Two-Stage Majoritarian Choice

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Abstract

We propose a class of decisive collective choice rules that rely on a linear ordering to partition the majority relation into two acyclic relations. The first of these relations is used to pare down the set of the feasible alternatives into a shortlist while the second is used to make a final choice from the shortlist.

Rules in this class are characterized by four properties: two classical rationality requirements (Sen's *expansion consistency* and Manzini and Mariotti's *weak WARP*); and adaptations of two classical collective choice requirements (Arrow's *independence of irrelevant alternatives* and Saari and Barney's *no preference reversal bias*). These rules also satisfy some other desirable properties, including an adaptation of May's *positive responsiveness.*

JEL Classification: D71, D72.

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1 Introduction

In many collective choice settings, rules that recommend more than one alternative are inappropriate. When it comes to selecting a public policy or passing legislation, for instance, it is essential to be decisive. May (1952) shows that majority voting is the only reasonable way to decide between two alternatives.[1] With more alternatives, no rule that is faithful to the majority can always choose rationally. The root of the problem is the Condorcet (1785) paradox: the majority relation may involve cycles. Arrow (1951) shows that this problem extends to non-majoritarian rules: barring dictatorship, there is no way to make rational and Pareto-efficient choices that satisfy the *independence of irrelevant alternatives* (IIA).

We take Arrow's result as good reason not to give up on majority rule, but rather to search for collective choice rules that are decisive, faithful to the majority view, and as rational as possible. Our emphasis on rationality is grounded in the view that, to gain broad legitimacy among the agents, a rule must exhibit some degree of consistency in choice. Following in the Arrovian tradition, we seek to achieve greater consistency by limiting irrational choice.

Two important properties necessary for rational choice, *expansion consistency* (Sen, 1971) and *weak WARP* (Manzini and Mariotti, 2007), are compatible with decisiveness and faithfulness to the majority. We propose a class of choice rules that satisfy these two properties as well as some other desiderata—including versions of Arrow's *IIA*, May's *positive responsiveness* and Saari and Barney's (2003) *no preference reversal bias*. Not least among the virtues of the rules we propose is their simplicity. Each uses a linear ordering to partition the majority relation into two acyclic relations. Then, as in Manzini and Mariotti's (2007) *rational shortlist methods*, the first relation is used to pare down the set of feasible alternatives into a shortlist before the second relation is used to make a final choice.

Since a higher ranking confers an advantage in terms of being chosen, we interpret the linear ordering associated with a given rule as a priority among the alternatives. While this priority is in principle exogenous, the choice setting frequently suggests a natural way to order the alternatives. In the public policy setting, for instance, it is sensible to prioritize policies that are less costly or, perhaps, more equitable. In legislative settings, it is customary to prioritize proposals by the order in which they were tabled or, in some jurisdictions, by their degree of divergence from the *status quo* legislation (Rasch, 2000, p. 15). Finally, in the committee setting, it may be appropriate to use the preference of the chair, which is conventionally used as a tie-breaking device (Robert, 2011, p. 405).

[1] In the sequel, we assume that the majority relation is decisive. This assumption is automatically satisfied when voter preferences are strict and the number of voters is odd. It is also fairly innocuous for large electorates.

2 The problem

Given a finite universe of social alternatives X, let $\mathcal{X} = \{A \in 2^X \mid 2 \leq |A|\}$ denote the set of *agendas* and $\mathcal{T}$ the set of *tournaments* on X. Formally, a tournament T is an *asymmetric* ($\nexists a, b$: aTb and bTa) and *total* ($\forall a, b$: aTb, bTa, or $a = b$) binary relation on X. We interpret each tournament $T \in \mathcal{T}$ to be the majority relation induced by an underlying profile of agent preferences over X (McGarvey, 1953). Given a tournament T and an agenda A, the problem is to recommend one alternative in A. Formally, the object of interest is a *choice rule*, that is a mapping $f : \mathcal{T} \times \mathcal{X} \to X$ such that $f(T; A) \in A$ for each $T \in \mathcal{T}$ and $A \in \mathcal{X}$.

We emphasize two aspects of our approach. First, we impose strong restrictions on what inputs are relevant for collective choice. Although the general voting model takes individual preferences as inputs, our rules only require the associated majority relation. While certainly restrictive, there is a rich tradition of rules called "tournament solutions" that take the same approach.[2] A key motivation for these rules is to maintain informational parsimony.

Second, we impose strong restrictions on what kind of output is permitted. While the general voting model allows a set of "acceptable" alternatives as output, we require our rules to be *decisive*. In our view, recommending more than one alternative is problematic. At best, it puts off the task of making a definite choice. At worst, it delegates the task of choosing among the acceptable alternatives, a choice which is quite likely to have welfare implications,[3] to an *ad hoc* and potentially undemocratic tie-breaking procedure.

As outlined, we focus on choice rules that are faithful to the majority for binary choices:

Faithfulness. *For all $T \in \mathcal{T}$ and $a, b \in X$: aTb implies $f(T; \{a, b\}) = a$.*

For a binary relation R on X, let $\max(R; A) := \{a \in A \mid \nexists b \in A : bRa\}$ denote the set of maximal elements of R in $A \in \mathcal{X}$. (When this set is a singleton, we write $\max(R; A) = a$ instead of $\max(R; A) = \{a\}$.) Let $\mathcal{P}$ denote the set of linear orderings on X.[4]

We note that the restriction of a choice rule f to any tournament $T \in \mathcal{T}$ defines a classical *choice function* $f(T; \cdot) : \mathcal{X} \to X$. The choice function $f(T; \cdot)$ is *rational* if there is a linear ordering $P \in \mathcal{P}$ such that $f(T; A) = \max(P; A)$ for all $A \in \mathcal{X}$. If f satisfies Faithfulness, then $f(T; \cdot)$ cannot be rational unless the tournament T is a linear ordering. The question is whether there are faithful choice rules for which $f(T; \cdot)$ is rational when T is a linear ordering and not *too* irrational otherwise.

Some of the simplest faithful choice rules from the literature use an exogenous linear

[2] For a comprehensive treatment of tournament solutions, see Laslier (1997).

[3] If T is induced by a profile of strict preferences, for instance, no two alternatives are Pareto indifferent.

[4] A linear ordering P is an asymmetric, total and *transitive* ($\forall a, b, c$: $aPbPc \Rightarrow aPc$) binary relation.

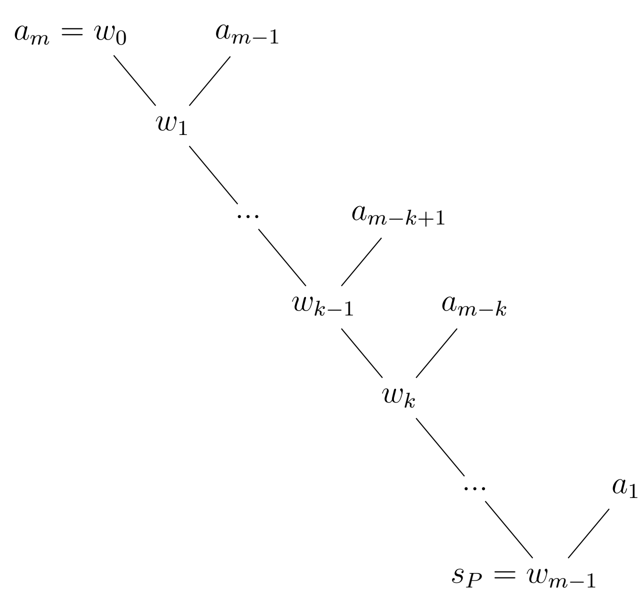
ordering $P \in \mathcal{P}$ to establish a *priority* among the alternatives. The basic idea is to give more of an "edge" to alternatives that are ranked higher by P and thus guarantee that choice is single-valued even when the alternatives are not easy to distinguish on principle (as in a Condorcet cycle $aTbTcTa$ among three alternatives $a, b, c \in X$).

One such approach uses the priority P as a tie-breaking device to make a selection from a Condorcet-consistent choice correspondence. Formally, $F : \mathcal{T} \times \mathcal{X} \to 2^X \setminus \{\emptyset\}$ is a *Condorcet-consistent correspondence* if, for all $T \in \mathcal{T}$ and $A \in \mathcal{X}$: (i) $F(T; A) \subseteq A$; and (ii) $F(T; A) = \{a\}$ if aTb for all $b \in A \setminus \{a\}$ (i.e., if a is the *Condorcet winner*).[5] The choice rule F_P generated by the Condorcet-consistent correspondence F and the priority $P \in \mathcal{P}$ is defined, for all $T \in \mathcal{T}$ and $A \in \mathcal{X}$, by $F_P(T; A) := \max(P; F(T; A))$.

Another approach uses the priority P to define a succession of binary elimination votes. For any agenda $A = \{a_1, ..., a_m\} \in \mathcal{X}$, label the alternatives so that $a_1 P ... P a_m$. Then, define $w_0(T; A) := a_m$ and, for $k = 1, ..., m-1$, define

$$w_k(T; A) := \begin{cases} w_{k-1}(T; A) & \text{if } w_{k-1}(T; A) T a_{m-k}, \\ a_{m-k} & \text{otherwise.} \end{cases}$$

The first vote eliminates either a_m or a_{m-1}. At any subsequent vote, the winner $w_{k-1}(T; A)$ from the previous vote is paired against the alternative a_{m-k}. The *successive elimination* rule s_P induced by the priority $P \in \mathcal{P}$ is then defined, for all $T \in \mathcal{T}$ and $A \in \mathcal{X}$, by $s_P(T; A) := w_{m-1}(T; A)$. This rule may be depicted as follows. (For ease of illustration, the dependence on $(T; A)$ has been suppressed):



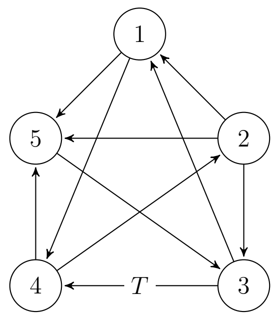
[5] In Fishburn (1977), Condorcet-consistent correspondences are called C1 social choice functions.

Both of these approaches induce choice rules that lack basic features of rationality:

Example 1 (Selection from the uncovered set). *One well-known Condorcet-consistent correspondence is the* uncovered set *correspondence* $UC : \mathcal{T} \times \mathcal{X} \to \mathcal{X} \setminus \{\emptyset\}$ *(Landau, 1951; Fishburn, 1977; Miller, 1980), which is defined, for all* $T \in \mathcal{T}$ *and* $A \in \mathcal{X}$*, by*

$$UC(T; A) := \{a \in A \mid \forall b \in A \setminus \{a\} : \text{(i) } aTb \text{ or (ii) } aTcTb \text{ for some } c \in A\}.$$

For the universe $X := \{1, 2, 3, 4, 5\}$*, consider the tournament* T *depicted below:*



By definition, $UC(T; \{1, 2, 3, 4\}) = \{2, 3, 4\}$, $UC(T; \{2, 5\}) = \{2\}$ *and* $UC(T; \{1, 2, 3, 4, 5\}) = \{1, 2, 3, 4\}$*. For the priority* $P = 1, ..., 5$ *(with the alternatives listed in decreasing order of* P*), it then follows that:*

$$UC_P(T; \{1, 2, 3, 4\}) = 2 = UC_P(T; \{2, 5\}) \text{ but } UC_P(T; \{1, 2, 3, 4, 5\}) = 1.$$

In other words, alternative 2 *is chosen from* $\{1, 2, 3, 4\}$ *and* $\{2, 5\}$ *but not their union.*[6] *Moreover, since* $UC(T; \{1, 2\}) = \{2\}$ *and* $UC(T; \{1, 2, 4\}) = \{1, 2, 4\}$*, it follows that:*

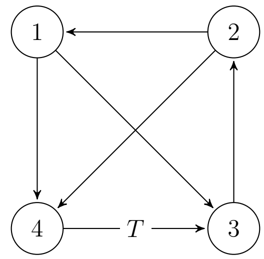
$$UC_P(T; \{1, 2\}) = 2 = UC_P(T; \{1, 2, 3, 4\}) \text{ but } UC_P(T; \{1, 2, 4\}) = 1.$$

So, 2 *is chosen over* 1 *from* $\{1, 2\}$ *and* $\{1, 2, 3, 4\}$ *but not the intermediate agenda* $\{1, 2, 4\}$*.*[7]

Example 2 (Successive elimination). *For* $X := \{1, 2, 3, 4\}$*, consider the tournament* $T \in \mathcal{T}$ *depicted below:*

[6] The same choice pattern can arise if we instead start with the *top cycle* correspondence TC (as defined in Section 4.2 below). If we modify the tournament T so that $4T'1$, then $TC_P(T'; \{1, 2, 3, 4\}) = 2 = TC_P(T'; \{2, 5\})$ but $TC_P(T'; \{1, 2, 3, 4, 5\}) = 1$.

[7] To see that this choice pattern cannot arise if we start with the top cycle correspondence TC, suppose $TC_P(T; A) = a = TC_P(T; \{a, b\})$ and $TC_P(T; B) = b$ for $\{a, b\} \subseteq B \subseteq A$. Since $TC_P(T; \{a, b\}) = a$ and $TC_P(T; B) = b$, bPa. Since $a \in TC(T; A)$ and $b = c_1T...Tc_n = a$ for some $c_1, ..., c_n \in B$, $b \in TC(T; A)$. Since bPa, this contradicts $TC_P(T; A) = a$.



For the successive elimination procedure s_P induced by the priority $P := 1, ..., 4$:

$$s_P(T; \{1,4\}) = s_P(T; \{1,2,3\}) = 1 \text{ but } s_P(T; \{1,2,3,4\}) = 2.$$

So, 1 is chosen from the agendas $\{1,4\}$ and $\{1,2,3\}$ but not their union. Moreover,

$$s_P(T; \{1,2\}) = s_P(T; \{1,2,3,4\}) = 2 \text{ but } s_P(T; \{1,2,3\}) = 1.$$

Thus, 2 is chosen over 1 from $\{1,2\}$ and $\{1,2,3,4\}$ but not the intermediate agenda $\{1,2,3\}$.[8]

The choice rules from Examples 1 and 2 both violate the following rationality properties:

Expansion Consistency. *For all $T \in \mathcal{T}$, $a \in X$, and $A, B \in \mathcal{X}$:*

$$f(T; A) = a = f(T; B) \text{ implies } f(T; A \cup B) = a.$$

Weak WARP. *For all $T \in \mathcal{T}$, distinct $a, b \in X$, and $A, B \in \mathcal{X}$ such that $\{a, b\} \subseteq B \subseteq A$:*

$$f(T; \{a, b\}) = a = f(T; A) \text{ implies } f(T; B) \neq b.$$

Expansion Consistency dates back to Sen (1971). Weak WARP was first introduced by Manzini and Mariotti (2007) and later studied by Cherepanov et al. (2013). Both properties weaken Samuelson's (1938) *weak axiom of revealed preference* (WARP), which requires $f(T; B) = a$ if $f(T; A) = a$ and $a \in B \subseteq A$. Since WARP characterizes rational choice in our setting, it is incompatible with the requirement that f satisfies Faithfulness.

[8]The same choice patterns can arise under the *amendment procedure* a_P (Miller, 1977, p. 779; Moulin, 1986, p. 287). Following our convention (that higher-ranked alternatives in P are more privileged), the linear ordering $P = 1, 2, 3, 4$ corresponds to the tree $\Gamma_4(4, 3, 2, 1)$ in Moulin. For the tournament T given in Example 2, the corresponding choice function gives $a_P(T; A) = s_P(T; A)$ for all $A \in \mathcal{X}$.

3 Two-stage majoritarian rules

We propose a class of choice rules that satisfy Faithfulness, Expansion Consistency and Weak WARP. Like the rules from Examples 1 and 2, each of our rules relies on an exogenous priority $P \in \mathcal{P}$. For our rules, the function of the linear ordering P is to partition the given tournament $T \in \mathcal{T}$ into two acyclic binary relations $T \cap P$ and $T \setminus P$. The first of these relations is used to obtain a preliminary shortlist of the feasible alternatives in the agenda $A \in \mathcal{X}$ while the second is used to make a final choice from the shortlist.

Formally, the *two-stage majoritarian choice rule f_P based on the priority* $P \in \mathcal{P}$ is defined, for all tournaments $T \in \mathcal{T}$ and agendas $A \in \mathcal{X}$, by

$$f_P(T; A) := \max(T \setminus P; \max(T \cap P; A)). \tag{1}$$

For each tournament $T \in \mathcal{T}$, the choice function $f_P(T; \cdot)$ is a *rational shortlist method* (RSM) in the sense of Manzini and Mariotti (2007), that is a choice function $c : \mathcal{X} \to X$ for which there is a pair of asymmetric binary relations (P_1, P_2) on X (called *rationales*) such that $c(A) = \max(P_2; \max(P_1; A))$ for all $A \in \mathcal{X}$. In general, the rationales P_1 and P_2 must satisfy nontrivial restrictions (see Lemma 2 of Dutta and Horan, 2015) to ensure that c is a well-defined choice function. It turns out that, for all $T \in \mathcal{T}$ and $P \in \mathcal{P}$, these restrictions are automatically satisfied when the rationales are $P_1 := T \cap P$ and $P_2 := T \setminus P$.

To see this, fix an agenda $A \in \mathcal{X}$. Since the binary relation $T \cap P$ is acyclic, the shortlist $M_A := \max(T \cap P; A)$ must be nonempty. The alternatives excluded from M_A are those dominated *both* in terms of the majority tournament T and the priority ordering P. In other words, the shortlist M_A consists of those alternatives that are not majority beaten by any higher priority alternatives. It follows that the restriction of the relation $T \setminus P$ to M_A must be a linear ordering. To see this, define the "reverse" linear ordering $P^{-1} := \{(a, b) \in X^2 \mid (b, a) \in P\}$ and observe that for all $a, b \in M_A$,

$$aTb \Leftrightarrow a(T \setminus P)b \Leftrightarrow a(T \cap P^{-1}) \Leftrightarrow aP^{-1}b.$$

This chain of equivalences shows that formula (1) can be re-written as follows:

$$f_P(T; A) = \max(T; M_A) \quad \text{or even} \quad f_P(T; A) = \max(P^{-1}; M_A).$$

In words, the alternative selected from the shortlist is the alternative most preferred by the majority. Equivalently, it is the shortlisted alternative with lowest priority.

Finally, note that if the tournament T disagrees with the priority P for all pairs of alternatives in X (i.e., $T \cap P = \emptyset$), then the shortlist M_A is just A itself. Since $T = P^{-1}$ is a linear ordering in that case, $f_P(T; A)$ must be the Condorcet winner of T in A. At the other extreme where the tournament T and the priority P coincide (i.e., $T = P$), the shortlist M_A contains only the Condorcet winner of T in A, which must be selected in the second stage.

The following example serves as further illustration of the rules that we propose.

Example 3 (Two-stage majoritarian rules). *For the tournament T from Example 2, the two rationales associated with the priority $P := 1, ..., 4$ are*

$$P_1 = T \cap P = \{(1,3),(1,4),(2,4)\} \quad \text{and} \quad P_2 = T \setminus P = \{(2,1),(3,2),(4,3)\}.$$

To understand the resulting two-stage majoritarian rule f_P, first consider the Condorcet cycle $A = \{1,2,3\}$. Since $1P_13$, the first stage eliminates alternative 3, which gives the shortlist $\{1,2\}$. Since $2P_21$, the second stage eliminates alternative 1 and $f_P(T; A) = 2$.

Letting $f_P^{-1}(T; x) := \{A \in \mathcal{X} \mid f(T; A) = x\}$, the same kind of reasoning establishes that:

$$\begin{aligned} f_P^{-1}(T;1) &= \{\{1,3\},\{1,4\}\}, \\ f_P^{-1}(T;2) &= \{\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,2,3,4\}\}, \\ f_P^{-1}(T;3) &= \{\{2,3\},\{2,3,4\}\}, \text{ and} \\ f_P^{-1}(T;4) &= \{\{3,4\}\}. \end{aligned}$$

By definition, every two-stage majoritarian rule f_P satisfies Faithfulness. Since the choice function $f_P(T; \cdot)$ is a rational shortlist method for each $T \in \mathcal{T}$, Manzini and Mariotti's characterization implies that f_P must satisfy Expansion Consistency and Weak WARP.[9]

Two-stage majoritarian rules also exhibit consistency properties *across* tournaments. One such property is an adaptation of Arrow's IIA to our setting (due to Moulin, 1986, p. 278). Let $T|_A$ denote the restriction of the tournament $T \in \mathcal{T}$ to the agenda $A \in \mathcal{X}$.

Choice IIA. *For all $T, T' \in \mathcal{T}$ and $A \in \mathcal{X}$ such that $T|_A = T'|_A$: $f(T; A) = f(T'; A)$.*

To paraphrase, the majority view of infeasible alternatives cannot affect choice. Besides two-stage majoritarian rules, this property is also satisfied by the rules from Examples 1 and 2 (as well as the variations of these rules discussed in footnotes 6 and 8).

Another inter-tournament consistency property satisfied by f_P, which is not satisfied by any of the other rules discussed in Section 2, is that choice must improve when all majority

[9] Rubinstein and Salant (2008) characterize the RSM model in terms of a different property called *Exclusion Consistency*, which can be adapted to our setting as follows: *for all $T \in \mathcal{T}$, $a \in X$, and $A, B \in \mathcal{X}$ such that $a \in B \setminus A$, $f(T; A \cup \{a\}) \notin \{f(T; A), a\}$ implies $f(T; B) \neq f(T; A)$.* In all of the subsequent analysis, this property can be used in place of Expansion Consistency and Weak WARP.

comparisons are reversed. Where $T^{-1} := \{(a,b) \in X^2 \mid (b,a) \in T\}$ denotes the reversal of a tournament $T \in \mathcal{T}$, this property can be stated as follows:

Reversal Improvement. *For all $T \in \mathcal{T}$ and $A \in \mathcal{X}$: $f(T;A)Tf(T^{-1};A)$.*

This property strengthens Faithfulness, which coincides with the special case where $|A| = 2$. It also strengthens a condition that Saari and Barney (2003, p. 17) proposed for the richer setting where collective choice may depend on individual preferences. Their condition requires the collective choice to change when all individual preferences are reversed.[10] In our setting, this amounts to the requirement that $f(T;A) \neq f(T^{-1};A)$.

Reversal Improvement further requires that reversing preferences must *improve* choice. What motivates us to strengthen Saari and Barney's condition in this way is the view that changes to the majority view ought to impact choice for the better. This makes Reversal Improvement similar in spirit to May's positive responsiveness (which we discuss at greater length in Section 4.2 below). The main difference is that May's condition relates to changes that reinforce the support for a particular choice. In contrast, our condition relates to changes that reverse all comparisons that led to a particular choice.

Combined with Expansion Consistency and Weak WARP, Choice IIA and Reversal Improvement characterize two-stage majoritarian rules. To state our result formally:

Theorem. *A choice rule $f : \mathcal{T} \times \mathcal{X} \to X$ is a two-stage majoritarian choice rule if and only if it satisfies Expansion Consistency, Weak WARP, Choice IIA and Reversal Improvement.*

Proof. (Necessity) The fact f_P satisfies Expansion Consistency and Weak WARP follows from Manzini and Mariotti (2007). Choice IIA is also immediate. To see that f_P satisfies Reversal Improvement, fix some $T \in \mathcal{T}$ and $A \in \mathcal{X}$. If $f_P(T^{-1};A)Pf_P(T;A)$, then $f_P(T;A)Tf_P(T^{-1};A)$ since $f_P(T;A) \in \max(T \cap P, A)$. Similarly, if $f_P(T;A)Pf_P(T^{-1};A)$, then $f_P(T^{-1};A)T^{-1}f_P(T;A)$. Finally, if $f_P(T;A) = f_P(T^{-1};A) = a$, then $a \in \max(T \cap P;A) \cap \max(T^{-1} \cap P;A)$. It follows that aPc for all $c \in A \setminus \{a\}$. Let b be the second-ranked alternative in A according to P. Since $f_P(T;A) = f_P(T^{-1};A) = a$, $b \notin \max(T \cap P;A) \cup \max(T^{-1} \cap P;A)$. So, aTb and $aT^{-1}b$, which is a contradiction.

(Sufficiency) Since the case $|X| = 2$ follows immediately from Reversal Improvement, suppose $|X| \geq 3$. Define the binary relation R on X such that, for all $x, y \in X$:

$$xRy \text{ if } xTzTyTx \text{ and } f(T;\{x,y,z\}) = x \text{ for some } T \in \mathcal{T} \text{ and } z \in X. \tag{2}$$

[10]Fishburn (1973, p. 157) earlier proposed a similar condition, which he called *Duality*.

Equivalently, by Reversal Improvement, it follows that

$$xRy \text{ if } xT'yT'zT'x \text{ and } f(T';\{x,y,z\}) = z \text{ for some } T' \in \mathcal{T} \text{ and } z \in X. \tag{3}$$

Finally, define the binary relation I on X such that, for all $x, y \in X$:

$$xIy \text{ if neither } xRy \text{ nor } yRx. \tag{4}$$

Step 1. *R is (i) asymmetric and (ii) transitive.*

(i) To the contrary, suppose $xRyRx$ for some $x, y \in X$. By definition, there are $c, d \in X \setminus \{x, y\}$ and $T, T' \in \mathcal{T}$ such that $xTyTcTx$, $xT'yT'dT'x$, $f(T;\{x,y,c\}) = c$, and $f(T';\{x,y,d\}) = y$. By Choice IIA, it must be that $c \neq d$. For $|X| = 3$, this yields a contradiction directly. For $|X| \geq 4$, consider $T^* \in \mathcal{T}$ such that $T^*|_C = T|_C$ for $C := \{x,y,c\}$, $T^*|_D = T'|_D$ for $D := \{x,y,d\}$, and cT^*d. By Faithfulness, $f(T^*;\{c,d\}) = c$ and $f(T^*;\{y,d\}) = y$. Since $f(T^*;C) = c$ and $f(T^*;D) = y$ by Choice IIA, Expansion Consistency leads to the following contradiction:

$$c = f(T^*; C \cup \{c,d\}) = f(T^*;\{x,y,c,d\}) = f(T^*; D \cup \{y,d\}) = y.$$

(ii) Suppose $xRyRz$. Consider $T \in \mathcal{T}$ such that $xTyTzTx$. If $f(T;\{x,y,z\}) \neq x$, yRx or zRy. Since $xRyRz$, this contradicts the asymmetry of R. So, $f(T;\{x,y,z\}) = x$ and xRz. □

Step 2. *There are exactly two distinct $a, b \in X$ such that aIb. Moreover, aRc and bRc for all $c \in X \setminus \{a, b\}$.*

Suppose there are pairs of distinct alternatives $\{x, y\}$ and $\{z, w\}$ (with $x \neq z, w$ and $z \neq x, y$) such that xIy and zIw. First, consider $T \in \mathcal{T}$ such that $xTzTwTx$ and its reversal T^{-1}. Since zIw, the definition of R implies zRx (and wRx). Next, consider $T \in \mathcal{T}$ such that $xTyTzTx$ and its reversal T^{-1}. Then, xRz since xIy. Since $xRzRx$ contradicts the asymmetry of R, it follows that aIb for at most one pair $\{a, b\}$.

If there is no such pair, then R is a linear ordering by Step 1. Suppose aRb and bRc for all $c \in X \setminus \{a, b\}$. Then, by definition of R, there is some $d \in X \setminus \{a, b\}$ such that dRb. Since this is a contradiction, there must be exactly one pair $\{a, b\}$ such that aIb. Finally, from the definition of R, it follows that aRc and bRc for all $c \in X \setminus \{a, b\}$. □

Complete R into a linear ordering P by defining aPb and xPy if xRy for $x, y \in X$.

Step 3. *For all $T \in \mathcal{T}$ and $A \in \mathcal{X}$, $f(T;A) = f_P(T;A)$.*

The proof is by strong induction on $|A|$. For the base cases $|A| = 2, 3$, the result follows from Reversal Improvement, Expansion Consistency, and the definition of R. For the induction step, suppose that the result holds for $2 \leq |A| < n$ and consider $|A| = n \geq 4$.

By the induction hypothesis, it suffices to show that $f(T;A) = f_P(T;A)$ for all $T \in \mathcal{T}$. Labelling the alternatives of $A = \{a_1, ..., a_n\}$ so that $a_1 P ... P a_n$, this is equivalent to showing that

$$f(T;A) = \begin{cases} a_n & \text{if } a_n = \max(T;A), \\ f(T;A \setminus \{a_n\}) & \text{otherwise.} \end{cases} \tag{5}$$

First, suppose $f(T;A \setminus \{a_n\}) = x$ and recursively define a sequence $\langle b_i \rangle_{i=0}^{m}$ in A such that:

(i) $b_0 := a_n$;
(ii) $B_{i+1} := \{y \in A \mid y(P \cap T)b_i\}$ and $b_{i+1} := \max(P; B_{i+1})$; and,
(iii) m is the smallest index such that $b_m P x$, $b_m = x$, or $B_{m+1} = \emptyset$.

Next, define $B := \{b_0, ..., b_m\}$. Since $f(T;A \setminus \{a_n\}) = x$, there are two possibilities. If $a_n T a_i$ for all $i = 1, ..., n-1$, then $B = \{a_n\} = \{b_0\}$. Otherwise, $B = \{b_0, ..., b_m\} \neq \{b_0\}$ with the features that: (a) $b_m(T \cap P)...(T \cap P)b_0$; and (b) $x(T \setminus P)b_m$ or $x = b_m$. This leaves three cases:

Case 1. If $B = \{a_n\}$, then $a_n = \max(T;A)$. By the induction hypothesis and the application of Expansion Consistency to $f(T;\{a_i, a_n\}) = a_n$ for $i = 1, ..., n-1$, it follows that $f(T;A) = a_n$.

Case 2. If $1 < |B \cup \{x\}| < n$, then $f(T;B \cup \{x\}) = x$ by (a)-(b) above and the induction hypothesis. So, $f(T;A) = f(T;(B \cup \{x\}) \cup (A \setminus \{a_n\})) = x$ by Expansion Consistency.

Case 3. If $B \cup \{x\} = A$, then $x \in \{a_1, a_2\}$ by (a)-(b) above. What is more, the definition of B implies: (c) $a_{i-1} T a_i$ for $i = 4, ..., n$; and (d) $a_i T a_j$ for $i = 4, ..., n$ and all $j < i-1$.

First, suppose $x = a_1$. By definition of B, $a_1 T a_2 T a_3 T a_1$. Given (c)-(d), $a_{i-2} T a_{i-1} T a_i T a_{i-2}$ for $i = 3, ..., n$. So, $f(T;\{a_{i-2}, a_{i-1}, a_i\}) = a_{i-2}$ by the induction hypothesis. Since $f(T;\{a_{i-2}, a_i\}) = a_i$ by the induction hypothesis, Weak WARP precludes $f(T;A) = a_i$. So, $f(T;A) \in \{a_1, a_2\}$.

To rule out $f(T;A) = a_2$, consider the reversal T^{-1} of T. Since $a_{i-2} T^{-1} a_i T^{-1} a_{i-1} T^{-1} a_{i-2}$ for $i = 3, ..., n$, the same kind of argument given for T implies $f(T^{-1};A) \in \{a_1, a_2\}$. Since $f(T;A) \in \{a_1, a_2\}$ and $a_1 T a_2$, Reversal Improvement then implies $f(T;A) = a_1 = x$.

Next, suppose $x = a_2$. From the definition of B and the fact that $f(T;(A \setminus \{a_n\}) = a_2$, it follows that $a_1 T a_3$ and $a_2 T a_1$. We distinguish two possibilities: (i) $a_3 T a_2$; and (ii) $a_2 T a_3$.

(i) Given $a_2Ta_1Ta_3Ta_2$ and (c), the same kind of argument used for $x = a_1$ establishes $f(T;A) = a_2$. (The difference is that $f(T;A) = a_3$ is ruled out by $a_2Ta_1Ta_3Ta_2$ while $f(T;A) = a_4$ is ruled out by $a_1Ta_3Ta_4Ta_1$. In turn, $f(T^{-1};A) = a_3$ is ruled out by $a_2T^{-1}a_3T^{-1}a_1T^{-1}a_2$ while $f(T^{-1};A) = a_4$ is ruled out by $a_1T^{-1}a_4T^{-1}a_3T^{-1}a_1$.)

(ii) By the induction hypothesis: $f(T;A\backslash\{a_1\}) = a_2$ given a_2Ta_3 and (c); and $f(T;\{a_1,a_2\}) = a_2$ given a_2Ta_1. So, $f(T;A) = f(T;(A\backslash\{a_1\})\cup\{a_1,a_2\}) = a_2 = x$ by Expansion Consistency. ■

Remark 1. The "revealed priority" R defined in the proof (see (2) and (3) above) is closely related to the "revealed rationales" for rational shortlist methods. In particular, $xTzTyTx$ and $f(T;\{x,y,z\}) = x$ reveal that, for every RSM representation of $f(T;\cdot)$, the second rationale *must* contain (y,x). Conversely, $xT'yT'zT'x$ and $f(T';\{x,y,z\}) = z$ reveal that, for every RSM representation of $f(T';\cdot)$, the first rationale *must* contain (x,y).

The proof shows that R admits exactly two completions into linear orderings, which are denoted P^a and P^b below. These priorities differ only in terms of how they rank the top two alternatives a and b from Step 2 of the proof. To see why this non-uniqueness is intrinsic to our model, note that formula (5) implies $f_{P^a}(T;\{a,b\}) = \max(T;\{a,b\}) = f_{P^b}(T;\{a,b\})$ for all $T \in \mathcal{T}$. As a result, $f_{P^a}(T;A) = f_{P^b}(T;A)$ for all $T \in \mathcal{T}$ and $A \in \mathcal{X}$.

4 Further remarks

4.1 Restricted choice settings

Our characterization of two-stage majoritarian rules relies on the full range of tournaments and the full range of agendas. This begs the question about what properties characterize two-stage majoritarian rules in settings that lack variability on one of these dimensions.[11]

4.1.1 The fixed tournament setting

For a fixed tournament $T \in \mathcal{T}$, the choice function $f_P(T;\,\cdot\,)$ is a rational shortlist method whose rationales $P_1 = T \cap P$ and $P_2 = T \setminus P$ are acyclic. Houy (2008) shows that acyclicity of the rationales limits the scope of potential choice behavior for rational shortlist methods. However, the rationales of $f_P(T;\,\cdot\,)$ are not merely acyclic. They are also linked through the priority P. The next example shows that this imposes additional restrictions on behavior.

[11] We are indebted to our two referees, each of whom encouraged us to think about one of these settings.

Example 4. (Acyclic RSM) *Suppose that* $X = \{1,2,3,4,5\}$*. Fix acyclic rationales*

$$\begin{aligned} P_1 &:= \{(1,2),(2,3),(4,1)\} \text{ and} \\ P_2 &:= \{(1,3),(1,5),(2,4),(3,4),(3,5),(5,2),(5,4)\}. \end{aligned}$$

Let $c := \max(P_2; \max(P_1; \cdot))$ *denote the associated rational shortlist method and* $T := P_1 \cup P_2$ *the associated tournament. To see that there is no priority* P *such that* $c = f_P(T; \cdot)$*, suppose otherwise. By revealed preference, it follows that:*

$$\begin{aligned} 1T2T4T1 \text{ and } c(\{1,2,4\}) = 4 &\Rightarrow (1,2) \in T \cap P; \\ 2T3T5T2 \text{ and } c(\{2,3,5\}) = 5 &\Rightarrow (2,3) \in T \cap P; \text{ and} \\ 1T3T4T1 \text{ and } c(\{1,3,4\}) = 3 &\Rightarrow (1,3) \in T \setminus P. \end{aligned}$$

Thus, $1P2P3P1$ *which contradicts the fact that* P *must be a linear ordering.*

The example suggests what is needed to characterize two-stage majoritarian choice in the fixed tournament setting. To elaborate, fix a choice function c. Let P_1^c and P_2^c denote the "revealed rationales" for rational shortlist methods – that is, the binary relations consisting of all pairs that *must* belong to the first and second rationales in any RSM representation of c, respectively.[12] Extending the logic of Example 4, it follows that $x \in X$ must have higher priority than $y \in X$ if the pair (x, y) belongs to the transitive closure of $P_1^c \cup (P_2^c)^{-1}$. It is not difficult to show that c reveals nothing more about the priority.

Combined with Proposition 2 of Dutta and Horan (2015), this observation establishes that *a choice function* c *is two-stage majoritarian if and only if it satisfies Expansion Consistency and the binary relation* $P_1^c \cup (P_2^c)^{-1}$ *is acyclic.* By way of comparison, Houy's characterization of acyclic rational shortlist methods uses Expansion Consistency and the weaker requirement that P_2^c is acyclic.[13]

4.1.2 The fixed agenda setting

There is a bijection between two-stage majoritarian rules in our setting and the setting with a fixed agenda X.[14] To elaborate, fix a tournament $T \in \mathcal{T}$ and an agenda $A \in \mathcal{X}$. Let T^A denote the tournament that coincides with T on A but puts each alternative in $X \setminus A$ below

[12] While it is not required to appreciate the subsequent discussion, Dutta and Horan (2015, Proposition 1) show that P_1^c and P_2^c admit simple definitions in terms of choice behavior. In particular: xP_1^cy if $c(B) = y$ and $c(B \cup \{x\}) \notin \{x, y\}$ for some $B \subset X$; and, xP_2^cy if $c(A) = x$ and $c(B) = y$ for some $\{x\} \subset A \subset B \subseteq X$.

[13] Since it is implied by the acyclicity of P_2^c, Weak WARP is not expressly required for either result.

[14] The same is also true for the choice rules considered by Apesteguia et al. (2014) and Horan (2021), which include the successive elimination rule from Example 2 and the amendment rule from footnote 8.

every alternative in A. For any two-stage majoritarian rule $f_P : \mathcal{T} \times \mathcal{X} \to X$, going from T^A to T is tantamount to removing the alternatives in $X \setminus A$, that is

$$f_P(T; A) = f_P(T^A; X). \tag{6}$$

From this identity, it follows that the usual projection $f(\,\cdot\,;\,\cdot\,) \mapsto f(\,\cdot\,; X)$ defines a bijection between two-stage majoritarian rules in our setting and the setting with a fixed agenda X.

By applying identity (6) to our axioms, it is also possible to "translate" our characterization to the fixed agenda setting. While Choice IIA becomes vacuous, the other three axioms continue to have bite. The drawback is that they become more difficult to interpret.

4.2 Flexibility and Pareto sub-optimality

The well-known *top cycle* correspondence $TC : \mathcal{T} \times \mathcal{X} \to \mathcal{X} \setminus \{\emptyset\}$ (Camion, 1959; Good, 1971; Schwartz, 1972; Smith, 1973; Fishburn, 1974) is defined, for all $T \in \mathcal{T}$ and $A \in \mathcal{X}$, by $TC(T; A) := \{a \in A \mid \forall b \in A \setminus \{a\} : a = c_1 T ... T c_n = b$ for some $c_1, ..., c_n \in A\}$. Just like the uncovered set, the top cycle correspondence is Condorcet-consistent.

For all $T \in \mathcal{T}$ and $A \in \mathcal{X}$, the set of alternatives that are chosen by some two-stage majoritarian rule coincides with the top cycle, that is $TC(T; A) = \{f_P(T; A) \mid P \in \mathcal{P}\}$. To see that $TC(T; A) \supseteq \{f_P(T; A) \mid P \in \mathcal{P}\}$, pick any $P \in \mathcal{P}$ and note that $f_P(T; A) = f_P(T; TC(T; A)) \in TC(T; A)$. For the reverse inclusion, pick any $a \in TC(T; A)$. By a standard result in graph theory, there is a path $a = a_1 T ... T a_m$ such that $\{a_1, ..., a_m\} = A$ (see e.g., Lemma 8.3.3 of Laslier, 1997). Fix a priority $P \in \mathcal{P}$ such that $a_1 P ... P a_m$. Since $\max(T \cap P; A) = a$ by construction, it follows that $f_P(T; A) = a$.

A classic result of Miller (1977) shows that successive elimination rules (Example 2) also trace out the top cycle, that is $TC(T; A) = \{s_P(T; A) \mid P \in \mathcal{P}\}$. As such, those rules provide exactly the same *flexibility* to the designer as two-stage majoritarian rules.

It is well known that, for some tournaments $T \in \mathcal{T}$ and agendas $A \in \mathcal{X}$ such that $|A| \geq 4$, the top cycle $TC(T; A)$ contains alternatives that are Pareto dominated at some preference profiles consistent with T. It follows that all two-stage majoritarian rules occasionally make Pareto sub-optimal choices. To illustrate, suppose $X := \{1, 2, 3, 4\}$ and consider the two-stage majoritarian rule f_P based on $P := 4, 3, 2, 1$. Suppose (as in Bordes, 1979, p. 188) that there are three agents with preferences $\succ_1 := 1, 4, 3, 2$, $\succ_2 := 2, 1, 4, 3$, and $\succ_3 := 3, 2, 1, 4$. Note that alternative 4 is Pareto-dominated by alternative 1. Since the majority tournament T for this profile coincides with the one from Example 2 however, $f_P(T; X) = 4$.

4.3 The connection to May

Two-stage majoritarian rules satisfy a natural adaptation of May's *positive responsiveness* to the tournament setting. To state this adaptation (originally formulated by Moulin, 1986, p. 285), say that a binary relation $R^{\uparrow a}$ on X *improves* an alternative $a \in X$ relative to another binary relation R on X if, for all $x, y \in X \setminus \{a\}$: (i) $aRx \Rightarrow aR^{\uparrow a}x$; and (ii) $xRy \Leftrightarrow xR^{\uparrow a}y$.

T-Monotonicity. *For all $T \in \mathcal{T}$, $a \in X$, $T^{\uparrow a} \in \mathcal{T}$ that improves a, and $A \in \mathcal{X}$:*

$$f(T; A) = a \textit{ implies } f(T^{\uparrow a}; A) = a.$$

Thus: improving the majority view of a chosen alternative must reinforce its choice.

To see that two-stage majoritarian rules satisfy this property, recall that $f_P(T; A)$ is the lowest priority alternative in A that beats all higher priority alternatives by majority. Improving $f_P(T; A)$ relative to T does nothing to change this: $f_P(T; A)$ still beats all higher priority alternatives; and every alternative with lower priority than $f_P(T; A)$ is still beaten by some alternative with higher priority.

It is well known that the rules from Example 2 (just like those from footnotes 6 and 8) also satisfy T-Monotonicity.[15] To see that the rules from Example 1 do not, consider $X := \{1, 2, 3, 4\}$ and $P := 4, 3, 2, 1$. Then, $UC_P(T; X) = 3$ for the tournament T from Example 2 while $UC_P(T'; X) = 4$ for the tournament T' that improves 3 relative to 1.

4.4 The role of the priority

A minor variation on the argument used to show that two-stage majoritarian rules satisfy T-Monotonicity also establishes that every rule f_P is monotonic with respect to the priority P. In other words, two-stage majoritarian rules satisfy the following property:

P-Monotonicity. *For all $P \in \mathcal{P}$, $a \in X$, $P^{\uparrow a} \in \mathcal{P}$ that improves a, $T \in \mathcal{T}$, and $A \in \mathcal{X}$:*

$$f_P(T; A) = a \textit{ implies } f_{P^{\uparrow a}}(T; A) = a.$$

This property formalizes the idea that higher ranked alternatives are more privileged.

The rules from Examples 1 and 2 (as well as the related rules from footnotes 6 and 8) satisfy an analogous property.[16] The difference is that the priority P plays a less intrusive

[15]See Exercise 9.4(c) of Moulin (1988, p. 250) for s_P and the Corollary to Theorem 9.5 (p. 247) for a_P. Horan (2021) shows that a much broader range of binary trees (which he calls "simple agendas") have the same feature. For TC_P, simply note that the top cycle cannot gain new members by improving one of its members.

[16]For UC_P and TC_P, the claim is straightforward. For s_P and a_P, see Exercise 9.5 of Moulin (1988, p.

role for two-stage majoritarian rules than it does for these other rules. To see this, first consider the successive elimination rules from Example 2. By definition, the chosen alternative $s_P(T;A)$ must defeat all higher priority alternatives in the agenda $A \in \mathcal{X}$. Because the same is true for $f_P(T;A)$, this means that, when the alternatives $s_P(T;A)$ and $f_P(T;A)$ differ, $f_P(T;A)$ must be preferred by a majority over $s_P(T;A)$.

The same reasoning shows that $f_P(T;A)$ must be weakly preferred by majority to the alternatives $TC_P(T;A)$ and $a_P(T;A)$ chosen by the top cycle selection rule (footnote 6) and the amendment rule (footnote 8). In fact, the same is also true for the uncovered set selection rule from Example 1 once differences in flexibility of the two rules are taken into account: if $f_P(T;A) \in UC(T;A)$, then $f_P(T;A)$ must be weakly preferred by a majority to $UC_P(T;A)$.

4.5 An extension

There may be settings where it is desirable to select the same alternative for a tournament and its reversal. To accommodate this possibility, it is necessary to weaken the conclusion of Reversal Improvement to allow $f(T;A) = f(T^{-1};A)$. Unlike Reversal Improvement, the resulting *Weak Reversal Improvement* property does not imply Faithfulness.

When combined with Faithfulness and the other requirements in our Theorem, Weak Reversal Improvement defines a much broader class of choice rules. The next example describes some rules in this class that share the same basic structure as two-stage majoritarian rules.

Example 5. (General two-stage majoritarian rules) *Let $\mathcal{R}_2$ denote the set of weak orderings*[17] *R on X such that, for any $x \in X$, the indifference class $I_R(x) := \{y \in X \mid xRyRx\}$ contains at most two alternatives. Given a weak ordering $R \in \mathcal{R}_2$, let g_R be the choice rule defined, for all $T \in \mathcal{T}$ and $A \in \mathcal{X}$, by*

$$g_R(T;A) := \max(T \setminus R; \max(T \cap R; A)).$$

To see that g_R does indeed define a choice rule, let R_T denote the linear ordering obtained by taking the lexicographic composition of R with a tournament $T \in \mathcal{T}$. Then, $g_R(T;A) = f_{R_T}(T;A)$ for all $A \in \mathcal{X}$. Not only does this show that g_R is well-defined, it shows that g_R is a two-stage majoritarian rule when R contains no indifferences (since, in that case, $R_T = R_{T'}$ for all $T, T' \in \mathcal{T}$). This is not true when $xRyRx$ for distinct $x, y \in X$. Then, x and y must be compared by the first rationale $T \cap R$ regardless of $T \in \mathcal{T}$, something which cannot occur for a two-stage majoritarian rule.

250). A much broader class of binary trees introduced by Horan (2021) (called "priority agendas") have the same feature.

[17] A weak ordering R is a complete ($\forall a, b : aRb$ or bRa) and transitive binary relation.

Besides Faithfulness, Expansion Consistency, Weak WARP, Choice IIA, and Weak Reversal Improvement, g_R satisfies T-Monotonicity and (the analog of) P-Monotonicity for any weak ordering $R \in \mathcal{R}_2$. To see that g_R may violate Reversal Improvement when $|X| \geq 3$, consider a weak ordering $R \in \mathcal{R}_2$ such that $1R2R3R2$ and a tournament $T \in \mathcal{T}$ such that $1T2T3T1$. Then, $g_R(T;A) = 1 = g_R(T^{-1};A)$ for the agenda $A := \{1,2,3\}$.

To close, we note that the rules from Example 4 provide the same flexibility as two-stage majoritarian rules, that is $TC(T;A) = \{g_{\sigma R}(T;A) \mid \sigma \text{ is a permutation on } X\}$ for all $R \in \mathcal{R}_2$, $T \in \mathcal{T}$, and $A \in \mathcal{X}$. As with two-stage majoritarian rules, the implication is that all of these rules occasionally make Pareto sub-optimal choices. This raises the question of whether an efficient choice rule can satisfy all of the desiderata listed in the last paragraph.

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