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In the Shadow of Shadow Banking: A Liquidity Perspective

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Liquidity requirements for commercial banks improve risk-sharing for depositors. Nevertheless, shadow banks, issuing securities with lower liquidity, operate outside such regulatory constraints. In an economy featuring shadow banks with a constant level of liquidity for shadow bank securities, higher liquidity requirements lead to a reduction in aggregate liquidity provision, owing to regulatory arbitrage incentives. Conversely, when the liquidity of shadow bank securities decreases with the market share of shadow banks, the incentive for regulatory arbitrage is reduced, and thus higher liquidity requirements could enhance aggregate liquidity provision.

KEYWORDS. Shadow Banking, Liquidity Requirements, Regulatory Arbitrage, Liquidity Shortage, Search and Matching.

JEL CLASSIFICATION. E40, E50, G20.

1. INTRODUCTION

Liquidity provision lies at the heart of the function of banks. Diamond and Dybvig (1983) show that banks can provide optimal liquidity insurance to consumers. However, Jacklin (1987) and Allen and Gale (2004) show that the liquidity role of banks relies on the premise that consumers are prohibited from engaging in trades in private markets. Farhi, Golosov, and Tsyvinski (2009) propose using liquidity requirements to restore the allocation efficiency of a banking system with private market trades. Requiring banks

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to hold more liquidity reserves reduces the interest rate in the private market and disincentivizes agents from deviating. However, financial innovations could help shadow banks bypass liquidity regulations. In this paper, we show that the existence of shadow banks could weaken or even invalidate the role of liquidity requirements, since shadow banks can again offer a high interest rate in the private market, while bank deposits are still constrained.

We characterize shadow banks as financial institutions that are free from liquidity requirements but produce debts less liquid than bank deposits. Regulatory arbitrage allows shadow banks to invest all received goods in long-term projects. Commercial banks are liquidity providers in the economy. When suffering liquidity shocks, consumers can sell a portion of their shadow bank securities in private markets to obtain liquidity. A reduced capacity to sell shadow bank securities indicates a higher degree of illiquidity of these assets.

Compared to commercial banks offering deposit contracts, shadow banks can bypass the liquidity requirement and make more investments, thus offering a higher long-term return. However, due to the illiquidity, shadow bank securities yield a lower short-term return. Based on this trade-off, consumers allocate their endowments to commercial banks and shadow banks. A higher liquidity requirement, while requiring commercial banks to hold more liquidity reserves, will make shadow banks more attractive, since commercial banks have to offer a lower long-term return under a higher liquidity requirement. Thus, although a higher liquidity requirement increases the liquidity provided by each unit of deposit, it also drives more funds to flow into illiquid shadow bank securities, which exacerbates the liquidity shortage problem since the market share of the liquidity providers (commercial banks) decreases. We show that this regulatory arbitrage effect will finally make liquidity requirements ineffective in improving aggregate liquidity provision.

When the liquidity requirement is lower than a certain threshold, only commercial banks exist in equilibrium. A higher liquidity requirement, by forcing commercial banks to hold more reserves, increases the aggregate liquidity provision. However, when the liquidity requirement is higher than the threshold, shadow banks emerge in equilibrium. The equilibrium thus features the coexistence of commercial and shadow banks. The regulatory arbitrage effect emerges. We show that the negative effect of the decrease in the amount of deposits is greater than the positive effect of the increase in the liquidity provided by each unit of deposit. Thus, a higher liquidity requirement will reduce aggregate liquidity provision with the coexistence of commercial and shadow banks.

The intuition is that when the liquidity requirement increases, if the aggregate liquidity provision does not change, then the private market return is so small that consumers would strictly prefer shadow bank securities. Thus, the aggregate liquidity provision needs to further decrease to make shadow bank securities less attractive. This result has important implications regarding liquidity regulation. In the presence of shadow banks, a higher liquidity requirement aiming at improving liquidity provision may instead exacerbate the liquidity shortage problem, since illiquid shadow bank securities substitute deposits. The regulators should therefore regulate commercial and shadow banks in a unified framework.

The optimal liquidity requirement for commercial banks thus depends on the liquidity of shadow bank securities. When shadow bank securities are very illiquid, the optimal liquidity requirement is identical to the no-shadow-banking case, as studied by Farhi, Golosov, and Tsyvinski (2009), and shadow banks do not arise in equilibrium. However, when shadow bank securities are sufficiently liquid, the same optimal liquidity requirement will give rise to shadow banks, and setting the liquidity requirement to be the threshold level at which shadow banks just do not emerge will help the economy to achieve a second-best allocation.

We also study how changes in the liquidity of shadow bank securities affect equilibrium allocation when the liquidity requirement is fixed. An increase in the liquidity of shadow bank securities has two effects. On the one hand, better shadow bank liquidity increases the attractiveness of shadow bank securities and thus decreases the supply of deposits. On the other hand, better liquidity means that more shadow bank securities can be sold in the private market, which demands more deposits to clear the market. The net effect varies with the liquidity requirement. When the liquidity requirement is relatively low, the market share of the shadow banks is very small, and thus a change in shadow bank liquidity only has a small impact on the demand for deposits. Thus, the decrease in deposit supply dominates and shadow banks will occupy a larger market share, which leads to lower aggregate liquidity provision. Conversely, when the liquidity requirement is very high, the market share of shadow banks is very large, and thus a change in shadow bank liquidity has a significant impact on the private market. In this case, the increase in deposit demand dominates, and commercial banks will occupy a larger market share, which leads to a higher liquidity provision.

The baseline model features a constant level of the liquidity of shadow bank securities. However, liquidity requirements may have a direct influence on the liquidity of shadow bank securities, since they can change the liquidity supply and demand. We thus extend the baseline model by endogenizing the liquidity of shadow bank securities in a search and matching framework. Search frictions serve as the micro-foundation of the illiquidity of shadow bank securities. Shadow bank securities, such as securitized assets, are traded predominantly in over-the-counter markets featuring search frictions (see Weill, 2020). Sellers need to search for buyers in these markets, and the sellers may fail to find buyers for all of their shadow bank securities. The liquidity supply and demand affect the difficulty of searching and thus determine the illiquidity of shadow bank securities.

We assume that there is an upper limit of the deposits that patient consumers can withdraw at date 1 to purchase shadow bank securities. Then, when the liquidity requirement is high, this constraint becomes binding and patient consumers cannot prematurely withdraw more deposits to purchase shadow bank securities. The probability of a successful matching will become smaller when the liquidity requirement further increases, because impatient consumers post more sell orders but the number of buyers does not increase. Thus, a higher liquidity requirement will reduce the liquidity of shadow bank securities and thus weaken the incentive for regulatory arbitrage. Although the market share of deposits still decreases with the liquidity requirement, the impact of liquidity requirements on aggregate liquidity provision may be quite different.

If the upper limit of patient consumers' early withdrawal is high, an increase in the liquidity requirement still reduces aggregate liquidity provision, and this result is the same as the baseline model. However, if the upper limit of patient consumers' early withdrawal is low, an increase in the liquidity requirement can result in higher aggregate liquidity provision. Liquidity requirements become effective in improving aggregate liquidity provision even in the presence of shadow banks. Intuitively, when the upper limit is low, patient consumers can only provide very limited liquidity to purchase shadow bank securities by withdrawing deposits. This imposes a strict constraint on the liquidity of shadow bank securities and significantly suppresses the incentive of regulatory arbitrage. Consequently, liquidity requirements become effective in adjusting aggregate liquidity provision.

Endogenizing the liquidity of shadow bank securities yields novel implications in terms of optimal liquidity regulations. When the liquidity of shadow bank securities is constant, shadow banks should not exist under the optimal liquidity requirement. By contrast, when the liquidity of shadow bank securities is endogenous, the optimal liquidity requirement may be achieved when commercial and shadow banks coexist, because a higher liquidity requirement can play a positive role in improving aggregate liquidity provision. Therefore, market structure and search frictions have an important impact on optimal liquidity regulations. Policymakers need to consider these factors when setting optimal liquidity regulations.

Evidently shown in our earlier discussion, our paper is most closely related to Farhi, Golosov, and Tsyvinski (2009). Another closely related paper is Arseneau, Rappoport W., and Vardoulakis (2020), which also studies how trade frictions affect liquidity provision. However, while they focus on the congestion externality in the OTC market that results in inefficient allocation of liquidity and how government intervention may correct it, in our paper, trade frictions may be ex ante welfare-improving since it acts as another ex ante commitment mechanism that disincentivizes agents from involving in inefficient regulatory arbitrage activities. Our paper also relates to He, Liu, and Xie (2023), which proposes a fiscal solution to the liquidity shortage problem when liquidity requirements are ineffective. While the focus of that paper is the alternative policy solution, this paper focuses on the mechanism through which the existence of shadow banking can render liquidity requirements ineffective. We analyze both scenarios where the liquidity of shadow bank securities is exogenously given or endogenously determined.

The costs and benefits of liquidity requirements come through risk-sharing for consumers. In our paper, the only risk that needs to be shared is the idiosyncratic preference shocks on consumers. However, pioneered by Allen and Gale (1998), literature has introduced aggregate risks on project returns and studied the corresponding optimal regulations. Allen and Gale (1998) show that in the presence of aggregate risks, standard deposit contracts that promise noncontingent payment cannot offer optimal risk sharing. In this case, a bank run can be welfare-improving as it provides payoffs contingent on the realized aggregate shock. They also find that liquidity requirements can be inefficient. However, their mechanism is that liquidity requirements exclude efficient bank runs that provide contingent payoffs, while our mechanism is that liquidity requirements can incur more severe regulatory arbitrage.

Following Allen and Gale (1998), an expanding body of literature employs the global game methodology to examine optimal government interventions when banks are subject to *endogenous* run risks. Allen et al. (2018) demonstrate that government guarantees, despite occasionally heightening the likelihood of bank runs, enhance overall welfare by bolstering liquidity provision. Kashyap, Tsomocos, and Vardoulakis (2023) solve a novel global game, where depositors choose whether to run based on conjectures about the liquidation value of long-term projects and beliefs about banks' monitoring. They show that to achieve the socially optimal allocation, a combination of three regulations, comprising a capital requirement, a liquidity requirement, and deposit or lending subsidies/taxes, are needed to correct the three distortions associated with banks' asset allocation, capital structure, and level of lending that is deposit funded. Schilling (2023) shows that aggressive interventions may actually render banks less stable, because depositors could preempt the regulator by running the banks. In contrast to these papers, we focus on equilibria with no bank runs, with liquidity regulations impacting consumers' incentives to invest in alternative assets free from regulations rather than their incentive to run the banks.

The regulatory arbitrage activities in our model are analogous to the *search-for-yield* behaviors that investors have incentive to invest in high-yield risky assets in a low-interest-rate environment (see Rajan, 2006). We examine the link between low interest rates and the incentive to invest in high-yield assets from a liquidity perspective, which is new compared to the existing literature (see Martinez-Miera and Repullo, 2017, Campbell and Sigalov, 2022, Allen, Barlevy, and Gale, 2022). In our model, a higher liquidity requirement not only limits the deposit contracts that banks can offer, but also provides more liquidity in the private market and reduces the private market return. Both of these two effects make high-yield shadow bank securities more attractive. Our policy implications have some similarities to Allen, Barlevy, and Gale (2022), who argue that macroprudential policies such as leverage restrictions can be counterproductive. However, the mechanism is different. In their paper, leverage restrictions discourage safe investments and shift resources toward riskier speculation. In our model, liquidity requirements discourage investments in high-liquidity deposits and shift resources toward less liquid shadow bank securities.

Finally, our paper is related to the strand of literature explaining the emergence of shadow banks. The existence of shadow banks can be motivated by the shortage of safe assets, and shadow banks can provide "quasi-safe" assets as imperfect substitutes for safe assets (see Gennaioli, Shleifer, and Vishny, 2013 and Moreira and Savov, 2017). The existence of shadow banks can also be motivated by regulatory arbitrage of capital requirements (see, for example, Plantin, 2014; Ordoñez, 2018; Huang, 2018; Kara and Ozsoy, 2020; and Farhi and Tirole, 2021). Plantin (2014) focuses on the impact of shadow banking on aggregate credit risk. He highlights that to reduce the overall credit risk of the economy, it may be optimal to relax capital requirements to suppress the incentive for regulatory arbitrage. Unlike these papers, we characterize shadow banks as regulatory arbitrage of liquidity requirements and thus are able to study the impact of shadow banks on aggregate liquidity provision. Grochulski and Zhang (2019) characterize shadow banks as intermediaries that can circumvent a tax on long-term investment but are subject to higher investment costs at date 0. By contrast, our paper characterizes shadow banks as intermediaries that can circumvent a liquidity requirement but have inferior liquidity at date 1 due to search frictions. This setting allows us to endogenize the cost of shadow bank securities and yields novel implications that the optimal liquidity requirement may be achieved when commercial and shadow banks coexist.

The remainder of the paper is organized as follows. Section 2 introduces the basic setup of the model. Section 3 characterizes the competitive equilibrium with shadow banks and studies the comparative statics. Section 4 studies the impact of liquidity requirements on aggregate liquidity provision when shadow bank liquidity is endogenous. Section 5 concludes.

2. The Model

Consider an economy extending over three dates ($t \in \{0, 1, 2\}$), and comprising three types of agents: a continuum of consumers with measure 1, a continuum of commercial banks, and a continuum of shadow banks. The economy features a single type of consumption goods that can be both consumed and used as an investment. There is a long-term production technology, capable of producing $R > 1$ units of consumption goods at date 2 for each unit invested at date 0. There is also a short-term storage technology that transfers one unit of consumption goods from date t to date $t+1$ without any losses. All agents have access to the storage technology, while only commercial banks and shadow banks have access to the production technology. If one unit of long-term investment is prematurely liquidated at date 1, a firm can collect a residual value of ν, where $0 < \nu < 1$.

Consumers are identical at date 0 and each consumer is endowed with one unit of consumption goods. At date 1, each consumer gets a draw of his own preference type after an idiosyncratic preference shock occurs. With probability λ, a consumer becomes impatient and has an urgent need to consume (denoted by type $\theta = i$); with probability $1-\lambda$, a consumer becomes patient (denoted by type $\theta = p$). The probability of a consumer becoming impatient, λ, is public information, A consumer's utility depends on his consumption of goods at date 1, date 2, and his type. The consumers' utility function has the following form:

$$U = \begin{cases} U\left(c_1\right) & \theta = i \\ U\left(c_1 + c_2\right) & \theta = p \end{cases},$$

where c_t is the consumption at date $t \in \{1, 2\}$. $U(\cdot)$ is a twice continuously differentiable, strictly increasing, and strictly concave function, and it satisfies Inada conditions—that is, $U'(\cdot) > 0$, $U''(\cdot) < 0$, $U'(0) = +\infty$, and $U'(\infty) = 0$. As in Diamond and Dybvig (1983), we assume that the relative risk aversion is greater than 1, that is, $-U''(c)c/U'(c) > 1$, $\forall c$. While an impatient consumer prefers consumption at date 1, the more efficient technology only yields outputs at date 2. A patient consumer, on the other hand, is indifferent between consumption at date 1 and date 2.

Consumers can deposit their goods in commercial banks, and banks use the deposited goods to invest. Commercial banks offer demand deposits, denoted as (d_{01}, d_{02}), which means that for each unit of deposited consumption goods, a consumer withdrawing at date 1 receives d_{01}, and a withdrawal at date 2 yields d_{02}. Consumers can withdraw a fraction of their deposits at date 1 and the rest of their deposits at date 2. Given that all agents have access to the storage technology, d_{01} and d_{02} cannot be less than 1 in any equilibrium. Note that a bank cannot provide payment conditional on the actual type of the consumers, because the ex-post type of each consumer is private information.

Besides, consumers can also purchase the securities issued by shadow banks, such as asset-backed securities (ABS), mortgage-backed securities (MBS), and collateralized loan obligations (CLO) in reality, and then shadow banks use the proceeds to invest. Unlike bank deposits, shadow bank securities only mature at date 2 and cannot be withdrawn at date 1. This assumption captures the fact that many shadow bank securities cannot be withdrawn or redeemed at any time. However, at date 1, if some consumers become impatient, they can sell their claims for shadow bank securities maturing at date 2 in exchange for consumption goods at date 1.

At date 0, each consumer allocates his wealth between deposits and shadow bank securities. We denote the proportion of goods invested in bank deposits by x, and then the proportion of goods invested in shadow bank securities is $1-x$.

At date 1, consumers have access to a private market after knowing their types, where they can sell their claims at date 2 in exchange for consumption goods at date 1. We use d_{12} to denote the *market gross return* from date 1 to date 2, which means that in the private market, when a buyer uses one unit of consumption goods to purchase claims, he can obtain a claim for d_{12} units of goods at date 2. In other words, the price of each unit of date-2 consumption goods is $1/d_{12}$. Consumers can both sell their claims for shadow bank securities and their claims for commercial bank deposits. The buyers do not distinguish between shadow bank securities and bank deposits in the private market, and thus there is only one price for date-2 consumption goods, that is, $1/d_{12}$. Since all agents can store goods without losses, d_{12} should be greater than 1 in any equilibrium.

In our model, commercial banks are subject to liquidity requirements, and shadow banks can bypass liquidity requirements. To maintain the "no question asked" property of their deposits (see Holmstrom, 2015) and acquire access to government safety nets, commercial banks must comply with liquidity requirements proposed by the regulator. The regulator sets the liquidity requirement ratio or equivalently, the reserve requirement ratio, to be $\underline{\ell}$, which means that for each unit of deposit, a commercial bank needs to hold at least $\underline{\ell}$ units of goods as liquidity reserves and can at most invest $1-\underline{\ell}$ units of goods. Denote the liquidity ratio of commercial banks by ℓ; that is, for each unit of deposit, a commercial bank chooses to hold ℓ units of goods as liquidity reserves and invest $1-\ell$ units of goods into the long-term production technology. The liquidity requirement constraint can be specified as

$$\ell \geq \underline{\ell}. \tag{1}$$

By contrast, shadow banks run similar businesses to commercial banks but are subject to less regulation. In our model, we consider an extreme case in which shadow banks are completely free from liquidity requirements, and thus they can invest all of their goods collected from investors. Since the shadow banking market is competitive, shadow banks earn zero profits. Therefore, for each unit of goods invested into shadow bank securities, shadow banks need to pay R units of goods to investors at date 2.

Bypassing liquidity requirements is a double-edged sword. Since shadow banks do not have access to the government safety net, such as deposit insurance, shadow bank securities are generally less liquid than deposits. We model the lower liquidity of shadow bank securities by assuming that a consumer can only sell $\alpha \in [0,1]$ proportion of his shadow bank securities in the private market at date 1 in exchange for consumption goods, while he can sell all of his deposits in the private market.

We use a search and matching framework to characterize the micro-foundation of this assumption. To sell the claims for deposits and shadow bank securities, buyers and sellers need to be matched. Bank deposits have better liquidity, and depositors can write checks from deposit accounts. Thus, commercial banks essentially help to match buyers and sellers, and there is no search friction when selling claims for deposits.

By contrast, shadow bank securities are traded predominantly in over-the-counter (OTC) markets. There is search friction in the OTC markets and the sellers may not be able to find the buyers when they want to sell the shadow bank securities (see Weill, 2020). α in our model measures this search friction. In particular, we follow Arseneau, Rappoport W., and Vardoulakis (2020) and assume that each impatient consumer sends a continuum of infinitesimal traders to the private market, where each trader is restricted to selling one unit of shadow bank securities. Since the proportion of goods invested in shadow bank securities is $1-x$, the mass of traders is also $1-x$. If the probability that a trader meets a buyer is α, then a consumer would end up selling α proportion of his shadow bank securities.

The private market provides a channel for arbitrage, as depositors can exchange any amount of date-2 claims for date-1 consumption goods. Thus, the present value of withdrawing at date 2, d_{02}/d_{12}, must be equal to the value of withdrawing at date 1, d_{01}. Thus, given d_{12}, commercial banks must offer contracts satisfying

$$d_{01} = \frac{d_{02}}{d_{12}}. \tag{2}$$

To see this, if $d_{01} > d_{02}/d_{12}$, then patient consumers would prematurely withdraw all deposits and invest in the private market to earn $d_{01}d_{12} > d_{02}$; if $d_{01} < d_{02}/d_{12}$, then instead of withdrawing d_{01} at date 1, impatient consumers will sell all of their deposits in the private market to earn d_{02}/d_{12}.

When the no-arbitrage condition is satisfied, impatient consumers can directly withdraw their deposits from commercial banks at date 1 and only need to sell their holdings of shadow bank securities in the private market. Then, in the private market, each impatient consumer sells α proportion of his shadow bank securities, which provides $\alpha(1-x)R$ units of goods at date 2. The price is $1/d_{12}$, and thus the total demand for consumption goods at date 1 is $\lambda\alpha(1-x)R/d_{12}$. This demand is satisfied by patient consumers, who prematurely withdraw their deposits. We denote the proportion of deposits prematurely withdrawn by *patient* consumers as β, and then the total supply of goods is $(1-\lambda)x\beta d_{01}$. Then, market clearing requires the demand and supply to be equal. In the baseline model, we assume α to be a constant, which equals $\bar{\alpha}$, and thus the market clearing condition becomes

$$\lambda (1-x)\,\bar{\alpha}\frac{R}{d_{12}} = (1-\lambda)\,x\beta d_{01}. \tag{3}$$

Later in Section 4, we will extend the model by endogenizing α to study the impact of financial regulations on the liquidity of shadow banks.

The timing of the economy is as follows.

Date 0: The central bank sets the liquidity requirement ratio as $\underline{\ell}$. Commercial banks offer a deposit contract to consumers. Shadow banks issue shadow bank securities to consumers. Consumers deposit a proportion x of their goods in commercial banks and use a proportion $1-x$ of their goods to buy shadow bank securities. Then commercial banks and shadow banks invest.

Date 1: Consumers' preference shocks are realized. Impatient consumers send a continuum of infinitesimal traders to the private market to sell their claims at date 2 in exchange for consumption goods at date 1, and the probability that a trader meets a buyer is $\bar{\alpha}$. Commercial banks pay depositors who come to withdraw according to the deposit contracts.

Date 2: The return on long-term investment is realized. Commercial banks and shadow banks use the investment output to pay consumers who have the claims. Consumers consume goods and then the economy closes.

2.1 *Optimization Problems of Consumers*

We first describe consumers' optimization problem. At date 1, after knowing his type θ, a consumer chooses to sell or buy claims for date-2 consumption goods in the private market to maximize his utility. A consumer could withdraw a fraction of his deposit, and exchange the rest in the private market. However, given the condition (2), an impatient consumer would find it optimal to withdraw all deposits at date 1. Moreover, the impatient consumer will sell an $\bar{\alpha}$ proportion of his shadow bank securities. Thus, the consumption of an impatient consumer is $c_1^i = d_{01}x + R\bar{\alpha}(1-x)/d_{12}$, where x is the share of deposits and $1-x$ is the share of shadow bank securities in the consumer's portfolio, which are determined at date 0.

A patient consumer always prefers to consume at date 2 since $d_{12} \geq 1$, and thus he will use all consumption goods he has at date 1 to buy claims for date-2 goods in the private market. He could prematurely withdraw a fraction of his deposits to invest in the private market and withdraw the rest of the deposits at date 2. Again, given the condition (2), both options offer the same payoff $d_{02} = d_{01}d_{12}$ at date 2. In addition, he would wait until the long-term project matures to earn the payment from shadow bank securities. Thus, given the share of deposits, x, a patient consumer would consume $c_1^p = 0$ at date 1 and $c_2^p = d_{02}x + R(1-x)$ at date 2.

Patient consumers always need to withdraw some deposits at date 1 in the presence of shadow banks, since they are the liquidity providers in the private market. Thus, as long as commercial and shadow banks coexist ($x < 1$), the share of prematurely withdrawn deposits (β) will be nonzero.

At date 0, given the expected market return d_{12} and the deposit contract (d_{01}, d_{02}), a consumer allocates his wealth between deposits and shadow bank securities to maximize the ex ante expected utility. That is, he solves

$$\begin{aligned} \max_{x\in[0,1]} \; & \lambda U\left(c_1^i\right)+(1-\lambda)\,U\left(c_1^p+c_2^p\right) \\ \text{s.t. } & c_1^i = d_{01}x+\frac{R}{d_{12}}\bar{\alpha}\,(1-x) \\ & c_1^p = 0, \quad c_2^p = d_{02}x+R\,(1-x) \end{aligned} \quad . \tag{4}$$

where x is the share of goods invested in deposits.

REMARK. Following Farhi, Golosov, and Tsyvinski (2009), since the focus of this paper is not on the concerns about individual intermediaries but rather on the inadequacy of aggregate liquidity provision, we neglect the possibility of bank runs considered by Diamond and Dybvig (1983) and Allen and Gale (1998) and only focus on the good equilibrium without bank runs. Implicitly, we are assuming that each consumer believes that other consumers' behaviors do not affect the deposit rates he can obtain. Thus, coordination issues will not arise in the model and patient consumers do not have any incentive to run the banks. The equilibrium is then uniquely determined.

2.2 *Optimization Problems of Commercial and Shadow Banks*

Taking the expected market return in the private market d_{12} as given, a commercial bank competitively offers a contract (d_{01}, d_{02}), and chooses the liquidity ratio ℓ to maximize its profits, provided that the ex ante expected utility delivered by the contract must be no less than the equilibrium utility. The problem is more tractable in its dual form, where the commercial bank maximizes consumers' expected utility at date 0, knowing that given interest rates, consumers optimally determine the demand for deposits and shadow bank securities, and the bank is subject to the liquidity requirement (1), the no-arbitrage condition (2), and the budget constraints. Thus, the commercial bank solves the following optimization problem:

$$\begin{aligned} \max_{d_{01},d_{02},\ell} \; & \lambda U\left(c_1^i\right)+(1-\lambda)\,U\left(c_1^p+c_2^p\right) \\ \text{s.t. } & c_1^i = d_{01}x+\frac{R}{d_{12}}\bar{\alpha}\,(1-x)\,, \quad c_1^p=0, \quad c_2^p = d_{02}x+R\,(1-x)\,, \\ & x \text{ solves problem (4);} \\ & d_{01}\geq 1, \quad d_{02}\geq 1, \quad d_{01}=\frac{d_{02}}{d_{12}}; \\ & [\lambda+(1-\lambda)\,\beta]\,d_{01}=\ell, \quad (1-\lambda)\,(1-\beta)\,d_{02}=R\,(1-\ell)\,; \\ & \ell\geq\underline{\ell}. \end{aligned} \tag{5}$$

Since there is no aggregate uncertainty, commercial banks can rationally and correctly expect β. Commercial banks will pay all of the liquidity reserves to consumers at date 1, because if there are some unused liquidity reserves, it would be more beneficial to invest them in the long-term projects at date 0.

For shadow banks, their decision problem is simple. Since the shadow banking market is also competitive, shadow banks will earn zero profits. Therefore, for each unit of goods invested into shadow bank securities, shadow banks pay R units of goods to the investors at date 2.

2.3 *Socially Optimal Allocation*

Since both commercial and shadow banks earn zero profits, social welfare is measured by the ex ante expected utility of consumers, which equals $\lambda U\left(c_1^i\right)+(1-\lambda)\,U\left(c_1^p+c_2^p\right)$. Let $\mathfrak{L}$ denote the aggregate liquidity provision of the economy, that is, the aggregate amount of liquidity reserves of the economy. The social planner chooses $\mathfrak{L}$ to maximize social welfare, subject to the budget constraint at date 1 $\lambda c_1^i+(1-\lambda)\,c_1^p=\mathfrak{L}$ and the budget constraint at date 2 $\lambda c_2^i+(1-\lambda)\,c_2^p=R\,(1-\mathfrak{L})$. The following lemma characterizes the socially optimal allocation.

LEMMA 1. *The socially optimal allocation satisfies* $c_1^i=\mathfrak{L}/\lambda, c_2^p=R\,(1-\mathfrak{L})/(1-\lambda), c_2^i=c_1^p=0$, *with the socially optimal aggregate liquidity provision* $\mathfrak{L}^*$ *satisfying:*

$$U'\left(\frac{\mathfrak{L}^*}{\lambda}\right)=RU'\left(\frac{R\,(1-\mathfrak{L}^*)}{1-\lambda}\right), \tag{6}$$

and we have $\mathfrak{L}^*>\lambda$, $c_1^i>1$ *and* $c_2^p<R$.

PROOF. See Appendix A.1. □

It is inefficient for impatient consumers to consume at date 2 and for patient consumers to consume at date 1. Then, the first-order condition determines the socially optimal aggregate liquidity provision. Finally, the standard assumption in Diamond-Dybvig type models that the consumers' relative risk aversion is greater than 1 implies that each impatient consumer should consume more than 1 unit of goods and each patient consumer should consume less than R units of goods.

3. COMPETITIVE EQUILIBRIUM

3.1 *Equilibrium Definition*

We can define our symmetric competitive equilibrium as follows.

DEFINITION 1. A competitive equilibrium with liquidity requirement $\underline{\ell}$ and the probability of a successful match $\bar{\alpha}$ is a set of allocations $\left(c_1^i,c_1^p,c_2^p\right)$, a deposit contract with a menu of two options (d_{01},d_{02}), the share of deposits in consumers' portfolio, x, the fraction of deposits prematurely withdrawn by patient consumers, β, and the market return in the private market at date 1, d_{12}, such that

(i) given β, d_{12} and $\underline{\ell}$, each commercial bank chooses (d_{01}, d_{02}) and ℓ to solve problem (5);

(ii) a consumer chooses x to solve problem (4);

(iii) The private market clears, that is, (3) is satisfied.

Since commercial banks are the only supplier of liquidity reserves in the economy, the aggregate liquidity provision of the economy is $\mathfrak{L} = x\ell$, where ℓ denotes commercial banks' liquidity ratio and x denotes the share of commercial banks.

3.2 *Equilibrium Characterization*

In this subsection, we characterize the competitive equilibrium. Regarding the market return in the private market, we only need to study the case with $d_{12} \leq R$, since $d_{12} > R$ cannot survive in equilibrium. This is because if $d_{12} > R$, both commercial and shadow banks will keep all goods at date 0 and use a fraction of goods to purchase claims for date-2 goods in the private market to fulfill date-2 repayment obligations. They will not invest any goods in the long-term investment technology at date 0. However, since no actual long-term investment is made, there are not sufficient goods to repay the loans in the private market at date 2.

3.2.1 *Commercial Banks' Deposit Contract* The next lemma characterizes the deposit contract and commercial banks' expectation on the proportion of patient consumers' premature withdrawal when the liquidity requirement $\underline{\ell}$ and the private market return d_{12} are taken as given.

LEMMA 2. *Given $\underline{\ell}$ and d_{12}, commercial banks' deposit contract and their expectation on the proportion of deposits prematurely withdrawn by patient consumers, β, satisfy:*

(i) *If $d_{12} = R$, then a bank voluntarily chooses a liquidity ratio $\ell > \underline{\ell}$ and offers $d_{01} = 1,\ d_{02} = R$, and it expects $\beta = (\ell - \lambda)/(1 - \lambda)$;*

(ii) *If $d_{12} < R$, then a bank chooses liquidity ratio $\ell = \underline{\ell}$ and offers $d_{01} = \underline{\ell} + R(1 - \underline{\ell})/d_{12}$, $d_{02} = \underline{\ell}d_{12} + (1 - \underline{\ell})R$, and it expects $\beta = (\underline{\ell}d_{12}/(R(1 - \underline{\ell}) + \underline{\ell}d_{12}) - \lambda)/(1 - \lambda)$.*

PROOF. See Appendix A.2. □

Taking the private market return d_{12} as given, a commercial bank offers a deposit contract to maximize the expected utility of consumers. Since the bank takes the market return d_{12} as given and $d_{02} = d_{01}d_{12}$, we know that regardless of x, the consumption of both impatient consumers (c_1^i) and patient consumers (c_2^p) monotonically increases with d_{01}. Therefore, the bank's problem is to simply maximize the short-term deposit rate d_{01}.

The optimal contract then depends on the comparison between the market return d_{12} and the return on the long-term technology R. If $d_{12} = R$, then it is indifferent between investing in the long-term technology and keeping liquidity reserves to invest in the private market. Thus, the liquidity requirement is not a binding constraint ($\ell > \underline{\ell}$). Since $d_{01} \geq 1$, the only feasible deposit contract is $d_{01} = 1$ and $d_{02} = R$. Any contracts with $d_{01} > 1$ cannot satisfy the bank's budget constraints. Moreover, the bank's binding budget constraints also provide information about the bank's expectation on the proportion of deposits prematurely withdrawn by patient consumers (β). When $d_{01} = 1$, we have $\beta = (\ell - \lambda)/(1 - \lambda)$. Intuitively, when the bank voluntarily holds more liquidity reserves than impatient consumers' withdrawal, they actually expect that the rest of the liquidity reserves will be withdrawn by patient consumers.

If $d_{12} < R$, i.e., the return of the long-term investment technology is strictly higher than the market return in the private market, the bank will minimize its holdings of liquidity reserves. This is because investing in the long-term technology and then selling the claims in the private market leads to a return of $R/d_{12} > 1$, which is strictly better than holding liquidity reserves. Thus, the liquidity requirement is a binding constraint ($\ell = \underline{\ell}$) and the short-term deposit rate (d_{01}) is strictly greater than 1.

3.2.2 *Supply of and Demand for Deposits* Given the private market return d_{12}, commercial banks offer deposit contracts (d_{01}, d_{02}) following Lemma 2. Then, given the private market return and bank deposit contract, we can characterize: (i) consumers' choice between bank deposits and shadow bank securities, and (ii) the market clearing condition of the private market, which will establish the supply of and demand for deposits, respectively.

The Supply of Deposits. We first study consumers' choice between bank deposits and shadow bank securities, which establishes *the supply of bank deposits*. At date 0, a consumer determines the proportion of goods to invest in deposits and shadow bank securities. He compares the expected marginal utility provided by those two assets to determine the allocation of his wealth. On the one hand, with probability λ, he becomes impatient, and bank deposits provide higher utility due to their better liquidity. Each unit of deposit provides an excess date-1 return of $d_{01} - \bar{\alpha}R/d_{12}$, compared to shadow bank securities. This excess return provides $\lambda U'\left(c_1^i\right)(d_{01} - \bar{\alpha}R/d_{12})$ units of expected additional utility. On the other hand, with probability $1 - \lambda$, he becomes patient, and shadow bank securities provide higher utility since they are free from liquidity regulations. Each unit of shadow bank security provides an excess date-2 return of $R - d_{02}$, compared to bank deposits. This excess return provides $(1-\lambda)\,U'\left(c_2^p\right)(R - d_{02})$ units of expected additional utility. The next lemma captures this trade-off.

LEMMA 3. *Consumers' incentive to invest in bank deposits increases with the private market return* d_{12}. *Moreover, there exists a threshold* $\hat{d}_{12}(\underline{\ell}) \leq R$ *such that*

(i) *When* $d_{12} < \hat{d}_{12}(\underline{\ell})$, *consumers invest in both bank deposits and shadow bank securities. The share of deposits* x *satisfies the following condition:*

$$\lambda U'\left(c_1^i\right)\left(d_{01} - \bar{\alpha}\frac{R}{d_{12}}\right) = (1-\lambda)\,U'\left(c_2^p\right)(R - d_{02}), \tag{7}$$

where $d_{01} = \underline{\ell} + R(1-\underline{\ell})/d_{12}$, $d_{02} = \underline{\ell}d_{12} + (1-\underline{\ell})\,R$, $c_1^i = (\underline{\ell} + R(1-\underline{\ell}-\bar{\alpha})/d_{12})\,x + \bar{\alpha}R/d_{12}$, *and* $c_2^p = \underline{\ell}\,(d_{12} - R)\,x + R$.

(ii) *Consumers strictly prefer deposits when* $d_{12} \geq \hat{d}_{12}(\underline{\ell})$. *In particular, if* $d_{12} = R$*, then consumers always prefer bank deposits regardless of* $\underline{\ell}$.

Proof. See Appendix A.3. □

Lemma 3 suggests that a higher private market return d_{12} will make bank deposits more attractive. Intuitively, the change in d_{12} has two types of effects on the relative utility of investing in bank deposits and shadow bank securities. The first one is *wealth effects*, which refer to the influence on consumption and marginal utility ($U'\left(c_1^i\right)$ and $U'\left(c_2^p\right)$). The second one is *substitution effects*, which refer to the influence on the relative return of bank deposits and shadow bank securities ($d_{01} - \bar{\alpha}R/d_{12}$ and $R - d_{02}$).

According to Lemma 2, an increase in d_{12} leads to an increase in the long-term deposit rate d_{02} and a decrease in the short-term deposit rate d_{01}. Then, $U'\left(c_2^p\right)$ decreases and $U'\left(c_1^i\right)$ increases; thus, wealth effects always make bank deposits more attractive. The substitution effects are ambiguous, because although $R - d_{02}$ always decreases, both d_{01} and $\bar{\alpha}R/d_{12}$ become lower, and thus the change in $d_{01} - \bar{\alpha}R/d_{12}$ is ambiguous. However, when the relative risk aversion is greater than 1, which is a standard assumption in Diamond and Dybvig (1983), the utility function will be sufficiently concave and the wealth effects will dominate, and an increase in the private market return d_{12} will always make bank deposits *more attractive*.

Finally, when $d_{12} = R$, bank deposits provide the same long-term return as shadow bank securities. Then consumers strictly prefer bank deposits to shadow bank securities due to bank deposits' better liquidity. With continuity of the problem, there exists a threshold $\hat{d}_{12}(\underline{\ell}) \leq R$, which is a function of the liquidity requirement, such that if d_{12} is higher than the threshold, consumers will not invest any goods in shadow banks. If d_{12} is lower than the threshold, then consumers invest in both bank deposits and shadow bank securities.

The Demand for Deposits. We now characterize the *demand for bank deposits*, arising from the market clearing condition of the private market.

Lemma 4. *Given the share of deposits* x*, the private market return* d_{12} *satisfies*

$$d_{12} = \begin{cases} R & \frac{\lambda\bar{\alpha}}{1+\lambda\bar{\alpha}-\lambda} \leq x < \frac{\lambda\bar{\alpha}}{\underline{\ell}+\lambda\bar{\alpha}-\lambda} \\ R\frac{\lambda\bar{\alpha}(1-x)+\lambda x(1-\underline{\ell})}{(1-\lambda)x\underline{\ell}} & x \geq \frac{\lambda\bar{\alpha}}{\underline{\ell}+\lambda\bar{\alpha}-\lambda} \end{cases}. \tag{8}$$

Proof. See Appendix A.4. □

Shadow bank securities are sold in the private market, which forms the demand for bank deposits. Patient consumers prematurely withdraw bank deposits to purchase these shadow bank securities. Since banks have rational expectations, the actual proportion of bank deposits prematurely withdrawn by patient consumers, β, that can clear the private market must be consistent with commercial banks' expectations in Lemma 2.

Given the share of bank deposits, x, if the liquidity requirement does not bind, then Lemma 2 implies $d_{12} = R$ in the private market. Commercial banks voluntarily choose a liquidity ratio that clears the private market, which equals $\ell = \lambda + \lambda\bar{\alpha}(1-x)/x$. This liquidity ratio determines a threshold of x. When x is smaller than the threshold, commercial banks must choose a high liquidity ratio to provide sufficient liquidity to clear the private market, which exceeds the liquidity requirement. Thus, the liquidity requirement does not bind and $d_{12} = R$. By contrast, if x is larger than the threshold, the liquidity ratio that clears the private market at $d_{12} = R$ is smaller than the liquidity requirement. The liquidity requirement becomes binding and forces commercial banks to hold more liquidity reserves, and thus the private market return d_{12} will decrease and become lower than R. Then, the relationship between d_{12} and x is given by equalizing patient consumers' share of the aggregate liquidity provision, $(1-\lambda)x\underline{\ell}$, and the market value of the total tradable assets of impatient consumers, $R(\lambda\bar{\alpha}(1-x) + \lambda x(1-\underline{\ell}))/d_{12}$.

Finally, since $d_{12} > R$ cannot survive in equilibrium, we only need to discuss the case with $x > \lambda\bar{\alpha}/(1+\lambda\bar{\alpha}-\lambda)$, because the voluntarily chosen liquidity ratio cannot exceed its upper limit 1. If $x < \lambda\bar{\alpha}/(1+\lambda\bar{\alpha}-\lambda)$, commercial banks cannot hold more liquidity as they have already kept all assets as liquid assets, and the shortage of liquid assets will drive the private market return d_{12} above R. Thus, the equilibrium cannot be sustained in this range of x.

Now, we obtain the supply of bank deposits from consumers' first-order condition (Lemma 3) and the demand for deposits from the market clearing condition of the private market (Lemma 4). The equilibrium is then determined by equalizing supply and demand.

PROPOSITION 1. *Given $\bar{\alpha} < 1$, there exists a threshold $\hat{\underline{\ell}} > \lambda$, such that*

(i) *when $\underline{\ell} < \lambda$, commercial banks voluntarily choose liquidity ratio $\lambda > \underline{\ell}$, the market share of commercial banks is $x = 1$, i.e. there are no shadow banks, and we have $\beta = 0$; $c_1^i = 1, c_1^p = 0, c_2^p = R$; $d_{12} = R$; $d_{01} = 1, d_{02} = R$.*

(ii) *when $\lambda \leq \underline{\ell} < \hat{\underline{\ell}}$, the market share of commercial banks is $x = 1$, i.e. there are no shadow banks, and we have $\beta = 0$; $c_1^i = \underline{\ell}/\lambda, c_1^p = 0, c_2^p = R(1-\underline{\ell})/(1-\lambda)$; $d_{12} = \lambda R(1-\underline{\ell})/[(1-\lambda)\underline{\ell}]$; $d_{01} = \underline{\ell}/\lambda$, $d_{02} = R(1-\underline{\ell})/(1-\lambda)$.*

(iii) *when $\underline{\ell} \geq \hat{\underline{\ell}}$, the market share of commercial banks is $x < 1$, i.e., commercial and shadow banks coexist. x is determined by*

$$\lambda U'\left(c_1^i\right)\left(d_{01} - \bar{\alpha}\frac{R}{d_{12}}\right) = (1-\lambda)U'\left(c_2^p\right)(R - d_{02}),$$

where $c_1^i = x\underline{\ell}/\lambda$,$c_1^p = 0$, $c_2^p = (Rx(1-\underline{\ell}) + (1-x)R[\lambda\bar{\alpha} + (1-\lambda)])/(1-\lambda)$; $d_{12} = R[\lambda(1-\underline{\ell})x + \bar{\alpha}\lambda(1-x)]/[\underline{\ell}(1-\lambda)x]$; $d_{01} = [\underline{\ell}(1-\underline{\ell})x + \underline{\ell}\bar{\alpha}\lambda(1-x)]/[\lambda(1-\underline{\ell})x + \bar{\alpha}\lambda(1-x)]$, $d_{02} = d_{01}d_{12}$; $\beta = \bar{\alpha}\lambda(1-x)/[(1-\underline{\ell})x + \bar{\alpha}\lambda(1-x)]$.

PROOF. See Appendix A.5. □

Since the supply of deposits is strictly upward-sloping and the demand for deposits is weakly downward-sloping, we conclude that there is always a unique equilibrium. Proposition 1 suggests that the equilibrium is one of the following three types (see Figure 1), depending on the liquidity requirement.

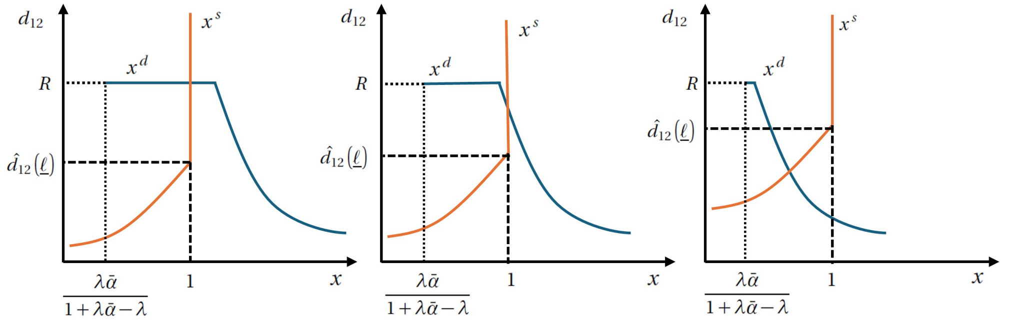

FIGURE 1. Three Types of Equilibrium

Note: The blue line represents the demand curve of deposits, and the orange line represents the supply curve of deposits.

The left figure of Figure 1 illustrates the equilibrium with $\underline{\ell} < \lambda$, in which the liquidity requirement does not bind. Consumers strictly prefer deposits because of their better liquidity. Each impatient consumer consumes 1 unit of goods, and each patient consumer consumes R units of goods. This is the standard result in the presence of a private market as in Jacklin (1987) and Allen and Gale (2004). Risk sharing is limited by the existence of the private market. The liquidity provided by the financial system is $\mathfrak{L} = \ell = \lambda$.

As the liquidity requirement $\underline{\ell}$ rises ($\ell \geq \lambda$), for a given private market return d_{12}, on the one hand, the market value of total tradable assets of impatient consumers is lower. Thus, it requires fewer deposits to buy all the assets, making the demand curve shift to the left. On the other hand, an increase in $\underline{\ell}$ lowers both short-term and long-term interest rates, resulting in a lower supply of deposits. Thus, the supply curve will shift up. An equilibrium with $\hat{d}_{12}(\underline{\ell}) < d_{12} < R$ and $x = 1$ would appear, as shown by the middle figure of Figure 1. In this range, since the liquidity requirement is only slightly higher than λ, the constraint on commercial banks is not very tight, and thus deposits still dominate shadow bank securities. Shadow banks do not emerge. The allocation is the same as the one in Farhi, Golosov, and Tsyvinski (2009): a liquidity requirement can effectively rectify the insufficient liquidity insurance problem in the presence of private markets, resulting in better liquidity insurance $\mathfrak{L} = \underline{\ell} > \lambda$.

Third, as the liquidity requirement further increases, the demand curve keeps moving left and the supply curve keeps moving up. Eventually, there will be a cutoff requirement $\hat{\underline{\ell}}$, above which the equilibrium is featured with $d_{12} < \hat{d}_{12}(\underline{\ell})$ and $x < 1$, as shown by the right figure of Figure 1. Commercial and shadow banks coexist for $\underline{\ell} > \hat{\underline{\ell}}$. Intuitively, when the liquidity requirement is sufficiently high, commercial banks are overly constrained and it becomes beneficial to circumvent the liquidity regulation by investing in shadow bank securities.

REMARK. When the liquidity requirement is binding ($\underline{\ell} > \lambda$), banks are forced to provide more liquidity, and date-2 claims become more valuable in the private market. The private market return d_{12} thus becomes lower than R. Although the no-arbitrage condition $d_{01}d_{12} = d_{02}$ still holds, the equilibrium deposit contract satisfies $d_{01} > 1$ and $d_{02} < R$. In this case, commercial banks still have an incentive to deviate by offering a lower d_{01} and a higher d_{02}, but liquidity requirements prevent them from doing that, since they cannot make more investments.

3.3 *Comparative Statics*

3.3.1 *The Impact of Liquidity Requirements* In this subsection, we will study the impact of liquidity requirements on the aggregate liquidity provision $\mathfrak{L}$. The aggregate liquidity provision of the economy equals the total amount of reserves at date 1, that is, $\mathfrak{L} = x\ell$. We only need to discuss the case where the liquidity requirement is effective, i.e., $\underline{\ell} \geq \lambda$. Then $\ell = \underline{\ell}$ holds by Proposition 1.

PROPOSITION 2. *Given $\bar{\alpha} < 1$,*

(i) *when $\lambda \leq \underline{\ell} < \hat{\underline{\ell}}$, (a) the liquidity provision $\mathfrak{L} = \underline{\ell}$, and it increases with the liquidity requirement; (b) the market share of commercial banks $x = 1$ and is unrelated to the liquidity requirement;*

(ii) *when $\underline{\ell} \geq \hat{\underline{\ell}}$, (a) the liquidity provision $\mathfrak{L} < \underline{\ell}$, and it decreases with the liquidity requirement; (b) the market share of commercial banks x decreases with the liquidity requirement.*

PROOF. See Appendix A.6. □

Proposition 2 suggests that the presence of shadow banks significantly changes the impact of liquidity requirements on the economy. When there are only commercial banks, the aggregate liquidity provision $\mathfrak{L}$ equals the liquidity requirement $\underline{\ell}$ set by the regulators. A higher liquidity requirement, by forcing commercial banks to hold more reserves, increases the aggregate liquidity provision.

However, raising liquidity requirements may lead to unintended consequences in the presence of shadow banks. When shadow banks exist, a higher liquidity requirement, although making commercial banks provide more liquidity to impatient consumers, also incurs a higher degree of regulatory arbitrage, as shadow banks are more attractive under a higher liquidity requirement. Thus, a higher liquidity requirement has two effects on the aggregate liquidity provision $\mathfrak{L} = x\underline{\ell}$. On the one hand, impatient consumers can obtain more liquidity from each unit of deposit, which alleviates the liquidity shortage problem (a higher $\underline{\ell}$). On the other hand, a higher requirement drives more funds to shadow banks and reduces the market share of commercial banks (a lower x), which exacerbates the liquidity shortage problem.

Proposition 2 shows that the effect of flight to shadow banks always outweighs the effect of increasing per unit deposit liquidity, and thus a higher liquidity requirement will reduce the aggregate liquidity provision and exacerbate the liquidity shortage problem. Figure 2 illustrates the mechanism. Denote the share of bank deposits in the original equilibrium as x^*. We use $x^* + dx$ to denote the share of bank deposits such that the aggregate liquidity provision $\mathfrak{L} = x^*\underline{\ell}$ remains *unchanged* after the liquidity requirement increases. Since an increase in liquidity requirement ($\underline{\ell} \to \underline{\ell} + d\underline{\ell}$, $d\underline{\ell} > 0$) will always reduce the share of deposits, we know $dx < 0$. We will show that to reach the new equilibrium, x must be further decreased to a point smaller than $x^* + dx$.

On the demand side, when the liquidity requirement increases, the demand curve shifts to the left as the market value of total tradable assets becomes lower. Since $x^*\underline{\ell}$ remains constant, the total investment in the long-term project $(1 - x^*\underline{\ell})$ will also be constant. However, since shadow bank securities are less liquid, only $\bar{\alpha} < 1$ proportion of impatient consumers' shadow bank securities are sold. Thus, there is an excess supply of liquidity, and the point with unchanged $x^*\underline{\ell}$ on the demand curve $(x^* + dx)$ is featured with a lower private market return, denoted by d_{12}^D.

On the supply side, when the liquidity requirement increases, the supply curve shifts to the left because a higher liquidity requirement reduces the attractiveness of bank deposits. Now, we will show that the point $\left(x^* + dx, d_{12}^D\right)$ is below the new supply curve; that is, denoting the private market return (d_{12}) at $x^* + dx$ on the new supply curve as d_{12}^S, we will show $d_{12}^S > d_{12}^D$. It is equivalent to showing that at $\left(x^* + dx, d_{12}^D\right)$, consumers have an incentive to deviate by allocating more endowments to shadow bank securities. Then, d_{12} must further increase to make consumers again become indifferent between investing in bank deposits and shadow bank securities, which reaches the new point on the supply curve.

Using the results in Lemma 3 and the fact that $d_{01} - \bar{\alpha}R/d_{12} = \left(c_1^i - \bar{\alpha}R/d_{12}\right)/x$ and $R - d_{02} = \underline{\ell}\,(R - d_{12})$, we can rewrite the first-order condition in the following form:

$$\lambda U'\left(c_1^i\right) \frac{c_1^i - \bar{\alpha}\dfrac{R}{d_{12}}}{x^*\underline{\ell}\,(R - d_{12})} = (1 - \lambda)\, U'\left(c_2^p\right). \tag{9}$$

The left-hand side of (9) represents the benefits of investing in bank deposits, and the right-hand side represents the benefits of investing in shadow bank securities. Then, similar to the discussions in Lemma 3, there are two effects.

Wealth effect ($U'\left(c_1^i\right)$ and $U'\left(c_2^p\right)$): Given that $x^*\underline{\ell}$ does not change, the fact that d_{12}^D clears the private market implies that impatient consumers would still consume all of the liquidity at date 1, i.e., $c_1^i = x^*\underline{\ell}/\lambda$ does not change. However, for patient consumers, the illiquidity of shadow bank securities implies that $1 - \bar{\alpha}$ proportion of the additional shadow bank investment is wasted, since these claims for date-2 goods cannot be sold to patient consumers, resulting in lower patient consumers' consumption. This wealth effect gives consumers a stronger incentive to invest in shadow bank securities to increase their consumption when becoming patient.

Substitution effect ($[c_1^i - \bar{\alpha}R/d_{12}]/[x^*\underline{\ell}\,(R - d_{12})]$): Since $x^*\underline{\ell}$ and c_1^i remain the same, the substitution effect depends on the change in the private market return d_{12}. We have shown that d_{12}^D is lower than the original private market return, which decreases the numerator and increases the denominator. Thus, the substitution effect also gives consumers a stronger incentive to invest in shadow bank securities.

Therefore, at $\left(x^* + dx, d_{12}^D\right)$, consumers have an incentive to allocate more endowments to shadow bank securities, and the private market return at $x^* + dx$ on the new supply curve d_{12}^S satisfies $d_{12}^S > d_{12}^D$. Then, to reach the new equilibrium, we need x to further decrease to reduce d_{12}^S and increase d_{12}^D, which means that the new equilibrium point must be on the left side of $x^* + dx$. Therefore, in the new equilibrium, the decrease in x must exceed the increase in $\underline{\ell}$, resulting in a lower $\mathfrak{L}$.

To summarize, if the aggregate liquidity provision does not change, the market return determined in the private market is so small that consumers would strictly prefer shadow bank securities. Thus, the decrease in the share of deposits must dominate the increase in the liquidity requirement, resulting in a decrease in the aggregate liquidity provision.

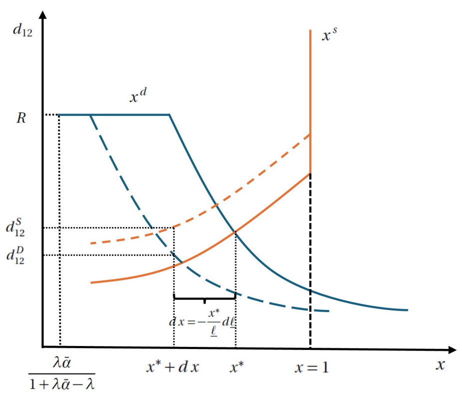

FIGURE 2. Mechanism Illustration

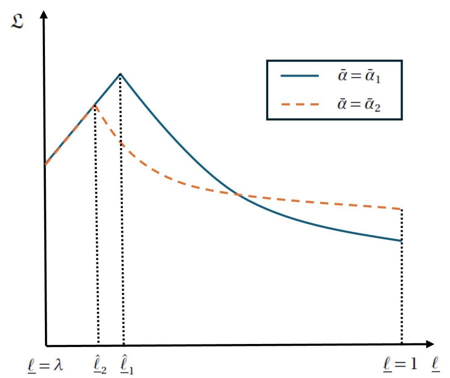

FIGURE 3. The Relations between $\mathfrak{L}$ and $\underline{\ell}$ under Different $\bar{\alpha}$ ($\bar{\alpha}_1 < \bar{\alpha}_2$)

Proposition 2 has important implications regarding liquidity regulations. In the presence of shadow banks, a higher liquidity requirement, while aimed at improving liquidity provision, may instead exacerbate the liquidity shortage problem, since illiquid shadow banks can bypass regulations and substitute deposits. This echoes the results that capital requirements may incur unintended consequences in the presence of shadow banks (see, for example, Plantin, 2015) and that macroprudential regulations such as leverage restrictions may be counterproductive (see, for example, Allen, Barlevy, and Gale, 2022). Our paper confirms that similar results apply to liquidity regulations. The unintended consequences will disappear if the regulators can effectively regulate shadow banks, and thus our paper again highlights the importance of regulating commercial and shadow banks in a unified framework.

3.3.2 *The Impact of the Liquidity of Shadow Bank Securities* Next, we will fix the liquidity requirement, and examine the effect of the liquidity of shadow bank securities $\bar{\alpha}$

on aggregate liquidity provision. We will focus on the case in which commercial and shadow banks coexist ($\underline{\ell} \geq \hat{\underline{\ell}}$), since when only commercial banks exist, resource allocations are independent of $\bar{\alpha}$.

PROPOSITION 3. *The cutoff liquidity requirement $\hat{\underline{\ell}}$ decreases with $\bar{\alpha}$. When $\underline{\ell} \geq \hat{\underline{\ell}}$, the market share of commercial banks x and aggregate liquidity provision $\mathfrak{L}$ have the following relation with $\bar{\alpha}$: (i) when $\underline{\ell}$ is close to $\hat{\underline{\ell}}$, x and $\mathfrak{L}$ decrease with $\bar{\alpha}$; (ii) when $\underline{\ell}$ is sufficiently large, x and $\mathfrak{L}$ increase with $\bar{\alpha}$.*

PROOF. See Appendix A.7. □

Proposition 3 suggests that when the liquidity requirement is relatively low, the improvement in the liquidity of shadow bank securities ($\bar{\alpha}$) will reduce aggregate liquidity provision and exacerbate the liquidity shortage problem; when the liquidity requirement is very high, the improvement in $\bar{\alpha}$ will increase aggregate liquidity provision and alleviate the liquidity shortage problem.

Intuitively, for any liquidity requirement, as $\bar{\alpha}$ increases, consumers have a stronger incentive to invest in shadow banks, shifting the supply curve up. Moreover, the increase in $\bar{\alpha}$ implies that more shadow bank securities can be sold in the private market, which demands more deposits for any private market return and shifts the demand curve to the right. Since the demand for bank deposits becomes higher but the supply of deposits becomes lower, the net effect is generally ambiguous. However, when the liquidity requirement is close to the cutoff value $\hat{\underline{\ell}}$, the market share of the shadow banks is very small, and thus a change in $\bar{\alpha}$ only has a small impact on the demand curve. Thus, the decrease in supply dominates and shadow banks will occupy a larger market share. Since the liquidity requirement is constant, a larger market share of shadow banks leads to lower aggregate liquidity provision. Conversely, when the liquidity requirement is sufficiently high, the market share of shadow banks is very large, and thus a change in $\bar{\alpha}$ has a significant impact on the private market. In this case, the increase in demand dominates, and commercial banks will occupy a larger market share, which leads to a higher liquidity provision.

One important implication of Proposition 3 is that an improvement in the liquidity of an individual asset may not improve the aggregate liquidity provision of the overall financial system, due to the substitution effect between different types of assets.

Figure 3 shows the relation between the aggregate liquidity provision and liquidity requirements under different shadow bank liquidity, which illustrates the results in both Proposition 2 and 3. The blue solid line describes the case with low shadow bank liquidity $\bar{\alpha}_1$, while the orange dashed line describes the case with high shadow bank liquidity $\bar{\alpha}_2$, that is, $\bar{\alpha}_1 < \bar{\alpha}_2$. When the liquidity of shadow bank securities is higher, shadow banks will emerge at a lower liquidity requirement ($\hat{\underline{\ell}}_2 < \hat{\underline{\ell}}_1$). The aggregate liquidity provision always decreases with the liquidity requirement after shadow banks emerge. When the liquidity requirement is close to $\hat{\underline{\ell}}$, the aggregate liquidity provision is higher when shadow bank liquidity is lower ($\bar{\alpha}_1$). When the liquidity requirement is sufficiently high, the aggregate liquidity provision is higher when shadow bank liquidity is higher ($\bar{\alpha}_2$).

3.4 *Optimal Liquidity Requirement*

In this subsection, we will characterize the optimal liquidity requirements that maximize social welfare. Without the presence of shadow banks, the optimal liquidity requirement is simply setting $\underline{\ell}^* = \mathfrak{L}^*$, where $\mathfrak{L}^*$ is the first-best aggregate liquidity provision. With the presence of shadow banks, we need to compare this $\underline{\ell}^*$ with the threshold $\hat{\underline{\ell}}$ at which shadow banks emerge. If $\underline{\ell}^* \leq \hat{\underline{\ell}}$, which means that shadow banks do not emerge at the first-best liquidity requirement, $\underline{\ell}^*$ can still help the economy to achieve the socially optimal allocation, which is similar to the results proposed by Farhi, Golosov, and Tsyvinski (2009).

By contrast, if $\underline{\ell}^* > \hat{\underline{\ell}}$, which means that shadow banks do emerge at the first-best liquidity requirement, then according to Proposition 2, $\underline{\ell}^*$ is no longer the optimal liquidity requirement because a higher liquidity requirement will reduce liquidity provision and social welfare in the presence of shadow banks.[1] Thus, setting the liquidity requirement as the threshold level that shadow banks do not emerge, that is, $\underline{\ell} = \hat{\underline{\ell}}$, will help the economy to achieve the second-best allocation.

According to Proposition 3, the threshold $\hat{\underline{\ell}}$ depends on the liquidity of shadow bank securities. Thus, we present the following proposition.

PROPOSITION 4. *There exists a cutoff value of $\bar{\alpha}$, denoted as $\bar{\alpha}^c$. When $\bar{\alpha} > \bar{\alpha}^c$, the optimal liquidity requirement is $\underline{\ell}^{opt} = \hat{\underline{\ell}}$; when $\bar{\alpha} \leq \bar{\alpha}^c$, the optimal liquidity requirement is $\underline{\ell}^{opt} = \underline{\ell}^*$.*

PROOF. See Appendix A.8. □

Figure 4 plots the relation between liquidity requirements and social welfare. The left figure illustrates the case with less liquid shadow bank securities. In this case, consumers have little incentive to invest in shadow banks, and setting the first-best liquidity requirement can achieve the socially optimal allocation. The right figure illustrates the case with more liquid shadow bank securities. In this case, shadow banks will emerge at the first-best liquidity requirement and prevent the economy from achieving the first-best allocation. The second-best allocation is achieved at the highest liquidity requirement at which no shadow banks exist.

4. Endogenous Liquidity of Shadow Bank Securities

In the baseline model, we assume that the liquidity of shadow bank securities (α) is a constant and use a search and matching framework to justify this assumption. However, liquidity requirements may influence α. As the liquidity requirement increases,

[1] Social welfare monotonically increases with the aggregate liquidity provision ($\mathfrak{L} = x\underline{\ell}$). When only commercial banks exist ($x = 1$), higher $x\underline{\ell}$ improves social welfare due to better risk sharing. When commercial and shadow banks coexist ($x < 1$), social welfare increases with the aggregate liquidity provision $x\underline{\ell}$ for two reasons: (i) higher $x\underline{\ell}$ increases the consumption of impatient consumers, thus improving risk sharing; (ii) higher $x\underline{\ell}$ means a lower aggregate investment $1 - x\underline{\ell}$, and Proposition 2 shows that $x\underline{\ell}$ is higher under a lower liquidity requirement, and thus the market share of deposits x is larger. This reduces the waste of shadow bank payoffs, since $\bar{\alpha} < 1$ and impatient consumers do not obtain utility from date-2 consumption.

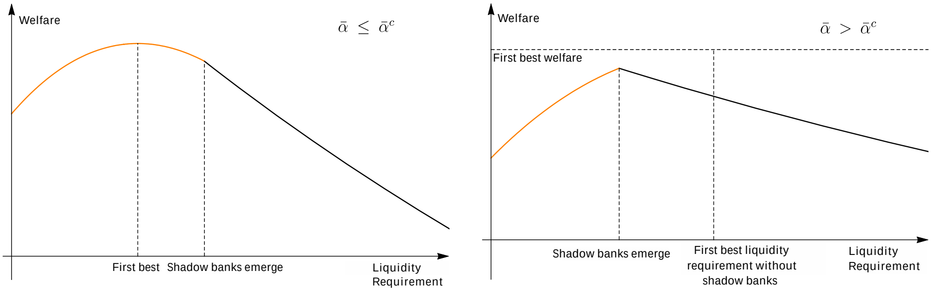

FIGURE 4. The Relation between Liquidity Requirements and Social Welfare

Note: The orange solid line represents the range in which shadow banks endogenously do not exist, and the black solid line represents the range in which commercial and shadow banks coexist.

more goods are invested in shadow banks at date 0, and the probability of successfully selling shadow bank securities may drop as impatient consumers post more sell orders. In this section, we will more explicitly model the search and matching mechanism to endogenize α and study the impact of liquidity requirements on the liquidity of shadow bank securities.

When α is a constant, as the liquidity requirement increases, the share of deposits x falls, and patient consumers need to prematurely withdraw more deposits to purchase the additional supply of shadow bank securities. Thus, a natural way to model the endogeneity of α is to assume that patient consumers are constrained from freely withdrawing deposits at date 1 to purchase claims for shadow bank securities. To maintain the perfect liquidity of deposits, we do not impose any constraints on purchasing claims for deposits.

Therefore, we assume that at date 1, claims for deposits can be sold in a centralized market, where consumers can sell *all* of their bank deposits in exchange for consumption goods at date 1 at the market price $1/d_{12}$. Patient consumers can freely withdraw their bank deposits and invest in the centralized market. By contrast, claims for shadow bank securities are traded in an OTC market. Patient consumers can withdraw *at most* $\bar{\beta}$ proportion of their bank deposits to invest in the OTC market. In reality, this assumption captures the fact that there is usually a delay in the settlement of large-scale money transactions.[2]

The price of shadow bank securities is determined through Nash bargaining. For simplicity, we assume that in a match, the seller (impatient consumer) has all bargaining power. Thus, the price of shadow bank securities equals that of bank deposits, which represents the outside option for the buyer (patient consumer).[3] That is, the price of

[2]For example, since the real-time gross settlement system (such as Fedwire) does not operate on weekends, wire transfers after the cut-off time on a Friday may not be processed until the following Monday. Another example is that it is often necessary to contact the bank in advance if a depositor wants to make a large withdrawal to ensure that the bank has the funds on hand. Then, if the settlement of a large payment is possible to be postponed, patient consumers may miss the opportunity to purchase claims for date-2 goods in the OTC market, which forces them to reduce their proportion of premature withdrawal to $\bar{\beta}$.

[3]All results will remain qualitatively unchanged should we assume that both the seller and the buyer have some bargaining power.

a shadow bank claim that delivers one unit of date-2 consumption goods is also $1/d_{12}$. Then, we can still use d_{12} to denote the gross return on *both* bank deposits and shadow bank securities from date 1 to date 2.

The no-arbitrage condition ($d_{01} = d_{02}/d_{12}$) in the baseline model continues to hold in the extended model, because patient consumers can still withdraw any amount of deposits to purchase deposit claims. Consequently, the change in the proportion of patient consumers' premature withdrawal $\bar{\beta}$ cannot be fully absorbed by the change in the price $1/d_{12}$, which necessitates a response in the probability of a successful matching α.

Then, α is determined by equalizing the matched sell orders $\lambda (1 - x)\, \alpha R/d_{12}$ and buy orders $(1 - \lambda)\, x\beta d_{01}$, which is the solution of the following equation:

$$\lambda (1 - x)\, \alpha \frac{R}{d_{12}} = (1 - \lambda)\, x\beta d_{01},$$

Note that this equation effectively substitutes $\bar{\alpha}$ with an endogenously determined α in the market clearing condition (3) of the benchmark case, which confirms that the current extension can serve as the micro-foundation of the benchmark model.

To be consistent with our previous analysis, we assume that there is an initial value of α, which equals $\bar{\alpha}$. For any $\bar{\alpha}$, there is a cutoff liquidity requirement $\tilde{\underline{\ell}}$ with which the proportion of patient consumers' premature withdrawal β reaches $\bar{\beta}$. When the liquidity requirement further increases, patient consumers cannot prematurely withdraw more bank deposits, and agents have common knowledge that β will stay unchanged at $\bar{\beta}$.[4] α will then decline and become smaller than the initial value $\bar{\alpha}$. Consequently, the proportion of shadow bank securities that can be sold in the OTC market declines, which reduces the liquidity of shadow bank securities. Lemma 5 characterizes the cutoff liquidity requirement $\tilde{\underline{\ell}}$.

LEMMA 5. *Given $\bar{\alpha}$ and $\bar{\beta}$, there exists a cutoff liquidity requirement $\tilde{\underline{\ell}}$, which is a function of $\bar{\alpha}$ and $\bar{\beta}$. When $\underline{\ell} \leq \tilde{\underline{\ell}}$, the endogenously determined proportion of patient consumers' premature withdrawal β is smaller than $\bar{\beta}$; when $\underline{\ell} > \tilde{\underline{\ell}}$, β is strictly greater than $\bar{\beta}$.*

PROOF. See Appendix A.9. □

When $\underline{\ell} \leq \tilde{\underline{\ell}}$, the constraint of the upper bound of β is not binding, and the equilibrium is identical to that in the previous section; that is, α remains constant at $\bar{\alpha}$, and β is endogenously determined. When $\underline{\ell} > \tilde{\underline{\ell}}$, the equilibrium will be different. β remains constant at $\bar{\beta}$, and α is endogenously determined. We provide the complete definition of the equilibrium as follows.

DEFINITION 2. Given an initial value of the probability of a successful matching with sellers $\bar{\alpha}$ and an upper bound of the proportion of patient consumers' premature withdrawal $\bar{\beta}$, a competitive equilibrium with liquidity requirement $\underline{\ell}$ is a set of allocations

[4] Arseneau, Rappoport W., and Vardoulakis (2020) assume a general matching function in the OTC market, which shares the same idea that when there are more sellers, the probability of a seller being matched will be lower. For tractability, we take the limits of the matching function and assume a constant α in the baseline model and a constant β in this section.

(c_1^i, c_1^p, c_2^p), a deposit contract with a menu of two options (d_{01}, d_{02}), the share of deposits in the consumer's portfolio x, the probability of a successful matching with sellers α, the proportion of patient consumers' premature withdrawal β, and the market return at date 1 d_{12}, such that

(i) When $\underline{\ell} \leq \tilde{\underline{\ell}}$, the equilibrium is identical to that defined in Definition 1.

(ii) When $\underline{\ell} > \tilde{\underline{\ell}}$, the equilibrium is defined as follows.

(a) given $\bar{\beta}$, d_{12} and $\underline{\ell}$, each commercial bank chooses (d_{01}, d_{02}) and ℓ to solve problem (5);

(b) consumers rationally anticipate α and choose x to solve problem (4);

(c) Given $\beta = \bar{\beta}$, the successfully matched sell orders are equal to buy orders, that is,

$$\lambda (1 - x) \alpha \frac{R}{d_{12}} = (1 - \lambda) x \bar{\beta} d_{01}. \tag{10}$$

4.1 *Equilibrium Characterization*

Since when $\underline{\ell} \leq \tilde{\underline{\ell}}$, the equilibrium is identical to that which has been studied in the previous section, here we only need to characterize the equilibrium when $\underline{\ell} > \tilde{\underline{\ell}}$.

When $\underline{\ell} > \tilde{\underline{\ell}}$, $\bar{\beta}$ affects both the price $1/d_{12}$ and the liquidity margin α of shadow bank securities. Specifically, the common knowledge about $\beta = \bar{\beta}$ directly determines the deposit contract by banks' budget constraints: $d_{01} = \underline{\ell}/(\lambda + (1 - \lambda) \bar{\beta})$ and $d_{02} = R(1 - \underline{\ell})/\left[(1 - \lambda)\left(1 - \bar{\beta}\right)\right]$. Thus, the gross return at date 1 equals $d_{12} = d_{02}/d_{01} = R(1 - \underline{\ell})\left(\lambda + (1 - \lambda)\bar{\beta}\right)/\left[\underline{\ell}(1 - \lambda)\left(1 - \bar{\beta}\right)\right]$. By substituting the deposit contracts and the gross return d_{12} into condition (10), we obtain the following relation between the probability of a successful matching α and the share of deposits x:

$$\alpha = \frac{\bar{\beta} x (1 - \underline{\ell})}{\lambda \left(1 - \bar{\beta}\right)(1 - x)}. \tag{11}$$

Intuitively, α and x have a positive correlation from the view of the demand for bank deposits. This is because a higher probability of a successful matching α means that there are more shadow bank securities to be sold in the OTC market, since the proportion of patient consumers' premature withdrawal is constant, more deposits are needed to match the sell orders.

Given d_{12}, the supply of deposits chosen by consumers depends on their rational expectations of α. If consumers expect a higher probability of successfully meeting a buyer in the OTC market, they will be able to sell a larger proportion of shadow bank securities and have less incentive to invest in deposits. This provides a relation between the probability of a successful matching α and the share of deposits x from the view of the supply of deposits, which is manifested in consumers' first-order condition:[5]

$$\lambda U'\left(c_1^i\right)\left(d_{01}-\alpha\frac{R}{d_{12}}\right)=(1-\lambda)\,U'\left(c_2^p\right)(R-d_{02})\,. \tag{12}$$

Similar to the previous discussions, when the demand for deposits equals the supply of deposits, the relations between α and x from the views of the demand for and supply of deposits simultaneously hold, which jointly determines α and x. The following proposition characterizes the equilibrium.

PROPOSITION 5. *There exists a unique equilibrium in which*

(i) *when $\underline{\ell}\le\tilde{\underline{\ell}}$, the resource allocations are identical to those characterized by Proposition 1;*

(ii) *when $\underline{\ell}\ge\tilde{\underline{\ell}}$, x is the solution to the first-order condition (12), where $c_1^i=x\underline{\ell}/\lambda$, $c_1^p=0$, $c_2^p=[Rx(1-\underline{\ell})/(1-\bar{\beta})+(1-\lambda)R(1-x)]/(1-\lambda)$; $d_{12}=[R(1-\underline{\ell})(\lambda+(1-\lambda)\bar{\beta})]/[\underline{\ell}(1-\lambda)(1-\bar{\beta})]$; $d_{01}=\underline{\ell}/[\lambda+(1-\lambda)\bar{\beta}]$, $d_{02}=R(1-\underline{\ell})/[(1-\lambda)(1-\bar{\beta})]$; $\alpha=\bar{\beta}x\,(1-\underline{\ell})/\left[\lambda\left(1-\bar{\beta}\right)(1-x)\right]$.*

PROOF. See Appendix A.10. □

In the extended model, the probability of a successful matching α is endogenously determined. An increase in the liquidity requirement, by driving more funds to the shadow banking system, will reduce the liquidity of shadow bank securities. This new effect weakens the incentive for regulatory arbitrage and may even change the impact of liquidity regulations on aggregate liquidity provision, which we will elaborate on in the next subsection.

4.2 *Comparative Statics and Optimal Liquidity Requirement*

The following proposition provides comparative statics regarding the share of commercial bank deposits and aggregate liquidity provision when $\underline{\ell}\ge\tilde{\underline{\ell}}$.

PROPOSITION 6. *When $\underline{\ell}\ge\tilde{\underline{\ell}}$,*

(i) *The share of commercial bank deposits x decreases with liquidity requirement $\underline{\ell}$.*

(ii) *When $\bar{\alpha}=1$, (a) the aggregate liquidity provision $\mathfrak{L}$ strictly decreases with $\underline{\ell}$ for any $\underline{\ell}>\tilde{\underline{\ell}}$ if $\bar{\beta}>\lambda U'(1)/[\lambda U'(1)+(1-\lambda)\,U'(R)\,R]$; (b) the aggregate liquidity provision $\mathfrak{L}$ strictly increases with $\underline{\ell}$ at $\underline{\ell}=\tilde{\underline{\ell}}$ if $\bar{\beta}<\lambda U'(1)/[\lambda U'(1)+(1-\lambda)\,U'(R)\,R]$.*

PROOF. See Appendix A.11. □

[5] Note that although α is endogenously determined in equilibrium, each single consumer takes the the probability of a successful matching α as given.

When the probability of a successful matching α is endogenously determined, a higher liquidity requirement has two effects. The first effect is that it directly reduces the date-1 return d_{12}, making bank deposits less attractive for any given α. This is the effect that we have discussed under a constant α in the previous section. The second effect is new. Since consumers invest more goods in shadow banks at date 0, impatient consumers have to post more sell orders and send more traders to the OTC market, resulting in a lower probability of successful matching and making shadow bank securities less attractive.

In terms of the market shares of bank deposits and shadow bank securities, we show that the first effect is dominant, because if the share of bank deposits does not decrease, the probability of a successful matching α determined in the OTC market is so large that consumers would strictly prefer shadow bank securities. Thus, the share of deposits needs to further decrease to reduce α and make consumers indifferent between bank deposits and shadow bank securities.

Although the second effect does not dominate the first one in terms of the determination of the market shares, it does partly offset the first effect and weaken the incentive for regulatory arbitrage. Thus, the same amount of increase in the liquidity requirement will lead to a smaller decrease in the share of bank deposits. This makes it unclear how the aggregate liquidity provision $\mathfrak{L} = x\underline{\ell}$ would change in response to an increase in the liquidity requirement. If the new second effect is small, then the decrease in deposit share x still outweighs the increase in the liquidity provided by each unit of deposit ($\underline{\ell}$), resulting in lower aggregate liquidity provision. However, if the new second effect is sufficiently large, then an increase in the liquidity requirement may only lead to a very small change in x, and the increase in $\underline{\ell}$ becomes dominant, resulting in higher aggregate liquidity provision. Liquidity requirements become effective in improving aggregate liquidity provision even in the presence of shadow banks.

To simplify the analysis of the relation between the aggregate liquidity provision and liquidity requirements under endogenous α, we consider a special case with the initial value of matching probability $\bar{\alpha} = 1$.[6] Then, Proposition 6 suggests that the aggregate liquidity provision $\mathfrak{L}$ strictly *decreases* with the liquidity requirement if $\bar{\beta}$ is large, and the aggregate liquidity provision $\mathfrak{L}$ strictly *increases* with the liquidity requirement around $\tilde{\underline{\ell}}$ when $\bar{\beta}$ is small. Intuitively, when $\bar{\beta}$ is large, patient consumers can still provide much liquidity in the OTC market by withdrawing bank deposits, and thus the liquidity of shadow bank securities will not significantly decrease. The new effect is small and does not significantly suppress regulatory arbitrage. The aggregate liquidity provision always decreases with the liquidity requirement, and the results in our baseline model with a constant $\bar{\alpha}$ do not change.

By contrast, when $\bar{\beta}$ is small, patient consumers can only provide very limited liquidity in the OTC market by withdrawing bank deposits, which imposes a strict constraint

[6]We consider the special case with $\bar{\alpha} = 1$ because it excludes the wealth effect and only keeps the substitution effect at $\underline{\ell} = \tilde{\underline{\ell}}$. To see this, in the first-order condition (12), $\alpha = 1$ at $\underline{\ell} = \tilde{\underline{\ell}}$ means the interest rates satisfy $d_{01} = 1, d_{02} = R, d_{12} = R$, and thus $d_{01} - \alpha R/d_{12} = R - d_{02} = 0$, which eliminates the impact of liquidity requirement on aggregate liquidity provision through $U'\left(c_1^i\right)$ and $U'\left(c_2^p\right)$.

on the liquidity of shadow bank securities and further suppresses the incentive for regulatory arbitrage. It becomes possible for the aggregate liquidity provision to increase with the liquidity requirement.

Therefore, in some cases, the results in the baseline model still hold with endogenous shadow bank liquidity; in some other cases, it becomes possible for the liquidity provision to increase with the liquidity requirement. This yields novel implications in terms of optimal liquidity regulations. In this example, if the liquidity requirement rises above λ, then funds will flow to shadow bank securities. Therefore, the cutoff liquidity requirement at which shadow banks emerge is $\hat{\underline{\ell}} = \lambda < \underline{\ell}^*$. Then, since $\hat{\underline{\ell}} < \underline{\ell}^*$, in the baseline model, the optimal liquidity regulation is to set $\underline{\ell} = \hat{\underline{\ell}}$; that is, setting the liquidity requirement to the threshold level that shadow banks do not emerge. However, when α is endogenous and $\bar{\beta}$ is small, in the next corollary, we will show that the optimal liquidity requirement must be higher than $\tilde{\underline{\ell}}$, under which commercial and shadow banks coexist.

COROLLARY 1. *If* $\bar{\alpha} = 1$ *and* $\bar{\beta} < \lambda\left(U'(1) - U'(R)R\right)/[\lambda U'(1) + (1-\lambda)U'(R)R]$, *the optimal liquidity requirement* $\underline{\ell}^{opt2}$ *satisfies* $\underline{\ell}^{opt2} > \tilde{\underline{\ell}} > \hat{\underline{\ell}}$, *and commercial and shadow banks coexist under the optimal liquidity requirement.*

PROOF. See Appendix A.12. □

Intuitively, endogenous matching probability α brings two new effects on social welfare. On the one hand, Proposition 6 has shown that an increase in the liquidity requirement can increase aggregate liquidity provision, which alleviates the liquidity shortage problem. On the other hand, a higher liquidity requirement also reduces the matching probability α, and impatient consumers can only sell a smaller proportion of their shadow bank securities. Since impatient consumers do not obtain any utility from date-2 consumption, this causes a waste of resources. The optimal liquidity regulation needs to balance these two effects.

In the previous section, we have shown that any liquidity requirements strictly smaller than the cutoff liquidity requirement at which shadow banks just emerge, $\hat{\underline{\ell}}$, cannot be the optimal choice. For any liquidity requirements in the range of $[\hat{\underline{\ell}}, \tilde{\underline{\ell}})$,[7] since β has not reached $\bar{\beta}$, α keeps constant at $\bar{\alpha} = 1$. Then, in this range, shadow bank securities do not have any liquidity frictions and thus the resource allocation is always $c_1^i = 1$ and $c_2^p = R$. Social welfare remains unchanged between $\hat{\underline{\ell}}$ and $\tilde{\underline{\ell}}$. Finally, Proposition 6 has shown that around $\tilde{\underline{\ell}}$, the aggregate liquidity provision $\mathfrak{L}$ increases with the liquidity requirement. The strength of the positive effect of improving aggregate liquidity provision $\mathfrak{L}$ decreases with $\bar{\beta}$, because a smaller $\bar{\beta}$ can more effectively deter regulatory arbitrage. Therefore, when $\bar{\beta}$ is sufficiently small, the effect of increasing aggregate liquidity provision dominates the effect of increasing resource waste, and social welfare

[7] $\tilde{\underline{\ell}} > \hat{\underline{\ell}}$ because $x = 1$ and $\beta = 0$ at $\hat{\underline{\ell}}$, while $\tilde{\underline{\ell}}$ makes $\beta = \bar{\beta} > 0$.

increases with the liquidity requirement at $\tilde{\underline{\ell}}$.[8] Consequently, the optimal liquidity requirement $\underline{\ell}^{opt2}$ satisfies $\underline{\ell}^{opt2} > \tilde{\underline{\ell}} > \hat{\underline{\ell}}$, and commercial and shadow banks coexist under the optimal liquidity requirement.

Corollary 1 demonstrates that market structure and search frictions have important impacts on optimal liquidity regulations. When shadow bank securities are traded in an OTC market, the degree of search frictions in the OTC market can significantly affect the demand for bank deposits and shadow bank securities, which leads to different optimal policies. Policymakers must consider the microstructure of the market when setting the optimal liquidity regulations.

5. Conclusion

We study the competition and complementarity between commercial and shadow banks in an economy where shadow banks can bypass liquidity requirements but suffer higher liquidity costs. With relatively high liquidity requirements, shadow banks emerge as a channel to circumvent the liquidity requirements.

Liquidity requirements may lead to unintended consequences in the presence of shadow banks. A higher liquidity requirement, while aimed at improving liquidity provision, may instead exacerbate the liquidity shortage and over-investment problem, since illiquid shadow banks can bypass regulations and substitute deposits. Optimal liquidity requirements should be set lower than the level that makes shadow banks emerge. An improvement in the liquidity of shadow bank securities may either improve or reduce aggregate liquidity provision under different liquidity regulation environments.

We also study the case with endogenous liquidity of shadow bank securities. An increase in the liquidity requirement, by driving more funds to the shadow banking system, reduces the liquidity of the shadow bank securities. This weakens the incentive for regulatory arbitrage. When the liquidity of shadow bank securities falls quickly enough as the regulations become more restrictive, liquidity requirements may become effective in improving aggregate liquidity provision even in the presence of shadow banks, and the optimal liquidity requirement can be achieved when commercial and shadow banks coexist.

Appendix A: Proofs

A.1 *Proof of Lemma 1*

It is inefficient for impatient consumers to consume at date 2 and for patient consumers to consume at date 1, and thus $c_2^i = c_1^p = 0$. The budget constraints then imply $c_1^i = \mathfrak{L}/\lambda, c_2^p = R(1-\mathfrak{L})/(1-\lambda)$. Substitute c_1^i and c_2^p into social welfare $\lambda U\left(c_1^i\right) + (1-\lambda) U\left(c_1^p + c_2^p\right)$ and take derivative with respect to $\mathfrak{L}$, we have the first-order condition that determines the socially optimal aggregate liquidity provision. Since the utility function satisfies $-U''(c)\,c/U'(c) > 1$, $U'(c)\,c$ must decrease in c, which implies

[8]Note that the previous condition of $\bar{\beta}$ in Proposition 6 (ii) can only ensure that aggregate liquidity provision $\mathfrak{L}$ increases with the liquidity requirement. To make the positive effect dominate the negative effect of resource wastes, a smaller $\bar{\beta}$ is needed.

$U'(1) > U'(R)R$. Thus, the first-order condition (6) implies $\mathfrak{L}^* > \lambda$. Moreover, the consumption satisfies $c_1^i > 1$ and $c_2^p < R$.

A.2 *Proof of Lemma 2*

Given d_{12}, a bank solves the problem (5). Note that the budget constraint for impatient and patient consumers are $c_1^i = d_{01}x + \alpha(1-x)R/d_{12}$ and $c_2^p = d_{02}x + R(1-x)$, and the no-arbitrage condition $d_{01} = d_{02}/d_{12}$ implies that the budget constraint for the patient consumers satisfies $c_2^p = d_{01}d_{12}x + R(1-x)$. Therefore, given d_{12}, the consumption for impatient and patient consumers increases monotonically with d_{01}, independent of x. In other words, the bank's problem is simply to maximize d_{01}, subject to all the constraints.

The bank's expectation of the proportion of deposits prematurely withdrawn by patient consumers must be consistent with the budget constraints, i.e., $\beta = (\ell/d_{01} - \lambda)/(1-\lambda)$. Moreover, we have

$$\begin{aligned} d_{01} &= [\lambda + (1-\lambda)\beta]\, d_{01} + (1-\lambda)(1-\beta)\, d_{01} \\ &= [\lambda + (1-\lambda)\beta]\, d_{01} + \frac{1}{d_{12}}(1-\lambda)(1-\beta)\, d_{02} \\ &= \ell + \frac{R}{d_{12}}(1-\ell), \end{aligned} \tag{A.1}$$

where the second equality uses the no-arbitrage condition. Therefore, when $d_{12} < R$, the short-term return decreases in the liquidity ratio ℓ. As a result, a bank would invest as much as possible, implying that the liquidity requirement binds. In this case, $\ell = \underline{\ell}$ implies the second part of the Lemma. When $d_{12} = R$, by (A.1) the short-term return is equal to 1, independent of ℓ. Thus, the liquidity constraint (1) may not bind. In this case, the above equation still holds with $d_{01} = 1$. Finally, while a bank can choose an arbitrary level $\ell > \underline{\ell}$, in a symmetric equilibrium all banks choose the same level, which is determined by the equilibrium allocation. This proves the first part of the Lemma.

A.3 *Proof of Lemma 3*

The expected marginal utility of deposit is $\lambda U'\left(c_1^i\right) d_{01} + (1-\lambda)U'\left(c_2^p\right) d_{02}$, and the expected marginal utility of shadow bank securities is $\lambda U'\left(c_1^i\right)\bar{\alpha}R/d_{12} + (1-\lambda)U'\left(c_2^p\right)R$. Thus, the trade-off depends on the comparison between $\lambda U'\left(c_1^i\right)(d_{01} - \bar{\alpha}R/d_{12})$ and $(1-\lambda)U'\left(c_2^p\right)(R - d_{02})$, with $c_1^i = d_{01}x + R\bar{\alpha}(1-x)/d_{12}$ and $c_2^p = d_{02}x + R(1-x)$.

By Lemma 2, we can describe deposit rates with d_{12}, which yields $d_{01} - \bar{\alpha}R/d_{12} = \underline{\ell} + R(1-\underline{\ell}-\bar{\alpha})/d_{12}$, $R - d_{02} = \underline{\ell}(R - d_{12})$, $c_1^i = (\underline{\ell} + R(1-\underline{\ell}-\bar{\alpha})/d_{12})x + \bar{\alpha}R/d_{12}$ and $c_2^p = \underline{\ell}(d_{12} - R)x + R$. Clearly $(1-\lambda)U'\left(c_2^p\right)(R - d_{02}) = (1-\lambda)U'\left(c_2^p\right)(\underline{\ell}(R - d_{12}))$ decreases in d_{12} since $U''(c) < 0$.

We rewrite $\lambda U'\left(c_1^i\right)(\underline{\ell} + R(1-\underline{\ell}-\bar{\alpha})/d_{12})$ as

$$\lambda U'\left(c_1^i\right) c_1^i \left(x + \bar{\alpha}R/[d_{12}\underline{\ell} + R(1-\underline{\ell}-\bar{\alpha})]\right)^{-1}.$$

Since $-U''(c)c/U'(c) > 1$, $U'\left(c_1^i\right)c_1^i$ decreases in c_1^i.

In addition, $(x+\bar{\alpha}R/[d_{12}\underline{\ell}+R(1-\underline{\ell}-\bar{\alpha})])^{-1}$ increases in d_{12} as well. Thus, it holds that $\lambda U'\left(c_1^i\right)(d_{01}-\bar{\alpha}R/d_{12})$ increases in d_{12}. Thus, the incentive to invest in deposits increases in d_{12}.

When $d_{12}=R$, $(1-\lambda)U'\left(c_2^p\right)(\underline{\ell}(R-d_{12}))=0$, and

$$\lambda U'\left(c_1^i\right)\left(d_{01}-\bar{\alpha}\frac{R}{d_{12}}\right)=\lambda U'\left(c_1^i\right)(1-\bar{\alpha})>0.$$

Then consumers prefer deposits for all $x\leq 1$ and $\underline{\ell}$, implying that consumers will choose $x=1$. Also, for $d_{12}=R(\underline{\ell}+\bar{\alpha}-1)/\underline{\ell}$, $d_{01}-\bar{\alpha}R/d_{12}=0$ and $R-d_{02}=R(1-\bar{\alpha})>0$. Then consumers strictly prefer shadow banks for all $x\leq 1$. Then, due to the fact that $\lambda U'\left(c_1^i\right)(d_{01}-\bar{\alpha}R/d_{12})$ decreases in x and $(1-\lambda)U'\left(c_2^p\right)(R-d_{02})$ increases in x, we can find a threshold $\hat{d}_{12}(\underline{\ell})\leq R$ such that

$$\lambda U'\left(c_1^i\right)\left(d_{01}-\bar{\alpha}\frac{R}{d_{12}}\right)>(1-\lambda)U'\left(c_2^p\right)(R-d_{02})$$

for all $x\leq 1$. Thus, if $d_{12}\geq\hat{d}_{12}(\underline{\ell})$, implying that consumers will choose $x=1$.

When $x<1$, we get a relationship between d_{12} and x by the first-order condition $\lambda U'\left(c_1^i\right)(\underline{\ell}+R(1-\underline{\ell}-\bar{\alpha})/d_{12})=(1-\lambda)U'\left(c_2^p\right)(\underline{\ell}(R-d_{12}))$.

A.4 *Proof of Lemma 4*

Since banks have rational expectations on β, we can substitute d_{01} and β in Lemma 2 into the private market clearing condition (3). When $d_{12}=R$, it satisfies $\lambda(1-x)\bar{\alpha}=x(\ell-\lambda)$, which implies $\ell=\lambda+\lambda\bar{\alpha}(1-x)/x$; i.e., a bank voluntarily chooses liquidity ratio $\ell=\lambda+\lambda\bar{\alpha}(1-x)/x$. Thus, it requires $\underline{\ell}<\lambda+\lambda\bar{\alpha}(1-x)/x$ since the liquidity requirement is not binding, or $x<\lambda\bar{\alpha}/(\underline{\ell}+\lambda\bar{\alpha}-\lambda)$.

When $d_{12}<R$, the liquidity requirement binds, i.e., $\underline{\ell}\geq\lambda+\lambda\bar{\alpha}(1-x)/x$, or $x\geq\lambda\bar{\alpha}/(\underline{\ell}+\lambda\bar{\alpha}-\lambda)$. In this case, we have $\beta=(\underline{\ell}d_{12}/(R(1-\underline{\ell})+\underline{\ell}d_{12})-\lambda)/(1-\lambda)$ and $d_{01}=\underline{\ell}+R(1-\underline{\ell})/d_{12}$. Thus the the private market clearing condition (3) implies $d_{12}=R[\lambda\bar{\alpha}(1-x)+\lambda x(1-\underline{\ell})]/[(1-\lambda)x\underline{\ell}]$.

Note that the voluntarily chosen liquidity ratio $\lambda+\lambda\bar{\alpha}(1-x)/x$ cannot exceed the upper limit 1, i.e., $\lambda+\lambda\bar{\alpha}(1-x)/x\leq 1$. Therefore it requires $x\geq\lambda\bar{\alpha}/(1+\lambda\bar{\alpha}-\lambda)$. If $x<\lambda\bar{\alpha}/(1+\lambda\bar{\alpha}-\lambda)$, banks cannot hold more liquidity as they have already kept all assets as liquid assets, and the shortage of liquid assets will drive the market return d_{12} above R. However, this cannot happen in any equilibrium, because if $d_{12}>R$, no one would invest any goods in the long-term investment technology at date 0 and all agents prefer investment in the private market. However, since no actual long-term investment is made, there are not sufficient goods to repay the loans in the private market at date 2.

A.5 *Proof of Proposition 1*

The first case is associated with $x=1$ and $d_{12}=R$. Thus, the voluntarily chosen liquidity ratio satisfies $\ell=\lambda+\lambda\bar{\alpha}(1-x)/x=\lambda$. In addition, by by (8) in Lemma 4, the cutoff value

$\lambda\bar{\alpha}/(\underline{\ell}+\lambda\bar{\alpha}-\lambda)$ must be greater than 1. Therefore we have $\underline{\ell}<\lambda$. By Lemma 2 we have $d_{01}=1, d_{02}=R$ and $\beta=1$. Thus, we have $c_1^i=d_{01}=1$ and $c_2^p=d_{02}=R$.

The second case is associated with $x=1$ and $\underline{\ell}\geq\lambda$. In this case, by Lemma 4 we know that $d_{12}=R\lambda\left(1-\underline{\ell}\right)/\left((1-\lambda)\underline{\ell}\right)$. Therefore by Lemma 2 we can solve $d_{01}=\underline{\ell}/\lambda, d_{02}=R\left(1-\underline{\ell}\right)/(1-\lambda)$ and $\beta=0$. Then, consumption satisfies $c_1^i=\underline{\ell}/\lambda, c_1^p=0, c_2^p=R\left(1-\underline{\ell}\right)/(1-\lambda)$.

Using the condition $d_{12}=R\lambda\left(1-\underline{\ell}\right)/\left((1-\lambda)\underline{\ell}\right)\geq\hat{d}_{12}\left(\underline{\ell}\right)$, we can determine the cut-off value $\hat{\underline{\ell}}$. Specifically, $\hat{d}_{12}\left(\underline{\ell}\right)$ is determined by

$$\lambda U'\left(c_1^i\right)\left(\underline{\ell}+\frac{R}{d_{12}}\left(1-\underline{\ell}-\bar{\alpha}\right)\right)=(1-\lambda)\,U'\left(c_2^p\right)\left(\underline{\ell}\left(R-d_{12}\right)\right),$$

where $c_1^i=\underline{\ell}+R\left(1-\underline{\ell}\right)/d_{12}$ and $c_2^p=d_{12}\underline{\ell}+R\left(1-\underline{\ell}\right)$. Substitute $d_{12}=R\lambda(1-\underline{\ell})/[(1-\lambda)\underline{\ell}]$ into the equation we get

$$U'\left(\frac{\underline{\ell}}{\lambda}\right)\frac{1-\underline{\ell}-\bar{\alpha}+\bar{\alpha}\lambda}{1-\underline{\ell}}=U'\left(\frac{R\left(1-\underline{\ell}\right)}{1-\lambda}\right)R\left(\frac{\underline{\ell}-\lambda}{\underline{\ell}}\right).$$

When $\underline{\ell}=\lambda$, the left-hand side is strictly greater; when $\underline{\ell}=1-\bar{\alpha}+\bar{\alpha}\lambda\in(\lambda,1)$, the right-hand is strictly greater. Thus, there exists an $\hat{\underline{\ell}}\in(\lambda,1-\bar{\alpha}+\bar{\alpha}\lambda)$ such that when $\lambda\leq\underline{\ell}<\hat{\underline{\ell}}$, the equilibrium is the second type, and when $\underline{\ell}\geq\hat{\underline{\ell}}$, it is the third type.

The third case is associated with $x<1$ and $\underline{\ell}\geq\hat{\underline{\ell}}$. Given d_{12} and x, we have $d_{01}=\underline{\ell}+R\left(1-\underline{\ell}\right)/d_{12},\ d_{02}=\underline{\ell}d_{12}+\left(1-\underline{\ell}\right)R$ and $c_1^i=\left(\underline{\ell}+R\left(1-\underline{\ell}-\bar{\alpha}\right)/d_{12}\right)x+R\bar{\alpha}/d_{12}$ and $c_2^p=\underline{\ell}\left(d_{12}-R\right)x+R$. And d_{12} and x are mutually determined by

$$\lambda U'\left(c_1^i\right)\left(\underline{\ell}+\frac{R}{d_{12}}\left(1-\underline{\ell}-\bar{\alpha}\right)\right)=(1-\lambda)\,U'\left(c_2^p\right)\left(\underline{\ell}\left(R-d_{12}\right)\right)$$
$$d_{12}=R\frac{\lambda\bar{\alpha}\left(1-x\right)+\lambda x\left(1-\underline{\ell}\right)}{(1-\lambda)\,x\underline{\ell}}.$$

We may substitute $d_{12}=R[\lambda\bar{\alpha}\left(1-x\right)+\lambda x\left(1-\underline{\ell}\right)]/[(1-\lambda)\,x\underline{\ell}]$ into the deposits and consumption, then the first-order condition may be described as a pure equation of x:

$$U'\left(c_1^i\right)\frac{\underline{\ell}\left[\left(1-\underline{\ell}\right)x+\bar{\alpha}\left(\lambda-x\right)\right]}{\left(1-\underline{\ell}\right)x+\bar{\alpha}\left(1-x\right)}=U'\left(c_2^p\right)R\frac{\left(\underline{\ell}-\lambda\right)x-\bar{\alpha}\lambda\left(1-x\right)}{x}$$

with interest rates $d_{01}=\left(\underline{\ell}\left(1-\underline{\ell}\right)x+\underline{\ell}\bar{\alpha}\lambda\left(1-x\right)\right)/\left(\lambda\left(1-\underline{\ell}\right)x+\bar{\alpha}\lambda\left(1-x\right)\right)$, and $d_{02}=R\left(\left(1-\underline{\ell}\right)x+\bar{\alpha}\lambda\left(1-x\right)\right)/\left((1-\lambda)\,x\right)$, and $d_{12}=d_{02}/d_{01}$, and consumption $c_1^i=x\underline{\ell}/\lambda$, $c_2^p=\left(xR\left(1-\underline{\ell}\right)+\left(1-x\right)R\left[\lambda\bar{\alpha}+\left(1-\lambda\right)\right]\right)/(1-\lambda)$. Solving x in the equation above, we can completely solve the equilibrium allocation.

A.6 *Proof of Proposition 2*

First, when $\lambda\leq\underline{\ell}<\hat{\underline{\ell}}$, $x=1$, which is unrelated to the liquidity requirement. Thus $\mathfrak{L}=\underline{\ell}$, which clearly increases in $\underline{\ell}$. This completes the proof of the first part.

For the second part, when $\underline{\ell}\geq\hat{\underline{\ell}}$, $x<1$. Thus $\mathfrak{L}=x\underline{\ell}<\underline{\ell}$. To study the change of $\mathfrak{L}$ in response to an increase in $\underline{\ell}$, we need to take into account the change of x. Using the following form of the first-order condition:

$$U'\left(c_1^i\right)\frac{\underline{\ell}\left[\left(1-\underline{\ell}\right)x+\bar{\alpha}\left(\lambda-x\right)\right]}{\left(1-\underline{\ell}\right)x+\bar{\alpha}\left(1-x\right)}=U'\left(c_2^p\right)R\frac{\left(\underline{\ell}-\lambda\right)x-\bar{\alpha}\lambda\left(1-x\right)}{x},$$

which by the fact that $c_1^i = x\underline{\ell}/\lambda$ can be rewritten as

$$\lambda U'\left(c_1^i\right)c_1^i\left(1-\frac{\left(1-\lambda\right)\bar{\alpha}}{\left(1-\bar{\alpha}\right)x+\bar{\alpha}-\underline{\ell}x}\right)=U'\left(c_2^p\right)R\left(\underline{\ell}x-\lambda x+\bar{\alpha}\lambda x-\bar{\alpha}\lambda\right).$$

in which the left-hand side decreases in x and $\underline{\ell}$ and the right-hand side increases in x and $\underline{\ell}$. Thus, when $\underline{\ell}$ increases, the left-hand side will be lower than the right-hand side. Thus, x has to decrease to restore the equilibrium. This proves the part 2(b).

To prove the (a) part, we need to further show that when $d\underline{\ell}>0$, $d\left(x\underline{\ell}\right)=xd\underline{\ell}+\underline{\ell}dx<0$. That is, $dx<-(x/\underline{\ell})d\underline{\ell}$. Our strategy is to show that if $dx=-(x/\underline{\ell})d\underline{\ell}$, then the left-hand side of the first-order condition will be lower than the right-hand side. Then x needs to further decrease to restore the equilibrium.

Note that

$$\left(1-\frac{\left(1-\lambda\right)\bar{\alpha}}{\left(1-\bar{\alpha}\right)x+\bar{\alpha}-\underline{\ell}x}\right)\cdot\frac{1}{\underline{\ell}x-\lambda x+\bar{\alpha}\lambda x-\bar{\alpha}\lambda}$$

satisfies that if $dx=-(x/\underline{\ell})d\underline{\ell}<0$, i.e., if $x\underline{\ell}$ is fixed, then it becomes smaller if $\bar{\alpha}<1$. Note that when $\bar{\alpha}=1$, it is irrelevant to x as long as $x\underline{\ell}$ does not change.

In addition, when $x\underline{\ell}$ does not change, $c_1^i=x\underline{\ell}/\lambda$ does not change as well. On the other hand, $c_2^p=\left(Rx\left(1-\underline{\ell}\right)+\left(1-x\right)R\left[\lambda\bar{\alpha}+\left(1-\lambda\right)\right]\right)/\left(1-\lambda\right)$ decreases because $\lambda\bar{\alpha}+1-\lambda<1$. Thus, $U'\left(c_1^i\right)c_1^i$ does not change and $U'\left(c_2^p\right)$ increases.

The above analysis has shown that if $dx=-(x/\underline{\ell})d\underline{\ell}$, then the left-hand side of the first-order condition will be lower than the right-hand side. Then x needs to further decrease to restore the equilibrium. In other words, $dx<-(x/\underline{\ell})d\underline{\ell}$ and $(d\mathfrak{L})/(d\underline{\ell})=x+\underline{\ell}(dx)/(d\underline{\ell})<0$. Essentially, we have shown that if the liquidity provision does not change, then the market return determined in the private market is so small that consumers would strictly prefer shadow bank securities. Thus, the decrease in the share of deposits must dominate the increase in the liquidity requirement, resulting in a decrease in the liquidity provision. This completes the proof of part (b).

A.7 *Proof of Proposition 3*

First, we show that $\hat{\underline{\ell}}$ decreases with $\bar{\alpha}$. In the proof of Proposition 1, the cutoff liquidity requirement $\hat{\underline{\ell}}$ satisfies

$$U'\left(\frac{\hat{\underline{\ell}}}{\lambda}\right)\frac{1-\hat{\underline{\ell}}-\bar{\alpha}+\bar{\alpha}\lambda}{1-\hat{\underline{\ell}}}=U'\left(\frac{R\left(1-\hat{\underline{\ell}}\right)}{1-\lambda}\right)R\left(\frac{\hat{\underline{\ell}}-\lambda}{\hat{\underline{\ell}}}\right),$$

in which the left-hand side decreases in $\hat{\underline{\ell}}$ while the right-hand side increases in $\hat{\underline{\ell}}$. Since the left-hand side decreases in $\bar{\alpha}$, it is clear that when $\bar{\alpha}$ increases, we need a lower cutoff value. In particular, when $\bar{\alpha}=1$, the cutoff value is $\hat{\underline{\ell}}=\lambda$.

When $\underline{\ell} \geq \hat{\underline{\ell}}$, we can analyze the relationship between $\mathfrak{L}$ and $\bar{\alpha}$ through the first-order condition, which can be rewritten as an equation of x:

$$\lambda U'\left(c_1^i\right) c_1^i \frac{1}{x} \frac{(1-\underline{\ell})\,x+\bar{\alpha}\,(\lambda-x)}{(1-\underline{\ell})\,x+\bar{\alpha}\,(1-x)} = U'\left(c_2^p\right) R \frac{(\underline{\ell}-\lambda)\,x-\bar{\alpha}\lambda\,(1-x)}{x},$$

with $c_1^i = x\underline{\ell}/\lambda, c_2^p = \left(Rx\,(1-\underline{\ell}) + (1-x)\,R\,[\lambda\bar{\alpha}+(1-\lambda)]\right)/(1-\lambda)$. Since $\lambda U'\left(c_1^i\right) c_1^i$ is independent of $\bar{\alpha}$, we analyze

$$U'\left(c_2^p\right) \frac{(\underline{\ell}-\lambda)\,x-\bar{\alpha}\lambda\,(1-x)}{x} \Big/ \left(\frac{1}{x}\frac{(1-\underline{\ell})\,x+\bar{\alpha}\,(\lambda-x)}{(1-\underline{\ell})\,x+\bar{\alpha}\,(1-x)}\right),$$

or

$$U'\left(c_2^p\right) [(\underline{\ell}-\lambda)\,x-\bar{\alpha}\lambda\,(1-x)] \left(1+\frac{(1-\lambda)\,\bar{\alpha}}{(1-\underline{\ell})\,x+\bar{\alpha}\,(\lambda-x)}\right).$$

When $\bar{\alpha}$ increases by $d\bar{\alpha}$, the formula changes by

$$\begin{aligned} &U'\left(c_2^p\right)\left[-\lambda\,(1-x)\frac{(1-\underline{\ell})\,x+\bar{\alpha}\,(1-x)}{(1-\underline{\ell})\,x+\bar{\alpha}\,(\lambda-x)} + \frac{[(\underline{\ell}-\lambda)\,x-\bar{\alpha}\lambda\,(1-x)]\,(1-\lambda)\,(1-\underline{\ell})\,x}{[(1-\underline{\ell})\,x+\bar{\alpha}\,(\lambda-x)]^2}\right] d\bar{\alpha} \\ &+ U''\left(c_2^p\right)\frac{(1-x)\,R\lambda}{1-\lambda}[(\underline{\ell}-\lambda)\,x-\bar{\alpha}\lambda\,(1-x)]\left(1+\frac{(1-\lambda)\,\bar{\alpha}}{(1-\underline{\ell})\,x+\bar{\alpha}\,(\lambda-x)}\right) d\bar{\alpha}\end{aligned}$$

The above formula becomes $U'\left(c_2^p\right)\left[\left(\hat{\underline{\ell}}-\lambda\right)(1-\lambda)\left(1-\hat{\underline{\ell}}\right)/\left[1-\hat{\underline{\ell}}+\bar{\alpha}\,(\lambda-1)\right]^2\right] d\bar{\alpha} > 0$ when $\underline{\ell} = \hat{\underline{\ell}}$, $x = 1$, implying that the increase in $\bar{\alpha}$ makes shadow bank securities more attractive. Thus, when $\bar{\alpha}$ increases, x will decrease and so will $\mathfrak{L} = x\underline{\ell}$ as $\underline{\ell}$ is fixed. When $\underline{\ell} = 1$, the above formula becomes

$$\left\{-U'\left(c_2^p\right) + U''\left(c_2^p\right)\frac{R}{1-\lambda}[(1-\lambda)\,x-\bar{\alpha}\lambda\,(1-x)]\right\}\lambda\frac{(1-x)^2}{\lambda-x}d\bar{\alpha} < 0,$$

implying that when $\bar{\alpha}$ increases, x will increase and so will $\mathfrak{L} = x\underline{\ell}$ as $\underline{\ell}$ is fixed.[9]

A.8 *Proof of Proposition 4*

In Section A.7 we have shown that $\hat{\underline{\ell}}$ decreases with $\bar{\alpha}$ monotonically. In particular, when $\bar{\alpha} = 1$, the cutoff value is $\hat{\underline{\ell}} = \lambda$. Denote the cutoff value of $\bar{\alpha}$ such that $\hat{\underline{\ell}} = \underline{\ell}^* > \lambda$ by $\bar{\alpha}^c$.

Suppose $\bar{\alpha} > \bar{\alpha}^c$, then $\hat{\underline{\ell}} < \underline{\ell}^*$, then setting $\underline{\ell} = \underline{\ell}^*$ implies the coexistence of commercial and shadow banks. Due to Proposition 2, for $\underline{\ell} \in (0, \hat{\underline{\ell}}]$, welfare increases since $\hat{\underline{\ell}} < \underline{\ell}^*$; for $\underline{\ell} \in (\hat{\underline{\ell}}, 1]$, an increase in $\underline{\ell}$ reduces welfare because (i) liquidity provision decreases in $\underline{\ell}$ due to Proposition 2, and (ii) more wasted resources due to lower share of commercial banks implied by Proposition 2.

[9]Since deposit contract weakly dominates shadow bank security in terms of short-term return, in any equilibrium it satisfies $d_{01} \geq \bar{\alpha}R/d_{12}$, which implies $(1-\underline{\ell}-\bar{\alpha})\,x+\bar{\alpha}\lambda \geq 0$. When $\underline{\ell} = 1$, it implies that $x \leq \lambda$.

Thus, the highest welfare is achieved at $\underline{\ell}=\hat{\underline{\ell}}$ when $\underline{\ell}^*>\hat{\underline{\ell}}$. On the other hand, suppose $\bar{\alpha}\leq\bar{\alpha}^c$, then $\underline{\ell}^*\leq\hat{\underline{\ell}}$, and the policy maker can achieve socially optimal allocation by setting $\underline{\ell}=\underline{\ell}^*$ without triggering regulatory arbitrage activities. In summary, the optimal regulation is $\underline{\ell}^{opt}=\min\left(\underline{\ell}^*,\hat{\underline{\ell}}\right)$.

A.9 *Proof of Lemma 5*

Suppose $\alpha=\bar{\alpha}$, then the equilibrium is the same as the one in the baseline model, and is fully characterized in Proposition 1. Note that when $\underline{\ell}=\hat{\underline{\ell}}$, shadow banks are just about to emerge in the economy, implying that $\beta=0<\bar{\beta}$. As $\underline{\ell}$ further increases, by Proposition 2, the share of deposits x decreases. Thus, by Proposition 1, the proportion of patient consumers' premature withdrawal $\beta=\bar{\alpha}\lambda(1-x)/\left((1-\underline{\ell})x+\bar{\alpha}\lambda(1-x)\right)$, which decreases in x and increases in $\underline{\ell}$, increases as well. Moreover, if $\underline{\ell}=1$, then $\beta=1\geq\bar{\beta}$. Therefore, there must be a liquidity requirement $\tilde{\underline{\ell}}$, such that the proportion of patient consumers' premature withdrawal determined in the baseline model equilibrium reaches $\bar{\beta}$, and it satisfies that when $\underline{\ell}\leq\tilde{\underline{\ell}}$, the endogenously determined proportion of patient consumers' premature withdrawal β is smaller than $\bar{\beta}$; when $\underline{\ell}>\tilde{\underline{\ell}}$, the endogenously determined proportion of patient consumers' premature withdrawal β is strictly greater than $\bar{\beta}$.

A.10 *Proof of Proposition 5*

Part (i) is the same as Proposition 1. As for part (ii), first, the interest rates and the market return are determined directly by Lemma 2 with $\beta=\bar{\beta}$. Consumption satisfies $c_1^i=x\underline{\ell}/\lambda, c_1^p=0, c_2^p=\left(xR(1-\underline{\ell})+(1-x)R(\lambda\alpha+(1-\lambda))\right)/(1-\lambda)$. By substituting (11) into c_2^p, we get $c_2^p=\left(Rx(1-\underline{\ell})/(1-\bar{\beta})+(1-\lambda)R(1-x)\right)/(1-\lambda)$. Finally, substitute all variables into the first-order condition (12), we have

$$\lambda U'\left(c_1^i\right)\frac{\underline{\ell}}{\lambda+(1-\lambda)\bar{\beta}}\left(1-\frac{(1-\lambda)\bar{\beta}x}{\lambda(1-x)}\right)=(1-\lambda)U'\left(c_2^p\right)\left(R-\frac{R(1-\underline{\ell})}{(1-\lambda)\left(1-\bar{\beta}\right)}\right)$$

Note that $d_{01}\geq 1$ implies that $\underline{\ell}\geq\lambda+(1-\lambda)\bar{\beta}$, or $(1-\underline{\ell})/(1-\bar{\beta})<1-\lambda$. Thus an increase in x lowers c_2^p and increases $U'\left(c_2^p\right)$. On the other hand, the left-hand side clearly decreases in x. Thus, if there is a solution, it must be unique. Now, when $x=0$, the left-hand side is lower than the right-hand side; when $x=\lambda/(\lambda+(1-\lambda)\bar{\beta})$, $1-\left((1-\lambda)\bar{\beta}x/(\lambda(1-x))\right)=0$ and the left-hand side is larger than the right-hand side. Thus, there must be a unique x such that the equation holds.

A.11 *Proof of Proposition 6*

First, by the first-order condition

$$\begin{aligned}&\lambda U'\left(c_1^i\right)\frac{\underline{\ell}}{\lambda+(1-\lambda)\bar{\beta}}\left(1-\frac{(1-\lambda)\bar{\beta}x}{\lambda(1-x)}\right)=(1-\lambda)U'\left(c_2^p\right)\left(R-\frac{R(1-\underline{\ell})}{(1-\lambda)\left(1-\bar{\beta}\right)}\right),\\ \Rightarrow&\lambda U'\left(c_1^i\right)c_1^i\frac{\lambda}{x}\frac{1}{\lambda+(1-\lambda)\bar{\beta}}\left(1-\frac{(1-\lambda)\bar{\beta}x}{\lambda(1-x)}\right)=(1-\lambda)U'\left(c_2^p\right)R\frac{\underline{\ell}-\left(\lambda+(1-\lambda)\bar{\beta}\right)}{(1-\lambda)\left(1-\bar{\beta}\right)}\end{aligned}$$

where $c_1^i = x\underline{\ell}/\lambda$ and $c_2^p = \left(Rx\left(1-\underline{\ell}\right)/(1-\bar{\beta}) + (1-\lambda) R (1-x)\right)/(1-\lambda)$, the left-hand side decreases in $\underline{\ell}$ since the relative risk aversion is greater than 1, and the right-hand side increases in $\underline{\ell}$. Thus a higher $\underline{\ell}$ makes the left-hand side smaller than the right-hand side, which requires x to decrease to restore the equality.

Second, we rewrite the first-order condition in the form

$$\lambda U'\left(c_1^i\right) \frac{1}{\lambda + (1-\lambda)\bar{\beta}} \frac{\lambda (1-x)\underline{\ell} - (1-\lambda)\bar{\beta}x\underline{\ell}}{\lambda (1-x)} \frac{(1-\lambda)\left(1-\bar{\beta}\right)}{\underline{\ell} - \left(\lambda + (1-\lambda)\bar{\beta}\right)} = (1-\lambda) U'\left(c_2^p\right) R.$$

If $x\underline{\ell}$ does not change, then c_1^i does not change and c_2^p falls, resulting in unchanged $U'\left(c_1^i\right)$ and higher $U'\left(c_2^p\right)$. Thus, the wealth effect makes consumers prefer shadow bank securities.

In terms of the substitution effect, we have to analyze the monotonicity of $\Phi = \left(\lambda(1-x)\underline{\ell} - (1-\lambda)\bar{\beta}x\underline{\ell}\right) / \left((1-x)\left[\underline{\ell} - \left(\lambda + (1-\lambda)\bar{\beta}\right)\right]\right)$, given $dx = -(x/\underline{\ell})d\underline{\ell}$, we can derive the change of the term with respect to the change $d\underline{\ell}$ and dx:

$$\begin{aligned} d\Phi \propto & \left[(1-x)\left(\underline{\ell} - \lambda - (1-\lambda)\bar{\beta}\right)\right](\lambda d\underline{\ell}) \\ & - \left[\lambda\underline{\ell}(1-x) - (1-\lambda)\bar{\beta}x\underline{\ell}\right]\left(d\underline{\ell} + \left(\lambda + (1-\lambda)\bar{\beta}\right)dx\right) \\ = & \left\{-\left(\lambda + (1-\lambda)\bar{\beta}\right)\left[\lambda(1-x)^2 + (1-\lambda)\bar{\beta}x^2\right] + (1-\lambda)\bar{\beta}x\underline{\ell}\right\} d\underline{\ell} \end{aligned}.$$

Since the first term $-\left(\lambda + (1-\lambda)\bar{\beta}\right)\left[\lambda(1-x)^2 + (1-\lambda)\bar{\beta}x^2\right]$ is negative and the second term $(1-\lambda)\bar{\beta}x\underline{\ell}$ is positive, whether $d\Phi > 0$ depends on the equilibrium allocation. To derive a sufficient condition, we notice that $\lambda(1-x)^2 + (1-\lambda)\bar{\beta}x^2 \geq \left(\lambda(1-\lambda)\bar{\beta}\right) / \left(\lambda + (1-\lambda)\bar{\beta}\right)$. Thus, if $x\underline{\ell} < \lambda$, then $d\Phi < -\lambda(1-\lambda)\bar{\beta} + (1-\lambda)\bar{\beta}x\underline{\ell} < 0$. In other words, a simple sufficient condition for $(d\mathfrak{L})/(d\underline{\ell}) < 0$ is $\mathfrak{L} < \lambda$. Note that since $(d\mathfrak{L})/(d\underline{\ell}) < 0$ when $\mathfrak{L} < \lambda$, it implies that once $\mathfrak{L} < \lambda$, $(d\mathfrak{L})/(d\underline{\ell}) < 0$ will always be true for any higher $\underline{\ell}$.

When $\bar{\alpha} = 1$, we first characterize the baseline equilibrium to characterize the cutoff value $\tilde{\underline{\ell}}$. The first-order condition in the baseline model (7) in this case satisfies $\lambda U'\left(c_1^i\right)(d_{01} - R/d_{12}) = (1-\lambda) U'\left(c_2^p\right)(R - d_{02})$, which by the fact that $d_{01} = d_{02}/d_{12}$ implies that $d_{02} = R$ must hold. Then by Lemma 2, $d_{12} < R$ cannot hold as in this case $d_{02} = \underline{\ell}d_{12} + (1-\underline{\ell}) R < R$. Therefore, $d_{12} = R$ must hold and we get $d_{01} = 1$. Thus, by the budget constraint of the bank, we have $\lambda + (1-\lambda)\beta = \underline{\ell}$ if $\underline{\ell} > \lambda$. In other words, $\beta > 0$ if $\underline{\ell} > \lambda$. Thus, the cutoff liquidity requirement at which shadow banks emerge is $\hat{\underline{\ell}} = \lambda$, and the cutoff liquidity requirement at which β reaches the upper limit $\bar{\beta}$ is $\tilde{\underline{\ell}} = \lambda + (1-\lambda)\bar{\beta}$. The allocation for $\underline{\ell} < \tilde{\underline{\ell}}$ is pretty simple. When $\bar{\alpha} = 1$, we have $c_1^i = d_{01}x + R\bar{\alpha}(1-x)/d_{12} = 1$ and $c_2^p = R$. Then, due to the resources constraint $c_1^i = x\underline{\ell}/\lambda$ we get $x = \lambda/\underline{\ell}$.

Now, we want to know dx at $\underline{\ell} = \tilde{\underline{\ell}} = \lambda + (1-\lambda)\bar{\beta}$. First, $x = \lambda/\underline{\ell} = \lambda/(\lambda + (1-\lambda)\bar{\beta})$. Second, using the FOC

$$\lambda U'\left(c_1^i\right) \frac{\underline{\ell}}{\lambda + (1-\lambda)\bar{\beta}} \left(1 - \frac{(1-\lambda)\bar{\beta}x}{\lambda(1-x)}\right) = (1-\lambda) U'\left(c_2^p\right) \left(R - \frac{R\left(1-\underline{\ell}\right)}{(1-\lambda)\left(1-\bar{\beta}\right)}\right),$$

we can derive $(dx)/(d\underline{\ell})$ from

$$\lambda U'\left(c_1^i\right)\left[\frac{1}{\lambda+(1-\lambda)\bar{\beta}}\left(1-\frac{(1-\lambda)\bar{\beta}x}{\lambda(1-x)}\right)d\underline{\ell}-\frac{\underline{\ell}}{\lambda+(1-\lambda)\bar{\beta}}\frac{(1-\lambda)\bar{\beta}}{\lambda(1-x)^2}dx\right],$$
$$=(1-\lambda)U'\left(c_2^p\right)\frac{R}{(1-\lambda)\left(1-\bar{\beta}\right)}d\underline{\ell}$$

where $c_1^i=x\underline{\ell}/\lambda=1$ and $c_2^p=\left(Rx(1-\underline{\ell})/(1-\bar{\beta})+(1-\lambda)R(1-x)\right)/(1-\lambda)=R$. Note that we do not have to consider the wealth effect $(\partial c_1^i)/(\partial\underline{\ell}),(\partial c_1^i)/(\partial x)$ and $(\partial c_2^p)/(\partial\underline{\ell}),(\partial c_2^p)/(\partial x)$ because we can show that $1-(1-\lambda)\bar{\beta}x/[\lambda(1-x)]=0$ and $R-\left(R(1-\underline{\ell})/\left((1-\lambda)\left(1-\bar{\beta}\right)\right)\right)=0$ at $\underline{\ell}=\tilde{\underline{\ell}}$, which is a special case when $\bar{\alpha}=1$. Thus, we have

$$\frac{dx}{d\underline{\ell}}=-\frac{x}{\underline{\ell}}\frac{U'(R)R(1-\lambda)\bar{\beta}}{U'(1)\lambda\left(1-\bar{\beta}\right)}.$$

To make $(d(x\underline{\ell}))/(d\underline{\ell})>0$, we need

$$x+\underline{\ell}\frac{dx}{d\underline{\ell}}=x\left[1-\frac{U'(R)R(1-\lambda)\bar{\beta}}{U'(1)\lambda\left(1-\bar{\beta}\right)}\right]>0,$$

which requires $\bar{\beta}<\lambda U'(1)/(\lambda U'(1)+(1-\lambda)U'(R)R)$.

On the other hand, if $\bar{\beta}>\lambda U'(1)/(\lambda U'(1)+(1-\lambda)U'(R)R)$, then $(d(x\underline{\ell}))/(d\underline{\ell})<0$, which means $\mathfrak{L}=x\underline{\ell}$ will drop below λ. Then, $(d\mathfrak{L})/(d\underline{\ell})<0$ will be true for any $\underline{\ell}>\tilde{\underline{\ell}}$.

A.12 *Proof of Corollary 1*

For $\underline{\ell}\geq\tilde{\underline{\ell}}$, the welfare measured by the expected utility for a consumer is $\lambda U\left(c_1^i\right)+(1-\lambda)U\left(c_2^p\right)$, where $c_1^i=x\underline{\ell}/\lambda$ and $c_2^p=\left(\left(Rx(1-\underline{\ell})/(1-\bar{\beta})\right)+(1-\lambda)R(1-x)\right)/(1-\lambda)$. When $\underline{\ell}$ increases, the welfare will change by

$$U'\left(c_1^i\right)(xd\underline{\ell}+\underline{\ell}dx)+U'\left(c_2^p\right)R\left(\left(\frac{1-\underline{\ell}}{1-\bar{\beta}}-(1-\lambda)\right)dx-\frac{x}{1-\bar{\beta}}d\underline{\ell}\right).$$

Suppose $\bar{\alpha}=1$, then $c_1^i=1$ and $c_2^p=R$ for any $\underline{\ell}\leq\tilde{\underline{\ell}}=\lambda+(1-\lambda)\bar{\beta}$. Thus, at $\underline{\ell}=\tilde{\underline{\ell}}$, the welfare will change by

$$U'(1)x\left[1-\frac{U'(R)R(1-\lambda)\bar{\beta}}{U'(1)\lambda\left(1-\bar{\beta}\right)}\right]d\underline{\ell}-U'(R)R\frac{x}{1-\bar{\beta}}d\underline{\ell}.$$

Therefore, to improve the welfare, it requires

$$U'(1)\left[1-\frac{U'(R)R(1-\lambda)\bar{\beta}}{U'(1)\lambda\left(1-\bar{\beta}\right)}\right]-U'(R)R\frac{1}{1-\bar{\beta}}\geq 0,$$

which implies $\bar{\beta}\leq(\lambda(U'(1)-U'(R)R))/(\lambda U'(1)+(1-\lambda)U'(R)R)$. Thus, when $\bar{\beta}<(\lambda(U'(1)-U'(R)R))/(\lambda U'(1)+(1-\lambda)U'(R)R)$, the welfare increases in $\underline{\ell}$ at $\tilde{\underline{\ell}}$. Moreover, since for $\bar{\alpha}=1$, $c_1^i=1$ and $c_2^p=R$ for any $\underline{\ell}\leq\tilde{\underline{\ell}}$. The welfare at $\tilde{\underline{\ell}}+d\underline{\ell}$ is strictly better than that for any liquidity requirement $\underline{\ell} \leq \tilde{\underline{\ell}}$. Therefore, the optimal liquidity requirement must be higher than $\tilde{\underline{\ell}}$, and thus higher than $\hat{\underline{\ell}}$ since $\tilde{\underline{\ell}} > \hat{\underline{\ell}} = \lambda$. Note that since the relative risk aversion is greater than 1, $U'(1) > U'(R)R$ holds, and the upper limit $(\lambda(U'(1) - U'(R)R)) / (\lambda U'(1) + (1-\lambda)U'(R)R)$ is strictly positive. Thus, we can always find a proper $\bar{\beta}$ that satisfies this condition.

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