CHOICE OR COMPETITION: DOES INTEGRATION BENEFIT EVERYONE?

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Abstract. Matching markets are often fragmented, organized at a small local level. While integration of matching markets may lead to welfare gains by expanding choice, it may also harm some market participants by increasing competition for the same resources. We show that every "good" mechanism fails the monotonicity requirement that no individuals be hurt by integration. Then we provide characterization results that identify conditions under which monotonicity becomes compatible with other desirable properties of matching mechanisms.

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1. Introduction

Allocation of resources such as daycare slots, school seats and vaccines are often conducted at small local levels. For example, in Tokyo, daycare slots and elementary school seats are allocated within each of the 23 small districts that partition the city.[1] Major cities in China such as Tianjin and Shanghai have an admission system for kindergartens where the cities are divided into small districts and a child in a given district can only be assigned to a school in the district. During the Covid-19 pandemic, Japanese government adopted the policy to first distribute vaccines to each municipality, such as each of the 23 small districts in Tokyo, which was then responsible for distributing the allotted vaccine to their residents. In the assignment of children into foster homes in Los Angeles County, CA, the assignment is conducted at an inefficiently fragmented level of regional offices (Robinson-Cortes, 2019).[2] Facing such *fragmentation* of the markets, one could hope for a welfare gain by the *integration* of the regions. What are the barriers against integration?

Integration entails two opposing effects for the individuals seeking for resources. On the one hand, it increases the *choice* for them because the resources in the integrated region become available. On the other hand, it increases the *competition* because the resources that were originally exclusive to the individuals in a given region become available to more individuals. The objective of this paper is to understand this tradeoff and characterize when the first effect dominates; that is, when individuals become better off by integration.

For this purpose, we consider a two-sided matching model, where we refer to agents in the two sides as students and schools.[3] A "region structure" partitions the set of students and schools, and we examine how a change in the region structure affects student welfare. In particular, we ask if a mechanism in consideration is *monotone*, meaning that integration always weakly improves student welfare. That is, we ask when the choice effect of the integration dominates the competition effect so that there is no barrier against integration in terms of social welfare.

Our first theorem (Theorem 1) shows that every "good" mechanism lacks monotonicity: No mechanism that is strategy-proof, Pareto efficient and individually rational is monotone. This result demonstrates that a policymaker designing a mechanism has to sometimes accommodate situations where competition overrides choice if they wish to

[1] There are some exceptions to this rule that allow for interdistrict transfers under limited scenarios, but such transfers are rarely implemented.

[2] See also Slaugh et al. (2016), who are, to our knowledge, the first to apply tools from matching theory to the problem of child adoption.

[3] See Gale and Shapley (1962), Roth (1984), Roth and Peranson (1999), and Abdulkadiroğlu and Sönmez (2003), among many others, for seminal work in two-sided matching markets.

maintain strategy-proofness, Pareto efficiency and individual rationality. Or, they have to abandon at least one of these three properties to retain monotonicity. Given this impossibility, we then consider mechanisms that are Pareto efficient and individually rational (while possibly being non-strategy-proof). We show that there exist monotone mechanisms that satisfy those properties if and only if the set of allowed region structures has a type of hierarchical structure (Theorem 2). This result demonstrates that there is a limit to monotonicity even when the requirement for strategy-proofness is lifted and completely characterizes such a limit.[4]

Intuitively, the effect of competition is present when schools in a given region prefer the students in other regions than the students in its own region. We investigate the validity of this intuition by considering well-known mechanisms in school choice. We show that (i) there exists a stable and monotone mechanism if and only if (ii) the deferred acceptance (DA) mechanism (applied to each region) is monotone if and only if (iii) the school preferences favor local students (Theorem 3). By contrast, we also prove that other well-known mechanisms such as the top trading cycles (TTC) mechanism (Shapley and Scarf, 1974) are not monotone even if school preferences favor locals. Our analysis demonstrates that integration improves welfare for every student in some practical scenarios under the DA mechanism while the same cannot be said for other well-known mechanisms. Hence, integration may face less public opposition under the DA mechanism than under those mechanisms.

The literature has studied fragmentation of markets via two approaches. The first is to explain it as an equilibrium outcome and the second is to analyze agents' behavior under each of fixed market structures (the fragmentation level) and compare the outcomes under different market structures. Under the first approach, Ellison, Fudenberg and Möbius (2004), for example, consider a model where traders choose auctions to participate, and show that multiple auctions can coexist in equilibrium because one trader moving from a market to another makes the latter market more competitive, disincentivizing the movement. Peivandi and Vohra (2021) consider a model where traders choose markets to participate and show that an integrated market can be blocked because strategic behavior of the traders causes inefficiency in the integrated market. Kaneko and Matsui (1999) consider a model of location choice of different types of individuals and show that discrimination can result in different types choosing different locations.[5] In contrast to these

[4]As we will mention when explaining Theorem 1, the existence of a monotone mechanism is straightforward when we lift Pareto efficiency or individual rationality instead.

[5]Peivandi and Vohra (2021) reviews other explanations of fragmentation.

models, it would be difficult for students to freely choose a region to belong to in our context. One could view our results that monotonicity is difficult to obtain as giving a novel explanation for the emergence of fragmentation: As long as we stick to other desiderata, it is impossible to have unanimous agreement for integration.

Under the second approach, Malamud and Rostek (2017), for example, consider a model that allows for coexistence of multiple "exchanges" where a given trader can participate in multiple markets and show that the market with multiple exchanges can be more efficient than the one with a single exchange because the aggregate risk portfolio in the latter is generally inefficient and fragmenting the market changes the agents' ability to diversify. In the context of matching, Allman et al. (2022) study the optimal design of geographical zones, from which students choose schools, to achieve a diversity goal. Integration in matching markets appears first in the seminal work by Ortega (2018), who studies monotonicity requirements that are more stringent than ours in the sense that they require improvement for agents on both sides of the market, with a restriction on the allowed region structures to the ones where there are distinct small regions and either (i) the only possible merger is where all the small regions merge, or (ii) all possible mergers are allowed. Due to those differences and others, his results are logically unrelated to ours.[6] Ortega (2018) and later studies such as Ortega (2019), Klein, Aue and Ortega (2024), and Gersbach and Haller (2022) study environments where both the competition and choice effects of integration exist and quantify those effects theoretically and empirically.

Monotonicity under integration is studied in the context of exchange economies: Chambers and Hayashi (2020) show that there is no social choice function that satisfy efficiency and their monotonicity requirement. The proof of this result is reminiscent of that of our result that there is no monotone mechanism that is Pareto efficient and individually rational when the set of allowed region structures does not have a type of hierarchical structure (the "only if" direction of our Theorem 2).

[6]More specifically, Ortega (2018) provides two theoretical results that might seem similar to some of our results. First, he shows that his notions of monotonicity are incompatible with stability. Recall that his monotonicity conditions are stronger than ours, and also note that stability is stronger than Pareto efficiency and individual rationality, which our impossibility result (Theorem 1) requires. The second result is that no Pareto efficient matching is monotone (in his sense). This result is reminiscent of the "only if" direction of our Theorem 2, which shows that if the allowed region structures do not form a weak hierarchy, then there does not exist a monotone mechanism (in our sense, which is weaker) that is Pareto efficient and individually rational (Ortega (2018) assumes that each agent on a given side deems all agents in the other side as acceptable).

There are other notions of monotonicity in the resource allocation literature. Population monotonicity (Thomson, 1983) requires that adding individuals make all existing individuals weakly worse off while resource monotonicity (Chun and Thomson, 1988) requires that adding resources make all individuals weakly better off.[7] These notions pertain to changes on one side of the market (population or resources) while holding the other side fixed, while our monotonicity notion pertains to the change on both sides. As a consequence, our monotonicity (almost) implies resource monotonicity, as expanding a region involves adding resources, namely new schools.[8] In light of our "choice versus competition" framing, the expanded choice due to integration corresponds to the effect associated with resource monotonicity and the increased competition corresponds to the effect associated with population monotonicity. Our monotonicity condition requires that the former outweighs the latter.

We note that Kamada and Kojima (2023*a*) also consider integration of multiple regions in a matching problem between students and schools and provide an approach complementary to the present paper. Specifically, their paper studies "partial integration" of regions, in the sense that the produced matching must satisfy the balancedness constraint: for each region, the total number of residents of other regions matched to schools in it must be equal to the total number of its residents matched to a school outside of the region.[9] That is, the paper takes as given the constraint that "full integration" of multiple regions is infeasible. The present paper, in contrast, studies whether, and to what extent, a full integration of regions is desirable. Their paper and ours are complementary in this sense.

At a high level, our paper shares the interest in integration in school choice with the literature, though in different contexts. For example, Doğan and Yenmez (2019) and Ekmekci and Yenmez (2019) compare an integrated school choice mechanism with a divided

[7] Sprumont (1990) also proposes a notion called population monotonicity in a context of cooperative games with transferable utility, requiring that each player's payoff weakly increases as the coalition becomes larger. Toda (2005, 2006) uses population monotonicity to characterize core in two-sided matching market models. We also note that some papers (e.g., Ehlers, Klaus and Pápai (2002)) use population monotonicity to mean that the changes of the individuals' welfare are all in a single direction: all existing individuals become weakly worse off or all of them become weakly better off.

[8] There are some technical differences which make the logical relationship incomplete. Most importantly, our model is endowed with a potentially restricted set of possible region structures, and the monotonicity requirement imposes restrictions only on those instances. This is in contrast with the resource monotonicity axiom which imposes restrictions for all possible configurations of available resources.

[9] Hafalir, Kojima and Yenmez (2022) introduced a balancedness constraint in the context of interdistrict school choice. A balancedness constraint across individual institutions was introduced by Dur and Ünver (2019).

choice mechanism in which the assignment of schools of different types is conducted separately. In their models, students can potentially be assigned to any school irrespective of the degree of integration, and thus, they do not entail the tradeoff between choice and competition, the issue that our paper studies. Another example is Hafalir, Kojima and Yenmez (2022), who consider interdistrict school choice under various constraints and policy objectives.[10]

This paper belongs to the literature in matching with constraints. Research in this literature include Abdulkadiroğlu (2005), Ergin and Sönmez (2006), Abraham, Irving and Manlove (2007), Biró et al. (2010), Hafalir, Yenmez and Yildirim (2013), Westkamp (2013), Goto et al. (2014), Kamada and Kojima (2015, 2017, 2018, 2023*b*), Kojima, Tamura and Yokoo (2018), Aygün and Turhan (2020) and Pathak et al. (2021). The main departure of the present paper is that we consider integration of multiple markets, while those earlier contributions treat the relevant market as given.

Our paper takes an axiomatic approach that has a long tradition in matching and resource allocation problems. Roth (1982), for instance, shows that there exists no stable mechanism that is strategy-proof for both students and schools. The method has been used to obtain impossibility results for different kinds of axioms (e.g., Sönmez (1997, 1999), Thomson (2011), Doğan (2016), and Chaudhury and Pápai (2025)), characterizing standard mechanisms (e.g., Pápai (2000), Sönmez and Ünver (2010), Kojima and Manea (2010), Ehlers and Klaus (2016), and Pycia and Ünver (2017)), and finding tight conditions under which mechanisms has desirable properties (e.g., Ergin (2002), Kesten (2006), and Hatfield, Kojima and Narita (2016)). The present study contributes to this literature by formalizing and analyzing the monotonicity axiom to study the issue of integration of matching markets.

Finally, benefit of integration as well as its possible cost has been a central issue in international economics for at least two centuries. Ricardo (1821) famously argued that opening up countries for international trade will benefit all countries through specialization and access to goods from abroad, broadening choice. Stolper and Samuelson (1941) offered a model in which, although trade improves overall welfare of a country, some sectors may be made worse off through competition. Our paper can be thought of as identifying and analyzing analogous forces of choice and competition in the context of matching problems.

[10]C.f., footnote 24.

2. Model

2.1. **Preliminary Definitions.** Let there be a finite set of students I and a finite set of schools S. Each student i has a strict preference relation $\succ_i$ over the set of schools and being unmatched (being unmatched is denoted by $\emptyset$). For any $s, s' \in S \cup \{\emptyset\}$, we write $s \succeq_i s'$ if and only if $s \succ_i s'$ or $s = s'$.

Each school $s \in S$ is endowed with a strict preference relation $\succ_s$ over the set of subsets of students (we use $\emptyset$ to denote the empty set with a slight abuse of notation). For any $I', I'' \subseteq I$, we write $I' \succeq_s I''$ if and only if $I' \succ_s I''$ or $I' = I''$. We denote by $\succ = (\succ_a)_{a \in I \cup S}$ the preference profile of all students and schools. For any $i, i' \in I \cup \{\emptyset\}$, we write $i \succeq_s i'$ if and only if $i \succ_s i'$ or $i = i'$.[11]

For each $s \in S$, fix a positive integer q_s. We assume that preference relation $\succ_s$ is **responsive with capacity** q_s (Roth, 1985), that is,

(1) For any $I' \subseteq I$ with $|I'| \leq q_s$, $i \in I \setminus I'$ and $i' \in I'$, $(I' \cup i) \setminus i' \succeq_s I'$ if and only if $i \succeq_s i'$, and
(2) For any $I' \subseteq I$ with $|I'| \leq q_s$ and $i' \in I'$, $I' \succeq_s I' \setminus i'$ if and only if $i' \succeq_s \emptyset$.
(3) $\emptyset \succ_s I'$ for any $I' \subseteq I$ with $|I'| > q_s$.

In words, we assume that the ranking of a student (or keeping a position vacant) is independent of her peers, and any set of students exceeding its capacity is unacceptable.

Student i is said to be **acceptable** to school s if $i \succ_s \emptyset$ (and unacceptable otherwise). Similarly, s is acceptable to i if $s \succ_i \emptyset$. It will turn out that only rankings of acceptable partners matter for our analysis, so we often write only acceptable partners to denote preferences and priorities. For example,

$$\succ_i : s, s'$$

means that school s is the most preferred, s' is the second most preferred, and s and s' are the only acceptable schools under preferences $\succ_i$ of student i. We also use analogous expressions for school preferences.

A **matching** μ is a mapping that satisfies (i) $\mu_i \in S \cup \{\emptyset\}$ for all $i \in I$, (ii) $\mu_s \subseteq I$ for all $s \in S$, and (iii) for any $i \in I$ and $s \in S$, $\mu_i = s$ if and only if $i \in \mu_s$. That is, a matching simply specifies which student is assigned to which school (if any).

A matching is **individually rational** if $\mu_a \succeq_a \emptyset$ for every $a \in I \cup S$.

2.2. **Regions.** Fix a **base of regions**, which is a partition R^0 of $I \cup S$. A region structure R is a partition of $I \cup S$ such that each $r \in R$ is of the form $r = r^1 \cup \cdots \cup r^k$ with

[11] We denote singleton set $\{x\}$ by x when there is no confusion.

$r^1, \ldots, r^k \in R^0$. An element $r \in R$ is called a region. That is, a region structure is a weakly coarser partition than the base of regions, and thus, for any region structure R and an element r^0 of the base of regions, $r \in R$ implies that we must have $r^0 \subseteq r$ or $r^0 \cap r = \emptyset$. Note that each s belongs to a single $r \in R$ and each i is a resident of a single $r \in R$. To simplify the exposition of some results, we hereafter assume that $|r \cap I| \geq 2$ and $|r \cap S| \geq 1$ hold for each $r \in R^0$. We denote by $\mathcal{Q}$ a nonempty subset of the set of all region structures.

We call tuple $(I, S, \mathcal{Q})$ an **environment**.

A matching μ is **feasible under** R if, for all $r \in R$ and $i \in r \cap I$, we have $\mu_i \in r \cup \{\emptyset\}$. A matching μ is **Pareto efficient under** R if (i) it is feasible under R and (ii) there exists no other matching μ' that is feasible under R and satisfies $\mu'_a \succeq_a \mu_a$ for every $a \in I \cup S$.[12]

Given a matching μ, a pair $(i, s) \in I \times S$ is called a **blocking pair** if $s \succ_i \mu_i$ and there is $I' \subseteq \mu_s \cup \{i\}$ such that $I' \succ_s \mu_s$. A matching μ is **stable under** R if (i) it is feasible under R, (ii) $s = \mu_i$ implies $s \succeq_i \emptyset$ and $i \in \mu_s$ implies $i \succeq_s \emptyset$, and (iii) it does not have any blocking pair (i, s) such that there exists $r \in R$ with $i, s \in r$. Gale and Shapley (1962) imply that there is a unique stable matching μ^* under R such that for every stable matching μ under R and every $i \in I$, we have $\mu^*_i \succeq_i \mu_i$. Call it a **student-optimal stable matching** (or, **SOSM**) **under** R.

A **mechanism** φ is a function from the set of preference profile-region structure pairs to the set of feasible matchings. That is, $\varphi(\succ, R)$ is a feasible matching under R.

Mechanism φ is **strategy-proof** if

$$\varphi_i(\succ, R) \succeq_i \varphi_i(\succ'_i, \succ_{-i}, R),$$

for every region structure $R \in \mathcal{Q}$, preference profile $\succ$, $i \in I$, and student preferences $\succ'_i$.[13]

Mechanism φ is **individually rational** if $\varphi(\succ, R)$ is individually rational for all $\succ$ and $R \in \mathcal{Q}$. Similarly, φ is **Pareto efficient** if $\varphi(\succ, R)$ is Pareto efficient under R for all $\succ$ and $R \in \mathcal{Q}$.

We say that φ is **stable** if, for any input $(\succ, R)$, the matching $\varphi(\succ, R)$ is stable under R given preference profile $\succ$. We say that φ is the **deferred acceptance mechanism** (or, the **DA mechanism**) if, for any input $(\succ, R)$, the matching $\varphi(\succ, R)$ is the SOSM under R given preference profile $\succ$.

[12]Notice that the notion of Pareto efficiency is two-sided, where we consider all agents in $I \cup S$. We provide a discussion on one-sided vs. two-sided Pareto efficiency after the statement of Theorem 1.

[13]We note that the definition requires reporting true preferences be a best reply for students only.

3. Limits of Monotone Mechanisms

We are now ready to introduce the key concept of this paper, monotonicity.

Definition 1. A mechanism φ is **student-welfare integration monotone**, or **monotone** for short, if, for all $R, R' \in \mathcal{Q}$, $r \in R$, $r' \in R'$ such that $r \subseteq r'$, $i \in r \cap I$, and $\succ$, we have $\varphi_i(\succ, R') \succeq_i \varphi_i(\succ, R)$.

In words, monotonicity requires that, when a region expands, all students in that region be made weakly better off. Two comments are in order. First, another possible definition would be to require only that the outcome of the mechanism not become Pareto inferior for students after multiple regions merge with each other. Such a requirement is weak and would be trivially satisfied by any mechanism that is Pareto efficient such as the TTC mechanism (where implementing a cycle would mean that each student receives the school that she points to along the cycle) and the Boston mechanism.[14] Second, we do not require schools be weakly better off. Our negative results (such as Theorem 1) clearly hold under a stronger requirement that all students and schools be made better off as a result of expansion.[15]

Definition 2. We say that $\mathcal{Q}$ **admits a merger** if there exist $R, R' \in \mathcal{Q}$, distinct $r_1, r_2 \in R$ and $r' \in R'$ such that $r_1 \cup r_2 \subseteq r'$.

We regard admitting a merger as a minimal requirement. The condition is satisfied if, for instance, $\mathcal{Q}$ includes the base of regions or the grand region structure (i.e., the partition consisting of a single cell) and contains at least two region structures.

Theorem 1. *Fix an environment $(I, S, \mathcal{Q})$ such that $\mathcal{Q}$ admits a merger. There exists no monotone mechanism that is strategy-proof, Pareto efficient, and individually rational.*

To prove this result, we consider an example of a market and any monotone mechanism that is Pareto efficient and individually rational. Pareto efficiency and individual rationality applied to regions before a merger limits the possible matchings after the merger through monotonicity. Such a limitation is shown to interfere with strategy-proofness. The formal proof is as follows.

Proof. Consider a monotone mechanism φ that is Pareto efficient and individually rational. We will show that φ is not strategy-proof.

[14] Indeed, the conclusion of Theorem 1 does not hold under this notion of monotonicity as, for instance, the TTC mechanism then satisfies all the requirements.

[15] A discussion on one-sided vs. two-sided monotonicity is given after the statement of Theorem 1 as well.

Because $\mathcal{Q}$ admits a merger, there exist $R, R' \in \mathcal{Q}$ with the following property: there exist distinct $r_1, r_2 \in R$ and $r' \in R'$ such that $r_1 \cup r_2 \subseteq r'$. Fix such (R, R', r_1, r_2, r') arbitrarily.

Let $\{s_1, i_1, i_1'\} \subseteq r_1$ and $\{s_2, i_2\} \subseteq r_2$: Such schools and students exist because regions are constructed from a base of regions. Consider a preference profile such that:

$$\succ_{i_1}: s_2, s_1, \quad \succ_{s_1}: i_2, i_1, i_1',$$
$$\succ_{i_1'}: s_2, s_1, \quad \succ_{s_2}: i_1, i_1', i_2,$$
$$\succ_{i_2}: s_1, s_2,$$

and the capacities of s_1 and s_2 are both one, while all other schools and students prefer $\emptyset$ the most.

By feasibility and the fact that $i_1, i_1' \notin r_2$, we have $\varphi_{i_1}(\succ, R) \neq s_2$ and $\varphi_{i_1'}(\succ, R) \neq s_2$. Similarly, we have $\varphi_{i_2}(\succ, R) \neq s_1$. These facts and the Pareto efficiency of φ imply $\varphi_{i_2}(\succ, R) = s_2$ and either $\varphi_{i_1}(\succ, R) = s_1$ or $\varphi_{i_1'}(\succ, R) = s_1$. Assume $\varphi_{i_1}(\succ, R) = s_1$—the proof for the case with $\varphi_{i_1'}(\succ, R) = s_1$ is symmetric.

Consider R'. Because of the monotonicity of φ and $r_1 \cup r_2 \subseteq r'$, it must be that $\varphi_{i_1}(\succ, R') \succeq_{i_1} \varphi_{i_1}(\succ, R) = s_1$ and $\varphi_{i_2}(\succ, R') \succeq_{i_2} \varphi_{i_2}(\succ, R) = s_2$. This and the Pareto efficiency of φ imply $\varphi_{i_1}(\succ, R') = s_2$ and $\varphi_{i_2}(\succ, R') = s_1$.

Now, consider another preference relation $\succ_{i_1}'$ of i_1 such that

$$\succ_{i_1}': s_2,$$

and let $\succ' := (\succ_{i_1}', \succ_{-i_1})$. Then, by the individual rationality of φ, we have $\varphi_{i_1}(\succ', R) = \emptyset$. This and Pareto efficiency of φ imply that $\varphi_{i_1'}(\succ', R) = s_1$ and $\varphi_{i_2}(\succ', R) = s_2$.

Now, consider R' again. Because of the monotonicity of φ and $r_1 \cup r_2 \subseteq r'$, it must be that $\varphi_{i_1'}(\succ', R') \succeq_{i_1'} \varphi_{i_1'}(\succ', R) = s_1$ and $\varphi_{i_2}(\succ', R') \succeq_{i_2} \varphi_{i_2}(\succ', R) = s_2$. This and the Pareto efficiency of φ imply $\varphi_{i_1'}(\succ', R') = s_2$ and $\varphi_{i_2}(\succ', R') = s_1$. Therefore, $\varphi_{i_1}(\succ', R') = \emptyset$.

Therefore, $\varphi_{i_1}(\succ, R') = s_2 \succ_{i_1}' \emptyset = \varphi_{i_1}(\succ', R')$, showing that φ is not strategy-proof. $\square$

This result demonstrates that every "good" mechanism lacks monotonicity. Specifically, as long as we require standard desiderata of strategy-proofness, Pareto efficiency, and individual rationality, the mechanism cannot be monotone. This result thus shows a limit to the policymakers aiming to achieve monotonicity.

One might wonder why we consider monotonicity for only "one side," i.e., students, while considering Pareto efficiency for "two sides," i.e., both students and schools. The answer is that those are the weaker conditions than the other alternatives. That is, the set of matchings satisfying (one-sided) monotonicity and (two-sided) Pareto efficiency is a superset of the set of matchings satisfying the two-sided version of monotonicity and the one-sided version of Pareto efficiency. Therefore, the impossibility result of Theorem 1 holds even if our (one-sided) monotonicity and/or (two-sided) Pareto efficiency are replaced with the two-sided version of monotonicity and/or the one-sided version of Pareto efficiency.

One might also wonder if it is more natural to require stability instead of Pareto efficiency. In response, we note that stability implies Pareto efficiency (recall that our Pareto efficiency is a "two-sided" notion). Therefore, the impossibility result of Theorem 1 holds when we replace Pareto efficiency with stability as well. As a consequence of imposing a weaker requirement, the result is not only applicable to stable mechanisms such as the DA mechanism but also to other standard ones such as the TTC mechanism.

We note that none of the conditions in Theorem 1 is extraneous: The DA mechanism and the TTC mechanism satisfy all conditions except for monotonicity. A mechanism that, for any region structure, produces the SOSM under the base of regions satisfies all conditions except for Pareto efficiency. A mechanism under which every student is matched to her first choice in her region satisfies all conditions except for individual rationality (for schools). A mechanism that satisfies all conditions except for strategy-proofness is analyzed in the next result. To do so, we begin by introducing a restriction on the region structures.

Definition 3. The region structures $\mathcal{Q}$ is **weakly hierarchical** if there exist no $R, R', R'' \in \mathcal{Q}$ such that there are $r \in R$, $r' \in R'$, and $r'' \in R''$ satisfying $r \cap r' \neq \emptyset$, $r \not\subseteq r'$, $r' \not\subseteq r$, and $r \cup r' \subseteq r''$.

Note that if $\mathcal{Q}$ satisfies the following property that we would call hierarchical, then it is also weakly hierarchical, hence the name. The property is that for all $R, R' \in \mathcal{Q}$, $r \in R$, and $r' \in R'$, we have $r \subseteq r'$, $r' \subseteq r$, or $r \cap r' = \emptyset$.

The main motivation for considering weak hierarchy is that it proves crucial in characterizing monotonicity. As such, we do not take a stance on whether weak hierarchy is a stringent requirement. Instead, we provide examples illustrating when regional structures are weakly hierarchical or not. For instance, if integration is possible only along an existing government structure, e.g., from districts within a municipality to the entire municipality, or from municipalities within a county to the entire county, then the region structures form a hierarchy, and thus a weak hierarchy. In contrast, suppose that there are three (mutually disjoint) municipalities A, B, and C, and A could be merged only with B or only with C or with both B and C. This case gives rise to region structures that are not weakly hierarchical. We note that, while weakly hierarchical region structures do not necessarily admit a merger or vice versa, any hierarchical region structures with cardinality of at least two admit a merger.

Theorem 2. *Fix an environment $(I, S, \mathcal{Q})$. There is a monotone mechanism that is Pareto efficient and individually rational if and only if $\mathcal{Q}$ is weakly hierarchical.*

The proof for the "only if" direction shows the contrapositive: It essentially considers regions A, B, and C in our previous example where student i in B and i' in C both like to be matched with school s in A that has the capacity of 1 (and they are mutually acceptable while there are no other acceptable student-school pairs). We then show that s must accommodate i when A is merged with B while it must accommodate i' when A is merged with C, but no mechanism can Pareto improve upon those matchings when all the three regions are merged.[16] The proof for the "if" direction is constructive, where we provide a procedure to compute the output of a mechanism. In the procedure, we start with the "smallest" regions and form a matching that respects Pareto efficiency and individual rationality for those regions. Then, we consider the "second smallest regions" and form a matching that improves upon the matchings formed for the smaller regions are respects Pareto efficiency. We "go up" in such a manner to consider larger and larger regions to form matchings for all regions, and this procedure is well defined when $\mathcal{Q}$ is weakly hierarchical. The formal proof is as follows.

Proof. "Only if" direction:

Consider a mechanism φ that is Pareto efficient and individually rational. We will show that φ is not monotone if $\mathcal{Q}$ is not weakly hierarchical.

Suppose that $\mathcal{Q}$ is not weakly hierarchical. Then, there must exist $R, R', R'' \in \mathcal{Q}$, $r \in R$, $r' \in R'$ and $r'' \in R''$ such that $r \setminus r'$, $r' \setminus r$ and $r \cap r'$ are all nonempty and $r \cup r' \subseteq r''$.

Take such (R, R', R'', r, r', r'') and take an arbitrary $s \in r \cap r' \cap S$, $i \in (r \setminus r') \cap I$ and $i' \in (r' \setminus r) \cap I$. Such a school and students exist because regions are constructed from a base of regions. Consider a preference profile such that:

$$\succ_i: s, \quad \succ_s: i, i',$$
$$\succ_{i'}: s,$$

and the capacity of school s is one, while all other schools and students prefer $\emptyset$ the most.

[16]This logic is reminiscent of the one in Chambers and Hayashi (2020) on the incompatibility between efficiency and their monotonicity requirement in exchange markets.

By feasibility, Pareto efficiency and the fact that $i, s \in r$ and $i', s \in r'$, we have $\varphi_i(\succ, R) = s$ and $\varphi_{i'}(\succ, R') = s$. However, since the capacity of s is one, the assumption that φ is individually rational implies that we must have either $\varphi_i(\succ, R'') \neq s$, which implies $\varphi_i(\succ, R'') = \emptyset$, or $\varphi_{i'}(\succ, R'') \neq s$, which implies $\varphi_{i'}(\succ, R'') = \emptyset$. This implies that either i is worse off under R'' compared to under R, or i' is worse off under R'' compared to under R'. Since $r \subseteq r''$ and $r' \subseteq r''$, this implies that φ is not monotone.

"If" direction:

Suppose that $\mathcal{Q}$ is weakly hierarchical. We construct a monotone mechanism φ that is Pareto efficient and individually rational.

For this purpose, let $\mathcal{R} = \bigcup_{R \in \mathcal{Q}} R$ and define a directed graph with the set of nodes being $\mathcal{R}$ and the set of edges being:

$$E = \{rr' | r, r' \in \mathcal{R}, r \subsetneq r' \text{ and } \nexists r'' \in \mathcal{R} \text{ s.t. } r \subsetneq r'' \subsetneq r'\}.$$

For every $r \in \mathcal{R}$, let $c(r)$ be the maximum length of a path in the graph that leads to r. Formally, $c : \mathcal{R} \to \{0\} \cup \mathbb{N}$ is a unique function that satisfies the following: (i) $c(r) = 0$ if there is no $\tilde{r} \in \mathcal{R}$ with $\tilde{r}r \in E$, and (ii) for any $r \in \mathcal{R}$ such that there is at least one $\tilde{r} \in \mathcal{R}$ with $\tilde{r}r \in E$,

$$c(r) = 1 + \max_{\tilde{r} \in \mathcal{R} \text{ s.t. } \tilde{r}r \in E} c(\tilde{r}).$$

Say that a matching μ is feasible for $r \in \mathcal{R}$ if $\mu_i \in r \cup \{\emptyset\}$ for every student $i \in r \cap I$ and $\mu_s \subseteq r$ for every school $s \in r \cap S$.

We define φ inductively as follows. Fix $\succ$.

Step 0: Consider r such that $c(r) = 0$. Take an arbitrary matching, denoted μ^r, that is feasible for r, Pareto efficient for r and individually rational.[17] (Such a matching exists because the set of all feasible and individually rational matchings is nonempty and finite.[18]) For every $a \in r$, we let $\varphi_a(\succ, R) = \mu^r_a$ for every $R \in \mathcal{Q}$ such that $r \in R$.

For any $n \geq 1$ such that there is $r \in \mathcal{R}$ such that $c(r) = n$, we define Step n as follows.

Step n: Consider r such that $c(r) = n$. Let $S(r) = \{\tilde{r} \in \mathcal{R} | \tilde{r}r \in E\}$. Since $\mathcal{Q}$ is weakly hierarchical, any two $\tilde{r}, \hat{r} \in S(r)$ are disjoint.

Consider a matching that is feasible for r, denoted by $\mu^{r,0}$, such that, for each $\tilde{r} \in S(r)$ and each $a \in \tilde{r}$, we set $\mu^{r,0}_a = \varphi_a(\succ, R)$ for some $R \in \mathcal{Q}$ satisfying $\tilde{r} \in R$ (the choice of

[17] Say that a matching is Pareto efficient for r if it is feasible for r and there exists no other matching μ' that is feasible for r such that $\mu'_a \succeq_a \mu_a$ for all $a \in r$.

[18] One way to find such a matching is to implement the DA mechanism for the students and schools in r.

R does not matter because $\varphi_a(\succ, R) = \varphi_a(\succ, R')$ for any $R, R' \in \mathcal{Q}$ satisfying $\tilde{r} \in R$ and $\tilde{r} \in R'$ from Steps $0, \dots, n-1$). Note that this $\mu_a^{r,0}$ is well defined due to Steps $0, \dots, n-1$ and the fact that any $\tilde{r}, \hat{r} \in S(r)$ are disjoint. Then, take an arbitrary matching, denoted μ^r, that is feasible for r and Pareto efficient for r and satisfies $\mu_a^r \succeq_a \mu_a^{r,0}$ for all $a \in I \cup S$ such that there exists $\tilde{r}$ with $a \in \tilde{r} \in S(r)$.[19] For every $a \in r$, we let $\varphi_a(\succ, R) = \mu_a^r$ for every $R \in \mathcal{Q}$ such that $r \in R$.

The above procedure pins down $\varphi_a(\succ, R)$ for all $a \in I \cup S$ and $R \in \mathcal{Q}$. Note that it follows from the construction that $\varphi(\succ, R)$ is a feasible matching and it is Pareto efficient. It is individually rational because at each n and any $r \in R$ such that $c(r) = n$, the matching $\mu^{r,0}$ is individually rational. Finally, φ is monotone because for any $r, r' \in \mathcal{R}$ such that $r \subsetneq r'$ and $i \in r \cap I$, the construction implies that there is a sequence $(r^1, \dots, r^K)$ for some K such that (i) $r^k \in \mathcal{R}$ for every $k = 1, \dots, K$, (ii) $rr^1, r^1r^2, \dots, r^{K-1}r^K, r^Kr' \in E$, and (iii) $\mu_i^{r'} \succeq_i \mu_i^{r^K} \succeq_i \cdots \succeq_i \mu^{r^1} \succeq_i \mu_i^r$.

This completes the proof. □

This result shows that there is a limit to monotonicity even when the requirement for strategy-proofness is lifted. Moreover, the result completely characterizes such a limit, providing a guidance to the policymaker about when one can guarantee an existence of a monotone mechanism that satisfies other desirable properties (for examples in which region structures are weakly hierarchical and not, respectively, refer to the illustrative paragraph subsequent to Definition 3).

4. When Is DA Monotone?

The preceding section showed senses in which monotonicity is hard to guarantee because of the competitive effect of integration. Intuitively, the effect of competition is present when schools in a given region prefer the students in other regions than the students in their own region. We investigate the validity of this intuition by considering a number of standard mechanisms in school choice. We find that this intuition is valid under the DA mechanism but not under other standard mechanisms.

We begin by defining basic concepts for this investigation.

Definition 4. Let $\succ_S$ be a profile of school preferences. A mechanism φ is **monotone at** $\succ_S$ if, for all $R, R' \in \mathcal{Q}$, $r \in R$, $r' \in R'$ such that $r \subseteq r'$, $i \in r \cap I$, and $\succ'$ such that $\succ'_S = \succ_S$, we have $\varphi_i(\succ', R') \succeq_i \varphi_i(\succ', R)$.

[19] Again, there is such a matching due to finiteness.

Definition 5. A school preference relation $\succ_s$ **favors locals** if there exist no $R, R' \in \mathcal{Q}, r \in R, r' \in R'$ with $s \in r \subseteq r'$, $i \in r \cap I$ with $i \succ_s \emptyset$, $I' \subseteq I$ with $I' \subseteq r'$, $I' \not\subseteq r$, and $|I'| = q_s$, such that $i' \succ_s i$ for all $i' \in I'$.

Intuitively, a school s fails to favor locals if a local student i is ranked lower than some non-local students in a manner that "matters" for matching. Specifically, we require that there be a set of competing students $I' \subseteq I$ such that (i) there are enough students in I' to fill the capacity of the school ($|I'| = q_s$), (ii) all students in I' are ranked higher by s than i ($i' \succ_s i$ for all $i' \in I'$), and (iii) some students in I' can compete for a seat with i only after the expansion of the region ($I' \subseteq r'$ and $I' \not\subseteq r$).

The main motivation for considering school preferences that favor locals is that it proves crucial in characterizing monotonicity of stable mechanisms. Just like for weak hierarchy, we do not take a stance on whether this condition is a stringent requirement but instead provide examples illustrating when school preferences favor locals or not. For instance, if schools are locally funded, as in public schools in the United States, "neighborhood priorities" are given to students over others, and it is often lexicographic in the sense that students from a school's neighborhood have higher priority over others irrespective of other characteristics of the students, resulting in school preferences that favor locals. In other cases, by contrast, priorities could be given to students who are not locals. For example, the assignment of daycare seats in Japan is conducted at the municipality level and it is so even in cases in which the municipality is a result of a merger of multiple municipalities: Japan experienced a large number of mergers in the early 21st century, with more than 3200 in 2001 to less than 1800 in 2011, and yet, to our knowledge, no municipalities provide priority depending on the applicant's original municipality of residence.

Theorem 3. *Fix an environment $(I, S, \mathcal{Q})$ and a profile of school preferences $\succ_S$. The following statements are equivalent.*

(1) *There exists a stable mechanism that is monotone at $\succ_S$.*
(2) *The DA mechanism is monotone at $\succ_S$.*
(3) *School preference relation $\succ_s$ favors locals for all $s \in S$.*

The intuition behind (1) implying (3) is that, if a school's preference relation does not favor locals, then the stability of a mechanism implies that the school may "kick out" some local students after a merger, violating monotonicity. To show that (3) implies (2), we consider the effect of expanding the region r to r' ($\supset r$) under the DA mechanism.

Specifically, we consider the effect to any student i in r of first adding students in $r' \setminus r$ and then adding schools in $r' \setminus r$. We argue that the former addition does not change the match for i as schools' preference relations favor locals, while the second addition make i weakly better off due to a comparative statics result by Crawford (1991). That is, expansion of the region does not intensify competition but expands choice. The formal proof of the theorem is as follows.

Proof. That (2) implies (1) is obvious. In the remainder, we will prove (1) $\Rightarrow$ (3) and (3) $\Rightarrow$ (2).

Proof of (1) $\Rightarrow$ (3): We begin by letting φ denote a stable mechanism. Suppose that there exists $s \in S$ such that $\succ_s$ does not favor locals. Then, there exist $R, R' \in \mathcal{Q}, r \in R, r' \in R'$ with $s \in r \subseteq r'$, $i \in r \cap I$ with $i \succ_s \emptyset$, $I' \subseteq r' \cap I$ with $|I'| = q_s$ and $I' \not\subseteq r$ such that $i' \succ_s i$ for all $i' \in I'$. Take such s, R, R', r, r', i, and I'.

Consider student preferences such that

(1) $s \succ_{i''} \emptyset \succ_{i''} s'$ for every $s' \in S \setminus s$ and $i'' \in \{i\} \cup I'$,

(2) $\emptyset \succ_{i''} s'$ for every $s' \in S$ and $i'' \in I \setminus (\{i\} \cup I')$.

First, consider R and region r. Because $|I' \cap r| \leq q_s - 1$ by $|I'| = q_s$ and $I' \not\subseteq r$, stability implies $\varphi_i(\succ, R) = s$. Next, consider R' and r'. Because $|I' \cap r'| = q_s$ and $i' \succ_s i$ for all $i' \in I'$, stability implies $\varphi_i(\succ, R') = \emptyset$. Therefore, we have shown $\varphi_i(\succ, R) = s \succ_i \emptyset = \varphi_i(\succ, R')$, so monotonicity is violated.

Proof of (3) $\Rightarrow$ (2): Suppose that $\succ_s$ favors locals for each $s \in S$, and let $R, R' \in \mathcal{Q}$, $r \in R$ and $r' \in R'$ be such that $r \subseteq r'$. First, consider the DA mechanism between all schools in r and all students in r'.[20] More specifically, consider a version of Gale and Shapley (1962)'s algorithm that outputs the outcome of the DA mechanism in which applications by students in $r' \setminus r$ are made only after all students in r either are tentatively matched or have been rejected by all schools that they find acceptable.[21] Note that, because $\succ_s$ favors locals for each $s \in r \cap S$, no student in r is rejected after students in $r' \setminus r$ begin to make applications.[22] Therefore, at the end of this algorithm, each student in r is matched

[20]Strictly speaking, we defined the DA mechanism only for each region structure R. However, it is straightforward to extend the definition to the one that operates between any set of students and any set of schools.

[21]We note that the outcome of Gale and Shapley (1962)'s algorithm does not depend on the order of applications (McVitie and Wilson, 1970).

[22]Otherwise, the condition in favoring locals is violated by setting I' as follows. Consider the first step after students in $r' \setminus r$ begin to make applications at which a student in r gets rejected, and let s be the school that made that rejection. Let I' be the set of all students that are tentatively accepted at s at that step.

to a school that she is matched with at the DA mechanism between all schools in r and all students in r. Now, because of the well-known comparative statics result that adding schools make students weakly better off under the DA mechanism (Crawford, 1991), the DA mechanism between all schools in r' and all students in r' places each student in r to a school that she weakly prefers, showing the monotonicity of the DA mechanism. □

The equivalence between (1) and (2) provides a certain justification of using the DA mechanism. Specifically, the set of school preference profiles at which the DA mechanism is monotone is no smaller than the set of school preferences at which there exists a stable and monotone mechanism. By contrast, some stable mechanisms may violate monotonicity even at school preferences at which the DA mechanism is monotone.[23]

The equivalence between (2) and (3) verifies the intuition that the negative effect of competition is caused precisely by schools that do not favor local students in the context of the DA mechanism.[24] Intuitively, when the school preferences favor locals, no student who is matched with her local school would be "kicked out" when students from other districts can make applications to the school.[25] Conversely, if school preferences do not favor locals, then there must be an instance where some students are kicked out when a region expands, violating monotonicity.

The conclusion of no kicking out when schools favor locals holds because we consider the DA mechanism, and indeed, other mechanisms may fail to have monotonicity even when schools favor locals. We discuss this point in Example 1 below as well as Examples 2-4 in the Appendix.

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[23]The following mechanism is not necessarily monotone even at such school preferences, i.e., even when school preferences favor locals (see the equivalence between (2) and (3)) : it produces the same outcome as the DA mechanism under some region structure while producing the outcome of the school-proposing deferred acceptance algorithm under another region structure.

[24] This result is reminiscent of a result by Hafalir, Kojima and Yenmez (2022), who find a condition under which integration makes every student weakly better off under the DA mechanism. Their setting, however, differs from ours in that they endow each region with a choice function and only consider region structures where all regions are separate or all regions are merged, which makes their result logically unrelated to ours. We also note that their study restricts attention to the DA mechanism, and thus contains no analogue of the equivalences between (1) and (2) or between (1) and (3).

[25]Being consistent with this intuition, the proof of (3) $\Rightarrow$ (2) in fact shows the following stronger result: For any $R \in \mathcal{Q}$ and $r \in R$, if school preference relation $\succ_s$ favors locals for all $s \in r$, then for any student $i \in r$ and a region structure R', the relation $r \subseteq r' \in R'$ implies that the DA mechanism gives i a weakly more preferred match under R' than under R.

For the TTC mechanism, the same proof as the one for "(1) $\Rightarrow$ (3)" of Theorem 3 shows that it is not monotone if $\succ_S$ does not favor locals. It is not necessarily monotone, however, even if $\succ_S$ favors locals.

Example 1 (Non-monotonicity of TTC under favoring locals)**.** Let $I = \{i_1, i_2, i'\}, S = \{s, s'\}$, $\mathcal{Q} = \{R, R'\}$, $R = \{r, r'\}$ where $r = \{i_1, i_2, s\}, r' = \{i', s'\}$, $R' = \{r \cup r'\}$.[26] Let

$$\succ_{i_1}: s', \qquad \succ_s: i_1, i_2, i',$$
$$\succ_{i_2}: s, \qquad \succ_{s'}: i', i_1, i_2,$$
$$\succ_{i'}: s, s'$$

where each school's capacity is 1. Note that $\succ_S$ favors locals. Under this preference profile, the TTC mechanism returns

$$\begin{pmatrix} s & s' & \emptyset \\ i_2 & i' & i_1, \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} s & s' & \emptyset \\ i' & i_1 & i_2 \end{pmatrix},$$

under region structures R and R', respectively. Hence, in particular, student i_2 is assigned to s under R and she is unmatched under R', while $s \succ_{i_2} \emptyset$. Thus, monotonicity is violated.

To get the intuition, first note that monotonicity of a mechanism may fail when merging regions result in a situation where a student who could secure a seat at a local school before a merger is displaced by some non-local student after the merger. Such displacement does not occur under the DA mechanism if school preference favor locals. However, the TTC mechanism is not stable, so such displacement may happen even if school preferences favor locals. In fact, other unstable mechanisms such as the Boston mechanism (Abdulkadiroğlu and Sönmez, 2003), the serial dictatorship, and the efficiency-adjusted DA mechanism (where every student consents; Kesten (2009)) also fail to be monotone even if school preferences favor locals for the same reason.[27] Overall, the intuition that the negative effect of competition is only caused by schools not favoring their local students is valid under the DA mechanism but not under other standard (but unstable) mechanisms.[28]

[26] Strictly speaking, our model assumes that each region is constructed from a base of regions and hence contains at least two students, a condition violated by r'. This is just for expositional simplicity, and it is straightforward to modify the present example such that the condition is satisfied.

[27] Specific examples are presented in the Appendix.

[28] We, however, note that there exist unstable mechanisms that are monotone when all schools' preferences favor locals. For example, a mechanism that treats a pre-determined subset of students $I' \subsetneq I$ as the set of all students and returns the output of the DA mechanism would do.

5. Conclusion

The present paper investigated the scope for welfare gain in integrating fragmented matching markets, identifying possible barriers against integration. Our analysis revealed difficulties with integrating markets in a "monotone" manner in the sense of leaving no students worse off. Specifically, we found that whether there exists a monotone mechanism depends on other properties of mechanisms to be required, the class of possible region structures, and school preferences. Further investigations are in order so as to better understand the implications of integrations as well as desirable matching mechanisms in the face of barriers to integration.

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Appendix A. Boston Mechanism, Serial Dictatorship, and Efficiency-Adjusted DA mechanism

We provide examples in which the Boston mechanism, the serial dictatorship and the efficiency-adjusted DA mechanism fail to be monotone although school preferences favor locals. The intuition for the lack of monotonicity is essentially the same as for the TTC mechanism (Example 1).

Example 2 (Non-monotonicity of Boston under favoring locals)**.** Let $I = \{i_1, i_2, i'\}, S = \{s_1, s_2, s'\}$, $\mathcal{Q} = \{R, R'\}$, $R = \{r, r'\}$ where $r = \{i_1, i_2, s_1, s_2\}, r' = \{i', s'\}$, $R' = \{r \cup r'\}$.[29] Let

$$
\begin{aligned}
&\succ_{i_1}: s_1, s_2 && \succ_{s_1}: i_1, i_2, i', \\
&\succ_{i_2}: s_1, s_2 && \succ_{s_2}: i_1, i_2, i', \\
&\succ_{i'}: s_2, s' && \succ_{s'}: i', i_1, i_2,
\end{aligned}
$$

where each school's capacity is 1. Note that $\succ_S$ favors locals. Under this preference profile, the Boston mechanism returns

$$
\begin{pmatrix} s_1 & s_2 & s' \\ i_1 & i_2 & i' \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} s_1 & s_2 & s' & \emptyset \\ i_1 & i' & \emptyset & i_2 \end{pmatrix},
$$

under region structures R and R', respectively. Hence, in particular, student i_2 is assigned to s_2 under R and she is unmatched under R', while $s_2 \succ_{i_2} \emptyset$. Thus, monotonicity is violated.

Example 3 (Non-monotonicity of serial dictatorship under favoring locals)**.** Let $I = \{i, i'\}, S = \{s, s'\}$, $\mathcal{Q} = \{R, R'\}$, $R = \{r, r'\}$ where $r = \{i, s\}, r' = \{i', s'\}$, $R' = \{r \cup r'\}$.[30] Let

$$
\begin{aligned}
&\succ_{i}: s && \succ_{s}: i, i', \\
&\succ_{i'}: s, s' && \succ_{s'}: i', i,
\end{aligned}
$$

where each school's capacity is 1. Note that $\succ_S$ favors locals. Consider serial dictatorship such that the serial order is i', i. Under this preference profile, the serial dictatorship returns

$$
\begin{pmatrix} s & s' & \emptyset \\ i & i' & \emptyset \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} s & s' & \emptyset \\ i' & \emptyset & i \end{pmatrix},
$$

[29] The same remark as in footnote 26 applies to this example as well.

[30] Again, the same remark as in footnote 26 applies to this example as well.

under region structures R and R', respectively. Hence, in particular, student i is assigned to s under R and she is unmatched at R', while $s \succ_i \emptyset$. Thus, monotonicity is violated.

Example 4 (Non-monotonicity of efficiency-adjusted DA mechanism under favoring locals)**.** Let $I = \{i_1, i_2, i_3\}, S = \{s_1, s_2, s'\}$, $\mathcal{Q} = \{R, R'\}$, $R = \{r, r'\}$ where $r = \{i_1, i_2, i_3, s_1, s_2\}, r' = \{s'\}$, $R' = \{r \cup r'\}$.[31] Let

$$\succ_{i_1}: s_2, s', s_1 \qquad\qquad \succ_{s_1}: i_1, i_3, i_2,$$

$$\succ_{i_2}: s_1, s_2 \qquad\qquad \succ_{s_2}: i_2, i_1,$$

$$\succ_{i_3}: s_1 \qquad\qquad \succ_{s'}: i_1,$$

where each schools' capacity is 1. Note that $\succ_S$ favors locals. Under this preference profile, the efficiency-adjusted DA mechanism (where every student consents) returns

$$\begin{pmatrix} s_1 & s_2 & s' & \emptyset \\ i_2 & i_1 & \emptyset & i_3 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} s_1 & s_2 & s' & \emptyset \\ i_3 & i_2 & i_1 & \emptyset \end{pmatrix},$$

under region structures R and R', respectively. Hence, in particular, student i_2 is assigned to s_1 under R and she is assigned to s_2 at R', while $s_2 \succ_{i_2} s_1$. Thus, monotonicity is violated.

The intuition for this example is as follows. Under R, the outcome of the DA mechanism is

$$\begin{pmatrix} s_1 & s_2 & s' & \emptyset \\ i_1 & i_2 & \emptyset & i_3 \end{pmatrix},$$

where i_3 is an interrupter for school s_1: i_3 makes s_1 reject i_2 and then i_1 makes s_1 reject i_3. The efficiency-adjusted DA mechanism then produces the outcome of the DA mechanism in the market without i_3. In particular, this improves the match for i_2 from s_2 to s_1. When the region expands and school s' becomes available, i_1 never applies to s_1 because she applies to and settles at s'. Hence, i_1 does not make s_1 reject i_3, and so i_3 is no longer regarded as an interrupter. This prevents the improvement that happened under R, and in particular i_2's match does not improve from s_2.

[31] Again, the same remark as in footnote 26 applies to this example as well.