Informed Intermediaries – Online Appendix

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1. Appendix - Proof of Proposition 2 (cont.)

1.1. Guesses.
Surplus sharing:

$$\beta(I, v_s, U, v_b) = \beta^I \geqslant \frac{1}{2} \qquad \beta(U, v_s, I, v_b) = 1 - \beta^I \leqslant \frac{1}{2}$$

$$\beta(I, v_s, I, v_b) = \frac{1}{2}$$

Efficient trading iff an informed agent is involved:

$$\mathcal{I}(i_s, v_s, i_b, v_b) = 1 \Leftrightarrow (i_s, i_b) \neq (U, U) \text{ and } V^{i_s}_{v_s 0} - V^{i_s}_{v_s 1} + V^{i_b}_{v_b 1} - V^{i_b}_{v_b 0} > 0.$$

1.2. Stationary Distribution.
Given the guesses for $\mathcal{I}$, the inflow equal to outflow equations for the stationary distribution become:

$$\mu^U_{L1}\left(\eta + \lambda(\mu^I_{H0} + \mu^I_{L0})\right) = \eta\mu^U_{L0} \tag{1}$$

$$\mu^U_{H0}\left(\eta + \lambda(\mu^I_{L1} + \mu^I_{H1})\right) = \eta\mu^U_{H1} \tag{2}$$

$$\mu^I_{L1}\left(\eta + \lambda(\mu^U_{H0} + \mu^I_{H0})\right) = (\eta + \lambda\mu^U_{L1})\mu^I_{L0} \tag{3}$$

$$\mu^I_{H0}\left(\eta + \lambda(\mu^U_{L1} + \mu^I_{L1})\right) = (\eta + \lambda\mu^U_{H0})\mu^I_{H1} \tag{4}$$

Combine (1) and (2), and using $\mu^U_{L1} + \mu^U_{L0} = \mu^U_{H0} + \mu^U_{H1} = \frac{1-\phi}{2}$ (since the half the uninformed agents have high valuation and half have low valuation), I get:

$$\mu^U_{L1}(2\eta + \lambda(\mu^I_{H0} + \mu^I_{L0})) = \mu^U_{H0}(2\eta + \lambda(\mu^I_{L1} + \mu^I_{H1})) = \frac{\eta(1-\phi)}{2} \tag{5}$$

Similarly use (3) and (4) and $\mu^I_{L1} + \mu^I_{L0} = \mu^I_{H0} + \mu^I_{H1}$ to get:

$$(\eta + \lambda\mu^U_{L1})(\mu^I_{H0} + \mu^I_{L0}) = (\eta + \lambda\mu^U_{H0})(\mu^I_{H1} + \mu^I_{L1}) \tag{6}$$

Use (5) and (6) to get $\mu^U_{L1} + \mu^I_{L1} + \mu^I_{H1} = \mu^U_{H0} + \mu^I_{H0} + \mu^I_{L0}$. Combining this with (5) yet again, conclude that $\mu^U_{L1} = \mu^U_{H0} \equiv \hat{\mu}^U$. This, along with (6), implies $(\mu^I_{H0} + \mu^I_{L0}) = (\mu^I_{L1} + \mu^I_{H1})$;

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and hence $\mu^I_{H0} = \mu^I_{L1} \equiv \hat{\mu}^I$. Rewrite the inflow equals outflow conditions now as:

$$\hat{\mu}^U(\eta + \lambda(\phi/2)) = \eta\frac{(1-\phi-2\hat{\mu}^U)}{2} \qquad \hat{\mu}^I(\eta + \lambda(\hat{\mu}^U + \hat{\mu}^I)) = \frac{\phi - 2\hat{\mu}^I}{2}(\eta + \lambda\hat{\mu}^U)$$

Solving these, I get:

$$\hat{\mu}^U = \frac{1-\phi}{4+\frac{\lambda}{\eta}} \qquad \hat{\mu}^I = -\frac{\eta+\lambda\hat{\mu}^U}{\lambda} + \sqrt{\left(\frac{\eta+\lambda\hat{\mu}^U}{\lambda}\right)^2 + \frac{\phi}{2}\frac{\eta+\lambda\hat{\mu}^U}{\lambda}}$$

1.3. **Unflagged Values.** Again taking into account the guesses for $\mathcal{I}$ and β, unflagged values are given by the system below

$$rV^I_{H0} = \eta(V^I_{H1} - V^I_{H0}) + \lambda\hat{\mu}^I\frac{V^I_{H1} - V^I_{H0} + V^I_{L0} - V^I_{L1}}{2} + \lambda\hat{\mu}^U\beta^I\left(V^I_{H1} - V^I_{H0} + V^U_{L0} - V^U_{L1}\right)$$

$$rV^I_{H1} = \delta_H + \eta(V^I_{H0} - V^I_{H1}) + \lambda\hat{\mu}^U\beta^I\left(V^I_{H0} - V^I_{H1} + V^U_{H1} - V^U_{H0}\right)$$

$$rV^I_{L1} = \delta_L + \eta(V^I_{L0} - V^I_{L1}) + \lambda\hat{\mu}^I\frac{V^I_{L0} - V^I_{L1} + V^I_{H1} - V^I_{H0}}{2} + \lambda\hat{\mu}^U\beta^I\left(V^I_{L0} - V^I_{L1} + V^U_{H1} - V^U_{H0}\right)$$

$$rV^I_{L0} = \eta(V^I_{L1} - V^I_{L0}) + \lambda\hat{\mu}^U\beta^I\left(V^I_{L1} - V^I_{00} + V^U_{L0} - V^U_{L1}\right)$$

$$\begin{aligned} rV^U_{H0} = \eta(V^U_{H1} - V^U_{H0}) &+ \lambda\hat{\mu}^I(1-\beta^I)\left(V^U_{H1} - V^U_{H0} + V^I_{L0} - V^I_{L1}\right) \\ &+ \lambda\hat{\mu}^I(1-\beta^I)\left(V^U_{H1} - V^U_{H0} + V^I_{H0} - V^I_{H1}\right) \end{aligned}$$

$$rV^U_{H1} = \delta_H + \eta(V^U_{H0} - V^U_{H1})$$

$$\begin{aligned} rV^U_{L1} = \delta_L + \eta(V^U_{L0} - V^U_{L1}) &+ \lambda\hat{\mu}^I(1-\beta^I)\left(V^U_{L0} - V^U_{L1} + V^I_{H1} - V^I_{H0}\right) \\ &+ \lambda\hat{\mu}^I(1-\beta^I)\left(V^U_{L0} - V^U_{L1} + V^I_{L1} - V^I_{L0}\right) \end{aligned}$$

$$rV^U_{L0} = \eta(V^U_{L1} - V^U_{L0})$$

In terms of the values of holding an asset, this system becomes:

$$\begin{aligned}
rS_H^I &= \delta_H - 2\eta S_H^I + \lambda\hat{\mu}^U\beta^I(S_H^U - S_H^I) + \lambda\hat{\mu}^I\frac{(S_L^I - S_H^I)}{2} + \lambda\hat{\mu}^U\beta^I(S_L^U - S_H^I) \\
rS_L^I &= \delta_L - 2\eta S_L^I + \lambda\hat{\mu}^U\beta^I(S_L^U - S_L^I) + \lambda\hat{\mu}^I\frac{(S_H^I - S_L^I)}{2} + \lambda\hat{\mu}^U\beta^I(S_H^U - S_L^I) \\
rS_H^U &= \delta_H - 2\eta S_H^U + \lambda\hat{\mu}^I(1-\beta^I)(S_L^I - S_H^U) + \lambda\frac{\phi - 2\hat{\mu}^I}{2}(1-\beta^I)(S_H^I - S_H^U) \\
rS_L^U &= \delta_L - 2\eta S_L^U + \lambda\hat{\mu}^I(1-\beta^I)(S_H^I - S_L^U) + \lambda\frac{\phi - 2\hat{\mu}^I}{2}(1-\beta^I)(S_L^I - S_L^U)
\end{aligned}$$

Add up the first two and the last two to get:

$$\begin{aligned}
(r+2\eta)(S_H^I + S_L^I) &= \delta_H + \delta_L + 2\lambda\hat{\mu}^U\beta^I(S_H^U + S_L^U) - 2\lambda\hat{\mu}^U\beta^I(S_H^I + S_L^I) \\
(r+2\eta)(S_H^U + S_L^U) &= \delta_H + \delta_L + \frac{\lambda\phi}{2}(1-\beta^I)(S_H^I + S_L^I) - \frac{\lambda\phi}{2}(1-\beta^I)(S_H^U + S_L^U)
\end{aligned}$$

These imply $(S_H^I + S_L^I) = (S_H^U + S_L^U) = \frac{\delta_H + \delta_L}{r+2\eta}$. Now from the original system, subtract the second equation from the first and the fourth from the third to find:

$$\begin{aligned}
(r+2\eta)(S_H^I - S_L^I) &= \delta_H - \delta_L - \lambda(2\hat{\mu}^U\beta^I + \hat{\mu}^I)(S_H^I - S_L^I) \\
(r+2\eta)(S_H^U - S_L^U) &= \delta_H - \delta_L - \frac{\lambda\phi}{2}(1-\beta^I)(S_H^U - S_L^U) + \left(\frac{\lambda\phi}{2} - \hat{\mu}^I\right)(1-\beta^I)(S_H^I - S_L^I)
\end{aligned}$$

Rearrange these to get the following expressions:

$$\begin{aligned}
&(S_H^I - S_L^I) = \hat{\alpha}^I(\delta_H - \delta_L) \\
&(S_H^I - S_L^I) = \hat{\alpha}^U(\delta_H - \delta_L) \\
&\text{where } \hat{\alpha}^I = \frac{1}{2(r + 2\eta + \lambda(2\hat{\mu}^U\beta^I + \hat{\mu}^I))} \\
&\text{and } \alpha^U = \hat{\alpha}^U = \left[\frac{r + 2\eta + \lambda\hat{\mu}^U + \frac{\lambda\phi}{4}}{r + 2\eta + \frac{\lambda\phi}{4}}\right]\hat{\alpha}^I
\end{aligned}$$

Which finally implies:

$$\begin{aligned}
S_H^i &= \left[\frac{1}{2(r+2\eta)}\right](\delta_H + \delta_L) + \frac{\hat{\alpha}^i}{2}(\delta_H - \delta_L) \\
S_L^i &= \left[\frac{1}{2(r+2\eta)}\right](\delta_H + \delta_L) - \frac{\hat{\alpha}^i}{2}(\delta_H - \delta_L)
\end{aligned}$$

1.4. **Flagged Values.** I solve for D^I_{H1}, D^I_{L1} and D^U_{H1}. The system defining $\left\{D^I_{va}\right\}$ is:

$$rD^I_{H1} = (\eta + \lambda\hat{\mu}^U)(D^I_{H0} - D^I_{H1})$$

$$rD^I_{L1} = (\eta + \lambda\hat{\mu}^U)(D^I_{L0} - D^I_{L1}) + \lambda\hat{\mu}^I \frac{S^I_H - S^I_L}{2}$$

$$rD^I_{L0} = (\eta + \lambda\hat{\mu}^U)(D^I_{L1} - D^I_{L0}) - \lambda\hat{\mu}^U \left(\beta^I S^U_L + (1-\beta^I)S^I_L\right)$$

$$rD^I_{H0} = (\eta + \lambda\hat{\mu}^U)(D^I_{H1} - D^I_{H0}) + \lambda\hat{\mu}^I \frac{S^I_H - S^I_L}{2} - \lambda\hat{\mu}^U \left(\beta^I S^I_H + (1-\beta^I)S^U_L\right)$$

Combining the first with the fourth and the second with the third:

$$r(D^I_{H1} - D^I_{H0}) = -2(\eta + \lambda\hat{\mu}^U)(D^I_{H1} - D^I_{H0}) - \lambda\hat{\mu}^I \frac{S^I_H - S^I_L}{2} + \lambda\hat{\mu}^U \left(\beta^I S^I_H + (1-\beta^I)S^U_L\right)$$

$$r(D^I_{L1} - D^I_{L0}) = -2(\eta + \lambda\hat{\mu}^U)(D^I_{L1} - D^I_{L0}) + \lambda\hat{\mu}^I \frac{S^I_H - S^I_L}{2} + \lambda\hat{\mu}^U \left(\beta^I S^U_L + (1-\beta^I)S^I_L\right)$$

Solve to find:

$$(D^I_{H1} - D^I_{H0}) = -\frac{\lambda\hat{\mu}^I}{(r + 2\eta + 2\lambda\hat{\mu}^U)} \frac{S^I_H - S^I_L}{2} + \frac{\lambda\hat{\mu}^U}{(r + 2\eta + 2\lambda\hat{\mu}^U)} \left(\beta^I S^I_H + (1-\beta^I)S^U_L\right)$$
$$(D^I_{L1} - D^I_{L0}) = \frac{\lambda\hat{\mu}^I}{(r + 2\eta + 2\lambda\hat{\mu}^U)} \frac{S^I_H - S^I_L}{2} + \frac{\lambda\hat{\mu}^U}{(r + 2\eta + 2\lambda\hat{\mu}^U)} \left(\beta^I S^U_L + (1-\beta^I)S^I_L\right)$$

Plug this back into the original system to get:

$$D^I_{H1} = \frac{\eta + \lambda\hat{\mu}^U}{r(r + 2\eta + 2\lambda\hat{\mu}^U)} \left[\lambda\hat{\mu}^I \frac{S^I_H - S^I_L}{2} - \lambda\hat{\mu}^U \left(\beta^I S^I_H + (1-\beta^I)S^U_L\right)\right] \tag{7}$$

$$D^I_{L1} = \frac{r + \eta + \lambda\hat{\mu}^U}{r(r + 2\eta + 2\lambda\hat{\mu}^U)} \lambda\hat{\mu}^I \frac{S^I_H - S^I_L}{2} - \frac{\lambda\hat{\mu}^U(\eta + \lambda\hat{\mu}^U)}{r(r + 2\eta + 2\lambda\hat{\mu}^U)} \left(\beta^I S^U_L + (1-\beta^I)S^I_L\right) \tag{8}$$

The system defining $\left\{D^U_{va}\right\}$ is:

$$
\begin{aligned}
rD^U_{H1} &= \eta(D^U_{H0} - D^U_{H1}) \\
rD^U_{H0} &= \eta(D^U_{H1} - D^U_{H0}) + \lambda\hat{\mu}^I(1-\beta^I)(S^U_H - S^I_L) + \lambda(\phi/2 - \hat{\mu}^I)(1-\beta^I)(S^U_H - S^I_H) \\
rD^U_{L1} &= \eta(D^U_{L0} - D^U_{L1}) + \lambda\hat{\mu}^I(1-\beta^I)(S^I_H - S^U_L) + \lambda(\phi/2 - \hat{\mu}^I)(1-\beta^I)(S^I_L - S^U_L) \\
rD^U_{L0} &= \eta(D^U_{L1} - D^U_{L0})
\end{aligned}
$$

Subtract the second from the first to get:

$$
r(D^U_{H1} - D^U_{H0}) = -2\eta(D^U_{H1} - D^U_{H0}) - \lambda\hat{\mu}^I(1-\beta^I)(S^U_H - S^I_L) - \lambda(\phi/2 - \hat{\mu}^I)(1-\beta^I)(S^U_H - S^I_H)
$$

$$
\Rightarrow (D^U_{H1} - D^U_{H0}) = -\frac{\lambda\hat{\mu}^I(1-\beta^I)}{r+2\eta}(S^U_H - S^I_L) - \frac{\lambda(\phi/2 - \hat{\mu}^I)(1-\beta^I)}{r+2\eta}(S^U_H - S^I_H)
$$

Finally plug back into the original system to get:

$$
D^U_{H1} = \frac{\eta\lambda\hat{\mu}^I(1-\beta^I)}{r(r+2\eta)}(S^U_H - S^I_L) + \frac{\eta\lambda(\phi/2 - \hat{\mu}^I)(1-\beta^I)}{r(r+2\eta)}(S^U_H - S^I_H) \tag{9}
$$