Optimal information structures in bilateral trade[*]

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Abstract

With the goal of maximizing expected gains from trade, this paper analyzes the jointly optimal information structure and mechanism in a bilateral trade setting. The difference in gains from trade between the optimal information structure and first best constitutes the minimal loss due to asymmetric information. With binary underlying types it is shown that more than 95% of first best can be achieved while the optimal mechanism without information design may achieve less than 90% of first best. For more general type distributions, the optimal information structure is a monotone partition of the type space and the optimal mechanism is deterministic. Necessary conditions for the optimal information structure are derived and a closed form solution is given for the binary type case.

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1. Introduction

Information asymmetries can lead to inefficiencies in economic transactions compared to a complete information benchmark, see, for example, Akerlof (1970); Mirrlees (1971); Baron and Myerson (1982). Myerson and Satterthwaite (1983) established this result in what is arguably the most basic economic setting: bilateral trade. In their model, a buyer holds *private* information about his valuation and a seller about his costs. Myerson and Satterthwaite establish an inefficiency

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result but also derive the trading mechanism maximizing expected gains from trade (EGT) in their setting. This paper generalizes their work by analyzing the pair of trading mechanism and information structure that jointly maximizes EGT.

More precisely, imagine that buyer and seller do not know their own valuation and costs perfectly but only have noisy and independent private signals, i.e. estimates, of these variables. This paper derives the information structure, i.e. a mapping from true valuation and costs to signals, and the trading mechanism, i.e. a mapping from signals to trading probability and price, that maximize EGT. EGT under this optimal information structure and optimal trading mechanism are consequently the maximal EGT that are attainable (by any information structure and trading mechanism) under the assumption that players will eventually hold the information they receive *privately*. Therefore, the difference in EGT attained by the solution of this paper and first best constitutes the EGT loss that can be attributed to information asymmetries. Any additional EGT loss has to be blamed on suboptimal institutions, i.e. either a suboptimal trading mechanism or a suboptimal information structure.

Limiting the information of a player has several effects.[1] Consider, for example, a buyer whose valuation is either high or low and suppose the information structure is such that he does not receive any information about which of the two valuations has realized. This makes it impossible to determine whether his valuation is above or below the costs of the seller and therefore less information directly harms efficiency. On the other hand, less information for a player also reduces his information rent. The latter effect relaxes the budget balance constraint and will therefore increase EGT. Section 1.1 illustrates this tradeoff using the canonical example with uniformly distributed costs and valuations. Section 1.2 summarizes the related literature and section 2 introduces the model formally. Section 3 presents the optimal trading mechanism for a given finite information structure. The main results of the paper are derived in section 4 which presents results on jointly optimal (information structure, mechanism) pairs. As another prominent example, binary signal/type distributions are discussed in section 5. Section 6 concludes. Proofs and derivations that are standard in the literature are relegated to the appendix.

[1]Throughout the paper, I use the setting of Myerson and Satterthwaite (1983), i.e. full information about types, as a benchmark and think of information design as a deviation from this full information benchmark. This differs from some other papers in information design that take the no information case, i.e. pooling all types on the same signal, as the benchmark with which the optimal information structure is compared.

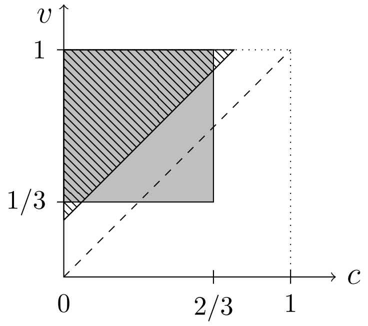
1.1. Example: Uniform type distribution

The canonical example in the bilateral trade literature assumes that the buyer's valuation (v) and the seller's cost (c) are uniformly distributed on $[0, 1]$. First best, i.e. trade if and only if valuation is above costs, then leads to EGT of $1/6 = 0.1\bar{6}$. Myerson and Satterthwaite (1983) showed that the second best trading mechanism implements trade if and only if $v - c \geq 1/4$ which leads to EGT of $9/64 = 0.140625$. Consequently, only 84.375% of first best EGT can be achieved without information design due to asymmetric information.

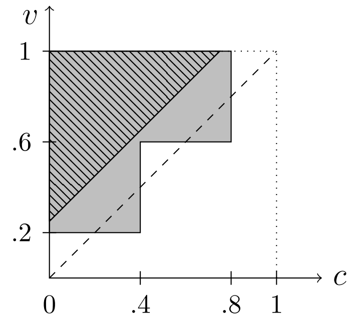
Now consider the following information structure: The buyer receives a high (low) signal if his valuation is above (below) 1/3. The buyer's expected valuation upon receiving the high (low) signal is therefore 2/3 (1/6). Similarly, the seller receives a low (high) signal if his cost is below (above) 2/3 leading to expected costs of 1/3 (5/6) in case of low (high) signal. Consider the trading mechanism that induces trade at price 1/2 if and only if the buyer receives the high and the seller receives the low signal. Otherwise, no trade takes place and no transfers are made. Clearly, this trading mechanism is incentive compatible and satisfies interim participation constraints. Expected gains from trade are $(2/3 - 1/3) * (2/3)^2 = 4/27 \approx 0.148$ or 88.9% of first best. This shows that a coarsening of the information structure can increase expected gains from trade. The reason is that less information reduces information rents which are at the heart of Myerson and Satterthwaite's inefficiency result.

Table 1 presents results of a numerical analysis in which – using the results of my paper – the optimal information structure with n buyer and n seller signals was derived.[2] For $n \geq 4$, the optimization algorithm used less than n types and EGT did not increase further. Information design closes almost the entire gap to first best in this example as 97.5% of first best EGT can be achieved in the optimal information structure. Figure 1 illustrates the results by comparing which types trade under full information, i.e. the Myerson-Satterthwaite setup, with the optimal two, three and four element information structures. For instance, figure 1c illustrates that the optimal information structure induces strictly more types

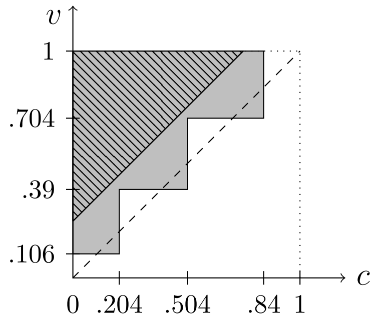
[2] The numerical analysis first creates a grid of possible information structures in the monotone partitional form, which is optimal by proposition 1, with n buyer and seller signals and maximizes EGT by brute force on this grid. The optimal information structure from this brute force method is used as a starting value for an optimization algorithm. The algorithm used is "MMA (Method of Moving Asymptotes)" as implemented in the NLopt package, see Johnson (2019) and Svanberg (2002). The code is available on the website of the author (`https://schottmueller.github.io/`). The usual disclaimer for numerical work applies, i.e. the solution is not exact and information partitions leading to even higher EGT are in principle possible.



(a) Full information (hatched) and optimal two element information structure (gray shaded).



(b) Full information (hatched) and optimal three element information structure (gray shaded).



(c) Full information (hatched) and optimal four element information structure (gray shaded).

Figure 1: Comparison of optimal trading mechanism under different information structures. Trade above (below) the 45° line is first best efficient (inefficient).

to trade than full information. Some of the additional trades are even inefficient (gray area below the 45° degree line). However, the additional efficient trades more than outweigh this negative effect in terms of EGT; see table 1. With three or four partition elements, the optimal trading mechanism is no longer a fixed price mechanism as different partition elements have distinct non-zero trading probabilities. It should, furthermore, be noted that the optimal four element information partition is asymmetric. A rough intuition for this asymmetry can be gained by considering the effect of coarsening information on potentially achievable EGT: consider the effect of moving from full information to no information (i.e. a one element partition) for the buyer. Abstracting from incentive problems, this reduces the potentially attainable EGT from $1/6$ to $(1/2 - 1/4)/2 = 1/8$ if the seller has full information but reduces potentially attainable EGT from $1/8$ to 0 if the seller has no information. The size of EGT losses from coarsening information depends on the amount of information the other party has. In particular, the potential EGT loss from coarser information is larger if the other player has coarse information himself. Section 5.1 discusses how this can lead to a convexity in the objective function which then in turn leads to asymmetric solutions.

n	EGT	EGT/EGT^{fb}	buyer signals	seller signals
1	0	0	0.5	0.5
2	0.148	.889	0.166, 0.666	0.333, 0.833
3	0.16	.967	0.1, 0.4, 0.8	0.2, 0.6, 0.9
4	0.163	.975	0.0503, 0.248, 0.547, 0.849	0.102, 0.354, 0.677, 0.925

Table 1: EGT in optimal information structures for uniform type distribution on $[0, 1]$ (numerical analysis, rounded to third digit)

1.2. Literature

The setting is similar to Myerson and Satterthwaite (1983) who derive the trading mechanism maximizing expected gains from trade in a bilateral trade setting in which (i) trade is voluntary, (ii) the budget has to be balanced and (iii) the buyer (seller) privately knows his valuation (costs). Retaining (i) and (ii), this paper modifies (iii) by determining the optimal (information structure, trading mechanism)-pair that maximizes expected gains from trade. Note that valuations and costs are independently distributed in Myerson and Satterthwaite (1983). As illustrated by an example in Myerson (1981, section 7) and shown in general by Cremer and McLean (1988), correlated types would allow to extract private information at no cost and thus allow for full efficiency. Information design could therefore achieve first best if the signal structures of buyer and seller were correlated.[3] Hence, this paper extends the assumption in Myerson and Satterthwaite (1983) that types are independent by requiring that also signals of buyer and seller have to be independent.

This paper is related to a recent literature on information design as surveyed in Bergemann and Morris (2019). Within this literature the following papers consider bilateral trade settings. Closely related is Gottardi and Mezzetti (2022) who study how a mediator can help to maximize expected gains from trade. They develop a "shuttle diplomacy" protocol in which a mediator travels back and forth between the two players providing more information to the initially uninformed players at each visit. Effectively, this protocol establishes a particular information structure through which buyer and seller learn their valuation and cost. Gottardi and Mezzetti show that first best efficiency can be achieved using their protocol.

[3]For example, informing each player of valuation and cost is one correlated information structure that eliminates private information as well as inefficiency entirely, see appendix B.

This is possible because Gottardi and Mezzetti's protocol correlates the information of buyer and seller. First best cannot be achieved in my setup as I extend the independence assumption of Myerson and Satterthwaite (1983) by assuming independent signals. Gottardi and Mezzetti (2022) also derive the information structure maximizing expected gains from trade if a fixed price mechanism is used.[4] In contrast, my paper employs the optimal trading mechanism which typically is not a fixed price mechanism. Yamashita (2018) covers a setting in which valuations and costs depend on a privately known, idiosyncratic component as well as a random state. He derives the EGT maximizing information structure with respect to this state under the assumption that signals about the state are public (there is no information design with respect to private signals). Assuming that for some values of the state trade is efficient for all possible realizations of private information, he shows that the optimal signal structure fully discloses low states but pools high states. In contrast, my paper is concerned with information design with respect to the private information of the players, i.e. signals are private, and there is no additional state. Less closely related are papers determining the consumer surplus maximizing information structure if a monopolist seller makes a take-it-or-leave-it offer (Roesler and Szentes, 2017; Condorelli and Szentes, 2020). My paper differs by having a different objective (consumer surplus vs. expected gains from trade) and private information for both players.

Bergemann and Pesendorfer (2007) is closest in terms of proof techniques. Their paper derives the information structure with independent signals that maximizes revenue in independent private value auctions. As in my paper, the optimal information structure turns out to be a monotone partition of the type space. Apart from the objective (revenue vs. gains from trade) and the setting (auction vs. bilateral trade), the main difference is that my paper deals with two-sided asymmetric information and overlapping type supports, i.e. gains from trade may be positive or negative depending on the type realization. Consequently, the problem of maximizing expected gains from trade is non-trivial and the budget balance constraint becomes relevant. In particular, the mechanism design problem cannot be written as an unconstrained maximization over virtual valuations.

Finally, Lang (2016, ch. 4) shows by means of an example that gains from trade can be higher if players have coarse information in a bilateral trade setting. However, he does not analyze information structures maximizing expected gains from trade.

[4] In the uniform example of section 1.1, the optimal fixed price mechanism derived in Gottardi and Mezzetti (2022) coincides with the optimal two element partition described in section 1.1.

The bilateral trade problem for a given information structure, i.e. without information design, was extensively studied in the eighties. Chatterjee and Samuelson (1983) derived an equilibrium in strictly increasing strategies in the double auction. Myerson and Satterthwaite (1983) showed that this equilibrium achieves the maximally possible expected gains from trade in the standard example with uniformly distributed types. Leininger et al. (1989) use the same uniform example but derived two families of equilibria in the double auction. These achieve expected gains from trade between zero and the second best level. In particular, the equilibria in step-functions appear on first sight related to the monotone partition information structures derived in this paper. The crucial difference between those is that in a coarse information structure all types pooled on the same signal have the same information and therefore there is only one – "average" – incentive compatibility constraint for all those types. In a step-function equilibrium, all pooled types make the same bid but have different valuations (respectively costs). Consequently, there is one incentive compatibility constraint for each type. Satterthwaite and Williams (1989) analyzed differentiable equilibira in a generalized double auction in which gains from trade are not split equally in case of trade. Cramton et al. (1987) showed that the inefficiency result of Myerson and Satterthwaite (1983) does not hold if property rights are more evenly distributed, i.e. if, instead of a seller owning the asset initially, both players are partners owning some share of the asset.[5]

2. Model

A single indivisible object may be traded between a buyer and a seller. The buyer's valuation for the object is distributed according to the cumulative distribution function (cdf) H_B with support on a bounded subset of $\mathbb{R}_+$. The buyer maximizes a linear utility function, i.e. he maximizes expected valuation minus expected payments. The seller's (opportunity) costs for procuring the object are distributed according to cdf H_S on a bounded subset of $\mathbb{R}_+$ and the seller maximizes expected payments minus expected costs. Gains from trade equal valuation minus costs if trade takes place and zero otherwise.[6]

[5] Another string of the literature that achieved an efficiency result considered markets with several buyers and sellers. Wilson (1985) showed that the double auction is efficient if the number of buyers and sellers is large. Rustichini et al. (1994) derived rates of convergence to efficiency as the number of market participants grows while Kojima and Yamashita (2017) propose a trading mechanism that is asymptotically efficient in a framework with interdependent valuations.

[6] Gains from trade equal welfare if (i) a possible budget surplus does not affect welfare and (ii) welfare denotes the sum of buyer and seller payoff.

To make the problem interesting, I assume that the supports of H_B and H_S are overlapping. In particular, I make the slightly stronger assumption that either the supports of both type distributions are identical intervals or that for each type of one player there are strictly positive gains from trade with some type of the other player:[7]

Assumption 1 (Overlapping support). *Either* $supp(H_S)$ *and* $supp(H_B)$ *are identical intervals or* $\min supp(H_S) < \min supp(H_B) < \max supp(H_S) < \max supp(H_B)$.

A signal structure for the buyer $F : supp(H_B) \to \Delta(\Sigma_v)$ maps each valuation to a probability distribution over a set of signals $\Sigma_v \subset \mathbb{R}$. As the buyer cares only about his expected valuation, it is without loss of generality to identify a signal with the expected valuation it induces. A signal v is thus understood to imply that the buyer has expected valuation v when receiving this signal. Using this convention (and a slight abuse of notation), a signal structure can be described by a probability distribution over a set of expected valuations. A signal structure F is then feasible if and only if H_B is a mean preserving spread of F. The same applies to the seller: A feasible signal structure for the seller can be described by a distribution G over expected costs such that H_S is a mean preserving spread of G. A signal structure is then described by a feasible F and a feasible G. Note that the two distributions F and G are required to be independent as otherwise first best could be achieved easily by essentially eliminating the information asymmetry between buyer and seller, see appendix B.

Without loss of generality, only incentive compatible direct revelation trading mechanisms are considered. A direct revelation trading mechanism assigns to each pair of signals (v, c) a probability of trade $y(v, c) \in [0, 1]$ and a transfer $t_B(v, c) \in \mathbb{R}$ the buyer pays as well as a transfer $t_S(v, c) \in \mathbb{R}$ the seller receives. Incentive compatibility requires that each player's expected utility is maximized by revealing his true signal given that the other player announces his signal truthfully, i.e.

$$\int vy(v, c) - t_B(v, c)\, dG(c) \geq \int vy(v', c) - t_B(v', c)\, dG(c) \quad \text{for all } v,\, v' \in supp(F) \tag{ICB}$$

$$\int t_S(v, c) - cy(v, c)\, dF(v) \geq \int t_S(v, c') - cy(v, c')\, dF(v) \quad \text{for all } c,\, c' \in supp(G). \tag{ICS}$$

[7]Assuming only overlapping support is sufficient to prove most of the results. However, lemma 1 and the results in section 5.1 make explicit use of the slightly stronger assumption below.

Participation is voluntary at the interim stage: in order to be feasible a trading mechanism must not only be incentive compatible but also yield an expected utility of at least zero conditional on any signal. Denoting the buyer's (seller's) interim utility by U (Π), this can be written as

$$U(v) = \int vy(v,c) - t_B(v,c)\, dG(c) \geq 0 \qquad \text{for all } v \in \mathit{supp}(F) \tag{PCB}$$

$$\Pi(c) = \int t_S(v,c) - cy(v,c)\, dF(v) \geq 0 \qquad \text{for all } c \in \mathit{supp}(G). \tag{PCS}$$

No outside source of funding is available and therefore the following ex post budget balance condition has to be satisfied:

$$t_B(v,c) = t_S(v,c) \qquad \text{for all } c \in \mathit{supp}(G) \text{ and } v \in \mathit{supp}(F). \tag{EPBB}$$

Note that due to the convention that signals are the corresponding expected values, EGT equal

$$\int\int y(v,c)(v-c)\, dG(c)\, dF(v).$$

The objective of this paper is to find the feasible signal structure (F and G) and feasible trading mechanism (y, t_B and t_S) that maximize expected gains from trade subject to (EPBB).

In the following, I will refer to an element of the support of H_B or H_S as *type* and to an element of the support of F or G as *signal.* I will call the buyer's (seller's) signal structure *fully informative* if $F = H_B$ ($G = H_S$) and *noisy* otherwise. Furthermore, a direct revelation trading mechanism will be referred to as *mechanism* while a pair consisting of such a mechanism and an signal structure will be called *grand mechanism.* I use the terms *signal structure* and *information structure* interchangeably.

3. Optimal mechanism for finite signal distributions

This section presents the optimal mechanism for a given finite signal structure. The derivation is relegated to appendix A as it is very similar to Myerson and Satterthwaite's (1983) well known analysis of the full information case with a continuum of types,[8].

To fix notation for the finite signal case, let the buyer have signal $v_i \in \{v_1, \ldots, v_n\}$ with probability ω_i and the seller have signal $c_j \in \{c_1, \ldots, c_m\}$ with

[8]Myerson and Satterthwaite's approach straightforwardly extends to the problem of finding the optimal mechanism in a given signal structure with a continuum of signals.

probability γ_j. Lower indices are assumed to denote lower signals. EGT equals

$$\sum_{i=1}^{n}\sum_{j=1}^{m} y(v_i, c_j)(v_i - c_j)\omega_i\gamma_j \tag{1}$$

where $y(v_i, c_j)$ is the probability of trade for v_i and c_j. Combining participation constraints, incentive compatibility constraints and budget balance (see appendix A), one can derive the following implementability condition where $W_i = \sum_{k=1}^{i} \omega_k$ and $\Gamma_j = \sum_{k=1}^{j} \gamma_k$:

$$\sum_{i=1}^{n}\sum_{j=1}^{m} y(v_i, c_j)\omega_i\gamma_j \left[v_i - (v_{i+1} - v_i)\frac{1 - W_i}{\omega_i} - c_j - (c_j - c_{j-1})\frac{\Gamma_{j-1}}{\gamma_j} \right] \geq 0. \tag{C}$$

This condition has the usual interpretation in terms of information rents: denote the expected probability of trade of a buyer with signal v_i by $Y_B(v_i) = \sum_{j=1}^{m} y(v_i, c_j)\gamma_j$ and, for illustration, concentrate on the valuation terms for i, i.e. $Y_B(v_i)(\omega_i v_i - (v_{i+1} - v_i)(1 - W_i))$. The first part, $Y_B(v_i)v_i$, is the willingness to pay of signal v_i which has probability ω_i. The second part, $Y_B(v_i)(v_{i+1} - v_i)(1 - W_i)$ is the information rent that v_i generates for higher signals. Namely signal v_{i+1} can earn a rent that is $Y_B(v_i)(v_{i+1} - v_i)$ higher than the one of v_i by misrepresenting as v_i. As downward incentive compatibility constraints between adjacent signals are binding, this information rent is generated not only for v_{i+1} but for all higher signals which have a total probability weight of $1 - W_i$. The total expected revenue that can be generated from the buyer equals the probability weighted sum of willingness to pay of all signals adjusted by the information rents they generate. The interpretation of the cost terms is similar.

The maximization problem of this paper can now be restated as maximizing EGT subject to (C) and standard monotonicty constraints on Y_S and Y_B that are necessary for incentive compatibility (see lemma 10 in appendix A).[9] Neglecting these monotonicity constraints, the mechanism design problem becomes maximizing (1) subject to (C). Hence, the optimal trading decision y must maximize the

[9] Y_S is defined analogously to Y_B as $Y_S(c_j) = \sum_{i=1}^{n} y(v_i, c_j)$.

Lagrangian

$$\mathcal{L}(y) = \sum_{i=1}^{n}\sum_{j=1}^{m} y(v_i, c_j)\omega_i\gamma_j \Bigg[(1+\lambda)v_i - \lambda(v_{i+1} - v_i)\frac{1-W_i}{\omega_i} \\ -(1+\lambda)c_j - \lambda(c_j - c_{j-1})\frac{\Gamma_{j-1}}{\gamma_j}\Bigg] \quad (2)$$

where $\lambda \geq 0$ is the Lagrange parameter of (C). As the Lagrangian is linear in y, the optimal decision rule is

$$y^*(v_i, c_j) \begin{cases} = 1 & \text{if } v_i - (v_{i+1} - v_i)\frac{\lambda}{1+\lambda}\frac{1-W_i}{\omega_i} > c_j + (c_j - c_{j-1})\frac{\lambda}{1+\lambda}\frac{\Gamma_{j-1}}{\gamma_j} \\ \in [0,1] & \text{if } v_i - (v_{i+1} - v_i)\frac{\lambda}{1+\lambda}\frac{1-W_i}{\omega_i} = c_j + (c_j - c_{j-1})\frac{\lambda}{1+\lambda}\frac{\Gamma_{j-1}}{\gamma_j} \\ = 0 & \text{else.} \end{cases} \quad (3)$$

This leaves us with two questions: First, is it possible that (C) is non-binding? Second, will y^* satisfy the neglected monotonicity conditions? Since the signal distribution will be chosen in order to maximize EGT, it is unclear whether the usual monotone hazard rate conditions apply to W and Γ. In the following, it will be shown that it is typically not optimal to choose the signal structure such that the monotonicity constraint is binding (in this case information is too fine) or such that (C) is slack (in this case information is too coarse).

4. Optimal grand mechanism

For most of this section I take the number of signals as given. That is, it is assumed that the support of F contains no more than n signals and the support of G contains no more than m signals.[10] This restriction is useful for several reasons. First, it simplifies notation and exposition. Second, two results will be shown at a later point in the paper that emphasize the relevance of finite signal structures. More precisely, finite signal structures turn out to be optimal if at least one of the two true type distributions H_B and H_S has finite support. Even if this is not the case, it will be shown that finite signal structures achieve EGT levels arbitrarily close to the levels infinite signal structures achieve. In the following, n and m will be assumed to be at least two. The justification for this is the following

[10] For notational convenience, I will then state and prove the results assuming that there are n (m) distinct buyer (seller) signals and all these signals have strictly positive probability, i.e. $\omega_i > 0$ and $\gamma_j > 0$ for all i and j. If it is optimal to use only $n^* < n$ ($m^* < m$) signals though n (m) signals are allowed, the results obviously still hold as the solution is equivalent to the solution with n^* (m^*) in place of n (m).

lemma which establishes that pooling all types on one signal is never optimal.

Lemma 1. *The support of the signal distribution in the EGT maximizing grand mechanism contains at least two elements for each player.*

Given the restriction to no more than n (m) buyer (seller) signals, I will show three main properties of the optimal grand mechanism: decision monotonicity, monotone partition structure and deterministic mechanism. Section 4.2 will then add a fourth property that holds if the type space is not too coarse: (C) binds.

Decision monotonicity refers to the standard monotonicity conditions for incentive compatibility: Y_S has to be decreasing and Y_B increasing. Note that $\lambda = 0$ in (3) would imply that y^* is the first mechanism and clearly this leads to monotone Y_S and Y_B (though not necessarily strictly monotone). In order to verify that neglecting the monotonicity constraints for Y_B and Y_S in the derivation of (3) was immaterial provided that the signal structure is optimal, it is therefore sufficient to concentrate on the case $\lambda \neq 0$; that is, the case where (C) binds. Suppose now the buyer's monotonicity constraint was binding, that is $Y_B(v_i) = Y_B(v_{i+1})$ for some $i \in \{1, \dots, n-1\}$. The proof of the following lemma shows that "merging the two signals into one" would not affect EGT but strictly relax the binding constraint (C) – thereby contradicting optimality of the grand mechanism. The intuition is that merging the signals leads to coarser information and therefore to lower information rents. However, there is no downside in terms of EGT as $Y_B(v_i) = Y_B(v_{i+1})$ implies that the additional information present in the original signal structure was not used to determine the efficient allocation.[11]

Lemma 2. *If (C) binds in the EGT maximizing grand mechanism with at most n buyer signals and m seller signals, then Y_S is strictly decreasing and Y_B is strictly increasing.*
If (C) does not bind in the EGT maximizing grand mechanism with at most n buyer signals and m seller signals, then either Y_S is strictly decreasing and Y_B is strictly increasing or there exists $\tilde{n} \leq n$ and $\tilde{m} \leq m$ such that (i) $\tilde{n} + \tilde{m} < n + m$ and (ii) EGT in the optimal grand mechanism with $\tilde{n}$ ($\tilde{m}$) buyer (seller) signals are no less than in the optimal grand mechanism with n (m) buyer (seller) signals.

Lemma 2 establishes that Y_B and Y_S are strictly monotone if (C) binds. If (C) does not bind, then there exists a coarser information structure that achieves

[11] The lemma is stated and proven for a finite number of signals. However, this is for notational convenience only and the result holds generally as "merging signals" for which the monotonicity constraint binds generally relaxes (C) without affecting EGT.

the same EGT and in which Y_B and Y_S are strictly monotone. In fact, the proof shows that this coarser information structure can be obtained by simply "merging signals", i.e. by assigning all signals v_i that have the same $Y_B(v_i)$ to the same new signal $\tilde{v}$.

The previous lemma established that the optimal mechanism is indeed characterized by (3) and neglecting the monotonicity constraints in its derivation is immaterial as Y_B and Y_S will be strictly monotone (or there exists another – coarser – information structure that is also optimal and in which Y_B and Y_S are strictly monotone). It is now worthwhile to return to (3). This optimality condition can be stated in terms of virtual valuations. That is, a buyer with signal v_i trades with a seller of signal c_j if his virtual valuaton exceeds the one of the seller. The virtual valuations are defined as

$$\begin{aligned} VV_B(v_i) &= v_i - (v_{i+1} - v_i)\frac{\lambda}{1+\lambda}\frac{1-W_i}{w_i} \\ VV_S(v_j) &= c_j + (c_j - c_{j-1})\frac{\lambda}{1+\lambda}\frac{\Gamma_{j-1}}{\gamma_j}. \end{aligned}$$

Strict monotonicity of Y_B implies that higher buyer signals have a higher probability to trade. Given the virtual valuation structure of the optimal mechanism, this implies that higher buyer signals must be associated with higher virtual valuations in the optimal grand mechanism. It also implies that between the virtual valuation of any two buyer signals there has to be the virtual valuation of a seller signal. The reason is that otherwise the two buyer signals would have the same probability of trade, i.e. the monotonicity constraint would hold with equality and it would be better to merge the signals. Hence, seller and buyer signals will alternate in terms of virtual valuations.

The main result of this paper establishes that the optimal information structure is a monotone partition of the type space. That is, each buyer signal v_i corresponds to a contiguous interval of types and the same is true for each seller signal c_j.[12]

Proposition 1. *The information structure in the EGT maximizing grand mechanism with at most n buyer and m seller signals is a monotone partition (up to a measure zero set).*

[12] If H_B is discrete, a monotone partition can assign a given type that has positive probability mass with some probability to v_i and some probability to v_{i+1}. It is then easier to think of a partition of $[0,1]$ where each signal v_i corresponds to an interval $(a_i, b_i] \subset [0,1]$ such that (i) signal v_i has probability mass $b_i - a_i$ in F and (ii) types in $H_B^{-1}((a_i, b_i])$ receive signal v_i where H_B^{-1} is the generalized inverse of H_B. The optimal information structure with no more than n elements can therefore be completely described by $n-1$ cutoffs.

The main idea behind the proof of proposition 1 is that an information structure that is not a monotone partition allows for both "mixing" and "demixing". Mixing refers here to the process of making an information structure coarser by moving two signals closer together. That is, if F assigns probabilities ω_i to v_i and ω_{i+1} to v_{i+1}, there is always a feasible information structure that uses the same probabilities but uses signals $v_i' > v_i$ and $v_{i+1}' < v_{i+1}$ instead of v_i and v_{i+1}. This can be achieved by sending the types that receive signal v_i (v_{i+1}) under F with some small probability the signal v_{i+1}' (v_i') instead (and v_i' (v_{i+1}') otherwise). If F is not a monotone partition, the opposite is possible as well: There is a feasible information structure that differs from F only by moving the signals v_i and v_{i+1} slightly apart from one another. The proof shows that EGT can always be improved by one of the two operations if both, mixing and demixing, are possible. It follows that the optimal information structure has to be a monotone partition where further demixing is impossible.

The previous results established properties of the information structure in the optimal grand mechanism. The following result establishes a property of the mechanism (3) in the optimal grand mechanism. In particular, it establishes that the mechanism is deterministic, i.e. that $y(v_i, c_j)$ is either 0 or 1 for all signal pairs (v_i, c_j) in the support of the optimal information structure. Note that the optimal mechanism for generic discrete information structures is not deterministic, i.e. $y(v_i, c_j) \in (0, 1)$ for typically one signal pair, because generically (C) does not hold with equality in deterministic mechanisms. In fact, the only reason for a stochastic mechanism is to relax (C) sufficiently so that (C) is just not violated. The following result demonstrates that information design is a more efficient way of achieving the goal of relaxing this constraint.

Proposition 2. *Assume that a grand mechanism with a fully informative signal structure cannot achieve first best EGT.*

In an optimal grand mechanism with at most n (m) buyer (seller) signals, the mechanism is deterministic if (C) binds.

If (C) does not bind, then there exists at least one optimal grand mechanism in which the mechanism is deterministic.

Proposition 2 establishes that it is without loss of generality to focus on deterministic mechanisms and consequently only such mechanisms will be considered in the remainder of the paper.

To illustrate proposition 2, consider the following example with $n = m = 2$.

Suppose $y(v_1, c_1) = 1/2$ while $y(v_2, c_1) = y(v_2, c_2) = 1$ and $y(v_1, c_2) = 0$.[13] This leads to information rents of $\omega_2\gamma_1(v_2 - v_1)/2$ for the buyer and $\gamma_1\omega_2(c_2 - c_1)$ for the seller. Now consider an alternative information structure in which half of the c_1 signals still receive the signal c_1 but the other half is merged with signal c_2 to the new signal $\tilde{c}_2 = (\gamma_1 c_1/2 + \gamma_2 c_2)/(\gamma_1/2 + \gamma_2)$. Also adjust the mechanism such that $y(v_1, c_1) = 1$ (while $\tilde{c}_2$ trades only with signal v_2). As the underlying types have the same trading probabilities as before, this change of the grand mechanism does not affect EGT. However, information rents in the new structure are now $\omega_2 \frac{\gamma_1}{2}(v_2 - v_1)$ for the buyer and $\frac{\gamma_1}{2}\omega_2(\tilde{c}_2 - c_1)$ for the seller. Hence, information rents for the seller are lower and (C) is strictly relaxed. Consequently, the initial situation cannot occur in the optimal grand mechanism if (C) binds.

A direct implication of proposition 2 is that generically a fully informative signal structure is not optimal if (i) the type set is finite and (ii) first best is not implementable. That is, the second best solution will typically not provide full information but make use of information design. To see this, suppose to the contrary that there was an optimal grand mechanism with a fully informative information structure although first best is not implementable. As first best is not implementable, (C) has to bind. With a discrete signal distribution and binding (C), the optimal trading mechanism is generically non-deterministic as one trading probability is just lowered sufficiently such that (C) holds with equality. But by proposition 2, the mechanism in the optimal grand mechanism is deterministic which contradicts that full information is optimal.

4.1. Necessary conditions for the optimal grand mechanism

This subsection derives a set of first order conditions that are satisfied by the information structure in the optimal grand mechanism with n buyer and m seller signals. To simplify notation, suppose that H_B and H_S have densities h_B and h_S that are strictly positive on a bounded and convex support. Monotonicity of the optimal information structure implies then that the optimal information structure can be represented by cutoff values $(k_0, k_1, \ldots, k_n)$ for the buyer where k_0 (k_n) is the infimum (supremum) of the support of H_B and $\omega_i = H_B(k_i) - H_B(k_{i-1})$ and $v_i = \int_{k_{i-1}}^{k_i} v \, dH_B(v)/\omega_i$. The seller's information structure can be represented by a set of cutoff values $(g_0, g_1, \ldots, g_m)$ such that g_0 (g_m) is the infimum (supremum) of the support of H_S and $\gamma_j = H_S(g_j) - H_S(g_{j-1})$ and $c_j = \int_{g_{j-1}}^{g_j} c \, dH_S(c)/\gamma_j$.

Note that the optimal cutoffs have to maximize the Lagrangian (2). That is,

[13] This is, for instance, the optimal mechanism if $c_1 = 0$, $c_2 = 0.9$, $v_1 = 0.1$, $v_2 = 1$, $\gamma_1 = 0.25$ and $\omega_2 = 0.5$ where (C) binds with $\lambda = 1/8$.

both $\partial\mathcal{L}/\partial k_i = 0$ and $\partial\mathcal{L}/\partial g_i = 0$ at the optimum. This gives a set of necessary conditions for the cutoffs $k_1, \ldots, k_{n-1}$ and $g_1, \ldots, g_{m-1}$. Namely,

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial k_i}/h_B(k_i) &= Y_B(v_i)\left[(1+\lambda)k_i - \lambda\left(\frac{v_{i+1}-k_i}{\omega_{i+1}} - \frac{k_i - v_i}{\omega_i}\right)(1-W_i) + \lambda(v_{i+1}-v_i)\right] \\ &+Y_B(v_{i+1})\left[-(1+\lambda)k_i + \lambda\frac{v_{i+1}-k_i}{\omega_{i+1}}(1-W_{i+1})\right] + Y_B(v_{i-1})\left[-\lambda\frac{k_i-v_i}{\omega_i}(1-W_{i-1})\right] \\ &+ \sum_{j=1}^{m}\left[y(v_i,c_j) - y(v_{i+1},c_j)\right]\left[-(1+\lambda)c_j\gamma_j - \lambda(c_j - c_{j-1}\Gamma_{j-1}\right] = 0 \quad (4)\end{aligned}$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial g_j}/h_S(g_j) &= Y_S(c_j)\left(-(1+\lambda)g_j - \lambda\frac{g_j - c_j}{\gamma_j}\Gamma_{j-1}\right) + Y_S(c_{j+2})\left[\lambda\frac{c_{j+1}-g_j}{\gamma_{j+1}}\Gamma_{j+1}\right] \\ &+ Y_S(c_{j+1})\left[-(1+\lambda)g_j - \lambda\left(\frac{c_{j+1}-g_j}{\gamma_{j+1}} - \frac{g_j - c_j}{\gamma_j}\right)\Gamma_j - \lambda(c_{j+1}-c_j)\right] \\ &+ \sum_{i=1}^{n}\left[y(v_i,c_j) - y(v_i,c_{j+1})\right]\left[(1+\lambda)\omega_i v_i - \lambda(v_{i+1}-v_i)(1-W_i)\right] = 0. \quad (5)\end{aligned}$$

One can substitute for w_i, v_i, W_i, c_i, γ_i, Γ_i the values above in order to express the conditions in terms of k_i and g_i. Nevertheless these first order conditions may, at first sight, not look useful as they still contain λ and Y_B as well as Y_S. However, note that (C) gives another equation which allows to solve for λ. While this still leaves Y_B and Y_S as unknowns, the previous results – in particular lemma 2 and proposition 2 – allow to limit the set of possible Y_B and Y_S in the optimal grand mechanism to very few candidate solutions: As Y_B is strictly increasing, signal v_{i+1} will trade with more seller signals than signal v_i. As Y_S is strictly decreasing, v_{i+1} will in fact trade with one more seller signal than v_i. By proposition 2, this implies that $Y(v_{i+1}) = Y(v_i) + \gamma_j$ where c_j is the one additional signal with which v_{i+1} trades. Following this logic, it is clear that only very few options have to be considered. For the buyer, the main question is whether signal v_1 never trades or trades with seller signal c_1. Depending on this $Y_B(v_1) = 0$ or $Y_B(v_1) = \gamma_1$. In the first case, $Y_B(v_2) = \gamma_1$ while in the second case $Y_B(v_2) = \gamma_1 + \gamma_2$. Proceeding inductively, Y_B can be fully constructed in each of the two cases. Note that this procedure also constructs Y_S. Hence, one is left with two options for Y_B and Y_S and the system of necessary first order conditions can be solved for both cases. Comparing maximal EGT in all solutions of the set of first order conditions yields the optimal grand mechanism with n (m) buyer (seller) signals.

As an example, consider the optimal grand mechanism if H_B and H_S are the uniform distribution on $[0,1]$ and $n = m = 2$. This implies that there is only one interior cutoff k (g) for the buyer (seller) and $v_1 = k/2$, $v_2 = (1+k)/2$, $c_1 = g/2$, $c_2 = (1+g)/2$, $\omega_1 = k = 1-\omega_2$ and $\gamma_1 = g = 1-\gamma_2$. Furthermore, $y(v_1, c_2) = 0$ as otherwise both players effectively have only one signal (and always trade). There are two options for $y(v_1, c_1)$ which can be either 1 or 0. The case $y(v_1, c_1) = 1$ is analyzed first. Strict monotonicity implies that $y(v_2, c_1) = 1$ and the assumption that there are two distinct buyer signals (and Y_B is strictly monotone) implies then $y(v_2, c_2) = 1$. Consequently, $Y_B(v_1) = g$, $Y_B(v_2) = 1$, $Y_S(c_1) = 1$ and $Y_S(c_2) = 1 - k$. Plugging these values in yields, after canceling terms,

$$\lambda g - (1+\lambda)k(1-g) + \frac{1+\lambda}{2}(1-g^2) = 0$$

for (4) and

$$-(1+\lambda)gk - (1-k)\lambda + \frac{1+\lambda}{2}k^2 = 0$$

for (5). Constraint (C) can be written as

$$2gk - 2g + k - k^2 + gk^2 - kg^2 \geq 0 \qquad \text{with "=" if } \lambda \neq 0.$$

Consequently, one obtains three equations in the three variables λ, g and k. The only feasible solution for $\lambda \neq 0$, in the sense of $k, g \in (0,1)$, of this system of equations is $k = 0.618034$ and $g = 0.381966$ which leads to EGT equal to 0.1459.[14] For $\lambda = 0$, one obtains $g = 1/3$ and $k = 2/3$. While this information structure satisfies (C), it only yields EGT of $1/9 < 0.1459$ and is therefore not optimal.

The second case is analyzed similarly and corresponds to $y(v_1, c_1) = 0$ which clearly implies $y(v_1, c_2) = 0$. By the strict monotonicity of Y_S and proposition 2, this implies $y(v_2, c_1) = 1$ and $y(v_2, c_2) = 0$. Note that this mechanism can be implemented with a fixed price mechanism and consequently (C) will not bind. Equations (4) and (5) become (after canceling terms)

$$k = g/2 \qquad\qquad g = (1+k)/2$$

which has the unique solution $g = 2/3$ and $k = 1/3$. EGT in this grand mechanism equal $4/27 = 0.\overline{148} > 0.1459$ and therefore the optimal grand mechanism with

[14] More precisely, the solution is $\lambda = \left(2\sqrt{5}-5\right)/5$ and $g = \left(6-2\sqrt{5}\right)/4$ and $k = \left(2\sqrt{5}-2\right)/4$.

$n = m = 2$ equals $v_1 = 1/6$, $v_2 = 2/3$, $\omega_1 = 1/3$, $\omega_2 = 2/3$, $c_1 = 1/3$, $c_2 = 5/6$, $\gamma_1 = 2/3$, $\gamma_2 = 1/3$.

4.2. Binding constraint (C)

It remains to clarify whether constraint (C) typically binds in the optimal information structure. As already pointed out in Myerson and Satterthwaite (1983), first best may be achievable (with full information) if the type space is coarse and in this case (C) is not binding. I present an example for this and another example in which (C) is slack due to the restriction to a coarse signal space. Eventually, I argue that (C) optimally binds if both type and signal space are not too coarse.

Example 1: *Let seller and buyer types be binary. More precisely, let H_B (H_S) assign probability $1/2$ to each element in $\{2, 4\}$ (respectively $\{1, 3\}$). Consider the fully informative signal structure and note that trade if and only if valuation is above cost is implementable with the following transfers: $t(2, 1) = 2$, $t(4, 3) = 3$, $t(2, 3) = 0$ and $t(4, 1) = 2.5$. It is straightforward to see that participation constraints are satisfied and incentive compatibility constraints are slack, i.e. (C) holds with inequality.*

The main problem considered in this section was the problem of finding the optimal signal structure *with no more than n (m) signals for the buyer (seller).* As a next step, I want to show that in the optimal information structure of *this problem* (C) may not bind even if the type space is a continuum, the signal structure is not fully informative and first best cannot be achieved.

Example 2: *Let H_S and H_B be uniform distributions on $[0, 1]$. The optimal information structure for $n = m = 2$ was derived in section 4.1. As trade takes place only between v_2 and $c_1 < v_2$, it is clear that (C) does not bind.*

The main result of this subsection states that whenever a situation as in example 2 occurs, increasing the number of signals will result in strictly higher EGT. As example 1 shows, this result cannot hold if the type space is coarse and therefore the following lemma is derived under the assumption that H_B and H_S have densities and their supports are identical intervals.

Lemma 3. *Let the support of H_B and H_S be identical intervals and let H_B and H_S be continuous. If (C) does not hold with equality in the optimal grand mechanism with at most n (m) buyer (seller) signals, then strictly higher EGT is obtained in the optimal grand mechanism with at most $n + 1$ ($m + 1$) buyer (seller) signals.*

The idea behind lemma 3 is simple: if (C) is slack, it is possible to introduce another cutoff close to the boundary of the support for one of the two players.

This allows to either enable additional efficient trades or avoid inefficient trades by the assumption that type spaces are identical intervals. Because (C) was initially slack, it will remain slack if the newly created signal is close enough to the support boundary (and therefore has very low probability). Lemma 3, therefore, illustrates that the information structure is typically too coarse if (C) does not bind.

4.3. Optimality of finite signal structures

So far, I restricted the information structure to contain no more than n (m) buyer (seller) types. This subsection contains two justifications for this simplifying approach.

First, the information structure in the optimal grand mechanism can in general be approximated arbitrarily closely by a finite monotone partition of the type space. The following lemma formalizes this approximation idea in a slightly more general manner by showing that whenever some grand mechanism achieves a certain EGT level $\overline{EGT}$, there is a grand mechanism with a finite signal structure that achieves an EGT level arbitrarily closely to $\overline{EGT}$.

Lemma 4. *Take any information structure (F, G) and denote EGT in this information structure (using the optimal mechanism given (F, G)) by $\overline{EGT}$. Then for any $\varepsilon > 0$ there exists an information structure (F_n, G_n) with finite support such that EGT under (F_n, G_n) (using the optimal mechanism given (F_n, G_n)) are at least $\overline{EGT} - \varepsilon$.*

Lemma 4 implies that the supremum of EGT attainable by grand mechanisms with finite information structures equals the supremum of EGT attainable by any grand mechanism. Admittedly, I did not show that there is a grand mechanism attaining this supremum, i.e. the existence of an optimal grand mechanism is unclear.

In the following, I show that in an important subclass of problems existence of an optimal grand mechanism is guaranteed and that the information structure in the optimal grand mechanism is finite. This is then also the second justification for focusing on finite information structures.

Proposition 1 implies that finite type distributions H_B and/or H_S lead to finite EGT maximizing information structures: a monotone partition of a type distribution with k elements in its support could lead to a signal structure with at most $2k - 1$ elements. Strict monotonicity of Y_B and Y_S then implies that also the other player's signal distribution has no more than $2k$ elements in its support. That is, the optimal grand mechanism with at most n (m) buyer (seller) signals is the same for all $n, m \geq 2k$. Combining this observation with lemma 4, leads to the conclusion that a grand mechanism with finite signal support has to be optimal. This is stated more formally in the following corollary.

Corollary 1. *Let the number of elements in the support of H_B (H_S) be finite and denote this number of elements by k. Then there exists an EGT maximizing grand mechanism with a signal structure for the buyer (seller) that contains at most $2k-1$ elements in its support while the signal structure for the seller (buyer) contains at most $2k$ elements in its support.*

5. Optimal binary signal structure

One important implication of corollary 1 for the binary case is that the information structure in the optimal grand mechanism has at most three elements in its support if the true type distribution is binary. In fact, it will be shown below that the support of the optimal signal distribution will be binary if the type distribution is binary. As binary type distributions do not only provide more structure but are also often used in the (applied) literature (Kamenica and Gentzkow, 2011; Taneva, 2019), it makes sense to investigate the binary case in more detail.

Before exploiting the binary structure of the type distribution, two results are stated that make only use of the restriction $n = m = 2$. In other words, lemmas 5 and 6 also hold if only the signal distribution is restricted to be binary while the type distribution may not be binary.

The first result is that the optimal grand mechanism enforces trade if and only if the expected value is above expected cost. The second result states that trade takes place between players with "good signals" but not between players with "bad signals". For simplicity, the signals concerning costs (value) are denoted in this section by c_l (v_l) and c_h (v_h) with $c_h > c_l$ ($v_h > v_l$).

Lemma 5. *The optimal grand mechanism for $n = m = 2$ enforces trade if and only if the buyer signal exceeds the seller signal.*

Lemma 6. *$y(v_l, c_h) = 0$ and $y(v_h, c_l) = 1$ in the optimal grand mechanism for $n = m = 2$.*

The "only if" part of lemma 5 holds by (3). To illustrate the "if" part of lemma 5 consider signals c_h and v_l: $c_h \geq v_l$ as otherwise a fixed price mechanism and pooling all types would be optimal but this is ruled out by lemma 1.

Similarly, lemma 6 has to be true as $y(v_l, c_h) > 0$ would imply that $v_l \geq c_h$ and therefore trade irrespective of the signal at price $(v_l+c_h)/2$ would be optimal. This is outcome equivalent to pooling all types and cannot be optimal as information design could be used to rule out some inefficient trades; see lemma 1. Furthermore, $y(v_h, c_l) = 1$ has to hold as otherwise maximal EGT would be zero which is impossible given assumption 1.

The method of proof used here, i.e. arguing through fixed price mechanisms, is admittedly specific to the binary signal case and naturally leads – in conjunction with the optimality of deterministic mechanisms – to the question whether fixed price mechanims are EGT maximizing in the optimal binary information structure.[15] As the following subsection illustrates, this is not the case and the restriction to fixed price mechanisms is with loss of generality even for binary type distributions.

5.1. Binary type distribution

This section considers the case where the true type distribution is binary. For this case, corollary 1 can be slightly strengthened and extended.

Corollary 2. *Let H_B and H_S have binary support. In the optimal grand mechanism the signal structure for the buyer (seller) has binary support and at least one element of the support is also an element of the support of H_B (H_S).*

By corollary 2, the optimal signal distribution is binary if types are binary and one of the valuation signals as well as one of the cost signals must be fully informative. To simplify notation in this restricted setup, denote the true cost and valuation types by $\underline{c} < \bar{c}$ and $\underline{v} < \bar{v}$. Assumption 1 can then be written as $\underline{c} < \underline{v} < \bar{c} < \bar{v}$.

Lemma 7. *Consider the optimal grand mechanism for binary type support. Then $y(v_l, c_l) = 1 = y(v_h, c_h)$ and both $v_h = \bar{v}$ and $c_l = \underline{c}$.*

Lemma 7 leaves only the option that trade occurs unless both signals are "bad". One consequence of this is that the optimal mechanism is not a fixed price mechanism. The inequality $c_h > v_l$ holds as otherwise pooling all types would increase EGT. As signals c_h and v_l trade if and only if the other player has the "good signal" no fixed price mechanism can be optimal as no fixed price can make both c_h and v_l indifferent between trading and not trading.

Lemma 7 does not entirely describe the information structure as it does not indicate with which probability $\bar{v}$ ($\underline{c}$) types receive the v_l (c_h) signal. However,

[15] The converse of this is certainly true: Restricting oneself to fixed price mechanisms, the only relevant information is whether the type is above or below the fixed price and therefore binary signals are optimal; see Gottardi and Mezzetti (2022).

it turns out that (C) has to be satisfied with equality unless a fully informative signal structure yields a budget surplus.

Lemma 8. *Consider the optimal grand mechanism for binary type support and assume that fully revealing signals do not achieve first best EGT. Constraint (C) will then bind in the optimal grand mechanism.*

Hence, the search for the optimal information structure is equivalent to a maximization problem over two variables with one constraint or – as the constraint can be solved explicitly for one of the variables – an optimization problem over one variable without constraint, see appendix D. It is even possible to show that the objective in the latter problem is convex and therefore the solution is a corner solution. This means that one of the two players will have a fully informative signal while the other's signal has just enough noise to ensure that (C) holds. To get some intuition for this result, consider EGT which following lemma 7 can be written as (see appendix D for details)

$$EGT(\omega_h,\gamma_l) = (\omega_h\underline{\gamma} + (\bar{\omega}-\omega_h)\gamma_l)(\bar{v}-\underline{c}) + \gamma_l(1-\bar{\omega})(\underline{v}-\underline{c}) + \omega_h(1-\underline{\gamma})(\bar{v}-\bar{c}).$$

Focus on the first of the three terms which is the only non-linear term. Types $\bar{v}$ and $\underline{c}$ trade if either the buyer gets signal v_h (as the high signal trades with both seller signals) which happens with probability $\omega_h\underline{\gamma}$ or if the buyer gets signal v_l but the seller receives signal c_l which happens with probability $(\bar{\omega}-\omega_h)\gamma_l$. What we see is that the efficient trade between $\bar{v}$ and $\underline{c}$ takes place with certainty if one of the two players has fully revealing information, i.e. if $\omega_h = \bar{\omega}$ or if $\gamma_l = \underline{\gamma}$. If both players receive noisy signals, however, this efficient trade will not take place with strictly positive probability. Hence, $EGT(\omega_h,\gamma_l)$ is convex and – although the shape of the implementability condition is admittedly relevant for this as well – it should not come as a surprise that corner solutions are optimal.

The information structure in the optimal grand mechanism is therefore one of the following two corner solutions[16]

1. *buyer revealing*: $v_h = \bar{v}$, $v_l = \underline{v}$, $c_l = \underline{c}$ and $c_h = \underline{c}\left(\underline{\gamma} - \gamma_l^{BB}(\bar{\omega})\right) / \left(1-\gamma_l^{BB}(\bar{\omega})\right) + \bar{c}\left(1-\underline{\gamma}\right) / \left(1-\gamma_l^{BB}(\bar{\omega})\right)$ while $\gamma_l = \gamma_l^{BB}(\bar{\omega})$ and $\omega_h = \bar{\omega}$
2. *seller revealing*: $v_h = \bar{v}$, $v_l = \bar{v}\left(\bar{\omega} - \omega_h^{BB}(\underline{\gamma})\right) / \left(1-\omega_h^{BB}(\underline{\gamma})\right) + \underline{v}\left(1-\bar{\omega}\right) / \left(1-\omega_h^{BB}(\underline{\gamma})\right)$, $c_l = \underline{c}$, $c_h = \bar{c}$ while $\gamma_l = \underline{\gamma}$ and $\omega_h = \omega_h^{BB}(\underline{\gamma})$.

[16] The function $\gamma_l^{BB}(\omega_h)$, which is defined in appendix D, gives the γ_l necessary to satisfy (C) with equality for a given ω_h. $\omega_h^{BB}(\gamma_l)$ is defined analogously.

It is straightforward to compute EGT in each of the two solution candidates above and the candidate achieving highest EGT is the optimal information structure. This comparison leads to the following result that completely describes the optimal grand mechanism in case of binary types.

Proposition 3. *Let the support of H_S and H_B be binary. Then the information structure in the optimal grand mechanism is* buyer revealing *if and only if*

$$\frac{(1-\underline{\gamma})(\bar{v}-\bar{c})}{(1-\bar{\omega})(\underline{v}-\underline{c})}\left[\bar{\omega}-\frac{1}{2}\left(1+\frac{\underline{\gamma}(\bar{v}-\underline{c})}{\bar{c}-\underline{\gamma}\underline{c}-(1-\underline{\gamma})\bar{v}}\right)\right.$$
$$\left.+\sqrt{\frac{1}{4}\left(1+\frac{\underline{\gamma}(\bar{v}-\underline{c})}{\bar{c}-\underline{\gamma}\underline{c}-(1-\underline{\gamma})\bar{v}}\right)^2-\frac{\underline{\gamma}\bar{\omega}(\bar{v}-\underline{v})+\underline{\gamma}(\underline{v}-\underline{c})}{\bar{c}-\underline{\gamma}\underline{c}-(1-\underline{\gamma})\bar{v}}}\right]$$
$$\geq \underline{\gamma}-\frac{1}{2}\left(1+\frac{\bar{\omega}(\bar{v}-\underline{c})}{\bar{\omega}\bar{v}-\underline{v}+(1-\bar{\omega})\underline{c}}\right)$$
$$+\sqrt{\frac{1}{4}\left(1+\frac{\bar{\omega}(\bar{v}-\underline{c})}{\bar{\omega}\bar{v}-\underline{v}+(1-\bar{\omega})\underline{c}}\right)^2-\frac{\bar{\omega}(\bar{v}-\bar{c})+\bar{\omega}\underline{\gamma}(\bar{c}-\underline{c})}{\bar{\omega}\bar{v}-\underline{v}+(1-\bar{\omega})\underline{c}}}$$

and seller revealing *if the reverse inequality holds.*

The resulting EGT can be compared to first best

$$EGT^{fb}=\bar{\omega}\bar{v}+(1-\bar{\omega})*\underline{\gamma}\underline{v}-\underline{\gamma}\underline{c}-(1-\underline{\gamma})\bar{\omega}\bar{c}.$$

The result of this comparison will generally depend on the values of the parameters $\bar{v}$, $\underline{v}$, $\underline{c}$, $\bar{c}$, $\underline{\gamma}$, $\bar{\omega}$. Note, however, that it is without loss of generality to set $\bar{v}=1$: Dividing all types by $\bar{v}$ will divide all constraints as well as the objective by $\bar{v}$ and therefore not affect the optimization problem. (Put differently, signals in the information structure of the optimal grand mechanism will be the previous optimal signals divided by $\bar{v}$. First and second best EGT will be divided by $\bar{v}$ as well.) With this normalization each of the remaining parameters, i.e. $\underline{v}$, $\underline{c}$, $\bar{c}$, $\underline{\gamma}$, $\bar{\omega}$, is in the compact set $[0,1]$ and consequently it is easy to numerically search for the parameter constellation in which the ratio of second best and first best EGT is minimal. Note that the just described normalization does not affect the ratio of second best and first best EGT. I computed this ratio numerically for all parameter values on a grid with stepsize 0.01, i.e. all parameter values $\bar{\omega},\underline{\gamma}\in\{0.01,0.02,\dots,0.99\}$ and $\underline{c}\in\{0.0,0.01,\dots,0.97\}$, $\underline{v}\in\{\underline{c}+0.01,\dots,0.98\}$, $\bar{c}\in\{\underline{v}+0.01,\dots,0.99\}$ are considered. The lowest ratio was 0.95417 which was achieved at $\bar{\omega}=\underline{\gamma}=0.04$, $\underline{c}=0$, $\bar{c}=0.99$, $\underline{v}=0.01$. This means that the combination of information and mechanism design can limit the loss due to asymmetric information to less than 5% in a binary type bilateral trade setting. The ratio of first best to second best EGT when using the optimal mechanism but not information design, i.e. assuming the fully informative signal structures for buyer and seller, is a natural comparison point. In this case, the lowest EGT ratio equals 0.89189 which was achieved at the same parameter constellation, i.e. $\bar{\omega} = \underline{\gamma} = 0.04$, $\underline{c} = 0$, $\bar{c} = 0.99$, $\underline{v} = 0.01$. This shows that information design can close more than half of the EGT gap left by mechanism design in a binary type bilateral trade setting.

As a final remark, note that in a symmetric setup, i.e. if $\underline{\gamma} = \bar{\omega}$ and $\bar{v} - \bar{c} = \underline{v} - \underline{c}$ hold, both buyer and seller revealing information structures are optimal. Clearly, the player whose information is revealed will have a higher expected payoff in this case as he receives a higher information rent. The asymmetry of the solution despite the symmetry of the setup is intuitively explained by the convexity of EGT explained above.

6. Conclusion

This paper characterizes the EGT maximizing grand mechanism, i.e. a pair of information structure and trading mechanism, in a bilateral trade setting. A closed form solution is derived for the special case in which the support of the true type distribution is binary. While the derivation is not straightforward, the resulting grand mechanism is strikingly simple in this binary type case: the optimal information structure is fully informative for one player and partially informative but still binary for the other player. The latter player receives either a signal fully revealing that he is a "good type" or a noisy signal. The optimal information structure renders the use of complicated mechanisms unnecessary: The optimal mechanism is deterministic and implements trade if and only if – conditional on the signals – expected value is above expected costs.

With more general finite type distributions, the information structure in the optimal grand mechanism is a monotone partition of the type space and the trading mechanism is deterministic. For type distributions with infinite support, EGT under the optimal information structure can be approximated arbitrarily closely by EGT in finite information structures that are monotone partitions of the type space.

EGT in the optimal grand mechanism can be interpreted as an upper bound on EGT achievable in light of asymmetric information by any institutional framework.

Consequently, the EGT loss compared to first best can be interpreted as the EGT loss that is fully attributable to information asymmetries. In the binary type setting, this information loss is less than 5% of first best. This is significantly less than the EGT loss without information design (while using the optimal mechanism as in Myerson and Satterthwaite (1983)) which can exceed 10%.

Appendix

A. Optimal mechanism for finite signal distribution

EGT equals

$$\sum_{i=1}^{n}\sum_{j=1}^{m} y(v_i, c_j)(v_i - c_j)\omega_i\gamma_j$$

where $y(v_i, c_j)$ is the probability of trade for v_i and c_j. A buyer of signal v_i has expected utility

$$U(v_i) = \sum_{j=1}^{m}(v_i y(v_i, c_j) - t_B(v_i, c_j))\gamma_j = v_i Y_B(v_i) - T_B(v_i) \tag{6}$$

where the expected transfer $\sum_j t_B(v_i, c_j)\gamma_j$ is denoted by $T_B(v_i)$ and the expected probability of trade is denoted by $Y_B(v_i) = \sum_j y(v_i, c_j)\gamma_j$. Similarly, the expected utility of the seller is

$$\Pi(c_j) = \sum_{i=1}^{n}(t_S(v_i, c_j) - c_j y(v_i, c_j))\omega_j = T_S(c_j) - c_j Y_S(c_j). \tag{7}$$

The goal is to determine the EGT maximizing y and transfer rules t_S and t_B subject to

- the participation constraints

$$U(v_i) \geq 0 \quad \text{for all } v_i \in \{v_1, \ldots, v_n\} \qquad \Pi(c_j) \geq 0 \text{ for all } c_j \in \{c_1, \ldots, c_m\}, \tag{8}$$

- the incentive compatibility constraints

$$v_i Y_B(v_i) - T_B(v_i) \geq v_i Y_B(v_k) - T_B(v_k) \quad \text{for all } v_i, v_k \in \{v_1, \ldots, v_n\}, \tag{IC$_\mathrm{B}$}$$

$$T_S(c_j) - c_j Y_S(c_j) \geq T_S(c_k) - c_j Y_S(c_k) \quad \text{for all } c_j, c_k \in \{c_1, \ldots, c_m\}, \tag{IC$_\mathrm{S}$}$$

- ex post budget balance

$$t_B(v_i, c_j) = t_S(v_i, c_j) \quad \text{for all } v_i \in \{v_1, \ldots, v_n\} \text{ and } c_j \in \{c_1, \ldots, c_m\}. \tag{9}$$

.

It is straightforward to show that in this setting every ex ante budget balanced mechanism can be made ex post budget balanced in the sense that starting from an ex ante budget balanced mechanism one can manipulate the transfer rules (without changing the decision rule y and therefore without changing EGT) in a way that the new mechanism satisfies ex post budget balance and incentive compatibility as well as participation constraints are nor affected. For this reason, it is without loss of generality to use the simpler (and in principle weaker) ex ante budget balance condition[17]

$$\sum_{i=1}^{n} \omega_i T_B(v_i) \geq \sum_{j=1}^{m} \gamma_j T_S(c_j). \tag{BB}$$

in inequality form instead of its ex post version in equality form. This standard result and its proof are given here for completeness:

Lemma 9. *Take a direct mechanism* (y, t_S, t_B) *that satisfies* (8), (IC$_\mathrm{B}$), (IC$_\mathrm{S}$) *and BB. Then there is a direct mechanism* $(y, \tilde{t}_S, \tilde{t}_B)$ *that satisfies* (8), (IC$_\mathrm{B}$), (IC$_\mathrm{S}$) *and the ex post budget balance constraint (9).*

Proof. If (BB) is satisfied with strict inequality, reducing t_B uniformly will keep all constraints satisfied and not affect EGT. Hence, it is without loss of generality to assume in the following that BB holds with equality under (y, t_S, t_B).

With a slight abuse of notation denote by $T_S(v_i) = \sum_{j=1}^{m} t_s(v_i, c_j)\gamma_j$ the expected transfer of the seller conditional on the buyer type being v_i. Now define the new payment rules

$$\begin{aligned} \tilde{t}_B(v_i, c_j) &= t_B(v_i, c_j) + [t_S(v_i, c_j) - t_B(v_i, c_j)] - [T_S(v_i) - T_B(v_i)] \\ \tilde{t}_S(v_i, c_j) &= t_S(v_i, c_j) - [T_S(v_i) - T_B(v_i)]\,. \end{aligned}$$

Clearly, $\tilde{t}_S(v_i, c_j) = \tilde{t}_B(v_i, c_j)$ and therefore ex post budget balance holds. Furthermore, $\tilde{T}_B(v_i) = T_B(v_i)$ for all v_i and similarly $\tilde{T}(c_j) = T(c_j)$ for all c_j by the assumption that (y, t_S, t_B) is ex ante budget balanced. As y – and therefore Y_S and Y_B – did not change, this implies that $(y, \tilde{t}_S, \tilde{t}_B)$ satisfies (8), (IC$_\mathrm{B}$), (IC$_\mathrm{S}$) because (y, t_S, t_B) did. □

Hence, I will without loss of generality use (BB) instead of (9) in the following. This allows to express the objective and all constraints in terms of interim transfers T_B and T_S or alternatively in terms of interim rents U and Π.

[17] Strictly speaking this is not a “budget balance” but a “no budget deficit” constraint. In order not to overload the presentation with too much terminology, I stick to the customary “budget balance” terminology.

The following lemma gives a simple characterization of incentive compatibility for the discrete case.

Lemma 10. ($\mathrm{IC_B}$) *is satisfied if and only if* Y_B *is increasing and*

$$U(v_i) = U(v_{i-1}) + \tilde{Y}_B(v_{i-1})(v_i - v_{i-1}) \quad \textit{for} \quad i = 2, \dots, n \tag{10}$$

where $Y_B(v_{i-1}) \leq \tilde{Y}_B(v_{i-1}) \leq Y_B(v_i)$. ($\mathrm{IC_S}$) *is satisfied if and only if* Y_S *is decreasing and*

$$\Pi(c_j) = \Pi(c_{j+1}) + \tilde{Y}_S(c_j)(c_{j+1} - c_j) \tag{11}$$

where $Y_S(c_j) \geq \tilde{Y}_S(c_j) \geq Y_S(c_{j+1})$.

Proof of lemma 10: *If:* Let (10) hold and Y_B be increasing. Take $i > k$. Iterating (10), yields

$$U(v_i) = U(v_k) + \sum_{j=k}^{i-1} \tilde{Y}_B(v_j)(v_{j+1} - v_j). \tag{12}$$

As $\tilde{Y}_B(v_j) \geq Y_B(v_j)$ and Y_B is increasing, this implies

$$\begin{aligned} U(v_i) &\geq U(v_k) + \sum_{j=k}^{i-1} Y_B(v_k)(v_{j+1} - v_j) \\ &= U(v_k) + Y_B(v_k)(v_i - v_k). \end{aligned}$$

Hence, ($\mathrm{IC_B}$) is satisfied for v_i and v_k. Similarly starting from (12), $\tilde{Y}_B(v_j) \leq \tilde{Y}_B(v_{j+1})$ and Y_B being increasing implies

$$U(v_i) \leq U(v_k) + \sum_{j=k}^{i-1} Y_B(v_i)(v_{j+1} - v_j)$$

and therefore $U(v_k) \geq U(v_i) + Y_B(v_i)(v_k - v_i)$ which means that ($\mathrm{IC_B}$) is satisfied for v_k and v_i.

Only if: Let ($\mathrm{IC_B}$) be satisfied. For $k = i - 1$, ($\mathrm{IC_B}$) is equivalent to $U(v_i) - U(v_{i-1}) \geq Y_B(v_{i-1})(v_i - v_{i-1})$. Using the incentive constraint that v_{i-1} does not want to misrepresent as v_i, ($\mathrm{IC_B}$) can be rearranged to $U(v_{i-1}) - U(v_i) \geq Y_B(v_i)(v_{i-1} - v_i)$. Taking these two inequalities together gives

$$Y_B(v_i) \geq \frac{U(v_i) - U(v_{i-1})}{v_i - v_{i-1}} \geq Y_B(v_{i-1}).$$

Hence, Y_B is increasing and (10) holds with $\tilde{Y}_B(v_{i-1}) = [U(v_i) - U(v_{i-1})]/[v_i - v_{i-1}]$.

The proof for the seller is analogous. □

(10) and (11) can be rewritten as[18]

$$\begin{aligned} U(v_i) &= U(v_1) + \sum_{k=1}^{i-1} \tilde{Y}_B(v_k)(v_{k+1} - v_k) \\ \Pi(c_j) &= \Pi(c_m) + \sum_{k=j}^{m-1} \tilde{Y}_S(c_k)(c_{k+1} - c_k). \end{aligned}$$

These conditions imply that the participation constraint for all types are implied by the participation constraints of v_1 and c_m. Furthermore, using these conditions to substitute rents for transfers in the budget balance constraint (BB) yields

$$\begin{aligned} &- U(v_1) + \sum_{i=1}^{n} \omega_i \left[v_i Y_B(v_i) - \sum_{k=1}^{i-1} \tilde{Y}_B(v_k)(v_{k+1} - v_k) \right] \\ &\qquad \geq \Pi(c_m) + \sum_{j=1}^{m} \gamma_j \left[c_j Y_S(c_j) + \sum_{k=j}^{m-1} \tilde{Y}_S(c_k)(c_{k+1} - c_k) \right] \end{aligned}$$

which is equivalent to[19]

$$\begin{aligned} &\sum_{i=1}^{n} \left[\omega_i Y_B(v_i) v_i - (v_{i+1} - v_i)\tilde{Y}_B(v_i)(1 - W_i) \right] \\ &\qquad \geq U(v_1) + \Pi(c_m) + \sum_{j=1}^{m} \left[\gamma_j Y_S(c_j) c_j + (c_{j+1} - c_j)\tilde{Y}_S(c_j)\Gamma_j \right] \end{aligned}$$

where $W_i = \sum_{k=1}^{i} \omega_k$ and $\Gamma_j = \sum_{k=1}^{j} \gamma_k$. In order to relax this constraint (without violating participation or incentive compatibility constraints), it is best to choose $U(v_1) = \Pi(c_m) = 0$ and $\tilde{Y}_S(c_j) = Y_S(c_{j+1})$ (recall that Y_S is decreasing and that $Y_S(c_j) \geq \tilde{Y}_S(c_j) \geq Y_S(c_{j+1})$) as well as $\tilde{Y}_B(v_i) = Y_B(v_i)$ (recall that Y_B is increasing and that $Y_B(v_{i+1}) \geq \tilde{Y}_B(v_i) \geq Y_B(v_i)$). Note that none of these variables is part of the objective (1) and therefore these choices are indeed optimal. With these

[18] Here I use the notational convention that $\sum_{k=j}^{0} \cdots = 0$ for any $j = 1, 2 \ldots$.

[19] Define $v_{n+1} = v_n$, $c_0 = c_1$ and $c_{m+1} = c_m$ for notational convenience and similarly $\tilde{Y}_b(v_{n+1}) = \tilde{Y}_S(c_{m+1}) = 0$.

choices the constraint can be written as

$$\sum_{i=1}^{n}\left[\omega_i Y_B(v_i)v_i-(v_{i+1}-v_i)Y_B(v_i)(1-W_i)\right]\geq\sum_{j=1}^{m}\left[\gamma_j Y_S(c_j)c_j+(c_{j+1}-c_j)Y_S(c_{j+1})\Gamma_j\right] \tag{13}$$

which is equivalent to (C).

Neglecting the monotonicity constraints on Y_S and Y_B for now, the mechanism design problem becomes maximizing (1) subject to (C). Hence, the optimal decision rule y must maximize the Lagrangian

$$\begin{aligned}\mathcal{L}(y)=\sum_{i=1}^{n}\sum_{j=1}^{m}y(v_i,c_j)\omega_i\gamma_j\Bigg[&(1+\lambda)v_i-\lambda(v_{i+1}-v_i)\frac{1-W_i}{\omega_i}\\&-(1+\lambda)c_j-\lambda(c_j-c_{j-1})\frac{\Gamma_{j-1}}{\gamma_j}\Bigg]\end{aligned}$$

where $\lambda\geq 0$ is the Lagrange parameter of the implementability constraint. As the Lagrangian is linear in y, the optimal decision rule is given by (3).

B. Correlated signals

In the bilateral trade setup of this paper it is straightforward to show that first best EGT are achievable if one considers correlated information structures. To this end, consider a signal structure that maps each pair of types (v, c) to itself, i.e. each player receives a signal equal to the true type vector (v, c). Amend this signal structure with the trading mechanism

$$\begin{aligned}y((v_B,c_B),(v_S,c_S)) &= \begin{cases}1 & \text{if } v_B=v_S\geq c_B=c_S\\ 0 & \text{else}\end{cases}\\ t_B((v_B,c_B),(v_S,c_S)) &= \begin{cases}(v_B+c_B)/2 & \text{if } v_B=v_S\geq c_B=c_S\\ 0 & \text{else}\end{cases}\end{aligned}$$

and $t_S((v_B,c_B),(v_S,c_S))=t_B((v_B,c_B),(v_S,c_S))$ where the first (second) argument in y, t_B and t_S is the reported buyer (seller) signal. It is straighforward to see that this trading mechanism is incentive compatible and satisfies the participation constraint. Most importantly, first best EGT is achieved by essentially eliminating the information asymmetry between buyer and seller.

C. Proofs of results in the text

Proof of lemma 1: Suppose to the contrary that all seller types are pooled on one signal $\mathbb{E}[c]$ in the EGT maximizing grand mechanism. In this case, the optimal mechanism is clearly a fixed price mechanism with price equal to $\mathbb{E}[c]$. Consequently, the optimal information structure for the buyer is without loss of generality binary: One signal v_l for all types below $\mathbb{E}[c]$ and one signal v_h for all types above $\mathbb{E}[c]$.

From here I have to distinguish two cases depending which version of assumption 1 holds. *First, I continue the proof under the assumption* $\min supp(H_S) < \min supp(H_B) < \max supp(H_S) < \max supp(H_B)$. This implies that v_h has positive probability mass denoted by ω_h. The argument now depends on whether v_l has positive probability mass.

As a first subcase assume that v_l has positive probability. I will change now the seller information structure and the mechanism in two steps and show that a EGT increasing improvement exists that still satisfies (C). In the first step, change the information structure of the seller to an information structure with two signals $c_l = v_l$ and $c_h \in (\mathbb{E}[c], v_h)$ while maintaining the mechanism $y(v_h, \cdot) = 1$ and $y(v_l, \cdot) = 0$. By assumption 1, such an information structure in which both c_l and c_h have positive probability exists.[20] Note that EGT are the same as before because the trading probabiltiy between any two types have not changed. Furthermore, (C) can be written as $\omega_h(v_h - c_h) > 0$, i.e. (C) is slack. In a second step, increase $y(v_l, c_l)$ from 0 to $\varepsilon > 0$ where ε is chosen small enough to keep (C), which reads $\omega_h(v_h - c_h) - \varepsilon\gamma_l(\omega_h(v_h - c_l) - v_l + c_l) \geq 0$, slack. As $v_l = c_l$, EGT are again unchanged. In a final step, change the seller's information structure such that γ_l, the probabiltiy of receiving the low signal, stays the same but $c_l = v_l - \varepsilon'$ and $c_h \in (\mathbb{E}[c], v_h)$ which is again possible by assumption 1 for $\varepsilon' > 0$ small enough. As $y(v_l, c_l) = \varepsilon \in (0, 1)$, this increases EGT. For $\varepsilon' > 0$ small enough (C) is not violated as it is continuous in ε' and was slack for $\varepsilon' = 0$. This establishes a grand mechanism satisfying (C), has Y_B in- and Y_S decreasing, and yielding strictly higher EGT than the initial structure in which the seller's types were pooled.

As second subcase assume that v_l has zero probability mass, i.e. $\mathbb{E}[c] \leq \min supp(H_B)$ and both seller and buyer types are pooled on a single signal each in the supposedly optimal information structure.[21] The optimal mechanism clearly enforces trade with probability 1 in this case. I will change the signal structure in several steps maintaining (C) in each step and (weakly) increasing EGT in each step. By assumption 1, there exists an $s \in [\min supp(H_S), \max supp(H_B)]$ and an

[20]For example, let $\gamma_l = H_S(v_l)/2$ where $H_S(v_l) > 0$ by assumption 1 as $v_l \geq \min supp(H_B) > \min supp(H_S)$.

[21]An argument analogous to the first case establishes also that $\mathbb{E}[v] \geq \max supp(H_S)$ in this case.

$\varepsilon > 0$ such that $H_B(s) > \varepsilon$ and $1 - H_S(s) > \varepsilon$. In a first step, change both players information structure to binary signals such that signal $c_h = v_l = s$ is sent with probability ε and the signals $c_l < c_h$ and $v_h > v_l$ are sent with probability $1 - \varepsilon$ (where c_l and v_h are chosen such that the expected value equals the expected value of the type distribution; by the definition of s and ε such a distribution is feasible). The new information structure leads to the same EGT when maintaining trade with probability 1 and is clearly budget balanced as a fixed price mechanism with price s is possible. In a second step, change the mechanism by setting $y(v_l, c_h) = 0$. As $v_l = c_h$, this does not affect EGT and as the change relaxes (C), this constraint holds now with strict inequality. In a final step, increase c_h slightly and decrease c_l slightly while both signals are still sent with probabilities ε and $1 - \varepsilon$ (the decrease in c_l is, of course, chosen such that the expected value is maintained). This is feasible as $1 - H_S(c_h) = 1 - H_S(s) > \varepsilon$ by the definition of ε. Since (C) is continuous in signals, a sufficiently small change will not violate this constraint. Furthermore, EGT are strictly increased as costs conditional on trade decreases – due to $Y_S(c_h) < Y_S(c_l)$ – and the probability of trade is unaffected. Clearly, Y_S is de- and Y_B is increasing.

The second case applies if assumption 1 is satisfied in the form that H_B and H_S have identical interval supports. This assumption implies that both v_h and v_l have positive probability mass denoted by ω_h and ω_l. Also note that clearly $v_l < \mathbb{E}[c] < v_h$.

I will change now the seller information structure and the mechanism in two steps and show that a EGT increasing improvement exists that satisfies (C). In the first step, change the information structure of the seller to an information structure with two signals $c_l = v_l$ and $c_h \in (\mathbb{E}[c], v_h)$ while maintaining the mechanism $y(v_h, \cdot) = 1$ and $y(v_l, \cdot) = 0$. By assumption 1 and $v_h > \mathbb{E}[c]$, such an information structure in which both c_l and c_h have positive probability exists.[22] Note that EGT are the same as before because the trading probabiltiy between any two types have not changed. Furthermore, constraint (C) can be written as $\omega_h(v_h - c_h) > 0$, i.e. (C) is slack. In a second step, increase $y(v_l, c_l)$ from 0 to $\varepsilon > 0$ where ε is chosen small enough to keep (C), which reads $\omega_h(v_h - c_h) - \varepsilon\gamma_l(\omega_h(v_h - c_l) - v_l + c_l) \geq 0$, slack. As $v_l = c_l$, EGT are again unchanged. In a final step, change the seller's information structure such that γ_l, the probabiltiy of receiving the low signal, stays the same but $c_l = v_l - \varepsilon'$ and $c_h \in (\mathbb{E}[c], v_h)$ which is again possible by assumption 1 for $\varepsilon' > 0$ small enough. As $y(v_l, c_l) = \varepsilon \in (0, 1)$, this increases EGT. For $\varepsilon' > 0$ small enough (C) is not violated as it is continuous in ε' and

[22] Think of γ_l, i.e. the probability of signal c_l, being very small.

was slack for $\varepsilon' = 0$. This establishes an information structure and mechanism satisfying (C) and yielding strictly higher EGT than the initial grand mechanism in which the seller's types were pooled. $\square$

Proof of lemma 2: Suppose to the contrary $Y_B(v_i) = Y_B(v_{i+1})$ for some $i \in \{1, \ldots, n-1\}$ in the optimal grand mechanism. In case the monotonicity constraint binds for more than two signals, let v_i be the lowest signal for which it binds. Now consider an information structure in which signals v_i and v_{i+1} are merged, that is, every type v that got either signal v_i or v_{i+1} will now get signal

$$\tilde{v} = \frac{\omega_i}{\omega_i + \omega_{i+1}} v_i + \frac{\omega_{i+1}}{\omega_i + \omega_{i+1}} v_{i+1}$$

and nothing changes for other types. Adapt the decision rules y by letting

$$\tilde{y}(\tilde{v}, c_j) = \frac{\omega_i}{\omega_i + \omega_{i+1}} y(v_i, c_j) + \frac{\omega_{i+1}}{\omega_i + \omega_{i+1}} y(v_{i+1}, c_j).$$

Note that this construction implies that $\tilde{Y_S} = Y_S$ and $\tilde{Y_B}(v_k) = Y_B(v_k)$ for all $k \in \{1, \ldots, i-1, i+2, \ldots, n\}$ and in particular $\tilde{Y_B}(\tilde{v}) = Y_B(v_i) = Y_B(v_{i+1})$. The objective (1) which can be written as $\sum_i \omega_i Y_B(v_i) v_i - \sum_j \gamma_j Y_S(c_j) c_j$ is therefore unchanged by the merging of signals. However, constraint (C) is strictly relaxed by the merging of signals: Note that (C) can be written as

$$\left\{ \sum_{i=1}^{n} Y_B(v_i)\omega_i \left[v_i - (v_{i+1} - v_i)\frac{1 - W_i}{\omega_i} \right] \right\} - \left\{ \sum_{j=1}^{m} Y_S(c_j)\gamma_j \left[c_j + (c_{j+1} - c_j)\frac{\Gamma_j}{\gamma_j} \right] \right\} \geq 0.$$

The merging of types affects only the two terms for v_i and v_{i+1} as Y_S and Y_B for other signals were not affected. Hence, the relevant two terms are (using the notation $\tilde{\omega} = \omega_i + \omega_{i+1}$)

$$
\begin{aligned}
& -Y_B(v_{i-1})v_i(1-W_{i-1}) + Y_B(v_i)\left[\omega_i v_i - (v_{i+1}-v_i)(1-W_i)\right] \\
& +Y_B(v_{i+1})\left[\omega_{i+1}v_{i+1} - (v_{i+2}-v_{i+1})(1-W_{i+1})\right] \\
= \; & -Y_B(v_{i-1})v_i(1-W_{i-1}) + \tilde{Y}_B(\tilde{v})\left[\tilde{v}\tilde{\omega} - (v_{i+1}-v_i)(1-W_i) - (v_{i+2}-v_{i+1})(1-W_{i+1})\right] \\
= \; & -Y_B(v_{i-1})\tilde{v}(1-W_{i-1}) + Y_B(v_{i-1})(\tilde{v}-v_i)(1-W_{i-1}) \\
& +\tilde{Y}_B(\tilde{v})\left[\tilde{v}\tilde{\omega} - (v_{i+1}-v_i)(1-W_{i+1}) - (v_{i+2}-v_{i+1})(1-W_{i+1})\right] - \tilde{Y}_B(\tilde{v})(v_{i+1}-v_i)\omega_{i+1} \\
= \; & -Y_B(v_{i-1})\tilde{v}(1-W_{i-1}) + Y_B(v_{i-1})(\tilde{v}-v_i)(1-W_{i+1}+\omega_i+\omega_{i+1}) \\
& +\tilde{Y}_B(\tilde{v})\left[\tilde{v}\tilde{\omega} - (v_{i+2}-v_i)(1-W_{i+1})\right] - \tilde{Y}_B(\tilde{v})(v_{i+1}-v_i)\omega_{i+1} \\
= \; & -Y_B(v_{i-1})\tilde{v}(1-W_{i-1}) + Y_B(v_{i-1})(\tilde{v}-v_i)(1-W_{i+1}) + Y_B(v_{i-1})(v_{i+1}-v_i)\omega_{i+1} \\
& +\tilde{Y}_B(\tilde{v})\left[\tilde{v}\tilde{\omega} - (v_{i+2}-\tilde{v})(1-W_{i+1})\right] - \tilde{Y}_B(\tilde{v})(v_{i+1}-v_i)\omega_{i+1} - \tilde{Y}_B(\tilde{v})(\tilde{v}-v_i)(1-W_{i+1}) \\
= \; & -Y_B(v_{i-1})\tilde{v}(1-W_{i-1}) + \tilde{Y}_B(\tilde{v})\left[\tilde{v}\tilde{\omega} - (v_{i+2}-\tilde{v})(1-W_{i+1})\right] \\
& +(Y_B(v_{i-1}) - \tilde{Y}_B(\tilde{v}))(\tilde{v}-v_i)(1-W_{i+1}) + (Y_B(v_{i-1}) - \tilde{Y}_B(\tilde{v}))(v_{i+1}-v_i)\omega_{i+1} \\
< \; & -Y_B(v_{i-1})\tilde{v}(1-W_{i-1}) + \tilde{Y}_B(\tilde{v})\left[\tilde{v}\tilde{\omega} - (v_{i+2}-\tilde{v})(1-W_{i+1})\right]
\end{aligned}
$$

where the first equality uses $Y_B(v_i) = Y_B(v_{i+1}) = \tilde{Y}_B(\tilde{v})$ and the definition of $\tilde{v}$, the inequality uses $\tilde{Y}_B(\tilde{v}) = Y_B(v_i) > Y(v_{i-1})$ (recall that i was the lowest pooled type). Note that the term we end up with is exactly the term referring to $\tilde{v}$ in (C) under the modified $\tilde{y}$. Consequently, the merging of signals strictly relaxed (C) without affecting the objective. If (C) binds, this contradicts the optimality of the initial grand mechanism. If (C) does not bind, then the information structure after the merging of types is coarser and also a solution as it is feasible, satisfies (C) and yields the same EGT as the initial information structure.

The proof for the seller is analogous. □

Proof of proposition 1: I show the result for the buyer. Suppose by way of contradiction that the $(v_i)_{i=1}^n$ and $(\omega_i)_{i=1}^n$ in the optimal grand mechanism do not form a monotone partition (up to a measure zero set). This implies that there exists some $i \in \{1, \ldots, n\}$ and a set of true valuation types N_i with some mass $\eta > 0$ that receives signal v_i and a set of true valuation types N_{i+1} with mass $\eta > 0$ that receives signal v_{i+1} such that $\mathbb{E}[v|v \in N_i] > \mathbb{E}[v|v \in N_{i+1}]$. We will return to these sets later.

Consider for now the optimization problem of maximizing EGT subject to (C): Maximize EGT, i.e. $\sum_i \sum_j \omega_i \gamma_j (v_i - c_j) y(v_i, c_j)$, over y, $(v_i)_{i=1}^n$, $(c_j)_{j=1}^m$, ω_i and γ_j subject to (C). Let the domain for y be $[0,1]$ and the domain for $(\omega_i)_{i=1}^n$, $(v_i)_{i=1}^n$ is the set of all distributions such that F is a mean preserving spread of these distributions. Respectively, the domain of $(\gamma_j)_{j=1}^m$, $(c_j)_{j=1}^m$ is such that G is a mean preserving spread of these distributions. Note that incentive compatibility and participation constraints will be automatically satisfied by the solution due to substituting the expressions from lemma 10 and participation contraints into the budget constraint in order to obtain (C). (By lemma 2 the monotonicity constraint is slack.) That is, the solution to this program will be the optimal information structure and mechanism if the number of buyer (seller) signals is restricted to no more than n (m).[23] Writing the Lagrangian for this optimization problem with Lagrange parameters λ for constraint (C) yields:

$$\mathcal{L} = \sum_{i=1}^{n}\sum_{j=1}^{m}\Bigg\{y(v_i,c_j)\omega_i\gamma_j\Bigg[(1+\lambda)v_i - \lambda(v_{i+1}-v_i)\frac{1-W_i}{\omega_i} \\ -(1+\lambda)c_j - \lambda(c_j - c_{j-1})\frac{\Gamma_{j-1}}{\gamma_j}\Bigg]\Bigg\}$$

A solution to this finite-dimesional problem exists by the Weierstrass theorem as the feasible set is compact and non-empty and the objective is continuous. Consider $\mathcal{L}$ evaluated at the solution values for y, $(\omega_i)_{i=1}^n$, $(\gamma_j)_{j=1}^m$ and $(c_j)_{j=1}^m$. Given that, the optimal values for $(v_i)_{i=1}^n$ have to maximize $\mathcal{L}$ (within the feasible set of v_i, i.e. all those $(v_i)_{i=1}^n$ that yield together with $(\omega_i)_{i=1}^n$ a distribution such that F is a mean preserving spread of it). Now consider the following family of buyer valuation distributions indexed by ε which I denote by $(\tilde{v}_i)_{i=1}^n$: Fix all valuations apart from some $\tilde{v}_i$ and $\tilde{v}_{i+1}$ at their optimal levels (i.e. at the values that are part of the solution of the maximization problem above) and let

$$\begin{aligned}\tilde{v}_i(\varepsilon) &= \frac{(\omega_i-\varepsilon)v_i + \varepsilon v_{i+1}}{\omega_i}\\ \tilde{v}_{i+1}(\varepsilon) &= \frac{(\omega_{i+1}-\varepsilon)v_{i+1} + \varepsilon v_i}{\omega_{i+1}}\end{aligned}$$

where v_i and v_{i+1} are the solution values in the maximization problem above. As $v_i(0) = v_i$ and $v_{i+1}(0) = v_{i+1}$, the auxiliary maximization problem of maximizing $\mathcal{L}$ over ε (where all variables apart from $\tilde{v}_i$ and $\tilde{v}_{i+1}$ are fixed at their optimal solution) must have a (local) maximum at $\varepsilon = 0$ if the information structure is feasible for ε in an open neighborhood around 0. The corresponding derivative of

[23]Strictly speaking one should also add constraints enforcing $v_{i+1} - v_i \geq 0$ and $c_{j+1} \geq c_j$ which will, however, not change the argument below and only clutter notation further.

$\mathcal{L}$ with respect to ε is

$$\frac{d\mathcal{L}}{d\varepsilon} = (v_{i+1} - v_i)\left[Y_B(v_i)(1+\lambda+\lambda(1-W_i)/\omega_i) - Y_B(v_{i-1})\lambda(1-W_{i-i})/\omega_i\right] \\ - (v_{i+1} - v_i)\left[Y_B(v_{i+1})(1+\lambda+\lambda(1-W_{i+1})/\omega_{i+1}) - Y_B(v_i)\lambda(1-W_i)/\omega_{i+1}\right] \tag{14}$$

Note that the derivative does not depend on ε, i.e. $\mathcal{L}$ in the auxiliary maximization problem is linear in ε. It is straightforward to see that $(\tilde{v}_i)_{i=1}^n$ is feasible for $\varepsilon \geq 0$ if $\varepsilon \geq 0$ is not too high. (Essentially $\tilde{v}_i$ and $\tilde{v}_{i+1}$ use the optimal information structure which is feasible and then swap the signal for ε of those types receiving signals v_i and v_{i+1} in the optimal information structure. Clearly, this does not change ω_i or ω_{i+1} and yields a new feasible information structure.) I will now show that $(\tilde{v}_i)_{i=1}^n$ are also feasible for $\varepsilon < 0$ (not too far from 0) if the optimal information structure is not a monotone partition. After ruling out that the slope of $\mathcal{L}$ in ε is zero, this will complete the proof as feasibility for ε in an open interval around 0 means that $\varepsilon = 0$ cannot maximize the linear $\mathcal{L}$ in the auxiliary problem. This contradiction establishes that the information structure (for the buyer) in the optimal grand mechanism must be a monotone partition.

To see that $\varepsilon < 0$ is feasible, consider changing the information structure by swapping the signal of mass $\tau < \eta$ in N_i and N_{i+1}, i.e mass $\tau < \eta$ of the types in N_i receives signal v_{i+1} (instead of v_i) and mass τ in N_{i+1} receives signal v_i (instead of v_{i+1}). This is clearly feasible and does not change ω_i or ω_{i+1} but the expected valuation when receiving signals v_i or v_{i+1} changes to

$$\begin{aligned} \tilde{v}_i(\tau) &= \frac{\omega_i v_i - \tau\left(\mathbb{E}[v|v\in N_i] - \mathbb{E}[v|v\in N_{i+1}]\right)}{\omega_i} \\ \tilde{v}_{i+1}(\tau) &= \frac{(\omega_{i+1} v_{i+1} + \tau\left(\mathbb{E}[v|v\in N_i] - \mathbb{E}[v|v\in N_{i+1}]\right)}{\omega_{i+1}}. \end{aligned}$$

Choosing $\tau = -\varepsilon(v_{i+1} - v_i)/\left(\mathbb{E}[v|v\in N_i] - \mathbb{E}[v|v\in N_{i+1}]\right)$ yields $\tilde{v}_i(\varepsilon)$ and $\tilde{v}_{i+1}(\varepsilon)$ for negative ε.

The last step is to rule out that $\mathcal{L}$ has slope 0 in ε (when fixing all variables apart from $\tilde{v}_i(\varepsilon)$ and $\tilde{v}_{i+1}(\varepsilon)$ at their optimal values). To get a contradiction suppose this was the case and note that this is only possible if $\lambda \neq 0$, see (14) and recall that $Y_B(v_{i+1}) > Y_B(v_i)$ by lemma 2. Then there exists an $\varepsilon' > 0$ such that

$$\tilde{v}_i(\varepsilon') - (\tilde{v}_{i+1}(\varepsilon') - \tilde{v}_i(\varepsilon'))\frac{\lambda}{1+\lambda}\frac{1-W_i}{\omega_i} = \tilde{v}_{i+1}(\varepsilon') - (v_{i+2} - \tilde{v}_{i+1}(\varepsilon'))\frac{\lambda}{1+\lambda}\frac{1-W_{i+1}}{\omega_{i+1}}, \tag{15}$$

i.e. the two signals have the same virtual valuation.[24] $\mathcal{L}$ evaluated for ε' is the same as when evaluated at the optimal solution by the assumption that its derivative in ε is zero. As a next step, change $y(v_i, \cdot)$ and $y(v_{i+1}, \cdot)$ by assigining the average trading probability, i.e. $\tilde{y}(\tilde{v}_i, c_j) = \tilde{y}(\tilde{v}_{i+1}, c_j) = y(v_i, c_j)\omega_i/(\omega_i + \omega_{i+1}) + y(v_{i+1}, c_j)\omega_{i+1}/(\omega_i + \omega_{i+1})$ for all $j = 1, \ldots, m$. As both $\tilde{v}_i(\varepsilon')$ and $\tilde{v}_{i+1}(\varepsilon')$ have the same virtual valuation and as $\mathcal{L}$ is linear in y, this does not change the value of $\mathcal{L}$. Finally, note that due to the argument in the proof of lemma 2, merging the two signals $\tilde{v}_i(\varepsilon')$ and $\tilde{v}_{i+1}(\varepsilon')$ into one signal will not affect EGT but relaxes (C). Hence, such a merging of signals will strictly increase $\mathcal{L}$. But this implies that $(v_i, v_{i+1}, y(v_i, \cdot), y(v_{i+1}, \cdot))$ do not jointly maximize $\mathcal{L}$ in an auxilliary problem in which we fix all other variables at their optimal values. This, however, contradicts the optimality of $(v_i, v_{i+1}, y(v_i, \cdot), y(v_{i+1}, \cdot))$.

The argument for the seller is analogous. □

Proof of lemma 4: Consider the hypothetical problem of maximizing EGT subject to (C) being violated by no more than η (through the choice of a grand mechanism). Denote the by $W^*(\eta)$ the value of this maximization problem (more formally, the supremum of EGT achievable by grand mechanisms that do not violate (C) by more than η). As both EGT and (C) are continuous, W^* is also continuous. Let $\tilde{\eta} < 0$ be such that $W^*(0) - W^*(\tilde{\eta}) < \varepsilon/3$. (Note that a negative η indicates a stricter constraint.)

Define the set of distributions $\mathcal{F}_\kappa$ as the set of distributions with cdfs F_κ such that (i) $\mathbb{E}_{F_\kappa}[v] \leq \mathbb{E}_{H_B}[v] - \kappa$ and (ii) $\int_{-\infty}^{x} F_\kappa(v)\, dv \leq \int_{-\infty}^{x} H_B(v + \kappa)\, dv - \kappa$ for all $x \in (-\infty, \max supp(H_B) - \kappa]$. Similarly, define the set $\mathcal{G}$ as the set of distributions with cdfs G_κ such that (i) $\mathbb{E}_{G_\kappa}[c] \geq \mathbb{E}_{H_S}[c] + \kappa$ and (ii) $\int_{-\infty}^{x} G_\kappa(c)\, dc \leq \int_{-\infty}^{x} H_S(c+\kappa)\, dc - \kappa$ for all $x \in (-\infty, \max supp(H_S) - \kappa]$. Note that $\mathcal{F}_0$ and $\mathcal{G}_0$ are the feasible sets of distributions in the EGT maximization problem of this paper as the set of mean preserving spreads of a distribution equals the set of distributions that have the same mean while also second order stochastically dominating the distribution, see Mas-Colell et al. (1995, ch. 6.D).

Consider now the problem of maximizing EGT subject to (C) being violated by no more than $\tilde{\eta}$ over the sets $\mathcal{F}_\kappa$ and $\mathcal{G}_\kappa$. Let F and G denote an information structure such that under this information structure and the optimal mechanism (i) (C) is violated by at most $\tilde{\eta}$, (ii) EGT are above $W^*(\tilde{\eta}) - \varepsilon/3$ and (iii) $F \in \mathcal{F}_{\tilde{\kappa}}$

[24]To be precise, such an ε' exists as the left hand side (LHS) of (15) is strictly below RHS for $\varepsilon' = 0$, LHS is strictly increasing in ε' while RHS is strictly decreasing in ε' and both LHS and RHS are continuous in ε'. Furthermore, $\tilde{v}_i(\varepsilon') = \tilde{v}_{i+1}(\varepsilon')$ if $\varepsilon' = (\omega_{i+1}\omega_i(v_{i+1} - v_i)/(v_{i+1}\omega_i - v_i\omega_{i+1} + i + 1\omega_{i+1} - v_i\omega_i))$ and RHS<LHS for this ε'. Consequently, the intermediate value theorem implies that an ε' exists at which LHS=RHS.

and $G \in \mathcal{G}_{\tilde{\kappa}}$ for some $\tilde{\kappa} > 0$. Such F, G and $\tilde{\kappa}$ exist by the definition of $\tilde{\eta}$ and as the conditions defining $\mathcal{F}_\kappa$ and $\mathcal{G}_\kappa$ are continuous in κ (while EGT and (C) are continuous in signals).

Approximate (F, G) by a series of distributions $(F_n, G_n)_{n=1}^{\infty}$ such that (i) the support of F_n and G_n have at most n elements and (ii) $F_n \to F$ almost everywhere and $G_n \to G$ almost everywhere.[25] Then F_n (G_n) converges to F (G) weakly and by the Helly-Bray theorem EGT and (C) under (F_n, G_n) converge to the corresponding values under (F, G).[26] Therefore for some sufficiently high n^* EGT under (F_{n^*}, G_{n^*}) are above $W^*(\tilde{\eta}) - 2\varepsilon/3 > W^*(0) - \varepsilon$ and (C) is violated by at most $\tilde{\eta}$. But this implies – by $\tilde{\eta} < 0$ – that under the finite information structure (F_{n^*}, G_{n^*}) EGT above $W^*(0) - \varepsilon$ are achievable without violating (C). Finally, define $F^*_{n^*}$ by "shifting F_{n^*} up" such that $F^*_{n^*}$ has expected value $\mathbb{E}_{H_B}[v]$, i.e. $F^*_{n^*}(x) = F_{n^*}(x - \mathbb{E}_{H_B}[v] + \mathbb{E}_{F_{n^*}}[v])$ and note that the definition of $\mathcal{F}_{\tilde{\kappa}}$ implies $\mathbb{E}_{H_B}[v] - \mathbb{E}_{F_{n^*}}[v] > 0$ (for n^* sufficiently high). Similarly, define $G^*_{n^*}(x) = F_{n^*}(x + \mathbb{E}_{H_S}[c] - \mathbb{E}_{G_{n^*}}[c])$. Note that shifting the distribution of buyer (seller) valuations up (down) by a constant, increases EGT and relaxes (C). Consequently, EGT under $(F^*_{n^*}, G^*_{n^*})$ are above $W^*(0) - \varepsilon$. Furthermore, H_B is a mean preserving spread of $F^*_{n^*}$ by the definition of $\mathcal{F}_{\tilde{\kappa}}$ and similarly H_S is a mean preserving spread of $G^*_{n^*}$. Consequently, EGT of at least $W^*(0) - \varepsilon$ can be achieved by a grand mechanism consisting of a feasible finite information structure and the optimal mechanism for this information structure. $\square$

Proof of proposition 2: The proof is by contradiction, i.e. I show that any grand mechanism such that $y(v_i, c_j) \in (0, 1)$ is not optimal. To do so consider the problem of maximizing the Lagrangian (2) over y, signals and probabilities. Optimality requires that there is no feasible grand mechanism achieving a higher Lagrangian value than the optimal one. For now, assume that the Lagrange parameter $\lambda \neq 0$. The proof exploits the following intermediate result in a number of ways:

Intermediate Result: In any grand mechanism maximizing the Lagrangian, $y(v_i, \cdot) \neq y(v_{i+1}, \cdot)$ for any two buyer signals v_i and v_{i+1} if $\lambda \neq 0$. Similarly,

[25] For readability and notational convenience, I assume in the following that F and G are continuous. If F and G already have finite support, this whole approximation step is, of course, unnecessary. If F and G are mixed with a finite number of mass points, the following approximation is understood to be applied only to the continuous part, i.e. F_n (G_n) have the same mass points as F and G but also discretize the continuous parts of F and G.

[26] We use the same mechanism y as under (F, G) here. For completeness, define $y(v, c) = sup_{v'<v, c'>c} y(v', c')$ for all (v, c) not in the support of (F, G) (and let $y(v, c) = 0$ if $y(v', c')$ is not defined for any $v' < v$ and $c' > c$). This ensures the monotonicity of Y_S and Y_B.

$y(\cdot, c_j) \neq y(\cdot, c_{j+1})$ for any two seller signals if $\lambda \neq 0$.[27]

Proof of the intermediate result: Follows directly from the argument in the proof of lemma 2. □

Suppose to the contrary of proposition 2 that $y(v_i, c_j) \in (0,1)$. Note that this implies that the derivative of $\mathcal{L}$ with respect to $y(v_i, c_j)$ equals zero as $\mathcal{L}$ is linear in $y(v_i, c_j)$. Hence, changing $y(v_i, c_j)$ to either 0 or 1 does not affect the value of the Lagrangian. If such a change results in two adjacent signals having the same mechanism y, the intermediate result above implies that optimality is violated as there exists another grand mechanism with at most n (m) buyer (seller) signals leading to a strictly higher value of the Lagrangian.

To see that such a change leads to two adjacent signals having the same mechanism y, note first that by the monotonicity of the virtual valuation $y(v_i, c_j) < 1$ implies $y(v_i, c_k) = 0$ for all $k > j$ and $y(v_l, c_j) = 0$ for all $l < i$. Furthermore, $y(v_i, c_j) > 0$ implies $y(v_i, c_k) = 1$ for all $k < j$ and $y(v_l, c_j) = 1$ for all $l > i$; see table 2 for an illustration. This implies that if $y(v_{i+1}, c_{j+1}) = 1$, then after changing $y(v_i, c_j)$ to zero $y(\cdot, c_j) = y(\cdot, c_{j+1})$. If, however, $y(v_{i+1}, c_{j+1}) = 0$, then after changing $y(v_i, c_j)$ to 1 $y(v_i, \cdot) = y(v_{i+1}, \cdot)$. If, $y(v_{i+1}, c_{j+1}) \in (0,1)$, then changing $y(v_{i+1}, c_{j+1})$ to zero and $y(v_i, c_j)$ to 1 will not affect the value of the Lagrangian but then again $y(v_i, \cdot) = y(v_{i+1}, \cdot)$. Finally, observe that if $i = n$ or $j = m$ (and therefore there is not v_{i+1} and c_{j+1}) similar steps can be undertaken with v_{i-1} and c_{j-1} instead of v_{i+1} and c_{j+1}.

	$\cdots$	c_{j-1}	c_j	c_{j+1}	$\cdots$
$\vdots$			$\vdots$	$\vdots$	$\cdots$
v_{i-1}			0	0	$\cdots$
v_i	$\cdots$	1	$y(v_i, c_j)$	0	$\cdots$
v_{i+1}	$\cdots$	1	1		
$\vdots$	$\cdots$	$\vdots$	$\vdots$		

Table 2: Implications of strictly monotone virtual valuation and $y(v_i, c_j) \in (0,1)$

Finally, consider $\lambda = 0$. In this case, $y(v_i, c_j) \in (0,1)$ implies $v_i = c_j$ by (3). Hence, all the steps above (in the $\lambda \neq 0$ case) will maintain the Lagrangian value and therefore EGT while – through the merging of types – strictly relax constraint (C). The resulting grand mechanism would then be optimal while (C) would be slack. □

[27] Even if $\lambda = 0$ the merging of types works but does not give a strict increase in the Lagrangian, i.e. there is always an optimal grand mechanism in which the result is true.

Proof of lemma 3: For concreteness let $n \leq m$ and assume that(C) holds with inequality under the optimal grand mechanism with at most n (m) buyer (seller) signals. Denote the information structure in the solution to this problem by the optimal cutoffs for the buyer $(\underline{s}, k_1, \ldots, k_{n-1}, \bar{s})$ and the seller $(\underline{s}, g_1, \ldots, g_{m-1}, \bar{s})$ where $\underline{s}$ and $\bar{s}$ are the minimum and maximum of the common support of H_S and H_B. Denote the optimal mechanism in this information structure by $y^*_{n,m}$.

To show that EGT is higher if one more signal is allowed, I will introduce an additional cutoff into either the buyer's or the seller's signal structure and show that this increases EGT without violating (C). To do so, I will distinguish two cases: First, the highest seller type sells with zero probability in $y^*_{n,m}$ and, second, the highest seller type sells with positive probability.

First, $y^*_{n,m}(v_n, c_m) > 0$. Then consider the cutoffs $(\underline{s}, g_1, \ldots, g_{m-1}, \ \bar{s} - \varepsilon, \bar{s})$ for the seller while the cutoffs for the buyer remain unchanged. Amend $y^*_{n,m}$ with $y(v_i, c_{m+1}) = 0$ for all v_i. For $\varepsilon = 0$ (implying $Y_S(c_{m+1}) = 0$), the information structure and also (C) are unchanged and therefore EGT is the same as above. As (C) is continuous in ε and held with inequality for $\varepsilon = 0$, it will still hold for $\varepsilon > 0$ small enough. Clearly, welfare is higher in the new information structure (for $\varepsilon > 0$ small enough) as inefficient trades between $v_n < \bar{s}$ and sellers with a type in $[\bar{s} - \varepsilon, \bar{s}]$ are avoided. (By the assumption that H_S is continuous with an interval as support, this event has positive probability.)

Second, $y^*_{n,m}(v_n, c_m) = 0$. Then consider the cutoffs $(\underline{s}, k_1, \ldots, k_{n-1}, \bar{x} - \varepsilon, \bar{s})$ for the buyer while the cutoffs for the seller remain unchanged. Amend $y^*_{n,m}$ with $y(v_{n+1}, c_j) = 1$ for all c_j. For $\varepsilon = 0$ (implying $\omega_{n+1} = 0$), the information structure and also (C) are unchanged and therefore EGT is the same as above. As (C) is continuous in ε and held with inequality for $\varepsilon = 0$, it will still hold for $\varepsilon > 0$ small enough. Clearly, welfare is higher in the new information structure (for $\varepsilon > 0$ small enough) as efficient trades between buyers with types in $[\bar{s} - \varepsilon, \bar{s}]$ and sellers with signal $c_m < \bar{s}$ are enabled. $\square$

Proof of lemma 5: By proposition 2, $y(v_l, c_l)$ and $y(v_h, c_h)$ are in $\{0, 1\}$. To show that trade takes place if and only if expected value is above expected cost note that (3) implies the "only if" part. For "if" consider first the case where either $y(v_l, c_l) = 0$ or $y(v_h, c_h) = 0$ (or both). In these cases, the optimal mechanism is a fixed price mechanism in which the fixed price can be chosen either $t = v_h$ or $t = c_l$ and clearly the result holds. The only remaining case is $y(v_h, c_h) = y(v_l, c_l) = 1$ and it remains to show $c_h > v_l$ in this case. Consider to the contrary $v_l \geq c_h$. But in this case a fixed price contract at price $t = c_h$ and trade with probability 1 would (weakly) increase EGT while being budget balanced, incentive compatible and satisfying the participation constraints. As in this case trade takes place regardless of signal, this outcome can be achieved by a totally uninformative information structure (i.e. one signal per player). The optimality of such a signal structure is, however, ruled out by lemma 1. □

Proof of lemma 6: The proofs will be by contradiction, i.e. I will show an improvement in EGT if the properties do not hold. First, suppose $y(v_l, c_h) > 0$. Note that this implies $y(v_i, c_j) = 1$ for all $(v_i, c_j) \neq (v_l, c_h)$ by monotonicity of the virtual valuation. By (3), $y(v_l, c_h) > 0$ implies $v_l \geq c_h$ (with strict inequality if either $v_h > v_l$ or $c_l < c_h$ have positive probability mass) and therefore EGT would be (weakly) higher if $y(v_l, c_h) = 1$, i.e. EGT would be higher if all buyer types bought from all seller types. As $v_l \geq c_h$ implies $\mathbb{E}[v] \geq \mathbb{E}[c]$ (again with strict inequality if either $v_h > v_l$ or $c_l < c_h$ have positive probability mass), trade with probability 1 is feasible by an information structure that sends signal $\mathbb{E}[v]$ to all buyers and $\mathbb{E}[c]$ to all sellers paired with a fixed price mechanism (where the fixed price is in $[\mathbb{E}[c], \mathbb{E}[v]]$.) As this information structure is not part of the optimal grand mechanism by lemma 1, there is no optimal grand mechanism in which $y(v_l, c_h) > 0$.

Second, suppose $y(v_h, c_l) < 1$. By the monotonicity of the virtual valuation, this implies $y(v_i, c_j) = 0$ for all $(v_i, c_j) \neq (v_h, c_l)$. By (3), $y(v_h, c_l) < 1$ implies $v_h \leq c_l$. EGT in this grand mechanism are therefore at most zero. Hence, it remains to show that there is an alternative grand mechanism yielding strictly positive EGT. By assumption 1, there exists a fixed price t such that the probability that $v \geq t$ as well as the probability that $c \leq t$ is strictly positive. Consider now the information structure that sends a high signal to buyers with valuation weakly above t and a low signal otherwise. Similarly, let the signal for sellers with $c \leq t$ be low and high otherwise. Pair this information structure with a mechanism enforcing trade if and only if the buyers signal is high and the sellers signal is low at price t. Clearly, this grand mechanism is incentive compatible, budget balanced, satisfies participation constraints and yields strictly positive EGT. □

Proof of corollary 2: As the information structure in the optimal grand mechanism is a monotone partition as a consequence of proposition 1, its support could have at most three elements in which case two of these elements would also be elements of the support of the true type distribution. The following result, stated as a separate lemma, rules this possibility out and therefore the support of the signal structure in the optimal grand mechanism can have at most two elements.

Lemma 11. *Let the true type distribution of buyer valuations H_B be discrete and*

let its support be $\{\hat{v}_1, \hat{v}_2, \dots\}$. If $\hat{v}_i$ and $\hat{v}_{i+1}$ are in the support of the signal distribution in the optimal grand mechanism with at most n (m) buyer (seller) signals and constraint (C) binds, then the optimal grand mechanism assigns zero probability to all signals in $(\hat{v}_i, \hat{v}_{i+1})$.
If constraint (C) does not bind, then either the optimal grand mechanism assigns zero probability to all signals in $(\hat{v}_i, \hat{v}_{i+1})$ or there exists an optimal information structure with less than n (m) buyer (seller) types.
(An analogous result holds for the seller.)

Proof of lemma 11: Suppose otherwise, i.e. let the information structure in the optimal grand mechanism put positive probability on types $v_{-i} < v_i < v_{i+1}$ and let v_{i-1} and v_{i+1} be neighboring elements in the support of H_B. Denote the corresponding probabilities in the optimal information structure by ω_{i-1}, ω_i and ω_{i+1}. We will consider the following alternative distributions indexed by ε:

$$\begin{aligned}\tilde{\omega}_{i-1}(\varepsilon) &= \omega_{i-1} - \varepsilon \frac{v_{i+1} - v_i}{v_{i+1} - v_{i-1}} \\ \tilde{\omega}_i(\varepsilon) &= \omega_i + \varepsilon \\ \tilde{\omega}_{i+1}(\varepsilon) &= \omega_{i+1} - \varepsilon \frac{v_i - v_{i-1}}{v_{i+1} - v_{i-1}}.\end{aligned}$$

(All other variables, e.g. cost types probabilities of trade and other valuation types, are fixed at their optimal levels.) Note that the expected valuation is not affected by changes in ε and as v_{i-1} and v_{i+1} are *neighboring* elements of the true valuation support positive as well as negative ε are feasible (if not too large in absolute value).

Now consider the Lagrangian $\mathcal{L}$ of the maximization problem maximizing EGT over ε subject to (C) (fixing all other variables at their optimal level). From the definition $\tilde{\omega}_{i-1}$, $\tilde{\omega}_i$ and $\tilde{\omega}_{i+1}$, it is clear the $\mathcal{L}$ is linear in ε. As ω_{i-1}, ω_i and ω_{i+1} are by assumption part of the optimal solution, $\mathcal{L}$ has to be maximized by $\varepsilon = 0$. As $\mathcal{L}$ is linear in ε and as ε in an open interval around 0 are feasible, this can only be the case if the derivative of $\mathcal{L}$ with respect to ε is zero everywhere. In the following it is shown that this is not possible if (C) binds.

Suppose the derivative of $\mathcal{L}$ with respect to ε is zero everywhere. For $\varepsilon = 0$, we have $VV(v_{i-1}, 0) < VV(v_i, 0) < VV(v_{i+1}, 0)$ by lemma 2 (where $VV(v_i, \varepsilon)$ denotes the virtual valuation of v_i for a given ε). As ε increases the virtual valuations change as $\tilde{\omega}_{i-1}$ and $\tilde{\omega}_{i+1}$ decrease while ω_i increases. Denote by $\varepsilon' > 0$ the lowest ε such that (at least) one of the following conditions is met

- $VV(v_i, \varepsilon) = VV(v_{i+1}, \varepsilon)$

- $\tilde{\omega}_{i-1}(\varepsilon) = 0$.

For concreteness, let the first condition be met at ε', i.e. $VV(v_i, \varepsilon') = VV(v_{i+1}, \varepsilon')$. Note that the value of $\mathcal{L}$ at $\varepsilon = \varepsilon'$ is the same as at $\varepsilon = 0$ as the derivative of $\mathcal{L}$ with respect to ε is supposed to be zero. As a next step (which will again not change $\mathcal{L}$), change $y(v_i, \cdot)$ and $y(v_{i+1}, \cdot)$ to $\tilde{y}(v_i, c_j) = \tilde{y}(v_{i+1}, c_j) = y(v_i, c_j)\omega_i/(\omega_i + \omega_{i+1}) + y(v_{i+1}, c_j)\omega_{i+1}/(\omega_i + \omega_{i+1})$ for $j = 1, \ldots, m$. This change will not affect $\mathcal{L}$ as $\mathcal{L}$ is linear in $y(v_i, c_j)$ with slope equal to the virtual valuation (plus a term that is constant across buyer signals and therefore unaffected) and both $\tilde{v}_i$ and $\tilde{v}_{i+1}$ had the same virtual valuation. As a last step, note that – following the proof of lemma 2 – merging types v_i and v_{i+1} to $v_i\omega_i/(\omega_i + \omega_{i+1}) + v_{i+1}\omega_{i+1}/(\omega_i + \omega_{i+1})$ with probability $\tilde{\omega}_i(\varepsilon') + \tilde{\omega}_{i+1}(\varepsilon')$ will not affect EGT but relax (C), see the proof of lemma 2. Hence, the value of $\mathcal{L}$ increases due to this change. However, this contradicts that at the optimal solution $\mathcal{L}$ is maximized by the "optimal" values v_{i-1}, v_i, v_{i+1} and ω_{i-1}, ω_i, ω_{i+1} (holding all other variables at their optimal values).

If the other conditions is met at ε', i.e. $\tilde{\omega}_{i-1}(\varepsilon') = 0$, the last step of the proof is similar. If $\tilde{\omega}_{i-1}(\varepsilon') = 0$, eliminating v_{i-1} will strictly increase $\mathcal{L}$ (as v_i's incentive compatibility constraint is strictly relaxed).

Finally, consider the case where (C) does not bind. Then the merging of signals in the steps above established that there is an information structure that (i) yields the same EGT, (ii) does not violate (C) and (iii) uses less signals than the initial optimal information structure. □

Proof of lemma 7: By lemmas 6 and 5, the only other possibilities are (i) $y(v_l, c_l) = 0 = y(v_l, c_h)$ while $y(v_h, c_l) = 1 = y(v_h, c_h)$, (ii) $y(v_l, c_l) = 1 = y(v_h, c_l)$ while $y(v_l, c_h) = 0 = y(v_h, c_h)$ and (iii) $y(v_l, c_l) = 0 = y(v_h, c_h) = y(v_l, c_h)$ while $y(v_h, c_l) = 1$. In (i) costs are not decision relevant and therefore it is without loss to have only one cost signal. In (ii) valuations are not decision relevant and it is without loss to have only one valuation signal. In both cases, the optimality of a single signal would contradict lemma 1. Therefore, only case (iii) remains to be ruled out which is done next.

Suppose, contrary to the lemma, that $y(v_l, c_l) = 0 = y(v_h, c_h) = y(v_l, c_h)$ while $y(v_h, c_l) = 1$, which means that trade occurs only between the high valuation and the low cost type, note that by lemma 5 $v_l \leq c_l$ and $v_h \leq c_h$. This immediately implies that $c_l > \underline{c}$ and $v_h < \bar{v}$ by assumption 1 and therefore $c_h = \bar{c}$ and $v_l = \underline{v}$ by corollary 2. The next step is to show $v_h = \bar{c}$. By lemma 5, $y(v_h, c_h) = 0$ implies $v_h \leq c_h = \bar{c}$. If $v_h < \bar{c}$, then increasing the probability that a $\bar{c}$ type receives a c_l

signal by $\varepsilon > 0$ will improve EGT as it reduces the probability of inefficient trade. The resulting information structure is clearly feasible for $\varepsilon > 0$ sufficiently small and budget balance still holds as a fixed price mechanism can be used. Hence, $v_h = \bar{c}$ has to hold. An analogous argument establishes $c_l = \underline{v}$. Note that as a consequence there are no gains from trade between a $\underline{v}$ type receiving signal v_h and a seller of signal c_l. I will now change first the information structure and then the mechanism to achieve higher EGT thereby contradicting the optimality of the original grand mechanism. First, EGT do not change if the buyer receives a fully informative signal (while holding the seller's information structure and y fix) because of the previous observation that there are zero gains from trade between a $\underline{v}$ type receiving v_h and a seller with signal c_l. But as $\bar{v} > \bar{c} \geq c_h$, EGT can be strictly increased from there by changing the mechanism y by setting $y(v_h, c_h) = 1$ instead of $y(v_h, c_h) = 0$. Again budget balance holds as the resulting mechanism can be implemented by a fixed price mechanism with price $t = c_h$.

Therefore, $y(v_l, c_l) = 1 = y(v_h, c_h)$ which implies that trade happens unless the cost signal is high and the valuation signal is low. I will hold the mechanism, i.e. y, fixed for the remainder of the proof and first focus on the buyer showing that $v_h = \bar{v}$ in the optimal information structure. By way of contradiction suppose $v_h < \bar{v}$ and note that by corollary 2 this implies $v_l = \underline{v}$. As $v_h < \bar{v}$, some buyers with true valuation $\underline{v}$ receive the signal v_h. Consider now moving ε of these buyers to signal v_l. Put differently, the following information structures are feasible for small $\varepsilon > 0$:

$$\tilde{v}_l(\varepsilon) = \underline{v} \qquad \tilde{v}_h(\varepsilon) = \frac{\omega_h - \varepsilon - \bar{\omega}}{\omega_h - \varepsilon}\underline{v} + \frac{\bar{\omega}}{\omega_h - \varepsilon}\bar{v} \qquad \tilde{\omega}_h(\varepsilon) = \omega_h - \varepsilon \qquad \tilde{\omega}_l(\varepsilon) = 1 - \omega_h + \varepsilon$$

where $\bar{\omega}$ is the share of $\bar{v}$ in the true buyer type distribution. Note that the original information structure is obtained for $\varepsilon = 0$. Constraint (C) in the binary case (with y fixed as above) can be written as

$$\gamma_l \tilde{v}_l(\varepsilon) + (1 - \gamma_l)\tilde{\omega}_h(\varepsilon)\tilde{v}_h(\varepsilon) - \tilde{\omega}_h(\varepsilon)c_h - \gamma_l(1 - \tilde{\omega}_h(\varepsilon))c_l \geq 0.$$

The derivative of the left hand side of this condition with respect to ε is $c_h - \underline{v} + \gamma_l(\underline{v} - c_l)$ which is positive as $c_h \geq \underline{v}$ and $\underline{v} \geq c_l$ by lemmas 6 and 5. Clearly, EGT are strictly increasing in ε as well as the moved types with valuation $\underline{v}$ no longer trade inefficiently with high cost sellers. This implies that EGT are strictly higher for $\varepsilon > 0$ while (C) is not violated and thereby optimality of the original grand mechanism is contradicted. Hence, $v_h = \bar{v}$ has to hold in the optimal grand mechanism.

The proof for $c_l = \underline{c}$ in the optimal grand mechanism is analogous. □

Proof of lemma 8: See appendix D below.

Proof of proposition 3: See appendix D below.

D. Derivations binary type distribution

By lemmas 6 and 7, $y(v_h, c_h) = y(v_l, c_l) = y(v_h, c_l) = 1$ while $y(v_l, c_h) = 0$ and $v_h = \bar{v}$ while $c_l = \underline{c}$. Let $\bar{\omega}$ ($\underline{\gamma}$) be the share of high (low) types in H_B (H_S). Then the optimization problem can be formulated in terms of the variables $\omega_h \in [0, \bar{\omega}]$ and $\gamma_l \in [0, \underline{\gamma}]$ and

$$\begin{aligned} v_l &= \frac{\bar{\omega} - \omega_h}{1 - \omega_h}\bar{v} + \frac{1 - \bar{\omega}}{1 - \omega_h}\underline{v} \\ c_h &= \frac{\underline{\gamma} - \gamma_l}{1 - \gamma_l}\underline{c} + \frac{1 - \underline{\gamma}}{1 - \gamma_l}\bar{c}. \end{aligned}$$

Constraint (C) can be written as

$$BB(\omega_h, \gamma_l) = \gamma_l \frac{\bar{\omega} - \omega_h}{1 - \omega_h}\bar{v} + \gamma_l \frac{1 - \bar{\omega}}{1 - \omega_h}\underline{v} + (1 - \gamma_l)\omega_h \bar{v} - \omega_h \frac{\underline{\gamma} - \gamma_l}{1 - \gamma_l}\underline{c} - \omega_h \frac{1 - \underline{\gamma}}{1 - \gamma_l}\bar{c} - \gamma_l(1 - \omega_h)\underline{c} \geq 0.$$

The objective, EGT, equals

$$EGT(\omega_h, \gamma_l) = (\omega_h \underline{\gamma} + (\bar{\omega} - \omega_h)\gamma_l)(\bar{v} - \underline{c}) + \gamma_l(1 - \bar{\omega})(\underline{v} - \underline{c}) + \omega_h(1 - \underline{\gamma})(\bar{v} - \bar{c}).$$

As $EGT(\omega_h, \gamma_l)$ is strictly increasing in both variables, BB holds with equality if and only if $BB(\bar{\omega}, \underline{\gamma}) < 0$: If BB held with inequality, increasing either γ_l or ω_h by a sufficiently small amount would increase EGT without violating BB.

Note at this point that it is possible to normalize the problem as described in the main text: the maximizing ω_h and γ_l in the original problem equal the maximizing choices in the normalized problem in which $\bar{v}^{normal} = 1$, $\underline{v}^{normal} = \underline{v}/\bar{v}$, $\underline{c}^{normal} = \underline{c}/\bar{v}$ and $\bar{c}^{normal} = \bar{c}/\bar{v}$. First/second best EGT in the original problem equals first/second best EGT in the normalized problem times $\bar{v}$. This is true as EGT, EGT^{fb}, and BB are linear in the types $\bar{v}$, $\underline{v}$, $\bar{c}$ and $\underline{c}$.

Solving the BB condition (holding with equality) for ω_h yields[28]

$$\omega_h^{BB}(\gamma_l) = \frac{1}{2}\left(1 + \frac{\gamma_l(\bar{v}-\underline{c})}{\frac{1-\gamma}{1-\gamma_l}(\bar{c}-\underline{c}) - (1-\gamma_l)(\bar{v}-\underline{c})}\right) - \sqrt{\frac{1}{4}\left(1 + \frac{\gamma_l(\bar{v}-\underline{c})}{\frac{1-\gamma}{1-\gamma_l}(\bar{c}-\underline{c}) - (1-\gamma_l)(\bar{v}-\underline{c})}\right)^2 - \frac{\gamma_l\bar{\omega}(\bar{v}-\underline{v}) + \gamma_l(\underline{v}-\underline{c})}{\frac{1-\gamma}{1-\gamma_l}(\bar{c}-\underline{c}) - (1-\gamma_l)(\bar{v}-\underline{c})}}$$

while solving the BB condition (holding with equality) for γ_l yields

$$\gamma_l^{BB}(\omega_h) = \frac{1}{2}\left(1 + \frac{\omega_h(\bar{v}-\underline{c})}{\frac{1-\bar{\omega}}{1-\omega_h}(\bar{v}-\underline{v}) - (1-\omega_h)(\bar{v}-\underline{c})}\right) - \sqrt{\frac{1}{4}\left(1 + \frac{\omega_h(\bar{v}-\underline{c})}{\frac{1-\bar{\omega}}{1-\omega_h}(\bar{v}-\underline{v}) - (1-\omega_h)(\bar{v}-\underline{c})}\right)^2 - \frac{\omega_h(\bar{v}-\bar{c}) + \omega_h\underline{\gamma}(\bar{c}-\underline{c})}{\frac{1-\bar{\omega}}{1-\omega_h}(\bar{v}-\underline{v}) - (1-\omega_h)(\bar{v}-\underline{c})}}.$$

$\omega_h^{BB}(\gamma_l)$ can be plugged into W in order to get a one-dimensional optimization problem over $\gamma_l \in [\gamma_l^{BB}(\bar{\omega}), \underline{\gamma}]$. I numerically verified that the resulting objective function is convex in γ_l (under the assumption that $BB(\bar{\omega}, \underline{\gamma}) < 0$).[29] Consequently the solution is either

- $\gamma_l = \gamma_l^{BB}(\bar{\omega})$ and therefore $\omega_h = \bar{\omega}$ or
- $\gamma_l = \underline{\gamma}$ and therefore $\omega_h = \omega_h^{BB}(\underline{\gamma})$.

Put differently, one player receives a perfectly informative signal and the other player a noisy signal. For concreteness, the relevant values $\gamma_l^{BB}(\bar{\omega})$ and $\omega_h^{BB}(\underline{\gamma})$

[28] The second solution of the quadratic equation is above 1 – as

$$\frac{\gamma_l(\bar{v}-\underline{c})}{\frac{1-\gamma}{1-\gamma_l}(\bar{c}-\underline{c}) - (1-\gamma_l)(\bar{v}-\underline{c})} > 1$$

by $\gamma_l \leq \underline{\gamma}$ – and therefore not relevant. Note that there always exists a solution in $(0,1)$ as (C) is slack if $\omega_h = 0$.

[29] The code is available on the website of the author (`https://schottmueller.github.io/`).

are given explicitly:

$$\begin{aligned}
\gamma_l^{BB}(\bar\omega) &= \frac{1}{2}\left(1+\frac{\bar\omega(\bar v-\underline{c})}{\bar\omega\bar v-\underline{v}+(1-\bar\omega)\underline{c}}\right)\\
&\quad -\sqrt{\frac{1}{4}\left(1+\frac{\bar\omega(\bar v-\underline{c})}{\bar\omega\bar v-\underline{v}+(1-\bar\omega)\underline{c}}\right)^2-\frac{\bar\omega(\bar v-\bar c)+\bar\omega\underline{\gamma}(\bar c-\underline{c})}{\bar\omega\bar v-\underline{v}+(1-\bar\omega)\underline{c}}}\\
\omega_h^{BB}(\underline{\gamma}) &= \frac{1}{2}\left(1+\frac{\underline{\gamma}(\bar v-\underline{c})}{\bar c-\underline{\gamma}\underline{c}-(1-\underline{\gamma})\bar v}\right)\\
&\quad -\sqrt{\frac{1}{4}\left(1+\frac{\underline{\gamma}(\bar v-\underline{c})}{\bar c-\underline{\gamma}\underline{c}-(1-\underline{\gamma})\bar v}\right)^2-\frac{\underline{\gamma}\bar\omega(\bar v-\underline{v})+\underline{\gamma}(\underline{v}-\underline{c})}{\bar c-\underline{\gamma}\underline{c}-(1-\underline{\gamma})\bar v}}.
\end{aligned}$$

To determine which of the two solutions yields higher EGT it is simplest to compare for both the difference to first best EGT. As $EGT(\bar\omega,\gamma_l)$ is linear in γ_l this difference can be expressed as

$$EGT(\bar\omega,\underline{\gamma})-EGT(\bar\omega,\gamma_l^{BB}(\bar\omega))=(1-\bar\omega)(\underline{v}-\underline{c})$$
$$\left(\underline{\gamma}-\frac{1}{2}\left(1+\frac{\bar\omega(\bar v-\underline{c})}{\bar\omega\bar v-\underline{v}+(1-\bar\omega)\underline{c}}\right)+\sqrt{\frac{1}{4}\left(1+\frac{\bar\omega(\bar v-\underline{c})}{\bar\omega\bar v-\underline{v}+(1-\bar\omega)\underline{c}}\right)^2-\frac{\bar\omega(\bar v-\bar c)+\bar\omega\underline{\gamma}(\bar c-\underline{c})}{\bar\omega\bar v-\underline{v}+(1-\bar\omega)\underline{c}}}\right)$$

$$EGT(\bar\omega,\underline{\gamma})-EGT(\omega_h^{BB}(\underline{\gamma}),\underline{\gamma}))=(1-\underline{\gamma})(\bar v-\bar c)$$
$$\left(\bar\omega-\frac{1}{2}\left(1+\frac{\underline{\gamma}(\bar v-\underline{c})}{\bar c-\underline{\gamma}\underline{c}-(1-\underline{\gamma})\bar v}\right)+\sqrt{\frac{1}{4}\left(1+\frac{\underline{\gamma}(\bar v-\underline{c})}{\bar c-\underline{\gamma}\underline{c}-(1-\underline{\gamma})\bar v}\right)^2-\frac{\underline{\gamma}\bar\omega(\bar v-\underline{v})+\underline{\gamma}(\underline{v}-\underline{c})}{\bar c-\underline{\gamma}\underline{c}-(1-\underline{\gamma})\bar v}}\right).$$

Consequently, $\gamma_l=\gamma_l^{BB}(\bar\omega)$ and therefore $\omega_h=\bar\omega$ in the optimal grand mechanism if and only if

$$\begin{aligned}
&\frac{(1-\underline{\gamma})(\bar v-\bar c)}{(1-\bar\omega)(\underline{v}-\underline{c})}\Bigg[\bar\omega-\frac{1}{2}\left(1+\frac{\underline{\gamma}(\bar v-\underline{c})}{\bar c-\underline{\gamma}\underline{c}-(1-\underline{\gamma})\bar v}\right)\\
&\qquad+\sqrt{\frac{1}{4}\left(1+\frac{\underline{\gamma}(\bar v-\underline{c})}{\bar c-\underline{\gamma}\underline{c}-(1-\underline{\gamma})\bar v}\right)^2-\frac{\underline{\gamma}\bar\omega(\bar v-\underline{v})+\underline{\gamma}(\underline{v}-\underline{c})}{\bar c-\underline{\gamma}\underline{c}-(1-\underline{\gamma})\bar v}}\Bigg]\\
&\qquad\qquad\geq \underline{\gamma}-\frac{1}{2}\left(1+\frac{\bar\omega(\bar v-\underline{c})}{\bar\omega\bar v-\underline{v}+(1-\bar\omega)\underline{c}}\right)\\
&\qquad\qquad+\sqrt{\frac{1}{4}\left(1+\frac{\bar\omega(\bar v-\underline{c})}{\bar\omega\bar v-\underline{v}+(1-\bar\omega)\underline{c}}\right)^2-\frac{\bar\omega(\bar v-\bar c)+\bar\omega\underline{\gamma}(\bar c-\underline{c})}{\bar\omega\bar v-\underline{v}+(1-\bar\omega)\underline{c}}}
\end{aligned}$$

and $\gamma_l = \underline{\gamma}$ and therefore $\omega_h = \omega_h^{BB}(\underline{\gamma})$ otherwise.

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