Wages as signals of worker mobility*

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February 2023

Abstract

We analyze a model in which workers direct their search on and off the job and employer-worker match productivities are private information. Employers can commit neither to post contracts such that wages are a function of tenure nor to disregard counteroffers. In this context, potential employers who do not observe workers' productivity in their current matches use wages as a signal of workers' willingness to switch jobs. In turn, this implies that the wage contracts that employers post in the market for entry jobs — the jobs unemployed workers search for — not only direct job search but also signal future worker mobility. When the costs of creating entry jobs are sufficiently small, the unique equilibrium supports the efficient allocation under full information. When the costs of creating entry jobs are sufficiently large, the efficient equilibrium may break down because match-specific risk gives rise to a holdup problem in the market for entry jobs. Then, the unique equilibrium may fail to reveal match productivities in the market for entry jobs. The non-revealing equilibrium features wage posting — pooling wage contracts — as well as counteroffers, which eliminates the holdup problem at the cost of distorting worker mobility.

JEL classification: D82; E24; J31.

Keywords: Competitive search equilibrium, private information, limited commitment, wage posting, counteroffers, worker mobility.

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1 Introduction

Economists increasingly recognize that worker mobility plays a key role in the allocation of labor in decentralized markets. Matching the right workers to the right jobs is a complex process and turnover provides a potential solution to otherwise large misallocation problems.[1] In this paper, we analyze a competitive search model of turnover. Like standard competitive search models, we assume that employers post contracts that are legally enforceable and direct search (e.g., Menzio and Shi, 2011). Unlike standard competitive search models, we assume that worker-employer match productivity is private information and that there is limited commitment.

We assume that commitment is limited in two important respects. First, firms cannot commit to disregard outside offers. Second, they cannot commit to post bilaterally efficient contracts. We allow firms to commit to contracts under which the wage is conditional on match productivity, which can take only one of two values. Thus, well matched and poorly matched workers may earn different wages. However, we assume that firms cannot commit to contracts under which the wage is a function of tenure. In this context, we show that the efficiency of turnover depends on the informational content of wage contracts in the market for entry jobs — which are the jobs that unemployed workers search for — which not only direct the job search of unemployed workers but also signal future worker mobility. The central issue is whether equilibrium wages reveal whether a worker is willing to move or is instead seeking to elicit a retention offer.

Our main results concern the existence and properties of competitive search equilibria. First, if the costs of creating entry jobs are small enough, there exists a revealing equilibrium with positive job-to-job quits that exhibits efficient turnover. This equilibrium simultaneously solves both the problem of directing the search of unemployed workers and the problem of signaling the potential mobility of employed workers. Efficient turnover requires that firms pay poorly matched workers their marginal productivity. This allows for maximization of the match surplus as it induces poorly matched workers to quit exactly when it is efficient to do so. Allocating the full match surplus to poorly matched workers, however, requires that firms recover the full costs of creating entry jobs from well matched workers, while poorly matched workers bear none of these costs. Workers who are well matched cannot avoid these

[1]Mortensen (1978) and Jovanovic (1979) are two early papers.

costs via search on the job because wages reveal productivity realizations to poaching firms, which renders well matched workers immobile.

The efficient equilibrium may break down when the costs of creating entry jobs are large enough. The reason is that match-specific risk gives rise to a holdup problem when contracts cannot be made contingent on tenure. This is because up front job creation costs are an *ex ante* investment on the part of firms. Workers, however, can credibly reject matches that deliver less than the value of unemployment, and can make this decision after observing the productivity realization. This implies that the delivery of positive *expected* surplus is not sufficient to ensure the creation of a job. The fact that efficiency requires that firms pay low productivity workers their marginal product implies that sufficiently high job creation costs may reduce the value of high productivity jobs to workers to a level below the value of unemployment, so workers will reject such offers. Hence, the efficient equilibrium solves the holdup problem, but only when job creation costs are sufficiently low. The existence of the efficient equilibrium depends on the ability of firms to offload all job creation costs to well matched workers. This implies that the efficiency of competitive search cannot be understood independently of the division of the match surplus.[2]

Next we show that if the costs of creating entry jobs are sufficiently large, there may exist an equilibrium that is both non-revealing and exhibits inefficient worker mobility. In a non-revealing equilibrium, firms post contracts under which wages are not contingent on productivity realizations, and so all entry jobs pay identical wages. By spreading the costs of creating entry jobs across matches, the non-revealing equilibrium solves the holdup problem in the market for entry jobs, but at the expense of distorting the market for non-entry jobs. Because entry jobs pay wages that fail to reveal match productivity, potential poaching firms cannot distinguish poorly matched from well matched workers. This creates adverse selection in the market for non-entry jobs: well matched workers have an incentive to search on the job, but solely to elicit a retention offer from their current employers rather than to change jobs. This congestion reduces the return to job creation in the market for non-entry jobs and, as a result, generates inefficient worker mobility.

That pooling wage contracts are an equilibrium outcome does not mean that

[2]Mortensen (1978) argues that turnover is independent of the division of the surplus if bilaterally efficient contracts are available. Stevens (2004) shows the first best contract specifies a hiring fee that workers pay on being hired. The worker is then paid marginal product while employed, which ensures any subsequent quit decision is jointly efficient.

the non-revealing equilibrium relies on unreasonable off-equilibrium beliefs. To rule out equilibria that rely on unduly pessimistic off-equilibrium beliefs, we propose a refinement that extends the one proposed by Guerrieri et al. (2010) in order to accommodate the possibility of non-revealing wages. This possibility arises in our setting because wage contracts in the market for entry jobs play both an informational and an allocative role. Our refinement restricts off-equilibrium beliefs in two distinct ways, in the spirit of the Intuitive Criterion and the D1 criterion, respectively (Cho and Kreps, 1987). First, they must place zero weight on types that can never gain from deviating from a fixed equilibrium outcome. Second, they must be supported on types that have the most to gain from deviating from a fixed equilibrium.[3]

The problem with pooling wage contracts is that they are the incorrect tool to retain workers unless the distribution of match productivities is non-degenerate. Accordingly, one might expect that choosing to commit to contracts that make wages contingent on productivity must always be better than committing to pooling contracts. However, this disregards the possibility of holdups and the (endogenous) informational content of wages. Once the signaling role of wages is taken into account, we show that posting pooling contracts in the market for entry jobs and subsequently responding to outside offers may in fact be an equilibrium outcome. This provides a novel perspective on both posted wages and counteroffers: they can be understood as complementary parts of a second-best solution to holdup problems.

Empirical evidence suggests that wage posting and counteroffers play a role in some markets. For example, Hall and Krueger (2012), in a survey of employed workers, document that nearly one third of workers knew exactly how much the job would pay at the time they were first interviewed. This finding is suggestive of wage posting. In a similar vein, Barron, Berger and Black (2006) find evidence from a survey of employers that suggests that employers are willing to match outside offers for roughly 41% of workers. We are not aware of any empirical work that examines the possible link between posted wages and counteroffers. From the perspective of our model, however, the two observations can be better understood jointly, rather than separately: by posting wages, firms anticipate that they will respond to outside offers. Our analysis illustrates how these observations arise from a common commitment

[3]Chang (2018) proposes a similar refinement in order to analyze separating and pooling competitive search equilibria in a model with multidimensional asymmetric information. Kurlat and Scheuer (2021) propose a refinement in the same spirit and apply it to a signaling model in which firms have heterogeneous information about workers' types.

failure, the source of which is twofold: firms can commit neither to post bilaterally efficient contracts nor to disregard counteroffers *ex post*.

1.1 Related literature

Our core analytical framework builds on previous work on competitive search equilibria with adverse selection (Guerrieri et al. 2010) to address turnover under incomplete information about workers' match productivity.[4] Our specification of search on the job combines elements of directed search that are standard in competitive search models (Menzio and Shi 2011) and elements of bargaining that are standard in random matching models (Cahuc, Postel-Vinay and Robin 2006). In particular, we allow employers to counter outside offers, which, combined with the fact that search on the job is directed, plays a crucial role in generating an adverse selection problem. The possibility of counteroffers renders our framework remarkably tractable by limiting the scope for job quits, thereby eliminating the sorts of wage ladders found in Delacroix and Shi (2006). This enables us to characterize pooling equilibria, which are neither block-recursive nor constrained efficient.[5]

Our paper is related to an existing literature that studies the signaling role of prices in directed search equilibria. For example, Delacroix and Shi (2013) show that competitive search equilibria can be inefficient when sellers post prices that not only direct buyers' search, but also signal product quality.[6] By contrast, we focus on job turnover and highlight an adverse selection problem, neither of which are present in Delacroix and Shi (2013). In our setting, employers' decisions to create jobs and workers' job search decisions take place *before* they posses any private information. Furthermore, match productivity is known to both parties *before* matching takes place and such information is contractible.

The importance of asymmetric information between employers is widely recog-

[4]Faig and Jerez (2005), Guerrieri (2008) and Moen and Rosen (2011) analyze competitive search equilibria in models with match-specific private information. Guerrieri and Shimer (2014) and Chang (2018) analyze competitive search equilibria with adverse selection in asset markets. Wright et al. (2021) provide an insightful survey of competitive search applications in economics.

[5]Shi (2009) analyzes block-recursive competitive search equilibria with search on the job.

[6]Menzio (2007) and Kim and Kircher (2015) examine the signaling role of prices when firms and workers can engage in pre-match communication, or cheap talk. Other work emphasizes the signaling role of promotions (Waldman, 1984), price announcements that differ from actual prices (Lester et al., 2017), or types of contracts (Stacey, 2016).

nized.[7] Waldman (1984) and Greenwald (1986) are two early examples. More recently, Carrillo-Tudela and Kaas (2015) consider on-the-job search with adverse selection under random matching, but they assume that wage contracts are not renegotiated when workers receive outside offers. By contrast, our focus is on the interaction between directed search and *ex post* renegotiation.

Our paper is also related to the literature that examines the efficiency properties of both wage posting and competitive search. It is well known that if employers can commit to general enough contracts, competitive search equilibrium is able to internalize a variety of externalities, including those that play an important role in our setting. For example, Acemoglu and Shimer (1999) show that competitive search solves holdup problems associated with pre-match investments. The interest in the efficiency properties of competitive search extends to the literature that studies on-the-job search and worker mobility. Burdett and Coles (2003) and Shi (2009) argue that on-the-job search induces firms to backload wages, thus making wages increase and quit rates decrease with tenure. Both papers, however, assume that the productivity of a match is public information and that a firm does not respond to outside offers. These restrictions hold in some markets, but they are clearly violated in others. Our paper seeks to understand the factors that determine when equilibrium matching offers do and do not arise.

The search literature tends to view commitment to wage posting as an efficient contract that exploits the role of directed search in attracting workers to the right jobs. Under this view, ex post bargaining tends to be seen as an alternative mechanism that is used to make ex post pay contingent on productivity. For example, Michelacci and Suarez (2006) argue that posting dominates bargaining when the allocative inefficiency of ex post bargaining is large enough that the benefits of posting wages in order to direct job search are correspondingly large. Conversely, when the benefits of directing job search are relatively small and the benefits from bargaining in terms of making pay responsive to productivity are large enough, bargaining dominates posting. We provide a very different perspective. Productivity-based pay requires commitment to contingent wages and it may be an efficient contract in terms of attracting and retaining workers. Posted wages are an inefficient way to solve holdup problems. Ex post bargaining and posted wages are not substitutes, but rather complementary tools that employers can use to attract and retain workers.

[7] Kahn (2013) is an interesting empirical study.

Our paper is related to a literature that examines the existence of counteroffers. Golan (2005) argues that counteroffers help achieve efficient job assignment in a model where there is no turnover. By contrast, we focus on worker mobility but disregard the issue of job assignment. Yamaguchi (2010) considers alternative sources of wage growth in the context of a standard random matching model with counteroffers. Postel-Vinay and Robin (2004) consider the incentives of employers to commit to non-matching of outside wage offers within a random matching model. By contrast, our focus is on the interaction between directed search and *ex post* renegotiation.

Estimating the returns to job tenure versus experience is notoriously difficult because productivity is unobservable, job turnover is endogenous, and there is enormous unobserved heterogeneity across workers, firms and matches (Abowd et al, 2006; Buchinsky et al., 2010). Yamaguchi (2010) and Bagger et al. (2014) interpret wage growth data through the lens of a random matching model, and emphasize the influence of on-the-job search on within-firm and between-firm wage growth. In particular, within-firm wage growth can be driven by experience, and by the exogenous arrival of counteroffers. Our analysis highlights the heterogeneity of equilibrium wage contracts across labor markets, which implies heterogeneous, and endogenous, observations of within-wage and between-firm wage growth. For example, in the spirit of Burdett and Coles (2003) and Shi (2009), firms in some markets may be able to commit to backload wages and offer separating wage contracts. By contrast, in the spirit of a non-revealing equilibrium of our model, firms in other markets may post pooling wage contracts and respond to future outside offers. Both of these strategies serve as a mechanism to backload wages, but their implications for job turnover, and wages, are different. Yet, in the spirit of the revealing equilibria of our model, other firms may offer separating contracts, but not be able to backload wages, in which case turnover may be efficient in some markets, but not in others.

2 The model

2.1 Environment

Time is discrete. There are *ex ante* homogeneous workers and *ex ante* homogeneous employers. All agents are risk neutral and discount the future at a rate $r > 0$. There is a unit measure of workers who are either employed or unemployed. An unemployed worker searches for a job and receives a flow benefit from unemployment equal to $b \geq 0$. All worker-employer matches produce y_h units of output with probability $\alpha \in (0,1)$ and y_l units of output with probability $1-\alpha$, where $b < y_l < y_h$. Subsequently, a separation shock makes an employed worker become unemployed with probability $\delta > 0$. An employed worker can search for a different job.

It will be convenient to assume that firms can post a vacancy for one of two types of jobs, indexed by $j = u, e$. We label them *entry* jobs, which cater to the unemployed ($j = u$), and *non-entry* jobs, which cater to employed workers ($j = e$). There is free entry of jobs and employers incur a cost $k_j > 0$ in order to post a vacancy. We allow the two types of vacancies to have different creation costs. For example, hiring employed versus unemployed workers may involve different screening costs. For simplicity, we assume that only unemployed workers can fill entry jobs and only employed workers can fill non-entry jobs.[8]

Each employer can post any feasible job, where a job x is defined below as a wage contract together with a job type. A submarket is defined by the job x posted in that submarket and the corresponding queue length q, which is defined as the ratio of workers searching for x to employers posting x. We refer to the submarket offering job x simply as "submarket x". Workers can direct their search across submarkets. Employed and unemployed workers have the same search intensity. Meetings are bilateral, so each employer meets at most one worker and vice versa. Workers who search in submarket x with queue length q meet an employer with probability $f(q)$ and employers in the same market meet a worker with probability $qf(q)$. We assume that $f(q)$ is twice differentiable, strictly decreasing and convex, with $f(0) = 1$ and $f(\infty) = 0$. We also assume that $qf(q)$ is strictly increasing and concave, approaching 1 as q converges to ∞. These assumptions ensure that the job-finding elasticity, given by $\eta(q) = -qf'(q)/f(q)$, is such that $0 = \eta(0) < \eta(1) \leq 1$, with $\eta'(q) > 0$. For simplicity, we also assume that $\eta(q)$ is concave, with $\eta(\infty) = 1$.[9]

When a worker (employed or not) and a potential employer meet, the latter observes the worker's labor market status and, if currently employed, her wage and

[8] One interpretation is that employers do not want to hire unemployed workers for non-entry jobs, because they lack the necessary experience, while they do not want to hire employed workers for entry-level positions, because overqualified workers are more likely to become dissatisfied with those jobs. It would be natural to assume that entry and non-entry jobs have different production technologies. For simplicity, however, we abstract away from such differences.

[9] An example of a meeting technology that satisfies these assumptions is $M(u,v) = uv/(u+v)$.

her job type. Then, the productivity of the potential match is drawn randomly and observed by both parties to the match. The match productivity, however, is not observed by outsiders to the match. For example, the current match productivity of an employed worker is not observed by potential new employers. Vice versa, the new match productivity at the poaching firm is not observed by the incumbent employer. Subsequently, employers decide whether or not to make formal offers. We assume that all employers make take-it-or-leave-it offers, that incumbent employers can counter outside offers, and that wages can only be renegotiated by mutual agreement. Finally, workers decide whether or not to accept any offers. To break ties, we assume that workers reject any outside offers that are matched by the incumbent employer. New matches start producing in the next period.

Contracts are specified in terms of fixed, though possibly productivity-contingent wages. The hiring and retention policies are not included as part of the contract, but firms and workers do take into account that fixed wages can be renegotiated by mutual agreement. A worker who gets a credible outside offer can choose to terminate the current wage contract and agree to a "new" contract with a different wage, which lasts until another outside offer arrives. If the outside offer is credible and if a better counteroffer is feasible, the incumbent employer then commits to make a new wage contract. Retention policies depend on history through the worker's current wage, which is a sufficient statistic for the payoff-relevant history of the current contract.

A job $x = (j, w_l, w_h)$ specifies the job type, $j \in \{u, e\}$, and a wage contract, $(w_l, w_h) \in [0, y_l] \times [0, y_h]$, both of which are observable. The job type (entry jobs versus non-entry jobs) conveys the necessary information to attract the right workers (employed versus unemployed workers).[10] The wage contract consists of the pair of fixed wages, w_l and w_h, to be paid when the realizations of match productivity in the new match are $y' = y_l$ and $y' = y_h$, respectively.

Neither workers nor employers can be forced to participate in a match before observing match productivity. That is, employers cannot commit to make a formal job offer and workers cannot commit to accept such an offer before observing the realized match productivity. In this sense, matches are pure inspection goods, rather than experience goods. These assumptions are made to highlight the role of incomplete information about workers' outside options.

[10] Marinescu and Wolthoff (2020) use data from CareerBuilder.com to argue that the informational content of job titles refers to the hierarchy, level of experience, and specialization of different jobs.

The adverse selection problem that arises from the combination of limited commitment and asymmetric information will play an important role in our analysis. Since match productivity is unobserved by third parties, a worker's current labor productivity is private information to the worker *vis-a-vis* potential new employers. Consequently, poaching offers cannot discriminate between workers with different outside options, unless (equilibrium) wages reveal match productivity. Since workers are unable to commit not to search on the job and employers are unable to commit to not countering outside offers, workers in high productivity matches have an incentive to seek outside offers solely to elicit retention offers from their current employers.

We assume that employers face a small cost of making a credible offer, so they will never make offers that they know will be rejected with certainty. We also assume that employed workers face a small search cost, so they will never search on the job unless they anticipate a profitable job offer will be forthcoming with positive probability. For simplicity, we assume these costs are negligible and so are not explicit about them.

2.2 Competitive search equilibrium

Let $s \in S$ denote a worker's payoff-relevant state. By convention, unemployed workers are associated with the state s_u. For all $s \neq s_u$, we let $s \equiv (w, y)$, where $w \in [0, y_h]$ denotes the worker's current wage and $y \in \{y_l, y_h\}$ denotes current match productivity. Thus, the feasible state space is given by $S = \{s_u\} \cup S_e$, where $S_e = [0, y_h] \times \{y_l, y_h\}$. To minimize clutter, we do not include the type j of the job an employed worker has in her payoff-relevant state. It will become clear that the worker's job type conveys no payoff-relevant information to potential employers, conditional on the worker's wage.

A stationary competitive search equilibrium (Moen, 1997) specifies a mapping Q from feasible jobs to submarket queues. Workers direct their search across all feasible jobs, taking as given the submarket queue length $Q(x)$ for all $x \in X$, where X is the set of feasible jobs. Workers' decisions must be optimal at any information set. This information set includes the worker's own state $s \in S$ as well as the distribution of workers across states, that is, the aggregate state of the economy $\psi : S \to [0, 1]$, where $\psi(s)$ is the proportion of state-s workers in the economy. It will become clear that competitive search equilibria need not be *block recursive*. That is, the agents' value functions and therefore equilibrium strategies may be a function of the aggregate state. However, in order to minimize clutter, we are not explicit about the potential dependence of the agents' value functions on the aggregate state ψ. We restrict attention to stationary equilibria throughout the paper.

For all $s \in S$, the value function of a worker in state s, denoted by $V(s)$, satisfies the following:

$$V(s) = w + \frac{1}{1+r}\left[\delta V(s_u) + (1-\delta)\left(V(s) + \max_{x \in X \cup \{x_\varnothing\}} U(s, x, Q(x))\right)\right], \quad (1)$$

where $x = x_\varnothing$ denotes the choice of not searching and where we use the convention that $w = b$ for unemployed workers. For employed workers, a match is destroyed with probability δ, in which case the worker becomes unemployed. The term $U(s, x, Q(x))$ denotes the expected surplus to a worker with current state s from searching in submarket x, with associated queue length $Q(x)$, evaluated next period. Formally,

$$U(s, x, Q(x)) = f(Q(x))\, \mathbb{E}_{y'}\left[g_h(x, w, s_o) \max\Big(0, V(s_o) - V(s)\Big)\right],$$

if $s = s_u$ and

$$U(s, x, Q(x)) = f(Q(x))\, \mathbb{E}_{y'}\left[g_h(x, w, s_o) \max\Big(0, V(s_o) - V(s), V(s_c) - V(s)\Big)\right],$$

if $s \neq s_u$, where $\mathbb{E}_{y'}$ denotes the expectation taken with respect to the random productivity draw y' and $g_h(x, w, s_o)$ denotes the hiring policy of the firm.

To understand these expressions, note that when searching for a job $x = (j, w_l', w_h')$, a worker meets an employer with probability $f(Q(x))$ in which case y' is realized and the wage offer w' is made, where $w' = w_l'$ if $y' = y_l$ and $w' = w_h'$ if $y' = y_h$. If the offer is accepted, then the worker's state becomes $s_o = (w', y')$. Workers with current state s reject any offer w' such that $V(s) > V(s_o)$. If $V(s) < V(s_o)$, an unemployed worker accepts the offer and her state becomes s_o, while an employed worker may be able to elicit a counteroffer from her current employer. When a counteroffer w_c is made, the worker decides whether to accept the outside offer w', in which case she her state becomes $s_o = (w', y')$, or to accept the counteroffer, in which case her state becomes $s_c = (w_c, y)$. Workers searching in submarket x anticipate the hiring policy $g_h(x, w, s_o)$ and the retention policy g_r such that $w_c = g_r(s, x, w')$. The employers' policies g_h and g_r are specified later.

A worker's optimal search policy is given by

$$g_x(s) \in \arg\max_{x \in X \cup \{x_\varnothing\}} U(s, x, Q(x)). \tag{2}$$

We now turn our attention to the workers' decision about whether or to not accept an offer. For simplicity, we restrict attention to pure acceptance policies and let g_a be such that

$$g_a(s, s_o, w_c) \in \begin{cases} \arg\max_{a \in \{0,1\}} \{aV(s_o) + (1-a)V(s)\} & \text{if} \quad s = s_u \\ \arg\max_{a \in \{0,1\}} \{aV(s_o) + (1-a)\max\{V(s), V(s_c)\}\} & \text{if} \quad s \neq s_u \end{cases} \tag{3}$$

for all $s \in S$, $s_o \in S_e$ and $w_c \in [0, y_h]$. For an employed worker in state $s = (w, y)$, $g_a(s, s_o, w_c) = 1$ if she accepts an offer from a new employer at the wage w', given that w_c is her current employer's counteroffer, with $s_o = (w', y')$. If the worker accepts the retention offer, then $g_a(s, s_o, w_c) = 0$ and her state becomes $s_c = (w_c, y)$. To break ties, we assume that the worker stays with her current employer and $g_a(s, s_o, w_c) = 0$ if $w_c = w'$. That is, matching outside offers is sufficient to retain workers.

It is easy to verify that for all $s = (w, y) \in S_e$ the present value of an ongoing match to the employer, denoted by $J_f(s)$, solves

$$\begin{aligned} \frac{J_f(s)}{1+r} &= \frac{y - w}{r + \delta + (1-\delta) f(Q(g_x(s))) \mathbb{E}_{y'}\{g_h(x, w, s_o)\}} \\ &+ \frac{(1-\delta) f(Q(g_x(s)))}{r + \delta + (1-\delta) f(Q(g_x(s))) \mathbb{E}_{y'}\{g_h(x, w, s_o)\}} \\ &\times \ \mathbb{E}_{y'}\left\{g_h(x, w, s_o) \max_{w_c}\left\{(1 - g_a(s, s_o, w_c)) \frac{J_f(s_c)}{1+r}\right\}\right\} \end{aligned} \tag{4}$$

subject to $w_c \geq w$,

where $s_o = (w', y')$ and $s_c = (w_c, y)$. The denominator on the right side reflects the three sources of discounting: the discount rate, the exogenous probability of job destruction and the probability that the worker receives an outside offer from a poaching firm. The incumbent employer keeps the worker if the counteroffer w_c is accepted, in which case the value of the match becomes $J_f(s_c)$.

The maximization problem in (4) has been written as if the incumbent employer knows the match realization y' associated with an outside offer. To see why, note that the employer needs to anticipate the worker's search policy (g_x), her acceptance policy (g_a) and the hiring policy of the worker's potential new employer (g_h). If $g_x(s) = x_\varnothing$, then the worker does not search on the job and $Q(x_\varnothing) = \infty$. If $g_x(s) \neq x_\varnothing$, then the worker searches in some submarket $x = (j, w'_l, w'_h)$. The employer anticipates that she searches in the market for non-entry jobs, that is, $j = e$, and also that when she meets a potential employer, she will get an outside offer w', where $w' = w'_l$ if $y' = y_l$ and $w' = w'_h$ if $y' = y_h$. Furthermore, the incumbent employer understands that all credible offers in the market for non-entry jobs must be such that $w'_h > y_l \geq w'_l$. This is because, via the hiring policy, outside offers are only made after the potential employer observes the productivity of the new match and because the incumbent employer can retain the worker by matching the outside offer. Hence, credible offers in the market for non-entry jobs always reveal the productivity realization in the new match, y'. In effect wage counteroffers are a function of y'. We have used this fact to write the max operator inside the expectation operator in the above equation, even though the incumbent firm does not observe the realization y' directly.

Let $g_r(s, x, w')$ be a solution to problem (4), for $s = (w, y) \in S_e$, $x = (j, w'_l, w'_h) \in X$ and $w' \in \{w'_l, w'_h\}$, where $g_r(s, x, w')$ specifies the best response w_c in order to retain a state-s worker who receives an offer w' after searching in submarket x.

Given $Q(x)$, workers searching for x do not need to account for the composition of workers in that submarket. By contrast, employers posting x need to anticipate not only the likelihood of meeting a worker, given by $Q(x) f(Q(x))$, but also the composition of the pool of workers searching for that contract. We let $\mu(\cdot|x)$ denote a probability distribution on S, for each $x \in X$. An employer posting x meets a worker with probability $Q(x) f(Q(x))$, in which case the expected surplus to the employer is given by $\mathbb{E}_s J(s, x)$, where $J(s, x)$ is the expected value of the employer's surplus conditional on meeting a state-s applicant and $\mathbb{E}_s$ is taken with respect to $\mu(\cdot|x)$. Thus, an employer is willing to post $x = (j, w'_l, w'_h)$ only if

$$k_j \leq Q(x) f(Q(x)) \mathbb{E}_s \{J(s, x) | x\},$$

where k_j is the cost of creating a type-j job, for $j \in \{u, e\}$, and where, for any

$s = (w, y)$ and any $x = (j, w'_l, w'_h)$,

$$J(s,x) = \mathbb{E}_{y'}\left\{\max_{h\in\{0,1\}}\left\{hg_a(s,s_o,g_r(s,x,w'))\frac{J_f(s_o)}{1+r}\right\}\right\}, \tag{5}$$

where $J_f(s_o)$ satisfies equation (4) and $s_o = (w', y')$, with $w' = w'_l$ if $y' = y_l$ and $w' = w'_h$ if $y' = y_h$. As shown in equation (5), potential employers anticipate the current acceptance policies of the workers they attract (g_a) and the retention policies of their current employers (g_r). In order to minimize clutter, we do not include these policies explicitly as arguments in the value function J.

Let $g_h(x, w, s_o)$ denote a solution to the problem in (5), for $s \neq s_u$ and write $g_h(x, b, s_o)$ for $s = s_u$, by convention. By assumption, employers posting entry jobs never make offers to employed workers, that is, $g_h((u, w'_l, w'_h), w, s_o) = 0$ if $w \neq b$. Similarly, employers posting non-entry jobs never make offers to unemployed workers, that is, $g_h((e, w'_l, w'_h), w, s_o) = 0$ if $w = b$. Recall, however, that while firms post contracts that direct search, hiring (and acceptance) decisions are made after meetings take place. Such limited commitment plays an important role. For example, our assumption that poaching firms never make offers that will be rejected with certainty means that firms sometimes meet with workers who they choose to not hire.

Definition 1 *A stationary equilibrium $\mathcal{E} = (X^*, S^*, V, J, g_x, g_a, g_h, g_r, Q, \mu, \psi)$ consists of a set of posted jobs $X^* \subseteq X$, a set of workers' states $S^* \subseteq S$, value functions $V : S \to R_+$ and $J : S \times X \to R_+$, policy functions $g_x : S \to X \cup \{x_\varnothing\}$, $g_a : S \times S_e \times [0, y_h] \to \{0, 1\}$, $g_h : X \times [0, y_h] \times S \to \{0, 1\}$ and $g_r : S_e \times X \times [0, y_h] \to [0, y_h]$, a function $Q : X \to R_+$, a distribution $\mu : S \times X \to [0, 1]$ and a distribution $\psi : S \to [0, 1]$, such that:*
(A) Workers' optimal behavior: V satisfies (1); g_x satisfies (2); g_a satisfies (3).
(B) Profit maximization: g_h, g_r and J solve (4) and (5). Moreover, for any $x = (j, w_l, w_h) \in X$, $Q(x) f(Q(x)) \int_S J(s,x)\, d\mu(s|x) \leq k_j$, with equality if $x \in X^*$.
(C) Consistent beliefs: For any $x \in X^*$,

$$\mu(s|x) = \frac{\psi(s)\,\mathbb{I}_x(g_x(s))}{\int_S \mathbb{I}_x(g_x(s))\, d\psi(s)}, \text{ with } \int_S \mathbb{I}_x(g_x(s))\, d\psi(s) > 0,$$

for all $s \in S$, where $\mathbb{I}_x(g_x(s)) = 1$ if $g_x(s) = x$ and $\mathbb{I}_x(g_x(s)) = 0$ if $g_x(s) \neq x$.

(D) Consistent allocations: for every $x \in X^*$, the mass of workers visiting submarket x divided by the mass of employers posting x is equal to $Q(x)$.
(E) Stationary distribution of workers: $\psi(s) = 0$, for any $s \notin S^*$;

$$\psi(s) = \int_{S^*} \Pr\left(s_{t+1} = s \mid s_t = s_0\right) d\psi(s_0),$$

for any $s \in S^*$, where $S^* = \{s \in S : \psi(s) > 0\}$ and where $\Pr\left(s_{t+1} = s \mid s_t = s_0\right)$ denotes the transition probability from state $s_0 \in S^*$ to state $s \in S^*$.

Part (A) of Definition 1 ensures that workers' search and acceptance policies are optimal for all states, taking as given the market queue length for all jobs. Part (B) ensures that employers' posting behavior and their subsequent retention policies are optimal, and employers posting equilibrium contracts make zero profits. Part (C) ensures that employers' beliefs are consistent with the workers' equilibrium search policies through Bayes' rule. It ensures that any contract that is posted in equilibrium attracts a positive mass of workers and that the distribution of workers searching for any equilibrium contract is exactly what the employers posting those contracts expect. Part (D) ensures the correct market clearing queue. Part (E) characterizes the stationary distribution of workers. The transition probabilities follow from the objects specified in $\mathcal{E}$. Without restrictions on the particular structure of a given equilibrium $\mathcal{E}$, a full characterization of the transition probabilities is straightforward but cumbersome and so it is not included in the definition. We refer to the pair (X^*, ψ), where S^* is the support of ψ, as an *equilibrium allocation.*

Below we show that there may be a separating equilibrium, which is block recursive, and a pooling equilibrium, which is not. Thus, in the separating equilibrium, the policy functions can be characterized without knowledge of the distribution of workers. In the pooling equilibrium, by contrast, the policy functions depend on the distribution of workers. Such a dependence may complicate greatly the characterization of an equilibrium in a general setting. However, it is worth noting up front that this is not the case in the present context. The reason is that the pooling equilibrium is such that the policy functions depend on the distribution of workers only through the exogenous distribution of match productivity. In particular, it will become clear that when a job $x_u \equiv \left(u, w_u^l, w_u^h\right)$ such that $w_u^l = w_u^h = w$ is posted in the market for entry jobs, the relevant policy functions depend on the distribution of workers ψ only through $\psi\left((w, y_l)\right) / \psi\left((w, y_h)\right) = (1-\alpha)/\alpha$. This makes pooling equilibria just as tractable as separating equilibria in our setting.

2.3 Equilibrium refinement

The definition of a competitive search equilibrium allows for more or less arbitrary off-equilibrium beliefs and so it allows for many equilibria, each of which is supported by particular beliefs in the markets where no trade takes place. The issue is that some contracts may not be traded because employers fear they would attract only undesirable types of workers. If workers expect the labor market queue associated with those contracts to be sufficiently high then those contracts would in fact not attract any workers and so the employers' pessimistic beliefs are never contradicted. To address this issue, we propose the following equilibrium refinement.

Definition 2 *An equilibrium $\mathcal{E} = (X^*, S^*, V, J, g_x, g_a, g_h, g_r, Q, \mu, \psi)$ is a refined equilibrium if:*

(i) for any $x \notin X^*$, $q \in R_+ \cup \{\infty\}$ and $s \in S$: $\mu(s|x) = 0$ if $U(s,x,q) < U(s, g_x(s), Q(g_x(s)))$;

(ii) there does not exist a job $x \notin X^*$, queue $q \in R_+ \cup \{\infty\}$ and beliefs μ' such that $qf(q) \int_S J(s,x)\, d\mu'(s|x) > k_j$, where μ' satisfies (i) and where, for any $s = (w, y_h)$ such that $U(s,x,q) \geq U(s, g_x(s), Q(g_x(s)))$:

$$\mu'((w,y_h)|x) = \begin{cases} 0 & \text{if} \quad U((w,y_h),x,q) = 0 \\ \left(\frac{\alpha}{1-\alpha}\right)\mu'((w,y_l)|x) & \text{if} \quad U((w,y_h),x,q) > 0. \end{cases}$$

An equilibrium is a refined equilibrium if there is no labor market queue such that a firm posting a deviating job can make positive profits and attract solely workers for whom this action is not equilibrium dominated. Part (i) is in the spirit of the Intuitive Criterion proposed in Cho and Kreps (1987), in that it requires off-equilibrium beliefs to place zero weight on types that can never gain from deviating from a fixed equilibrium outcome. Part (ii) is in the spirit of the D1 criterion (Cho and Kreps, 1987) in that it requires that off-equilibrium beliefs be supported on types that have the most to gain from deviating from a fixed equilibrium. Specifically, our refinement builds on the concepts proposed in Gale (1992, 1996) and Guerrieri et al. (2010), by requiring that deviating firms anticipate that an off-equilibrium contract would attract those workers who are willing to endure the highest labor market queue. Our refinement is a natural extension to allow for both separating and pooling equilibria.

Part (i) of the refinement implies that employers posting an off–equilibrium job x must believe that the only workers the job would ever attract must be indifferent between x and their preferred equilibrium job. For a fixed equilibrium allocation, this requirement pins down the queue $Q(x)$, for all $x \notin X^*$. Part (ii) plays a role because the employers' arbitrary pessimism could render any *ad hoc* submarket inactive, in which case the employers' unduly pessimistic beliefs about the composition of workers are never contradicted. It requires that employers be reasonably optimistic in order to ensure they believe that each submarket attracts the type that is in fact willing to endure the highest labor market queue.[11] This requirement has bite in both the market for entry jobs and the market for non-entry jobs.

To see why, consider first a deviating job in the market for non-entry jobs. Because wage contracts are observable, a potential employer who meets a worker knows whether the worker's current wage w is associated with a pooling or a separating wage contract. If it is a separating contract, then the employer will never make a job offer to a worker whose current state is (w, y_h). Thus, $U((w, y_h), x, q) = 0$, in which case Part (ii) rules out equilibria that are supported by off-equilibrium beliefs such that $\mu((w, y_h)|x) > 0$. In those unrefined equilibria, employers are willing to enter submarket x only if $Q(x)$ is high, while workers are reluctant to search in that submarket because $Q(x)$ is high. However, under the reasonable belief that $\mu((w, y_h)|x) = 0$, there is a queue $q < Q(x)$ such that the job x would attract workers and be profitable to the firm. Hence, those equilibria do not survive our refinement. If it is a pooling contract, then the employer will make an offer to any worker they meet. Thus, $U((w, y_h), x, q) > 0$. In this case, Part (ii) rules out equilibria that are supported by off-equilibrium beliefs such that $\mu((w, y_l)|x) / \mu((w, y_h)|x) < (1 - \alpha)/\alpha$. In those unrefined equilibria, there is a queue $q < Q(x)$ such that employers posting the deviating job x would in fact attract workers and make profits, if only they believed that $\mu((w, y_l)|x) / \mu((w, y_h)|x) \geq (1 - \alpha)/\alpha$.

Now consider the market for entry jobs. Employers posting a deviating job x understand that they will only meet unemployed workers, who are identical. However, the value of entering submarket x to the worker depends on the value of future search on the job, which in turn depends on the employers' beliefs in submarkets

[11] In this sense our refinement is in the same spirit as the one in Dubey and Geanakoplos (2002).

where non-entry jobs are traded. If x involves a separating contract, our refinement implies that unemployed workers anticipate that they will search on the job if they are poorly matched but not otherwise. If x involves a pooling contract, our refinement implies that unemployed workers anticipate that they will search on the job regardless of match quality. In both cases, the equilibrium refinement rules out equilibria in which a contract fails to attract unemployed workers simply because they have unduly pessimistic beliefs about the returns from (future) on-the-job search.

3 Equilibrium worker mobility

In this section we analyze the equilibrium properties of the model presented in Section 2. Our analysis highlights the role of wages as a public signal of worker mobility.

We begin by noting that our assumptions impose a lot of structure on the problem. Recall that we assume that incumbent firms can retain workers by matching outside offers and that poaching firms never make offers that will be rejected with certainty. Hence, since outside offers are made after observing the productivity of the new match, poaching firms never make an offer to a worker with whom they would form a low productivity match. Recall that we also assume that workers never search on the job unless they anticipate a profitable job offer will be forthcoming with positive probability. Hence, workers with high match productivity do not search if they know that poaching firms can infer their current match productivity and will, therefore, never make them a poaching offer.

This shapes the set of possible job and wage transitions as follows: all job switches occur when a worker in a low productivity match meets a firm with which she has a high productivity match. A worker in a high productivity match (who can search on the job by pooling) who meets a firm with which she would also form a high productivity match will elicit both a job offer from the poaching firm and a retention offer from the incumbent firm. Therefore, all job switches and wage changes reveal that a worker is now employed in a high productivity match. Hence, the wage ladder has at most two rungs.

Our model shares the well known property that the allocation supported by a competitive search equilibrium can be characterized as the solution of a corresponding constrained optimization problem. Our focus below is on equilibrium allocations that exhibit positive job-to-job quits. First, we state the relevant optimization problem

((P1)–(P2)). It maximizes the value of unemployment subject to the constraint that firms make non-negative profits as well as additional participation constraints that ensure that workers are willing to form both types of matches in the market for entry jobs. Then, we formalize the tight relationship between a solution to this problem and an equilibrium allocation (Proposition 1). Finally, we provide conditions under which there exists a refined equilibrium involving separating wage contracts in all active submarkets (Section 3.1) and conditions under which there exists a refined equilibrium involving pooling contracts in the market for entry jobs (Section 3.2).

Let $\rho \in \{1-\alpha, 1\}$ denote the fraction of poorly matched workers among all on-the-job searchers. Let $F_e(s, \rho)$ be the value of an entry job to a worker in state $s = (w, y) \neq s_u$ for a given value of ρ, and let

$$F_e(s,\rho) = w + \frac{1}{1+r}\Big[\delta F_u + (1-\delta)\Big(F_e(s,\rho) + G(s,\rho)\Big)\Big], \tag{P1}$$

where

$$\frac{G(s,\rho)}{1+r} = \max_{(w',q')}\left\{f(q')\,\alpha\max\left\{0, \frac{w'}{r+\delta} + \left(\frac{\delta}{r+\delta}\right)\frac{F_u}{1+r} - \frac{F_e(s,\rho)}{1+r}\right\}\right\}$$

subject to

$$\begin{aligned}
&k_e \leq q' f(q')\,\alpha\rho\left(\frac{y_h - w'}{r+\delta}\right),\\
&w' > y_l,\\
&q' = \infty \ \text{ if } \ (s,\rho) = ((w, y_h), 1).
\end{aligned}$$

Denote a solution to Problem (P1) by $(w_e(s,\rho), q_e(s,\rho))$.

The term $G(s,\rho)/(1+r)$ represents the option value to an employed worker of on-the-job search in the market for non-entry jobs. As discussed above, the structure of our counteroffer game implies that an employed worker whose wage reveals her to be in a high productivity match cannot profit from on-the-job search. Accordingly, we adopt the convention that $q_e((w, y_h), 1) = \infty$. Workers in low productivity matches and workers who are indistinguishable from them can search on the job, and the option value of this search is given by the constrained optimization problem above.

Workers who can profit from on-the-job search face a relatively straightforward competitive search problem. The first constraint imposes that firms that post a non-entry job must make non-negative expected profits. It accounts for the fact that the workers they hire will not get any future outside offers. It also accounts for the fact that employers never make a poaching offer unless it is to form a high productivity match. When $\rho = 1$, the constraint is written as if all poaching offers are accepted by workers. Below we show that this outcome corresponds to the case where equilibrium wages reveal match productivity, in which case workers searching on the job from high productivity matches do not crowd out workers searching from low productivity matches. When $\rho = 1 - \alpha$, the constraint is written as if poaching offers are accepted by workers with probability $1 - \alpha$. This outcome corresponds to the case in which equilibrium wages do not reveal productivity, high productivity workers search on the job, and a fraction $1 - \alpha$ of applicants to poaching firms reject job offers in favor of retention offers. The other constraint, $w' > y_l$, reflects the assumption that poachers recognize the fact that employed workers can only be recruited if the poaching offer exceeds the worker's current productivity.

Note that a solution to Problem (P1) has $w_e\left(\left(w, y_h\right), 1-\alpha\right) = w_e\left(\left(w, y_l\right), 1-\alpha\right)$ and $q_e\left(\left(w, y_h\right), 1-\alpha\right) = q_e\left(\left(w, y_l\right), 1-\alpha\right)$. This captures the fact that workers searching on the job from high and low productivity matches have identical incentives in an equilibrium where wages in entry jobs do not reveal match productivities. Both types of workers compete for the same outside offers, where subsequent retention offers elicited by well matched workers will just match the outside offers that will be accepted by poorly matched workers. Thus, the usual single crossing condition does not hold in this case.

Let $s_l = (w_l, y_l)$ and $s_h = (w_h, y_h)$, and let F_u be the value of unemployment to a worker, where

$$F_u = b + \frac{1}{1+r}\left[F_u + \max_{(w_l, w_h, q)} f\left(q\right)\left(\alpha F_e\left(s_h, \rho\right) + \left(1-\alpha\right) F_e\left(s_l, \rho\right) - F_u\right)\right] \tag{P2}$$

subject to

$$k_u \leq q f(q)\left[\alpha\left(\frac{y_h}{r+\delta} - \frac{w_e\left(s_h, \rho\right)}{r+\delta} + \frac{w_e(s_h, \rho) - w_h}{r+\delta+(1-\delta)\alpha f(q_e(s_h, \rho))}\right)\right.$$
$$\left. + \left(1-\alpha\right)\left(\frac{y_l - w_l}{r+\delta+\left(1-\delta\right)\alpha f\left(q_e\left(s_l, \rho\right)\right)}\right)\right],$$

$$\rho = \begin{cases} 1 & \text{if } w_h \neq w_l \\ 1-\alpha & \text{if } w_h = w_l, \end{cases}$$

$$F_u \leq \min\Big\{F_e\left(s_l, \rho\right), F_e\left(s_h, \rho\right)\Big\},$$

$$w_l \leq y_l \text{ , } w_h \leq y_h.$$

Denote a solution to Problem (P2) by $\left(w_u^l, w_u^h, q_u\right)$.

The last constraint reflects the fact that employers cannot commit to pay wages that exceed the worker's marginal product. The previous constraint requires that unemployed workers be willing to accept all matches. Proposition 1 below implies that this participation constraint must hold along the equilibrium path in any refined equilibrium with positive job quits.

The first constraint is the non-negative profits constraint, incorporating all possibilities for on-the-job search allowed under our assumptions. The term within the parentheses in the second line represents the profits a firm enjoys when if forms a low productivity match with an unemployed job seeker. Such workers are always able to search on the job, never elicit retention offers, and quit whenever they meet a poaching firm with which they form a high productivity match. The two terms within the parentheses in the first line reflect the profits an employer enjoys when it forms a high productivity match with an unemployed job seeker. The first term is the expected discounted value of the profits received if the employer were to pay the future retention offer $w_e(s_h, \rho)$. The second term reflects the temporary extra profits due to the fact that the entry wage w_h of a high productivity worker is lower than the retention offer the worker will elicit as soon as she receives an outside offer. The denominator reflects the sources of discounting: the discount rate (r), the exogenous probability of job destruction (δ), and the probability that such a worker receives an outside offer ($(1-\delta)\alpha f(q_e(s_h, \rho))$), in which case the incumbent firm will match and the worker's wage will change.

In principle, the solution to Problem (P2) may be associated with either separating ($w_u^h \neq w_u^l$) or pooling ($w_u^h = w_u^l$) contracts. In turn, this determines the relevant value of ρ that is necessary to assess the solution to the relevant version of Problem (P1). This information is essential and so it is explicitly stated in the second constraint. Note that the non-negative profits constraint accounts for the fact that workers in high productivity matches will never receive the future retention offer $w_e(s_h, 1)$ when contracts are separating. Formally, if $w_u^h \neq w_u^l$, then $\rho = 1$, in which case $q_e(s_h, 1) = \infty$ in Problem (P1) and the two terms involving $w_e(s_h, \rho)$ cancel each other out.

Proposition 1 *(i) Consider a refined equilibrium* $(X^*, S^*, V, J, g_x, g_a, g_h, g_r, Q, \mu, \psi)$ *with positive job quits. If* $x = (u, w_l, w_h) \in X^*$ *and* $x' = (e, w_l', w_h') \in X^*$*, then* $(w_l, w_h, Q(x))$ *solves Problem (P2) and* $(w_h', Q(x))$ *solves Problem (P1) for* $s = (w_l, y_l)$*. (ii) Conversely, if* (w_l, w_h, q) *solves Problem (P2) and* (w_h', q') *solves Problem (P1) for* $s = (w_l, y_l)$*, with* $q' \in (0, \infty)$*, then there exists a refined equilibrium with positive job quits such that* $x = (u, w_l, w_h) \in X^*$ *and* $x' = (e, w_l', w_h') \in X^*$*.*

If a refined equilibrium has positive job quits, then the equilibrium allocation solves Problem (P1)–(P2). Conversely, any solution to Problem (P1)–(P2) that exhibits positive job quits is part of a refined equilibrium.

The objective function in Problem (P2) is not generally concave in (w_l, w_h, q). The main complication arises because employers do not take workers' future quit rates as given, but rather they understand that workers' future quit rates are a function of their current wages. To see why, consider how a worker' current wage affects her trade-off between quit rates and future wages. For a given wage, a worker is willing to quit at a relatively slower rate only in exchange for relatively higher future wages. The higher her current wage, the lower the *ex post* surplus she can obtain from a given wage and thus, the lower the worker's quit rate. Since a given (future) wage represents a smaller proportional share of the wage gain in the worker expected surplus for workers with higher current wages, a worker's quit rate declines with her current wage at a decreasing rate. While this property is as one would expect, it implies that the worker's value function $F_e(s, \rho)$, for $s = (w, y)$, may not be a concave function of w. In general, it is unclear whether the properties of $q_e(s, \rho)$ ensure that both the worker's surplus and the employer's surplus are well-behaved with respect to w.

It is well known that the above problem complicates significantly the analysis of competitive search on the job (e.g., Delacroix and Shi, 2006). We are able to address this technical problem in the Appendix by viewing the solution to (P1) as a mapping from the workers' quit rates to their current wages, rather than the reverse.

3.1 Revealing equilibria

In this section we characterize refined equilibria in which all employers post separating contracts and so wages in both entry and non-entry jobs reveal match productivity.

We employ the terminology of the traditional rational expectations equilibrium literature and refer to this kind of equilibrium as (fully) revealing. We begin by providing necessary and sufficient conditions for existence of a constrained-efficient steady-state allocation with positive job quits (Proposition 2). Then, we provide sufficient conditions under which such an allocation can be supported by a refined equilibrium (Proposition 3) and sufficient conditions under which it cannot (Proposition 4). We also provide sufficient conditions under which the unique refined equilibrium is revealing and it fails to support the efficient allocation (Proposition 5).

An allocation is constrained-efficient (efficient, for short) if it maximizes the present value of aggregate production net of search costs under full information. In the Appendix we characterize the efficient steady-state allocation as the solution to a planning problem. Proposition 2 provides necessary and sufficient conditions for existence of an efficient steady-state allocation in which non-entry jobs are created and so there are positive job quits.

Proposition 2 *Let*

$$K_u = \alpha \frac{y_h - b}{r+\delta} + (1-\alpha)\left(\frac{y_h - b}{r+\delta} - \frac{y_h - y_l}{r+\delta+(1-\delta)\alpha}\right).$$

For each $k_u \in (0, K_u)$ *there are two numbers* $\kappa_e \in \big(0, \alpha\,(y_h - y_l)\,/\,(r+\delta)\big]$ *and* $\kappa_u \in \big[0, \alpha\,(y_h - y_l)\,/\,(r+\delta)\big)$ *such that there exists an efficient allocation with positive job quits if and only if: (i)* $k_u \in (\kappa_u, K_u)$ *and (ii)* $k_e \in (0, \kappa_e)$.

Condition (i) in Proposition 2 requires that the costs of creating *entry* jobs are neither too large nor too small. Condition (ii) requires that the costs of creating *non-entry* jobs are small enough.

To understand the role of these conditions it is useful to note the properties of an efficient allocation with positive job quits. First, in addition to high-productivity matches, low-productivity matches are formed in the market for entry jobs. Second, workers search on the job if and only if they are currently employed in low-productivity matches. Third, they switch jobs if the new match has high-productivity.

Now consider an allocation with these properties. First, the costs of creating *non-entry* jobs must be small enough to ensure that the creation of those jobs generates positive surplus. It is easy to see that $k_e < \alpha\,(y_h - y_l)\,/\,(r+\delta)$ is a necessary and sufficient condition. Furthermore, the creation of entry jobs must generate positive surplus when both good (i.e., high productivity) and bad (i.e., low productivity) matches are formed. In the Appendix we show that this is the case if and only if both $k_u \in (0, K_u)$ and $k_e \in (0, \kappa_e)$, where the precise number κ_e depends on k_u.

Lastly, the costs of creating entry jobs must be large enough to ensure that it is efficient to form bad matches, rather than waiting to form a good match, in the market for *entry* jobs. In the Appendix we show that for each (k_u, k_e) such that $k_u \in (0, K_u)$ and $k_e \in (0, \kappa_e)$, it is efficient to form bad matches in the market for entry jobs if and only if $k_u \geq \kappa_u$, where κ_u is some number such that $\kappa_u < \alpha (y_h - y_l) / (r + \delta)$. We also prove the following corollary to Proposition 2.

Corollary 1 *Consider the value κ_u given in Proposition 2.*
(i) If $(y_h - b) / (y_h - y_l) \leq (r + \delta + \alpha) / (r + \delta + (1 - \delta) \alpha)$, then $\kappa_u > 0$, for $k_e > 0$.
(ii) If $(y_h - b) / (y_h - y_l) > (r + \delta + \alpha) / (r + \delta + (1 - \delta) \alpha)$, then there is a number $\kappa > 0$ such that $\kappa_u = 0$ if and only if $k_e \leq \kappa$.

The condition in Part (i) of the corollary ensures that the formation of bad matches is *inefficient* when k_u is small enough. The condition in Part (ii) ensures that it is *efficient* to form bad matches whenever both k_u and k_e are small enough. Intuitively, when k_u is sufficiently small, then k_e must be small enough and the productivity of bad matches must be sufficiently higher than the flow benefit from unemployment to ensure that it is efficient to form bad matches in the market for entry jobs.

Proposition 3 *Let $(y_h - b) / (y_h - y_l) > (r + \delta + \alpha) / (r + \delta + (1 - \delta) \alpha)$. There is a number $\kappa'_e \in \big(0, \alpha (y_h - y_l) / (r + \delta)\big)$ such that for each $k_e \in (0, \kappa'_e)$ there is a number $\kappa'_u \in \big(0, \alpha (y_h - y_l) / (r + \delta)\big)$ such that (i) there exists a refined equilibrium with positive job quits for any $k_e \in (0, \kappa'_e)$ and $k_u \in (0, \kappa'_u)$; (ii) any refined equilibrium supports the efficient allocation; (iii) the equilibrium distribution of workers ψ is unique and it is such that workers in low productivity matches are paid their marginal product.*

In the Appendix we show that there exists an efficient equilibrium with positive job quits if and only if (1) the efficient allocation exhibits positive job quits and (2) unemployed workers are willing to form good matches. The formation of bad matches, which is necessary for positive job quits, is efficient only if the costs of creating entry jobs are sufficiently large (Proposition 2). However, unemployed workers are willing to form good matches only if the costs of creating entry jobs are sufficiently small.

Proposition 3 provides sufficient conditions that ensure that it is efficient to form bad matches and, simultaneously, unemployed workers are willing to accept good matches. Specifically, note that, as shown in Corollary 1, the condition $(y_h - b) / (y_h - y_l) > (r + \delta + \alpha) / (r + \delta + (1 - \delta)\alpha)$ ensures that there exists an efficient allocation with positive job quits whenever the costs of creating entry and non-entry jobs are both sufficiently small.

Intuitively, a refined equilibrium that supports an efficient allocation with positive job quits must be fully revealing. In the Appendix, we show that an equilibrium allocation with $w_l \neq w_h$ in the market for entry jobs must be such that $V(w_l, y_l) \geq V(s_u)$. We also show that it must be such that $V(w_h, y_h) > V(w_l, y_l)$ if the costs of creating entry jobs are sufficiently small, as assumed in the proposition.[12] In turn, this ensures that $V(w_h, y_h) \geq V(s_u)$, which is needed for unemployed workers to be willing to accept equilibrium job offers regardless of the realization of match productivity. Furthermore, the conditions in Proposition 3 ensure that there is a unique solution to Problem (P1)–(P2) and so Proposition 1 ensures that any two refined equilibria support essentially the same equilibrium allocation (X^*, ψ). The only element of non-uniqueness is irrelevant in that a job $x' = (e, w'_l, w'_h) \in X^*$ offers a wage w'_l that is indeterminate simply because low productivity matches are never formed in the market for non-entry jobs. In particular, the distribution of workers ψ is unique.

In the efficient equilibrium with positive job quits the wage distribution has three mass points: one wage for each productivity realization for workers who find jobs out of unemployment, and one wage for workers who find jobs via on-the-job search. Equilibrium transitions are as follows: All job offers made to unemployed workers are accepted. Unemployed workers who meet a firm with which they form a low productivity match conduct on-the-job search. These workers change jobs upon meeting another firm with which they form a high productivity match. Our equilibrium refinement implies that workers in high productivity matches will not crowd out poorly matched workers in a revealing equilibrium. Accordingly, such workers do not profit from search on the job and never change jobs. Jobs are destroyed both exogenously and, for low productivity matches, endogenously by quits.

Poorly matched workers are paid their marginal product. This is needed if the equilibrium is to support the efficient allocation. Intuitively, the *ex ante* match surplus

[12] Our arguments in the proof of Proposition 3 also imply that one can find values of k_u for which there exists an efficient equilibrium such that $V(w_h, y_h) < V(w_l, y_l)$.

is maximized when the employer assigns all of the match surplus to poorly matched workers *ex post*, in which case they quit exactly when it is efficient to do so. Such a surplus division is optimal from the viewpoint of employers, because they are able to maximize surplus extraction when workers are well matched *ex post*. Here, it is worth stressing the role of directed search. For example, if search were undirected, a firm may offer a starting wage equal to the flow value of unemployment (b) and then wait to match the worker's outside offers later. By contrast, directed search induces $w_l > b$ precisely because the wage $w_l = b$ is suboptimal for an unemployed applicant's tradeoff between the wage and the meeting probability.

The constrained-efficient equilibrium is such that (w_e, q_e) is the unique pair (w', q') that satisfies the usual zero profit and matching efficiency conditions in the market for employed workers. That is, the expected value of a vacancy to potential poachers equals the cost of posting the vacancy, which implies that employers are willing to offer higher wages and suffer reductions in the net present value of their profits only if they expect to fill their vacancies at a faster rate. The matching-efficiency condition implies that the ratio of the worker's surplus to the firm's surplus in new matches equals the ratio of their matching elasticities.

Similarly, we show in the Appendix that $\left(w_u^h, q_u\right)$ is the unique pair (w, q) that satisfies the usual zero profit and matching efficiency conditions in the market for unemployed workers, together with the Bellman equation in Problem (P2). It should be noted that the latter is completely standard because of the result that, in equilibrium, firms earn no profit from bad matches. Consequently, the firm's match surplus is entirely a function of the profits it makes from good matches. Since well matched workers cannot profit from on-the-job search in a revealing equilibrium, employers have no incentive to set wages in order to manipulate their quit rates.

Since the costs of job creation are paid entirely by well matched workers, the existence of an efficient equilibrium with positive quits rests on the costs of creating entry jobs being small enough that unemployed workers are willing to accept good matches. However, if the cost of creating entry jobs is sufficiently high, then there may be no separating contract that implements the efficient allocation. The next proposition provides sufficient conditions under which this is in fact the case.

Proposition 4 *Suppose there is an efficient allocation with positive job quits. For each $k_e > 0$ and $k_u > \alpha\left(y_h - y_l\right) / \left(r + \delta\right)$, there is a number $\beta < y_l$ such that there is no refined equilibrium that supports the efficient allocation for any (b, k_u, k_e) such* *that* $b \in (\beta, y_l)$, $k_u \in \left(\alpha\left(y_h - y_l\right)/\left(r+\delta\right), K_u\right)$ *and* $k_e \in (0, \kappa_e)$, *where* K_u *and* κ_e *are given in Proposition 2.*

Thus, there is no constrained-efficient equilibrium with positive job quits if the costs of creating entry jobs and the flow benefit from unemployment are sufficiently high. This result would not be surprising if the efficient allocation did not involve any job quits. However, the proposition restricts attention to the case where the efficient allocation does have positive job quits and so the result reflects the non-trivial breakdown of the constrained-efficient equilibrium.

The problem is that the employer must recover the entire cost of job creation by extracting surplus from high productivity matches. But this may lower the value of employment in good matches below the value of unemployment when the costs of creating entry jobs are sufficiently high. If contracts can only be made contingent on match productivity and outside offers, entry jobs are subject to a holdup problem. The reason is that unemployed workers are able to credibly reject any offer associated with good matches whose value does not exceed the value of unemployment. Accordingly, employers will be unable to solve the holdup problem without distorting job creation if the cost of creating entry jobs is high enough. The holdup problem would disappear if employers could commit to the bilaterally efficient contract, which pays poorly matched workers a lower wage in the first period, and it pays their marginal product later on. Then, refined equilibria would be necessarily efficient.

Propositions 3 and 4 imply that there are feasible parameters $(y_l, y_h, \alpha, r, b, \delta, k_e)$ and $k_u = \kappa$ such that any refined equilibrium supports an efficient allocation with positive quits where unemployed workers are indifferent between accepting and rejecting good matches. In the Appendix, we show that there is a value κ' such that refined equilibria are revealing and do not implement the efficient allocation if $k_u \in (\kappa, \kappa')$.

Proposition 5 *There are feasible parameter values* $(y_l, y_h, \alpha, r, b, \delta, k_e)$ *and a non-empty interval* (κ, κ') *such that: (i) there exists a refined equilibrium with positive job quits for all* $k_u \in (\kappa, \kappa')$*; (ii) any refined equilibrium is revealing and the equilibrium distribution of workers* ψ *is unique; (iii) either* $w_l < y_l$*, in which case the quits rate is inefficiently high and the rate at which unemployed workers find entry jobs is inefficiently low, or* $w_l = y_l$*, in which case the quits rate is efficient, but the rate at which unemployed workers find entry jobs is inefficiently high.*

Now the equilibrium allocation is such that unemployed workers are indifferent between accepting and rejecting good matches. There are two ways in which this participation constraint can be met. Either workers in bad matches are paid less than their marginal product (an interior allocation), or they are paid their marginal product (a corner allocation). In the former case, workers in the market for non-entry jobs have outside options that are too low and so they elicit poaching offers that involve inefficiently low wages and inefficiently high labor market queues. In the latter case, job turnover is efficient and it is instead the creation of entry jobs that is excessive. In principle, either distortion can be an equilibrium outcome. However, while we show that either job turnover or the creation of entry jobs must be excessive in an inefficient revealing equilibrium, we have not been able to prove that both distortions do in fact arise in some equilibrium.

3.2 Non-revealing equilibrium

In this section we provide conditions under which there exists a refined equilibrium such that pooling contracts, which do not reveal match productivity, are posted in the market for entry jobs. We refer to this kind of equilibria as non-revealing.[13] Below we argue that the existence of the non-revealing equilibrium is particularly interesting, because it sheds new light on the role of both wage posting and counteroffers.

In the non-revealing equilibrium the wage distribution has two mass points. All entry jobs pay an identical wage. Unemployed workers accept all offers, and all workers employed in entry jobs search on the job, with workers who are well matched *ex post* mimicking the on-the-job search behavior of workers who are poorly matched. As a result of pooling, all workers searching on the job face the same matching probabilities. Poorly matched workers change jobs upon meeting another employer with which they form a high productivity match. Well matched workers receive retention offers upon meeting another employer with which they form a high productivity match. Thus, all workers employed in entry jobs have identical incentives to search on the job, so within-firm and between-firm wage mobility result in identical wages.

Consider the problem of a worker who is currently employed in an entry-level job earning a wage w and searching for a non-entry job. It is easy to verify that an interior solution of Problem (P1), (w', q'), satisfies the familiar zero-profit and matching

[13] Examples of pooling equilibria are found in Shi (2002) and Shimer (2005) in the context of labor markets and Chang (2018) and Guerrieri and Shimer (2014) in the context of asset markets.

efficiency conditions in the market for employed workers. The latter condition is identical to the corresponding matching efficiency condition in the revealing equilibrium, though the entry wage (w) is generally different. Furthermore, in a non-revealing equilibrium, potential poachers need to anticipate that a fraction $1 - \alpha$ of their pool of applicants are poorly matched in their current jobs, and so a fraction α will turn down their job offers because they are only searching to elicit a retention offer.

Just as before, solving Problem (P2) is non-trivial because the objective function is not generally concave in (w, q). Once again we are able to address this problem by viewing the solution to Problem (P1) as a mapping from the workers' quit rates to their entry wages, rather than the reverse, and then treat current and future quit rates as the relevant choice variables in Problem (P2). This approach allows us to characterize the allocation in the non-revealing equilibrium and prove the following.

Proposition 6 *There is a number $\alpha_0 \in (0,1)$ such that for each $\alpha \in (\alpha_0, 1)$ there are numbers $\beta_0 \in (0, y_l)$ and $\kappa_0 \in \big(0, (1-\alpha)\,\alpha\,(y_h - y_l)\,/\,(r+\delta)\big)$ such that (i) there exists a refined equilibrium with positive job quits for any (b, k_u, k_e) such that $b \in (\beta_0, y_l)$, $k_u \in \big[\alpha\,(y_h - y_l)\,/\,(r+\delta)\,, \alpha\,(y_h - b)\,/\,(r+\delta)\big)$ and $k_e \in (0, \kappa_0)$; (ii) any refined equilibrium is non-revealing and the equilibrium distribution of workers ψ is unique; (iii) the rate at which unemployed workers find entry jobs and the rate at which poorly matched workers quit are both inefficiently low.*

The proof of the proposition proceeds as follows. First, we provide sufficient conditions under which there is a unique interior solution to Problem (P1)–(P2) when we restrict attention to the class of entry jobs offering pooling contracts. We begin by showing that there is a number $\kappa_0 \in \big(0, (1-\alpha)\,\alpha\,(y_h - y_l)\,/\,(r+\delta)\big)$ that ensures that the solution to Problem (P1) subject to $\rho = 1 - \alpha$ exhibits positive quits for all $k_e \in (0, \kappa_0)$. Then, we show that there is a unique contract that solves Problem (P2) for all $k_u \in \big[\alpha\,(y_h - y_l)\,/\,(r+\delta)\,, \alpha\,(y_h - b)\,/\,(r+\delta)\big)$ when we restrict attention to the class of pooling contracts, and that the posted wage is such that $w_u^* < y_l$. Intuitively, the costs of creating entry jobs are high enough that the optimal pooling contract is such that the costs of job creation are shared between good and bad matches, that is, $w_u^* < y_l$. The problem is that while employers can lower the workers' future quit rates by raising entry wages, workers with higher entry wages elicit higher outside offers. Since well matched workers cannot be prevented from seeking outside offers, the allocation of surplus at the margin is allocated disproportionately to the worker. Accordingly, employers do not typically have an incentive to raise entry wages all the way to y_l.

The second part of the proof provides conditions under which the unique solution to Problem (P1)–(P2) indeed involves a pooling contract in the market for entry jobs. Proposition 1 then implies that all refined equilibria are essentially the same in that they all support exactly the same equilibrium distribution of workers. Formally, first we show that for each $\alpha \in (0, 1)$ there is a number $\beta_0 \in (0, y_l)$ such that the efficient separating contract will attract no unemployed workers for all $b \in (\beta_0, y_l)$ and all $k_u \geq \alpha (y_h - y_l) / (r + \delta)$. The logic is the same as the one underlying the breakdown of efficient revealing equilibria. When the cost of job creation is sufficiently high, then it is possible that the value of a good match to a worker is dominated by the value of unemployment. When that happens, unemployed workers reject good matches unless the costs of job creation are shared across types of matches, which is inefficient. Second, we show that the surplus of a separating contract such that workers are indifferent between forming a good match and staying unemployed becomes negligible as the probability of a good match approaches one. By contrast, we show that the surplus associated with the pooling contract that maximizes the value of unemployment subject to non-negative profits remains bounded away from zero, provided that the cost of creating non-entry jobs, k_e, is sufficiently close to zero. The latter condition is needed because otherwise the gains from trade in the market for non-entry jobs necessarily vanish as the probability of a good match approaches one.

Pooling wage contracts in the market for entry jobs solve the hold up problem associated with separating contract precisely because they are non-revealing. However, this creates adverse selection in the market for non-entry jobs. The problem is that well matched workers cannot commit to not searching on the job. Instead, they have an incentive to search, but solely to elicit a retention offer from their current employers rather than to change jobs. This congestion reduces the return to job creation in the market for non-entry jobs and, as a result, generates inefficient turnover.

It is easy to see that worker mobility is necessarily too low, conditional on the wages workers are paid in entry jobs. However, as shown in Proposition 5, the fact that those wages are lower than y_l alone tends to increase job turnover, relative to the efficient allocation. Proposition 6 provides sufficient conditions under which the equilibrium quits rate of poorly matched workers is in fact lower than the quits rate under the efficient allocation. Furthermore, under the same conditions, the rate at which unemployed workers find entry jobs is inefficiently low.

4 Discussion and conclusion

The central message of this paper is that non-revealing wages and counteroffers can be understood as complementary features of a second-best market solution to a holdup problem that is associated with revealing wages when there is limited commitment. Holdup problems can arise when employers incur costs to create entry jobs but must share the surplus of the worker-employer match with the worker. The holdups we emphasize in this paper are associated with the impact of match-specific risk on (future) worker mobility. Bilaterally efficient worker mobility requires that workers who are expected to leave their current job enjoy the full surplus of the worker-employer match. This implies that the costs of creating entry jobs (e.g., capital costs, job training costs) must fall disproportionately on those workers whom employers expect to retain, that is, those employed in good matches. If these costs are sufficiently large, and if employers cannot commit to a bilaterally efficient wage-tenure contract, the competitive search equilibrium must involve distortions.

Commitment to a bilaterally efficient wage-tenure contract would allow the firms to share job creation costs across types of matches without distorting wage mobility. Long-term wage-tenure contracts allow firms to recover the cost of job creation in the earlier period while generating a sufficient surplus for the workers later. With efficient wage-tenure contracts, employers could reduce the wage workers earn in low-productivity matches during the first period and then pay them their marginal product, so their on-the-job search decisions maximize the joint surplus of the match. The first-period earnings could even be negative if the costs of creating entry jobs are sufficiently high. In this context, internships might be viewed as a mechanism to backload worker compensation. Employer-provided pension plans can be understood as a solution to a version of this problem. More generally, mechanisms that allow the firm to credibly promise future increases in worker compensation provide a potential solution to holdup problems in the market for entry jobs. This view is different from, but complementary to, the common view that backloading compensation via increasing wage-tenure profiles and pension plans is a mechanism to retain workers.

Our results imply that posted wages and counteroffers can be understood as resulting from a common commitment failure. In particular, the inability of firms to commit to not making counteroffers is central to the existence of the non-revealing equilibrium. If firms can commit to not matching outside offers, then recruiting firms know that they will be able to attract an employed applicant by posting a higher wage. But then the holdup and adverse selection problems both disappear.

In our setting, counteroffers are associated with informational problems, so they only arise as part of inefficient equilibria. However, they also have a productive role: they allow for an equilibrium with worker mobility when job creation costs are high enough that the efficient equilibrium breaks down. Non-revealing equilibria are the only equilibria that exhibit within-firm wage mobility, in addition to between-firm wage mobility. The latter is associated with productivity increases. The former is driven by retention offers, without the need for productivity increases, employer learning, or new public information. To the extent that counteroffers are not uncommon in many markets, disregarding them leads to overestimating the contribution of factors such as employer learning, and worker experience, to wage growth.

From the perspective of our model, the lack of counteroffers in some markets is not the result of firms' ability to commit to not making counteroffers. Rather, it is a symptom of a revealing equilibrium, which is in fact supported by the inability of firms to commit to not making counteroffers. The reason why counteroffers are not observed is that they serve no purpose along the equilibrium path. The insight here is that the *threat* of retention offers precludes on-the-job search for workers in entry jobs with high productivity. The fact that wage contracts reveal match productivity in the market for entry jobs can be viewed as a commitment device that prevents well matched workers from searching on the job, which would be inefficient.

The existence of non-revealing equilibria depends on whether the costs of creating entry jobs are large *relative* to the expected surplus of good versus bad matches, as reflected in the term $\alpha\,(y_h - y_l)\,/\,(r+\delta)$. Thus, all else equal, non-revealing equilibria are more likely when the costs of creating entry jobs are larger, or when the dispersion in match productivities is larger. One implication is that counteroffers are not necessarily restricted to high-paying jobs.[14] Our analysis also suggests that

[14]For example, the fact that "no-poaching of workers" clauses preventing managers from hiring employees that have worked elsewhere in the same chain have been the norm in the U.S. fast-food industry until recently is suggestive of the potential relevance of counteroffers in low-wage markets. Using data on franchise contracts in 2016 by 156 of the largest franchise chains in the U.S., Krueger and Ashenfelter (2022) report that 58 percent include no-poaching clauses. They also find that these clauses are more common for franchises in low-wage and high-turnover industries.

younger workers — who are more likely to be employed in entry-level jobs — and workers at smaller establishments — where employers may be unable to commit to bilaterally efficient wage contracts — are likely to be paid lower wages but are more likely to receive counteroffers. Similarly, jobs with higher up front creation costs, such as capital intensive jobs, are more likely to suffer from the hold up problem we identify. This is consistent with available evidence on counteroffers (Barron et al., 2006) and on performance pay (MacLeod and Parent, 1999).

Most broadly, our analysis illustrates the potential for inefficient turnover when wages must both direct search and signal worker mobility. On the one hand, posted wages, which we characterized as pooling wage contracts, create adverse selection in the market for non-entry jobs. The problem is that well matched workers cannot be identified and, therefore, have an incentive to search on the job in order to elicit retention offers from their current employers. This alone tends to depress worker mobility. On the other hand, inefficiently low posted wages tend to lower the workers' outside option. This mechanism alone tends to generate excessive worker mobility.

Proposition 6 provided sufficient conditions under which there exist non-revealing equilibria that exhibit too little worker mobility as well as too little creation of entry jobs. Proposition 5 showed that either job turnover or the creation of entry jobs must be excessive in an inefficient revealing equilibrium. More generally, we conjecture that competitive search equilibria can have too much or too little worker mobility and may exhibit too much or too little creation of entry jobs. We believe that the potential for a variety of labor market distortions is important. For instance, the common perception is that relatively high worker mobility rates are a symptom of efficient worker mobility from bad to good matches. The problem with this interpretation is that observed mobility may be high simply because the economy endogenously created too many bad matches in the first place.

Appendix

Proof of Proposition 1

To prove Part (i), let $(X^{*}, S^{*}, V, J, g_x, g_a, g_h, g_r, Q, \mu, \psi)$ be a refined equilibrium with positive job quits. Then, there must be a job $x = (u, w_l, w_h) \in X^{*}$ such that $Q(x) \in (0, \infty)$ and a job $x' = (e, w_l', w_h') \in X^{*}$ such that $Q(x') \in (0, \infty)$. First, we prove that $(w_h', Q(x'))$ must solve Problem (P1) for any state $s = (w_l, y_l) \in S^{*}$. Then, we prove that $(w_l, w_h, Q(x))$ must solve Problem (P2).

Consider the market for non-entry jobs. Profit maximization ensures that the non-negative profits constraint in Problem (P1) is satisfied, where consistency of equilibrium beliefs requires that $\rho = 1 - \alpha$ if $w_h = w_l$ and $\rho = 1$ if $w_h \neq w_l$. Our discussion in the main text makes it clear that if $w_h \neq w_l$, then workers employed in high productivity matches do not search on the job. Thus, the last constraint in Problem (P1) must be satisfied as well. Next, note that $w_h' > y_l$. Otherwise $Q(x') \notin (0, \infty)$.

Suppose, by contradiction, that $(w_h', Q(x'))$ does not solve Problem (P1) for some state $s = (w_l, y_l) \in S^{*}$. Then, $G(s) > U(s, x', Q(x'))$, for a fixed value $F_u = V(s_u)$. Accordingly, there must be a job $\widehat{x} = (e, \widehat{w}_l, \widehat{w}_h)$ and a queue $\widehat{q}$ satisfying the constraints in Problem (P1) such that

$$f(\widehat{q})\, \alpha \left[\frac{\widehat{w}_h}{r+\delta} + \left(\frac{\delta}{r+\delta} \right) \frac{V(s_u)}{1+r} - \frac{F_e(s, \rho)}{1+r} \right] > U(s, x', Q(x')).$$

Since $(X^{*}, S^{*}, V, J, g_x, g_a, g_h, g_r, Q, \mu, \psi)$ is a refined equilibrium, we have

$$f(Q(\widehat{x}))\, \alpha \left[\frac{\widehat{w}_h}{r+\delta} + \left(\frac{\delta}{r+\delta} \right) \frac{V(s_u)}{1+r} - \frac{V(s)}{1+r} \right] \leq U(s, x', Q(x')).$$

Noting that $U(s, x', Q(x')) > 0$ because $w_h' > y_l \geq w_l$, and noting that we are assuming by contradiction that $F_e(s, \rho) > V(s)$, the above inequalities imply that $f(\widehat{q}) > f(Q(\widehat{x}))$, hence $\widehat{q} < Q(\widehat{x})$. If either $\rho = 1 = \mu((w_l, y_l) | \widehat{x})$ or $\rho = 1 - \alpha \leq \mu((w_l, y_l) | \widehat{x})$, then

$$\widehat{q} f(\widehat{q})\, \alpha \rho \left(\frac{y_h - \widehat{w}_h}{r+\delta} \right) < Q(\widehat{x}) f(Q(\widehat{x}))\, \alpha \mu((w_l, y_l) | \widehat{x}) \left(\frac{y_h - \widehat{w}_h}{r+\delta} \right) \leq k_e,$$

where the weak inequality follows from profit maximization. Hence, posting $\widehat{x}$ violates the non-negative profits constraint in Problem (P1). Accordingly, if the equilibrium is such that either $w_h \neq w_l$ and $\mu\left((w_l, y_l)\,|\widehat{x}\right) = 1$, or $w_h = w_l$ and $\mu\left((w_l, y_l)\,|\widehat{x}\right) \geq 1-\alpha$, then $(w_h', Q(x'))$ must solve Problem (P1) for any state $s = (w_l, y_l) \in S^*$.

Therefore, if $(w_h', Q(x'))$ does not solve Problem (P1) for some state $s = (w_l, y_l) \in S^*$, then either $w_h \neq w_l$ and $\mu\left((w_l, y_l)\,|\widehat{x}\right) < 1$, or $w_h = w_l$ and $\mu\left((w_l, y_l)\,|\widehat{x}\right) < 1 - \alpha$. But then our arguments immediately imply that the proposed equilibrium violates Part (ii) of the equilibrium refinement and so it cannot be a refined equilibrium. We conclude that $(w_h', Q(x'))$ must solve Problem (P1) for any state $s = (w_l, y_l) \in S^*$.

Next, consider the market for entry jobs. Profit maximization ensures that the non-negative profits constraint in Problem (P2) is satisfied. Furthermore, $V(s_u) \leq V(s_l)$. Otherwise, the equilibrium cannot exhibit positive job quits.

If the equilibrium has $w_h = w_l$, then $V(s_h) = V(s_l)$, hence $V(s_u) \leq V(s_h) = V(s_l)$. Otherwise, the equilibrium cannot exhibit positive job quits. If the equilibrium has $w_h \neq w_l$ and $V(s_u) > V(s_h)$, then it is easy to see that there is a number $\epsilon > 0$ and a deviating job $(u, w_l, \widehat{w}_h)$ such that $\widehat{w}_h = y_h - \epsilon > y_l \geq w_l$, with $V\left((\widehat{w}_h, y_h)\right) > V(s_l) \geq V(s_u)$, and a queue, that would attract workers and be profitable. Hence, the constraint $V(s_u) \leq V(s_h)$ must be satisfied.

Suppose, by contradiction, that $(w_l, w_h, Q(x))$ does not solve Problem (P2). Then there must be an entry job $\widehat{x} = (u, \widehat{w}_l, \widehat{w}_h)$ and a queue $\widehat{q}$ satisfying the constraints in Problem (P2) such that

$$f(\widehat{q})\Big(\alpha F_e\left((\widehat{w}_h, y_h), \rho\right) + (1-\alpha) F_e\left((\widehat{w}_l, y_l), \rho\right) - F_u\Big) > U\left(s_u, x, Q(x)\right). \quad (6)$$

Since $(X^*, S^*, V, J, g_x, g_a, g_h, g_r, Q, \mu, \psi)$ is a refined equilibrium, we have

$$f(Q(\widehat{x}))\Big(\alpha V\left((\widehat{w}_h, y_h)\right) + (1-\alpha) V\left((\widehat{w}_l, y_l)\right) - V(s_u)\Big) \leq U\left(s_u, x, Q(x)\right). \quad (7)$$

Suppose that $F_e(s, \rho) = V(s)$, for all $s \in \{(\widehat{w}_l, y_l), (\widehat{w}_h, y_h)\}$. If $F_u > V(s_u)$, then the above inequalities imply that $f(\widehat{q}) > f(Q(\widehat{x}))$. Hence, $\widehat{q} < Q(\widehat{x})$. But then, it follows from the argument above that posting $\widehat{x}$ violates the non-negative profits constraint in Problem (P2). Therefore, it must be that $F_u = V(s_u)$ as well.

Now suppose that $F_e(s, \rho) > V(s)$, for some $s \in \{(\widehat{w}_l, y_l), (\widehat{w}_h, y_h)\}$. If $F_u > V(s_u)$ and $F_e(s, \rho) - F_u \geq V(s) - V(s_u)$, then it must be that $G(s) > U(s, x', Q(x'))$, for $x' = (e, w_l', w_h') = g_x(s)$. But then, there must be a non-entry job $\widehat{x}' = (e, \widehat{w}_l', \widehat{w}_h')$ and a queue $\widehat{q}'$ satisfying the constraints in Problem (P1) such that

$$f(\widehat{q}')\,\alpha\left[\frac{\widehat{w}_h'}{r+\delta}-\left(\frac{r}{r+\delta}\right)\frac{F_u}{1+r}-\frac{F_e(s,\rho)-F_u}{1+r}\right]>U\left(s,\widehat{x}',Q(\widehat{x}')\right).$$

Since $(X^*,S^*,V,J,g_x,g_a,g_h,g_r,Q,\mu,\psi)$ is a refined equilibrium, we have

$$f(Q(\widehat{x}'))\,\alpha\left[\frac{\widehat{w}_h'}{r+\delta}-\left(\frac{r}{r+\delta}\right)\frac{V(s_u)}{1+r}-\frac{V(s)-F_u}{1+r}\right]\leq U\left(s,\widehat{x}',Q(\widehat{x}')\right).$$

Since we are assuming, by contradiction, that $F_u>V(s_u)$ and $F_e(s,\rho)-F_u\geq V(s)-V(s_u)$, the above inequalities imply that $\widehat{q}'<Q(\widehat{x}')$. But then, it follows from the argument above that posting the non-entry job $\widehat{x}'$ violates the non-negative profits constraint in Problem (P1). In turn, this implies that it cannot be that $F_e(s,\rho)-F_u\geq V(s)-V(s_u)$ for $s\in\{(\widehat{w}_l,y_l),(\widehat{w}_h,y_h)\}$. But then, inequalities (6) and (7) imply that $\widehat{q}<Q(\widehat{x})$. Now it follows from the argument above that posting the entry job $\widehat{x}$ must violate the non-negative profits constraint in Problem (P2). Therefore, it must be that $F_u=V(s_u)$.

To prove Part (ii), suppose that (w_l,w_h,q) solves Problem (P2) and (w_h',q'), with $q'\in(0,\infty)$, solves Problem (P1) for $s=(w_l,y_l)$. To construct an equilibrium, let $Q(x)$ satisfy

$$U\left(s_u,x,Q(x)\right)=\max_{(w_l,w_h,q)}f(q)\Big(\alpha F_e(s_h,\rho)+(1-\alpha)F_e(s_l,\rho)-F_u\Big)$$

for any entry job $x=(u,w_l,w_h)\in X$ and

$$U\left(s_l,x',Q(x')\right)=\max_{(w',q')}\left\{f(q')\,\alpha\max\left\{0,\frac{w'}{r+\delta}+\left(\frac{\delta}{r+\delta}\right)\frac{F_u}{1+r}-\frac{F_e(s_l,\rho)}{1+r}\right\}\right\},$$

for any non-entry job $x'=(e,w_l',w_h')\in X$, where $s_l=(w_l,y_l)$, $s_h=(w_h,y_h)$, $\rho=1$ if $w_h\neq w_l$ and $\rho=1-\alpha$ if $w_h=w_l$, or $Q(x)=0$ if there is no solution to the equation. Note that these conditions uniquely pin down $Q(x)$ for all $x\in X$. It is immediate that the value function V and the associated policy functions are such that the workers' behavior is optimal.

Next we show that profit maximization is never violated for any feasible job x, queue q and beliefs μ satisfying the equilibrium refinement. To that end, let $\mu((w,y_h)|x)=0$ if $U((w,y_h),x,q)=0$ and let $\mu((w,y_h)|x)=\alpha\mu((w,y_l)|x)/(1-\alpha)$

if $U\left(\left(w, y_h\right), x, q\right) > 0$, for any feasible w, x and q. Suppose, by contradiction, that profit maximization is violated for some non-entry job $x' = (e, w'_l, w'_h)$ and queue $Q(x')$. Then, it must be that

$$Q(x') f\left(Q(x')\right) \alpha\mu\left(\left(w, y_l\right)|x\right)\left(\frac{y_h - w'_h}{r+\delta}\right) > k_e.$$

It follows that there is some $q' \in (0, Q(x'))$ such that

$$q' f\left(q'\right) \alpha\mu\left(\left(w, y_l\right)|x\right)\left(\frac{y_h - w'_h}{r+\delta}\right) = k_e.$$

But then, the construction of $Q(x)$ implies that $V(s_u) < F_u$, a contradiction. A similar argument applies to any entry job $x = (u, w_l, w_h) \in X$. **QED**

Proof of Proposition 2

We say an allocation is constrained efficient (efficient, for short) if it maximizes the present value of aggregate production net of search costs under full information. We characterize the efficient allocation as the solution to a planning problem and provide necessary and sufficient conditions for an efficient allocation to have positive job quits.

First, note that the state of the economy at the beginning of each period can be summarized by $\{u, m\}$, where $u \in [0, 1]$ is the measure of unemployed workers, and $m : \{y_l, y_h\} \to [0, 1]$, where $m(y)$ denotes the measure of employed workers with match productivity y. Let $p(y)$ denote the probability with which a match has productivity realization y. Let $z_u(y)$ denote the probability with which a meeting between an unemployed worker and a job is turned into a match given the productivity realization y, and $z_e(y'|y)$ denote the probability with which a meeting between a worker and a job with productivity realization y' is turned into a match given that the worker is currently employed in a job with match productivity y. Finally, let q_u denote the labor market queue where unemployed workers search for jobs, and $q_e(y)$ denote the labor market queue where employed workers search given that they are currently employed in jobs with productivity y.

Aggregate output can be written as:

$$Y(u, m) = bu + \sum_y y m(y) - k_u \frac{u}{q_u} - (1-\delta) k_e \sum_y \frac{m(y)}{q_e(y)}. \tag{8}$$

Denote by $\widehat{u}$ the measure of unemployed workers one period ahead, and by $\widehat{m}(y)$ the measure of employed workers with match productivity y one period ahead. Then,

$$\widehat{u} = \left(1 - \sum_y f(q_u) z_u(y)\right) u + \delta \sum_y m(y) \tag{9}$$

and

$$\begin{aligned} \widehat{m}(y) &= p(y) f(q_u) z_u(y) u + (1-\delta) m(y) \left[1 - p(y') f(q_e(y)) z_e(y'|y)\right] \\ &\quad + (1-\delta) m(y') p(y) f(q_e(y')) z_e(y|y'). \end{aligned} \tag{10}$$

The allocation that maximizes aggregate output net of search costs can be characterized as the solution to the planning problem:

$$J(u,m) = \max_{q_u, z_u, q_e, z_e} \left\{ Y(u,m) + \frac{J(\widehat{u}, \widehat{m})}{1+r} \right\}, \tag{11}$$

subject to equations (8)–(10). $J(u,m)$ is the unique solution to the planner's problem and can be written as:

$$J(u,m) = J_u u + \sum_y m(y) J_e(y),$$

where

$$J_u = \max_{q_u, z_u} \left\{ b - \frac{k_u}{q_u} + \sum_y p(y) f(q_u) z_u(y) \frac{J_e(y)}{1+r} + \left(1 - \sum_y p(y) f(q_u) z_u(y)\right) \frac{J_u}{1+r} \right\} \tag{12}$$

and

$$J_e(y) = \max_{z_e, q_e} \left\{ y - (1-\delta)\frac{k_e}{q_e(y)} + \delta\frac{J_u}{1+r} \right.$$
$$+ (1-\delta)\left(1 - \sum_{y'} p(y')f(q_e(y))z_e(y'|y)\right)\frac{J_e(y)}{1+r} \tag{13}$$
$$\left. +(1-\delta)\sum_{y'} p(y')f(q_e(y))z_e(y'|y)\frac{J_e(y')}{1+r} \right\}.$$

It is easy to verify that at the optimum $q_e(y_h) = \infty$. This implies:

$$J_e(y_h) = y_h + \delta\frac{J_u}{1+r} + (1-\delta)\frac{J_e(y_h)}{1+r} > J_e(y_l). \tag{14}$$

It is also easy to verify that $z_e(y_h|y_l) = 1$ and $z_e(y_l|y_l) \in [0,1]$ at the optimum. This means that the planner's problem has multiple solutions, all of which yield the same optimal value. The multiplicity concerns the probability with which the planner instructs workers to accept or reject lateral job moves. We characterize the solution when $z_e(y_l|y_l) = 0$.

The necessary condition of (13) with respect to $q_e(y_l)$ can be written:

$$\frac{J_e(y_h)}{1+r} - \frac{J_e(y_l)}{1+r} = \frac{k_e}{\alpha q_e(y_l)f(q_e(y_l))\eta(q_e(y_l))} \tag{15}$$

and the Bellman equation for $J_e(y_l)$ gives:

$$\frac{J_e(y_l)}{1+r} = \frac{1}{r+\delta+(1-\delta)f(q_e(y_l))\alpha}\left(y_l - y_h - (1-\delta)\frac{k_e}{q_e(y_l)}\right) + \frac{J_e(y_h)}{1+r}. \tag{16}$$

Equations (15) and (16) imply that $q_e(y_l) = q_b \in (q_a, \infty)$ at an optimum, where $q_a > 0$ solves

$$q_a f\left(q_a\right)\alpha\left(\frac{y_h - y_l}{r+\delta}\right) = k_e \tag{17}$$

and q_b solves

$$\frac{y_h - y_l}{r+\delta} = \left(\frac{k_e}{q_b f\left(q_b\right)\alpha}\right)\left(1 + \left(\frac{1-\eta\left(q_b\right)}{\eta\left(q_b\right)}\right)\left(\frac{r+\delta+(1-\delta)\,\alpha f\left(q_b\right)}{r+\delta}\right)\right). \tag{18}$$

The right side of (18) is strictly decreasing in q_b, it converges to ∞ as q_b approaches 0 and it converges to k_e/α as q approaches ∞. Hence, there is a unique solution $q_b \in (q_a, \infty)$ if and only if

$$k_e < \alpha \left(\frac{y_h - y_l}{r + \delta} \right). \tag{19}$$

Conjecture that $z_u(y) = 1$ for $y = \{y_l, y_h\}$. The necessary condition of (12) with respect to q_u can be written:

$$\frac{r}{r+\delta}\frac{J_u}{1+r} = \frac{y_h}{r+\delta} - \frac{k_u}{q_u f(q_u)\eta(q_u)} - \frac{(1-\alpha)k_e}{\eta(q_b) q_b f(q_b)\alpha}. \tag{20}$$

From the Bellman equation for J_u:

$$\frac{rJ_u}{1+r} = b + \frac{k_u}{q_u}\left(\frac{1-\eta(q_u)}{\eta(q_u)} \right). \tag{21}$$

These two equations imply that an optimal q_u satisfies

$$\frac{y_h - b}{r+\delta} - \frac{(1-\alpha)\, k_e}{\eta\,(q_b)\, q_b f\,(q_b)\,\alpha} = \frac{k_u}{\eta\,(q_u)\, q_u f\,(q_u)} + \left(\frac{1-\eta\,(q_u)}{\eta\,(q_u)} \right) \frac{k_u}{(r+\delta)\, q_u}. \tag{22}$$

The right side of (22) is strictly decreasing in q_u, it converges to ∞ as q_u approaches 0 and it converges to k_u as q_u approaches ∞. Hence, there is a unique solution $q_u \in (0, \infty)$ that solves the equation if and only if

$$\frac{y_h - b}{r+\delta} - \frac{(1-\alpha)\, k_e}{\eta\,(q_b)\, q_b f\,(q_b)\,\alpha} > k_u. \tag{23}$$

Combine equations (14) and (20) to obtain

$$\frac{J_e(y_h)}{1+r} - \frac{J_u}{1+r} = \frac{k_u}{q_u f(q_u)\eta(q_u)} + \frac{(1-\alpha)\, k_e}{\alpha q_b f(q_b)\eta(q_b)} \tag{24}$$

and, use equation (15) to obtain

$$\frac{J_e(y_l)}{1+r} - \frac{J_u}{1+r} = \frac{k_u}{q_u f(q_u)\eta(q_u)} - \frac{k_e}{q_b f(q_b)\eta(q_b)}, \tag{25}$$

which is non-negative if and only if

$$\frac{k_u}{q_u f(q_u)\eta(q_u)} \geq \frac{k_e}{q_b f(q_b)\eta(q_b)}. \tag{26}$$

Hence, $z_u(y) = 1$ for $y = \{y_l, y_h\}$, if and only if equation (26) is satisfied. It follows that there exists an efficient steady-state allocation with positive job quits if and only if equations (19), (23) and (26) are all satisfied, where q_b and q_u are the unique solutions to equations (18) and (22).

Differentiate equation (18) to verify that

$$\frac{\partial q_b}{\partial k_e} > 0 \text{ and } \frac{\partial}{\partial k_e}\left(\frac{k_e}{\eta\left(q_b\right) q_b f\left(q_b\right)}\right) > 0,$$

with

$$\frac{k_e}{\eta\left(q_b\right) q_b f\left(q_b\right) \alpha} \in \left(\frac{y_h - y_l}{r + \delta + (1-\delta)\alpha}, \frac{y_h - y_l}{r+\delta}\right), \tag{27}$$

for all $k_e < \alpha\left(y_h - y_l\right) / \left(r + \delta\right)$. It follows that $k_u < K_u$ is necessary for (23), where K_u is given in Proposition 2. Thus, for each $k_u \in (0, K_u)$ there is a number $\kappa_e \in \left(0, \alpha\left(y_h - y_l\right) / \left(r+\delta\right)\right]$ such that conditions (19) and (23) are satisfied if and only if both $k_u \in (0, K_u)$ and $k_e \in (0, \kappa_e)$.

Differentiating equation (22) to verify that

$$\frac{\partial q_u}{\partial k_u} > 0 \text{ and } \frac{\partial}{\partial k_u}\left(\frac{k_u}{q_u f\left(q_u\right) \eta\left(q_u\right)}\right) > 0,$$

for all $k_e < \alpha\left(y_h - y_l\right) / \left(r + \delta\right)$ and $k_u < \left(y_h - b\right) / \left(r+\delta\right) - \left(1-\alpha\right)\left(y_h - y_l\right) / \left(r+\delta\right)$, and using (27), it follows that for each (k_u, k_e) such that conditions (19) and (23) are satisfied there exists a number $\kappa_u < \alpha\left(y_h - y_l\right) / \left(r+\delta\right)$ such that condition (26) is satisfied if and only if $k_u \geq \kappa_u$. This concludes the proof of Proposition 2. **QED**

Proof of Corollary 1

Consider the value κ_u given in Proposition 2. Note that (22) implies that

$$\lim_{k_u \to 0} \frac{k_u}{\eta\left(q_u\right) q_u f\left(q_u\right)} = \left(\frac{y_h - b}{r+\delta} - \frac{(1-\alpha) k_e}{\eta\left(q_b\right) q_b f\left(q_b\right) \alpha}\right)\left(\frac{r+\delta}{1+r+\delta}\right).$$

Hence, for any $k_e \in (0, \kappa_e)$, one can verify that $\kappa_u = 0$ if and only if

$$y_h - b \geq (\alpha + r + \delta) \frac{k_e}{\eta(q_b) q_b f(q_b) \alpha}.$$

The right side of the inequality is increasing in k_e. Hence, there is a number κ such that $\kappa_u = 0$ if and only if $k_e \leq \kappa$. Parts (i) and (ii) of Corollary 1 follow immediately from (27). **QED**

Proof of Proposition 3

First, we assume that $\rho = 1$ and characterize an interior solution to Problem (P1)–(P2), which disregards both the participation constraints and the informational constraints associated with pooling contracts. Then, we provide necessary and sufficient conditions under which the candidate interior solution supports an efficient steady-state allocation with positive job quits. Finally, we provide sufficient conditions under which the candidate interior solution is the unique solution to Problem (P1)–(P2).

Proposition 1 implies that a revealing equilibrium is such that $F_u = V(s_u)$ and $F_e(s, 1) = V(s)$, for $s \in \{(w_l, y_l), (w_h, y_h)\}$, where $x = (u, w_l, w_h) \in X^*$. Thus, we characterize the solution to Problem (P1)–(P2) with $\rho = 1$ in terms of the value function V in order to minimize clutter.

We begin by characterizing the solution to (P1) as a function of a worker's wage.

Lemma 1 *Let $k_e \in \big(0, \alpha(y_h - y_l)/(r+\delta)\big)$. For any $w \in [0, y_l]$, $(w_e(s), q_e(s))$, for $s = (w, y_l)$, is given by the unique pair (w', q') with $y_l < w' < y_h$ and $0 < q_a < q' \leq q_b < \infty$ that solves the following conditions:*

$$q' f(q') \alpha \left(\frac{y_h - w'}{r + \delta}\right) = k_e,$$

$$\frac{\frac{y_h - w'}{r+\delta}}{\frac{y_h - w'}{r+\delta} + \frac{w' - w}{r + \delta + (1-\delta)\alpha f(q')}} \leq \eta(q')$$

and $q' \leq q_b$ with complementary slackness, where q_a is given by (17) and $q_b > q_a$ is given by (18).

Proof: The first-order conditions for an interior solution to (P1) with $\rho = 1$ are:

$$\lambda q' = 1,$$

where λ is the relevant Lagrange multiplier, and

$$\frac{w'}{r+\delta}+\left(\frac{\delta}{r+\delta}\right)\frac{V\left(s_u\right)}{1+r}-\frac{V\left(\left(w,y_l\right)\right)}{1+r}=\lambda q'\left(\frac{1-\eta\left(q'\right)}{\eta\left(q'\right)}\right)\left(\frac{y_h-w'}{r+\delta}\right),$$

together with the zero-profit constraint

$$q'f\left(q'\right)\alpha\left(\frac{y_h-w'}{r+\delta}\right)=k_e.$$

This is the first condition stated in the lemma. The second condition follows from combining the first two first-order conditions above and the fact that the Bellman equation implies that a solution to the problem must be such that

$$\frac{w'}{r+\delta}+\left(\frac{\delta}{r+\delta}\right)\frac{V\left(s_u\right)}{1+r}-\frac{V\left(\left(w,y_l\right)\right)}{1+r}=\frac{w'-w}{r+\delta+\left(1-\delta\right)\alpha f\left(q'\right)}.$$

Clearly, $w_e\left(\left(w,y_l\right)\right)>y_l$ if and only if $q_e\left(\left(w,y_l\right)\right)>q_a$. The assumption in the lemma ensures that $0<q_a<\infty$.

Combining the two conditions stated in the proposition implies that an interior solution $q_e\left(\left(w,y_l\right)\right)$ is the unique value of q' that solves

$$\frac{y_h-w}{r+\delta}=\left(\frac{k_e}{q'f\left(q'\right)\alpha}\right)\left(1+\left(\frac{1-\eta\left(q'\right)}{\eta\left(q'\right)}\right)\left(\frac{r+\delta+\left(1-\delta\right)\alpha f\left(q'\right)}{r+\delta}\right)\right). \qquad (28)$$

It follows that $w\leq y_l$ implies that $q_e\left(\left(w,y_l\right)\right)\leq q_b$. Clearly, $\infty>q_b>q_a>0$. **QED**

Invert (28) to express the worker's current wage as a function of q':

$$W\left(q'\right)\equiv y_h-\left(\frac{k_e}{q'f\left(q'\right)\alpha}\right)\left(r+\delta+\left(\frac{1-\eta\left(q'\right)}{\eta\left(q'\right)}\right)\left(r+\delta+\left(1-\delta\right)\alpha f\left(q'\right)\right)\right), \qquad (29)$$

for all $q'\in\left(q_a,q_b\right]$, and note the following.

Lemma 2 *$W\left(q'\right)$ and $V\left(\left(W\left(q'\right),y_l\right)\right)$ are strictly increasing and concave functions of q' on $\left(q_a,q_b\right]$.*

Proof: It is easy to verify that the Bellman equation for $V((w, y_l))$ implies that

$$\begin{aligned}\frac{V((W(q'), y_l))}{1+r} &= \left(\frac{\delta}{r+\delta}\right)\frac{V(s_u)}{1+r} + \left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha f(q')}\right)\frac{W(q')}{r+\delta} \\ &+ \left(1 - \frac{r+\delta}{r+\delta+(1-\delta)\alpha f(q')}\right)\frac{w_e((W(q'), y_l))}{r+\delta}\end{aligned} \tag{30}$$

and, using the first-order conditions stated in Lemma 1, one can write

$$\frac{V((W(q'), y_l))}{1+r} = \frac{y_h}{r+\delta} + \left(\frac{\delta}{r+\delta}\right)\frac{V(s_u)}{1+r} - \frac{k_e}{\eta(q')\,q' f(q')\,\alpha}. \tag{31}$$

One can verify that

$$\frac{\partial}{\partial q}\left(\frac{V((W(q'), y_l))}{1+r}\right) = \frac{k_e}{q' f(q')\,\alpha}\left(\frac{\eta'(q')}{(\eta(q'))^2} + \frac{1}{q'}\left(\frac{1-\eta(q')}{\eta(q')}\right)\right),$$

which is positive on $(q_a, q_b]$. A sufficient condition for it to be strictly decreasing on $(q_a, q_b]$ is that $\eta'(q')/(\eta(q'))^2$ is a decreasing function, which follows from the concavity of η. Hence $V((W(q'), y_l))$ is strictly concave on $(q_a, q_b]$, as required.

Next, differentiating equation (28) with respect to w and q' one can verify that

$$\frac{\partial W(q')}{\partial q'} = (r+\delta+(1-\delta)\alpha f(q'))\frac{\partial}{\partial q'}\left(\frac{V((W(q'), y_l))}{1+r}\right),$$

which is positive and strictly decreasing on $(q_a, q_b]$, because both f and $\partial V/\partial q'$ are positive and strictly decreasing on $(q_a, q_b]$. Hence, W is strictly increasing and concave *on* $(q_a, q_b]$. **QED**

Let $M(s)$ denote the match surplus as a function of the worker's state and note that

$$\frac{M((w, y_h))}{1+r} = \frac{V((w, y_h))}{1+r} - \frac{V(s_u)}{1+r} + \frac{y_h - w}{r+\delta} \tag{32}$$

and

$$\frac{M((W(q'), y_l))}{1+r} = \frac{V((W(q'), y_l))}{1+r} - \frac{V(s_u)}{1+r} + \frac{y_l - W(q')}{r+\delta+(1-\delta)\alpha f(q')}. \tag{33}$$

Lemma 3 *$M((w, y_h))$ is independent of w; $M((W(q'), y_l))$ is a strictly concave function of q' on $(q_a, q_b]$ and it is maximized at $q' = q_b$; $M((W(q'), y_l)) - V((W(q'), y_l))$ is a strictly decreasing and convex function of q' on $(q_a, q_b]$.*

Proof: Fix $V(s_u)$. Noting that

$$\frac{V((w, y_h))}{1+r} = \frac{w}{r+\delta} + \frac{\delta}{r+\delta}\frac{V(s_u)}{1+r}$$

one can write

$$\frac{M((w, y_h))}{1+r} = \frac{y_h}{r+\delta} - \frac{r}{r+\delta}\frac{V(s_u)}{1+r},$$

which is independent of q'. Using (31), together with (29) and (33), one can write

$$\begin{aligned}\frac{M((W(q'), y_l))}{1+r} = {} & \frac{y_h}{r+\delta} - \frac{r}{r+\delta}\frac{V(s_u)}{1+r} - \frac{r+\delta}{r+\delta+(1-\delta)\alpha f(q')}\left(\frac{y_h - y_l}{r+\delta}\right) \\ & - \left(1 - \frac{r+\delta}{r+\delta+(1-\delta)\alpha f(q')}\right)\left(\frac{k_e}{q' f(q')\alpha}\right).\end{aligned} \tag{34}$$

where $M((w, y_h)) > M((W(q'), y_l))$ whenever $y_h > y_l$. Differentiating equation (34) one can verify that

$$\begin{aligned}\frac{\partial}{\partial q'}\left(\frac{M((W(q'), y_l))}{1+r}\right) = {} & \left(\frac{1-\delta}{(q')^2\left[r+\delta+(1-\delta)\alpha f(q')\right]}\right)\Bigg((1-\eta(q'))k_e \\ & - \left(\frac{(r+\delta)\eta(q')}{r+\delta+(1-\delta)\alpha f(q')}\right)\left(q' f(q')\alpha\left(\frac{y_h - y_l}{r+\delta}\right) - k_e\right)\Bigg).\end{aligned}$$

The term in the first line is decreasing in q' since $q' f(q')$ is strictly increasing on $(q_a, q_b]$. The terms in the second line are also decreasing in q', since f is decreasing, η is increasing and $q' f(q')$ is increasing on $(q_a, q_b]$, and $q' f(q')\alpha(y_h - y_l) \geq (r+\delta)k_e$ for $q' > q_a$. Hence $M((W(q'), y_l))$ is strictly concave on $(q_a, q_b]$. It is now easy to verify that equation (18) is a necessary and sufficient condition for $\partial M((W(q'), y_l))/\partial q' = 0$. Hence $M((W(q'), y_l))$ is maximized at $q' = q_b$.

Using equations (31) and (34) one can write

$$\frac{M(s)-\Big(V(s)-V(s_u)\Big)}{1+r} = \left(\frac{k_e}{q'f(q')\alpha}\right)\left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha f(q')}+\frac{1-\eta(q')}{\eta(q')}\right) - \left(\frac{y_h-y_l}{r+\delta+(1-\delta)\alpha f(q')}\right),$$

for $s=(W(q'),y_l)$, and differentiating this equation one can verify that

$$\frac{\partial}{\partial q}\left(\frac{M(s)-(V(s)-V(s_u)))}{1+r}\right)$$
$$= \left(\frac{(1-\delta)\alpha f'(q')}{[r+\delta+(1-\delta)\alpha f(q')]^2}\right)\left(y_h-y_l-\frac{(r+\delta)k_e}{q'f(q')\alpha}\right)$$
$$-\left(\frac{k_e}{q'f(q')\alpha}\right)\left(\left(\frac{1-\eta(q')}{q'}\right)\left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha f(q')}+\frac{1-\eta(q')}{\eta(q')}\right)+\frac{\eta'(q')}{(\eta(q'))^2}\right)$$

for $s=(W(q),y_l)$. The term in the first line of the right side is negative since $f'<0$ and $q'f(q')\alpha(y_h-y_l)\geq(r+\delta)k_e$ for $q>q_a$. The term subtracted in the second line is positive since $\eta(q)<1$ and $\eta'>0$. Hence $M(s)-(V(s)-V(s_u))$, for $s=(W(q'),y_l)$, is a strictly decreasing function of q' on $(q_a,q_b]$. Moreover, the term in the first line of the right side is an increasing function of q', because f' is increasing, $q'f(q')$ is increasing and f is decreasing. The term subtracted in the second line is a decreasing function of q', since $q'f(q')$ and η are increasing and f and $\eta'(q')/(\eta(q'))^2$ are decreasing. Hence, $M(s)-(V(s)-V(s_u))$, for $s=(W(q'),y_l)$, is a strictly convex function of q'. **QED**

Maintain the assumption that $\rho=1$. Consider the following problem, which disregards the constraints $V(s_u)\leq\min\{V((W(q'),y_l)),V(w,y_h)\}$ and $w\neq W(q')$.

$$V(s_u)=b+\frac{1}{1+r}\left[V(s_u)+\max_{w,q,q'}f(q)\Big(V_0(w,q')-V(s_u)\Big)\right], \tag{P3}$$

subject to

$$k_u \leq qf(q)\left(\frac{M_0(w,q') - \left(V_0(w,q') - V(s_u)\right)}{1+r}\right),$$

$$q' \in (q_a, q_b]\ ,\ w \leq y_h,$$

where

$$V_0(w,q') = \alpha V((w,y_h)) + (1-\alpha)V((W(q'),y_l)),$$

and

$$M_0(w,q') = \alpha M((w,y_h)) + (1-\alpha)M((W(q'),y_l)).$$

Let $\left(w_u^h, q_u, q_e^l\right)$ denote a solution to Problem (P3). Even though the objective is not concave in (w,q,q'), we provide conditions below under which its solution is unique, and it has $V(s_u) < \min\{V((W(q'),y_l)), V(w,y_h)\}$ and $w_u^h \neq W\left(q_e^l\right)$. It is then easy to see that $\left(w_u^h, W\left(q_e^l\right), q_u\right)$ solves problem (P2), with $\rho = 1$, since $q_e^l = q_e\left(\left(W\left(q_e^l\right), y_l\right)\right)$. One can readily verify that an *interior* solution to Problem (P3) is such that the total surplus of the match is maximized. Specifically, it must be that $\partial M_0(w,q')/\partial q' = 0$, which requires that $\partial M((W(q'),y_l))/\partial q' = 0$. Hence, Lemma 3 implies that $q_e^l = q_b$, thus

$$q_e((W(q_b),y_l)) = q_b, \tag{35}$$

and Lemma 1 then implies that

$$q_b f(q_b)\alpha\left(\frac{y_h - w_e((W(q_b),y_l))}{r+\delta}\right) = k_e, \tag{36}$$

where q_b is given by equation (18). Comparing (18) and (28), it follows that $W\left(q_e^l\right) = y_l$, thus

$$w_u^l = y_l. \tag{37}$$

Next note that $\left(w_u^h, q_u\right)$ is given by any pair (w,q) that satisfies the zero-profit condition

$$qf(q)\alpha\left(\frac{y_h-w}{r+\delta}\right)=k_u \tag{38}$$

and the standard condition for matching efficiency in the market for unemployed workers, given by

$$\frac{\alpha\left(\frac{y_h-w}{r+\delta}\right)}{\alpha\left(\frac{y_h-w}{r+\delta}\right)+\left(\alpha\frac{V((w,y_h))}{1+r}+(1-\alpha)\frac{V((y_l,y_l))}{1+r}-\frac{V(s_u)}{1+r}\right)}=\eta\left(q\right) \tag{39}$$

together with the Bellman equation in Problem (P3). Note that

$$\begin{aligned} V\left(s_u\right) &= b+\left(1-f\left(q_u\right)\right)\frac{V\left(s_u\right)}{1+r}+f\left(q_u\right)\frac{V_0\left(w_u^h,q_b\right)}{1+r} \\ &= b+\left(1-f\left(q_u\right)\right)\frac{V\left(s_u\right)}{1+r}+f\left(q_u\right)\left(\frac{V\left(s_u\right)}{1+r}+\left(\frac{1-\eta\left(q_u\right)}{\eta\left(q_u\right)}\right)\frac{k_u}{q_uf\left(q_u\right)}\right), \end{aligned}$$

where the first equality comes from the Bellman equation in Problem (P3) and the second equality follows from (39) and (38). It follows that

$$\frac{rV\left(s_u\right)}{1+r}=b+\left(\frac{1-\eta\left(q_u\right)}{\eta\left(q_u\right)}\right)\frac{k_u}{q_u}. \tag{40}$$

Using this equation, together with equations (38) and (39) and the fact that

$$\frac{V_0\left(w_u^h,q_e^l\right)}{1+r}-\frac{V\left(s_u\right)}{1+r}=\alpha\frac{w_u^h}{r+\delta}+(1-\alpha)\left(\frac{y_h}{r+\delta}-\frac{k_e}{\eta\left(q_b\right)q_bf\left(q_b\right)\alpha}\right)-\left(\frac{r}{r+\delta}\right)\frac{V\left(s_u\right)}{1+r}, \tag{41}$$

it follows that q_u is given by (22). Recall that there is a unique value $q_u \in (0,\infty)$ that solves (22) if and only if condition (23) holds. In turn, (23) implies that $k_u < y_h/(r+\delta)$ and so there must be a number $w \in (0, y_h)$ that solves equation (38).

Lemma 4 *Generically, an efficient steady-state allocation with positive job quits can be supported by a refined equilibrium if and only if*

$$\frac{(1-\alpha)\,k_e}{\alpha q_bf(q_b)\eta(q_b)}+\frac{k_u}{q_uf(q_u)\eta(q_u)}\geq\frac{k_u}{\alpha q_uf\left(q_u\right)},$$

where q_b and q_u are the unique solutions to equations (18) and (22).

Proof: Assume the conditions in Proposition 2 hold, so there exists an efficient steady-state allocation with positive job quits. We show that, generically, the condition in the lemma is both necessary and sufficient for the efficient allocation to be part of the unique solution to Problem (P1)–(P2). The only exception is the non-generic case in which the efficient allocation requires that the solution to (38) has exactly $w_u^h = y_l$. The lemma then follows immediately from Proposition 1.

Consider a solution to Problem (P3). Note that

$$V((y_l, y_l)) - V(s_u) = V_0\left(w_u^h, q_b\right) - V(s_u) - \alpha\left[V\left(\left(w_u^h, y_h\right)\right) - V((y_l, y_l))\right],$$

$$V\left(\left(w_u^h, y_h\right)\right) - V(s_u) = V_0\left(w_u^h, q_b\right) - V(s_u) + (1-\alpha)\left[V\left(\left(w_u^h, y_h\right)\right) - V((y_l, y_l))\right],$$

where $V_0\left(w_u^h, q_b\right) = \alpha V\left(\left(w_u^h, y_h\right)\right) + (1-\alpha) V((y_l, y_l))$, and use the fact that

$$\frac{V_0\left(w_u^h, q_b\right)}{1+r} - \frac{V(s_u)}{1+r} = \left(\frac{1-\eta(q_u)}{\eta(q_u)}\right)\frac{k_u}{q_u f(q_u)}$$

and the fact that

$$\frac{V\left(\left(w_u^h, y_h\right)\right)}{1+r} - \frac{V((y_l, y_l))}{1+r} = \frac{k_e}{\eta(q_b) q_b f(q_b) \alpha} - \frac{k_u}{\alpha q_u f(q_u)} \tag{42}$$

to write

$$\frac{V((y_l, y_l))}{1+r} - \frac{V(s_u)}{1+r} = \frac{k_u}{\eta(q_u) q_u f(q_u)} - \frac{k_e}{\eta(q_b) q_b f(q_b)}. \tag{43}$$

Note that the right sides of (25) and (43) are identical and so

$$\frac{V((y_l, y_l))}{1+r} - \frac{V(s_u)}{1+r} = \frac{J_e(y_l)}{1+r} - \frac{J_u}{1+r},$$

which is positive whenever the efficient allocation has positive job quits.

Using (24), (42) and (43), it follows that

$$\frac{V\left(\left(w_u^h, y_h\right)\right)}{1+r} - \frac{V(s_u)}{1+r} = \frac{J_e(y_h)}{1+r} - \frac{J_u}{1+r} - \frac{k_u}{\alpha q_u f(q_u)}, \tag{44}$$

which is positive if and only if the condition stated in the lemma is satisfied.

Now, suppose the condition in the lemma is satisfied. Note that the right sides of (21) and (40) are identical, so $V(s_u) = J_u$. Since Problem (P3) is simply an unconstrained version of Problem (P2), it follows that the unique solution to Problem (P2) is such that $\rho = 1$, as conjectured. **QED**

Next, differentiate equation (22) to verify that

$$\frac{\partial q_u}{\partial k_u} > 0 \text{ and } \frac{\partial}{\partial k_u}\left(\frac{k_u}{q_u f(q_u)}\right) > 0,$$

for all $k_e < \alpha(y_h - y_l)/(r+\delta)$ and $k_u < (y_h - b)/(r+\delta) - (1-\alpha)(y_h - y_l)/(r+\delta)$, with

$$\lim_{k_u \to 0} \frac{k_u}{q_u f(q_u)} = 0.$$

It follows from (42) that for any $k_e \in \big(0, \alpha(y_h - y_l)/(r+\delta)\big)$ there exists a number $\kappa_0 \in \big(0, \alpha(y_h - y_l)/(r+\delta)\big)$ such that $V\left(\left(w_u^h, y_h\right)\right) - V((y_l, y_l)) > 0$, with $w_l = y_l < w_h < y_h$, and, furthermore, $k_u < K_u$, for all $k_u \in (0, \kappa_0)$. Since $w_u^h \neq y_l$, equilibrium wages reveal the current productivity of employed workers.

Assuming that $(y_h - b)/(y_h - y_l) > (r+\delta+\alpha)/(r+\delta+(1-\delta)\alpha)$, Part (i) of Proposition 3 follows from Corollary 1. Since the solution to Problem (P1)–(P2) is unique, parts (ii) and (iii) follow from Proposition 1. It is straightforward to characterize ψ. The unemployment rate is

$$\psi(s_u) = \frac{\delta}{\delta + f(q_u)}.$$

The wage distribution has three mass points: $\left(w_u^l, w_u^h, w_e\left(\left(w_u^l, y_l\right)\right)\right)$, where $w_u^l = y_l$. The mass of workers earning the wage w_u^l is

$$\psi\left(\left(w_u^l, y_l\right)\right) = \left(\frac{(1-\alpha) f(q_u)}{\delta + (1-\delta)\alpha f(q_b)}\right)\psi(s_u),$$

where $q_b \equiv q_e\left(\left(w_u^l, y_l\right)\right)$. The mass of workers earning the wage w_u^h is

$$\psi\left(\left(w_u^h, y_h\right)\right) = \left(\frac{\alpha f(q_u)}{\delta}\right)\psi(s_u)$$

and the mass of workers earning the wage $w_e\left(\left(w_u^l, y_l\right)\right)$ is

$$\psi\left(\left(w_e\left(\left(w_u^l, y_l\right)\right), y_h\right)\right) = \left(\frac{(1-\delta)\,\alpha f\left(q_b\right)}{\delta}\right)\psi\left(\left(w_u^l, y_l\right)\right). \textbf{ QED}$$

Proof of Proposition 4

Suppose, to the contrary, that there is a refined equilibrium that supports an efficient allocation with positive job quits. From equation (38), $k_u > \alpha\left(y_h - y_l\right)/\left(r+\delta\right)$ implies that $w_u^h < y_l$. It follows that for each $k_e > 0$ and $k_u > \alpha\left(y_h - y_l\right)/\left(r+\delta\right)$, there is a number $\beta \in \left(0, y_l\right)$ such that $w_u^h < b$ for any $b \in \left(\beta, y_l\right)$. But then it must be that $V\big(\left(w_h, y_h\right)\big) - V\left(s_u\right) < 0$; a contradiction. To conclude the proof, note that Proposition 2 implies that the efficient allocation has positive job quits for any $\left(b, k_u, k_e\right)$ such that $b \in \left(\beta, y_l\right)$, $k_u \in \left(\alpha\left(y_h - y_l\right)/\left(r+\delta\right), K_u\right)$ and $k_e \in \left(0, \kappa_e\right)$. **QED**

Proof of Proposition 5

Consider the following analog of Problem (P3):

$$V\left(s_u\right) = b + \frac{1}{1+r}\left[V\left(s_u\right) + \max_{w,q,q'} f\left(q\right)\Big(V_0\left(w, q'\right) - V\left(s_u\right)\Big)\right], \tag{P4}$$

subject to

$$k_u \leq q f\left(q\right)\left(\frac{M_0\left(w, q'\right) - \Big(V_0\left(w, q'\right) - V\left(s_u\right)\Big)}{1+r}\right),$$

$$q' \in \left(q_a, q_b\right]\ ,\ w \leq y_h,$$

$$V\left(\left(w, y_h\right)\right) = V\left(s_u\right).$$

One can verify that a solution to Problem (P4) satisfies the following conditions:

$$\lambda q \leq \frac{\partial V_0/\partial q'}{\partial V_0/\partial q' - \partial M_0/\partial q'}, \tag{45}$$

with equality if $q' < q_b$,

$$\lambda q\left(\frac{1-\eta\left(q\right)}{\eta\left(q\right)}\right)\frac{k_u}{q}\left(\frac{1}{f(q)} + \frac{1-\alpha}{r+\delta}\right) = \left(1-\alpha\right)\left(\frac{y_h - b}{r+\delta} - \frac{k_e}{\eta\left(q'\right)q' f\left(q'\right)\alpha}\right), \tag{46}$$

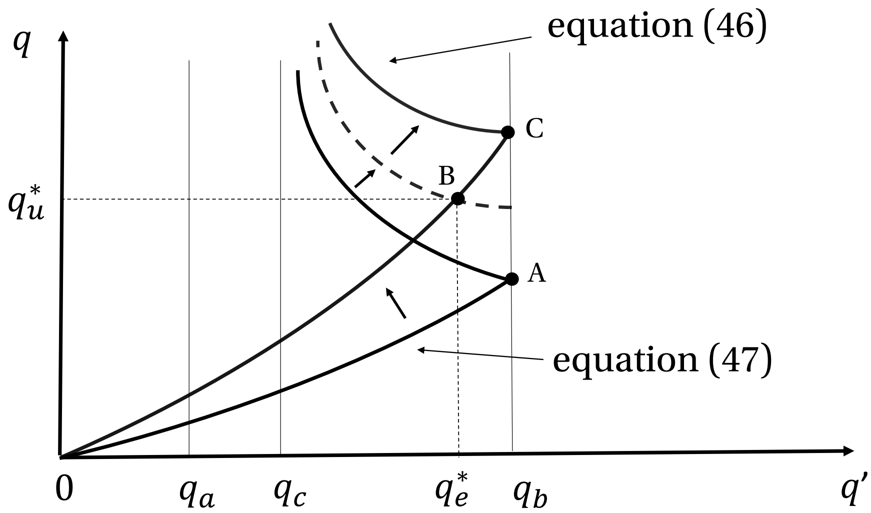

Figure 1: existence and uniqueness of a solution to Problem (P4)

and

$$\alpha\left[\left(\frac{r+\delta}{r+\delta+(1-\alpha)f(q)}\right)\frac{y_h-b}{r+\delta}+\left(1-\frac{r+\delta}{r+\delta+(1-\alpha)f(q)}\right)\frac{k_e}{\eta\left(q'\right)q'f\left(q'\right)\alpha}\right]$$
$$-\frac{k_u}{qf(q)}=\frac{(1-\alpha)(r+\delta)}{r+\delta+(1-\delta)f(q')\alpha}$$
$$\times\left[\frac{y_h-y_l}{r+\delta}-\frac{k_e}{q'f\left(q'\right)\alpha}\left(1+\left(\frac{1-\eta\left(q'\right)}{\eta\left(q'\right)}\right)\left(\frac{r+\delta+(1-\delta)\,\alpha f\left(q'\right)}{r+\delta}\right)\right)\right]. \quad (47)$$

Equation (46) characterizes q as a decreasing function of q' on (q_c, q_b), with $\lim_{q'\to q_c+} q=\infty$, where q_c is given by

$$\frac{y_h-b}{r+\delta}-\frac{k_e}{\eta\left(q_c\right)q_cf\left(q_c\right)\alpha}=0, \quad (48)$$

while (47) characterizes q as an increasing function of q' on (q_c, q_b), with $\lim_{q'\to 0+} q=0$.

Propositions 3 and 4 imply that there exist parameter values $(y_l, y_h, \alpha, r, b, \delta, k_e)$ and a number $\kappa\in(0,K_u)$ such that: (1) if $k_u=\kappa$, then an efficient allocation with positive job quits is supported by a refined equilibrium and it has $V\left((w,y_h)\right)=V\left(s_u\right)$ and (2) there is a non-empty interval $(\kappa,\kappa')\subset(0,K_u)$ such that an efficient allocation with positive job quits cannot be supported by a refined equilibrium for any $k_u\in(\kappa,\kappa')$.

One can verify that equations (46) and (47) are equivalent to equations (18) and (22) if $k_u = \kappa$. To that end, use (46) to write the left side of (47) as

$$\alpha \left[\frac{y_h - b}{r+\delta} - \lambda q \left(\frac{1-\eta(q)}{\eta(q)} \right) \frac{k_u}{q(r+\delta)} \right] - \frac{k_u}{qf(q)}$$

and note that (18) implies that the right side of (47) is equal to 0 if and only if $q' = q_b$. Writing (46) as

$$\begin{aligned} \frac{y_h - b}{r+\delta} - \frac{(1-\alpha)k_e}{\eta(q')\,q'f(q')\,\alpha} =& \lambda q \left[\left(\frac{1-\eta(q)}{\eta(q)} \right) \frac{k_u}{q(r+\delta)} + \frac{k_u}{\eta(q)qf(q)} \right] \\ &+ \alpha \left[\frac{y_h - b}{r+\delta} - \lambda q \left(\frac{1-\eta(q)}{\eta(q)} \right) \frac{k_u}{q(r+\delta)} \right] - \lambda q \left(\frac{k_u}{qf(q)} \right), \end{aligned}$$

it follows that equations (46) and (47) are equivalent to equations (18) and (22) if and only if $\lambda q = 1$ and $q' = q_b$. In this case, the unique solution to Problem (P4) is shown as point A in Figure 1. Recall that this interior solution holds at $q' = q_b$.

Next, note that for $k_u \in (\kappa, \kappa')$ the unique solution to Problem (P4) can be interior, as illustrated in point B, or a corner, as in point C. If $q_e^* < q_b$, then equations (46) and (47) solve for (q_u^*, q_e^*), where (45) holds with equality and pins down λq_u^*. If $q_e^* = q_b$, then equation (47) evaluated at $q' = q_b$ determines q_u^*, whereas equation (46) evaluated at $q = q_u^*$ and $q' = q_b$ determines λq_u^*.

Noting that the number κ' can be chosen arbitrarily close to κ, it follows that the solution to Problem (P4), for all $k_u \in (\kappa, \kappa')$, can be arbitrarily close to the efficient allocation. The arguments we used in Proposition 3 now imply that κ' can be chosen so the unique solution to Problem (P2) is given by the solution to Problem (P4).

Parts (i) and (ii) of the proposition follow immediately from Proposition 1. Part (iii) follows from comparing equations (46) and (47) with equations (18) and (22), for fixed $k_u \in (\kappa, \kappa')$, and noting that $\lambda q > 1$ if $q' < q_b$, whereas $\lambda q < 1$ if $q' = q_b$. **QED**

Proof of Proposition 6

We first characterize the solution to Problem (P1)–(P2) subject to $\rho = 1 - \alpha$. Then, we provide conditions under which it is the unique solution to Problem (P1)–(P2). As we did above, we characterize the solution to Problem (P1)–(P2) with $\rho = 1 - \alpha$ in terms of the value function V in order to minimize clutter.

Suppose $\rho = 1 - \alpha$. The first part of the proof parallels that of Proposition 3. An interior solution of Problem (P1) satisfies the familiar matching efficiency condition

$$\frac{\frac{y_h-w'}{r+\delta}}{\frac{y_h-w'}{r+\delta}+\frac{w'-w}{r+\delta+(1-\delta)\alpha f(q')}}=\eta\left(q'\right),$$

and the zero-profit condition

$$q'f\left(q'\right)\left(1-\alpha\right)\alpha\left(\frac{y_h-w'}{r+\delta}\right)=k_e, \tag{49}$$

where potential poachers anticipate that a fraction α will turn down their job offers.

Because the objective function in Problem (P2) is not generally concave in (w,q), we adopt the same strategy we followed in the proof of Proposition 3. That is, we view the solution to Problem (P1) as a mapping from the workers' quit rates to their entry wages, rather than the reverse, and then treat current and future quit rates as the relevant choice variables in Problem (P2). To that end, use the above first-order conditions to express the worker's entry wage as a function of q':

$$\widetilde{W}\left(q'\right)\equiv y_h-\left(\frac{k_e}{q'f\left(q'\right)\left(1-\alpha\right)\alpha}\right)\left(r+\delta+\left(\frac{1-\eta\left(q'\right)}{\eta\left(q'\right)}\right)\left(r+\delta+\left(1-\delta\right)\alpha f\left(q'\right)\right)\right). \tag{50}$$

We have that $q_e\left(s\right)=q_e\left(\left(w,y_l\right)\right)=q_e\left(\left(w,y_h\right)\right)$ and $q_e\left(s\right)\in\left(\widehat{q}_a,\widehat{q}_b\right]$, where $w_e\left(s\right)>y_l$ if and only if $q_e\left(s\right)>\widehat{q}_a$ and $\widetilde{W}\left(q_e\left(s\right)\right)\leq y_l$ if and only if $q_e\left(s\right)\leq\widehat{q}_b$ and where $\widehat{q}_a$ and $\widehat{q}_b$ are given by

$$\widehat{q}_af\left(\widehat{q}_a\right)\left(1-\alpha\right)\alpha\left(\frac{y_h-y_l}{r+\delta}\right)=k_e \tag{51}$$

and

$$\frac{y_h-y_l}{r+\delta}=\left(\frac{k_e}{\widehat{q}_bf\left(\widehat{q}_b\right)\left(1-\alpha\right)\alpha}\right)\left(1+\left(\frac{1-\eta\left(\widehat{q}_b\right)}{\eta\left(\widehat{q}_b\right)}\right)\left(\frac{r+\delta+\left(1-\delta\right)\alpha f\left(\widehat{q}_b\right)}{r+\delta}\right)\right). \tag{52}$$

Clearly, $\infty>\widehat{q}_b>\widehat{q}_a>0$ if $k_e<\left(1-\alpha\right)\alpha\left(y_h-y_l\right)/\left(r+\delta\right)$.

Consider the following analog of Problem (P4):

$$V\left(s_u\right)=b+\frac{1}{1+r}\left[V\left(s_u\right)+\max_{q,q'}f\left(q\right)\left(\widetilde{V}_0\left(q'\right)-V\left(s_u\right)\right)\right], \tag{P5}$$

subject to

$$k_u \leq qf(q)\left(\frac{\widetilde{M}_0(q') - \left(\widetilde{V}_0(q') - V(s_u)\right)}{1+r}\right),$$

$$q' \in (\widehat{q}_a, \widehat{q}_b].$$

$\widetilde{V}_0(q')$ denotes the *ex ante* value of a match to a worker as a function of q':

$$\begin{aligned}\widetilde{V}_0(q') &\equiv \alpha V\left(\left(\widetilde{W}(q'), y_h\right)\right) + (1-\alpha) V\left(\left(\widetilde{W}(q'), y_l\right)\right)\\ &= V\left(\left(\widetilde{W}(q'), y_h\right)\right) = V\left(\left(\widetilde{W}(q'), y_l\right)\right)\end{aligned}$$

and $\widetilde{M}_0(q')$ denotes the *ex ante* surplus associated with the match:

$$\widetilde{M}_0(q') \equiv \alpha \widetilde{M}\left(\left(\widetilde{W}(q'), y_h\right)\right) + (1-\alpha)\widetilde{M}\left(\left(\widetilde{W}(q'), y_l\right)\right),$$

where $\widetilde{M}\left(\left(\widetilde{W}(q'), y_l\right)\right)$ is the *ex post* surplus associated with a low productivity match:

$$\frac{\widetilde{M}\left(\left(\widetilde{W}(q'), y_l\right)\right)}{1+r} = \frac{V\left(\left(\widetilde{W}(q'), y_l\right)\right) - V(s_u)}{1+r} + \frac{r+\delta}{r+\delta+(1-\delta)\alpha f(q')}\frac{y_l - \widetilde{W}(q')}{r+\delta}$$

and $\widetilde{M}\left(\left(\widetilde{W}(q'), y_h\right)\right)$ is the *ex post* surplus associated with a high productivity match:

$$\begin{aligned}\frac{\widetilde{M}\left(\left(\widetilde{W}(q'), y_h\right)\right)}{1+r} &= \frac{V\left(\left(\widetilde{W}(q'), y_h\right)\right) - V(s_u)}{1+r}\\ &\quad + \left(1 - \frac{r+\delta}{r+\delta+(1-\delta)\alpha f(q')}\right)\frac{y_h - w_e\left(\left(\widetilde{W}(q'), y_h\right)\right)}{r+\delta}\\ &\quad + \left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha f(q')}\right)\frac{y_h - \widetilde{W}(q')}{r+\delta},\end{aligned}$$

which reflects the fact that *ex post* well matched workers will search for outside offers solely to elicit a retention offer from their current employer.

Noting that

$$\frac{\widetilde{V}_0\left(q'\right)-V\left(s_u\right)}{1+r}=\frac{y_h}{r+\delta}-\left(\frac{r}{r+\delta}\right)\frac{V\left(s_u\right)}{1+r}-\frac{k_e}{\eta\left(q'\right)q'f\left(q'\right)\left(1-\alpha\right)\alpha} \tag{53}$$

and using (49)–(50), one can verify that

$$\begin{aligned}\frac{\widetilde{M}_0\left(q'\right)}{1+r}&=\frac{y_h}{r+\delta}-\left(\frac{r}{r+\delta}\right)\frac{V\left(s_u\right)}{1+r}\\ &\quad-\left(1-\alpha\right)\left(\frac{r+\delta}{r+\delta+\left(1-\delta\right)\alpha f\left(q'\right)}\right)\left(\frac{y_h-y_l}{r+\delta}\right)\\ &\quad-\left(1-\alpha\right)\left(1-\frac{r+\delta}{r+\delta+\left(1-\delta\right)\alpha f\left(q'\right)}\right)\left(\frac{k_e}{q'f\left(q'\right)\left(1-\alpha\right)\alpha}\right).\end{aligned}$$

Lemma 5 *(i) $\widetilde{W}\left(q'\right)$ and $\widetilde{V}_0\left(q'\right)$ are strictly increasing and concave functions of q' on $(\widehat{q}_a,\widehat{q}_b]$. (ii) $\widetilde{M}_0\left(q'\right)$ is a strictly concave function of q' on $(\widehat{q}_a,\widehat{q}_b]\subset(0,\infty)$ and it is maximized at $q'=\widehat{q}_b$; $\widetilde{M}_0\left(q'\right)-\widetilde{V}_0\left(q'\right)$ is a strictly decreasing and convex function of q' on $[\widehat{q}_a,\widehat{q}_b]$.*

Proof: It replicates the arguments in Proposition 3 with minor changes. **QED**

The first-order conditions for an interior solution of Problem (P5) are given by

$$\frac{\widetilde{V}_0\left(q'\right)-V\left(s_u\right)}{1+r}=\lambda q\left(\frac{1-\eta\left(q\right)}{\eta\left(q\right)}\right)\frac{k_u}{qf\left(q\right)}, \tag{54}$$

$$\lambda=\frac{f\left(q\right)\partial\widetilde{V}_0/\partial q'}{qf\left(q\right)\left(\partial\widetilde{V}_0/\partial q'-\partial\widetilde{M}_0/\partial q'\right)} \tag{55}$$

and

$$qf\left(q\right)\left(\frac{\widetilde{M}_0\left(q'\right)-\left(\widetilde{V}_0\left(q'\right)-V\left(s_u\right)\right)}{1+r}\right)=k_u, \tag{56}$$

where λ is the multiplier associated with the employer's zero-profit constraint, given by (56). Equation (54) coincides with the standard matching efficiency condition if and only if the multiplier equals $1/q$. Consider equation (55). The multiplier is the expected value of surplus to the worker associated with a higher labor market queue at the margin $(f(q)\,\partial\widetilde{V}_0/\partial q')$ evaluated in terms of the employer's surplus $\left(qf(q)\left(\partial\widetilde{V}_0/\partial q' - \partial\widetilde{M}_0/\partial q'\right)\right)$. The expected surplus of a match is maximized at $\partial\widetilde{M}_0/\partial q' = 0$, which implies that $\lambda = 1/q$. Lemma 5 implies that this happens exactly at the corner when $\widetilde{W}(q') = y_l$.

Following similar steps as in the proof of Proposition 3 one can verify that an interior solution to Problem (P5) satisfies

$$\frac{y_h - b}{r+\delta} - \frac{k_e}{\eta(q')\,q'f(q')\,(1-\alpha)\,\alpha} = \lambda q\left(\frac{1-\eta(q)}{\eta(q)}\right)\frac{k_u}{q}\left(\frac{1}{f(q)} + \frac{1}{r+\delta}\right), \tag{57}$$

where λ is given by (55), and

$$\begin{aligned}\frac{k_u}{qf(q)} = &-(1-\alpha)\left(\frac{r+\delta}{r+\delta+(1-\delta)\,\alpha f(q')}\right)\left(\frac{y_h-y_l}{r+\delta}\right)\\ &+\left(\frac{k_e}{q'f(q')\,(1-\alpha)\,\alpha}\right)\left(\frac{(1-\alpha)(r+\delta)}{r+\delta+(1-\delta)\,\alpha f(q')} + \frac{1-\eta(q')}{\eta(q')} + \alpha\right).\end{aligned} \tag{58}$$

Lemma 6 *There is a number $\kappa_0 \in \big(0, (1-\alpha)\,\alpha\,(y_h - y_l)/(r+\delta)\big)$ such that equations (55), (57) and (58) have a unique solution $(\lambda, q, q') = (\lambda^*, q_u^*, q_e^*)$, with $q_u^* \in (0,\infty)$, $q_e^* \in (\widehat{q}_a, \widehat{q}_b)$, and $\lambda^* q_u^* > 1$, for all (k_u, k_e) such that $k_e \in (0, \kappa_0)$ and $k_u \in \big[\alpha\,(y_h - y_l)/(r+\delta), \alpha\,(y_h - b)/(r+\delta)\big)$, where $\widehat{q}_a$ is given by (51) and $\widehat{q}_b$ is given by (52).*

Proof: Differentiating equation (55) one can verify that the following inequality is necessary and sufficient for $\partial(\lambda q)/\partial q' < 0$:

$$\frac{-\partial^2\widetilde{M}_0/\partial q'^2}{-\partial^2\widetilde{V}_0/\partial q'^2} > \frac{\partial\widetilde{M}_0/\partial q'}{\partial\widetilde{V}_0/\partial q'}.$$

The left side of the inequality is greater than one, since $\widetilde{M}_0 - \widetilde{V}_0$ *is a strictly convex function of* q'. The right side is smaller than one, since $\widetilde{M}_0 - \widetilde{V}_0$ *is a strictly decreasing function of* q'. Hence, $\partial(\lambda q)/\partial q' < 0$. Moreover, note that $\lambda q \geq 1$ if and only if $\partial\tilde{M}_0/\partial q' \geq 0$.

Equation (57) characterizes q as a decreasing function of q' on $(\widehat{q}_c, \widehat{q}_b)$, where $\widehat{q}_c$ is

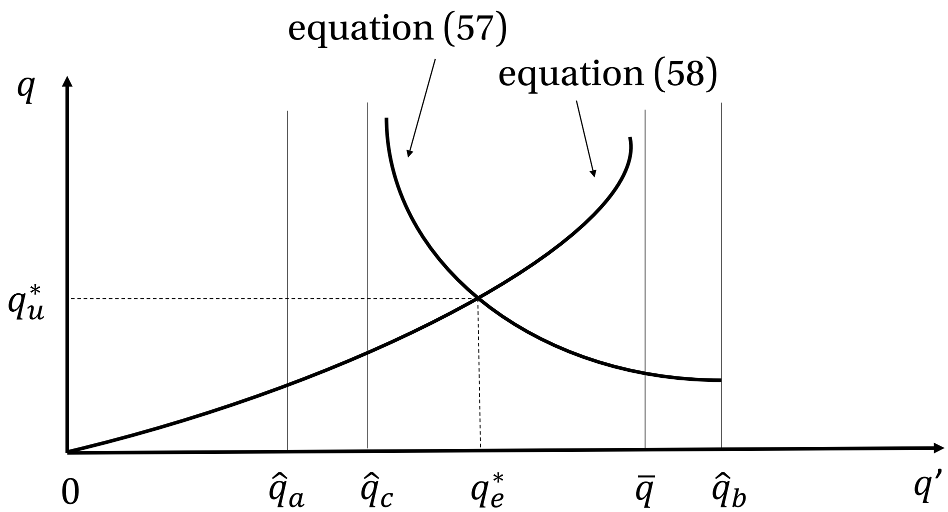

Figure 2: existence and uniqueness of an interior solution to Problem (P5)

given by

$$\frac{y_h - b}{r+\delta} - \frac{k_e}{\eta\left(\widehat{q}_c\right)\widehat{q}_c f\left(\widehat{q}_c\right)(1-\alpha)\alpha} = 0. \tag{59}$$

To verify that $\widehat{q}_c < \widehat{q}_b$, note that (52) implies that

$$\frac{k_e}{\eta\left(\widehat{q}_b\right)\widehat{q}_b f\left(\widehat{q}_b\right)(1-\alpha)\alpha} < \frac{y_h - y_l}{r+\delta}, \tag{60}$$

for all $k_e < (1-\alpha)\alpha(y_h - y_l)/(r+\delta)$, which, together with the fact that $y_l > b$, implies that $\widehat{q}_b > \widehat{q}_c$, as required. It is evident that $\widehat{q}_c > 0$. Furthermore, as illustrated in Figure 2, q approaches ∞ as q' approaches $\widehat{q}_c$, whereas q approaches some number in $(0,\infty)$ as q' approaches $\widehat{q}_b$.

Equation (58) characterizes q as an increasing function of q' on a subset of $(\widehat{q}_c, \widehat{q}_b)$. Specifically, note that q approaches zero as q' approaches zero. Hence, q approaches some number in $(0,\infty)$ as q' approaches $\widehat{q}_c$.

Next, writing (52) as

$$\frac{\alpha k_e}{\eta\left(\widehat{q}_b\right)\widehat{q}_b f\left(\widehat{q}_b\right)(1-\alpha)\alpha} = -(1-\alpha)\left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha f\left(\widehat{q}_b\right)}\right)\left(\frac{y_h-y_l}{r+\delta}\right) + \left(\frac{k_e}{\widehat{q}_b f\left(\widehat{q}_b\right)(1-\alpha)\alpha}\right)\left(\frac{(1-\alpha)(r+\delta)}{r+\delta+(1-\delta)\alpha f\left(\widehat{q}_b\right)}+\frac{1-\eta\left(\widehat{q}_b\right)}{\eta\left(\widehat{q}_b\right)}+\alpha\right),$$

and comparing this with (58), it follows that

$$\frac{\alpha k_e}{\eta\left(\widehat{q}_c\right)\widehat{q}_c f\left(\widehat{q}_c\right)(1-\alpha)\alpha} > k_u \geq \frac{\alpha k_e}{\eta\left(\widehat{q}_b\right)\widehat{q}_b f\left(\widehat{q}_b\right)(1-\alpha)\alpha}$$

is necessary and sufficient to ensure that there is a number $\bar{q} \in (\widehat{q}_c, \widehat{q}_b]$ such that q approaches ∞ as q' approaches $\bar{q}$. To see why, note that the first inequality is necessary and sufficient to ensure that the right side of (58) becomes larger than k_u as q' approaches $\widehat{q}_c$. Then note that the second inequality is necessary and sufficient to ensure that the right side of (58) becomes smaller than k_u as q' approaches $\widehat{q}_b$.

It follows from (59) and (60) that

$$\alpha\frac{y_h-b}{r+\delta} > k_u \geq \alpha\frac{y_h-y_l}{r+\delta} \tag{61}$$

is sufficient to ensure that there is a number $\bar{q} \in (\widehat{q}_c, \widehat{q}_b)$ such that q approaches ∞ as q' approaches $\bar{q}$. Hence, (61) is sufficient to ensure that equations (55), (57) and (58) have a unique solution $(\lambda, q, q') = (\lambda^*, q_u^*, q_e^*)$, with $q_u^* \in (0, \infty)$, $q_e^* \in (\widehat{q}_c, \widehat{q}_b)$ and $\lambda^* q_u^* > 1$. Figure 2 illustrates this.

It remains to provide conditions under which $q_e^* > \widehat{q}_a$. Comparing the definition of $\widehat{q}_a$ in (51) and that of $\widehat{q}_c$ in (59), it follows that there is a number $\kappa_0 \in \big(0, (1-\alpha)\alpha(y_h-y_l)/(r+\delta)\big)$ such that $\widehat{q}_a \leq \widehat{q}_c$, for all $k_e \in (0, \kappa_0)$. Hence, $q_e^* \in (\widehat{q}_a, \widehat{q}_b)$ for all $k_e \in (0, \kappa_0)$, as required. **QED**

Next, we provide sufficient conditions under which the unique solution to Problem (P2) is given by the solution to Problem (P5). To that end, consider a deviation in the market for entry jobs. First, suppose the deviating contract $\left(w_l^d, w_h^d\right)$ is such that $w_h^d \neq w_l^d = y_l$. Then, the profits of the deviating firm must be such that

$$q^d f\left(q^d\right)\alpha\left(\frac{y_h-w_h^d}{r+\delta}\right) \geq k_u,$$

where q^d is the corresponding queue. Accordingly, it must be that $w_h^d < y_l$ for all $k_u \geq \alpha\left(y_h - y_l\right)/\left(r+\delta\right)$. Next note that

$$\frac{V\left(\left(w^d, y_h\right)\right)}{1+r} - \frac{V(s_u)}{1+r} = \frac{w^d}{r+\delta} - \left(\frac{r}{r+\delta}\right)\frac{V(s_u)}{1+r},$$

where $V(s_u)$ is the value of unemployment that solves Problem (P5). Since $b < rV(s_u)/\left(1+r\right)$, for all $b \in (0, y_l)$, it follows that for each $\alpha \in (0,1)$ there is a number $\beta_0 \in (0, y_l)$ such that $V\left(\left(w_h^d, y_h\right)\right) - V\left(s_u\right) < 0$ for all $b \in (\beta_0, y_l)$ and all $k_u \geq \alpha\left(y_h - y_l\right)/\left(r+\delta\right)$. When this is the case, unemployed workers who meet a deviating firm are better off rejecting any job offer associated with a high productivity match. But then a deviating contract such that $w_h^d \neq w_l^d = y_l$ cannot be profitable for the firm.

Now suppose that the deviating contract $\left(w_l^d, w_h^d\right)$ is such that $w_h^d \neq w_l^d \neq y_l$. There are two cases to consider. If the deviating contract is such that $V\left(\left(w_h^d, y_h\right)\right) - V\left(s_u\right) < 0$, then the contract attracts unemployed workers if and only if

$$f\left(q^d\right)\left[(1-\alpha)V\left(\left(w_l^d, y_l\right)\right) - V\left(s_u\right)\right] > f\left(q_u^*\right)\left(\widetilde{V}_0\left(q_e^*\right) - V\left(s_u\right)\right),$$

and if $V\left(\left(w_h^d, y_h\right)\right) - V\left(s_u\right) = 0$, then the contract attracts unemployed workers if and only if

$$f\left(q^d\right)(1-\alpha)\left[V\left(\left(w_l^d, y_l\right)\right) - V\left(s_u\right)\right] > f\left(q_u^*\right)\left(\widetilde{V}_0\left(q_e^*\right) - V\left(s_u\right)\right).$$

In either case one can verify that there is a number $\alpha_0 \in (0,1)$ such that for each $\alpha \in (\alpha_0, 1)$ no deviating job will attract unemployed workers, for any (b, k_u, k_e) such that $b \in (\beta_0, y_l)$ — for β_0 as defined above — $k_u \in \left[\alpha\left(y_h - y_l\right)/\left(r+\delta\right), \alpha\left(y_h - b\right)/\left(r+\delta\right)\right)$ and $k_e \in \left(0, (1-\alpha)\alpha\left(y_h - y_l\right)/\left(r+\delta\right)\right)$. To see why, for given values of α and k_e satisfying the assumptions of the proposition, let $k_e = m(1-\alpha)\alpha$, with

$$m = q_e^* f\left(q_e^*\right)\left(\frac{y_h - w_e^*}{r+\delta}\right).$$

Maintaining $k_e = m(1-\alpha)\alpha$, for the fixed value of m, we have

$$\lim_{(\alpha, k_e)\to(1,0)} f\left(q_u^*\right)\left(\widetilde{V}_0\left(q_e^*\right) - V\left(s_u\right)\right) > 0,$$

which is well defined for some feasible values of k_u. In particular, note that there is a value of k_u satisfying (61) such that there is an interior solution with all the properties specified in Lemma 6, for all $\alpha \in (0,1)$ and $k_e < (1-\alpha)\,\alpha\,(y_h - y_l)\,/\,(r+\delta)$. One example is $k_u = \alpha\,(y_h - y_l)\,/\,(r+\delta)$.

It follows that there is a number $\alpha_0 \in (0,1)$ such that for each $\alpha \in (\alpha_0, 1)$ the unique solution to Problem (P2) is given by the solution to Problem (P5), for all (b, k_u, k_e) such that $b \in (\beta_0, y_l)$, $k_u \in \big[\alpha\,(y_h - y_l)\,/\,(r+\delta)\,,\alpha\,(y_h - b)\,/\,(r+\delta)\big)$ and $k_e \in (0, \kappa_0)$, where β_0 is given above and κ_0 is given in Lemma 6.

Parts (i) and (ii) of the proposition follow immediately from Proposition 1. Letting $w_u^* \equiv \widetilde{W}\,(q_e^*)$ and $w_e^* \equiv w_e\,((w_u^*, y_h)) = w_e\,((w_u^*, y_h))$, it follows that $w_e^*\,(s) > y_l$ and $w_u^* < y_l$, since $q_e^* > \widehat{q}_a$ and $q_e^* < \widehat{q}_b$. It is straightforward to characterize ψ. The unemployment rate is given by

$$\psi\,(s_u) = \frac{\delta}{\delta + f\,(q_u^*)}.$$

The wage distribution has two mass points: $\left(w_u^*, w_e^*\right)$. The mass of workers earning the wage w_u^* is given by $\psi\,((w_u^*, y_l)) + \psi\,((w_u^*, y_h))$, where

$$\psi\,((w_u^*, y_l)) = (1-\alpha)\left(\frac{f\,(q_u^*)}{\delta + (1-\delta)\,\alpha f\,(q_e^*)}\right)\psi\,(s_u)\,,$$

and

$$\psi\,((w_u^*, y_h)) = \alpha\left(\frac{f\,(q_u^*)}{\delta + (1-\delta)\,\alpha f\,(q_e^*)}\right)\psi\,(s_u)\,,$$

with $q_e^* \equiv q_e\,((w_u^*, y_l)) = q_e\,((w_u^*, y_h))$. The mass of workers earning the wage w_e^* is

$$\psi\,((w_e^*, y_h)) = \left(\frac{(1-\delta)\,\alpha f\,(q_e^*)}{\delta}\right)\Big(\psi\,((w_u^*, y_l)) + \psi\,((w_u^*, y_h))\Big).$$

To prove that the quits rate of poorly matched workers is inefficiently low suppose, by contradiction, that $\alpha f\,(q_e^*) > \alpha f\,(q_b)$. Comparing (18) and (50), it follows that $(1-\alpha)\,\widetilde{W}\,(q_e^*) + \alpha y_h < y_l$. But then, one can choose α_0 in the proposition so that this inequality is violated for all $\alpha \in (\alpha_0, 1)$.

To prove that the rate at which unemployed workers find entry jobs is inefficiently low, let $k_e = m(1-\alpha)\alpha$, where m is defined above, and write (57)–(58) in the limit as $\beta \to y_l$, $\alpha \to 1$ and $k_e \to 0$ as

$$\frac{y_h - y_l}{r+\delta} - \lim_{(\alpha,k_e)\to(1,0)} \frac{k_e}{\eta(q')\, q' f(q')\,(1-\alpha)\,\alpha} = \lim_{(\alpha,k_e)\to(1,0)} \lambda q \left(\frac{1-\eta(q)}{\eta(q)}\right) \frac{k_u}{q} \left(\frac{1}{f(q)} + \frac{1}{r+\delta}\right), \quad (62)$$

where λ is given by (55), and

$$\frac{k_u}{q f(q)} = \lim_{(\alpha,k_e)\to(1,0)} \frac{k_e}{q' f(q')\,(1-\alpha)\,\alpha\eta(q')}. \quad (63)$$

Use (63) to write (62) as

$$\frac{y_h - y_l}{r+\delta} = \frac{k_u}{q f(q)} + \lim_{(\alpha,k_e)\to(1,0)} \lambda q \left(\frac{1-\eta(q)}{\eta(q)}\right) \frac{k_u}{q} \left(\frac{1}{f(q)} + \frac{1}{r+\delta}\right). \quad (64)$$

Similarly, let $k_e = m(1-\alpha)\alpha$, where m is defined above, and write (22) in the limit as $\beta \to y_l$, $\alpha \to 1$ and $k_e \to 0$ as

$$\frac{y_h - y_l}{r+\delta} = \frac{k_u}{q_u f(q_u)} + \left(\frac{1-\eta(q_u)}{\eta(q_u)}\right) \frac{k_u}{q_u} \left(\frac{1}{f(q_u)} + \frac{1}{r+\delta}\right). \quad (65)$$

Comparing (64), with $q = q_u^*$, and (65), noting that $\lim_{(\alpha,k_e)\to(1,0)} \lambda q > 1$, it follows that $q_u < q_u^*$. This concludes the proof. **QED**

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