SUPPLEMENT TO "EFFICIENT DYNAMIC MECHANISMS IN ENVIRONMENTS WITH INTERDEPENDENT VALUATIONS: THE ROLE OF CONTINGENT TRANSFERS"

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In this online appendix, we discuss budget balance and surplus extraction in dynamic mechanisms with interdependent valuations. We also study efficient dynamic mechanisms when agents' signals evolve independently. Finally, we provide proofs that are omitted in the main paper.

1. Budget-Balanced Mechanisms

We first consider budget-balanced mechanisms when time horizon is infinite $(T = \infty)$.[1] The mechanism $\Gamma = (\Theta_t, a_t, p_t)_{t=1}^T$ is *ex ante budget balanced* if

$$\mathbb{E}\left[\sum_{t=1}^T \delta^{t-1} \sum_{i=1}^N p_t^i\right] \geq 0.$$

The mechanism is *budget balanced* if for each t,

$$\mathbb{E}\left[\sum_{i=1}^N p_t^i\right] = 0.$$

The mechanism is *ex post budget balanced* if for each t,

$$\sum_{i=1}^N p_t^i \equiv 0.$$

These notions are related to the mechanism designer's financing abilities. When the designer has access to long-term outside financing, an ex ante balanced budget means that the expected present value of all transfers from agents is non-negative. If the financing ability is limited, the relevant notion is budget-balance, which says that in each period the designer

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[1]For the finite-horizon case, the same approach adopted in this subsection yields a balanced budget in all but the last period.

breaks even on average. Without any outside financing, ex post budget-balance requires that agents' transfers sum to zero in each period for any realized signal profile.

As we mentioned in the paper, one problem with the constructed efficient dynamic mechanisms is that they run large deficits subsidizing agents in each period. Budget balance requires these subsidies to be financed by the participants. An important insight from Athey and Segal [1] is that the problem of contingent deviations needs to be carefully addressed when signals are persistent, since transfers in each period to be calculated based on the conditional distribution of signals in order to balance the budget. However, the conditional distributions are manipulable by agents through their previous reported signals. The balanced team mechanism proposed by Athey and Segal [1] is not applicable in our settings with interdependent valuations and information correlation.

We first show that ex ante budget balanced mechanisms can be constructed by introducing participation fees to the original efficient dynamic mechanism in the first period. After observing the first period's signal θ_1^i, each agent i pays a proportion of the expected discounted sum of other agents' total subsidies, an amount that is independent of her current signal θ_1^i. In expectation, the total amount of participation fees is equal to the total amount of future subsidies. Specifically, let $\{p_t^i\}$ denote the transfers in efficient dynamic mechanism constructed in Theorem 3.1 in the paper. Note that for each i, $p_1^i \equiv 0$. For each $\theta_1 \in \Theta_1$, every agent's equilibrium payoff in the efficient mechanism is $W(\theta_1)$. So the expected discounted sum of subsidies for agent i is

$$\mathbb{E}\left[\sum_{t \geq 1} \delta^{t-1} p_t^i\right] = \mathbb{E}\left[W(\theta_1) - \sum_{t \geq 1} \delta^{t-1} u^i(a_t^*(\theta_t), \theta_t)\right],$$

where the expectation is over the entire sequence of signal profiles. For each i and θ_1^i, define

$$\eta^i(\theta_1^i) \triangleq -\mathbb{E}\left[\sum_{t \geq 1} \delta^{t-1} p_t^i \,\Big|\, \theta_1^i\right] = -\mathbb{E}\left[W(\theta_1) - \sum_{t \geq 1} \delta^{t-1} u^i(a_t^*(\theta_t), \theta_t) \,\Big|\, \theta_1^i\right].$$

Then for each agent i, consider the transfers $\{\tilde{p}_t^i\}$ defined as

$$\tilde{p}_1^i(\theta_1) = \frac{1}{N-1} \sum_{j \neq i} \eta^j(\theta_1^j),$$

and $\tilde{p}_t^i = p_t^i$ for $t \geq 2$. Note that $\tilde{p}_1^i$ is independent of agent i's report, so $\{\tilde{p}_t^i\}$ is also periodic ex post incentive compatible. Moreover, by the law of iterated expectations, the expected

sum of transfers satisfies

$$\mathbb{E}\left[\sum_{i=1}^{N}\sum_{t\geq 1}\delta^{t-1}\tilde{p}_t^i\right] = \mathbb{E}\left[\sum_{i=1}^{N}\tilde{p}_1^i + \sum_{i=1}^{N}\sum_{t\geq 2}\delta^{t-1}\tilde{p}_t^i\right]$$

$$= \mathbb{E}\left[\sum_{i=1}^{N}\left(\eta^i(\theta_1^i) + \sum_{t\geq 1}\delta^{t-1}p_t^i\right)\right]$$

$$= \sum_{i=1}^{N}\mathbb{E}\left[-\mathbb{E}\left[\sum_{t\geq 1}\delta^{t-1}p_t^i \,\middle|\, \theta_1^i\right] + \sum_{t\geq 1}\delta^{t-1}p_t^i\right]$$

$$= 0.$$

Suppose next that the designer has limited instruments for intertemporal financing. We now construct a budget balanced mechanism under which the expected sum of transfers in each period is zero. For each i, t, a_t and θ_t, define

$$\xi^i(a_t, \theta_t) \triangleq u^i(a_t, \theta_t) - \frac{1}{N}\sum_{j=1}^{N}u^j(a_t, \theta_t)$$

to be the deviation of agent i's flow utility from the average flow utility. Since $\xi^i(a_t, \theta_t)$ is bounded, by the argument in the proof of Theorem 3.1, if Assumption 2 holds, there exist transfers $\hat{p}_{t+1}^i : \Theta_{t+1}^{-i} \times \Theta_t^i \times A_t \times \Theta_t^{-i} \to \mathbb{R}$ such that for each a_t, θ_t^{-i} and each pair (θ_t^i, r_t^i), we have

$$\xi^i(a_t, \theta_t) = \delta \sum_{\theta_{t+1}\in\Theta_{t+1}} \hat{p}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i})\mu_{t+1}(\theta_{t+1}|a_t, \theta_t)$$

$$\leq \delta \sum_{\theta_{t+1}\in\Theta_{t+1}} \hat{p}_{t+1}^i(\theta_{t+1}^{-i}, r_t^i; a_t, \theta_t^{-i})\mu_{t+1}(\theta_{t+1}|a_t, \theta_t).$$

Set $\hat{p}_1^i \equiv 0$ for each i and consider the dynamic mechanism $\{a_t^*, \hat{p}_t\}$. The expected sum of transfers in period $t+1$ under the truthful strategies is

$$\sum_{\theta_{t+1}\in\Theta_{t+1}}\sum_{i=1}^{N}\hat{p}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t^*(\theta_t), \theta_t^{-i})\mu(\theta_{t+1}|a_t^*(\theta_t), \theta_t) = \sum_{i=1}^{N}\xi^i(a_t^*(\theta_t), \theta_t) = 0.$$

Moreover, if Assumption 3 holds, then similar to the logic in Theorem 3.2, there are transfers $\tilde{p}_{t+1}^i : \Theta_{t+1}^{-i} \times A_t \times \Theta_t^{-i} \to \mathbb{R}$ such that for each a_t and θ_t, we have

$$\xi^i(a_t, \theta_t) = \delta \sum_{\theta_{t+1}\in\Theta_{t+1}} \tilde{p}_{t+1}^i(\theta_{t+1}^{-i}; a_t, \theta_t^{-i})\mu_{t+1}(\theta_{t+1}|a_t, \theta_t),$$

and hence a balanced budget

$$\sum_{\theta_{t+1}\in\Theta_{t+1}}\sum_{i=1}^{N}\tilde{p}_{t+1}^{i}(\theta_{t+1}^{-i};a_{t}^{*}(\theta_{t}),\theta_{t}^{-i})\mu_{t+1}(\theta_{t+1}|a_{t}^{*}(\theta_{t}),\theta_{t})=0.$$

Therefore, we only need to show that either $\{\hat{p}_t\}$ or $\{\tilde{p}_t\}$ achieves incentive compatibility. The result is summarized in the next proposition.

Proposition 1.1. *Suppose $T = \infty$. Under either Assumption 2 or 3, there exists an efficient dynamic mechanism that is periodic ex post incentive compatible and balances the budget in the truthful equilibrium.*

Note that the above transfers, $\{\hat{p}_t\}$ and $\{\tilde{p}_t\}$, only balance the budget on the equilibrium path. More assumptions on the joint distributions of signals are needed for ex post budget balance along the line of analysis in Kosenok and Severinov [6] and Hörner, Takahashi and Vieille [5]. Since this question is beyond the scope of the current paper, we leave it for future research.

2. Efficient Mechanisms without Correlation

If the correlation conditions are violated, the construction in the proof of Theorem 3.1 and Theorem 3.2 may not work for some utility functions. In this section, we drop the assumption that signal spaces are finite but restrict our attention to one-dimensional environments and the evolution of private information is independent across agents. We construct a transfer schedule that extends the generalized VCG mechanism to dynamic settings.

We say that a transfer $\{p_t\}_{t=1}^{T}$ or a mechanism $\{a_t^*, p_t\}_{t=1}^{T}$ is *history-independent* if for each t and θ_t, and for any two public histories h_t and h_t',

$$p_t(h_t, \theta_t) = p_t(h_t', \theta_t).$$

That is, a history-independent transfer p_t depends only on the reported profile $r_t \in \Theta_t$ in period t. Under a history-independent mechanism, agent i's period-t continuation payoff depends only on her private signal θ_t^i, i.e.,

$$V_t^i(\theta_t^i) = \max_{r_t^i\in\Theta_t^i}\mathbb{E}\left[u^i(a_t^*(r_t^i,\theta_t^{-i}),\theta_t)-p_t^i(r_t^i,\theta_t^{-i})+\delta V_{t+1}^i(\theta_{t+1}^i)\right].$$

In this case, we also define $V_t^i(a_t, \theta_t)$ as

$$V_t^i(a_t, \theta_t) = u^i(a_t, \theta_t) + \delta \mathbb{E}\left[V_{t+1}^i(\theta_{t+1}^i)|a_t, \theta_t\right].$$

Assumption 5 (One-dimensional private signals) For each i and each t, $\Theta_t^i = [0, 1]$.

Under Assumption 5, we can generalize the monotonicity condition in static model studied by Bergemann and Välimäki [2]. To save notation, assume that for each t, $A_t = A \equiv \{a^1, \ldots, a^K\}$. For any i, t and θ_t^{-i}, define the set $\Theta_t^{i,k} \subset \Theta_t^i$ as

$$\Theta_t^{i,k} = \left\{ \theta_t^i \in \Theta_t^i \left| \begin{array}{l} \sum_i u^i(a^k, \theta_t) + \delta \mathbb{E}\left[W_{t+1}(\theta_{t+1})|a^k, \theta_t\right] \\ \geq \sum_i u^i(a^l, \theta_t) + \delta \mathbb{E}\left[W_{t+1}(\theta_{t+1})|a^l, \theta_t\right], \end{array} \right. \forall a^l \neq a^k \right\}.$$

We say that the collections of sets $\{\Theta_t^{i,k}\}_{k=1}^K$ satisfies *monotonicity* if for each k, $\theta_t^i, \tilde{\theta}_t^i \in \Theta_t^{i,k}$ implies that for each $\lambda \in [0, 1]$, $\lambda \theta_t^i + (1 - \lambda)\tilde{\theta}_t^i \in \Theta_t^{i,k}$. Under monotonicity, there exists an efficient allocation a_t^* in period t such that after relabeling the social alternatives, Θ_t^i can be partitioned into successive intervals $\{S_t^{i,1}, \ldots, S_t^{i,K}\}$ and each a^k is chosen if and only if $\theta_t^i \in S_t^{i,k}$. Then for each i, t and θ_t^{-i}, there is a linear order $\prec_t^i$ (which also depends on θ_t^{-i}) on A:

$$a^1 \prec_t^i \ldots \prec_t^i a^K.$$

Assumption 6 (Independent transitions) For $t = 1$, $\mu_1 = \prod_{i=1}^N \mu_1^i$, where for each i, $\mu_1^i \in \Delta(\Theta_1^i)$. For each $t > 1$, $\mu_t(\theta_t|a_{t-1}, \theta_{t-1}) = \prod_{i=1}^N \mu_t^i(\theta_t^i|a_{t-1}, \theta_{t-1}^i)$, where for each i, $\mu_t^i : A \times \Theta_{t-1}^i \to \Delta(\Theta_t^i)$ is a transition probability.

Suppose $a_t^*(\theta_t) = a^k$, then consider the following *history-independent* transfer

$$(1) \qquad p_t^{i*}(\theta_t) = \sum_{\kappa=1}^k \sum_{j \neq i} \left[u^j(a^{\kappa-1}, x_t^i(\kappa, \theta_t^{-i}), \theta_t^{-i}) - u^j(a^\kappa, x_t^i(\kappa, \theta_t^{-i}), \theta_t^{-i}) \right]$$

$$+ \sum_{\kappa=1}^k \delta \mathbb{E}\left[W_{t+1}(\theta_{t+1}) - V_{t+1}^i(\theta_{t+1})|a^{\kappa-1}, x_t^i(\kappa, \theta_t^{-i}), \theta_t^{-i}\right]$$

$$- \sum_{\kappa=1}^k \delta \mathbb{E}\left[W_{t+1}(\theta_{t+1}) - V_{t+1}^i(\theta_{t+1})|a^\kappa, x_t^i(\kappa, \theta_t^{-i}), \theta_t^{-i}\right],$$

where $x_t^i(\kappa, \theta_t^{-i}) \triangleq \inf\{\theta_t^i : a_t^*(\theta_t^i, \theta_t^{-i}) = a^\kappa\}$. Note that $p_t^{i*}(\theta_t)$ does not depend directly on θ_t^i under Assumption 6.

Finally, recall that $W_t(\theta_t)$ is the continuation social surplus given period-t signal profile θ_t. For each a_t and θ_t, define $W_t(a_t, \theta_t)$ as

$$W_t(a_t, \theta_t) = \sum_{i=1}^{N} u^i(a_t, \theta_t) + \delta \mathbb{E}\left[W_{t+1}(\theta_{t+1}) | a_t, \theta_t\right].$$

The next theorem shows that the transfer constructed in (1) is periodic ex post incentive compatible under some restrictions on the primitives. Therefore, it extends of the generalized VCG mechanism to dynamic environments with interdependent valuations.

Proposition 2.1. *Suppose that Assumptions 5 and 6 hold. There exists a periodic ex post incentive compatible mechanism $\{a_t^*, p_t\}$ with history-independent transfers if for each t, i and θ_t^{-i}, there exists an order on the allocation space A such that*

(1) $W_t(a_t, \theta_t^i, \theta_t^{-i})$ *is single-crossing in* (a_t, θ_t^i),
(2) $V_t^i(a_t, \theta_t^i, \theta_t^{-i})$ *has increasing difference in* (a_t, θ_t^i).

Remark 2.2. The transfer schedule (1) can also be viewed a generalization of the dynamic pivot mechanism constructed by Bergemann and Välimäki [3]. To see this, suppose that each utility function u^i does not depend on θ_t^{-i} and that private information is statistically independent across agents, then (1) can be written as

$$p_t^{i*}(\theta_t) = \sum_{j \neq i} \left[u^j(a_t^*(\underline{\theta}^i, \theta^{-i}), \theta_t^{-i}) - u^j(a_t^*(\theta_t), \theta_t^{-i}) \right]$$

$$+ \delta \mathbb{E}\left[W^{-i}(\theta_{t+1}) | a_t^*(\underline{\theta}^i, \theta^{-i}), \theta_t\right] - \delta \mathbb{E}\left[W^{-i}(\theta_{t+1}) | a_t^*(\theta_t), \theta_t\right],$$

where

$$W^{-i}(\theta_t) = W(\theta_t) - V^i(\theta_t) = \max_{\{a_s\}_{s \geq t}} \mathbb{E}\left[\sum_{s \geq t} \delta^{s-t}\left(u^i(a_s, \underline{\theta}^i) + \sum_{j \neq i} u^j(a_s, \theta_s^j)\right)\right].$$

Therefore each agent i's transfer p_t^{i*} in every period t is the flow externality cost that she imposes on other agents.

3. Surplus Extraction

In this section, we consider the problem of full surplus extraction in the infinite-horizon $(T = \infty)$ case. We assume that each agent's utility function is non-negative and normalize each agent's outside option from any period onward to zero. To simplify notations, we also

assume that the transition probabilities are *stationary*, i.e., for each t, $\Theta_t = \Theta_{t+1}$, $A_{t+1} = A_t$, and $\mu_{t+1}(\theta_{t+1}|a_t, \theta_t) = \mu(\theta_{t+1}|a_t, \theta_t)$. We will show that the designer can always extract all the expected surplus from the agents by exploiting the intertemporal correlation of private information. We also emphasize that intertemporal correlation plays a key role in surplus extraction as it does in efficient mechanisms. In contrast, the attempt of generalizing Crémer and McLean [4] and McAfee and Reny [7] based on correlation of intra-period signals fails due to the possibility of belief manipulations by agents.

Formally, we say that a dynamic mechanism $\{a_t, p_t\}$ achieves *full-surplus extraction* if

$$\mathbb{E}\left[W(\theta_1) - \sum_{i=1}^{N}\sum_{t=1}^{\infty}\delta^{t-1}p_t^i\right] = 0.$$

That is, the expected discounted total transfer collected by the designer is equal to the expected maximal social surplus. The following result shows that a simple modification of the efficient dynamic mechanism in Theorem 3.1 ensures full surplus extraction and agents' participation constraints in the first period.

Proposition 3.1. *Suppose $T = \infty$. Under Assumptions 1 and 2 (or Assumptions 1 and 3), there exists a periodic ex post incentive compatible dynamic mechanism that achieves full surplus extraction.*

In effect, the dynamic surplus extraction mechanism in Proposition 3.1 asks each agent to pay a fixed and bounded participation fee and choose from a collection of lotteries in each period, followed by announcing her current signal as in the efficient dynamic mechanism in Theorem 3.1. The outcome of each lottery is revealed in the next period, depending on other agents' reports in *both* periods. All lotteries pay bonuses to the agent, thereby ensuring agents to participate in the mechanism in each period. The upfront participation fees, which can be thought as prices of entering any such lotteries, serve to extract the surplus from agents.

Remark 3.2. Another notion of surplus extraction would require that the designer obtains the entire continuation social surplus after *each* history. While in our mechanism each agent collects zero expected surplus from the beginning of their interactions, her continuation payoff after any nontrivial history is in fact positive as she obtains bonuses from the lottery purchased in the previous round. Thus, the mechanism does not satisfy this stronger version

of surplus extraction. We conjecture that given agents' interim participation constraints in each period, it is impossible to achieve surplus extraction after each history.

4. Appendix A: Proofs of Results in Sections 1–3

4.1. Proof of Proposition 1.1.

Proof. Since budget balance under either $\{\hat{p}_t\}$ or $\{\tilde{p}_t\}$ is established in the main text, we only need to show that both mechanisms, $\{a_t^*, \hat{p}_t\}$ and $\{a_t^*, \tilde{p}_t\}$, are periodic ex post incentive compatible. By the one-shot deviation principle, it suffices to prove that truth-telling is incentive compatible for agent i in period t after any history, if all agents report truthfully from period $t+1$ onward. Here we prove the result for $\{a_t^*, \tilde{p}_t\}$. The proof for $\{a_t^*, \hat{p}_t\}$ is similar and hence omitted.

Fix any $h_t = \{h_{t-1}, \theta_{t-1}, a_{t-1}\}$, we need to show that for each i and θ, $r_t^i = \theta_t^i$ is a solution to the following maximization problem

$$
(2) \quad \max_{r_t^i} \left\{ u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t) - \tilde{p}_t^i(\theta_t^{-i}; a_{t-1}, \theta_{t-1}^{-i}) \right.
$$
$$
\left. + \delta \sum_{\theta_{t+1}} \left[\frac{1}{N} W(\theta_{t+1}) - \tilde{p}_{t+1}^i(\theta_{t+1}^{-i}; a_t^*(r_t^i, \theta_t^{-i}), \theta_t^{-i}) \right] \mu_{t+1}(\theta_{t+1} | a_t^*(r_t^i, \theta_t^{-i}), \theta_t) \right\}.
$$

By construction, we have

$$
\delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \tilde{p}_{t+1}^i(\theta_{t+1}^{-i}; a_t^*(r_t^i, \theta_t^{-i}), \theta_t^{-i}) \mu_{t+1}(\theta_{t+1} | a_t^*(r_t^i, \theta_t^{-i}), \theta_t)
$$
$$
= u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t) - \frac{1}{N} \sum_{i=1}^N u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t).
$$

So the problem (2) is equivalent to

$$
(3) \quad \max_{r_t^i} \left\{ u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t) - \tilde{p}_t^i(\theta_t^{-i}; a_{t-1}, \theta_{t-1}^{-i}) - u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t) \right.
$$
$$
\left. + \frac{1}{N} \sum_{i=1}^N u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t) + \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \frac{1}{N} W(\theta_{t+1}) \mu_{t+1}(\theta_{t+1} | a_t^*(r_t^i, \theta_t^{-i}), \theta_t) \right\}.
$$

Since the second term in the objective function of (3), $\tilde{p}_t^i(\theta_t^{-i}; a_{t-1}, \theta_{t-1}^{-i})$, is independent of r_t^i, solutions to problem (3) are also solutions to the following problem

$$(4) \quad \max_{r_t^i} \left\{ \frac{1}{N} \sum_{i=1}^{N} u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t) + \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \frac{1}{N} W(\theta_{t+1}) \mu_{t+1}(\theta_{t+1} | a_t^*(r_t^i, \theta_t^{-i}), \theta_t) \right\}.$$

The result then follows from the definition of a_t^*. $\qquad\square$

4.2. Proof of Proposition 2.1.

Proof. The proof is by backward induction on t. For each t, the argument follows the same lines as the proof of Proposition 3 in Bergemann and Välimäki [2] (pages 1029–1030) with the transfers defined in (1).

$\qquad\square$

4.3. Proof of Proposition 3.1.

Proof. The proof is similar to that of Theorem 3.1. For each t and i, agent i's current signal θ_t^i is correlated with other agents' signals θ_{t+1}^{-i} in the next period as in Assumption 2, there exists a function $\tilde{q}_{t+1}^i : \Theta_{t+1}^{-i} \times \Theta_t^i \times A_t \times \Theta_t^{-i} \to \mathbb{R}$ such that for each a_t, θ_t^{-i} and each pair (θ_t^i, r_t^i),

$$u^i(a_t, \theta_t) = \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \tilde{q}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i}) \mu(\theta_{t+1} | a_t, \theta_t)$$

$$\leq \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \tilde{q}_{t+1}^i(\theta_{t+1}^{-i}, r_t^i; a_t, \theta_t^{-i}) \mu(\theta_{t+1} | a_t, \theta_t).$$

By the stationarity assumption, we have $\tilde{q}_{t+1}^i = \tilde{q}_t^i$ for each $t \geq 2$. For each a_t, θ_t^{-i}, let $K^i(a_t, \theta_t^{-i}) \in \mathbb{R}$ be an upper bound of $|\tilde{q}_{t+1}^i|$, i.e.,

$$K^i(a_t, \theta_t^{-i}) > \sup_{\theta_{t+1}^{-i}, \theta_t^i} \left| \tilde{q}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i}) \right|.$$

Let $K^i = \max_{a_t, \theta_t^{-i}} K^i(a_t, \theta_t^{-i})$. We also set $\tilde{q}_1^i \equiv 0$ for all i. Note that the dynamic mechanism $\{a_t^*, \tilde{q}_t\}$ is well-defined.

We first show that $\{a_t^*, \tilde{q}_t\}$ is periodic ex post incentive compatible. Again assume all agents other than agent i report truthfully. If agent i reports truthfully in period t, i.e.,

$r_t^i = \theta_t^i$, her continuation payoff is

$$u^i(a_t^*(\theta_t), \theta_t) - \tilde{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i)$$

$$- \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \tilde{q}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t^*(\theta_t), \theta_t^{-i}) \mu(\theta_{t+1} | a_t^*(\theta_t), \theta_t)$$

$$= - \tilde{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i).$$

Suppose agent i deviates to a message r_t^i such that $a_t^*(r_t^i, \theta_t^{-i}) = a_t^*(\theta_t)$, then her continuation payoff satisfies

$$u^i(a_t^*(\theta_t), \theta_t) - \tilde{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i)$$

$$- \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \tilde{q}_{t+1}^i(\theta_{t+1}^{-i}, r_t^i; a_t^*(\theta_t), \theta_t^{-i}) \mu(\theta_{t+1} | a_t^*(\theta_t), \theta_t)$$

$$\leq u^i(a_t^*(\theta_t), \theta_t) - \tilde{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i)$$

$$- \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \tilde{q}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t^*(\theta_t), \theta_t^{-i}) \mu(\theta_{t+1} | a_t^*(\theta_t), \theta_t)$$

$$= - \tilde{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i).$$

Suppose agent i deviates to a message r_t^i such that $a_t^*(r_t^i, \theta_t^{-i}) = a' \neq a_t^*(\theta_t)$, then her continuation payoff satisfies

$$u^i(a', \theta_t) - \tilde{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i) - \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \tilde{q}_{t+1}^i(\theta_{t+1}^{-i}, r_t^i; a', \theta_t^{-i}) \mu(\theta_{t+1} | a', \theta_t)$$

$$\leq u^i(a', \theta_t) - \tilde{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i) - \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \tilde{q}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a', \theta_t^{-i}) \mu(\theta_{t+1} | a', \theta_t)$$

$$= - \tilde{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i).$$

Thus, after any history h_t, truth-telling is optimal for agent i provided that other agents also report their signals truthfully. The transfers $\tilde{q}_{t+1}^i$ can be viewed as lottery *payments* in period $t + 1$ that agent i commits to fulfill in period t. Since each agent in every period on average pays her flow utility in the previous period, it is straightforward to verify that the designer extracts all surplus with the mechanism $\{a_t^*, \tilde{q}_t\}$.

Although agent i's participation constraint in period 1 is satisfied under the mechanism $\{a_t^*, \tilde{q}_t\}$ as we have $\tilde{q}_1^i \equiv 0$, her participation constraints in any subsequent period could be violated. To see this, note that the above reasoning also shows that agent i's equilibrium

continuation payoff after history h_t is $-\tilde{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i)$, which may be less attractive than her outside option from period t onward.

This problem can be resolved by replacing the transfers $\tilde{q}_{t+1}^i$ in period $t+1$ with an upfront charge in period t and lottery *bonuses* in period $t+1$. Recall that for each a_t, θ_t^{-i}, $K^i(a_t, \theta_t^{-i})$ is an upper bound of $q_{t+1}^i(\cdot, \cdot; a_t, \theta_t^{-i})$. For each t and i, define a new transfer function $\hat{q}_{t+1}^i : \Theta_{t+1}^{-i} \times \Theta_t^i \times A_t \times \Theta_t^{-i} \to \mathbb{R}$ by

$$\hat{q}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i}) \equiv \tilde{q}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i}) - K^i(a_t, \theta_t^{-i}).$$

Also define $\hat{q}_1^i \equiv 0$. Note that by construction $q_{t+1}^i \leq 0$ for each t and i. Thus, $\hat{q}_{t+1}^i$ can be viewed as lottery bonuses for agent i. Set $c_t^i(a_t, \theta_t^{-i}) = \delta K^i(a_t, \theta_t^{-i})$ to be the entrance fee or "price" of the lottery $\{\hat{q}_{t+1}^i\}$ that agent i pays in period t.

Finally, for each agent i, define a sequence of transfers $\hat{p}_t^i$ as follows: (a) in the first period, agent i pays an entrance fee $\hat{p}_1^i(\theta_1) = c_1^i(a_1^*(\theta_1^i, \theta_1^{-i}), \theta_1^{-i})$; (b) in each subsequent periods, agent i collects the lottery bonus and pays another entrance fee, i.e., $\forall t \geq 1$,

$$\hat{p}_{t+1}^i(h_t, \theta_{t+1}) = \hat{q}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i}) + c_{t+1}^i(a_{t+1}^*(\theta_{t+1}^i, \theta_{t+1}^{-i}), \theta_{t+1}^{-i}).$$

Under the mechanism $\{a_t^*, \hat{p}_t\}$, after any history h_t agent i's continuation payoff from truthtelling is well-defined and given by

$$u^i(a_t^*(\theta_t), \theta_t) - \hat{p}_t^i(h_t, \theta_t) - \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \hat{q}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t^*(\theta_t), \theta_t^{-i}) \mu(\theta_{t+1}|a_t^*(\theta_t), \theta_t)$$

$$- \lim_{T \to \infty} \delta^{T-t+1} \mathbb{E}\left[K^i(a_T^*(\theta_T), \theta_T^{-i})|a_t^*(\theta_t), \theta_t\right]$$

$$= u^i(a_t^*(\theta_t), \theta_t) - \hat{p}_t^i(h_t, \theta_t) - \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \hat{q}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t^*(\theta_t), \theta_t^{-i}) \mu(\theta_{t+1}|a_t^*(\theta_t), \theta_t)$$

$$= -\hat{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i) \geq 0.$$

On the other hand, agent i's continuation payoff from lying in period t is no greater than $-\hat{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i)$. Since the expected discounted sum of transfers satisfies

$$\mathbb{E}\left[\sum_t \delta^{t-1} \hat{p}_t^i\right] = \mathbb{E}\left[\sum_t \delta^{t-1} \tilde{q}_t^i\right],$$

it follows that the mechanism $\{a_t^*, \hat{p}_t\}$ is periodic ex post incentive compatible and achieves full surplus extraction.

□

5. Appendix B: Proofs Omitted in the Paper

In this section, we first prove Lemma A.2 in the paper and then complete the proofs of Theorem 3.1 and Theorem 3.2 in the paper.

5.1. **Proof of Lemma A.2.** It follows directly from the following lemma (Lemma 5.1).

Let S and T be finite sets. For each $t \in T$, let $\mu(\cdot|t) \in \Delta(S)$ be a probability distribution over S.

Lemma 5.1. *Suppose for each $t \in T$, we have*

$$dist_2 \left(\mu(\cdot|t) - Conv \left\{ \mu(\cdot|t') \right\}_{t' \neq t} \right) \geq \epsilon,$$

for some $\epsilon > 0$. Then for any function $u : T \to \mathbb{R}$, there exists a function $p : S \times T \to \mathbb{R}$ such that for any t and t',

$$u(t) = \sum_{s \in S} p(s,t)\mu(s|t) \leq \sum_{s \in S} p(s,t')\mu(s|t),$$

and

$$\max_{s,t} |p(s,t)| \leq \left(1 + \frac{4}{\epsilon} \right) \cdot \max_t |u(t)|.$$

Proof. For each t, let $\nu(\cdot|t) \equiv (\nu(s|t))_{s \in S} = \min_{\tilde{\nu}(\cdot|t) \in Conv\{\mu(\cdot|t')\}_{t' \neq t}} \|\mu(\cdot|t) - \tilde{\nu}(\cdot|t)\|_2.$[2] Define

$$d(s,t) \equiv \frac{\mu(s|t) - \nu(s|t)}{\|\mu(\cdot|t) - \nu(\cdot|t)\|_2}.$$

Note that $\|d(\cdot,t)\|_2 = 1$. By construction, for any t and t', we have

$$\sum_{s' \in S} d(s',t) \left(\mu(s'|t') - \nu(s'|t) \right) \leq 0$$

and

$$\sum_{s' \in S} d(s',t) \left(\mu(s'|t) - \nu(s'|t) \right) = \|\mu(\cdot|t) - \nu(\cdot|t)\|_2 \geq \epsilon.$$

Define

$$e(s,t) \equiv \sum_{s' \in S} d(s',t)\mu(s'|t) - d(s,t).$$

Then for any t and t', we have

$$\sum_{s' \in S} e(s',t)\mu(s'|t) = 0 < \epsilon \leq \sum_{s' \in S} e(s',t)\mu(s'|t').$$

---

[2] $\| \cdot \|_2$ is the Euclidean norm.

Define

$$p(s, t) \equiv u(t) + c \cdot e(s, t)$$

where $c = 2 \max_t |u(t)|/\epsilon$. Then for any t and t', we have

$$u(t) = \sum_{s \in S} p(s, t) \mu(s|t)$$

and

$$\sum_{s \in S} p(s, t') \mu(s|t) = u(t') + c \sum_{s \in S} e(s, t') \mu(s|t) \geq u'(t) + c\epsilon \geq u(t).$$

Finally, since $\max_s |e(s, t)| \leq \|e(\cdot, t)\|_2 \leq 2$, we have

$$\max_s p(s, t) \leq |u(t)| + 2c,$$

which implies that

$$\max_{s,t} |p(s, t)| \leq \left(1 + \frac{4}{\epsilon}\right) \cdot \max_t |u(t)|.$$

$\square$

5.2. **Proof of Theorem 3.1 (Finite horizon).** Let $W_t(\theta_t)$ denote the expected period-t continuation social surplus given signal profile θ_t, i.e.,

$$W_t(\theta_t) = \mathbb{E}\left[\sum_{s=t}^{T} \delta^{s-t} \sum_{i=1}^{N} u^i(a_t^*(\theta_t), \theta_t) \Big| \theta_t\right].$$

First consider the problem in period T. By Assumption 4, there exists an ex post incentive compatible transfer $p_T : \Theta_T \to \mathbb{R}^N$ that implements the efficient allocation a_T^*. Given (a_T^*, p_T), the payoff V_T^i for each agent i in the truth-telling equilibrium is given by

$$V_T^i(\theta_T) = u^i(a_T^*(\theta_T), \theta_T) - p_T^i(\theta_T),$$

for each θ_T.

Next consider agent i's incentive problem in period $T - 1$ with an arbitrary public history $h_{T-1} = (r_1, a_1, r_2, a_2, \ldots, r_{t-1}, a_{t-1})$. Suppose that agents other than i always report truthfully. For each pair (a_{T-1}, θ_{T-1}), define

$$\pi_{T-1}^i(a_{T-1}, \theta_{T-1}) = \sum_{j \neq i} u^j(a_{T-1}, \theta_{T-1}) + \delta\mathbb{E}\left[W(\theta_T) - V_T^i(\theta_T)|a_{T-1}, \theta_{T-1}\right].$$

By Lemma A.1 in the paper there exists a function $\tilde{p}_T^i(\theta_T^{-i}, \theta_{T-1}^i; a_{T-1}, \theta_{T-1}^{-i})$ such that for every a_{T-1}, θ_{T-1}^{-i}, θ_{T-1}^i and r_{T-1}^i, we have

$$\pi_{T-1}^i(a_{T-1}, \theta_{T-1}) = \delta \sum_{\theta_T \in \Theta_T} \tilde{p}_T^i(\theta_T^{-i}, \theta_{T-1}^i; a_{T-1}, \theta_{T-1}^{-i}) \mu_T(\theta_T | a_{T-1}, \theta_{T-1}),$$

and

$$\sum_{\theta_T \in \Theta_T} \tilde{p}_T^i(\theta_T^{-i}, r_{T-1}^i; a_{T-1}, \theta_{T-1}^{-i}) \mu_T(\theta_T | a_{T-1}, \theta_{T-1})$$

$$\leq \sum_{\theta_T \in \Theta_T} \tilde{p}_T^i(\theta_T^{-i}, \theta_{T-1}^i; a_{T-1}, \theta_{T-1}^{-i}) \mu_T(\theta_T | a_{T-1}, \theta_{T-1})$$

Define a new period-T transfer $\bar{p}_T^i : \Theta_{T-1}^{-i} \times \Theta_{T-1}^i \times A_{T-1} \times \Theta_T \to \mathbb{R}$ for agent i as

$$\bar{p}_T^i(\theta_{T-1}^{-i}, \theta_{T-1}^i; a_{T-1}, \theta_T) = p_T^i(\theta_T) - \tilde{p}_T^i(\theta_T^{-i}, \theta_{T-1}^i; a_{T-1}, \theta_{T-1}^{-i}).$$

Note that $\tilde{p}_T^i$ is independent of θ_T^i, so agent i still finds it optimal to report truthfully in period T under this new transfer $\bar{p}_T^i$. By construction, given that other agents always report truthfully, it follows that for every realized signal θ_{T-1}^i, it is optimal for agent i to report $r_{T-1}^i = \theta_{T-1}^i$. Also note that for every signal profile θ_{T-1}, agent i's continuation payoff V_{T-1}^i in the truth-telling equilibrium is

$$V_{T-1}^i(\theta_{T-1}) = W_{T-1}(\theta_{T-1}).$$

Now for any $t < T$, suppose that there exist transfer schedules $\{\bar{p}_{s+1}^i\}_{s=t}^{T-1}$ for each agent i such that truth-telling consists of a periodic ex post equilibrium from any period $s = t, \ldots, T$ and each agent i's continuation payoff in the truth-telling equilibrium is $V_t^i(\theta_t) = W_t(\theta_t)$ for all θ_t. We need to construct a transfer $\bar{p}_t^i : \Theta_t^{-i} \times \Theta_{t-1}^i \times A_{t-1} \times \Theta_{t-1}^{-i} \to \mathbb{R}$ for each agent i such that for all a_{t-1}, θ_{t-1}^{-i}, θ_{t-1}^i and r_{t-1}^i,

$$-\sum_{j \neq i} u^j(a_{t-1}, \theta_{t-1}) = \delta \sum_{\theta_t \in \Theta_t} \bar{p}_t^i(\theta_t^{-i}, \theta_{t-1}^i; a_{t-1}, \theta_{t-1}^{-i}) \mu_t(\theta_t | a_{t-1}, \theta_{t-1}),$$

and

$$\sum_{\theta_t \in \Theta_t} \bar{p}_t^i(\theta_t^{-i}, \theta_{t-1}^i; a_{t-1}, \theta_{t-1}^{-i}) \mu_t(\theta_t | a_{t-1}, \theta_{t-1}) \leq \sum_{\theta_t \in \Theta_t} \bar{p}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^{-i}) \mu_t(\theta_t | a_{t-1}, \theta_{t-1})$$

The existence of $\bar{p}_t^i$ again follows from Lemma 1. Since $\bar{p}_t^i$ is independent of θ_t^i, incentive constraints for truth-telling in periods $s = t, \ldots, T$ still hold.

By construction, if other agents always report truthfully, then it is optimal for agent i to report $r^i_{T-1} = \theta^i_{T-1}$. Also note that in period $t-1$, agent i's continuation payoff in the truth-telling equilibrium is

$$V^i_{t-1}(\theta_{t-1}) = W_{t-1}(\theta_{t-1}),$$

for all signal profiles θ_{t-1}.

Finally, inducting on t backwards, we have a sequence of transfers $\{\bar{p}_t\}^T_{t=1}$, where $\bar{p}^i_1 \equiv 0$ for each i. Therefore, truth-telling consists of a periodic ex post equilibrium under the efficient dynamic mechanism $\{a^*_t, \bar{p}_t\}^T_{t=1}$.

5.3. Proof of Theorem 3.2 (Infinite horizon).

Assume all agents other than i report their signals truthfully and focus on agent i's incentive problem. Fix a socially efficient allocation rule a^*_t. By Assumptions 1 and 3, for each i and t, there exists a contingent transfer $p^i_{t+1}(\theta^{-i}_{t+1}; a_t, \theta^{-i}_t)$ that satisfies

$$-\sum_{j \neq i} u^j(a_t, \theta_t) = \delta \sum_{\theta^{-i}_{t+1} \in \Theta^{-i}_{t+1}} p^i_{t+1}(\theta^{-i}_{t+1}; a_t, \theta^{-i}_t) \mu^{-i}_{t+1}(\theta^{-i}_{t+1}|a_t, \theta_t),$$

for every a_t and θ_t. Set $p^i_1 \equiv 0$. Furthermore, since for any $t \geq \bar{T}$, the matrix

$$M^{-i}_{t+1}(a_t, \theta^{-i}_t) \equiv \left[\mu^{-i}_{t+1}(\theta^{-i}_{t+1}|a_t, \theta^i_t, \theta^{-i}_t) \right]_{|\Theta^{-i}_{t+1}| \times |\Theta^i_t|}$$

satisfies

$$\left\| \left(M^{-i}_{t+1}(a_t, \theta^{-i}_t) \right)^+ \right\| \leq \bar{D},$$

we can set the transfer $p^i_{t+1}(\theta^{-i}_{t+1}; a_t, \theta^{-i}_t)$ as

$$\overrightarrow{p}^i_{t+1}(\cdot; a_t, \theta^{-i}_t) = \frac{1}{\delta} \left(M^{-i}_{t+1}(a_t, \theta^{-i}_t) \right)^+ \overrightarrow{u}^{-i}(\cdot; a_t, \theta^{-i}_t),$$

where $\overrightarrow{p}^i_{t+1}(\cdot; a_t, \theta^{-i}_t) = \left(p^i_{t+1}(\theta^{-i}_{t+1}; a_t, \theta^{-i}_t) \right)_{\theta^{-i}_{t+1}}$ and $\overrightarrow{u}^{-i}(\cdot; a_t, \theta^{-i}_t) = \left(-\sum_{j \neq i} u^j(a_t, \theta^i_t, \theta^{-i}_t) \right)_{\theta^i_t}$ are column vectors. It follows that

$$\left\| \overrightarrow{p}^i_{t+1}(\cdot; a_t, \theta^{-i}_t) \right\|_\infty \leq \frac{1}{\delta} \left\| \left(M^{-i}_{t+1}(a_t, \theta^{-i}_t) \right)^+ \right\| \cdot \left\| \overrightarrow{u}^{-i}(\cdot; a_t, \theta^{-i}_t) \right\|_\infty \leq \frac{\bar{D}}{\delta} \left\| \overrightarrow{u}^{-i}(\cdot; a_t, \theta^{-i}_t) \right\|_\infty,$$

that is,

$$\max_{\theta^{-i}_{t+1}} \left| p^i_{t+1}(\theta^{-i}_{t+1}; a_t, \theta^{-i}_t) \right| \leq \frac{\bar{D}}{\delta} \max_{\theta^i_t} \left| \sum_{j \neq i} u^j(a_t, \theta_t) \right|.$$

Thus, for any sequence $(a_t, \theta_t)_{t \geq 1}$, we have

$$
\sum_{t=1}^{\infty} \delta^{t-1} \left| u^i(a_t, \theta_t) - p_t^i(\theta_t^{-i}; a_{t-1}, \theta_{t-1}^{-i}) \right|
$$

$$
= \sum_{t=1}^{\bar{T}} \delta^{t-1} \left| u^i(a_t, \theta_t) - p_t^i(\theta_t^{-i}; a_{t-1}, \theta_{t-1}^{-i}) \right| + \sum_{t=\bar{T}}^{\infty} \delta^t \left| u^i(a_{t+1}, \theta_{t+1}) - p_{t+1}^i(\theta_{t+1}^{-i}; a_t, \theta_t^{-i}) \right|
$$

$$
\leq L^i + \sum_{t=\bar{T}}^{\infty} \delta^t \left[\left| u^i(a_{t+1}, \theta_{t+1}) \right| + \frac{\bar{D}}{\delta} \max_{\theta_t^i} \left| \sum_{j \neq i} u^j(a_t, \theta_t) \right| \right]
$$

$$
\leq L^i + \max\{1, \bar{D}\} \cdot \left(\sum_{j=1}^{N} \max_{(a_t, \theta_t)_{t \geq 1}} \sum_{t=1}^{\infty} \delta^{t-1} |u^j(a_t, \theta_t)| \right)
$$

where $L^i = \max_{(a_t, \theta_t)_{t=1}^{\bar{T}}} \sum_{t=1}^{\bar{T}} \delta^{t-1} \left| u^i(a_t, \theta_t) - p_t^i(\theta_t^{-i}; a_{t-1}, \theta_{t-1}^{-i}) \right| < \infty$. Hence, by Assumption 1, agent i's discounted payoffs under the transfers $p_{t+1}^i(\theta_{t+1}^{-i}; a_t, \theta_t^{-i})$ are always well-defined. Applying the one-shot deviation principle, we only need to show that after any public history up to period t, agent i does not benefit from deviating to $r_t^i \neq \theta_t^i$ and $r_s^i = \theta_s^i$ for $s > t$.

If agent i reports truthfully in period t, i.e., $r_t^i = \theta_t^i$, her continuation payoff is

$$
u^i(a_t^*(\theta_t), \theta_t) - p_t^i(\theta_t^{-i}; a_{t-1}, \theta_{t-1}^{-i})
$$
$$
+ \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \left[W(\theta_{t+1}) - p_{t+1}^i(\theta_{t+1}^{-i}; a_t^*(\theta_t), \theta_t^{-i}) \right] \mu_{t+1}(\theta_{t+1} | a_t^*(\theta_t), \theta_t)
$$
$$
= W(\theta_t) - p_t^i(\theta_t^{-i}; a_{t-1}, \theta_{t-1}^{-i}).
$$

Suppose agent i deviates to a message r_t^i such that $a_t^*(r_t^i, \theta_t^{-i}) = a_t^*(\theta_t)$, then her continuation payoff remains the same. Thus, deviating to a message r_t^i without changing the allocation is not profitable.

If agent i deviates to a message r_t^i such that $a_t^*(r_t^i, \theta_t^{-i}) = a' \neq a_t^*(\theta_t)$, then her continuation payoff satisfies

$$
u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t) - p_t^i(\theta_t^{-i}; a_{t-1}, \theta_{t-1}^{-i})
$$
$$
+ \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \left[W(\theta_{t+1}) - p_{t+1}^i(\theta_{t+1}^{-i}; a_t^*(r_t^i, \theta_t^{-i}), \theta_t^{-i}) \right] \mu_{t+1}(\theta_{t+1} | a_t^*(r_t^i, \theta_t^{-i}), \theta_t)
$$
$$
= u^i(a', \theta_t) - p_t^i(\theta_t^{-i}; a_{t-1}, \theta_{t-1}^{-i})
$$
$$
+ \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \left[W(\theta_{t+1}) - p_{t+1}^i(\theta_{t+1}^{-i}; a', \theta_t^{-i}) \right] \mu_{t+1}(\theta_{t+1} | a', \theta_t)
$$
$$
\leq W(\theta_t) - p_t^i(\theta_t^{-i}; a_{t-1}, \theta_{t-1}^{-i}),
$$

where the inequality follows from the definition of a_t^*. Thus, deviating to a message r_t^i which changes the allocation is not profitable either. Therefore, we conclude that truth-telling consists of a periodic ex post equilibrium.

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