Exchange rates and monetary spillovers[*]

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Abstract

When do flexible exchange rates prevent monetary and financial conditions from spilling over across currencies? We examine a model in which international investors strategically supply capital to a small inflation-targeting economy with flexible exchange rates. For some combination of parameters, the unique equilibrium exhibits the observed empirical feature of prolonged episodes of capital inflows and appreciation of the domestic currency, followed by reversals where capital outflows go hand-in-hand with currency depreciation, a rise in domestic interest rates, and inflationary pressure. Arbitrarily small shocks to global financial conditions suffice to trigger these dynamics.

1 Introduction

The flexible exchange rate version of the Mundell-Fleming model (Fleming (1962), Mundell (1963)) lays out the case for how flexible exchange rates allow monetary authorities to pursue domestic macroeconomic objectives in a world of free capital flows. Post-crisis discussions of monetary spillovers have revisited this classic proposition. The BIS report on global liquidity (BIS (2011)) is a recent exposition of cross-border monetary spillovers and it has been followed by an active literature which has examined the extent to which floating exchange rates fail to insulate monetary policy from external developments (see, for instance, Agrippino and Rey (2015), Rey, (2013, 2015), Bruno and Shin (2015a, 2015b)).

The broad picture emerging from this literature is that of a significant global comovement in leverage and asset prices that is related to US monetary policy. This global factor is associated with surge and sudden stops in capital flows, and with exchange rate fluctuations that deviate from uncovered interest parity (UIP).

This paper offers a theoretical model in which the differential between the interest rate on an international funding currency and on that of a small open economy generates excessive fluctuations in leverage, bond prices, and inflation in the small open economy. This instability is generated by the capital flows of global investors seeking to reap self-justified rents from carry trades. The rents are self-justified in the sense that capital inflows (outflows) generate positive (negative) abnormal returns on carry trades that justify the flows in the first place.

We contribute to the theoretical literature on self-fulfilling international crises pioneered by Obstfeld (1996) along two dimensions. First, we write down a fully dynamic coordination game among global investors in infinite horizon in which investors' beliefs about each others' future positions are uniquely determined along the equilibrium path. Shocks to the interest-rate differential serve as their coordination device and affect their collective beliefs in a non-linear fashion. This way, we show that coordination games are not only useful to model snapshot crisis episodes, but can also help understand more protracted developments within the financial system. Second, we embed this coordination game in a simple but standard monetary model of a small open economy. This enables us to identify the set of primitive parameters of the domestic economy under which it lends itself to such destabilizing

speculation.

We proceed in two steps. We first couch the novel economic mechanism through which global investors' portfolio choice generates monetary spillovers in a small open economy in the simplest possible environment: a perfect-foresight model. Monetary spillovers stem from two ingredients. First, the central bank in the small open economy uses an interest-rate rule that responds to global investors' inflows only insofar as they affect the price level, but that does not track their direct impact on asset prices (and thus on the real rate). In other words, the central bank "thinks like the FED," as if it was in a large economy.[1] As a result inflows (outflows) are deflationary (inflationary). The second ingredient is the assumption (borne out by the data) that the non tradable goods of the small economy have more rigid prices than the tradable ones. This implies that the inflationary impact of capital flows must operate through the prices of tradable goods, and thus leads to large fluctuations in the nominal exchange rate. We show that for some parameter values, there are two stable steady-state solutions — one associated with capital inflows and the other with capital outflows. The steady state with capital inflows is associated with an appreciation of the domestic currency and a failure of uncovered interest parity (UIP) yielding an abnormal positive return on carry trades, and the steady state with capital outflows is its mirror image.

In the second step of our analysis, we build a stochastic version of this perfect-foresight model. We introduce exogenous shocks to the dollar interest rate and adapt the iterated-dominance techniques of Frankel and Pauzner (2000) to refine the outcome of the model to a unique solution. We show that the state space can be partitioned into two regions— a region where all global investors pile into the local currency bond, and one in which they short these bonds to be long US dollar-denominated assets. The transition between the two regions can be triggered by small fluctuations in the US dollar interest rate and by the endogenous changes in domestic financial conditions. An easing of US monetary conditions typically creates a prolonged episode of capital inflows, benign domestic financial conditions, and appreciation of the currency for the small economy. Subsequent small increases in the US rate do not immediately reverse the up-phase of the cycle but will reverse it when a "tantrum"

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[1]An anonymous referee suggested this nice formulation of our assumption.

boundary is reached. Hitting the "tantrum" boundary triggers currency depreciation, capital outflows, an increase in domestic bond yields, and inflationary pressure. These features are reminiscent of that experienced by a number of emerging economies during the 2013 "taper tantrum" episode that followed the announcement of a possible tapering of the highly accommodative US monetary policy.

Related Literature

Our approach is most closely related to models of financial instability which involve coordination problems and self-fulfilling speculative episodes. In a similar spirit, Farhi and Tirole (2012) and Schneider and Tornell (2004) offer models of "collective moral hazard" in which the government bails out speculators if their aggregate losses are sufficiently large, thereby inducing a coordination motive among speculators. We formalize the dynamic coordination game among investors using the dynamic extension of global-game methods developed by Frankel and Pauzner (2000) and Burdzy, Frankel, and Pauzner (2001) to obtain a unique equilibrium outcome. We show that these global-game tools can be adapted to the situation where coordination motives coexist with congestion effects. This is important because most financial models with coordination motives also feature congestion effects. In a model of bank run, Goldstein and Pauzner (2004) adapt static global-game techniques to the case in which strategic complementarities similarly fail to hold everywhere. In a model of sovereign-debt refinancing, He, Krishnamurthy, and Milbradt (2016) also apply global-game techniques in a context in which a large debt size comes at the benefit of smaller congestion effects but at the cost of a higher rollover risk.

Most closely related to our work, He and Xiong (2012) apply the equilibrium -selection techniques developed by Burdzy, Frankel, and Pauzner (2001) in a dynamic financial context — the roll-over of short-term debt.

We also relate to the theoretical literature that seeks to model both crises and the build up of fragility that precedes them. Lorenzoni (2008) builds a model in which commitment problems on both lending and borrowing sides lead to excessive borrowing ex-ante and excessive volatility ex-post. Sannikov (2014) endogenizes the build up of fragility by assuming

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that heterogeneous investors have access to assets of varying "liquidity" defined as follows. An illiquid asset generates more value than a liquid one when held by expert investors, and less value when it is in the hands of the unsophisticated ones. Fragility builds up because the value destroyed when this illiquid asset changes hands has no impact on the steady-state target leverage of experts, so that endogenous illiquidity-risk taking by experts in quiet times alone can lead to large crises even absent large fundamental risk.

Our paper also relates to the literature on portfolio choice in incomplete markets. In a recent contribution, Gabaix and Maggiori (2015) introduce financial intermediaries that operate in incomplete global financial markets by intermediating gains from trade between countries. We also model global investors as financial institutions exploiting the incompleteness of global markets.[2] Garleanu, Panageas and Yu (2015) present a model in which investors face costs to extend their participation in markets located on a circle for diversification purposes. Our result that the profitability of investment increases in the weight of others' participation bears similarities with their finding that participation and leverage reinforce each other, possibly leading to multiple equilibria.

We also relate to the large literature that studies sudden stops in international capital flows and their effects on the exchange rate and the real economy. Obstfeld and Rogoff (2007) expect that the unwinding of the US current account deficit would lead to a major depreciation of the US dollar. They argue that if the Federal Reserve is willing to tolerate any exchange rate movement to maintain price stability, then tradable goods prices must increase to accommodate a balanced current account and this should lead to a large impact on the exchange rate. A related relationship between relative domestic prices, inflation targeting, and exchange rate is at the core of our analysis. Mendoza (2010) seeks to explain the large and protracted impact of sudden stops to the real economy by introducing occasionally binding collateral constraints that create balance-sheet externalities in an international DSGE model. Although we do not offer quantitative insights as he does, we also seek to explain important nonlinearities in international capital flows with the presence of complementarities among investors. In our model, complementarities do not stem from balance-sheet externalities, but

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[2] We discuss interesting differences between their conclusions and ours below in Section 2.5.

rather from the exploitation of the domestic interest rule by global investors.

Finally, our results complement the recent work on the risk-taking channel of currency appreciation, introduced by Bruno and Shin (2015a, 2015b) in the context of cross-border banking, whereby currency mismatches on borrowers' balance sheets lead to credit supply effects of exchange rate fluctuations. Whereas these models of the risk-taking channel are static, our global-game model solves for the dynamic path of the key macro variables.

2 A simple perfect-foresight model

Time is discrete and is indexed by t. There is a single tradable good that has a fixed unit price in US dollars. We study the interactions between two types of agents with different investment opportunities, households populating a small open economy and global investors. The households can only trade a domestic nominal bond whereas global investors have access both to this bond and to dollar-denominated assets. We show that a domestic monetary policy that does not respond sufficiently aggressively to the capital flows of global investors opens up arbitrage opportunities for them. We describe in turn the households, the global investors, and then the monetary frictions in the domestic economy.

2.1 Households

The households live in a small open economy. They use a domestic currency that trades at S_t dollars per unit at date t, where the exchange rate S_t will be determined in equilibrium.[3]

At each date, a unit mass of households are born. Households live for two dates, consume when young and old. They set firms and supply labor inelastically when old. Each household receives an initial endowment at birth with nominal value $P_t W \geq 0$, where P_t is the domestic price level.[4]

The cohort that is born at date t has quasi-linear preferences over bundles of consumption

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[3]Both US dollar and this domestic currency serve only as units of account ("cashless economy").

[4]If the endowment of young households is zero, then the global investors introduced below cannot have an aggregate short position in domestic bonds. A strictly positive endowment plays no other role than allowing such short positions.

and labor (C_t, C_{t+1}, N_{t+1})

$$U\left(C_t, C_{t+1}, N_{t+1}\right) = \ln C_t + \frac{C_{t+1} - N_{t+1}^{1+\chi}}{R}, \tag{1}$$

where $\chi > 0$ and $R > 1$ is the subjective discount rate.

Domestic consumption services C_t are produced combining the tradable good C_t^T and a nontradable good C_t^N according to the technology

$$C_t = \frac{\left(C_t^N\right)^\alpha \left(C_t^T\right)^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}, \tag{2}$$

where $\alpha \in (0,1)$. As is well-known, optimal spending across goods by households implies that the domestic price level of consumption services satisfies

$$P_t = (P_t^N)^\alpha (P_t^T)^{1-\alpha}, \tag{3}$$

where P_t^T and P_t^N are the respective prices of the tradable and nontradable goods.

Domestic firms set by old households use labor input to produce. Due to quasi-linear preferences, our results do not depend on the specification of the firms' production functions. All that is needed is that the nontradable good is produced in finite, non zero quantities at each date. Households collect labor income and the profits from their firms when old.

Households have access to the domestic bond market, in which risk-free one-period bonds denominated in the domestic currency are available in zero net supply. The nominal interest rate on these bonds, I_{t+1}, is set by the domestic central bank according to a rule to be described below.

2.2 Global investors

A unit mass of global investors have access to both the local-currency bond market and to US dollar-denominated one-period bonds. The exogenous nominal return on US dollar bonds is denoted by $I^* > 0$. Global investors consume outside the local economy, and their utility is increasing in the consumption of the tradable good.[5]

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[5]Whether they also derive utility from consuming other goods, and the curvature of their utility function are immaterial. This is only true because the economy is deterministic, and will no longer be so in Section 3.

In forming their financial portfolios, global investors face limits on the size of their exposures in domestic bonds, reflecting leverage constraints or exposure caps imposed by internal risk limits. We assume that the position in domestic bonds of any investor must lie in the interval $[P_t L^-, P_t L^+]$, where these limits are denominated in the domestic currency and

$$L^- > -W,$$

which ensures that households always consume positively.[6]

The return to a global investor from investing in the local currency bond market relative to the return on dollar bonds is given by

$$\Theta_{t+1} = \frac{S_{t+1}}{S_t} \frac{I_{t+1}}{I^*} \tag{4}$$

We may interpret Θ_{t+1} as the return to a carry-trade position in which the investor borrows dollars at rate I^* and then invests the proceeds in the local currency bond yielding I_{t+1}. Uncovered interest parity (UIP) holds when $\Theta_{t+1} = 1$.

2.3 Capital flows and portfolio choices

We denote $L_t \in [L^-, L^+]$ the real net aggregate borrowing by young households from global investors at date t (possibly negative). In equilibrium, these capital flows result from the following optimal portfolio decisions by global investors and households.

Global investors. Since the economy is deterministic, optimal portfolio choice by global investors implies that L_t must satisfy:

$$L_t \begin{cases} = L^+ & \text{if } \Theta_{t+1} > 1, \\ = L^- & \text{if } \Theta_{t+1} < 1, \\ \in (L^-, L^+) & \text{if } \Theta_{t+1} = 1. \end{cases} \tag{5}$$

In words, global investors choose corner portfolios unless they are indifferent between investing in US-dollar denominated assets or in domestic bonds.

Households. The real net aggregate borrowing L_t also solves in equilibrium

$$C_t = L_t + W, \tag{6}$$

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[6]Setting lending limits in real terms simplifies the exposition but is not crucial. Nominal rigidities in trading limits would actually amplify our results.

where C_t, the equilibrium consumption of the date-t representative young household, solves

$$\max_{\{C_t, C_{t+1}\}} \left\{ \ln C_t + \frac{C_{t+1}}{R} \right\} \tag{7}$$

s.t.

$$C_t + \frac{C_{t+1}P_{t+1}}{I_{t+1}P_t} = W + \frac{\Omega_{t+1}P_{t+1}}{I_{t+1}P_t}, \tag{8}$$

where Ω_{t+1} is the household's (endogenous) income at date $t+1$, comprised of labor income and domestic firms' profits.[7] At the equilibrium, (6) and the first-order condition from $\{(7); (8)\}$ yield the Euler equation:

$$I_{t+1} = \frac{P_{t+1}}{P_t} \frac{I_{t+1}}{(L_t + W)}. \tag{9}$$

2.4 Monetary policy rule and nominal rigidities

Interest-feedback rule. We suppose that the domestic monetary authority sets the nominal interest rate between t and $t+1$, I_{t+1}, following the interest-rate feedback rule:

$$I_{t+1} = R \left(\frac{P_t}{P_{t-1}} \right)^{1+\Phi} \tag{10}$$

where

$$\Phi > 0 \tag{11}$$

The interest rate rule (10) follows the Taylor principle from (11) in that the nominal interest rate reacts more than one-for-one to the price level change. Setting the target inflation rate to zero is only a normalization.

Nominal rigidities. An important ingredient of the model is that the price of the nontradable good is less flexible than that of the tradable good.[8] We formalize this very

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[7]For brevity we do not impose that C_{t+1} be positive. Since households' debt will always be bounded in equilibrium, this holds if Ω_{t+1} is sufficiently large or if so is W and the government can make intergenerational transfers.

[8]This is consistent with evidence documented by Burstein, Eichenbaum, and Rebelo (2005).

simply as follows. We assume that the tradable good has a flexible price P_t^T in the domestic currency, and that the law of one price holds ("PPP at the docks"), implying

$$P_t^T S_t = 1, \tag{12}$$

whereas the nontradable good has a fixed price that we normalize to 1 without loss of generality:

$$P_t^N = 1. \tag{13}$$

2.5 Steady-state solution

We are now equipped to solve for the perfect-foresight equilibria of this economy. A perfect-foresight equilibrium is a sequence $(L_t, P_t^T, P_t^N, P_t, S_t, I_t)$ correctly anticipated by households and global investors such that

- Households optimally allocate consumption across goods ((3) holds) and dates ((9) holds);

- Global investors form optimal portfolios ((5) holds);

- The central bank applies the interest rule (10);

- (12) and (13) hold.

We introduce the following notation.

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\begin{aligned}
r &= \ln R, \\
\delta &= \ln\left(\frac{R}{I^*}\right), \\
\theta_t &= \ln \Theta_t, \\
i_t &= \ln I_t, \\
s_t &= \ln S_t, \\
l_t &= \ln(L_t + W), \\
\pi_{t+1} &= \ln\left(\frac{P_{t+1}}{P_t}\right).
\end{aligned}
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The Euler equation (9) and the interest-rate rule (10) can be gathered as follows:

$$i_{t+1} = r - l_t + \pi_{t+1}, \tag{14}$$

$$i_{t+1} = r + (1 + \Phi) \pi_t. \tag{15}$$

Together, they define a linear-difference equation for the path of inflation:

$$\pi_t = \frac{\pi_{t+1} - l_t}{1 + \Phi} \tag{16}$$

which has a unique non-exploding solution:

$$\pi_t = -\sum_{k \geq 0} \frac{l_{t+k}}{(1 + \Phi)^{k+1}}. \tag{17}$$

Equation (17) shows that current inflation is affected by current and future capital inflows $(l_{t+k})_{k \geq 0}$. The Taylor principle ensures that π_t is well-defined, since l_{t+k} is bounded and $\Phi > 0$.

This expression for π_t highlights that our model shares the generic feature of standard interest-rule based monetary models that inflation reflects anticipated future "shocks." In our context, the "shocks" are not the usual exogenously assumed policy shocks, but rather are the equilibrium consequence of optimal portfolio choice by global investors.

Using (3) and (12), we have:

$$\pi_{t+1} = -(1 - \alpha)(s_{t+1} - s_t). \tag{18}$$

Equation (18) expresses inflation in terms of exchange-rate depreciation. One can rewrite (4) as

$$\theta_{t+1} = s_{t+1} - s_t + i_{t+1} - \ln I^*,$$
$$= -\frac{1}{1 - \alpha} \pi_{t+1} + \pi_{t+1} - l_t + \delta, \tag{19}$$
$$= \frac{\alpha}{1 - \alpha} \sum_{k \geq 0} \frac{l_{t+k+1}}{(1 + \Phi)^{k+1}} - l_t + \delta, \tag{20}$$

where (19) follows from (18) and the Euler equation (14) , and (20) follows from the expression for π_{t+1} given by (17).

Expression (20) reveals two key properties of the model. First, individual trading profit is decreasing in today's aggregate inflow l_t; the more the other traders invest in domestic bonds, the lower the profit of an individual trader. This is akin to strategic substitution. Second, the trading profit is increasing in future inflows l_{t+k}; the more future traders invest in domestic bonds, the higher the individual profit today. This is the standard strategic complementarity that a coordination model requires. Trading limits prevents strategic substitution from working fully in the current period. We shall now see that this opens an opportunity for multiple self-confirming paths.

We focus on the steady-states in which the debt level l is constant over time. We introduce

$$\underline{l} \equiv \ln(W + L^-), \tag{21}$$

$$\bar{l} \equiv \ln(W + L^+). \tag{22}$$

For brevity the remainder of the paper focusses on the case in which

$$\underline{l} < 0 < \bar{l}.$$

Proposition 1 (Multiplicity of steady-states) *Suppose there exists* $l^* \in (\underline{l}, \bar{l})$ *such that*

$$\frac{\alpha - \Phi(1 - \alpha)}{\Phi(1 - \alpha)} l^* + \delta = 0.$$

Then $l = l^*$ *is a steady state in which uncovered interest parity (UIP) holds. If* $\Phi(1-\alpha) > \alpha$, *there is no other steady-state solution. However, if* $\Phi(1 - \alpha) < \alpha$, *there are two further steady-state solutions; there is a steady-state with maximum capital inflows* $(l = \bar{l})$, *and there is a steady-state with maximum capital outflows* $(l = \underline{l})$.

Proof of Proposition 1. First, note that for l fixed, the relative return to investing in the local currency bond given by (20) can be written as

$$\theta = \frac{\alpha - \Phi(1 - \alpha)}{\Phi(1 - \alpha)} l + \delta \tag{23}$$

Let $l^* \in (\underline{l}, \bar{l})$ be such that $\theta = 0$. For such an l^* investors are indifferent between investing in the local currency bond or the dollar bond. Hence, l^* is a steady-state solution of our

model. If $\Phi(1-\alpha) > \alpha$, this is the only steady-state solution, since θ is decreasing in l: More (less) foreign lending makes foreign lending unprofitable (profitable). This case corresponds in particular to the fully flexible benchmark ($\alpha=0$).

Now consider the case where $\alpha > \Phi(1-\alpha)$. In this case, we have two further steady-state solutions corresponding to the corner solutions $l = \bar{l}$ and $l = \underline{l}$.

If all investors choose to invest in the local currency bond, then $l = \bar{l}$, so that $\theta > 0$. This implies that investing in the local currency bond is strictly better than investing in the dollar bond and vindicates investors' choice $l = \bar{l}$. Conversely, if all investors choose to invest in the dollar bond ($l = \underline{l}$), then $\theta < 0$, implying that investing in the dollar bond is optimal. ∎

Proposition 1 highlights the possibility of both self-fulfilling capital flow surges and outflows as extremal steady-state solutions of our model. These steady-states correspond to binding risk limits for global investors, failure of UIP ($\theta \neq 1$), and off-target inflation. The intuition behind the multiplicity of steady-states is as follows.

The first key ingredient is the fact, established in (17), that current and future capital inflows (outflows) lead inflation to be below (above) target:

$$\pi_t = -\sum_{k \geq 0} \frac{l_{t+k}}{(1+\Phi)^{k+1}}.$$

This owes in turn to the assumption, discussed in more detail in Section 2.6 below, that the central bank "thinks like the FED." The interest-rate rule (10) uses a constant target real rate R whereas capital flows affect the real rate that prevails in the economy. The wedge between the target R and the actual real rate is akin to a policy shock leading to equilibrium inflation straying away from target (normalized to 1).

Second, it is instructive to decompose the return on carry trades into the (nominal) exchange rate appreciation $s_{t+1} - s_t$ and the (nominal) interest-rate differential (the "carry") $i_{t+1} - \ln I^*$:

$$\theta_{t+1} = s_{t+1} - s_t + i_{t+1} - \ln I^*, \tag{24}$$

with

$$s_{t+1} - s_t = -\frac{\pi_{t+1}}{1 - \alpha}, \tag{25}$$

$$i_{t+1} - \ln I^* = \pi_{t+1} - l_t + \delta. \tag{26}$$

Expression (26) shows that capital inflows negatively affect the interest-rate differential through two channels. First, current capital inflows reduce the real rate by increasing demand for domestic bonds (term $-l_t$). Second, future capital inflows also affect the nominal rate through their negative impacts on inflation (term π_{t+1}).

On the other hand, (25) shows that future capital inflows lead to an appreciation of the nominal exchange rate. The assumption of complete pass-through (12) implies that a reduction in the price level of the tradable good is exactly offset by an appreciation of the nominal exchange rate. Since the nontradable good has a sticky price, the overall deflationary impact of future capital inflows on the CPI operates through the price of the tradable good, with a multiplier $1/(1 - \alpha)$ that decreases in the share of tradables in consumption. This leads to an appreciation of the nominal exchange rate that is therefore larger than the overall CPI deflation (same multiplier $1/(1 - \alpha)$ in (25)).

Foreign inflows therefore have an ambiguous net effect on the return on carry trades. The negative effect of inflows on the interest-rate differential makes carry trades akin to strategic substitutes. Carry trades by other global investors reduce the appeal of carry trades for a given investor because it makes domestic bonds more expensive in the domestic currency. This effect is stabilizing and leads to a unique steady-state in which UIP holds when prevalent. Conversely, the effect of inflows on exchange-rate appreciation introduces strategic complementarities among investors, and is destabilizing. If this effect more than offsets the stabilizing impact of inflows on the interest-rate differential, the anticipation of future large capital inflows in the small open economy is self-fulfilling and leads to an extremal steady-state with corner portfolios and failure of UIP. The model is essentially symmetric, so everything also works the mirror-image way in the steady-state with extremal outflows.

Formally, strategic complementarities are sufficiently strong to allow for destabilizing carry trades when the condition $\alpha > \Phi(1 - \alpha)$ is satisfied. This occurs when nominal rigidities are sufficiently important (α sufficiently large) and monetary policy sufficiently passive (Φ

sufficiently small). Both conditions imply that small changes in inflation expectations are consistent with large swings in the nominal exchange rate. Otherwise stated, the domestic monetary authority could eliminate extremal steady-states by committing to a sufficiently large Φ, which means committing to a sufficiently large reduction in the policy rate in the presence of large capital inflows and/or large appreciation of the exchange rate.

It is interesting to contrast this mechanism generating excess returns for global investors with that in Gabaix and Maggiori (2015). In their setup, tighter financial constraints lead to larger excess returns on carry trades because it forces global investors to leave carry-trade profits on the table. Here, conversely, if one interprets tighter financial constraints as narrower trading limits, then such tighter constraints lead to smaller excess returns on carry trades. This is because these excess returns are generated by the destabilizing impact of foreign capital flows in or out of the small economy. This impact increases in the size of the carry trade.

2.6   Comments

The conjunction of two ingredients gives rise to multiple steady-states: heterogeneous nominal rigidities and an interest-rate rule (11) that does not fully track the impact of foreign flows on the real rate. It is instructive to discuss the respective roles of these ingredients in a straightforward extension of the model in which the domestic monetary authority targets an inflation rate e^{π^*}, the domestic monetary authority uses the rule

$$i_{t+1} = r - \psi l_t + \pi^* + (1 + \Phi)(\pi_t - \pi^*), \tag{27}$$

and the inflation rate of the nontradable price is of the form $e^{\pi^N(l_t)}$, where $\pi^N(.)$ is an increasing function. This captures in a reduced form that capital inflows may lead the nontradable sector to overheat, for example through mortgage and real estate booms.

The baseline model corresponds to the case $\pi^* = \pi^N(.) = \psi = 0$. The case $\psi = 1$ corresponds to the situation in which the interest-rule perfectly tracks the real rate of the domestic economy. A straightforward adaptation of the derivations leading to Proposition 1

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shows that capital flows affect inflation relative to target:

$$\pi_t = \pi^* - (1 - \psi) \sum_{k \geq 0} \frac{l_{t+k}}{(1 + \Phi)^{k+1}}, \qquad (28)$$

and that the expected return on carry trades for a fixed $l \in [\underline{l}, \bar{l}]$ given by (23) becomes:

$$\theta = \underbrace{\frac{\alpha(1 - \psi) - (1 - \alpha)\Phi}{(1 - \alpha)\Phi}}_{A} l + \underbrace{\frac{\alpha(\pi^N(l) - \pi^*)}{1 - \alpha}}_{B} + \delta. \qquad (29)$$

Expression (29) yields two interesting insights.

First, from (A), steady-state multiplicity arises when

$$\alpha(1 - \psi) > (1 - \alpha)\Phi. \qquad (30)$$

For a fixed value of α, the minimum value of Φ ensuring that there is a unique steady-state decreases with respect to ψ. Otherwise stated, a monetary authority that is committed to discourage destabilizing carry trades must respond all the more aggressively to CPI fluctuations because it is reluctant to respond to asset-price fluctuations. Notice that the central bank can repel carry traders in this framework in two other ways: using a measure of inflation that is tilted towards tradables, or adding to the interest-rate rule a term that is sufficiently decreasing in the exchange-rate appreciation. Such policies play the same stabilizing role as an increase in Φ.

Second, from term (B), the expected return on carry trades is higher when $\pi^N(l) - \pi^*$ is large, or, when inflows create significant above-trend inflation in the nontradable sector. This overheating must be offset by below-trend inflation in the tradable sector which translates in turn into a positive trend in the nominal exchange rate regardless of the capital flows. Term (B) reflects this trend. This is broadly consistent with the casual observation that the recipient of carry trade inflows often are economies that are committed to inflation targeting and experience overheating. Typical examples prior to the unusual post-2008 monetary environment include Iceland or New Zealand. A full-fledged model of such overheating in the nontradable sector is an interesting route for future research.

The purpose of this perfect-foresight analysis is to present our novel mechanism for self-justified destabilizing capital flows in the simplest and most transparent environment. Yet this perfect-foresight analysis raises two obvious issues:

- The multiplicity of steady-states leaves unclear how agents can coordinate on any equilibrium behavior at all.

- If carry traders hold the same position forever, then the prices of non tradables and the real-rate target of the central bank should eventually adjust.

We now turn to a stochastic version of our benchmark model and employ global-game techniques to tie down a unique dynamic solution that solves both issues. The goal of the analysis is to provide the theoretical foundations of the dynamics of an open economy in which surges of capital inflows can be explained alongside the reversals that happen in practice.

3   Stochastic model

We will proceed to develop a stochastic version of our model, and then solve for the uniquely determined time paths by using perturbation methods that resemble global-game methods, but which are better suited for dynamic contexts. More precisely, we introduce two modifications to the perfect-foresight model developed in Section 2 that jointly generate equilibrium uniqueness. First, we assume that the US dollar rate is subject to exogenous shocks. Second, we posit that global investors supply slowly-moving capital to the domestic economy.

Shocks to the US dollar rate. We assume that the interest rate on US dollar-denominated bonds between two dates t and $t+1$ is given by

$$I_{t+1}^* = R\left(1 - w_t\right), \tag{31}$$

where $(w_t)_{t \in \mathbb{N}}$ is a stochastic process with increments that are independent and normally distributed with zero mean and standard deviation σ.[9] Recall that R is the discount factor of households.

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[9]We will discuss more general stochastic processes below. Proofs are simpler in this case of a random walk.

Slow-moving capital. Second, we assume that each global investor can revise his investment strategy with probability q only at each date, where $q \in (0, 1]$. The occurence of these switching dates are independent across investors, and so a mass q of them can revise their positions at each date. In between two switching dates, each global investor commits to a strategy and thus to lend to (or borrow from) households the amount decided at the previous switching date, within $[P_t L^-, P_t L^+]$ (in units of the domestic currency).

Unlike in the perfect-foresight environment of Section 2, we now need to specify global investors' preferences in order to characterise their portfolio choice. For simplicity, we suppose that they are a unit mass of long-lived risk-neutral domestic agents that discount future consumption at the rate R. They are penniless but can form zero-cost portfolios in bonds denominated in either currency. We still suppose that

$$\ln(L^- + W) \equiv \underline{l} < 0 < \ln(L^+ + W) \equiv \bar{l}. \tag{32}$$

A natural empirical counterpart of these agents are domestic banks that channel funds between domestic households and global capital markets.

The perfect-foresight model studied in Section 2 corresponds to the particular case of this stochastic model in which

$$\sigma = 0, \tag{33}$$

$$q = 1. \tag{34}$$

Risk-neutrality implies that global investors choose corner portfolios unless the carry trades yields a zero expected return. We deem "long" a global investor who committed to maximum lending L^+ at his last switching date, and "short" one who committed to the maximum borrowing L^-. We let x_t denote the fraction of long global investors at date t. The aggregate real net lending L_t to the cohort of households born at date t is then equal to

$$L_t = x_t L^+ + (1 - x_t)L^-. \tag{35}$$

Suppose that a global investor has a chance to revise his position at a date t. Denoting T_s his next switching date, the expected unit return from the carry trade — the expected value from committing to lend one additional real unit to each future cohort until T_s — is

$$\Pi_t = E_t\left[\sum_{n\geq 0}\frac{1_{\{T_s>t+n\}}}{R^n}\frac{P_{t+n}S_{t+n}}{P_{t+n+1}S_{t+n+1}}\left[\frac{S_{t+n+1}I_{t+n+1}}{RS_{t+n}}-(1-w_{t+n})\right]\right]. \tag{36}$$

Expression (36) states that the global investor earns the carry-trade return associated with each cohort of households born before he gets a chance to revise his position.[10]

The evolution of the economy is then fully described by two state variables, the exogenous state variable w_t and the endogenous state variable x_t. The exogenous state variable w_t directly affects only the expected return on carry trade Π_t while the endogenous one x_t directly affects both the carry trade return and the equilibrium variables (L_t, I_t, P_t, S_t) of the domestic economy. We are now equipped to define an equilibrium.

An equilibrium is characterized by a process x_t that is adapted to the filtration of w_t such that:

$$x_t = \begin{cases} (1-q)x_{t-1} & \text{if } \Pi_t < 0, \\ (1-q)x_{t-1}+q & \text{if } \Pi_t > 0, \end{cases} \tag{37}$$

where

$$\Pi_t = E_t\left[\sum_{n\geq 0}\frac{1_{\{T_s>t+n\}}}{R^n}\frac{P_{t+n}S_{t+n}}{P_{t+n+1}S_{t+n+1}}\left[\frac{S_{t+n+1}I_{t+n+1}}{RS_{t+n}}-(1-w_{t+n})\right]\right],$$

$$I_{T_{n+1}} = R\left(\frac{P_{T_n}}{P_{T_{n-1}}}\right)^{1+\Phi}, \tag{38}$$

$$E_{T_n}\left[\frac{I_{T_{n+1}}P_{T_n}}{P_{T_{n+1}}}\right] = \frac{R}{L_{T_n}+W}, \tag{39}$$

$$\frac{S_{T_{n+1}}}{S_{T_n}} = \left(\frac{P_{T_{n+1}}}{P_{T_n}}\right)^{\frac{-1}{1-\alpha}}. \tag{40}$$

Exactly as in the perfect-foresight case, equilibrium in the domestic economy is characterized by the Taylor rule (38), households' Euler equation (39), and equation (40) linking the nominal exchange rate to the price level. Equation (37) states that global investors make

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[10]To arrive at (36), note that investing one real unit at arrival date $t+n$ costs USD $P_{t+n}S_{t+n}$. The net rate of return $(S_{t+n+1}/S_{t+n})I_{t+n+1} - I^*_{t+n+1}$ applies to this dollar amount. The resulting consumption at date $t+n+1$ is then this USD profit divided by $P_{t+n+1}S_{t+n+1}$. Re-arranging and discounting these terms yields (36).

optimal portfolio choices. They become long at switching dates at which the expected return on the carry trade is positive (or remain long if this was their previous positions), and short if this is negative (or remain short if this was their previous positions).

Note that relations (38) and (40) are identical to their counterparts in the perfect-foresight case, and are in particular log-linear in $L_t + W$. Conversely, the Euler equation (39) now features an expectation over the inverse of inflation given the stochastic environment. As a result, the system of equations defining the equilibrium is no longer log-linear in $L_t + W$. Similarly the expected profit Π_t is not log-linear. For the remainder of the paper, we will solve for a linearized version of these equilibrium equations:

Linear approximation. We solve for an equilibrium process x_t that satisfies the first-order expansions of (38), (39), (40), and (37) in $\underline{l}$ and $\bar{l}$.

This boils down to normalizing $W = 1$[11] and assuming that L^+ and L^- are sufficiently small that global investors have a small impact per period on the domestic real rate, and thus on the rate of inflation and on the appreciation/depreciation of the nominal exchange rate. This log-linearization in L_t around 0 is akin to repeating the log-linearization around the steady-state in the standard New Keynesian model.

All the results in the balance of the paper are up to this log-linearization. We have:

Proposition 2 *(Unique equilibrium) If*

$$\alpha > q + \Phi\left(1 - \alpha\right), \tag{41}$$

there exists a unique equilibrium defined by a decreasing function f such that

$$x_t = x_{t-1} + q(\mathbb{1}_{\{w_t > f(x_{t-1})\}} - x_{t-1}), \tag{42}$$

where $\mathbb{1}_{\{.\}}$ denotes the indicator function.

Equilibrium dynamics. The frontier f divides the (w, x)-space into two regions. Proposition 2 states that in the unique equilibrium, any investor decides to be long when the system is to the right of the frontier f at his switching date, and short when it is on the left

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[11]so that the real rate is R when $L_t = 0$.

x

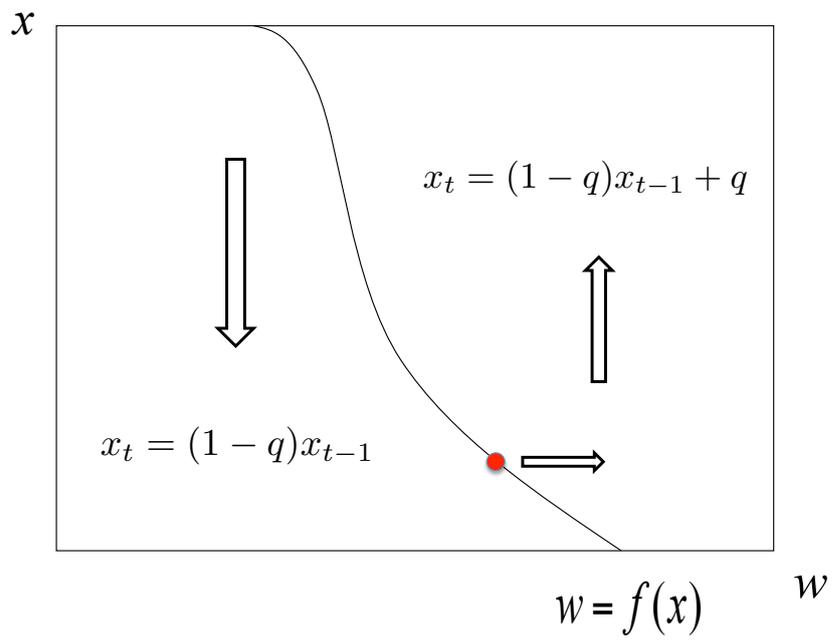$x_t = (1-q)x_{t-1} + q$

$x_t = (1-q)x_{t-1}$

$w = f(x)$

w

Figure 1: **Equilibrium dynamics.** The frontier f divides the *(w,x)*-space into two regions. In the unique equilibrium, lending increases in the right-hand region and declines in the left.

of the frontier. Thus, net lending (and therefore the nominal exchange rate) will tend to rise in the right-hand region, and tend to fall in the left-hand region, as indicated by the arrows in Figure 1.

The main features of these dynamics can be seen from Figure 1. Starting from the red dot on the frontier, a positive shock on w will pull the system to the right of it. Unless the path of w_t is such that a larger negative shock brings it back on the frontier immediately, a more likely scenario is that lending grows for a while so that x_t becomes close to 1. If cumulative negative shocks on w eventually lead the system back to the left of the frontier, then there are outflows. These dynamics therefore correspond to prolonged episodes of appreciation of the domestic currency, large cumulated capital inflows, and benign domestic financial conditions following a negative shock on the US interest rate. Subsequent small increases in the US interest rate do not reverse these dynamics until a tipping point is reached. This point triggers a currency depreciation, capital outflows, an increase in bond yields, domestic inflation, and a tightening of domestic monetary policy. After a long build-up of carry trades and thus a high value of x, these dynamics are asymmetric around the tipping point as carry trades increase before the bifurcation at the rate $q(1 - x)$, whereas they decrease at the relatively much larger rate qx right after it. These features of sudden stops correspond to that experienced by the "Fragile Five" (Brazil, Indonesia, India, South Africa and Turkey) following Bernanke's testimony about the possible "tapering" of the highly accommodative US monetary policy (see Aoki, Benigno, and Kiyotaki, 2015).

It is admittedly not surprising that the equilibrium displays periods of capital inflows or outflows given that investors' positions are assumed to be sticky and bounded. The interesting part of these dynamics lies in our view in the subtle nonlinear impact of the fundamental (the US interest rate) on investors' coordination. After they have reaped positive excess returns on carry trades for a long time, only a large accumulation of negative news can lead investors to switch beliefs about each others' future positions and thus about the profitability of carry trades. When the tipping point is reached, however, a small incremental negative news has a disproportionate impact on investors' position. We consider this to be a signature pattern of episodes of destabilizing speculation that this model captures parsimoniously.

Expected return on carry trades. The expected return on the carry trade at date t, Π_t, is zero if and only if $w_t = f(x_{t-1})$. It is positive if (w_t, x_{t-1}) is on the right of the frontier f in the (w, x)-space and negative if it is on the left of f. Thus, carry trades exhibit abnormal expected returns that increase in the net open interest. They are risky trades though, as an investor can be stuck in a position that generates losses when other investors revert their trade before his trade unwinds.

Slow-moving capital. Recall that condition $\alpha > \Phi(1 - \alpha)$ was generating multiple steady-states in the perfect-foresight case. Condition (41) embeds the additional assumption of slow-moving capital — that q be sufficiently small. We will discuss this assumption in detail in Section 4. Its role can be briefly explained at this stage as follows. Recall from the perfect-foresight analysis that foreign inflows, by making domestic bonds more expensive in the domestic currency, make carry trades akin to strategic substitutes. This stabilizing effect is more than offset by the exchange-rate appreciation induced by carry trades when $\alpha > \Phi(1-\alpha)$, thereby opening up the possibility of extremal steady-states. But $\alpha > \Phi(1-\alpha)$ is sufficient to ensure that the stabilizing effect of carry trades is always dominated by the destabilizing one *only in the case of steady-states with constant carry-trade size x studied in Section 2.* The stronger condition (41) ensures that this is actually the case over all possible paths of x_t.

Proof of Proposition 2

We now proceed with the proof of the proposition. It first proves convenient to characterize the aggregate behavior of global investors as follows. The future carry-trade sizes from date t on are fully characterized by the date-$(t-1)$ carry-trade size x_{t-1}, and by a stochastic process $(\epsilon_{t+s})_{s \geq 0}$ that is adapted to the filtration of w and takes values in $[0, 1]$. The realization of ϵ_{t+s} denotes the fraction of global investors who decide to be long (or to stay long if this was their position at the previous switching date) among those who have a chance to switch

their position at date $t + s$. The size of the carry trade at date $t + s$ is then

$$x_{t+s} = (1 - q)x_{t+s-1} + q\epsilon_{t+s} \tag{43}$$

$$= (1 - q)^{s+1}x_{t-1} + q\sum_{k=0}^{s}(1 - q)^{s-k}\epsilon_{t+k}. \tag{44}$$

For such an aggregate behavior $\{x_{t-1}; (\epsilon_{t+s})_{s\geq 0}\}$, we denote $\Pi(x_{t-1}, (\epsilon_{t+s})_{s\geq 0}, w_t)$ the expected profit from committing to the carry trade at date t as defined in (36). The central result leading to Proposition 2 is:

Lemma 3 *(Global strategic complementarities)* *Condition (41) implies that for all* $\{x_{t-1}; (\epsilon_{t+s})_{s\geq 0}, w_t\}$ *and* $\{x'_{t-1}; (\epsilon'_{t+s})_{s\geq 0}, w'_t\}$ *such that*

$$x_{t-1} \geq x'_{t-1}, \tag{45}$$

$$w_t > w'_t, \tag{46}$$

and almost surely,

$$\epsilon_{t+s} \geq \epsilon'_{t+s}, \tag{47}$$

we have

$$\Pi(x_{t-1}, (\epsilon_{t+s}), w_t) > \Pi(x'_{t-1}, (\epsilon'_{t+s}), w'_t). \tag{48}$$

Proof. See the appendix. ∎

Lemma 3 states that if (41) holds, then the current expected return on carry trades increases in past, current, and future long positions. Past positions are summarized by x_{t-1}, whereas the process ϵ characterizes current and future evolutions of the carry-trade size. Otherwise stated, under such conditions, long positions by global investors at all dates are akin to strategic complements. As detailed in Section 4 below, if q is sufficiently close to 1 other things being equal, then this is no longer true. In this case, past and current long positions hurt current expected returns because the increase in bond prices that they induce becomes a dominant effect. Conversely, for q sufficiently small, investors expect such past and current long positions to persist and positively affect future exchange-rates.

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The balance of the paper posits that (41) holds. We can then show the existence of a unique equilibrium using iterated-dominance techniques that are similar to that developed in Frankel and Pauzner (2000).[12]

First, Lemma 3 implies that for given values of x_{t-1} and w_t, the current and future trades such that

$$\epsilon_{t+s} = 0 \tag{49}$$

correspond to a lower bound on the expected carry-trade return. When ϵ is minimum this way, there exists a frontier f_0 such that

$$w_t = f_0(x_{t-1}) \implies \Pi(x_{t-1}, (0), w_t) = 0. \tag{50}$$

The frontier f_0 is decreasing from Lemma 3. An admissible equilibrium behavior must be such that investors who have a chance to switch when the system is on the right of f_0 become long.

Define now f_1 such that

$$w_t = f_1(x_{t-1}) \implies \Pi(x_{t-1}, (\epsilon^1_{t+s}), w_t) = 0 \tag{51}$$

where the process ϵ_1 obeys

$$\epsilon^1_{t+s} = \mathbb{1}_{\{w_t > f_0(x_{t+s-1})\}}. \tag{52}$$

That is, f_1 is such that an investor is indifferent between being long or short when the system is on f_1 at his switching date if he believes that other investors become long if and only if they are on the right of f_0. This function f_1 must be decreasing. Suppose otherwise that two points (w', x') and (w'', x'') on f_1 satisfy

$$x'' > x',$$
$$w'' > w'.$$

[12]Frankel and Pauzner (2000) have developed this elegant solution technique in a continuous-time model. An earlier version of our paper was also written in continuous time. The adaptation of their technique to the discrete-time model studied here is straightforward.

For a given path of shocks to the US interest rate, it is easy to see that the path starting from (w'', x'') will always be on the right of f_0 in the (w, x) plane whenever so is the path starting from (w', x'). Lemma 3 then implies that the expected return on the carry trade is strictly larger at (w'', x'') than at (w', x'), a contradiction.

By iterating this process, we obtain a limit f_∞ of the sequence of frontiers $(f_n)_{n \geq 0}$ that is decreasing as a limit of decreasing functions.[13] The process

$$x_t = x_{t-1} + q(\mathbb{1}_{\{w_t > f_\infty(x_{t-1})\}} - x_{t-1}) \tag{53}$$

is an admissible equilibrium since by construction, if all investors switch to being short to the left of f_∞ and to being long to the right, the indifference point for an investor also lies on f_∞. We now show that this is the only equilibrium process.

Consider a translation to the left of the graph of f_∞ in (w, x) so that the whole of the curve lies in a region where w_t is sufficiently small that being short is dominant regardless of the dynamics of x_t. Call this translation f_0'. To the left of f_0', going short is dominant. Then construct f_1' as the rightmost translation of f_0' such that an investor must choose to be short to the left of f_1' if he believes that other investors will play according to f_0'. By iterating this process, we obtain a sequence of translations to the right of f_0'. Denote by f_∞' the limit of the sequence. Refer to Figure 2.

The boundary f_∞' does not necessarily define an equilibrium strategy, since it was merely constructed as a translation of f_0'. However, we know that if all others were to play according to the boundary f_∞', then there is at least one point A on f_∞' where the investor is indifferent. If there were no such point as A, this would imply that f_∞' is not the rightmost translation, as required in the definition.

We claim that f_∞' and f_∞ coincide exactly. The argument is by contradiction. Suppose that we have a gap between f_∞' and f_∞. Then, choose point B on f_∞ such that A and B have the same height - i.e. correspond to the same x. But then, since the shape of the boundaries of f_∞' and f_∞ and the values of x are identical, the paths starting from A must have the same distribution as the paths starting from B up to the constant difference in the initial values of w. This contradicts the hypothesis that an investor is indifferent between the two

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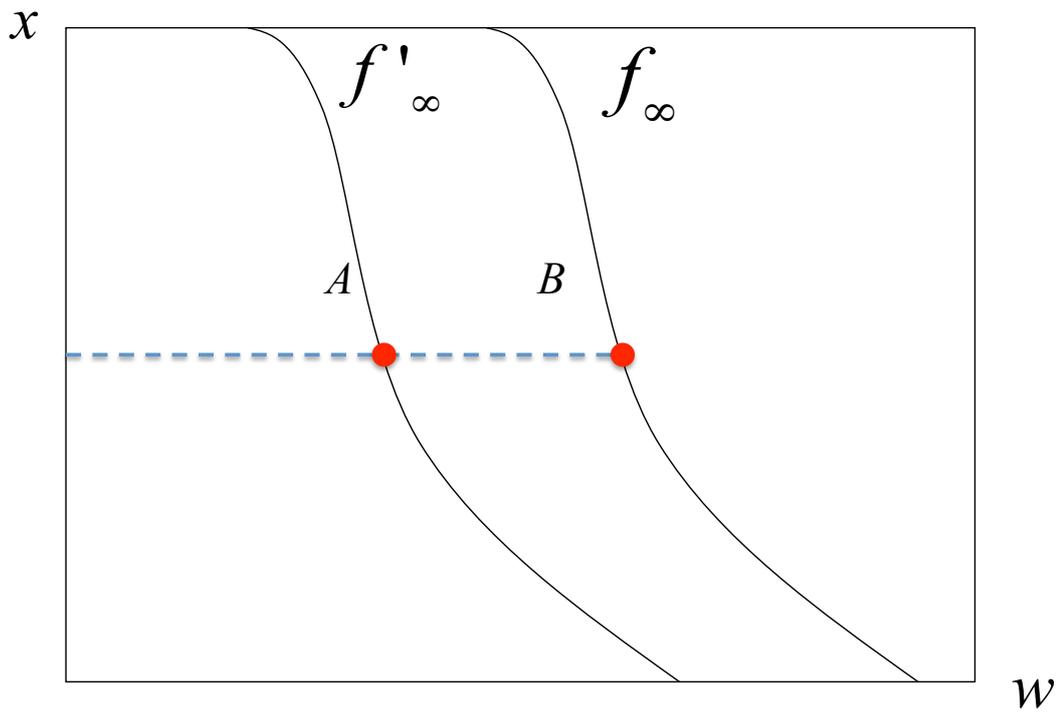[13]We also show in the appendix that f_∞ is Lipschitz continuous.

Figure 2: **Uniqueness of the limiting boundary.** This figure illustrates the argument for the uniqueness of the equilibrium boundary separating the two regions. The boundary f'_∞ coincides with f_∞.

actions both at A and at B. If he were indifferent at A, he would strictly prefer being long at B, and if he is indifferent at B, he would strictly prefer being short when in A. But we constructed A and B so that investors are indifferent in both A and B. Thus, there is only one way to make everything consistent, namely to conclude that $A = B$. Thus, there is no "gap," and we must have $f'_\infty = f_\infty$. ∎

4 Comments and extensions

4.1 Slow moving capital

In order to better grasp the role of the assumption of slow-moving capital (q sufficiently small), it is worthwhile comparing our setup with that in which Burdzy, Frankel, and Pauzner (BFP) develop the iterated-dominance solution that we adapted in the above.

BFP develop a model in which a unit mass of long-lived agents play at each date a two-action—R and S, say—coordination game. Each player's date-t net payoff from playing R over S is increasing in the fraction of the other players who play R at this date (this fraction is the equivalent of x_t here). It is also increasing in an exogenous stochastic parameter (the equivalent of w_t here). Action R (S) is dominant for sufficiently large (small) values of the exogenous parameter, whereas this stage game has multiple equilibria for intermediate parameter values. If agents are free to choose their action at each date (the equivalent of $q = 1$ here), then any sequence of equilibria of the stage games is an equilibrium since the games are completely unrelated. BFP show that the assumption that agents must commit to a particular action for several dates (the equivalent of $q < 1$ here) generates equilibrium uniqueness, however. The broad intuition is that if one action is not dominant now but very likely to be so in the future because the exogenous parameter is close to a dominance value, agents who have a current switching date are willing to commit to this action, and this expands the dominance areas of each action by contagion.

The trading game that global investors play here is fundamentally different from the sequence of static coordination games with independent payoffs studied in BFP. Even when $q = 1$, beliefs about all future actions affect the payoff of the current stage game in our forward looking monetary model. This is transparent from expression (20) of the expected

return on the date-t carry trade in the perfect-foresight case. The reason we need to assume slow-moving capital is thus very different from that in BFP. It is meant to ensure that past, current, and future trading decisions are akin to strategic complements, so that we obtain the same unique equilibrium with stochastic bifurcations as in BFP. If q is sufficiently close to 1 other things being equal, then past and current long positions reduce the expected return on the carry trade by making domestic bonds more expensive. For q sufficiently small, this congestion effect is more than offset by the fact that a high current value of x means that x will also be large in the future, thereby warranting large exchange-rate appreciations.

To sum up, in BFP, agents' actions are strategic complements by assumption even when agents move fast, and the assumption that agents move slowly serves to make current decisions dependent on beliefs about the future actions of other agents. In our setup, current decisions depend on such beliefs about future actions by construction—even when capital is arbitrarily fast. The assumption of slow-moving capital serves to ensure that actions are strategic complements. Otherwise stated, the game that we study has a formally similar payoff to that in BFP only when agents move slowly, whereas both games are actually very different when agents move fast.

Although we have not been able to fully characterize equilibria when (41) does not hold, it is relatively easy to see that they have very different qualitative properties in this case. Consider for simplicity the case $q = 1$. In this case,

$$x_t = \epsilon_t,$$

and the fraction of investors who decide to be long at date t, $x(w_t)$, fully characterizes Markov equilibria. We deem monotone equilibrium an equilibrium such that this mapping is monotonic. We have:

Lemma 4 (Congestion effects when $q = 1$) *Any monotone equilibrium is such that $x(w_t)$ is continuous. This implies that i) the expected return on the carry trade is zero over an interval of values of w; ii) the carry-trade size remains bounded away from its extremal values for sufficiently small fluctuations of w_t.*

Proof. See the appendix. ∎

Lemma 4 showcases the stabilizing role of current trades when $q = 1$. The congestion effects in this case imply that for sufficiently small shocks, carry trades are not "destabilizing." For such shocks, uncovered interest parity holds (the expected return on carry trades is zero) and the size of the carry trade remains commensurate with that of the shocks. This stands in sharp contrast with the dynamics described in Proposition 2. We have seen that in the case of sufficiently slowly moving capital, the expected return on carry trades is always abnormal except on a negligible set (the bifurcation frontier). Paths that feature arbitrarily small fluctuations of w_t can thus lead to arbitrarily large carry-trade sizes as long as they remain on one side of the frontier.

Empirical relevance. Although this setup is too stylized to lend itself to a meaningful calibration, we believe that condition (41) does not require that capital be moving implausibly slowly. If, following Burstein et al. (2005), one argues that "pure" tradables account for 15% of the CPI, then assuming rigid nontradables yields $\alpha = 85\%$.[14] With $\Phi = 0.5$, condition (41) still holds when $q = 75\%$ with such a value of α. A value $q = 50\%$ is compatible with a lower $\alpha = 2/3$. Still, $q < 1$ implies that a subset of investors in foreign-exchange markets do not seek to actively manage their holdings, or/and that imperfect liquidity prevents them from doing so. The imperfect liquidity of foreign-exchange markets is well-documented in the microstructure literature (see, e.g., Evans and Lyons (2002), Lyons (2001), or Sarno and Taylor (2001)), particularly so for small currencies. Using the bid-ask spreads that prevail in practice, Burnside et al. (2006) find that transactions costs do put significant limits on the optimal size and profitability of foreign-exchange carry trades.

Regarding limited active management, Bacchetta and van Wincoop (2010) claim that the financial institutions that actively manage foreign exchange positions, such as hedge funds, manage only a tiny fraction of cross border financial holdings. In fact, Bacchetta and van Wincoop (2010) use a calibration in which investors rebalance their investments only every other year in order to rationalize a number of exchange-rate patterns. Also, a number of popular pre-crisis carry trades notoriously involved a significant fraction of retail investors

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[14]The assumption of fully rigid nontradable prices is obviously too strong. On the other hand, we assume a complete pass through ("PPP at the docks"), whereas an incomplete pass through of, say, 50% as in Obstfeld and Rogoff (2007) would considerably reinforce our results. It would play the same role as an increase in α in weakening the link between the nominal exchange rate and domestic price levels.

(epitomized, for example, as the "Belgian dentist" investing in "glacier" bonds denominated in Icelandic krona, or the "Japanese widow" investing in New-Zealand dollars denominated uridashi bonds) who are likely to pay limited attention to market fluctuations.

4.2 The role of trading limits

It is interesting to discuss the role of the exogenous trading limits $[P_{T_n}L^-, P_{T_n}L^+]$ both in the perfect-foresight and stochastic versions of the model. In Section 2, it is clearly necessary to impose trading limits regardless of investors' preferences because carry trades are textbook arbitrage opportunities—portfolios with positive payoffs at all times in all states with at least a strictly positive payoff. When they coordinate on extremal trading positions, global investors basically create two different risk-free rates for themselves. Absent trading limits they would therefore always want to take infinitely large positions no matter how risk averse. Whereas this paper does not offer microfoundations for such trading limits, they could result from agency problems within globally investing firms such as, for example, a cash-flow diversion problem.

The analysis is more complex in the stochastic environment of Section 3. Holding the trading strategies of other investors fixed, a carry trade generates losses with a non-zero probability in equilibrium given the shocks to the US rate. Thus, any risk-averse investor would form an interior portfolio pinned down by a marginal indifference condition given other investors' trading strategies. Whether this implies that our framework could accommodate unconstrained portfolio choice by agents with smooth preferences is unclear. Assuming away any bound on trading strategies eliminates the dominance areas that are necessary to ignite the iterated-dominance solution applied here, as sufficiently large adverse positions by other investors could make any position by a given investor undesirable. In fact, Frankel, Morris, and Pauzner (2003) solve a static global game in which the action space is very general and need not be countable using iterated-dominance techniques similar to that used here. But they still need a bounded action space.

One way to preserve the existence of dominance areas with a milder assumption than individual trading limits would be to ensure that the unit expected return on carry trades

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cannot vary without bounds with respect to the aggregate position of global investors. This could be justified by the assumption that sufficiently extreme aggregate carry-trade positions trigger a regime change— adjustment of non tradable prices, of the policy rate, or the implementation of policies such as capital controls— that put a cap on how these positions can affect the return on carry trades.

4.3 The case of small shocks and time intervals

The qualitative description of equilibrium dynamics under the conditions stated in Proposition 2 leaves it unclear whether large fluctuations of the carry-trade size are frequent events. Starting from the frontier ($w_t = f(x_{t-1})$), is the system likely to head off immediately towards extreme values of x, or is it more likely to experience small fluctuations around the frontier for many periods?

Applying to our model the results established in Burdzy, Frankel, and Pauzner (2001), we show that bifurcations towards extreme values of x are actually the most likely outcome in the case of small shocks and small time intervals. To tackle this case, we parameterize the time elapsing between two dates as $\tau \in (0, 1]$. We redefine the standard deviation of shocks to the US rate as $\sigma\sqrt{\tau}$, the per period discount rate as $1 + (R - 1)\tau$, and the probability of receiving a switching date as $q\tau$. The case $\tau = 1$ therefore corresponds to the model studied thus far.

We first establish a technical result ensuring that the condition on α, Φ, and q leading to a unique equilibrium with decreasing frontier in Proposition 2 can be made independent of the parameters σ and τ. We have

Lemma 5 *If (41) holds, then for all $\tau \in (0, 1]$ and $\sigma > 0$ there exists a unique equilibrium characterized by a strictly decreasing frontier $f_{\sigma,\tau}(.)$ as in Proposition 2.*

Proof. See the appendix. ∎

We denote the frontier $f_{\sigma,\tau}$ so as to stress its dependence on these two parameters. Suppose that (41) holds . Fix $x_{t-1} \in (0, 1)$ and $\delta < min\{x_{t-1}; 1 - x_{t-1}\}$. Suppose the system is on the frontier ($w_t = f_{\sigma,\tau}(x_{t-1})$). We deem δ-bifurcation a path whereby the system stays on the same side of the frontier until x becomes within δ of 0 or 1. We have:

Lemma 6 *(**Small shocks and time intervals**)* *Fix $\eta > 0$. There exists $\overline{\sigma}$ and a function $\overline{\tau}(.)$ such that for all $\sigma \leq \overline{\sigma}$ and $\tau \leq \overline{\tau}(\sigma)$, a δ-bifurcation occurs before date η/q with probability at least $1 - \eta$. The probability that the bifurcation is upwards is within η of $1 - x_{t-1}$.*

Proof. Theorem 2 and Lemma 2 in Burdzy, Frankel, and Pauzner (2001). ∎

Lemma 6 shows that in this case at least, the equilibrium dynamics whereby the system experiences large bifurcations with rare but abrupt reversals are likely scenarios.

4.4 More general shocks

Bounded shocks to the US rate We model the interest-rate differential as a pure random walk for expositional simplicity. It is easy to see that we could write it as $d(w_t)$, where w_t is a standard Brownian motion, and d a Lipschitz increasing function, possibly bounded as long as there are still dominant actions for w_t sufficiently large or small.

Transitory shocks to the US rate While a strong persistence in shocks to the US rate is undoubtedly realistic, extensions of this framework can also accommodate for various forms of mean-reversion (Burdzy, Frankel, and Pauzner, 2001, or Frankel and Burdzy, 2005).

5 Concluding remarks

The independence of monetary policy under liberalised capital flows and floating exchange rates has been a benchmark principle in international finance. In our paper, we have explored a parsimonious model of global investors facing each other in a dynamic global game and found that under plausible conditions, the model generates boom bust cycles associated with coordinated capital inflows and outflows. In such a setting, monetary conditions depend on the coordination outcome of investors who have access to the domestic bond market, as well as on the economic fundamentals. Thus, we qualify the proposition that a floating exchange rate guarantees monetary autonomy by showing that as capital flows more smoothly into a small open economy, then a commitment to a more aggressive monetary response to capital flows is required in order to discourage destabilizing carry trades.

Assuming that households are risk-neutral over late consumption dramatically simplifies the analysis. With strictly concave preferences, the current real rate would depend on consumption growth, so that we could no longer abstract from the impact of foreign lending on quantities and thus production in the domestic economy as we are able to do here. We find it useful to derive our novel mechanism for self-fulfilling profitable carry trades in a highly tractable framework that describes the interplays of the ingredients at work in a fully transparent fashion. An interesting avenue for future research, which would be more simulation-based, is the study of the impact of such carry trades on quantities under more standard preferences.

A Appendix

A.1 Proof of Lemma 3

The first-order expansion of the Euler equation (39):

$$\ln I_{t+1} + \ln E_t \left[\frac{P_t}{P_{t+1}} \right] = \ln R - \ln(L_t + W) \tag{54}$$

in $\underline{l}$, $\bar{l}$ yields:

$$\ln I_{t+1} - E_t \left[\ln \frac{P_{t+1}}{P_t} \right] = \ln R - l_t \tag{55}$$

where

$$l_t = \underline{l}(1 - x_t) + \bar{l}x_t. \tag{56}$$

Combined with the Taylor rule (38), this yields domestic inflation as a function of future expected inflows as in the perfect-foresight case:

$$\ln \frac{P_t}{P_{t-1}} = -\sum_{k \geq 0} \frac{E_t \left[l_{t+k} \right]}{(1 + \Phi)^{k+1}}. \tag{57}$$

As in the perfect-foresight case, (40) yields in turn:

$$E_t \left[\ln \frac{S_{t+1}I_{t+1}}{RS_t} \right] = \frac{\alpha}{1 - \alpha} \sum_{k \geq 0} \frac{E_t \left[l_{t+k+1} \right]}{(1 + \Phi)^{k+1}} - l_t. \tag{58}$$

One can write (36) as

$$\Pi_t = E_t \left[\sum_{n \geq 0} \frac{1_{\{T_s > t+n\}}}{R^n} E_{t+n} \left[\frac{P_{t+n}S_{t+n}}{P_{t+n+1}S_{t+n+1}} \left(\frac{S_{t+n+1}I_{t+n+1}}{RS_{t+n}} - 1 + w_{t+n} \right) \right] \right]. \tag{59}$$

At first-order w.r.t. $\underline{l}$, $\bar{l}$,

$$E_{t+n} \left[\frac{P_{t+n}S_{t+n}}{P_{t+n+1}S_{t+n+1}} \left(\frac{S_{t+n+1}I_{t+n+1}}{RS_{t+n}} - 1 \right) \right] = E_{t+n} \left[\ln \frac{S_{t+n+1}I_{t+n+1}}{RS_{t+n}} \right] \tag{60}$$

$$= \frac{\alpha}{1 - \alpha} \sum_{k \geq 0} \frac{E_{t+n} \left[l_{t+n+k+1} \right]}{(1 + \Phi)^{k+1}} - l_{t+n}. \tag{61}$$

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Thus,

$$\Pi_t = E_t \left[\sum_{n\geq 0} \frac{(1-q)^n}{R^n} \left(\frac{\alpha}{1-\alpha} \sum_{k\geq 0} \frac{l_{t+n+k+1}}{(1+\Phi)^{k+1}} - l_{t+n} \right) \right] + \frac{Rw_t}{R+q-1}, \tag{62}$$

re-arranging yields

$$\Pi_t = -l_t + E_t \left[\sum_{n\geq 1} \left(\frac{1-q}{R} \right)^n l_{t+n} \left[\frac{\alpha}{1-\alpha} \sum_{i=1}^{n} \left[\frac{R}{(1-q)(1+\Phi)} \right]^i - 1 \right] \right] + \frac{Rw_t}{R+q-1}. \tag{63}$$

From (56) and (44), this implies

$$\Pi(x_{t-1}, (\epsilon_{t+s}), w_t) = \underline{l}\kappa + (\bar{l} - \underline{l})\pi(x_{t-1}, (\epsilon_{t+s})) + \frac{Rw_t}{R+q-1}, \tag{64}$$

where

$$\kappa = \sum_{n\geq 1} \left(\frac{1-q}{R} \right)^n \left[\frac{\alpha}{1-\alpha} \sum_{i=1}^{n} \left[\frac{R}{(1-q)(1+\Phi)} \right]^i - 1 \right] - 1, \tag{65}$$

$$\pi(x_{t-1}, (\epsilon_{t+s})) = -(1-q)x_{t-1} - q\epsilon_t + \sum_{n\geq 1} \left[\begin{array}{c} \left(\frac{1-q}{R} \right)^n \left[(1-q)^{n+1}x_{t-1} + q\sum_{k=0}^{n}(1-q)^{n-k}E_t[\epsilon_{t+k}] \right] \\ \times \left[\frac{\alpha}{1-\alpha} \sum_{i=1}^{n} \left(\frac{R}{(1-q)(1+\Phi)} \right)^i - 1 \right] \end{array} \right]. \tag{66}$$

One can write the term $\pi(x_{t-1}, (\epsilon_{t+s}))$ as:

$$\pi(x_{t-1}, (\epsilon_{t+s})) = A(q)[(1-q)x_{t-1} + q\epsilon_t] + \sum_{k\geq 1} B_k(q)qE_t[\epsilon_{t+k}], \tag{67}$$

where

$$A(q) = -1 + \sum_{n\geq 1} \left[\frac{(1-q)^2}{R} \right]^n \left[\frac{\alpha}{1-\alpha} \sum_{i=1}^{n} \left(\frac{R}{(1-q)(1+\Phi)} \right)^i - 1 \right], \tag{68}$$

$$B_k(q) = \frac{1}{(1-q)^k} \sum_{n\geq k} \left[\frac{(1-q)^2}{R} \right]^n \left[\frac{\alpha}{1-\alpha} \sum_{i=1}^{n} \left(\frac{R}{(1-q)(1+\Phi)} \right)^i - 1 \right]. \tag{69}$$

Straightforward computations show that

$$A(0) \geq 0, B_k(0) \geq 0 \tag{70}$$

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for all k iff

$$\alpha \geq (1-\alpha)\Phi. \tag{71}$$

In addition, $A(q)$ and $(1-q)^k B_k(q)$ for given values of R and Φ are equal to $A(0)$ and $B_k(0)$ for

$$R' = \frac{R}{(1-q)^2}, \tag{72}$$

$$1 + \Phi' = \frac{1+\Phi}{1-q}, \tag{73}$$

which yields the result.

A.2 Complement to the proof of Proposition 2

We prove here that f_∞ is Lipschitz decreasing. First, it is transparent from (66) that f_0 is affine. We show that f_1 is Lipschitz with a constant that is smaller than that of f_0, that we denote K_0. Suppose by contradiction that two points (w_t, x_t) and (w'_t, x'_t) on f_1 satisfy

$$x' > x, \tag{74}$$

$$\frac{x'_t - x_t}{w_t - w'_t} < \frac{1}{K_0}. \tag{75}$$

We compare the paths x'_{t+u} and x_{t+u} corresponding to pairs of paths of w'_{t+u} and w_{t+u} that satisfy for all $u \geq 0$

$$w_{t+u} - w'_{t+u} = w_t - w'_t. \tag{76}$$

It must be that for such pairs of paths:

$$x'_{t+u} - x_{t+u} \leq (x'_t - x_t)(1-q)^u. \tag{77}$$

Otherwise it would have to be the case that (w', x') can be on the right of f_0 when (w, x) is not. Suppose by contradiction that this can be. Let T denote the first time at which this occurs. It must be that for some $T' \in [0, T]$,

$$K_0(1-q)^{T'}(x'_t - x_t) \geq w_{t+T'} - w'_{t+T'} = w_t - w'_t, \tag{78}$$

a contradiction with (75).

Thus along such paths of $w'_{t+u} - w_{t+u}$, $x'_{t+u} - x_{t+u}$ shrinks at least as fast as when investors switch to being short all the time. Together with (75), this implies that the expected return on the carry trade cannot be the same in (w_t, x_t) and (w'_t, x'_t), a contradiction.

That f_∞ is also Lipschitz-continuous with a smaller constant than K_0 then follows by induction.

A.3 Proof of Lemma 4

The expected return on carry trades (63) becomes for $q = 1$:

$$\Pi_t = -l_t + \frac{\alpha}{1-\alpha} \sum_{n \geq 1} \frac{E_t[l_{t+n}]}{(1+\Phi)^n} + w_t. \tag{79}$$

Suppose the mapping $l(w)$ from $\mathbb{R}$ into $[\underline{l}, \overline{l}]$ defines a Markov monotone equilibrium. It must be increasing given the existence of dominance regions. We show that it must be continuous. Suppose by contradiction a discontinuity at some value w^0. An inspection of (79) shows that the expected return on carry trade should be discontinuously decreasing in the neighborhood of w_0 since the therm $-l_t$ is discontinuous whereas the expectation is continuous. This cannot be because an increasing l must correspond to an increasing expected return.

The mapping l is thus continuous. It cannot be constant over $\mathbb{R}$ given the existence of dominance regions. Thus l takes values within $(\underline{l}, \overline{l})$ over an interval of values of w. The expected return on carry trades must be zero for such interior portfolio choices.

Finally, that small fluctuations of w lead to small fluctuations of l is a direct consequence of continuity.

A.4 Proof of Lemma 5

Note that the conditions on α, Φ, and q in Proposition 2 serve only to establish Lemma 3. The proof of Lemma 3 shows in turn that these conditions ensure that the coefficients of x_{t-1} and $E_t[\epsilon_{t+k}]$ for all $k \geq 0$ in (66) are strictly positive. These coefficients do not depend on σ, and only on α, Φ, and $q\tau$. If (41) holds then these coefficients are therefore positive for all $\tau \leq 1$.

References

Agrippino, Silvia Miranda and Helene Rey (2015) "World Asset Markets and the Global Financial Cycle," working paper

Albagli, Elias, Christian Hellwig and Aleh Tsyvinski (2015) "A Theory of Asset Prices based on Heterogeneous Information," working paper

Aoki, Kosuke, Gianluca Benigno and Nobuhiro Kiyotaki (2015) "Monetary and Financial Policies in Emerging Markets," working paper

Bacchetta Philippe and Eric van Wincoop (2010) "Infrequent Portfolio Decisions: A Solution to the Forward Discount Puzzle," *American Economic Review*, 100, 870-904

Bank for International Settlements (2011) "Global liquidity – concept, measurement and policy implications" CGFS Papers 45, Committee on the Global Financial System http://www.bis.org/publ/cgfs45.pdf

Bruno, Valentina and Hyun Song Shin (2015a) "Cross-Border Banking and Global Liquidity," *Review of Economic Studies,* 82 (2), 535-564

Bruno, Valentina and Hyun Song Shin (2015b) "Capital flows and the risk-taking channel of monetary policy" *Journal of Monetary Economics*, 71, 119-32

Burdzy, Krzysztof, David Frankel and Ady Pauzner (2001) "Fast Equilibrium Selection by Rational Players Living in a Changing World," *Econometrica,* 68, 163-190

Burnside, Craig, Martin Eichenbaum, Isaac Kleshchelski and Sergio Rebelo (2006) "The Returns to Currency Speculation," NBER Working Paper No. 12489

Burstein, Ariel, Martin Eichenbaum and Sergio Rebelo (2005) "Large Devaluations and the Real Exchange Rate," *Journal of Political Economy*, Vol. 113, No. 4, 742-784

Evans, Martin and Richard K. Lyons (2002) "Order Flow and Exchange Rate Dynamics," *Journal of Political Economy,* 110, 170-80

Farhi, Emmanuel and Xavier Gabaix (2015) "Rare Disasters and Exchange Rates", forthcoming in the *Quarterly Journal of Economics*

Farhi, Emmanuel and Jean Tirole (2012) "Collective Moral Hazard, Maturity Mismatch and Systemic Bailouts," *American Economic Review*, vol. 102, 60-93

Fleming, J. Marcus (1962) "Domestic financial policies under fixed and floating exchange rates" *IMF Staff Papers* 9, 369–379

Frankel, David and Krzysztof Burdzy (2005) "Shocks and Business Cycles," *Advances in Theoretical Economics,* v. 5 iss. 1 paper no. 2

Frankel, David, Stephen Morris and Ady Pauzner (2003) "Equilibrium selection in global games with strategic complementarities," *Journal of Economic Theory,* 108 (1), 1-44

Frankel, David and Ady Pauzner (2000) "Resolving Indeterminacy in Dynamic Settings: The Role of Shocks," *Quarterly Journal of Economics,* Vol. 115, 283-304

Gabaix, Xavier and Matteo Maggiori (2015) "International Liquidity and Exchange Rate Dynamics," forthcoming in the *Quarterly Journal of Economics*

Garleanu, Nicolae, Stavros Panageas and Jianfeng Yu (2015)"Financial Entanglement: A Theory of Incomplete Integration, Leverage, Crashes, and Contagion," forthcoming in the *American Economic Review*

Goldstein, Itay and Ady Pauzner (2005) "Demand Deposit Contracts and the Probability of Bank Runs," *Journal of Finance*, 60(3), 1293-1328

He, Zhiguo, Arvind Krishnamurthy and Konstantin Milbradt (2016) "A Model of Safe Asset Determination," forthcoming in the *American Economic Review*

He, Zhiguo and Wei Xiong (2012) "Dynamic Debt Runs," *Review of Financial Studies*, 25, 1799-1843

Lorenzoni, Guido (2008) "Inefficient Credit Booms," *Review of Economic Studies,* 75 (3), 809-833

Lyons, Richard K. (2001) "The Microstructure Approach to Exchange Rates," Cambridge, Mass.: MIT Press.

Mendoza, Enrique G. (2010) "Sudden Stops, Financial Crises, and Leverage." *American Economic Review,* 100(5): 1941-66

Mundell, Robert A. (1963): "Capital mobility and stabilisation policy under fixed and flexible exchange rates" *Canadian Journal of Economic and Political Science* 29(4), 475–485

Obstfeld, Maurice (1996) "Models of currency crises with self-fulfilling features," *European Economic Review*, vol. 40(3-5), 1037-1047

Obstfeld, Maurice and Kenneth Rogoff (2007) The Unsustainable U S Current Account Position Revisited. G7 Current Account Imbalances: Sustainability and Adjustment. Chicago: University of Chicago Press. NBER volume

Rey, Hélène (2013) "Dilemma not Trilemma: The global financial cycle and monetary policy independence" proceedings of the Federal Reserve Bank of Kansas City Economic Symposium at Jackson Hole

Rey, Hélène (2015) "International Channels of Transmission of Monetary Policy and the Mundellian Trilemma"Mundell Fleming Lecture 2014, *IMF Economic Review*

Sarno, Lucio and Mark Taylor (2001) "The Microstructure of the Foreign Exchange Market, A Selective Survey of the Literature," Princeton Studies in International Economics, No. 89 Sannikov, Yuliy (2014) "Quiet Times and Financial Fragility," working paper

Schneider, Martin and Aaron Tornell (2004), "Balance Sheet Effects, Bailout Guarantees and Financial Crises," *Review of Economic Studies,* 71, 883-913