# COMMON ENROLLMENT IN SCHOOL CHOICE

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ABSTRACT. Increasingly, more school districts across the US are using centralized admissions for charter, magnet, and neighborhood schools in a common enrollment system. We first show that, across all school-participation patterns, full participation in the common (or unified) enrollment system leads to the most preferred outcome for students. Second, we show that, in general, participation by all schools may not be achievable because schools have incentives to stay out. This may explain why some districts have not managed to attain full participation. We also consider some specific settings where full participation can be achieved and propose two schemes that can be used by policymakers to achieve full participation in general settings.

## 1. Introduction

Market design has been successful in proposing methods for clearinghouses to allocate scarce resources in different contexts, such as matching students with schools, doctors with residency positions, and patients with donated organs. Thus far, most of the focus has been on the properties of the proposed methods. However, a typical assumption that all agents in the market would voluntarily participate in a clearinghouse may be violated, since an agent may deem it more advantageous to not participate and to find better matches by staying outside of the system. In this paper, we consider the school choice setting and analyze student welfare and participation incentives for schools in a clearinghouse.[1]

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[1]See Abdulkadiroğlu and Sönmez (2003), who study school choice as a market-design problem.

Even though the voluntary-participation assumption is innocuous for district-run schools, it is problematic for charter, magnet, and private schools, which can choose not to participate. While some school districts have clearinghouses to assign students to district-run schools, almost all charter schools run their admissions systems independently.[2] However, there has been recent interest in a small but growing number of districts in having a clearinghouse that assigns students to all public schools. This system is usually referred to as *common enrollment* (also called *unified* or *universal*). Currently, there are at least eight school districts that have a common enrollment system: Camden (NJ), Chicago (IL), Denver (CO), the District of Columbia, Indianapolis (IN), New Haven (CT), New Orleans (LA), and Newark (NJ).[3]

We propose a framework to study the participation incentives of schools in a common enrollment system and the implications of full participation on student welfare. Furthermore, we explore the ability of common enrollment systems to give the necessary incentives to all schools to join the system rather than to evade it. This leads us to study a school's incentives to unilaterally evade the system. When there is no school that has incentives to unilaterally evade the system, we say that the environment is *integration compatible* (or that full participation is achievable), and when there is at least one school that has strict incentives to unilaterally evade the system, we say that the environment is *integration incompatible* (or that full participation is not achievable).

In our model, each school decides whether to join the common enrollment system or evade it. If a school joins the system, then it participates in the clearinghouse. The clearinghouse, regardless of which schools participate in it, uses the student-proposing or school-proposing deferred-acceptance algorithm (DA) of Gale and Shapley (1962) to assign students to the participating schools. On the other hand, if a school evades the system, it runs its own admissions program. We observe that a school's incentives to evade the system depend on the timing of the evading school's admissions relative to that of the clearinghouse used by the system. Therefore, we consider three natural timing scenarios: The evading school runs its admissions program either before, simultaneously with, or after the clearinghouse.

We first investigate the implications of a common enrollment system on student welfare. We show that across all three timing scenarios, all students are weakly better off

[2]Charter schools are publicly funded but they have more freedom than district-run public schools. In particular, they can run their own admissions, and they can structure their curriculum. In the 2017-18 school year, more than 7,000 charter schools enrolled nearly 3.2 million students (National Alliance for Public Charter Schools, 2017).

[3]For Camden's system, see `https://www.camdenenrollment.org`, for Chicago's `https://go.cps.edu/`, for Denver's `http://schoolchoice.dpsk12.org/`, for Indianapolis's `https://enrollindy.org/`, for New Haven's `http://choice.nhps.net`, for New Orleans's `http://enrollnola.org/`, for Newark's `http://www.newarkenrolls.org/`, and for Washington D.C.'s `http://www.myschooldc.org/` (all accessed on 12/6/2018).

when all schools join, compared to when one school unilaterally evades, the system (Theorem 1). Ideally, we would like to argue that the result is true when multiple schools evade the system and run their admissions independently, but we do not explicitly model the decentralized market when multiple schools evade. However, if the outcome of the decentralized admissions process among the evading schools coincides with that of DA,[4] then a more general result holds (Lemma 1): Students are better off under the full-participation outcome compared to the outcome when some schools evade. This further motivates the investigation of policies that may achieve full participation.

Next, we study the incentives of schools to unilaterally evade the system when the evading school runs its admissions program after the clearinghouse. We show that every school weakly prefers to evade the system if all other schools have joined it (Theorem 2). Therefore, unless each school is indifferent between joining or evading, it is impossible to achieve full participation. The intuition for Theorem 2 is as follows. When a school evades the system, all students are assigned to less preferred schools in the system because there is more competition for each school seat. In particular, students who would have been matched with the evading school get strictly less preferred schools. As a result, all of these students, and potentially more, apply to the evading school. Consequently, by evading, a school gets a more preferred set of students. Furthermore, we show, via an example, that the failure to achieve full participation persists for all stable matching algorithms (Remark 2).

We further evaluate a school's incentives to unilaterally evade the system across all timing scenarios. We show that an evading school prefers to run its admissions program as late as possible: It prefers running admissions after to running simultaneously with the clearinghouse, and it prefers running simultaneously to running before the clearinghouse (Theorem 3). This result establishes that incentives to unilaterally evade are minimized if the evading schools are restricted to running their admissions only before the clearinghouse. Therefore, the set of problems in which full participation is achievable when evading schools run admissions before the clearinghouse is a superset of the set of problems in which full participation is achievable when evading schools run admissions simultaneously with or after the clearinghouse. Consequently, restricting evading schools to run admissions before the clearinghouse may sometimes be effective in achieving full participation. However, we also establish that even when the evading schools run their admissions before the clearinghouse, full participation may not be achievable (Remark 3).

[4]Note that if there is a single evading school, then the outcome of the admissions process of this school is identical to the DA outcome. If the students are vertically differentiated, as in college admissions in some countries such as Brazil, China, Tunisia, and Turkey, then there is a unique stable matching that coincides with the outcome of serial dictatorship by the students and DA.

Since full participation is not achievable in general, we consider specific environments where full participation can be achieved. We show that when schools are *vertically differentiated*, i.e., when students have a common ranking of schools, full participation is achievable for all timing scenarios. Furthermore, when schools are not vertically differentiated, then full participation is not guaranteed. More explicitly, if students do not have the same preference ranking, then there exist school choice rules for which at least one school is strictly better off by unilaterally evading the system when the student-proposing DA is used in the clearinghouse (Theorem 4).

In another specific environment, schools are *revenue maximizers*. Charter schools get public funding per student, and in many cases their admissions criteria are imposed by law or by the school district. In particular, they have to run lotteries to admit students when they are over demanded. As a result, charter schools may not have actual preferences. However, charter schools maximize their revenue by admitting as many students as possible without going over their capacity. When schools are revenue maximizers, we show that full participation is achievable if the admissions timing of evading schools is restricted to the preemption case (Theorem 5). We further argue that full participation is not guaranteed in the other two timing scenarios (Remark 5). These results give further support for a policy that restricts the admissions timing of evading schools.

Theorems 2 and 4 highlight the difficulty of achieving full participation. But are there policies that can help achieve full participation, which improves student welfare? One obvious policy is to impose mandatory participation by new legislation. This may be possible in some states where there is strong support by public officials for common enrollment. However, new legislation may not be possible when charter schools have strong public support, such as in Washington D.C., where around 45% of the overall student population is served by charter schools.[5] Even if such legislation is possible, it may not be imposed on already existing charters with grandfathered rights to run their own admissions. In addition, imposing such legislation may require officials to use a significant amount of political capital. Therefore, even when new legislation is possible, integrating schools via other means is a more desirable option. In the rest of the paper, we use market-design tools that help achieve full participation and suggest two possible remedies.

A potential remedy for achieving full participation is imposing a no-poaching policy: A student who is matched in the clearinghouse cannot attend an evading school. Such a policy puts a restriction on the evading schools' admissions only if the evading schools move after the clearinghouse; therefore, we focus only on this scenario to examine the effectiveness of the no-poaching policy. A variant of this policy is used in the National

[5] See `http://www.dcpcsb.org/facts-and-figures-market-share` (accessed on 12/6/2018).

Resident Matching Program.[6] However, we show that the no-poaching policy does not always guarantee full participation by providing an example in which a school is strictly better off by unilaterally evading the system (Remark 6). We further investigate the effectiveness of the no-poaching policy when schools are revenue maximizers. We show that the no-poaching policy is effective in achieving full participation if all schools are revenue maximizers (Theorem 6). These results highlight that the effectiveness of the no-poaching policy depends on the domain of school preferences. In practice, the no-poaching policy can be implemented by obstructing funding to an evading school for any student matched in the clearinghouse.

Finally, we propose a second remedy to help achieve full participation. If some schools evade the system, the clearinghouse adds a virtual copy of each evading school and runs its algorithm as if all schools participated. The set of students who get matched with a virtual school is unmatched whereas any other student gets their assigned school. The idea behind the virtual-school algorithm is to *internalize the external competition* created by evading schools. In the after timing scenario, if a school unilaterally evades the system, then all the students who are matched with the virtual copy of the evading school in the clearinghouse, and potentially others, apply to the evading school. When the algorithm is stable, the evading school admits the set of students assigned to the virtual school, i.e., the same set of students that it would have gotten had it joined the system. Hence, full participation is achievable in the after case when the clearinghouse uses a stable virtual-school algorithm. In the other two timing scenarios, it is no longer true that a school gets the same set of students by joining and unilaterally evading the system. In fact, unilateral evasion always leads to a weakly less preferred outcome and sometimes to a strictly less preferred outcome. Thus, full participation is achievable under all three timing scenarios (Theorem 7).

The implementation of the virtual-school mechanism requires two seemingly strong assumptions. First, it requires knowledge of the choice rules of the schools that evade the system. However, this assumption is innocuous in our setting for charter schools, since their choice rules are determined by law or by the school district. Second, students have to rank all schools, including the schools that evade the system. In particular, if a student ranks an evading school and is matched with the virtual copy of it in the clearinghouse, she is unmatched. However, this student will be admitted by the evading school after the clearinghouse.

We note that our analysis is restricted to analyzing the incentives of a school to unilaterally evade the system. This analysis allows us to examine when full participation

[6] See `http://www.nrmp.org/match-participation-agreements/` for more details (accessed on 12/6/2018).

is achievable. However, due to the generality we assume in school choice rules and because we do not explicitly model the decentralized admissions process when multiple schools evade, we are unable to present general predictions about school behavior when full participation is not achievable.

To study school behavior more generally, we present a stylized game played among schools in Section 9. In this game, students are vertically differentiated, and choice rules of schools are identical. Typically, students are vertically differentiated when each student has a test score, and schools prefer a student with a higher test score to a student with a lower test score. In the game, there is a public school that always participates in the system and two charter schools that simultaneously decide whether to join the system or evade it. Schools that evade run their admissions after the clearinghouse. We explain in detail matching outcomes when all schools join, when only one charter school evades, and when both charter schools evade. Evading schools get an advantage over joining the system. This is because the clearinghouse operates without anticipating that some of the students who are assigned to the schools will leave for the evading school(s), and because schools accept only students with high test scores. Therefore, the evading schools admit students with high scores who prefer them, and some of the students with moderate test scores who did not get matched in the clearinghouse. In the unique pure-strategy Nash equilibrium, both charter schools evade the system, and the students are worse off in equilibrium compared to the outcome when all schools join.

The related literature discussion is in Section 10.

## 2. Model

**2.1. Preliminary Definitions.** A ***student-assignment problem*** $P$ is a tuple $(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})$ where

- $\mathcal{S} = \{s_1, \dots, s_n\}$ is a set of students,
- $\mathcal{C} = \{c_1, \dots, c_m\}$ is a set of schools,[7]
- $\succ_{\mathcal{S}} = (\succ_{s_1}, \dots, \succ_{s_n})$ is a list of strict student preferences, and
- $Ch_{\mathcal{C}} = (Ch_{c_1}, \dots, Ch_{c_m})$ is a list of school choice rules.

For any student $s$, $\succ_s$ is a strict preference relation over $\mathcal{C} \cup \{s\}$ where $c \succ_s s$ means that student $s$ strictly prefers school $c$ to being unmatched. In that case, school $c$ is ***acceptable*** to student $s$. Let $\succeq_s$ be the "at least as good as" relation induced by $\succ_s$. For any school $c$, $Ch_c$ is a choice rule over all sets of students where $Ch_c(S)$ is the chosen subset for any

[7] We do not distinguish between private, district-run public, or charter schools. But whenever we consider a school that can choose to join the system, then we implicitly assume that it is not a district-run public school.

set of students $S$.[8] Throughout the paper, we assume that choice rules of schools are path independent:

**Definition 1.** *School $c$'s choice rule is **path independent** if, for every set of students $S$ and $S'$,*

$$Ch_c(S \cup S') = Ch_c(S \cup Ch_c(S')).$$

If a choice rule is path independent, then any set of students can be divided into (not necessarily disjoint) subsets and the choice rule can be applied to these subsets in any order without changing the final choice. This concept was first introduced informally by Arrow (1951), and formally by Plott (1973).[9] Path-independent choice rules arise naturally, for instance, when schools have diversity considerations.[10]

A school $c$ ***revealed prefers*** a set of students $S$ to another set of students $S'$ if it chooses $S$ when all students in both $S$ and $S'$ are available. More formally, there exists $\tilde{S}$ such that $\tilde{S} \supseteq S \cup S'$ and $Ch_c(\tilde{S}) = S$. Furthermore, if $S$ and $S'$ are different, then school $c$ ***strictly revealed prefers*** set $S$ to set $S'$. When the choice rule is path independent, revealed preference becomes a *partial order*.

Choice rule of school $c$ is ***responsive*** if there exist a strict preference relation $\succ_c$ over $\mathcal{S} \cup \{c\}$ and a capacity $q_c \in \mathbb{N}$ such that, for any set of students $S$, $Ch_c(S)$ is constructed as follows. Say that student $s$ is ***acceptable*** for school $c$ if $s \succ_c c$. Let $k$ be the minimum of the capacity $q_c$ and the number of acceptable students in $S$. If $k = 0$, then $Ch_c(S) = \emptyset$. If $k \geq 1$, then $Ch_c(S) = \bigcup_{i=1}^{k} \{s_i^*\}$, where $s_i^*$ is the $i$-th highest-ranked student in $S$ with respect to $\succ_c$. In words, the choice from a set is the union of the highest-ranked acceptable students up to the capacity of the school. Responsive choice rules are path independent.[11]

The outcome of a student-assignment problem is a ***matching*** between students and schools. Formally, a matching $\mu$ is a function on the set of all agents such that

- for any student $s$, $\mu(s) \in \mathcal{C} \cup \{s\}$,
- for any school $c$, $\mu(c) \subseteq \mathcal{S}$, and
- for any student $s$ and school $c$, $\mu(s) = c$ if, and only if, $s \in \mu(c)$.

The first two conditions require that a student is either matched with a school or left unmatched and that a school is matched with a set of students. The last condition requires

[8]Taking school choice rules as primitives of the model rather than their preferences has many advantages, see Chambers and Yenmez (2017). For example, choice rules are useful to study matching with externalities (Pycia and Yenmez, 2014).

[9]Path independence is equivalent to a mild consistency condition and the *substitutability* condition (Aizerman and Malishevski, 1981).

[10]See, for example, Hafalir et al. (2013), Ehlers et al. (2014), and Echenique and Yenmez (2015).

[11]Chambers and Yenmez (2018) provide an axiomatic characterization of responsive choice rules using path independence.

that a school is matched with a student if, and only if, the student is in the set of students matched with the school. This condition ensures the feasibility of the matching.

The student-proposing deferred acceptance algorithm (DA) of Gale and Shapley (1962) is commonly used in clearinghouses to assign students to schools. We provide a description when a subset of schools, say $C$, participates.

**Step 1:** Each student proposes to her most preferred acceptable school in $C$ if such a school exists. Suppose that $S_c^1$ is the set of students who proposes to school $c$. School $c$ tentatively accepts students in $Ch_c(S_c^1)$ and permanently rejects the rest. If there are no rejections, then stop.

**Step k:** Each student who was rejected in Step $k-1$ proposes to her next preferred acceptable school in $C$, if such a school exists. Suppose that $S_c^k$ is the union of the set of students who were tentatively accepted by school $c$ in Step $k-1$ and of the set of students who just proposed to school $c$. School $c$ tentatively accepts students in $Ch_c(S_c^k)$ and permanently rejects the rest. If there are no rejections, then stop.

This algorithm ends in finite time since there can only be a finite number of proposals. Denote the outcome of the student-proposing DA by SPDA$(\mathcal{S}, C, \succ_{\mathcal{S}}, Ch_C)$, where $C$ is the set of participating schools. When all schools participate, the outcome is denoted by SPDA$(P)$.

The clearinghouse can use an alternative algorithm, the school-proposing DA, in which schools make the proposals instead of students. We provide a definition in Appendix A. Its outcome is denoted by CPDA$(\mathcal{S}, C, \succ_{\mathcal{S}}, Ch_C)$, where $C$ is the set of participating schools. When all schools participate, the outcome is denoted by CPDA$(P)$.

Both SPDA and CPDA produce a stable matching when school choice rules are path independent:

**Definition 2.** *A matching $\mu$ is **stable** if*

*(1) (individual rationality for students) for every student $s$, $\mu(s) \succeq_s s$,*
*(2) (individual rationality for schools) for every school $c$, $Ch_c(\mu(c)) = \mu(c)$, and*
*(3) (no blocking) there exists no $(c, s)$ such that $c \succ_s \mu(s)$ and $s \in Ch_c(\mu(c) \cup \{s\})$.*

Furthermore, when choice rules of schools are path independent, the SPDA produces the **student-optimal stable matching**: Each student weakly prefers the outcome of this algorithm to any other stable matching (Hatfield and Milgrom, 2005; Chambers and Yenmez, 2017).

**2.2. Mechanisms and Integration Compatibility.** We consider an environment in which schools can join or evade a common enrollment system and the matching outcome depends on the set of participating schools. In this environment, a ***mechanism*** is a profile of functions $(\mathcal{M}_C)_{C \subseteq \mathcal{C}}$ that maps each student-assignment problem into a matching based on the set of participating schools. For a problem $P$, when $C$ is the set of schools that join the system, the matching outcome is $\mathcal{M}_C(P)$, which specifies the set of students assigned to each school, including those that evade the system. The following property of mechanisms formalizes the notion of achieving full participation.

**Definition 3.** *Fix a set of schools $\mathcal{C}$. A mechanism $(\mathcal{M}_C)_{C\subseteq\mathcal{C}}$ is **integration compatible** for a problem $P=(\mathcal{S},\mathcal{C},\succ_{\mathcal{S}},Ch_{\mathcal{C}})$, if every school $c_i\in\mathcal{C}$ revealed prefers its outcome in $\mathcal{M}_{\mathcal{C}}(P)$ to its outcome in $\mathcal{M}_{\mathcal{C}\setminus\{c_i\}}(P)$. It is **integration incompatible** for a problem $P$ if there exists a school $c_i$ that strictly revealed prefers its outcome in $\mathcal{M}_{\mathcal{C}\setminus\{c_i\}}(P)$ to its outcome in $\mathcal{M}_{\mathcal{C}}(P)$.*

If a mechanism $(\mathcal{M}_C)_{C\subseteq\mathcal{C}}$ is integration compatible for a problem $P$, then every school revealed prefers its outcome when all schools join to its outcome from unilateral evasion. However, when it is integration incompatible, there exists a school that strictly revealed prefers the latter to the former. To check whether a mechanism is integration compatible or incompatible, we only need to consider the outcomes when either all schools join the system, i.e., $\mathcal{M}_{\mathcal{C}}(P)$, or when exactly one school evades it, i.e., $\mathcal{M}_{\mathcal{C}\setminus\{c_i\}}(P)$ for every school $c_i$. Therefore, for the rest of the paper, we do not explicitly provide a description of the matching produced by a mechanism when at least two schools evade.

We interpret integration compatibility or incompatibility of a mechanism as a test to see whether we could expect voluntary participation by all schools in the system. If a mechanism is integration compatible, we say that full participation is achievable, and when it is integration incompatible, we say that full participation is not achievable.

**2.3. Mechanisms under Different Timing Scenarios.** In a student-assignment problem, each school decides whether to join a common enrollment system that uses a clearinghouse to assign students to participating schools, or evade it and run its own admissions program. In other words, each school has two actions: 'join' or 'evade'. The clearinghouse employs either version of the deferred-acceptance algorithm unless otherwise noted. The evading schools run their admissions either before, simultaneously with, or after the clearinghouse. We consider all three admissions timing scenarios below.

Each admissions timing scenario results in a different mechanism. The clearinghouse uses the same version of the deferred-acceptance algorithm regardless of the set of schools that participate in it, which we denote by DA. Since our focus is on integration compatibility, we specify the mechanism when a school unilaterally evades the system under different timing scenarios. If there is no evading school, the matching outcome is denoted by $\mu^{int}$. Likewise, when there is one evading school $c$, the matching outcome is denoted by $\mu_c^p$, $\mu_c^s$, and $\mu_c^a$, for the preemption, simultaneous with, and after scenarios, respectively.

**Preemption:** In this scenario, the evading school $c$ runs its admissions before the clearinghouse. All students who prefer $c$ to their outside option apply to $c$, i.e., $S_c \equiv \{s | c \succ_s s\}$ is the set of applicants to $c$. School $c$ admits $Ch_c(S_c)$. Students learn whether they got in or not before participating in the clearinghouse. If any student has been admitted by $c$ and assigned to another school in the clearinghouse, then the student accepts her preferred school among these two. The rest of the students are assigned to their matches if they have been admitted to a school; otherwise, they remain unmatched.

A student admitted to the evading school $c$ participates in the clearinghouse as well. Potentially, this student can submit either her preferences over all acceptable schools in the clearinghouse or she can rank only those schools that are preferred to $c$. In other words, the student can truncate her preference at $c$. We assume that students admitted to the evading school submit truncated preferences that rank the schools that are more preferred than $c$ truthfully. The alternative, that students rank all acceptable schools in the clearinghouse, makes the preemption case the same as the case in which the evading school runs its admissions simultaneously with the clearinghouse, a scenario which we consider next.

Now, we formally describe the matching outcome of the mechanism for the preemption case when only a single school, $c$, evades, i.e., we describe $\mu_c^p$. School $c$ admits the set of students $Ch_c(S_c)$. Then students participate in the clearinghouse, where DA is used with the profile of truncated student preferences, denoted as $\succ'_{\mathcal{S}}$. If $s \in Ch_c(S_c)$, then $\succ'_s$ is defined as follows:

- $c' \succ'_s c''$ if and only if $c' \succ_s c''$, and
- $c' \succ'_s s$ if and only if $c' \succ_s c$.

If $s \notin Ch_c(S_c)$, then $\succ'_s = \succ_s$.

The outcome of the clearinghouse is $\text{DA}(\mathcal{S}, \mathcal{C} \setminus \{c\}, \succ'_{\mathcal{S}}, Ch_{\mathcal{C} \setminus \{c\}})$. Each student admitted to two schools is matched with her preferred school. Therefore, the set of students who enroll in school $c$ is the admitted students who prefer school $c$ to the matching in the clearinghouse: $\mu_c^p(c) = \{s \in Ch_c(S_c) | c \succ_s \text{DA}(\mathcal{S}, \mathcal{C} \setminus \{c\}, \succ'_{\mathcal{S}}, Ch_{\mathcal{C} \setminus \{c\}})(s)\}$. For any other school $c'$, the set of students matched to this school is $\mu_c^p(c') = \text{DA}(\mathcal{S}, \mathcal{C} \setminus \{c\}, \succ'_{\mathcal{S}}, Ch_{\mathcal{C} \setminus \{c\}})(c') \setminus \mu_c^p(c)$.

**Simultaneous:** In this scenario, the evading school $c$ runs its admissions simultaneously with the clearinghouse. In this case, all students participate in the clearinghouse and those who find $c$ acceptable also apply to $c$. If any student has been admitted by $c$ and assigned to another school in the clearinghouse, then the student is matched with her preferred school. The rest of the students are assigned to their matches if they have been admitted to a school; otherwise, they remain unmatched.

We formally describe the matching outcome of the mechanism for the simultaneous case when only a single school, $c$, evades, i.e., we describe $\mu_c^s$. The outcome of the clearinghouse is $\text{DA}(\mathcal{S}, \mathcal{C} \setminus \{c\}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}\setminus\{c\}})$. School $c$ admits the set of students $Ch_c(S_c)$, where $S_c = \{s | c \succ_s s\}$. Therefore, students who are matched with school $c$ are those who have been admitted by $c$ and who prefer $c$ to their assigned schools in the clearinghouse: $\mu_c^s(c) = \{s \in Ch_c(S_c) | c \succ_s \text{DA}(\mathcal{S}, \mathcal{C} \setminus \{c\}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}\setminus\{c\}})(s)\}$. For any other school $c'$, the set of students matched to this school is $\mu_c^s(c') = \text{DA}(\mathcal{S}, \mathcal{C} \setminus \{c\}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}\setminus\{c\}})(c') \setminus \mu_c^s(c)$.

**After:** In this scenario, the evading school $c$ runs its admissions after the clearinghouse. First, all students participate in the clearinghouse. Then they learn their assigned schools and decide whether to apply to $c$ or not. We assume that only students who prefer $c$ to their assigned schools apply.[12] School $c$ admits students from the set of applicants. The admitted students then are permanently matched with $c$ since they all prefer $c$ to their assigned schools in the clearinghouse. The rest of the students are matched with their assigned schools in the clearinghouse or remain unmatched.

We formally describe the matching outcome of the mechanism for the after case when only a single school, $c$, evades, i.e., we describe $\mu_c^a$. The outcome of the clearinghouse is $\text{DA}(\mathcal{S}, \mathcal{C} \setminus \{c\}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}\setminus\{c\}})$. A student applies to $c$ if she prefers $c$ to her assigned school. Therefore, the set of students that applies to school $c$ is $\{s | c \succ_s \text{DA}(\mathcal{S}, \mathcal{C} \setminus \{c\}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}\setminus\{c\}})(s)\}$. Hence, the set of students that school $c$ is matched with is $\mu_c^a(c) = Ch_c(\{s | c \succ_s \text{SPDA}(\mathcal{S}, \mathcal{C} \setminus \{c\}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}\setminus\{c\}})(s)\})$. For any other school $c'$, the set of students matched to this school is $\mu_c^a(c') = \text{SPDA}(\mathcal{S}, \mathcal{C} \setminus \{c\}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}\setminus\{c\}})(c') \setminus \mu_c^a(c)$.

## 3. The Case for Integration: Student Welfare

In this section, we analyze student welfare based on whether all schools join the common enrollment system or not. In particular, we show that students are better off when all schools participate.

**Theorem 1.** *For every student, the matching outcome when all schools join the system, $\mu^{int}$, is more preferred than the matching outcome when one school $c$ unilaterally evades the system, regardless of when $c$ admits its students (i.e., the matching outcomes in $\mu_c^p$, $\mu_c^s$, and $\mu_c^a$).*

One important implication is that public policies that incentivize schools to join a common enrollment system are desirable from the student-welfare perspective.

To prove Theorem 1, we establish a more general result and study one possible scenario of decentralized admissions when there are two separate clearinghouses, each running the same algorithm (SPDA or CPDA). Each school participates in only one clearinghouse.

[12] This assumption is justified, for example, if there is a small application fee or if the student has to exert costly effort to apply. It mirrors the assumption in the preemption case that students submit their truncated preferences.

Denote the set of schools participating in the first and second clearinghouses by $\mathcal{C}^1$ and $\mathcal{C}^2$, respectively. First students rank schools in $\mathcal{C}^1$ and get assigned to a school in $\mathcal{C}^1$ or remain unmatched. Then they participate in the second clearinghouse. They can either submit their full ranking of schools in $\mathcal{C}^2$ or just submit the ranking of schools in $\mathcal{C}^2$ that are preferred to their assigned school in the first clearinghouse. In other words, students can truncate their preferences at their assigned school in the first clearinghouse.[13] Finally, each student who has been assigned to a school in both clearinghouses gets matched with her preferred school. The rest of the students are matched with the school that they have been admitted to or remain unmatched. We call this the **divided system**. The alternative student matching system is a clearinghouse with all the schools that uses the same algorithm. We call this the **integrated system**.

**Lemma 1.** *Every student weakly prefers the matching outcome in the integrated system to the matching outcome in the divided system.*

Note that Theorem 1 follows from Lemma 1. This is because, when there is only one evading school, both versions of the deferred-acceptance algorithm are equivalent to the school choosing from the set of available students.

## 4. School Incentives for Integration

To explore when a mechanism is integration compatible, we study the incentives of a school to join a common enrollment system when all other schools have joined it. We first consider the timing scenario when the evading school admits students after the clearinghouse.

**Theorem 2.** *Every school $c$ revealed prefers the matching outcome when $c$ unilaterally evades the system and admits students after the clearinghouse, $\mu_c^a$, to the matching outcome when all schools join the system, $\mu^{int}$.*

The intuition for this result is as follows. Removing school $c$ in the deferred acceptance algorithm makes all students weakly worse off because there is more competition for school seats. Therefore, students who get matched with $c$ in $\mu^{int}$ are strictly worse off in the clearinghouse when $c$ is removed because student preferences over schools are strict. Consequently, all of these students in $\mu^{int}(c)$ apply to $c$ when it unilaterally evades the system. Therefore, when $c$ unilaterally evades the system and admits students after the clearinghouse, it considers a set of students including the ones in $\mu^{int}(c)$. Consequently, $c$ revealed prefers the set of students that it gets in $\mu_c^a$ to the set of students that it gets in $\mu^{int}$.

[13]The case when students submit the same preferences to multiple clearinghouses that use SPDA has also been analyzed in Doğan and Yenmez (2018). See also Manjunath and Turhan (2016).

One implication of Theorem 2 is the following:

**Corollary 1.** *Suppose that the evading school runs its admissions after the clearinghouse. Then a mechanism is integration compatible for a problem if, and only if, for every school c the set of students that c gets when all schools join, $\mu^{int}(c)$, is equal to the set of students that c gets when it unilaterally evades, $\mu^a_c(c)$.*

In other words, integration compatibility requires each school to be indifferent between joining or evading when others have joined. This is a stringent requirement in many problems. In the next section, we study the domain of problems when it can be satisfied.

*Remark* 1. Theorem 2 does not hold when the clearinghouse uses other algorithms, even when they produce stable matchings: We provide an example in Appendix C.1 where a school strictly prefers to join the system when it uses the school-proposing DA if all schools join and the student-proposing DA if only one school evades.

*Remark* 2. Theorem 2 shows that when the clearinghouse uses a version of DA, a school weakly prefers to evade the system. This opens up the question of whether the clearinghouse can use algorithms that produce stable outcomes such that the resulting mechanism is integration compatible. In Appendix C.2, we provide an example that demonstrates that no such mechanism is integration compatible. Specifically, there exists a school in the example such that there is a unique stable matching when all schools join or this school unilaterally evades the system. But this school strictly prefers the outcome when it unilaterally evades to the outcome when all schools join.

We next compare the set of students that a school gets by unilaterally evading the system under the three timing scenarios.

**Theorem 3.** *Every school c revealed prefers the matching outcome of the after case, $\mu^a_c$, to the matching outcome of the simultaneous case, $\mu^s_c$. It also revealed prefers the matching outcome of the simultaneous case, $\mu^s_c$, to the matching outcome of the preemption case, $\mu^p_c$.*

This result shows that an evading school prefers to run its admissions as late as possible. It further implies that a policy that restricts admissions timing of evading schools to the preemption case can achieve full participation more often than the other timing scenarios: More specifically, if a mechanism is integration compatible for a problem in the after case or simultaneous case, then it is also integration compatible for this problem in the preemption case. Likewise, if a mechanism is integration compatible for a problem in the after case, then it is also integration compatible for the problem in the simultaneous case.

*Remark* 3. When a school unilaterally evades the system and runs its admissions before or simultaneously with the clearinghouse, it can be worse off than the integration outcome because admitted students are not guaranteed to enroll in the school. The following example demonstrates this possibility. In this example, the clearinghouse always uses SPDA.

Consider the following student-assignment problem $P_1$. Suppose that there are two schools $c_1, c_2$ and two students $s_1, s_2$. Schools have responsive choice rules with capacity one. Agent preferences are as follows: $\succ_{c_1}$: $s_1 \succ s_2 \succ c_1$, $\succ_{c_2}$: $s_1 \succ s_2 \succ c_2$, $\succ_{s_1}$: $c_2 \succ c_1 \succ s_1$ and $\succ_{s_2}$: $c_1 \succ c_2 \succ s_2$. This information is summarized in Table 1A.

| $\succ_{c_1}$ | $\succ_{c_2}$ | $\succ_{s_1}$ | $\succ_{s_2}$ |
|---|---|---|---|
| $s_1$ | $s_1$ | $c_2$ | $c_1$ |
| $s_2$ | $s_2$ | $c_1$ | $c_2$ |
| $c_1$ | $c_2$ | $s_1$ | $s_2$ |

(A) Problem $P_1$

| $\succ_{c_1}$ | $\succ_{c_2}$ | $\succ_{s_1}$ | $\succ_{s_2}$ |
|---|---|---|---|
| $s_1$ | $s_2$ | $c_2$ | $c_1$ |
| $s_2$ | $s_1$ | $c_1$ | $c_2$ |
| $c_1$ | $c_2$ | $s_1$ | $s_2$ |

(B) Problem $P_2$

TABLE 1. Problem $P_1$ is integration compatible while Problem $P_2$ is integration incompatible in both the preemption and simultaneous cases.

The integration outcome for problem $P_1$ is determined by SPDA when all schools participate. Student $s_1$ is matched with $c_2$ and $s_2$ is matched with $c_1$. In the preemption case, when $c_1$ unilaterally evades the system and admits students before the clearinghouse, both students apply to $c_1$. School $c_1$ accepts $s_1$ and rejects $s_2$. In the clearinghouse, $s_2$ applies to $c_2$ because $c_2$ is acceptable and $s_1$ applies to $c_2$ because $c_2$ is more preferred than $c_1$. School $c_2$ accepts $s_1$ and rejects $s_2$. Since $s_1$ strictly prefers $c_2$ to $c_1$, she is matched with $c_2$. School $c_1$ is unmatched. Likewise, the matching outcome when $c_1$ unilaterally evades the system and admits students simultaneously with the clearinghouse is the same. Therefore, $c_1$ strictly revealed prefers the integration outcome $\mu^{int}$ to the matching outcomes $\mu^p_{c_1}$ and $\mu^s_{c_1}$. In a similar exercise, where $c_2$ unilaterally evades the system, it is easy to verify that $c_2$ is matched with $s_1$ in all three possibilities $\mu^{int}$, $\mu^p_{c_2}$, and $\mu^s_{c_2}$. Therefore, the mechanisms for the preemption case and simultaneous case are integration compatible for problem $P_1$.

However, it is also possible that a school prefers unilateral evasion outcome to the integration outcome under these two timing scenarios. To illustrate this, we modify problem $P_1$ by changing the preference relation of $c_2$ so that $s_2$ is ranked higher than $s_1$. Table 1B summarizes the preference profile. We call this problem $P_2$.

The integration outcome for problem $P_2$ is the same as the integration outcome for problem $P_1$. In the preemption case, when $c_1$ unilaterally evades the system and admits students before the clearinghouse, both students apply to $c_1$. It accepts $s_1$ and rejects $s_2$. In the clearinghouse, $s_2$ applies to $c_2$ because $c_2$ is acceptable and $s_1$ applies to $c_2$ because

$c_2$ is more preferred than $c_1$. School $c_2$ accepts $s_2$ and rejects $s_1$. In this preemption case, $s_1$ is matched with $c_1$ and $s_2$ is matched with $c_2$. The matching outcome when $c_1$ unilaterally evades the system and admits students simultaneously with the clearinghouse is the same. Under both timing scenarios, $c_1$ strictly revealed prefers the evasion outcome, which is $\{s_1\}$, to the integration outcome, which is $\{s_2\}$. Therefore, the mechanisms for the preemption case and the simultaneous case are integration incompatible for problem $P_2$.

## 5. Vertically Differentiated Schools

As we have shown in the previous section, in general settings, the mechanisms that we study are integration incompatible for some problems. Therefore, a natural question to ask is for which environments these mechanisms are integration compatible. Next, we provide an answer to this question.

**Theorem 4.** *Suppose that students have the same preferences over schools. Then the mechanisms induced by the three timing scenarios are integration compatible. Conversely, if students find all schools acceptable but they do not have the same preferences over them and the clearinghouse uses SPDA, then there exist school choice rules such that these mechanisms are integration incompatible.*

We relegate the proof to Appendix B. When all students have the same preferences over schools, i.e., when schools are vertically differentiated, there is a unique stable matching. This stable matching can be produced by a serial dictatorship of schools in which schools choose the set of students that they like using the order in student preferences. Therefore, there is a clear hierarchy of schools, and, as a result, there is no real competition between schools. Consequently, regardless of whether a school participates in the clearinghouse or not, a school chooses from the same set of students: The set of students who are unmatched after the higher-ranked schools admit their students. Since each school is indifferent between joining the system when others have joined and unilaterally evading it, the mechanisms are integration compatible by Corollary 1.

When students find all schools acceptable but do not have the same preferences over them, then there exist two students and two schools such that the two students rank these schools differently. To prove the second statement in the theorem, we construct choice rules such that when one of these two schools unilaterally evades the system, the outcome is the school-optimal stable matching, regardless of when the evading school runs its admissions. When they all join the system, the outcome is the student-optimal stable matching because SPDA is used. As a result, the mechanisms are integration incompatible for this problem.

*Remark* 4. We have shown that when schools are vertically differentiated, the mechanisms are integration compatible. Does a similar result hold if students are vertically differentiated? When schools have responsive choice rules with the same preferences over students, the mechanism induced by the after timing scenario is integration incompatible for some problem. We provide such an example in Appendix C.3. This example also demonstrates that the mechanism is integration incompatible when schools only care about the number of students that they get. Such school preferences are realistic when choice rules are enforced by the school district or law (see the discussion in Section 6).

The results of Theorem 4 suggest that a clearinghouse in which all schools participate is hard to establish unless schools are vertically differentiated, which is a restriction on student preferences. Next we consider a natural restriction on school preferences and study if full participation is achievable under this assumption.

## 6. Revenue Maximizing Charter Schools

Even though charter schools are subject to fewer rules and regulations than district-run public schools, most of the states in the US have strict admission policies.[14] For example, in New York charter schools have to give higher priorities in admissions to returning students, siblings of students already enrolled in the school, and students who reside in the school district. Furthermore, when there is excess demand, schools may have to use lotteries to choose their students.[15]

On the other hand, charter schools are publicly funded and usually get funding per student. Motivated by this, we assume that charter schools would like to admit as many students as possible without violating their capacity. We formalize such choice rules as follows.

**Definition 4.** *Choice rule of school $c$ is* ***capacity filling*** *if there exists a capacity $q_c$ such that for every set of students $S$, $|Ch_c(S)| = \min\{q_c, |S|\}$.*

If a school has a capacity-filling choice rule, it admits all applicants if the number of applicants is less than its capacity, and it fills its capacity when the number of applicants is weakly more than the capacity.[16] For example, responsive choice rules satisfy this property.

We study the integration compatibility of mechanisms for revenue-maximizing schools. We first consider the preemption case.

[14] Magnet schools, which are public schools with specialized curricula, can choose their students based on exam scores, interviews, or auditions. Therefore, it is safe to assume that choice rules present the actual preferences for public magnet schools as well as private schools.

[15] A school that uses its own admission criteria may be subject to probation and closure.

[16] Alkan (2001) introduces this property and calls it *quota filling*.

**Theorem 5.** *Suppose that school choice rules are capacity filling, schools only care about the number of students, and they prefer to have more students (up to their capacity). Then the mechanism induced by the preemption case is integration compatible.*

When a school unilaterally evades the system in the preemption case, the evading school takes a risk of admitting a student who will then be admitted to a more preferred school in the clearinghouse. When the school choice rule is capacity filling, such a school would strictly prefer to evade the system only if this school does not fill its capacity in the integration outcome. In this case, a key observation we make is that the set of students a school gets by evading preemptively is a subset of the students it gets under the integration outcome, when in the latter case the school does not fill its capacity. This observation delivers the result.

*Remark* 5. We also consider the other two timing scenarios. When the evading school runs its admissions simultaneously with or after the clearinghouse, the mechanism induced may not be integration compatible even when schools are revenue maximizing and they have capacity-filling choice rules. To see this, modify problem $P_2$ in Remark 3, presented in Table 1B, so that $c_1$'s capacity is two. Then $c_1$ can increase the number of students that it gets by unilaterally evading when it runs its admissions simultaneously with or after the clearinghouse, which uses either version of DA.

In the rest of this section, we discuss the implications of imposing the assumption that schools are revenue maximizing on our earlier results. In Theorem 2, we show that a school revealed prefers the set of students that it gets by unilaterally evading and running its admissions after the clearinghouse to the set of students that it gets by joining the system. When the choice rule is capacity filling, the school receives weakly more students by evading the clearinghouse. Therefore, the result that the school weakly prefers to evade remains valid even when the school only cares about the number of students or, equivalently, revenue. In practice, not all charter schools can fill their capacities.

The first statement in Theorem 4, which shows that the three mechanisms are integration compatible if students have the same preferences over schools, remains true. In this case, each school is indifferent between joining or unilaterally evading the system, so a school admits the same number of students in both cases.

Next we investigate the effectiveness of some familiar policies in achieving full participation for general environments.

## 7. Remedy: No Poaching

The National Resident Matching Program has a "binding commitment" policy:

```
All Match commitments are binding. The ranking of applicants by
a program director and the ranking of programs by an applicant
establishes a binding commitment to offer or to accept an
appointment if a match results.
```

In other words, whenever a match results between a program and a doctor who have ranked each other, the residency program has to make an offer to the doctor and the doctor has to accept the offer. This policy is implemented to stop the programs from making offers to doctors after they learn the outcome of the Match.

In our setting, we can impose a similar "no poaching" policy by not giving public funds to the evading school for any student that it enrolls who was matched in the clearinghouse. Obviously, this policy restricts the evading schools only when admissions are done after the clearinghouse, so in what follows we focus on this timing scenario.

Intuitively, such a policy discourages schools from evading the clearinghouse as a school that evades the system can only admit unmatched students. Indeed, when schools are revenue maximizing and they have capacity-filling choice rules, the mechanism induced by the after case and the no-poaching policy is integration compatible.

**Theorem 6.** *Suppose school choice rules are capacity filling, schools only care about the number of students, and they prefer to have more students (up to their capacity). Then the mechanism induced by the after case and the no-poaching policy is integration compatible.*

This result is easy to see. Suppose that students find all schools acceptable. Consider the case when school $c$ unilaterally evades the system. Then school $c$ can only admit the unmatched students. Note that a student is unmatched only when all the schools in the clearinghouse fill their seats. But if school $c$ joins the system, then it will be able to admit at least the same number of students, if not more.[17]

However, when schools are not revenue maximizers, the mechanisms that we consider may not be integration compatible for some problems.

*Remark* 6. Suppose that the evading school runs its admissions after the clearinghouse and that the no-poaching policy is adopted. The mechanism when the clearinghouse uses SPDA is integration incompatible for Problem P2 in Remark 3 (see Table 1B). The mechanism when the clearinghouse uses CPDA is not integration compatible for the problem introduced in Appendix C.2 (see Table 3). We verify these results in Appendix C.4.

[17] In a subsequent work, Afacan (2016) shows that even when students may find some schools unacceptable, the same result holds.

## 8. Remedy: Internalizing the External Competition via Virtual Schools

A second approach to incentivize schools to join the common enrollment system is to use a different algorithm in the clearinghouse when some schools evade the system. We consider the following algorithm and the mechanism induced by it.

The input to the virtual-school algorithm is the student-assignment problem and the set of participating schools. The algorithm first uses either version of DA on the whole problem as if all schools were participating, effectively adding a virtual copy of each evading school. Students matched with the virtual schools are unmatched. Other students are matched with their DA outcomes. We call this algorithm **the virtual-school DA**.

Since integration compatibility of a mechanism depends only on the outcomes when at most one school evades the system, we formally describe the mechanism induced by the virtual-school DA in each timing scenario when at most one school evades. When all schools participate, the clearinghouse uses DA as before.

**Preemption:** Suppose that school $c$ unilaterally evades the system and runs its admissions before the clearinghouse. Then $c$ first admits from the set of students who find it acceptable, i.e., $Ch_c(S_c)$ where $S_c = \{s | c \succ_s s\}$. The students admitted by $c$ submit truncated preferences as in Section 2.3, so the student preference profile used in the clearinghouse is $\succ'_{\mathcal{S}}$. Therefore, $\check{\mu}^p_c(c) \equiv \{s \in Ch_c(S_c) | c \succeq_s \text{DA}(\mathcal{S}, \mathcal{C}, \succ'_{\mathcal{S}}, Ch_{\mathcal{C}})(s)\}$ is the set of students that $c$ is matched with. For any other school $c'$, $\check{\mu}^p_c(c') \equiv \text{DA}(\mathcal{S}, \mathcal{C}, \succ'_{\mathcal{S}}, Ch_{\mathcal{C}})(c')$ is the set of students $c'$ is matched with.

**Simultaneous:** Suppose that school $c$ unilaterally evades the system and runs its admissions simultaneously with the clearinghouse. Then $c$ first admits from the set of students who find it acceptable, i.e., $Ch_c(S_c)$. The clearinghouse uses the virtual-school DA (with virtual school $c$). Then $\check{\mu}^s_c(c) \equiv \{s \in Ch_c(S_c) | c \succeq_s \text{DA}(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})(s)\}$ is the set of students that $c$ is matched with. For any other school $c'$, $\check{\mu}^s_c(c') \equiv \text{DA}(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})(c') \setminus \check{\mu}^s_c(c)$ is the set of students that $c'$ is matched with.

**After:** Suppose that school $c$ unilaterally evades the system and runs its admissions after the clearinghouse. Then DA is run with virtual school $c$. The set of students matched with the virtual school will remain unmatched in the clearinghouse. The set of students who apply to school $c$ is $\tilde{S} = \{s : c \succ_s \text{DA}(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})(s)\} \cup \text{DA}(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})(c)$. Here $\text{DA}(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})(c)$ is the set of students matched with the virtual school, so they are unmatched and they want to apply to school $c$. Hence, $\tilde{S}$ is equivalent to $\{s : c \succeq_s \text{DA}(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})(s)\}$. The set of students who get matched with school $c$ is then $\check{\mu}^a_c(c) \equiv Ch_c(\tilde{S}) = Ch_c(\{s : c \succeq_s \text{DA}(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})(s)\})$. For any other school $c'$, $\check{\mu}^a_c(c') \equiv \text{DA}(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})(c') \setminus \check{\mu}^a_c(c)$.

The virtual-school DA requires that students rank all schools, including the evading schools, and it also requires knowledge of the choice rules of evading schools. The first requirement can be implemented by adding the outside schools as options when families rank schools. The second requirement is innocuous because charter school choice rules are determined by the school district or by law.[18]

**Theorem 7.** *For every timing scenario, the mechanism induced by the virtual-school DA and the timing scenario is integration compatible for all problems.*

We provide the key observation for the mechanism induced by the after case. If a school evades the system and runs its admissions after the clearinghouse, the set of students who would like to be matched with that school is the same set of students that the school gets in the integration outcome. Therefore, the mechanism is integration compatible.

Now, we explain the intuition of this result. When a school evades the system, it avoids competition within the algorithm, which makes all students weakly worse off. Furthermore, in the after case, after the outcome of the clearinghouse is finalized, it can choose from the set of students who would like to go to this school. Since the remaining schools cannot make or receive additional offers at this point, the evading school is better off. The virtual-school mechanism takes away the advantage of the evading school by creating competition within the clearinghouse: When some students are matched with the virtual school, the remaining schools will still be able to make or receive offers in the clearinghouse. In other words, the virtual school mechanism *internalizes the external competition*.

In the simultaneous case, the evading school may be strictly worse off in comparison to joining the system. This is because the evading school is matched with a subset of the set of students it would have been matched with if it joined the system. In the preemption case, the students admitted by the evading school submit their truncated preferences to the clearinghouse. We use a comparative statics result to show that even in the preemption case, the evading school is weakly worse off in general, and may be strictly worse off for some problems.

*Remark 7.* Consider the setting of Section 6 where schools only care about the number of students and choice rules of schools are capacity filling. Under this assumption, for every timing scenario, the mechanism induced by the virtual-school DA and the timing scenario is still integration compatible.

## 9. A Simple Participation Game

We now present an illustrative example of a school participation game. The players are two charter schools. There are also a (district-run) public school and a continuum of students who are assumed to be nonstrategic and their behavior is exogenously given.

[18]This assumption is satisfied, for example, if students take a standardized exam that determines their ranking for different schools.

At the beginning of the game, charter schools simultaneously choose whether to join a common enrollment system or to run their admissions independently. The public school is assumed to always join the system. In this example, we assume that a charter school that evades the system moves after the clearinghouse.

A school is denoted by $i \in \{1,2,3\}$, where $i = 1$ is the public school and $i \in \{2,3\}$ are the charter schools. Each student has a strict preference ranking over the schools. Because there are three schools, there are six possible student preferences. Let $\mathcal{P}$ denote the set of all school rankings, with a generic element denoted by $p$. For each preference ranking $p$, there is a unit mass of students whose preference ranking is $p$. Each student also has a test score, $t \in [0,1]$. We assume that test scores are uniformly distributed over $[0,1]$, and this distribution is independent from student preferences. Each student is represented by the tuple $(p,t)$. Let $F$ denote the induced measure of students on $\mathcal{P} \times [0,1]$. For instance, the measure of students who prefer school 1 to school 2 to school 3, and receive a score larger than 0.5 is 0.5. Figure 1 depicts student preferences and scores. The total measure of students is six, while each school has a capacity that can accommodate a group of students with measure two.

Choice rules of schools over the students depend only on the observable test score, and every school prefers students with higher scores to those with lower scores. These choice rules are responsive with capacity of measure two. The underlying preference is that $(p_1,t_1) \succ (p_2,t_2)$ if $t_1 > t_2$. Therefore, if a student with score $t$ is chosen from a set, then all students with scores higher than $t$ are also chosen from the same set. Formally, the resulting choice rule for school $i$ is as follows: For any measurable set $A \subseteq \mathcal{P} \times [0,1]$, $Ch_i(A) = S$, where $S$ is the largest set $S' \subseteq A$ with $F(S') \leq 2$, and $(p_1,t_1),(p_2,t_2) \in A$, $t_1 > t_2$, and $(p_2,t_2) \in S'$ imply $(p_1,t_1) \in S'$.

Recall that the players in the game are the charter schools. The students and public school are not strategic agents. We describe the matching outcomes depending on the behavior of the charter schools.

**Both schools join.** In this environment, if all three schools participate in the clearinghouse that runs SPDA, then each student is placed in their top choice, because capacity constraints when students are placed in their top choices are not binding.

**School 3 evades.** We now analyze what happens if schools 1 and 2 join the system while school 3 evades the system and admits students after the clearinghouse. In the first stage, all students apply to their top choices among $\{1,2\}$. Because there is excess demand of students, each school $i \in \{1,2\}$ accepts students who apply to them with scores $t \geq \frac{1}{3}$. In the second stage, the rejected students with scores $t < \frac{1}{3}$ apply to their second choices, but since each school has accepted students with higher scores, they get rejected again. SPDA ends here since there are no more schools that rejected students can apply to.

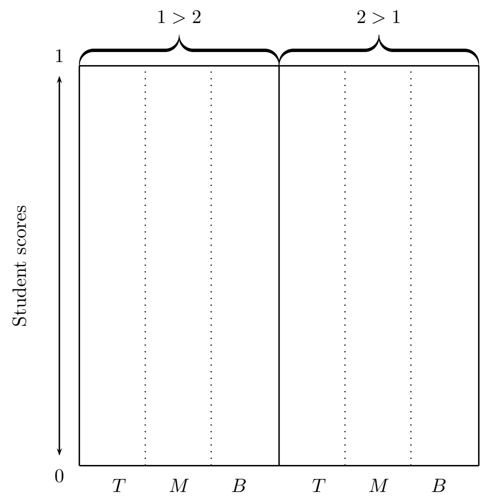


FIG. 1. This figure illustrates student preference rankings and scores, where each sub-column is one student preference profile. The first sub-column denotes students whose ranking is $3 > 1 > 2$, the second one $1 > 3 > 2$, and the third one $1 > 2 > 3$. The fourth, fifth, and sixth columns represent $3 > 2 > 1$, $2 > 3 > 1$, and $2 > 1 > 3$, respectively.

Next, all students that rank school 3 as their top choice, and all other students that are unmatched in the clearinghouse, i.e., students with scores $t < \frac{1}{3}$, apply to school 3. Hence, the demand for school 3 has measure

$$2 + 4 \times \frac{1}{3} = \frac{10}{3} > 2,$$

i.e., school 3 is over demanded. Therefore, school 3 accepts all students with scores greater than $t_3$ where

$$t_3 : 2(1 - t_3) + 4\left(\frac{1}{3} - t_3\right) = 2,$$

i.e., $t_3 = \frac{2}{9}$.

The final allocation is the following: Schools $i \in \{1, 2\}$ get students who rank $i$ as their top choices and whose score is not less than $\frac{1}{3}$, while school 3 gets all remaining students with scores not less than $\frac{2}{9}$. Figure 2 illustrates the final allocation in this case.

School 3 revealed prefers its final allocation to its allocation in the integration outcome, because it fills its capacity and the measure of students with scores smaller than $\frac{2}{9}$ and whose top choice is school 3 are replaced by students with scores between $\frac{2}{9}$ and $\frac{1}{3}$.[19] Schools 1 and 2 are worse off because they get students with scores not less than $\frac{1}{3}$ who rank them as their top choice, i.e., these schools have unmatched seats compared to the allocation that would prevail if all three schools join the system. Students are also worse

[19]More precisely, school 3's choice from the union of the set of students it gets by evading and the set of students it gets by joining would be equal to the set of students it gets by evading.

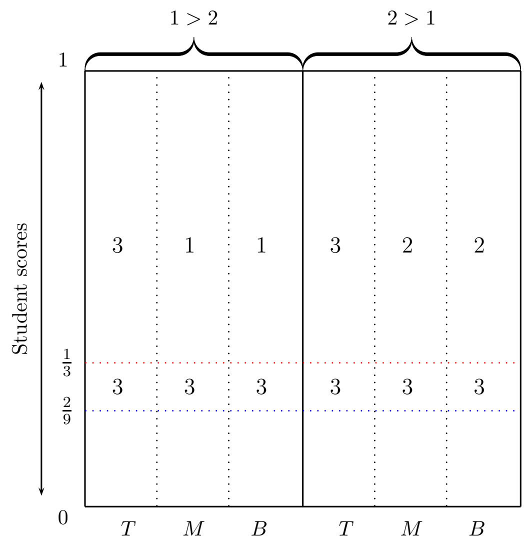


FIG. 2. This figure illustrates the final allocation when school 2 joins but school 3 evades.

off since in the integration outcome, each student gets her top choice, whereas when school 3 evades, some students get placed in their second or third choices and students with scores less than $\frac{2}{9}$ are unmatched.

**Schools 2 and 3 evade.** Suppose now that schools 2 and 3 evade the system. One caveat is that when there are multiple schools who run their own admissions in a decentralized market, the details of the procedure by which the schools and students move in the extensive form matter. To simplify the analysis, we assume that students in the aftermarket apply to their most preferred schools only.[20]

In the clearinghouse, all students apply to school 1, and the SPDA algorithm stops after one round with school 1 accepting all students with scores not less than $\frac{2}{3}$.

Next, all unmatched students and all students whose top choice is not school 1 apply to either school 2 or school 3. School $i \in \{2,3\}$ accepts, among all students applying, those that prefer school $i$ to school $j$ ($j \in \{2,3\} \setminus \{i\}$) and whose score is not less than $t_i$, where

$$t_i : 2(1 - t_i) + \left(\frac{2}{3} - t_i\right) = 2,$$

[20] Such an assumption is made for simplicity, and can be dispensed with if, for instance, there is a small application fee, or if students have to exert costly effort in applying to schools. Likewise, if the admissions in the aftermarket is run via a serial dictatorship by the students, then the final allocation is identical to that in which students apply to their top choices. Such an allocation can be also implemented by a protocol where students with higher test scores get admitted to their preferred schools among those schools whose capacity has not been filled yet. See Andersson et al. (2018) for the details of such a protocol.

so, $t_i = \frac{2}{9}$. Figure 3 below depicts the final allocation.

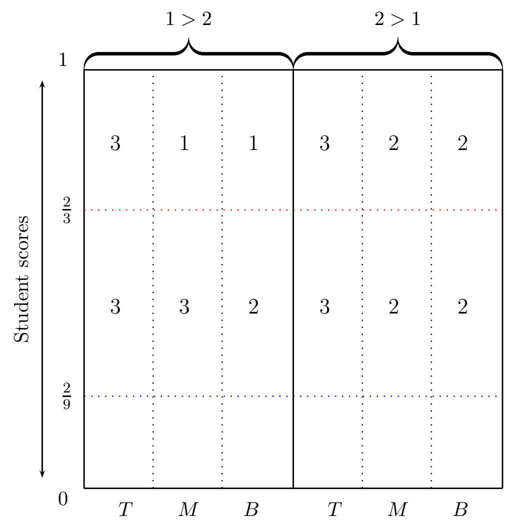


FIG. 3. This figure illustrates the final allocation when schools 2 and 3 both evade.

In this case, each school $i \in \{2,3\}$ revealed prefers the outcome when both $i$ and $j$ ($j \in \{2,3\} \setminus \{i\}$) evade to the outcome when only $j$ evades. This is because, when $j$ evades and $i$ joins the system, $i$ matches with students that have scores of at least $\frac{1}{3}$ and for whom $i$ is the top choice. If $i$ also evades, then $i$ is matched with all these students, the students who have preferences $i > 1 > j$ or $i > j > 1$ and have scores in $[\frac{2}{9}, \frac{1}{3})$, and also with the students that have scores in $[\frac{2}{9}, \frac{2}{3})$ and have preferences $1 > i > j$.[21] Students are worse off compared to the integration outcome, since in the integration outcome all students are matched with their top choices, whereas when at least one school evades, some students get unmatched, and some students are matched with a school that is not their top choice.

**Equilibrium:** Suppose that at time $t = -1$, both charter schools $i \in \{2,3\}$ simultaneously choose one of two actions, $a_i \in \mathcal{A} \equiv \{join, evade\}$. The matching outcomes for each action profile are described above. An action profile $(a_2, a_3)$ is a pure-strategy Nash equilibrium if each school $i$ revealed prefers the matching outcome induced by $(a_i, a_j)$ to the matching outcome induced by $(\mathcal{A} \setminus \{a_i\}, a_j)$.

This game admits a unique pure-strategy Nash equilibrium in which both schools evade. This is because a charter school gets a strictly more preferred set of students by evading the system regardless of the decision of the other charter school.

[21] School $i$ revealed prefers this set to the set of students that it gets when only $j$ evades because the latter set is a subset of the former set.

**No-poaching policy:** If the no-poaching policy is implemented, then when a school unilaterally evades the system, it can only accept students that are unmatched in the clearinghouse. These are the students with test scores less than $1/3$. Hence, a school revealed prefers the set of students it gets by joining to the set of students it gets by evading. So, with the no-poaching policy, full participation is a pure-strategy Nash equilibrium. We now argue that full participation is the unique pure-strategy Nash equilibrium. Suppose school $i \in \{2,3\}$ evades. Then, by evading, school $j \in \{2,3\} \setminus \{i\}$, gets students with scores less than $2/3$ who rank school $j$ as their top choice. Whereas, by joining, school $j$ gets students with scores at least $1/3$ that prefer school $j$ to school 1. Therefore, school $j$ revealed prefers joining the system to evading it, regardless of school $i$'s action. Hence, full participation is the unique pure-strategy Nash equilibrium.

**Virtual Schools:** If a school $i \in \{2,3\}$ unilaterally evades the system, the clearinghouse adds a virtual copy of school $i$ and runs SPDA. Therefore, only the students that rank school $i$ as their top choice are unmatched, and all the other students are matched with their top choice. Therefore, only the unmatched students apply to school $i$, and school $i$ is matched with only those students that rank $i$ as their top choice. Therefore, a school is indifferent between unilaterally evading and joining. Hence, full participation is a pure-strategy Nash equilibrium. Furthermore, if both schools evade, then the resulting outcome is still the same as the integration outcome. Therefore, the virtual-school mechanism replicates the integration outcome regardless of the participation decisions of schools. Notice that, in this example, the virtual-school mechanism achieves full participation rather trivially. But, with the other timing scenarios, the evading school may strictly prefer to join the system to unilaterally evading it.

## 10. Related Literature

In this section, we discuss the related literature on manipulation and sequential mechanisms in matching.

The literature on manipulation via pre-arranged matches (Sönmez, 1999) and capacity manipulation (Sönmez, 1997; Konishi and Ünver, 2006), is related to our work. Manipulation via pre-arranged matching is a scheme that a school can use to admit one student before the clearinghouse is run, as in the preemption case of our model. However, in our preemption case, unlike the above scheme, when a school evades the system, it cannot participate in the clearinghouse, whereas students matched with the evading school still participate. Furthermore, we also consider the other timing scenarios when an evading school can also admit students simultaneously with or after the clearinghouse.

In capacity manipulation, schools can underreport their capacities to get a better set of students. In our model, a school, when evading the system, causes two things: first, it affects the outcome of the matching at the clearinghouse, by not being a part of it, and this is similar to a manipulation scheme by declaring zero capacity. There is, however, a second effect that the evading school imposes on the outcome of the final matching: The evading school becomes an *outside option* for the students who are considering whether to accept their match in the clearinghouse or to reject their match there and match with the evading school. This latter effect of evasion from a central system is absent in the capacity manipulation scheme where the school that manipulates its capacity gets its students entirely from the clearinghouse.

The second related literature is on sequential matching mechanisms. Blum and Rothblum (2002) and Boyle and Echenique (2009) study sequential bargaining in matching markets.[22] They model this as a process in which agents make new offers after each new entry, which then can be accepted or rejected, and a stable outcome is reached before the next agent enters the market. They show that there is an advantage to entering later in the market. In this literature, there is no strategic decision by agents; the order in which agents enter the market is exogenously fixed. Furthermore, our game is very different as there is a clearinghouse and each school decides whether they want to join or evade. In contrast to this literature, our main focus is to come up with institutions that yield participation by all schools.

Manjunath and Turhan (2016) consider an alternative setting in which two clearinghouses run in parallel. They provide a mechanism based on repeated DA that produces a stable matching. They do not study the incentives of schools to join a particular system; in contrast, they assume that the set of schools participating in each clearinghouse is given exogenously. In another work, Dur and Kesten (2018) study sequential matching mechanisms and show that these mechanisms do not have desirable fairness and efficiency properties. In a subsequent work to ours, Andersson et al. (2018) study a school choice setting with private and public schools and study properties of dynamic mechanisms when private-school admissions are done before or after public-school admissions. Even though their main focus is the welfare properties of these mechanisms, they also consider the incentives of a private school to join the public-school admissions system. They show that a private school weakly prefers to join the public-school admissions system when private school admissions are done before public school admissions. In their setting, schools have the same priority ranking over students. In our setting, a school may prefer to evade the system even when it runs its admissions before the clearinghouse, as we do not restrict school priorities over students (Remark 3).

[22]Blum and Rothblum (2002) consider the one-to-one matching problem whereas Boyle and Echenique (2009) analyze the many-to-one matching problem with responsive preferences.

In a coalition-formation setting, Pycia (2012) studies extensive form games and shows that every stable coalition structure can be supported as an equilibrium under some assumptions on games and agent preferences. Different from this work, we focus on certain extensive form games for student admissions and analyze school incentives to join the system.

In a recent work, Ashlagi and Roth (2014) study the incentives of hospitals to include all incompatible patient-donor pairs in the kidney exchange market. They show that in finite markets, hospitals do not fully disclose incompatible pairs for algorithms that maximize the number of transplants.[23] However, they also show that the inefficiency loss in large markets is small. This work is closely related to the capacity-manipulation literature that we discuss above, even though the kidney-exchange market has different objectives and properties than the school choice problem.

## 11. Conclusion

Increasingly, more school districts are adopting a common enrollment system for student assignment. We provide a framework to study the participation incentives of schools in such a system. We first show that the outcome when all schools participate in the system is preferred by students to the outcome when only a subset of the schools participate. Second, we show that, for general environments, full participation is not achievable.

In our model, we assume that schools can only choose to join or evade the system and that students are not strategic agents. Using our framework, the analysis can be expanded to situations in which students are strategic agents and schools have more actions. In addition, we do not conduct a full-equilibrium analysis that would allow us to make predictions about the school-participation rate based on the environment. These are important and interesting research questions left for future research.

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## Appendix A. The School-Proposing Deferred-Acceptance Algorithm

Given a set of schools $C$ that participates in the clearinghouse, the school-proposing deferred-acceptance algorithm (CPDA) works as follows.

**Step 1:** Each school $c \in C$ proposes to the students in $Ch_c(\mathcal{S})$. Each student who has been proposed to tentatively accepts the most preferred school among the schools that have proposed to her if it is an acceptable school and permanently rejects the rest. Let $S_c^1$ be the set of students who has rejected the proposal of school $c$. If there are no rejections, then stop.

**Step k,** $k \geq 2$**:** Each school $c \in C$ proposes to the students in $Ch_c(\mathcal{S} \setminus S_c^{k-1})$. Each student who has been proposed to tentatively accepts the most preferred school among the schools that have proposed to her if it is an acceptable school and permanently rejects the rest. Let $S_c^k$ be the set of students who has rejected the proposal of school $c$ in one of the first $k$ steps. If there are no rejections, then stop.

The algorithm ends in finite time since there can only be a finite number of rejections. Let $K$ be the final step. The outcome of the algorithm for school $c$ is $Ch_c(\mathcal{S} \setminus S_c^K)$.

## Appendix B. Omitted Proofs

In this appendix, we provide the omitted proofs.

**Proof of Lemma 1.** Let DA denote either version of the deferred-acceptance algorithm. First, for any student $s$, $\text{DA}(\mathcal{S}, \mathcal{C}^1 \cup \mathcal{C}^2, \succ_{\mathcal{S}}, Ch_{\mathcal{C}^1 \cup \mathcal{C}^2})(s)$ is weakly more preferred than $\text{DA}(\mathcal{S}, \mathcal{C}^1, \succ_{\mathcal{S}}, Ch_{\mathcal{C}^1})(s)$ and $\text{DA}(\mathcal{S}, \mathcal{C}^2, \succ_{\mathcal{S}}, Ch_{\mathcal{C}^2})(s)$ by Corollary 1 of Chambers and Yenmez (2017). Therefore, for each student $s$,

$$\mathrm{DA}(\mathcal{S},\mathcal{C}^1\cup\mathcal{C}^2,\succ_{\mathcal{S}},Ch_{\mathcal{C}^1\cup\mathcal{C}^2})(s)\succeq_s \max_{\succeq_s}\ \{\mathrm{DA}(\mathcal{S},\mathcal{C}^1,\succ_{\mathcal{S}},Ch_{\mathcal{C}^1})(s),\\ \mathrm{DA}(\mathcal{S},\mathcal{C}^2,\succ_{\mathcal{S}},Ch_{\mathcal{C}^2})(s)\}.$$

This shows that each student $s$ weakly prefers the integrated system to the divided one when students do not truncate their preferences at the second clearinghouse.

We next consider the case when students truncate their preferences at the second clearinghouse. For each student $s$, let $\succ_s'$ be the truncation of her preferences at $\mathrm{DA}(\mathcal{S},\mathcal{C}^1,\succ_{\mathcal{S}},Ch_{\mathcal{C}^1})(s)$, as we have defined in Section 2. Therefore, as in the previous paragraph, we get that

$$\mathrm{DA}(\mathcal{S},\mathcal{C}^1\cup\mathcal{C}^2,\succ_{\mathcal{S}}',Ch_{\mathcal{C}^1\cup\mathcal{C}^2})(s)\succeq_s' \max_{\succeq_s'}\ \{\mathrm{DA}(\mathcal{S},\mathcal{C}^1,\succ_{\mathcal{S}}',Ch_{\mathcal{C}^1})(s),\\ \mathrm{DA}(\mathcal{S},\mathcal{C}^2,\succ_{\mathcal{S}}',Ch_{\mathcal{C}^2})(s)\}.$$

Note that truncating student preference rankings at the DA outcome and then running DA again does not change the outcome, so $\mathrm{DA}(\mathcal{S},\mathcal{C}^1,\succ_{\mathcal{S}}',Ch_{\mathcal{C}^1})(s)=\mathrm{DA}(\mathcal{S},\mathcal{C}^1,\succ_{\mathcal{S}},Ch_{\mathcal{C}^1})(s)$. Likewise $\mathrm{DA}(\mathcal{S},\mathcal{C}^1\cup\mathcal{C}^2,\succ_{\mathcal{S}}',Ch_{\mathcal{C}^1\cup\mathcal{C}^2})(s)=\mathrm{DA}(\mathcal{S},\mathcal{C}^1\cup\mathcal{C}^2,\succ_{\mathcal{S}},Ch_{\mathcal{C}^1\cup\mathcal{C}^2})(s)$ because truncating the preferences below the DA outcome, which follows from Corollary 1 of Chambers and Yenmez (2017), does not change the DA outcome. Therefore, the displayed inequality above can be rewritten as

$$\mathrm{DA}(\mathcal{S},\mathcal{C}^1\cup\mathcal{C}^2,\succ_{\mathcal{S}},Ch_{\mathcal{C}^1\cup\mathcal{C}^2})(s)\succeq_s' \max_{\succeq_s'}\ \{\mathrm{DA}(\mathcal{S},\mathcal{C}^1,\succ_{\mathcal{S}},Ch_{\mathcal{C}^1})(s),\\ \mathrm{DA}(\mathcal{S},\mathcal{C}^2,\succ_{\mathcal{S}}',Ch_{\mathcal{C}^2})(s)\}.$$

Now we consider two cases.

**Case 1:** $\mathrm{DA}(\mathcal{S},\mathcal{C}^1\cup\mathcal{C}^2,\succ_{\mathcal{S}},Ch_{\mathcal{C}^1\cup\mathcal{C}^2})(s)=s$. In this case, $\mathrm{DA}(\mathcal{S},\mathcal{C}^1,\succ_{\mathcal{S}},Ch_{\mathcal{C}^1})(s)=s$ as well, since the former is weakly more preferred than the latter, and the latter is weakly more preferred than $s$ with respect to $\succ_s$. For the displayed inequality to be true, we also need that $\mathrm{DA}(\mathcal{S},\mathcal{C}^2,\succ_{\mathcal{S}}',Ch_{\mathcal{C}^2})(s)=s$. Therefore, the outcome in the integrated system, which is $\mathrm{DA}(\mathcal{S},\mathcal{C}^1\cup\mathcal{C}^2,\succ_{\mathcal{S}},Ch_{\mathcal{C}^1\cup\mathcal{C}^2})(s)=s$, is weakly more preferred than the outcome in the divided system with respect to $\succeq_s$, which is $\max_{\succeq_s}\{\mathrm{DA}(\mathcal{S},\mathcal{C}^1,\succ_{\mathcal{S}},Ch_{\mathcal{C}^1})(s),\mathrm{DA}(\mathcal{S},\mathcal{C}^2,\succ_{\mathcal{S}}',Ch_{\mathcal{C}^2})(s)\}=s$.

**Case 2:** $\mathrm{DA}(\mathcal{S},\mathcal{C}^1\cup\mathcal{C}^2,\succ_{\mathcal{S}},Ch_{\mathcal{C}^1\cup\mathcal{C}^2})(s)=c$ for some $c\in\mathcal{C}$. Then we get that $\mathrm{DA}(\mathcal{S},\mathcal{C}^1\cup\mathcal{C}^2,\succ_{\mathcal{S}},Ch_{\mathcal{C}^1\cup\mathcal{C}^2})(s)$ is weakly more preferred than the outcome in the divided system with respect to $\succeq_s$, which is $\max_{\succeq_s}\{\mathrm{DA}(\mathcal{S},\mathcal{C}^1,\succ_{\mathcal{S}},Ch_{\mathcal{C}^1})(s),\mathrm{DA}(\mathcal{S},\mathcal{C}^2,\succ_{\mathcal{S}}',Ch_{\mathcal{C}^2})(s)\}=s$ by the displayed inequality because $\succeq_s'$ is a truncation of $\succeq_s$.

□

**Auxiliary Lemma.** The following lemma is helpful in some of the proofs.

**Lemma 2.** *Let $c$ be a school. Suppose that $\mu_{\mathcal{C}}$ and $\mu_{\mathcal{C}\setminus\{c\}}$ are the matchings produced by the clearinghouse when $\mathcal{C}$ and $\mathcal{C}\setminus\{c\}$ are the set of participating schools, respectively. If*

$$\{s : c \succ_s \mu_{\mathcal{C}\setminus\{c\}}(s)\} \supseteq \mu_{\mathcal{C}}(c),$$

*then $c$ revealed prefers $Ch_c(\{s : c \succ_s \mu_{\mathcal{C}\setminus\{c\}}(s)\})$ to $\mu_{\mathcal{C}}(c)$.*

*Proof.* Let $\tilde{S} \equiv \{s : c \succ_s \mu_{\mathcal{C}\setminus\{c\}}(s)\}$, $S \equiv Ch_c(\{s : c \succ_s \mu_{\mathcal{C}\setminus\{c\}}(s)\})$, and $S' \equiv \mu_{\mathcal{C}}(c)$. Then by construction, $Ch_c(\tilde{S}) = S$, and, by assumption, $\tilde{S} \supseteq S' \cup S$. Therefore, $S$ is revealed preferred to $S'$ by $c$. □

**Proof of Theorem 2.** Let DA denote either version of the deferred-acceptance algorithm.

By Corollary 1 of Chambers and Yenmez (2017), for every student $s$, $\mu^{int}(s) \succeq_s \text{DA}(\mathcal{S},\mathcal{C}\setminus\{c\},\succ_{\mathcal{S}},Ch_{\mathcal{C}\setminus\{c\}})(s)$. Therefore, for any student $s \in \mu^{int}(c)$, we have $c \succeq_s \text{DA}(\mathcal{S},\mathcal{C}\setminus\{c\},\succ_{\mathcal{S}},Ch_{\mathcal{C}\setminus\{c\}})(s)$. Since $\text{DA}(\mathcal{S},\mathcal{C}\setminus\{c\},\succ_{\mathcal{S}},Ch_{\mathcal{C}\setminus\{c\}})(s)$ cannot be $c$ and $\succ_s$ is a strict preference ranking, we get $c \succ_s \text{DA}(\mathcal{S},\mathcal{C}\setminus\{c\},\succ_{\mathcal{S}},Ch_{\mathcal{C}\setminus\{c\}})(s)$. Therefore, $s \in \mu^{int}(c)$ implies $s \in \{s' : c \succ_{s'} \text{DA}(\mathcal{S},\mathcal{C}\setminus\{c\},\succ_{\mathcal{S}},Ch_{\mathcal{C}\setminus\{c\}})(s')\}$ or equivalently $\{s' : c \succ_{s'} \text{DA}(\mathcal{S},\mathcal{C}\setminus\{c\},\succ_{\mathcal{S}},Ch_{\mathcal{C}\setminus\{c\}})(s)\} \supseteq \mu^{int}(c)$. Since $\mu^a_c(c) = Ch_c(\{s' | c \succ_{s'} \text{DA}(\mathcal{S},\mathcal{C}\setminus\{c\},\succ_{\mathcal{S}},Ch_{\mathcal{C}\setminus\{c\}})(s')\})$, by Lemma 2, we conclude that $c$ revealed prefers $\mu^a_c(c)$ to $\mu^{int}(c)$.

**Proof of Theorem 3.** Recall that $\mu^a_c(c) = Ch_c(\{s | c \succ_s \text{DA}(\mathcal{S},\mathcal{C}\setminus\{c\},\succ_{\mathcal{S}},Ch_{\mathcal{C}\setminus\{c\}})(s)\})$, $\mu^s_c(c) = \{s \in Ch_c(S_c) | c \succ_s \text{DA}(\mathcal{S},\mathcal{C}\setminus\{c\},\succ_{\mathcal{S}},Ch_{\mathcal{C}\setminus\{c\}})(s)\}$, and $\mu^p_c(c) = \{s \in Ch_c(S_c) | c \succ_s \text{DA}(\mathcal{S},\mathcal{C}\setminus\{c\},\succ'_{\mathcal{S}},Ch_{\mathcal{C}\setminus\{c\}})(s)\}$.

Now let us prove the first claim that $c$ revealed prefers $\mu^a_c(c)$ to $\mu^s_c(c)$. Consider student $s \in \mu^s_c(c)$. By construction, $c \succ_s \text{DA}(\mathcal{S},\mathcal{C}\setminus\{c\},\succ_{\mathcal{S}},Ch_{\mathcal{C}\setminus\{c\}})(s)$. Therefore, this student applies to $c$ in the after case. As a result, $\mu^a_c(c)$ is chosen from a set that includes $\mu^s_c(c)$, which means that $c$ revealed prefers $\mu^a_c(c)$ to $\mu^s_c(c)$.

Next we prove the second statement that $c$ revealed prefers $\mu^s_c(c)$ to $\mu^p_c(c)$. Let $s \in \mu^p_c(c)$. By construction, $s \in Ch_c(S_c)$ and $c \succ_s \text{DA}(\mathcal{S},\mathcal{C}\setminus\{c\},\succ'_{\mathcal{S}},Ch_{\mathcal{C}\setminus\{c\}})(s)$, which implies that $\text{DA}(\mathcal{S},\mathcal{C}\setminus\{c\},\succ'_{\mathcal{S}},Ch_{\mathcal{C}\setminus\{c\}})(s) = s$. By Corollary 1 of Chambers and Yenmez (2017), $s$ weakly prefers (with respect to $\succ'_s$) the outcome of $\text{DA}(\mathcal{S},\mathcal{C}\setminus\{c\},\succ'_{\mathcal{S}},Ch_{\mathcal{C}\setminus\{c\}})(s) = s$ to $\text{DA}(\mathcal{S},\mathcal{C}\setminus\{c\},(\succ_{\mathcal{S}\setminus\{s\}},\succ'_s),Ch_{\mathcal{C}\setminus\{c\}})(s)$. Therefore, the latter must be the outside option $s$. Furthermore, if $s$ submits $\succ_s$ instead of $\succ'_s$ while others submit $\succ_{\mathcal{S}\setminus\{s\}}$, $s$ is either unmatched or matched with a school that is less preferred than $c$. More precisely, $c \succ_s \text{DA}(\mathcal{S},\mathcal{C}\setminus\{c\},\succ_{\mathcal{S}},Ch_{\mathcal{C}\setminus\{c\}})(s)$, which implies that $s \in \mu^s_c(c)$ because $s \in Ch_c(S_c)$. This shows that $\mu^p_s(c) \subseteq \mu^s_c(c)$. Therefore, $Ch_c(\mu^s_c(c) \cup \mu^p(c)) = Ch_c(\mu^s_c(c)) = \mu^s_c(c)$, where the second equality follows from path independence because $\mu^s_c(c) \subseteq Ch_c(S_c)$. Hence, $c$ revealed prefers $\mu^s_c(c)$ to $\mu^p_c(c)$. □

**Proof of Theorem 4.** We start with the first claim that when students have the same preference ranking and either version of DA is used in the clearinghouse, the three mechanisms are integration compatible. In light of Theorem 3, which shows that the evading school prefers to run its admissions as late as possible, we prove the claim for only the case when the evading school runs its admissions after the clearinghouse.

Since students have the same preference ranking over schools, there is a unique stable matching. This stable matching can be produced by a serial dictatorship of schools in which schools choose their students one by one. The order is determined by the common student preference ranking.

Consider the school whose ranking is $k$ where $1 \leq k \leq m$, say $c_k$. When all schools join the system, let $S_i$ denote the set of students matched with school $c_i$, $1 \leq i \leq m$. Then, for every $i$, $S_i = Ch_i(\mathcal{S} \setminus \bigcup_{j=1}^{i-1} S_j)$. In other words, $c_i$ chooses from students who remain unmatched after the first $i-1$ schools. In particular, $S_k = Ch_k(\mathcal{S} \setminus \bigcup_{j=1}^{k-1} S_j)$.

When all schools except $c_k$ join the system, the unique stable matching can be produced by letting these schools, excluding $c_k$, choose students one by one in the same order. Therefore, $c_1, \ldots, c_{k-1}$ get matched with the same set of students. The remaining students get matched with one of the remaining schools that are strictly worse than $c_k$. Therefore, the set of students that apply to $c_k$ is $\mathcal{S} \setminus \bigcup_{j=1}^{k-1} S_j$. Therefore, $c_k$ admits $Ch_k(\mathcal{S} \setminus \bigcup_{j=1}^{k-1} S_j)$, which is $S_k$ by construction.

Hence, $c_k$ is matched with the same set of students in $\mu^{int}$ and $\mu^a_{c_k}$ for every $k$. By Corollary 1, the mechanism for the after case is integration compatible.

To show the second claim, suppose that students find all schools acceptable but do not have the same preferences over them. Then there exist students $s_1, s_2$ and schools $c_1, c_2$ such that $c_1 \succ_{s_1} c_2$ and $c_2 \succ_{s_2} c_1$. We construct choice rules such that at least one school strictly prefers evading the system to joining it when other schools have joined it for all timing scenarios.

Suppose that all schools have responsive choice rules with capacity one. Consider the following preferences over students: $\succ_{c_1}$: $s_2 \succ_{c_1} s_1 \succ_{c_1} c_1$ and $\succ_{c_2}$: $s_1 \succ_{c_2} s_2 \succ_{c_2} c_2$. For schools $c_1$ and $c_2$, there are no other acceptable students. For the rest of the schools, there are no acceptable students. Therefore, in any stable matching, all students other than $s_1$ and $s_2$ are unmatched, and all schools other than $c_1$ and $c_2$ are unmatched. Hence, we focus on these two students and two schools.

When all schools join the system, SPDA is used. At the first step, $s_i$ proposes to $c_i$ for $i = 1, 2$. Since $s_i$ is acceptable to $c_i$, there are no rejections, and the algorithm ends at this step.

Suppose now that $c_1$ evades the system. Regardless of the admissions timing, $c_1$ is matched with $s_2$. Hence, $c_1$ is matched with its top-ranked student. Therefore, these mechanisms are integration incompatible. $\square$

**Proof of Theorem 5.** We need to show that $|\mu_c^p(c)| \leq |\mu^{int}(c)|$. Suppose, for contradiction, that $|\mu_c^p(c)| > |\mu^{int}(c)|$. Since the choice rule of school $c$ is capacity filling, $|\mu_c^p(c)| \leq q_c$, so $|\mu^{int}(c)| < q_c$.

We first consider SPDA. Recall that $\mu_c^p(c) = \{s \in Ch_c(S_c) | c \succ_s \text{SPDA}(\mathcal{S}, \mathcal{C} \setminus \{c\}, \succ'_{\mathcal{S}}, Ch_{\mathcal{C}\setminus\{c\}})(s)\}$. Because $c$ cannot fill its capacity in the integration outcome, it never rejects any students in SPDA because its choice rule is capacity filling. As a result, $\{s | c \succ_s \text{SPDA}(\mathcal{S}, \mathcal{C} \setminus \{c\}, \succ'_{\mathcal{S}}, Ch_{\mathcal{C}\setminus\{c\}})(s)\} = \text{SPDA}(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})(c)$ because a student prefers $c$ to the outcome in the clearinghouse when $c$ does not participate, and students submit truncated preferences if, and only if, the student is matched with $c$ in the clearinghouse when all schools participate. Therefore, $\mu_c^p(c)$ is a subset of $\text{SPDA}(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})(c) = \mu^{int}(c)$. This is a contradiction to the assumption that $|\mu_c^p(c)| > |\mu^{int}(c)|$.

We next consider CPDA. The set of students that the school can get in the preemption case is $\mu_c^p(c) = \{s \in Ch_c(S_c) | c \succ_s \text{CPDA}(\mathcal{S}, \mathcal{C} \setminus \{c\}, \succ'_{\mathcal{S}}, Ch_{\mathcal{C}\setminus\{c\}})(s)\}$. Therefore, a student $s$ is in $\mu_c^p(c)$ if and only if $s \in Ch_c(S_c)$ and $\text{CPDA}(\mathcal{S}, \mathcal{C} \setminus \{c\}, \succ'_{\mathcal{S}}, Ch_{\mathcal{C}\setminus\{c\}})(s) = s$ since each student who gets an offer from $c$ truncates her preferences at this school. Because school choice rules are path independent and capacity filling, if a student is unmatched in a stable matching then the student is unmatched in all stable matchings, which is an implication of the rural-hospitals theorem (Hatfield and Milgrom, 2005). Therefore, a student $s$ is in $\mu_c^p(c)$ if, and only if, $s \in Ch_c(S_c)$ and $\text{SPDA}(\mathcal{S}, \mathcal{C} \setminus \{c\}, \succ'_{\mathcal{S}}, Ch_{\mathcal{C}\setminus\{c\}})(s) = s$, i.e., $\mu_c^p(c) = \{s \in Ch_c(S_s) | c \succ_s \text{SPDA}(\mathcal{S}, \mathcal{C} \setminus \{c\}, \succ'_{\mathcal{S}}, Ch_{\mathcal{C}\setminus\{c\}})(s)\}$. Then, as we have argued in the previous paragraph, $\mu_c^p(c)$ is a subset of $\text{SPDA}(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})(c)$. Therefore, $|\mu_c^p(c)| \leq |\text{SPDA}(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})(c)|$. But the rural hospitals theorem states that $|\text{CPDA}(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})(c)| = |\text{SPDA}(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})(c)|$. We conclude that $|\mu_c^p(c)| \leq |\text{CPDA}(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})(c)| = |\mu^{int}(c)|$, which is a contradiction to the assumption that in the preemption case $c$ gets more students. $\square$

**Proof of Theorem 7.** Recall that the integration outcome for school $c$ is $\mu^{int}(c) = \text{DA}(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})(c)$.

**After case:** Suppose that $c$ unilaterally evades the system and runs its admissions after the clearinghouse. Then the same DA is run with virtual school $c$. The set of students matched with virtual school $c$ are unmatched in the clearinghouse. Now, let us identify the set of students who would like to switch to $c$: $\tilde{S} = \{s : c \succ_s \mu^{int}(s)\} \cup \mu^{int}(c)$. Here $\mu^{int}(c)$ is the set of students matched with the virtual school, so they are unmatched and they want to apply to $c$. Hence, $\tilde{S}$ is equivalent to $\{s : c \succeq_s \mu^{int}(s)\}$. The set of students who get matched with $c$ is then $\breve{\mu}_c^a(c) = Ch_c(\tilde{S}) = Ch_c(\{s : c \succeq_s \mu^{int}(s)\})$. Since $\mu^{int}$ is a stable matching, $\check{\mu}_c^a(c) = Ch_c(\{s : c \succeq_s \mu^{int}(s)\}) = \mu^{int}(c)$, so $\check{\mu}_c^a(c) = \mu^{int}(c)$. Therefore, the mechanism induced by the after case is integration compatible.

**Simultaneous case:** Suppose that $c$ unilaterally evades the system and runs its admissions simultaneously with the clearinghouse. Then $c$ admits $Ch_c(S_c)$. The outcome of the clearinghouse is DA with virtual school $c$. Then $\check{\mu}_c^s(c) = \{s \in Ch_c(S_c) | c \succeq_s \text{DA}(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})(s)\}$. But since DA is stable, $\mu^{int}(c) = Ch_c(\{c \succeq_s \text{DA}(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})(s)\})$, where $\{c \succeq_s \text{DA}(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})(s)\}$ is a superset of $\check{\mu}_c^s(c)$. Therefore, c revealed prefers $\mu^{int}(c)$ to $\check{\mu}_c^s(c)$, which implies that the mechanism induced by the simultaneous case is integration compatible.

**Preemption case:** Suppose that $c$ unilaterally evades the system and runs its admissions before the clearinghouse. Then $c$ admits $Ch_c(S_c)$. The students admitted by $c$ submit truncated preferences to the clearinghouse. Therefore, $\check{\mu}_c^p(c) = \{s' \in Ch_c(S_c) | c \succeq_{s'} \text{DA}(\mathcal{S}, \mathcal{C}, \succ'_{\mathcal{S}}, Ch_{\mathcal{C}})(s')\}$. Since students admitted by $c$ truncate their preferences, this set can also be written as $\check{\mu}_c^p(c) = \{s' \in Ch_c(S_c) | \text{DA}(\mathcal{S}, \mathcal{C}, \succ'_{\mathcal{S}}, Ch_{\mathcal{C}})(s') = s'\}$.

Let $s \in \check{\mu}_c^p(c)$, then $s = \text{DA}(\mathcal{S}, \mathcal{C}, (\succ_{\mathcal{S}\setminus\{s\}}, \succ'_s), Ch_{\mathcal{C}})(s)$ by an immediate application of Corollary 1 in Chambers and Yenmez (2017) because other students rank more schools. Therefore, if $s$ submits her full ranking $\succ_s$ instead of the truncated preferences $\succ'_s$, she will be matched with $c$ or a school that she likes less: $c \succeq_s \text{DA}(\mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}})(s) = \mu^{int}(s)$. This is because, by the same comparative statics, when a student ranks more schools each school gets a weakly revealed preferred set. Hence, no school ranked higher than $c$ can be matched with $s$. Therefore, $s \in \tilde{S} \equiv \{s' | c \succeq_{s'} \mu^{int}(s')\}$, which implies $\check{\mu}_c^p(c) \subseteq \tilde{S}$. Since $\mu^{int}(c) = Ch_c(\tilde{S})$, $c$ revealed prefers $\mu^{int}(c)$ to $\check{\mu}_c^p(c)$, which means that the mechanism induced by the preemption case is integration compatible.

□

## Appendix C. Examples

**C.1. Theorem 2 does not hold for every stable matching mechanism.** Theorem 2 shows that when the clearinghouse implements either version of DA, then each school $c$ revealed prefers $\mu_c^a(c)$ to $\mu^{int}(c)$. In this example, we show that the same conclusion does not hold for every stable matching algorithm used by the clearinghouse. In the following example, the clearinghouse implements the school-proposing DA when all schools join, but it switches to the student-proposing DA when a school evades.

Suppose that there are three schools $c_1, c_2, c_3$ and three students $s_1, s_2, s_3$. Schools $c_2$ and $c_3$ have responsive choice rules. They both have a capacity of one. Agents' preferences are as follows: $\succ_{c_2}$: $s_2 \succ s_1 \succ s_3 \succ c_2$, $\succ_{c_3}$: $s_3 \succ s_1 \succ s_2 \succ c_3$, $\succ_{s_1}$: $c_2 \succ c_1 \succ c_3 \succ s_1$, $\succ_{s_2}$: $c_1 \succ c_2 \succ c_3 \succ s_2$, and $\succ_{s_3}$: $c_1 \succ c_2 \succ c_3 \succ s_3$. This information is summarized in Table 2 below.

Table 2

| | $\succ_{c_2}$ | $\succ_{c_3}$ | $\succ_{s_1}$ | $\succ_{s_2}$ | $\succ_{s_3}$ |
|---|---|---|---|---|---|
| | $s_2$ | $s_3$ | $c_2$ | $c_1$ | $c_1$ |
| | $s_1$ | $s_1$ | $c_1$ | $c_2$ | $c_2$ |
| | $s_3$ | $s_2$ | $c_3$ | $c_3$ | $c_3$ |
| | $c_2$ | $c_3$ | $s_1$ | $s_2$ | $s_3$ |
| capacities | $q_{c_2} = 1$ | $q_{c_3} = 1$ | | | |

School $c_1$ has the following choice rule: $Ch_{c_1}(\{s_1, s_2, s_3\}) = Ch_{c_1}(\{s_1, s_2\}) = Ch_{c_1}(\{s_1, s_3\}) = \{s_1\}$, $Ch_{c_1}(\{s_2, s_3\}) = \{s_2, s_3\}$, and $Ch_{c_1}(\{s_i\}) = \{s_i\}$ for every $i = 1, 2, 3$.

Consider the case in which all schools join the system. Then the clearinghouse uses the school-proposing DA. At the first step of the algorithm, $c_1$ applies to $s_1$, $c_2$ applies to $s_2$, and $c_3$ applies to $s_3$. All of the schools are tentatively accepted. Since there are no rejections, the algorithm ends after this step and the tentative matching is made permanent. The final matching is $\mu^{int} = \{(c_1, \{s_1\}), (c_2, \{s_2\}), (c_3, \{s_3\})\}$.

Consider the case in which $c_3$ unilaterally evades the system. The clearinghouse uses the student-proposing DA. Student $s_1$ proposes to $c_2$, and students $s_2$ and $s_3$ propose to $c_1$. All of the students are tentatively accepted. Since there are no rejections, the algorithm ends. School $c_1$ is matched with $s_2$ and $s_3$, and $c_2$ is matched with $s_1$. Since all students prefer this matching outcome to $c_3$, none of them applies to $c_3$. The final matching is $\mu^a_{c_3} = \{(c_1, \{s_2, s_3\}), (c_2, \{s_1\}), (c_3, \emptyset)\}$.

Since $c_3$ revealed prefers $\{s_3\}$ to $\emptyset$, it prefers to join the system rather than evading it and admitting students after the clearinghouse.

**C.2. No stable matching mechanism guarantees integration.** In Theorem 2, we have shown that, regardless of which version of DA is used, every school $c$ revealed prefers $\mu^a_c(c)$ to $\mu^{int}(c)$. To investigate whether there exists an integration compatible mechanism with a clearinghouses that uses a stable algorithm, we consider the following example that has a unique stable matching regardless of whether $c_2$ joins the system or evades it. Therefore, every stable algorithm has to produce the unique stable matching under these two scenarios.

Suppose that there are two schools $c_1, c_2$ and three students $s_1, s_2, s_3$. School $c_1$'s capacity is one, whereas $c_2$'s capacity is two. Both schools have responsive choice rules with the following preferences: $\succ_{c_1}$: $s_1 \succ s_2 \succ s_3 \succ c_1$, $\succ_{c_2}$: $s_2 \succ s_1 \succ s_3 \succ c_2$. Students' preferences are as follows: $\succ_{s_1}$: $c_2 \succ c_1 \succ s_1$, $\succ_{s_2}$: $c_1 \succ c_2 \succ s_2$, and $\succ_{s_3}$: $c_1 \succ c_2 \succ s_3$. This information is summarized in Table 3 below.

Table 3

| | $\succ_{c_1}$ | $\succ_{c_2}$ | $\succ_{s_1}$ | $\succ_{s_2}$ | $\succ_{s_3}$ |
|---|---|---|---|---|---|
| | $s_1$ | $s_2$ | $c_2$ | $c_1$ | $c_1$ |
| | $s_2$ | $s_1$ | $c_1$ | $c_2$ | $c_2$ |
| | $s_3$ | $s_3$ | $s_1$ | $s_2$ | $s_3$ |
| | $c_1$ | $c_2$ | | | |
| capacities | $q_{c_1} = 1$ | $q_{c_2} = 2$ | | | |

Consider the case in which both schools join the system. In any stable matching, $c_2$ and $s_1$ must be matched. Otherwise, they would form a blocking pair. Given this pair, $c_1$ and $s_2$ must also be matched in any stable matching. Finally, $c_2$ and $s_3$ must be matched as well. Therefore, there is a unique stable matching: $\mu^{int} = \{(c_1, \{s_2\}), (c_2, \{s_1, s_3\})\}$.

Consider the case in which $c_2$ evades the system. Since there is only one school in the clearinghouse, $c_1$ must be matched with $s_1$ in any stable algorithm. Since all students prefer $c_2$ to the outcome of the algorithm, they all apply to $c_2$. School $c_2$ accepts students $s_1$ and $s_2$. The final matching is $\mu^a_{c_2} = \{(c_1, \emptyset), (c_2, \{s_1, s_2\})\}$.

School $c_2$ strictly revealed prefers $\{s_1, s_2\}$ to $\{s_1, s_3\}$. Therefore, the mechanism is integration incompatible.

**C.3. Common school priorities do not guarantee integration.** Suppose that there are three schools $c_1, c_2, c_3$ and two students $s_1, s_2$. School $c_1$'s capacity is two, $c_2$'s capacity is one, and $c_3$'s capacity is two. Every $c_i$ has a responsive choice rule with the following preference $s_1 \succ s_2 \succ c_i$, for $i = 1, 2, 3$. Students' preferences are as follows: $\succ_{s_1}$: $c_1 \succ c_2 \succ c_3 \succ s_1$ and $\succ_{s_2}$: $c_2 \succ c_1 \succ c_3 \succ s_2$. This information is summarized in Table 4 below.

Table 4

| | $\succ_{c_1}$ | $\succ_{c_2}$ | $\succ_{c_3}$ | $\succ_{s_1}$ | $\succ_{s_2}$ |
|---|---|---|---|---|---|
| | $s_1$ | $s_1$ | $s_1$ | $c_1$ | $c_2$ |
| | $s_2$ | $s_2$ | $s_2$ | $c_2$ | $c_1$ |
| | $c_1$ | $c_2$ | $c_3$ | $c_3$ | $c_3$ |
| | | | | $s_1$ | $s_2$ |
| capacities | $q_{c_1} = 2$ | $q_{c_2} = 1$ | $q_{c_3} = 2$ | | |

Consider the case in which all schools join the system. Then there exists a unique stable matching: $\mu^{int} = \{(c_1, \{s_1\}), (c_2, \{s_2\}), (c_3, \emptyset))\}$. This is the outcome of the clearinghouse.

Consider the case in which $c_1$ unilaterally evades the system. For the matching market without $c_1$, there is a unique stable matching, which is the outcome of the clearinghouse: $\{(c_2, \{s_1\}), (c_3, \{s_2\}))\}$. Since both students prefer $c_1$ to the outcome of the clearinghouse, they apply to $c_1$. School $c_1$ accepts them both since it has a capacity of two. The final matching is $\mu^a_{c_1} = \{(c_1, \{s_1, s_2\}), (c_2, \emptyset), (c_3, \emptyset)\}$.

School $c_1$ strictly revealed prefers $\{s_1, s_2\}$ to $\{s_1\}$. Therefore, the mechanism is integration incompatible.[24]

**C.4. No-poaching policy does not guarantee integration compatibility.** For SPDA, consider Problem P2 in Remark 3, presented in Table 1B. The integration outcome is $\{(c_1, \{s_2\}), (c_2, \{s_1\})\}$.

Consider the case in which $c_2$ unilaterally evades the system. In the clearinghouse, both students propose to $c_1$. School $c_1$ accepts $s_1$ and rejects $s_2$. Afterwards, $s_2$ applies to $c_2$. School $c_2$ accepts $s_2$. The final matching is $\{(c_1, \{s_1\}), (c_2, \{s_2\})\}$.

Since $c_2$ revealed prefers $s_2$ to $s_1$, the mechanism is integration incompatible.

For CPDA, consider the example in Appendix C.2. The integration outcome is $\{(c_1, \{s_2\}), (c_2, \{s_1, s_3\})\}$.

Consider the case in which $c_2$ unilaterally evades the system. School $c_1$ proposes to $s_1$. Student $s_1$ accepts $c_1$'s offer. Then the unmatched students apply to $c_2$. School $c_2$ accepts both students. The final matching is $\{(c_1, \{s_1\}), (c_2, \{s_2, s_3\})\}$.

Since $c_2$ does not revealed prefer $\{s_1, s_3\}$ to $\{s_2, s_3\}$, the mechanism is not integration compatible.

[24] This example shows that even the *acyclicity* condition of Ergin (2002) does not guarantee integration compatibility.