Informed Intermediaries – Online Appendix

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1. APPENDIX - PROOF OF PROPOSITION 2 (CONT.)

1.1. Guesses. Surplus sharing:

\[
\beta(I, v_s, U, v_b) = \beta^I \geq \frac{1}{2} \quad \beta(U, v_s, I, v_b) = 1 - \beta^I \leq \frac{1}{2}
\]

\[
\beta(I, v_s, I, v_b) = \frac{1}{2}
\]

Efficient trading iff an informed agent is involved:

\[
\mathcal{I}(i_s, v_s, i_b, v_b) = 1 \iff (i_s, i_b) \neq (U, U) \text{ and } V^i_{v_s0} - V^i_{v_b1} + V^i_{v_s1} - V^i_{v_b0} > 0.
\]

1.2. Stationary Distribution. Given the guesses for \( \mathcal{I} \), the inflow equal to outflow equations for the stationary distribution become:

(1) \[ \mu^U_{L1}(\eta + \lambda(\mu^U_{H0} + \mu^U_{L0})) = \eta\mu^U_{L0} \]

(2) \[ \mu^U_{H0}(\eta + \lambda(\mu^U_{L1} + \mu^U_{H1})) = \eta\mu^U_{H1} \]

(3) \[ \mu^U_{L1}(\eta + \lambda(\mu^U_{H0} + \mu^U_{H1})) = (\eta + \lambda\mu^U_{L1})\mu^U_{L0} \]

(4) \[ \mu^U_{H0}(\eta + \lambda(\mu^U_{L1} + \mu^U_{H1})) = (\eta + \lambda\mu^U_{H0})\mu^U_{H1} \]

Combine (1) and (2), and using \( \mu^U_{L1} + \mu^U_{L0} = \mu^U_{H0} + \mu^U_{H1} = \frac{1-\phi}{2} \) (since the half the uninformed agents have high valuation and half have low valuation), I get:

(5) \[ \mu^U_{L1}(2\eta + \lambda(\mu^U_{H0} + \mu^U_{L0})) = \mu^U_{H0}(2\eta + \lambda(\mu^U_{L1} + \mu^U_{H1})) = \frac{\eta(1-\phi)}{2} \]

Similarly use (3) and (4) and \( \mu^I_{L1} + \mu^I_{L0} = \mu^I_{H0} + \mu^I_{H1} \) to get:

(6) \[ (\eta + \lambda\mu^U_{L1})(\mu^I_{H0} + \mu^I_{L0}) = (\eta + \lambda\mu^U_{H0})(\mu^I_{H1} + \mu^I_{L1}) \]

Use (5) and (6) to get \( \mu^U_{L1} + \mu^U_{L1} + \mu^U_{H1} = \mu^U_{H0} + \mu^U_{H0} + \mu^U_{L0} \). Combining this with (5) yet again, conclude that \( \mu^U_{L1} = \mu^U_{H0} \equiv \mu^U \). This, along with (6), implies \( \mu^I_{H0} + \mu^I_{L0} = (\mu^I_{L1} + \mu^I_{H1}) \);

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and hence \( \mu_{H0}^{I} = \mu_{L1}^{I} \equiv \hat{\mu}^{I} \). Rewrite the inflow equals outflow conditions now as:

\[
\hat{\mu}^{U}(\eta + \lambda(\phi/2)) = \eta \left(1 - \phi - 2\hat{\mu}^{U}\right) / 2 \quad \hat{\mu}^{I}(\eta + \lambda(\hat{\mu}^{U} + \hat{\mu}^{I})) = \frac{\phi - 2\hat{\mu}^{I}}{2}(\eta + \lambda\hat{\mu}^{U})
\]

Solving these, I get:

\[
\hat{\mu}^{U} = \frac{1 - \phi}{4 + \frac{\lambda}{\eta}} \quad \hat{\mu}^{I} = -\frac{\eta + \lambda\hat{\mu}^{U}}{\lambda} + \sqrt{\left(\frac{\eta + \lambda\hat{\mu}^{U}}{\lambda}\right)^2 + \frac{\phi \eta + \lambda\hat{\mu}^{U}}{2}}
\]

1.3. Unflagged Values. Again taking into account the guesses for \( I \) and \( \beta \), unflagged values are given by the system below:

\[
rV_{H0}^{I} = \eta(V_{H1}^{I} - V_{H0}^{I}) + \lambda\hat{\mu}^{I}V_{H1}^{I} - V_{H0}^{I} + V_{L0}^{I} - V_{L1}^{I} / 2 + \lambda\hat{\mu}^{U}\beta^{I} (V_{H1}^{I} - V_{H0}^{I} + V_{L0}^{U} - V_{L1}^{U})
\]

\[
rV_{H1}^{I} = \delta_{H} \eta (V_{H0}^{I} - V_{H1}^{I}) + \lambda\hat{\mu}^{U}\beta^{I} (V_{H0}^{I} - V_{H1}^{I} + V_{H1}^{U} - V_{H0}^{U})
\]

\[
rV_{L1}^{I} = \delta_{L} \eta (V_{L0}^{I} - V_{L1}^{I}) + \lambda\hat{\mu}^{I}V_{L0}^{I} - V_{L1}^{I} + V_{H1}^{I} - V_{H0}^{I} / 2 + \lambda\hat{\mu}^{U}\beta^{I} (V_{L0}^{I} - V_{L1}^{I} + V_{H1}^{U} - V_{H0}^{U})
\]

\[
rV_{L0}^{I} = \eta(V_{L1}^{I} - V_{L0}^{I}) + \lambda\hat{\mu}^{U}\beta^{I} (V_{L1}^{I} - V_{00}^{I} + V_{L0}^{U} - V_{L1}^{U})
\]

\[
rV_{H0}^{U} = \eta(V_{H1}^{U} - V_{H0}^{U}) + \lambda\hat{\mu}^{I}(1 - \beta^{I}) (V_{H1}^{U} - V_{H0}^{U} + V_{L0}^{I} - V_{L1}^{I}) + \lambda\hat{\mu}^{I}(1 - \beta^{I}) (V_{H1}^{U} - V_{H0}^{U} + V_{H0}^{I} - V_{H1}^{I})
\]

\[
rV_{H1}^{U} = \delta_{H} \eta (V_{H0}^{U} - V_{H1}^{U})
\]

\[
rV_{L1}^{U} = \delta_{L} \eta (V_{L0}^{U} - V_{L1}^{U}) + \lambda\hat{\mu}^{I}(1 - \beta^{I}) (V_{L0}^{U} - V_{L1}^{U} + V_{H1}^{U} - V_{H0}^{U}) + \lambda\hat{\mu}^{I}(1 - \beta^{I}) (V_{L0}^{U} - V_{L1}^{U} + V_{L1}^{U} - V_{L0}^{U})
\]

\[
rV_{L0}^{U} = \eta(V_{L1}^{U} - V_{L0}^{U})
\]
In terms of the values of holding an asset, this system becomes:

\[
\begin{align*}
    rS_H^i &= \delta_H - 2\eta S_H^i + \lambda \mu U^i \beta^I (S_U^i - S_H^i) + \lambda \mu I (S_L^i - S_H^i) \\
    rS_L^i &= \delta_L - 2\eta S_L^i + \lambda \mu U^i \beta^I (S_U^i - S_L^i) + \lambda \mu I (S_H^i - S_L^i) \\
    rS_H^U &= \delta_H - 2\eta S_H^U + \lambda \mu I (1 - \beta^I)(S_L^i - S_H^i) + \lambda \mu U^I (1 - \beta^I)(S_U^i - S_H^i) \\
    rS_L^U &= \delta_L - 2\eta S_L^U + \lambda \mu I (1 - \beta^I)(S_H^i - S_L^i) + \lambda \mu U^I (1 - \beta^I)(S_L^i - S_U^i)
\end{align*}
\]

Add up the first two and the last two to get:

\[
\begin{align*}
    (r + 2\eta)(S_H^i + S_L^i) &= \delta_H + \delta_L + 2\lambda \mu U^i \beta^I (S_U^i + S_L^i) - 2\lambda \mu U^I (S_H^i + S_L^i) \\
    (r + 2\eta)(S_H^U + S_L^U) &= \delta_H + \delta_L + \frac{\phi}{2}(1 - \beta^I)(S_H^i + S_L^i) - \frac{\phi}{2}(1 - \beta^I)(S_U^i + S_L^U)
\end{align*}
\]

These imply \((S_H^i + S_L^i) = (S_H^U + S_L^U) = \frac{\delta_H + \delta_L}{r + 2\eta}\). Now from the original system, subtract the second equation from the first and the fourth from the third to find:

\[
\begin{align*}
    (r + 2\eta)(S_H^i - S_L^i) &= \delta_H - \delta_L - \lambda(2\mu U^I \beta^I + \mu I)(S_H^i - S_L^i) \\
    (r + 2\eta)(S_H^U - S_L^U) &= \delta_H - \delta_L - \frac{\lambda \phi}{2}(1 - \beta^I)(S_H^i - S_L^i) + \left(\frac{\lambda \phi}{2} - \mu I\right)(1 - \beta^I)(S_H^i - S_L^i)
\end{align*}
\]

Rearrange these to get the following expressions:

\[
\begin{align*}
    (S_H^i - S_L^i) &= \hat{\alpha}^I (\delta_H - \delta_L) \\
    (S_H^U - S_L^U) &= \hat{\alpha}^U (\delta_H - \delta_L)
\end{align*}
\]

where \(\hat{\alpha}^I = \frac{1}{2(r + 2\eta + \lambda(2\mu U^I \beta^I + \mu I))}\)

and \(\hat{\alpha}^U = \hat{\alpha}^U = \left[\frac{r + 2\eta + \lambda \mu U^I + \frac{\lambda \phi}{4}}{r + 2\eta + \frac{\lambda \phi}{4}}\right] \hat{\alpha}^I\)

Which finally implies:

\[
\begin{align*}
    S_H^i &= \left[\frac{1}{2(r + 2\eta)}\right] (\delta_H + \delta_L) + \frac{\hat{\alpha}^I}{2}(\delta_H - \delta_L) \\
    S_L^i &= \left[\frac{1}{2(r + 2\eta)}\right] (\delta_H + \delta_L) - \frac{\hat{\alpha}^I}{2}(\delta_H - \delta_L)
\end{align*}
\]
1.4. **Flagged Values.** I solve for \( D_{H1}^I, D_{L1}^I \) and \( D_{H1}^U \). The system defining \( \{ D_{va}^I \} \) is:

\[
\begin{align*}
r D_{H1}^I &= (\eta + \lambda \hat{\mu}^U)(D_{H0}^I - D_{H1}^I) \\
r D_{L1}^I &= (\eta + \lambda \hat{\mu}^U)(D_{L0}^I - D_{L1}^I) + \lambda \hat{\mu}^I \frac{S_{H}^I - S_{L}^I}{2} \\
r D_{L0}^I &= (\eta + \lambda \hat{\mu}^U)(D_{L1}^I - D_{L0}^I) - \lambda \hat{\mu}^U (\beta^I S_{H}^I + (1 - \beta^I)S_{L}^I) \\
r D_{H0}^I &= (\eta + \lambda \hat{\mu}^U)(D_{H1}^I - D_{H0}^I) + \lambda \hat{\mu}^I \frac{S_{H}^I - S_{L}^I}{2} - \lambda \hat{\mu}^U (\beta^I S_{H}^I + (1 - \beta^I)S_{L}^I)
\end{align*}
\]

Combining the first with the fourth and the second with the third:

\[
\begin{align*}
r(D_{H1}^I - D_{H0}^I) &= -2(\eta + \lambda \hat{\mu}^U)(D_{H1}^I - D_{H0}^I) - \lambda \hat{\mu}^I \frac{S_{H}^I - S_{L}^I}{2} + \lambda \hat{\mu}^U (\beta^I S_{H}^I + (1 - \beta^I)S_{L}^I) \\
r(D_{L1}^I - D_{L0}^I) &= -2(\eta + \lambda \hat{\mu}^U)(D_{L1}^I - D_{L0}^I) + \lambda \hat{\mu}^I \frac{S_{H}^I - S_{L}^I}{2} + \lambda \hat{\mu}^U (\beta^I S_{L}^I + (1 - \beta^I)S_{L}^I)
\end{align*}
\]

Solve to find:

\[
\begin{align*}
(D_{H1}^I - D_{H0}^I) &= -\frac{\lambda \hat{\mu}^I}{(r + 2 \eta + 2 \lambda \hat{\mu}^U)} \frac{S_{H}^I - S_{L}^I}{2} + \frac{\lambda \hat{\mu}^U}{(r + 2 \eta + 2 \lambda \hat{\mu}^U)} (\beta^I S_{H}^I + (1 - \beta^I)S_{L}^I) \\
(D_{L1}^I - D_{L0}^I) &= \frac{\lambda \hat{\mu}^I}{(r + 2 \eta + 2 \lambda \hat{\mu}^U)} \frac{S_{H}^I - S_{L}^I}{2} + \frac{\lambda \hat{\mu}^U}{(r + 2 \eta + 2 \lambda \hat{\mu}^U)} (\beta^I S_{L}^I + (1 - \beta^I)S_{L}^I)
\end{align*}
\]

Plug this back into the original system to get:

\[
\begin{align*}
(7) \quad D_{H1}^I &= \frac{\eta + \lambda \hat{\mu}^U}{r(r + 2 \eta + 2 \lambda \hat{\mu}^U)} \left[ \lambda \hat{\mu}^I \frac{S_{H}^I - S_{L}^I}{2} - \lambda \hat{\mu}^U (\beta^I S_{H}^I + (1 - \beta^I)S_{L}^I) \right] \\
(8) \quad D_{L1}^I &= \frac{r + \eta + \lambda \hat{\mu}^U}{r(r + 2 \eta + 2 \lambda \hat{\mu}^U)} \lambda \hat{\mu}^I \frac{S_{H}^I - S_{L}^I}{2} - \frac{\lambda \hat{\mu}^U (\eta + \lambda \hat{\mu}^U)}{r(r + 2 \eta + 2 \lambda \hat{\mu}^U)} (\beta^I S_{L}^I + (1 - \beta^I)S_{L}^I)
\end{align*}
\]

The system defining \( \{ D_{va}^U \} \) is:
\[ \begin{align*}
    r D_{H1}^U &= \eta(D_{H0}^U - D_{H1}^U) \\
    r D_{H0}^U &= \eta(D_{H1}^U - D_{H0}^U) + \lambda \hat{\mu} I (1 - \beta I)(S_H^U - S_L^I) + \lambda(\phi/2 - \hat{\mu} I)(1 - \beta I)(S_H^U - S_H^I) \\
    r D_{L1}^U &= \eta(D_{L0}^U - D_{L1}^U) + \lambda \hat{\mu} I (1 - \beta I)(S_H^I - S_L^U) + \lambda(\phi/2 - \hat{\mu} I)(1 - \beta I)(S_L^I - S_L^U) \\
    r D_{L0}^U &= \eta(D_{L1}^U - D_{L0}^U) \\
\end{align*} \]

Subtract the second from the first to get:
\[ \begin{align*}
    r(D_{H1}^U - D_{H0}^U) &= -2\eta(D_{H1}^U - D_{H0}^U) - \lambda \hat{\mu} I (1 - \beta I)(S_H^U - S_L^I) - \lambda(\phi/2 - \hat{\mu} I)(1 - \beta I)(S_H^U - S_H^I) \\
    \Rightarrow (D_{H1}^U - D_{H0}^U) &= -\frac{\lambda \hat{\mu} I (1 - \beta I)}{r + 2\eta}(S_H^U - S_L^I) - \frac{\lambda(\phi/2 - \hat{\mu} I)(1 - \beta I)}{r + 2\eta}(S_H^U - S_H^I) \\
\end{align*} \]

Finally plug back into the original system to get:
\[ \begin{align*}
    D_{H1}^U &= \frac{\eta \lambda \hat{\mu} I (1 - \beta I)}{r(r + 2\eta)}(S_H^U - S_L^I) + \frac{\eta \lambda(\phi/2 - \hat{\mu} I)(1 - \beta I)}{r(r + 2\eta)}(S_H^U - S_H^I) \\
\end{align*} \]