Ex-post approaches to prioritarianism and sufficientarianism*

Matthew D. Adler
Duke Law School
210 Science Drive
Durham, NC 27708
USA
adler@law.duke.edu

Walter Bossert
Department of Economics
University of Montreal
P.O. Box 6128, Station Downtown
Montreal QC H3C 3J7
Canada
walter.bossert.wb@gmail.com

Susumu Cato
Institute of Social Science
University of Tokyo
7-3-1, Hongo, Bunkyo-ku
Tokyo 113-0033
Japan
susumu.cato@gmail.com

Kohei Kamaga
Faculty of Economics
Sophia University
7-1, Kioi-cho, Chiyoda-ku
Tokyo 102-8554
Japan
kohei.kamaga@sophia.ac.jp

Abstract. Although sufficientarianism has been gaining interest as a theory of distributive justice in recent years, it has not been examined in the presence of risk. We propose an ex-post approach to sufficientarianism that has a strong link to ex-post prioritarianism. Both ex-post criteria are based on an axiom that we refer to as prospect independence of the unconcerned, a natural extension of the independence axiom known from the literature that focuses on situations with no risk. We characterize a class of ex-post prioritarian orderings as well as the corresponding class of ex-post sufficientarian orderings. In addition, we point out some important differences between these two ex-post criteria, and we examine how they fare when assessed in terms of specific ex-ante Paretian axioms.

Keywords: sufficientarianism, prioritarianism, risk, ex-post approach, prospect independence of the unconcerned, axiomatic characterization

JEL Classification: D31, D63, D71, D81

This version: January 30, 2025

* We thank Youngsub Chun, Conchita D'Ambrosio, Marc Fleurbaey, Biung-Ghi Ju, Marcus Pivato, Arunava Sen, Stéphane Zuber, and three reviewers for helpful comments. We also extend our gratitude to Rakesh Vohra, co-editor of this journal, for his valuable suggestions. The paper was presented at the 17th meeting of the Society for Social Choice and Welfare in Paris, at the 5th East Asia Game Theory Conference in Jeju, at Seoul National University, at Tohoku University, and at the Online Social Choice and Welfare Seminar. Financial support from KAKENHI through grants Nos. JP20H01446, JP20K01565, JP22K01387, JP22H05083, JP22H05086, and JP23K01316 is gratefully acknowledged.

1 Introduction

The concept of prioritarianism has been much discussed in the literature on philosophy and formal ethics over the last thirty years, beginning with an influential 1991 lecture by philosopher Derek Parfit (published as Parfit, 2000). Parfit describes prioritarianism as a non-egalitarian alternative to utilitarianism. Unlike utilitarianism (which focuses on total well-being), prioritarianism gives extra weight to the well-being of the worse off. Prioritarianism is usually represented as the sum of transformed utilities, where the common transformation that is applied to individual utilities is increasing and strictly concave. Prioritarianism satisfies the Pigou-Dalton transfer principle in utility (well-being) space (see Pigou, 1912, and Dalton, 1920), while utilitarianism does not. The axiomatic difference between prioritarianism and egalitarianism concerns an independence axiom. The prioritarian ranking of any two utility vectors is independent of the utility levels of unaffected individuals, while the egalitarian ranking is not. On prioritarianism, see generally Rabinowicz (2002), McCarthy (2008), Adler (2018, 2019), Adler and Holtug (2019), and Adler and Norheim (2022).

The ethical debates between utilitarianism, prioritarianism, and egalitarianism have in turn fueled interest in a fourth approach, sufficientarianism. The pioneering work on sufficientarianism was undertaken by Frankfurt (1987) and Crisp (2003). Its distinctive feature is the use of a threshold that represents sufficiency. The threshold is a utility level such that an individual is deemed to have enough if and only if his or her utility reaches this level. Roughly speaking, the primary concern of this ethical theory is to minimize insufficiency among individuals. Since the work of Frankfurt (1987) and Crisp (2003), a large philosophical literature on sufficientarianism has arisen (Brown, 2005; Benbaji, 2005, 2006; Casal, 2007; Huseby, 2010, 2020; Shields, 2012, 2016; Axelsen and Nielsen, 2015; Hirose, 2016; Segall, 2016; Herlitz, 2018; Nielsen, 2019; Hassoun, 2021; Knight, 2022; Timmer, 2022). Axiomatic foundations of sufficientarian theories have recently been provided in contributions such as Alcantud, Mariotti, and Veneziani (2022), Bossert, Cato, and Kamaga (2022, 2023), Chambers and Ye (2024), and Nakada and Sakamoto (2024).

The underlying motivation for sufficientarianism is well expressed by Frankfurt (1987) and Crisp (2003). Frankfurt (1987, pp. 21–22) writes that "[w]hat is important from the point of view of morality is not that everyone should have *the same* but that each should have *enough*." The notion of 'having enough' is identified with a level of well-being (utility) that is at or above a given threshold. Crisp (2003) notes that prioritarianism is willing to impose large costs on badly-off individuals for the sake of small benefits to better-off individuals, if the better-off individuals are sufficiently numerous. The sufficientarian threshold blocks this implication of prioritarianism: benefits to above-threshold individuals will never justify losses to below-threshold individuals.

The identification of the threshold is of crucial importance to sufficientarianism. Indeed, one common criticism of sufficientarianism is that determining a plausible threshold is difficult, leading to concerns about its arbitrariness (Timmer, 2022). We think of it as being equivalent to a poverty line; this seems to reflect a natural interpretation of the notion of sufficiency. The predominant choice in the literature is to assume that the threshold is externally given—that is, it does not depend on the distribution of the variable under consideration. This implicit convention seems to rest on a sound conceptual foundation, parallel to the use of poverty lines. If, instead of being given as an independently determined minimally acceptable standard of well-being, an endogenous method (such as a percentage of the median) is employed, counterintuitive conclusions may emerge: a country with a very high median income may end up being classified as poorer than a country with a very low median income. A fixed threshold avoids shortcomings of this nature. In addition, the threshold constitutes an important policy parameter that provides an intuitively appealing and clear focal point for policy debates, and sensitivity analyses regarding alternative parameter choices provide a transparent tool for their assessment.

There are, of course, alternatives to expressing a threshold in terms of utility. However, it seems difficult to think of them as being entirely divorced from concerns regarding people's levels of well-being. For instance, using income or consumption rather than utility seems to be nothing but an approximation based on the assumption that higher income or higher consumption levels are beneficial for the members of a society.

A social ordering of utility distributions based on sufficientarian principles is used to inform the decisions of policy makers. Among the utility distributions that correspond to feasible policy choices, a selection is made that is best according to this ordering.

Traditional sufficientarian approaches do not pay much attention to the presence of risk. However, most public-policy choices involve considerable risk as far as the outcomes that eventually materialize are concerned and, therefore, there appears to be a need to go beyond the riskless case. We propose to do so by utilizing a framework of social evaluation of state-contingent alternatives. This framework includes a fixed probability distribution over the set of states. Ex-post utility distributions that occur in each state are assumed to be variable, and a matrix composed of the utilities of all individuals in all states is called a prospect. We examine an ordering (that is, a complete and transitive binary relation) defined on the set of prospects. Fleurbaey (2010) provides a new ex-post welfare criterion relying on this framework; see also Fleurbaey and Zuber (2013). Related approaches built on this framework can be found in Blackorby, Bossert, and Donaldson (2002, 2005) and Blackorby, Donaldson, and Weymark (1999, 2008). Originally introduced by Arrow (1953; 1964) in the context of individual decision-making, Blackorby, Davidson, and Donaldson (1977) establish an axiomatic foundation of the expected-utility hypothesis with state-contingent alternatives.

The sufficientarian principles that we primarily examine in this paper are instances of what we refer to as *ex-post sufficientarianism*. These theories emphasize the depth of insufficiency from an ex-post viewpoint rather than focusing on the expected-utility level evaluated from an ex-ante perspective. Clearly, the expected insufficiency of ex-post utilities is significantly different from the insufficiency of individual expected utilities. If one considers the insufficiency of expected utilities, it does not matter if the utility is significantly below the threshold as long as the associated probability is very small. Under ex-post sufficientarianism, as long as some ex-post utilities fall below the threshold, it is always considered a significant problem.

Ex-post sufficientarianism is closely related to ex-post prioritarianism. According to *ex-post prioritarianism*, a prospect is better when the expected value of the sum of transformed ex-post utilities is higher. This thought is advocated by Rabinowicz (2002) and Adler and Sanchirico (2006); see also Adler (2012). To the best of our knowledge, there has been no axiomatic characterization of ex-post prioritarianism so far. In the context of Harsanyi's (1955) impartial observer theorem, Grant, Kajii, Polak, and Safra (2010) provide an axiomatic characterization of the evaluation of product lotteries composed of an outcome lottery and an identity lottery (not state-contingent alternatives) that takes the form of the weighted sum of concave transformations of an individual's expected utility. Aside from the difference in the analytical framework, the criterion they characterize corresponds to an ex-ante approach to prioritarianism, not an ex-post approach.

In prioritarian approaches, the difference between evaluating equality at the level of ex-ante expected utility versus ex-post utilities can lead to significant policy divergences. However, a key issue arises when comparing prioritarian and sufficientarian approaches to social risk evaluation. Neither ex-post nor ex-ante prioritarianism considers policies with a very small probability of severe insufficiency as a major concern. In contrast, ex-post sufficientarianism treats such cases as critically important. This fundamental difference highlights how incorporating risk into sufficientarian frameworks offers a novel perspective on social risk evaluation. By emphasizing the importance of avoiding any instance of insufficiency, regardless of its probability, ex-post sufficientarianism provides a unique approach to addressing societal risks and inequalities.

We employ a unified method to characterize ex-post prioritarianism and ex-post sufficientarianism. Our key axiom is what we call prospect independence of the unconcerned, the ex-post variant of well-established independence properties that are familiar from the literature on social evaluation without risk. Individuals who face the same risk in two prospects are called unconcerned, and the axiom requires that the social comparison of these two prospects is independent of unconcerned individuals.

Our first main result consists of a characterization of the class of ex-post prioritarian orderings by combining prospect independence of the unconcerned with strong Pareto for no risk, continuity, the Pigou-Dalton transfer principle for no risk, and the social expected-utility hypothesis. Strong Pareto for no risk, continuity, and the Pigou-Dalton transfer principle for no risk are standard axioms, which are commonly used when characterizing prioritarian orderings in a framework with no risk. The social expected-utility hypothesis requires the existence of a social von Neumann-Morgenstern function such that prospects are ranked in terms of the expected values of ex-post social welfare.

We then proceed to a characterization of ex-post sufficientarianism. In addition to prospect independence of the unconcerned, there is another key axiom for the characterization that is intended to capture the distinctive nature of sufficientarianism. Sufficientarian theories are primarily concerned with changes in utilities below the threshold but that does not mean that utilities above the threshold do not matter and, therefore, sufficientarian theories can very well be compatible with Paretian axioms. Sufficientarianism puts unequivocal priority on the utilities below the threshold and uses those above the threshold as a tie-breaking device. We formalize this attribute as an axiom that we label ex-post absolute priority. This axiom is a natural extension of the axiom of absolute priority proposed by Bossert, Cato, and Kamaga (2022, 2023).

Ex-post absolute priority is not compatible with the social expected-utility hypothesis, a fundamental property of ex-post prioritarianism, given strong Pareto for no risk. This incompatibility is caused by the lexicographic priority assigned to those below the threshold by the axiom of ex-post absolute priority. In other words, the existence of a sufficiency threshold does not allow us to apply the social expected-utility hypothesis across this threshold. However, if the social expected-utility hypothesis is restricted to utilities below the threshold and above the threshold separately, it is compatible with ex-post absolute priority.

We characterize the class of ex-post sufficientarian orderings by using prospect independence of the unconcerned, ex-post absolute priority, and the restricted social expected-utility hypothesis, in addition to strong Pareto for no risk, anonymity, and two restricted continuity axioms. It is well-known that most sufficientarian theories are not compatible with full continuity and, thus, only restricted versions such as continuity below the threshold and continuity above the threshold can be satisfied; see Roemer (2004) and Bossert, Cato, and Kamaga (2022, 2023) for detailed discussions.

A comparison with our characterization of ex-post prioritarianism highlights a fundamental trade-off within the sufficientarian approach. While ex-post prioritarian orderings satisfy the social expected-utility hypothesis and continuity, our ex-post sufficientarian orderings fail to do so. This implies that ex-post sufficientarian orderings may exhibit sudden large policy changes in response to small changes in the social environment. However, if we abandon the unconditional form of strong Pareto for no risk, it is possible to satisfy both the social expected-utility hypothesis and continuity in the sufficientarian approach. As we demonstrate later, requiring these two axioms and restricting strong Pareto for no risk to utilities below the threshold, we obtain a characterization of what we call *ex-post upper-limit sufficientarianism*, another formulation of ex-post sufficientarianism. A distinctive feature of ex-post upper-limit sufficientarian orderings is that they minimize the sum of transformed shortfalls from the threshold and ignore the well-being levels of those above the threshold. This trade-off between these properties is based on the extent to which ex-post sufficientarian approaches can diverge from ex-post prioritarianism, which satisfies all of the properties mentioned above.

To put our contribution into perspective, we note first that the issue of social evaluation with risk has been an important topic since the pioneering contribution of Harsanyi (1955) who provides a formal foundation of utilitarianism; see also Blackorby, Donaldson, and Weymark (1999). Diamond (1967) raises an ex-ante equality issue that applies to Harsanyi's arguments. Hammond (1983) and Broome (1991) provide early observations on ex-post criteria, which are substantially developed by Rabinowicz (2002), Adler and Sanchirico (2006), and Adler (2012) as ex-post prioritarianism.

Notably, both ex-post prioritarian and ex-post sufficientarian orderings violate the ex-ante weak Pareto principle, which requires that a prospect is better than another if each individual's expected utility in the former is higher than in the latter. Fleurbaey (2010) proposes a weakening of the ex-ante weak Pareto principle, weak Pareto for equal risk, according to which the ex-ante Pareto principle applies to prospects where all individuals face the same risk. Using this axiom, Fleurbaey (2010) provides a characterization of what is called the class of expected equally-distributed-equivalent (EDE) social orderings. We highlight some differences between expected EDE social orderings and ex-post social orderings. The axiomatic analysis of Fleurbaey (2010) is extended by Fleurbaey and Zuber (2013); see also Fleurbaey, Gajdos, and Zuber (2015) as well as Mongin and Pivato (2015). In particular, Fleurbaey and Zuber (2013) use an independence property similar to ours to provide a joint characterization of the utilitarian ordering and a specific multiplicative form.

Section 2 introduces the formal setting employed in this paper. Our basic axioms are defined and discussed in Section 3. Section 4 contains our results on ex-post prioritarian social evaluation, and Section 5 is devoted to ex-post sufficientarian criteria. Sections 6 and 7 examine the relationship with ex-ante Paretian requirements. Section 8 concludes. The independence of the axioms used in our main characterization results is established in the Appendix.

2 Setting

For $r \in \mathbb{N}$, we use $\mathbf{1}_r$ to denote the r-dimensional vector composed of r ones. We consider a framework of social evaluation of state-contingent alternatives. Let $S = \{1, \ldots, m\}$ be the finite set of $m \geq 2$ states and $(\pi^s)_{s \in S}$ be an exogenously given fixed probability distribution on the states $s \in S$. We assume that $\pi^s > 0$ for all $s \in S$ and $\sum_{s \in S} \pi^s = 1$. This assumption involves no loss of generality as long as there are at least two states with positive probabilities because any state with a probability of zero may be dropped. The finite set of individuals is given by $N = \{1, \ldots, n\}$, where $n \geq 3$ is assumed.

Let u_i^s denote the utility level of individual i in state s. Social alternatives to be evaluated are given by prospects. A prospect is an $n \times m$ matrix $u = (u_i^s)_{i \in N, s \in S}$, and the set of all prospects is $\mathcal{D} = \mathbb{R}^{n \times m}$. Given a prospect $u \in \mathcal{D}$, $u^s = (u_1^s, \ldots, u_i^s, \ldots, u_n^s) \in \mathbb{R}^n$ is the utility distribution realized in state $s \in S$ and, analogously, $u_i = (u_i^1, \ldots, u_i^s, \ldots, u_i^m) \in \mathbb{R}^m$ is the m-tuple composed of the utility levels experienced by individual $i \in N$ in each state. For all $u \in \mathcal{D}$ and for all $i \in N$, let $E(u_i)$ be the expected value $E(u_i) = \sum_{s \in S} \pi^s u_i^s$ of individual i's ex-post utilities.

A subdomain of $\mathcal{D}$ is also considered in our analysis. A prospect u such that $u^s = u^{s'}$ for all $s, s' \in S$ does not include any risk. Such a prospect is called *riskless*. Let $\mathcal{D}^c$ be the set of riskless prospects. For all $u \in \mathcal{D}$ and for all $s \in S$, let $[u^s] = (u^s, \ldots, u^s) \in \mathcal{D}^c$ denote a riskless prospect such that u^s occurs in each state $s' \in S$. We note that, for each riskless prospect $u \in \mathcal{D}^c$, there exists a prospect $u^s \in \mathbb{R}^n$ such that $[u^s] = u$. Furthermore, if $u \in \mathcal{D}^c$, then $E(u_i) = u_i^s$ for all $s \in S$ and for all $i \in N$.

The sufficiency threshold $\theta \in \mathbb{R}$ is an exogenously given threshold level of utility. A given threshold θ is common to all states $s \in S$ and applies to ex-post utility levels in all states. Its interpretation is that, for each state $s \in S$, those individuals whose ex-post utilities are on or above the threshold are deemed to have enough. For all $u \in \mathcal{D}$ and for all $s \in S$, we define the sets of those individuals whose utility is lower than and higher than the threshold θ in state s by

$$L(u^s) = \{i \in N \mid u_i^s < \theta\};$$

$$H(u^s) = \{i \in N \mid u_i^s > \theta\}.$$

A social ordering for prospects is a complete and transitive binary relation R on $\mathcal{D}$. We use the definition of completeness that encompasses reflexivity; that is, completeness requires that any two (not necessarily distinct) prospects u and v can be compared by R. For two prospects $u, v \in \mathcal{D}$, we write uRv instead of $(u, v) \in R$ to indicate that u is at least as good as v. The asymmetric and symmetric parts of R are denoted by P and I.

A function $g\colon \mathbb{R} \to \mathbb{R}$ is increasing if and only if, for all $x, y \in \mathbb{R}$, $x > y$ implies $g(x) > g(y)$. The social ordering R is *ex-post generalized utilitarian* if and only if there exists a continuous and increasing function $g\colon \mathbb{R} \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}$, uRv if and only if

$$\sum_{s \in S} \pi^s \sum_{i \in N} g(u_i^s) \geq \sum_{s \in S} \pi^s \sum_{i \in N} g(v_i^s).$$

An important subclass of these principles consists of the ex-post prioritarian orderings. A social ordering R is *ex-post prioritarian* if and only if it is an ex-post generalized utilitarian ordering associated with an increasing and strictly concave (and, thus, continuous) function $g\colon \mathbb{R} \to \mathbb{R}$. Ex-post prioritarianism is a very natural extension of prioritarianism to the evaluation of risky situations because a prioritarian evaluation applies to each state $s \in S$.

An alternative special case of ex-post generalized utilitarianism is *ex-post utilitarianism,* which is associated with a linear transformation g. Harsanyi (1955) characterizes ex-post utilitarianism as an ordering defined on the set of lotteries. Utilitarianism for prospects is characterized by Blackorby, Bossert, and Donaldson (2005) who employ an ex-ante approach to evaluating prospects.

Brown (2005), Hirose (2016), and Bossert, Cato, and Kamaga (2022, 2023) develop sufficientarian orderings in a framework that does not involve risk. Their sufficientarian principles are based on a lexicographic procedure. The primary criterion employed consists of the total gap between (transformed) utilities and the sufficiency threshold for those below the threshold. If these gaps are equal for two distributions, the corresponding gap for those above the threshold is consulted. These orderings are compatible with the Pareto principle. We extend their formulation of sufficientarianism to the evaluation of risky situations, which applies sufficientarianism (Bossert, Cato, and Kamaga, 2022, 2023) to each state.

A social ordering R is *ex-post sufficientarian* if and only if there exists a continuous and increasing function $g\colon \mathbb{R} \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}$, uRv if and only if

$$\sum_{s\in S} \pi^s \sum_{i\in L(u^s)} (g(u_i^s) - g(\theta)) > \sum_{s\in S} \pi^s \sum_{i\in L(v^s)} (g(v_i^s) - g(\theta))$$

or

$$\sum_{s\in S} \pi^s \sum_{i\in L(u^s)} (g(u_i^s) - g(\theta)) = \sum_{s\in S} \pi^s \sum_{i\in L(v^s)} (g(v_i^s) - g(\theta)) \text{ and}$$
$$\sum_{s\in S} \pi^s \sum_{i\in H(u^s)} (g(u_i^s) - g(\theta)) \geq \sum_{s\in S} \pi^s \sum_{i\in H(v^s)} (g(v_i^s) - g(\theta)).$$

We note that if there is no risk (that is, if $u, v \in \mathcal{D}^c$), any ex-post sufficientarian ordering coincides with the corresponding sufficientarian ordering proposed by Bossert, Cato, and Kamaga (2022).

There is an extensive literature on establishing rankings on sets of matrices; see, for instance, Marshall and Olkin (1979) and, more recently, Dahl, Guterman, and Shteyner (2018) for matrix majorization. To the best of our knowledge, the sufficientarian orderings that we discuss are not mentioned in this literature. This can be attributed to the special nature of our matrices. The influence of a threshold and the resulting lexicographic structure specific to the context of sufficientarianism are, as far as we are aware, absent in other areas of research. For instance, in the context of equality of opportunity, matrices have been studied extensively as well; see Fleurbaey and Maniquet (2011, Section 4) for a comprehensive survey.

On the other hand, the ex-post prioritarian orderings are related to matrix majorization. For any prospect $u, v \in \mathcal{D}$, u is said to be majorized by v if $u = Bv$ holds for some $n \times n$ doubly stochastic matrix B (that is, a matrix $B = [b_{ij}]_{1 \leq i,j \leq n} \in \mathbb{R}_+^{n \times n}$ such that $\sum_{j=1}^{n} b_{ij} = 1$ for all rows $i \in \{1, \ldots, n\}$ and $\sum_{i=1}^{n} b_{ij} = 1$ for all columns $j \in \{1, \ldots, n\}$); see Marshall and Olkin (1979). Note that $u = Bv$ means that u^s is equal or less spread out than v^s for all $s \in S$. Marshall, Olkin, and Arnold (2011) and Karlin and Rinott (1983) establish the following characterization result: for all $u, v \in \mathcal{D}$, u is majorized by v if and only if

$$\sum_{i \in N} f(u_i) \geq \sum_{i \in N} f(v_i)$$

for all continuous and concave functions $f \colon \mathbb{R}^m \to \mathbb{R}$. Note that, for any increasing and strictly concave function $g \colon \mathbb{R} \to \mathbb{R}$, the function $f \colon \mathbb{R}^m \to \mathbb{R}$ defined by $f(u_i) = \sum_{s \in S} \pi^s g(u_i^s)$ is continuous and concave. Thus, if a prospect u is majorized by a prospect v, then u is declared at least as good as v by all ex-post prioritarian orderings because

$$\sum_{s \in S} \pi^s \sum_{i \in N} g(u_i^s) = \sum_{i \in N} \sum_{s \in S} \pi^s g(u_i^s) \geq \sum_{i \in N} \sum_{s \in S} \pi^s g(v_i^s) = \sum_{s \in S} \pi^s \sum_{i \in N} g(v_i^s)$$

holds for all increasing and strictly concave functions $g \colon \mathbb{R} \to \mathbb{R}$.

3 Basic axioms

First, we introduce the strong Pareto principle defined for the evaluation of riskless prospects.

Strong Pareto for no risk: For all $u, v \in \mathcal{D}^c$, if, for all $s \in S$, $u_i^s \geq v_i^s$ for all $i \in N$ and $u_i^s > v_i^s$ for some $i \in N$, then uPv.

The continuity axiom is a robustness condition. It requires that small changes in a prospect do not lead to large changes in the social ordering.

Continuity: For all $u \in \mathcal{D}$, the sets $\{v \in \mathcal{D} \mid vRu\}$ and $\{v \in \mathcal{D} \mid uRv\}$ are closed in $\mathcal{D}$.

Anonymity is an uncontroversial and fundamental impartiality property. It requires that all individuals' ex-post utilities be treated equally.

Anonymity: For all $u, v \in \mathcal{D}$ and for all bijections $\rho \colon N \to N$, if $v_i = u_{\rho(i)}$ for all $i \in N$, then uIv.

Our next axiom requires that a social ordering satisfy the expected-utility hypothesis. More precisely, we assume that there exists a social von Neumann-Morgenstern function W such that prospects are ranked by the comparison of the expected values of ex-post social welfare.

Social expected-utility hypothesis: There exists a function $W \colon \mathbb{R}^n \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}$,

$$uRv \;\Leftrightarrow\; \sum_{s \in S} \pi^s W(u^s) \geq \sum_{s \in S} \pi^s W(v^s).$$

The social expected-utility hypothesis implies statewise dominance, a property that is familiar from the literature on decision theory.

Statewise dominance: For all $u, v \in \mathcal{D}$, if $[u^s]P[v^s]$ for all $s \in S$, then uPv.

Define, for all non-empty strict subsets M of N and for all $u \in \mathcal{D}$, $u_M = (u_i)_{i \in M}$ and $u_{N \setminus M} = (u_i)_{i \in N \setminus M}$. Using this notation, we now introduce an independence axiom that has considerable intuitive appeal. The condition requires that a social ordering be independent of the ex-post utilities of those who are unconcerned in every state.

Prospect independence of the unconcerned: For all $u, u', v, v' \in \mathcal{D}$ and for all non-empty $M \subsetneq N$,

$$(u_M, v_{N \setminus M}) R (u'_M, v_{N \setminus M}) \;\Leftrightarrow\; (u_M, v'_{N \setminus M}) R (u'_M, v'_{N \setminus M}).$$

Independence properties of this nature are ubiquitous not only in the literature on social evaluation but, more generally, in numerous approaches in economics and political philosophy. The underlying intuition is very transparent and allows for a powerful defense of the requirement. In the statement of the axiom, those in $N \setminus M$ are unconcerned—the choice of prospects to be compared does not affect their ex-post utilities in any state. It seems only natural that the resulting comparisons do not depend on these utility levels. That this separability property is highly plausible becomes apparent especially if a comprehensive notion of who is included in the overall population N is employed. It is usually assumed (at least implicitly) that a utility distribution (or, in our case, a prospect) represents a full history, from the remote past to the distant future, of the lifetime well-being of those who ever live. This includes individuals whose lives are long over, such as Cleopatra or Aristotle—and, more importantly, less prominent persons about whose lives very little (if anything) is known. If a comparison of two prospects were to depend on the ex-post utilities of the long dead, serious difficulties could not but emerge immediately. For instance, newly discovered evidence from archaeological excavations—such as proof that Cleopatra lived a miserable life due to illness and disability—certainly should not influence today's public-policy choices. Although we think that this independence property can be defended with some plausible arguments, it certainly is not entirely uncontroversial; see also our remarks to that effect in Section 6. This is, however, also the case for other independence properties if risky choices are being assessed.

Because we work within a fixed overall population in this paper, prospect independence of the unconcerned is the only primary separability condition considered here. There are several versions of separability in our fixed-population setting because risk is present, but this version is considered to be plausible for examining ex-post welfare criteria. Adler (2022, pp. 66–75) introduces prospect independence of the unconcerned under the label of policy separability, and provides a detailed normative defense. In a variable-population setting, additional versions that are just as plausible can be considered; see Blackorby, Bossert, and Donaldson (2005, Chapter 5) for an extensive discussion.

We note that Fleurbaey and Zuber (2013) use a similar separability property which they label independence of the utilities of the sure. This property restricts $v_{N\setminus M}$ and $v'_{N\setminus M}$ to those whose utility levels are constant across states. Thus, their condition (which is stated formally in the Appendix) requires that a social ranking is independent of the ex-post utilities of those who are unconcerned in every state and bear no risk. This property is obviously weaker than our condition. Notably, independence of the utilities of the sure is not enough to establish our main characterization results; see the Appendix for a counterexample.

Finally, we present a version of the Pigou-Dalton transfer principle (Pigou, 1912; Dalton, 1920), which formalizes an equity consideration. The variant that we employ merely requires that a progressive transfer is desirable for prospects with no risk.

Pigou-Dalton transfer principle for no risk: For all $u, v \in \mathcal{D}^c$, if, for all $s \in S$, there exist $i, j \in N$ and $\delta \in \mathbb{R}_{++}$ such that $v_i^s = u_i^s - \delta \geq u_j^s + \delta = v_j^s$ and $u_k^s = v_k^s$ for all $k \in N \setminus \{i, j\}$, then vPu.

4 Ex-post prioritarianism

The main result of this section characterizes the class of ex-post prioritarian orderings. We begin with the following lemma, which is restricted to prospects with no risk.

Lemma 1. If a social ordering R satisfies strong Pareto for no risk, continuity, anonymity, and prospect independence of the unconcerned, then there exists a continuous and increasing function $g\colon \mathbb{R} \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}^c$,

$$uRv \;\Leftrightarrow\; \sum_{i \in N} g(u_i^s) \geq \sum_{i \in N} g(v_i^s).$$

Proof. Let $u, v \in \mathcal{D}^c$. Since u and v are riskless, letting $s \in S$, we can define the ordering R^s on $\mathbb{R}^n$ such that, for all $u, v \in \mathcal{D}^c$,

$$u^s R^s v^s \;\Leftrightarrow\; uRv.$$

Note that strong Pareto for no risk, anonymity, continuity, and prospect independence of the unconcerned imply that R^s satisfies the corresponding properties. Since $n \geq 3$, there exists a continuous and increasing function $g\colon \mathbb{R} \to \mathbb{R}$ such that

$$u^s R^s v^s \;\Leftrightarrow\; \sum_{i \in N} g(u_i^s) \geq \sum_{i \in N} g(v_i^s);$$

see Debreu (1959, pp. 56–59) and Blackorby, Bossert, and Donaldson (2005, Theorem 4.7). Combining these equivalences, the lemma is proved. ■

That there is a formal link between independence properties and additively separable structures is certainly not new. In this context, the contributions of Debreu (1959) and Gorman (1968) are of particular importance; see also Blackorby, Primont, and Russell (1978, Section 4.4) for a detailed discussion and proof of Gorman's fundamental result on overlapping separable sets of variables. Of course, we do not claim any originality regarding the mathematical underpinnings of the result of Lemma 1; rather, it is the application of these earlier observations to our specific framework that constitutes the novelty of the lemma. This is parallel to the use of Gorman's theorem in numerous earlier contributions such as that of Blackorby and Donaldson (1984) in the context of population ethics.

To present the next lemma, we need some additional notation and definitions. Given a continuous and increasing function $g\colon \mathbb{R} \to \mathbb{R}$, let Y denote the set of attainable values of the sum of transformed utilities $\sum_{i \in N} g(u_i^s)$ in state $s \in S$. The set Y is a non-degenerate open interval because g is continuous and increasing and $\mathbb{R}$ is connected.

We now show that a generalized class of ex-post criteria is obtained if the social expected-utility hypothesis is added to the axioms that appear in Lemma 1; see Theorem 3 of Blackorby, Bossert, and Donaldson (1998) for a related result that is established for a social ordering defined on lotteries.

Lemma 2. If a social ordering R satisfies strong Pareto for no risk, continuity, anonymity, the social expected-utility hypothesis, and prospect independence of the unconcerned, then there exist continuous and increasing functions $g\colon \mathbb{R} \to \mathbb{R}$ and $\psi\colon Y \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}$,

$$uRv \;\Leftrightarrow\; \sum_{s \in S} \pi^s \psi \left(\sum_{i \in N} g(u_i^s) \right) \geq \sum_{s \in S} \pi^s \psi \left(\sum_{i \in N} g(v_i^s) \right).$$

Proof. By the social expected-utility hypothesis, there exists a function $W\colon \mathbb{R}^n \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}$,

$$uRv \;\Leftrightarrow\; \sum_{s \in S} \pi^s W(u^s) \geq \sum_{s \in S} \pi^s W(v^s). \tag{1}$$

Since $\mathcal{D}^c \subset \mathcal{D}$, it follows that, for all $u, v \in \mathcal{D}^c$,

$$uRv \;\Leftrightarrow\; W(u^s) \geq W(v^s). \tag{2}$$

Since R satisfies continuity, W can be chosen to be continuous. Lemma 1 implies that there exists a continuous and increasing function $g\colon \mathbb{R} \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}^c$,

$$uRv \;\Leftrightarrow\; \sum_{i \in N} g(u_i^s) \geq \sum_{i \in N} g(v_i^s). \tag{3}$$

From (2) and (3), we obtain that, for all $u^s = (u_1^s, \dots, u_n^s), v^s = (v_1^s, \dots, v_n^s) \in \mathbb{R}^n$,

$$W(u^s) \geq W(v^s) \Leftrightarrow \sum_{i \in N} g(u_i^s) \geq \sum_{i \in N} g(v_i^s).$$

Therefore, there exists an increasing function $\psi \colon Y \to \mathbb{R}$ such that, for all $u^s = (u_1^s, \dots, u_n^s) \in \mathbb{R}^n$,

$$W(u^s) = \psi\left(\sum_{i \in N} g(u_i^s)\right).$$

Since W is continuous, ψ can be chosen to be continuous. By (1), the lemma is proved. ■

In the following theorem, we provide a characterization of ex-post generalized utilitarianism using the axioms of Lemma 2. As its proof shows, the function ψ that appears in the statement of Lemma 2 must be affine.

Theorem 1. A social ordering R satisfies strong Pareto for no risk, continuity, anonymity, the social expected-utility hypothesis, and prospect independence of the unconcerned if and only if R is an ex-post generalized utilitarian ordering.

Proof. It is straightforward to prove the 'if' part of the theorem statement. To prove the 'only if' part, observe first that Lemma 2 implies the existence of continuous and increasing functions $g \colon \mathbb{R} \to \mathbb{R}$ and $\psi \colon Y \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}$,

$$uRv \Leftrightarrow \sum_{s \in S} \pi^s \psi\left(\sum_{i \in N} g(u_i^s)\right) \geq \sum_{s \in S} \pi^s \psi\left(\sum_{i \in N} g(v_i^s)\right). \tag{4}$$

To show that ψ is affine, let $(\gamma^1, \gamma^2) \in Y^2$. Since ψ is continuous and increasing on Y, there exists $(\tilde{\gamma}^1, \tilde{\gamma}^2) \in Y^2$ with $\gamma^1 > \tilde{\gamma}^1$ and $\gamma^2 < \tilde{\gamma}^2$ such that

$$\sum_{s=1}^{2} \pi^s \psi(\gamma^s) = \sum_{s=1}^{2} \pi^s \psi(\tilde{\gamma}^s). \tag{5}$$

Step 1. Assume first that n is even. We show that, for any $a \in (0,1)$,

$$\sum_{s=1}^{2} \pi^s \psi(\gamma^s) = \sum_{s=1}^{2} \pi^s \psi(a\gamma^s + (1-a)\tilde{\gamma}^s). \tag{6}$$

Let $(\bar{\gamma}^1, \bar{\gamma}^2)$ denote the midpoint of (γ^1, γ^2) and $(\tilde{\gamma}^1, \tilde{\gamma}^2)$ in Y^2. Formally, for each $s = 1, 2$,

$$\bar{\gamma}^s = \frac{\gamma^s + \tilde{\gamma}^s}{2}.$$

We begin by showing that

$$\sum_{s=1}^{2} \pi^s \psi(\bar{\gamma}^s) = \sum_{s=1}^{2} \pi^s \psi(\gamma^s). \tag{7}$$

Since $(\bar{\gamma}^1, \bar{\gamma}^2) \in Y^2$, there exist $u, v \in \mathcal{D}$ such that, for each $s = 1, 2$,

$$\begin{aligned} &u_i^s = g^{-1}(\gamma^s/n) \;\text{ and }\; v_i^s = g^{-1}(\tilde{\gamma}^s/n) \text{ for all } i \in \{1, \ldots, n/2\}, \\ &u_j^s = v_j^s = g^{-1}(\gamma^s/n) \text{ for all } j \in \{n/2+1, \ldots, n\}, \end{aligned}$$

and $u_i^s = v_i^s$ for all $s \in S \setminus \{1,2\}$ and for all $i \in N$. Furthermore, there exist $\hat{u}, \hat{v} \in \mathcal{D}$ such that, for each $s = 1, 2$,

$$\begin{aligned} &u_i^s = \hat{u}_i^s \text{ and } v_i^s = \hat{v}_i^s \text{ for all } i \in \{1, \ldots, n/2\}, \\ &\hat{u}_j^s = \hat{v}_j^s = g^{-1}(\tilde{\gamma}^s/n) \text{ for all } j \in \{n/2+1, \ldots, n\}, \end{aligned}$$

and $\hat{u}_i^s = \hat{v}_i^s = u_i^s$ for all $s \in S \setminus \{1,2\}$ and for all $i \in N$. Note that, for each $s = 1, 2$,

$$\sum_{i \in N} g(u_i^s) = n \cdot \frac{\gamma^s}{n} = \gamma^s, \;\; \sum_{i \in N} g(v_i^s) = \sum_{i \in N} g(\hat{u}_i^s) = \bar{\gamma}^s,$$

and

$$\sum_{i \in N} g(\hat{v}_i^s) = n \cdot \frac{\tilde{\gamma}^s}{n} = \tilde{\gamma}^s.$$

Since R satisfies prospect independence of the unconcerned, we obtain

$$uRv \;\Leftrightarrow\; \hat{u}R\hat{v} \;\text{ and }\; vRu \;\Leftrightarrow\; \hat{v}R\hat{u}.$$

Thus, if uPv holds, then $\hat{u}P\hat{v}$ follows and we obtain by Lemma 2 that

$$\sum_{s=1}^{2} \pi^s \psi(\gamma^s) > \sum_{s=1}^{2} \pi^s \psi(\bar{\gamma}^s) \;\text{ and }\; \sum_{s=1}^{2} \pi^s \psi(\bar{\gamma}^s) > \sum_{s=1}^{2} \pi^s \psi(\tilde{\gamma}^s),$$

and we obtain a contradiction to (5). Similarly, if vPu holds, it follows that

$$\sum_{s=1}^{2} \pi^s \psi(\gamma^s) < \sum_{s=1}^{2} \pi^s \psi(\bar{\gamma}^s) < \sum_{s=1}^{2} \pi^s \psi(\tilde{\gamma}^s),$$

a contradiction. Hence, uIv must hold, and $\hat{u}I\hat{v}$ follows as well. Thus, by (4), we obtain (7). Since ψ is continuous, applying the above argument repeatedly, we obtain that (6) holds for any $a \in (0,1)$.

Step 2. Now suppose that n is odd. We show that (6) holds for any $a \in (0,1)$. For all $t \in \mathbb{N} \setminus \{1\}$ and for all $\ell \in \{1, \ldots, t-1\}$, define $\bar{\gamma}^s(\ell, t) \in \mathbb{R}$ by, for each $s = 1, 2$,

$$\bar{\gamma}^s(\ell, t) = \frac{\ell}{t}\tilde{\gamma}^s + \frac{t-\ell}{t}\gamma^s.$$

For $t \in \mathbb{N}$ and for all $\ell \in \{2, \ldots, t-1\}$, we obtain

$$\lim_{a \to \infty} n \cdot g(a) > \gamma^1 > \bar{\gamma}^1(\ell - 1, t) > \bar{\gamma}^1(\ell, t)$$

and

$$\lim_{a \to -\infty} n \cdot g(a) < \gamma^2 < \bar{\gamma}^2(\ell - 1, t) < \bar{\gamma}^2(\ell, t).$$

Thus, there exists $t^1 \in \mathbb{N} \setminus \{1\}$ such that, for all $t \geq t^1$, there exists $(u_1^1, \ldots, u_n^1) \in \mathbb{R}^n$ such that

$$\begin{aligned} g(u_i^1) &= g(u_j^1) > \frac{\gamma^1}{n} \text{ for all } i, j \in \{1, \ldots, n-1\}, \\ g(u_n^1) &= \frac{\bar{\gamma}^1(1,t)}{n} < \frac{\gamma^1}{n} \text{ and } \sum_{i \in N} g(u_i^1) = \gamma^1. \end{aligned}$$

Moreover, there exists $t^2 \in \mathbb{N} \setminus \{1\}$ such that, for all $t \geq t^2$, there exists $(u_1^2, \ldots, u_n^2) \in \mathbb{R}^n$ such that

$$\begin{aligned} g(u_i^2) &= g(u_j^2) < \frac{\gamma^2}{n} \text{ for all } i \in \{1, \ldots, n-1\}, \\ g(u_n^2) &= \frac{\bar{\gamma}^2(1,t)}{n} > \frac{\gamma^2}{n} \text{ and } \sum_{i \in N} g(u_i^2) = \gamma^2. \end{aligned}$$

We now define $t^* = 2 \cdot \max\{t^1, t^2\}$. Then, there exist $u, v \in \mathcal{D}$ such that, for each $s = 1, 2$,

$$\begin{aligned}
&u_i^s = u_j^s \text{ for all } i, j \in \{1, \dots, n-1\}, \\
&g(u_n^s) = \frac{\bar{\gamma}^s(2, t^*)}{n} \quad \text{and} \quad \sum_{i \in N} g(u_i^s) = \gamma^s, \\
&g(v_i^s) = \frac{\bar{\gamma}^s(2, t^*)}{n} \text{ for all } i \in \{1, \dots, (n-1)/2\} \cup \{n\}, \\
&v_j^s = u_j^s \text{ for all } j \in \{(n-1)/2 + 1, \dots, n-1\},
\end{aligned}$$

and $u_i^s = v_i^s$ for all $s \in S \setminus \{1, 2\}$ and for all $i \in N$. Furthermore, there exist $\hat{u}, \hat{v} \in \mathcal{D}$ such that, for each $s = 1, 2$,

$$\begin{aligned}
&\hat{u}_i^s = u_i^s \text{ and } \hat{v}_i^s = v_i^s \text{ for all } i \in \{1, \dots, (n-1)/2\} \cup \{n\}, \\
&g(\hat{u}_j^s) = g(\hat{v}_j^s) = \frac{\bar{\gamma}^s(2, t^*)}{n} \text{ for all } j \in \{(n-1)/2 + 1, \dots, n-1\},
\end{aligned}$$

and $\hat{u}_i^s = \hat{v}_i^s = u_i^s$ for all $s \in S \setminus \{1, 2\}$ and for all $i \in N$. Note that, for each $s = 1, 2$,

$$\begin{aligned}
\sum_{i \in N} g(v_i^s) &= \frac{n+1}{2} \cdot \frac{\bar{\gamma}^s(2, t^*)}{n} + \frac{n-1}{2} g(u_1^s) \\
&= \frac{n+1}{2} \cdot \frac{\bar{\gamma}^s(2, t^*)}{n} + \frac{n-1}{2} \cdot \frac{1}{n-1} \cdot \left(\gamma^s - \frac{\bar{\gamma}^s(2, t^*)}{n}\right) \\
&= \frac{\bar{\gamma}^s(2, t^*) + \gamma^s}{2} \\
&= \bar{\gamma}^s(1, t^*), \\
\sum_{i \in N} g(\hat{u}_i^s) &= \frac{n-1}{2} \cdot \frac{1}{n-1} \cdot \left(\gamma^s - \frac{\bar{\gamma}^s(2, t^*)}{n}\right) + \frac{n+1}{2} \cdot \frac{\bar{\gamma}^s(2, t^*)}{n} \\
&= \bar{\gamma}^s(1, t^*),
\end{aligned}$$

and

$$\sum_{i \in N} g(\hat{v}_i^s) = \bar{\gamma}^s(2, t^*).$$

Since R satisfies prospect independence of the unconcerned, applying the argument employed in Step 1, we obtain the following three cases.

$$\text{(a)} \sum_{s=1}^{2} \pi^s \psi(\gamma^s) > \sum_{s=1}^{2} \pi^s \psi(\bar{\gamma}^s(1, t^*)) > \sum_{s=1}^{2} \pi^s \psi(\bar{\gamma}^s(2, t^*)),$$
$$\text{(b)} \sum_{s=1}^{2} \pi^s \psi(\gamma^s) < \sum_{s=1}^{2} \pi^s \psi(\bar{\gamma}^s(1, t^*)) < \sum_{s=1}^{2} \pi^s \psi(\bar{\gamma}^s(2, t^*)),$$
$$\text{(c)} \sum_{s=1}^{2} \pi^s \psi(\gamma^s) = \sum_{s=1}^{2} \pi^s \psi(\bar{\gamma}^s(1, t^*)) = \sum_{s=1}^{2} \pi^s \psi(\bar{\gamma}^s(2, t^*)).$$

We show by contradiction that case (c) holds. First, suppose that case (a) holds. Then, we can find $u, v \in \mathcal{D}$ such that, for each $s = 1, 2$,

$$\begin{aligned}
&u_i^s = u_j^s \text{ for all } i, j \in \{1, \ldots, n-1\}, \\
&g(u_n^s) = \frac{\bar{\gamma}^s(3, t^*)}{n} \text{ and } \sum_{i \in N} g(u_i^s) = \bar{\gamma}^s(1, t^*), \\
&g(v_i^s) = \frac{\bar{\gamma}^s(3, t^*)}{n} \text{ for all } i \in \{1, \ldots, (n-1)/2\} \cup \{n\}, \\
&v_j^s = u_j^s \text{ for all } j \in \{(n-1)/2 + 1, \ldots, n-1\},
\end{aligned}$$

and $u_i^s = v_i^s$ for all $s \in S \setminus \{1, 2\}$ and for all $i \in N$. Furthermore, there exist $\hat{u}, \hat{v} \in \mathcal{D}$ such that, for each $s = 1, 2$,

$$\begin{aligned}
&\hat{u}_i^s = u_i^s \text{ and } \hat{v}_i^s = v_i^s \text{ for all } i \in \{1, \ldots, (n-1)/2\} \cup \{n\}, \\
&g(\hat{u}_j^s) = g(\hat{v}_j^s) = \frac{\bar{\gamma}^s(3, t^*)}{n} \text{ for all } j \in \{(n-1)/2 + 1, \ldots, n-1\},
\end{aligned}$$

and $\hat{u}_i^s = \hat{v}_i^s = u_i^s$ for all $s \in S \setminus \{1, 2\}$ and for all $i \in N$. Note that, for each $s = 1, 2$,

$$\sum_{i \in N} g(v_i^s) = \bar{\gamma}^s(2, t^*) = \sum_{i \in N} g(\hat{u}_i^s) \text{ and } \sum_{i \in N} g(\hat{v}_i^s) = \bar{\gamma}^s(3, t^*).$$

Thus, it follows from (4) and prospect independence of the unconcerned that the inequality

$$\sum_{s=1}^{2} \pi^s \psi(\bar{\gamma}^s(1, t^*)) > \sum_{s=1}^{2} \pi^s \psi(\bar{\gamma}^s(2, t^*))$$

implies the inequality

$$\sum_{s=1}^{2} \pi^s \psi(\bar{\gamma}^s(2, t^*)) > \sum_{s=1}^{2} \pi^s \psi(\bar{\gamma}^s(3, t^*)).$$

Applying this argument repeatedly, we obtain that

$$\sum_{s=1}^{2} \pi^s \psi(\gamma^s) > \sum_{s=1}^{2} \pi^s \psi(\tilde{\gamma}^s).$$

However, this is a contradiction to (5). Similarly, if case (b) holds, we obtain a contradiction. Therefore, case (c) must hold.

Applying the argument that we used to show a contradiction in case (a), we obtain that, for each $\ell \in \{2, \ldots, t^* - 1\}$,

$$\sum_{s=1}^{2} \pi^s \psi(\bar{\gamma}^s(\ell - 1, t^*)) = \sum_{s=1}^{2} \pi^s \psi(\bar{\gamma}^s(\ell, t^*)).$$

Since ψ is continuous, it follows from the same argument as in Step 1 that (6) holds for any $a \in (0, 1)$.

Step 3. Applying the argument used to derive the implication of case (c) in Step 2, we can extend the result that (6) holds for any $a \in (0, 1)$ to any parameter $a \in (-\infty, 0) \cup (1, \infty)$. Therefore, we can conclude that, for any $(\bar{\gamma}^1, \bar{\gamma}^2) \in Y^2$ that lies on the straight line passing through (γ^1, γ^2) and $(\tilde{\gamma}^1, \tilde{\gamma}^2)$,

$$\sum_{s=1}^{2} \pi^s \psi(\gamma^s) = \sum_{s=1}^{2} \pi^s \psi(\bar{\gamma}^s).$$

This implies that there exists $(\alpha^1, \alpha^2) \in \mathbb{R}^2_{++}$ such that, for all $(\gamma^1, \gamma^2) \in Y^2$,

$$\sum_{s=1}^{2} \pi^s \psi(\gamma^s) = \sum_{s=1}^{2} \alpha^s \gamma^s.$$

Thus, given a fixed $\gamma^2 = \bar{\gamma}^2$, it follows that, for all $\gamma^1 \in Y$,

$$\psi(\gamma^1) = \frac{\alpha^1}{\pi^1}\gamma^1 + \frac{\alpha^2\bar{\gamma}^2 - \pi^2\psi(\bar{\gamma}^2)}{\pi^1}.$$

Therefore, ψ is affine. Consequently, we can assume that ψ in (4) is given by $\psi(a) = a$ for all $a \in Y$. ■

If we require the Pigou-Dalton transfer principle for no risk in addition to the axioms of Theorem 1, the utility transformation g that ex-post generalized utilitarianism employs must be strictly midpoint-concave (that is, $g((x +$

$y)/2) > [g(x) + g(y)]/2$ for all $x, y \in \mathbb{R}$ with $x \neq y$). Since any continuous, strictly midpoint-concave function is strictly concave, only the class of ex-post prioritarian orderings is permissible. To present a characterization of ex-post prioritarianism, however, we no longer need to assume all of the axioms of Theorem 1. Specifically, anonymity becomes redundant. To see this, we first state a variant of Lemma 1 of Fleurbaey and Zuber (2013), which shows that if the Pigou-Dalton transfer principle for no risk is added to the other three axioms of Theorem 1, anonymity is implied. The axioms of Fleurbaey and Zuber's (2013) lemma are slightly different from ours; to be precise, the social rationality and independence axioms that they employ are weaker than ours but their transfer axiom is stronger than ours. However, as they state in their discussion (Fleurbaey and Zuber, 2013, p. 685), the Pigou–Dalton transfer principle for no risk suffices to prove their lemma. Thus, we state the following lemma without a proof.

Lemma 3. If a social ordering R satisfies strong Pareto for no risk, continuity, the social expected-utility hypothesis, prospect independence of the unconcerned, and the Pigou-Dalton transfer principle for no risk, then R satisfies anonymity.

The following theorem axiomatizes ex-post prioritarianism.

Theorem 2. A social ordering R satisfies strong Pareto for no risk, continuity, the social expected-utility hypothesis, prospect independence of the unconcerned, and the Pigou-Dalton transfer principle for no risk if and only if R is an ex-post prioritarian ordering.

Proof. 'If.' Suppose that R is an ex-post prioritarian ordering. This implies that R is ex-post generalized utilitarian and, by Theorem 1, R satisfies all axioms other than the Pigou-Dalton transfer principle for no risk. It is easy to show that the Pigou-Dalton transfer principle for no risk is also satisfied; see, for example, Table 4.2 of Blackorby, Bossert, and Donaldson (2005, p. 82).

'Only if.' Assume that a social ordering R satisfies strong Pareto for no risk, continuity, the social expected-utility hypothesis, prospect independence of the unconcerned, and the Pigou-Dalton transfer principle for no risk. By Lemma 3, R satisfies anonymity. Theorem 1 implies that R is ex-post generalized utilitarian. As mentioned above, g must be strictly concave because of the Pigou-Dalton transfer principle for no risk. Thus, R must be ex-post prioritarian. ■

In the above theorem, the social expected-utility hypothesis cannot be weakened to the following alternative social rationality axiom.

Sensitivity to risk: For all $u, v \in \mathcal{D}$, if there exist $s, s' \in S$ with $\pi_s > \pi_{s'}$ such that

$$[u^s]P[v^s], \; u^{s'} = v^s, \; v^{s'} = u^s, \text{ and } u^t = v^t \text{ for all } t \in S \setminus \{s, s'\},$$

then uPv.

A social ordering R is *ex-post prioritarian with probability weighing* if and only if there exist an increasing and strictly concave function $g\colon \mathbb{R} \to \mathbb{R}$ and an increasing and continuous function $\phi\colon (0,1) \to \mathbb{R}_{++}$ such that, for all $u, v \in \mathcal{D}$,

$$uRv \;\Leftrightarrow\; \sum_{s \in S} \phi(\pi^s) \sum_{i \in N} g(u_i^s) \geq \sum_{s \in S} \phi(\pi^s) \sum_{i \in N} g(v_i^s).$$

This ordering satisfies strong Pareto for no risk, continuity, anonymity, statewise dominance, prospect independence of the unconcerned, the Pigou-Dalton transfer principle for no risk, and sensitivity to risk. Unless the function ϕ is a homogeneous linear function, these orderings do not satisfy the social expected-utility hypothesis.

5 Ex-post sufficientarianism

According to sufficientarianism, absolute priority is assigned to utility levels below the sufficiency threshold θ. This suggests that sufficientarian theories primarily care about changes below the threshold. Thus, as an auxiliary step, it is helpful to introduce censored prospects at the level of the threshold θ. For each $u \in \mathcal{D}$, let

$$u_L = (\min\{u_i^s, \theta\})_{i \in N, s \in S}$$

and

$$u_H = (\max\{u_i^s, \theta\})_{i \in N, s \in S}.$$

Typically, when considering two prospects u and v, sufficientarian orderings first compare u_L and v_L. If required, a comparison between u_H and v_H is employed as a tie-breaking criterion. The idea that absolute priority should be given to those below the threshold constitutes the core of sufficientarianism; see, for example, Crisp (2003), Brown (2005), and Casal (2007). The following axiom is a natural extension of the fundamental property of sufficientarianism to the evaluation of prospects.

Ex-post absolute priority: For all $u, v \in \mathcal{D}$,

$$u_L P v_L \;\Rightarrow\; u P v$$

and

$$u_L I v_L \;\Rightarrow\; [u R v \Leftrightarrow u_H R v_H].$$

Ex-post absolute priority puts limitations on the permissible form of social rationality. Specifically, from its definition, the lexicographic treatment embodied by ex-post absolute priority applies to the subdomain $\mathcal{D}^c$ of riskless prospects. On the other hand, on the subdomain $\mathcal{D}^c$, the social expected-utility hypothesis requires that a social ordering R be represented by a real-valued function $W \colon \mathbb{R}^n \to \mathbb{R}$. As is well-known, a lexicographic ordering cannot be represented by a real-valued function on a continuum (Debreu, 1954). Indeed, as the following theorem shows, once we require a social ordering to satisfy strong Pareto for no risk, ex-post absolute priority forces us to give up the realization of the social expected-utility hypothesis. Thus, when combined with strong Pareto for no risk, ex-post absolute priority implies that we must abandon the conventional use of social welfare measures as real-valued functions, which are typically used for public policy evaluations.

Theorem 3. There exists no social ordering R that satisfies strong Pareto for no risk, the social expected-utility hypothesis, and ex-post absolute priority.

Proof. Suppose that the social ordering R satisfies strong Pareto for no risk, the social expected-utility hypothesis, and ex-post absolute priority. For all $a \in [\theta - 1, \theta]$, let $u(a)$ and $v(a)$ be the riskless prospects in $\mathcal{D}^c$ defined by letting, for all $s \in S$,

$$u(a)^s_1 = a, \; u(a)^s_2 = \cdots = u(a)^s_n = \theta + 1$$

and

$$v(a)^s_1 = a, \; v(a)^s_2 = \cdots = v(a)^s_n = \theta + 2.$$

Then, for all $a, b \in [\theta - 1, \theta]$ with $a > b$, the three axioms together imply that $W(v(a)^s) > W(u(a)^s) > W(v(b)^s) > W(u(b)^s)$. Therefore, the non-degenerate intervals

$$I(a) = [W(u(a)^s), W(v(a)^s)] \;\text{ and }\; I(b) = [W(u(b)^s), W(v(b)^s)]$$

are mutually disjoint. Since the interval $[\theta - 1, \theta]$ is uncountable, each of uncountably many intervals $I(a)$ contains a rational number. This is a contradiction because the set of rational numbers is countable. ■

The incompatibility between ex-post absolute priority and the social expected-utility hypothesis, given strong Pareto for no risk, is due to the impossibility of a numerical representation of a lexicographic ordering on a continuum. In this sense, ex-post absolute priority does not unduly impose constraints on the permissible form of social rationality of risk evaluation. Indeed, ex-post absolute priority is compatible with statewise dominance. This can be verified by the modification of an ex-post sufficientarian ordering using probability weighing in analogy to ex-post prioritarianism with probability weighing we presented in the last section; see the ordering R^{10} in the Appendix for its formal definition.

In view of Theorem 3, we need to weaken the social expected-utility hypothesis if we are to respect the fundamental property of sufficientarianism. When absolute priority is given to ex-post utilities below the sufficiency threshold, it seems reasonable to postulate social rationality only for the cases where every individual's ex-post utilities are either equal to or below, or equal to or above, the sufficiency threshold. To do so, we define two subdomains of $\mathcal{D}$. Let

$$\mathcal{D}_L = \{u \in \mathcal{D} \mid u_i^s \leq \theta \text{ for all } i \in N \text{ and for all } s \in S\}$$

and

$$\mathcal{D}_H = \{u \in \mathcal{D} \mid u_i^s \geq \theta \text{ for all } i \in N \text{ and for all } s \in S\}.$$

Note that, for any $u \in \mathcal{D}$, $u_L \in \mathcal{D}_L$ and $u_H \in \mathcal{D}_H$.

The following axiom restricts the social expected-utility hypothesis to censored prospects.

Restricted social expected-utility hypothesis: There exists a function $W\colon \mathbb{R}^n \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}_L$,

$$uRv \;\Leftrightarrow\; \sum_{s \in S} \pi^s W(u^s) \geq \sum_{s \in S} \pi^s W(v^s)$$

and, for all $u, v \in \mathcal{D}_H$,

$$uRv \;\Leftrightarrow\; \sum_{s \in S} \pi^s W(u^s) \geq \sum_{s \in S} \pi^s W(v^s).$$

The restricted social expected-utility hypothesis is compatible with ex-post absolute priority. Moreover, the conjunction of ex-post absolute priority and the restricted social expected-utility hypothesis can equivalently be represented by the following single concise requirement of social rationality in which the sufficientarian equity consideration is embedded.

Expected sufficientarian hypothesis: There exists a function $W\colon \mathbb{R}^n \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}$,

$$\sum_{s\in S} \pi^s W(u_L^s) > \sum_{s\in S} \pi^s W(v_L^s) \;\Rightarrow\; uPv$$

and

$$\sum_{s\in S} \pi^s W(u_L^s) = \sum_{s\in S} \pi^s W(v_L^s) \;\Rightarrow\; \left[uRv \;\Leftrightarrow\; \sum_{s\in S} \pi^s W(u_H^s) \geq \sum_{s\in S} \pi^s W(v_H^s) \right].$$

Lemma 4. A social ordering R satisfies the expected sufficientarian hypothesis if and only if R satisfies ex-post absolute priority and the restricted social expected-utility hypothesis.

Proof. 'Only if.' Assume that R satisfies the expected sufficientarian hypothesis. First, we show that the restricted social expected-utility hypothesis is satisfied. Let W be a function that satisfies the requisite property stated in the expected sufficientarian hypothesis. Let $u, v \in \mathcal{D}_L$. Note that $u = u_L$ and $v = v_L$. Thus, the expected sufficientarian hypothesis implies that

$$\sum_{s\in S} \pi^s W(u^s) > \sum_{s\in S} \pi^s W(v^s) \;\Rightarrow\; uPv.$$

Because u_H and v_H are empty in this case, the equality $\sum_{s\in S} \pi^s W(u^s) = \sum_{s\in S} \pi^s W(v^s)$ implies uIv. Therefore,

$$\sum_{s\in S} \pi^s W(u^s) \geq \sum_{s\in S} \pi^s W(v^s) \;\Leftrightarrow\; uRv.$$

The proof of the second part of the restricted social expected-utility hypothesis is analogous.

Next, we show that ex-post absolute priority is satisfied. Let $u, v \in \mathcal{D}$. As shown above, the expected sufficientarian hypothesis implies the restricted social expected-utility hypothesis. Thus, if $u_L P v_L$, then

$$\sum_{s\in S} \pi^s W(u_L^s) > \sum_{s\in S} \pi^s W(v_L^s),$$

since $u_L, v_L \in \mathcal{D}_L$. Now the expected sufficientarian hypothesis implies that uPv. Similarly, if $u_L I v_L$, it follows that

$$\sum_{s \in S} \pi^s W(u_L^s) = \sum_{s \in S} \pi^s W(v_L^s).$$

The expected sufficientarian hypothesis implies

$$\sum_{s \in S} \pi^s W(u_H^s) \geq \sum_{s \in S} \pi^s W(v_H^s) \;\Leftrightarrow\; uRv.$$

By the restricted social expected-utility hypothesis,

$$u_H R v_H \;\Leftrightarrow\; \sum_{s \in S} \pi^s W(u_H^s) \geq \sum_{s \in S} \pi^s W(v_H^s).$$

Thus, combining these equivalences, we obtain

$$u_H R v_H \;\Leftrightarrow\; uRv$$

so that R satisfies ex-post absolute priority.

'If.' Suppose that R satisfies ex-post absolute priority and the restricted social expected-utility hypothesis. Let W be a function that satisfies the requisite property stated in the restricted social expected-utility hypothesis. Let $u, v \in \mathcal{D}$. First, assume that

$$\sum_{s \in S} \pi^s W(u_L^s) > \sum_{s \in S} \pi^s W(v_L^s).$$

Note that $u_L, v_L \in \mathcal{D}_L$. Thus, it follows from the restricted social expected-utility hypothesis that

$$u_L P v_L.$$

From ex-post absolute priority, uPv follows. Next, we assume that

$$\sum_{s \in S} \pi^s W(u_L^s) = \sum_{s \in S} \pi^s W(v_L^s).$$

The restricted social expected-utility hypothesis implies that $u_L I v_L$. By ex-post absolute priority, we obtain

$$u_H R v_H \;\Leftrightarrow\; uRv.$$

From the restricted social expected-utility hypothesis, it follows that

$$\sum_{s\in S}\pi^s W(u_H^s) \geq \sum_{s\in S}\pi^s W(v_H^s) \;\Leftrightarrow\; u_H R v_H.$$

Thus, combining these equivalences, we obtain

$$\sum_{s\in S}\pi^s W(u_H^s) \geq \sum_{s\in S}\pi^s W(v_H^s) \;\Leftrightarrow\; uRv.$$

Therefore, R satisfies the expected sufficientarian hypothesis. ■

As pointed out by Roemer (2004) and echoed by Theorem 3 in terms of functional representability, sufficientarianism cannot be entirely continuous, but it is conditionally continuous; see Bossert, Cato, and Kamaga (2022, 2023) for a discussion of this issue in a deterministic setting. Motivated by arguments that parallel those employed in the case of the restricted expected-utility hypothesis, the following conditional continuity axioms require that R be continuous on the subdomains $\mathcal{D}_L$ and $\mathcal{D}_H$, respectively.

Continuity below the threshold: For all $u \in \mathcal{D}_L$, the sets $\{v \in \mathcal{D}_L \mid vRu\}$ and $\{v \in \mathcal{D}_L \mid uRv\}$ are closed in $\mathcal{D}_L$.

Continuity above the threshold: For all $u \in \mathcal{D}_H$, the sets $\{v \in \mathcal{D}_H \mid vRu\}$ and $\{v \in \mathcal{D}_H \mid uRv\}$ are closed in $\mathcal{D}_H$.

Ex-post sufficientarianism is characterized by replacing the social expected-utility hypothesis and continuity in Theorem 1 with the expected sufficientarian hypothesis and the two conditional continuity axioms.

Theorem 4. A social ordering R satisfies strong Pareto for no risk, continuity above the threshold, continuity below the threshold, anonymity, the expected sufficientarian hypothesis, and prospect independence of the unconcerned if and only if R is an ex-post sufficientarian ordering.

To prove this theorem, we employ two lemmas that use Theorem 1. The first of these states that ex-post generalized utilitarianism must be applied to prospects below the threshold.

Lemma 5. If a social ordering R satisfies strong Pareto for no risk, continuity below the threshold, anonymity, the expected sufficientarian hypothesis, and prospect independence of the unconcerned, then there exists a continuous and increasing function $g_L\colon (-\infty, \theta] \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}_L$,

$$uRv \;\Leftrightarrow\; \sum_{s\in S} \pi^s \sum_{i\in N} g_L(u_i^s) \geq \sum_{s\in S} \pi^s \sum_{i\in N} g_L(v_i^s). \tag{8}$$

Proof. Suppose that R satisfies strong Pareto for no risk, continuity below the threshold, anonymity, the expected sufficientarian hypothesis, and prospect independence of the unconcerned. By Lemma 4, R satisfies the restricted social expected-utility hypothesis. Let R_L be the restriction of R on $\mathcal{D}_L$, that is, for all $u, v \in \mathcal{D}_L$,

$$uR_Lv \;\Leftrightarrow\; uRv.$$

Note that R_L satisfies strong Pareto for no risk, continuity, anonymity, prospect independence of the unconcerned, and the social expected-utility hypothesis on $\mathcal{D}_L$. Applying Theorem 1 to R_L, there exists a continuous and increasing function $g_L\colon (-\infty, \theta] \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}_L$,

$$uR_Lv \;\Leftrightarrow\; \sum_{s\in S} \pi^s \sum_{i\in N} g_L(u_i^s) \geq \sum_{s\in S} \pi^s \sum_{i\in N} g_L(v_i^s).$$

This establishes the statement of the lemma. ■

The next lemma states an analogous result for prospects above the threshold. Its proof is analogous to that of Lemma 5.

Lemma 6. If a social ordering R satisfies strong Pareto for no risk, continuity above the threshold, anonymity, the expected sufficientarian hypothesis, and prospect independence of the unconcerned, then there exists a continuous and increasing function $g_H\colon [\theta, \infty) \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}_H$,

$$uRv \;\Leftrightarrow\; \sum_{s\in S} \pi^s \sum_{i\in N} g_H(u_i^s) \geq \sum_{s\in S} \pi^s \sum_{i\in N} g_H(v_i^s). \tag{9}$$

Proof of Theorem 4. It is straightforward to verify that all axioms are satisfied by any ex-post sufficientarian ordering.

Conversely, suppose that R satisfies the axioms of the theorem statement. Lemma 4 implies that R satisfies ex-post absolute priority. From Lemmas 5 and 6, it follows that there exist continuous and increasing functions $g_L\colon (-\infty, \theta] \to \mathbb{R}$ and $g_H\colon [\theta, \infty) \to \mathbb{R}$ that satisfy (8) and (9), respectively. Define the function $g\colon \mathbb{R} \to \mathbb{R}$ by

$$g(a) = \begin{cases} g_L(a) - g_L(\theta) + g_H(\theta) & \text{if } a \in (-\infty, \theta), \\ g_H(a) & \text{if } a \in [\theta, +\infty). \end{cases}$$

This function is obviously increasing on $\mathbb{R}$. It is also continuous because

$$\lim_{a \to \theta^-} g_L(a) - g_L(\theta) + g_H(\theta) = g_H(\theta).$$

We show that R is the ex-post sufficientarian ordering associated with g. Let $u, v \in \mathcal{D}$. We first assume that

$$\sum_{s \in S} \pi^s \sum_{i \in L(u^s)} (g(u_i^s) - g(\theta)) > \sum_{s \in S} \pi^s \sum_{i \in L(v^s)} (g(v_i^s) - g(\theta)).$$

Letting $u_L = w$ and $v_L = z$, this implies that

$$\sum_{s \in S} \pi^s \sum_{i \in N} g_L(w_i^s) > \sum_{s \in S} \pi^s \sum_{i \in N} g_L(z_i^s).$$

From Lemma 5, we obtain $u_L P v_L$. By ex-post absolute priority (which is implied by the expected sufficientarian hypothesis; see Lemma 4), uPv follows.

Next, we assume that

$$\sum_{s \in S} \pi^s \sum_{i \in L(u^s)} (g(u_i^s) - g(\theta)) = \sum_{s \in S} \pi^s \sum_{i \in L(v^s)} (g(v_i^s) - g(\theta)) \text{ and}$$
$$\sum_{s \in S} \pi^s \sum_{i \in H(u^s)} (g(u_i^s) - g(\theta)) \geq \sum_{s \in S} \pi^s \sum_{i \in H(v^s)} (g(v_i^s) - g(\theta)).$$

Applying an analogous argument to u_L, v_L, u_H, and v_H, it follows from Lemmas 5 and 6 that $u_L I v_L$ and $u_H R v_H$. By ex-post absolute priority, uRv follows. ■

From Lemma 4, we obtain the following corollary to Theorem 4.

Corollary 1. A social ordering R satisfies strong Pareto for no risk, continuity above the threshold, continuity below the threshold, anonymity, the restricted social expected-utility hypothesis, ex-post absolute priority, and prospect independence of the unconcerned if and only if R is an ex-post sufficientarian ordering.

The Pigou-Dalton transfer principle for no risk can be amended in the context of ex-post sufficientarian orderings. As is the case for the version used to characterize ex-post prioritarianism, it is sufficient to restrict attention to prospects with no risk. In analogy to the approach followed in Bossert, Cato, and Kamaga (2022, Section V), two versions of the Pigou-Dalton transfer principles for no risk can be employed—one that applies below the threshold, one that applies above the threshold. If these two principles are added to the axioms of Theorem 4 (or of Corollary 1), the restrictions of the transformation g to utility values less than or equal to θ and to utility values greater than or equal to θ are strictly concave. Note, however, that this does not imply the strict concavity of g on its entire domain. See Bossert, Cato, and Kamaga (2022, Section V) for a detailed discussion.

Thus far, we have focused on a specific form of ex-post sufficientarianism. However, various classes of sufficientarian orderings have been explored within the framework of no risk. Early sufficientarians (most notably, Frankfurt, 1987) advocate the headcount approach, which maximizes the number of individuals above the threshold. A natural extension for scenarios with risk is the *ex-post headcount* ordering, defined by letting, for all $u, v \in \mathcal{D}$, uRv if and only if

$$\sum_{s \in S} \pi^s |N \setminus L(u^s)| \geq \sum_{s \in S} \pi^s |N \setminus L(v^s)|.$$

This ordering evaluates social outcomes based on the expected number of individuals at or above the threshold. We note that the ex-post headcount ordering satisfies ex-post absolute priority, continuity above the threshold, anonymity, the expected sufficientarian hypothesis, and prospect independence of the unconcerned, while it does not satisfy continuity below the threshold. Importantly, it violates strong Pareto for no risk. However, the ex-post headcount ordering adheres to the social expected-utility hypothesis, which ex-post sufficientarianism does not. This divergence is clearly explained by Theorem 3, which demonstrates the incompatibility of strong Pareto for no risk, the social expected-utility hypothesis, and ex-post absolute priority.

The headcount ordering and our ex-post sufficientarian orderings are not continuous. However, if strong Pareto for no risk is relaxed and the other axioms of Theorem 4 are retained, a continuous candidate emerges. A social ordering R is *ex-post upper-limit sufficientarian* if and only if there exists a continuous and increasing function $g\colon (-\infty, \theta] \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}$, uRv if and only if

$$\sum_{s \in S} \pi^s \sum_{i \in L(u^s)} (g(u_i^s) - g(\theta)) \geq \sum_{s \in S} \pi^s \sum_{i \in L(v^s)} (g(v_i^s) - g(\theta)).$$

We note that the term upper-limit sufficientarianism is adopted from Shields (2012, 2016). Clearly, these orderings are continuous. Moreover, they satisfy all axioms of Theorem 4, except for strong Pareto for no risk. The following characterization of the ex-post upper-limit sufficientarian orderings shows that these orderings are the only possibility if we employ strong Pareto for no risk on the subdomain of $\mathcal{D}_L$.

Strong Pareto for no risk below the threshold: For all $u, v \in \mathcal{D}^c \cap \mathcal{D}_L$, if, for all $s \in S$, $u_i^s \geq v_i^s$ for all $i \in N$ and $u_i^s > v_i^s$ for some $i \in N$, then uPv.

Theorem 5. A social ordering R satisfies strong Pareto for no risk below the threshold, continuity, anonymity, the expected sufficientarian hypothesis, and prospect independence of the unconcerned if and only if R is an ex-post upper-limit sufficientarian ordering.

Proof. Since the 'if' part is straightforward, we focus on the 'only-if' part. By Lemma 5 and the expected sufficientarian hypothesis, there exists a continuous and increasing function $g\colon (-\infty, \theta] \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}$,

$$\sum_{s \in S} \pi^s \sum_{i \in N} g(\bar{u}_i^s) > \sum_{s \in S} \pi^s \sum_{i \in N} g(\bar{v}_i^s) \;\Rightarrow\; uPv, \tag{10}$$

where $\bar{u} = u_L$ and $\bar{v} = v_L$. Thus, it suffices to show that

$$\sum_{s \in S} \pi^s \sum_{i \in N} g(\bar{u}_i^s) = \sum_{s \in S} \pi^s \sum_{i \in N} g(\bar{v}_i^s) \;\Rightarrow\; uIv.$$

To this end, we suppose that

$$\sum_{s \in S} \pi^s \sum_{i \in N} g(\bar{u}_i^s) = \sum_{s \in S} \pi^s \sum_{i \in N} g(\bar{v}_i^s)$$

and show that vRu. The proof that uRv holds is analogous.

First, consider the case where $H(u^{s'}) \neq N$ for some $s' \in S$ (that is, there exists $j \in N$ and $s' \in S$ such that $u_j^{s'} \leq \theta$). Let $\delta > 0$ and define $\hat{u}(k)$ as follows. For each $s \in S$,

$$\hat{u}_i^s(k) = u_i^s - \frac{\delta}{k} \;\text{ for all } i \in N.$$

By (10), $vP\hat{u}(k)$ for all $k \in \mathbb{N}$. Thus, continuity implies that vRu. Therefore, it follows that, for all $u, v \in \mathcal{D}$,

$$H(u^{s'}) \neq N \text{ for some } s' \in S \text{ and } \sum_{s \in S} \pi^s \sum_{i \in N} g(\bar{u}_i^s) = \sum_{s \in S} \pi^s \sum_{i \in N} g(\bar{v}_i^s) \\ \Rightarrow\; vRu, \tag{11}$$

where $\bar{u} = u_L$ and $\bar{v} = v_L$.

Now consider the case where $H(u^s) = N$ for all $s \in S$ (that is, everyone experiences a utility above θ for every state). Note that

$$u_L = (\theta\mathbf{1}_m, \ldots, \theta\mathbf{1}_m).$$

To show that vRu, we establish $u_L R u$ as an auxiliary step. To the contrary, assume that uPu_L. Take any $i^* \in N$. We define $\tilde{u} \in \mathcal{D}$ by

$$\tilde{u}_{i^*} = u_{i^*} \;\text{ and }\; \tilde{u}_i = \theta\mathbf{1}_m \;\text{ for all } i \in N \setminus \{i^*\}.$$

For $\tilde{u}$ and u_L, the antecedent of (11) holds. Hence, $u_L R \tilde{u}$. Since $u P u_L$ and $u_L R \tilde{u}$, transitivity implies that $u P \tilde{u}$. Since $u_{i^*} = \tilde{u}_{i^*}$ and $u P \tilde{u}$, prospect independence of the unconcerned implies that

$$(u_{-i^*}, \theta \mathbf{1}_m) P (\tilde{u}_{-i^*}, \theta \mathbf{1}_m).$$

For $(u_{-i^*}, \theta \mathbf{1}_m)$ and $(\tilde{u}_{-i^*}, \theta \mathbf{1}_m)$, the antecedent of (11) holds. Thus, we obtain

$$(\tilde{u}_{-i^*}, \theta \mathbf{1}_m) R (u_{-i^*}, \theta \mathbf{1}_m),$$

a contradiction. Thus, $u_L R u$ follows. Finally, we show that $v R u$. To the contrary, assume that $u P v$. Combining this with $u_L R u$, transitivity implies that $u_L P v$. However, this contradicts (11). Thus, we obtain $v R u$. ■

Since the ex-post upper-limit sufficientarian orderings satisfy the social expected-utility hypothesis, Lemma 4 implies the following result.

Corollary 2. A social ordering R satisfies strong Pareto for no risk below the threshold, continuity, anonymity, the social expected-utility hypothesis, ex-post absolute priority, and prospect independence of the unconcerned if and only if R is an ex-post upper-limit sufficientarian ordering.

This result highlights the advantages of ex-post upper-limit sufficientarian orderings over ex-post headcount orderings. Like the ex-post headcount ordering, ex-post upper-limit sufficientarian orderings satisfy the social expected-utility hypothesis. However, they have the advantage of partially adhering to the strong Pareto principle for no risk and the Pigou-Dalton transfer principle for no risk (when g is strictly concave), principles that the ex-post headcount ordering violates. Consequently, ex-post upper-limit sufficientarian orderings align more closely with the fundamental criteria of efficiency and equity.

Table 1 compares five orderings: ex-post sufficientarianism, the ex-post headcount ordering, ex-post upper-limit sufficientarianism, ex-post prioritarianism, and ex-ante prioritarianism. All orderings in this table satisfy prospect independence of the unconcerned. If the function g is strictly concave, the ex-post sufficientarian orderings achieve efficiency and equity, though at the cost of continuity and the social expected-utility hypothesis. Observe that ex-post prioritarianism satisfies all the axioms listed in this table. In comparison, none of the orderings defined in terms of a sufficiency threshold comply with all of these axioms. We also note that ex-ante prioritarianism examined by Grant, Kajii, Polak, and Safra (2010) violates the social expected-utility hypothesis. This table clearly articulates the fundamental similarities and differences between possible forms of prioritarianism and sufficientarianism.

Table 1: Comparison

Ordering	SPNR	PDNR	Continuity	SEUH
Ex-post sufficientarianism	✓	✓	$\mathcal{D}_L$ and $\mathcal{D}_H$	$\mathcal{D}_L$ and $\mathcal{D}_H$
Ex-post headcount ordering			$\mathcal{D}_L$	✓
Ex-post upper-limit sufficientarianism	$\mathcal{D}_L$	$\mathcal{D}_L$	✓	✓
Ex-post prioritarianism	✓	✓	✓	✓
Ex-ante prioritarianism	✓	✓	✓	

The function g is assumed to be strictly concave. SPNR stands for strong Pareto for no risk; PDNR is the Pigou-Dalton transfer principle for no risk; SEUH represents the social expected-utility hypothesis.

6 Weak Pareto for equal risk

Our axiomatizations of ex-post prioritarianism and ex-post sufficientarianism employ prospect independence of the unconcerned. Although the members of these classes satisfy strong Pareto for no risk, none of them satisfy the axiom of weak Pareto for equal risk that Fleurbaey (2010) employs. To define this axiom, we consider another subdomain of $\mathcal{D}$. A prospect $u \in \mathcal{D}$ is *egalitarian* if $u_i = u_j$ for all $i, j \in N$. Let $\mathcal{D}^e$ be the set of egalitarian prospects. Note that, if $u \in \mathcal{D}^e$, then $E(u_i) = E(u_j)$ for all $i, j \in N$. Weak Pareto for equal risk postulates the weak Pareto principle for egalitarian prospects.

$$
\begin{tikzcd}[ampersand replacement=\&]
u' \& \bar{u} \& u'' \\
\begin{bmatrix} 2 & 0 \\ 2 & 0 \end{bmatrix} \arrow[r, leftrightarrow] \arrow[d, leftrightarrow, shift right] \& \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \arrow[r, leftrightarrow] \& \begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix} \arrow[d, leftrightarrow, shift right] \arrow[d, dotted, shift left] \\
\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \arrow[u, dotted, shift right] \arrow[ur] \& \& \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \arrow[ul] \\
v' \& \& v''
\end{tikzcd}
$$

Figure 1: Trilemma between separability, ex-ante efficiency, and ex-post equity

Weak Pareto for equal risk: For all $u, v \in \mathcal{D}^e$, if $E(u_i) > E(v_i)$ for all $i \in N$, then uPv.

Note that, combined with continuity, weak Pareto for equal risk implies the following restricted Pareto indifference condition.

Pareto indifference for equal risk: For all $u, v \in \mathcal{D}^e$, if $E(u_i) = E(v_i)$ for all $i \in N$, then uIv.

Weak Pareto for equal risk (and Pareto indifference for equal risk) by itself is compatible with prospect independence of the unconcerned; for example, ex-post utilitarianism satisfies these axioms. However, weak Pareto for equal risk cannot be satisfied by an ex-post prioritarian ordering. As we will show, this impossibility result generalizes to the trilemma between prospect independence of the unconcerned, Pareto indifference for equal risk, and the following variant of the Pigou-Dalton transfer principle.

Ex-post equalization principle: For all $u \in \mathcal{D}^e$ and for all $v \in \mathcal{D} \setminus \mathcal{D}^e$, if $\sum_{i \in N} u_i^s = \sum_{i \in N} v_i^s$ for all $s \in S$, then uPv.

As is straightforward to verify, the ex-post prioritarian orderings satisfy the ex-post equalization principle.

The trilemma between the above-mentioned three axioms can be seen as a formal generalization of Fleurbaey's (2010, Section V) argument why we should abandon prospect independence of the unconcerned when weak Pareto for equal risk is employed. Thus, it is useful to illustrate the trilemma using the prospects that are analogous to those used in his argument. Consider the five prospects, u', $\bar{u}$, u'', v', and v'', of two persons and two equally probable states that are presented in Figure 1. Pareto indifference for equal risk implies $u'I\bar{u}$ and $\bar{u}Iu''$, as indicated by a bidirectional arrow in Figure 1. The ex-post equalization principle implies $\bar{u}Pv'$ and $\bar{u}Pv''$; the implication is represented by a unidirectional arrow in Figure 1.

Therefore, if we endorse the weak ex-ante Paretian consideration and the ex-post equity embodied by these axioms, we conclude, by transitivity, that $u'Pv'$ and $u''Pv''$. Fleurbaey (2010) argues for these evaluations on the grounds that there is perfect positive correlation between individuals across states in u' and u'' as opposed to perfect negative correlation in v' and v''; see also Fleurbaey and Zuber (2013, 2015).

On the other hand, prospect independence of the unconcerned implies that the evaluation cannot depend on such reasoning because it requires that $u'Rv'$ if and only if $v''Ru''$. Specifically, if $u'Pv'$, we must conclude that $v''Pu''$, represented by a dotted unidirectional arrow in Figure 1. Thus, in this case, a preference for perfect positive correlation between individuals is switched to quite the opposite evaluation in favor of perfect negative correlation. This observation indicates that the axiom of prospect independence of the unconcerned is not without its problems—a comment that applies to numerous independence properties in the context of choice under risk. Meanwhile, if $u'Iv'$ (as prescribed by ex-post prioritarianism), it follows that u'' and v'' are equally good. This is illustrated by means of a bidirectional arrow in Figure 1. In either case, the evaluation violates transitivity, and we end up with the trilemma between the three axioms.

We now formally state the trilemma of the three axioms, which implies that an ex-post prioritarian ordering cannot satisfy weak Pareto for equal risk because it satisfies the other two axioms and continuity.

Theorem 6. There exists no social ordering R that satisfies Pareto indifference for equal risk, prospect independence of the unconcerned, and the ex-post equalization principle.

Proof. For simplicity, we assume that there are three individuals and two states; it is straightforward to extend the proof to the general case. The first state occurs with probability p and the second with probability $1-p$. By Pareto indifference for equal risk, there exists $\delta \in \mathbb{R}_{++}$ such that

$$\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} I \begin{bmatrix} 0 & \delta \\ 0 & \delta \\ 0 & \delta \end{bmatrix}. \tag{12}$$

That is, $p = (1-p)\delta$ holds. Define v, v', and v'' by

$$v = \begin{bmatrix} 0 & \delta \\ 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad v' = \begin{bmatrix} 1 & 0 \\ 0 & \delta \\ 1 & 0 \end{bmatrix}, \quad \text{and} \quad v'' = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & \delta \end{bmatrix}.$$

Because $p = (1-p)\delta$, it follows that $p(2/3) + (1-p)(\delta/3) = p$. Thus, Pareto indifference for equal risk and the ex-post equalization principle imply, respectively,

$$\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} I \begin{bmatrix} 2/3 & \delta/3 \\ 2/3 & \delta/3 \\ 2/3 & \delta/3 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 2/3 & \delta/3 \\ 2/3 & \delta/3 \\ 2/3 & \delta/3 \end{bmatrix} P \begin{bmatrix} 0 & \delta \\ 1 & 0 \\ 1 & 0 \end{bmatrix}.$$

By transitivity,

$$\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} P \begin{bmatrix} 0 & \delta \\ 1 & 0 \\ 1 & 0 \end{bmatrix}. \tag{13}$$

By the same argument as above, we obtain for v' and v'' that

$$\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} P \begin{bmatrix} 1 & 0 \\ 0 & \delta \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} P \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & \delta \end{bmatrix}. \tag{14}$$

From prospect independence of the unconcerned, (14) implies

$$\begin{bmatrix} 0 & \delta \\ 1 & 0 \\ 1 & 0 \end{bmatrix} P \begin{bmatrix} 0 & \delta \\ 0 & \delta \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 & \delta \\ 0 & \delta \\ 1 & 0 \end{bmatrix} P \begin{bmatrix} 0 & \delta \\ 0 & \delta \\ 0 & \delta \end{bmatrix}.$$

Thus, by (13), we obtain

$$\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} P \begin{bmatrix} 0 & \delta \\ 1 & 0 \\ 1 & 0 \end{bmatrix} P \begin{bmatrix} 0 & \delta \\ 0 & \delta \\ 1 & 0 \end{bmatrix} P \begin{bmatrix} 0 & \delta \\ 0 & \delta \\ 0 & \delta \end{bmatrix}.$$

By transitivity and (12), this is a contradiction. ■

As can be seen from the proof of Theorem 6, the result is true even on the restricted domains $\mathcal{D}_L$ and $\mathcal{D}_H$. Let us consider the following redistribution principles, which are weaker than the ex-post equalization principle.

Ex-post equalization principle below the threshold: For all $u \in \mathcal{D}^e \cap \mathcal{D}_L$ and for all $v \in \mathcal{D}_L \setminus \mathcal{D}^e$, if $\sum_{i \in N} u_i^s = \sum_{i \in N} v_i^s$ for all $s \in S$, then uPv.

Ex-post equalization principle above the threshold: For all $u \in \mathcal{D}^e \cap \mathcal{D}_H$ and for all $v \in \mathcal{D}_H \setminus \mathcal{D}^e$, if $\sum_{i \in N} u_i^s = \sum_{i \in N} v_i^s$ for all $s \in S$, then uPv.

Analogously, we can define the corresponding weaker versions of Pareto indifference for equal risk as follows.

Pareto indifference for equal risk below the threshold: For all $u, v \in \mathcal{D}^e \cap \mathcal{D}_L$, if $E(u_i) = E(v_i)$ for all $i \in N$, then uIv.

Pareto indifference for equal risk above the threshold: For all $u, v \in \mathcal{D}^e \cap \mathcal{D}_H$, if $E(u_i) = E(v_i)$ for all $i \in N$, then uIv.

These axioms are implied by the conjunction of weak Pareto for equal risk and the conditional continuity axioms on the subdomains $\mathcal{D}_L$ and $\mathcal{D}_H$.

Using the restricted versions of Pareto indifference for equal risk and the ex-post equalization principle, we obtain the following corollary. It implies that weak Pareto for equal risk is incompatible with any ex-post sufficientarian ordering associated with a transformation g that is strictly concave on $\mathcal{D}_L$ or on $\mathcal{D}_H$.

Corollary 3. There exists no social ordering R that satisfies Pareto indifference for equal risk below (above) the threshold, prospect independence of the unconcerned, and the ex-post equalization principle below (above) the threshold.

The incompatibility between weak Pareto for equal risk and the entire class of ex-post sufficientarian orderings can be explained from another perspective that is different from the trilemma stated in Corollary 3. As alluded to earlier, ex-post sufficientarianism considers the existence of people below the threshold a significant problem regardless of the exogenously given probability associated with that state, which is embodied as the ex-post absolute priority axiom. Giving absolute priority to ex-post utilities below the sufficiency threshold, an ex-post sufficientarian evaluation of prospects will be inconsistent with ex-ante efficiency of the evaluation for which ex-post utilities above the threshold also matter. As the following theorem shows, ex-post absolute priority and weak Pareto for equal risk are incompatible.

Theorem 7. There exists no social ordering R that satisfies weak Pareto for equal risk and ex-post absolute priority.

Proof. Let $a, b \in \mathbb{R}$ be such that $a < \theta < b$ and

$$\pi^1 a + \sum_{s \in \{2,\ldots,m\}} \pi^s b > \theta.$$

Consider the distributions $u, v, w \in \mathcal{D}^e$ defined by

$$u_i = (a, b\mathbf{1}_{m-1}), \quad v_i = \theta\mathbf{1}_m, \quad \text{and} \quad w_i = (a, \theta\mathbf{1}_{m-1})$$

for all $i \in N$. Note that $w = u_L$ and $v, w \in \mathcal{D}_L$. Thus, it follows from weak Pareto for equal risk that vPw because

$$E(v_i) = \theta > \pi^1 a + \sum_{s \in \{2,\ldots,m\}} \pi^s \theta = E(w_i)$$

for all $i \in N$. Ex-post absolute priority implies that vPu. On the other hand, weak Pareto for equal risk implies that uPv since

$$E(u_i) = \pi^1 a + \sum_{s \in \{2,\ldots,m\}} \pi^s b > \theta = E(v_i)$$

for all $i \in N$. This is a contradiction. ■

In view of Theorem 7, any principle of ex-post sufficientarianism needs to abandon weak Pareto for equal risk, as long as it satisfies ex-post absolute priority. From Corollary 1, this applies to all ex-post sufficientarian orderings.

The two impossibility results established in Theorems 6 and 7 are related to Theorem 1 of Fleurbaey (2010), who proposes an alternative approach to ex-post prioritarianism. Consider an increasing and strictly concave function $h\colon \mathbb{R} \to \mathbb{R}$, and define the function $\Xi_h^n\colon \mathbb{R}^n \to \mathbb{R}$ by letting

$$\Xi_h^n(x) = h^{-1}\left(\frac{1}{n}\sum_{i \in N} h(x_i)\right)$$

for all $x \in \mathbb{R}^n$. The number $\Xi_h^n(u^s)$ is called the equally-distributed-equivalent (EDE) utility for u^s, provided that the ex-post evaluation of each state $s \in S$ is performed by the prioritarian evaluation $\sum_{i \in N} h(u_i^s)$. A social ordering is *expected EDE prioritarian* if and only if there exists an increasing and strictly concave function $h\colon \mathbb{R} \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}$,

$$uRv \;\Leftrightarrow\; \sum_{s \in S} \pi^s \Xi_h^n(u^s) \geq \sum_{s \in S} \pi^s \Xi_h^n(v^s). \tag{15}$$

See Fleurbaey (2010) for a more general definition of an expected EDE criterion. According to his result, the general class of expected EDE criteria is axiomatized by statewise dominance, weak Pareto for no risk, weak Pareto for equal risk, and continuity; see also Fleurbaey and Zuber (2013) for a characterization of a subclass. We note that the expected EDE criterion coincides with ex-post utilitarianism whenever EDE utility is equal to average utility for each state. From Theorem 6, if an expected EDE criterion satisfies the ex-post equalization principle, it is incompatible with prospect independence of the unconcerned. Furthermore, from Theorem 7, the expected EDE criteria are incompatible with ex-post absolute priority. This means that there exists a fundamental tension between the ex-post sufficientarian approach and the expected EDE approach to assessing prospects.

7 Interchangeability for equally probable states

Although ex-post prioritarianism and ex-post sufficientarianism cannot comply with weak Pareto for equal risk, they are nevertheless capable of respecting the individuals' ex-ante evaluations in a different way. To illustrate this observation, consider a prospect u such that both Ann and Bob obtain utility levels of 2 in state 1 and zero in state 2; see Table 2. Now consider a different prospect v such that Bob gets zero in state 1 and 2 in state 2, all other things being equal. That is, Bob's ex-post utility levels are interchanged between the two states. Note that the prospects of Ann and Bob are exactly the same as the prospects u' and v' we presented in Figure 1. Assuming that the two states are equally probable, the following axiom states that this change does not affect the relative goodness of these prospects. That is, the original prospect is as good as the prospect that is generated by this interchange.

Interchangeability for equally probable states: Suppose that there exist $s, s' \in S$ with $\pi_s = \pi_{s'}$. For all $u, v \in \mathcal{D}$, if there exist $i \in N$ such that

$$u_i^s = v_i^{s'}, v_i^s = u_i^{s'},\ v_j^s = u_j^s,\ v_j^{s'} = u_j^{s'} \text{ for all } j \in N \setminus \{i\},$$
$$\text{and } u^t = v^t \text{ for all } t \in S \setminus \{s, s'\},$$

then uIv.

This axiom can be seen as a restricted version of ex-ante Pareto indifference; note that the interchange in question does not affect anyone's claims or

Table 2: Interchangeable prospects

Prospect u		
	State 1 (0.5)	State 2 (0.5)
Ann	2	0
Bob	2	0

Prospect v		
	State 1 (0.5)	State 2 (0.5)
Ann	2	0
Bob	0	2

interests. For the individual whose utility levels are interchanged, in which state he or she receives the higher utility level 2 is just a matter of labeling states and, thus, is irrelevant to his or her ex-ante evaluation of the two prospects. For the other individuals, nothing changes. Consequently, this interchange does not affect anyone's ex-ante utilities, and $E(u_k) = E(v_k)$ holds for all $k \in N$.

Note that prospect independence of the unconcerned and the social expected-utility hypothesis together imply this axiom. To demonstrate this, assume that $N = \{1, 2, 3\}$ and $S = \{1, 2\}$ for simplicity, and suppose that $i = 1$, $s = 1$, and $s' = 2$ in the statement of the axiom. The prospects u and v are given by

$$u = \begin{bmatrix} u_1^1 & u_1^2 \\ u_2^1 & u_2^2 \\ u_3^1 & u_3^2 \end{bmatrix} \quad \text{and} \quad v = \begin{bmatrix} u_1^2 & u_1^1 \\ u_2^1 & u_2^2 \\ u_3^1 & u_3^2 \end{bmatrix}.$$

Now consider the prospects u' and v' that are constructed by interchanging u_i^1 and u_i^2 for all $i \in N \setminus \{1\}$ in u and v, that is,

$$u' = \begin{bmatrix} u_1^1 & u_1^2 \\ u_2^2 & u_2^1 \\ u_3^2 & u_3^1 \end{bmatrix} \quad \text{and} \quad v' = \begin{bmatrix} u_1^2 & u_1^1 \\ u_2^2 & u_2^1 \\ u_3^2 & u_3^1 \end{bmatrix}.$$

The social expected-utility hypothesis implies that uIv' and vIu' because $W(u^s) = W(v'^s)$ and $W(v^s) = W(u'^s)$ for each $s \in S$. Prospect independence of the unconcerned implies that uRv if and only if $u'Rv'$. Therefore, we conclude that uIv because R is transitive.

From Theorem 2, ex-post prioritarian orderings satisfy interchangeability for equally probable states. However, expected EDE prioritarianism is incompatible with this axiom. To see this, consider the prospects u and v of Table 2. Assuming that everyone other than Ann and Bob receives $a \in \mathbb{R}$ in both states, we obtain

$$\begin{aligned}
\Xi_h^n(u^{s_1}) + \Xi_h^n(u^{s_2}) &= h^{-1}\left(\frac{2h(2)+(n-2)h(a)}{n}\right) + h^{-1}\left(\frac{2h(0)+(n-2)h(a)}{n}\right) \\
&> 2h^{-1}\left(\frac{h(2)+h(0)+(n-2)h(a)}{n}\right) \\
&= \Xi_h^n(v^{s_1}) + \Xi_h^n(v^{s_2})
\end{aligned}$$

because of the strict convexity of h^{-1}. This means that expected EDE prioritarianism concludes that u is better than v. Therefore, expected EDE prioritarianism cannot be neutral to the interchange that does not affect anyone's ex-ante utility and, in this sense, it cannot respect the individuals' ex-ante utilities. Consequently, the advantage of expected EDE prioritarianism in respecting individuals' ex-ante utilities is not as strong as it may appear to be.

Ex-post sufficientarianism also satisfies interchangeability for equally probable states. More generally, prospect independence of the unconcerned and the expected sufficientarian hypothesis together imply this axiom. Indeed, if the sufficientarian threshold θ is such that $\theta \geq \max\{u_i^s, u_i^{s'}\}$ or $\theta \leq \min\{u_i^s, u_i^{s'}\}$, the argument employed to show that prospect independence of the unconcerned and the social expected-utility hypothesis imply the axiom applies. Furthermore, the same argument works if $u_i^s > \theta > u_i^{s'}$. These observations follow because the censored prospects $u_L = \tilde{u}$ and $v_L = \tilde{v}$ satisfy

$$\tilde{u}_i^s = \theta = \tilde{v}_i^{s'}, \tilde{v}_i^s = \tilde{u}_i^{s'},\ \tilde{v}_j^s = \tilde{u}_j^s,\ \tilde{v}_j^{s'} = \tilde{u}_j^{s'} \text{ for all } j \in N \setminus \{i\},$$
$$\text{and } \tilde{u}^t = \tilde{v}^t \text{ for all } t \in S \setminus \{s, s'\},$$

and u_H and v_H satisfy an analogous property. Although ex-post prioritarianism and ex-post sufficientarianism do not satisfy weak Pareto for equal risk, they respect individuals' ex-ante utilities in a way that expected EDE prioritarianism does not.

8 Concluding remarks

In this paper, we employ a unified method to characterize ex-post welfare criteria over state-contingent alternatives. Our key axiom is prospect independence of the unconcerned, which is a risk-dependent variant of a well-established separability property. Adding a set of standard requirements leads to a characterization of ex-post prioritarianism. Utilizing this axiomatization, we characterize ex-post sufficientarianism. In the latter result, the axiom of ex-post absolute priority appears in addition to prospect independence of the unconcerned.

There are several tasks that remain to be addressed in future work. We focus on the case where the population is fixed but, evidently, there is considerable uncertainty regarding the size and the composition of future populations. In many countries, it is an urgent problem to address uncertainty related to well-being and population through public policies. Extending our framework to a variable-population setting may yield an important analytical tool to deal with population issues. The independence axioms introduced by Blackorby and Donaldson (1984) and by Blackorby, Bossert, and Donaldson (2005) play a significant role in population ethics under certainty. The extension of these axioms to prospects may constitute a promising path towards the examination of variable-population extensions of ex-post prioritarianism and ex-post sufficientarianism.

We assume in this paper that there is a single exogenously given threshold. This appears to be a quite natural assumption, given that the sufficiency threshold is associated with a particular level of well-being that may correspond to the basic needs of individuals—just as is the case for a poverty line. However, as alluded to in some earlier literature such as Casal (2007), it may very well be the case that there are multiple thresholds. For instance, a lower threshold may represent basic needs, whereas a higher threshold indicates an affluent life. Nakada and Sakamoto (2024) axiomatize a general class of social orderings that may be associated with multiple thresholds, including the possibility of a countably infinite number of thresholds. Huseby (2020) suggests that an increase in individual well-being levels does not morally matter above the highest threshold. This means that some Paretian axioms may be violated—and this observation applies even to weak Pareto for no risk. Exploring multiple thresholds within the framework of this paper constitutes another plausible direction for future research.

Appendix

Independence of the axioms in Theorem 1

Consider an increasing and continuous function $g\colon \mathbb{R} \to \mathbb{R}$. Define the ordering R^1 as follows. For all $u, v \in \mathcal{D}$, uRv if and only if

$$\sum_{s\in S} \pi^s \sum_{i\in N} g(u_i^s) \leq \sum_{s\in S} \pi^s \sum_{i\in N} g(v_i^s).$$

This ordering satisfies all axioms other than strong Pareto for no risk.

Define $g\colon \mathbb{R} \to \mathbb{R}$ by

$$g(x) = \begin{cases} -e^{-x} \text{ if } x < 0, \\ 1 - e^{-x} \text{ if } x \geq 0. \end{cases}$$

This is an increasing function that is discontinuous at zero. Furthermore, g is strictly concave on $(-\infty, 0)$ and on $[0, \infty)$. Define the ordering R^2 as follows.

For all $u, v \in \mathcal{D}$, uR^2v if and only if

$$\sum_{s \in S} \pi^s \sum_{i \in N} g(u_i^s) \geq \sum_{s \in S} \pi^s \sum_{i \in N} g(v_i^s).$$

This ordering satisfies all axioms other than continuity.

Consider n continuous and increasing functions $g_i \colon \mathbb{R} \to \mathbb{R}$ for all $i \in N$ with the property that there exist $j, k \in N$ such that g_k is not an affine transformation of g_j. Define the ordering R^3 as follows. For all $u, v \in \mathcal{D}$, uR^3v if and only if

$$\sum_{s \in S} \pi^s \sum_{i \in N} g_i(u_i^s) \geq \sum_{s \in S} \pi^s \sum_{i \in N} g_i(v_i^s).$$

This ordering satisfies all axioms other than anonymity.

An ordering that is prioritarian with probability weighing such that the function ϕ in its definition is not linear satisfies all axioms other than the social expected-utility hypothesis.

Define the function $W^* \colon \mathbb{R}^n \to \mathbb{R}$ by

$$W^*(x) = \min\{x_1, \ldots, x_n\} + \sum_{i=1}^{n} x_i$$

for all $x \in \mathbb{R}^n$. Now define the ordering R^4 as follows. For all $u, v \in \mathcal{D}$, uR^4v if and only if

$$\sum_{s \in S} \pi^s W^*(u^s) \geq \sum_{s \in S} \pi^s W^*(v^s).$$

This ordering satisfies all axioms other than prospect independence of the unconcerned.

Independence of the axioms in Theorem 4

Let $g \colon \mathbb{R} \to \mathbb{R}$ be an increasing and continuous function. Define R^5 by letting, for all $u, v \in \mathcal{D}$, uR^5v if and only if

$$\sum_{s \in S} \pi^s \sum_{i \in L(u^s)} (g(u_i^s) - g(\theta)) < \sum_{s \in S} \pi^s \sum_{i \in L(v^s)} (g(v_i^s) - g(\theta))$$

or

$$\sum_{s\in S}\pi^s\sum_{i\in L(u^s)}(g(u_i^s)-g(\theta))=\sum_{s\in S}\pi^s\sum_{i\in L(v^s)}(g(v_i^s)-g(\theta)) \text{ and}$$
$$\sum_{s\in S}\pi^s\sum_{i\in H(u^s)}(g(u_i^s)-g(\theta))\leq\sum_{s\in S}\pi^s\sum_{i\in H(v^s)}(g(v_i^s)-g(\theta)).$$

This ordering satisfies all axioms other than strong Pareto for no risk.

Let $g\colon \mathbb{R}\to\mathbb{R}$ be an increasing function that is not continuous at a point below θ. Define R^6 by letting, for all $u,v\in\mathcal{D}$, uR^6v if and only if

$$\sum_{s\in S}\pi^s\sum_{i\in L(u^s)}(g(u_i^s)-g(\theta))>\sum_{s\in S}\pi^s\sum_{i\in L(v^s)}(g(v_i^s)-g(\theta))$$

or

$$\sum_{s\in S}\pi^s\sum_{i\in L(u^s)}(g(u_i^s)-g(\theta))=\sum_{s\in S}\pi^s\sum_{i\in L(v^s)}(g(v_i^s)-g(\theta)) \text{ and}$$
$$\sum_{s\in S}\pi^s\sum_{i\in H(u^s)}(g(u_i^s)-g(\theta))\geq\sum_{s\in S}\pi^s\sum_{i\in H(v^s)}(g(v_i^s)-g(\theta)).$$

This ordering satisfies all axioms other than continuity below the threshold.

Let $g\colon \mathbb{R}\to\mathbb{R}$ be an increasing function that is not continuous at a point above θ. Define R^7 in analogy to R^6. Clearly, R^7 satisfies all axioms other than continuity above the threshold.

Consider n continuous and increasing functions $g_i\colon \mathbb{R}\to\mathbb{R}$ for all $i\in N$ with the property that there exist $j,k\in N$ such that g_k is not an affine transformation of g_j. Define the ordering R^8 by letting, for all $u,v\in\mathcal{D}$, uR^8v if and only if

$$\sum_{s\in S}\pi^s\sum_{i\in L(u^s)}(g_i(u_i^s)-g_i(\theta))>\sum_{s\in S}\pi^s\sum_{i\in L(v^s)}(g_i(v_i^s)-g_i(\theta))$$

or

$$\sum_{s\in S}\pi^s\sum_{i\in L(u^s)}(g_i(u_i^s)-g_i(\theta))=\sum_{s\in S}\pi^s\sum_{i\in L(v^s)}(g_i(v_i^s)-g_i(\theta)) \text{ and}$$
$$\sum_{s\in S}\pi^s\sum_{i\in H(u^s)}(g_i(u_i^s)-g_i(\theta))\geq\sum_{s\in S}\pi^s\sum_{i\in H(v^s)}(g_i(v_i^s)-g(\theta)).$$

This ordering satisfies all axioms other than anonymity.

Define R^9 by letting, for all $u, v \in \mathcal{D}$,

$$uR^9v \Leftrightarrow \sum_{s\in S} \pi^s \Xi_h^n(u_L^s) > \sum_{s\in S} \pi^s \Xi_h^n(v_L^s) \text{ or } \left[\sum_{s\in S} \pi^s \Xi_h^n(u_L^s) = \sum_{s\in S} \pi^s \Xi_h^n(v_L^s) \text{ and } \sum_{s\in S} \pi^s \Xi_h^n(u_H^s) \geq \sum_{s\in S} \pi^s \Xi_h^n(v_H^s)\right].$$

This ordering satisfies all axioms other than prospect independence of the unconcerned.

Let $g\colon \mathbb{R} \to \mathbb{R}$ be an increasing and continuous function, and let $\phi\colon (0,1) \to \mathbb{R}_{++}$ be an increasing and continuous function that is not linear. Define R^{10} by letting, for all $u, v \in \mathcal{D}$, $uR^{10}v$ if and only if

$$\sum_{s\in S} \phi(\pi^s) \sum_{i\in L(u^s)} (g(u_i^s) - g(\theta)) > \sum_{s\in S} \phi(\pi^s) \sum_{i\in L(v^s)} (g(v_i^s) - g(\theta))$$

or

$$\sum_{s\in S} \phi(\pi^s) \sum_{i\in L(u^s)} (g(u_i^s) - g(\theta)) = \sum_{s\in S} \phi(\pi^s) \sum_{i\in L(v^s)} (g(v_i^s) - g(\theta)) \text{ and}$$
$$\sum_{s\in S} \phi(\pi^s) \sum_{i\in H(u^s)} (g(u_i^s) - g(\theta)) \geq \sum_{s\in S} \phi(\pi^s) \sum_{i\in H(v^s)} (g(v_i^s) - g(\theta)).$$

This ordering satisfies all axioms other than the expected sufficientarian hypothesis. Furthermore, it satisfies ex-post absolute priority but violates the restricted expected-utility hypothesis. Thus, the restricted expected-utility hypothesis is independent of the other axioms in Corollary 1. As a remark aside, this ordering satisfies statewise dominance.

Independence of the utilities of the sure

The independence axiom used by Fleurbaey and Zuber (2013) is formally stated as follows.

Independence of the utilities of the sure: For all $u, v \in \mathcal{D}$, for all $u', v' \in \mathcal{D}^c$, and for all non-empty $M \subsetneq N$,

$$(u_M, v_{N\setminus M})R(u'_M, v_{N\setminus M}) \Leftrightarrow (u_M, v'_{N\setminus M})R(u'_M, v'_{N\setminus M}).$$

According to this axiom, social evaluations are not affected by unconcerned individuals whose utility levels are constant across states. It is obvious that independence of the utilities of the sure is logically weaker than prospect independence of the unconcerned.

One might ask if this weaker axiom is sufficient to establish our characterization of ex-post generalized utilitarianism (or sufficientarianism). The answer is no.

Let $g\colon \mathbb{R} \to \mathbb{R}$ be an increasing and continuous function, and define the function $\Lambda_g\colon \mathbb{R}^n \to \mathbb{R}$ by letting

$$\Lambda_g(x) = K^{\sum_{i \in N} g(x_i)}$$

for all $x \in \mathbb{R}^n$, where K is a constant larger than one. Define R by letting, for all $u, v \in \mathcal{D}$,

$$uRv \Leftrightarrow \sum_{s \in S} \pi_s \Lambda_g(u^s) \geq \sum_{s \in S} \pi_s \Lambda_g(v^s).$$

The ordering R satisfies strong Pareto for no risk, continuity, anonymity, the social expected-utility hypothesis, and independence of the utilities of the sure. However, prospect independence of the unconcerned is not satisfied. A similar example can be used to show that independence of the utilities of the sure is not sufficient to characterize ex-post sufficientarianism.

Which independence axiom to impose on the social ordering R—prospect independence of the unconcerned or the weaker axiom of independence of the utilities of the sure—is a normative question. Addressing that question in detail lies beyond the scope of this article, but we believe that a good case can be made for the stronger axiom. The choice of axiom depends upon the interpretation of prospects and states. In decision theory, a mapping from states to outcomes represents a possible choice (action) for a decision-maker in some choice situation. Probabilities assigned to states encode the decision-maker's uncertainty. "Each state ... is a compilation of all characteristics/factors about which [the decision-maker] is uncertain and which are relevant to the consequences that will ensue from his choice." (Kreps, 1988, p. 34). A prospect, in the decision-theoretic interpretation, represents a possible social choice, mapping each state into a vector of utilities for everyone in the population. The choices available to a social decision-maker are represented by the corresponding set of prospects.

The idea behind prospect independence of the unconcerned is that the comparison of two possible social choices, represented by prospects u and v, should be independent of the utility of anyone whose well-being is unaffected by the choice—that is, anyone whose utility is the same in u and in v. But the social planner may not know for certain what the utility level of an unaffected person is. Consider individuals who are already dead at the time of choice. Although the social planner can be sure that the lifetime well-being of the dead will be unaffected by her decision, she may not (and very likely will not) know what their well-being levels were. In other words, the dead may be among the unconcerned while not being among the sure. It is very compelling to exclude the influence of the dead when making policy choices; see our discussion following the definition of prospect independence of the unconcerned. Thus, a strong case can be made in favor of the more powerful independence axiom.

Independence of the unconcerned for no risk

A substantially weaker independence axiom can be formalized as follows.

Independence of the unconcerned for no risk: For all $u, u', v, v' \in \mathcal{D}^c$ and for all non-empty $M \subsetneq N$,

$$(u_M, v_{N\setminus M})R(u'_M, v_{N\setminus M}) \Leftrightarrow (u_M, v'_{N\setminus M})R(u'_M, v'_{N\setminus M}).$$

This applies the separability requirement only in the case with no risk. It is obvious that independence of the unconcerned for no risk is logically weaker than independence of the utilities of the sure.

If prospect independence of the unconcerned is replaced with independence of the unconcerned for no risk in the axioms of Theorem 1, the following characterization result is obtained.

Theorem A. A social ordering R satisfies strong Pareto for no risk, continuity, anonymity, the social expected-utility hypothesis, and independence of the unconcerned for no risk if and only if there exist continuous and increasing functions $g\colon \mathbb{R} \to \mathbb{R}$ and $\psi\colon Y \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}$,

$$uRv \Leftrightarrow \sum_{s\in S} \pi^s \psi \left(\sum_{i\in N} g(u_i^s) \right) \geq \sum_{s\in S} \pi^s \psi \left(\sum_{i\in N} g(v_i^s) \right).$$

Proof. 'If.' It is easy to verify that all axioms listed in the theorem statement are satisfied when there are such g and ψ.

'Only if.' Lemmas 1 and 2 hold even if prospect independence the unconcerned is replaced with independence of the unconcerned for no risk. This implies the claim. ■

The characterized orderings constitute a general class of ex-post principles. We note that this covers both ex-post prioritarian orderings and expected EDE prioritarian orderings. Theorem A highlights a novel implication of Theorem 1 or Theorem 2, which essentially demonstrates that the linearity of ψ is imposed by prospect independence of the unconcerned.

Some numerical examples

We provide some numerical examples in order to demonstrate how our sufficientarian principles differ. Consider the three prospects defined by

$$u = \begin{bmatrix} 0 & 30 \\ 0 & 30 \end{bmatrix}, u' = \begin{bmatrix} 5 & 30 \\ 5 & 30 \end{bmatrix}, u'' = \begin{bmatrix} 5 & 35 \\ 5 & 35 \end{bmatrix}.$$

First, these three prospects are pairwise indifferent according to the ex-post headcount ordering. Second, according to the ex-post upper-limit sufficientarian orderings, u' is better than u, while u'' is indifferent to u'. Third, u'' is better than u' and u' is better than u according to the ex-post sufficientarian orderings.

Now consider the four prospects given by

$$v = \begin{bmatrix} 0 & 0 \\ 30 & 30 \end{bmatrix}, v' = \begin{bmatrix} 5 & 5 \\ 25 & 25 \end{bmatrix}, v'' = \begin{bmatrix} 10 & 10 \\ 20 & 20 \end{bmatrix}, v''' = \begin{bmatrix} 15 & 15 \\ 15 & 15 \end{bmatrix}.$$

First, v''' is indifferent to v'', v'' is better than v', and v' is indifferent to v, according to the ex-post headcount ordering. Second, according to the ex-post upper-limit sufficientarian orderings, v''' is better than v'', v'' is better than v', and v' is indifferent to v. Third, v''' is better than v'', v'' is better than v', and v' is better to v, according to the ex-post sufficientarian orderings.

References

[1] Adler, Matthew D. (2012). Well-Being and Fair Distribution: Beyond Cost-Benefit Analysis. Oxford: Oxford University Press.

[2] Adler, Matthew D. (2018). Prioritarianism: room for desert? Utilitas, 30, 172–197.

[3] Adler, Matthew D. (2019). Measuring Social Welfare: An Introduction. Oxford: Oxford University Press.

[4] Adler, Matthew D. (2022). Theory of prioritarianism. In: Matthew D. Adler and Ole F. Norheim, eds. Prioritarianism in Practice. Cambridge: Cambridge University Press, 37–127.

[5] Adler, Matthew D. and Nils Holtug (2019). Prioritarianism: a response to critics. Politics, Philosophy & Economics, 18, 101–144.

[6] Adler, Matthew D. and Ole F. Norheim, eds. (2022). Prioritarianism in Practice. Cambridge: Cambridge University Press.

[7] Adler, Matthew D. and Chris William Sanchirico (2006). Inequality and uncertainty: theory and legal applications. University of Pennsylvania Law Review, 155, 279–377.

[8] Alcantud, José Carlos R., Marco Mariotti, and Roberto Veneziani (2022). Sufficientarianism. Theoretical Economics, 17, 1529–1557.

[9] Arrow, Kenneth J. (1953). Le rôle des valeurs boursières pour la répartition la meilleure des risques. In: Econométrie. Paris: Centre National de la Recherche Scientifique, 41–47. Translated and republished as Arrow, Kenneth J. (1964). The role of securities in the optimal allocation of risk-bearing. Review of Economic Studies, 31, 91–96.

[10] Axelsen, David V. and Lasse Nielsen (2015). Sufficiency as freedom from duress. Journal of Political Philosophy, 23, 406–426.

[11] Benbaji, Yitzhak (2005). The doctrine of sufficiency: a defence. Utilitas, 17, 310–332.

[12] Benbaji, Yitzhak (2006). Sufficiency or priority? European Journal of Philosophy, 14, 327–348.

[13] Blackorby, Charles, Walter Bossert, and David Donaldson (1998). Uncertainty and critical-level population principles. Journal of Population Economics, 11, 1–20.

[14] Blackorby, Charles, Walter Bossert, and David Donaldson (2002). Utilitarianism and the theory of justice. In: Kenneth J. Arrow, Amartya K. Sen, and Kotaro Suzumura, eds. Handbook of Social Choice and Welfare, Volume 1. Amsterdam: North-Holland, 543–596.

[15] Blackorby, Charles, Walter Bossert, and David Donaldson (2005). Population Issues in Social Choice Theory, Welfare Economics, and Ethics. New York: Cambridge University Press.

[16] Blackorby, Charles, Russell Davidson, and David Donaldson (1977). A homiletic exposition of the expected utility hypothesis. Economica, 44, 351–358.

[17] Blackorby, Charles and David Donaldson (1984). Social criteria for evaluating population change. Journal of Public Economics, 25, 13–33.

[18] Blackorby, Charles, David Donaldson, and John A. Weymark (1999). Harsanyi's social aggregation theorem for state-contingent alternatives. Journal of Mathematical Economics, 32, 365–387.

[19] Blackorby, Charles, David Donaldson, and John A. Weymark (2008). Social aggregation and the expected utility hypothesis. In: Marc Fleurbaey, Maurice Salles, and John A. Weymark, eds. Justice, Political Liberalism, and Utilitarianism: Themes from Harsanyi and Rawls. New York: Cambridge University Press, 136–183.

[20] Blackorby, Charles, Daniel Primont, and R. Robert Russell (1978). Duality, Separability, and Functional Structure: Theory and Economic Applications. New York: North-Holland.

[21] Bossert, Walter, Susumu Cato, and Kohei Kamaga (2022). Critical-level sufficientarianism. Journal of Political Philosophy, 30, 434–461.

[22] Bossert, Walter, Susumu Cato, and Kohei Kamaga (2023). Thresholds, critical levels, and generalized sufficientarian principles. Economic Theory, 75, 1099–1139.

[23] Broome, John (1991). Weighing Goods: Equality, Uncertainty and Time. Oxford: Blackwell.

[24] Brown, Campbell (2005). Priority or sufficiency . . . or both? Economics & Philosophy, 21, 199–220.

[25] Casal, Paula (2007). Why sufficiency is not enough. Ethics, 117, 296–326.

[26] Chambers, Christopher P., and Siming Ye (2024). Haves and have-nots: a theory of economic sufficientarianism. Journal of Economic Theory, 217, 105805.

[27] Crisp, Roger (2003). Equity, priority, and compassion. Ethics, 113, 745–763.

[28] Dahl, Geir, Alexander Guterman, and Pavel Shteyner (2018). Majorization for matrix classes. Linear Algebra and its Applications, 555, 201–221.

[29] Dalton, Hugh (1920). The measurement of the inequality of incomes. Economic Journal, 30, 348–361.

[30] Debreu, Gerard (1954). Representation of a preference ordering by a numerical function. In Robert M. Thrall, Clyde H. Coombs, and Robert L. Davis, eds. Decision Processes. 159–165. New York: Wiley.

[31] Debreu, Gerard (1959). Theory of Value: An Axiomatic Analysis of Economic Equilibrium. New York: Wiley.

[32] Diamond, Peter A. (1967). Cardinal welfare, individualistic ethics, and interpersonal comparison of utility: comment. Journal of Political Economy, 75, 765–766.

[33] Fleurbaey, Marc (2010). Assessing risky social situations. Journal of Political Economy, 118, 649–680.

[34] Fleurbaey, Marc, Thibault Gajdos, and Stéphane Zuber (2015). Social rationality, separability, and equity under uncertainty. Mathematical Social Sciences, 73, 13–22.

[35] Fleurbaey, Marc and François Maniquet (2011). Compensation and responsibility. In: Kenneth J. Arrow, Amartya K. Sen, and Kotaro Suzumura, eds. Handbook of Social Choice and Welfare, Vol. II. Amsterdam: North-Holland, 508–604.

[36] Fleurbaey, Marc and Stéphane Zuber (2013). Inequality aversion and separability in social risk evaluation. Economic Theory, 54, 675–692.

[37] Fleurbaey, Marc and Stéphane Zuber (2015). Discounting, beyond utilitarianism. Economics: The Open-Access, Open-Assessment E-Journal, 9, 1–52.

[38] Frankfurt, Harry (1987). Equality as a moral ideal. Ethics, 98, 21–43.

[39] Gorman, William M. (1968). The structure of utility functions. Review of Economic Studies, 35, 367–390.

[40] Grant, Simon, Atsushi Kajii, Ben Polak, and Zvi Safra (2010). Generalized utilitarianism and Harsanyi's impartial observer theorem. Econometrica, 78, 1939–1971.

[41] Hammond, Peter J. (1983). Ex-post optimality as a dynamically consistent objective for collective choice under uncertainty. In: Prasanta K. Pattanaik and Maurice Salles, eds. Social Choice and Welfare. Amsterdam: North-Holland, 175–205.

[42] Harsanyi, John C. (1955). Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility. Journal of Political Economy, 63, 309–321.

[43] Hassoun, Nicole (2021). Sufficiency and the minimally good life. Utilitas, 33, 321–336.

[44] Herlitz, Anders (2018). The indispensability of sufficientarianism. Critical Review of International Social and Political Philosophy, 22, 929–942.

[45] Hirose, Iwao (2016). Axiological sufficientarianism. In: Carina Fourie and Annette Rid, eds. What is Enough? Sufficiency, Justice, and Health. Oxford: Oxford University Press, 51–68.

[46] Huseby, Robert (2010). Sufficiency: restated and defended. Journal of Political Philosophy, 18, 178–197.

[47] Huseby, Robert (2020). Sufficiency and the threshold question. Journal of Ethics, 24, 207–223.

[48] Karlin, Samuel and Yosef Rinott (1983). Comparison of measures, multivariate majorization, and applications to statistics. In: Samuel Karlin, Takeshi Amemiya, and Leo A. Goodman, eds. Studies in Econometrics, Time Series, and Multivariate Statistics. New York: Academic Press, 465–489.

[49] Knight, Carl (2022). Enough is too much: the excessiveness objection to sufficientarianism. Economics & Philosophy, 38, 275–299.

[50] Kreps, David M. (1988). Notes on the Theory of Choice. Boulder: Westview Press.

[51] Marshall, Albert W. and Ingram Olkin (1979). Inequalities: Theory of Majorization and Its Applications. New York: Academic Press.

[52] Marshall, Albert W., Ingram Olkin, and Barry C. Arnold (2011). Inequalities: Theory of Majorization and Its Applications, 2^{nd} edition. New York: Springer.

[53] McCarthy, David (2008). Utilitarianism and prioritarianism II. Economics & Philosophy, 24, 1–33.

[54] Mongin, Philippe and Marcus Pivato (2015). Ranking multidimensional alternatives and uncertain prospects. Journal of Economic Theory, 157, 146–171.

[55] Nakada, Satoshi and Norihito Sakamoto (2024). The multi-threshold generalized sufficientarianism and level-oligarchy. Research Center for Normative Economics Discussion Paper 13, Institute of Economic Research, Hitotsubashi University.

[56] Nielsen, Lasse (2019). What is wrong with sufficiency? Res Publica, 25, 21–38.

[57] Parfit, Derek (2000). Equality or priority? In: Matthew Clayton and Andrew Williams, eds. The Ideal of Equality. London: Macmillan, 81–125.

[58] Pigou, Arthur C. (1912). Wealth and Welfare. London: Macmillan.

[59] Rabinowicz, Wlodek (2002). Prioritarianism for prospects. Utilitas, 14, 2–21.

[60] Roemer, John E. (2004). Eclectic distributional ethics. Politics, Philosophy & Economics, 3, 267–281.

[61] Segall, Shlomi (2016). What is the point of sufficiency? Journal of Applied Philosophy, 33, 36–52.

[62] Shields, Liam (2012). The prospects for sufficientarianism. Utilitas, 24, 101–117.

[63] Shields, Liam (2016). Just Enough: Sufficiency as a Demand of Justice. Edinburgh: Edinburgh University Press.

[64] Timmer, Dick (2022). Justice, thresholds, and the three claims of sufficientarianism. Journal of Political Philosophy, 30, 298–323.