Time-consistent fair social choice*

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Abstract

In this paper, we study intertemporal social welfare evaluations when agents have heterogeneous preferences that are interpersonally noncomparable. We first show that, even if all agents share the same preferences, there is a conflict between the axioms of Pareto principle, time consistency, and equity requiring society to reduce inequality regardless of the past. We argue that responsibility for past choices should be taken into account and thus the equity axiom is not compelling. Then, we introduce another form of equity that takes the past into consideration and is compatible with time consistency. Using this form of equity and time consistency, we characterize maximin and leximin social welfare criteria that are history-dependent.

Keywords: Time consistency; equity; efficiency; responsibility; social welfare

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1 Introduction

Many economic decisions are intertemporal: people save, borrow, and choose consumption plans for the present and the future. Such decisions vary among agents because their preferences are heterogeneous. Moreover, agents may enjoy different levels of well-being in different periods of their lives. Many public policies, including social security reforms and policies to reduce greenhouse gases emissions, have intertemporal consequences. Thus, it is important to have criteria for evaluating intertemporal allocations.

In this paper, we study how allocations of lifetime consumption streams should be evaluated. An important aspect of this study is considering how the past should be taken into account in terms of responsibility. For instance, consider two agents, Al and Bill, who are the same age. Suppose that Al was born into a rich family and lived a secure and contented life, whereas Bill was born into a poor family and led a deprived life. Now, however, Al is poor because he wasted his wealth, whereas Bill prudently saved for future consumption, and is more affluent than Al. Then, considering the past situations of Al and Bill, it is not obvious whether to redistribute from Bill to Al, because Al could be considered responsible for his past profligacy.

The above example shows that the evaluation of lifetime consumption allocations could depend on the past. In dynamics, most studies consider time-invariant decision rules, that is, decisions that are independent of history. This property can be desirable for individual decision-making, but is not necessarily desirable for social evaluations (Hayashi, 2016; Millner and Heal, 2018). Theories of equality of opportunities have proposed that agents should be held responsible for their choices and that society should compensate only for inequalities that are not in some way traceable to the agent's choices (Cohen, 1989; Roemer, 1998).[1] Taking the example above, it could be argued, on the one hand, that the social planner should take into account the information that Al consumed all of his wealth while Bill saved for the future. On the other hand, it may be considered too harsh to hold Al

[1] See Fleurbaey (2008) for a survey of the literature.

fully responsible for his past choices and to leave him living in poverty (Fleurbaey, 1995, 2008; Anderson, 1999). To reconcile these conflicting views, it is important to analyze the relationship between normative conditions and social attitudes toward history.

In this paper, we analyze this problem using a social choice framework, in which agents have heterogeneous lifetime utility functions that are interpersonally noncomparable. We derive a criterion for interpersonal comparisons using the fair social choice approach (Fleurbaey and Maniquet, 2011). Our domain is restricted to the set of additively separable lifetime utility functions with exponential discounting.

We study social welfare criteria satisfying equity and time consistency, as well as the Pareto principle (*Weak Pareto* and *Pareto Indifference*), which requires that unanimous evaluations should be respected. These principles are essential for intertemporal social decisions. In particular, in the context of intertemporal choice, time consistency is a basic rationality postulate. This axiom insists that decisions at different points in time should not be contradictory.

Our first result (Proposition 1 in Section 3) shows that even if all agents share the same preferences, no social ordering exists that satisfies *Pareto Indifference*, time consistency, and an equity axiom called *Transfer to the Worst-Off*. The equity axiom requires that, given any history, if allocations are *constant*, that is, they do not change over time from the present, then a transfer from a better-off agent to the worst-off agent (with the same preferences) should be socially desirable. The intuition of the result is as follows. We consider the allocations in Table 1. For instance, $\boldsymbol{x}_A$ and $\boldsymbol{x}_B$ are the lifetime consumption streams of Al and Bill, respectively, where Al and Bill consume 20 and 0 in the initial period ($t = 0$), and 0 and 10 in the next period and thereafter, respectively. Suppose that Al and Bill are in $t = 1$. Then, by *Transfer to the Worst-Off*, $(\boldsymbol{y}_A, \boldsymbol{y}_B)$ is socially preferred to $(\boldsymbol{x}_A, \boldsymbol{x}_B)$ at $t = 1$. Note that the equity axiom requires the reduction of inequality *regardless of the agents' past consumptions*. However, time consistency requires that $(\boldsymbol{y}_A, \boldsymbol{y}_B)$ should be considered better than $(\boldsymbol{x}_A, \boldsymbol{x}_B)$ not only in $t = 1$ but also in $t = 0$. We can construct a common preference for them such that, at $t = 0$, Al is indifferent between $\boldsymbol{x}_A$ and $\boldsymbol{x}'_A$

t	0	1	2	3	⋯
$\boldsymbol{x}_A$	20	0	0	0	⋯
$\boldsymbol{x}_B$	0	10	10	10	⋯
$\boldsymbol{x}'_A$	10	10	10	10	⋯
$\boldsymbol{x}'_B$	5	5	5	5	⋯

t	0	1	2	3	⋯
$\boldsymbol{y}_A$	20	2	2	2	⋯
$\boldsymbol{y}_B$	0	8	8	8	⋯
$\boldsymbol{y}'_A$	11	11	11	11	⋯
$\boldsymbol{y}'_B$	4	4	4	4	⋯

Table 1: The allocations in Proposition 1

and between $\boldsymbol{y}_A$ and $\boldsymbol{y}'_A$, whereas Bill is indifferent between $\boldsymbol{x}_B$ and $\boldsymbol{x}'_B$ and between $\boldsymbol{y}_A$ and $\boldsymbol{y}'_B$. Then, by *Pareto Indifference* and transitivity, $(\boldsymbol{y}'_A, \boldsymbol{y}'_B)$ is considered better than $(\boldsymbol{x}'_A, \boldsymbol{x}'_B)$. However, *Transfer to the Worst-Off* claims that $(\boldsymbol{x}'_A, \boldsymbol{x}'_B)$ should be better than $(\boldsymbol{y}'_A, \boldsymbol{y}'_B)$, which is a contradiction.

While time consistency and the Pareto principle are compelling, *Transfer to the Worst-Off* may not be because it requires redistribution without any concern for the past. This paper provides possibility results using other equity axioms. In Section 4, we introduce *Hammond Equity on Constant Allocations*, which is based on Hammond's (1976) equity axiom and requires that, on constant allocations from the initial period, any reduction of inequality between two agents should be socially acceptable because the inequality exists from the initial period and is due to factors for which they are not responsible. Then, by the Pareto principle, *Time Consistency*, *Hammond Equity on Constant Allocations*, and a continuity axiom, Theorem 2 characterizes a maximin social ordering, where agents' situations are assessed by equivalent consumption levels giving the same normalized lifetime utilities as the actual consumption streams from the initial period. Note that this criterion is history-dependent, as it is important to take past choices into consideration when comparing allocations. We also discuss how the social evaluations should vary with history.

Our result is in contrast with classical egalitarian social criteria. These criteria are problematic because they are time-inconsistent and history-independent, and therefore ignore responsibility for past choices. In contrast, this paper proposes an egalitarian criterion that is time-consistent and history-dependent.

Although *Hammond Equity* is often used in the welfare economics literature, it is strong because its implication is an infinite inequality aversion. Therefore, we derive *Hammond Equity on Constant Allocations* using a weaker axiom, *Equity for Equals*, which requires a transfer of constant consumption from the better-off to the worse-off agent when their preferences are the same and the past consumptions are equal (and therefore there is no difference in terms of responsibility). We also present an invariance axiom which requires that agents' time preferences should not matter for comparing constant allocations because we do not have to care about discounting future utilities or consumption smoothing. Then, Lemma 3 in Section 5 shows that the Pareto principle, *Time Consistency*, *Equity for Equals* and the invariance axiom together imply *Hammond Equity on Constant Allocations*. From this result, we can obtain another characterization of the maximin criterion in Theorem 2.

We also characterize the leximin version of our criterion. We introduce another equity axiom named *Suppes Indifference for Equals*. This axiom is based on the Suppes grading principle (Suppes, 1966), and requires to treat agents equally if they have the same time preferences and past consumptions because there is no difference in terms of responsibility among them. Using this axiom, the *Strong Pareto* principle and the other axioms in Lemma 3, we axiomatize the leximin criterion.

2 The Model

We adopt a discrete time model. Time is indexed by $t \in \mathcal{T}$, where $\mathcal{T}$ is the set of all non-negative integers. The set of agents is denoted by $N = \{1, \cdots, n\}$, where $n \geq 2$. Agents' lives span all of $\mathcal{T}$. We use $x_{it} \in \mathbb{R}_+$ to denote agent i's consumption in period t, where $\mathbb{R}_+$ is the set of nonnegative real numbers. Let $\boldsymbol{x}_t = (x_{it})_{i \in N} \in \mathbb{R}_+^N$ be an allocation of consumption at $t \in \mathcal{T}$. In addition, we denote $\boldsymbol{x}_i^T = (x_{it})_{t \geq T}$ given $T \in \mathcal{T}$.

In this paper, we consider situations where agents may have heterogeneous time preferences. Given any $\boldsymbol{x}_i^T$, agent i's lifetime consumption streams are evaluated by

$$U_i(\boldsymbol{x}_i^T) \equiv \sum_{t \geq T} \delta_i^{t-T} u_i(x_{it}),$$

where $u_i : \mathbb{R}_+ \to \mathbb{R}$ and $\delta_i \in (0, 1)$ are agent i's instantaneous utility function and exponential discount factor, respectively. We assume that u_i is continuous and increasing. Let U_i denote agent i's lifetime utility function. The set of such lifetime utility functions is denoted by $\mathcal{U}$.[2]

Given $T \geq 1$, $\boldsymbol{h}^T = (\boldsymbol{x}_t)_{t<T} \in \mathbb{R}_+^{NT}$ is a history of allocations at evaluation period T. The initial period is $T = 0$ with the null history $\boldsymbol{h}^0 = \phi$. Let $H_T = \{(\boldsymbol{x}_t)_{t<T} | \boldsymbol{x}_t \in \mathbb{R}_+^N\}$ be the set of histories at T. We define the set of histories as $H = \bigcup_{T \in \mathcal{T}} H_T$.

Given any history $\boldsymbol{h}^T \in H$, the set of allocations of consumption streams from T is isomorphic to $(\mathbb{R}_+^N)^\infty$. Hence, slightly abusing the notation, we define the set of allocations of consumption streams as $X = \{\boldsymbol{x}^T \in (\mathbb{R}_+^N)^\infty | T \in \mathcal{T},\ \sup_{t \geq T} x_{it} < \infty\ (\forall i \in N)\}$.

A consumption stream $\boldsymbol{x}_i^T = (x_{it})_{t \geq T}$ is said to be *constant* if $x_{it} = x_{it'}$ for all $t, t' \geq T$. Hereafter, agent i's constant consumption stream from T is denoted by $\bar{\boldsymbol{x}}_i^T$. Without any risk of confusion, the instantaneous consumption level in $\bar{\boldsymbol{x}}_i^T$ is denoted by x_i. A *constant allocation* is an allocation of constant consumption streams denoted by $\bar{\boldsymbol{x}}^T = (\bar{\boldsymbol{x}}_i^T)_{i \in N}$. Let $\bar{X}$ be the set of constant allocations. Moreover, given $\boldsymbol{x}^T \in X$ and $\boldsymbol{w} \in \mathbb{R}_+^N$, $(\boldsymbol{w}, \boldsymbol{x}^T) \in X$ is an allocation where $\boldsymbol{x}^T$ follows $\boldsymbol{w}$.

Let $\mathcal{D} = \mathcal{U}^N$ be the set of profiles of lifetime utility functions. $\mathcal{D}^E$ is the subset of $\mathcal{D}$ where all agents have the same lifetime utility functions.

The social planner's problem is to rank allocations of consumption streams from T, given agents' lifetime utility functions and histories. A social welfare criterion $\succsim$ is a mapping that determines a binary relation over allocations of consumption streams for every history and profile of lifetime utility functions. Given $U = (U_i)_{i \in N} \in \mathcal{D}$ and $\boldsymbol{h}^T \in H$, $\succsim_{\boldsymbol{h}^T}^U$ denotes a binary relation over X. Unless otherwise noted, we assume that $\succsim_{\boldsymbol{h}^T}^U$ is a *quasi-ordering*,

[2] Our results hold even if u_i is further assumed to be concave.

i.e., reflexive and transitive. It is called an *ordering* if it is complete and transitive. The asymmetric and symmetric parts of $\succsim^U_{\boldsymbol{h}^T}$ are $\succ^U_{\boldsymbol{h}^T}$ and $\sim^U_{\boldsymbol{h}^T}$, respectively.

3 Efficiency, Equity, and Time Consistency

In this section, we introduce axioms of efficiency, equity, and time consistency. We also show that these axioms are incompatible.

The first axiom is the standard Pareto principle.

Weak Pareto. For all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

if $U_i(\boldsymbol{x}_i^T) \geq U_i(\boldsymbol{y}_i^T)$ for all $i \in N$, then $\boldsymbol{x}^T \succsim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T$, and

if $U_i(\boldsymbol{x}_i^T) > U_i(\boldsymbol{y}_i^T)$ for all $i \in N$, then $\boldsymbol{x}^T \succ^U_{\boldsymbol{h}^T} \boldsymbol{y}^T$.

This axiom implies that a unanimous improvement should be socially preferred. Note that this axiom implies *Pareto Indifference*, which requires that if $U_i(\boldsymbol{x}_i^T) = U_i(\boldsymbol{y}_i^T)$ for all $i \in N$, then $\boldsymbol{x}^T \sim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T$.

Next, we introduce an equity axiom.

Transfer to the worst-off. For all $U \in \mathcal{D}^E$, all $\boldsymbol{h}^T \in H$, all $\bar{\boldsymbol{x}}^T, \bar{\boldsymbol{y}}^T \in \bar{X}$ and all $\varepsilon > 0$, if there exist $j, k \in N$ such that $x_k = y_k + \varepsilon < y_j - \varepsilon = x_j$ where $y_k = 0 < y_i$ for all $i \neq k$, and $x_i = y_i$ for all $i \neq j, k$, then $\bar{\boldsymbol{x}}^T \succ^U_{\boldsymbol{h}^T} \bar{\boldsymbol{y}}^T$.

In situations where all agents have the same lifetime utility functions, this requirement implies that, in constant allocations, it should be socially preferable to transfer to the worst-off agent with zero consumption regardless of the past. Considering the example of Al and Bill in the Introduction, this axiom requires a transfer from Bill to Al without considering Al's responsibility for past consumption. Although Al was profligate, he is now and will be in poverty with zero consumption. It would be harsh to leave him in that severe situation and, thus, the axiom requires the social planner to help him. A history-independent social criterion can satisfy this axiom if it is preferable to help the worst-off on constant allocations.

The third axiom is a well-known axiom of time consistency.

Time Consistency. For all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, all $\boldsymbol{w} \in \mathbb{R}^N_+$, and all $\boldsymbol{x}^{T+1}, \boldsymbol{y}^{T+1} \in X$,

$$(\boldsymbol{w}, \boldsymbol{x}^{T+1}) \succsim^U_{\boldsymbol{h}^T} (\boldsymbol{w}, \boldsymbol{y}^{T+1}) \Longleftrightarrow \boldsymbol{x}^{T+1} \succsim^U_{(\boldsymbol{h}^T, \boldsymbol{w})} \boldsymbol{y}^{T+1}.$$

This axiom states that decisions at periods T and $T+1$ should be consistent. This is necessary for the social decision to be credible over time. Repeated applications of this axiom imply that social decisions at all $t \leq T-1$ and T are consistent: For arbitrary $U \in \mathcal{D}$, $\boldsymbol{h}^T \in H$, and $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\boldsymbol{x}^T \succsim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T \Longleftrightarrow (\boldsymbol{h}^T, \boldsymbol{x}^T) \succsim^U_{\boldsymbol{h}^0} (\boldsymbol{h}^T, \boldsymbol{y}^T).$$

Time Consistency is violated by history-independent egalitarian criteria such as the following one: For all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\boldsymbol{x}^T \succsim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T \Longleftrightarrow \min_{i \in N} \mu(\boldsymbol{x}^T_i, U) \geq \min_{i \in N} \mu(\boldsymbol{y}^T_i, U),$$

where $\mu(\boldsymbol{z}^T_i, U) = u_i^{-1}\big((1-\delta_i)\sum_{t \geq T} \delta_i^{t-T} u_i(z_{it})\big)$ for each $\boldsymbol{z}^T_i \in X$. Note that this ordering satisfies *Transfer to the Worst-Off*, and helps the worst-off agent without considering responsibility for past consumption. Those history-independent egalitarian criteria do not satisfy *Time Consistency* because the identity of the worst-off agent changes over time.

The following Proposition 1 shows that those axioms are incompatible even if all agents have the same lifetime utility function.

Proposition 1. On $\mathcal{D}^E$, there exists no $\succsim$ satisfying *Weak Pareto, Transfer to the Worst-off,* and *Time Consistency.*

Proof.[3] We show the result using a two-person example. Let $N = \{1, 2\}$, and $U \in \mathcal{D}$ be such that $u_i(x) = x$ and $\delta_i = \frac{1}{2}$ for all $i \in N$. We consider $\bar{\boldsymbol{x}}^{T+1}, \bar{\boldsymbol{y}}^{T+1} \in \bar{X}$ such that $\boldsymbol{x}_t = (10, 0)$ and $\boldsymbol{y}_t = (8, 2)$ for all $t \geq T+1$. Then, by *Transfer to the Worst-off*, we

[3] The author is grateful to Noriaki Kiguchi for helpful comments on the proof.

have $\bar{\boldsymbol{y}}^{T+1} \succ^U_{\hat{\boldsymbol{h}}^{T+1}} \bar{\boldsymbol{x}}^{T+1}$ for all $\hat{\boldsymbol{h}}^T \in H$, and thus $\bar{\boldsymbol{y}}^{T+1} \succ^U_{(\boldsymbol{h}^T, \boldsymbol{w})} \bar{\boldsymbol{x}}^{T+1}$ with $(\boldsymbol{h}^T, \boldsymbol{w}) \in H$ and $\boldsymbol{w} = (0, 20)$. *Time Consistency* implies that

$$\bar{\boldsymbol{y}}^{T+1} \succ^U_{(\boldsymbol{h}^T, \boldsymbol{w})} \bar{\boldsymbol{x}}^{T+1} \Longleftrightarrow (\boldsymbol{w}, \bar{\boldsymbol{y}}^{T+1}) \succ^U_{\boldsymbol{h}^T} (\boldsymbol{w}, \bar{\boldsymbol{x}}^{T+1}).$$

Note that

$$U_1(0, \bar{\boldsymbol{x}}_1^{T+1}) = 0 + \sum_{t=T+1}^{\infty} \frac{1}{2^{t-T}} 10 = 10,\ U_1(0, \bar{\boldsymbol{y}}_1^{T+1}) = 0 + \sum_{t=T+1}^{\infty} \frac{1}{2^{t-T}} 8 = 8,$$
$$U_2(20, \bar{\boldsymbol{x}}_2^{T+1}) = 20 + \sum_{t=T+1}^{\infty} \frac{1}{2^{t-T}} 0 = 20,\ U_2(20, \bar{\boldsymbol{y}}_2^{T+1}) = 20 + \sum_{t=T+1}^{\infty} \frac{1}{2^{t-T}} 2 = 22.$$

Now consider $\bar{\boldsymbol{x}}'^{T+1}, \bar{\boldsymbol{y}}'^{T+1} \in \bar{X}$ such that $\boldsymbol{x}'_t = (2, 12)$ and $\boldsymbol{y}'_t = (0, 14)$ for all $t \geq T+1$. Moreover, let $\boldsymbol{x}'^T, \boldsymbol{y}'^T$ be such that $\boldsymbol{x}_1'^T = (8, \bar{\boldsymbol{x}}_1'^{T+1})$, $\boldsymbol{x}_2'^T = (8, \bar{\boldsymbol{x}}_2'^{T+1})$, $\boldsymbol{y}_1'^T = (8, \bar{\boldsymbol{y}}_1'^{T+1})$, $\boldsymbol{y}_2'^T = (8, \bar{\boldsymbol{y}}_2'^{T+1})$. Then, we have

$$U_1(\boldsymbol{x}_1'^T) = 8 + \sum_{t=T+1}^{\infty} \frac{1}{2^{t-T}} 2 = 10,\ U_1(\boldsymbol{y}_1'^T) = 8 + \sum_{t=T+1}^{\infty} \frac{1}{2^{t-T}} 0 = 8,$$
$$U_2(\boldsymbol{x}_2'^T) = 8 + \sum_{t=T+1}^{\infty} \frac{1}{2^{t-T}} 12 = 20,\ U_2(\boldsymbol{y}_2'^T) = 8 + \sum_{t=T+1}^{\infty} \frac{1}{2^{t-T}} 14 = 22.$$

It follows from *Weak Pareto* that $\boldsymbol{x}'^T \sim^U_{\boldsymbol{h}^T} (\boldsymbol{w}, \bar{\boldsymbol{x}}^T)$ and $\boldsymbol{y}'^T \sim^U_{\boldsymbol{h}^T} (\boldsymbol{w}, \bar{\boldsymbol{y}}^T)$. Therefore, by $(\boldsymbol{w}, \bar{\boldsymbol{y}}^T) \succ^U_{\boldsymbol{h}^T} (\boldsymbol{w}, \bar{\boldsymbol{x}}^T)$ and transitivity, we have $(\boldsymbol{w}', \bar{\boldsymbol{y}}'^{T+1}) = \boldsymbol{y}'^T \succ^U_{\boldsymbol{h}^T} \boldsymbol{x}'^T = (\boldsymbol{w}', \bar{\boldsymbol{x}}'^{T+1})$, where $\boldsymbol{w}' = (8, 8)$. Then, *Time Consistency* implies $\bar{\boldsymbol{y}}'^{T+1} \succ^U_{(\boldsymbol{h}^T, \boldsymbol{w}')} \bar{\boldsymbol{x}}'^{T+1}$, which contradicts with *Transfer to the Worst-off*. $\square$

Proposition 1 shows that it is impossible to help the currently worst-off agent by completely forgiving past profligacy while taking responsibility into account. Actually, *Transfer to the Worst-Off* would not be compelling because the axiom requires fully ignoring past consumptions. If responsibility for past choices is important for social evaluation and the evaluation should be regarded as credible over time, we should consider another equity axiom that takes the past into account. We deal with this problem in the next section.

4 Axiomatization

In this section, we introduce another equity axiom and characterize the maximin social welfare criterion. First, we present the following equity axiom.

Hammond Equity on Constant Allocations. For all $U \in \mathcal{D}$, $\bar{\boldsymbol{x}}^0 = (x_i)_{i\in N}$, $\bar{\boldsymbol{y}}^0 = (y_i)_{i\in N} \in \bar{X}$, if there are $j, k \in N$ such that $y_k < x_k < x_j < y_j$ and $x_i = y_i$ for all $i \neq j, k$, then $\bar{\boldsymbol{x}}^0 \succsim^U_{\boldsymbol{h}^0} \bar{\boldsymbol{y}}^0$.

This axiom argues that, on constant allocations from the initial period, any reduction of inequality between a better-off and a worse-off agent should be socially weakly preferred. Note that the agents do not differ in terms of responsibility because their consumption levels are unequal from the initial period. This is a version of Hammond's (1976) equity axiom, which is a widely accepted but strong property. In Lemma 3, we derive this axiom using a much weaker equity axiom.

We also require a continuity axiom to obtain the characterization of the maximin criterion. Let us define a quasi-ordering R over $\mathbb{R}^N_+$ such that, for all $U \in \mathcal{D}$, for all $\bar{\boldsymbol{x}}^0 = (x_i)_{i\in N}, \bar{\boldsymbol{y}}^0 = (y_i)_{i\in N} \in \bar{X}$,

$$(x_i)_{i\in N} R (y_i)_{i\in N} \Longleftrightarrow \bar{\boldsymbol{x}}^0 \succsim^U_{\boldsymbol{h}^0} \bar{\boldsymbol{y}}^0.$$

In this case, we say that R is generated from $\succsim$ on $\bar{X}$.[4] The asymmetric and symmetric parts of R are denoted by P and I, respectively.

Continuity on Constant Allocations. For all $\boldsymbol{x} = (x_i)_{i\in N} \in \mathbb{R}^N_+$, $\{\boldsymbol{y} \in \mathbb{R}^N_+ | \boldsymbol{y} R \boldsymbol{x}\}$ and $\{\boldsymbol{y} \in \mathbb{R}^N_+ | \boldsymbol{x} R \boldsymbol{y}\}$ are closed, where R is generated from $\succsim$ on $\bar{X}$.

Here, we introduce a standard for interpersonal comparison used in our social welfare criterion. For each $\boldsymbol{h}^T_i = (h_{i0}, \cdots, h_{i,T-1}), \boldsymbol{x}^T_i$, and U_i, define

$$\psi(\boldsymbol{h}^T_i, \boldsymbol{x}^T_i, U_i) = u_i^{-1}\left((1-\delta_i)\left[\sum_{t=0}^{T-1} \delta_i^t u_i(h_{it}) + \sum_{t\geq T} \delta_i^t u_i(x_{it})\right]\right).$$

[4]The formulation is essentially introduced by Hayashi (2016).

This is the consumption level giving the agent the same utility as the normalized lifetime utility from the initial period.

Then, we obtain the following result.

Theorem 2. $\succsim$ satisfies *Weak Pareto, Hammond Equity on Constant Allocations, Time Consistency,* and *Continuity on Constant allocations* if and only if for all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\boldsymbol{x}^T \succsim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T \Longleftrightarrow \min_{i \in N} \psi(\boldsymbol{h}^T_i, \boldsymbol{x}^T_i, U_i) \geq \min_{i \in N} \psi(\boldsymbol{h}^T_i, \boldsymbol{y}^T_i, U_i).$$

Proof. The proof of the "if" part is straightforward. Hence, we only prove the "only if" part.

First, we show that for all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\min_{i \in N} \psi(\boldsymbol{h}^T_i, \boldsymbol{x}^T_i, U_i) > \min_{i \in N} \psi(\boldsymbol{h}^T_i, \boldsymbol{y}^T_i, U_i) \Longrightarrow \boldsymbol{x}^T \succ^U_{\boldsymbol{h}^T} \boldsymbol{y}^T.$$

By *Time Consistency*, we have $\boldsymbol{x}^T \succsim_{\boldsymbol{h}^T} \boldsymbol{y}^T$ if and only if $(\boldsymbol{h}^T, \boldsymbol{x}^T) \succsim_{\boldsymbol{h}^0} (\boldsymbol{h}^T, \boldsymbol{y}^T)$. Thus, we want to show $(\boldsymbol{h}^T, \boldsymbol{x}^T) \succ^U_{h^0} (\boldsymbol{h}^T, \boldsymbol{y}^T)$.

Let $\bar{\boldsymbol{x}}^0 = (x_i)_{i \in N}, \bar{\boldsymbol{y}}^0 = (y_i)_{i \in N} \in \bar{X}$ be constant allocations such that, in every period, each i has

$$x_i = \psi(\boldsymbol{h}^T_i, \boldsymbol{x}^T_i, U_i), y_i = \psi(\boldsymbol{h}^T_i, \boldsymbol{y}^T_i, U_i).$$

By *Weak Pareto*, $\bar{\boldsymbol{x}}^0 \sim^U_{\boldsymbol{h}^0} (\boldsymbol{h}^T, \bar{\boldsymbol{x}}^T)$ and $\bar{\boldsymbol{y}}^0 \sim^U_{\boldsymbol{h}^0} (\boldsymbol{h}^T, \bar{\boldsymbol{y}}^T)$, and therefore by transitivity, it suffices to prove $\bar{\boldsymbol{x}}^0 \succ^U_{\boldsymbol{h}^0} \bar{\boldsymbol{y}}^0$.

If $x_i > y_i$ for all $i \in N$, it is straightforward to have the desired result by *Weak Pareto*. Thus, we assume that $y_i \geq x_i$ for some i. Without loss of generality, and by *Weak Pareto* (to have only strict inequalities), we can assume that $y_1 < y_2 < \cdots < y_n$. By repeated applications of *Hammond Equity on Constant Allocations*, for $\bar{\boldsymbol{z}}^0 \in \bar{X}$ such that

$$y_1 < z_1 < z_i < \min\{y_2, \min_i x_i\},$$

we obtain $\bar{\boldsymbol{z}}^0 \succsim^U_{\boldsymbol{h}^0} \bar{\boldsymbol{y}}^0$. *Weak Pareto* implies $\bar{\boldsymbol{x}}^0 \succ^U_{\boldsymbol{h}^0} \bar{\boldsymbol{z}}^0$. Therefore, by transitivity, we have the desired result.

Next, we show that

$$\min_{i\in N}\psi(\boldsymbol{h}_i^T,\boldsymbol{x}_i^T,U)=\min_{i\in N}\psi(\boldsymbol{h}_i^T,\boldsymbol{y}_i^T,U)\Longrightarrow \boldsymbol{x}^T\sim^U_{\boldsymbol{h}^T}\boldsymbol{y}^T.$$

Define $\boldsymbol{x}(\varepsilon)^T\in X$ and $\bar{\boldsymbol{x}}(\varepsilon)^0=(x_i(\varepsilon))_{i\in N},\bar{\boldsymbol{y}}^0=(y_i)_{i\in N}\in\bar{X}$ such that, for all $i\in N$,

$$\begin{aligned}&x_{it}(\varepsilon)=x_{it}+\varepsilon\ \text{ for each } t\geq T,\\&x_i(\varepsilon)=\psi(\boldsymbol{h}_i^T,\boldsymbol{x}_i(\varepsilon)^T,U),\\&y_i=\psi(\boldsymbol{h}_i^T,\boldsymbol{y}_i^T,U).\end{aligned}$$

Then, by the result above, we obtain $\bar{\boldsymbol{x}}(\varepsilon)^0\succ^U_{\boldsymbol{h}^0}\bar{\boldsymbol{y}}^0$ when $\varepsilon>0$, and $\bar{\boldsymbol{y}}^0\succ^\delta_{\boldsymbol{h}^0}\bar{\boldsymbol{x}}(\varepsilon)^0$ when $\varepsilon<0$. We also have $(\boldsymbol{h}^T,\boldsymbol{x}(\varepsilon)^T)\sim^U_{\boldsymbol{h}^0}\bar{\boldsymbol{x}}(\varepsilon)^0$ for all $\varepsilon\in\mathbb{R}$ and $(\boldsymbol{h}^T,\boldsymbol{y}^T)\sim^U_{\boldsymbol{h}^0}\bar{\boldsymbol{y}}^0$ by *Time Consistency* and *Weak Pareto*.

Here we consider R generated from $\succsim$ on $\bar{X}$. Then, we obtain

$$(x_i(\varepsilon))_{i\in N}R(y_i)_{i\in N}\Longleftrightarrow\bar{\boldsymbol{x}}(\varepsilon)^0\succsim^\delta_{\boldsymbol{h}^0}\bar{\boldsymbol{y}}^0.$$

Let $\varepsilon>0$. By the argument above, we have $\bar{\boldsymbol{x}}(\varepsilon)^0\succ^U_{\boldsymbol{h}^0}\bar{\boldsymbol{y}}^0$, and hence $(x_i(\varepsilon))_{i\in N}P(y_i)_{i\in N}$ for all $\varepsilon>0$. Letting $\varepsilon\to0$, it follows from *Continuity on Constant allocations* that $(x_i(0))_{i\in N}R(y_i)_{i\in N}$, which implies $\bar{\boldsymbol{x}}(0)^0\succsim^U_{\boldsymbol{h}^0}\bar{\boldsymbol{y}}^0$. Similarly, considering the case of $\varepsilon<0$, $\bar{\boldsymbol{y}}^0\succ^U_{\boldsymbol{h}^0}\bar{\boldsymbol{x}}(\varepsilon)^0$, and hence $(y_i)_{i\in N}P(x_i(\varepsilon))_{i\in N}$. Then, letting $\varepsilon\to0$, it follows from *Continuity on Constant allocations* that $(y_i)_{i\in N}R(x_i(0))_{i\in N}$, which implies $\bar{\boldsymbol{y}}^0\succsim^U_{\boldsymbol{h}^0}\bar{\boldsymbol{x}}(0)^0$. Therefore, we have $\bar{\boldsymbol{x}}(0)^0\sim^U_{\boldsymbol{h}^0}\bar{\boldsymbol{y}}^0$. *Weak Pareto* implies $\bar{\boldsymbol{x}}(0)^0\sim^U_{\boldsymbol{h}^0}(\boldsymbol{h}^T,\boldsymbol{x}(0)^T)=(\boldsymbol{h}^T,\boldsymbol{x}^T)$. By *Time Consistency* and transitivity, we obtain the desired result. $\square$

5 Further Discussion

In this section, we provide further discussions relevant to our social criterion. The maximin social criterion is based on the strongest preference for smoothing consumption in society. Moreover, it is egalitarian and history-dependent in the sense that it may not demand redistribution from an agent with higher consumption to another with lower consumption in the present if the former was worse-off in the past. For instance, in the case of Al and Bill, even if $x_{At} < y_{At} < y_{Bt} < x_{Bt}$ for all $t \geq T$, reducing the inequality in $\boldsymbol{x}^T$ may not be desirable for the social criterion if Al's situation was much better than Bill's in the past. This is because *Time Consistency* requires taking the past allocation into account, and *Equity for Equals* requires the redistribution when the agents have the same lifetime utility function and past consumption levels. A redistribution from Bill to Al in $\boldsymbol{x}^T$ is supported by the criterion if $\psi(\boldsymbol{h}_B^T, \boldsymbol{x}_B^T, U_B) > \psi(\boldsymbol{h}_A^T, \boldsymbol{x}_A^T, U_A)$. Note that, even when Al's past consumptions were higher than Bill's ($\boldsymbol{h}_A^T \gg \boldsymbol{h}_B^T$), the maximin criterion may demand redistribution from Bill to Al if Al's consumption levels in the present and future are sufficiently lower than Bill's. We believe that this is reasonable because it is too harsh to leave agents in severe poverty even if they were not prudent.

Although our maximin criterion is history-dependent, none of our axioms describes a restriction on how the evaluation of two allocations should vary depending on past consumptions. Here, we present two axioms of history-dependence in terms of responsibility that are satisfied by the maximin criterion.

Supporting Transfer under Equal Responsibility. For all $U \in \mathcal{D}^E$, all $\boldsymbol{h}^T \in H$ such that $T \geq 1$, all $\bar{\boldsymbol{x}}^T, \bar{\boldsymbol{y}}^T \in \bar{X}$, and all $\varepsilon > 0$, such that $T \geq 1$ and $x_i = y_i + (n-1)\varepsilon < y_j - \varepsilon = x_j$ for all $j \neq i$ and $\bar{\boldsymbol{y}} \succ^U_{\boldsymbol{h}^T} \bar{\boldsymbol{x}}$, $\bar{\boldsymbol{x}} \succ^U_{\tilde{\boldsymbol{h}}^T} \bar{\boldsymbol{y}}$ where $\tilde{\boldsymbol{h}}_j^T = \sum_{k \in N} \boldsymbol{h}_k^T / n$ for all $j \in N$.

Opposing Transfer under Profligacy. For all $U \in \mathcal{D}^E$, all $\boldsymbol{h}^T \in H$ such that $T \geq 1$, all $\bar{\boldsymbol{x}}^T, \bar{\boldsymbol{y}}^T \in \bar{X}$, and all $\varepsilon > 0$, such that $x_i = y_i + (n-1)\varepsilon < y_j - \varepsilon = x_j$ for all $j \neq i$ and $\bar{\boldsymbol{x}} \succ^U_{\boldsymbol{h}^T} \bar{\boldsymbol{y}}$, there exists $\tilde{\boldsymbol{h}}^T$ such that $\tilde{\boldsymbol{h}}_j^T \ll \tilde{\boldsymbol{h}}_i^T$ for all $j \neq i$ and $\bar{\boldsymbol{y}} \succ^U_{\tilde{\boldsymbol{h}}^T} \bar{\boldsymbol{x}}$.

These axioms consider a redistribution from the better-off agents to the worst-off agent on constant allocations from T where their preferences are the same. The first axiom insists that, even if the redistribution is not considered good in the case where the worst-off agent was prodigal, the redistribution becomes desirable if all agents' past consumptions are the same and there is no difference among agents in terms of responsibility. Similarly, the second axiom argues that, even if the redistribution is supported under a history, it is denied if the worst-off agent was profligate. Both axioms describe the relations between the evaluation of redistribution at T and the past consumptions, taking responsibility into account. These axioms are satisfied by our maximin criterion.[5]

Although *Hammond Equity on Constant Allocations* can only be applied on constant allocations, our maximin satisfies another equity axiom as follows.

Dominance Aversion for Equals. For all $U \in \mathcal{D}^E$, all $\boldsymbol{h}^T \in H$, all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$, if there exist $j, k \in N$ such that $\boldsymbol{h}_j^T \leq \boldsymbol{h}_k^T$, $\boldsymbol{y}_j \ll \boldsymbol{x}_j \leq \boldsymbol{x}_k \ll \boldsymbol{y}_k$, and $\boldsymbol{x}_i = \boldsymbol{y}_i$ for all $i \neq j, k$, then $\boldsymbol{x}^T \succsim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T$

This axiom states that if two agents have the same preferences and one has more consumption than the other in the past, more equal consumptions are socially weakly preferred.[6]

Next, we derive *Hammond Equity on Constant Allocations* using a weaker equity axiom and an invariance axiom. The equity axiom is as follows.

Equity for Equals. For all $U \in \mathcal{D}^E$, all $\boldsymbol{h}^T \in H$, all $\bar{\boldsymbol{x}}^T, \bar{\boldsymbol{y}}^T \in \bar{X}$ and all $\varepsilon > 0$, if there exist $j, k \in N$ such that $\boldsymbol{h}_j^T = \boldsymbol{h}_k^T$, $x_j = y_j + \varepsilon < y_k - \varepsilon = x_k$, and $x_i = y_i$ for all $i \neq j, k$, then $\bar{\boldsymbol{x}}^T \succsim^U_{\boldsymbol{h}^T} \bar{\boldsymbol{y}}^T$.

This axiom argues that a transfer from a better-off to a worse-off agent on constant allocations should be accepted when their lifetime utility functions are the same and the past allocations are equal. The idea is that, if agents have identical preferences and consumed equally in the past, there is no difference among them in terms of responsibility, and thus inequality is caused by factors for which they are not responsible. Then, redistribution to reduce such inequality should be socially acceptable. For later discussion, we introduce the following strict version.

[5] We consider only redistributions over constant allocations for simplicity. It is possible to formulate similar axioms of history-dependence for nonconstant allocations.

[6] We do not require the principle of dominance aversion for agents with different preferences because it is incompatible with *Weak Pareto* under *Time Consistency*. See Fleurbaey and Trannoy (2003) for a similar impossibility result in the static model.

Strict Equity for Equals. For all $U \in \mathcal{D}^E$, all $\boldsymbol{h}^T \in H$, all $\bar{\boldsymbol{x}}^T, \bar{\boldsymbol{y}}^T \in \bar{X}$ and all $\varepsilon > 0$, if there exist $j, k \in N$ such that $\boldsymbol{h}_j^T = \boldsymbol{h}_k^T$, $x_j = y_j + \varepsilon < y_k - \varepsilon = x_k$, and $x_i = y_i$ for all $i \neq j, k$, then $\bar{\boldsymbol{x}}^T \succ_{\boldsymbol{h}^T}^U \bar{\boldsymbol{y}}^T$.

We also introduce an independence axiom.[7] This axiom insists that the social evaluation should be independent of discount factors and instantaneous utility functions when comparing constant allocations from the initial period.

Invariance on Constant Allocations. For all $U, U' \in \mathcal{D}$, for all $\bar{\boldsymbol{x}}^0, \bar{\boldsymbol{y}}^0 \in \bar{X}$,

$$\bar{\boldsymbol{x}}^0 \succsim_{\boldsymbol{h}^0}^U \bar{\boldsymbol{y}}^0 \Longleftrightarrow \bar{\boldsymbol{x}}^0 \succsim_{\boldsymbol{h}^0}^{U'} \bar{\boldsymbol{y}}^0.$$

The intuition behind this axiom is as follows. For each agent, the instantaneous utility function represents the degree of preference for consumption smoothing, and the discount factor indicates the weight of the future utility relative to the current utility. When evaluating constant allocations, the consumptions are completely smoothed and the utility levels are constant over time. Then, discount factors and instantaneous utility functions are irrelevant to the evaluation because they do not play any role, and constant consumption can be regarded as a measure of living standards because the consumption level corresponds one-to-one to the lifetime utility level. Therefore, the evaluation of the constant allocations should be independent of the lifetime utility functions.

Then, we obtain the following Lemma 3.

Lemma 3. *Weak Pareto, Time Consistency, Equity for Equals*, and *Invariance on Constant Allocations* together imply *Hammond Equity on Constant Allocations*

The proof is given in the Appendix.

We can provide another axiomatization of the maximin criterion, which immediately follows from Theorem 2 and Lemma 3.

[7] A similar axiom is introduced by Chambers and Echenique (2012) in the context of preference aggregation under risk.

Theorem 4. $\succsim$ satisfies *Weak Pareto, Equity for Equals, Time Consistency, Invariance on Constant allocations,* and *Continuity on Constant allocations* if and only if for all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\boldsymbol{x}^T \succsim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T \iff \min_{i \in N} \psi(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T, U_i) \geq \min_{i \in N} \psi(\boldsymbol{h}_i^T, \boldsymbol{y}_i^T, U_i).$$

A weakness of the maximin criterion is that it violates *Strong Pareto*. This axiom requires that, given any history, if at least one person's lifetime utility increases without a decrease in others' lifetime utilities, it should be considered a social improvement.

Strong Pareto. For all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,
if $U_i(\boldsymbol{x}_i^T) \geq U_i(\boldsymbol{y}_i^T)$ for all $i \in N$, then $\boldsymbol{x}^T \succsim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T$, and
if in addition $U_j(\boldsymbol{x}_j^T) > U_j(\boldsymbol{y}_j^T)$ for some $j \in N$, then $\boldsymbol{x}^T \succ^U_{\boldsymbol{h}^T} \boldsymbol{y}^T$.

We note that, in Lemma 3, *Hammond Equity on Constant Allocations* is obtained with strict preference if *Weak Pareto* is replaced with *Strong Pareto*. This can be easily checked by just replacing *Weak Pareto* with *Strong Pareto* in the proof of Lemma 3.

We introduce one more axiom to obtain the characterization.

Suppes Indifference for Equals. For all $U \in \mathcal{D}^E$ and all $\boldsymbol{h}^T \in H$ such that $\boldsymbol{h}_i = \boldsymbol{h}_j$ for all $i, j \in N$, for all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$, if there exists a permutation $\pi : N \to N$ such that $x_i = y_{\pi(i)}$ for each $i \in N$, then $\boldsymbol{x}^T \sim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T$.

This axiom is based on the grading principle of justice by Suppes (1966). If agents have the same lifetime utility functions and past consumptions, there is no difference among them in terms of responsibility. This axiom requires the social planner to treat agents impartially in such situations.

To define the leximin criterion, we introduce some notations. Given an n-vector $\boldsymbol{a} \in \mathbb{R}^N_+$, let $a_{(i)}$ be the ith lowest element in the vector. $\geq_{lex}$ denotes the leximin criterion defined as follows. For all $\boldsymbol{a}, \boldsymbol{b} \in \mathbb{R}^N_+$, $\boldsymbol{a} >_{lex} \boldsymbol{b}$ if there exists $r \leq n$ such that $a_{(i)} = b_{(i)}$ for all $i \leq r-1$ and $a_{(r)} > b_{(r)}$, and $\boldsymbol{a} =_{lex} \boldsymbol{b}$ if $a_{(i)} = b_{(i)}$ for all $i \leq n$. We denote $\boldsymbol{a} \geq_{lex} \boldsymbol{b}$ if $\boldsymbol{a} >_{lex} \boldsymbol{b}$ or $\boldsymbol{a} =_{lex} \boldsymbol{b}$.

Now we have the following result.

Theorem 5. $\succsim$ satisfies *Strong Pareto, Time Consistency, Equity for Equals*, *Invariance on Constant Allocations*, and *Suppes Indifference for Equals* if and only if , for all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\big(\psi(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T, U_i)\big)_{i\in N} \geq_{lex} \big(\psi(\boldsymbol{h}_i^T, \boldsymbol{y}_i^T, U_i)\big)_{i\in N} \Longleftrightarrow \boldsymbol{x}^T \succsim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T.$$

Proof. Since the proof of the "if" part is straightforward, we only prove the "only if" part.

First, we claim the following fact: For all $U \in \mathcal{D}$, $\bar{\boldsymbol{x}}^0 = (x_i)_{i\in N}, \bar{\boldsymbol{y}}^0 = (y_i)_{i\in N} \in \bar{X}$, if there exists a permutation $\pi : N \to N$ such that $x_i = y_{\pi(i)}$ for each $i \in N$, then $\bar{\boldsymbol{x}}^0 \sim^U_{\boldsymbol{h}^0} \bar{\boldsymbol{y}}^0$. By *Invariance on Constant Allocations*, for any $\bar{U} \in \mathcal{D}^E$, we have $\bar{\boldsymbol{x}}^0 \succsim^{\bar{U}}_{\boldsymbol{h}^0} \bar{\boldsymbol{y}}^0$ if and only if $\bar{\boldsymbol{x}}^0 \succsim^U_{\boldsymbol{h}^0} \bar{\boldsymbol{y}}^0$. Then, we can apply *Suppes Indifference for Equals* to obtain $\bar{\boldsymbol{x}}^0 \sim^{\bar{U}}_{\boldsymbol{h}^0} \bar{\boldsymbol{y}}^0$, which implies $\bar{\boldsymbol{x}}^0 \sim^U_{\boldsymbol{h}^0} \bar{\boldsymbol{y}}^0$.

Next, we prove that R on $\mathbb{R}^N_+$ generated from $\succsim$ is represented by the leximin criterion $\geq_{lex}$. By the assumption, for all $U \in \mathcal{D}$, $\bar{\boldsymbol{x}}^0 = (x_i)_{i\in N}, \bar{\boldsymbol{y}}^0 = (y_i)_{i\in N} \in \bar{X}$,

(1) if $x_i \geq y_i$ for all $i \in N$ and $x_j > y_j$ for some $j \in N$, it follows that $U_i(\bar{\boldsymbol{x}}_i^0) \geq U_i(\bar{\boldsymbol{y}}_i^0)$ for all $i \in N$ and $U_j(\bar{\boldsymbol{x}}_j^0) > U_j(\bar{\boldsymbol{y}}_j^0)$ for some $j \in N$, and thus $\bar{\boldsymbol{x}}^0 \succ^U_{\boldsymbol{h}^0} \bar{\boldsymbol{y}}^0$ by *Strong Pareto*, which implies $(x_i)_{i\in N} P (y_i)_{i\in N}$;

(2) if $y_k > x_k > x_j > y_j$ for some j, k and $x_i = y_i$ for all $i \neq j, k$, as mentioned after the statement of *Strong Pareto*, the Lemma 3 with *Strong Pareto* implies $\bar{\boldsymbol{x}}^0 \succ^U_{\boldsymbol{h}^0} \bar{\boldsymbol{y}}^0$ by *Hammond Equity for Constant Allocations* with the strict prefefrence, which implies $(x_i)_{i\in N} P (y_i)_{i\in N}$;

(3) if there exists a permutation $\pi : N \to N$ such that $x_i = y_{\pi(i)}$ for all $i \in N$, we have $\bar{\boldsymbol{x}}^0 \succsim^U_{\boldsymbol{h}^0} \bar{\boldsymbol{y}}^0$ by the above fact, which implies $(x_i)_{i\in N} I (y_i)_{i\in N}$.

(1)–(3) together mean that, as an evaluation criterion over n-vectors, R satisfies *Strong Pareto, Hammond Equity*, and *Suppes Indifference*, respectively. Then, it follows from Theorem 4 and 5 of Hammond (1979) that R is represented by $\geq_{lex}$ on $\mathbb{R}^N_+$.[8] From this result, we have the following. For all $U \in \mathcal{D}$, $\bar{\boldsymbol{x}}^0 = (x_i)_{i\in N}, \bar{\boldsymbol{y}}^0 = (y_i)_{i\in N} \in \bar{X}$,

$$(x_i)_{i\in N} = \big(\psi(\boldsymbol{h}_i^0, \bar{\boldsymbol{x}}_i^0, U_i)\big)_{i\in N} \geq_{lex} \big(\psi(\boldsymbol{h}_i^0, \bar{\boldsymbol{x}}_i^0, U_i)\big)_{i\in N} = (y_i)_{i\in N} \Longleftrightarrow \bar{\boldsymbol{x}}^0 \succsim^U_{\boldsymbol{h}^0} \bar{\boldsymbol{y}}^0.$$

Now we show the result. It follows from *Time Consistency* and *Strong Pareto* that, for all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$, there exist $\bar{\boldsymbol{x}}^0 = (x_i)_{i\in N}, \bar{\boldsymbol{y}}^0 = (y_i)_{i\in N} \in \bar{X}$ such that $x_i = \psi(\boldsymbol{h}_i^T, \boldsymbol{y}_i^T, U_i)$ and $y_i = \psi(\boldsymbol{h}_i^T, \boldsymbol{y}_i^T, U_i)$ and

$$\boldsymbol{x}^T \succsim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T \Longleftrightarrow \big(\psi(\boldsymbol{h}_i^0, \bar{\boldsymbol{x}}_i^0, U_i)\big)_{i\in N} = \bar{\boldsymbol{x}}^0 \succsim^U_{\boldsymbol{h}^0} = \bar{\boldsymbol{y}}^0 \big(\psi(\boldsymbol{h}_i^0, \bar{\boldsymbol{y}}_i^0, U_i)\big)_{i\in N}.$$

From the above argument, the evaluation over constant allocations is represented by the leximin criterion. Therefore, we have

$$\boldsymbol{x}^T \succsim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T \Longleftrightarrow \big(\psi(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T, U_i)\big)_{i\in N} \geq_{lex} \big(\psi(\boldsymbol{h}_i^T, \boldsymbol{y}_i^T, U_i)\big)_{i\in N},$$

which is the desired result. □

We close this section by discussing some generalizations. First, although we use the infinite horizon model, similar results can be obtained in a finite horizon model with almost the same proof. Second, with suitable modifications (admitting a constant consumption equivalence), we can obtain similar results even if lifetime utility functions are not additively separable functions. On this general domain, we can flexibly change the preferences on constant allocations, and thus can use almost the same technique as the proof of Lemma 3.

6 Independence of the Axioms in Lemma 3

In this section, we discuss the roles for the axioms in Lemma 3. These axioms are crucial for our characterization results because they together imply the strict ranking in Theorem

[8] Hammond (1979) considers n-vectors on $\mathbb{R}^n$, but the proofs are valid also on $\mathbb{R}^n_+$.

2, which is consistent with both the maximin and leximin social criteria. Using examples of social criteria violating *Hammond Equity on Constant Allocations*, we also argue that each axiom in Lemma 3 is indispensable for the result.

Weak Pareto is obviously important. The following criterion satisfies *Invariance on Constant Allocations*, *Equity for Equals*, and *Time Consistency*, but violates *Weak Pareto*. For all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\boldsymbol{x}^T \tilde{\succsim}^U_{\boldsymbol{h}^T} \boldsymbol{y}^T \Longleftrightarrow \sum_{i \in N}(1-\delta_i)\left[\sum_{t=0}^{T-1} \delta_i^t h_{it} + \sum_{t \geq T} \delta_i^t x_{it}\right] \geq \sum_{i \in N}(1-\delta_i)\left[\sum_{t=0}^{T-1} \delta_i^t h_{it} + \sum_{t \geq T} \delta_i^t y_{it}\right].$$

Without the Pareto principle, we have social criteria that do not respect agents' preferences.

To see the necessity of *Equity for Equals*, consider the following criterion $\hat{\succsim}$: For all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\boldsymbol{x}^T \hat{\succsim}^U_{\boldsymbol{h}^T} \boldsymbol{y}^T \Longleftrightarrow \sum_{i \in N} \psi(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T, U_i) \geq \sum_{i \in N} \psi(\boldsymbol{h}_i^T, \boldsymbol{y}_i^T, U_i).$$

This criterion obviously satisfies *Strong Pareto, Time Consistency*, and *Invariance on Constant Allocations*. However, $\hat{\succsim}$ violates *Equity for Equals*. To show this, consider a two-person situation $N = \{1, 2\}$ where U is such that

$$u_1(x) = u_2(x) = \begin{cases} 2x \text{ for } x \in [0, 100], \\ 200 + (x - 100) = 100 + x \text{ for } x > 100, \end{cases}$$

and $\delta_1 = \delta_2 = 1/2$. Let $\boldsymbol{h}^1$ be such that $h_{10} = h_{20} = 50$, and $\bar{\boldsymbol{x}}^1, \bar{\boldsymbol{y}}^1 \in \bar{X}$ be such that $(x_1, x_2) = (400, 0)$ and $(y_1, y_2) = (220, 180)$. Then, *Equity for Equals* requires that $\bar{\boldsymbol{y}}^1$ should be socially weakly preferred to $\bar{\boldsymbol{x}}^1$ under U and $\boldsymbol{h}^1$. However, we have

$$(1-\delta_1)U_1(\boldsymbol{h}_1^1, \bar{\boldsymbol{x}}_1^1) = \frac{100+500}{2} = 300,\ (1-\delta_2)U_2(\boldsymbol{h}_2^1, \bar{\boldsymbol{x}}_2^1) = \frac{100+0}{2} = 50,$$
$$(1-\delta_1)U_1(\boldsymbol{h}_1^1, \bar{\boldsymbol{y}}_1^1) = \frac{100+320}{2} = 210,\ (1-\delta_2)U_2(\boldsymbol{h}_2^1, \bar{\boldsymbol{y}}_2^1) = \frac{100+280}{2} = 190,$$

and hence, we obtain

$$\psi(\boldsymbol{h}_1^1, \bar{\boldsymbol{x}}_1^1, U_1) = 300 - 100 = 200,\ \psi(\boldsymbol{h}_2^1, \bar{\boldsymbol{x}}_2^1, U_2) = 50/2 = 25,$$
$$\psi(\boldsymbol{h}_1^1, \bar{\boldsymbol{y}}_1^1, U_1) = 210 - 100 = 110,\ \psi(\boldsymbol{h}_2^1, \bar{\boldsymbol{y}}_2^1, U_2) = 190/2 = 95.$$

This means that $\bar{\boldsymbol{x}}^1 \hat{\succ}^U_{\boldsymbol{h}^1} \bar{\boldsymbol{y}}^1$.[9]

Time Consistency is also necessary for Lemma 3. To see this, consider the ordering $\bar{\succsim}$ such that for all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\boldsymbol{x}^T \bar{\succsim}^U_{\boldsymbol{h}^T} \boldsymbol{y}^T \Longleftrightarrow \sum_{i \in N} \mu(\boldsymbol{x}^T_i, U_i) \geq \sum_{i \in N} \mu(\boldsymbol{y}^T_i, U_i),$$

where $\mu(\boldsymbol{x}^T_i, U) = u_i^{-1}\big((1-\delta_i)\sum_{t \geq T} \delta_i^{t-T} u_i(x_{it})\big)$. This criterion obviously satisfies *Strong Pareto* and *Invariance on Constant Allocations*. It also satisfies *Equity for Equals* if the instantaneous utility functions are concave. However, it violates *Time Consistency*: Consider a two-person situation $N = \{1, 2\}$ where U is such that

$$u_1(x) = u_2(x) = \begin{cases} 2x \text{ for } x \in [0, 3], \\ 6 + (x - 3) = 3 + x \text{ for } x > 3, \end{cases}$$

and $\delta_1 = \delta_2 = 1/2$. Let $\boldsymbol{h}^1$ be such that $h_{10} = 0$, $h_{20} = 1$, and $\bar{\boldsymbol{x}}^1, \bar{\boldsymbol{y}}^1 \in \bar{X}$ be such that $(x_1, x_2) = (9, 1)$ and $(y_1, y_2) = (1, 9)$. We obviously have $\bar{\boldsymbol{x}}^1 \bar{\sim}^U_{\boldsymbol{h}^1} \bar{\boldsymbol{y}}^1$. However,

$$(1-\delta_1)U_1(\boldsymbol{h}^1_1, \bar{\boldsymbol{x}}^1_1) = \frac{0+12}{2} = 6,\ (1-\delta_2)U_2(\boldsymbol{h}^1_2, \bar{\boldsymbol{x}}^1_2) = \frac{2+2}{2} = 2,$$
$$(1-\delta_1)U_1(\boldsymbol{h}^1_1, \bar{\boldsymbol{y}}^1_1) = \frac{0+2}{2} = 1,\ (1-\delta_2)U_2(\boldsymbol{h}^1_2, \bar{\boldsymbol{y}}^1_2) = \frac{2+12}{2} = 7,$$

and hence, we obtain

$$\mu(\boldsymbol{h}^1_1, \bar{\boldsymbol{x}}^1_1, U_1) = 3,\ \mu(\boldsymbol{h}^1_2, \bar{\boldsymbol{x}}^1_2, U_2) = 1,$$
$$\mu(\boldsymbol{h}^1_1, \bar{\boldsymbol{y}}^1_1, U_1) = 0.5,\ \mu(\boldsymbol{h}^1_2, \bar{\boldsymbol{y}}^1_2, U_2) = 4.$$

This means that $(\boldsymbol{h}^1, \bar{\boldsymbol{y}}^1) \bar{\succ}^U_{\boldsymbol{h}^0} (\boldsymbol{h}^1, \bar{\boldsymbol{x}}^1)$ and *Time Consistency* is violated.

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[9] When the instantaneous utility functions are linear, $\hat{\succsim}$ satisfies *Equity for Equals*. This can be checked by $\psi(\boldsymbol{h}^T_i, \boldsymbol{x}^T_i, U_i) = (1-\delta_i)[\sum_{t=0}^{T-1} \delta_i^t h_{it} + \sum_{t \geq T} \delta_i^t x_{it}]$ in this case. However, our example above shows that $\hat{\succsim}$ violates *Equity for Equals* under the more concave utility function. This is contrary to the usual argument that the more concave are the agent utility functions, the more inequality averse are the social welfare evaluations. This is because, as discussed later, social criteria is ordinal under *Pareto Indifference*, *Time Consistency*, and *Invariance on Constant Allocations*. The invariance axiom is especially essential for this point.

The above examples show that each of the three axioms play crucial roles in deriving the infinite inequality aversion in the Lemma 3. If we drop any of the three axioms, we have social criteria that are not inequality averse. The proof of Lemma 3 shows that, for any two constant allocations satisfying the axioms of *Hammond Equity on Constant Allocations*, there exists a preference profile under which the three axiom together imply that the allocation with the more equal consumptions between the two agents is socially weakly preferred. Hence, the combination of the three axioms has a strong implication of equality. By *Invariance on Constant Allocations*, such social preferences for more equal constant consumptions are invariant of the lifetime utility functions.

Next, we examine *Invariance on Constant Allocations* in more detail. There are reasonable social criteria that satisfy *Weak Pareto*, *Equity for Equals* and *Time Consistency*, but violate the invariance axiom. One example is an intertemporal version of the Ω-equivalent maximin criterion (Fleurbaey and Maniquet, 2011), where $\Omega \in \mathbb{R}^{\infty}_{++}$ is the exogenously given endowment stream. This criterion is defined as follows. For all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\boldsymbol{x}^T \succsim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T \iff \min_{i \in N} \omega(\boldsymbol{h}^T_i, \boldsymbol{x}^T_i, U_i) \geq \min_{i \in N} \omega(\boldsymbol{h}^T_i, \boldsymbol{y}^T_i, U_i),$$

where

$$\omega(\boldsymbol{h}^T_i, \boldsymbol{x}^T_i, U_i) = \inf \left\{ a \in \mathbb{R}_+ \middle| U_i(a\Omega) \geq \sum_{t=0}^{T-1} \delta_i^t u_i(h_{it}) + \sum_{t \geq T} \delta_i^t u_i(x_{it}) \right\}.$$

This criterion is different from the maximin criterion in Theorem 2 if Ω is not constant. Another example is an intertemporal version of the r-budget-equivalent criterion (Fleurbaey and Maniquet, 2011), where $r \in (0, 1)$ is the exogenously given price. This criterion is defined as follows. For all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\boldsymbol{x}^T \succsim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T \iff \min_{i \in N} \lambda_r(\boldsymbol{h}^T_i, \boldsymbol{x}^T_i, U_i) \geq \min_{i \in N} \lambda_r(\boldsymbol{h}^T_i, \boldsymbol{y}^T_i, U_i),$$

where

$$\lambda_r(\boldsymbol{h}^T_i, \boldsymbol{x}^T_i, U_i) = \inf \left\{ \sum_{t \in \mathcal{T}} r^t z_{it} \middle| U_i(\boldsymbol{z}^0_i) \geq \sum_{t=0}^{T-1} \delta_i^t u_i(h_{it}) + \sum_{t \geq T} \delta_i^t u_i(x_{it}) \right\}.$$

The leximin versions of these two criteria are examples satisfying *Strong Pareto* and *Strict Equity for Equals* but violating *Invariance on Constant Allocations*.

Under *Pareto Indifference*, *Time Consistency* and *Invariance on Constant Allocations*, we can work with equivalent constant allocations $\big(\psi(\boldsymbol{h}_i^T, \boldsymbol{y}_i^T, U_i)\big)_{i\in N}$ and social criteria rely only on ordinal information on agent preferences. By the proof of Theorem 2, it follows from *Weak Pareto, Time Consistency* and *Invariance on Constant Allocations* that, for all $U, U' \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\begin{aligned}\boldsymbol{x}^T \succsim_{\boldsymbol{h}^T}^U \boldsymbol{y}^T &\Longleftrightarrow \big(\psi(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T, U_i)\big)_{i\in N} \succsim_{\boldsymbol{h}^0}^U \big(\psi(\boldsymbol{h}_i^T, \boldsymbol{y}_i^T, U_i)\big)_{i\in N} \\ &\Longleftrightarrow \big(\psi(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T, U_i)\big)_{i\in N} \succsim_{\boldsymbol{h}^0}^{U'} \big(\psi(\boldsymbol{h}_i^T, \boldsymbol{y}_i^T, U_i)\big)_{i\in N} \\ &\Longleftrightarrow \big(\psi(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T, U_i)\big)_{i\in N} R \big(\psi(\boldsymbol{h}_i^T, \boldsymbol{y}_i^T, U_i)\big)_{i\in N},\end{aligned}$$

where R is the quasi-ordering generated from $\succsim$. Then, for each $i \in N$ and for any strictly increasing function $f_i : \mathbb{R} \to \mathbb{R}$, $f_i(U_i(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T)) = f_i(U_i(\bar{\boldsymbol{x}}_i^0))$ if and only if $U_i(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T) = U_i(\bar{\boldsymbol{x}}_i^0)$ where $x_i = \psi(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T, U_i)$, which implies that $\psi(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T, U_i) = \psi(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T, f_i \circ U_i)$ for all $\boldsymbol{h}_i^T, \boldsymbol{x}_i^T, U_i$. This means that social criteria satisfying the three axioms are invariant to strictly increasing transformations of the agent utility functions, and hence they are ordinal.

Invariance on Constant Allocations is especially important to derive the ordinality above. As an example that satisfies *Strong Pareto* and *Time Consistency* but is not ordinal, consider utilitarian social criteria based on $\sum_{i\in N} V_i(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T)$ where V_i is a lifetime utility function representing the agent lifetime preference and

$$V_i(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T) = \sum_{t=0}^{T-1} \delta_i^t v_i(h_{it}) + \sum_{t\geq T} \delta_i^t v_i(x_{it}).$$ [10]

The utilitarian criteria also satisfy *Equity for Equals* if the v_i functions are concave. We can also consider generalized utilitarian criteria based on $\sum_{i\in N} g\big(V_i(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T)\big)$, where $g : \mathbb{R} \to \mathbb{R}$

[10] In a static model of production economies, Fleurbaey and Maniquet (2006, p. 65) consider these social criteria as an example violating an invariance axiom named *Hansson Independence*. Although these criteria need more information on the utility functions than the social criteria in this paper, we can nevertheless consider which of our axioms they satisfy.

is a continuous, strictly increasing, and strictly concave function. These criteria satisfy *Strict Equity for Equals* if the v_i functions are concave. However, these criteria do not satisfy *Invariance on Constant Allocations* and, thus, are not ordinal. We can also see that, if the invariance axiom is violated, we have social criteria that are not infinitely inequality averse like maximin criteria. To obtain maximin or leximin criteria, we can require ordinality.

As mentioned above, *Invariance on Constant Allocations* rules out many plausible criteria. It is worth considering the structure of social criteria that satisfy *Weak Pareto*, *Time Consistency*, and inequality aversion but do not necessarily satisfy the invariance axiom. We have already introduced examples of such social criteria above. They have common properties, which we describe as follows. There exists a quasi-ordering R^* on $\mathbb{R}^N$ and $C : H \times \mathbb{R}_+^\infty \times \mathcal{U} \to \mathbb{R}$ such that, for all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\boldsymbol{x}^T \succsim_{\boldsymbol{h}^T}^U \boldsymbol{y}^T \iff \big(C(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T, U_i)\big)_{i\in N} R^* \big(C(\boldsymbol{h}_i^T, \boldsymbol{y}_i^T, U_i)\big)_{i\in N}.$$

Each agent i's situation in $\boldsymbol{x}^T$ under $\boldsymbol{h}^T$ is evaluated by $C(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T, U_i)$. *Time Consistency* implies that the evaluation depends not only on $\boldsymbol{x}_i^T$ but also on $\boldsymbol{h}_i^T$. Let P^* and I^* be the asymmetric and symmetric parts of R^*, respectively. By *Weak Pareto*, $U_i(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T) \geq U_i(\boldsymbol{h}_i^T, \boldsymbol{y}_i^T)$ for all i implies that $\big(C(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T, U_i)\big)_{i\in N} R^* \big(C(\boldsymbol{h}_i^T, \boldsymbol{y}_i^T, U_i)\big)_{i\in N}$, and $U_i(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T) > U_i(\boldsymbol{h}_i^T, \boldsymbol{y}_i^T)$ for all i implies that $\big(C(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T, U_i)\big)_{i\in N} P^* \big(C(\boldsymbol{h}_i^T, \boldsymbol{y}_i^T, U_i)\big)_{i\in N}$. If $\succsim$ satisfies *Equity for Equals* (resp. *Strict Equity for Equals*), then for $U \in \mathcal{U}^E$ and $\bar{\boldsymbol{x}}^T, \bar{\boldsymbol{y}}^T \in \bar{X}$ such that $x_j = y_j + \varepsilon < y_k - \varepsilon = x_k$ and $x_i = y_i$ for all $i \neq j, k$, $\big(C(\boldsymbol{h}_i^T, \bar{\boldsymbol{x}}_i^T, U_i)\big)_{i\in N} R^* \big(C(\boldsymbol{h}_i^T, \bar{\boldsymbol{y}}_i^T, U_i)\big)_{i\in N}$ (resp. $\big(C(\boldsymbol{h}_i^T, \bar{\boldsymbol{x}}_i^T, U_i)\big)_{i\in N} P^* \big(C(\boldsymbol{h}_i^T, \bar{\boldsymbol{y}}_i^T, U_i)\big)_{i\in N}$).

We can consider another type of equity as follows.

Strict Equity on Constant Allocations. For all $U \in \mathcal{D}^E$, all $\bar{\boldsymbol{x}}^0, \bar{\boldsymbol{y}}^0 \in \bar{X}$ and all $\varepsilon > 0$, if there exist $j, k \in N$ such that $x_j = y_j + \varepsilon < y_k - \varepsilon = x_k$ and $x_i = y_i$ for all $i \neq j, k$, then $\bar{\boldsymbol{x}}^0 \succ_{\boldsymbol{h}^0}^U \bar{\boldsymbol{y}}^0$.

This axiom insists that, on constant allocations, if agents have the same preferences, a transfer from a better-off person to a worse-off person should be socially preferred. For instance, if the u_i functions are concave, this axiom is satisfied by the following criterion. For all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\boldsymbol{x}^T \succsim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T \Longleftrightarrow \sum_{i \in N} g\big(C(\boldsymbol{h}^T_i, \boldsymbol{x}^T_i, U_i)\big) \geq \sum_{i \in N} g\big(C(\boldsymbol{h}^T_i, \boldsymbol{y}^T_i, U_i)\big),$$

where $g : \mathbb{R} \to \mathbb{R}$ is a continuous, strictly increasing, and strictly concave function. Note that this criterion is in the class described by R^* above. These arguments show that if we drop *Invariance on Constant Allocations*, there are many possibilities of Paretian, equitable, and time-consistent social criteria. Characterizing such a general class of social criteria remains a topic for future research.

7 Related Literature

In this section, we discuss the related literature. First, we discuss consistency axioms. In the literature on intertemporal decision-making, aside from *Time Consistency*, the two axioms below are often required.

Stationarity (Koopmans, 1960). For all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, all $\boldsymbol{w} \in \mathbb{R}^N_+$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\boldsymbol{x}^T \succsim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T \Longleftrightarrow (\boldsymbol{w}, \boldsymbol{x}^T) \succsim^U_{\boldsymbol{h}^T} (\boldsymbol{w}, \boldsymbol{y}^T).$$

Time Invariance. For all $U \in \mathcal{D}$, all $\boldsymbol{h}^T, \tilde{\boldsymbol{h}}^\tau \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\boldsymbol{x}^T \succsim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T \Longleftrightarrow \boldsymbol{x}^\tau \succsim^U_{\tilde{\boldsymbol{h}}^\tau} \boldsymbol{y}^\tau,$$

where $\boldsymbol{x}^T = \boldsymbol{x}^\tau$ and $\boldsymbol{y}^T = \boldsymbol{y}^\tau$.

Stationarity says that social evaluations are independent of the same first periodic allocations.[11] *Time Invariance* requires that decision-making should be independent of what

[11]It is straightforward to see that an impossibility result similar to our Proposition 1 can be obtained by replacing *Time Consistency* with *Stationarity*.

happened in the past. Halevy (2015) shows that any two of the three axioms—*Time Consistency, Stationarity,* and *Time Invariance* —imply the remaining one. Note that the social ordering in Theorem 2 satisfies *Time Consistency* but violates *Time Invariance* and *Stationarity.*

Zuber (2011) shows that when agents have the common discount factors, continuous social welfare functions satisfying the strong Pareto principle, *Time Consistency*, and *Time Invariance* should be additive. If discount factors are heterogeneous, there exists no social welfare function satisfying the axioms. In the setting where all agents have the same consumption levels in every period, Jackson and Yariv (2015) show the impossibility of a "time-consistent" social preference satisfying the strong Pareto principle being dictatorial. Millner and Heal (2018) argue that the time consistency axiom of Jackson and Yariv (2015) is the conjunction of *Time Invariance* and *Stationarity.* By Halevy's (2015) result mentioned above, Zuber (2011) and Jackson and Yariv (2015) actually have all the three rationality axioms. In particular, under *Stationarity*, a social criterion satisfying the Pareto principle is dictatorial (Millner and Heal, 2018, pp.161–162). Therefore, under these axioms, it is difficult to aggregate heterogeneous discount factors. In contrast, we only require *Time Consistency*, and can obtain non-dictatorial social functions satisfying the Pareto principle and equity axioms.

Hayashi (2016) argues that *Time Invariance* is a good property for individual decision-making, but not for social decisions, especially when responsibility is important for evaluating intertemporal allocations. Millner and Heal (2018) also argue that for dynamic social decision-making by a group of agents, time invariance is a problematic feature for social preferences, both normatively and positively, and that time consistency is more suitable. They also claim that the three consistency axioms are conflated in the literature. In this paper, therefore, following these arguments, we require only the time consistency axiom.

Another relevant study is that of Feng and Ke (2018). They consider the aggregation of heterogeneous expected utility functions and discount factors in an intergenerational setting, where each agent in each generation lives for one period and altruistically cares about the descendants' (discounted) expected utilities. The social planner's objective function is assumed to be an exponential discounting utility function with the social discount factor. This objective function satisfies *Time Consistency, Time Invariance*, and *Stationarity*. They propose a weaker Pareto principle called *Intergenerational Pareto* to avoid the impossibility result. This axiom requires that, in each period, if all agents in the current and future generations prefer one consumption stream to another, the former should be socially preferred in that period. This is weaker than the Pareto axioms by Zuber (2011) and Jackson and Yariv (2015) because descendants' expected utilities are aggregated separately. Then, they show that under certain axioms, the planner's objective function satisfies *Intergenerational Pareto* and is strictly increasing in each agent's expected utility (and, hence, nondictatorial) if and only if the social discount factor is sufficiently high. While they do not introduce any equity property, Chambers and Echenique (2018) consider intergenerationally equitable social criteria in a related problem.

Next, we discuss equity. To our knowledge, equity axioms have received little attention in the context of intertemporal social decision-making. One exception is Bommier and Zuber (2012, Theorem 4), who show that any two time-consistent and strongly Paretian social evaluation functions cannot have different degrees of inequality aversion based on *Hammond Equity*. Another exception is Hayashi (2016), who characterizes a class of social orderings by the strong Pareto principle, *Time Consistency*, equity based on convexity of the social welfare function, separability (of irrelevant agents), and some invariance axioms. Hayashi's (2016) social ordering consistently updates weights on utilities depending on the past allocations.

In the literature, it is common to assume that utilities are interpersonally comparable. This assumption is stringent and does not provide any view on how to assess agents' situations. In contrast, we derive a criterion for assessing the agents' situations using the fair social choice approach (Fleurbaey and Maniquet, 2011). To our knowledge, this paper is the first to study the problem of time-consistent fair social evaluation with taking into account responsibility for past choices. However, Fleurbaey et al. (2014) study a relevant but different problem in a dynamic setting where agents have a risk of being short-lived. They characterize a maximin social ordering using a principle of compensating short-lived agents. Although they argue that short-lived agents cannot be regarded responsible for their short lives, they do not consider the agents' responsibility for choices that may shorten their lives. In contrast, responsibility for past choices is central to our analysis. Another difference is that Fleurbaey et al. (2014) consider a broad domain of continuous preference orderings and *Hansson Independence*, which requires that social preferences over two allocations depend only on the agent indifference curves at these allocations. This setting brings a flexibility to the analysis. Our domain is very restrictive and we cannot use the standard techniques in the literature of fair social ordering. In the proof of Lemma 3 below, we construct a specific lifetime utility function to derive the results.

8 Appendix

Proof of Lemma 3. Given any $U \in \mathcal{U}$. Let $\bar{\boldsymbol{x}}^0 = (x_i)_{i \in N}, \bar{\boldsymbol{y}}^0 = (y_i)_{i \in N} \in \bar{X}$ be such that $y_k < x_k < x_j < y_j$ for some $j, k \in N$ and $x_i = y_i$ for all $i \neq j, k$. To prove that $\succsim$ satisfies *Equality on Constant Allocations*, we prove $\bar{\boldsymbol{x}}^0 \succsim^U_{\boldsymbol{h}^0} \bar{\boldsymbol{y}}^0$. If $x_k - y_k \geq y_j - x_j$, the proof is straightforward by *Equity for Equals*, *Invariance for Constant allocations*, and *Weak Pareto*. Thus, in the following, we assume $x_k - y_k < y_j - x_j$.

Since $\bar{\boldsymbol{x}}^0$ and $\bar{\boldsymbol{y}}^0$ are constant from the initial period, by *Invariance on Constant allocations*, we can arbitrarily modify the lifetime utility functions. To construct a convenient utility function, we introduce parameters $a, b, c, e, \varepsilon \in \mathbb{R}_{++}$ and $d \in (0, 1)$ such that

$$a(x_k - c - y_k) + y_j - (x_k - c) = (1 - d)\big[a(x_k - c - y_k) + e - (x_k - c)\big], \tag{1}$$

$$a(x_k - c - y_k) = (1 - d)\big[a(x_k - c - y_k) + e - (x_k - c)\big] - dba\varepsilon, \tag{2}$$

$$a(x_k - 2c - y_k) = (1 - d)\big[a(x_k - c - y_k) + e - (x_k - c)\big] - 2dba\varepsilon, \tag{3}$$

$$0 = (1 - d)\big[a(x_k - c - y_k) + e - (x_k - c)\big] - 3dba\varepsilon. \tag{4}$$

Define $U_0 \in \mathcal{U}$ as

$$u_0(x) = \begin{cases} ba(x - y_k) \text{ for } x < y_k, \\ a(x - y_k) \text{ for } x \in \bigl[y_k, x_k - c\bigr], \\ a(x_k - c - y_k) + x - (x_k - c) \text{ for } x > x_k - c, \end{cases}$$

and $\delta_0 = d \in (0, 1)$.[12] As discussed later, $a > 1$ and $b > 1$, and thus u_0 is concave. Let $U \in \mathcal{D}$ denote the preference profile where all agents have U_0.

Define $\boldsymbol{z}^0, \boldsymbol{w}^0 \in X$ such that

$$\begin{aligned} &z_{k0} = e,\ z_{kt} = y_k \text{ for all } t \geq 1, \\ &z_{j0} = e,\ z_{jt} = y_k - 3\varepsilon \text{ for all } t \geq 1, \\ &w_{k0} = e,\ w_{kt} = y_k - \varepsilon \text{ for all } t \geq 1, \\ &w_{j0} = e,\ w_{jt} = y_k - 2\varepsilon \text{ for all } t \geq 1, \\ &\boldsymbol{z}_i^0 = \boldsymbol{w}_i^0 = \bar{\boldsymbol{x}}_i^0 = \bar{\boldsymbol{y}}_i^0 \text{ for all } i \neq j, k. \end{aligned}$$

Then, from (1)–(4), we can see that

$$\begin{aligned} u_0(y_j) &= a(x_k - c - y_k) + y_j - (x_k - c) \\ &= (1 - d)\bigl(a(x_k - c - y_k) + e - (x_k - c)\bigr) \\ &= (1 - d)\bigl(a(x_k - c - y_k) + e - (x_k - c)\bigr) + dba(y_k - y_k) \\ &= (1 - d)U_0(\boldsymbol{z}_j), \end{aligned}$$

$$\begin{aligned} u_0(x_k - c) &= a(x_k - c - y_k) \\ &= (1 - d)\bigl(a(x_k - c - y_k) + e - (x_k - c)\bigr) - bad\varepsilon \\ &= (1 - d)\bigl(a(x_k - c - y_k) + e - (x_k - c)\bigr) + dba(y_k - \varepsilon - y_k) \\ &= (1 - d)U_0(\boldsymbol{w}_j), \end{aligned}$$

[12] A similar form was introduced by Fleurbaey and Zuber (2017) in a different setting, although the parameters are different.

$$
\begin{aligned}
u_0(x_k - 2c) &= a(x_k - 2c - y_k) \\
&= (1-d)\big(a(x_k - c - y_k) + e - (x_k - c)\big) - 2bad\varepsilon \\
&= (1-d)\big(a(x_k - c - y_k) + e - (x_k - c)\big) + dba(y_k - 2\varepsilon - y_k) \\
&= (1-d)U_0(\boldsymbol{w}_k),
\end{aligned}
$$

$$
\begin{aligned}
u_0(y_k) &= 0 \\
&= (1-d)\big(a(x_k - c - y_k) + e - (x_k - c)\big) - 3bad\varepsilon \\
&= (1-d)\big(a(x_k - c - y_k) + e - (x_k - c)\big) + dba(y_k - 3\varepsilon - y_k) \\
&= (1-d)U_0(\boldsymbol{z}_k),
\end{aligned}
$$

Here, we specify a, b, c, e. From (1) and (2), we obtain

$$
y_j - (x_k - c) = dba\varepsilon. \tag{5}
$$

From (3) and (4), we have

$$
a(x_k - 2c - y_k) = dba\varepsilon,\ \ x_k - 2c - y_k = db\varepsilon. \tag{6}
$$

From (2) and (3),

$$
ac = dba\varepsilon,\ \ c = db\varepsilon. \tag{7}
$$

From (1) and (4), we have

$$
a(x_k - c - y_k) + y_j - (x_k - c) = 3dba\varepsilon. \tag{8}
$$

From (6) and (7), we have

$$
c = \frac{x_k - y_k}{3}. \tag{9}
$$

Then, from (7) and (9), we obtain

$$
b = \frac{x_k - y_k}{3d\varepsilon}. \tag{10}
$$

$b > 1$ if ε is sufficiently close to 0. By (5), (9), and (10), we have

$$a = \frac{3(y_j - x_k)}{x_k - y_k} + 1. \tag{11}$$

$a > 1$ because $y_j > x_j > x_k > y_k$. We have to show $e > x_k - c$ so that U_0 is well-defined. From (4) and (7),

$$e = \frac{1+2d}{1-d} ac + x_k - c, \tag{12}$$

and hence $e > x_k - c$. By (9) and (11), we obtain

$$e = \frac{1+2d}{1-d}\left(y_j - x_k + \frac{x_k - y_k}{3} \right) + x_k - \frac{x_k - y_k}{3}. \tag{13}$$

Now we prove the result. *Weak Pareto* implies $\boldsymbol{z}^0 \succsim^U_{\boldsymbol{h}^0} \bar{\boldsymbol{y}}^0$. Let $\boldsymbol{h}^1$ be such that

$$\begin{aligned} h_{k0} &= z_{k0} = w_{k0} = z_{j0} = w_{j0} = f, \\ h_{i0} &= z_{i0} = w_{i0} \text{ for all } i \neq j, k. \end{aligned}$$

Since $\boldsymbol{z}^1, \boldsymbol{w}^1 \in \bar{X}$ by definition, it follows from *Equity for Equals* that $\boldsymbol{z}^1 \succsim^U_{\boldsymbol{h}^1} \boldsymbol{w}^1$. Then, we obtain $\boldsymbol{z}^0 \succsim^U_{\boldsymbol{h}^0} \boldsymbol{w}^0$ by *Time Consistency*. Moreover, $\bar{\boldsymbol{x}}^0 \succsim^U_{\boldsymbol{h}^0} \boldsymbol{z}^0$ follows from *Weak Pareto*. By transitivity, we obtain the desired result. □

Independence of the Axioms.

We first show the independence of the axioms in Theorem 2.

Dropping Weak Pareto. Consider the ordering $\succsim$ such that for all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$, $\boldsymbol{x}^T \sim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T$.

Dropping Time Consistency. Consider the ordering $\succsim$ such that for all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\boldsymbol{x}^T \succsim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T \iff \min_{i \in N} \mu(\boldsymbol{x}^T_i, U) \geq \min_{i \in N} \mu(\boldsymbol{y}^T_i, U),$$

where $\mu(\boldsymbol{x}^T_i, U) = u_i^{-1}\big((1-\delta_i) \sum_{t \geq T} \delta_i^{t-T} u_i(x_{it})\big)$.

Dropping Hammond Equity on Constant Allocations. Consider the ordering $\succsim$ such that for all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\boldsymbol{x}^T \succsim_{\boldsymbol{h}^T}^U \boldsymbol{y}^T \Longleftrightarrow \sum_{i \in N} \psi(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T, U) \geq \sum_{i \in N} \psi(\boldsymbol{h}_i^T, \boldsymbol{y}_i^T, U).$$

Dropping Continuity on Constant Allocations. Consider the ordering $\succsim$ such that for all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\boldsymbol{x}^T \succsim_{\boldsymbol{h}^T}^U \boldsymbol{y}^T \Longleftrightarrow \big(\psi(\boldsymbol{h}^T, \boldsymbol{x}^T, U)\big)_{i \in N} \geq_{lex} \big(\psi(\boldsymbol{h}^T, \boldsymbol{y}^T, U)\big)_{i \in N},$$

where $\geq_{lex}$ is the lexicographic order over $\mathbb{R}_+^N$.

We next show the independence of the axioms in Theorem 4.

Dropping Equity for Equals. Consider the criterion to drop *Hammond Equity on Constant Allocations* above.

Dropping Invariance on Constant Allocations. Consider the ordering $\succsim$ such that for all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\boldsymbol{x}^T \succsim_{\boldsymbol{h}^T}^U \boldsymbol{y}^T \Longleftrightarrow \min_{i \in N} \psi^1(\boldsymbol{h}^T, \boldsymbol{x}^T, U) \geq \min_{i \in N} \psi^1(\boldsymbol{h}^T, \boldsymbol{y}^T, U),$$

where $\psi^1(\boldsymbol{h}^T, \boldsymbol{x}^T, U) = u_i^{-1}\left((1-\delta_i)\left[2u_i(h_{i0}) + \sum_{t=1}^T \delta_i^t u_i(h_{it}) + \sum_{t \geq T} \delta_i^t u_i(x_{it})\right]\right)$.

The remaining axioms can be dropped by the same examples as those of Theorem 2 above.

Lastly, we prove the independence of the axioms in Theorem 5.

Dropping Suppes Indifference for Equals. Consider the ordering $\succsim$ such that for all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

(1) if $\left(\psi(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T, U_i)\right)_{i\in N} >_{lex} \left(\psi(\boldsymbol{h}_i^T, \boldsymbol{y}_i^T, U_i)\right)_{i\in N}$, then $\boldsymbol{x}^T \succ^U_{\boldsymbol{h}^T} \boldsymbol{y}^T$;

(2) if $\left(\psi(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T, U_i)\right)_{i\in N} =_{lex} \left(\psi(\boldsymbol{h}_i^T, \boldsymbol{y}_i^T, U_i)\right)_{i\in N}$, and

$\psi(\boldsymbol{h}_1^T, \boldsymbol{x}_1^T, U_1) < \psi(\boldsymbol{h}_i^T, \boldsymbol{x}_i^T, U_i)$ for all $i \neq 1$,

$\psi(\boldsymbol{h}_2^T, \boldsymbol{y}_2^T, U_2) < \psi(\boldsymbol{h}_i^T, \boldsymbol{y}_i^T, U_i)$ for all $i \neq 2$,

then $\boldsymbol{x}^T \succ^U_{\boldsymbol{h}^T} \boldsymbol{y}^T$;

(3) $\boldsymbol{x}^T \sim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T$ in all other cases.

Dropping Time Consistency. Consider the ordering $\succsim$ such that for all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\boldsymbol{x}^T \succsim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T \Longleftrightarrow \left(\mu(\boldsymbol{x}_i^T, U)\right)_{i\in N} \geq_{lex} \left(\mu(\boldsymbol{y}_i^T, U)\right)_{i\in N}.$$

Dropping Invariance on Constant Allocations. Consider the ordering $\succsim$ such that for all $U \in \mathcal{D}$, all $\boldsymbol{h}^T \in H$, and all $\boldsymbol{x}^T, \boldsymbol{y}^T \in X$,

$$\boldsymbol{x}^T \succsim^U_{\boldsymbol{h}^T} \boldsymbol{y}^T \Longleftrightarrow \left(\psi^1(\boldsymbol{h}^T, \boldsymbol{x}^T, U)\right)_{i\in N} \geq \left(\psi^1(\boldsymbol{h}^T, \boldsymbol{y}^T, U)\right)_{i\in N},$$

where $\psi^1(\boldsymbol{h}^T, \boldsymbol{x}^T, U) = u_i^{-1}\left((1-\delta_i)\left[2u_i(h_{i0}) + \sum_{t=1}^T \delta_i^t u_i(h_{it}) + \sum_{t\geq T} \delta_i^t u_i(x_{it})\right]\right)$.

The remaining axioms can be dropped by the same examples as those of Theorem 2 and 2 above.

References

[1] Anderson, Elizabeth (1999), "What is the point of equality?" *Ethics*, 109, 287-337.

[2] Bommier, Antone, and Stèphane Zuber (2012), "The Pareto principle of optimal inequality." *International Economic Review*, 53-2, 593-608.

[3] Chambers, Cristopher P., and Federico Echenique (2012), "When does aggregation reduce risk aversion?" *Games and Economics Behavior*, 76, 582-595.

[4] Chambers, Cristopher P., and Federico Echenique (2018), "On multiple discount rates." *Econometrica*, 86, 1325-1346.

[5] Cohen, Gerald A. (1989), "On the currency of egalitarian justice." *Ethics* 99, 906-944.

[6] Feng, Tangren, and Shaowei Ke (2018), "Social discounting and intergenerational Pareto." *Econometrica* 86, 1537-1567.

[7] Fleurbaey, Marc (1995), "Equality and Responsibility." *European Economic Review*, 39, 683-689.

[8] Fleurbaey, Marc (2008), *Fairness, Responsibility, and Welfare.* Oxford University Press.

[9] Fleurbaey Marc, and Francçis Maniquet (2006) "Fair income tax." *Review of Economic Studies*, 73, 55-83.

[10] Fleurbaey, Marc, and Francçis Maniquet (2011), *A Theory of Fairness and Social Welfare.* Econometric Society Monograph, Cambridge University Press.

[11] Fleurbaey, Marc, Marie-Louise Leroux, and Gregory Ponthiere (2014), "Compensating the dead." *Journal of Mathematical Economics*, 51, 28-41.

[12] Fleurbaey, Marc, Zuber, Stèphane (2017), "Fair management of social risk." *Journal of Economic Theory*, 169, 666-706.

[13] Hammond, Peter J. (1976), "Equity, Arrow's conditions, and Rawls' difference principle." *Econometrica*, 44-4, 793-804.

[14] Hammond, Peter J. (1979), "Equity in two person situations: Some consequences." *Econometrica*, 47, 1127-1135.

[15] Halevy, Yoram (2015), "Time consistency: stationarity and time invariance." *Econometrica*, 83-1, 335-352.

[16] Hayashi, Takashi (2016), " Consistent updating of social welfare functions." *Social Choice and Welfare*, 46-3, pp. 569-608.

[17] Jackson, Matthew O., and Leeat Yariv (2015), "Collective dynamic choice: the necessity of time consistency." *American Economic Journal: Microeconomics*, 7-4, 150-178.

[18] Koopmans, Tjalling C. (1960), "Stationary ordinal utility and impatience." *Econometrica* 28-2, 287-309.

[19] Millner, Antony, and Geoffrey Heal (2018), "Time consistency and time invariance in collective intertemporal choice." *Journal of Economic Theory*, 176, 158-169.

[20] Roemer, John E. (1998), *Equality of Opportunity.* Harvard University Press.

[21] Suppes, Patrick (1966), "Some formal models of grading principles." *Synthese*, 6, 284-306.

[22] Zuber, Stèphane (2011), "The aggregation of preferences: Can we ignore the past?" *Theory and Decision*, 70-3, 367-384.