Supplementary Material

Supplement to “Informed intermediaries”
(Theoretical Economics, Vol. 17, No. 1, January 2022, 57–87)

PAULA ONUCHIC
Department of Economics, New York University

APPENDIX: PROOF OF PROPOSITION 2 (CONTINUED)

S.1 GUESSES

Surplus sharing:

\[
\begin{align*}
\beta(I, v_s, U, v_b) &= \beta^I \geq \frac{1}{2}, \\
\beta(U, v_s, I, v_b) &= 1 - \beta^I \leq \frac{1}{2}, \\
\beta(I, v_s, I, v_b) &= \frac{1}{2}.
\end{align*}
\]

Efficient trading if and only if an informed agent is involved:

\[I(i_s, v_s, i_b, v_b) = 1 \iff (i_s, i_b) \neq (U, U)\quad \text{and} \quad V_{i_s} - V_{i_b} + V_{i_b} - V_{i_s} > 0.\]

S.2 STATIONARY DISTRIBUTION

Given the guesses for \(I\), the inflow equal to outflow equations for the stationary distribution become

\[
\begin{align*}
\mu^U_{L1}(\eta + \lambda(\mu^U_{H0} + \mu^I_{L0})) &= \eta \mu^U_{L0}, \quad \text{(S.1)} \\
\mu^U_{H0}(\eta + \lambda(\mu^U_{L1} + \mu^I_{H1})) &= \eta \mu^U_{H1}, \quad \text{(S.2)} \\
\mu^I_{L1}(\eta + \lambda(\mu^U_{H0} + \mu^I_{H0})) &= (\eta + \lambda \mu^U_{L1}) \mu^I_{L0}, \quad \text{(S.3)} \\
\mu^I_{H0}(\eta + \lambda(\mu^U_{L1} + \mu^I_{L1})) &= (\eta + \lambda \mu^U_{H0}) \mu^I_{H1}. \quad \text{(S.4)}
\end{align*}
\]

Combine (S.1) and (S.2), and using \(\mu^U_{L1} + \mu^U_{L0} = \mu^U_{H0} + \mu^U_{H1} = \frac{1 - \phi}{2}\) (since half of the uninformed agents have high valuation and half have low valuation), I get

\[\mu^U_{L1}(2\eta + \lambda(\mu^I_{H0} + \mu^I_{L0})) = \mu^U_{H0}(2\eta + \lambda(\mu^I_{L1} + \mu^I_{H1})) = \frac{\eta(1 - \phi)}{2}. \quad \text{(S.5)}\]

Similarly use (S.3) and (S.4) and \(\mu^I_{L1} + \mu^I_{L0} = \mu^I_{H0} + \mu^I_{H1}\) to get

\[(\eta + \lambda \mu^U_{L1})(\mu^I_{H0} + \mu^I_{L0}) = (\eta + \lambda \mu^U_{H0})(\mu^I_{H1} + \mu^I_{L1}). \quad \text{(S.6)}\]
Use (S.5) and (S.6) to get $\mu^U_{L1} + \mu^I_{L1} + \mu^I_{H1} = \mu^U_{H0} + \mu^I_{H0} + \mu^I_{L0}$. Combining this with (S.5) yet again, conclude that $\mu^U_{L1} = \mu^U_{H0} \equiv \hat{\mu}^U$. This, along with (S.6), implies $(\mu^I_{H0} + \mu^I_{L0}) = (\mu^I_{L1} + \mu^I_{H1})$; hence, $\mu^I_{H0} = \mu^I_{L1} \equiv \hat{\mu}^I$. Rewrite the inflow equals outflow conditions now as

$$\hat{\mu}^U (\eta + \lambda(\phi/2)) = \eta \frac{(1 - \phi - 2\hat{\mu}^U)}{2} \hat{\mu}^I, \quad (\eta + \lambda(\hat{\mu}^U + \hat{\mu}^I)) = \frac{\phi - 2\hat{\mu}^I}{2} (\eta + \lambda\hat{\mu}^U).$$

Solving these, I get

$$\hat{\mu}^U = \frac{1 - \phi}{4 + \frac{\lambda}{\eta}}, \quad \hat{\mu}^I = -\eta + \lambda, \hat{\mu}^U = + \sqrt{\left(\eta + \lambda\hat{\mu}^U \frac{\lambda}{\eta}\right)^2 + \frac{\phi}{2} \eta + \lambda\hat{\mu}^U \frac{\lambda}{\eta}}.$$

### S.3 Unflagged values

Again taking into account the guesses for $I$ and $\beta$, unflagged values are given by the system

$$rV^I_{H0} = \eta(V^I_{H0} - V^I_{H1}) + \lambda\hat{\mu}^I V^I_{H1} - V^I_{H0} + V^I_{L0} - V^I_{L1} + \lambda\hat{\mu}^U \beta^I (V^I_{H0} - V^I_{H1} + V^I_{L0} - V^I_{L1}),$$

$$rV^I_{H1} = \delta_H + \eta(V^I_{H0} - V^I_{H1}) + \lambda\hat{\mu}^U \beta^I (V^I_{H0} - V^I_{H1} + V^I_{L0} - V^I_{H0}),$$

$$rV^I_{L1} = \delta_L + \eta(V^I_{L0} - V^I_{L1}) + \lambda\hat{\mu}^I V^I_{L0} - V^I_{L1} + V^I_{H1} - V^I_{H0} + \lambda\hat{\mu}^U \beta^I(V^I_{L0} - V^I_{L1} + V^I_{H0} - V^I_{H1}),$$

$$rV^U_{L0} = \eta(V^U_{L0} - V^U_{L1}) + \lambda\hat{\mu}^I (1 - \beta^I) (V^U_{L0} - V^U_{L1} + V^U_{H1} - V^U_{H0}),$$

$$rV^U_{L1} = \delta_L + \eta(V^U_{L0} - V^U_{L0}) + \lambda\hat{\mu}^I (1 - \beta^I) (V^U_{L0} - V^U_{L1} + V^U_{H1} - V^U_{H0}) + \lambda\hat{\mu}^I (1 - \beta^I)(V^U_{L0} - V^U_{L1} + V^U_{H1} - V^U_{H0}),$$

$$rV^U_{L0} = \eta(V^U_{L1} - V^U_{L0}).$$

In terms of the values of holding an asset, this system becomes

$$rS^I_{H} = \delta_H - 2\eta S^I_{H} + \lambda\hat{\mu}^U \beta^I (S^I_{H} - S^I_{H}) + \lambda\hat{\mu}^I (\frac{S^I_{L} - S^I_{H}}{2}) + \lambda\hat{\mu}^U \beta^I (S^I_{H} - S^I_{H}),$$

$$rS^U_{L} = \delta_L - 2\eta S^I_{L} + \lambda\hat{\mu}^U \beta^I (S^I_{L} - S^I_{H}) + \lambda\hat{\mu}^I (\frac{S^I_{H} - S^I_{L}}{2}) + \lambda\hat{\mu}^U \beta^I (S^I_{H} - S^I_{L}),$$

$$rS^U_{H} = \delta_H - 2\eta S^I_{H} + \lambda\hat{\mu}^I (1 - \beta^I) (S^I_{L} - S^I_{H}) + \lambda\frac{\phi - 2\hat{\mu}^I}{2} (1 - \beta^I)(S^I_{H} - S^I_{H}).$$
\[ rS_H^U = \delta_L - 2\eta S_L^U + \lambda \hat{\mu} (1 - \beta^I)(S_H^I - S_L^I) + \frac{\lambda - 2\hat{\mu}^I}{2} (1 - \beta^I)(S_L^I - S_L^U). \]

Add up the first two lines and the last two lines to get

\[ (r + 2\eta)(S_H^I + S_L^I) = \delta_H + \delta_L + 2\lambda \hat{\mu}^U \beta^I (S_H^U + S_L^U) - 2\lambda \hat{\mu}^U \beta^I (S_H^I + S_L^I), \]

\[ (r + 2\eta)(S_H^U + S_L^U) = \delta_H + \delta_L + \frac{\lambda \hat{\mu}}{2} (1 - \beta^I)(S_H^I + S_L^I) - \frac{\lambda \hat{\mu}^I}{2} (1 - \beta^I)(S_H^U + S_L^U). \]

These imply \((S_H^I + S_L^I) = (S_H^U + S_L^U) = \frac{\delta_H + \delta_L}{r + 2\eta} \). Now from the original system, subtract the second equation from the first and the fourth from the third to find

\[ (r + 2\eta)(S_H^I - S_L^I) = \delta_H - \delta_L - \lambda (2\hat{\mu}^U \beta^I + \hat{\mu}^I)(S_H^I - S_L^I), \]

\[ (r + 2\eta)(S_H^U - S_L^U) = \delta_H - \delta_L - \frac{\lambda \hat{\mu}^I}{2} (1 - \beta^I)(S_H^I - S_L^I) \]

\[ + \left( \frac{\lambda \hat{\mu}}{2} - \hat{\mu}^I \right) (1 - \beta^I)(S_H^I - S_L^I). \]

Rearrange these to get the expressions

\[ (S_H^I - S_L^I) = \hat{\alpha}^I (\delta_H - \delta_L), \]

\[ (S_H^I - S_L^I) = \hat{\alpha}^U (\delta_H - \delta_L), \]

where

\[ \hat{\alpha}^I = \frac{1}{2(r + 2\eta + \lambda (2\hat{\mu}^U \beta^I + \hat{\mu}^I))}, \]

\[ \hat{\alpha}^U = \hat{\alpha}^U = \left[ \frac{r + 2\eta + \lambda \hat{\mu}^U + \frac{\lambda \hat{\mu}^I}{4}}{r + 2\eta + \frac{\lambda \hat{\mu}^I}{4}} \right] \hat{\alpha}^I, \]

which finally implies

\[ S_H^I = \left[ \frac{1}{2(r + 2\eta)} \right] (\delta_H + \delta_L) + \frac{\hat{\alpha}^I}{2} (\delta_H - \delta_L), \]

\[ S_L^I = \left[ \frac{1}{2(r + 2\eta)} \right] (\delta_H + \delta_L) - \frac{\hat{\alpha}^I}{2} (\delta_H - \delta_L). \]

S.4 Flagged values

I solve for \( D_{H1}^I, D_{L1}^I, \) and \( D_{H1}^U \). The system defining \( D_{va}^I \) is

\[ rD_{H1}^I = (\eta + \lambda \hat{\mu}^U)(D_{H0}^I - D_{H1}^I), \]

\[ rD_{L1}^I = (\eta + \lambda \hat{\mu}^U)(D_{L0}^I - D_{L1}^I) + \lambda \hat{\mu}^I S_H^I - S_L^I. \]
\[ rD_{L0}^I = (\eta + \lambda \hat{\mu}^U)(D_{L1}^I - D_{L0}^I) - \lambda \hat{\mu}^U (\beta^I S_H^U - S_L^U) + (1 - \beta^I)S_L^U, \]
\[ rD_{H0}^I = (\eta + \lambda \hat{\mu}^U)(D_{H1}^I - D_{H0}^I) + \lambda \hat{\mu}^I \left( S_H^I - S_L^I \right) + \lambda \hat{\mu}^U (\beta^I S_H^U - S_L^U). \]

Combining the first equality with the fourth and the second equality with the third yields
\[ r(D_{H1}^I - D_{H0}^I) = -2(\eta + \lambda \hat{\mu}^U)(D_{H1}^I - D_{H0}^I) - \lambda \hat{\mu}^I \left( S_H^I - S_L^I \right) + \lambda \hat{\mu}^U (\beta^I S_H^U + (1 - \beta^I)S_L^U), \]
\[ r(D_{L1}^I - D_{L0}^I) = -2(\eta + \lambda \hat{\mu}^U)(D_{L1}^I - D_{L0}^I) + \lambda \hat{\mu}^I \left( S_H^I - S_L^I \right) + \lambda \hat{\mu}^U (\beta^I S_H^U + (1 - \beta^I)S_L^U). \]

Solve to find
\[ (D_{H1}^I - D_{H0}^I) = -\frac{\lambda \hat{\mu}^I}{(r + 2\eta + 2\lambda \hat{\mu}^U)} S_H^I - S_L^I + \frac{\lambda \hat{\mu}^U}{(r + 2\eta + 2\lambda \hat{\mu}^U)} (\beta^I S_H^U + (1 - \beta^I)S_L^U), \]
\[ (D_{L1}^I - D_{L0}^I) = \frac{\lambda \hat{\mu}^I}{(r + 2\eta + 2\lambda \hat{\mu}^U)} S_H^I - S_L^I + \frac{\lambda \hat{\mu}^U}{(r + 2\eta + 2\lambda \hat{\mu}^U)} (\beta^I S_H^U + (1 - \beta^I)S_L^U). \]

Plug this back into the original system to get
\[ D_{H1}^I = \frac{\eta + \lambda \hat{\mu}^U}{(r + 2\eta + 2\lambda \hat{\mu}^U)} \left[ \lambda \hat{\mu}^I S_H^I - S_L^I + \lambda \hat{\mu}^U (\beta^I S_H^U + (1 - \beta^I)S_L^U) \right], \] (S.8)
\[ D_{L1}^I = \frac{r + \eta + \lambda \hat{\mu}^U}{(r + 2\eta + 2\lambda \hat{\mu}^U)} \left[ \lambda \hat{\mu}^I S_H^I - S_L^I \right] + \frac{\lambda \hat{\mu}^U}{(r + 2\eta + 2\lambda \hat{\mu}^U)} (\beta^I S_H^U + (1 - \beta^I)S_L^U). \] (S.9)

The system defining \( D_{\nu \lambda}^U \) is
\[ rD_{H1}^U = \eta(D_{H0}^U - D_{H1}^U), \]
\[ rD_{H0}^U = \eta(D_{H1}^U - D_{H0}^U) + \lambda \hat{\mu}^I (1 - \beta^I)(S_H^U - S_L^U) + \lambda(\phi/2 - \mu^I)(1 - \beta^I)(S_H^U - S_L^U), \]
\[ rD_{L1}^U = \eta(D_{L0}^U - D_{L1}^U) + \lambda \hat{\mu}^I (1 - \beta^I)(S_H^U - S_L^U) + \lambda(\phi/2 - \mu^I)(1 - \beta^I)(S_H^U - S_L^U), \]
\[ rD_{L0}^U = \eta(D_{L1}^U - D_{L0}^U). \]

Subtract the second equality from the first to get
\[ r(D_{H1}^U - D_{H0}^U) = -2\eta(D_{H1}^U - D_{H0}^U) - \lambda \hat{\mu}^I (1 - \beta^I)(S_H^U - S_L^U) \]
\[ - \lambda(\phi/2 - \mu^I)(1 - \beta^I)(S_H^U - S_L^U) \]
\[ \Rightarrow (D_{H1}^U - D_{H0}^U) = -\frac{\lambda \hat{\mu}^I (1 - \beta^I)}{r + 2\eta} (S_H^U - S_L^U) - \frac{\lambda(\phi/2 - \mu^I)(1 - \beta^I)}{r + 2\eta} (S_H^U - S_L^U). \]
Finally plug back into the original system to get

\[
D_H^{U} = \frac{\eta \lambda \tilde{\mu} (1 - \beta I)}{r(r + 2 \eta)} (S_H^{U} - S_L^{I}) + \frac{\eta \lambda (\phi/2 - \tilde{\mu} I)}{r(r + 2 \eta)} (S_H^{U} - S_H^{I}). \tag{S.10}
\]

Co-editor Florian Scheuer handled this manuscript.

Manuscript received 11 December, 2019; final version accepted 8 March, 2021; available online 17 March, 2021.