Buying voters with uncertain instrumental preferences

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Abstract

We analyze a vote-buying model where the members of a committee vote on a proposal important to a vote buyer. Each member incurs a privately-drawn disutility if the proposal passes. We characterize the cheapest combination of bribes that guarantees the proposal passes in all equilibria. When members vote simultaneously, the number of bribes is at least 50% larger than the number of votes required to pass the proposal (vote threshold). The number of bribes increases with the dispersion of the disutility distribution and all members are bribed with sufficient dispersion. A proportional increase in the number of members and the vote threshold leads to a less-than-proportional increase in capture cost, and the cost may increase with the vote threshold. With sequential voting and disutility distribution $U[0,1]$, all members are bribed and bribes are equal. Finally, sequential voting increases capture cost in small committees and decreases it in large committees.

Keywords. Vote buying, legislatures, political economy.

JEL Classification. D71, D72

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1 Introduction

Governments often introduce bills that go against the interests of parliament members, such as a law limiting dual mandates.[1] To overcome members' opposition, the government can offer rewards to those who support the bill, e.g., investments in legislative districts.

We develop a vote-buying model to analyze these situations. A committee votes on a proposal that favors the interests of a vote buyer. However, committee members prefer the proposal not to pass. To gain support, the vote buyer offers bribes to members in exchange for their votes. Our key innovation is introducing uncertainty to members' preferences. For instance, in the case of a dual mandate prohibition, this uncertainty reflects each member's uncertain future (re)election prospects. Our first example illustrates how a vote buyer exploits the implications of this uncertainty for pivotal probabilities.

Example 1. A three-member committee votes on a proposal. The proposal passes if at least two members vote for it. In this example, members vote simultaneously. Members dislike the proposal. Crucially, each member draws his disutility v_i privately at the beginning of the game: $v_i \overset{iid}{\sim} U[0,1]$.

A vote buyer (feminine pronoun) is interested in the proposal passing. Before the vote, she publicly commits to paying a bribe to some members if they individually vote for the proposal. We assume the value of the bribe is $b \geq 0$ and that it is the same for all bribed members. The vote buyer knows the distribution of members'

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[1] A dual mandate, or double jobbing, is the practice in which elected officials serve in more than one public position simultaneously. For example, more than 80% of parliament members in France held another office before a law prohibiting dual mandates was passed in 2013.

disutilities but does not observe their realizations. The proposal is important to her so she wants to guarantee that it passes with certainty in all equilibria of the voting subgame. Subject to this condition, she minimizes the cost of bribes.

We compare two strategies for the vote buyer. First, suppose she bribes two members. We assume the unbribed member plays his weakly dominant strategy and votes against the proposal. The proposal passes with certainty if the two bribed members vote for regardless of their disutility. This strategy profile is clearly the unique equilibrium if $b > 1$ because voting for is a dominant strategy for all disutilities. However, if $b < 1$, bribed members with disutility $v_i > b$ would deviate and the strategy profile is not an equilibrium. Moreover, as will be established, there exists an equilibrium where the proposal is rejected with a positive probability. Thus, the cheapest bribe such that the proposal passes with certainty in any equilibrium is $b = 1$ to the two members, which yields a cost of 2.

Instead, suppose the vote buyer bribes all three members. We will show that if $b > \frac{8}{27}$, there is no equilibrium where the proposal is rejected with positive probability; that is, buying a third member is cheaper for the vote buyer. For each member, voting for the proposal guarantees bribe payment. However, if the member is pivotal (i.e., if exactly one other member votes for the proposal), it also leads to the passing of the proposal. Denoting the pivotal probability by π, member i votes for if $b > v_i \times \pi$.

The equilibrium of the voting subgame takes a cutoff form: a member votes for the proposal if his disutility is below a threshold. For now, focus on symmetric strategies and call the common cutoff v. Then $\pi(v) = 2v(1 - v)$ and an equilibrium cutoff $\overline{v} \in (0, 1)$ satisfies

$$b = \overline{v}\pi(\overline{v}).$$

In Figure 1, we plot the right-hand side of this equation. For small bribes like b^1, two equilibria exist with cutoffs $\overline{v}_1$ and $\overline{v}_2$. Moreover, there is a third equilibrium where all members accept the bribe: committee members are not pivotal and have no incentive to deviate. Throughout the paper, we assume committee members play the equilibrium in which the proposal is rejected with the highest probability. For instance, faced with b^1, they would play $\overline{v}_1$ as this lower cutoff implies the lowest probability of passing.

When b is larger than the maximum of $v\pi(v)$, the third equilibrium, where the

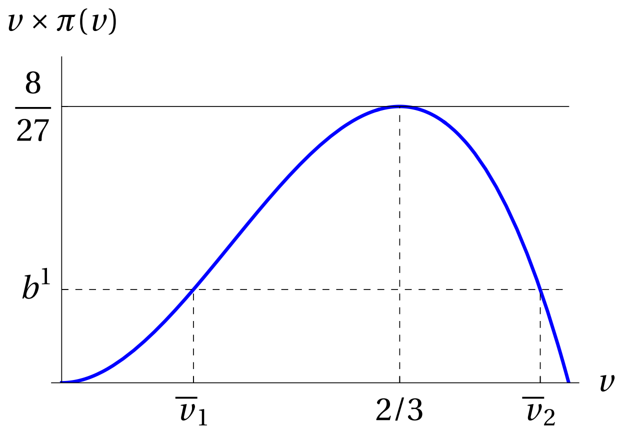


Figure 1: The Structure of Equilibrium in the Voting Subgame

Notes: As the cutoff used by other members changes, so does the value of $v\pi(v)$ (solid blue). The value of the maximum is $\frac{8}{27}$ (reached for $v = \frac{2}{3}$). For a given bribe b^1 below $\frac{8}{27}$, there are three equilibria of the voting subgame: one with cutoff $\overline{v}_1$, one with cutoff $\overline{v}_2$, and one in which all members vote for the proposal. For bribes above $\frac{8}{27}$ only the latter exists.

proposal passes with certainty, is the only equilibrium of the voting subgame. Here, the maximum is $\frac{8}{27}$. Thus, it is sufficient to pay slightly more than $\frac{8}{9}$, which is the cost of bribing all three members, to guarantee there is no equilibrium where the proposal is rejected with positive probability. Intuitively, bribing more members reduces members' pivotal probabilities, forcing them to accept smaller bribes. $\triangle$

We characterize the cheapest combination of bribes required for passing the proposal with certainty in all equilibria. We consider various factors such as the disutility distribution, the number of committee members, and the vote threshold. Specifically, we examine simultaneous voting in Section 2. First, in Section 2.1, we assume symmetric strategies and equal bribes for all members bribed. Our main finding is that the cheapest capture always involves a number of bribes at least 50% higher than the vote threshold. Furthermore, the number of bribes increases with dispersion, and all members are bribed when there is enough dispersion. As for the capture cost, increasing the vote threshold and the number of members proportionally results in a less-than-proportional increase in cost (because members are less likely to be pivotal in a large committee), while increasing only the vote threshold increases the capture cost if more than half of the members must vote for to pass the proposal.

In Section 2.2, members may play asymmetric strategies. Depending on the distribution, there may exist asymmetric equilibria where the proposal can be rejected when members receive the bribes of Section 2.1. This is the case when the disutility dispersion is small, but not when it is large. Section 2.3 considers unequal bribes. With large dispersion, we show with an example that unequal bribes can yield a lower capture cost. However, we establish that if $v_i \overset{iid}{\sim} U[0, 1]$ as in Example 1, the capture cost is minimized by equal bribes.

We study sequential voting with $v_i \overset{iid}{\sim} U[0, 1]$ in Section 3. The vote buyer also exploits pivotal considerations and the cheapest capture requires offering the same bribe to all members. Finally, Section 4 shows that compared to sequential voting, simultaneous voting yields a higher capture cost if the committee is large, while the opposite is true for small committees or very high or very low vote thresholds.

The model has a variety of applications. Our setup primarily applies to decision-making in organizations. For example, a CEO may want to persuade board members to make a decision favoring his interests. If board members expect the decision to be approved regardless of their vote, the CEO can obtain their support in exchange for small favors. Alternatively, consider the application of Genicot and Ray (2006) in which a raider takes over a company. In such a case, the post-takeover value of non-tendered shares could be diluted, harming all shareholders.[2] Nevertheless, if shareholders expect the takeover to happen regardless of their selling decision, shares could be bought at little cost. Finally, our model has implications for lobbying and vote-buying in committees of experts (like FDA committees) or juries.

We contribute to the vote-buying literature by combining a single vote buyer with committee members who care about the vote's outcome but are uncertain about each other's preferences. The combination is novel, though literature on each ingredient exists.

Several papers study vote-buying when members have publicly known preferences over outcomes. Dal Bo (2007) shows that a vote buyer bribes a committee at no

[2]For instance, this happens in Grossman and Hart (1980) where the raider uses the dilution to force atomistic shareholders to sell, but Bagnoli and Lipman (1988) show that dilution does not necessarily happen with a finite number of shareholders.

cost by conditioning the bribes on the complete voting profile. She offers to pay an infinitesimal amount if members are not pivotal and a large bribe if votes are decisive. By contrast, we exclude any contracts based on the joint realization of votes. Moreover, the models of Rasmusen and Ramseyer (1994) and Dahm and Glazer (2015) feature some equilibria unfavorable to committee members in which a supermajority accepts small bribes because no member is pivotal. Instead, we allow members to coordinate on their preferred equilibrium. Cheap capture also occurs in Genicot and Ray (2006) and Chen and Zápal (2020) where the vote buyer approaches members sequentially and exploits the timing of offers. On the contrary, the vote buyer makes all offers at the same time in our model, both in simultaneous and sequential voting.

We focus on the probability of a vote being decisive and do not consider information aggregation (Feddersen and Pesendorfer 1996, 1997, 1998). Feddersen and Pesendorfer (1998) highlight that unanimity, which in our setup maximizes capture cost, makes information harder to aggregate. Henry (2008) and Felgenhauer and Grüner (2008) combine vote-buying and information aggregation. In Henry (2008), each committee member receives a signal about the quality of a common value proposal. Bribes determine the number of members who vote informatively, shaping members' inferences conditional on being pivotal. Similarly, in Ekmekci and Lauermann (2019) an election organizer chooses turnout to manipulate the information aggregated. These papers consider a common value proposal while we focus on private values.

The mechanism exploited by the vote buyer in our model relies on pivotality and is not present in the literature on vote-buying with expressive preferences: e.g., in Zápal (2017), members' responses to a bribe are uncertain, but pivotal considerations are absent because members do not take into account the effect of their vote on the outcome. Groseclose and Snyder (1996), Banks (2000), Dekel et al. (2008), Morgan and Várdy (2011) and Iaryczower and Oliveros (2017) also assume expressive preferences and introduce a second vote buyer. They find that the first mover bribes a large coalition to increase the cost for the follower.

Our paper proposes a new explanation for the high empirical frequency of supermajorities. While early theories of coalition formation predicted minimal winning coalitions (Axelrod, 1970), some later papers predict supermajorities (Koehler, 1975; Weingast, 1979; Shepsle and Weingast, 1981; Baron and Diermeier, 2001). The closest to us is Carrubba and Volden (2000), in which a larger-than-necessary coalition ensures no member can prevent the costly passing of other members' bills. Supermajorities are also found in the literature on legislative bargaining (Volden and Wiseman, 2007; Tsai and Yang, 2010; Dahm et al., 2014); for an overview, see Eraslan and Evdokimov (2019). For instance, Norman (2002) characterizes the non-symmetric equilibria of the classical model of Baron and Ferejohn (1989) and shows that some proposals can be unanimously approved.

Chen and Eraslan (2013, 2014) look at the other side of the problem and study a vote-selling model where members with uncertain preferences send messages to the vote buyer to influence the proposal.

Finally, we are also related to the larger literature on unique implementation with moral hazard. In Winter (2004) and Winter (2006), agents separately perform individual tasks for a project that succeeds if all agents succeed. In case of success, the principal rewards agents who support the project. Contributions are simultaneous in Winter (2004) and sequential in Winter (2006). Our paper differs as members' preferences are uncertain. Winter's principal aims to prevent asymmetric equilibria where the project fails and Winter establishes that discriminatory rewards can be optimal. With sufficient uncertainty, we find that asymmetric equilibria cannot be sustained, and equal bribes may be preferred.

2 Simultaneous Voting

We consider a committee of n members voting simultaneously on a proposal. The vote threshold m is the minimum number of votes for required to pass the proposal. We exclude $m = n$ (unanimity required to pass the proposal) and $m = 1$ (unanimity required to reject it).[3] At the beginning of the game, committee members draw their disutilities from the passing of the proposal. These disutilities are drawn privately and independently from a common distribution: $v_i \overset{iid}{\sim} F(\cdot)$, where v_i is the disutility of member i. $F(\cdot)$ has support $[v_{min}, v_{max}]$ with $v_{min} \geq 0$ and v_{max} finite. We assume that the disutility distribution $F(\cdot)$ is continuously differentiable on (v_{min}, v_{max}), and

[3] We discuss unanimous vote thresholds at the end of subsection 2.1: they require technical modifications of the results but do not affect our conclusions.

has an increasing generalized hazard rate:

$$\frac{\partial}{\partial v}\left(\frac{vF'(v)}{1-F(v)}\right) \geq 0.$$

Other models (e.g. Lariviere, 2006) use this assumption which is satisfied by all Uniform and Beta distributions.

Before the voting subgame, a vote buyer who favors the proposal publicly offers bribes $(b_1, ..., b_n)$, where $b_i \geq 0$ is member i's payment if he votes for the proposal. We assume the proposal is important to the vote buyer, so she minimizes the capture cost, i.e., the amount spent on bribes, subject to the proposal passing with certainty.

We focus on Bayesian Nash equilibria for the voting subgame. When multiple equilibria exist, we assume committee members play one of the equilibria where the proposal passes with the smallest probability. This assumption is in the spirit of Winter (2004) and Genicot and Ray (2006). First, it rules out equilibria where the proposal passes with arbitrarily small bribes because all bribed members accept and are not pivotal. Second, it follows naturally if a vote buyer to whom the proposal is important is uncertain about which equilibrium will be played. Third, it selects an equilibrium preferred by committee members.

The game's timing is as follows. First, committee members privately observe their disutility. Then, the vote buyer offers bribes $(b_1, ..., b_n)$. Members observe the bribes and simultaneously choose whether to vote for or against the proposal. Finally, the proposal passes if at least m members vote for it.

2.1 Symmetric Voting Strategies and Equal Bribes

This subsection focuses on equal bribes: the vote buyer bribes k members who all receive the same bribe b. Thus, the combination of bribes is characterized by (b, k). For committee member i, a strategy $\sigma_i : v_i \to [0, 1]$ is a mapping from disutility v_i into a probability of voting for the proposal. We only consider members to whom the vote buyer offers a bribe; unbribed members are assumed to use their weakly dominant strategy and vote against the proposal. We focus on symmetric equilibria, i.e., equilibria in which bribed members play the same strategy.

We first solve the voting subgame. Given a combination of bribes (b, k), if a member is not pivotal, the payoff difference between voting for and against is the bribe's value. If he is pivotal, a vote for the proposal makes it pass and he incurs his disutility. We denote the pivotal probability of committee member i by π_i. He accepts the bribe and votes for the proposal if $b > v_i\pi_i$, where $v_i\pi_i$ is the expected cost of voting for the proposal. Moreover, he votes against if $b < v_i\pi_i$ and can vote for with any probability if $b = v_i\pi_i$. Thus, equilibrium strategies take a cutoff form. Since we focus on symmetric equilibria, all members vote for the proposal if their disutility is smaller than some cutoff $\overline{v}$ determined in equilibrium.

Our first lemma characterizes the equilibrium of the voting subgame where the proposal passes with the smallest probability. When fewer members are bribed than the vote threshold ($k < m$), there exists an equilibrium of the voting subgame where the proposal is always rejected, and hence no bribe can guarantee that the proposal passes with certainty in any equilibrium. As a result, we focus on $k \in \{m, ..., n\}$.

Lemma 1. *Suppose the vote buyer offers a bribe $b > 0$ to k members with $m \leq k \leq n$. In the symmetric equilibrium of the voting subgame in which the proposal passes with the smallest probability,*

(a) If $b \leq \max_{v\in[v_{min},v_{max}]} v\pi^k(v)$, bribed members vote for the proposal if their disutility is smaller than a cutoff $\overline{v}$ that satisfies $\overline{v} = \min\{v \in [v_{min}, v_{max}] : b = v\pi^k(v)\}$ where

$$\pi^k(v) = \binom{k-1}{m-1} F(v)^{m-1}\big(1 - F(v)\big)^{k-m}.$$

Moreover, they vote against if their disutility is larger than $\overline{v}$ and a member with $v_i = \overline{v}$ can vote for with any probability.

(b) If $b > \max_{v\in[v_{min},v_{max}]} v\pi^k(v)$, all bribed members vote for the proposal regardless of their disutility: $\overline{v} > v_{max}$.

(Proof in Appendix A.2.) First, consider the case where m members are bribed. $v\pi^m(v)$ is increasing in v and $\pi^m(v) \to 1$ as $v \to v_{max}$. For $b \leq v_{max}$, Lemma 1.a characterizes the unique equilibrium cutoff and the proposal is rejected with positive probability. For $b > v_{max}$, the strategy profile described in Lemma 1.b is the unique equilibrium, and the proposal passes with certainty.

Now consider $k > m$. As established in Lemma A.2.1 in Appendix A.2, increasing generalized hazard rates imply that $v\pi^k(v)$ is single-peaked in v for $v \in [v_{min}, v_{max}]$. By the intermediate value theorem, the equation $v\pi^k(v) = b$ admits two solutions if $b < \max_{v\in[v_{min},v_{max}]} v\pi^k(v)$, one if $b = \max_{v\in[v_{min},v_{max}]} v\pi^k(v)$ and none otherwise. Thus, equilibrium cutoffs are illustrated by Figure 1. The equilibrium where the proposal passes with the smallest probability is associated with the smallest cutoff, and this cutoff is characterized by Lemma 1.a. If we let $v_k^* := \arg\max_{v\in[v_{min},v_{max}]} v\pi^k(v)$, the smallest bribe such that the proposal passes with certainty in any symmetric equilibrium is $b_k^* = v_k^*\pi^k(v_k^*)$.[4] For Example 1 and hence in Figure 1, $b_3^* = \frac{8}{27}$.

We now turn to the vote buyer's problem. As just established, if the vote buyer offers k bribes, she needs to offer b_k^* to make the proposal pass with certainty. Hence, her cost $c(k)$ is determined by the equilibrium where the cutoff is v_k^*:

$$c(k) = k \times \max_{v\in[v_{min},v_{max}]} v\pi^k(v) = k \times v_k^*\pi^k(v_k^*) = k \times b_k^*.$$

We want to determine a cost-minimizing number of bribes $\arg\min_{k\in\{m,\dots,n\}} c(k)$.[5] Intuitively, while bribing additional members requires paying more bribes, it also makes it harder for committee members to be pivotal with a high probability. Hence, it decreases b_k^*. Which effect dominates depends on the number of bribes, on the vote threshold, and on the disutility distribution. Our main result characterizes $\arg\min_{k\in\{m,\dots,n\}} c(k)$:

Proposition 1.

(a) For any disutility distribution, any cost-minimizing number of bribes is at least $\min\{\frac{3}{2}m - 1, n\}$,

(b) For any number of bribes $k \in \mathbb{N}$ *such that* $\min\{\frac{3}{2}m + 1, n\} \leq k \leq n$, *there exists a disutility distribution such that k is a cost-minimizing number of bribes.*

(Proof in Appendix A.2.) Proposition 1 implies that the vote buyer always wants to offer a number of bribes substantially larger than the vote threshold. If she could

[4] More precisely, b_k^* is the "smallest number above $v_k^*\pi^k(v_k^*)$", which is not defined because bribes are on a continuum, but makes sense as the limit of a grid.

[5] As k has to be an integer, there are distributions for which $\arg\min_{k\in\{m,\dots,n\}} c(k)$ is not unique. In particular, $c(k)$ can be minimized for two consecutive integers.

offer any number of bribes, she would choose at least $k = \frac{3}{2}m - 1$, which for m large represents a number of bribes 50% larger than the vote threshold. However, the number of bribes cannot exceed the number of members. As a result, when there are fewer members than $\frac{3}{2}m - 1$, this constraint binds and the vote buyer bribes all members. With more than $\frac{3}{2}m$ members, it can still be the case that all members are bribed, but it depends on the disutility distribution. This is true even when the number of members is arbitrarily large: for some distributions, the vote buyer's cost is always decreasing in the number of bribes and she offers as many bribes as possible.

We now show Proposition 1 in three steps. First, we establish that the vote buyer bribes more members when the disutility distribution is more dispersed. Second, we show that even when dispersion is small, any cost-minimizing number of bribes is at least $\frac{3}{2}m - 1$. Finally, we demonstrate that with a sufficiently dispersed distribution, all members are bribed regardless of their number. The definition of dispersion used for this analysis is from Shaked and Shanthikumar (2007, p.213):

Definition 1. *$\tilde{F}(\cdot)$ is more dispersed than $F(\cdot)$ if the ratio of the inverse CDFs, $\tilde{F}^{-1}(q)/F^{-1}(q)$, is nondecreasing in q for all $q \in (0,1)$. In such a case, we write $F \leq_* \tilde{F}$.*[6]

An example of distributions ranked in this order are $U[\frac{1}{2} - \alpha, \frac{1}{2} + \alpha]$ with $\alpha \in (0, \frac{1}{2}]$, which become more dispersed as α increases.[7] We use these uniform distributions to simulate the cost-minimizing number of bribes $\arg\min_{k \in \{m,\dots,n\}} c(k)$ in Figure 2. In line with Proposition 1, the smallest cost-minimizing number of bribes is approximately $\frac{3}{2}m$ and is obtained for small dispersion ($\alpha \to 0$). Moreover, the cost-minimizing number of bribes increases with dispersion, which is not specific to uniform distributions: for all distributions that can be ranked in our dispersion order,

[6]The increasing generalized hazard rate assumption implies that $F'(v) > 0$, so the CDF is strictly monotone. Hence, $F^{-1}(q)$ is well-defined for $q \in (0, 1)$.

[7]These distributions are centered around $\frac{1}{2}$, but some distributions with different means can also be dispersion ranked. In particular, moving the uniform support to the right on the real line decreases the variance relative to the mean, which results in less dispersion. However, note that the $\leq_*$ order is not complete and some distributions cannot be ranked.

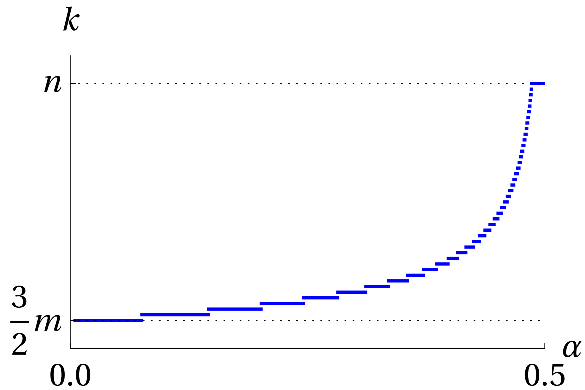


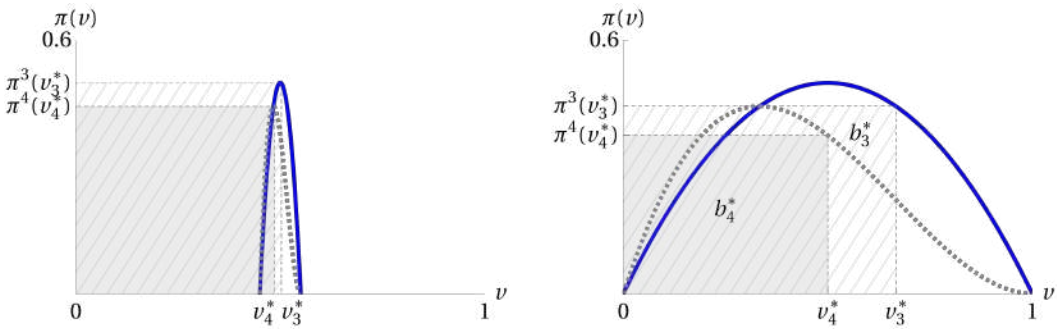
Figure 2: Cost-Minimizing Number of Bribes as Dispersion Varies

Notes: We depict the solutions to the cost-minimization problem $\arg\min_{k\in\{m,\ldots,n\}} c(k)$ when $v_i \overset{iid}{\sim} U[\frac{1}{2}-\alpha, \frac{1}{2}+\alpha]$ as we vary the dispersion α. The simulation assumes $(m,n) = (8,54)$; n is large compared to m so that the constraint $k \leq n$ does not bind.

Lemma 2. *The set of cost-minimizing numbers of bribes with a distribution $\tilde{F}(\cdot)$ dominates in the strong set order the set of cost-minimizing numbers of bribes with a less dispersed distribution $F(\cdot)$.*

(Proof in Appendix A.2.) Lemma 2 simply states that the vote buyer bribes more members when the distribution is more dispersed. However, it needs to be expressed in terms of dominating sets because $\arg\min_{k\in\{m,\ldots,n\}} c(k)$ is not necessarily a singleton. It is an application of Theorem 5 of Milgrom and Shannon (1994). Formally, if Y' and Y are subsets of $\mathbb{R}$, set Y' dominates Y in the strong set order if for any $x' \in Y'$ and $x \in Y$, we have $\max\{x', x\} \in Y'$ and $\min\{x', x\} \in Y$.

The committee consists of members who each make an individually rational decision governed by Lemma 1. However, to build intuition, we abstract from the behavior of individual members and pretend the committee, as a single player, is choosing the cutoff that maximizes the value of the bribes and hence the vote buyer's cost. With this approach, the vote buyer and committee are playing a two-player game: the vote buyer moves first and chooses the number of bribes k to minimize $c(k)$. Then, the committee chooses the cutoff v that maximizes $v\pi^k(v)$. We refer to any member who draws $v_i = v$ as a cutoff member. Hence, $v\pi^k(v)$ corresponds to a cutoff member's expected cost of voting for the proposal. This cost can be represented in the $(v, \pi^k(v))$



(a) $U[0.45, 0.55]$ (less dispersed)

(b) $U[0, 1]$ (more dispersed)

Figure 3: The Effect of Additional Bribes As Dispersion Varies

Notes: We depict $\pi^3(v)$ (solid) and $\pi^4(v)$ (dashed) for less (left) and more (right) dispersed disutility distributions. The hashed area rectangle represents $b_3^* = v_3^* \pi^3(v_3^*)$ and the grey area represents $b_4^* = v_4^* \pi^4(v_4^*)$. For both panels, $m = 2$.

graph: it is the area of the rectangle spanned by the origin and a point $(v, \pi^k(v))$. In each panel of Figure 3, we plot $\pi^k(v)$ for $k \in \{3, 4\}$ with vote threshold $m = 2$. The cutoff v affects a cutoff member's expected cost through two channels. Firstly, a larger cutoff implies a higher disutility for a cutoff member, and hence a larger width of the rectangle. Secondly, the cutoff determines the probability that a cutoff member is pivotal, which corresponds to the height of the rectangle. Increasing the cutoff increases this probability up to a point and then decreases it. A cutoff member's largest expected cost of voting for the proposal is $b_k^* = \max_{v \in [v_{min}, v_{max}]} v\pi^k(v)$, the area of the largest possible rectangle.

Turning to the vote buyer, we can see that offering four bribes instead of three decreases b_k^* through both channels. Firstly, $v_4^* < v_3^*$: if an additional member can vote for, the committee maintains a high pivotal probability by decreasing the probability that each member votes for, which amounts to decreasing the cutoff. Secondly, $\pi^4(v_4^*) < \pi^3(v_3^*)$: the lower cutoff is not sufficient to prevent a decrease in the pivotal probability. Hence, the additional bribe reduces the width as well as the height of the largest possible rectangle. The vote buyer trades off the decrease in b_k^* against the cost of the additional bribe.

Crucially, this trade-off depends on dispersion. The two panels of Figure 3 illustrate that b_k^* decreases more with a dispersed distribution. Hence, the incentive to offer an additional bribe increases with dispersion. The distribution is less dispersed in Figure 3a, where $U[\frac{1}{2}-\alpha,\frac{1}{2}+\alpha]$ with a small dispersion parameter α, than in Figure 3b, which illustrates the case of $v_i \overset{iid}{\sim} U[0,1]$. Firstly, with arbitrarily small dispersion, $v_3^* \to \frac{1}{2}$ and $v_4^* \to \frac{1}{2}$ and the width of the rectangle is (almost) independent of k. By contrast, with more dispersion, the committee relies more on a cutoff member's disutility to increase his cost of voting for the proposal. Hence, v_k^* is more sensitive to k, and offering a fourth bribe has a larger effect on the width of the rectangle.

Moreover, we now argue that the pivotal probability at the cutoff v_k^*, which is the height $\pi^k(v_k^*)$ of the largest rectangle, is also more sensitive to k with dispersion. To begin with, dispersion induces reliance on disutilities, which implies lower pivotal probabilities: both $\pi^3(v_3^*)$ and $\pi^4(v_4^*)$ are lower in Figure 3b than in Figure 3a. The pivotal probability is lower, however, because, members are more likely to vote for with dispersion. This, in turn, implies that the pivotal probability is also more sensitive to changes in the number of bribes k: bribing an additional member implies an additional vote against the proposal is needed for the pivotal event. As the additional bribed member is less likely to vote against the proposal with dispersion, bribing an additional member lowers the pivotal probability more when the distribution is dispersed.

To summarize, dispersion implies that both v_k^* and $\pi^k(v_k^*)$ are more sensitive to k. Hence, the vote buyer offers fewer bribes when the distribution is not dispersed. As dispersion is bounded below by a Dirac measure, we can now derive the lower bound for any cost-minimizing number of bribes stated in Proposition 1.a:

Lemma 3. *Any cost-minimizing number of bribes is at least* $\min\{\frac{3}{2}m-1,n\}$.

(Proof in Appendix A.2.) Figure 3a illustrates that when $v_i \overset{iid}{\sim} U[\frac{1}{2}-\alpha,\frac{1}{2}+\alpha]$, for all $k\in\{m,...,n\}$, $v_k^* \to \frac{1}{2}$ as $\alpha \to 0$. This is not specific to uniform distributions and $v_k^* \to \frac{1}{2}$ for any distribution for which the density concentrates around $\frac{1}{2}$. In such cases, the cutoff member's disutility (and, hence, the width of the rectangle) has to be close to $\frac{1}{2}$. As a result, the committee can only affect the cutoff member's cost through the pivotal probability (the height of the rectangle) by choosing the probability that a member votes for. Denoting this probability by p, we have for $v_i \overset{iid}{\sim} U[\frac{1}{2}-\alpha,\frac{1}{2}+\alpha]$

and $\alpha \to 0$:[8]

$$b_k^* \to \frac{1}{2} \max_{p\in[0,1]} \pi^k(p) = \frac{1}{2} \max_{p\in[0,1]} \binom{k-1}{m-1} p^{m-1}(1-p)^{k-m}.$$

Hence, the game played by the committee and the vote buyer simplifies as follows: the committee (who plays second) chooses p to maximize the pivotal probability; the vote buyer (who plays first) chooses k to minimize

$$c(k) = k \times \frac{1}{2} \max_{p\in[0,1]} \binom{k-1}{m-1} p^{m-1}(1-p)^{k-m}.$$

Jointly, p and k determine the leave-one-out vote tally distribution, i.e., the distribution Binomial$(k-1, p)$ of the number of *other* members voting for the proposal, evaluated from the perspective of any given bribed member. Hence, the effects of p and k on Binomial$(k-1, p)$ are key to understanding their effect on the pivotal probability, which is the probability mass of Binomial$(k-1, p)$ at $m-1$.

Solving the game backward, if the vote buyer offered k bribes, the committee maximizes the pivotal probability by choosing $p = (m-1)/(k-1) = p_k^*$. This implies that the expectation as well as the mode of Binomial$(k-1, p_k^*)$ are $m-1$. Hence, the pivotal probability is the mass of Binomial$(k-1, p)$ at its mode. Anticipating p_k^*, the vote buyer's choice of k does not affect the mode. However, it affects the mass at the mode through the variance of Binomial$(k-1, p_k^*)$. Intuitively, the more variance, the less mass at the mode. As k increases, so do the number of trials as well as the variance. However, by the Poisson limit theorem, the mass at the mode is approaching that of a Poisson random variable with the same mean as k becomes large. Hence, increases in k eventually barely affect the variance but still increase the cost by requiring more bribes. Thus, any cost-minimizing number of bribes is finite.

For large committees, there is a closed form that links the variance of Binomial$(k-1, p_k^*)$ to the mass at its mode. We can use it to show that $\arg\min_{k\in\{m,\ldots,n\}} c(k) \approx \frac{3}{2}m$. By any central limit theorem, as $k \to \infty$, Binomial$(k-1, p_k^*)$ approximates $\mathrm{N}\big((k-1)p_k^*, \sigma = \sqrt{(k-1)p_k^*(1-p_k^*)}\big)$. Evaluated at its mean $(k-1)p_k^* = m-1$, this approximation gives a pivotal probability $\phi(0)/\sigma$. Using $\sqrt{k-1} \approx \sqrt{k}$ (true for large

[8]Technically, this follows from Berge's maximum theorem; see the proof of Proposition 1.

k), the vote buyer's problem can be rewritten as a variance maximization

$$\arg\min_{k\in\{m,\dots,n\}} kb_k^* \approx \arg\min_{k\in\{m,\dots,n\}} k \times \frac{\phi(0)}{\sqrt{kp_k^*(1-p_k^*)}} = \arg\max_{k\in\{m,\dots,n\}} \frac{p_k^*(1-p_k^*)}{k}.$$

The realization of an individual vote is determined by Bernoulli(p_k^*). Hence, the variance of an individual vote is $p_k^*(1-p_k^*)$. Dividing by k, we obtain the variance of the share of votes for among bribed members. Maximizing this variance with respect to k is equivalent to the vote buyer's problem. The value of k affects the variance through two channels. Firstly, increasing k decreases the variance through the denominator: the vote share becomes more predictable when the number of trials increases. Secondly, k affects the individual vote variance $p_k^*(1-p_k^*)$ through its impact on p_k^*, and $p_k^*(1-p_k^*)$ is maximized when $p_k^* = \frac{1}{2}$. Since $p_k^* \approx m/k$ for large m and k, increasing k pushes p_k^* towards $\frac{1}{2}$ for $k < 2m$ and away from $\frac{1}{2}$ for $k \geq 2m$. The optimal k lies between $k = m$ and $k = 2m$, with the solution being approximately $\frac{3}{2}m$. Thus, the optimal k trades off these two channels, half-way between the smallest number of bribes that can make the proposal pass and the point above which k decreases the individual vote variance.

We have now established that any cost-minimizing number of bribes is at least $\frac{3}{2}m - 1$ and that it increases with dispersion. Our next result shows that there is no upper bound on the number of bribes: for sufficiently dispersed distributions, $\arg\min_{k\in\{m,\dots,n\}} c(k) = n$ for any number of committee members n.

Lemma 4. *If the disutility distribution is $U[0,1]$, all members are bribed.*

(Proof in Appendix A.2.) We now consider $v_i \overset{iid}{\sim} U[0,1]$, which was depicted in Figure 3b and corresponds to the case where the vote buyer offers more bribes. With this distribution, we have $F(v) = v$. Thus, the cutoff chosen by the committee is equal to the probability that a member votes for. This allows us to reexpress a cutoff member's expected cost of voting for the proposal:

$$\begin{aligned} v\pi^k(v) = v\binom{k-1}{m-1}v^{m-1}(1-v)^{k-m} &= \frac{m}{k}\binom{k}{m}v^m(1-v)^{k-m} \\ &= \frac{m}{k}\mathbb{P}\Big(m \text{ votes for} \mid k \text{ have cutoff } v\Big). \end{aligned}$$

Hence, the game played by the committee and the vote buyer is as follows: the committee maximizes the probability of m votes for and the vote buyer chooses k to minimize

$$c(k) = k \times \max_{v\in[v_{min},v_{max}]} v\pi^k(v) = m \times \max_{v\in[v_{min},v_{max}]} \mathbb{P}\Big(m \text{ votes for} \mid k \text{ have cutoff } v\Big).$$

Notice that the number of bribes k cancels out when we factor out m/k in $v\pi^k(v)$. This final expression for the cost contrasts with the discussion of Lemma 3: with $v_i \sim U[0,1]$, the vote buyer chooses k to minimize the probability of m votes for amongst k members, not k times the probability of $m-1$ votes for amongst $k-1$ members. Hence, the key component is the distribution of the number of votes for among the k bribed members, which is Binomial(k, v). Again, we can analyze the effects of v and k on this distribution to understand the result.

If the vote buyer offered k bribes, the committee maximizes the probability of m votes for by choosing $v = v_k^* = m/k$. Hence, the mode of Binomial(k, v_k^*) is m. Anticipating this, the vote buyer chooses k to minimize the probability of this modal event. As explained in the discussion of Lemma 3, this amounts to maximizing the variance of Binomial(k, v_k^*), which is $k \times m/k\,(1-m/k) = m\,(1-m/k)$. As this expression is strictly increasing in k, the vote buyer chooses k as large as feasible: $\arg\min_{k\in\{m,\dots,n\}} c(k) = n$.

To summarize, we have established that the cost-minimizing number of bribes $\arg\min_{k\in\{m,\dots,n\}} c(k)$ is close to $\frac{3}{2}m$ when the distribution is not dispersed, that it increases with dispersion, and that all members are bribed with a sufficiently dispersed distribution. Finally, the proof of Proposition 1.b uses disutility distributions $U[\frac{1}{2}-\alpha, \frac{1}{2}+\alpha]$ to establish that, as illustrated by Figure 2, all numbers of bribes $k \in \mathbb{N}$ such that $\min\{\frac{3}{2}m+1, n\} \leq k \leq n$ are a cost-minimizing number of bribes for some value of α.

We now move on to comparative statics for the capture cost, which we define as the cost paid by the vote buyer with a cost-minimizing number of bribes:

$$C^{sim}(m,n) = \min_{k\in\{m,\dots,n\}} c(k).$$

The capture cost depends on the structure of the committee, and in particular on the number of committee members n and on the vote threshold m. Trivially, $C^{sim}(m, n)$ (weakly) decreases with n: n does not affect b_k^* for any k, and increasing n only relaxes the constraint $k \leq n$. Our next result considers the less trivial effects of a proportional increase in m and n, as well as the effect of m:

Proposition 2.

(a) Proportional increases in vote threshold m and number of committee members n raise capture cost sub-proportionally: $C^{sim}(\lambda m, \lambda n) < \lambda C^{sim}(m, n)$ with $\lambda \in \mathbb{N}_+$.

(b) Suppose only a majority can pass the proposal, i.e., $n \leq 2m - 1$. Then, for any number of bribes k, b_k^* and $C^{sim}(m, n)$ increase in the vote threshold m.

(Proof in Appendix A.3.) Considering Proposition 2.a, suppose we multiply both m and n by the same scalar $\lambda \in \mathbb{N}_+$. The proportional vote threshold m/n is unaffected and for all $k \in \{m, ..., n\}$, $\lambda k \in \mathbb{N}$. Intuitively, holding the share of members bribed constant, pivotal probabilities are smaller in a larger committee. Thus, b_k^* in a committee (m, n) is larger than $b_{\lambda k}^*$ in a committee $(\lambda m, \lambda n)$. If k minimizes the capture cost for a committee (m, n), the vote buyer can bribe λk members in a committee $(\lambda m, \lambda n)$, even if doing so does not necessarily minimize the capture cost. Hence, as $b_{\lambda k}^* < b_k^*$, the capture cost is multiplied by less than λ.

We turn to Proposition 2.b, which shows that when more than half of the members must vote for to pass the proposal, an increase in the vote threshold m leads to larger bribes, and thus to an increase in the cost. The effect of m on the capture cost depends on how it affects b_k^* for the cost-minimizing number of bribes. However, as $\arg\min_{k \in \{m,...,n\}} c(k)$ has no closed-form characterization, we need to consider the effect of m on b_k^* for all $k \in \{m, ..., n\}$.

The effect of m on b_k^* goes through the same two channels as the effect of k, which we illustrated in Figure 3. Firstly, m has an effect opposite to k on v_k^*: if an additional vote for is needed, the committee increases the probability of a vote for to maintain a high pivotal probability. This increase amounts to choosing a higher v_k^* and raises b_k^*.

However, turning to the second channel, the effect of m on the pivotal probability $\pi^k(v_k^*)$ depends on the relationship between k and m. Importantly, when $k > 2m - 1$, $\pi^k(v_k^*)$ may decrease with m and hence the net effect of m on b_k^* is ambiguous. To see this, focus on the small dispersion case and consider the effect of m on

Binomial$(k-1, p_k^*)$. As explained in the discussion of Lemma 3, this distribution determines the pivotal probability, which decreases with its variance $(k-1) \times p_k^*(1-p_k^*)$. We recognize the individual vote variance $p_k^*(1-p_k^*)$. If $p_k^* = (m-1)/(k-1) < \frac{1}{2}$, raising m increases the individual vote variance and $\pi^k(p_k^*)$ decreases with m. Thus, if $n > 2m-1$, b_k^* could decrease with m for some $k \in \{2m, \ldots, n\}$. In a previous version of the paper (Louis-Sidois and Musolff 2023), we proposed an example that confirms that such cases exist. Hence, we need the restriction $n \leq 2m-1$ to rule out $k > 2m-1$.

We conclude with a discussion of unanimous vote thresholds. When unanimity is required to pass the proposal ($m = n$), all members need to be bribed to make the proposal pass with certainty. Hence, Proposition 1 is trivially true. However, the vote buyer must offer $b_m^* = v_{max}$ to all n members. Thus, multiplying both m and n by the same scalar $\lambda \in \mathbb{N}_+$ would multiply the capture cost by exactly λ, which contrasts with Proposition 2.a: requiring unanimity to pass the proposal uniquely protects the committee from outside influence by severing the pivotal channel.

Moreover, requiring unanimity to reject the proposal ($m = 1$) also alters the pivotal channel. $\pi^k(v)$ is not single-peaked but rather decreasing in v, with $\pi^k(v_{min}) = 1$: all members are pivotal if they all vote against. Hence, if $b < v_{min}$, there is an equilibrium of the voting subgame where all bribed members vote against and the proposal is always rejected. Thus, Lemma 1 would have to be modified, but it does not affect other key findings: the number of bribes trivially exceeds $\frac{3}{2}m - 1 = \frac{1}{2}$. Moreover, for $v_i \overset{iid}{\sim} U[0,1]$, $v_{min} = 0$ and we cannot have $b < v_{min}$. Hence, the argument of Lemma 4 implies that all members are bribed. To summarize, unanimous vote thresholds require some modifications of the results but do not affect the conclusions of the paper.

2.2 Asymmetric Voting Strategies

If members could play asymmetric strategies, would there be an equilibrium of the voting subgame where the proposal is rejected with positive probability if the vote buyer offers the cost-minimizing bribes derived with symmetric strategies? Example 2 shows that the focus on symmetric strategies is indeed not always without loss of generality.

Example 2. Let $(m,n) = (2,3)$ and all members have the same disutility: $v_i = \frac{1}{2}$ for all i.[9] Suppose the vote buyer offers b to all three members. With symmetric strategies, all members vote for with the same probability p (the equilibrium is formally derived in the proof of Lemma A.2.3). An equilibrium probability $\bar{p} \in [0,1]$ solves:

$$b = \frac{1}{2}\pi(\bar{p}) = \bar{p}(1-\bar{p}).$$

This expression is single-peaked and its maximum is $\frac{1}{4}$. As long as $b \leq \frac{1}{4}$, there is an equilibrium with $\bar{p} < 1$ where the proposal is rejected with a positive probability. Thus, if the vote buyer offers (slightly more than) $b = \frac{1}{4}$, she pays $\frac{3}{4}$ and the proposal passes with certainty in all equilibria where members play symmetric strategies.

Now we allow for asymmetric strategies when the three members receive $b = \frac{1}{4}$. There is an equilibrium where one member accepts his bribe with a probability of one and the other two decline with a probability of one. Thus, the focus on symmetric strategies is not without loss. Indeed, with no dispersion, the cheapest bribes such that the proposal passes with certainty in all equilibria are $b = \frac{1}{2}$ offered to two members.$\triangle$

For sufficiently dispersed distributions, however, there is no equilibrium of the voting subgame where the proposal is rejected with a positive probability if the vote buyer offers the cost-minimizing bribes derived in Section 2.1:

Proposition 3. *Suppose the distribution is at least as dispersed as $U[0,1]$. Offering b_n^* to n members, which minimizes the capture cost if members use symmetric strategies, ensures the proposal passes with certainty in any equilibrium of the voting subgame.*

(Proof in Appendix A.3.) Intuitively, dispersion makes the behavior of other members harder to predict, which prevents the existence of asymmetric equilibria. Formally, the proposition's proof relies on an iterated deletion of strictly dominated strategies. Member i's pivotal probability is maximized if others split suitably between two extreme cutoffs. In particular, if $m-1$ other members always accept (cutoff at v_{max}) and $n-m$ always decline (cutoff at v_{min}), member i is pivotal with certainty. Even then, member i still votes for if $v_i < b_n^*$. Hence, cutoffs below b_n^* are not rationalizable.

[9] In this example, we relax the assumption that $F(\cdot)$ is continuously differentiable and has an increasing generalized hazard rate to provide the clearest illustration.

Once those strategies have been eliminated, member i cannot anticipate being pivotal with certainty, and another set of cutoffs is not rationalizable. For distributions at least as dispersed as $U[0,1]$, eventually, no cutoff below v_{max} is rationalizable, and the proposal passes with certainty in all equilibria.

We use $U[0,1]$ as a benchmark to provide a lower bound on dispersion for the proposition to be true. However, not all distributions are ranked in our dispersion order. Thus, this lower bound is sufficient but not necessary and there exist other distributions for which offering b_n^* to n members would also ensure that the proposal passes with certainty in any equilibrium; e.g., see Example 3.

2.3 Unequal Bribes

Can unequal bribes reduce the capture cost? Example 3 illustrates that the vote buyer can 'divide and conquer' for some distributions, and hence that unequal bribes can yield a lower capture cost.

Example 3. Let $(m,n) = (2,3)$ and $v_i \overset{iid}{\sim}$ Bernoulli($\frac{1}{2}$).[10] For bribed member i, a strategy consists of a probability of accepting the bribe if $v_i = 0$, and a probability of accepting if $v_i = 1$. We first characterize the cost-minimizing bribes for $k = 3$ and $k = 2$ if the vote buyer offers the same bribe to all bribed members. Next, we show that there exist unequal bribes that yield a lower capture cost.

Suppose the vote buyer offers the same bribe $b > 0$ to all three members. Each accepts if $v_i = 0$. Hence, the pivotal probability of member i would be maximized if the other two members vote against if their disutility is 1. Then, $\pi_i = \frac{1}{2}$. Thus, if $b < \frac{1}{2}$, there is an equilibrium where members vote for if their disutility is 0 and vote against if their disutility is 1. As a result, the bribe needs to be at least $b = \frac{1}{2}$. If it is (slightly more than) $\frac{1}{2}$, an iterated deletion of dominated strategies proves that no equilibrium of the voting subgame exists where the proposal is rejected with positive probability. A member with disutility $v_i = 0$ votes for. Thus, no member can be pivotal with probability larger than $\frac{1}{2}$, and all of them accept. Hence, capture costs $\frac{3}{2}$.

Now, suppose two members are bribed. The unbribed member votes against. Thus, a bribed member would always be pivotal if the other accepts regardless of his

[10] In this example, we relax the assumption that $F(\cdot)$ is continuously differentiable and has an increasing generalized hazard rate to provide the clearest illustration.

disutility. As a result, the vote buyer needs to offer 1 to both members to guarantee that the proposal passes with certainty in all equilibria. Hence, capture costs 2.

However, the unequal bribes $(b_1, b_2, b_3) = (0.51, 0.51, 0.01)$ yield a lower capture cost. All members vote for if their disutility is 0, and no pivotal probability can exceed $\frac{1}{2}$. Thus, members 1 and 2 always accept. In turn, member 3 is not pivotal and also accepts. As a result, the proposal is accepted with certainty in all equilibria. $\triangle$

Nevertheless, equal bribes can be preferred for other distributions. In particular, equal bribes do minimize capture cost in Example 1:

Example 4. Consider $(m, n) = (2, 3)$ with $v_i \overset{iid}{\sim} U[0, 1]$ and allow the vote buyer to offer unequal bribes (b_1, b_2, b_3). Denoting by $\overline{v}_i$ the equilibrium cutoff of member i, an equilibrium of the voting subgame where all cutoffs are in $(0, 1)$ satisfies:[11]

$$\begin{aligned} \overline{v}_1 \pi_1(\overline{v}_2, \overline{v}_3) &= b_1, \\ \overline{v}_2 \pi_2(\overline{v}_1, \overline{v}_3) &= b_2, \\ \overline{v}_3 \pi_3(\overline{v}_1, \overline{v}_2) &= b_3. \end{aligned} \tag{1}$$

When bribes are large enough, an equilibrium satisfying (1) does not exist and all committee members voting for regardless of their disutility is the unique equilibrium. Thus, the vote buyer offers the cheapest (b_1, b_2, b_3) such that (1) has no solution. To identify these bribes, it is useful to look at the Jacobian of (1):

$$J = \begin{pmatrix} \overline{v}_2(1-\overline{v}_3) + (1-\overline{v}_2)\overline{v}_3 & \overline{v}_1(1-2\overline{v}_3) & \overline{v}_1(1-2\overline{v}_2) \\ \overline{v}_2(1-2\overline{v}_3) & \overline{v}_1(1-\overline{v}_3) + (1-\overline{v}_1)v_3 & (1-2\overline{v}_1)\overline{v}_2 \\ (1-2\overline{v}_2)\overline{v}_3 & (1-2\overline{v}_1)\overline{v}_3 & \overline{v}_1(1-\overline{v}_2) + (1-\overline{v}_1)\overline{v}_2 \end{pmatrix}$$

For given bribes (b_1, b_2, b_3), suppose there exist $(\overline{v}_1, \overline{v}_2, \overline{v}_3)$ solving (1). As long as J is non-singular, following any ϵ-perturbation of (b_1, b_2, b_3) there also exists a solution to (1). By contrast, when the determinant of J is 0, we can find an ϵ-perturbation

[11] Appendix A.4 considers cases where some cutoffs are 0 or 1. They do not affect our conclusion: if there exists a local perturbation of the bribes that guarantees there is no equilibrium where the proposal can be rejected, then the bribes are more expensive than $b_i = \frac{8}{27}$ for all members.

of the bribes such that (1) has no solution within a neighborhood of $(\overline{v}_1, \overline{v}_2, \overline{v}_3)$ and, potentially, no solution at all.

The cost cannot be minimized if some bribes are larger than needed to pass the proposal. Therefore, the cheapest bribes such that the proposal passes with certainty in equilibrium must be arbitrarily close to a (b_1, b_2, b_3) for which there is an ϵ-perturbation of the bribes such that (1) has no solution (the sufficient condition). This can only be the case when the determinant of J is zero (the necessary condition). In the following, we first establish that the bribes of Example 1 ($b_i = \frac{8}{27}$ for all members) satisfy the necessary condition, and then show they also satisfy the sufficient condition. Finally, we argue that they are the cheapest bribes satisfying the necessary condition.

To begin with, computing the determinant of J gives $2\overline{v}_1\overline{v}_2\overline{v}_3(2-\overline{v}_1-\overline{v}_2-\overline{v}_3)$, i.e., the matrix is singular if $\overline{v}_1+\overline{v}_2+\overline{v}_3 = 2$.[12] Thus, if a combination of bribes is associated with an equilibrium satisfying $\sum_{i=1}^{3} \overline{v}_i = 2$, then there exists an ϵ-perturbation of the bribes that ensures that (1) has no solution within a neighborhood of $(\overline{v}_1, \overline{v}_2, \overline{v}_3)$. In particular, $\sum_{i=1}^{3} \overline{v}_i = 2$ is satisfied for $\overline{v}_1 = \overline{v}_2 = \overline{v}_3 = \frac{2}{3}$. Plugging these values into (1) shows that these cutoffs are an equilibrium if $b_i = \frac{8}{27}$ for all members; hence, these bribes satisfy the necessary condition.

Moreover, the iterated deletion of dominated strategies used for Proposition 3 guarantees that if all bribes are (slightly more than) $\frac{8}{27}$, no cutoff in $[0, 1)$ is rationalizable. Hence, the ϵ-perturbation consisting in marginally increasing all bribes guarantees that (1) has no solution, and $b_i = \frac{8}{27}$ for all members satisfies the sufficient condition.

Finally, we establish that $b_i = \frac{8}{27}$ for all members are the cheapest bribes satisfying the necessary condition. We look for $(\overline{v}_1, \overline{v}_2, \overline{v}_3) \in [0, 1]^3, \sum_{i=1}^{3} \overline{v}_i = 2$ that minimizes $\sum_{i=1}^{3} b_i$. Without loss of generality, suppose $\overline{v}_1 < \overline{v}_2 < \overline{v}_3$. We now show that decreasing the difference between cutoffs decreases $\sum_{i=1}^{3} b_i$. In particular, let us increase $\overline{v}_1$ and decrease $\overline{v}_3$ by the same amount: $d\overline{v}_1 = 1$, $d\overline{v}_3 = -1$ and $d\overline{v}_2 = 0$,

[12]The determinant is also 0 if some cutoffs are 0. As $\overline{v}_i \geq b_i$, $\overline{v}_i = 0$ can only be part of an equilibrium if $b_i = 0$. But then, the proposal passes with probability 1 if the two other members receive a bribe of 1, which does not minimize the capture cost.

which keeps $\sum_{i=1}^{3} \overline{v}_i$ constant. Using the Jacobian, the change in the capture cost is

$$\sum_{i=1}^{3} db_i = (\overline{v}_1 - \overline{v}_3)(6\overline{v}_2 - 2),$$

which is negative because $\overline{v}_1 < \overline{v}_3$ and $\overline{v}_2 > 1/3$.[13] As a result, under the constraint $\sum_{i=1}^{3} \overline{v}_i = 2$, $\overline{v}_1 = \overline{v}_2 = \overline{v}_3 = \frac{2}{3}$, minimize $\sum_{i=1}^{3} b_i$. These cutoffs are an equilibrium if $b_i = \frac{8}{27}$ for all members. Thus, the equal bribes identified in Example 1 do minimize the capture cost. △

Is it generally true that the vote buyer offers the same bribes when $v_i \overset{iid}{\sim} U[0,1]$? Example 4 established this for $(m,n) = (2,3)$ by demonstrating that the Jacobian of (1) is not invertible if and only if $\sum_{i=1}^{n} \overline{v}_i = m$. We find the same condition for $(m,n) = (1,2)$, $(1,3)$, $(3,4)$ and $(1,4)$. Hence, in all these cases, equal bribes minimize the capture cost. However, we could not find a general formula for the determinant and prove the result for any (m,n).

Even if Example 3 showed that the restriction to equal bribes is not always without loss of generality, Example 4 indicates that the main model generates economic insights going beyond the equal bribes assumption. To see this, notice that equal bribes did not prevent the vote buyer from setting $k < n$ (i.e., offering some bribes of zero). Restricted to offering equal bribes, the vote buyer did not want to exploit this extreme form of inequality with $v_i \overset{iid}{\sim} U[0,1]$ and instead bribed all members. Example 4 further establishes that for $(m,n) = (2,3)$, she never benefits from *any* form of inequality in the bribes.

3 Sequential Voting

We now consider a committee voting sequentially. The proposal passes if at least m of n members vote for it. Members draw their disutilities from the passing of the proposal at the beginning of the game: $v_i \overset{iid}{\sim} U[0,1]$. The order of votes is known in advance and members observe previous votes, like in the US Senate where members

[13] As $\overline{v}_1 + \overline{v}_2 + \overline{v}_3 = 2$ and $\overline{v}_3 < 1$, we obtain $\overline{v}_1 + \overline{v}_2 \geq 1$. Combining with $\overline{v}_2 > \overline{v}_1$, we must have $\overline{v}_2 > 1/3$.

vote in alphabetical order. The vote buyer minimizes the capture cost subject to the proposal passing with certainty. Bribes are simultaneously and publicly offered to all members before the vote begins, and a bribe is paid if a member votes for the proposal. Bribes can be unequal, but they cannot depend on the number of votes still needed to pass the proposal when the member votes.[14]

This section shows that the vote buyer also offers a number of bribes larger than the vote threshold to exploit pivotal considerations with sequential voting:

Proposition 4. *When voting is sequential and $v_i \stackrel{iid}{\sim} U[0,1]$, the vote buyer bribes all members equally, offering $b = 1/(n-(m-1))$ to all n members.*

(Proof in text below.) Hence, with $v_i \stackrel{iid}{\sim} U[0,1]$, bribing all members equally ensures that the proposal passes with certainty in all equilibria at the lowest possible cost for both simultaneous and sequential voting.

To establish this result, we first characterize the equilibrium of the voting subgame, which itself has to be decomposed into multiple subgames. Without loss of generality, we focus on members who receive strictly positive bribes. Define $S(x,y)$ as the subgame where x votes are needed to pass the proposal and y members still have to vote. The voting subgame begins in $S(m,n)$. If in $S(x,y)$ the member votes for the proposal, $S(x-1,y-1)$ is reached while a vote against leads to $S(x,y-1)$. Members and bribes b_y are now indexed by $y \in \{1,...,n\}$, the number of members still to vote.

Members use backward induction to infer their pivotal probability as in Spenkuch et al. (2018). Let $v(x,y)$ be the cutoff played in $S(x,y)$ and $p(x,y)$ be the probability that the proposal passes given that $S(x,y)$ is reached. We jointly characterize $v(x,y)$ and $p(x,y)$ to find the equilibrium of the voting subgame, beginning with two-member committees. Table 1 gives an example for the expressions of $v(x,y)$ in the left and

[14] As in Genicot and Ray (2006), this assumption rules out bribes which depend on the number of other members accepting. However, it has very different implications because we consider bribes offered before the vote, while members can be approached sequentially in Genicot and Ray (2006). Therefore, in their setup, bribes may depend on the number of votes still needed to pass the proposal. In our model, this would imply that the vote buyer offers 1 to all remaining members if all of their votes are required to pass the proposal, and small bribes when there are more members. If $n > m$, all members would vote for and receive small bribes on the equilibrium path.

$p(x, y)$ in the right panel. When a member votes for the proposal, the subgame located North-West is reached; if he votes against, we move North.

	$v(x, y)$			$p(x, y)$		
	$x = 0$	$x = 1$	$x = 2$	$x = 0$	$x = 1$	$x = 2$
$y = 1$	1	b_1	1	1	b_1	0
$y = 2$	1	$\frac{b_2}{1-b_1}$	$\frac{b_2}{b_1}$	1	$b_1 + b_2$	b_2

Table 1: Equilibrium of the Voting Subgame, An Example

Notes: Equilibrium cutoffs (left panel) and passing probabilities (right panel) for $b_2 \leq b_1$ and $b_1 + b_2 \leq 1$. x is the number of votes required to pass the proposal and y the number of members still to vote.

Example 5. Let $(m, n) = (1, 2)$ and assume members receive positive bribes with $b_1 + b_2 \leq 1$. We solve the game backward and start with the last member, $y = 1$. If member $y = 2$ voted for, $y = 1$ votes in $S(0, 1)$. The proposal passes regardless of the vote of member $y = 1$, who accepts with certainty. Thus, $v(0, 1) = 1$ and $p(0, 1) = 1$. If member $y = 2$ voted against, member $y = 1$ votes in $S(1, 1)$, where he is pivotal. He votes for if $b_1 > v_1$ and we have $v(1, 1) = p(1, 1) = b_1$.

Moving backwards, member $y = 2$ starts in $S(1, 2)$. A vote for passes the proposal. Alternatively, if he votes against, $S(1, 1)$ is reached, where the proposal passes with probability b_1. Thus, member $y = 2$ votes for if

$$b_2 - v_2 > -v_2 b_1 \iff v_2 < \frac{b_2}{1 - b_1},$$

so that $v(1, 2) = b_2/(1 - b_1)$ and the proposal passes with probability

$$p(1, 2) = \frac{b_2}{1 - b_1} + \left(1 - \frac{b_2}{1 - b_1}\right) b_1 = b_1 + b_2.$$

Thus, bribes are substitutes from the perspective of the vote buyer: the proposal passes with certainty for any bribes s.t. $b_1 + b_2 = 1$ at a cost of 1. $\triangle$

Using a recursive characterization of $v(1, y)$ and $p(1, y)$, this substitutability of bribes generalizes when one vote is needed to pass the proposal:

Lemma 5. *In equilibrium,* $p(1,y) = \min\left\{\sum_{s=1}^{y} b_s, 1\right\}$.

(Proof in Appendix B.) With $v_i \overset{iid}{\sim} U[0,1]$, the vote buyer is exactly indifferent between bribing the first member to vote or a member voting later. For general distributions, the problem is not tractable, but there is still a tradeoff. On the one hand, the first member can always determine the passing of the proposal: as $x = 1$, this member is pivotal. On the other hand, a member who votes late is less likely to be pivotal (the proposal may be already accepted), but his cutoff affects members who vote earlier: they forecast that voting against is less likely to make the proposal rejected when later members receive higher bribes. Hence, their cutoffs also increase, as we can see in Example 5 where the cutoff of the first member $v(1,2) = b_2/(1-b_1)$ increases with the bribe of the second member b_1.

The next example shows that when more than one vote is needed to pass the proposal, bribes are not perfect substitutes.

Example 6. Let $(m,n) = (2,2)$ and assume $b_1 + b_2 \leq 1$. We start with $b_2 < b_1$. First, consider member $y = 1$. If member $y = 2$ voted for, $S(1,1)$ is reached, for which we have established $v(1,1) = p(1,1) = b_1$. If member $y = 2$ voted against, the proposal will be rejected and member $y = 1$ votes for. Turning to member $y = 2$, he starts in $S(2,2)$ and votes for if

$$b_2 - v_2 b_1 > 0 \iff v_2 < \frac{b_2}{b_1}$$

so that $v(2,2) = b_2/b_1$ and the proposal passes with probability

$$p(2,2) = v(2,2)p(1,1) = b_2.$$

Bribes are not substitutes anymore: only b_2, the smaller of the two bribes, affects the probability of passing.

Instead, suppose $b_2 \geq b_1$. The strategy of member $y = 1$ is as before and member $y = 2$ votes for if $v_2 < b_2/b_1$. As $b_2/b_1 > 1$, he always votes for and the proposal passes with probability $p(1,1) = b_1$. Again, only the smaller of the bribes affects the probability of passing.

As a result, the vote buyer offers equal bribes. If not, the largest bribe does not affect the probability of passing and should be decreased. Finally, given that the probability of passing is equal to the smaller bribe, this bribe must be 1 to make the proposal pass with certainty. Hence, capture cost is minimized when $b_2 = b_1 = 1$. $\triangle$

The example generalizes as follows:[15]

Lemma 6. *Let $b_y^{(s)}$ be the s-th order statistic (i.e., the s-th lowest value) amongst $\{b_1, \dots, b_y\}$. Then, for $x \geq 1$, in equilibrium $p(x, y) = \min\left\{\sum_{s=1}^{y-(x-1)} b_y^{(s)}, 1\right\}$.*

(Proof in Appendix B.) Hence, the probability of passing is the sum of the $n-(m-1)$ smallest bribes. Intuitively, for given bribes, a member is more likely to accept if he votes early. For instance, in Example 6, when the member who receives the largest bribe votes first ($b_2 \geq b_1$), he accepts regardless of his disutility and free-rides on the second member, relying on him to reject the proposal. This finding generalizes: in $S(x, y)$ with $x > 1$, if b_y is one of the $x - 1$ largest bribes among members still to vote, member y accepts regardless of his disutility. Now, suppose the first $m - 1$ members receive the largest bribes. They accept regardless of their disutility and $S(1, n-(m+1))$ is reached with a probability of one. We have the same pattern when all bribes are equal: the $m - 1$ first members accept and free-ride on the $n - (m - 1)$ last members, who can potentially reject the proposal. Then, we have $x = 1$ and, by Lemma 5, the probability of passing is the sum of the remaining bribes, which are the $n - (m - 1)$ lowest bribes.

Instead, if the member who receives the largest bribe votes last in Example 6 ($b_2 < b_1$), both members decline if their disutility is large enough. In such cases, increasing the largest bribe b_1 has two countervailing effects on the probability of passing. On the one hand, increasing b_1 directly increases the probability of passing because it raises the cutoff of the second member $v(1, 1) = b_1$. On the other hand, b_1 decreases the probability of passing through the first member: he forecasts that voting for is more likely to make the proposal pass, and hence becomes more likely to decline and make the proposal rejected. When $v_i \overset{iid}{\sim} U[0, 1]$, the two effects cancel out and an increase in b_1 does not affect the probability of passing. This mechanism generalizes and the probability of passing is also the sum of the $n - (m - 1)$ lowest bribes if the members who receive the largest bribes do not vote first.

[15] This Lemma uses the convention that empty sums evaluate to zero.

We can now consider the problem of the vote buyer. Given that voting starts in subgame $S(m,n)$, the vote buyer chooses the cheapest combination $\{b_y\}_{y=1}^n$ such that $p(m,n)=1$. Using Lemma 6, we can write this problem as

$$\min_{\{b_y\}_{y=1}^n} \sum_{y=1}^n b_y \quad \text{s.t.} \quad \sum_{s=1}^{n-(m-1)} b_y^{(s)} \geq 1.$$

The $m-1$ largest bribes do not affect the probability of passing. Thus, the cost is minimized when the $m-1$ largest bribes are equal to $b_y^{(n-(m-1))}$, the maximum of the $n-(m-1)$ smallest bribes. As the sum of these bribes must be 1 to make the proposal pass with certainty, the smallest $b_y^{(n-(m-1))}$ is achieved when they are all equal to $1/(n-(m-1))$. Therefore, the lowest bribes are $1/(n-(m-1))$, which implies that all members are bribed. Furthermore, the $m-1$ largest bribes are also equal to the $n-(m-1)$ smallest bribes. As a result, all bribes are equal and we obtain Proposition 4.

We conclude this section with comparative statics for the capture cost. Given that the vote buyer pays $b_n^* = 1/(n-(m-1))$ to n members, the resulting cost is

$$C^{seq}(m,n) \quad = \quad \frac{n}{n-(m-1)}.$$

Comparative statics are similar to Proposition 2. If we multiply m and n by the same scalar λ satisfying $(\lambda m, \lambda n) \in \mathbb{N}_+^2$, the cost is multiplied by less than λ. Moreover, the effect of m is now clearly positive. Finally, the capture cost decreases with n.

4 Cost Comparison

We compare the capture costs under simultaneous and sequential voting for $v_i \overset{iid}{\sim} U[0,1]$. With simultaneous voting, all members are bribed and the cost is

$$C^{sim}(m,n) = n\binom{n-1}{m-1}\left(\frac{m}{n}\right)^m \left(1-\frac{m}{n}\right)^{n-m}.$$

Which voting timing minimizes capture cost depends on the number of members n and on the vote threshold m. The result of the comparison, illustrated in Figure 4, is as follows:

Proposition 5. *Suppose* $v_i \overset{iid}{\sim} U[0,1]$.

(a) If it takes one or all but one votes to pass the proposal, the capture cost is lower with simultaneous voting: $C^{sim}(m,n) < C^{seq}(m,n)$ *for* $m = 1$ *and* $m = n-1$.

(b) If unanimity is not required to pass the proposal (i.e., $m < n$*), there is a* λ^* *such that* $C^{seq}(\lambda m, \lambda n) < C^{sim}(\lambda m, \lambda n)$ *with* $\lambda > \lambda^*$ *and* $(\lambda m, \lambda n) \in \mathbb{N}_+^2$.

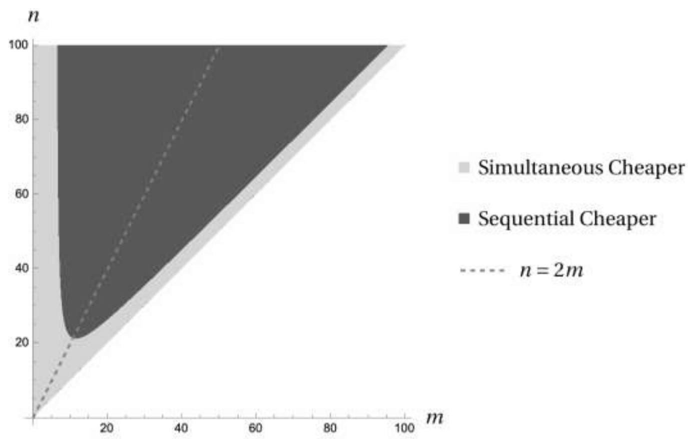


Figure 4: Cost Comparison

Notes: Simulation of the lowest cost as a function of m and n.

(Proof in Appendix C.) The models with sequential and simultaneous voting differ in multiple aspects. However, the equilibrium structure of the voting subgame with sequential voting provides an intuition for the cost comparison. When all bribes are equal to b_n^*, all members accept on the equilibrium path, which shuts down some interactions and prevents a clear exposition of the underlying mechanisms. Instead, suppose members receive equal bribes slightly lower than b_n^*. The cost of these bribes is close to the capture cost, but some members can vote against on the equilibrium path. As explained after Lemma 6, the group of the $m-1$ first members accept their bribes and rely on the group of the $n-(m-1)$ last members to potentially reject the proposal. Intuitively, the first group free-rides on the second group and this free-riding decreases the capture cost. Moreover, free-riding is particularly pronounced if both groups are large: there should be both members who free-ride and members to free-ride on to make the capture cost lower under sequential voting.

If one of the two groups is small, there is a limited effect of free-riding and we find that the capture cost is smaller with simultaneous voting. In particular, if $m = 1$, one vote is sufficient to pass the proposal and the group of free-riders is empty. Indeed, we have $C^{seq}(1, n) = 1$. Meanwhile, $C^{sim}(1, n) = (1 - 1/n)^{n-1}$ is $\frac{1}{2}$ for $n = 2$ and decreases in n.[16] Thus, $C^{seq}(1, n) > C^{sim}(1, n)$ as stated in Proposition 5.a.

Now consider a vote threshold close to unanimity $m = n - 1$ (for $m = n$, pivotal considerations cannot be exploited for both sequential and simultaneous voting and the capture cost is m either way). The group of the $n - (m - 1)$ last members is empty and there is no one to free-ride on. Hence, we also find that the capture cost is lower with simultaneous voting in Proposition 5.a. Formally, $C^{seq}(n - 1, n) = n/2$ and $C^{sim}(n - 1, n) = n \times (1 - 1/n)^n$. As $(1 - 1/n)^n$ is increasing in n, and $\frac{1}{2} > e^{-1} = \lim_{n\to\infty}(1 - 1/n)^n$, we have $C^{seq}(n - 1, n) > C^{sim}(n - 1, n)$.

Turning to Proposition 5.b, the result simply states that the capture cost is smaller under sequential voting if m and n are sufficiently large and the vote threshold is not one of the extreme cases already discussed. This result can also be explained with free-riding: the size of the two groups is limited when m and n are small, and Figure 4 confirms that the capture cost is smaller with simultaneous voting. As we multiply both m and n by a given λ such that $(\lambda m, \lambda n) \in \mathbb{N}^2_+$, the size of the two groups increases and the capture cost becomes eventually smaller with sequential voting because of free-riding. In the limit, we have:

$$\lim_{\lambda\to\infty} C^{seq}(\lambda m, \lambda n) = \frac{1}{1 - \frac{m}{n}} < \infty = \lim_{\lambda\to\infty} C^{sim}(\lambda m, \lambda n).$$

Hence, free-riding even implies that the cost grows bounded with sequential voting, which is not the case with simultaneous voting. With sequential voting, the cost depends on the share of members in the two groups. The proposal is accepted with certainty if the sum of the bribes in the group of the $n - (m - 1)$ last members is one. These bribes represent a share $(n - (m - 1))/n$ of the capture cost because all

[16] As $v_{min} = 0$, the assumption $m > 1$ in Section 2.1 plays no role and $C^{sim}(m, n)$ accurately defines the capture cost for $m = 1$.

members receive the same bribe, and the total cost is the inverse of this share. As λ becomes large, this share converges to $(n-m)/n$ and free-riding implies that the cost is bounded.

5 Concluding Remarks

When members have uncertain preferences, the vote buyer bribes supermajorities to exploit pivotal considerations. As we considered bribes conditioned on individual voting decisions, we conclude with a discussion of other contractual environments.

If bribes are conditioned on the passing of the proposal, a pivotal vote also decides the payment of the bribes. Thus, it is a weakly dominant strategy to vote against if the disutility exceeds the bribe. The vote buyer cannot exploit pivotal considerations with such contracts: to make the proposal pass with certainty, she has to offer v_{max} to m members. Hence, conditioning on passing is bad for the vote buyer. Instead, suppose bribes depend on the number of votes for. A vote matters for the bribe even when it is not pivotal for the passing of the proposal. In a previous version of the paper (Louis-Sidois and Musolff 2023), we proposed an example where the vote buyer exploited pivotal considerations: she bribed a number of members larger than the vote threshold and paid less than when she only conditioned on passing.

In an unrestricted contractual environment, bribes can be contingent on the entire vector of votes. In such a case, Dal Bo (2007) has established that capture occurs at no cost: the vote buyer promises a bribe v_{max} if a member is pivotal and an arbitrarily small bribe otherwise. Voting for is then a dominant strategy, and when more than m members receive such offers, the proposal always passes. If such contracts are allowed, our solution is still relevant for a budget-constrained vote buyer: even if members are never pivotal in equilibrium, the vote buyer must be able to pay the large pivotal bribes for Dal Bo's strategy to be credible. Therefore, while our solution is more expensive (as the vote buyer actually pays the bribes), it would nevertheless be feasible for lower budget constraints.

We have considered offers visible to all. If offers are privately communicated to each member, the vote buyer cannot credibly claim to have bribed more members than necessary and the number of bribes is equal to the vote threshold in equilibrium.

Each bribe is equal to v_{max}, and capture is more expensive with private offers. To see why offering more bribes than the vote threshold is not credible, consider a voting profile where more than m members vote for with certainty. The vote buyer would deviate and propose exactly m bribes. This deviation cannot be detected by members who continue receiving the bribe, but, in equilibrium, a bribed member cannot believe there are more than $m-1$ other bribes. The bribe must be equal to v_{max} for him to always vote for.

Finally, our model can be reinterpreted with punishments for members who vote against instead of bribes. For the vote buyer, enforcing punishment is likely to be costly, which implies she effectively pays for members who vote against the proposal. If she has enough resources for punishment, she uses the strategy of this paper. Capture is costless because all approached members vote for. The capture cost we computed corresponds to the minimum resources needed to secure certain passing of the proposal.

Appendix A: Proofs (Simultaneous Voting)

A.1 Helpful Facts

Before proceeding to the proofs, we establish some necessary prerequisites. Recall that $\Gamma(\cdot)$ is a continuous extension of the factorial function. In particular, $\Gamma(x) = (x-1)!$ for $x \in \mathbb{N}$. Thus,

$$\binom{k}{m} = \exp\Big(\log\Gamma(k+1) - \log\Gamma(m+1) - \log\Gamma(k-m+1)\Big).$$

The digamma function is defined as $\psi(x) = \frac{\partial}{\partial x}\log\Gamma(x)$. Hence,

$$\frac{\partial \log\binom{k}{m}}{\partial k} = \psi(k+1) - \psi(k-m+1), \quad \frac{\partial \log\binom{k}{m}}{\partial m} = \psi(k-m+1) - \psi(m+1).$$

To characterize these derivatives, our proofs below will make use of the fact that $\psi(x+1) = \psi(x) + \frac{1}{x}$ and hence $\psi(b) - \psi(a) = \sum_{c=a}^{b-1}\frac{1}{c}$ for $b > a$. For these and more facts about $\psi(\cdot)$, see Abramowitz and Stegun (1972, p.258).

A.2 Proof of Proposition 1

We build towards the proof of Proposition 1 via several intermediate results.

Lemma A.2.1. *If $k > m$ and $F(\cdot)$ has an increasing generalized hazard rate, $v\pi^k(v)$ is single-peaked in $v \in [v_{min}, v_{max}]$.*

Proof. Writing out the pivotal probability, we want to show the single-peakedness of

$$v\pi^k(v) = v\binom{k-1}{m-1}F(v)^{m-1}\big(1-F(v)\big)^{k-m},$$

with associated log derivative

$$\frac{d\log v\pi^k(v)}{dv} = \frac{1}{v}\left(1 + \frac{vF'(v)}{1-F(v)}\frac{m-1-(k-1)F(v)}{F(v)}\right).$$

As $F'(v) \geq 0$, $\lim_{v\to v_{min}} \frac{d\log v\pi^k(v)}{dv} > 0$. We now establish $\lim_{v\to v_{max}} \frac{d\log v\pi^k(v)}{dv} < 0$. Suppose (for later contradiction) that $\frac{F'(v)}{1-F(v)}$ is bounded above by some A. Then

$$-\log(1-F(v)) = \int_{v_{min}}^{v} \frac{F'(x)}{1-F(x)}dx \leq A(v - v_{min}).$$

But this would imply that the LHS is finite as $v \to v_{max}$, a contradiction. Hence $\frac{F'(v)}{1-F(v)}$ is unbounded, and as $\frac{vF'(v)}{1-F(v)}$ is increasing, $\lim_{v\to v_{max}} \frac{vF'(v)}{1-F(v)} = \infty$. Finally, this implies $\lim_{v\to v_{max}} \frac{d\log v\pi^k(v)}{dv} < 0$.

Thus, as $\frac{d\log v\pi^k(v)}{dv}$ is continuous in v, there has to be at least one solution to FOC $\frac{d\log v\pi^k(v)}{dv} = 0$. Furthermore, we can rearrange this FOC to yield

$$\left[(k-1) - \frac{m-1}{F(v)}\right]^{-1} = \frac{vF'(v)}{1-F(v)}. \tag{2}$$

The RHS is the generalized hazard rate. It is weakly increasing (by assumption) and positive. The LHS is negative for $F(v) < \frac{m-1}{k-1}$, so we can restrict attention to $F(v) \geq \frac{m-1}{k-1}$. On this domain, the LHS is strictly decreasing. Thus, (2) has a unique solution. As the FOC has a unique solution and $v\pi^k(v)$ is initially increasing and eventually decreasing in v, it must be single-peaked in v. □

Lemma 1. *Suppose the vote buyer offers a bribe $b > 0$ to k members with $m \leq k \leq n$. In the symmetric equilibrium of the voting subgame in which the proposal passes with the smallest probability,*

(a) If $b \leq \max_{v \in [v_{min}, v_{max}]} v\pi^k(v)$, bribed members vote for the proposal if their disutility is smaller than a cutoff $\overline{v}$ that satisfies $\overline{v} = \min\{v \in [v_{min}, v_{max}] : b = v\pi^k(v)\}$ where

$$\pi^k(v) = \binom{k-1}{m-1} F(v)^{m-1} \big(1 - F(v)\big)^{k-m}.$$

Moreover, they vote against if their disutility is larger than $\overline{v}$ and a member with $v_i = \overline{v}$ can vote for with any probability.

(b) If $b > \max_{v \in [v_{min}, v_{max}]} v\pi^k(v)$, all bribed members vote for the proposal regardless of their disutility: $\overline{v} > v_{max}$.

Proof. In a symmetric equilibrium, bribed members' strategies σ_i depend only on their disutility v_i, i.e., $\sigma_i = \sigma(v_i)$. As established in the main text, σ takes a cutoff form, i.e., $\sigma(v_i) = 1$ if $v_i < \overline{v}$, $\sigma(\overline{v}) \in [0, 1]$, and $\sigma(v_i) = 0$ if $v_i > \overline{v}$. $\sigma(\overline{v})$ does not affect pivotal probabilities as $F(\cdot)$ is continuous. Furthermore, we assume unbribed members vote against. We consider the possible values of $\overline{v}$:

1. If $\overline{v} \in [v_{min}, v_{max}]$, $0 \leq \pi^k(\overline{v}) \leq 1$ and $\overline{v}$ is an equilibrium if $\overline{v}\pi^k(\overline{v}) = b$. Lemma A.2.1 guarantees that there are at most two such equilibria, so that either $\min\{v \in [v_{min}, v_{max}] : v\pi^k(v) = b\}$ is well-defined or there is no such equilibrium.
2. If $\overline{v} > v_{max}$, every bribed member votes for regardless of his disutility.
   (i) If $k > m$, members are pivotal with a probability of zero, and all members voting for is an equilibrium.
   (ii) If $k = m$, members are pivotal with a probability of one if they all vote for. Hence, all members voting for is an equilibrium iff $b \geq v_{max}$. □

Lemma A.2.2. *Suppose $\tilde{F}(\cdot)$ and $F(\cdot)$ have increasing generalized hazard rates, where $\tilde{F}(\cdot)$ is more dispersed than $F(\cdot)$. Then, for any number of bribed members $k \in \{m, \dots, n\}$, $\tilde{F}(\tilde{v}_k^*) \geq F(v_k^*)$, i.e., in equilibrium each bribed member is more likely to vote for when the distribution is more dispersed.*

Proof. Here v_k^* and $\tilde{v}_k^*$ refer to the solutions to FOC (2) with F and $\tilde{F}$ respectively. We can express this FOC in terms of $p = F(v)$ as

$$\left[(k-1) - \frac{m-1}{p}\right]^{-1} = \frac{F^{-1}(p)F'(F^{-1}(p))}{1-p}. \tag{3}$$

Consider (3) separately for the two distributions F and $\tilde{F}$. The LHSs are identical and, for $p \geq \frac{m-1}{k-1}$, decreasing in p while the RHSs are increasing. We now establish that for $p \in [0,1]$ the RHS is smaller for $\tilde{F}$ than for F using the definition of dispersion:

$$\begin{aligned}\frac{\partial}{\partial p}\left(\frac{\tilde{F}^{-1}(p)}{F^{-1}(p)}\right) \geq 0 \quad &\Rightarrow \quad \frac{1}{\tilde{F}'(\tilde{F}^{-1}(p))}F^{-1}(p) \geq \tilde{F}^{-1}(p)\frac{1}{F'(F^{-1}(p))} \\ &\Rightarrow \quad F^{-1}(p)F'(F^{-1}(p)) \geq \tilde{F}^{-1}(p)\tilde{F}'(\tilde{F}^{-1}(p)) \\ &\Rightarrow \quad \frac{F^{-1}(p)F'(F^{-1}(p))}{1-p} \geq \frac{\tilde{F}^{-1}(p)\tilde{F}'(\tilde{F}^{-1}(p))}{1-p}\end{aligned}$$

It follows that $\tilde{F}(\tilde{v}_k^*) \geq F(v_k^*)$. □

Lemma A.2.3. *For any number of bribed members $k \in \{m, \ldots, n\}$, let p_k^* be the probability of voting for that maximizes a cutoff member's expected cost of voting for when $v_i = \delta \in \mathbb{R}_+$ for all i, i.e., the disutility distribution has no dispersion:*

$$p_k^* := \arg\max_{p\in[0,1]} \delta\pi^k(p) = \arg\max_{p\in[0,1]} \binom{k-1}{m-1} p^{m-1}(1-p)^{k-m}.$$

For any disutility distribution $F(\cdot)$ that is continuously differentiable on $[v_{min}, v_{max}]$ and has an increasing generalized hazard rate, $F(v_k^*) \geq p_k^*$.

Proof. When $v_i = \delta \in \mathbb{R}_+$ for all i, in an equilibrium where bribed members play the same strategy, they mix and vote for with a common probability p. Assuming k members are bribed, the pivotal probability is:

$$\pi^k(p) = \binom{k-1}{m-1} p^{m-1}(1-p)^{k-m}.$$

This function is single-peaked in p. Furthermore, members vote for if $b > \delta\pi^k(p)$, against if $b < \delta\pi^k(p)$ and are indifferent if $b = \delta\pi^k(p)$. If $b = \delta\pi^k(p)$ for some p, then all members voting for with probability p is an equilibrium in which the proposal is not accepted with certainty. Hence, to make the proposal pass with certainty in any equilibrium of the voting subgame, b must be at least $b_k^* = \max_{p\in[0,1]} \delta\pi^k(p)$, and the vote buyer's cost is kb_k^*. $\delta\pi^k(p)$ is maximized if $\frac{d\delta\pi^k(p)}{dp} = 0$, which is satisfied if:

$$(m-1) - (k-1)p = 0. \tag{4}$$

Hence, $p_k^* = \arg\max_{p\in[0,1]} \delta\pi^k(p) = \frac{m-1}{k-1}$. By contrast, the FOC defining $F(v_k^*)$ for a distribution with dispersion is (3), with $p = F(v)$. The p solving (3) is above $\frac{m-1}{k-1}$: for $p < \frac{m-1}{k-1}$, the LHS of (3) is negative and the RHS is positive. Hence, $F(v_k^*) \geq p_k^*$. $\square$

Lemma 2. *The set of cost-minimizing numbers of bribes with a distribution $\tilde{F}(\cdot)$ dominates in the strong set order the set of cost-minimizing numbers of bribes with a less dispersed distribution $F(\cdot)$.*

Proof. As a prerequisite, we find a simple expression for $\frac{d\log c}{dk}$. Note

$$\begin{aligned} \log\big(c(k)\big) &= \log(k) + log(v_k^*) + \log\binom{k-1}{m-1} + (m-1)\log F(v_k^*) \\ &\quad + (k-m)\log\Big(1 - F(v_k^*)\Big). \end{aligned}$$

Interpreting k as a real number, taking the total derivative w.r.t. k and applying the envelope theorem:

$$\begin{aligned} \frac{d\log\big(c(k)\big)}{dk} &= \frac{\partial\log\big(c(k)\big)}{\partial k} + \underbrace{\frac{\partial\log\big(c(k)\big)}{\partial v}\bigg|_{v=v_k^*}}_{0} \times \frac{dv_k^*}{dk} \\ &= \frac{1}{k} + \psi(k) - \psi(k-m+1) + \log\Big(1 - F(v_k^*)\Big). \end{aligned} \tag{5}$$

We can now prove the result. Consider distributions $\bar{F}(\cdot)$ and $\tilde{F}(\cdot)$, with $\tilde{F}(\cdot)$ more dispersed. Making the dependence of cost on distribution F explicit for this proof only, for any $k_1, k_2 \in \{m, \ldots, n\}$, $k_1 < k_2$,

$$[\log c(k_2, F) - \log c(k_1, F)] = \int_{k_1}^{k_2} \frac{d \log c(k, F)}{dk} dk$$

$$= \int_{k_1}^{k_2} \frac{1}{k} + \psi(k) - \psi(k - m + 1) + \log\Big(1 - F(v_k^*)\Big) dk. \tag{6}$$

From Lemma A.2.2, $\tilde{F}(\tilde{v}_k^*) \geq \bar{F}(\bar{v}_k^*)$ and thus $\frac{d \log c(k,\tilde{F})}{dk} \leq \frac{d \log c(k,\bar{F})}{dk}$, i.e., the cost decreases faster (or increases slower) in k for more dispersed distributions. Hence, if $c(k_2, \tilde{F}) \geq c(k_1, \tilde{F})$, then also $c(k_2, \bar{F}) \geq c(k_1, \bar{F})$ (and the same for strict inequalities). We indicate usage of this fact and its contrapositive by $\Rightarrow_*$ below. Suppose (A) $\bar{k} \in \arg\min_{k \in \{m,\ldots,n\}} c(k, \bar{F})$ and (B) $\tilde{k} \in \arg\min_{k \in \{m,\ldots,n\}} c(k, \tilde{F})$. Then:

1. $\boxed{\min\{\bar{k}, \tilde{k}\} \in \arg\min_{k \in \{m,\ldots,n\}} c(k, \bar{F}).}$ If $\bar{k} \leq \tilde{k}$, this follows from (A). If $\bar{k} > \tilde{k}$, by (B), $c(\bar{k}, \tilde{F}) \geq c(\tilde{k}, \tilde{F}) \Rightarrow_* c(\bar{k}, \bar{F}) \geq c(\tilde{k}, \bar{F})$. Thus, $\tilde{k}$ also minimizes $c(k, \bar{F})$ as required.
2. $\boxed{\max\{\bar{k}, \tilde{k}\} \in \arg\min_{k \in \{m,\ldots,n\}} c(k, \tilde{F}).}$ If $\bar{k} \leq \tilde{k}$, this follows from (B). If $\bar{k} > \tilde{k}$, by (A), $c(\tilde{k}, \bar{F}) \geq c(\bar{k}, \bar{F}) \Rightarrow_* c(\tilde{k}, \tilde{F}) \geq c(\bar{k}, \tilde{F})$. Thus, $\bar{k}$ also minimizes $c(k, \tilde{F})$ as required. □

Lemma A.2.4. *When the disutility distribution has no dispersion*

(a) any cost-minimizing number of bribes weakly exceeds $\min\{\frac{3}{2}m - 1, n\}$,

(b) any cost-minimizing number of bribes is at most $\min\{\frac{3}{2}m + 1, n\}$.

Proof. We establish that $c(k)$ is decreasing for $k \leq \frac{3}{2}m - \frac{1}{2}$ and increasing for $k \geq \frac{3}{2}m + \frac{1}{2}$. Recall that with no dispersion, $v_i = \delta \in \mathbb{R}_+$ for all members $i \in \{1, \ldots, n\}$. The solution to (4) is $p_k^* = \frac{m-1}{k-1}$, giving a cost for the vote buyer of

$$c(k) = k \times \delta \binom{k-1}{m-1} \left(\frac{m-1}{k-1}\right)^{m-1} \left(\frac{k-m}{k-1}\right)^{k-m}.$$

The log derivative of this expression w.r.t. k is

$$\frac{\partial \log c(k)}{\partial k} = \frac{1}{k} + [\psi(k) - \log(k-1)] - [\psi(k-m+1) - \log(k-m)] =: \xi(k, m)$$

where ψ is the Digamma function. We employ the following inequality from Qi et al. (2005, p.305, Corollary 1) to bound ξ:

$$\frac{1}{2x} - \frac{1}{12x^2} < \psi(x+1) - \log(x) < \frac{1}{2x}.$$

In particular, we have $\underline{\xi}(k,m) < \xi(k,m) < \overline{\xi}(k,m)$, where

$$\begin{aligned}
\underline{\xi}(k,m) &= \frac{1}{k} + \frac{1}{2(k-1)} - \frac{1}{12(k-1)^2} - \frac{1}{2(k-m)}, \\
\overline{\xi}(k,m) &= \frac{1}{k} + \frac{1}{2(k-1)} + \frac{1}{12(k-m)^2} - \frac{1}{2(k-m)}.
\end{aligned}$$

We use these bounds to establish the two claims.

(a) $\boxed{c(k) \text{ is decreasing for } k \leq (3/2)m - (1/2).}$

(i) For $m = 2$, there is nothing to prove as only $k = 2$ satisfies $k \leq \frac{3}{2}m - \frac{1}{2}$.

(ii) For $m = 3$, $\frac{3}{2}m - \frac{1}{2} \leq m + 1$ so it suffices to show $c(m+1) < c(m)$:

$$c(m+1) = \delta(m+1) \times \left(\frac{m-1}{m}\right)^{m-1} < \delta(m+1) \times \frac{m}{m+1} = c(m).$$

This inequality did not use $m = 3$ and implies that $c(k)$ decreases for $k \leq m + 1$.

(iii) For $m > 3$, we also require that for $k \in [m+1, \frac{3}{2}m - \frac{1}{2}]$, $\xi(k,m) < \overline{\xi}(k,m) < 0$. But note

$$\overline{\xi}\left(\frac{3}{2}m - \frac{1}{2}, m\right) = \frac{3-m}{3(m-1)^2(3m-1)} < 0,$$

where the inequality follows as $m > 3$. Furthermore,

$$\frac{\partial \overline{\xi}(k,m)}{\partial k} = \frac{1}{6}\left(-\frac{6}{k^2} + \frac{3}{(k-m)^2} + \frac{1}{(m-k)^3} - \frac{3}{(k-1)^2}\right) > 0.$$

where the inequality follows for $k \in [m+1, \frac{3}{2}m - \frac{1}{2}]$. To conclude, $\overline{\xi}(k,m)$ increases on $k \in [m+1, \frac{3}{2}m - \frac{1}{2}]$ and is negative for $k = \frac{3}{2}m - \frac{1}{2}$; it must therefore be negative on the interval. Hence, $\xi(k,m)$ is also negative, and

$c(k)$ is decreasing in k.

(b) $\boxed{c(k) \text{ is increasing for } k \geq (3/2)m + (1/2).}$ We now establish that $\underline{\xi}(k, m) > 0$ for $k \geq \frac{3}{2}m + \frac{1}{2}$. Consider $k(k-m)\underline{\xi}(k, m)$, which has the sign of $\underline{\xi}(k, m)$ as $k > m$. We have:

$$\frac{\partial[k(k-m)\underline{\xi}(k, m)]}{\partial k} = \frac{k(5m+32) + 12k^2(k-3) - 7m - 6}{12(k-1)^3}.$$

The denominator is positive. The numerator has value $\frac{5+3m(2+m(-22+27m))}{2} > 0$ for $k = \frac{3}{2}m + \frac{1}{2}$ and increases with k (the derivative w.r.t. k is $32+36(k-2)k+5m$, which is positive as $k \geq 2$). Thus, $\partial[k(k-m)\underline{\xi}(k, m)]/\partial k > 0$ for $k \geq 3m/2+1/2$. Thus, if $k(k-m)\underline{\xi}(k, m)$, and hence $\underline{\xi}(k, m)$ are positive for some $\bar{k}$, then $\underline{\xi}(k, m) > 0$ for $k > \bar{k}$. Moreover:

$$\underline{\xi}\left(\tfrac{3}{2}m + \tfrac{1}{2}, m\right) = \frac{m(69m-28) - 1}{3(1-3m)^2(m+1)(3m+1)} > 0.$$

Thus, $\underline{\xi}(k, m) > 0$ for $k \geq \frac{3}{2}m + \frac{1}{2}$. As a result, $\xi(k, m) > 0$ and the cost increases for $k \geq \frac{3}{2}m + \frac{1}{2}$.

Finally, notice that if m is even, $\frac{3}{2}m - 1$, $\frac{3}{2}m$ and $\frac{3}{2}m + 1$ are integers. If m is odd, $\frac{3}{2}m - \frac{1}{2}$ or $\frac{3}{2}m + \frac{1}{2}$ are integers. Thus, any integer minimizing $c(k)$ weakly exceeds $\frac{3}{2}m - 1$ and is at most $\frac{3}{2}m + 1$. □

Lemma 3. *Any cost-minimizing number of bribes is at least* $\min\{\frac{3}{2}m - 1, n\}$.

Proof. First of all, the set of cost-minimizing numbers of bribes with a distribution $F(\cdot)$ that is continuously differentiable on $[v_{min}, v_{max}]$ and has an increasing generalized hazard rate dominates in the strong set order the set of cost-minimizing numbers of bribes when the disutility distribution has no dispersion. This follows from the proof of Lemma 2. Consider (6). With no dispersion, $F(v_k^*)$ has to be replaced by p_k^*. By Lemma A.2.3, $F(v_k^*)$ exceeds p_k^*. Hence, the cost decreases faster (or increases slower) in k with dispersion than without. Thus, the proof of Lemma 2 establishes set order dominance. Moreover, with no dispersion, every element of $\arg\min_{k \in \{m,\dots,n\}} c(k)$ is at least $\min\{\frac{3}{2}m - 1, n\}$ (Lemma A.2.4). As a result, every element of $\arg\min_{k \in \{m,\dots,n\}} c(k)$ is at least $\min\{\frac{3}{2}m - 1, n\}$. □

Lemma 4. *If the disutility distribution is $U[0,1]$, all members are bribed.*

Proof. If $v_i \overset{iid}{\sim} U[0,1]$, we have $F(v_k^*) = \frac{m}{k}$. Plugging into (5) yields

$$\frac{\partial \log c(k)}{\partial k} = \log\left(\frac{k-m}{k}\right) + \psi(k) - \psi(k-m+1) + \frac{1}{k},$$

Using the properties of the digamma function given in Appendix A.1 and noting that for any decreasing function $\sum_{s=a+1}^{b} g(s) < \int_a^b g(s)ds$,

$$\begin{aligned} \psi(k) - \psi(k-m+1) + \frac{1}{k} &= \sum_{s=1}^{m} \frac{1}{k-m+s} \\ &< \int_0^m \frac{1}{k-m+s} ds = -log\Big(\frac{k-m}{k}\Big). \end{aligned}$$

Thus, the cost strictly decreases in k so that $\arg\min_{k\in\{m,\dots,n\}} c(k) = n$. □

Lemma A.2.5. *For $v_i \sim U\left[\frac{1}{2}-\alpha, \frac{1}{2}+\alpha\right]$, the cost function $c(k)$ has a unique global real minimizer.*

Proof. Recall that

$$\frac{d \log c(k)}{dk} = \frac{1}{k} + \psi(k) - \psi(k-m+1) + \log(1 - F(v_k^*)).$$

We establish that this expression crosses the horizontal axis at most once, and necessarily from below. To do so, we will show that when the FOC is satisfied (i.e., when $\frac{d \log c(k)}{dk} = 0$), $\frac{d^2 \log c(k)}{dk^2} > 0$.[17]

1. Consider:

$$\frac{d^2 \log c(k)}{dk^2} = -\frac{1}{k^2} + \psi'(k) - \psi'(k-m+1) - \frac{F'(v_k^*)}{1-F(v_k^*)}\frac{dv_k^*}{dk}.$$

[17] One may worry that there could be multiple local minima. However, this cannot be the case as the derivative is continuous: v_k^* is continuous, and hence so is $\frac{d \log c(k)}{dk}$. This means $c(k)$ has (at most) one local minimum (otherwise it would have to have at least one local maximum, which is ruled out by the proof).

The expression has the same sign as:

$$k\left(-\frac{1}{k^2}+\psi'(k)-\psi'(k-m+1)-\frac{F'(v_k^*)}{1-F(v_k^*)}\frac{dv_k^*}{dk}\right). \tag{7}$$

We now argue that for $v_i \sim U\left[\frac{1}{2}-\alpha, \frac{1}{2}+\alpha\right]$, (7) is increasing in α so that if we want to show that it is positive, we only need to do so for $\alpha \to 0$.

2. For $v_i \sim U\left[\frac{1}{2}-\alpha, \frac{1}{2}+\alpha\right]$, the FOC for the choice of v_k^* reduces to

$$\frac{2(k-m)}{2\alpha-2v_k^*+1}=\frac{2(m-1)}{2\alpha+2v_k^*-1}+\frac{1}{v_k^*} \tag{8}$$

Hence,

$$v_k^*=\frac{\sqrt{(4\alpha k-8\alpha m+4\alpha-2k-2)^2-16\left(1-4\alpha^2\right)k}-4\alpha k+8\alpha m-4\alpha+2k+2}{8k}. \tag{9}$$

Furthermore, we can implicitly differentiate (8) to get an expression for $\frac{dv_k^*}{dk}$ and plug the value of v_k^* from (9) into $\frac{F'(v_k^*)}{1-F(v_k^*)}$ to find:

$$-\frac{F'(v_k^*)}{1-F(v_k^*)}\frac{dv_k^*}{dk}=\frac{m\sqrt{4\left(4\alpha^2-1\right)k+(-2\alpha(k-2m+1)+k+1)^2}-((2\alpha-1)k(m-2))+\alpha m(4m-2)+m}{2k(k-m)\sqrt{4\left(4\alpha^2-1\right)k+(-2\alpha(k-2m+1)+k+1)^2}}. \tag{10}$$

The derivative of this last expression with respect to α is

$$\frac{d}{d\alpha}\left(-\frac{F'(v_k^*)}{1-F(v_k^*)}\frac{dv_k^*}{dk}\right)=\frac{8(m-1)(1-2\alpha)}{\left(4\left(4\alpha^2-1\right)k+(-2\alpha(k-2m+1)+k+1)^2\right)^{3/2}}>0.$$

Hence it is increasing in α, and so is (7).

3. For $\alpha \to 0$, (10) implies:

$$-\frac{F'(v_k^*)}{1-F(v_k^*)}\frac{dv_k^*}{dk}\to\frac{m-1}{(k-1)(k-m)}$$

and (7) becomes:

$$k\left(-\frac{1}{k^2}+\psi'(k)-\psi'(k-m+1)+\frac{m-1}{(k-1)(k-m)}\right).$$

We only need to show this expression is positive for k such that the FOC ($\frac{d \log c(k)}{dk} = 0$) is satisfied. We establish a strictly stronger claim: we prove that a different expression (Z below) that reduces to this expression when the FOC is satisfied is positive for all k:

$$\begin{aligned} Z &= \psi(k) - \psi(k-m+1) + \log(1 - \frac{m-1}{k-1}) \\ &\quad + k \left(\psi'(k) - \psi'(k-m+1) + \frac{m-1}{(k-1)(k-m)} \right) \end{aligned}$$

For $\alpha \to 0$, $F(v_k^*) \to \frac{m-1}{k-1}$ and the FOC is satisfied if $\frac{1}{k} + \psi(k) - \psi(k-m+1) + \log(1 - \frac{m-1}{k-1}) = 0$. In this case, $Z = (7)$.

4. To show $Z > 0$, we first utilize the bounds of Qi et al (2005) to provide a lower bound for the first line (and, then, for Z). To this end, note that their Corollary 8 implies

$$\frac{1}{2x} + \log(x) - \frac{1}{12x^2} \leq \psi(x+1) \leq \frac{1}{2x} + \log(x),$$

and hence

$$\begin{aligned} &\psi(k) - \psi(k-m+1) + \log(k-m) - \log(k-1) \\ &\geq -\frac{1}{2(k-m)} - \frac{1}{12(k-1)^2} + \frac{1}{2(k-1)} \\ &\geq \frac{1}{k-1} - \frac{1}{k-m}. \end{aligned}$$

Plugging this back into Z, it now suffices to show

$$\frac{1}{k-1} - \frac{1}{k-m} + k \left(\psi'(k) - \psi'(k-m+1) + \frac{m-1}{(k-1)(k-m)} \right) \geq 0.$$

This simplifies to

$$\frac{m-1}{k(k-m)} + \psi'(k) - \psi'(k-m+1) \geq 0.$$

We again utilize bounds from Corollary 8 of Qi et al (2005), this time

$$\frac{1}{x} - \frac{1}{2x^2} + \frac{1}{6x^3} - \frac{1}{30x^5} \leq \psi'(x+1) \leq \frac{1}{x} - \frac{1}{2x^2} + \frac{1}{6x^3}.$$

Thus, we need to show

$$\frac{\frac{1}{m-k} - 1}{k} + \frac{1}{30}\left(\frac{15}{(k-m)^2} + \frac{5}{(m-k)^3} - \frac{15}{(k-1)^2} + \frac{5}{(k-1)^3} - \frac{1}{(k-1)^5} + \frac{30}{k-1}\right) \geq 0.$$

This holds for $k \geq m+1$. As we have shown in the proof of Lemma A.2.4 that the cost decreases between $k = m$ and $k = m+1$, the FOC cannot be satisfied for $k \in [m, m+1]$. Hence, whenever $\frac{d \log c(k)}{dk} = 0$, $\frac{d^2 \log c(k)}{dk^2} > 0$. □

Proposition 1.

(a) For any disutility distribution, any cost-minimizing number of bribes is at least $\min\{\frac{3}{2}m - 1, n\}$*,*

(b) For any number of bribes $k \in \mathbb{N}$ *such that* $\min\{\frac{3}{2}m + 1, n\} \leq k \leq n$*, there exists a disutility distribution such that* k *is a cost-minimizing number of bribes.*

Proof. Notice that this proof does not follow the order of the text: it builds on Lemmata 2, 3 and 4.

(a) See Lemma 3.

(b) We establish the claim using uniform distributions: $v_i \overset{iid}{\sim} U\left[\frac{1}{2} - \alpha, \frac{1}{2} + \alpha\right]$. For this proof only, let $k^* := \arg\min_{k \in [m,n]} c(k)$ be the real (as opposed to integer) number of bribes that minimizes the cost; this number is unique by Lemma A.2.5. We first show that k^* is continuous in α. Then, we establish that $\lim_{\alpha \to 0} k^* \leq \frac{3}{2}m + \frac{1}{2}$ and $\lim_{\alpha \to \frac{1}{2}} k^* = n$. Hence, by the intermediate value theorem, for all $k \geq \frac{3}{2}m + \frac{1}{2}$, there exists an $\alpha \in (0, \frac{1}{2})$ such that $k^* = k$. Finally, for any integer $k \in \{m, \ldots, n\}$, $k^* = k$ implies $k \in \arg\min_{\kappa \in \{m,\ldots,n\}} c(\kappa)$. Hence, if m is odd, $\frac{3}{2}m + \frac{1}{2} \in \mathbb{N}$ and for all $k \in \mathbb{N}$ such that $\min\{\frac{3}{2}m + \frac{1}{2}, n\} \leq k \leq n$, there exists an α such that $k \in \arg\min_{\kappa \in \{m,\ldots,n\}} c(\kappa)$. If m is even, $\frac{3}{2}m + 1 \in \mathbb{N}$ and for all $k \in \mathbb{N}$ such that $\min\{\frac{3}{2}m + 1, n\} \leq k \leq n$, there exists an α such that $k \in \arg\min_{\kappa \in \{m,\ldots,n\}} c(\kappa)$.

(i) $\boxed{k^* \text{ is continuous in } \alpha.}$ To begin with, v_k^* is continuous in α as the roots of a polynomial are continuous functions of its coefficients and (2), the FOC defining v_k^*, simplifies to a polynomial when $v_i \overset{iid}{\sim} U\left[\frac{1}{2}-\alpha, \frac{1}{2}+\alpha\right]$:

$$4\alpha^2 - 1 = (-2 + 4\alpha - 2k + 4\alpha k - 8\alpha m)v_k^* + 4k(v_k^*)^2.$$

This, in turn, implies that $c(k)$ is continuous in α, which implies via Berge's maximum theorem that its minimizer k^* is continuous in α.

(ii) $\boxed{\lim_{\alpha\to 0} k^* \leq (3/2)m + 1/2.}$ As $v_i \sim U\left[\frac{1}{2}-\alpha, \frac{1}{2}+\alpha\right]$ implies $F(v) = \frac{v-(1/2-\alpha)}{2\alpha}$ for $v \in \left[\frac{1}{2}-\alpha, \frac{1}{2}+\alpha\right]$, we have $F^{-1}(p) = \frac{1}{2}-\alpha+2\alpha p$ for $p \in [0,1]$ and we can express the maximization defining b_k^* in terms of $p = F(v)$:

$$b_k^* = \max_{p\in[0,1]} \left[\frac{1}{2} - \alpha + 2\alpha p\right] \times \binom{k-1}{m-1} p^{m-1}(1-p)^{k-m}.$$

As $\alpha \to 0$, we have $\left[\frac{1}{2}-\alpha+2\alpha p\right] \to \frac{1}{2}$ for all $p \in [0,1]$: as the distribution converges to a mass point, all its quantiles converge to this point. By Berge's maximum theorem, this convergence implies the convergence of the maximum:

$$b_k^* \to \max_{p\in[0,1]} \frac{1}{2} \times \binom{k-1}{m-1} p^{m-1}(1-p)^{k-m}.$$

Hence, all b_k^* converge to the values they have in Lemma A.2.3 (with $\delta = \frac{1}{2}$) where the disutility distribution has no dispersion. By Berge's maximum theorem, k^* thus converges to the minimizer of the cost under no dispersion as $\alpha \to 0$. Finally, Lemma A.2.4 establishes that the cost under no dispersion increases for $k \geq \frac{3}{2}m + \frac{1}{2}$. Hence, we also have $k^* \leq \frac{3}{2}m + \frac{1}{2}$ when $\alpha \to 0$.

(iii) $\boxed{\lim_{\alpha\to 1/2} k^* = n.}$ This follows from the proof of Lemma 4, which establishes that $c(k)$ decreases with k when $\alpha = \frac{1}{2}$. □

A.3 Other Simultaneous Voting Proofs

Proposition 2.

(a) Proportional increases in vote threshold m and number of committee members n

raise capture cost sub-proportionally: $C^{sim}(\lambda m, \lambda n) < \lambda C^{sim}(m,n)$ *with* $\lambda \in \mathbb{N}_+$*.*

(b) Suppose only a majority can pass the proposal, i.e., $n \leq 2m-1$*. Then, for any number of bribes* k*,* b_k^* *and* $C^{sim}(m,n)$ *increase in the vote threshold* m*.*

Proof.

(a) Suppose the vote buyer bribes k members in a committee (m,n). As $\lambda \in \mathbb{N}_+$, she can bribe λk members in a committee $(\lambda m, \lambda n)$. While λk bribes need not minimize cost in the larger committee, they give an upper bound on its minimized value. Thus, recalling $p = F(v)$, it suffices to show that if

$$b(\lambda, p) = \frac{F^{-1}(p)}{p} \times \binom{\lambda k - 1}{\lambda m - 1} [p^m (1-p)^{k-m}]^{\lambda},$$

then $b(\lambda, p^*) = \max_{p \in [0,1]} b(\lambda, p)$ decreases in λ. The log derivative of $b(\lambda, p^*)$ is:

$$\begin{aligned}
\frac{d \log b(\lambda, p^*)}{d\lambda} &= \frac{\partial \log b(\lambda, p^*)}{\partial \lambda} + \left.\frac{\partial \log b(\lambda, p)}{\partial p}\right|_{p=p^*} \frac{dp^*}{d\lambda} \\
&=_{(A)} \frac{\partial \log b(\lambda, p^*)}{\partial \lambda} \\
&= \log\left((p^*)^m (1-p^*)^{k-m}\right) - (k-m)\psi(\lambda k - \lambda m + 1) + k\psi(\lambda k) - m\psi(\lambda m) \\
&=_{(B)} \log\left((p^*)^m (1-p^*)^{k-m}\right) - (k-m)\psi(\lambda k - \lambda m + 1) \\
&\quad + k\psi(\lambda k + 1) - m\psi(\lambda m + 1) \\
&\leq_{(C)} \log\left(\left(\frac{m}{k}\right)^m \left(1 - \frac{m}{k}\right)^{k-m}\right) - (k-m)\psi(\lambda k - \lambda m + 1) \\
&\quad + k\psi(\lambda k + 1) - m\psi(\lambda m + 1) \\
&=_{(D)} mg(m) - kg(k) \\
&<_{(E)} 0
\end{aligned}$$

where

(A) letting $p^* = \arg\max_{p \in [0,1]} b(\lambda, p)$, $\left.\frac{\partial \log b(\lambda, p)}{\partial p}\right|_{p=p^*} = 0$ by the envelope theorem,

(B) uses $x\psi(\lambda x) = x[\psi(\lambda x + 1) - \frac{1}{\lambda x}] = x\psi(\lambda x + 1) - 1/\lambda$,

(C) uses the fact that $p^m(1-p)^{k-m} \leq \max_{p \in [0,1]} p^m (1-p)^{k-m} = \left(\frac{m}{k}\right)^m \left(1 - \frac{m}{k}\right)^{k-m}$,

(D) defines $g(x) := [\log(x) - \log(k-m)] - [\psi(\lambda x+1) - \psi(\lambda k - \lambda m + 1)]$, and

(E) follows because $g(\cdot)$ is increasing. To see this, note

$$\begin{aligned} g'(x) &= \frac{1}{x} - \lambda\psi'(\lambda x+1) \\ &=_{(i)} \frac{1}{x} - \lambda\psi'(\lambda x) + \frac{\lambda}{\lambda^2 x^2} \\ &>_{(ii)} \frac{1}{x} - \lambda\left[\frac{1}{\lambda x} + \frac{1}{\lambda^2 x^2}\right] + \frac{\lambda}{\lambda^2 x^2} = 0, \end{aligned}$$

where (i) uses $\psi'(u+1) = \psi'(u) - \frac{1}{u^2}$ and (ii) uses $\psi'(u) < \frac{1}{u} + \frac{1}{u^2}$ from Guo and Qi (2010, Lemma 3, p.107).

(b) If $n \leq 2m-1$, we show that b_k^* is increasing in m for all $k \in \{m, ..., n\}$. Hence, $\min_{k\in\{m,...,n\}} c(k)$ must also be increasing in m as $c(k) = kb_k^*$. By the envelope theorem, $\left.\frac{\partial \log b_k^*}{\partial v}\right|_{v=v_k^*} = 0$ and

$$\begin{aligned} \frac{d\log b_k^*}{dm} &= \frac{\partial \log b_k^*}{\partial m} + \left.\frac{\partial \log b_k^*}{\partial v}\right|_{v=v_k^*}\frac{dv}{dm} = \frac{\partial \log b_k^*}{\partial m} \\ &= \psi(k-m+1) - \psi(m) + \log F(v_k^*) - \log\big(1 - F(v_k^*)\big). \end{aligned}$$

Lemma A.2.3 implies $F(v_k^*) \geq \frac{m-1}{k-1}$. Thus,

$$\begin{aligned} \frac{d\log b_k^*}{dm} &\geq \psi(k-m+1) - \psi(m) + \log\left(\frac{m-1}{k-1}\right) - \log\left(\frac{k-m}{k-1}\right) \\ &= [\log(m-1) - log(k-m)] - [\psi(m) - \psi(k-m+1)]. \end{aligned}$$

$n \leq 2m-1$ implies $m-1 \geq k-m$. Then

$$\begin{aligned} [\log(m-1) - log(k-m)] &= \int_{s=k-m}^{m-1} \frac{1}{s} ds \\ &> \sum_{s=k-m+1}^{m-1} \frac{1}{s} = \psi(m) - \psi(k-m+1), \end{aligned}$$

where the last equality results from the property of the digamma function at the beginning of the proof section. As a result, $\frac{d\log b_k^*}{dm} > 0$. □

Proposition 3. *Suppose the distribution is at least as dispersed as $U[0,1]$. Offering b_n^* to n members, which minimizes the capture cost if members use symmetric strategies, ensures the proposal passes with certainty in any equilibrium of the voting subgame.*

Proof. Recall that Footnote 4 defines b_n^* as the smallest number above $v_n^* \pi^n(v_n^*)$. For this proof, we assume there exists a fixed minimum currency $\epsilon > 0$, so that $b_n^* = v_n^* \pi_n(v_n^*) + \epsilon$. We use a simultaneous iterated deletion of strictly dominated strategies to argue that when n members are bribed with b_n^*, the proposal passes with certainty in any equilibrium.

We eliminate cutoffs in increasing order. Let ℓ_i^t be the smallest rationalizable cutoff for member i after iteration t. Then, ℓ_i^{t+1} is the smallest rationalizable cutoff for member i when no other member j plays a cutoff below ℓ_j^t. At each iteration, we simultaneously eliminate cutoffs for all members. We have $\forall i : \ell_i^0 = v_{min}$ and, as the disutility distribution is the same for all members, the same set of cutoffs is eliminated for all members at each step. Thus, $\ell_i^t = \ell^t \; \forall i$.

Let $f_x(\mathbf{v}^y)$ be the probability that among all members but y there are exactly x votes for (writing $\mathbf{v}^y = (\overline{v}_1, \ldots, \overline{v}_{y-1}, \overline{v}_{y+1}, \ldots, \overline{v}_n)$ for the vector of cutoffs of all members other than y). Without loss of generality, consider member 1. Let $\pi^{max}(\ell)$ denote the maximal pivotal probability he can expect if every other member has a cutoff of at least ℓ:

$$\pi^{max}(\ell) := \max_{\{(\overline{v}_2,\ldots,\overline{v}_n) \,:\, \forall i \in \{2,\ldots,n\} \; \ell \leq \overline{v}_i \leq v_{max}\}} f_{m-1}(\mathbf{v}^1).$$

Then, the smallest rationalizable cutoff ℓ^{t+1} at iteration $t+1$ solves

$$\ell^{t+1} \times \pi^{max}(\ell^t) = b_n^*.$$

The remainder of the proof shows that if $\ell^t < v_{max}$, then

$$(\ell^{t+1} - \ell^t)\pi^{max}(\ell^t) = b_n^* - \ell^t \pi^{max}(\ell^t) \geq \epsilon, \tag{11}$$

whence $\ell^{t+1} \geq \ell^t + \epsilon$ (as $\pi^{max} \leq 1$) and all cutoffs smaller than v_{max} are eventually eliminated. We proceed by bounding $\ell^t \pi^{max}(\ell^t)$.

1. For any $i \neq 1$, we can rewrite $f_{m-1}(\mathbf{v}^1)$ as:

$$f_{m-1}(\mathbf{v}^1) = F(\overline{v}_i) f_{m-2}(\mathbf{v}^{1,i}) + (1 - F(\overline{v}_i)) f_{m-1}(\mathbf{v}^{1,i}),$$

   where $\mathbf{v}^{1,i} = (\overline{v}_2, \ldots, \overline{v}_{i-1}, \overline{v}_{i+1}, \ldots, \overline{v}_n)$ is the vector of cutoffs of all members other than 1 and i. Hence, the sign of $\frac{\partial f_{m-1}(\mathbf{v}^1)}{\partial \overline{v}_i}$ is independent of $\overline{v}_i$. Thus, there is a solution to the maximization problem $\pi^{max}(\ell^t)$ with $\overline{v}_i \in \{\ell^t, v_{max}\}$ for all i. In light of this, let $\pi^{n,h}(\ell^t)$ be the value of the pivotal probability if exactly h of the $n-1$ other bribed members choose a cutoff of v_{max} and $n-1-h$ choose a cutoff of ℓ^t; then

$$\pi^{max}(\ell^t) = \max_{h \in \{0,\ldots,n-1\}} \pi^{n,h}(\ell^t).$$

2. To bound $\ell^t \max_h \pi^{n,h}(\ell^t)$ from above, note

$$\ell^t \max_{h \in \{0,\ldots,n-1\}} \pi^{n,h}(\ell^t) \leq \max_{h \in \{0,\ldots,n-1\},\, \ell^t \in [v_{min}, v_{max}]} \ell^t \pi^{n,h}(\ell^t) = \max_{h \in \{0,\ldots,n-1\}} \hat{b}_{n,h},$$

   where $\hat{b}_{n,h}$ is such that when offering any amount strictly above $\hat{b}_{n,h}$ to $n-h$ members with vote threshold $m-h$, there is no equilibrium where members vote against the proposal with positive probability. Thus, $\hat{b}_{n,h} = 0$ for $h \geq m$ and else

$$\hat{b}_{n,h} = \max_{v \in [v_{min}, v_{max}]} v \underbrace{\binom{n-h-1}{m-h-1} [F(v)]^{m-1-h} [1-F(v)]^{n-m}}_{\pi^{n,h}(v)}.$$

3. By definition, $\hat{b}_{n,0} = b_n^* - \epsilon$. When the distribution is more dispersed than $U[0,1]$, we now show that $\hat{b}_{n,h} < b_n^* - \epsilon$ for all $h \in \{1, ..., m\}$. We have:

$$\frac{\partial \log \hat{b}_{n,h}}{\partial h} = -\left\{ \psi(n-h) - \psi(m-h) + \log[F(v_{n,h}^*)] \right\}, \tag{12}$$

   where ψ is the Digamma function and we define $v_{n,h}^* := \arg\max_{v \in [v_{min}, v_{max}]} v \pi^{n,h}(v)$.

- If $v_i \overset{iid}{\sim} U[0,1]$, $F(v^*_{n,h}) = \frac{m-h}{n-h}$ and (12) is

$$-\left\{[\psi(n-h)-\psi(m-h)]-[\log(n-h)-\log(m-h)]\right\}.$$

  Using the properties of the digamma function given in Appendix A.1 and noting that for any decreasing function $\sum_{s=a}^{b-1} g(s) > \int_a^b g(s)ds$,

$$\begin{aligned}\psi(n-h)-\psi(m-h) &= \sum_{s=m}^{n-1}\frac{1}{s-h}\\ &> \int_m^n \frac{1}{s-h}ds = \log(n-h)-\log(m-h).\end{aligned}$$

  Therefore, (12) is negative for $U[0,1]$.

- By Lemma A.2.2, $F(v^*_{n,h})$ is larger for more dispersed distributions. Thus, (12) must also be negative for distributions more dispersed than $U[0,1]$.

Putting these steps together,[18]

$$\begin{aligned}\ell^t \times \pi^{max}(\ell^t) &=_{(1)} \ell^t \max_{h\in\{0,\ldots,n-1\}} \pi^{n,h}(\ell^t)\\ &\leq_{(2)} \max_{h\in\{0,\ldots,n-1\}} \hat{b}_{n,h}\\ &=_{(3)} \hat{b}_{n,0}\\ &=_{(3)} b^*_n - \epsilon.\end{aligned}$$

Plugging this into (11) indeed yields $\ell^{t+1} - \ell^t \geq \epsilon$. □

A.4 Boundary Solutions in Example 4

When some members have cutoffs at the boundary, i.e., when for at least one $i \in \{1,2,3\}$, $\overline{v}_i \in \{0,1\}$, the nonsingularity of the Jacobian is not informative about the existence of nearby equilibria for any local perturbation of the bribes: while local perturbations such that Equation (1) continues to hold must exist, these local perturbations may take some $\overline{v}_i$ outside of the feasible region $[0,1]$. To address this

[18] The index on equalities/inequalities refers to the relevant step in the proof.

concern, we consider bribes associated with an equilibrium with cutoffs at the boundary and such that a local perturbation of the bribes guarantees there is no equilibrium where the proposal can be rejected. We show that such bribes are more expensive than $b_i = \frac{8}{27}$ for all members

- Cutoffs at 1.
  - $(\overline{v}_1, \overline{v}_2, \overline{v}_3) = (1, 1, 1)$ is an equilibrium for any (b_1, b_2, b_3), but it is irrelevant for the existence of other equilibria.
  - $(\overline{v}_2, \overline{v}_3) = (1, 1)$. $v_1 < 1$ only if $b_1 = 0$. Wlog, suppose $b_2 \leq b_3$. Then, if $b_2 < 1$, $(\overline{v}_1, \overline{v}_2, \overline{v}_3) = (0, b_2, 1)$ is an equilibrium. Hence, the cost is at least 2 to make the proposal pass with certainty.
  - $\overline{v}_3 = 1$, $\overline{v}_1 \leq \overline{v}_2 < 1$. Then $(\overline{v}_1, \overline{v}_2)$ satisfy:

$$\overline{v}_1(1 - \overline{v}_2) = b_1 \quad ; \quad \overline{v}_2(1 - \overline{v}_1) = b_2. \tag{13}$$

    The vote buyer minimizes $b_1 + b_2 + b_3$. Notice $\overline{v}_3 = 1$ requires $b_3 \geq \pi_3 = \overline{v}_1(1 - \overline{v}_2) + \overline{v}_2(1 - \overline{v}_1) = b_1 + b_2$. Thus, $b_1 + b_2 + b_3$ is at least

$$2(b_1 + b_2) = 2(\overline{v}_1 + \overline{v}_2 - 2\overline{v}_1\overline{v}_2). \tag{14}$$

    The Jacobian of (13) is:

$$J = \begin{pmatrix} 1 - \overline{v}_2 & -\overline{v}_1 \\ -\overline{v}_2 & 1 - \overline{v}_1 \end{pmatrix}.$$

    The determinant is $1 - \overline{v}_1 - \overline{v}_2$, which is 0 if $\overline{v}_1 + \overline{v}_2 = 1$. Under this condition, the cost in (14) is minimized for $\overline{v}_1 = \overline{v}_2 = \frac{1}{2}$ for which it is equal to 1. Thus, the cheapest bribes such that (13) has no solution are necessarily more expensive than $\frac{8}{9}$ (the capture cost with equal bribes.)

- Cutoffs at 0.

- $\overline{v}_i = 0$ only if $b_i = 0$. With two or three cutoffs (and hence bribes) at 0, the proposal never passes.
- $\overline{v}_1 = 0$ and $0 < \overline{v}_2 \leq \overline{v}_3$ requires $b_1 = 0$, $b_2 > 0$, and $b_3 > 0$. An equilibrium would have to satisfy $\overline{v}_2\overline{v}_3 = b_2$ and $\overline{v}_3\overline{v}_2 = b_3$. If $b_2 \neq b_3$, this system does not have a solution. Suppose $b_2 = b_3$. Then, if $b_2 < 1$, $(\overline{v}_1, \overline{v}_2, \overline{v}_3) = (0, b_2, 1)$ is an equilibrium. Hence, with one cutoff at 0, the cost is at least 2 to make the proposal pass with certainty.

Appendix B: Proofs (Sequential Voting)

Lemma 5. *In equilibrium,* $p(1,y) = \min\left\{\sum_{s=1}^{y} b_s, 1\right\}$.

Proof. In general, if member y votes for, his expected utility is $b_y - v_y p(x-1, y-1)$ while a vote against gives $-v_y p(x, y-1)$. Therefore, in $S(x,y)$, the member votes for of the proposal if his disutility is larger than a cutoff $v(x,y)$ defined by:

$$v(x,y) = \min\left\{\frac{b_y}{p(x-1,y-1) - p(x,y-1)}, 1\right\}.$$

For all y, $p(0, y-1) = 1$. Thus, the cutoff of member y in $S(1,y)$ is

$$v(1,y) = \min\left\{\frac{b_y}{1 - p(1,y-1)}, 1\right\}.$$

If $\frac{b_y}{1-p(1,y-1)} < 1$, the probability of passing is

$$\begin{aligned}
p(1,y) &= v(1,y)p(0,y-1) + (1 - v(1,y))p(1,y-1) \\
&= \frac{b_y}{1-p(1,y-1)} \times 1 + \left(1 - \frac{b_y}{1-p(1,y-1)}\right) \times p(1,y-1) \\
&= b_y + p(1,y-1)
\end{aligned}$$

We use an induction to complete the proof. Notice that the Lemma holds for $y = 1$ and assume that it holds for $y-1$. Then $p(1,y-1) = \min\left\{\sum_{s=1}^{y-1} b_s, 1\right\}$ and we do have $p(1,y) = \min\left\{\sum_{s=1}^{y} b_s, 1\right\}$, which proves the claim. $\square$

Lemma 6. *Let $b_y^{(s)}$ be the s-th order statistic (i.e., the s-th lowest value) amongst $\{b_1, \ldots, b_y\}$. Then, for $x \geq 1$, in equilibrium $p(x, y) = \min\left\{ \sum_{s=1}^{y-(x-1)} b_y^{(s)}, 1\right\}$.*

Proof. We proceed by induction on x. Lemma 5 proves the base case ($x = 1$). Suppose the result holds for $x - 1$. We prove that it also holds for x. To do so, we use an induction on y.

- Base Case: $y = 1$. If $x \geq 2$, $p(x, 1) = 0$; if $x = 1$ then $p(x, 1) = b_1$ as required (recall we use the convention that empty sums evaluate to zero).

- Inductive Step. Rearranging the equation for a member's cutoff and then employing the inductive hypothesis, we have

$$\begin{aligned} v(x,y) &= \min\left\{\frac{b_y}{p(x-1,y-1)-p(x,y-1)}, 1\right\} \\ &= \begin{cases} \min\left\{\frac{b_y}{b_{y-1}^{(y-(x-1))}}, 1\right\} & \text{if } p(x-1,y-1) < 1, \\ \min\left\{\frac{b_y}{1-\sum_{s=1}^{y-x} b_{y-1}^{(s)}}, 1\right\} & \begin{array}{l}\text{if } p(x-1,y-1) = 1 \\ \ \text{and } p(x,y-1) < 1,\end{array} \\ 1 & \text{if } p(x,y-1) = 1. \end{cases} \end{aligned}$$

  We now verify the expression for $p(x, y)$ following these cases.

  - Assume $p(x - 1, y - 1) < 1$.
    - Assume $b_y > b_{y-1}^{(y-(x-1))}$. Then $v(x, y) = 1$ so that

$$\begin{aligned} p(x,y) &= p(x-1,y-1) \\ &= \min\left\{\sum_{s=1}^{y-(x-1)} b_{y-1}^{(s)}, 1\right\} \\ &= \min\left\{\sum_{s=1}^{y-(x-1)} b_y^{(s)}, 1\right\}. \end{aligned}$$

* Assume $b_y \leq b_{y-1}^{(y-(x-1))}$. Then

$$\begin{aligned} p(x,y) &= p(x,y-1) + v(x,y)[p(x-1,y-1) - p(x,y-1)] \\ &= \sum_{s=1}^{y-x} b_{y-1}^{(s)} + \frac{b_y}{b_{y-1}^{(y-(x-1))}} b_{y-1}^{(y-(x-1))} \\ &= \sum_{s=1}^{y-x} b_{y-1}^{(s)} + b_y = \sum_{s=1}^{y-(x-1)} b_y^{(s)}. \end{aligned}$$

- Assume $p(x-1,y-1) = 1$ and $p(x,y-1) < 1$.
  * Assume $b_y \geq 1 - \sum_{s=1}^{y-x} b_{y-1}^{(s)}$. Then $v(x,y) = 1$ and hence $p(x,y) = p(x-1,y-1) = 1$. Thus, we need to show that

$$\sum_{s=1}^{y-(x-1)} b_y^{(s)} \geq 1.$$

If $b_y < b_{y-1}^{(y-(x-1))}$, this follows from $b_y \geq 1 - \sum_{s=1}^{y-x} b_{y-1}^{(s)}$; if not, it follows from $p(x-1,y-1) = 1$.
  * Assume $b_y < 1 - \sum_{s=1}^{y-x} b_{y-1}^{(s)}$. Then

$$\begin{aligned} p(x,y) &= p(x,y-1) + v(x,y)[1 - p(x,y-1)] \\ &= b_y + \sum_{s=1}^{(y-1)-(x-1)} b_{y-1}^{(s)} \end{aligned}$$

If $b_y < b_{y-1}^{(y-(x-1))}$, we are done. Moreover, $b_y \geq b_{y-1}^{(y-(x-1))}$ contradicts the case assumption: it implies

$$\sum_{s=1}^{y-(x-1)} b_y^{(s)} = \sum_{s=1}^{y-(x-1)} b_{y-1}^{(s)},$$

which is at least 1 as $p(x-1,y-1) = 1$. But also

$$b_y + \sum_{s=1}^{y-x} b_{y-1}^{(s)} \geq \sum_{s=1}^{y-(x-1)} b_y^{(s)}$$

This is a contradiction with $b_y < 1 - \sum_{s=1}^{y-x} b_{y-1}^{(s)}$.

– Assume $p(x, y-1) = 1$. Then $p(x, y) = 1$, so we need to show that

$$\sum_{s=1}^{y-(x-1)} b_y^{(s)} \geq 1.$$

First notice that $p(x, y-1) = 1$ implies

$$\sum_{s=1}^{y-x} b_{y-1}^{(s)} \geq 1.$$

If $b_y > b_{y-1}^{(y-(x-1))}$, the desired result follows as $\sum_{s=1}^{y-x} b_y^{(s)} = \sum_{s=1}^{y-x} b_{y-1}^{(s)}$. Otherwise, it follows by adding b_y to $\sum_{s=1}^{y-x} b_{y-1}^{(s)}$. □

Appendix C: Proofs (Cost Comparison)

Proposition 5. *Suppose $v_i \overset{iid}{\sim} U[0,1]$.*

(a) If it takes one or all but one votes to pass the proposal, the capture cost is lower with simultaneous voting: $C^{sim}(m,n) < C^{seq}(m,n)$ for $m = 1$ and $m = n-1$.

(b) If unanimity is not required to pass the proposal (i.e., $m < n$), there is a λ^* such that $C^{seq}(\lambda m, \lambda n) < C^{sim}(\lambda m, \lambda n)$ with $\lambda > \lambda^*$ and $(\lambda m, \lambda n) \in \mathbb{N}_+^2$.

Proof.

(a) $\boxed{m = 1.}$ Recall $C^{seq}(1, n) = 1$. Meanwhile, $C^{sim}(1, n) = \left(1 - \frac{1}{n}\right)^{n-1}$ is $\frac{1}{2}$ for $n = 2$ and decreases in n. Thus, $C^{seq}(1, n) > C^{sim}(1, n)$.

$\boxed{m = n-1.}$ Recall $C^{seq}(n-1, n) = \frac{n}{2}$ and $C^{sim}(n-1, n) = n \times \left(1 - \frac{1}{n}\right)^n$. As $(1 - \frac{1}{n})^n$ is increasing in n, and $\frac{1}{2} > e^{-1} = \lim_{n\to\infty}(1 - \frac{1}{n})^n$, we have $C^{seq}(n-1, n) > C^{sim}(n-1, n)$.

(b) We have $\lim_{\lambda\to\infty} C^{seq}(\lambda m, \lambda n) = \frac{1}{1-\frac{m}{n}} < \infty = \lim_{\lambda\to\infty} C^{sim}(\lambda m, \lambda n)$. □

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