Information Aggregation in Poisson-Elections*

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Abstract

The modern Condorcet jury theorem states that under weak conditions, when voters have common interests, elections will aggregate information when the population is large, in any equilibrium. Here, we study the performance of large elections with population uncertainty. We find that the modern Condorcet jury theorem holds if and only if the expected number of voters is independent of the state. If the expected number of voters depends on the state, then additional equilibria exist in which information is not aggregated. The main driving force is that, everything else equal, voters are more likely to be pivotal if the population is small.

Elections are said to be effective in aggregating information that is dispersed among citizens, for example, about uncertainty regarding future economic prospects, costs and benefits of a public good, or the political ramifications of a trade deal. This belief has been justified by the so-called Condorcet Jury theorem (see Ladha (1992)), which asserts that large electorates choose correct outcomes, and in its modern form by Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1997, 1998), Wit (1998), Duggan and Martinelli (2001), and others. Precisely, the modern Condorcet jury theorem states that under weak conditions, in a large voting game with common

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values, *all* responsive and symmetric Nash equilibria aggregate information. The Condorcet jury theorem is one motivation for using elections for making collective choices. In its modern form, it provides "*a rational choice foundation for the claim that majorities invariably 'do better' than individuals*" (at least, for large electorates); see Austen-Smith and Banks (1996).

Most of these earlier contributions assume that the number of voters is deterministic and known. Myerson (1998a) observed that the size of the electorate is often uncertain. Importantly, this uncertainty may not be independent of the underlying state of the world. Turnout is affected by many factors: For example, the (opportunity) cost of participation may depend on the perceived economic prospects, something we formalize in an extension in this paper. Similarly, the awareness of elections may depend on the competency or the motivation of the current office holders because of its effect on news coverage and general political engagement. Finally, election turnouts are often subject to manipulation by interested parties who may choose to influence turnout strategically and differently across states by facilitating or obstructing voter participation; see Ekmekci and Lauermann (2020). Many of these factors naturally correlate with the relevant state, and it would be an extraordinary conincidence for all the many factors determining turnout to exactly equalize across states.

Here, we abstract from these particular sources of state-dependent participation and study whether the modern Condorcet Jury Theorem is robust to such population uncertainty. To do so, we use the model by Myerson (1998a): Voters have to choose among two alternatives (two policies). They share common values that depend on an unknown, binary state of nature. The number of voters is Poisson distributed and the mean of the distribution may depend on the state. Each voter observes a private, conditionally independent signal.

To start, note that any asymmetry in the expected number of voters itself contains additional information about the state of the world, hence, there is one more source of information—in addition to the private signals of the voters—that the electorate could use to aggregate information. However, as we argue below, even though there is more information that could be used, large electorates may fail to aggregate any information, that is, the modern Condorcet Jury Theorem is not robust to population uncertainty.

Our first theorem restates the result from Myerson (1998a): If voters have noisy but informative signals about the state of the world, then large electorates in which the population size is state dependent admit at least one Nash equilibrium that aggregates information, i.e., that chooses the correct outcome with a probability close to one. So, one part of the Condorcet Jury theorem survives: Large electorates are able to aggregate information.

Our main theorem shows, however, that the second part of the Condorcet Jury theorem fails: there are plausible equilibria that fail to aggregate information when the population is state dependent. In such equilibria, the majority of voters vote as if the state is the one in which there are fewer voters. Therefore, the policy wins that is preferred in the state in which there are fewer voters. Such equilibria are responsive, and when sufficiently informative signals are possible, these equilibria are stable.

Thus, our main finding is that the modern Condorcet Jury theorem holds with population uncertainty if and only if this uncertainty is independent from the state. Otherwise, if the population is statistically state dependent, additional responsive equilibria exist that fail to aggregate information.

The key force that helps sustain such equilibria is a "**participation curse**". A vote is more likely to change the outcome of the election, i.e., to be *pivotal*, when there are fewer voters, all else being equal. Therefore, a majority of voters—but not all voters—vote as if the state is the one with fewer voters.

We then explore whether strategic abstention can help eliminate such "bad" equilibria. Krishna and Morgan (2012) showed that voluntary voting improves on compulsory voting and induces sincere voting outcomes when there are binary signals. In Feddersen and Pesendorfer (1997), abstention allows the uninformed players to participate in a rate that cancels out the effect of partisans who cast their votes in one way independently of their signals. Hence, one may hope strategic abstention to help the electorate "undo" the asymmetry in the population size induced by exogenous factors. However, we show that allowing abstention does not eliminate responsive equilibria that fail to aggregate information.

We also show that the additional equilibria that fail to aggregate information can be stable. Finally, we provide an example where state-dependent turnout endogenously arises by the participation decisions of the voters due to cost distributions that depend on the state.

Related Literature: Our results relate to three contributions. Ekmekci and Lauermann (2020) also consider a setting in which the number of voters is state-dependent (but the number of voters is deterministic conditional on the state) and show that information aggregation may fail. That paper focuses on the actions of an organizer who determines the turnout. Myerson (1998a) introduces a model of Poisson elections in which the expected number of voters may be state-dependent and voting is compulsory. He shows that there exist equilibria that aggregate information. Krishna and Morgan (2012) study a model of Poisson elections in which the expected number of voters is independent of the state and abstention is allowed, showing that for the case with binary signals, voting is sincere in all equilibria and information can be aggregated.[1] Relative to Ekmekci and Lauermann (2020), we consider Poisson elections and allow abstention; relative to Myerson (1998a), we show the existence of additional equilibria and allow abstention; and relative to Krishna and Morgan (2012), we allow continuous signals and show that there are additional equilibria when the expected number of voters depends on the state.

The literature has identified other circumstances in which information may fail to aggregate. Feddersen and Pesendorfer (1997) show such a failure in an extension (Section 6 of their paper) when the aggregate distribution of preferences remains uncertain conditional on the realized state. Mandler (2012) demonstrates a similar failure if the aggregate distribution of signals remains uncertain. In these settings, the effective state is multi-dimensional. Intuitively, this implies an invertibility problem from the relevant order statistic of the vote shares to payoff-relevant states. A similar problem is identified by Bhattacharya (2013), who observes the necessity of preference monotonicity for information aggregation; see also Bhattacharya (2018) and Ali, Mihm, and Siga (2017). Barelli, Bhattacharya, and Siga (2018) study what conditions on the joint distributions of states and voters' signals make information aggregation feasible with two or more alternatives. Bouton and Castanheira (2012) show that, in a Poisson election with more than two alternatives, information aggregation fails with most voting rules except for approval voting. Gul and Pesendorfer (2009) show that information aggregation fails when some voters do not observe a candidate's policy choice. In our setting, conditional on the state, there is no aggregate uncertainty (in the sense that the mean of the Poisson distribution is known), preferences over policies are monotone in the state, and there is no policy uncertainty.

[1] Their paper considers costly voting, which is the main focus of their analysis.

1 Model

The model setup follows Myerson (1998a). Voters have to decide between two policies, A and B. There are two states, α and β, with prior probability

$$\pi = \Pr\{\alpha\},$$

with $0 < \pi < 1$ and $\Pr\{\beta\} = 1 - \pi$. The voters have common values: Each voter receives a payoff of 1 if the policy matches the state, and payoff of 0 otherwise.

However, voters do not know the realized state. Instead, voters observe noisy signals $x \in [\underline{x}, \bar{x}]$. Conditional on the state, the signals are independent and identically distributed. The c.d.f. of the signal distribution is $G(\cdot|\omega)$. The distribution is atomless and admits a continuous density. Without loss of generality, signals are ordered so the weak MLRP holds,

$$\frac{g(x|\alpha)}{g(x|\beta)} \text{ is weakly decreasing in } x.$$

In addition, $g(x|\omega) > 0$ for all $x \in (\underline{x}, \bar{x})$. This, together with G being atomless, rules out that voters receive perfectly revealing signals with positive probability. We define $\lim_{x\to\underline{x}} \frac{g(x|\alpha)}{g(x|\beta)} =: \frac{g(\underline{x}|\alpha)}{g(\underline{x}|\beta)} \in \mathbb{R}_+ \cup \{\infty\}$ and $\lim_{x\to\bar{x}} \frac{g(x|\alpha)}{g(x|\beta)} =: \frac{g(\bar{x}|\alpha)}{g(\bar{x}|\beta)} \in \mathbb{R}_+$. Signals contain some information, meaning, $\frac{g(\underline{x}|\alpha)}{g(\underline{x}|\beta)} > 1 > \frac{g(\bar{x}|\alpha)}{g(\bar{x}|\beta)}$. As in Duggan and Martinelli (2001) and Krishna and Morgan (2012), we assume that

$$\frac{\pi}{1-\pi}\frac{g(\underline{x}|\alpha)}{g(\underline{x}|\beta)} > 1 > \frac{\pi}{1-\pi}\frac{g(\bar{x}|\alpha)}{g(\bar{x}|\beta)}. \tag{1}$$

With this assumption, based on their own signal alone, a voter with the strongest signal for α would prefer policy A and a voter with the strongest signal for β would prefer policy B. The assumption holds if the prior is uniform. The assumption also holds if signals are sufficiently informative.

The number of voters is Poisson distributed in each state, with an expected number of $n_\alpha = n$ and $n_\beta = \theta n$; so, the probability that there are t voters in state ω is

$$\Pr\{t|\omega\} = \frac{(n_\omega)^t e^{-n_\omega}}{t!}.$$

The policy is decided by simple majority rule among submitted votes. If there is a tie, then a fair coin flip decides. Abstention is not possible for now.

We consider pure and type-symmetric voting strategies.[2] A voting strategy is a function $a : [\underline{x}, \bar{x}] \to [0,1]$, with $a(x)$ the probability to vote for A.

Let $U(x, W; a, n)$ be the expected utility for a voter having signal x who votes for $W \in \{A, B\}$, given that all other voters use strategy a and the expected number of voters is $(n, \theta n)$ in states α and β, respectively. We often drop a and n.

We study voting strategies that form a (Bayesian) Nash equilibrium. A voting strategy a is a Nash equilibrium if and only if $U(x, A; a, n) > U(x, B; a, n)$ implies $a(x) = 1$ and $U(x, A; a, n) < U(x, B; a, n)$ implies $a(x) = 0$.

2 Preliminary Characterization

In addition to learning from their own signal, voters obtain information about the total number of participants (and, hence, the state) via their own participation. In particular, the likelihood ratio of the two states conditional on having signal x and participating is[3]

$$\frac{\Pr(\alpha|x)}{\Pr(\beta|x)} = \frac{\pi}{1-\pi}\frac{n}{\theta n}\frac{g(x|\alpha)}{g(x|\beta)}. \tag{2}$$

Let T denote the event that the number of A and B votes is the same, $T-1$ the event that there is one less A vote than B votes, and $T+1$ the event that there is one more A vote. Then, the difference $U(x, A; a, n) - U(x, B; a, n)$ is equal to

$$\begin{aligned} &\Pr(\alpha|x)\left(\Pr[T-1|\alpha]\frac{1}{2} + \Pr[T|\alpha] + \Pr[T+1|\alpha]\frac{1}{2}\right) \\ &- \Pr(\beta|x)\left(\Pr[T-1|\beta]\frac{1}{2} + \Pr[T|\beta] + \Pr[T+1|\beta]\frac{1}{2}\right). \end{aligned} \tag{3}$$

Voting A versus voting B changes the payoffs only in the events $T-1$, T, and $T+1$. In the first event, voting A rather than B increases the probability of A winning from 0 to 1/2, in the second event it increases the probability from 0 to 1, and in the third event it increases the probability from 1/2 to 1.

[2] Myerson (1998a, p. 377) argues that in a Poisson election, all equilibria should be type-symmetric.

[3] See Milchtaich (2004) for a discussion of updating in Poisson games with state-dependent participation; see also our example in Section 6.

The probability that the decision to vote A versus B turns out to be *pivotal* is

$$\Pr(\text{Piv}|\omega) = \frac{1}{2}\Pr[T-1|\omega] + \Pr[T|\omega] + \frac{1}{2}\Pr[T+1|\omega].$$

It is then easy to see from (2) and (3) that voting for A is a best response for a voter having signal x if

$$\gamma(x;a,n) := \frac{\pi}{1-\pi}\frac{1}{\theta}\frac{g(x|\alpha)}{g(x|\beta)}\frac{\Pr(\text{Piv}|\alpha)}{\Pr(\text{Piv}|\beta)} \geq 1,$$

where γ denotes the *critical likelihood ratio.*

A strategy a is a cutoff strategy if for some $\hat{x}$, we have $a(x) = 1$ if $x > \hat{x}$ and $a(x) = 0$ if $x < \hat{x}$. We state without proof that cutoff strategies are without loss of generality. This is immediate from γ being nonincreasing in x.

Lemma 1. *If a strategy forms a Nash equilibrium, it is equivalent to a cutoff strategy.*[4]

Our generic notation is $\hat{x}$ for the strategy: "Vote for A if $x \in (\underline{x}, \hat{x})$, Vote for B if $x \in (\hat{x}, \bar{x})$." A cutoff strategy is said to be *responsive* if the cutoff is interior, $\hat{x} \in (\underline{x}, \bar{x})$; it is *non-responsive* if either $\hat{x} = \underline{x}$ or $\hat{x} = \bar{x}$, that is, if all voters are supporting the same alternative.

Abusing notation, let $\gamma(x;\hat{x},n)$ be the critical likelihood ratio given cutoff $\hat{x}$. Note that the map $x \to \gamma(x;x,n)$ is continuous; thus, $\hat{x} \in (\underline{x}, \bar{x})$ is an interior Nash equilibrium if and only if

$$\gamma(\hat{x};\hat{x},n) = 1.$$

Moreover, γ being continuous immediately implies that an equilibrium always exists.

Lemma 2. *There exists a Nash equilibrium.*

If $\gamma(t;t,n) > 1$ for all t, then $\hat{x} = \underline{x}$ is an equilibrium and if $\gamma(t;t,n) < 1$ for all t, then $\hat{x} = \bar{x}$ is an equilibrium. Finally, if $\gamma(t';t',n) \geq 1 \geq \gamma(t'';t'',n)$ for some t' and t'', then there is some $\hat{x}$ between t' and t'' such that $\gamma(\hat{x};\hat{x},n) = 1$.

Note that in a Poisson election, the probability of being pivotal is strictly positive, whatever strategy the other voters use. In particular, in Poisson elections there are

[4] If the likelihood ratio is constant on some interval, every voting strategy is equivalent to a voting strategy in cutoffs (because we can reorder votes on that interval). If the likelihood ratio is strictly increasing, every voting strategy is in cutoffs.

no "trivial" equilibria, in contrast to elections with a deterministic number of voters. Thus, the Poisson distribution acts as a tremble that refines away certain equilibria; see Bouton and Castanheira (2012, p. 61).

We also use the following approximation of the critical likelihood ratio.

Lemma 3. *Consider a sequence of voting games in which the expected number of participants is* $(n, \theta n)$ *in states* α *and* β*, respectively, and* $n \to \infty$*. Given a sequence of cutoffs* $(\hat{x}^n)_{n \in \mathbb{N}}$*, let*

$$M(\omega) := 1 - \lim_{n\to\infty} 2\sqrt{G(\hat{x}^n|\omega)(1 - G(\hat{x}^n|\omega))},$$

whenever the limit exists. If $\lim_{n\to\infty} \hat{x}^n \in (\underline{x}, \bar{x})$*, then*

$$\lim_{n\to\infty} \frac{\Pr[Piv|\alpha; \hat{x}^n, n]}{\Pr[Piv|\beta; \hat{x}^n, n]} = \begin{cases} \infty & \text{if} \quad M(\alpha) < \theta M(\beta), \\ 0 & \text{if} \quad M(\alpha) > \theta M(\beta). \end{cases}$$

For $\theta = 1$*, this simplifies to*

$$\lim_{n\to\infty} \frac{\Pr[Piv|\alpha; \hat{x}^n, n]}{\Pr[Piv|\beta; \hat{x}^n, n]} = \begin{cases} \infty & \text{if} \quad \lim\limits_{n\to\infty} \left|G(\hat{x}^n|\alpha) - \frac{1}{2}\right| < \left|G(\hat{x}^n|\beta) - \frac{1}{2}\right|, \\ 0 & \text{if} \quad \lim\limits_{n\to\infty} \left|G(\hat{x}^n|\alpha) - \frac{1}{2}\right| > \left|G(\hat{x}^n|\beta) - \frac{1}{2}\right|. \end{cases}$$

This lemma follows from standard approximations to pivot probabilities; see Krishna and Morgan (2012). The proof of the lemma is in Section A.1 in the appendix, where we re-state these general approximations and this and other lemmas for our purposes.

To interpret the result, note that $G(\hat{x}^n|\omega)$ is the expected vote share of A in state ω. Thus, in the case $\theta = 1$, the result says that the state in which the election is closer to being tied in expectation becomes arbitrarily more likely conditional on the election being actually tied. For general θ, note that $M(\omega) = 0$ if and only if $\lim G(\hat{x}^n|\omega) = \frac{1}{2}$, and otherwise $0 < M(\omega) \leq 1$. Thus, if the election is expected to be tied in one state, this state becomes arbitrarily more likely conditional on being pivotal.

3 Information Aggregation and the Modern Condorcet Jury Theorem

The modern Condorcet theorem states that in large elections, all equilibria aggregate information. More formally, we say a sequence of equilibria $\{\hat{x}^n\}_{n=1}^{\infty}$ aggregates information if A wins with probability converging to 1 in state α and B wins in state β. For a deterministic number of voters with common values, the modern Condorcet Jury theorem is shown by Feddersen and Pesendorfer (1998).[5] For an uncertain number of voters, it is shown by Krishna and Morgan (2012) for Poisson elections in which the expected number of voters is independent of the state, $\theta = 1$, and signals are binary. For Poisson elections with a state-dependent number of voters, Myerson (1998a) shows that, for all $\theta > 0$, there exists at least some sequence of equilibria that aggregates information.

Theorem 1. *Consider a sequence of voting games in which the expected number of participants is* $(n, \theta n)$ *in states* α *and* β*, respectively, and* $n \to \infty$*.*

1. Myerson (1998a). *For all* $\theta \in (0, \infty)$*, there exists a sequence of equilibria that aggregates information.*

2. *Krishna and Morgan (2012). All sequences of equilibria aggregate information if* $\theta = 1$ *(there is no imbalance) and (1) holds (signals are sufficiently informative).*

Myerson (1998a) provides a direct proof of Item 1, as we do here. An alternative method of proof utilizes the common interest structure, following McLennan (1998); see the working paper version Ekmekci and Lauermann (2018).

For deterministic elections, a result that is analogous to Item 2 holds only for symmetric and responsive equilibria. For Poisson elections, symmetry holds by construction (Myerson, 1998a, p. 377) and there are no non-responsive equilibria if (1) holds; hence, the result is stronger: If $\theta = 1$, *all* Nash equilibria aggregate information.[6]

[5] They show this for symmetric and responsive equilibria with binary signals; see also Wit (1998). Duggan and Martinelli (2001) show the result for a continuum of signals.

[6] We emphasize the importance of (1) for these stronger conclusions. If the condition fails, then it can be easily seen that nonresponsive equilibria exist. These equilibria are not "trivial" (the pivotal probability is not 0), which contrasts with the existence of trivial equilibria in elections with a deterministic number of voters. Moreover, one can also show that if the condition fails, then there are responsive equilibrium sequences that do not aggregate information.

4 Failure of Condorcet Jury Theorem

We now show that the modern Condorcet theorem fails if $\theta \neq 1$.

Theorem 2. *Consider a sequence of voting games in which the expected number of participants is $(n, \theta n)$ in states α and β, respectively, and $n \to \infty$.*

- *If $\theta < 1$, then there exists a sequence of responsive Nash equilibria in which B wins in both states with a probability converging to 1.*
- *If $\theta > 1$, then there exists a sequence of responsive Nash equilibria in which A wins in both states with a probability converging to 1.*

The proof is in the appendix in Section A.3. Define the median signals as

$$x_\alpha : G(x_\alpha|\alpha) = 1/2 \text{ and } x_\beta : G(x_\beta|\beta) = 1/2.$$

Because of the MLRP, the signal distribution in state β first-order stochastically dominates the distribution in state α; thus, $x_\alpha < x_\beta$. Note that, when $\hat{x} = x_\omega$, then the election is expected to be tied in state ω.

Consider the case with $\theta < 1$. The main observations used in the proof are that

$$\lim_{n\to\infty} \gamma(x_\alpha; x_\alpha, n) = \infty, \tag{4}$$

and that, given $\theta < 1$, for any x_R small enough, with $\underline{x} < x_R < x_\alpha$,

$$\lim_{n\to\infty} \gamma(x_R; x_R, n) = 0. \tag{5}$$

Since the map $x \to \gamma(x, x; n)$ is continuous, the intermediate value theorem implies that, for all n large enough, there exists some $\hat{x}^n \in (x_R, x_\alpha)$ such that $\gamma(\hat{x}^n; \hat{x}^n, n) = 1$. We verify that $\lim_{n\to\infty} \hat{x}^n < x_\alpha$, and, hence, B wins with probability converging to 1 in both states by the law of large numbers.

The critical observations (4) and (5) follow from Lemma 3. For (4), note that for a cutoff $\hat{x} = x_\alpha$, the election is tied in expectations in state α while B wins with certainty in state β. So, $0 = M(\alpha) < M(\beta)$, and, by Lemma 3, conditional on being pivotal, a voter becomes certain that the state is α; thus, (4) holds.

Now, consider some x_R close to $\underline{x}$. Then, $M(\alpha)$ and $M(\beta)$ are both close to 1. Therefore, $\theta < 1$ implies $M(\alpha) > \theta M(\beta)$, and, hence, by Lemma 3, conditional on being pivotal, a voter becomes certain that the state is β; thus, (5) holds.

For further insights, note that when x_R is close to $\underline{x}$, then B will win in both states. Moreover, the margin of victory as a proportion of votes is larger in state β than in α, that is, $\frac{1}{2} - G(x_R|\beta) > \frac{1}{2} - G(x_R|\alpha)$. However, when $\theta < 1$ and x_R is close to $\underline{x}$, then the margin of victory in the absolute number of voters is *smaller* in state β than in α, that is, $n\theta\left(\frac{1}{2} - G(x_R|\beta)\right) < n\left(\frac{1}{2} - G(x_R|\alpha)\right)$. Hence, it turns out that a voter is more likely to be pivotal in state β.

At the heart of the aggregation failure is the fact that there are fewer voters in state β when $\theta < 1$. Because the number is smaller, a voter is more likely to be pivotal in that state, and, given that sophisticated voters condition on being pivotal, they tend to support B, even if their signals are strongly in favor of A.

5 Robustness

We now investigate the robustness of the main finding of this paper, that is, unbalanced state-dependent participation gives rise to equilibria that fail to aggregate information. First, we investigate whether such equilibria survive a further stability refinement. Second, we investigate whether voluntary voting eliminates such equilibria.

Most of our analysis in this section will be for the case when the signals are unboundedly informative, that is,

$$\frac{g(\underline{x}|\alpha)}{g(\underline{x}|\beta)} = \infty \text{ and } \frac{g(\overline{x}|\alpha)}{g(\overline{x}|\beta)} = 0.$$

This is needed for some of the arguments, and it simplifies the analysis of voluntary voting. Also, the failure of information aggregation is maybe more stark when there are arbitrarily informative signals.

5.1 Stability

For this discussion, consider $\theta < 1$. Recall that, in this case, equation (5) holds for some x_R and all n large enough. Further, if signals are unboundedly informative, it is immediate that for any fixed n, there exists some x_L^n with $\underline{x} < x_L^n < x_R$ such that[7]

$$\gamma\left(x_L^n; x_L^n, n\right) > 1. \tag{6}$$

Together with (5), inequality (6) implies that there exists some equilibrium with $\hat{x}_s^n \in (x_L^n, x_R)$ for all n large enough by the intermediate value theorem. Thus, when signals are unboundedly informative, there are at least two interior equilibria in which information aggregation fails, this one and the previous one with $\hat{x}^n \in (x_R, x_\alpha)$.

This argument implies also that, when signals are unboundedly informative and $\theta < 1$, there is one "stable" equilibrium that fails to aggregate information by the following argument: From (5) and (6), for all n large enough, $\tilde{\gamma}(x) = \gamma(x; x, n)$ cuts 1 from above, at least once at some point $\hat{x}_s < x_R$. If $\hat{x}_s$ is the only equilibrium cutoff in some neighborhood, then $\hat{x}_s$ is an equilibrium cutoff that is expectationally stable in the sense of Fey (1997). In particular, this cutoff has the property that it is the outcome of a dynamic best-response iteration: If such a process starts in a neighborhood of the equilibrium cutoff, then the process will eventually converge back to it.[8]

Unstable Equilibria. Applying the same argument to the case with boundedly informative signals, it follows from (4) and (5) that there exists at least one point $\hat{x}_r \in (x_L, x_\alpha)$ at which $\tilde{\gamma}$ crosses 1 from below. Thus, when signals are boundedly informative, there exists at least one equilibrium cutoff $\hat{x}_r$ that is unstable.[9]

Non-Responsive Equilibria. As observed by Myerson (1998a), for all $\theta \neq 1$, if signals are boundedly informative, then there are also non-responsive equilibria when n is large enough. Consider $\theta < 1$ and suppose $\hat{x} = \underline{x}$, so that all voters support B. In this case, a voter is pivotal whenever there is either no other voter or just one. The likelihood ratio of being pivotal is therefore

$$\frac{\Pr\left[\mathrm{Piv}|\alpha; \underline{x}, n\right]}{\Pr\left[\mathrm{Piv}|\beta; \underline{x}, n\right]} = \frac{e^{-n}\left(1+n\right)}{e^{-\theta n}\left(1+\theta n\right)} \approx e^{-n(1-\theta)}\frac{1}{\theta} \rightarrow_{n\rightarrow\infty} 0.$$

[7] This holds because, for any fixed n, the ratio $\frac{\Pr[Piv|\alpha;x,n]}{\Pr[Piv|\beta;x,n]}$ is bounded.

[8] It may be that for some $\varepsilon > 0$, and $\tilde{x}_0 < \tilde{x}_1$, we have $\tilde{\gamma}(x) > 1$ for $x \in (\tilde{x}_0 - \varepsilon, \tilde{x}_0)$, $\tilde{\gamma}(x) = 1$ for $x \in [\tilde{x}_0, \tilde{x}_1]$ and $\tilde{\gamma}(x) < 1$ for $x \in (\tilde{x}_1, \tilde{x}_1 + \varepsilon)$. We may call such a set of equilibria "pseudo-stable," with singletons being a special case.

[9] On the other hand, even though there must be some equilibrium that is not stable, we cannot rule out that there are stable equilibria. In fact, our previous discussion implies that it is easily possible to construct stable equilibria for some boundedly informative signals.

Thus, given that signals are boundedly informative, it is a best response for a voter to vote for B independently of her signal, for n large enough. Non-responsive equilibria are stable. In particular, these equilibria are not "trivial" because each voter is still pivotal with a strictly positive probability—in contrast to analogous non-responsive equilibria with a deterministic number of voters.

5.2 Voluntary Voting (Abstention)

We now consider the possibility of abstention or "voluntary voting." Feddersen and Pesendorfer (1996) noted that voters may have a strict incentive to abstain because of the "swing voter's curse." Moreover, the possibility of abstention necessarily increases the expected payoff of a representative agent in the best equilibrium relative to compulsory voting.

In addition, Krishna and Morgan (2012) observe that with abstention and a binary signal, there is no longer a conflict between voting strategically and voting sincerely that is often present with compulsory voting even when $\theta = 1$.[10] Thus, abstention may help eliminate the equilibria that we identified before since these equilibria relied on voters with a strong signal towards state α to nevertheless vote B. Thus, we now ask whether abstention may help eliminate the bad equilibria.

With abstention, our generic notation is (y, z) for the strategy: "Vote for A if $x \in (\underline{x}, y)$, Abstain if $x \in (y, z)$, Vote for B if $x \in (z, \bar{x})$." Analogously, we call a voting strategy (y, z) *non-responsive* if either $z = \underline{x}$ or $y = \bar{x}$, so, either all participants vote B or all vote A. Otherwise, an equilibrium is *responsive*.

Theorem 3. *Suppose voting is voluntary and signals are unboundedly informative. Consider a sequence of voting games in which the expected number of participants is $(n, \theta n)$ in states α and β, respectively, and $n \to \infty$.*

1. *If $\theta < 1$, then there is a sequence of responsive Nash equilibria in which B wins in both states with probability converging to 1.*

2. *If $\theta > 1$, then there is a sequence of responsive Nash equilibria in which A wins in both states with probability converging to 1.*

The proof is in the appendix. The basic idea is this. Consider an auxiliary game $\Gamma(x_R, n)$ with a parameter $x_R > \underline{x}$ in which voters with signals $x \geq x_R$ must vote

[10] Also, if voting is costly, abstention allows to reduce overall voting costs.

for B but which otherwise remains unchanged. By a standard argument, this game has an equilibrium. Then, for n large enough, this equilibrium is shown to be also an equilibrium of the original game if x_R is small enough, in particular, if $x_R < x_\alpha$. The critical argument for this proof is that for x_R small enough, given any strategy profile with $y \leq z \leq x_R$, the probability of state β conditional on being pivotal (for either candidate) converges to 1.[11] Thus, voters with signals around $x_R > \underline{x}$ will optimally vote for B—and hence this restriction does not bind. Thus, this is an equilibrium of the original game. Moreover, the equilibrium is responsive: Since signals are unboundedly informative, for every given n, voters will optimally vote A for some signal sufficiently close to $\underline{x}$.

With boundedly informative signals there are additional technical issues that require the development of new tools. In the working paper version Ekmekci and Lauermann (2018), we considered the case of a binary signal as in Krishna and Morgan (2012) and showed the existence of analogous equilibria.

6 Example: State-Dependent Participation Costs

We now present a scenario where state-dependent participation arises endogeneously in an equilibrium of a costly voting model with a state-dependent cost distribution. As an example, suppose some citizens vote on an economic reform. Then, it is natural to assume that the opportunity costs of participation depend on the citizens' personal economic situations, which may be correlated with the reform's desirability.[12]

Specifically, suppose the number of citizens is Poisson-distributed with mean m. Further suppose that each citizen draws a voting cost according to the state-dependent, discrete probability distribution $\Pr(c|\omega)$, whose support is $\{0, \bar{c}\}$, with $\bar{c} > 0$. We assume that costs and the state are correlated, i.e., $\Pr(c|\alpha) \neq \Pr(c|\beta)$ for $c \in \{0, \bar{c}\}$. Otherwise, the model remains as before. In particular, each citizen also observes a signal $x \in [\underline{x}, \bar{x}]$. Conditional on the state, the cost and the signal are independent for a given citizen as well as across the citizens. After observing her own cost c and signal x, each citizen decides whether to vote at costs c or abstain. The voting rule is simple majority rule, with ties broken by a fair coin flip.[13]

[11] Note that this argument is very similar to the one used in (5) for compulsory voting.

[12] In Ekmekci and Lauermann (2020) we consider another scenario with endogeneous state-dependent turnout. There, an informed election organizer recruits voters.

[13] One may view this example as a stylized version of Krishna and Morgan (2012) with a state-

Now consider the costless voting model from Section 5.2 where the signal is unboundedly informative. Suppose that the expected number of participants is $m\Pr(0|\alpha)$ and $m\Pr(0|\beta)$ in states α and β, respectively. Let (y, z) be the equilibrium identified in Theorem 3. We now argue that, when m is large, there is an equilibrium of our costly voting model where citizens with high costs $\bar{c}$ abstain and citizens with low costs behave according to (y, z), voting for A if $x < y$ and B if $x > z$, and abstaining if $x \in (y, z)$. To understand why, note that the expected number of votes cast in the equilibria identified in Theorem 3 grows without bound as m grows. This implies that the probabilities of being pivotal vanish, and hence, it is optimal for citizens with high-cost citizens to abstain. Given this behavior, it is optimal for low-cost citizens to behave according to (y, z) because, given the high-cost voters' abstention, they face the same problem as in the costless voting model.

Thus, when m is large, there is an equilibrium of a costly voting game with exactly the same outcome as the equilibrium of the costless voting game with exogenously state-dependent number of participants.[14] Note also that voters learn from their own participation cost, implying that participants' beliefs are different from the prior.

7 Conclusion

We study the set of equilibria of Poisson elections when the expected number of voters is state-dependent. We show that large Poisson elections robustly aggregate information—in the sense that *all* equilibria imply the correct choice with probability converging to one—if and only if the expected number of voters is constant across states. If the expected number of voters is different, then there are additional responsive equilibria that fail to aggregate information. The basic reason for this is that voters are more likely to be pivotal when the electorate is smaller. This pivotal inference leads to equilibria in which voters systematically vote for the policy that is optimal in the state with fewer expected voters. When signals are sufficiently informative, these equilibria can be chosen to be stable. Abstention does not eliminate the additional equilibria.

There are many reasons for participation to be state-dependent: In Ekmekci and

dependent cost distribution.

[14] However, there may also be equilibria in which only voters with $c = \bar{c}$ participate, and they do so with a small probability. These equilibria also fail to aggregate information, in this case because of the low turnout.

Lauermann (2020), an election organizer has private information about the state and recruits voters with a different intensity across states. In the example in Section 6, state-dependent participation results from the correlation between participation costs and the state. In general, such correlation may also reflect a varying awareness across the population about an election or referendum and its specifics. A comprehensive analysis of costly voting with state-dependent cost distributions is a promising avenue for future research.

A stark feature of our model is that the outcome is deterministic when the number of voters is large. The reason is that in our model, there is only idiosyncratic uncertainty. However, in reality, election outcomes often seem to be fundamentally difficult to predict even for professional election forecasters, with "surprises" happening frequently. One simple way to introduce aggregate uncertainty is via an exogenous random share of partisans who vote for a certain candidate independently of their signals. In the working paper version, Ekmekci and Lauermann (2018), we show that our results continue to hold qualitatively when this aggregate uncertainty is small. Again, a comprehensive analysis is left for future research.

A Appendix

Sequences and Limits. When taking limits, we mean with respect to subsequences for which a limit exists (in the extended reals). In the context of our proofs, such subsequences can always be found and proving statements for all converging subsequences will be sufficient for the desired claims. We also drop the delimiter from lim when the limit is taken with respect to $n \to \infty$.

A.1 Toward a Proof of Lemma 3 (Auxiliary Results)

In the appendix, we denote the expected number of A and B votes in state α as

$$\sigma_A(\hat{x}) = nG(\hat{x}|\alpha) \text{ and } \sigma_B(\hat{x}) = n(1 - G(\hat{x}|\alpha)).$$

Similarly, for state β,

$$\tau_A(\hat{x}) = \theta nG(\hat{x}|\beta) \text{ and } \tau_B(\hat{x}) = \theta n(1 - G(\hat{x}|\beta)).$$

We often drop the arguments from σ_W and τ_W.

We approximate the pivotal probabilities. Given any sequence $\{x^k\}_{k=1}^{\infty}$ and two functions f and g, we say

$$f \approx g \qquad \text{if} \qquad \lim_{k\to\infty} \frac{f\left(x^k\right)}{g\left(x^k\right)} = 1.$$

To improve readability, we suppress the sequence index k in the following statement.

We first prove an intermediary approximation result.

Lemma 4. *If $\sigma_A\sigma_B \to \infty$, then*

$$\begin{aligned}
\Pr[T|\alpha] &\approx e^{-\sigma_A-\sigma_B}\frac{e^{2\sqrt{\sigma_A\sigma_B}}}{\sqrt{2\pi 2\sqrt{\sigma_A\sigma_B}}}, \\
\Pr[T\pm 1|\beta] &\approx e^{-\sigma_A-\sigma_B}\frac{e^{2\sqrt{\sigma_A\sigma_B}}}{\sqrt{2\pi 2\sqrt{\sigma_A\sigma_B}}}\left(\frac{\sigma_A}{\sigma_B}\right)^{\pm 1/2}.
\end{aligned}$$

If $\sigma_A\sigma_B \to k \in (0,\infty)$, then

$$\begin{aligned}
\Pr[T-1|\alpha] &\approx e^{-\sigma_A-\sigma_B}\sigma_B\frac{I_1\left(2\sqrt{k}\right)}{\sqrt{k}}, \\
\Pr[T|\alpha] &\approx e^{-\sigma_A-\sigma_B}I_0\left(2\sqrt{k}\right), \\
\Pr[T+1|\alpha] &\approx e^{-\sigma_A-\sigma_B}\sigma_A\frac{I_1\left(2\sqrt{k}\right)}{\sqrt{k}},
\end{aligned}$$

with $I_0 : \mathbb{R}_+ \to \mathbb{R}_+$ a continuous, strictly positive function with $\lim_{z\to\infty} I_0(z) = I_1(z) = \infty$, $I_0(0) = 1$ and $I_1 : \mathbb{R}_+ \to \mathbb{R}_+$ a continuous function that is strictly positive on $(0,\infty)$ but $\lim_{z\to 0}\frac{I_1(z)}{z} = 1/2$.

If $\sigma_A\sigma_B \to 0$, then

$$\begin{aligned}
\Pr[T-1|\alpha] &\approx e^{-\sigma_A-\sigma_B}\sigma_B, \\
\Pr[T|\alpha] &\approx e^{-\sigma_A-\sigma_B}, \\
\Pr[T+1|\alpha] &\approx e^{-\sigma_A-\sigma_B}\sigma_A.
\end{aligned}$$

All analogous approximations hold for state β, after substituting τ_W for σ_W.

Proof of Lemma 4. The Lemma follows immediately from observations from Krishna and Morgan (2012), equations (4) and (5), namely,

$$\Pr\left[T|\alpha\right] = e^{-\sigma_A-\sigma_B} I_0\left(2\sqrt{\sigma_A\sigma_B}\right), \tag{7}$$

$$\Pr\left[T\pm 1|\alpha\right] = e^{-\sigma_A-\sigma_B}\left(\frac{\sigma_A}{\sigma_B}\right)^{\pm 1/2} I_1\left(2\sqrt{\sigma_A\sigma_B}\right), \tag{8}$$

where I_0 and I_1 are the so-called "modified Bessel functions." The approximations then use properties of the modified Bessel functions, namely, that

$$\lim_{z\to\infty}\frac{\frac{e^z}{\sqrt{2\pi z}}}{I_0\left(z\right)} = \lim_{z\to\infty}\frac{\frac{e^z}{\sqrt{2\pi z}}}{I_1\left(z\right)} = 1,$$

and that

$$\lim_{z\to 0}\frac{I_1\left(z\right)}{z} = \frac{1}{2} \Rightarrow \frac{\left(\frac{\sigma_B}{\sigma_A}\right)^{1/2} I_1\left(2\sqrt{\sigma_A\sigma_B}\right)}{\sigma_B} = \frac{I_1\left(2\sqrt{\sigma_A\sigma_B}\right)}{\sqrt{\sigma_A\sigma_B}} \to 1.$$

Now, the approximations follow. $\square$

Proof of Lemma 3. Recall that

$$\Pr\left(Piv|\omega\right) = \frac{1}{2}\Pr\left[T-1|\omega\right] + \Pr\left[T|\omega\right] + \frac{1}{2}\Pr\left[T+1|\omega\right].$$

From $\lim \hat{x}^n \in (\underline{x}, \bar{x})$, we have $\sigma_A\sigma_B \to \infty$ and $\tau_A\tau_B \to \infty$. So, Lemma 4 implies

$$\begin{aligned} & \frac{1}{2}\Pr\left[T-1|\alpha\right] + \Pr\left[T|\alpha\right] + \frac{1}{2}\Pr\left[T+1|\alpha\right] \\ \approx\; & e^{-\sigma_A-\sigma_B}\frac{e^{2\sqrt{\sigma_A\sigma_B}}}{\sqrt{2\pi 2\sqrt{\sigma_A\sigma_B}}}\frac{1}{2}\left(2+\left(\frac{\sigma_A}{\sigma_B}\right)^{+1/2}+\left(\frac{\sigma_A}{\sigma_B}\right)^{-1/2}\right) \\ =\; & \frac{e^{-n+2\sqrt{\sigma_A\sigma_B}}}{\sqrt{2\pi 2\sqrt{\sigma_A\sigma_B}}}\frac{1}{2}\left(2+\left(\frac{\sigma_A}{\sigma_B}\right)^{+1/2}+\left(\frac{\sigma_A}{\sigma_B}\right)^{-1/2}\right). \end{aligned}$$

Furthermore, $\lim \hat{x}^n \in (\underline{x}, \bar{x})$ implies that

$$0 < \lim \frac{\sqrt[4]{\tau_A\tau_B}}{\sqrt[4]{\sigma_A\sigma_B}}\frac{2+\sqrt{\frac{\sigma_B}{\sigma_A}}+\sqrt{\frac{\sigma_A}{\sigma_B}}}{2+\sqrt{\frac{\tau_B}{\tau_A}}+\sqrt{\frac{\tau_A}{\tau_B}}} =: K < \infty.$$

This is because with $\hat{x} = \lim \hat{x}^n$,

$$\lim \frac{\sqrt[4]{\tau_A \tau_B}}{\sqrt[4]{\sigma_A \sigma_B}} = \lim \frac{\sqrt[4]{\theta^2 n^2 G\left(\hat{x}^n|\beta\right)\left(1 - G\left(\hat{x}^n|\beta\right)\right)}}{\sqrt[4]{n^2 G\left(\hat{x}^n|\alpha\right)\left(1 - G\left(\hat{x}^n|\alpha\right)\right)}} = \frac{\sqrt{\theta}\sqrt[4]{G\left(\hat{x}|\beta\right)\left(1 - G\left(\hat{x}|\beta\right)\right)}}{\sqrt[4]{G\left(\hat{x}|\alpha\right)\left(1 - G\left(\hat{x}|\alpha\right)\right)}},$$

and

$$\lim \frac{2 + \sqrt{\frac{\sigma_B}{\sigma_A}} + \sqrt{\frac{\sigma_A}{\sigma_B}}}{2 + \sqrt{\frac{\tau_B}{\tau_A}} + \sqrt{\frac{\tau_A}{\tau_B}}} = \frac{2 + \sqrt{\frac{(1-G(\hat{x}|\alpha))}{G(\hat{x}|\alpha)}} + \sqrt{\frac{G(\hat{x}|\alpha)}{(1-G(\hat{x}|\alpha))}}}{2 + \sqrt{\frac{(1-G(\hat{x}|\beta))}{G(\hat{x}|\beta)}} + \sqrt{\frac{G(\hat{x}|\beta)}{(1-G(\hat{x}|\beta))}}}.$$

So,

$$\begin{aligned} \lim \frac{\Pr\left[Piv|\alpha\right]}{\Pr\left[Piv|\beta\right]} &= \lim \frac{e^{-n+2\sqrt{\sigma_A \sigma_B}}}{e^{-\theta n+2\sqrt{\tau_A \tau_B}}} \frac{\sqrt[4]{\tau_A \tau_B}}{\sqrt[4]{\sigma_A \sigma_B}} \frac{2 + \sqrt{\frac{\sigma_B}{\sigma_A}} + \sqrt{\frac{\sigma_A}{\sigma_B}}}{2 + \sqrt{\frac{\tau_B}{\tau_A}} + \sqrt{\frac{\tau_A}{\tau_B}}} \\ &= K \lim e^{n\left(2\sqrt{G(\hat{x}^n|\alpha)(1-G(\hat{x}^n|\alpha))}-1-\theta\left(2\sqrt{G(\hat{x}^n|\beta)(1-G(\hat{x}^n|\beta))}-1\right)\right)}. \end{aligned}$$

and the lemma now follows. □

A.2 Proof of Theorem 1

Proof of Item 1: Existence. Recall that the median signals solve $G\left(x_\alpha|\alpha\right) = 1/2$ and $G\left(x_\beta|\beta\right) = 1/2$. By the MLRP, $G\left(\cdot|\beta\right)$ first-order stochastically dominates $G\left(\cdot|\alpha\right)$. Therefore, $x_\alpha < x_\beta$, and

$$G\left(x_\alpha|\beta\right) < \frac{1}{2} < G\left(x_\beta|\alpha\right).$$

We show that

$$\lim \gamma\left(x_\alpha; x_\alpha, n\right) = \infty, \tag{9}$$

and

$$\lim \gamma\left(x_\beta; x_\beta, n\right) = 0. \tag{10}$$

Note that $\left|G\left(x_\alpha|\alpha\right) - \frac{1}{2}\right| = 0 < \left|G\left(x_\alpha|\beta\right) - \frac{1}{2}\right|$. So, using Lemma 3, $M\left(\alpha\right) = 0 < M\left(\beta\right)$ implies

$$\lim \frac{\Pr\left[Piv|\alpha; x_\alpha, n\right]}{\Pr\left[Piv|\beta; x_\alpha, n\right]} = \infty,$$

proving (9). Equation (10) follows analogously.

Given (9) and (10), the intermediate value theorem implies the existence of some

$\hat{x}^n \in (x_\alpha, x_\beta)$ for all n large enough. By the same argument as above, it cannot be that $\hat{x}^n \to x_\alpha$ or $\hat{x}^n \to x_\beta$, observing that $M(\alpha) = 0 < M(\beta)$ in the first case and $M(\alpha) > 0 = M(\beta)$ in the second case.

Thus, there exists a sequence of equilibria with $\lim \hat{x}^n \in (x_\alpha, x_\beta)$, and the weak law of large numbers implies that information aggregates.

Proof of Item 2: Information Aggregation in all Sequences if $\theta = 1$.
Given $\theta = 1$, we first show that for any sequence of cutoffs $\{x^n\}_{n=1}^{\infty}$,

$$\lim \frac{\Pr[Piv|\alpha; x^n, n]}{\Pr[Piv|\beta; x^n, n]} = \begin{cases} \infty & \text{if } \ \underline{x} < \lim x^n \leq x_\alpha, \\ 0 & \text{if } \ x_\beta \leq \lim x^n < \bar{x}. \end{cases}$$

This rules out that such sequences are Nash equilibria, of course. Consider $x_\beta \leq \lim x^n < \bar{x}$. From the MLRP, $1/2 \leq \lim G(\hat{x}^n|\beta) < \lim G(\hat{x}^n|\alpha) < 1$. Now, the claim follows from Lemma 3.

We now rule out equilibria in which x^n is close to $\bar{x}$. By assumption (1), there is some $x_r > x_\beta$ such that for $x > x_r$

$$1 > \frac{\pi}{1-\pi} \frac{g(x|\alpha)}{g(x|\beta)}.$$

Now, if $x \in (x_r, \bar{x})$, then $1/2 < \lim G(\hat{x}^n|\beta) < \lim G(\hat{x}^n|\alpha) < 1$ implies that the probability $\Pr[Piv|\beta; x, n] > \Pr[Piv|\alpha; x, n]$. To see this, consider a fixed voter and suppose the realized number of other voters is m and each of the m other voters supports A with i.i.d. probability $G(x|\omega)$. If $m = 0$, then the voter is pivotal in both states with equal likelihood. If $m > 0$ is even, then the fixed voter affects the election if and only if exactly $\frac{m}{2}$ other voters support A and B. The probability that exactly $\frac{m}{2}$ voters support each policy is strictly larger in state B since $G(x|\beta)(1 - G(x|\beta)) > G(x|\alpha)(1 - G(x|\alpha))$ by $1/2 < \lim G(\hat{x}^n|\beta) < \lim G(\hat{x}^n|\alpha) < 1$. If m is odd, then the fixed voter affects the election if and only if she votes A and $\frac{m-1}{2}$ other voters support A and $\frac{m+1}{2}$ support B. Similarly, a vote for B changes the outcome if $\frac{m+1}{2}$ support A and $\frac{m-1}{2}$ support B. With $\frac{m-1}{2} =: r$ and $q_\omega := G(x|\omega)$, the sum of these two probabilities is

$$\begin{aligned}
&\binom{2r+1}{r}(q_\omega)^r(1-q_\omega)^{r+1}+\binom{2r+1}{r+1}(q_\omega)^{r+1}(1-q_\omega)^r \\
= &\binom{2r+1}{r}(q_\omega)^r(1-q_\omega)^r(q_\omega+(1-q_\omega))=\binom{2r+1}{r}(q_\omega)^r(1-q_\omega)^r.
\end{aligned}$$

Again, $G(x|\beta)(1-G(x|\beta))>G(x|\alpha)(1-G(x|\alpha))$ implies that this probability is higher in state β. Thus, conditional on any realization of the number of other voters (either even or odd), the probability to affect the election is higher in state β. Hence, for all $\bar{x}>x>x_r$,

$$\frac{\Pr[Piv|\alpha;x,n]}{\Pr[Piv|\beta;x,n]}<1.$$

Thus, for all n and $x>x_r$,

$$\frac{\pi}{1-\pi}\frac{g(x|\alpha)}{g(x|\beta)}\frac{n}{n\theta}\frac{\Pr[Piv|\alpha;x,n]}{\Pr[Piv|\beta;x,n]}<\frac{\pi}{1-\pi}\frac{g(x|\alpha)}{g(x|\beta)}<1.$$

There can be no equilibrium with a cutoff $x\in(x_r,\bar{x})$ for any n. A symmetric argument rules out equilibria with cutoffs x close to $\underline{x}$.

Finally, from $\theta=1$, we have for a cutoff $\bar{x}$ that

$$\begin{aligned}
\frac{\Pr[Piv|\alpha;\bar{x},n]}{\Pr[Piv|\beta;\bar{x},n]} &= \frac{\frac{1}{2}\Pr[T-1|\alpha]+\Pr[T|\alpha]+\frac{1}{2}\Pr[T+1|\alpha]}{\frac{1}{2}\Pr[T-1|\beta]+\Pr[T|\beta]+\frac{1}{2}\Pr[T+1|\beta]} \\
&= \frac{0+e^{-n}+\frac{1}{2}e^{-n}n}{0+e^{-n}+\frac{1}{2}e^{-n}n}=1,
\end{aligned}$$

which follows because A cannot be behind if the cutoff is $\bar{x}$ (all vote A), a tie occurs only if no voter participates, and A is one ahead if there is exactly one voter. Thus,

$$\begin{aligned}
\gamma(\bar{x};\bar{x},n) &= \frac{\pi}{1-\pi}\frac{g(\bar{x}|\alpha)}{g(\bar{x}|\beta)}\frac{n}{n\theta}\frac{\Pr[Piv|\alpha;\bar{x},n]}{\Pr[Piv|\beta;\bar{x},n]} \\
&= \frac{\pi}{1-\pi}\frac{g(\bar{x}|\alpha)}{g(\bar{x}|\beta)}<1,
\end{aligned}$$

and so by the continuity of $g(\cdot|\alpha)/g(\cdot|\beta)$, $\gamma(x;\bar{x},n)<1$ for all $x<\bar{x}$. There can be no equilibrium with cutoff $\bar{x}$ for any n. Similarly, there can be no equilibrium with cutoff $\underline{x}$ for any n. $\square$

A.3 Proof of Theorem 2

We prove the theorem for $\theta < 1$. The argument for $\theta > 1$ is analogous and omitted. The proof uses the intermediate value theorem, utilizing our previous developments. In particular, we already observed in (9) that

$$\lim \gamma\left(x_\alpha; x_\alpha, n\right) = \infty. \tag{11}$$

Since $\theta < 1$, there exists some $x_R \in (\underline{x}, x_\alpha)$ small enough such that

$$1 - 2\sqrt{G\left(x_R|\alpha\right)\left(1 - G\left(x_R|\alpha\right)\right)} > \theta\left[1 - 2\sqrt{G\left(x_R|\beta\right)\left(1 - G\left(x_R|\beta\right)\right)}\right],$$

noting that the left side approaches 1 for $x_R \to \underline{x}$ and the right side approaches θ. Using Lemma 3, we can conclude that

$$\lim \frac{\Pr\left[Piv|\alpha; x_R, n\right]}{\Pr\left[Piv|\beta; x_R, n\right]} = 0.$$

Since $\frac{g(x_R|\alpha)}{g(x_R|\beta)} < \infty$, this implies

$$\lim \gamma\left(x_R; x_R, n\right) = 0. \tag{12}$$

Given (11) and (12), the existence of an interior Nash equilibrium $\hat{x}^n$ with $\hat{x}^n \in (x_R, x_\alpha)$ and $\gamma\left(\hat{x}^n; \hat{x}^n, n\right) = 1$ for all n large enough follows from the intermediate value theorem. By Lemma 3, the conclusion of (11) also holds if $\hat{x}^n \to x_\alpha$. Thus, $\lim \hat{x}^n < x_\alpha$. So, B wins with probability converging to 1 by the weak law of large numbers, proving the claim for $\theta < 1$. $\square$

A.4 Proofs for Section 5.2 (Abstention)

Notation. The pivotality probabilities with abstention are

$$\Pr\left[PivA|\alpha\right] = \frac{1}{2}\Pr\left[T|\alpha\right] + \frac{1}{2}\Pr\left[T-1|\alpha\right]$$

and

$$\Pr\left[PivB|\alpha\right] = \frac{1}{2}\Pr\left[T|\alpha\right] + \frac{1}{2}\Pr\left[T+1|\alpha\right].$$

We denote the critical likelihood ratio at the cutoff types as

$$\gamma_A(y,z,n) = \frac{\pi}{1-\pi}\frac{1}{\theta}\frac{g(y|\alpha)}{g(y|\beta)}\frac{\Pr[PivA|\alpha;y,z,n]}{\Pr[PivA|\beta;y,z,n]},$$

and

$$\gamma_B(y,z,n) = \frac{\pi}{1-\pi}\frac{1}{\theta}\frac{g(z|\alpha)}{g(z|\beta)}\frac{\Pr[PivB|\alpha;y,z,n]}{\Pr[PivB|\beta;y,z,n]}.$$

A strategy profile (y,z) is an interior Nash equilibrium if $\underline{x} < y \leq z < \bar{x}$ and

$$1 = \gamma_A(y,z,n) = \gamma_B(y,z,n).$$

Throughout this section, we consider the case with $\theta < 1$, and signals are unboundedly informative, in particular,

$$\frac{g(\underline{x}|\alpha)}{g(\underline{x}|\beta)} = \infty.$$

Let $\Gamma(x_R,n)$ be an auxiliary game in which voters with signals $x \geq x_R$ must vote B, while voters below x_R can choose between voting A, B, or abstaining as before. We will show that $\Gamma(x_R,n)$ has an equilibrium that satisfies the properties of the theorem and for which the constraint at x_R does not bind.

Note that $\Gamma(x_R,n)$ has an equilibrium (y,z) by standard arguments for all x_R; see Myerson (1998b).

We use the following Lemma, proven at the end of this section.

Lemma 5. *Suppose $\theta < 1$, abstention is possible, and signals are unboundedly informative. There exists some $x_R \in (\underline{x}, x_\alpha)$ such that for any sequence (y^n, z^n) with $y^n \leq z^n \leq x_R$,*

$$\lim \frac{\Pr[PivA|\alpha;y^n,z^n,n]}{\Pr[PivA|\beta;y^n,z^n,n]} = \lim \frac{\Pr[PivB|\alpha;y^n,z^n,n]}{\Pr[PivB|\beta;y^n,z^n,n]} = 0.$$

Now, suppose that (y^n,z^n) is an equilibrium sequence of $\Gamma(x_R,n)$, for x_R chosen to satisfy Lemma 5. It cannot be that $z^n \to x_R$. Suppose otherwise. If $z^n \to x_R$, then

$$\begin{aligned}\lim \gamma_B\left(y^n, z^n, n\right) &= \lim \frac{\pi}{1-\pi} \frac{g\left(z^n|\alpha\right)}{g\left(z^n|\beta\right)} \frac{\Pr\left[PivB|\alpha\right]}{\Pr\left[PivB|\beta\right]} \\ &= \frac{\pi}{1-\pi} \frac{g\left(x_R|\alpha\right)}{g\left(x_R|\beta\right)} \lim \frac{\Pr\left[PivB|\alpha\right]}{\Pr\left[PivB|\beta\right]} \\ &= 0.\end{aligned}$$

Thus, for any x', any voter having a signal in (x', x_R) would have a strict preference to vote for B. Thus, it must be that $\lim z^n < x_R$. But this implies that all voters with signals in $(\lim z^n, x_R)$ prefer voting B to voting A or abstaining. By the MLRP, this implies that in particular all voters with signals $x \geq x_R$ prefer voting B. Hence, the initial restriction of $\Gamma\left(x_R, n\right)$ relative to the original game does not bind. Therefore, for large n, (y^n, z^n) is also an equilibrium of the original game. Clearly, from $\lim z^n < x_R < x_\alpha$, policy B is chosen with probability converging to one. This proves the claim of the theorem.

Proof of Lemma 5.

There exists a signal $x_R \in (\underline{x}, x_\alpha)$ such that for all $y \leq z \leq x_R$,

$$\sqrt{1-G\left(z|\alpha\right)} - \sqrt{G\left(y|\alpha\right)} > \sqrt{\theta}\left(\sqrt{1-G\left(z|\beta\right)} - \sqrt{G\left(y|\beta\right)}\right).$$

Hence, for all $y^n \leq z^n \leq x_R$,

$$\lim \frac{e^{-\left(\sqrt{\sigma_A}-\sqrt{\sigma_B}\right)^2}}{e^{-\left(\sqrt{\tau_A}-\sqrt{\tau_B}\right)^2}} = 0. \tag{13}$$

Moreover,

$$\lim \frac{e^{-\sigma_A-\sigma_B}}{e^{-\tau_A-\tau_B}} = 0.$$

This follows from

$$\begin{aligned}-\sigma_A-\sigma_B+\tau_A+\tau_B &= \tau_A-\sigma_A+\tau_B-\sigma_B \\ &= n\left(\theta G\left(y|\beta\right) - G\left(y|\alpha\right) + \theta\left(1-G\left(z|\beta\right)\right) - \left(1-G\left(z|\alpha\right)\right)\right),\end{aligned}$$

and

$$\left(\theta G\left(y|\beta\right) - G\left(y|\alpha\right) + \theta\left(1-G\left(z|\beta\right)\right) - \left(1-G\left(z|\alpha\right)\right)\right) < 0,$$

from $\theta G(y|\beta) < G(y|\alpha)$ (by the MLRP) and $\theta(1-G(z|\beta)) < (1-G(z|\alpha))$ (which is necessary by $z \leq x_R$ for our choice of x_R).

Case 1. Suppose $\tau_A\tau_B \to \infty$. Then, $\sigma_A\sigma_B \to \infty$, which follows from $\lim \frac{\sigma_B}{\tau_B} > 0$ and $\sigma_B \to \infty$ (by $z^n \leq x_R < \bar{x}$) and $\sigma_A > \tau_A$ (by the MLRP). Then, from Lemma 4,

$$\lim \frac{\Pr[PivA|\alpha]}{\Pr[PivA|\beta]} = \lim \frac{e^{-\left(\sqrt{\sigma_A}-\sqrt{\sigma_B}\right)^2}}{e^{-\left(\sqrt{\tau_A}-\sqrt{\tau_B}\right)^2}} \frac{\sqrt{2\pi 2\sqrt{\tau_A\tau_B}}}{\sqrt{2\pi 2\sqrt{\sigma_A\sigma_B}}} \frac{1+\frac{\sqrt{\sigma_B}}{\sqrt{\sigma_A}}}{1+\frac{\sqrt{\tau_B}}{\sqrt{\tau_A}}}.$$

If $\lim y^n > \underline{x}$, then we are done because of (13) and the last fractions are bounded. Suppose $\lim y^n = \underline{x}$. Then,

$$\lim \frac{\sqrt{2\pi 2\sqrt{\tau_A\tau_B}}}{\sqrt{2\pi 2\sqrt{\sigma_A\sigma_B}}} \frac{1+\frac{\sqrt{\sigma_B}}{\sqrt{\sigma_A}}}{1+\frac{\sqrt{\tau_B}}{\sqrt{\tau_A}}} < \infty,$$

since $\lim \frac{\tau_B}{\sigma_B} \in (0,\infty)$ and $\frac{\tau_A}{\sigma_A} \leq 1$.

Similarly,

$$\lim \frac{\Pr[PivB|\alpha]}{\Pr[PivB|\beta]} = \lim \frac{e^{-\left(\sqrt{\sigma_A}-\sqrt{\sigma_B}\right)^2}}{e^{-\left(\sqrt{\tau_A}-\sqrt{\tau_B}\right)^2}} \frac{\sqrt{2\pi 2\sqrt{\tau_A\tau_B}}}{\sqrt{2\pi 2\sqrt{\sigma_A\sigma_B}}} \frac{1+\frac{\sqrt{\sigma_A}}{\sqrt{\sigma_B}}}{1+\frac{\sqrt{\tau_A}}{\sqrt{\tau_B}}},$$

and

$$\lim \frac{\sqrt{2\pi 2\sqrt{\tau_A\tau_B}}}{\sqrt{2\pi 2\sqrt{\sigma_A\sigma_B}}} \frac{1+\frac{\sqrt{\sigma_A}}{\sqrt{\sigma_B}}}{1+\frac{\sqrt{\tau_A}}{\sqrt{\tau_B}}} < \infty,$$

follows from $\lim \frac{\sqrt{\sigma_A}}{\sqrt{\sigma_B}} < \infty$, $\lim \frac{\tau_B}{\sigma_B} \in (0,\infty)$, and $\frac{\tau_A}{\sigma_A} \leq 1$.

Case 2a. Suppose $\tau_A\tau_B \to k < \infty$ and $z = \lim \sigma_A\sigma_B < \infty$. This requires $y^n \to \underline{x}$. Then, from Lemma 4,

$$\begin{aligned} \Pr[PivA|\beta] &\approx e^{-\tau_A-\tau_B}\left(I_0\left(2\sqrt{k}\right)+\tau_B\frac{I_1\left(2\sqrt{k}\right)}{\sqrt{k}}\right), \\ \Pr[PivB|\beta] &\approx e^{-\tau_A-\tau_B}\left(I_0\left(2\sqrt{k}\right)+\tau_A\frac{I_1\left(2\sqrt{k}\right)}{\sqrt{k}}\right), \end{aligned}$$

with $\frac{I_1(2\sqrt{k})}{\sqrt{k}} = 1$ if $k = 0$. Similarly, from $z = \lim \sigma_A \sigma_B < \infty$, we have

$$\begin{aligned}
\Pr[PivA|\alpha] &\approx e^{-\sigma_A - \sigma_B}\left(I_0\left(2\sqrt{z}\right) + \sigma_B \frac{I_1\left(2\sqrt{z}\right)}{\sqrt{z}}\right), \\
\Pr[PivB|\alpha] &\approx e^{-\sigma_A - \sigma_B}\left(I_0\left(2\sqrt{z}\right) + \sigma_A \frac{I_1\left(2\sqrt{z}\right)}{\sqrt{z}}\right).
\end{aligned}$$

So, if $\lim \sigma_A \sigma_B < \infty$ then

$$\lim \frac{\Pr[PivA|\alpha]}{\Pr[PivA|\beta]} = \lim \frac{e^{-\sigma_A - \sigma_B}}{e^{-\tau_A - \tau_B}} \frac{I_0\left(2\sqrt{z}\right) + \sigma_B \frac{I_1\left(2\sqrt{z}\right)}{\sqrt{z}}}{I_0\left(2\sqrt{k}\right) + \tau_B \frac{I_1\left(2\sqrt{k}\right)}{\sqrt{k}}} \to 0,$$

since $\frac{e^{-\sigma_A - \sigma_B}}{e^{-\tau_A - \tau_B}} \to 0$ by (13), $I_0\left(2\sqrt{k}\right) > 0$, $\frac{I_1\left(2\sqrt{k}\right)}{\sqrt{k}} > 0$ and $\lim \frac{\sigma_B}{\tau_B} < \infty$. Analogously,

$$\lim \frac{\Pr[PivB|\alpha]}{\Pr[PivB|\beta]} = \lim \frac{e^{-\sigma_A - \sigma_B}}{e^{-\tau_A - \tau_B}} \frac{I_0\left(2\sqrt{z}\right) + \sigma_A \frac{I_1\left(2\sqrt{z}\right)}{\sqrt{z}}}{I_0\left(2\sqrt{k}\right) + \tau_A \frac{I_1\left(2\sqrt{k}\right)}{\sqrt{k}}} \to 0,$$

since $\lim \sigma_A = \lim \tau_A = 0$.

Case 2b. Suppose $\tau_A \tau_B \to k < \infty$ and $\lim \sigma_A \sigma_B = \infty$. Then, from Lemma 4,

$$\lim \frac{\Pr[PivA|\alpha]}{\Pr[PivA|\beta]} = \lim \frac{e^{-\sigma_A - \sigma_B}}{e^{-\tau_A - \tau_B}} \frac{e^{2\sqrt{\sigma_A \sigma_B}}\left(\frac{\left(1 + \frac{\sqrt{\sigma_B}}{\sqrt{\sigma_A}}\right)}{\sqrt{2\pi 2\sqrt{\sigma_A \sigma_B}}}\right)}{I_0\left(2\sqrt{k}\right) + \tau_B \frac{I_1\left(2\sqrt{k}\right)}{\sqrt{k}}}.$$

Now, observe that

$$\lim e^{2\sqrt{\tau_A \tau_B}} \frac{e^{-\sigma_A - \sigma_B}}{e^{-\tau_A - \tau_B}} \frac{e^{2\sqrt{\sigma_A \sigma_B}}}{e^{2\sqrt{\tau_A \tau_B}}} = e^{2\sqrt{k}} \lim \frac{e^{-\left(\sqrt{\sigma_A} - \sqrt{\sigma_B}\right)^2}}{e^{-\left(\sqrt{\tau_A} - \sqrt{\tau_B}\right)^2}} = 0.$$

Moreover, from $I_0\left(2\sqrt{k}\right) \in (0, \infty)$, $\frac{I_1\left(2\sqrt{k}\right)}{\sqrt{k}} \in (0, \infty)$, $\tau_B \to \infty$, and $\sigma_A \sigma_B \to \infty$

$$\lim \frac{\left(\frac{\left(1 + \frac{\sqrt{\sigma_B}}{\sqrt{\sigma_A}}\right)}{\sqrt{2\pi 2\sqrt{\sigma_A \sigma_B}}}\right)}{I_0\left(2\sqrt{k}\right) + \tau_B \frac{I_1\left(2\sqrt{k}\right)}{\sqrt{k}}} \le \lim \frac{\frac{\sqrt{\sigma_B}}{\sqrt{\sigma_A}}}{\tau_B} \le \lim \frac{\frac{\sqrt{\sigma_B}}{\sqrt{\sigma_A}}}{\sigma_B} = \frac{1}{\sqrt{\sigma_A \sigma_B}} \to 0.$$

This proves the result. □

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