert: the number of Y recommendations among other experts is between $qn-1$ and qn, and that the agent is going to undergo the operation with her if she recommends Y. In an informative equilibrium, piv is necessarily a positive-probability event. Thus, an individual expert is willing to recommend Y if and only if her signal s leads to $\Pr(\omega = 1|s, piv) \geq \frac{l}{1+l}$. It is clear that $\Pr(\omega = 1|s, piv)$ is increasing in s, leading the expert to adopt a cutoff strategy of recommending Y if and only if s (weakly) exceeds a cutoff. Moreover, as all experts face the same problem, they necessarily adopt a symmetric cutoff strategy in an informative equilibrium. In sum, in an informative equilibrium of the voting mechanism with n experts, the symmetric cutoff, denoted by $s_n^* \in (\underline{s}, \bar{s})$, is characterized by the following equation:

$$\Pr(\omega = 1|s_n^*, piv) = \frac{l}{1+l}. \tag{24}$$

We first show that if n is sufficiently large, a solution $s_n^* \in (\underline{s}, \bar{s})$ to equation (24) exists. The conditional probability $\Pr(\omega = 1|s^*, piv)$ is given by

$$\Pr(\omega = 1|s^*, piv) = \left(1 + \frac{1}{s^*} \frac{1-\pi}{\pi} \frac{1-F(s^*|1)}{1-F(s^*|0)} \left(\left(\frac{1-F(s^*|0)}{1-F(s^*|1)}\right)^q \left(\frac{F(s^*|0)}{F(s^*|1)}\right)^{1-q}\right)^n\right)^{-1}.$$

It is immediate that $\Pr(\omega = 1|s^*, piv)$ is continuous in s^*. Moreover, fixing a s^* sufficiently close to $\bar{s}$, $\Pr(\omega = 1|s^*, piv) > \frac{l}{1+l}$ for n sufficiently large. On the other hand, fixing a s^* sufficiently close to $\underline{s}$, $\Pr(\omega = 1|s^*, piv) < \frac{l}{1+l}$ for n sufficiently large. The existence of a solution to equation (24) then follows from the intermediate value theorem.

Next, we show that $\lim_{n \to \infty} s_n^*$ exists. Let $\{s_{n_k}^*\}$ be a convergent subsequence of $\{s_n^*\}$. It is necessarily that the sequence $\left\{\left(\left(\frac{1-F(s_{n_k}^*|0)}{1-F(s_{n_k}^*|1)}\right)^q \left(\frac{F(s_{n_k}^*|0)}{F(s_{n_k}^*|1)}\right)^{1-q}\right)^{n_k}\right\}$ converges, so $\lim_{k \to \infty} \left(\frac{1-F(s_{n_k}^*|0)}{1-F(s_{n_k}^*|1)}\right)^q \left(\frac{F(s_{n_k}^*|0)}{F(s_{n_k}^*|1)}\right)^{1-q} =$

[39] The last equality makes use of the observation that $\lim_{s^* \to 0} \frac{\ln(1-F(s^*|0))}{s^*} = -f(0|0) < 0$.

1. Consequently, the limit $s^* \equiv \lim_{k\to\infty} s^*_{n_k}$ is characterized by the equation

$$\left(\frac{1-F(s^*|0)}{1-F(s^*|1)}\right)^q \left(\frac{F(s^*|0)}{F(s^*|1)}\right)^{1-q} = 1. \tag{25}$$

The left-hand side of equation (25) is decreasing in s^*, larger than 1 for s^* sufficiently close to $\underline{s}$, and smaller than 1 for s^* sufficiently close to $\bar{s}$. As a result, there exists a unique solution to equation (25), and thus a unique subsequential limit for $\{s_n\}$. Therefore, $\lim_{n\to\infty} s^*_n$ exists and is given by the solution to equation (25).

Finally, we show that $1-F(s^*|0) < q < 1-F(s^*|1)$, which in turn implies, by the law of large numbers, that the probability that the agent makes the correct operation decision converges to one as the number of consulted experts goes to infinity. Denote by F_ω^{-1} the inverse of $F(s|\omega)$. For each ω, the function $(1-F(s|\omega))^q (F(s|\omega))^{1-q}$ is inverted U-shaped in s with a maximum of $q^q(1-q)^{1-q}$, which occurs at $F_\omega^{-1}(1-q)$. As $F_1^{-1}(1-q) > F_0^{-1}(1-q)$, the solution to equation (25) necessarily occurs in the interval $\left(F_0^{-1}(1-q), F_1^{-1}(1-q)\right)$. Q.E.D.

Appendix A.7: Discussion of Conservative Experts

In this appendix, suppose $l \geq L$. Following the appendices above, let $s^*_n \equiv s^*(c_n)$.

First, in the proof of Proposition 2, Lemma 5 still holds. The proof for the existence of informative equilibrium is still valid, provided that $\pi \in \left(\max\left\{\frac{l}{l+\bar{s}}, \frac{L}{1+L}\right\}, \frac{l}{l+\underline{s}}\right)$. The extra condition $\pi > \frac{l}{l+\bar{s}}$ ensures that the fixed point of the correspondence Z does not occur at the boundary point $\bar{s}$.

Next, we explain that if $\pi \in \left(\max\left\{\frac{l}{l+\bar{s}}, \frac{L}{1+L}\right\}, \frac{l}{l+\underline{s}}\right)$, it is possible to pick a sequence of informative-equilibrium cutoff such that $s^*_n \to \underline{s}$. Let $\varepsilon \in \left(0, \frac{\bar{s}-\underline{s}}{2}\right)$. We modify the definition of Z in the proof of Proposition 2 as follows. Let $Z' : [\underline{s}, \underline{s}+\varepsilon] \rightrightarrows [\underline{s}, \underline{s}+\varepsilon]$ be a correspondence defined by

$$Z'(\hat{s}) \equiv \{\max\{\underline{s}, \min\{\underline{s}+\varepsilon, x(\hat{s}, b)\}\} : b \in \Psi(\hat{s})\}.$$

Replacing $\bar{s}$ with $\underline{s}+\varepsilon$ in the proof of Proposition 2 shows that Z' admits a fixed point. It suffices to show that the fixed point does not occur at $\underline{s}+\varepsilon$ when c is sufficiently small. To see this, first substituting $\hat{s} = \underline{s}+\varepsilon$ and $c = c_n$ into the agent's value function (8) and then taking limit, we get equation (12), to which $V(p) = p$ is the unique solution. Thus, as $c_n \to 0$, the agent's best response has $p_1(\underline{s}+\varepsilon)$ arbitrarily close to 1. Using (5) and (7), an individual expert's best response satisfies $x(s^*_n, b) \leq \left(\frac{1}{p_1(\underline{s}+\varepsilon)} - 1\right) \frac{1-F(\underline{s}+\varepsilon|1)}{1-F(\underline{s}+\varepsilon|0)} l$, which is strictly less than $\underline{s}+\varepsilon$ for n sufficiently large. As a result, there exists an $N' \in \mathbb{N}$ such that for all $n > N'$, the correspondence Z', and hence Z, has a fixed point

$s_n^* < \underline{s}+\varepsilon$. It is clear that this sequence $\{s_n^*\}_{n>N'}$ does not converge to $\bar{s}$. By the argument in the proof of part (i) of Lemma 2, it must converge to $\underline{s}$.

Finally, we show that if $s_n^* \to \bar{s}$, then $\limsup_{n\to\infty} p_1(s_n^*) \leq \frac{l}{1+l}$. By substituting (5) into (7), we get inequality (19). The right-hand side of the inequality converges to $\frac{l}{l+1}$ as $s_n^* \to \bar{s}$, giving $\limsup_{n\to\infty} p_1(s_n^*) \leq \frac{l}{1+l}$. As a result, we can apply the argument in the proof of Proposition 3 to obtain upper bounds on payoffs in the limit along such a sequence. If $\pi < \frac{l}{1+l}$, then $\limsup_{n\to\infty} U(c_n) \leq \pi\frac{1}{\frac{l}{1+l}}\left(-L+(1+L)\frac{l}{1+l}\right) = \pi\left(1-\frac{L}{l}\right)$. If $\pi \geq \frac{l}{1+l}$, $\limsup_{n\to\infty} U(c_n) \leq -L+\pi(1+L)$. For experts' payoff, if $\pi < \frac{l}{1+l}$, then $\limsup_{n\to\infty} T(c_n) = 0$; if $\pi \geq \frac{l}{1+l}$, $\limsup_{n\to\infty} T(c_n) \leq -l+\pi(1+l)$. It is straightforward that these bounds are lower than the respective upper bounds identified in part (i) of Proposition 3.

Appendix A.8: Discussion of Search with Recall

In this appendix, we explain more formally why the probability of the state conditional on being pivotal is still given by equation (4) when recall of experts is allowed.

With recall, an expert is pivotal if, given her recommendation Y, the agent decides to undergo the operation and, in particular, he picks her for the operation. Let $y(h)$ be the number of Y recommendations in history h. Dropping the dependence of q_ω on strategies to simplify notation, the likelihood ratio of this pivotal event $piv(b)$ can be written as

$$\frac{\Pr(piv(b)\,|\omega=0;s^*,b)}{\Pr(piv(b)\,|\omega=1;s^*,b)} = \frac{\sum_{h\in H} q_0(h)\left[\frac{b_1(h,Y)}{y(h,Y)}+\sum_{h'\in H} q_0(h'|h,Y)(1-F(s^*|0))\frac{b_1(h,Y,h',Y)}{y(h,Y,h',Y)}\right]}{\sum_{h\in H} q_1(h)\left[\frac{b_1(h,Y)}{y(h,Y)}+\sum_{h'\in H} q_1(h'|h,Y)(1-F(s^*|1))\frac{b_1(h,Y,h',Y)}{y(h,Y,h',Y)}\right]}, \tag{26}$$

where $q_\omega(h'|h,Y)$ stands for the ex-ante probability that history (h,Y,h') realizes at the beginning of a period, conditional on the realization of the history (h,Y) by the end of some previous period. To understand (26), fix an expert j and a state ω. Suppose an agent with history h shows up for consultation. If $b_1(h,Y)>0$, then the agent stops and takes the operation with positive probability given a Y recommendation by expert j. In this case, expert j is pivotal with probability $\frac{b_1(h,Y)}{y(h,Y)}$, as the agent will choose, with equal probabilities, among the experts who recommended Y to carry out the operation. If $b_1(h,Y)<1$ (and thus $q_0(\varnothing|h,Y)>0$), then agent may leave expert j for further consultations. With a conditional probability $q_\omega(h'|h,Y)$, the agent arrives at a history (h,Y,h') prior to consulting some other experts. In this case, he takes the operation with probability $(1-F(s^*|\omega))\,b_1(h,Y,h',Y)$ by the end of the period, and in particular, he returns to expert j for operation with probability $(1-F(s^*|\omega))\frac{b_1(h,Y,h',Y)}{y(h,Y,h',Y)}$. The bracketed terms in (26) therefore are the probabilities, for the respective state ω, that expert j is pivotal conditional on the agent's history being h.

The numerator and denominator in (26) can be simplified as follows:

$$\begin{aligned}
& \sum_{h\in H} q_\omega(h)\left[\frac{b_1(h,Y)}{y(h,Y)}+\sum_{h'\in H} q_\omega(h'|h,Y)(1-F(s^*|\omega))\frac{b_1(h,Y,h',Y)}{y(h,Y,h',Y)}\right] \\
= & \sum_{h\in H}\left\{q_\omega(h)\frac{b_1(h,Y)}{y(h,Y)}+q_\omega(h)\left[\sum_{h'\in H}\frac{q_\omega(h,Y,h')}{q_\omega(h)(1-F(s^*|\omega))}(1-F(s^*|\omega))\frac{b_1(h,Y,h',Y)}{y(h,Y,h',Y)}\right]\right\} \\
= & \sum_{h\in H}\left[q_\omega(h)\frac{b_1(h,Y)}{y(h,Y)}+\sum_{h'\in H} q_\omega(h,Y,h')\frac{b_1(h,Y,h',Y)}{y(h,Y,h',Y)}\right] \\
= & \sum_{h\in H} q_\omega(h)\, b_1(h,Y).
\end{aligned}$$

The first equality follows from definitions and Bayes' rule. The last equality is obtained by noting that a history (h,Y,h',Y) is counted exactly $y(h,Y,h',Y)$ times in the summations.

By the computation above, the likelihood ratio of the pivotal event with recall is the same as that in equation (4). Intuitively, the term $\sum_{h\in H} q_\omega(h)\, b_1(h,Y)$ is the expected number of pivotal experts in state ω, which does not depend on whether the agent sticks with the last expert or goes back to some previously consulted expert for the operation.

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