Information transmission in persuasion models with imperfect verification*

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Abstract

I study a persuasion game between a privately informed agent and a decision maker (DM) who can imperfectly verify the statements made by the agent by observing a signal that is correlated with the agent's information. I find that whether or not the DM benefits from communicating with the agent depends on whether the DM's signal and the agent's private information satisfy a weak affiliation condition. I then discuss the significance of this result to the debate over the use of self-appraisals in business. I argue that, in general, self-appraisals are only useful when the workers' abilities are multidimensional.

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1 Introduction

Consider the following classic problem of economic theory. A decision maker (DM, she) must choose one of two alternatives. An agent (he), known to strictly prefer one of the alternatives, has some information about the value of each alternative to the DM. If the agent cannot commit to a reporting strategy prior to learning his private information/type, there is no scope for communication between the two players to make the DM better off; the DM might as well ignore the agent.[1] I consider the case of a DM who is able to imperfectly verify the agent's private information by observing a correlated signal. In practice, these signals can take the form of product reviews available to customers prior to a purchasing decision, performance reports available to managers who determine whether to promote their workers, criminal evidence available to judges, etc.

A priori, it is not clear whether the DM can learn anything from the agent. Clearly, if the DM's signal was perfectly correlated with the agent's type, there would be nothing to learn from the agent. It is also clear that, if the DM's signal was independent of the agent's type, no information would be transmitted, because the DM would have no way of judging the agent's credibility. This paper studies the conditions under which imperfect correlation leads to communication being valuable to the DM if she has commitment power. I find that whether communication is valuable or not depends on whether the DM's signal and the agent's private information satisfy a "weak affiliation" condition.

Let me present the results using the example of a firm (DM) that decides whether to promote a worker (agent). The firm only wants to promote the worker if his ability $\theta \in \mathbb{R}$ is larger than some (normalized) threshold equal to 0. Both players privately observe a signal correlated with θ: the firm observes $s \in \mathbb{R}$ (e.g., the worker's sales' record), while the worker observes $v \in \mathbb{R}$. I assume that the worker is better informed than the firm in that $E(\theta|v) = E(\theta|v,s)$ for all (v,s); an assumption that is compatible with simply assuming that the worker knows his ability, i.e., $v = \theta$. The prior distribution of v is denoted by q, while the conditional distribution of s is denoted by $p(s|\theta)$. Distribution p is called *ordered* if, perhaps after reordering s, the likelihood ratio $\frac{p(s|v')}{p(s|v'')}$

[1] When the DM has commitment power, the optimal mechanism is to pre-select a subset of (lotteries over) alternatives and then delegate the decision to the agent (Alonso and Matouschek, 2008), which does not help the DM in this setting. The cheap talk literature (Crawford and Sobel, 1982) studies the case where the DM cannot commit and confirms the same result. If the agent has commitment power, the Bayesian persuasion literature (Kamenica and Gentzkow, 2011) has shown that communication is possible.

is increasing with s for all pairs (v', v'') such that $v' > 0$ and $v'' < 0$.

The first result of the paper (Proposition 1) is that, regardless of the prior q, communication is not valuable whenever p is ordered. As the reader will note, p being ordered is a weaker version of the condition that v and s be affiliated.[2] Therefore, an immediate corollary is that, when the players' signals are affiliated, communication is never valuable. Affiliation seems like a natural assumption for a signal of ability to have; the higher is the worker's ability, the more likely it is that the worker's sales' record is better. Therefore, this first result suggests that firms learn nothing by asking their workers for input when evaluating them. Indeed, this is in line with some of the criticism over the use of self-appraisals in business, which makes the point that workers have no reason not to exaggerate when asked to rate their own performance (e.g., Thornton III, 1980, and of Campbell and Lee, 1988).

When distribution p is not ordered, communication is very much possible. Proposition 2 states two general results aimed at arguing that it is likely that the DM gains from communicating with the agent when p is not ordered: a sufficient condition over pair (q, p) for which communication is valuable, and a broad condition over p for which there is always some prior distribution q for which communication is valuable. I then discuss three applications where not only is it natural for p not to be ordered, but also the conditions of Proposition 2 are satisfied and communication is valuable.

The first application is a slightly more general version of the firm/worker application from above except that the worker's ability is multi-dimensional. Say, for example, that $\theta = \theta_1 + \theta_2$, where θ_1 represents the worker's technical skill and θ_2 represents the worker's social skill. The worker observes both his skills - $v = (v_1, v_2) = (\theta_1, \theta_2)$ - while the firm observes $s = (s_1, s_2)$, where each s_i is positively affiliated with v_i for $i = 1, 2$. It can be verified that, if θ_1 is independent of θ_2, v and s are not affiliated; indeed p is not ordered.

In the text, I show that, if for every v_1, there is some v_2 for which it is profitable for the firm to hire the worker, then communication is valuable. I then describe a mechanism that increases the firm's expected payoff by making use of the worker's input.

The mechanism is as follows. The worker is asked to pick one of various "evaluation tracks". One of the tracks, aimed at workers with better technical skills, is a track that only considers s_1 in the worker's evaluation; specifically, the worker is promoted if and only if s_1 is sufficiently large, regardless of the value of s_2. Another track works the

[2] Indeed, p being ordered corresponds exactly to v and s being affiliated if the support of v is partially ordered by $\succcurlyeq$, where $v' \succ v''$ if and only if $v' > 0$ and $v'' < 0$.

same but for for social skills (i.e., a worker who picks this track is promoted if and only if s_2 is sufficiently large). In general, different tracks lead to different combinations of s_1 and s_2 that lead to promotion. Workers are incentivized to pick different tracks depending on their abilities precisely because they may have different relative strengths. A worker with good technical skills distinguishes himself from bad workers by doing well in technical tasks and not necessarily in tasks that require a high level of social ability. In that sense, for a firm, it makes sense to allow a more technically skilled worker to be evaluated in a way that suits him best. Indeed, this application is useful in providing some intuition as to why communication is not valuable when p is ordered, which happens when ability is unidimensional. In that case, every good worker distinguishes himself from every bad worker in the same way; by having a high s.

Multidimensionality is not the only reason why distribution p might not be ordered. In the second application, I make this point by analyzing a setting where a buyer (DM) decides whether to purchase a product sold by a seller (agent). The seller knows the product's (uni-dimensional) quality $\theta \in \mathbb{R}$ (so that $v = \theta$), while the buyer has access to an imperfect verification technology that, in the spirit of the lie detection literature (Balbuzanov, 2019, Dziuda and Salas, 2019), makes uniform mistakes in identifying the product's quality with positive probability. In this setting, in addition to showing that communication has value, I completely characterize the optimal mechanism for the buyer. For some parameter values, the optimal mechanism is such that, whenever the quality is good enough for the buyer to want to buy the product, the seller announces the true quality level and the buyer buys the product if and only if the signal matches the announcement. Despite being completely biased towards persuading the buyer to buy the product, the seller has an incentive not to exaggerate out of fear that his announcement will not match the signal.

In the final application, p might not be ordered if the agent's signal v contains information about the DM's signal s in addition to the information about θ. In this application, an agent is evaluated by a DM who relies on the evaluation of various experts/referees. In addition to knowing his value, the agent also has information on which referee is the most capable of providing an accurate evaluation. I find that, once again, communication has value for the DM. Indeed, in the optimal mechanism for the DM, the agent recommends the referee that he finds the most capable, while the DM follows the recommendation of the referee chosen by the agent.

Finally, it is worth noting that *the sufficient conditions for communication to be valuable provided in this paper do not rely on the DM having commitment power*. Specifically, it can be proven that there is always a perfect Bayesian equilibrium of a cheap talk game, where the agent sends a cheap talk message (without knowing s) before the DM determines an action, that implements an optimal allocation.[3] This has important implications for the above discussion over the usefulness of self-appraisals. When ability is multi-dimensional, not only is it the case that the firm does better by seeking input from its workers; this can simply be done by asking workers for input and then interpreting that input in a sequentially rational manner. In that sense, this paper suggests that, when ability is multi-dimensional, simple self-appraisals, where workers just discuss their relative strengths and weaknesses, are actually useful for the firm.

The paper proceeds as follows. After discussing the related literature, I present the model in section 2; I then discuss a simple example that illustrates some of the main theoretical ideas behind the results in section 3; in section 4, I present the general results of the paper (Propositions 1 and 2); in section 5, I discuss the three applications mentioned above; finally, in section 6, I conclude.

1.1 Related literature

As mentioned above, the setting I consider is one where the DM cannot gain from communicating if she does not have a private signal of her own (or, equivalently, if her signal is either independent or perfectly correlated to the agent's). The reason is that the agent always prefers to report whatever increases the probability of his favourite alternative being chosen, regardless of his private information. This result holds regardless of whether the DM can commit (as in Alonso and Matouschek, 2008) or not (as in Crawford and Sobel, 1992). Indeed, absent some way of verifying the agent's report, the DM can only gain from communicating if there are more than two alternatives available (Lipnowski and Ravid, 2020) or if the sender can commit to a reporting strategy (Kamenica and Gentzkow, 2011).[4]

The novelty in this paper relative to the vast literature on delegation and on cheap talk is the ability of the DM to verify the agent's reports. A branch of the economic theory literature has modelled this verification ability by studying models of *hard evidence*, where either the DM is able to verify (possibly at a cost) whether some of the statements made by the agent are true or false (e.g., Baron and Besanko, 1984, Ben-Porath, Dekel and Lipman, 2014, and Mylovanov and Zapechelnyuk, 2017), or the

[3] This follows directly from a more general result proven in Ben-Porath, Dekel and Lipman (2019).

[4] Guo and Shmaya (2019) study an identical model to this paper's except that the agent can commit to a reporting strategy prior to observing his signal. They find that communication has value even when types are affiliated.

agent himself is able to prove some of the statements he makes (e.g., Green and Laffont, 1986, Bull and Watson, 2007, Deneckere and Severinov, 2008, and Strausz, 2017). In principal-agent models with hard evidence, it has been shown that there might be value for the DM in communicating with the agent even when the DM chooses between only two alternatives and the agent favors one of them (e.g., Glazer and Rubinstein, 2004, Lai, 2014, Ball and Gao, 2019, Carroll and Egorov, 2019). This paper expands on these by considering the conditions under which communication is valuable under probabilistic verification.

In models of probabilistic verification, the verification technology of the DM is imperfect, so that she cannot say for certain whether each statement made by the agent is true or false. As in this paper, probabilistic verification can be modelled by allowing the DM to observe a signal that is correlated with the agent's type.[5] Silva (2019a) and Siegel and Strulovici (2021) show that, if the DM chooses between more than two alternatives and (at least) one of the players is risk averse, communication has value to the DM when both players' signals are affiliated. By contrast, under the same affiliation assumption, this paper proves that communication has no value when the players are risk neutral (or when there are only two alternatives available to the DM). Silva (2019b) and Pereyra and Silva (2021) show that communication may be valuable under risk neutrality if there are multiple agents with possibly correlated types.

Kattwinkel (2019) studies a similar model - two alternatives and correlated signals - but assumes the agent's signal is not a sufficient statistic of the DM's signal.[6] In that sense, Kattwinkel (2019) is not, strictly speaking, a pure model of imperfect verification, because the DM's signal provides information that goes beyond the verification of the agent's information. Nevertheless, the comparison between the two papers is interesting as very different predictions emerge under affiliation. While in this paper's model of (pure) imperfect verification, communication has no value under affiliation, in Kattwinkel (2019), communication might have value even if signals are affiliated. I go through the theoretical reasons behind this difference in section 4. From a practical perspective, whether affiliation implies that communication is valuable or not depends on whether the DM's signal informs the DM beyond the agent's type. Kattwinkel's motivating example is one where the DM observes the cost of assigning an object to the agent, while the agent observes the (correlated) value of the object. While the cost of the object might serve to help the DM ascertain the truthfulness of the agent's

[5] A general treatment of probabilistic verification can be found in Silva (2020) and in Ball and Kattwinkel (2022).

[6] The same applies to Bloch, Dutta and Dziubinski (2023) who study a model with multiple agents.

report about the value of the object, it clearly does more than that, as both the value and the cost are relevant to determine whether the DM wants to assign the object to the agent.

Also related is the literature on cheap talk which discusses whether the existence of an exogenous signal that is observed by the receiver reduces or enhances communication from the sender.[7] The relevance to this paper is stronger than what may appear because of the result mentioned in the Introduction that the DM can implement her favourite allocation through a cheap talk mechanism. In the setting I consider, where the sender's preferences are state-independent and the receiver chooses between two alternatives, the existence of an exogenous signal cannot harm the receiver even if she cannot commit, because, without the signal, there would be no information transmitted at all. In the paper, I discuss the conditions under which the receiver being informed actually generates communication.[8]

Finally, while there are no transfers allowed in the model, the idea that correlated signals help the DM is reminiscent of Cremer and McLean (1988). The common ground between the two papers is that, when the DM's signal is correlated with the agent's signal, each of the agent's types have different beliefs over the DM's signal realization and the DM can explore this to induce truthful reporting. If transfers were allowed, the DM's gains from communicating would be larger and communication might have value even under affiliated types.[9]

2 Model

There is a DM and an agent. The DM must choose $x \in [0, 1]$ and her payoff is given by θx, where $\theta \in \mathbb{R}$ represents the unknown *state*. The agent's payoff is x. Variable x might represent the probability of choosing the agent's favorite alternative in case there are two alternatives available (e.g., the probability of hiring the agent), or it might represent an arbitrary continuous policy (e.g., the salary to pay the agent), provided both players are risk neutral.

Both players observe discrete signals that are correlated with the state θ: the agent

[7]In Chen (2012), de Barreda (2013) and Ishida and Shimizu (2016), it is the former, while in Ishida and Shimizu (2019) it is the latter.

[8]Watson (1996) also considers state-independent preferences but, while I determine the distributions over signals for which *some* information is transmitted, Watson does a similar analysis to determine when *all* of the agent's information is transmitted.

[9]Relatedly, in an information design model, Krahmer (2021) shows that the first-best allocation would be attainable by the use of small punishments and/or transfers if the DM could privately randomize over the information observed by the agent.

observes $v \in V$ while the DM observes $s \in S$. I assume that $E(\theta|v,s) = E(\theta|v) \equiv \theta_v$ for all $v \in V$ and $s \in S$. This means that knowing s is irrelevant (in terms of the conditional expectation) if one already knows v. The point of this assumption is to make the agent unequivocally better informed than the DM, so that s is only used as a tool to imperfectly verify the agent's report. A natural special case of the model is when $v = \theta_v$ in which case the agent *only* knows the (expected) value of θ.[10] It is also possible that v contains additional information correlated with s (e.g., $v = (\theta_v, y)$, where y is a random variable correlated with s).[11]

I denote the prior distribution of v by $q \in \Delta V$ and assume it has full support. The conditional distribution of s is $p(s|v) \in \Delta S$ for each $v \in V$, where $p(s|v) > 0$ for all $(v,s) \in V \times S$. Notice that if the DM was able to directly observe v, she would reward the agent ($x = 1$) if $\theta_v > 0$ and would not ($x = 0$) if $\theta_v < 0$. To facilitate the exposition of the results, I assume that $\theta_v \neq 0$ for all $v \in V$, so that the DM has strict preferences for each $v \in V$. Let

$$\overline{V} \equiv \{v \in V : \theta_v > 0\}$$

and

$$\underline{V} \equiv \{v \in V : \theta_v < 0\}.$$

I refer to elements of $\overline{V}$ as "high" types and elements of $\underline{V}$ as "low" types. If either set is empty, the problem is trivial (the DM either blindly rewards the agent or she does not), so I assume that neither set is empty.

Definition 1 *Distribution p is **ordered** if and only if there is a linear order $\succcurlyeq$ such that, for all $(s, s') \in S \times S$,*

$$s \succ s' \Rightarrow \frac{p(s|\overline{v})}{p(s|\underline{v})} \geq \frac{p(s'|\overline{v})}{p(s'|\underline{v})}$$

for all $\overline{v} \in \overline{V}$ and $\underline{v} \in \underline{V}$.

In words, if p is ordered, then it is possible to reorder S such that the likelihood ratio between any high type and any low type is (weakly) increasing.[12]

An allocation is denoted by $h : V \times S \to [0,1]$ and is incentive compatible if and

[10] If $v = \theta_v$, it is without loss of generality to just assume that $v = \theta$.

[11] In section 5.3, I discussion one such application.

[12] A similar assumption called "group monotonicity" is made in Deb and Stewart (2018), where the authors study how a firm should determine the tasks its workers perform in order to best determine their ability. Despite the similarity of the two conditions, group monotonicity is neither implied nor implies that p is ordered.

only if

$$E\left(h\left(v,s\right)|v\right)\geq E\left(h\left(v',s\right)|v\right)$$

for all $v'\in V$ and $v\in V$. Notice that this definition of incentive compatibility assumes that the agent reports his type without knowing the realization of the DM's signal s. That is how incentives are given to the agent; the agent might have different beliefs about s depending on the realization of v.[13] By the revelation principle (Myerson, 1979), if the DM has commitment power, she can implement any incentive compatible allocation h by simply asking the agent to report his type v truthfully and then assigning him reward $h\left(v,s\right)$. The DM's payoff for any allocation h is $U\left(h\right)\equiv E\left(\theta h\left(v,s\right)\right)$. An optimal allocation maximizes U among the set of incentive compatible allocations.

One property of U that the reader should keep in mind for the analysis of the following sections is that it can be written as a linear function of each type's expected payoff:

$$U(h)=E(\theta h(v,s))=E_v[\theta_v E(h(v,s)|v)].$$

This means that if there are two allocations h' and h'' such that

$$E\left(h'\left(v,s\right)|v\right)=E\left(h''\left(v,s\right)|v\right)$$

for all $v\in V$, then $U\left(h'\right)=U\left(h''\right)$. Moreover, if

$$E\left(h'\left(\overline{v},s\right)|\overline{v}\right)\geq E\left(h''\left(\overline{v},s\right)|\overline{v}\right)$$

for all $\overline{v}\in\overline{V}$ and

$$E\left(h'\left(\underline{v},s\right)|\underline{v}\right)\leq E\left(h''\left(\underline{v},s\right)|\underline{v}\right)$$

for all $\underline{v}\in\underline{V}$, then $U\left(h'\right)\geq U\left(h''\right)$.

Finally, let allocation h^* be such that $h^*(v,s)=\mathbf{1}\{E(\theta|s)\geq 0\}$ for all $(v,s)\in V\times S$ and notice that it is optimal among those that are independent of v. I say that *communication has value* for the DM when allocation h^* is not optimal.

[13] If the agent was able to observe s prior to reporting, then the DM could not gain from communicating with the agent unless he was able to commit to a reporting strategy.

3 Example

In this example, I try to convey some of the intuition behind the importance of p being ordered in determining whether communication has value to the DM.

Let us consider a simple three-type example where $v = \theta_v \in \{-1, 1, 2\}$, so that there are two high types and one low type. Assume that $S = \{L, R\}$ and that p is given by the following table, where, for convenience, I assume that $\alpha < 3/4$ and $\beta < 3/4$.

$p(s \mid \theta_v)$	$s = L$	$s = R$
$\theta_v = 2$	$\frac{3}{4}$	$\frac{1}{4}$
$\theta_v = 1$	α	$1-\alpha$
$\theta_v = -1$	β	$1-\beta$

Notice that, given these restrictions, p is ordered if and only if $\alpha \geq \beta$. Assume also that q is such that $E(\theta|L) > 0 > E(\theta|R)$, so that allocation h^* is as follows:

h^*	$s = L$	$s = R$
$\theta_v = 2$	1	0
$\theta_v = 1$	1	0
$\theta_v = -1$	1	0

Consider the following allocation, which perturbs allocation h^* as described in the table below:

	$s = L$	$s = R$
$\theta_v = 2$	1	0
$\theta_v = 1$	$1-\varepsilon$	δ
$\theta_v = -1$	1	0

Notice that the DM would like to increase δ from 0, because that would increase the payoff of high type $\theta_v = 1$. However, in order to increase δ while preventing type $\theta_v = -1$ from mimicking $\theta_v = 1$, the DM must also raise ε. The issue is then whether the combination of the two effects leaves type $v = 1$ better off or not. For type $v = -1$ not to want to deviate, it must be that

$$\beta(-\varepsilon) + (1-\beta)\delta \leq 0 \Leftrightarrow \frac{\delta}{\varepsilon} \leq \frac{\beta}{1-\beta}. \tag{1}$$

For type $v = 1$ to be better off, it must be that

$$\alpha(-\varepsilon) + (1-\alpha)\delta > 0 \Leftrightarrow \frac{\delta}{\varepsilon} > \frac{\alpha}{1-\alpha} \tag{2}$$

It immediately follows that conditions (1) and (2) can only simultaneously hold if $\alpha < \beta$. Moreover, if $\alpha < \beta$ and condition (1) holds, type $\theta_v = 2$ would prefer not to deviate, because

$$\frac{\frac{3}{4}}{\frac{1}{4}} \geq \frac{\beta}{1-\beta} \geq \frac{\delta}{\varepsilon}.$$

Therefore, it follows that a pair (ε, δ) can be found such that the perturbed allocation is incentive compatible and an improvement for the DM if and only if p is ordered. The reason why this happens is that, when p is ordered, shifting rewards from signal L to signal R is relatively better for low type $\theta_v = -1$ than for high type $\theta_v = 1$. But if p is not ordered, a similar shift becomes relatively better for high type $\theta_v = 1$.

More broadly, when p is not ordered, different high types distinguish themselves from low types differently. In this example, if the DM only faced the pair of types $\theta_v = 2$ and $\theta_v = -1$, she would prefer to reward the agent more when $s = L$, because L would have been more likely to have been generated by $\theta_v = 2$ (type $\theta_v = 2$ "distinguishes himself" from type $\theta_v = -1$ by obtaining signal $s = L$). But if $\alpha < \beta$ and the DM faced only the pair of types $\theta_v = 1$ and $\theta_v = -1$, it would be the opposite; she would prefer to reward the agent more when $s = R$. When that happens, the DM might benefit from offering a menu of reward functions that the agent self-selects into depending on his type.

In the text, I build on some of the insights of this example. I start by proving that, if p is ordered, communication is never valuable. The argument is more involved than in this example as all possible alternative incentive compatible allocations need to be considered; not just perturbations of h^*. Moreover, in general settings, one needs to account for the possibility of high types and low types mimicking each other.

I then proceed to study under which conditions communication is valuable when p is not ordered. As the example suggests, whether or not communication is valuable if p is not ordered may depend on the prior distribution q (the argument used the assumption that $E(\theta|L) > 0 > E(\theta|R)$). However, I find that, if p is not ordered, it is natural/likely that communication is valuable. I argue this in two ways. First, I provide sufficient conditions over p and q for which communication has value; specifically, Proposition 2 shows that, under fairly general conditions over p, a prior q can always be found for which communication is valuable. Second, I provide three applications where, not only is it natural for p not to be ordered, but also the conditions under which communication is valuable are quite plausible.

4 General Results

In this section, I discuss the general conditions under which communication has value for the DM. To that end, I start by introducing a simpler problem where the DM only chooses how to reward high types and allows low types to copy the high types they want. In Lemma 1, I prove that this is actually optimal for the DM. This implies that communication has value if and only if there is a solution to the simpler problem that generates a strictly larger expected payoff for the DM than the optimal allocation without communication h^*.

Let

$$\widetilde{U}(\eta) \equiv \sum_{\overline{v}\in\overline{V}} q(\overline{v})\theta_{\overline{v}} E(\eta(\overline{v},s)|\overline{v}) + \sum_{\underline{v}\in\underline{V}} q(\underline{v})\theta_{\underline{v}} \max_{\overline{v}'\in\overline{V}} E(\eta(\overline{v}',s|\underline{v})$$

for all $\eta : \overline{V} \times S \to [0,1]$.

Lemma 1 *For every η that maximizes $\widetilde{U}$, there is an optimal allocation $\widetilde{h}$ such that $\widetilde{h}(\overline{v},\cdot) = \eta(\overline{v},\cdot)$ for all $\overline{v} \in \overline{V}$ and $\widetilde{h}(\underline{v},\cdot) = \eta(\omega(\underline{v},\cdot))$ for all $\underline{v} \in \underline{V}$, where*

$$\omega(\underline{v}) \in \arg\max_{\overline{v}\in\overline{V}} E(\eta(\overline{v},\cdot|\underline{v})).$$

Proof.

Take any η that maximizes $\widetilde{U}$ (which trivially exists due to the objective function being continuous over a compact set) and consider allocation $\widetilde{h}$ as defined in the statement. I start by proving that allocation $\widetilde{h}$ is incentive compatible. All incentive constraints of every low type are satisfied by construction, as each low type's reward function is equal to the reward function of their best deviation. By way of contradiction, suppose some of the incentive constraints of high types are violated. Specifically, say that there is a pair $\overline{v}', \overline{v}'' \in \overline{V}$ such that

$$E\left(\eta\left(\overline{v}',s\right)|\overline{v}'\right) < E\left(\eta\left(\overline{v}'',s\right)|\overline{v}'\right).$$

Construct $\eta' : \overline{V} \times S \to [0,1]$ as follows: $\eta'(\overline{v},\cdot) = \eta(\overline{v},\cdot)$ for all $\overline{v} \neq \overline{v}'$, and $\eta'(\overline{v}',\cdot) = \eta(\overline{v}'',\cdot)$. It follows that $\widetilde{U}(\eta') > \widetilde{U}(\eta)$, a contradiction to the optimality of η.

Now, I prove that $\widetilde{h}$ is an optimal allocation. By way of contradiction, suppose there is some other incentive compatible allocation h'' such that $U(h'') > U(\widetilde{h})$. Let $\eta'' : \overline{V} \times S \to [0,1]$ be such that $\eta''(\overline{v},\cdot) = h''(\overline{v},\cdot)$. By construction of $\widetilde{U}$, it follows that $\widetilde{U}(\eta'') \geq U(h'')$, because h'' is incentive compatible (so that low types that the

DM wants to punish are weakly better off reporting truthfully over mimicking a high type). But then,

$$U(\widetilde{h}) = \widetilde{U}(\eta) \geq \widetilde{U}(\eta'') \geq U(h'') > U(\widetilde{h}),$$

which is a contradiction.[14]

■

In proving Lemma 1, it is crucial that both players have linear preferences over x, so that the DM wants to maximize each high type's expected payoff and minimize each low type's expected payoff. In particular, the property that it is optimal for each low-type agent to be given the same reward function as some high-type agent would be violated in more general settings as is shown in Silva (2019a) and in Siegel and Strulovici (2021).[15]

Lemma 1 is used to prove the first general result of the paper: for communication to be valuable p must not be ordered.

Proposition 1 *If p is ordered, communication has no value for the DM for any prior $q \in \Delta V$.*

Proof. Let $\eta^* : \overline{V} \times S \to [0,1]$ be such that $\eta^*(\overline{v}, \cdot) = h^*(\overline{v}, \cdot)$. By Lemma 1, it is enough to prove that η^* maximizes $\widetilde{U}$. Suppose not, so that there is some $\widehat{\eta} : \overline{V} \times S \to [0,1]$ that maximizes $\widetilde{U}$ such that $\widetilde{U}(\widehat{\eta}) > \widetilde{U}(\eta^*)$. Because p is ordered, there is a linear order $\succcurlyeq$ over s such that the likelihood ratio $\frac{p(s|\overline{v})}{p(s|\underline{v})}$ is increasing with s for any $(\overline{v}, \underline{v}) \in \overline{V} \times \underline{V}$. Consider $\widetilde{\eta} : \overline{V} \times S \to [0,1]$ as follows: for all $(\overline{v}, s) \in \overline{V} \times S$,

$$\widetilde{\eta}(\overline{v}, s) = \begin{cases} 1 \, if \; s \succ \alpha(\overline{v}) \\ \beta(\overline{v}) \, if \; s = \alpha(\overline{v}), \\ 0 \, if \; s \prec \alpha(\overline{v}) \end{cases}$$

where each $\alpha(\overline{v}) \in S$ and $\beta(\overline{v}) \in [0,1]$ are such that $E(\widetilde{\eta}(\overline{v}, s)|\overline{v}) = E(\widehat{\eta}(\overline{v}, s)|\overline{v})$.[16] I prove in Appendix A that, while, by construction, the expected payoff of every high type stays the same, p being ordered implies that each low type is made (weakly) worse.

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[14] I am grateful to an anonymous referee for suggesting a simpler proof.

[15] To get a sense of the argument, let us say that the agent's payoff function is $u(x) : [0,1] \to [0,1]$, where u is some increasing but strictly concave function. Fix some low type $\underline{v} \in \underline{V}$ and the high type $\omega(\underline{v}) \in \overline{V}$ he would prefer to mimic. If $\widetilde{h}(\omega(\underline{v}), \cdot)$ is not constant, the DM would be strictly better off choosing $\widetilde{h}(\underline{v}, s) = ce \in [0,1]$ for all s, where ce (certainty equivalent) is such that $u(ce) = E\left(u(\widetilde{h}(\omega(\underline{v}), s))|\underline{v}\right)$. In this way, low type $\underline{v}$ would be indifferent to mimicking type $\omega(\underline{v})$ but the DM would be made better off, because $ce < E\left(\widetilde{h}(\omega(\underline{v}), s)|\underline{v}\right)$.

[16] Notice that $\alpha(\overline{v})$ and $\beta(\overline{v})$ exist because $E(\widehat{\eta}(\overline{v}, s)|\overline{v}) \in [0,1]$ for each $\overline{v} \in \overline{V}$.

The intuition can be grasped from the example of the previous section; by ordering rewards according to $\succcurlyeq$, one benefits high types more than low types. Therefore, if one keeps high types indifferent, low types cannot be made better off.

This observation implies that $\widetilde{U}(\widetilde{\eta}) \geq \widetilde{U}(\widehat{\eta}) > \widetilde{U}(\eta^*)$, which, in turn, implies that $U(\widetilde{h}) > U(h^*)$, where $\widetilde{h} : V \times S \to [0.1]$ is such that $\widetilde{h}(\overline{v}, \cdot) = \widetilde{\eta}(\overline{v}, \cdot)$ and $\widetilde{h}(\underline{v}, \cdot) = \widetilde{\eta}(\omega(\underline{v}, \cdot))$ for all $\underline{v} \in \underline{V}$, where

$$\omega(\underline{v}) \in \arg\max_{\overline{v} \in \overline{V}} E(\widetilde{\eta}(\overline{v}, \cdot | \underline{v})).$$

By Lemma 1, allocation $\widetilde{h}$ is incentive compatible, which implies that $\alpha(\cdot)$ and $\beta(\cdot)$ must be constant; if not, every type would prefer the lowest α and, conditional on α, the lowest β. This implies that allocation $\widetilde{h}$ is independent of v and strictly preferred by the DM to allocation h^*, which is a contradiction.

■

A simple corollary of Proposition 1 is that communication has no value when the agent's signal and the DM's signal are affiliated, a standard assumption in Economics.[17] To gain some intuition on why affiliation prevents informative communication, consider the application discussed in the Introduction, where a firm wonders whether it is worth to ask its worker to complete a self-appraisal in order to determine his promotion. Say that the worker knows his ability, while the firm only observes the worker's sales' record. In principle, a self-report could be used to select an evaluation method; some reports would lead to promotion for some sales' values, while some other reports would lead to promotion for other sales' values. However, if ability and sales are affiliated, then each high-ability worker would prefer a rule that leads to promotion if the sales' record is good enough, because that is how high-ability workers distinguish themselves from low-ability workers; better workers distinguish themselves from worse workers by having good sales' records, not by having mediocre sales' records.

It is worth emphasizing that a key assumption in Proposition 1 is that v is a sufficient statistic of s, so that $E(\theta|v, s)$ is constant with s for all v. Under this

[17]Formally, s and v are affiliated if there is a linear order $\succcurlyeq$ such that, for all $(s, s') \in S \times S$,

$$s \succ s' \Rightarrow \frac{p(s|v')}{p(s|v'')} \geq \frac{p(s'|v')}{p(s'|v'')}$$

for all $v', v'' \in V$ such that $\theta_{v'} > \theta_{v''}$. Affiliation is a common assumption in moral hazard (e.g., Holmstrom, 1979, Grossman and Hart, 1983, Lambert, 1983), auctions (e.g., Milgrom and Weber, 1982, Persico, 2000, Pinkse and Tan, 2005, etc.) and principal-agent/sender-receiver models (e.g., Ottaviani and Prat, 2001, Kattwinkel, 2019, Guo and Shmaya, 2019, etc.).

assumption, the DM never loses by making each type's reward function a threshold rule over the linear order $\succcurlyeq$; as the proof of Proposition 1 shows with the construction of $\tilde{h}$, this reduces the low types' incentives to mimic and has no impact on the DM's expected payoff.

By contrast, if $E(\theta|v,s)$ was decreasing with s (and s is real-valued), imposing threshold rules over s would reduce the DM's expected payoff, because, for each fixed type v, a larger s would be more likely when θ is lower. Indeed, Kattwinkel (2019) shows that, when $\theta = v - s$ (so that both v and s are real-valued) and there is positive affiliation, communication might have value for the DM.[18] If, however, $E(\theta|v,s)$ is increasing with s, positive affiliation leads to communication not being valuable, because the enforcement of the aforementioned threshold rules would even increase the DM's expected payoff (a higher s would be more likely when θ is higher for any fixed v) in addition to dissuading misreports from lower types.

I now turn to the conditions under which communication is valuable to the DM. Below, I provide two sufficient conditions for communication to be valuable: a joint condition over p and q and a condition only on p for which there is always some q for which communication is valuable.

Proposition 2 *Fix p.*

i) Given $q \in \Delta V$, communication has value if there is some $\overline{v}' \in \overline{V}$ and some $s', s'' \in S$ such that

$$E(\theta|s') > 0 > E(\theta|s'')$$

and

$$\frac{p(s'|\overline{v}')}{p(s'|\underline{v})} < \frac{p(s''|\overline{v}')}{p(s''|\underline{v})}$$

for all $\underline{v} \in \underline{V}$.

ii) There is $q \in \Delta V$ such that communication has value if there are $\underline{v} \in \underline{V}$, $\overline{v}', \overline{v}'' \in \overline{V}$ and $s', s'' \in S$ such that

$$\frac{p\left(s'|\overline{v}'\right)}{p\left(s'|\underline{v}\right)} < \frac{p\left(s''|\overline{v}'\right)}{p\left(s''|\underline{v}\right)} \text{ and } \frac{p\left(s'|\overline{v}''\right)}{p\left(s'|\underline{v}\right)} > \frac{p\left(s''|\overline{v}''\right)}{p\left(s''|\underline{v}\right)}. \tag{3}$$

[18] In Kattwinkel (2019), in the optimal mechanism for the DM, if the agent reports a low v, then he is rewarded if and only if s is low enough. However, if the agent reports a high v, he is rewarded if and only if s is neither too high nor too low. The DM foregoes rewarding the agent when v is high and s is low to dissuade low types from mimicking high types - relative to high types, low types believe a low s is more likely due to positive affiliation.

Proof. To prove i), let

$$\rho \equiv \max_{\underline{v} \in \underline{V}} \frac{p(s''|\underline{v})}{p(s'|\underline{v})}$$

and take any $\varepsilon \in (0,1]$ and $\delta \in (0,1]$ such that $\varepsilon = \delta\rho$. Construct allocation h' as follows: for all $(v,s) \in V \times S$,

$$h'(v,s) = \begin{cases} g(s) \; if \; E(g(s)|v) > E(h^*(v,s)|v), \\ h^*(v,s) \; otherwise, \end{cases}$$

where function $g : S \to [0,1]$ is such that

$$g(s) = \begin{cases} 1 - \varepsilon \; if \; s = s', \\ \delta \; if \; s = s'', \\ h^*(v,s) \; otherwise. \end{cases}$$

By construction, allocation h' is incentive compatible. Moreover, the fact that

$$\frac{p(s''|\overline{v}')}{p(s'|\overline{v}')} > \rho \geq \frac{p(s''|\underline{v})}{p(s'|\underline{v})}$$

for all $\underline{v} \in \underline{V}$ implies that $E(g(s)|\overline{v}') > E(h^*(v,s)|\overline{v}')$ and that $E(g(s)|\underline{v}) \leq E(h^*(v,s)|\underline{v})$ for any $\underline{v} \in \underline{V}$, which, in turn, implies that $U(h') > U(h^*)$.

For ii), take any prior $q \in \Delta V$ such that

$$E(\theta|s') > 0 > E(\theta|s'')$$

that places a small probability on low types that are different than $\underline{v}$. For example, consider any q such that $q(v) = 0$ for all $v \neq \overline{v}', \overline{v}'', \underline{v}$, where $q(\overline{v}')$, $q(\overline{v}'')$ and $q(\underline{v})$ satisfy

$$\frac{p(s'|\overline{v}')}{p(s'|\underline{v})} q(\overline{v}')\,\overline{v}' + \frac{p(s'|\overline{v}'')}{p(s'|\underline{v})} q(\overline{v}'')\,\overline{v}'' > -q(\underline{v})\,\underline{v} > \frac{p(s''|\overline{v}')}{p(s''|\underline{v})} q(\overline{v}')\,\overline{v}' + \frac{p(s''|\overline{v}'')}{p(s''|\underline{v})} q(\overline{v}'')\,\overline{v}''.$$

Then, it follows that $U(h') > U(h^*)$, provided the measure of low types who prefer g over $h^*(v,\cdot)$ is sufficiently small (as in the q provided). ■

As the reader will note, condition ii) is satisfied in the Example of section 3 if $\alpha < \beta < 3/4$; specifically, $\overline{v}'' = 2$, $\overline{v}' = 1$ and $\underline{v} = -1$ with $s' = L$ and $s'' = R$. Condition i) is also satisfied if $\alpha < \beta < 3/4$, provided one assumes that q is such that $E(\theta|L) > 0 > E(\theta|R)$.

Finally, it is possible for p not to be ordered and for condition (3) not to hold simultaneously. Condition (3) fails to hold if and only if, for each low type $\underline{v} \in \underline{V}$, there is a linear order $\succcurlyeq_{\underline{v}}$ over s such that

$$s' \succ_{\underline{v}} s'' \Rightarrow \frac{p(s'|\overline{v})}{p(s'|\underline{v})} \geq \frac{p(s''|\overline{v})}{p(s''|\underline{v})}$$

for all $\overline{v} \in \overline{V}$. Let the set of such linear orders be denoted by $\Gamma(\underline{v})$. Then, for p not to be ordered, it must be that $\bigcap_{\underline{v} \in \underline{V}} \Gamma(\underline{v}) = \emptyset$. In that case, it is possible that communication has no value for any prior q but it might also happen that a prior can be found where communication is valuable.[19]

5 Applications

In this section, I discuss three applications where communication has value to the DM. The first application shows how the multidimensionality of the players' signals is likely to lead to communication being valuable. The second application shows that unidimensional signals might lead to communication being valuable as well when the DM has a faulty verification technology. The third application shows that communication might also be valuable when the agent has access to information about the DM's signal directly (and not just information about θ).

5.1 On multidimensionality

As before, the agent takes the role of a worker who is evaluated by the firm he works for (the DM). The agent's productivity θ is a (random) function of the agent's various skills. The agent privately knows the value of each of these, while the DM only observes imperfectly correlated signals. Specifically, let $v = (v_1, ..., v_J) \in V \equiv \prod_{j=1}^{J} V_j$ denote the multidimensional private signal observed by the agent, where each $v_j \in V_j \subset \mathbb{R}$ represents how good the agent is at skill $j = 1, ..., J$. Then, let $\theta_v = \sum_{j=1}^{J} v_j$ represent the agent's expected productivity. The DM observes multidimensional signal $s = (s_1, ..., s_J) \in S \equiv \prod_{j=1}^{J} S_j$, where each $s_j \in S_j \subset \mathbb{R}$ is positively affiliated with each v_j and conditionally independent across dimensions. Assume that each S_j and each V_j is finite and let the maximum and minimum of each set V_j be denoted by $\overline{v}_j$ and

[19] For example, communication is not valuable when there is a single high type with positive density or when the various high types have the same conditional distribution over s. In Appendix B, I provide an example where communication does help the DM.

$\underline{v}_j$ respectively. It is straightforward to verify that s and v are not affiliated (nor p is ordered) if $J > 1$. Indeed, below I use Proposition 2 to prove that, under fairly general conditions, communication is valuable to the firm.

Condition A: There is some dimension j^* and a pair $s'_{j^*}, s''_{j^*} \in S_{j^*}$ such that $s'_{j^*} < s''_{j^*}$ and

$$E(\theta|s'_{j^*}, \underline{s}_{-j^*}) < E(\theta|s''_{j^*}, \underline{s}_{-j^*}) < 0 < E(\theta|s'_{j^*}, \overline{s}_{-j^*}) < E(\theta|s''_{j^*}, \overline{s}_{-j^*}),$$

where $\underline{s}_{-j^*}$ and $\overline{s}_{-j^*}$ represent the case where every dimension but j^* reaches its lowest value and highest value respectively. Notice that the affiliation assumption implies that $E(\theta|s)$ is increasing in each s_j. Therefore, condition A is no more than a nondegeneracy condition that rules out trivial solutions and $J = 1$.

Condition B: For all $j = 1, ..., J$,

$$\overline{v}_j + \sum_{j' \neq j} \underline{v}_{j'} > 0.$$

Condition B requires the DM to want to hire the agent provided that at least one of the j dimensions reaches its maximum value.

Proposition 3 *If conditions A and B hold, communication has value.*

Proof. Let $s' \equiv (s'_{j^*}, \overline{s}_{-j^*})$ and $s'' \equiv (s''_{j^*}, \underline{s}_{-j^*})$. By condition A), it follows that $E(\theta|s') > 0 > E(\theta|s'')$. Define type $\overline{v}' \equiv (\overline{v}_{j^*}, \underline{v}_{-j^*})$ and notice that, by condition B), $\theta_{\overline{v}'} > 0$. Notice also that, for every type v that the DM would not want to hire (i.e., every low type), $v_{j^*} < \overline{v}_{j^*}$. Therefore,

$$\frac{p(s''|\overline{v}')}{p(s'|\overline{v}')} > \frac{p(s''|v)}{p(s'|v)}$$

for all v such that $\theta_v < 0$. It is then sufficient to apply Proposition 2, part i) to conclude that communication has value. ■

The reason why communication is valuable when there is more than one dimension is that high-type agents have different relative strengths. Therefore, the way in which high-type agents distinguish themselves from low-type agents is not constant; some high-type agents distinguish themselves from low type-agents by being exceptional in some dimensions rather than in others. The DM can use the agent's input in order to determine which evaluation scheme suits better each high-type agent.

Consider, as an example, the following mechanism. The DM asks the agent to select one of $J+1$ "evaluation tracks". If the agent chooses track $j=0$, he is promoted if and only if $E(\theta|s) \geq 0$ (i.e., the default is to be evaluated as if there was no communication). By contrast, if the agent chooses track $j=1,...,J$, whether or not the agent gets promoted only depends on s_j (i.e., the agent gets promoted if and only if s_j is sufficiently large). In Appendix C, I prove that such a mechanism can be constructed where high-type agents select different tracks based on their relative strengths. In particular, "specialists" pick the new track $j>0$ that suits their best skill, while high-type agents with a more balanced profile stick with track 0. More importantly, I prove that this mechanism does strictly better than simply promoting the agent as a function of s, because it improves the probability that "specialists" are promoted without increasing the probability of any low-type agent being promoted.[20]

5.2 Faulty verification technology

A seller (the agent), privately informed of the quality of the product he wants to sell ($v=\theta_v=\theta$), communicates with a buyer (the DM) who observes an imperfect signal of quality $s \in V \subseteq \mathbb{R}$. Assume that

$$p(s|v) = \begin{cases} \lambda \, if \; s=\theta \\ \frac{1-\lambda}{|V|-1} \, if \; s \neq \theta \end{cases},$$

where $\lambda \geq \frac{1}{|V|}$. In words, each product quality θ is more likely to generate a signal $s=\theta$ than any other signal, which is assumed to be equally likely. For example, it might be that the DM asks a third party to verify the product's quality but that third party only does so with some probability. As before, the DM's signal allows her to imperfectly verify the statements made by the agent.

Notice that communication is not valuable when $\lambda \in \left\{\frac{1}{|V|}, 1\right\}$. The case of $\lambda = \frac{1}{|V|}$ is the case where the third party never verifies the product's quality; s would then be independent of v and no information would be transmitted. If $\lambda=1$, the product

[20]This mechanism is reminiscent of cheap talk equilibria in models with multidimensional signals. In Chakraborty and Harbaugh (2007) and Che, Dessein and Kartik (2013) multidimensionality partially aligns the agent and the DM's preferences enough to sustain equilibria where the agent makes comparative statements about various alternatives. If the agent has state-independent preferences, Chakraborty and Harbaugh (2007, 2010) show that multidimensionality may also enable communication if the agent is able to trade-off the various dimensions in order to be kept indifferent. In a setting with only two alternatives that is not possible. In this paper, it is the fact that the DM is privately informed that allows for communication. The role of multidimensionality is to generate a non-ordered p, which is what enables communication.

is always verified, so the DM always knows exactly the quality of the product and does not require the information that the agent has. I show that, for a large set of parameters, communication is valuable when $\lambda \in \left(\frac{1}{|V|}, 1\right)$.

Proposition 4 *Communication is valuable if $E(\theta|s)$ is increasing with $s \in S$ and there are $s', s'' \in S$ such that $s' > s'' > 0$ for which $E(\theta|s') > 0 > E(\theta|s'')$.*

Proof. Let $\overline{v}'' \equiv s'' > 0$ and notice that

$$\frac{p(s''|\overline{v}'')}{p(s'|\overline{v}'')} > \frac{p(s''|\underline{v})}{p(s'|\underline{v})}$$

for all $\underline{v} < 0$. Therefore, the statement follows by Proposition 2, part i). ■

Neither assumption in the statement of Proposition 4 is too demanding: $E(\theta|s)$ is increasing with s if, for example, v is uniformly distributed, while the existence of signals s' and s'' can be guaranteed if it is likely that the product's quality is negative.

In this application, in addition to demonstrating that communication is valuable, it is possible to go further and characterize the optimal mechanism for the DM. I prove in Appendix D that, in the optimal mechanism, the agent always announces a positive product quality level $\widehat{\theta} > 0$. Depending on the prior q, the DM's strategy is one of the following. For some prior distributions, the DM buys the product with some probability $\tau \in (0, 1]$ whenever her signal matches the announcement $(s = \widehat{\theta})$ and does not otherwise. For the other prior distributions, the DM buys the product with certainty if her signal matches the announcement $(s = \widehat{\theta})$, buys with probability $\tau \in (0, 1)$ if $s > 0$ even though it does not match the announcement, and chooses not buy it at all if $s < 0$. If the product's quality is positive $(\theta > 0)$, the agent prefers to announce it truthfully $(\widehat{\theta} = \theta)$, because he fears that the DM's signal will not match the quality announced if he exaggerates. If it is negative, the agent randomizes over the set of positive announcements $\widehat{\theta} > 0$.

This application is also useful because it can be used to illustrate how the commitment result in Ben-Porath, Dekel and Lipman (2019) applies. As the reader will note, it does not matter for the DM how exactly the agent randomizes when the product's quality is negative; any corresponding allocation is optimal, because the DM's expected payoff only depends on the interim utility of the various agent's types. Indeed, that is key in proving that the DM requires no commitment power. Because it does not matter how the agent randomizes when the product's quality is negative, it is possible to find a randomization distribution for the agent such that the DM's posterior beliefs upon observing the agent's announcement and her own signal are consistent with her strategy. Specifically, take the second set of prior distributions described above. There is a randomization distribution for the agent such that the DM believes the expected quality of the product to be positive if $\widehat{\theta} = s > 0$ (i.e., the DM would prefer to buy), equal to 0 if $\widehat{\theta} \neq s > 0$ (i.e., the DM would be indifferent) and negative if $s < 0$ (i.e., the DM would prefer not to buy).

In addition to illustrating how uni-dimensional signals might lead to communication being valuable, this application is also interesting in and of itself because of its relation to the literature on lie detection (Balbuzanov, 2019, Dziuda and Salas, 2019). In that literature, agents who make reports that are different from their observed signals might trigger a lie detector, which returns a verdict of true or false. The benefits of modelling lie detection in a game where the DM observes a correlated signal are that, on the one hand, it allows for "false positives"; in the literature on lie detection, true statements never trigger the lie detector. Furthermore, agents have an arbitrarily wide array of statements at their disposal, which, in addition to its added realism, allows for the use of the revelation principle.

5.3 Multiple sources

The author of a scientific article (the agent) sends his paper for publication at some journal, while the editor (the DM) obtains a signal s about the paper's contribution by consulting J referees. Assume that $s = (s_1, s_2, ..., s_J) \in S \equiv \prod_{j=1}^{J} S_j$ and that each $s_j \in S_j$ represents referee j's opinion. The agent observes $v = (\theta, y)$ so that, in addition to observing the value of the article $\theta \in \Theta$, the agent observes a second random variable $y \in \{1, 2, ..., J\}$ that represents which of the referees is the most able. Specifically, assume that, conditional on v, the distribution of each s_j is independent across j and given by

$$p_j\left(s_j|\theta, y\right) = \begin{cases} f^y\left(s_j|\theta\right)\ if\ y = j \\ f^0\left(s_j\right)\ if\ y \neq j \end{cases},$$

where each $f^y\left(s_j|\theta\right)$ is such that $\frac{f^y(\cdot|\theta')}{f^y(\cdot|\theta'')}$ is increasing for all $\theta' > \theta''$. This means that, if $y = j$, then s_j is affiliated with θ; if not, then s_j and θ are independent. Throughout, assume that Θ and each S_j are finite and denote the minimum element of Θ by $\underline{\theta}$.

If the DM does not communicate with the agent, she simply chooses to accept the paper whenever $E\left(\theta|s\right) \geq 0$. The problem with that is that there is a lot of noise in s, because all but one referee provide reports that are independent of the paper's quality.

As in the previous application, it is possible to find the optimal allocation under some general assumption over the distributions of the signals. Specifically, let us assume that the agent prefers to select the most able referee over any of the referees who have an independent signal even when the paper's quality θ is the lowest: for all $y \in \{1, ..., J\}$,

$$\sum_{s_y \in S_y} \mathbf{1}\left\{E\left(\theta|s_y, y\right) \geq 0\right\} f^y\left(s_y|\underline{\theta}\right) \geq \max_{j \in \{1,..,J\}} \sum_{s_j \in S_j} \mathbf{1}\left\{E\left(\theta|s_j, j\right) \geq 0\right\} f^0\left(s_j\right). \quad (4)$$

I show in Appendix E that, if condition (4) holds, an optimal allocation can be implemented by the agent (truthfully) reporting his preferred referee r to the DM and the DM accepting the paper if and only if $E\left(\theta|s_r, r\right) \geq 0$. The intuition is that, even though the agent cannot be given enough incentives to report θ directly (because of Proposition 1), he reports everything else (the y in this case), which helps the DM make as good of a decision as possible.

6 Conclusion

The paper discusses the conditions under which communication between a privately informed agent and a DM is beneficial for the DM when she can imperfectly verify the agent's report. As is well known, in the two extreme cases of independent signals and perfectly correlated signals, communication does not help the DM. The paper finds that, when signals are imperfectly correlated, whether communication helps the DM depends, to a large extent, on whether the signals satisfy a weak notion of affiliation (i.e., if p is ordered).

The paper makes three broad points. First, if the signals do satisfy weak affiliation, communication is never valuable to the DM. Second, there are various natural settings in which weak affiliation is not satisfied. The paper provides three such examples: multidimensional signals, imperfect verification with uniform "lie-detection" technology, and the existence of belief-types on the side of the agent. Third, when weak affiliation is not satisfied, it is natural that communication has some value to the DM. The paper argues this through Proposition 2 and by deriving sufficient conditions for information to be valuable in each of the three applications discussed. The results in Ben-Porath, Dekel and Lipman (2019) further complement the strength of this last point, because they show that the DM does not require commitment power to obtain all of the value generated by communication. Indeed, in sections 5.2. and 5.3., I explicitly characterize optimal mechanisms for the DM which do not require her to have commitment power.

From an applied point of view, these results contribute to the discussion over the usefulness of self-appraisals for firms. They suggest that in unidimensional jobs, where there is only one way of doing the job well, self-appraisals are indeed pointless as has been suggested by the business literature. But for multidimensional jobs - jobs that require multidimensional skills - self-appraisals are generally useful for firms. Moreover, firms do not even have to design specific evaluation mechanisms and then have the power to enforce them. It is sufficient to ask workers for input before the correlated signal is realized and then use that input in a sequentially optimal way.

7 Appendix

7.1 Appendix A

In this appendix, I complete the proof of Proposition 1 by showing that, for any $\overline{v} \in \overline{V}$ and $\underline{v} \in \underline{V}$,

$$E(\widetilde{\eta}(\overline{v}, s)|\underline{v}) \leq E(\widehat{\eta}(\overline{v}, s)|\underline{v}).$$

This follows by repeatedly applying the lemma below, which, in words, states that, if the DM moves the rewards towards the signals s with larger ratio $\frac{p(s|\overline{v})}{p(s|\underline{v})}$ while keeping the high type $\overline{v}$ indifferent, the low type is made worse off.

Lemma 2 *Consider any* $(\overline{v}, \underline{v}) \in \overline{V} \times \underline{V}$*, any pair* $s', s'' \in S$ *such that* $\frac{p(s''|\overline{v})}{p(s''|\underline{v})} \geq (>) \frac{p(s'|\overline{v})}{p(s'|\underline{v})}$ *and any reward function* $g : S \to [0,1]$ *such that* $g(s') > 0$ *and* $g(s'') < 1$*. Consider any reward function* $g' : S \to [0,1]$ *such that*

$$g'(s) = \begin{cases} g(s) - \varepsilon \; if \; s = s' \\ g(s) + \delta \; if \; s = s'' \\ g(s) \; otherwise \end{cases}.$$

If $\varepsilon \in (0, g(s')]$ *and* $\delta \in (0, 1 - g(s'')]$ *are such that*

$$E(g'(s)|\overline{v}) = E(g(s)|\overline{v}),$$

then

$$E(g'(s)|\underline{v}) \leq (<) \; E(g(s)|\underline{v}).$$

Proof. In order for type $\overline{v}$ to be indifferent, it must be that

$$\frac{\varepsilon}{\delta} = \frac{p(s''|\overline{v})}{p(s'|\overline{v})}.$$

Notice that

$$\frac{p(s''|\overline{v})}{p(s''|\underline{v})} \geq (>) \frac{p(s'|\overline{v})}{p(s'|\underline{v})} \Leftrightarrow \frac{p(s''|\overline{v})}{p(s'|\overline{v})} = \frac{\varepsilon}{\delta} \geq (>) \ \frac{p(s''|\underline{v})}{p(s'|\underline{v})},$$

which implies that

$$E\left(g'(s)\,|\underline{v}\right) \leq (<) \ E\left(g(s)\,|\underline{v}\right).$$

■

7.2 Appendix B

Below, I provide the example mentioned in footnote 19. Let $v = \theta_v \in \left\{-2, -\frac{6}{5}, \frac{2}{5}, \frac{18}{5}\right\}$ and $s \in \{L, M, H\}$ with q being uniform and

$p(s|v) =$

	L	M	H
$\theta_v = 18/5$	0.24	0.26	0.5
$\theta_v = 2/5$	0.15	0.25	0.6
$\theta_v = -6/5$	0.1	0.2	0.7
$\theta_v = -2$	0.4	0.4	0.2

It can be shown that the optimal allocation h is given by the table below:

$h(v, s) =$

	L	M	H
$\theta_v = 18/5$	1	0	1
$\theta_v = 2/5$	0.85	1	0.077
$\theta_v = -6/5$	1	0	1
$\theta_v = -2$	1	0	1

which is incentive compatible and strictly better than h^*, where $h^*(v, s) = \mathbf{1}\{s \in \{L, H\}\}$.

7.3 Appendix C

Start by defining

$$\widetilde{v}_j = \min\left\{v_j \in V_j : v_j + \sum_{i \neq j} \underline{v}_i \geq 0\right\}$$

as the lowest value that the agent might have at skill j for the DM to want to hire him even when his other skills have minimum value. Notice that type $v^j \equiv \left(\underline{v}_1, ..., \widetilde{v}_j, \underline{v}_{j+1}, ..., \underline{v}_J\right)$ is a high type.

Consider the mechanism described in the text where each track $j > 0$ is such that the probability the agent is promoted is $g_j(s_j)$, where

$$g_j(s_j) = \begin{cases} 1\, if\ s_j > \alpha_j \\ \beta_j\, if\ s_j = \alpha_j \\ 0\, if\ s_j < \alpha_j \end{cases}$$

and $\alpha_j \in S_j$ and $\beta_j \in [0, 1]$ are such that type v^j is indifferent between choosing track j and choosing track 0.

It is sufficient to prove that some high types strictly benefit from picking $j > 1$, while every low type's favourite track is $j = 0$. To that end, notice that, for all $v = \left(\underline{v}_1, ..., v_j, \underline{v}_{j+1}, ..., \underline{v}_J\right)$,

$$\frac{p(s'|v)}{p(s'|v^j)} > \frac{p(s''|v)}{p(s''|v^j)}$$

if and only if

$$\frac{p_j\left(s_j'|v_j\right)}{p\left(s_j'|\widetilde{v}_j\right)} > \frac{p_j\left(s_j''|v_j\right)}{p\left(s_j''|\widetilde{v}_j\right)}.$$

Notice also that each track j is such that some of the weight on the signals s for which $E(\theta|s) > 0$ are shifted towards signals with larger values of s_j. Therefore, by construction of each track j and by the repeated use of Lemma 2 (stated in Appendix A), it follows that type v strictly prefers track j if $v_j > \widetilde{v}_j$ and strictly prefers track 0 if $v_j < \widetilde{v}_j$. In other words, types that have minimum skills for all other dimensions will choose track j if and only if they are a high type (i.e, specialists in skill j will choose track j). Moreover, the fact that such type $v = \left(\underline{v}_1, ..., v_j', \underline{v}_{j+1}, ..., \underline{v}_J\right)$ with $v_j' < \widetilde{v}_j$ prefers track 0 implies that any type v' with $v_j = v_j'$ will also prefer track 0, because the expected payoff of choosing track j is the same for both types while the expected payoff of reporting track 0 is larger for type v' than it is for type v. Therefore, every low type prefers to choose track 0.

7.4 Appendix D

For each high type $\overline{v} \in \overline{V}$, divide $S(= V)$ into three sets:

$$A(\overline{v}) \equiv \{\overline{v}\},$$

$$B(\overline{v}) = \left\{s \in \overline{V} : s \neq \overline{v}\right\}$$

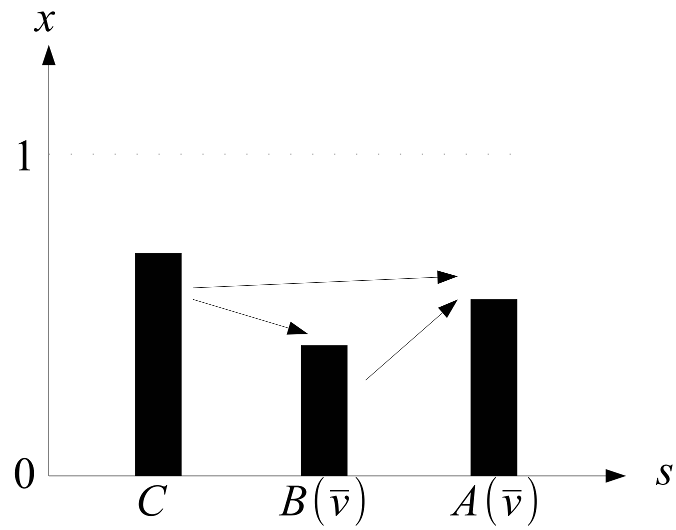

Figure 1: Shifting rewards to the top

and

$$C = \underline{V}.$$

Notice that, for any pair $(\overline{v}, \underline{v}) \in \overline{V} \times \underline{V}$ and for any $s_a \in A(\overline{v})$, $s_b \in B(\overline{v})$ and $s_c \in C$,

$$\frac{p(s_a|\overline{v})}{p(s_a|\underline{v})} > \frac{p(s_b|\overline{v})}{p(s_b|\underline{v})} > \frac{p(s_c|\overline{v})}{p(s_c|\underline{v})}.$$

By defining $\eta^* : \overline{V} \times S \to [0.1]$ as some maximizer of $\widetilde{U}$, construct $\widehat{\eta} : \overline{V} \times S \to [0.1]$ as follows: for each high type $\overline{v} \in \overline{V}$ shift rewards first from sets $B(\overline{v})$ and C towards set $A(\overline{v})$ and then from set C to set $B(\overline{v})$ as Figure 1 illustrates in such a way that

$$E(\widehat{\eta}(\overline{v}, s)|\overline{v}) = E(\eta^*(\overline{v}, s)|\overline{v}).$$

By repeatedly invoking Lemma 2, it follows that $\widehat{\eta}$ is also a maximizer of $\widetilde{U}$ (because each high-type's interim utility stays the same, while each low type's interim utility becomes weakly lower). Then, use $\widehat{\eta}$ to construct $\widetilde{\eta} : \overline{V} \times S \to [0,1]$, where, for each $\overline{v} \in \overline{V}$, $\widetilde{\eta}(\overline{v}, s) = \widehat{\eta}(\overline{v}, s)$ for all $s \in A(\overline{v}) \cup C$, and

$$\widetilde{\eta}(\overline{v}, s) = \frac{\sum_{s' \in B(\overline{v})} \widehat{\eta}(\overline{v}, s')}{|B(\overline{v})|}$$

for all $s \in B(\overline{v})$. By construction, $\widetilde{\eta}$ maximizes $\widetilde{U}$ (because every type's interim utility stays the same).

Define allocation $\widetilde{h} : V \times S \to [0,1]$ such that $\widetilde{h}(\overline{v}, \cdot) = \widetilde{\eta}(\overline{v}, \cdot)$ for all $\overline{v} \in \overline{V}$ and $\widetilde{h}(\underline{v}) = \widetilde{\eta}(\omega(\underline{v}))$ for all $\underline{v} \in \underline{V}$, where

$$\omega(\underline{v}) \in \arg\max_{\overline{v} \in \overline{V}} E(\widetilde{\eta}(\overline{v}, \cdot)|\underline{v}).$$

By Lemma 1, allocation $\widetilde{h}$ is an optimal incentive compatible allocation.

The proof is completed by demonstrating the following two claims, which, combined, demonstrate that allocation $\widetilde{h}$ has the properties described in the text.

Claim 1: *For all $\overline{v} \in \overline{V}$, $\widetilde{\eta}(\overline{v}, s) = 0$ for all $s < 0$.*

Proof. Suppose not, so that there is some pair $(\overline{v}', \underline{v}') \in \overline{V} \times \underline{V}$ such that $\widetilde{\eta}(\overline{v}', \underline{v}') > 0$. By construction, that means that $\widetilde{\eta}(\overline{v}', s) = 1$ for all $s \in \overline{V}$, which, by incentive compatibility of $\widetilde{h}$, implies that, for all $\overline{v} \in \overline{V}$, $\widetilde{h}(\overline{v}, s) = 1$ for all $s \in \overline{V}$. That every report is rewarded if $s \in \overline{V}$ implies that the expected utility of every high type is the same, i.e.,

$$E(\widetilde{\eta}(\overline{v}, s)|\overline{v}) = E(\widetilde{\eta}(\overline{v}, s)|\overline{v}')$$

for all $\overline{v} \in \overline{V}$. As a result, it also follows that allocation $\overline{h} : V \times S \to [0,1]$ is an optimal allocation, where $\overline{h}$ is such that $\overline{h}(v, \cdot) = \widetilde{\eta}(\overline{v}', \cdot)$ for all $v \in V$. Seeing as allocation $\overline{h}$ does not depend on the seller's report of $v \in V$, it follows that allocation h^* defined in the text (which is optimal among those that are independent of v) is also optimal, which contradicts Proposition 5.

■

Claim 2: *For all $\overline{v}, \overline{v}' \in \overline{V}$, $E(\widetilde{\eta}(\overline{v}, s)|\overline{v}) = E(\widetilde{\eta}(\overline{v}', s)|\overline{v}')$.*

Proof. Suppose not, so that there are some $\overline{v}'', \overline{v}' \in \overline{V}$ such that

$$E(\widetilde{\eta}(\overline{v}'', s)|\overline{v}'') > E(\widetilde{\eta}(\overline{v}', s)|\overline{v}').$$

Consider $\overline{\eta} : \overline{V} \times S \to [0.1]$ such that i) $\overline{\eta}(\overline{v}, \cdot) = \widetilde{\eta}(\overline{v}, \cdot)$ for all $\overline{v} \neq \overline{v}'$, ii) $\overline{\eta}(\overline{v}', \overline{v}') = \widetilde{\eta}(\overline{v}'', \overline{v}'')$, iii) for all $s > 0$ such that $s \neq \overline{v}'$, $\overline{\eta}(\overline{v}', s) = \widetilde{\eta}(\overline{v}'', \overline{v}')$ and iv) for all $s < 0$, $\overline{\eta}(\overline{v}', s) = \widetilde{\eta}(\overline{v}'', s) = 0$. By construction, $\widetilde{U}(\overline{\eta}) > \widetilde{U}(\widetilde{\eta})$, which is a contradiction. ■

7.5 Appendix E

Consider the following relaxed problem R', where the only incentive constraints considered are as follows:

$$E\left(h\left(\theta, y\right)|\theta, y\right) \geq E\left(h\left(\theta', y\right)|\theta, y\right)$$

for all θ, θ'. In words, the agent is not allowed to misreport over y. By Proposition 1, it follows that the allocation $\widehat{h}$ that solves this relaxed problem is as follows:

$$\widehat{h}\left(\left(\theta, y\right), s\right) = \begin{cases} 1\, if\ E\left(\theta | s_y, y\right) \geq c \\ 0\, if\ E\left(\theta | s_y, y\right) < c \end{cases}$$

for all $(\theta, y) \in V$ and $s \in S$. Therefore, allocation $\widehat{h}$ is optimal whenever it is incentive compatible, which happens when condition (4) holds.

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