Sophisticated Banking Contracts and Fragility When Withdrawal Information is Public*

Xuesong Huang†

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Abstract

I study whether self-fulfilling bank runs can occur when banks use sophisticated contracts and withdrawal decisions are public information. In a finite-agent version of Diamond and Dybvig (1983) with correlated types, I first present an example in which a bank run perfect Bayesian equilibrium exists. However, its existence relies on off-path beliefs that are unreasonable in terms of forward induction. To discipline beliefs, I use forward induction equilibrium (Cho, 1987) as the solution concept. I show that, whenever the allocation rule is strictly incentive compatible, the truth-telling strategy is the unique forward induction equilibrium in the withdrawal game, and no bank run occurs. Therefore, with forward induction, sophisticated contracts can prevent bank runs when there is public information about withdrawal decisions.

Keywords: Bank runs; Sophisticated contracts; Public information; Forward induction; Correlated types.

JEL Codes: D82, D83, G21

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†Lingnan College, Sun Yat-sen University, xuesong.huangll@gmail.com

1 Introduction

In the modern financial system, intermediaries play a vital role in facilitating the flow of resources toward more productive projects. Much like traditional commercial banks, a wide range of intermediaries are in the business of maturity transformation, issuing short-maturity debts to finance long-maturity investments. Such institutions include structured investment vehicles (SIVs), money market funds (MMFs), open-end mutual funds, etc. However, as observed in the past 15 years, those financial intermediaries are susceptible to sudden increases in withdrawals that resemble a bank run, which can strain the financial system and have repercussions in the entire economy.[1] As a consequence, how to make those financial intermediaries less fragile remains a timely and essential question.

In the seminal paper by Diamond and Dybvig (1983), the phenomenon of a bank run is characterized as a Nash equilibrium outcome in the withdrawal game played by depositors. In this framework, a bank run equilibrium is a coordination failure. If each depositor fears that other depositors will rush to withdraw from the bank, it is in his own best interest to rush to withdraw as well before the bank runs out of funds. Several papers following Diamond and Dybvig propose a *mechanism design* approach to study bank runs. Early works include Wallace (1988, 1990), Green and Lin (2003) and Peck and Shell (2003). They explore whether the design of the banking contract could eliminate bank runs as equilibrium outcomes. The banking contracts studied in this literature are *sophisticated* in the sense that payments to depositors are dynamically adjusted contingent on past withdrawals. Green and Lin (2003) show that, under certain conditions, the use of such sophisticated banking contracts can prevent bank runs and implement efficient allocations, while Ennis and Keister (2009) give conditions under which a bank run equilibrium still exists.

A common criticism of this literature is that sophisticated banking contracts are rare in reality. One reason for this may be moral hazard problems that arise when the bank operator can adjust payments based on withdrawal demand that is not observed by depositors.[2] However, with the advent of blockchain and its embedded smart-

[1]For example, see Schmidt et al. (2016) and Li et al. (2021) for the run on money market funds during the 2008 financial crisis and the 2020 COVID crisis, respectively.

[2]See Andolfatto and Nosal (2008) for a more formal discussion on this point.

contract technology, there is a natural solution to this problem. One can program a sophisticated banking contract as a smart contract on a (public) blockchain, which will execute automatically without a third party like the bank operator.[3] Furthermore, the irreversible nature of blockchain guarantees that no one can tamper with the contract.

However, this solution of using smart contracts on a blockchain brings in a new issue. That is, because of the *information transparency* inherent in a public blockchain, any depositor's transactions with the banking (smart) contract are public and, therefore, fully observable by other depositors. In other words, withdrawal by a given depositor will be observed by all other depositors, which sharply contrasts with the normal operations in traditional banks. These developments, therefore, raise a new research question: *Can sophisticated banking contracts eliminate bank runs when information about withdrawals is publicly available?*

I study this question in a finite-agent version of Diamond-Dybvig with correlated liquidity types and a formal sequential service constraint as in Green and Lin (2000). The banking contract asks each depositor to make a withdrawal decision, and the bank's payment to each depositor is contingent on the withdrawal decisions of depositors earlier in the line. Furthermore, each depositor's withdrawal decision is perfectly observable by other depositors. Therefore, given the banking contract and public information about withdrawal decisions, depositors are playing a (sequential) *full-information withdrawal game*, which shares some features of a signaling game. In particular, each depositor's withdrawal decision can serve as a signal of their private liquidity type for depositors later in the sequence.

First, I show by an example that a bank run perfect Bayesian equilibrium can exist in the full-information withdrawal game. Here, the assumption of correlated types is crucial. To see why, first note that, in the full-information withdrawal game, each depositor can update their belief about earlier depositors' types after observing their withdrawal decisions. When types are independent, each depositor's updated belief about earlier depositors' types does not affect their inference about the remaining depositors' types. However, when types are correlated, each depositor's updated belief about earlier depositors' types *does* affect their inference about the remaining

[3] A recent paper by Routledge and Zetlin-Jones (2022) provides a practical guide on GitHub that shows how to program such contracts using Solidity code on the Ethereum Network.

depositors' types. As a result, each depositor's belief matters when types are correlated. Since perfect Bayesian equilibrium does not restrict the choice of off-path beliefs, I can pick *any* off-path beliefs that rationalize a bank run perfect Bayesian equilibrium.

However, in the example of a bank run perfect Bayesian equilibrium that I construct, the choice of off-path beliefs is unreasonable in terms of forward induction. It is easy to see why forward induction is a natural criterion in this environment. Any depositor who chooses not to withdraw immediately from the bank intends to consume later on. However, a depositor of the impatient type wants to consume as early as possible to meet his liquidity needs. Therefore, by forward induction, anyone who chooses not to withdraw immediately from the bank cannot be of the impatient type. To formally incorporate such a restriction on off-path beliefs, I use forward induction equilibrium introduced in Cho (1987) as the equilibrium concept for the full-information withdrawal game.[4] The main result shows that, whenever the allocation/payment rule is strictly incentive compatible, there is a unique forward induction equilibrium in the associated full-information withdrawal game, and no bank run occurs. Therefore, sophisticated banking contracts can eliminate fragility when withdrawal decisions are publicly observed.

Lastly, I consider two additional issues related to this result. First, I study under what conditions efficient allocations are strictly incentive compatible. In particular, I identify an additional condition on the correlation structure across types such that the first-best allocation rule is strictly incentive compatible, which provides a generalization of Lemma 5 in Green and Lin (2003). Combined with the main result, this additional result gives a sufficient condition for the first-best allocation to be uniquely implementable by a sophisticated banking contract in this setting. Second, I use the indirect mechanism studied in Andolfatto et al. (2017) to illustrate that introducing indirect mechanisms cannot weaken the strict incentive compatibility condition in the main result. Therefore, in contrast to the case when withdrawal decisions are not observed, there appears to be no gain in considering indirect mechanisms in the environment studied in this paper.

[4] In classical signaling games, forward induction equilibrium boils down to the "intuitive criterion" studied in Cho and Kreps (1987).

2 Related literature

This paper is most closely related to the mechanism design literature on bank runs following Green and Lin (2003). Table 1 provides a brief summary of the main results in this literature and highlights how the existence of bank run equilibria depends on the assumptions in the underlying environment.[5] This paper makes two contributions to this literature. First, parallel to the contributions in Ennis and Keister (2009), this paper shows that, if one relaxed the assumption of independent types to allow correlated types, there could exist a bank run perfect Bayesian equilibrium in the full-information withdrawal game, in contrast to the no bank run result in Andolfatto et al. (2007) with independent types. Second, as indicated in Table 1, this paper shows that, with forward induction, the no bank run result can be re-established even when types are correlated.

	Independent types	Correlated types
No information	There exists a bank run Bayes Nash equilibrium (Peck and Shell, 2003)	
Information about position	No bank run exists as a Bayes Nash equilibrium (Green and Lin, 2003)	There exists a bank run Bayes Nash equilibrium (Ennis and Keister, 2009)
Information about position and withdrawal decisions	No bank run exists as a perfect Bayesian equilibrium (Andolfatto et al., 2007)	No bank run exists as a *forward induction equilibrium* (This paper)

Table 1: Summary of main results in the existing literature

The papers in Table 1 all focus on direct mechanisms in which depositors are asked to report their types. Some recent papers instead study indirect mechanisms in which depositors are asked to report additional messages, and they show that those indirect mechanisms can prevent bank runs. For example, Andolfatto et al. (2017) study an indirect mechanism in which depositors are asked to report not only their types but also their beliefs regarding whether there is a bank run going on. Another paper by Payne and Weiss (2020) studies a different indirect mechanism in which each depositor is allowed to ask to hold shares of the intermediary's long-term assets. Cavalcanti and Monteiro (2016) study an indirect mechanism in which depositors

[5]See Ennis and Keister (2010) for a detailed survey on this literature.

are asked to report their types twice, once before receiving consumption from the intermediary and once after. In this paper, instead of asking for more information *from* depositors as in those indirect mechanisms, I show that the intermediary can prevent bank runs by providing more information *to* depositors.

In the Diamond-Dybvig framework, several papers have studied information revelation about depositors' withdrawal information. Nosal and Wallace (2009) study how information revelation about depositors' withdrawal decisions affects the set of implementable allocation rules. This paper, in contrast, takes as given that information about withdrawal decisions is fully revealed, as would occur with smart contracts on a public blockchain. I show that any allocation rule that is strictly incentive compatible in this environment can be uniquely implemented by a sophisticated contract. Kinateder and Kiss (2014) also study the full-information withdrawal game in a finite-agent version of Diamond-Dybvig. They assume *aggregate certainty*, i.e., the total number of impatient depositors is known, and show that there is a unique perfect Bayesian equilibrium in the full-information withdrawal game. In contrast, this paper assumes *aggregate uncertainty*, which is a more natural assumption with a finite number of depositors, and shows that multiple perfect Bayesian equilibria could exist in the full-information withdrawal game. Ennis and Keister (2016) study a different setup in which only decisions of withdrawal are revealed to the bank and depositors. They show by examples that there are bank run equilibria under the optimal banking contract. In comparison, this paper focuses on the setup in which decisions of both withdrawal and no withdrawal are revealed to the bank and depositors. I show that, under the optimal banking contract, no bank run can occur as a forward induction equilibrium.

Lastly, this paper is related to the literature that applies forward induction reasoning to eliminate unreasonable equilibria in various settings such as advertising (Milgrom and Roberts, 1986), matching models of money (Nosal and Wallace, 2007), stable matching under incomplete information (Pomatto, 2021), and credit market competition with monitoring (Goldstein et al., 2022). This paper contributes to this literature by identifying a new application of forward induction reasoning in a model of bank runs.

The rest of the paper is organized as follows. The next section sets up the environment, the full-information withdrawal game, and the planner's problem for the first-best allocation rule. Section 4 introduces two examples with correlated types to illustrate the signaling aspect of the full-information withdrawal game and the problem of allowing arbitrary off-path beliefs in perfect Bayesian equilibrium. Section 5 defines forward induction equilibrium and presents the main result. Section 6 discusses two issues related to the main result: efficiency and indirect mechanisms. Section 7 concludes with a discussion of implications for asset-backed stablecoins and directions for future research.

3 The setup

In this section, I first introduce the environment, which is an extension of Green and Lin (2003) to allow correlated types as in Ennis and Keister (2009). Next, I give a formal description of the full-information withdrawal game played by depositors and define perfect Bayesian equilibrium. Lastly, I set up the planner's problem for the first-best allocation rule and explain how the problem can be reduced to a dynamic programming problem.

3.1 The environment

The economy consists of two time periods, indexed by $t \in \{0, 1\}$, a finite number of depositors, indexed by $i \in \mathbf{I} = \{1, 2, \cdots, I\}$, and a single consumption good that can be consumed in both periods. Each depositor i learns his private type $\omega_i \in \Omega = \{0, 1\}$ in period 0, and his preference is given by[6]

$$u_i(a_i, \omega_i) = \begin{cases} v(a_i^0) & \text{if } \omega_i = 0 \\ \rho v(a_i^0 + a_i^1) & \text{if } \omega_i = 1, \end{cases} \tag{1}$$

where $a_i = (a_i^0, a_i^1)$ denotes the consumption of depositor i in each period, $\rho > 0$, and, as in Diamond and Dybvig (1983), the function $v : \mathbb{R}_+ \to \mathbb{R}$ is assumed to satisfy the following properties:

[6] This formulation of preferences is exactly the same as in Diamond and Dybvig (1983). It reduces to the formulation in Green and Lin (2003) by letting $\rho = 1$.

Assumption 1. *(i) v is strictly increasing, twice continuously differentiable and strictly concave; (ii) v satisfies the Inada conditions where* $\lim_{c\to 0} v'(c) = \infty$ *and* $\lim_{c\to\infty} v'(c) = 0$*; and (iii) relative risk aversion of v is greater than or equal to 1 everywhere, i.e., for any $c \in \mathbb{R}_{++}$,*

$$-\frac{cv"(c)}{v'(c)} \geq 1.$$

If $\omega_i = 0$, the depositor is *impatient* and only values consumption in period 0. If $\omega_i = 1$, the depositor is *patient* and values consumption in both periods. Denote $\omega = (\omega_1, \omega_2, \cdots, \omega_I) \in \prod_{i=1}^{I} \Omega_i = \prod_{i=1}^{I} \Omega = \Omega^I$ as depositors' type profile. Furthermore, let $\omega_{1:i-1} = (\omega_1, \cdots, \omega_{i-1}) \in \prod_{k=1}^{i-1} \Omega = \Omega^{i-1}$ and $\omega_{i+1:I} = (\omega_{i+1}, \cdots, \omega_I) \in \prod_{k=i+1}^{I} \Omega = \Omega^{I-i}$. The common prior $\mathcal{P}$ is given by

$$\mathcal{P}(\omega) = \frac{p(\theta(\omega))}{C(I, \theta(\omega))},$$

where $C(I,\theta) = \frac{I!}{\theta!(I-\theta)!}$, $p : \{0, 1, \cdots, I\} \to (0, 1]$ is an exogenous probability mass function satisfying $\sum\limits_{\theta=0}^{I} p(\theta) = 1$, and $\theta(\omega) = \sum\limits_{i=1}^{I} \omega_i$ is the total number of patient depositors in type profile ω. One can interpret this formulation as follows: Nature first chooses the total number of patient depositors θ according to the probability mass function $p(\cdot)$. Then θ depositors are chosen at random (with each depositor equally likely to be chosen) and assigned to be patient. The remaining depositors are assigned to be impatient. Under this approach, each depositor has the same *ex ante* probability of being patient. Note that this formulation allows depositors' types to be correlated. Furthermore, it includes independent types as a special case by letting the probability mass function $p(\cdot)$ be a binomial distribution, i.e.,

$$p(\theta) = C(I, \theta)(1 - \pi)^{\theta} \pi^{I-\theta},$$

where π is the probability of each depositor being impatient.

There is an intermediary that allocates consumption goods to depositors. The intermediary is endowed with I units of consumption goods in period 0, and has the access to an investment technology transforming each unit of the good that is not consumed in period 0 into R units of the good in period 1.[7] Throughout the paper, I assume that $\rho R > 1$.[8] As in Wallace (1988), depositors are isolated from each other and from the intermediary at the beginning of period 0, but each depositor has an opportunity to contact the intermediary in each period in order to receive goods. Goods are nonstorable and must be consumed immediately after contacting the intermediary.[9] The (ex-post) allocation bundle $a = (a_1, a_2, \cdots, a_I)$ distributed by the intermediary must satisfy the following feasibility condition:

$$\sum_{i=1}^{I} (a_i^0 + \frac{a_i^1}{R}) \leq I.$$

In other words, the present value of total consumption goods the intermediary can allocate should not exceed its initial endowment. Let $\mathbb{A}$ denote the set of all feasible allocations. As in Green and Lin (2003), I allow the intermediary to use *sophisticated contracts* that allocate consumption goods contingent on depositors' (reported) type profile. Formally, an *allocation rule*, denoted as α, maps any type profile in Ω^I into a feasible allocation in $\mathbb{A}$. Let $\mathbb{F} = \{\alpha : \Omega^I \to \mathbb{A}\}$ denote the set of allocation rules.

Lastly, the *sequential service constraint* requires depositors to contact the intermediary in a fixed order given by the index i, beginning with depositor 1 and ending with depositor I.[10] As a result, the intermediary must determine the period-0 consumption of depositor i based on the partial history $\omega_{1:i} = (\omega_1, \cdots, \omega_i)$ since the remaining depositors have not contacted the intermediary yet. Formally, an allocation rule satisfies the sequential service constraint if

$$\alpha_i^0(\omega) = \alpha_i^0(\hat{\omega}) \text{ for all } \omega, \hat{\omega} \in \Omega^I \text{ such that } \omega_{1:i} = \hat{\omega}_{1:i}. \tag{2}$$

[7] The intermediary's endowment can also be interpreted as the total deposits from depositors, each of whom is endowed with one unit of the good. This setup simplifies the exposition by starting the analysis with these endowments already deposited in the intermediary. For analysis of depositors' choice of whether to deposit, see Peck and Setayesh (2022) and Shell and Zhang (2020).

[8] This is the assumption made in Diamond and Dybvig (1983). When $\rho = 1$, this condition reduces to $R > 1$, which is the assumption in Green and Lin (2003).

[9] This assumption eliminates the possibility for trade among depositors after contacting the intermediary. See Jacklin (1987) and Wallace (1988) on this point.

[10] Green and Lin (2003) assume that depositors have imperfect knowledge about the order in which they contact the intermediary. In this paper, I follow Green and Lin (2000) and Ennis and Keister (2009) in assuming that each depositor exactly knows their position in the sequence.

In other words, the sequential service constraint is a measurability condition on the period-0 allocation rule. Consequently, the period-0 allocation rule for depositor i can be simplified to a function mapping from Ω^i to $\mathbb{R}_+$, i.e., $\alpha_i^0(\cdot) : \Omega^i \to \mathbb{R}_+$. Note that the period-1 allocation is made after all reports have been made in period 0, and, as a result, the sequential service constraint does not apply to $\alpha_i^1(\cdot)$. Denote the set of allocation rules that satisfy the sequential service constraint as

$$\mathbb{F}^s = \{\alpha \in \mathbb{F} | \text{ condition (2) holds for all } i\}.$$

3.2 Full-information withdrawal game

In the full-information withdrawal game, each depositor i reports a private type in period 0, $r_i \in \Omega = \{0, 1\}$, which may or may not be his true type ω_i, after observing the report history before him, i.e., $h_i = (r_1, r_2, \cdots, r_{i-1})$.[11] Let H_i be the set of all possible report histories prior to depositor i. A behavior (mixed) strategy for depositor i is defined as $\sigma_i : H_i \times \Omega \to \Delta(\Omega)$, where $\Delta(\Omega)$ is the set of all possible distributions over Ω. A pure strategy for depositor i is defined as $\Gamma_i : H_i \times \Omega \to \Omega$. In particular, Γ_i is the behavior strategy σ_i satisfying $\sigma_i(\Gamma_i(h_i, \omega_i)|h_i, \omega_i) = 1$. Let $\sigma = (\sigma_1, \sigma_2, \cdots, \sigma_I)$ be a behavior strategy profile, and $\Gamma = (\Gamma_1, \Gamma_2, \cdots, \Gamma_I)$ be a pure strategy profile. ***The truth-telling strategy (profile)*** is a pure strategy profile Γ in which $\Gamma_i(h_i, \omega_i) = \omega_i$ for any history h_i and type ω_i. Following the literature, ***a (bank) run strategy (profile)*** is a behavior strategy profile σ in which there exists a depositor i of the patient type, i.e., $\omega_i = 1$, and a history h_i such that $\sigma_i(r_i = 0|h_i, \omega_i = 1) > 0$.

Depositor i's information sets are labelled by $(h_i, \omega_i) \in H_i \times \Omega$, and his belief at information set (h_i, ω_i) is denoted as $\mu_i(\cdot|h_i, \omega_i) \in \Delta(\Omega^{i-1})$. In particular, $\mu_i(\omega_{1:i-1}|h_i, \omega_i)$ specifies depositor i's belief about the probability of previous depositors' true type profile being $\omega_{1:i-1}$, after knowing his own true type ω_i and observing the report history prior to him h_i. Let $\mu = (\mu_1, \mu_2, \cdots, \mu_I)$ denote a system of beliefs.

[11] Because of the structure of depositors' preferences, in the full-information withdrawal game induced by any efficient allocation rule, if a depositor reports to be impatient, he will receive consumption right away in the early period. In other words, his report of being impatient is equivalent to the colloquial saying of "withdraw from the bank." If a depositor reports to be patient, he will only receive consumption in the late period, and his report of being patient is equivalent to the colloquial saying of "not withdraw from the bank." I will use these terms interchangeably.

Then, under any belief $\mu_i(\cdot|h_i, \omega_i) \in \Delta(\Omega^{i-1})$, depositor i's induced probability of the remaining depositors' true types being $\omega_{i+1:I} = (\omega_{i+1}, \cdots, \omega_I)$ is given by

$$\phi_i^{\mu_i}(\omega_{i+1:I}|h_i, \omega_i) = \sum_{\omega'_{1:i-1} \in \Omega^{i-1}} \mathcal{P}(\omega_{i+1:I}|\omega'_{1:i-1}, \omega_i)\mu_i(\omega'_{1:i-1}|h_i, \omega_i). \tag{3}$$

Remark 1. If depositors' types are independent as in Andolfatto et al. (2007), i.e., $\mathcal{P}(\omega_{i+1:I}|\omega_{1:i-1}, \omega_i) = \mathcal{P}(\omega_{i+1:I})$, it follows from (3) that

$$\begin{aligned}\phi_i^{\mu_i}(\omega_{i+1:I}|h_i, \omega_i) &= \sum_{\omega'_{1:i-1}} \mathcal{P}(\omega_{i+1:I})\mu_i(\omega'_{1:i-1}|h_i, \omega_i) \\ &= \mathcal{P}(\omega_{i+1:I}) \sum_{\omega'_{1:i-1}} \mu_i(\omega'_{1:i-1}|h_i, \omega_i) = \mathcal{P}(\omega_{i+1:I}),\end{aligned}$$

which is independent of h_i and ω_i. In words, with independent types, depositor i's induced probability distribution over the remaining depositors' types is independent of what previous $i-1$ depositors have reported and of depositor i's true type. However, with correlated types, depositor i's induced probability distribution over the remaining depositors' types *does* depend on his true type and his belief about previous depositors' true types given their reports.

Lastly, each depositor's final payoff in the full-information withdrawal game is determined by the allocation rule.[12] Specifically, in the full-information withdrawal game *induced by* the allocation rule $\alpha \in \mathbb{F}^s$, given h_i and ω_i, depositor i's period-0 allocation from reporting r_i is $\alpha_i^0(h_i, r_i)$, and his period-1 allocation is $\alpha_i^1(h_i, r_i, r_{i+1:I})$, which depends on the remaining depositors' reports. Note that, given the remaining depositors' behavior strategy profile $\sigma_{i+1:I} = (\sigma_{i+1}, \cdots, \sigma_I)$, the remaining depositors' reports are dependent on h_i, r_i and the realization of $\omega_{i+1:I}$. Therefore, given $h_i, \omega_{i:I}$ and $\sigma_{i+1:I}$, let

$$u_i(\alpha_i^0(h_i, r_i), \alpha_i^1(h_i, r_i, \sigma_{i+1:I}(h_i, r_i, \omega_{i+1:I}), \omega_i))$$

denote depositor i's expected payoff of reporting r_i. This expected utility can be

[12] For the rest of the paper, to simplify the presentation, I only focus on allocation rules that satisfy the following properties: $\alpha_i^0(h_i, r_i = 0) > 0, \alpha_i^0(h_i, r_i = 1) = 0, \alpha_i^1(h_i, r_i = 0, r_{i+1:I}) < \alpha_i^1(h_i, r_i = 1, r_{i+1:I})$. Given the structure of depositors' preferences, these conditions are minimal requirements for any efficient allocation to satisfy.

written as

$$\sum_{r_{i+1:I}\in\Omega^{I-i}} u_i(\alpha_i^0(h_i,r_i),\alpha_i^1(h_i,r_i,r_{i+1:I}),\omega_i)\sigma_{i+1}(r_{i+1}|h_i,r_i,\omega_{i+1})\cdots\sigma_I(r_I|h_i,r_i,\cdots,r_{I-1},\omega_I).$$

As a result, given any strategy profile σ and any belief μ_i, depositor i's expected payoff of choosing σ_i at information set (h_i,ω_i) is given by

$$\begin{aligned}&U_i^{\mu_i}(\sigma_i,\sigma_{i+1:I}|h_i,\omega_i)\\ &=\sum_{\omega_{i+1:I}}[\sum_{r_i\in\Omega}\sigma_i(r_i|h_i,\omega_i)u_i(\alpha_i^0(h_i,r_i),\alpha_i^1(h_i,r_i,\sigma_{i+1:I}(h_i,r_i,\omega_{i+1:I})),\omega_i)]\phi_i^{\mu_i}(\omega_{i+1:I}|h_i,\omega_i),\end{aligned}$$

where $\phi_i^{\mu_i}$ is determined by μ_i following (3). I say $\sigma_i(\cdot|h_i,\omega_i)$ is ***sequentially rational*** under the belief $\mu_i(\cdot|h_i,\omega_i)$ if

$$\sigma_i(\cdot|h_i,\omega_i)\in\underset{\sigma_i'\in\Delta(\Omega)}{\operatorname{argmax}}\,U_i^{\mu_i}(\sigma_i',\sigma_{i+1:I}|h_i,\omega_i).$$

Remark 2. If $\omega_i=0$, for any history h_i and any belief μ_i, $\sigma_i(r_i=0|h_i,\omega_i=0)=1$ is sequentially rational under μ_i.

This remark is a direct consequence of the preferences in (1), since the impatient type does not value consumption in period 1. In words, it says that if depositor i is truly impatient he will always report truthfully.

Furthermore, a history $h_i=(r_1,r_2,\cdots,r_{i-1})$ is ***on-path given strategy profile*** σ if for each $k=1,2,\cdots,i-1$ there exists a ω_k such that

$$\sigma_k(r_k|r_1,\cdots,r_{k-1},\omega_k)>0.$$

Otherwise, a history h_i is ***off-path*** **given strategy profile** σ. Next, I introduce the definition of perfect Bayesian equilibrium.

Definition 1. *A strategy profile σ is a **perfect Bayesian equilibrium (PBE)** if there exists a system of beliefs μ such that, for any i, any ω_i, and any history $h_i=(r_1,\cdots,r_{i-1})$, the following hold:*

1. *If h_i is on-path given σ, then $\mu_i(\cdot|h_i,\omega_i)\in\Delta(\Omega^{i-1})$ satisfies Bayes' rule: For*

any $\omega_{1:i-1}$,

$$\mu_i(\omega_{1:i-1}|h_i,\omega_i)=\frac{\mathcal{P}(\omega_{1:i-1})\prod_{j=1}^{i-1}\sigma_j(r_j|r_1,\cdots,r_{j-1},\omega_j)}{\sum_{\hat{\omega}_{1:i-1}}\mathcal{P}(\hat{\omega}_{1:i-1})\prod_{j=1}^{i-1}\sigma_j(r_j|r_1,\cdots,r_{j-1},\hat{\omega}_j)}; \tag{4}$$

2. $\sigma_i(\cdot|h_i,\omega_i)$ *is sequentially rational under* $\mu_i(\cdot|h_i,\omega_i)$.

3.3 The first-best allocation rule

In this section, I focus on the first-best allocation rule with complete information which has served as the main benchmark in Diamond and Dybvig (1983) and its following literature.[13] The first best allocation rule, denoted as $\mathbf{a}\in\mathbb{F}^s$, is the solution to the following social welfare maximization problem:

$$\max_{\alpha\in\mathbb{F}^s}\mathbb{E}_\omega\left[\sum_{i\in\mathbf{I}}u_i(\alpha_i^0(\omega_{1:i}),\alpha_i^1(\omega),\omega_i)\right]. \tag{5}$$

Note that, under the preferences in (1), the first-best allocation rule $\mathbf{a}$ must satisfy the following: for any $\omega\in\Omega$,

$$\mathbf{a}_i^0(\omega_{1:i})=0 \quad \text{if } \omega_i=1 \quad \text{and} \quad \mathbf{a}_i^1(\omega)=0 \quad \text{if } \omega_i=0. \tag{6}$$

In other words, the first-best allocation rule $\mathbf{a}$ requires that impatient depositors only consume in period 0 and patient depositors only consume in period 1. Furthermore, since patient depositors are risk averse, the first-best allocation rule $\mathbf{a}$ requires that they evenly divide the remaining resources in period 1, i.e., for any $\omega\in\Omega$,

$$\mathbf{a}_i^1(\omega)=\frac{R(I-\sum_{i=1}^{I}\mathbf{a}_i^0(\omega_{1:i}))}{\theta(\omega)} \quad \text{if } \omega_i=1. \tag{7}$$

Then what is left to solve is the period-0 allocation of each depositor i if he is impatient, $\mathbf{a}_i^0(\omega_{1:i})$, as a function of the partial history $\omega_{1:i}$. By Lemma 1 in Ennis and Keister (2009), these allocations can be found by using the results above to formulate

[13] All examples in Section 4 also use the first-best allocation rule.

(5) as a finite dynamic programming problem, which can be solved recursively.[14]

4 Examples

In this section, I study two examples with correlated types. First, I reconsider the main example in Ennis and Keister (2009), which studies the withdrawal game in which each depositor only has information regarding his position in the sequence and no information about earlier depositors' reports. They show that there exist multiple Bayes-Nash equilibria including a bank run equilibrium.[15] Interestingly, I will show that, in their example, the full-information withdrawal game has a unique perfect Bayesian equilibrium, and no bank run occurs. Here, the signaling aspect of the full-information withdrawal game plays an important role of eliminating bank run equilibria. However, as is usually the case with signaling games, the second example highlights the necessity of imposing reasonable restrictions over off-equilibrium beliefs, which motivates the introduction of forward induction equilibrium in section 5.

Example 1 (Ennis and Keister (2009)): Consider a 4-depositor environment with CRRA utility functions, i.e.,

$$v(x) = \frac{x^{1-\gamma}}{1-\gamma}. \tag{8}$$

Types are correlated and $p(\cdot)$ is defined as

$$p(2) = 1 - \varepsilon, p(\theta) = \frac{\varepsilon}{4} \text{ for } \theta = 0, 1, 3, 4, \tag{9}$$

where $p(\theta)$ is the probability of the set of type profiles in which exactly θ depositors are patient and set $\varepsilon = 0.04\%$. Therefore, by (9), it is highly likely that there will be two patient depositors in the economy. Other parameter values are $\rho = 1$, $R = 2$ and $\gamma = 6$.

Figure 1 depicts the possible period-0 consumption levels in the first-best alloca-

[14] All examples in Section 4 use CRRA utility functions as in Ennis and Keister (2009). To accommodate the introduction of ρ, I use a modified version of the recursive algorithm in their Proposition 1 to solve for the period-0 allocation in the first-best allocation rule.

[15] In their withdrawal game, since depositors have no information about previous depositors' reports, the set of Bayes-Nash equilibria coincides with the set of perfect Bayesian equilibria.

tion rule for each depositor i if he is impatient. In the figure, the circles correspond to partial histories in which depositor 1 is impatient, while the triangles correspond to histories in which depositor 1 is patient. For example, there are two possible period-0 allocations for impatient depositor 2, depending on depositor 1's type. When depositor 1 is impatient, the period-0 allocation for impatient depositor 2 (the circle) is almost the same as the period-0 allocation for him when depositor 1 is patient (the triangle). Then for impatient depositor 3, there are four possible period-0 allocations depending on the first two depositors' types. Figure 1 shows that the period-0 allocation for impatient depositor 3 is significantly lower following $\omega_{1:2} = (0, 0)$ than following other partial histories. In general, there are 2^{i-1} possible period-0 allocations for impatient depositor i, each corresponding to a specific realization of the types of depositors earlier in the sequence.

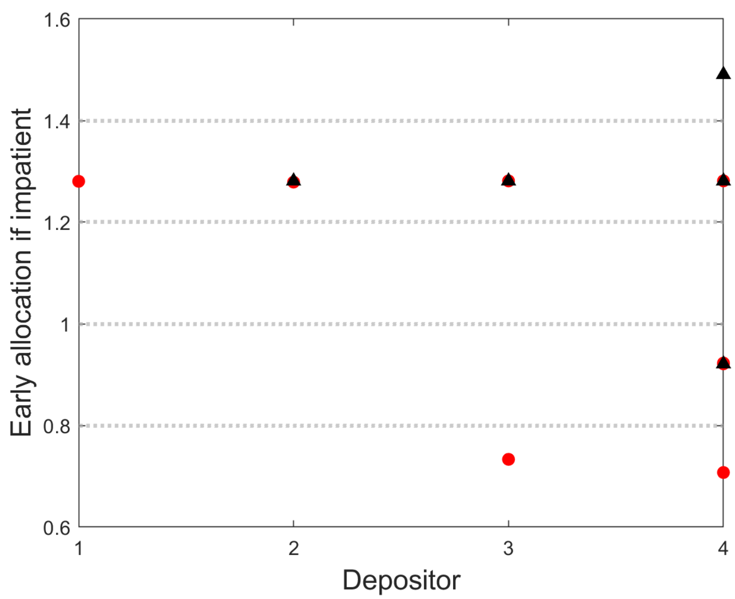

Figure 1: Period-0 allocation in the first-best allocation rule

First, recall the bank run equilibrium in Ennis and Keister: The first two depositors always report to be impatient while the last two depositors report truthfully.[16] The reason why the first two depositors choose to run is that (i) they expect the other to run and (ii) they worry that the last two depositors might both be impatient, in

[16]Under the first-best allocation rule, it is straightforward to show that the last two depositors always choose to report truthfully.

which case there will be more early withdrawals than expected (since $p(2) \approx 1$) and the remaining payoffs will be low.

In the full-information withdrawal game, depositor 2 can perfectly observe depositor 1's report. What if depositor 2 observes that depositor 1 has reported to be patient? Then he knows, if he also chooses to report to be patient, there will be no more than two early withdrawals, in which case reporting "patient" is preferred. Formally, let us compare patient depositor 2's expected payoff of reporting 0 ("impatient") and 1 ("patient") after history $h_2 = 1$:[17]

$$\textbf{Report 0:} \quad U_2^{\mu_2}(r_2 = 0, \Gamma_{3:4} | h_2 = 1, \omega_2 = 1) = v(\mathbf{a}_2^0(1, 0)) = v(1.28);$$
$$\textbf{Report 1:} \quad U_2^{\mu_2}(r_2 = 1, \Gamma_{3:4} | h_2 = 1, \omega_2 = 1) \geq v(\mathbf{a}_2^1(1, 1, 0, 0)) = v(1.44).$$

Here, $\mathbf{a}_2^0(1, 0)$ is the period-0 allocation for patient depositor 2 if he reports "impatient" given depositor 1's report of being patient. If he reports "patient", the inequality follows from the fact that $\mathbf{a}_2^1(1, 1, 0, 0)$ is patient depositor 2's worst possible period-1 allocation, i.e., when the last two depositors are both impatient. Note that this inequality holds for any μ_2. Therefore, patient depositor 2 always chooses to report truthfully after history $h_2 = 1$. Anticipating this, patient depositor 1 also wants to report to be patient. Since depositor 1 reports truthfully, patient depositor 2 will believe that depositor 1's report of being impatient is made by the truly impatient type. As a consequence, he also chooses to report truthfully after history $h_2 = 0$. Therefore, the truth-telling strategy is the unique PBE in the full-information withdrawal game.

Discussion. This example shows that, with public information about withdrawal decisions, no bank runs can occur as a PBE equilibrium outcome in the withdrawal game. Here, the key is that each depositor's withdrawal decision has a signaling aspect. In particular, in this example, depositor 1 can "notify" depositor 2 that he is not withdrawing, which he knows will affect depositor 2's decision in a favorable way.

However, the unique PBE result in this example relies on the fact that patient depositor 2's belief after history $h_2 = 1$ does not matter. This property is not true

[17] Since the last two depositors always report truthfully, $\Gamma_3(h_3, \omega_3) = \omega_3$ for all h_3, ω_3, and $\Gamma_4(h_4, \omega_4) = \omega_4$ for all h_4, ω_4.

in general. In particular, note that, in condition 1 of definition 1, Bayes' rule is only relevant for on-path histories to pin down depositor i's beliefs about previous $i-1$ depositors' types. It says nothing about depositor i's beliefs at off-path histories. In fact, PBE allows any belief for an off-path history. What if, at an off-path history, a depositor believes that a report of being patient is actually a lie made by the truly impatient type? The next example shows that this type of off-path belief can indeed rationalize a run PBE in the full-information withdrawal game.[18]

Example 2 (A run PBE): Consider a 6-depositor environment with the CRRA utility function in equation (8). Let $\rho = 0.7, \gamma = 6, R = 1.5$, and set

$$p(1) = \frac{1-\varepsilon}{100}, p(4) = \frac{4(1-\varepsilon)}{100}, p(5) = \frac{95(1-\varepsilon)}{100}; p(\theta) = \frac{\varepsilon}{4}, \text{ for } \theta = 0, 2, 3, 6, \quad (10)$$

where $\varepsilon = 0.0004\%$. Here, similar to the "one-peak" feature in example 1, the probabilities in (10) imply that, with very high probability, there will be 5 patient depositors in the economy. However, different from example 1, there are also nonnegligible probabilities that the economy has 1 or 4 patient depositors, which plays an important role in constructing a run PBE in this example. Figure 2 depicts the possible period-0 consumption levels in the first-best allocation rule for each depositor i if he is impatient.

There are at least two PBE in the full-information withdrawal game induced by the first-best allocation rule. First, it is straightforward to show that the truth-telling strategy is a PBE. Second, I claim that the following run strategy is also a PBE:

$$\Gamma_1(\emptyset, \omega_1) = 0 \text{ for all } \omega_1;\ \Gamma_2(0, \omega_2) = \omega_2, \Gamma_2(1, \omega_2) = 0 \text{ for all } \omega_2;$$

$$\Gamma_3(h_3, \omega_3) = \begin{cases} 0 & \text{if } h_3 = (1,1) \\ \omega_3 & \text{if } h_3 \neq (1,1) \end{cases} \text{ for all } \omega_3;$$

$$\Gamma_4(h_4, \omega_4) = \begin{cases} 0 & \text{if } h_4 = (1,1,1) \text{ or } (1,1,0) \\ \omega_4 & \text{o.w.} \end{cases} \text{ for all } \omega_4; \quad (11)$$

[18] Andolfatto et al. (2007) in their last section mention the possibility of constructing examples with a run PBE in the setup with correlated types. However, they do not provide such an example due to the complexity of solving the constrained optimization problem for the second-best allocation rule. Here, I circumvent their problem by focusing on the unconstrained optimization for the first-best allocation rule and adopt the numerical algorithm developed in Ennis and Keister (2009).

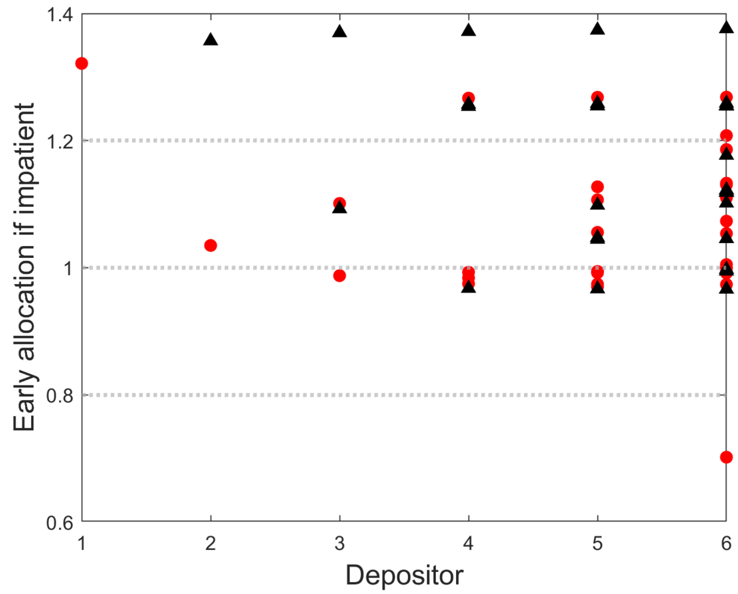

Figure 2: Period-0 allocation in the first-best allocation rule

$$\Gamma_5(h_5, \omega_5) = \omega_5 \text{ for all } h_5 \text{ and } \omega_5;\ \Gamma_6(h_6, \omega_6) = \omega_6 \text{ for all } h_6 \text{ and } \omega_6.$$

In words, patient depositor 1 chooses to misreport, and misreporting behavior by the remaining depositors only occur at some off-path histories. Therefore, the equilibrium outcome of this run PBE is that only the first depositor runs while the remaining depositors all report truthfully.

Next, I provide an informal discussion about why this run strategy is a PBE. Detailed computations are delegated to Appendix A. First, it is straightforward to show that the last two depositors always report truthfully. Then, let us focus on patient depositor 4's off-path history $h_4 = (1, 1, 1)$. Choose $\mu_4(\cdot|h_4 = (1, 1, 1), \omega_4 = 1)$ to be

$$\mu_4(\omega_{1:3} = (0, 0, 0)|h_4 = (1, 1, 1), \omega_4 = 1) = 1. \tag{12}$$

That is, patient depositor 4 believes that, when the first three depositors all report to be patient, these reports are all lies and he is the only patient depositor among the first four. With only two depositors left, for patient depositor 4, the maximal number of patient depositors in the economy can only be three. Note that, by the probabilities in (10), $p(2)$ and $p(3)$ are almost negligible compared to $p(1)$. Therefore, patient depositor 4 believes almost surely that he will be the only patient depositor in the economy and the remaining two depositors should both be impatient, which corresponds to the period-1 allocation $\mathbf{a}_4^1(1,1,1,1,0,0) = 1.27$. However, if patient depositor 4 reports to be impatient, he will receive a larger period-0 allocation $\mathbf{a}_4^0(1,1,1,0) = 1.37$. Hence, it is sequentially rational for patient depositor 4 to misreport at off-path history $h_4 = (1,1,1)$ under (12). The same argument applies to patient depositor 4's off-path history $h_4 = (1,1,0)$.

Then, at patient depositor 3's off-path history $h_3 = (1,1)$, pick $\mu_3(\cdot|h_3 = (1,1), \omega_3 = 1)$ to be

$$\mu_3(\omega_{1:2} = (0,0)|h_3 = (1,1), \omega_3 = 1) = 1. \tag{13}$$

Here, patient depositor 3 also believes that the report of being patient is a lie made by the impatient type, which makes him believe that he is the only patient depositor among the first three depositors. With three depositors left, for patient depositor 3, the maximal number of patient depositors in the economy can only be 4. Note that by the probabilities in (10), $p(2)$ and $p(3)$ are almost negligible compared to $p(1)$ and $p(4)$. Therefore, patient depositor 3 believes almost surely that there will be two possible scenarios for the remaining depositors: all of them are impatient or all of them are patient. Under such belief, as shown in Appendix A, it is sequentially rational for patient depositor 3 to misreport at off-path history $h_3 = (1,1)$.

Similarly, at patient depositor 2's off-path history $h_2 = 1$, let $\mu_2(\omega_1 = 0|h_2 = 1, \omega_2 = 1) = 1$. I show in Appendix A that it is sequentially rational for patient depositor 2 to misreport at $h_2 = 1$ given this belief. Lastly, anticipating misreporting behavior by the remaining depositors if he reports to be patient, patient depositor 1 also chooses to misreport. Therefore, the run strategy in (11) is indeed a PBE.

Discussion. Example 2 relies on depositors believing that the observed reports of "patient" are lies made by the impatient type. However, since by Remark 2 impatient depositors always report truthfully, depositors should never believe that the report of being patient is a lie made by the impatient type. In this sense, the beliefs μ_i I pick for off-path histories in example 2 are not reasonable, which raises the following question: Given any strategy profile σ, what is a reasonable way to pin down $\mu_i(\cdot|h_i, \omega_i)$ for an off-path history h_i? The next section provides one answer to this question by introducing a refinement of PBE called forward induction equilibrium.

5 Main results

In this section, I first introduce the formal definition of forward induction equilibrium and explain why the run PBE in example 2 fails to be a forward induction equilibrium. Then I state the unique forward induction equilibrium result.

5.1 Forward induction equilibrium

I adapt the general definition of forward induction equilibrium in Cho (1987) to the full-information withdrawal game. First, I introduce the definition of *introspectively consistent belief* and show that the off-path beliefs I pick in example 2 are not introspectively consistent.

Given any mixed strategy profile σ, I say r_i is a ***bad deviation*** from σ for depositor i at information set (h_i, ω_i) if the following are true:

$$\sigma_i(r_i|h_i, \omega_i) = 0 \text{ and } \max_{\tilde{\sigma}_{i+1:I}} U_i^{\mu_i}(r_i, \tilde{\sigma}_{i+1:I}|h_i, \omega_i) < U_i^{\mu_i}(\sigma_i, \sigma_{i+1:I}|h_i, \omega_i),$$

for any $\mu_i(\cdot|h_i, \omega_i)$. In other words, r_i is a bad deviation from σ at (h_i, ω_i) if, first, r_i is indeed a deviation and, second, the best possible expected payoff of choosing r_i is less than the status quo payoff, i.e., the expected payoff of following the given strategy σ, under whatever belief depositor i has about previous depositors' types.

For example, consider the bank run PBE Γ in example 2. Choose any impatient depositor i, i.e., $\omega_i = 0$, and any history h_i. First note that $r_i = 1$ is a deviation from Γ at information set $(h_i, \omega_i = 0)$. Then, by the preferences in (1) and the CRRA utility function in equation (8), $\max_{\tilde{\sigma}^I_{i+1}} U_i^{\mu_i}(r_i = 1, \tilde{\sigma}_{i+1:I}|h_i, \omega_i) = -\infty$ for any $\mu_i(\cdot|h_i, \omega_i)$. As a consequence, $r_i = 1$ is a bad deviation from Γ at information set $(h_i, \omega_i = 0)$. In other words, the report of being patient is a bad deviation for impatient depositor i from the given run PBE. However, it is important to note that the same argument does not apply to the report of being impatient for patient depositor i. This is due to the fact that, for patient depositor i, there may be beliefs under which a report of being impatient yields a higher expected payoff than the status quo payoff.

Next, given any strategy profile σ and any history $h_i = (r_1, \cdots, r_{i-1})$, define

$B(h_i|\sigma) = \{\omega_{1:i-1} \in \Omega^{i-1}|$ there exists a $j \in \{1, \cdots, i-1\}$ such that r_j is a bad deviation from σ for depositor j at information set $(h_j = (r_1, \cdots, r_{j-1}), \omega_j)\}$.

In words, $B(h_i|\sigma)$ is the set of all previous $i-1$ depositors' type profiles such that h_i contains at least one bad deviation made by the first $i-1$ depositors from the given strategy profile σ. Again, consider the run strategy PBE Γ in example 2 and history $h_4 = (1, 1, 1)$. Then, $\omega_{1:3} = (0, 0, 0)$ is in $B(h_4 = (1, 1, 1)|\Gamma)$ since there exists a $j = 1$ such that $r_1 = 1$ is a bad deviation from Γ for depositor 1 at information set $(h_1 = \emptyset, \omega_1 = 0)$. Also, $\omega_{1:3} = (1, 1, 0)$ is in $B(h_4 = (1, 1, 1)|\Gamma)$ since there exists a $j = 3$ such that $r_3 = 1$ is a bad deviation from Γ for depositor 3 at information set $(h_3 = (1, 1), \omega_3 = 0)$. In fact, the only $\omega_{1:3}$ that is not in $B(h_4 = (1, 1, 1)|\Gamma)$ is $\omega_{1:3} = (1, 1, 1)$.

To complete the definition of introspectively consistent belief, I need one more piece of notation. Let Π^0 be the set of all completely mixed strategies that assign positive probabilities to all reports at all histories, and define the set

$$\Psi^0 = \{(\sigma, \mu)|\sigma \in \Pi^0, \mu \text{ is defined via Bayes' rule in (4)}\}.$$

Definition 2. *A system of beliefs μ is* ***introspectively consistent*** *with respect to a given strategy profile σ if there exists a sequence $\{(\sigma^n, \mu^n) \in \Psi^0\}_{n=1}^{\infty}$ satisfying*

$$\mu_i^n(B(h_i|\sigma)|h_i, \omega_i) \to 0 \text{ as } n \to \infty, \tag{14}$$

for any i, h_i, ω_i, and

$$\lim_{n\to\infty} (\sigma^n, \mu^n) = (\sigma, \mu). \tag{15}$$

Given a strategy profile σ, an introspectively consistent belief μ_i puts zero probability over any $\omega_{1:i-1}$ that involves bad deviations and is the limit of a sequence of beliefs for completely mixed strategies that converge to σ.[19] Consider the run PBE Γ in example 2 discussed above. Since only $\omega_{1:3} = (1, 1, 1)$ is not in $B(h_4 = (1, 1, 1)|\Gamma)$, it follows from (14) that, at the off-path history $h_4 = (1, 1, 1)$, an introspectively consistent belief with respect to Γ should put probability one over $\omega_{1:3} = (1, 1, 1)$. That is, depositor 4 should always believe that all previous reports of being patient

[19] In an earlier version of this paper, I constructed examples showing that the introduction of a sequence of completely mixed strategies is also necessary to exclude unreasonable off-path beliefs.

are made by the truly patient type. Therefore, the off-path beliefs I pick in example 2 are not introspectively consistent.

Now, I am ready to state the definition of forward induction equilibrium.

Definition 3. *A strategy profile σ is a* ***forward induction equilibrium (FIE)*** *if there exists an introspectively consistent belief μ with respect to σ such that, for any i, any ω_i and any history h_i, $\sigma_i(\cdot|h_i, \omega_i)$ is sequentially rational under $\mu_i(\cdot|h_i, \omega_i)$.*

In other words, a strategy profile σ **fails to be** a forward induction equilibrium if, for any introspectively consistent belief μ with respect to σ, σ is not sequentially rational, i.e., there exists an information set (h_i, ω_i) of depositor i in which $\sigma_i(\cdot|h_i, \omega_i)$ is not sequentially rational under $\mu_i(\cdot|h_i, \omega_i)$. The run PBE Γ in example 2 fails to be a FIE because, when we restrict to introspectively consistent beliefs, the strategy profile is no longer sequentially rational at some information sets that consist of off-path histories.

5.2 Unique forward induction equilibrium

Using forward induction equilibrium as the equilibrium concept, this section identifies conditions on the allocation rule under which the truth-telling strategy is the unique FIE in its induced full-information withdrawal game. It turns out that the key condition is a notion of strict incentive compatibility.

First, I say that an allocation rule α is ***incentive compatible (IC)*** if and only if the truth-telling strategy is a FIE in the full-information withdrawal game induced by α. The following proposition provides a further characterization of incentive compatible allocation rules.

Proposition 1. *An allocation rule α is IC if and only if the following are true: For any i and $\omega_{1:i-1} \in \Omega^{i-1}$,*

$$v(\alpha_i^0(\omega_{1:i-1}, 0)) \leq \sum_{\omega_{i+1:I}} v(\alpha_i^1(\omega_{1:i-1}, 1, \omega_{i+1:I}))\mathcal{P}(\omega_{i+1:I}|\omega_{1:i-1}, 1). \tag{16}$$

The formal proof is in Appendix D. In words, (16) formalizes the idea that a patient depositor should report his type truthfully if, first, he believes that all previous reports are indeed truthful, and, second, all remaining depositors will report truthfully.

Next, I present a stronger notion of incentive compatibility.

Definition 4. *An allocation rule α is **strictly incentive compatible (SIC)** if and only if, for any i and $\omega_{1:i-1} \in \Omega^{i-1}$,*

$$v(\alpha_i^0(\omega_{1:i-1}, 0)) < \sum_{\omega_{i+1:I}} v(\alpha_i^1(\omega_{1:i-1}, 1, \omega_{i+1:I}))\mathcal{P}(\omega_{i+1:I}|\omega_{1:i-1}, 1). \tag{17}$$

Here, the only difference between inequality (17) and inequality (16) is that (17) has a strict inequality. Note that, when types are independent, i.e., $\mathcal{P}(\omega_{i+1:I}|\omega_{1:i-1}, 1) = \mathcal{P}(\omega_{i+1:I})$, (17) reduces to the following: for any i and $\omega_{1:i-1}$,

$$v(\alpha_i^0(\hat{\omega}_{1:i-1}, 0)) < \sum_{\omega_{i+1:I}} v(\alpha_i^1(\omega_{1:i-1}, 1, \omega_{i+1:I}))\mathcal{P}(\omega_{i+1:I}), \tag{18}$$

which is exactly the inequality in Lemma 5 of Green and Lin (2003). Similarly, as explained by Green and Lin, (18) formalizes the idea that a patient depositor should strictly prefer to report truthfully if all remaining depositors are going to report truthfully. Therefore, based on (18), Green and Lin use an argument similar to *backward induction reasoning* to prove their unique equilibrium result.

When types are correlated, the issue becomes more complicated. In particular, as illustrated in example 2, if a patient depositor believes someone earlier in the sequence has lied about their type, he might choose to lie as well despite the fact that all remaining depositors will report truthfully. This is where *forward induction reasoning* kicks in. As shown in the following theorem, forward induction reasoning could correctly align a patient depositor's (off-path) belief in a way that makes lying an inferior choice for him.

Theorem 1. *For any allocation rule α that is SIC, the truth-telling strategy is the unique FIE in the full-information withdrawal game induced by α.*

In other words, when an allocation rule is SIC, there always exists a unique FIE in its induced full-information withdrawal game, and bank runs do not occur. The proof, which is presented in Appendix B, adopts an iterative deletion process that combines elements of backward induction and forward induction. To illustrate this deletion process, let us consider the simplest nontrivial case, where $I = 3$. To show the truth-telling strategy is the unique FIE, I need to show that any strategy other than the truth-telling strategy fails to be a FIE.

First, since by Remark 2 impatient depositors never misreport, I can focus on all run strategies.[20] Denote the set of all run strategies as D^0. Note for depositor 3, since he is the last depositor, it follows from (17) that patient depositor 3 always chooses to report truthfully at any history h_3. Then any run strategy in D^0 that has patient depositor 3 misreporting fails to be a FIE. Deleting those run strategies leaves us a subset of run strategies in which misreporting only happens between the first two depositors. Denote this subset of D^0 as D^1.

Going backward, next consider patient depositor 2 at history $h_2 = 1$. A key observation is that, given any run strategy in D^1, patient depositor 2's introspectively consistent belief at information set $(h_2 = 1, \omega_2 = 1)$ is unique and given by $\mu_2(\omega_1 = 1|h_2 = 1, \omega_2 = 1) = 1$ (Lemma 1 in Appendix B). Under such a belief, patient depositor 2 believes that depositor 1's report of being patient is a truthful report. Since for any run strategy in D^1 depositor 3 reports truthfully, it follows from (17) that it is sequentially rational for patient depositor 2 to report truthfully at history $h_2 = 1$ (Lemma 3 in Appendix B). Therefore, any run strategy in D^1 that has patient depositor 2 misreporting at history $h_2 = 1$ fails to be a FIE. Deleting those run strategies leaves us with a subset of run strategies in which the only possible misreporting is by patient depositor 1 or by patient depositor 2 at history $h_2 = 0$. Denote this subset of D^1 as D^2.

Next, given any run strategy in D^2, since all remaining depositors report truthfully after $r_1 = 1$, it follows from (17) that misreporting by patient depositor 1 is not sequentially rational. Therefore, any run strategy in D^2 that involves patient depositor 1 misreporting fails to be a FIE. Deleting those run strategies leaves us with a subset of run strategies in which only patient depositor 2 misreports at history $h_2 = 0$. Denote this subset of D^2 as D^3.

Lastly, note that D^3 only has run strategies in which misreporting occurs on

[20] Recall that, in the full-information withdrawal game, a run strategy is a behavior strategy profile in which some patient depositors choose to misreport with strictly positive probability at some of their information sets.

patient depositor 2 at history $h_2 = 0$. Note that, given any run strategy in D^3, patient depositor 2's introspectively consistent belief at $h_2 = 0$ can be uniquely pinned down by Bayes' rule and is given by $\mu_2(\omega_1 = 0|h_2 = 0, \omega_2 = 1) = 1$ (Lemma 2 in Appendix B). In other words, since depositor 1 would have reported truthfully if he were patient, his report of being patient must also be truthful. Since depositor 3 will report truthfully, it follows from (17) that it is sequentially rational for patient depositor 2 to report truthfully at history $h_2 = 0$. Therefore, any run strategy in D^3 also fails to be a FIE. Then deleting those run strategies leaves us with an empty set, meaning that no run strategy can be a FIE. This completes the iterative deletion process.[21]

Remark 3. As illustrated above, the iterative deletion process starts with the last depositor, which resembles backward induction reasoning. Then forward induction reasoning is used to pin down beliefs. In particular, at the penultimate depositor's history $h_{I-1} = (1, \cdots, 1)$, forward induction reasoning allows us to directly pin down the unique introspectively consistent belief with respect to any run strategy.

Remark 4. The reason why SIC is needed is to break ties when a depositor is indifferent between lying and telling the truth. An alternative approach that leads to the same outcome is to use (weak) incentive compatibility and assume, as in Andolfatto et al. (2007), that an indifferent depositor always reports truthfully.

Remark 5. In example 2, it is straightforward to show that the first-best allocation rule $\mathbf{a}$ is SIC. Therefore, by Theorem 1, the truth-telling strategy is not only a FIE but the unique FIE in the full-information withdrawal game induced by $\mathbf{a}$. In other words, under the parameter values in example 2, the first-best allocation rule is uniquely implementable as a FIE outcome, and no bank runs occur.

6 Discussion

In this section, I discuss two issues related to the main result. First, I study under what conditions the "efficient" allocation satisfies SIC and, hence, can be implemented

[21]When $I = 3$, since the last depositor always reports truthfully, a full-information withdrawal game boils down to a signaling game between the first two depositors, and FIE reduces to the "intuitive criterion" studied in Cho and Kreps (1987). Andolfatto et al. (2007), in their last section, offer similar informal arguments using the "intuitive criterion" with $I = 3$ when types are correlated.

as the unique FIE. Second, I ask whether the use of indirect mechanisms can weaken the SIC condition in the main result.

6.1 Efficiency

First, consider a benevolent planner/intermediary who wants to maximize the following weighted welfare function:

$$\mathbb{E}_\omega[\sum_{i\in\mathbf{I}} W_i u_i(\alpha_i^0(\omega_{1:i}), \alpha_i^1(\omega), \omega_i)],$$

where $W_i \in \mathbb{R}_{++}$ is depositor i's Pareto weight, subject to the following incentive constraints:

$$\sum_{\omega_{i+1:I}} v(\alpha_i^1(\omega_{1:i-1}, 1, \omega_{i+1:I}))\mathcal{P}(\omega_{i+1:I}|\omega_{1:i-1}, 1) - v(\alpha_i^0(\omega_{1:i-1}, 0)) \geq \delta \text{ for all } i \text{ and } \omega_{1:i-1},$$

where $\delta > 0$.[22] Let $W = (W_1, W_2, \cdots, W_N)$ denote the vector of depositors' Pareto weights and $\alpha^*(W, \delta) = (\alpha_0^*(W, \delta), \alpha_1^*(W, \delta))$ the solution to the above maximization problem. Then, for any (W, δ) that admits a solution to the above maximization problem, it is straightforward to see that $\alpha^*(W, \delta)$ is SIC since it must satisfy inequality (17). As a result, it follows from Theorem 1 that, for any $\alpha^*(W, \delta)$, the truth-telling strategy is the unique FIE in the full-information withdrawal game induced by $\alpha^*(W, \delta)$. An important observation based on this result is that the constrained efficient allocation, i.e., the solution to the above maximization problem with equal Pareto weights and $\delta = 0$, can be approximately implemented. That is, for any $\varepsilon > 0$, there exists a uniquely implementable allocation $\alpha^*(\delta)$ with δ close to zero such that $\alpha^*(\delta)$ is within ε of the constrained efficient allocation .[23]

Next, I focus on the first-best allocation rule, the solution to the unconstrained

[22]With equal Pareto weights for all depositors, Andolfatto et al. (2017) study a similar constrained welfare maximization problem. In their environment, since each depositor has no extra information besides their private type, the welfare maximization problem only has one incentive constraint parametrized by δ. As a result, the maximum welfare in their problem can be higher than the problem studied in this paper. See Sultanum (2022) for a concrete example illustrating this point.

[23]This result is a direct consequence of applying Berge's maximum theorem to the constrained welfare maximization problem with equal Pareto weights and $\delta \geq 0$ being the parameter. One can state a similar result for the maximum welfare as well.

welfare maximization problem (5) with equal Pareto weights for all depositors, and study the following question: Under what conditions on the underlying environment is the first-best allocation rule SIC, i.e., satisfies inequality (17), and therefore implementable as the unique FIE? When types are independent, Green and Lin (2003) show that, under Assumption 1 and the following assumption, the first-best allocation rule satisfies inequality (18), the independent-type version of inequality (17).

Assumption 2. *Absolute risk aversion of v is non-increasing everywhere, i.e., for any $c \in \mathbb{R}_+$,*

$$\frac{d}{dc}\frac{v"(c)}{v'(c)} \geq 0.$$

When types are correlated, consider the benevolent intermediary's optimization problem (5). Suppose that it is depositor i's turn to approach the intermediary and the first $i-1$ depositors' types are $\hat{\omega}_{1:i-1}$. Since types are correlated, for the intermediary's problem, the probability distribution over the remaining depositors' types is *dependent* on depositor i's type. In particular, if depositor i is of the impatient type, the probability distribution over the remaining depositors' types is $\mathcal{P}(\cdot|\hat{\omega}_{1:i-1}, 0) \in \Delta^{I-i}$. If depositor i is of the patient type, the probability distribution over the remaining depositors' types is $\mathcal{P}(\cdot|\hat{\omega}_{1:i-1}, 1) \in \Delta^{I-i}$. Note that, if $\mathcal{P}(\cdot|\hat{\omega}_{1:i-1}, 0) = \mathcal{P}(\cdot|\hat{\omega}_{1:i-1}, 1)$, the problem is exactly the same as in Green and Lin. However, when $\mathcal{P}(\cdot|\hat{\omega}_{1:i-1}, 0) \neq \mathcal{P}(\cdot|\hat{\omega}_{1:i-1}, 1)$, the problem is more complicated, and some sort of order needs to be established on $\mathcal{P}(\cdot|\hat{\omega}_{1:i-1}, 0)$ and $\mathcal{P}(\cdot|\hat{\omega}_{1:i-1}, 1)$ for inequality (17) to hold. The following assumption provides a sufficient condition.

Assumption 3. *The function $p : \{0, 1, \cdots, I\} \to [0, 1]$ satisfies a monotonicity condition, i.e., for any $k \in \{0, 1, \cdots, I-2\}$,*

$$\frac{p(k)/C(I,k)}{p(k+1)/C(I,k+1)} \geq \frac{p(k+1)/C(I,k+1)}{p(k+2)/C(I,k+2)}. \tag{19}$$

Or equivalently,

$$\frac{\mathcal{P}(\omega : \theta(\omega) = k)}{\mathcal{P}(\omega : \theta(\omega) = k+1)} \geq \frac{\mathcal{P}(\omega : \theta(\omega) = k+1)}{\mathcal{P}(\omega : \theta(\omega) = k+2)}.$$

To better understand this assumption, let us consider two examples. First, consider the case with independent types as in Green and Lin (2003). Let the probability of being impatient be π. Then the total number of patient depositors in the economy follows the Binomial distribution, and its probability mass function is given by $p(k) = C(I,k)(1-\pi)^k \pi^{I-k}$ for all $k \in \{0, 1, \cdots, I\}$. It is straightforward to check that Assumption 3 is satisfied with equality in this case.

Second, consider a special class of correlated types. Suppose that there is a finite number of unobserved states in the economy indexed by $\kappa_j \in K = \{\kappa_1, \cdots, \kappa_m\}$, and nature chooses state κ_j with probability p_j. Conditional on the event that the state is κ_j, each depositor's type is independently distributed, and the probability of being impatient is π_j. Here, depositors' types are correlated because of those unobserved states.[24] In this example, the probability mass function of the total number of patient depositors in the economy is given by $p(k) = \sum_{j=1}^{m} C(I,k)(1-\pi_j)^k \pi_j^{I-k} p_j$, for all $k \in \{0, 1, \cdots, I\}$. The following proposition shows that this $p(\cdot)$ also satisfies Assumption 3. The formal proof is delegated to Appendix D.

Proposition 2. *Suppose that* $p(k) = \sum_{j=1}^{m} C(I,k)(1-\pi_j)^k \pi_j^{I-k} p_j$, *for all* $k \in \{0, 1, \cdots, I\}$. *Then Assumption 3 holds.*

As shown by Lemma 4 in Appendix C, the monotonicity condition in Assumption 3 yields a property that, if depositor i turns out to be patient, the remaining depositors are more likely to be patient. As a consequence, a benevolent intermediary will allocate more goods to patient depositors in period 1. Therefore, this property together with the proof technique in Green and Lin's Lemma 5 allow me to prove the following proposition.

Proposition 3. *Suppose that Assumption 1,2 and 3 hold. Then the first-best allocation rule is SIC.*

The formal proof of Proposition 3 is in Appendix C. The next theorem is a direct consequence of Proposition 3 and Theorem 1.

Theorem 2. *Suppose that Assumption 1,2 and 3 hold. The truth-telling strategy is the unique FIE in the full-information withdrawal game induced by the first-best allocation rule.*

[24] If there is only one state in the economy, i.e., $K = \{\kappa\}$, this example boils down to the case with independent types.

Therefore, when types are correlated, Assumption 3 is the extra sufficient condition for the first-best allocation rule to be implementable as the unique FIE.[25]

6.2 Indirect mechanisms

This paper so far focuses on direct mechanisms $\{M, \alpha\}$ with full information disclosure where each depositor's message space is their type space, i.e., $M = \Omega$, and the allocation rule is $\alpha \in \mathbb{F}^s$. Theorem 1 shows that the direct mechanism $\{M, \alpha\}$ with full information disclosure admits a unique FIE whenever the allocation rule α is SIC. What if one considers indirect mechanisms with full information disclosure?[26] Can the sufficient condition for the unique FIE result be weakened to require only (weak) IC? Next, I want to use the indirect mechanism studied in Andolfatto et al. (2017) to illustrate how indirect mechanisms also require the allocation rule to be SIC.

Following Andolfatto et al. (2017), I construct an indirect mechanism $\{\hat{M}, \hat{\alpha}\}$ based on the direct mechanism $\{M, \alpha\}$ studied in the paper. In particular, let $\hat{M} = M \cup \{seg\}$, i.e., each depositor has one more possible choice of message, and construct $\hat{\alpha}$ as follows. If depositor i reports $r_i = 0$, then

$$\hat{\alpha}_i^0(r_{1:i-1}, 0) = \begin{cases} \alpha_i^0(r_{1:i-1}, 0) & \text{if } r_j \in \{0, 1\} \text{ for all } j < i \\ 0 & \text{if } r_j = seg \text{ for some } j < i, \end{cases}$$

and

$$\hat{\alpha}_i^1(r_{1:i-1}, 0, r_{i+1:I}) = 0.$$

If depositor i reports $r_i = seg$, then

$$\hat{\alpha}_i^0(r_{1:i-1}, seg) = 0 \quad \text{and} \quad \hat{\alpha}_i^1(r_{1:i-1}, seg, r_{i+1:I}) = \hat{\alpha}_i^0(r_{1:i-1}, 0) + \varepsilon,$$

where $\varepsilon > 0$ is an arbitrarily small number. Lastly, if depositor i reports $r_i = 1$, then

[25] Assumption 3 is only a sufficient condition for the first-best allocation rule to be implementable as the unique FIE. In example 2, Assumption 3 is violated but the first-best allocation rule is still SIC and, therefore, uniquely implementable.

[26] I would like to thank one referee for pointing out this direction.

$\hat{\alpha}_i^0(r_{1:i-1}, 1) = 0$ and

$$\hat{\alpha}_i^1(r_{1:i-1}, 1, r_{i+1:I}) = \frac{R[I - \sum_{j=1}^{I} \hat{\alpha}_j^0(r_{1:j})] - \sum_{j=1}^{I} \hat{\alpha}_j^1(r_{1:i-1}, 1, r_{i+1:I}) \mathbb{1}_{\{r_j = seg\}}}{\theta(r_{1:i-1}, 1, r_{i+1:I})},$$

where $\theta(r_{1:I})$ denotes the number of depositors who reported $r = 1$ in $r_{1:I}$.

On one hand, this indirect mechanism has the feature that it is a strictly dominated action for a patient depositor to report 0, since reporting seg always gives a strictly higher payoff. Therefore, with forward induction, if a depositor observed a report $r = 0$, he should believe with probability 1 that such a report is made by a truly impatient depositor. In addition, since impatient depositors never report 1 or seg, if a depositor observed a report $r \in \{1, seg\}$, he should believe with probability 1 that such a report is made by a truly patient depositor. Therefore, in the full-information withdrawal game induced by this indirect mechanism, beliefs are correctly aligned at all information sets under forward induction.

On the other hand, in the full-information withdrawal game induced by this indirect mechanism, there are more possible previous report histories for each depositor i. Let H_i^d denote the set of report histories made by the first $i-1$ depositors that are the same as in the full-information withdrawal game induced by the direct mechanism. For any $h_i \in H_i^d$, since beliefs are correctly aligned under forward induction, we can compare patient depositor i's payoff of reporting seg and 1:

$$\begin{aligned} \textbf{Report } \boldsymbol{seg}\textbf{:} \quad & \rho v(\hat{\alpha}_i^0(h_i, seg)) = \rho v(\alpha_i^0(h_i, 0) + \varepsilon); \qquad (20) \\ \textbf{Report 1:} \quad & \mathbb{E}_{\omega_{i+1:I}}[\rho v(\alpha_i^1(\omega_{1:i-1}, 1, \omega_{i+1:I})) | \omega_{1:i-1} = h_i, \omega_i = 1]. \end{aligned}$$

To make reporting 1 sequentially rational for patient depositor i, I need the strict inequality in SIC because of the extra ε term in (20). If SIC was weakened to IC and IC was satisfied with equality at h_i, reporting seg would be more profitable than reporting 1 no matter how small ε is. In this case, truth-telling is no longer sequentially rational for patient depositor i after history h_i, which results in a bank run FIE.

Therefore, SIC cannot be weakened to IC for the full-information withdrawal game induced by the indirect mechanism studied above to admit a unique FIE. Here, the key observation is that SIC is necessary to show truth-telling at information sets that coincide with the full-information withdrawal game induced by the direct mechanism. This observation applies to other indirect mechanisms as well. Furthermore, another interesting observation is that the unique FIE result relies on SIC more strongly under the indirect mechanism studied above. Recall that, in Theorem 1, assuming SIC avoids the discussion on tie-breaking rules for patient depositors. One can instead assume IC and whenever a patient depositor is indifferent he always reports truthfully. Then the unique FIE result in Theorem 1 still holds. However, under the indirect mechanism studied above, even if one assumes the same tie-breaking rule, IC is not enough for the unique FIE result because of the extra ε term in (20). Therefore, in this sense, instead of weakening the sufficient condition in Theorem 1, using indirect mechanisms such as the one studied above can in fact tighten it.

7 Conclusion

This paper shows that, with forward induction, sophisticated banking contracts can implement efficient allocations while preventing bank runs when there is public information about withdrawal decisions. Therefore, using smart contracts on public blockchains is a promising way to provide more stable and efficient intermediary services that involve maturity transformation. One place where smart contracts seem particularly likely to be useful is for asset-backed stablecoins such as Tether and USDC, which promise a stable conversion rate between a crypto currency and a fiat currency.[27] When there is aggregate uncertainty, it is not feasible to keep the value of an intermediary's liabilities fixed in all states of the world. Stablecoins nevertheless make this type of "promise" and leave it unspecified how they will meet the obligation in some states. This type of setup seems prone to runs especially during the time of broader market stress.[28] This paper suggests that runs can be prevented by instead taking advantage of smart contracts to commit to a sophisticated payment schedule.

[27]The recent collapse of the third largest stablecoin Terra in May 2022 brought (algorithmic) stablecoins into the spotlight and raised general concerns for the potential instability of stablecoins among market participants and regulators. For example, see https://www.ft.com/content/48d82c7a-495f-4d5e-a87a-a56bea58e760.

[28]After the collapse of Terra, the largest stablecoin Tether depegged from $1 and tumbled as low as $0.94. See https://www.cnbc.com/2022/05/12/tether-usdt-stablecoin-drops-below-1-peg.html.

This approach would deliver a "run-proof coin", which might be more desirable than a so-called stablecoin.

The emergence of distributed ledger system and its accompanied smart-contract technology open up more possibilities to practically implement sophisticated contracts that have only existed in theory in the past (Townsend, 2020). However, as pointed out in Sultanum (2022), a downside of information transparency inherent in many distributed ledger systems is that it tends to tighten incentive compatibility constraints and shrink the set of incentive compatible allocations. This paper instead highlights a positive side: Within the set of (strictly) incentive compatible allocations, information transparency and forward induction imply that sophisticated contracts can eliminate bank runs. Whether and how information transparency affects financial stability in other settings is an interesting area for future research.

Appendices

A Omitted computations in example 2

Example 2 (Continued): At patient depositor 4's off-path history $h_4 = (1,1,0)$, pick $\mu_4(\cdot|h_4 = (1,1,0), \omega_4 = 1)$ to be

$$\mu_4(\omega_{1:3} = (0,0,0)|h_4 = (1,1,0), \omega_4 = 1) = 1. \tag{21}$$

That is, patient depositor 4, after observing history $h_4 = (1,1,0)$, believes that the report of being patient is a lie made by the impatient type. Since ε is almost negligible, patient depositor 4's induced probability distribution over the remaining depositors' types is

$$\phi_4^{\mu_4}(\omega_{5:6} = (0,0)|h_4 = (1,1,0), \omega_4 = 1) = \mathcal{P}(\omega_{5:6} = (0,0)|\omega_{1:3} = (0,0,0), \omega_4 = 1) \approx 1.$$

Compare patient depositor 4's expected payoff of reporting 0 and 1 after $h_4 = (1,1,0)$:

Report 0: $U_4^{\mu_4}(r_4 = 0, \Gamma_{5:6}|h_4 = (1,1,0), \omega_4 = 1) = \rho v(\mathbf{a}_4^0(1,1,0,0)) = \rho v(1.26)$;

Report 1: $U_4^{\mu_4}(r_4 = 1, \Gamma_{5:6}|h_4 = (1,1,0), \omega_4 = 1) \approx \rho v(\mathbf{a}_4^1(1,1,0,1,0,0)) = \rho v(1.13) < \rho v(1.26)$.

Therefore, $\Gamma_4((1,1,0),1) = 0$ is sequentially rational under belief (21).

At patient depositor 3's off-path history $h_3 = (1,1)$, choose $\mu_3(\cdot|h_3 = (1,1), \omega_3 = 1)$ to be

$$\mu_3(\omega_{1:2} = (0,0)|h_3 = (1,1), \omega_3 = 1) = 1. \tag{22}$$

That is, patient depositor 3 believes that the report of being patient is a lie made by the impatient type. Since ε is almost negligible, patient depositor 3's induced probability distribution over the remaining depositors' types is

$$\begin{aligned}
\phi_3^{\mu_3}(\omega_{4:6} = (0,0,0)|h_3 = (1,1), \omega_3 = 1) &= \mathcal{P}(\omega_{4:6} = (0,0,0)|\omega_{1:2} = (0,0), \omega_3 = 1) \\
&\approx \frac{\mathcal{P}(0,0,1,0,0,0)}{\mathcal{P}(0,0,1,0,0,0) + \mathcal{P}(0,0,1,1,1,1)} \\
&= \frac{\frac{1}{6}p(1)}{\frac{1}{6}p(1) + \frac{1}{15}p(4)} = 0.38; \\
\phi_3^{\mu_3}(\omega_{4:6} = (1,1,1)|h_3 = (1,1), \omega_3 = 1) &= \mathcal{P}(\omega_{4:6} = (1,1,1)|\omega_{1:2} = (0,0), \omega_3 = 1) \\
&\approx \frac{\mathcal{P}(0,0,1,1,1,1)}{\mathcal{P}(0,0,1,0,0,0) + \mathcal{P}(0,0,1,1,1,1)} \\
&= \frac{\frac{1}{15}p(4)}{\frac{1}{6}p(1) + \frac{1}{15}p(4)} = 0.62.
\end{aligned}$$

Compare patient depositor 3's expected payoff of reporting 0 and 1 after $h_3 = (1,1)$:

Report 0: $U_3^{\mu_3}(r_3 = 0, \Gamma_{4:6}|h_3 = (1,1), \omega_3 = 1) = \rho v(\mathbf{a}_3^0(1,1,0)) = \rho v(1.37) = -0.03$;

Report 1: $U_3^{\mu_3}(r_3 = 1, \Gamma_{4:6}|h_3 = (1,1), \omega_3 = 1)$

$$\begin{aligned}
&\approx 0.38\rho v(\mathbf{a}_3^1(1,1,1,0,0,0)) + 0.62\rho \underbrace{v(\mathbf{a}_3^1(1,1,1,0,1,1))}_{\Gamma_4((1,1,1),1)=0} \\
&= 0.38\rho v(1.13) + 0.62\rho v(1.39) = -0.05 < -0.03.
\end{aligned}$$

Therefore, $\Gamma_3((1,1),1) = 0$ is sequentially rational under belief (22).

At patient depositor 2's off-path history $h_2 = 1$, choose $\mu_2(\cdot|h_2 = 1, \omega_2 = 1)$ to be

$$\mu_2(\omega_1 = 0|h_2 = 1, \omega_2 = 1) = 1. \tag{23}$$

Following similar calculations, patient depositor 2's induced probability distribution over the remaining depositors' types is

$$\phi_2^{\mu_2}(\omega_{3:6} = (0,0,0,0)|h_2 = 1, \omega_2 = 1) = \mathcal{P}(\omega_{3:6} = (0,0,0,0)|\omega_1 = 0, \omega_2 = 1) \approx 0.01;$$
$$\phi_2^{\mu_2}(\omega_{3:6}|h_2 = 1, \omega_2 = 1) = \mathcal{P}(\omega_{3:6}|\omega_1 = 0, \omega_2 = 1) \approx 0.015, \text{ for any } \omega_{3:6} \text{ such that } \theta(\omega_{3:6}) = 3;$$
$$\phi_2^{\mu_2}(\omega_{3:6} = (1,1,1,1)|h_2 = 1, \omega_2 = 1) = \mathcal{P}(\omega_{3:6} = (1,1,1,1)|\omega_1 = 0, \omega_2 = 1) \approx 0.93.$$

Compare patient depositor 2's expected payoff of reporting 0 and 1 after $h_2 = 1$:

Report 0: $U_2^{\mu_2}(r_2 = 0, \Gamma_{3:6}|h_2 = 1, \omega_2 = 1) = \rho v(\mathbf{a}_2^0(1,0)) = \rho v(1.36);$

Report 1: $U_2^{\mu_2}(r_2 = 1, \Gamma_{3:6}|h_2 = 1, \omega_2 = 1)$

$$\begin{aligned}
&\approx 0.01\rho v(\mathbf{a}_2^1(1,1,0,0,0,0)) + \underbrace{0.015\rho v(\mathbf{a}_2^1(1,1,0,0,1,0))}_{\Gamma_3((1,1),1)=0\&\Gamma_4((1,1,0),1)=0} + \underbrace{0.015\rho v(\mathbf{a}_2^1(1,1,0,0,0,1))}_{\Gamma_3((1,1),1)=0\&\Gamma_4((1,1,0),1)=0} \\
&+ \underbrace{0.015\rho v(\mathbf{a}_2^1(1,1,0,0,1,1))}_{\Gamma_3((1,1),1)=0\&\Gamma_4((1,1,0),1)=0} + \underbrace{0.015\rho v(\mathbf{a}_2^1(1,1,0,0,1,1))}_{\Gamma_3((1,1),1)=0\&\Gamma_4((1,1,0),1)=0} + \underbrace{0.93\rho v(\mathbf{a}_2^1(1,1,0,0,1,1))}_{\Gamma_3((1,1),1)=0\&\Gamma_4((1,1,0),1)=0} \\
&= 0.01\rho v(1.01) + 0.015\rho v(1.13) + 0.015\rho v(1.17) + 0.96\rho v(1.27) \\
&< \rho v(1.36).
\end{aligned}$$

Therefore, $\Gamma_2(1,1) = 0$ is sequentially rational under belief (23).

Lastly, for patient depositor 1, note that, for any $\omega_{2:6}$ such that $\theta(\omega_{2:6}) \leq 3$, $\mathbf{a}_1^1(1, \omega_{2:6}) < \mathbf{a}_1^0(0)$. Therefore, I will focus on the last nonnegligible case where $\theta(\omega_{2:6}) = 4$. There are $C(5,4) = 5$ possible realizations for $\theta(\omega_{2:6}) = 4$, and all have the same probability. Then compare patient depositor 1's expected payoff of reporting 0 and 1:

Report 0: $U_1(r_1 = 0, \Gamma_{2:6}|h_1 = \emptyset, \omega_1 = 1) = \rho v(\mathbf{a}_1^0(0)) = \rho v(1.32) = -0.035;$

Report 1: $U_1(r_1 = 1, \Gamma_{2:6}|h_1 = \emptyset, \omega_1 = 1)$

$$\begin{aligned}
&< \frac{1}{5}\rho \underbrace{v(\mathbf{a}_1^1(1,0,1,1,1,0))}_{\Gamma_2(1,1)=0} + \frac{1}{5}\rho \underbrace{v(\mathbf{a}_1^1(1,0,1,1,0,1))}_{\Gamma_2(1,1)=0} + \frac{1}{5}\rho \underbrace{v(\mathbf{a}_1^1(1,0,1,0,1,1))}_{\Gamma_2(1,1)=0} \\
&+ \frac{1}{5}\rho \underbrace{v(\mathbf{a}_1^1(1,0,0,1,1,1))}_{\Gamma_2(1,1)=0} + \frac{1}{5}\rho v(\mathbf{a}_1^1(1,0,1,1,1,1)) \\
&= \frac{1}{5}\rho v(1.27) + \frac{1}{5}\rho v(1.27) + \frac{1}{5}\rho v(1.27) + \frac{1}{5}\rho v(1.33) + \frac{1}{5}\rho v(1.39)
\end{aligned}$$

$$= -0.038 < -0.035.$$

Therefore, $\Gamma_1(\emptyset, 1) = 0$ is sequentially rational. To complete the proof, it is straightforward to check that the rest of strategies in (11) are indeed parts of a PBE.

B Proof of Theorem 1

Proof of Theorem 1. Choose any allocation rule α that is SIC. To show the truth-telling strategy is the unique FIE in the full-information withdrawal game induced by α, it is equivalent to show that any strategy profile other than the truth-telling strategy fails to be a FIE. First, by Remark 2, an impatient type never report to be patient. Therefore, any strategy profile that involves an impatient type misreporting fails to be a FIE. As a result, I can focus on the set of all run strategies:

$$D^0 = \{\sigma | \sigma_i(0|h_i, 0) = 1 \text{ for all } i \text{ and } h_i \ \& \ \sigma_i(0|h_i, 1) > 0 \text{ for some } i \text{ and } h_i\}.$$

I will adopt an iterated procedure of eliminating run strategies in D^0 and show that any run strategy in D^0 fails to be a FIE. First, I prove the following two lemmas which state that, given any run strategy in D^0, the introspectively consistent beliefs at some information sets are unique and have the feature that depositors believe all previous reports to be truthful.

Lemma 1. *Given any run strategy $\sigma \in D^0$ and any depositor i of type ω_i, his introspectively consistent belief at information set $(h_i = (1, \cdots, 1), \omega_i)$ with respect to σ is unique and given by*

$$\mu_i(\omega_{1:i-1} = (1, \cdots, 1) | h_i = (1, \cdots, 1), \omega_i) = 1. \tag{24}$$

Proof. Choose any run strategy $\sigma \in D^0$ and any depositor i of type ω_i. The history $h_i = (1, \cdots, 1)$ can be either on-path or off-path given strategy σ. If it is on path, then the strategy σ must satisfy the following: For any $j \in \{1, \cdots, i-1\}$,

$$\sigma_j(1 | h_j = (1, \cdots, 1), 1) > 0.$$

It follows from $\sigma \in D^0$ that for any $j \in \{1, \cdots, i-1\}$, $\sigma_j(1|h_j = (1, \cdots, 1), 0) = 0$. Therefore, the introspectively consistent belief at information set $(h_i = (1, \cdots, 1), \omega_i)$ is uniquely pinned down by Bayes' rule in (4) and given by (24). Then consider the case when $h_i = (1, \cdots, 1)$ is off-path given σ. Since only $\omega_{1:i-1} = (1, \cdots, 1)$ is not in $B(h_i|\sigma)$, for any sequence of $\{(\sigma^n, \mu^n) \in \Psi^0\}_{n=1}^{\infty}$ satisfying $\lim_{n\to\infty}(\sigma^n, \mu^n) = (\sigma, \mu)$, it follows from (14) that the introspectively consistent belief at information set $(h_i = (1, \cdots, 1), \omega_i)$ is uniquely given by (24), which completes the proof. □

Lemma 2. *Given any run strategy $\sigma \in D^0$, define*

$$T(\sigma) = \{(j, h_j)|j \in \mathbf{I}, \sigma_j(1|h_j, 1) = 1\}.$$

If $T(\sigma) \neq \emptyset$, then for any depositor i of type ω_i and any history $h_i = (r_1, \cdots, r_{i-1})$ in which, for any $j \in \{1, \cdots, i-1\}$, $r_j = 0$ if and only if $(j, (r_1, \cdots, r_{j-1})) \in T(\sigma)$, his introspectively consistent belief at information set (h_i, ω_i) with respect to σ is unique and given by

$$\mu_i(\omega_{1:i-1} = h_i|h_i, \omega_i) = 1. \tag{25}$$

Proof. Choose any run strategy $\sigma \in D^0$ such that $T(\sigma) \neq \emptyset$, any depositor i of type ω_i and any history $h_i = (r_1, \cdots, r_{i-1})$ in which, for any $j \in \{1, \cdots, j-1\}$, $r_j = 0$ if and only if $(j, (r_1, \cdots, r_{j-1})) \in T(\sigma)$. Note the history h_i can either be on-path or off-path given strategy σ.

If it is on-path, then the strategy σ must also satisfy the following: For any $j \in \{1, \cdots, i-1\}$ such that $r_j = 1$, $\sigma_j(1|h_j, 1) > 0$ with h_j being a partial history of h_i. Then, for any $j \in \{1, \cdots, i-1\}$ such that $r_j = 0$, $(j, (r_1, \cdots, r_{j-1})) \in T(\sigma)$ implies that $\sigma_j(1|h_j, 1) = 1$ with h_j being a partial history of h_i. Furthermore, it follows from $\sigma \in D^0$ that, for any $j \in \{1, \cdots, i-1\}$, $\sigma_j(1|h_j, 0) = 0$ with h_j being a partial history of h_i. Therefore, the introspectively consistent belief at h_i is uniquely pinned down by Bayes' rule in (4) and given by (25).

If it is off-path, then

$$\{\omega_{1:i-1}|\omega_j = 1 \text{ for any } j \in \{1, \cdots, i-1\} \text{ such that } r_j = 1\} \not\subseteq B(h_i|\sigma). \tag{26}$$

Choose any sequence of $\{(\sigma^n, \mu^n) \in \Psi^0\}_{n=1}^{\infty}$ satisfying $\lim_{n\to\infty}(\sigma^n, \mu^n) = (\sigma, \mu)$. Since

$r_j = 0$ if and only if $(j, (r_1, \cdots, r_{j-1})) \in T(\sigma)$, together with (26) and (14), the introspectively consistent belief at (h_i, ω_i) is unique and given by (25). $\square$

The next lemma says that, if patient depositor i believes that all previous reports are truthful reports and all remaining depositors will report truthfully, it is sequentially rational for him to also report truthfully.

Lemma 3. *For any patient depositor i, history h_i and run strategy $\sigma \in D^0$, suppose the following hold:*

1. $\mu_i(\omega_{1:i-1} = h_i | h_i, \omega_i = 1) = 1$;
2. *For $j \in \{i+1, \cdots, I\}$, $\sigma_j(\omega_j | h_j, \omega_j) = 1$ for all h_j including $(h_i, 1)$ as a partial history.*

Then $\Gamma_i(h_i, 1) = 1$ is sequentially rational under μ_i.

Proof. Choose any patient depositor i, history h_i, and run strategy $\sigma \in D^0$. It follows from $\mu_i(\omega_{1:i-1} = h_i | h_i, \omega_i = 1) = 1$ that $\phi_i^{\mu_i}(\omega_{i+1:I} | h_i, \omega_i) = \mathcal{P}(\omega_{i+1:I} | \omega_{1:i-1} = h_i, \omega_i)$. Compare patient depositor i's expected payoff of reporting 0 and 1 at history h_i:

$$\begin{aligned}
&\textbf{Report 0:} \quad U_i^{\mu_i}(r_i = 0, \sigma_{i+1:I} | h_i, \omega_i = 1) = \rho v(\alpha_i^0(h_i, 0));\\
&\textbf{Report 1:} \quad U_i^{\mu_i}(r_i = 1, \sigma_{i+1:I} | h_i, \omega_i = 1)\\
&\qquad\qquad\qquad = \mathbb{E}_{\omega_{i+1:I}}[\rho v(\alpha_i^1(\omega_{1:i-1}, 1, \omega_{i+1:I})) | \omega_{1:i-1} = h_i, \omega_i = 1],
\end{aligned}$$

where the equality follows from the condition that $\sigma_j(\omega_j | h_j, \omega_j) = 1$ for all $j \in \{i+1, \cdots, I\}$ and all h_j including $(h_i, 1)$ as a partial history. Since the allocation rule α is SIC, i.e., satisfies inequality (17), $\Gamma_i(h_i, 1) = 1$ is sequentially rational under μ_i. $\square$

Now I will begin the iterative elimination process from the last depositor, depositor I. Since the allocation rule α is SIC, depositor I always report truthfully at any history h_I. Therefore, any run strategy in D^0 that has depositor I misreporting fails to be a FIE. Eliminating those run strategies gives us

$$D^1 = D^0 \setminus \{\sigma \in D^0 | \sigma_I(0 | h_I, 1) > 0 \text{ for all } h_I\}.$$

Next, consider patient depositor $I-1$ after history $h_{I-1} = (1, \cdots, 1)$. By Lemma 1, for any run strategy in $D^1 \subseteq D^0$, his introspectively consistent belief at information set $(h_{I-1} = (1, \cdots, 1), \omega_{I-1} = 1)$ is unique and given by

$$\mu_{I-1}(\omega_{1:I-2} = (1, \cdots, 1) | h_{I-1} = (1, \cdots, 1), \omega_{I-1} = 1) = 1.$$

Since for any run strategy in D^1, $\sigma_I(\omega_I | (1, \cdots, 1), \omega_I) = 1$, it follows from Lemma 3 that, after history $h_{I-1} = (1, \cdots, 1)$, $\Gamma_{I-1}(h_{I-1}, 1) = 1$ is sequentially rational under any introspective consistent belief. Therefore, what is left to check is in the following subset of D^1:

$$D^2 = D^1 \setminus \{\sigma \in D^1 | \sigma_{I-1}(0 | (1, \cdots, 1), 1) > 0\}.$$

Given any run strategy in D^2, consider patient depositor $I-2$ after history $h_{I-2} = (1, \cdots, 1)$. Again, by Lemma 1, his introspectively consistent belief at information set $(h_{I-2} = (1, \cdots, 1), \omega_{I-2} = 1)$ is unique and given by

$$\mu_{I-2}(\omega_{1:I-3} = (1, \cdots, 1) | h_{I-2} = (1, \cdots, 1), \omega_{I-2} = 1) = 1.$$

Note that for any run strategy in D^2, $\sigma_j(\omega_j | h_j, \omega_j) = 1$ for all h_j including $(h_{I-2} = (1, \cdots, 1), 1)$ as a partial history. Then it follows from Lemma 3 that, after history $h_{I-2} = (1, \cdots, 1)$, $\Gamma_{I-2}(h_{I-2}, 1) = 1$ is sequentially rational under any introspective consistent belief. Therefore, what is left to check is in the following subset of D^2:

$$D^3 = D^2 \setminus \{\sigma \in D^1 | \sigma_{I-2}(0 | (1, \cdots, 1), 1) > 0\}.$$

Given any run strategy in D^3, before moving to depositor $I-3$, I need to check depositor $I-1$ at history $h_{I-1} = (1, \cdots, 1, 0)$. Here, at $h_{I-1} = (1, \cdots, 1, 0)$, $r_{I-2} = 0$, and for any run strategy σ in D^3, $\sigma_{I-2}(1 | h_{I-2} = (1, \cdots, 1), 1) = 1$, i.e., $(j = I - 2, h_{I-2} = (1, \cdots, 1)) \in T(\sigma)$. Then by Lemma 2, his introspectively consistent belief at information set $(h_{I-1} = (1, \cdots, 1, 0), \omega_{I-1} = 1)$ is unique and given by

$$\mu_{I-1}(\omega_{1:I-2} = (1, \cdots, 1, 0) | h_{I-1} = (1, \cdots, 1, 0), \omega_{I-1} = 1) = 1.$$

Since for any run strategy in $D^3 \subseteq D^1$, $\sigma_I(\omega_I | (1, \cdots, 1), \omega_I) = 1$, it follows from

Lemma 3 that, after history $h_{I-1} = (1, \cdots, 1, 0)$, $\Gamma_{I-1}(h_{I-1}, 1) = 1$ is sequentially rational under any introspective consistent belief. Therefore, what is left to check is in the following subset of D^3:

$$D^4 = D^3 \setminus \{\sigma \in D^3 | \sigma_{I-1}(0|(1, \cdots, 1, 0), 1) > 0\}.$$

Repeating this elimination procedure to go through the rest of histories leads to an empty set, meaning that no run strategy in D^0 can be a FIE. Therefore, the truth-telling strategy is the unique FIE in the full-information withdrawal game. □

C Proof of Proposition 3

Before proving Proposition 3, I first show the following lemma.

Lemma 4. *Suppose that Assumption 3 holds. Then for any $i \in \mathbf{I}$ and $k \in \{0, 1, \cdots, i-2\}$,*

$$\mathcal{P}(\omega_i = 1|\theta(\omega_{1:i-1}) = k) \leq \mathcal{P}(\omega_i = 1|\theta(\omega_{1:i-1}) = k + 1). \tag{27}$$

Proof of Lemma 4. Suppose that Assumption 3 holds. First, I show by induction that the following inequality holds for any $N \in \mathbb{Z}_+$:

$$\frac{\sum_{i=0}^{N} C(N, i)\mathcal{P}(\omega : \theta(\omega) = k + i)}{\sum_{i=0}^{N} C(N, i)\mathcal{P}(\omega : \theta(\omega) = k + 1 + i)} \geq \frac{\sum_{i=0}^{N} C(N, i)\mathcal{P}(\omega : \theta(\omega) = k + 1 + i)}{\sum_{i=0}^{N} C(N, i)\mathcal{P}(\omega : \theta(\omega) = k + 2 + i)}. \tag{28}$$

First, by (19), (28) holds for $N = 0$. Suppose (28) holds for $N = n - 1$. By the reduction property of combinations, I have for any $a_i \in \mathbb{R}$, $i = 0, \cdots, n$,

$$\sum_{i=0}^{n} C(n, i)a_i = \sum_{i=0}^{n-1} C(n - 1, i)a_i + \sum_{i=1}^{n} C(n - 1, i - 1)a_i.$$

Therefore, it follows that

$$\begin{aligned}
&\frac{\sum_{i=0}^{n} C(n, i)\mathcal{P}(\omega : \theta(\omega) = k + i)}{\sum_{i=0}^{n} C(n, i)\mathcal{P}(\omega : \theta(\omega) = k + 1 + i)} \\
&= \frac{\sum_{i=0}^{n-1} C(n - 1, i)\mathcal{P}(\omega : \theta(\omega) = k + i) + \sum_{i=1}^{n} C(n - 1, i - 1)\mathcal{P}(\omega : \theta(\omega) = k + i)}{\sum_{i=0}^{n-1} C(n - 1, i)\mathcal{P}(\omega : \theta(\omega) = k + 1 + i) + \sum_{i=1}^{n} C(n - 1, i - 1)\mathcal{P}(\omega : \theta(\omega) = k + 1 + i)}
\end{aligned}$$

$$= \frac{\sum_{i=0}^{n-1} C(n-1,i)\mathcal{P}(\omega:\theta(\omega)=k+i) + \sum_{i=0}^{n-1} C(n-1,i)\mathcal{P}(\omega:\theta(\omega)=k+1+i)}{\sum_{i=0}^{n-1} C(n-1,i)\mathcal{P}(\omega:\theta(\omega)=k+1+i) + \sum_{i=0}^{n-1} C(n-1,i)\mathcal{P}(\omega:\theta(\omega)=k+2+i)}$$
$$= \frac{1 + \frac{\sum_{i=0}^{n-1} C(n-1,i)\mathcal{P}(\omega:\theta(\omega)=k+i)}{\sum_{i=0}^{n-1} C(n-1,i)\mathcal{P}(\omega:\theta(\omega)=k+1+i)}}{1 + \frac{\sum_{i=0}^{n-1} C(n-1,i)\mathcal{P}(\omega:\theta(\omega)=k+2+i)}{\sum_{i=0}^{n-1} C(n-1,i)\mathcal{P}(\omega:\theta(\omega)=k+1+i)}},$$

where $\sum_{i=0}^{n-1} C(n-1,i)\mathcal{P}(\omega:\theta(\omega)=k+1+i) = \sum_{j=1}^{n} C(n-1,j-1)\mathcal{P}(\omega:\theta(\omega)=k+j)$ by letting $j=i+1$ and $\sum_{i=1}^{n} C(n-1,i-1)\mathcal{P}(\omega:\theta(\omega)=k+1+i) = \sum_{j=0}^{n-1} C(n-1,j)\mathcal{P}(\omega:\theta(\omega)=k+2+j)$ by letting $j=i-1$. Similarly,

$$\frac{\sum_{i=0}^{n} C(n,i)\mathcal{P}(\omega:\theta(\omega)=k+1+i)}{\sum_{i=0}^{n} C(n,i)\mathcal{P}(\omega:\theta(\omega)=k+2+i)} = \frac{1 + \frac{\sum_{i=0}^{n-1} C(n-1,i)\mathcal{P}(\omega:\theta(\omega)=k+1+i)}{\sum_{i=0}^{n-1} C(n-1,i)\mathcal{P}(\omega:\theta(\omega)=k+2+i)}}{1 + \frac{\sum_{i=0}^{n-1} C(n-1,i)\mathcal{P}(\omega:\theta(\omega)=k+3+i)}{\sum_{i=0}^{n-1} C(n-1,i)\mathcal{P}(\omega:\theta(\omega)=k+2+i)}}.$$

Since (28) holds for $N=n-1$, it follows that

$$\frac{\sum_{i=0}^{n} C(n,i)\mathcal{P}(\omega:\theta(\omega)=k+i)}{\sum_{i=0}^{n} C(n,i)\mathcal{P}(\omega:\theta(\omega)=k+1+i)} \geq \frac{\sum_{i=0}^{n} C(n,i)\mathcal{P}(\omega:\theta(\omega)=k+1+i)}{\sum_{i=0}^{n} C(n,i)\mathcal{P}(\omega:\theta(\omega)=k+2+i)}.$$

Therefore, (28) holds for $N=n$, which completes the induction proof. Next, choose any $i \in \mathbf{I}$ and $k \in \{0,1,\cdots,i-2\}$. Note that

$$\begin{aligned}\mathcal{P}(\omega_i=1|\theta(\omega_{1:i-1})=k) &= \frac{\sum_{\omega_{i+1:I}} \mathcal{P}(\omega_{1:i-1},1,\omega_{i+1:I})}{\sum_{\omega_{i+1:I}} \mathcal{P}(\omega_{1:i-1},1,\omega_{i+1:I}) + \sum_{\omega_{i+1:I}} \mathcal{P}(\omega_{1:i-1},0,\omega_{i+1:I})} \\ &= \frac{1}{1 + \frac{\sum_{\omega_{i+1:I}} \mathcal{P}(\omega_{1:i-1},0,\omega_{i+1:I})}{\sum_{\omega_{i+1:I}} \mathcal{P}(\omega_{1:i-1},1,\omega_{i+1:I})}},\end{aligned}$$

where

$$\frac{\sum_{\omega_{i+1:I}} \mathcal{P}(\omega_{1:i-1},0,\omega_{i+1:I})}{\sum_{\omega_{i+1:I}} \mathcal{P}(\omega_{1:i-1},1,\omega_{i+1:I})} = \frac{\sum_{j=0}^{I-i} C(I-i,j)\mathcal{P}(\omega:\theta(\omega)=k+j)}{\sum_{j=0}^{I-i} C(I-i,j)\mathcal{P}(\omega:\theta(\omega)=k+1+j)}.$$

Similarly,

$$\mathcal{P}(\omega_i=1|\theta(\omega_{1:i-1})=k+1) = \frac{1}{1 + \frac{\sum_{j=0}^{I-i} C(I-i,j)\mathcal{P}(\omega:\theta(\omega)=k+1+j)}{\sum_{j=0}^{I-i} C(I-i,j)\mathcal{P}(\omega:\theta(\omega)=k+2+j)}}.$$

It follows from (28) by letting $N = I - i$ that

$$\mathcal{P}(\omega_i = 1|\theta(\omega_{1:i-1}) = k) \leq \mathcal{P}(\omega_i = 1|\theta(\omega_{1:i-1}) = k+1),$$

which completes the proof. □

Proof of Theorem 2. Suppose Assumption 1, 2 and 3 hold. By Lemma 1 in Ennis and Keister (2009), the problem of solving the period-0 allocation of each depositor i if impatient in the first-best allocation rule can be formulated as a finite dynamic programming problem. For any depositor i, write down the following Bellman equations:

$$V_i^0(\theta_i, y_i; \pi_{i+1}^I(\theta_i)) = \max_{a_i^0} v(a_i^0) + \pi_{i+1}(\theta_i)V_{i+1}^1(\theta_i, y_i - a_i^0; \pi_{i+2}^I(\theta_i + 1)) \\ + (1 - \pi_{i+1}(\theta_i))V_{i+1}^0(\theta_i, y_i - a_i^0; \pi_{i+2}^I(\theta_i)),$$

and

$$V_i^1(\theta_i, y_i; \pi_{i+1}^I(\theta_i + 1)) = \pi_{i+1}(\theta_i + 1)V_{i+1}^1(\theta_i + 1, y_i; \pi_{i+2}^I(\theta_i + 2)) \\ + (1 - \pi_{i+1}(\theta_i + 1))V_{i+1}^0(\theta_i + 1, y_i; \pi_{i+2}^I(\theta_i + 1)).$$

Here V_i^0 is the value function conditional on depositor i being impatient, i.e., $\omega_i = 0$, and V_i^1 is the value function conditional on depositor i being patient, i.e., $\omega_i = 1$. θ_i is the total number of patient depositors among the first $i-1$ depositors, and y_i is the remaining resources. Also, $\pi_j^I(\theta) = (\pi_j(\theta), \pi_{j+1}(\theta), \pi_{j+1}(\theta+1), \cdots, \pi_I(\theta), \cdots, \pi_I(\theta + I - j))$, where $\pi_j(\theta) = \mathcal{P}(\omega_j = 1|\theta(\omega_{1:j-1}) = \theta)$. Next, I prove the following Lemma:

Lemma 5. *For any depositor i, any θ_i and any y_i, the following inequality holds:*

$$\frac{\partial V_i^1(\theta_i, y_i; \pi_{i+1}^I(\theta_i + 1))}{\partial y_i} \leq \frac{\partial V_i^0(\theta_i, y_i; \pi_{i+1}^I(\theta_i))}{\partial y_i}. \tag{29}$$

Proof. The proof is by induction. First, consider the case when $i = I$. Then the Bellman equation conditional on $\omega_I = 0$ is

$$V_I^0(\theta_I, y_I) = \max_{a_I^0} v(a_I^0) + \theta_I \rho v(\frac{R(y_I - a_I^0)}{\theta_I}).$$

The first-order condition is

$$v'(a_I^0) = \rho R v'(\frac{R(y_I - a_I)}{\theta_I}).$$

It follows from Assumption 1 and $\rho R > 1$ that

$$\frac{y_I - a_I^0}{\theta_I} \leq a_I^0 \leq \frac{R(y_I - a_I^0)}{\theta_I} \Rightarrow \frac{y_I}{\theta_I + 1} \geq \frac{y_I - a_I^0}{\theta_I}. \tag{30}$$

By the envelope theorem,

$$\frac{\partial V_I^0(\theta_I, y_I)}{\partial y_I} = \rho R v'(\frac{R(y_I - a_I^0)}{\theta_I}).$$

The Bellman equation conditional on $\omega_I = 1$ is

$$V_I^1(\theta_I, y_I) = (\theta_I + 1)\rho v(\frac{R y_I}{\theta_I + 1}).$$

Taking partial derivatives with respect to y_I on both sides yields

$$\frac{\partial V_I^1(\theta_I, y_I)}{\partial y_I} = \rho R v'(\frac{R y_I}{\theta_I + 1}).$$

It follows from Assumption 1 and (30) that

$$\frac{\partial V_I^1(\theta_I, y_I)}{\partial y_I} \leq \frac{\partial V_I^0(\theta_I, y_I)}{\partial y_I}.$$

Therefore, inequality (29) holds for $i = I$. Next, suppose that (29) holds for all $i = k+1, \cdots, I$. I need to show that (29) also holds for $i = k$. The Bellman equation conditional on $\omega_k = 0$ is

$$\begin{aligned} V_k^0(\theta_k, y_k; \pi_{k+1}^I(\theta_k)) = \max_{a_k^0} v(a_k^0) &+ \pi_{k+1}(\theta_k) V_{k+1}^1(\theta_k, y_k - a_k^0; \pi_{k+2}^I(\theta_k + 1)) \\ &+ (1 - \pi_{k+1}(\theta_k)) V_{k+1}^0(\theta_k, y_k - a_k^0; \pi_{k+2}^I(\theta_k)). \end{aligned}$$

The first-order condition is

$$v'(a_k^0) = \pi_{k+1}(\theta_k)\frac{\partial V_{k+1}^1(\theta_k, y_k - a_k^0; \pi_{k+2}^I(\theta_k+1))}{\partial y_{k+1}} + (1-\pi_{k+1}(\theta_k))\frac{\partial V_{k+1}^0(\theta_k, y_k - a_k^0; \pi_{k+2}^I(\theta_k))}{\partial y_{k+1}}.$$

Define the function

$$\begin{aligned} H_k(a_k^0, \pi_{k+1}^I(\theta_k)) = v(a_k^0) + \pi_{k+1}(\theta_k)V_{k+1}^1(\theta_k, y_k - a_k^0; \pi_{k+2}^I(\theta_k+1)) \\ + (1-\pi_{k+1}(\theta_k))V_{k+1}^0(\theta_k, y_k - a_k^0; \pi_{k+2}^I(\theta_k)). \end{aligned}$$

Since (29) holds for all $i = k+1, \cdots, I$, the function $H_k(a_k^0, \pi_{k+1}^I(\theta_k))$ has increasing differences in $(a_k^0, \pi_{k+1}^I(\theta_k))$. Therefore, it follows from Theorem 6.1 in Topkis (1978) that $a_k^0(\pi_{k+1}^I(\theta_k)) \leq a_k^0(\hat{\pi}_{k+1}^I(\theta_k))$ whenever $\pi_{k+1}^I(\theta_k) \leq \hat{\pi}_{k+1}^I(\theta_k)$. As a result, by the first-order condition and the envelope theorem,

$$\frac{\partial V_k^0(\theta_k, y_k; \pi_{k+1}^I(\theta_k))}{\partial y_k} \geq \frac{\partial V_k^0(\theta_k, y_k; \hat{\pi}_{k+1}^I(\theta_k))}{\partial y_k}, \tag{31}$$

whenever $\pi_{k+1}^I(\theta_k) \leq \hat{\pi}_{k+1}^I(\theta_k)$. Note that the Bellman equation conditional on $\omega_k = 1$ is

$$\begin{aligned} V_k^1(\theta_k, y_k; \pi_{k+1}^I(\theta_k+1)) =& \pi_{k+1}(\theta_k+1)V_{k+1}^1(\theta_k+1, y_k; \pi_{k+2}^I(\theta_k+2)) \\ &+ (1-\pi_{k+1}(\theta_k+1))V_{k+1}^0(\theta_k+1, y_k; \pi_{k+2}^I(\theta_k+1)). \end{aligned} \tag{32}$$

Consider the following constructed Bellman equation conditional on $\omega_i = 0$:

$$\begin{aligned} V_k^0(\theta_k, y_k; \pi_{k+1}^I(\theta_k+1)) = \max_{a_k^0} v(a_k^0) + \pi_{k+1}(\theta_k+1)V_{k+1}^1(\theta_k, y_k - a_k^0; \pi_{k+2}^I(\theta_k+2)) \\ + (1-\pi_{k+1}(\theta_k+1))V_{k+1}^0(\theta_k, y_k - a_k^0; \pi_{k+2}^I(\theta_k+1)), \end{aligned} \tag{33}$$

which has the same transition probabilities as in (32). Then I can apply the proof technique of Lemma 5 in Green and Lin (2003) to the problem (33) and (32), and get

$$\frac{\partial V_k^1(\theta_k, y_k; \pi_{k+1}^I(\theta_k+1))}{\partial y_k} \leq \frac{\partial V_k^0(\theta_k, y_k; \pi_{k+1}^I(\theta_k+1))}{\partial y_k}. \tag{34}$$

Lastly, By Lemma 4, $\pi^I_{k+1}(\theta_k + 1) \geq \pi^I_{k+1}(\theta_k)$. Then it follows from (31) that

$$\frac{\partial V^0_k(\theta_k, y_k; \pi^I_{k+1}(\theta_k + 1))}{\partial y_k} \leq \frac{\partial V^0_k(\theta_k, y_k; \pi^I_{k+1}(\theta_k))}{\partial y_k}.$$

Combining with (34) leads to

$$\frac{\partial V^1_k(\theta_k, y_k; \pi^I_{k+1}(\theta_k + 1))}{\partial y_k} \leq \frac{\partial V^0_k(\theta_k, y_k; \pi^I_{k+1}(\theta_k))}{\partial y_k}.$$

Therefore, (29) holds for $i = k$, which completes the induction argument. $\square$

Then it follows from Lemma 5 and the envelope theorem that

$$\begin{aligned}
&\mathbb{E}_{\omega_{i+1:I}}[v'(\frac{R(y - \sum_{k=i}^{I} \mathbf{a}^0_k(\omega_{1:i-1}, 0, \omega_{i+1:I}))}{\theta(\omega_{1:i-1}) + \theta(\omega_{i+1:I})})|\omega_{1:i-1}, \omega_i = 0] \\
&\qquad \geq \mathbb{E}_{\omega_{i+1:I}}[v'(\frac{R(y - \sum_{k=i+1}^{I} \mathbf{a}^0_k(\omega_{1:i-1}, 1, \omega_{i+1:I}))}{\theta(\omega_{1:i-1}) + 1 + \theta(\omega_{i+1:I})})|\omega_{1:i-1}, \omega_i = 1], \qquad (35)
\end{aligned}$$

for any depositor i, any $\omega_{1:i-1}$ and any y. Here, (35) is the correlated types analogue of (A.7) in Green and Lin (2003) with different conditional expectations on each side. The rest of the proof is exactly the same as in the proof of Lemma 5 in Green and Lin. By $\rho R > 1$, the first-order condition and the envelope theorem of the optimization problem for $\omega_i = 0$, I have

$$v'(\mathbf{a}^0_i(\omega_{1:i-1}, 0)) > \mathbb{E}_{\omega_{i+1:I}}[v'(\frac{R(y - \sum_{k=i}^{I} \mathbf{a}^0_k(\omega_{1:i-1}, 0, \omega_{i+1:I}))}{\theta(\omega_{1:i-1}) + \theta(\omega_{i+1:I})})|\omega_{1:i-1}, \omega_i = 0].$$

It follows from (35) that

$$v'(\mathbf{a}^0_i(\omega_{1:i-1}, 0)) > \mathbb{E}_{\omega_{i+1:I}}[v'(\frac{R(y - \sum_{k=i+1}^{I} \mathbf{a}^0_k(\omega_{1:i-1}, 1, \omega_{i+1:I}))}{\theta(\omega_{1:i-1}) + 1 + \theta(\omega_{i+1:I})})|\omega_{1:i-1}, \omega_i = 1].$$

By Assumption 2, there exists a function $\mu : \mathbb{R} \to \mathbb{R}_+$ such that $\mu' < 0$, $\mu'' > 0$, and

$$v'(c) = \mu(v(c)).$$

Then it follows that

$$
\begin{aligned}
\mu(v(\mathbf{a}_i^0(\omega_{1:i-1},0))) &> \mathbb{E}_{\omega_{i+1:I}}[\mu(v(\frac{R(y-\sum_{k=i+1}^{I}\mathbf{a}_k^0(\omega_{1:i-1},1,\omega_{i+1:I}))}{\theta(\omega_{1:i-1})+1+\theta(\omega_{i+1:I})}))|\omega_{1:i-1},\omega_i=1] \\
&> \mu(\mathbb{E}_{\omega_{i+1:I}}[v(\frac{R(y-\sum_{k=i+1}^{I}\mathbf{a}_k^0(\omega_{1:i-1},1,\omega_{i+1:I}))}{\theta(\omega_{1:i-1})+1+\theta(\omega_{i+1:I})})|\omega_{1:i-1},\omega_i=1]) \quad (\text{By } \mu''>0).
\end{aligned}
$$

Since $\mu' < 0$,

$$
\begin{aligned}
v(\mathbf{a}_i^0(\omega_{1:i-1},0)) &< \mathbb{E}_{\omega_{i+1:I}}[v(\frac{R(y-\sum_{k=i+1}^{I}\mathbf{a}_k^0(\omega_{1:i-1},1,\omega_{i+1:I}))}{\theta(\omega_{1:i-1})+1+\theta(\omega_{i+1:I})})|\omega_{1:i-1},\omega_i=1] \\
&= \mathbb{E}_{\omega_{i+1:I}}[v(\mathbf{a}_i^1(\omega_{1:i-1},1,\omega_{i+1:I}))|\omega_{1:i-1},\omega_i=1],
\end{aligned}
$$

which completes the proof. □

D Omitted proofs

Proof of Proposition 1. First, the "only if" part is straightforward. Next, for the "if" part, suppose that (16) holds for any i and $\omega_{1:i-1} \in \Omega^{i-1}$. Note that all histories are on-path given the truth-telling strategy. Therefore, the system of beliefs given the truth-telling strategy is uniquely pinned down by Bayes' rule. In particular, choose any depositor i and information set (h_i, ω_i). It follows from (4) that

$$
\mu_i(\omega_{1:i-1}|h_i,\omega_i) = \begin{cases} 1 & \text{if } \omega_{1:i-1} = h_i \\ 0 & \text{if } \omega_{1:i-1} \neq h_i. \end{cases}
$$

Then by (3), $\phi_i^{\mu_i}(\cdot|h_i,\omega_i)$ is given by

$$
\phi_i^{\mu_i}(\omega_{i+1:I}|h_i,\omega_i) = \sum_{\omega_{1:i-1}} \mathcal{P}(\omega_{i+1:I}|\omega_{1:i-1},\omega_i)\mu_i(\omega_{1:i-1}|h_i) = \mathcal{P}(\omega_{i+1:I}|\omega_{1:i-1}=h_i,\omega_i),
$$

for any $\omega_{i+1:I}$. Since by Remark 2 impatient depositor i always reports truthfully, let us consider patient depositor i's problem. Given that all depositors after depositor i will report truthfully, i.e., for any $j \in \{i+1,\cdots,I\}, \Gamma_j(h_j,\omega_j) = \omega_j$, compare his payoff of reporting 0 (lying) and 1 (truth-telling) at history h_i under μ_i:

Report 0: $U_i^{\mu_i}(r_i = 0, \Gamma_{i+1:I}|h_i, \omega_i) = \rho v(\alpha_i^0(h_i, 0))$;

Report 1: $U_i^{\mu_i}(r_i = 1, \Gamma_{i+1:I}|h_i, \omega_i) = \mathbb{E}_{\omega_{i+1:I}}[\rho v(\alpha_i^1(h_i, 1, \omega_{i+1:I}))|\omega_{1:i-1} = h_i, \omega_i = 1]$.

Since $h_i \in \Omega^{i-1}$, it follows from (16) that $\Gamma_i(h_i, 1) = 1$ is sequentially rational for patient depositor i at history h_i. Therefore, the truth-telling strategy is a FIE in the full-information withdrawal game induced by α, i.e., α is IC. $\square$

Proof of Proposition 2. Choose any $k \in \{0, 1, \cdots, I-2\}$. To show Assumption 3 holds, it is equivalent to show the following:

$$[\sum_{j=1}^{m}(1-\pi_j)^k \pi_j^{I-k} p_j][\sum_{j=1}^{m}(1-\pi_j)^{k+2} \pi_j^{I-k-2} p_j] - [\sum_{j=1}^{m}(1-\pi_j)^{k+1} \pi_j^{I-k-1} p_j]^2 \geq 0,$$

which can be rewritten as

$$\begin{aligned}
&\sum_{i \neq j}[(1-\pi_i)^k \pi_i^{I-k}(1-\pi_j)^{k+2}\pi_j^{I-k-2} + (1-\pi_j)^k \pi_j^{I-k}(1-\pi_i)^{k+2}\pi_i^{I-k-2}]p_i p_j \\
&\geq \sum_{i \neq j}[2(1-\pi_i)^{k+1}\pi_i^{I-k-1}(1-\pi_j)^{k+1}\pi_j^{I-k-1}]p_i p_j. \qquad (36)
\end{aligned}$$

Note that for any $i \neq j$, the following is true:

$$\{[(1-\pi_i)^k \pi_i^{I-k}(1-\pi_j)^{k+2}\pi_j^{I-k-2}]^{\frac{1}{2}} + [(1-\pi_j)^k \pi_j^{I-k}(1-\pi_i)^{k+2}\pi_i^{I-k-2}]^{\frac{1}{2}}\}^2 \geq 0.$$

Therefore, for any $i \neq j$,

$$(1-\pi_i)^k \pi_i^{I-k}(1-\pi_j)^{k+2}\pi_j^{I-k-2} + (1-\pi_j)^k \pi_j^{I-k}(1-\pi_i)^{k+2}\pi_i^{I-k-2} \geq 2(1-\pi_i)^{k+1}\pi_i^{I-k-1}(1-\pi_j)^{k+1}\pi_j^{I-k-1}.$$

Then it implies that (36) holds, which completes the proof. $\square$

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